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Modeling collaborative data service provision around an open source platform under uncertainty with stochastic provision games ☆

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ABSTRACT

This paper uses concepts taken from Cooperative Game Theory to model the incentives to join forces among a group of agents involved in collaborative provision of a mobile app under uncertainty around an open source platform. Demand uncertainty leads the agents to reach a noncooperative equilibrium by offering low quality apps. This can be avoided by introducing a coordination scheme through a common platform that eliminates the effects of lack of information. Coordination is achieved by providing a revenue sharing scheme enforcing the stability of the collaboration but also defined in a "fair" way, depending on the importance of the resources that each provider supplies to the app. To this aim, we introduce the concept of *Stochastic Provision Games*. This coordination leads both to higher app quality and improved profitability for the participants.

1. Introduction

In the mobile industry, developers and distributors participate in collaborative provision around what is called a platform, *i.e.* an entity coordinating interactions between groups of stakeholders. Revenues stemming by the delivery of the apps are shared according to predefined rules. A notable example is the 70/30 splitting ratio introduced by Apple, where developers get 70% of the revenue, and the platform takes 30%. This model was popularized by Apple with the launch of the App Store in 2008.[1] Apple's adoption of the 70/30 split set a precedent that many other digital service providers have followed, including Google's Play Store[2] and various digital game distribution platforms like Steam. Nevertheless, a steadily growing share of developers has in the past complained about the rigidity of application of such a business model [1], and videogame platforms such as Steam or Microsoft have tried to define different sharing schemes depending on the relative importance of the developer. We are focused on the analysis of potential drivers reflecting the relative market power of the platform with respect to developers, which should be taken into account when defining such revenue shares. More specifically, we develop a framework that will be used to evaluate a case study and analyze it under the following research questions:

- What is the impact of the size of the platform customer base on the relative profitability and revenue sharing between platform and developer(s)?
- Is it preferable to cooperate with developers producing simple apps or with the ones producing complex apps?
- What is the difference between cooperating with large developers and with small ones in terms of profitability and revenue sharing?

To tackle these questions we provide a framework focusing on the definition of economic incentives needed to ensure collaboration for the provision of mobile apps. This is done by devising an appropriate scheme of revenue shares suited to achieve coordination of the Quality level of Service (QoS) offered by each developer to a composite mobile app. In our setup, the revenue sharing scheme should be designed so to account for the synergies or redundancies among the contributions of each provider to the total revenues of the platform. To this aim, we will use concepts drawn from both Noncooperative and Cooperative Game Theory (CGT). The noncooperative layer of the framework is used to derive the profits of the agents when they do not engage in cooperation with the platform. The profits computed in the noncooperative layer are then used in the cooperative layer to derive a system of revenue shares to be granted to the participants to incentivize them to operate in a coordinated way. The solution method that we propose computes the system of revenue shares as the closest $\mathcal{L}_1$ element in the Core to the

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[2] https://www.mobiiworld.com/blog/what-you-should-know-about-mobile-apps-revenue-split-for-ios-and-android/

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Shapley value [2] of the Cooperative Game. While our framework does not aim to deliver theoretical guidelines on defining revenue shares, it offers a tool for analyzing revenue-sharing mechanisms on a case-by-case basis. Through the lens of a case study on the collaborative provision of a mobile app, we demonstrate the utility of this analytical tool in dissecting specific scenarios. Further, by undertaking sensitivity analyses, detailed in one of the appendices, we fortify our findings, providing deeper insights into the collaborative architecture. These analyses ensure that, while our immediate conclusions are drawn from a distinct case, the methodology laid out can be applied broadly to scrutinize similar partnerships and revenue-sharing problems.

The literature classifies two kinds of business models for collaborative provision of services around platforms [3]: a *brokered model* where the provision is hierarchically coordinated by one of the agents who aims at extracting the largest possible share of value out of the provision, and a *distributed model*, where the distribution of bargaining power is balanced among the agents involved. The brokered model can be mathematically described by utilizing the concept of Stackelberg Game, as done in [4–9] and solved by means of bilevel coordination problems as in [10–14]. Recent research has introduced risk management in the coordination of a single seller and single purchaser [15].

Most of the existing economic analyses on platforms focus on the study of the optimal price structure under different assumptions about platform governance and conditions of competition [16–18]. Quantitative economic research on the topic of ICT service provision has attracted increasing attention only afterwards [19,20], and it is mainly tailored to physical production. The approach we take in this article is based on a different viewpoint; we study the relationship between non-homogeneous service providers that supply different components to the bundle of services provided by the platform.

Besides, Section 2 presents a brief survey of the relevant research literature while the general modeling framework is reported and analyzed in Sections 3–6, followed by a case study in Section 7. Finally, Section 8 concludes the paper.

2. Literature review

Most of the literature on CGT is focused on transferable utility games [21], where different solution concepts have been developed, such as the concept of core, as in [22,23], and the Shapley value (see [2]). A comprehensive exposition of CGT can be found in the book by [24].

One of the earliest contributions useful to model supply chain coordination is given by the theory of linear production games, introduced in [25] and analyzed in many papers on CGT [26–29].

Cooperative allocation games, such as the one studied in this paper, have several common points with the newsvendor problem. Examples of applications of Game Theory to the analysis of the cooperation between supply chain agents can be found in [19,30–36].

A case of application of CGT for coordination between Platform Operator and developer is in [37]. The article, though, does not consider the Nash-Game on the quality of the provision when the developers do not sign binding Service Level Agreements (SLAs) with the Platform Operator. An approach where cooperative and non-cooperative Game Theory are used in a common framework for defining a revenue sharing mechanism can be found in [38]. Their approach is different from the one used in this paper as the computation of the Nash equilibrium is only done as a post processing step to check the sustainability of a given predetermined revenue sharing scheme. An analysis using a combination of non-cooperative and cooperative game theory has been introduced in [39]. In this case the decision makers are the downstream buyers which decide whether to replenish through intermediaries or directly from the producers by forming coalitions.

A strand of literature modeling collaboration between multiple vendors using CGT has focused on joint inventory management. [40–42] study the coordination of a decentralized inventory sharing system with n retailers who non-cooperatively determine their order quantities but cooperatively share their inventory. [43] analyze the problem of cost sharing for the usage of common inventory and [44,45] study the pooling of spare parts between multiple companies maintaining capital goods. [46] discusses the need for a dynamic implementation of the revenue sharing between platform and providers. More recent studies have increased the focus towards business modeling of the platform operations and their interactions with third party service providers. [47] study the problem of choosing logistic services for e-commerce platforms. In the analyzed framework, a developer can choose to deliver its services using the in-built platform delivery services or via a third party contractor. The authors investigate how the relative market power levels between the platform and the third party logistic service providers lead to different cooperation/competition configurations. A systematic review of recent developments of solution concepts in CGT and their application in operations management can be found in [48]. Some of the points raised by the authors include that the coexistence of cooperation and competition among firms needs to be considered in research, along with the incorporation of transaction costs and market risks in cooperative models, that the stability of the cooperation when using the Shapley value requires consideration of the agents' risk attitudes, and the treatment of uncertainty should be factored in the analysis of the cooperative patterns. An interesting contribution considering the allocation of profit under uncertainty using CGT has been introduced in [49] who define the side payments with incomplete information on the contribution of each actor to the grand coalition. An interesting contribution that could serve as a foundational reference for extending this work is from [50], who investigate the dynamics between two mobile app platforms and a single app developer, focusing on the co-creation of app quality under revenue-sharing contracts. The authors find that an increase in the developer's revenue share from one platform leads to a decrease in the share offered by the other, which can benefit as a free-rider on the quality improvements. Among recent contributions to collaborative service provision around platforms, [51] examine how fairness concerns from the platform or developer influence bargaining in the mobile app supply chain. In contrast, our model prescribes strategies for stable and efficient supply chain cooperation. [52] present a study more aligned with ours, using CGT to explore platform-content developer coordination. They find that fixed subscription fees for a polarized user base jeopardize platform stability. In contrast, our approach considers variable, app-based pricing, which maintains stability.

