

Practical Experiences from Application of a Comprehensive Simulator for Pumped Storage Hydropower Investment Decisions on a Real Investment Case

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Abstract—The future electricity system is expected to include a higher share of production from renewable energy sources. This will lead to increased price volatility and thus a higher demand for short-term balancing by hydropower systems, which will have to operate closer to their maximum flexibility limits. In order to perform an investment analysis it is thus important to model physical characteristics and limitations of hydropower systems as close to reality as possible. A simulator based on medium- and short-term hydropower modelling is used to carry out an investment analysis for a Norwegian hydropower producer. The results demonstrate how detailed modelling can provide new insight into the implications of short-term effects in pumped storage hydropower investment decisions.

Index Terms—Hydropower Scheduling, Investment Analysis, Optimization, Pumped Storage

I. INTRODUCTION

The increasing share of production from variable renewable energy sources that are currently being deployed in the electricity system, primarily solar- and wind power, is causing increased price volatility [1]. The hydropower producers benefit from this development due to their capability of rapidly adjusting production and reversing turbines at low prices to store the energy for later use. A consequence of the new production patterns is more start-stop and production near the maximum capacity where the physical limitations in the watercourse are limiting the production. Hydropower producers are typically using short term models (STMs) with a relatively high level of detail for scheduling the daily operation, while medium term models (MTMs) are used for strategic decisions and investment analyses due to their ability to analyse longer time horizons under different uncertainties such as inflow and price. However, state dependent relationships are to a lesser extent included in MTMs due to mathematical limitations. Examples of such relationships that must be considered in the daily operation are head dependent discharge capacity, weir flows and tunnel flows. The main motivation of this work is

to include many of these physical limitations in an investment analysis building on the simulation framework described in [2], [3].

The Sira-Kvina hydropower system, located in the southwestern part of Norway, consists of two river systems with a total of 8 hydropower plants. The river systems combine at Tonstad, the power plant with the highest annual production in Norway. With its location in the connection point of a subsea DC cable to the European continent and proximity to the North Sea, it has a strategic beneficial location for balancing variable renewable power production. Increasing both pumping capacity and flexibility in the system is thus of direct importance to the integration and balancing of new renewable production and the European shift towards a fossil-free electricity supply. However, the watercourse is complex to operate due to the high inflow to Tonstad, whose upstream reservoirs are relatively small. The watercourse is also significantly restricted by weirs and transfer tunnels, as well as rigorous pressure restrictions in the headrace tunnel of Tonstad [4]. Thus, the production at Tonstad must be carefully coordinated with the upstream plants to maintain high production capacity while keeping the risk of overflow low.

The simulator uses an MTM based on stochastic dual dynamic programming (SDDP) [5]. SDDP is state of the art for solving multi-stage stochastic problems with a high number of states, and is widely used for the hydropower scheduling problem. The method requires a convex problem formulation, where purely linear formulations are preferred due to computational benefits. Several methods for including non-convex relations in SDDP have been proposed, such as convexification of bi-linear terms using McCormix envelopes [6], including integer variables through alternative decomposition methods [7], and stochastic dynamic programming for including price uncertainties [8]. Alternatively, additional details can be added in final simulations that are performed for specific scenarios after convergence [9]. However, global optimality cannot be guaranteed for the simulations since the model formulation deviates from the one used for finding the strategy.

The idea behind this simulator is to use a detailed state-of-the-art STM in the simulation phase. The STM SHOP is used for the simulation and the MTM ProdRisk is used for the strategy as described in [3]. The STM uses a combination of mixed-integer linear programming (MILP) and successive linear programming for managing non-linearities and non-convexities in the problem formulation, see [10]–[12] for details. The MTM provides an operating strategy through water values depending on reservoir storage, inflow and price, so-called cuts, that are used as input to the STM. Both models consider risk neutral price takers, the STM being deterministic and the MTM with stochastic price and inflow. Both models are in operational use by a significant number of producers in Nordic and central European countries, and a similar setup has been used to benchmark historical operation [13].