Our framework is different from the approaches we mentioned earlier. Unlike those, which enhance production typically by pooling resources, ours requires a balanced input from providers with varied expertise. This is because, as mentioned, developers may have specialized skills that are not transferable across different providers. Instead of focusing on resource pooling, our framework encourages providers to collaborate, aiming to improve coordination in the quality of services offered. This eliminates the need for providers to form anticipations on the quality level their peers will deliver. In our approach, the key benefit of collaborative provision is this enhanced coordination of quality supply.

3. Cooperative model of modular service provision

We consider a mobile app created bundling basic components which can be sold on or off-platform. We model the collaborative provision of the app through binding contracts. Providers decide quality levels and negotiate revenue redistribution within the coalition. We assume that agents make decisions within a risk-neutral framework. While exploring the behavior of risk-averse agents would indeed enrich the range of possible analyses, it introduces additional complexities. For example, it raises questions about how individual risks are pooled among actors within coalitions.

Quality is considered as the ability of a component to meet the user's requirements by performing a set of tasks [53]. Following [54–57], higher quality investment leads to better app quality and increased potential demand. In our framework, app quality improves linearly with the number of development hours invested. The choice to assume a linear quality-work allocation relation might not necessarily reflect how its measure develops as the effort grows and, in many cases, it might be more appropriate to assume a marginally decreasing quality development. Our choice to employ a linear model aims to leverage the computational efficiency and clarity afforded by linear programming in defining coalition behaviors. This approach significantly simplifies the explanation of outcomes in our case study, where parameter variations could yield complex effects not easily interpreted through a nonlinear approach.

We define a *cooperative provision setup* as a tuple

$$\mathcal{I},\ \mathcal{P},\ (W_i)_{i\in\mathcal{I}},\ (p_j)_{j\in\mathcal{P}},\ (c_i)_{i\in\mathcal{I}},\ (\lambda_{ij})_{i\in\mathcal{I},\ j\in\mathcal{P}},\ (d_j)_{j\in\mathcal{P}}$$

where $\mathcal{I}$ is the set of developers/components; $\mathcal{P}$ is the set of mobile apps; W_i is the provision capacity for the component supplied by developer $i \in \mathcal{I}$; p_j is the reference price for the delivered app j; c_i is the hourly development cost for producing the component for the ith developer; λ_{ij} is the amount of working hours needed by the developer to produce component i to bundle mobile app $j \in \mathcal{P}$ with the highest level of quality; d_j is the random quality demand for mobile app $j \in \mathcal{P}$. We consider uncertainty in quality demand to model developers offering different quality levels in response to identical uncertainties, highlighting the adaptive strategies developers might employ based on their unique conditions in terms of costs and capacity. We model this uncertainty through the use of scenarios $s \in \Omega$, with each scenario defining a different demand level. This approach is chosen for its simplicity in modeling two-stage linear stochastic programming problems. It also offers the flexibility to incorporate a broad spectrum of probability distributions, not limited to those definable by a closed-form density function.

We assume that there is an estimation of the maximum possible sales for the application $D_j^{\max}$, while D_{js} represents the market demand for the application under scenario s. We define $d_{js} = D_{js}/D_j^{\max}$ as the relative market demand. We sort the demand based on quality requirements, from low to high. Users who are willing to purchase a low-quality app are also willing to buy it at a higher quality level for the same price. Therefore d_{js}, ranging from zero to one, represents the maximum demanded quality for mobile app j in scenario s. The supplied quality of each component is proportional to the used development time. Let q_{ij} define the number of hours spent by the developer in developing component i. We then define the quality delivered by the developer as the ratio $\frac{q_{ij}}{\lambda_{ij}} = Q_{ij}$. We assume that every developer contributes to the overall app quality proportionally to the number of hours that they need to allocate to develop their component. Therefore, the increase in quality supplied by developer i is given as $\frac{\lambda_{ij}Q_{ij}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}$ where $Q_{ij} \in [0,1]$ expresses the chosen quality level by developer i. The ratio denotes the marginal quality contribution of provider i as the number of hours that she allocates over the total number of hours needed to build all the components necessary to bundle the mobile app. The marginal revenue contribution by developer i to mobile app j is $p_j D_j^{\max}\frac{\lambda_{ij}\min\{Q_{ij},d_{js}\}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}$. This reflects the fact that the provided quality will only be purchased for the proposed price by customers not demanding a higher quality. In collaborative service provision, an agent's workload decisions depend on the anticipated quality of complementary components supplied by other developers. Without prior quality level negotiations, each agent develops her component based on the anticipated quality delivered by the complementary providers. We define the overall app quality as the quality level provided by the worst component. The measurable quality of service j delivered to users under scenario s is given by $\min_{i\in\mathcal{I}_j}\{Q_{ij}\}$.

Given that Q_{kj} is the level of quality that component providers $k \in \mathcal{I}_j$ decides to satisfy, the satisfied quality demand for mobile app j under scenario s is

$$\min\left\{d_{js}, \min_{k\in\mathcal{I}_j} Q_{kj}\right\}.$$

The previous formula models the fact that the agents provide their component deciding the quality level based on their anticipation of other actors' supply, and on the distribution of demand for quality. If we define π_s as the probability for scenario s, the ith developer work allocation problem when everyone is acting on her own is the following

$$\max_{Q_{ij}} \sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s \min\left\{d_{js}, \min_{k\in\mathcal{I}_j} Q_{kj}\right\} - c_i\lambda_{ij}Q_{ij}\right) \tag{1}$$

subject to

$$\sum_{j\in\mathcal{P}_i}\lambda_{ij}Q_{ij} \le W_i \tag{2}$$

$$Q_{ij} \ge 0. \tag{3}$$

where constraint (2) defines the capacity constraint in working hours.