The investment analysis using the proposed simulator on the Sira-Kvina hydropower system has revealed several challenges that must be addressed carefully when combining two separate models with varying aggregation level that are solved with different methods. For example, the MTM uses an aggregated plant description through a piece-wise linear relation between discharge and production, while the STM models each individual turbine with detailed efficiency curves for both generator and turbine. Moreover, the MTM uses an aggregated reservoir description, while the STM divides into multiple reservoirs with physical flow descriptions between them.

The contributions of this paper are: *a)* practical experiences from running a comprehensive simulator with an aggregated MTM and a detailed STM, *b)* methods for ensuring consistency between the STM and the MTM model description, and *c)* numerical results of the performed investment analysis.

II. SIMULATOR FRAMEWORK

The simulator is implemented in Python and distributed as a Python package [14]. Both the MTM ProdRisk and the STM SHOP have their own Python interfaces, namely `pyprodrisk` [15] and `pyshop` [16]. These are used to build and modify the respective models and to acquire the optimization results.

The simulation framework requires a system description as well as price and inflow scenarios. The simulator first runs the MTM to determine a strategy formulated as linear cutting planes for each individual week. The planning horizon for the optimization is typically 104–208 weeks [3], depending on the size of the largest reservoir. Once the strategy from the MTM is known, the STM is used to run forward simulations to get a detailed evaluation of the proposed strategy and system. The STM is run sequentially week by week and scenario by scenario using the cuts from the MTM as illustrated in Fig. 1. The initial value for the very first STM simulation is taken from the MTM. The end reservoir volume from a STM simulation is used as initial condition for the next run. The total number of sequential simulations with the STM is 52 weeks times the number of scenarios.

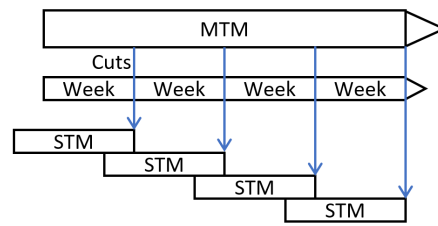


Fig. 1. Simulator framework.

A. Medium term model

The MTM formulates a linear multi-stage stochastic program that is solved using SDDP, where inflow and power price are stochastic variables. It is usually run with weekly stages, where each week is represented by a decision problem formulated as a linear programming problem. See [17] for further description of the method. The strategy from the MTM is inherited by the STM through water-value cuts. The MTM uses a less detailed model description compared to the STM, where e.g. generators in a plant are aggregated into one unit. It is run with a time resolution of three hours.

B. Short term model

The STM used for detailed evaluation of the strategy is a deterministic short-term hydropower optimization program based on successive MILP, see [10], [18]. With the STM, more details about the system can be provided, such as modelling each individual generating unit. The STM captures more details with respect to plant efficiency and production capacity [12], flows in tunnel networks and pressure limitations [19], and physical flow in rivers and over weirs. Experience from daily operation shows that these are becoming important limiting factors due to the increased price volatility, and they should therefore be considered in investment decisions. The STM is run with hourly resolution.

III. THE SIRA-KVINA HYDROPOWER SYSTEM

The Sira-Kvina hydropower system, shown in Fig. 3 in the Appendix, includes 7 plants, 18 turbines and has a total installed capacity of 1770 MW. The annual production is around 6800 GWh, covering around 5% of the total demand in Norway. The system comprises two river systems, Sira and Kvina, that meet at Tonstad. There is currently pumping capability at Duge, and adding pumps at Tjørhom is under consideration.

One of the main challenges of the system is that Tonstad, the largest plant with 960 MW capacity, has very small intake reservoirs and several complex physical factors limiting its production capacity. The tunnels from the two intake reservoirs, Ousdalsvann and Homstølvann, meet above Tonstad as illustrated in Fig. 2. The tunnel to Ousdalsvann is about twice as long as the tunnel to Homstølvann, and thus it has a higher head loss. When Tonstad is producing near maximum capacity, the head difference between the reservoirs can be several meters despite them being hydraulically coupled. Therefore, in

situations with high inflow, a high head level in Homstølvann can easily cause spillage at Ousdalsvann.