Suppose that some developers involved in the collaborative provision of apps $j \in \mathcal{P}$ have decided to join coalitions. The coalitions in this case are associated with elements of the partitions Ξ of set $\mathcal{I}$, which are composed of the subsets C of $\mathcal{I}$ such that $C \cap C' = \emptyset$ if $C, C' \in \Xi,\ C \neq C'$ and for any $i \in \mathcal{I}$ there exists $C \in \Xi$ such that $i \in C$. Some of these coalitions may consist of just a single developer. The highest level of quality for mobile app j that a developer $i \in C$ will satisfy is Q_j^C, commonly decided by the members of coalition C by means of SLAs. Thus, the amount of hours allocated by developer i who decides to join the coalition C is

$$q_{ij} = \lambda_{ij}Q_j^C,\ i \in C$$

The profit of a coalition is the sum of the incremental profits provided by members under reference prices p_j. These profits are proportional to the increase in quality level provided by each developer. In this case, if the level of quality that coalition C decides to satisfy is given by Q_j^C for mobile app j, the total supplied quality is

$$\min\left\{d_{js}, \min_{C\in\Xi} Q_j^C\right\}. \tag{4}$$

Therefore, the work allocation problem that coalition C belonging to the set of coalitions Ξ faces under fixed decisions of other coalitions $\mathcal{K} \in \Xi$ is the following

$$\max_{Q_j^C} \sum_{i\in C}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s \min\left\{d_{js}, \min_{\mathcal{K}\in\Xi} Q_j^{\mathcal{K}}\right\} - c_i\lambda_{ij}Q_j^C\right) \tag{5}$$

subject to

$$\sum_{j\in\mathcal{P}_i}\lambda_{ij}Q_j^C \le W_i,\ i \in C \tag{6}$$

$$Q_j^C \ge 0. \tag{7}$$

It is easy to recast problem (5)–(7) as the deterministic equivalent of a linear stochastic programming problem with recourse (see [58,59]) with Q_j^C as the first stage variables and y_{ijs} as recourse variables.

$$\max_{y_{ijs},Q_j^C} \sum_{i\in C}\left[\sum_{j\in\mathcal{P}_i}\lambda_{ij}\left(p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}} - c_i\right)Q_j^C - \sum_{j\in\mathcal{P}_i}\sum_{s\in\Omega}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\pi_s y_{ijs}\right] \tag{8}$$

subject to

$$\sum_{j\in\mathcal{P}_i}\lambda_{ij}Q_j^C \le W_i, \qquad i \in C \tag{9}$$

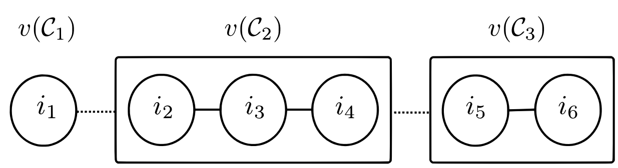

Fig. 1. Interplay between coalitions of developers.

$$\lambda_{ij} Q_j^C - y_{ijs} \le \lambda_{ij} d_{js}, \qquad i \in C, \quad j \in \mathcal{P}_i, \quad s \in \Omega \tag{10}$$

$$Q_j^C \le Q_j^{\mathcal{K}}, \qquad i \in C, \quad \mathcal{K} \in \Xi \setminus C \quad j \in \mathcal{P}_i \tag{11}$$

$$y_{ijs}, Q_j^C \ge 0, \qquad i \in C, \quad j \in \mathcal{P}_i, \quad s \in \Omega. \tag{12}$$

Notice how this problem is not the typical resource pooling problem often found in literature about cooperative games, where different agents pool their resources to extend their production possibilities. In fact, in problem (8)–(12), resources are not summed over the different coalition participants, but the related constraints are repeated for each agent. This means that the improvements in profitability only stem from the possibility of coordinating quality levels, based on constraint (11). As the coalition C grows, there will be less need to worry about the uncoordinated actions of other providers outside the coalition. Instead, the coalition will increasingly decide on the quality level internally. This aspect of our model is even more important when looking at the grand coalition $\mathcal{I}$. Here, because everyone agrees on quality levels internally, we do not need constraint (11).

4. Game theoretical analysis of cooperative service provision

This section provides the analysis and solution to the cooperative provision problem. Namely, starting from the introduced work allocation problem (5)–(7) we study how revenues can be shared among the participating agents. This analysis consists of two layers.

1. We consider a noncooperative game between coalitions with payoffs (5) and strategy set (6) , (7). The analysis uses the concept of dominant Nash equilibrium, defined as follows:

Definition 1. Let x' and $\bar{x}$ with $x' \neq \bar{x}$ be Nash equilibria for actors i with payoff function $F_i(x)$; $\bar{x}$ is a dominant Nash equilibrium if $F_i(x') < F_i(\bar{x})$ for all i.

We show that this game has a unique dominant Nash equilibrium [60] that yields equilibrium levels $Q_j^{C_k}$ of quality that the coalitions want to satisfy. We assume that the coalitions can choose this equilibrium.

2. After obtaining the equilibrium profits of the coalitions, we apply the framework of CGT to assess the stability of the grand coalition $\mathcal{I}$.

Combined, these two layers constitute a *stochastic provision game*, where stochasticity is due to the random nature of demand for quality. Fig. 1 illustrates the interactions among providers in our framework. The example showcases three coalitions. Dashed lines represent the non-cooperative layer, while bold lines represent the cooperative layer. Within a coalition, agents sign binding contracts (continuous lines) to coordinate their supply. The different coalitions do not coordinate their provision through contracts but rely on anticipations of other coalitions' behavior.

4.1. Noncooperative layer of the stochastic provision game

We start with presenting a general result that describes the Nash equilibrium of a class of noncooperative games which the game (5)–(7) between coalitions belongs to.

Let us consider $i = 1 : N$ actors with actions $x^i \in X^i \subseteq \mathbb{R}^n$. The set X^i of admissible actions is defined as follows:

$$X^i = \left\{ x^i \mid x^i \in V^i,\ F_k^i\left(x^i\right) \le 0,\ k = 1 : K \right\}$$

where V^i is the natural domain for the decisions x^i and $F_k^i\left(x^i\right) \le 0,\ k = 1 : K$ is a set of additional constraints that the agent i needs to satisfy. The payoff function $F_0^i(x)$, $x = \left(x^1, \dots, x^N\right)$ of agent i has the following form:

$$F_0^i(x) = F_0^i(x^i, z(x)),\ z(x) = \left(z_1(x), \dots, z_n(x)\right),\ z_j(x) = \min_i x_j^i. \tag{13}$$

Let us define the collection $\bar{x} = \left(\bar{x}^1, \dots, \bar{x}^N\right)$ of the agent's strategies as follows

$$\bar{x}^k = \left(\bar{x}_1, \dots, \bar{x}_n\right),\ \bar{x}_j = \min_i \hat{x}_j^i,\ \forall k \tag{14}$$

where $\hat{x}^i$ is the solution of the problem

$$\max_{x^i \in X^i} F_0^i(x^i, x^i) \tag{15}$$

The following proposition shows when $\bar{x}$ is the dominant Nash equilibrium point.

Proposition 1. *Suppose that the following conditions are satisfied.*

1. Functions $F_k^i\left(x^i\right)$ are nondecreasing for $x^i \in X^i$, namely if $y^i, u^i \in X^i$, with $y^i \ge u^i$ componentwise, then

$$F_k^i\left(y^i\right) \ge F_k^i\left(u^i\right)$$

2. Sets $V^i \subseteq \mathbb{R}^+$ are monotone, namely if $u^i \in V^i$ and $0 \le y^i \le u^i$, then $y^i \in V^i$.

3. Function $F_0^i(x^i, z)$ is nonincreasing with respect to $x^i \in X^i$, namely if $y^i \ge u^i$ then $F_0^i(y^i, z) \le F_0^i(u^i, z)$.