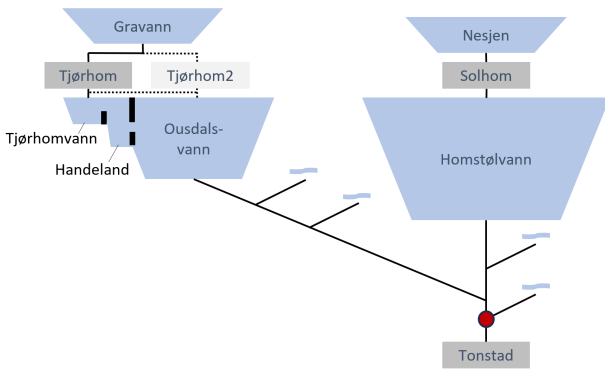


Fig. 2. Detailed topology of the Tonstad intake reservoirs and proposed new investment at Tjørhom.

Tjørhom and Solhom, that discharge into Ousdalsvann and Homstølvann, have lower discharge capacity than Tonstad. Due to this, in situations with lasting high prices, Tonstad tends to empty the intake reservoirs. If the pressure into Tonstad becomes too low, the discharge capacity is limited due to the risk of air pockets in the tunnel system. Therefore, the production into Ousdalsvann and Homstølvann must be coordinated accurately to ensure high production capacity available and Tonstad when the price is high. Moreover, the tunnel system above Tonstad has multiple creek intakes that affect the pressure and production capacity. The plant Tjørhom discharges into the reservoir Tjørhomvann, where the water further flows to Handeland over a crest before a tunnel connects Handeland to Ousdalsvann. The flow between these reservoirs is also a limiting factor that must be considered in the operation of Tonstad.

The challenges at Tonstad described above can be reduced by installing new generation capacity at Tjørhom, with pumping capability and a new outlet tunnel directly to Ousdalsvann. Increased generation capacity discharging directly to Ousdalsvann should make it easier to supply Tonstad with enough water when the price is high. Additionally, in high inflow periods, spillage can be avoided by using the pump. This work has focused on using the extensive simulation framework to evaluate this investment option.

IV. PRACTICAL EXPERIENCES

The main idea with the simulation framework is to embed the operation strategy from the MTM with the STM in order to precisely evaluate the performance of different investment options considering both long term strategic decisions and operational limitations due to state-dependent short-term effects. This section will visit some of the differences between the models and explain some of the practical challenges that have been managed during the development of the simulator.

A. Water values for small reservoirs

The water in larger reservoirs is typically valued higher than for small reservoirs since they can withstand high inflow

and draught for longer periods. The water in the reservoirs above Tonstad, that are relatively small and inflexible, is therefore valued relatively low. However, the water still has a strategic value for two reasons that are not properly described in the MTM: *a)* High reservoir levels increase the plant's net head which means more power out of each unit of water. *b)* The pressure restriction above Tonstad limits the maximum production capacity when the reservoir level is low, which in turn can lead to a situation where the opportunity to produce at a high price is lost. Therefore, higher reservoir levels increases the maximum production capacity. As a consequence, the water value at Tonstad is often underestimated and thus the reservoirs are often emptied towards the end of the STM optimization horizon. To counteract this effect, the STM time horizon has been extended from 7 to 10 days and the initial values for each STM simulation are taken from the end of day 7 in the previous simulation. This leads to smoother transitions for the small reservoirs in the sequential STM simulations.

B. Reservoir aggregation

Between the plants Duge and Tjørhom, there are four lakes that have merged into one due to artificial dams. The flow between these lakes is restricted by natural crests, and is highly state dependent. The same applies for Tjørhomvann and Handeland, while Handeland and Ousdalsvann are connected with a tunnel. Since the flow between the reservoirs is state dependent and difficult to approximate with linear relaxations, it is not included in the MTM. It is possible to model these individual lakes as separate reservoirs where the flows between them are decision variables in the MTM. However, an unwanted side effect from this is that the hydraulically connected reservoirs may get different water values from the MTM. Therefore, reservoirs that are connected with tunnels and channels where the flow can not be controlled are modelled as one in the MTM, while the STM models each reservoir individually. The relation between head and volume for the detailed STM description is automatically aggregated by the simulation framework and applied to the MTM.