4. Function $F_0^i(x^i, x^i)$ is uniformly unimodal on X^i, namely if $y^i, u^i \in X^i$ are such that all their components coincide except $y_j^i \neq u_j^i$ then $F_0^i(y^i, y^i) \le F_0^i(u^i, u^i)$ if $y_j^i \le u_j^i \le \hat{x}^i$ and $F_0^i(y^i, y^i) \ge F_0^i(u^i, u^i)$ if $\hat{x}_j^i \le y_j^i \le u_j^i$.

Then

1. The collection $\bar{x}$ of agent's strategies defined in (14) –(15) is a Nash equilibrium.

2. If solutions $\hat{x}^i$ of problems (15) are unique and inequalities in Conditions 3,4 above are strict if y^i is different from u^i then $\bar{x}$ is the dominant Nash equilibrium point.

Proof. See appendix

Suppose now that the partition Ξ of the set of actors $\mathcal{I}$ into a coalitions is fixed. Proposition 1 allows to characterize the dominant Nash equilibrium played in the noncooperative game by the coalitions from Ξ with payoffs defined by (5) –(6).

Proposition 2. *Suppose that $c_i, \lambda_{ij} > 0$ and Q_j^C is the unique solution of problem*

$$\max_{Q \ge 0} \sum_{i \in C} \sum_{j \in \mathcal{P}_i} \left(\lambda_{ij} p_j \frac{D_j^{max}}{\sum_{k \in \mathcal{I}} \lambda_{kj}} \sum_{s \in \Omega} \pi_s \min\left\{d_{js}, Q_j\right\} - c_i \lambda_{ij} Q_j \right) \tag{16}$$

subject to constraint (6) and

$$Q_j\left(\Xi\right) = \min_{C \in \Xi} Q_j^C,\ j \in \mathcal{P} \tag{17}$$

Then $Q\left(\Xi\right) = \left\{Q_j\left(\Xi\right),\ j \in \mathcal{P}\right\}$ is the unique dominant Nash equilibrium point for the noncooperative game with payoffs defined by (5) –(6).

Proof. See appendix

The economic implications of these propositions indicate that the quality of components provided for app j will match the quality set by the provider offering the lowest quality. This occurs because if any other provider opts for a higher quality, it will not be reflected in the overall quality of the app, according to the definition for the overall app quality provided in (4).

4.2. Cooperative layer of the stochastic provision game

We will now explore the potential benefits of all providers coming together to form a grand coalition comprising all actors $i \in \mathcal{I}$. This analysis assumes that no agent assumes the role of a platform operator. Developers have the choice to engage in collaborative development of a mobile app that is sold off-platform. We will describe the solution approach for this setup and then consider the presence of a platform operator. It will be demonstrated that a profit division within the grand coalition exists, which makes it unprofitable for any subgroup of actors to leave the coalition. In order to do this we shall cast the problem (16), (6) with Nash equilibrium from (17) into the framework of Cooperative Game Theory with transferable utility. For the sake of completeness of the exposition we shall introduce the notions of this theory where necessary, following [24].

Definition 2 (*[24], Definition 2.1.1*)**.** A cooperative game with transferable utility (a TU game) is a pair (N, v) where N is a set of actors and v is a coalition function that associates a real number $v(S)$ with each subset S of N.

Based on this definition, we consider the set $\mathcal{I}$ of developers as the set of actors. The coalition function $v(C)$ is taken directly from Proposition 2 as the equilibrium profit $F^C(Q(\Xi))$:

$$\sum_{i\in C}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s\min\left\{d_{js},Q_j(\Xi)\right\}-c_i\lambda_{ij}Q_j(\Xi)\right)$$

where $Q_j(\Xi)$ is defined as in (17). However, its direct usage results in ambiguity because $F^C(Q(\Xi))$ depends, through (17), on the coalition set Ξ. In order to resolve this ambiguity we will assume that the actors that form coalition C assume that all other actors form the opposing coalition $\mathcal{I}\setminus C$. Then the possible subcoalitions that we consider are the partitions of the grand coalition Ξ_C such that

$$\Xi_C = \{C\}\cup\{\mathcal{I}\setminus C\}. \tag{18}$$

In this way we can unambiguously define the characteristic function for coalition C, $v(C)$ as

$$v(C) = F^C\left(Q\left(\Xi_C\right)\right). \tag{19}$$

The pair $(\mathcal{I}, v)$ with $v(C)$ from (19) defines a cooperative game in the sense of Definition 2. The results of CGT can be utilized for its analysis. The question that interests us is whether it is possible to divide the common profit of the grand coalition $\mathcal{I}$ among its members in such a way that no group $C \subset \mathcal{I}$ will be incentivized to abandon this coalition. Let us denote by x_i the payoff that actor i obtains after the division of coalition profits among its members. The concept of the *core* defines the payoffs that make the grand coalition stable.

Definition 3 (*[24], Definition 2.3.2*)**.** The core of a game (N, v) is such set of payoffs $x = \{x_i,\ i\in N\}$ that

$$\sum_{i\in N}x_i \le v(N),\ \sum_{i\in S}x_i \ge v(S),\forall S\subseteq N.$$

We shall prove that the stochastic provision game we have introduced has nonempty core. In order to do this we utilize the criterion that was found by [22,61].

Theorem 1 (*[24], Theorem 3.1.4*)**.** *A necessary and sufficient condition that the core of a game (N, v) is not empty is that for each collection $\mathcal{B}\subseteq 2^N$ of the set of actors N and for each collection of positive numbers δ_S, $S\in\mathcal{B}$ such that*

$$\sum_{S\in\mathcal{B}}\delta_S\chi_k(S) = 1,\ \forall k\in N,\ \chi_k(S) = \begin{cases}1 & \text{if } k\in S\\ 0 & \text{otherwise}\end{cases} \tag{20}$$

it is satisfied

$$v(N) \ge \sum_{S\in\mathcal{B}}\delta_S v(S). \tag{21}$$

The following result confirms that the cooperative provision game $(\mathcal{I}, v)$ has nonempty core.

Proposition 3. *The cooperative provision game $(\mathcal{I}, v)$ with coalition function (19) has nonempty core for the partition Ξ_C.*

Proof. See appendix

From an economic perspective, having a nonempty core ensures that providers can always reach an agreement to collaborate around a platform that is more profitable than operating individually.

5. Game solution combining the core with the shapley value

In this section we develop a specific way to define the system of side payments x_i that is negotiated by the actors in order to divide the profits of the cooperative provision game. It combines the concepts of core and the Shapley value to single out a vector x belonging to the core.

Definition 4 (*[24], Corollary 8.1.5*)**.** The Shapley value of cooperative game $(\mathcal{I}, v)$ is given by

$$\phi_i(\mathcal{I},v) = \sum_{C\subseteq\mathcal{I}\setminus\{i\}}\frac{|C|!(|\mathcal{I}|-|C|-1)!}{|\mathcal{I}|!}\{v(C\cup\{i\})-v(C)\} \tag{22}$$

for every $i\in\mathcal{I}$, where $|A|$ is the number of elements in set A.

The criterion that we adopt is the closest $\mathcal{L}_1$ distance from the Shapley value. It involves two steps:

1. *Compute the coalition function $v(C)$ of each coalition $C\subseteq\mathcal{I}$.*

This is done by solving the problem

$$\max_{y_{ijs},Q_j}\sum_{i\in\mathcal{K}}\left[\sum_{j\in\mathcal{P}_i}\lambda_{ij}\left(p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}-c_i\right)Q_j-\sum_{j\in\mathcal{P}_i}\sum_{s\in\Omega}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\pi_s y_{ijs}\right] \tag{23}$$

subject to

$$\sum_{j\in\mathcal{P}_i}\lambda_{ij}Q_j\le W_i,\qquad i\in\mathcal{K} \tag{24}$$

$$\lambda_{ij}Q_j-y_{ijs}\le\lambda_{ij}d_{js},\qquad i\in\mathcal{K},\quad j\in\mathcal{P}_i,\quad s\in\Omega \tag{25}$$

$$y_{ijs},Q_j\ge 0,\qquad i\in\mathcal{K},\quad j\in\mathcal{P}_i,\quad s\in\Omega,$$

with $\mathcal{K}=C$ and $\mathcal{K}=\mathcal{I}\setminus C$. This results in two optimal solutions Q^C and $Q^{\mathcal{I}\setminus C}$. Take

$$\hat{Q}_j^C = \min\left\{Q_j^C, Q_j^{\mathcal{I}\setminus C}\right\}$$

and compute

$$v(\mathcal{K}) = \sum_{i\in\mathcal{K}}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s\min\left\{d_{js},\hat{Q}_j^C\right\}-c_i\lambda_{ij}\hat{Q}_j^C\right)$$

for $\mathcal{K}=C,\ \mathcal{I}\setminus C$.

2. *Compute the Shapley value $\phi_i(\mathcal{I}, v)$* for each actor $i\in\mathcal{I}$.
3. *Compute the element of the core x_i closest to the Shapley value ϕ_i* in $\mathcal{L}_1$ norm.

This is done by solving the problem

$$\min_{x_i\ge 0}\sum_{i\in\mathcal{I}}|\,x_i-\phi_i(\mathcal{I},v)\,|$$

$$\text{s.t.}\ \sum_{i\in C}x_i\ge v(C),\qquad C\subset\mathcal{I}, C\ne\emptyset$$

$$\sum_{i\in\mathcal{I}}x_i = v(\mathcal{I}),$$

which can be easily linearized.

Our framework prioritizes stability by ensuring that any formed coalition remains robust against any subset of participants opting to work independently, which is fundamental for the effectiveness of cooperative endeavors. This stability is guaranteed by focusing on payoff distributions within the game's core, where no group of participants would benefit from deviating. Within this stable structure, we then seek to incorporate fairness, secondary to stability but vital for the collaborative spirit of an open source platform. This is achieved by aligning the selected payoff vector as closely as possible to the Shapley value.

6. Modeling the platform operator

The platform operator is assigned index $i = 1$. Unlike other developers, the platform operator does not consider a work allocation problem but rather serves as a demand enhancer. Selling the app on the platform or off-platform impacts the number of potential customers that developers can reach. The value function for the Platform Operator alone is always zero [62]. More formally, we introduce the cooperative provision game $(\tilde{v}, \mathcal{I})$ with the properties $\tilde{v}(\{1\}) = 0$, $\tilde{v}(\mathcal{I} \setminus \{1\}) < v(\mathcal{I} \setminus \{1\})$ and $\tilde{v}(C) = v(C), \quad C \neq \{1\}, C \neq \mathcal{I} \setminus \{1\}$. The reason why $\tilde{v}(\mathcal{I} \setminus \{1\}) < v(\mathcal{I} \setminus \{1\})$ is related to the change of demand level in the game $(\tilde{v}, \mathcal{I})$, when the platform does not participate in a coalition. This structure ensures that the developer is willing to engage in collaboration with the Platform Operator in order to benefit of its customer base.

If no coalition has direct cooperation with the platform, no one can benefit of the marketplace for selling the app, with the consequence that the total available demand for the app will be given by D_j^{noPO}, with $D_j^{\max} > D_j^{\text{noPO}}$. This is the case of a developer using her website for delivering her app. Let us define the demand considered by each coalition C as

$$D_j^C = \begin{cases} D_j^{\max} & \text{if the Platform Operator is part of the coalition} \\ D_j^{\text{noPO}} & \text{otherwise} \end{cases}$$

and with D_j^{noPO} defining the maximum customer base that the coalition can reach without the collaboration of the Platform Operator.

The problem up to a given coalition C is $\tilde{v}(C) =$

$$\max_{Q_j^C, y_{ijs}} \sum_{i\in C} \sum_{j\in\mathcal{P}_i} \sum_{s\in\Omega} \pi_s p_j \frac{D_j^C}{\sum_{k\in\mathcal{I}} \lambda_{kj}} \left(\lambda_{ij} Q_j^C - y_{ijs} \right) - \sum_{i\in C} \sum_{j\in\mathcal{P}_i} c_i \lambda_{ij} Q_j^C \tag{26}$$

$$\text{s.t. } \lambda_{ij} Q_j^C - y_{ijs} \le \lambda_{ij} d_{js} \qquad j \in \mathcal{P}_i, \quad i \in C, \quad s \in \Omega \tag{27}$$

$$\sum_{j\in\mathcal{P}_i} \lambda_{ij} Q_j^C \le W_i \qquad i \in C \tag{28}$$

$$Q_j^C \le Q_j^{*\mathcal{I}\setminus C} \qquad j \in \mathcal{P}_i, \quad i \in C \tag{29}$$

$$Q_j^C, y_{ijs} \ge 0$$

where $Q_j^{*\mathcal{I}\setminus C}$ is the optimal response of the complementary coalition.

The Platform Operator has $\lambda_{1j} = 0$, since her contribution does not add quality to the app. The only contribution of the Platform Operator to the coalition is the expansion of the customer base, represented by $D_j = D_j^{\max}$.

Similarly to problem (8)–(11) and defining $q_{ij} = \lambda_{ij} Q_j$, the centralized problem is obtained as

$\tilde{v}(\mathcal{I}) =$

$$\max_{Q_j^\mathcal{I}, y_{ijs}} \sum_{i\in \mathcal{I}} \sum_{j\in\mathcal{P}_i} \sum_{s\in\Omega} \pi_s p_j \frac{D_j^{\max}}{\sum_{k\in\mathcal{I}} \lambda_{kj}} \left(\lambda_{ij} Q_j^\mathcal{I} - y_{ijs} \right) - \sum_{i\in \mathcal{I}} \sum_{j\in\mathcal{P}_i} c_i \lambda_{ij} Q_j^\mathcal{I} \tag{30}$$

$$\text{s.t. } \lambda_{ij} Q_j^\mathcal{I} - y_{ijs} \le \lambda_{ij} d_{js} \qquad j \in \mathcal{P}_i, \quad i \in \mathcal{I}, \quad s \in \Omega \tag{31}$$

$$\sum_{j\in\mathcal{P}_i} \lambda_{ij} Q_j^\mathcal{I} \le W_i \qquad i \in \mathcal{I} \tag{32}$$

$$Q_j^\mathcal{I}, y_{ijs} \ge 0$$

The definition of the value functions for every subcoalition of $\mathcal{I}$ equips the collaborative provision game $(\tilde{v}, \mathcal{I})$ with the following property

Corollary 1. *The provision game described $(\tilde{v}, \mathcal{I})$ has non-empty core.*

Proof. See appendix

Definition 5 (*[24], Definition 2.1.6*)**.** A game (N, v) is convex if

$$v(S \cup \{i\}) - v(S) \le v(T \cup \{i\}) - v(T) \qquad \forall S \subseteq T \subseteq N \setminus \{i\}.$$

In the case where the mobile app provision game involves only two actors, the developer and the Platform Operator, the game is convex. Convex games are characterized by having the Shapley value as an element of the core.