C. Consistent production and discharge capacities

The STM describes each generating unit individually, and constructs a curve describing the relation between production and discharge, a so-called production-discharge (PQ)-curve, based on head dependent turbine efficiency curves and generator efficiency curves for each individual unit and time-step. The maximum discharge capacity of each plant is a detrimental factor for the MTM strategy, and thus consistent description of the PQ-curves between the models is important.

The PQ-curves for the MTM are found in the following way: Each plant is optimized using the STM for a fixed head for different predetermined discharge levels spanning the whole operating range of the plant such that the corresponding production values are calculated. The resulting set of PQ-points is then convexified, meaning points breaking the convexity of the curve are removed, and the resulting curve is given as input to the MTM. An equivalent method is applied for pumps.

For several plants, the discharge capacity is reduced for low head. Head dependent discharge capacity is currently not included in the MTM, but is accounted for in the STM by head dependent efficiency curves with individual discharge limits.

D. Bypass costs

Since the discharge capacities of Tjørhom and Solhom are undersized compared to Tonstad, it can in situations with lasting high price be profitable to bypass water from Nesjen and Gravann to maintain high production at Tonstad. In reality, this is unwanted and is therefore penalized using bypass costs in both models, so that bypass is limited to situations where it is highly necessary.

E. Model robustness

A severe challenge when running a high number of sequential simulations, in this case 1560, is that if the STM fails in one of the simulations it will impact the results of all consecutive simulations. A significant effort has been put into creating an STM that is robust for all the different numerical combinations of inputs. Several state dependent restrictions have been implemented using MILP formulations, such as head dependent minimum pump height and head dependent minimum production height at Duge, as well as state dependent weir flow between Tjørhom and Ousdalsvann.

Several of the constraints in the STM are soft, meaning that penalty variables are used to avoid infeasible problems. For example, if a reservoir runs empty, artificial water can be used at a very high cost. It is easier to debug the case when an invalid solution is penalized instead of being infeasible. If large penalties are observed for an STM run, this can be a consequence of lacking convergence, which sometimes can be solved by running additional iterations.

MILP models are typically solved within a predefined tolerance, which in this case was 2000 €. Situations where the model can not be solved within a reasonable time limit can occur, and the consequent solutions may include significant penalty costs due to violations of physical restrictions. In these situations, the step length of the optimization model is increased by a factor 2, which reduces the number of variables with 50%. This is repeated until a solution is achieved within the solver's time-limit. Although some precision is lost, these solutions are preferred over solutions that are not within the acceptable tolerance and thus not good enough to give reasonable initial state for the next simulation.

It is useful to be able to investigate the results before the whole simulation is done, to be able to detect issues as early on as possible. The simulator saves the results from each STM simulation continuously so that they can be accessed immediately, including after a system crash. It is also possible to resume the STM simulations from any given point, which can be useful in such situations.

F. Comparison of results with few scenarios

The MTM selects initial reservoir levels for the first scenario such that a sequential simulation of all 30 scenarios will arrive

at those levels at the end of the final scenario. Therefore, when all 30 scenarios have been run, the change in reservoir levels from the beginning to the end of the simulation period should be zero. However, when comparing results with fewer scenarios, the difference between start and end reservoir levels can be significant, due to some years being dry and others wet. To compensate for this, it is assumed that the residual volume between the beginning of the first scenario and the end of the final scenario are valued based on the the average obtained price for the respective reservoirs. This additional value/cost is distributed between all evaluated scenarios.

V. RESULTS

The simulator has been used to evaluate the *base* case and the investment option *tjørhom*, the latter including a new 90 MW plant with pumping capabilities as described earlier. The investment option has been evaluated using 30 historical weather scenarios from 1986 to 2015, together with a price prognosis based on these weather scenarios for the expected power system in 2035 delivered by Volue Insight [20]. The price prognosis can not be published in this paper.

The simulations were performed on a server running Ubuntu with an *AMD Ryzen Threadripper 3990X 64-Core Processor*. Up to four cases were run in parallel, with each case assigned to 16 cores. Due to recently updated information, the challenges with model robustness, and time constraints there was not enough time to finish running through all scenarios at the time of writing. The final simulation included 11 out of 30 scenarios. The MTM takes around 4 hours to finish and the STM takes around 3-5 minutes to run each case, meaning running all 30 scenarios takes around 5 days.

Table I summarizes the results of the investment analysis with the *tjørhom* case relative to the *base* case for the analysed period of 11 scenarios, together with the MTM results for all 30 scenarios. The net income refers to the total income minus the total expense for all plants. The corrected net income is corrected for differences in water consumption, as described earlier. The estimated value of the total difference in water volume as a percentage of the net income was for the STM 1.34% for *base* and 0.57% for *tjørhom*, and for the MTM 0.64% for *base* and 0.13% for *tjørhom*. Both the MTM and the STM showed an increase in net income for the *tjørhom* investment case. The MTM showed an increase in net income corrected for change in reservoir levels of 2.9%, while the STM showed a difference of 1.9% for the analysed scenarios. For all scenarios the MTM showed an increase of 3.2%. The relative change in corrected net income is also shown per plant. The pump investment led to significantly more income at Tjørhom, where the additional capacity was installed. It also led to a slight increase at Tonstad, due to the increased ability to sustain high production for longer time. Additionally, the *tjørhom* investment case has less bypass. Since bypassing water is generally not desirable, although it might be profitable, this is an additional benefit that should be considered in the investment decision.

TABLE I
SUMMARY OF RESULTS, RELATIVE TO BASE CASE

	STM [%]	MTM [%]	MTM all [%]
Net Income corrected	2.02	3.01	3.16
Net Income	1.22	2.48	3.16
Net Production	-1.80	-0.83	-0.20
Bypass	-18.32	2.73	-12.77
Net income per plant			
Tjørhom	15.33	20.62	21.79
Tonstad	0.51	1.20	1.48
Solhom	0.44	0.67	1.52
Kvinen	0.10	0.24	1.62
Roskrepp	-0.36	1.75	-1.24
Duge	3.70	4.49	0.11

VI. DISCUSSION AND CONCLUSION

The final simulation with the STM ensures that the strategy from the MTM is applicable to shorter time horizons and increased short-term variations. With the shift towards more renewable energy sources, short-term effects are important to take into account when performing investment analyses. The simulator is thus a highly relevant tool for investigating how suited different investment options are for the future energy system. Since the STM model formulation used in the simulation deviates from the MTM model used for finding the strategy, global optimality can not be guaranteed. However, the STM supplements the MTM results and provides more information than the MTM would on its own.

Throughout the development of the simulator, several practical challenges had to be managed. Due to the complexity of the Sira-Kvina system, the main challenge was to ensure sufficient robustness of the model. This was accomplished by imposing state-dependent restrictions for the STM, using soft constraints and penalty variables, and re-solving cases with increased time step length if needed. Continuous saving of results allowed for early detection of issues during the development of the simulator. Additional adaptations include adjusting bypass costs and overlapping STM runs to ensure smooth transitions between runs for small reservoirs.

To ensure consistency between the MTM and the STM, hydraulically coupled reservoirs were modelled as one in the MTM and the STM was used to generate PQ curves for the MTM. The results demonstrate an adequate consistency between the STM and the MTM, in particular for the largest reservoirs, which is an indicator of reasonable results that are applicable to both shorter and longer time horizons. The goal is not identical results between the models, but the MTM should provide a long term target for the STM such that the MTM decisions can be refined.

The differences between *base* and *tjørhom* were fairly similar for the MTM and the STM, but with a larger difference in income from the MTM. This could be due to more detailed restrictions in the STM. The results from the MTM were fairly similar for the 11 scenarios and the full run, which together with the small estimated value of the total difference in water

volume shows that the results are useful even though only 11 scenarios were included.