Proposition 4. *The Cooperative Provision Game $(\tilde{v}, \mathcal{I})$ with two actors is convex.*

Proof. See appendix

A game is convex when the addition of an agent to a coalition not only enhances its marginal benefits but does so increasingly as the coalition grows larger. In our case, the Platform operator alone cannot realize gains from the app, and a single app developer faces limited off-platform market size. In either case, the marginal benefits that they can provide on their own (against the coalition with no actors) is smaller than the marginal benefits that they can provide when one of the actors is already in the coalition. While adding a second actor is clearly beneficial, the impact of a third actor depends on existing coalition quality. For instance, imagine an app developer selling her product through a platform without mandatory user identification. Now, imagine a regulatory change necessitating user profiles for all app users, prompting the inclusion of a third party responsible for managing these profiles within the collaboration. If this profile management service is subpar, causing app instability, the collaboration's overall quality suffers. Opting for this extended collaboration may reduce the developer's contribution compared to collaborating solely with the platform operator, despite coalition expansion.

7. Case study

We will analyze the revenue sharing mechanism between the Platform Operator and two developers collaborating to provide an app that can process audio and video information in real time, mimicking human interaction. It sends this data to a server-based AI model for processing, then receives and vocalizes the textual output, engaging users conversationally. A developer focuses on real-time video integration, and the other on audio, both interfacing with a central server that provides AI services for a monthly fee but is not part of the collaborative provision. The video segment requires six months of development, equivalent to roughly 1000[3] hours at 7 h a day, 25 days a month, with developer costs at \$150[4] per hour, taxes included. The audio segment is expected to take 600 h (about three months), costing \$200 per hour. We assume that the video developer can dedicate up to 2520 person-hours, while the audio developer has a 2500 person-hour maximum capacity for this project. Potential global downloads are projected at 140000 if sold via the platform, dropping to 70000 if the components are sold separately for off-platform integration by users. The platform operator is assumed to only enhance the demand, and therefore we do not include any cost, capacity or effort for this actor. More formally, if we index the real-time app as $j = 1$, the Platform as $i = 1$, the video component developer as $i = 2$ and the audio component developer as $i = 3$, the parameters used for modeling the case study are therefore defined as follows:

[3] https://solveit.dev/blog/how-long-does-it-take-to-develop-an-app

[4] https://www.pixelcrayons.com/blog/software-development/software-development-hourly-rates/

$\lambda_{2,1} = 1000,\ \lambda_{3,1} = 600$
$c_2 = 150,\ c_3 = 200$
$p_1 = 4.2$
$W_2 = 2520,\ W_3 = 2500$
$D_1^{\max} = 140000,\ D_1^{\text{noPO}} = 70000$

We also sample values for the interest for quality from a standard uniform distribution, *i.e.*, $d_j \in U\{0,1\}$, by simulating 100 realizations, each with a probability of 0.01, and assigning the values to parameter d_{js}. In our analysis, we assume two levels of maximum demand: *high* with an official platform and *low* off-platform. The app development costs serve as the potential price the developer could ask if working as a third-party consultant.

We conduct the following analyses:

1. *Position readjustment of providers due to increase of platform customer base.*

Is collaboration more favorable on a small or large platform? Fig. 2(a) illustrates that the side payment resulting from cooperation with a platform with a progressively larger customer base increases for developers based on their contribution to coalition profitability. Both developers and the platform mutually benefit from the larger customer base, and the revenue shares stabilize quickly, reflecting the agents' relative influence on platform revenues. Considering the horizontal nature of the collaboration, the revenue shares do not change as the number of customers increases, as depicted in Fig. 2(b).

2. *Position readjustment of providers with increasing complexity in the app development.*

Is it better to develop simple or complex apps? Complex apps require extensive time for development, testing, and bug fixing. With a limited working capacity, the resulting quality may decrease, leading to a reduction of sales. In our scenario, we assume that the video developer must allocate a growing amount of work to develop its component, while the audio developer does not require additional time for its add-on. To maintain a high QoS, the side payment extracted by the video developer and the platform increases at the expense of the side payment granted to the audio developer, as depicted in Fig. 3(a). The video developer experiences improved profitability because the video component's significance grows in defining the overall bundle quality. Additionally, the platform operator increases profits because, with complex apps, developers are more inclined to collaborate and supply their components within the platform, benefiting from the larger customer base it provides, and from the added value of coordination. The audio developer experiences a decline in profitability as its component becomes relatively easier to deliver. Once the profits of all actors reach the same levels, they decline together due to a steady decrease in app quality caused by longer required development times. The revenue shares mirror the pattern of the side payments and converge to 33%, granted to each agent (Fig. 3(b)). Hence, a platform can benefit from collaborating with developers offering complex components because these developers are more motivated to collaborate with the platform. This gives the platform operator an advantage over the developers, leading to increased profitability. However, this advantage disappears if the cost of app development becomes excessively high due to its complexity.

3. *Position readjustment of providers due to different developer size.*

Is it better to work with large or small developers? Figs. 4(a) and 4(b) illustrate how the relative bargaining power between the platform and developers changes with the size of the video developer. As size increases, developers can offer higher quality. Payments to developers and the platform grow proportionally until supply meets the maximum quality demand. Larger developers have more incentive to join a platform for a larger customer base, which leads to higher profits. The platform operator's contribution to the coalition is stronger when developers serve a large market. This is reflected by the fact that the Platform Operator is able to extract a slightly larger portion of the revenue share before settling at a fixed share, as shown in Fig. 4(b).

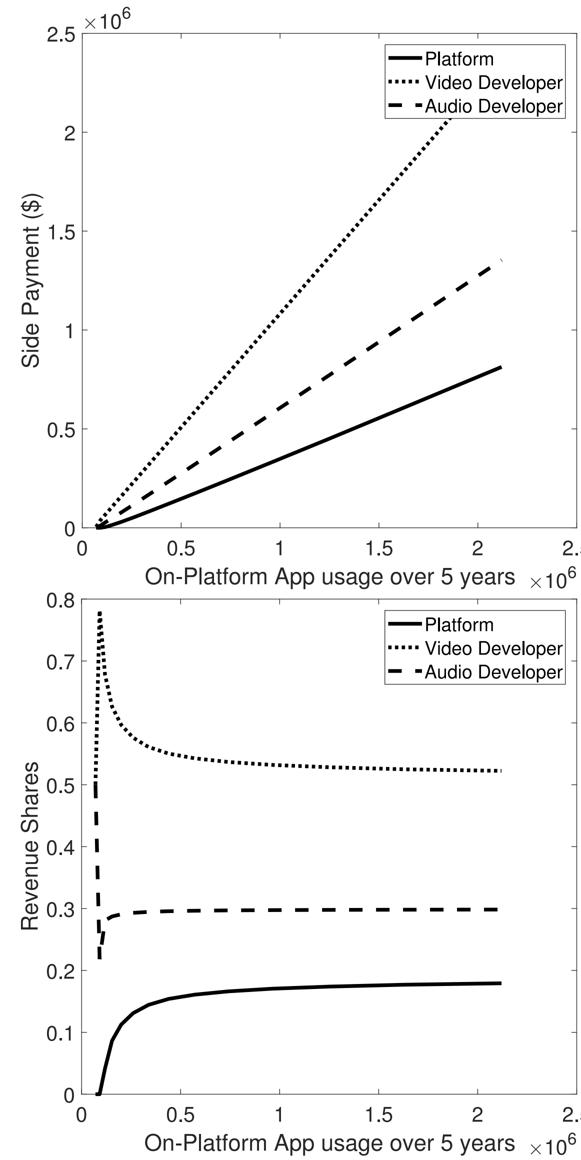

Fig. 2. Dependence of the side payments (a) and revenue shares (b) on the cumulated on-platform usage of the app.

Although large developers receive a smaller revenue share compared to small developers, the overall payment is higher. In order to show the robustness of the findings of the case study, a sensitivity analysis of the main results discussed in this section is reported in Appendix II.

8. Conclusions

In this paper, we have established a framework to analyze complex patterns of competition and collaboration among multiple actors engaged in collaborative service provision with demand uncertainty. The business model under investigation considers all participants equally important in terms of their roles in providing services to end users. Thus, coordination is achieved by designing an incentive scheme based on each actor's contribution to the overall partnership value. A key aspect of this research is defining a system of side payments for cooperative game participants, which we compute as the point in the core with the minimum $\mathcal{L}^1$-distance from the Shapley value.

The conducted analyses provide several insights. Firstly, the platform's coordination role reduces the risk in quality provision by facilitating information sharing among developers. This results in higher

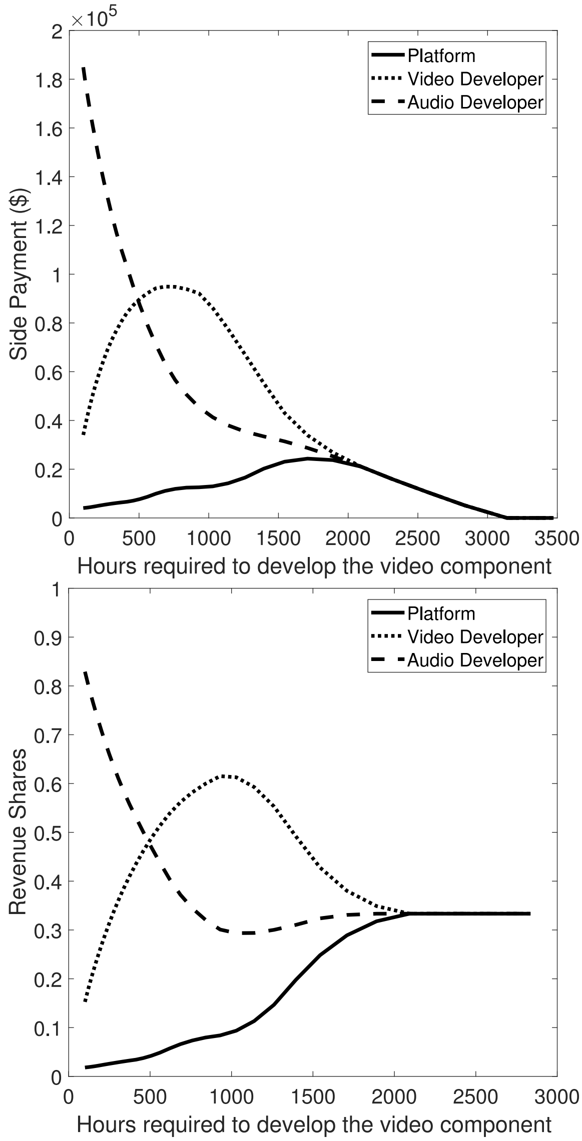

Fig. 3. Dependence of the side payments (a) and revenue shares (b) on the hours required to develop the video component with maximum quality.

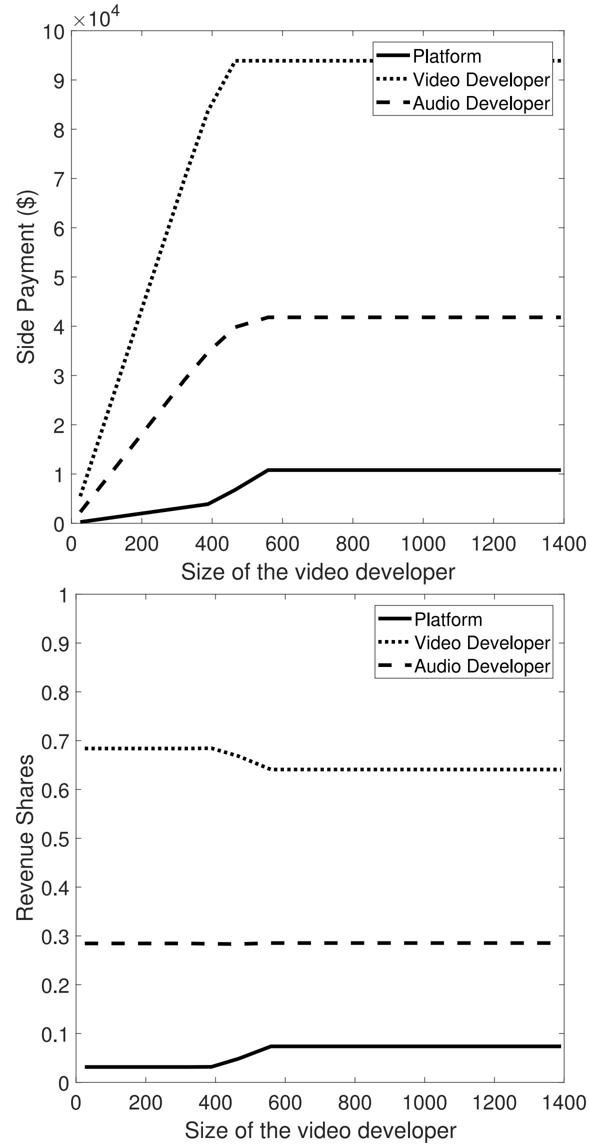

Fig. 4. Dependence of the side payments (a) and revenue shares (b) on the size of the video developer.

app quality and improved profitability for all participants. Additionally, negotiating revenue shares based on specific characteristics of business partners, such as size, expected user expenditure, or app complexity, can enhance platform profitability. Moreover, there is a moderate relationship between revenue share and side payment granted to each partner. In some cases, the platform operator may opt for a smaller revenue share that actually reflects a larger side payment. Our study highlights that the size of the platform's customer base does impact on the profits of all the participants in proportion to their contribution to the overall sales leaving the revenue shares unchanged. While the platform greatly increases the customer base for the developers, it is also true that the platform cannot sell content without the developers. In a peer-to-peer, horizontal collaboration scheme, this leads to a side payment that is aligned with the synergies and redundancies that each agent brings to the overall coalition. The answer to the second research question is that the developers producing complex apps should prefer distributing them under a platform, due to the possibility to reach a larger customer basis. Moreover the share of revenue granted to the developer increases with the complexity of the app. The reason is that the platform needs to provide incentives to develop the app with a high quality standard, so to reach the largest possible base of customers. Nevertheless, very complex apps might lead to development difficulties that decrease the profitability for the coalition. This levels off the share of revenues granted from the platform operator to the developers, leading to an equal sharing of the revenues. Also, while all developers have incentive to join a platform, the platform will manage to retain a larger revenue share from large developers because small developers might not necessarily need to appeal to a large base of users, while large ones need to reach more customers and are willing to sacrifice a small part of their share in exchange for a larger market. Future research will delve into analyzing this problem considering different distributions of market power between the platform and developers.