Sometimes, the optimal solution found by the simulator deviates from the desired behaviour, e.g. bypassing water to maintain high production at Tonstad at high prices. However, often when the simulator is forced to avoid these solutions through artificial penalty costs, other issues will appear. Fig. 4 in the Appendix illustrates an example of this. A high penalty was set to avoid bypassing water from Gravann to Tonstad. With high inflow and price, *tjørhom* is able to keep the Gravann reservoir level down and thus produce at maximum capacity at Duge. Meanwhile, in the STM, Gravann is overflowing due to limited discharge capacity at Tjørhom hence *base* has to reduce the production at Duge to limit the overflow at Gravann. Duge cannot pump when the downstream head is below 651 or 655 masl (depending on the time of year) due to license regulations. These limitations are not accounted for in the MTM. When later the inflow is reduced while the prices remain high, *base* has more water left at Svartevann and is thus able to maintain production at high effect to exploit the high pricing, while *tjørhom* has to produce at a lower effect, a disadvantage that is not considered in the MTM strategy. In summary, *base* is forced to risk overflow at Gravann, which in this case happened to be profitable due to the production limitations at Duge. This can partly explain why the profit from the STM is less than from the MTM. It might be more reasonable to avoid penalizing the bypass since it can lead to unexpected side effects, and rather see the bypass as additional information to the income results.

A. Further work

Running the simulator on the model setup with more extreme price scenarios might prove the final STM simulation to be even more important. This is highly relevant for the investment analysis as the future energy market will see increased price volatility.

It would be interesting to investigate how the level of detail in the STM affects the simulation time versus how much it affects the final results. Perhaps the STM model description could be simplified or the time resolution could be reduced to allow for shorter computation time while still providing sufficient accuracy.

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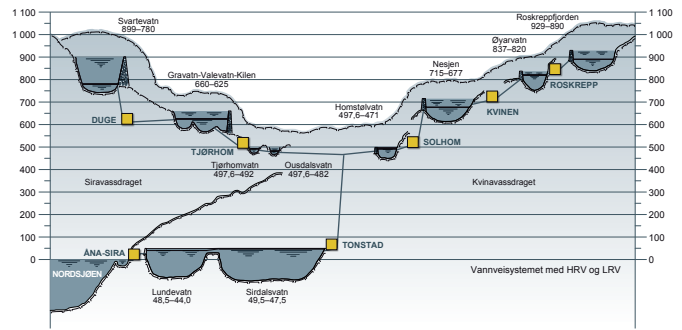


Fig. 3. The Sira-Kvina hydropower system. Power plants are represented by yellow squares. Figure obtained with permission from Sira-Kvina Kraftselskap (2024).

VII. APPENDIX

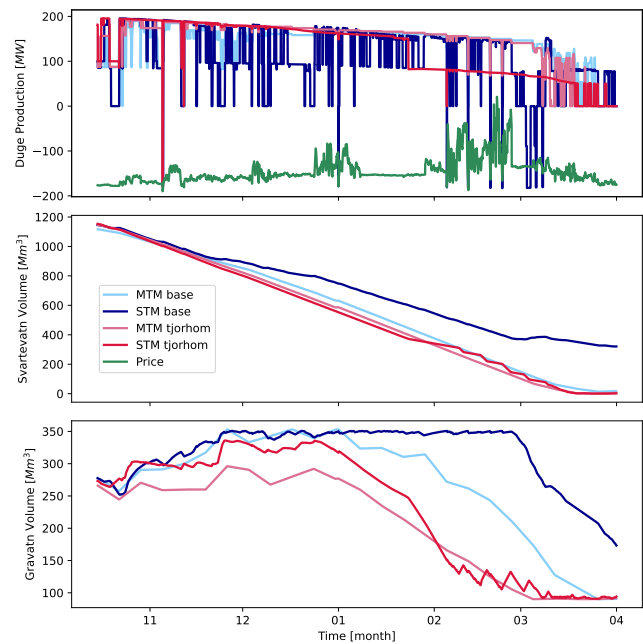


Fig. 4. Selected results from scenarios 8 and 9. The upper subplot shows the price together with the production at Duge for the STM and the MTM for base and tjorhom. The middle subplot shows the respective reservoir volumes at Svarlevatn, while the lower subplot show the reservoir volumes at Gravann.