CRediT authorship contribution statement

Paolo Pisciella: Writing – review & editing, Writing – original draft, Software, Methodology, Conceptualization. **Alexei A. Gaivoronski:** Writing – review & editing, Methodology, Conceptualization.

Declaration of competing interest

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The authors declare that there is no conflict of interest regarding the publication of this paper.

Data availability

Sources for data are defined in links into the paper as well as stated in the case study, where the used data is also reported.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT4 in order to improve language and readability. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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Appendix A. Appendix I

Proof (*Proposition 1*)**.** Let us select an arbitrary i. From (14) follows that $\bar{x}^i \leq \hat{x}^i$. Besides, $\hat{x}^i_j \in V^i$ as the solution of (15). Therefore $\bar{x}^i \in V^i$ due to Condition 2. Since $\hat{x}^i \in X^i$ as the solution of (15) then $F^i_k\left(\hat{x}^i\right) \leq 0$. Since $F^i_k\left(x^i\right)$ is nondecreasing on V^i and $\bar{x}^i \leq \hat{x}^i$ this yields $F^i_k\left(\bar{x}^i\right) \leq 0$ for all k. Thus, $\bar{x}^i$ is feasible for all i.

Suppose now that $\bar{x}$ is not a Nash equilibrium. This means that for some i there exists $y^i \in X^i$, $y^i \neq \bar{x}^i$ such that

$$F^i_0(y^i, z\left(\bar{x}_y\right)) > F^i_0(\bar{x}^i, z(\bar{x})), \tag{A.1}$$

where $\bar{x}_y = \left(\bar{x}^1, \dots, \bar{x}^{i-1}, y^i, \bar{x}^{i+1}, \dots, \bar{x}^N\right)$. Let us assume that there exist components $\bar{x}^i_j$ and y^i_j of vectors $\bar{x}^i$ and y^i such that $\bar{x}^i_j < y^i_j$ and take $\tilde{y}^i = \left(y^i_1, \dots, y^i_{j-1}, \bar{x}^i_j, \; y^i_{j+1}, \dots, y^i_n\right)$, $\tilde{x}_y = \left(\bar{x}^1, \dots, \bar{x}^{i-1}, \tilde{y}^i, \bar{x}^{i+1}, \dots, \bar{x}^N\right)$. Then $\tilde{y}^i \leq y^i$ componentwise and therefore $\tilde{y}^i \in X^i$ due to Conditions 1,2. From (13), (14) follows that $z_j(\bar{x}) = z_j(\bar{x}_y) = z_j(\tilde{x}_y)$ and, consequently, $z(\bar{x}_y) = z(\tilde{x}_y)$ because $\bar{x}_y$ and $\tilde{x}_y$ differ only in the jth component of the strategy of agent i which in both cases is not smaller than this component of the strategies of other agents, and the strategies of the other agents in both $\bar{x}_y$ and $\tilde{x}_y$ are equal. Thus, Condition 3 yields:

$$F^i_0(y^i, z\left(\bar{x}_y\right)) = F^i_0(y^i, z\left(\tilde{x}_y\right)) \leq F^i_0(\tilde{y}^i, z\left(\tilde{x}_y\right))$$

In other words, by substituting the point y^i by the point $\tilde{y}^i$ that coincides with y^i with the exception of component y^i_j that is changed to $\min\left\{y^i_j, \bar{x}^i_j\right\}$ the agent i will obtain a feasible strategy with at least as good value of his objective when the strategies of other agents do not change. Making the same substitution for all other components y^i_k : $\bar{x}^i_k < y^i_k$ we shall obtain a feasible point $u^i = \min\left\{y^i, \bar{x}^i\right\}$ such that for $\bar{x}_u = \left(\bar{x}^1, \dots, \bar{x}^{i-1}, u^i, \bar{x}^{i+1}, \dots, \bar{x}^N\right)$

$$F^i_0(u^i, z\left(\bar{x}_u\right)) \geq F^i_0(y^i, z\left(\bar{x}_y\right)) > F^i_0(\bar{x}^i, z(\bar{x})) \tag{A.2}$$

according to our original assumption (A.1). Then there exists a component u^i_j such that $u^i_j < \bar{x}^i_j$, otherwise (A.2) would be impossible. Let us take

$$\tilde{u}^i = \left(u^i_1, \dots, u^i_{j-1}, \bar{x}^i_j, u^i_{j+1}, \dots, u^i_n\right), \; \tilde{x}_u = \left(\bar{x}^1, \dots, \bar{x}^{i-1}, \tilde{u}^i, \bar{x}^{i+1}, \dots, \bar{x}^N\right).$$

Again, $\tilde{u}^i \in X^i$ due to $\tilde{u}^i \leq \bar{x}^i$ and Conditions 1,2. Due to the definition of operator $z(x)$ from (13) we have $F^i_0(u^i, z\left(\bar{x}_u\right)) = F^i_0(u^i, u^i)$, $F^i_0(\tilde{u}^i, z^i\left(\tilde{x}_u\right)) = F^i_0(\tilde{u}^i, \tilde{u}^i)$ since $u^i \leq \bar{x}^i$, $\tilde{u}^i \leq \bar{x}^i$. In addition, $u^i_j \leq \tilde{u}^i_j \leq \hat{x}^i_j$. This, together with the unimodality from Condition 4 yields:

$$F^i_0(u^i, z^i\left(\bar{x}_u\right)) = F^i_0(u^i, u^i) \leq F^i_0(\tilde{u}^i, \tilde{u}^i) = F^i_0(\tilde{u}^i, z^i\left(\tilde{x}_u\right))$$

In other words, by substituting the point u^i by the point $\tilde{u}^i$ that coincides with u^i with the exception of component $u^i_j < \bar{x}^i_j$ that is changed to $\bar{x}^i_j$ the agent i will obtain a feasible strategy with at least as good value of his objective. Performing this substitution for all components u_k : $\bar{x}^i_k > u_k$ we obtain a contradiction with (A.2). The first assertion of this proposition is proved.

Suppose now that x' is some other Nash equilibrium point, different from $\bar{x}$. Suppose that there exist i and k such that $x'^i_j > x'^k_j$ for some fixed j. Let us consider $\tilde{x}^i = \left(x'^i_1, \dots, x'^i_{j-1}, x'^k_j, x'^i_{j+1}, \dots, x'^i_n\right)$, $\tilde{x} = \left(x'^1, \dots, x'^{i-1}, \tilde{x}^i, x'^{i+1}, \dots, x'^N\right)$. Since $\tilde{x}^i \leq x'^i$ we have $\tilde{x}^i \in X^i$ due to Conditions 1,2. Due to (13) we have $z_j(\tilde{x}) = z_j(x')$, $z(\tilde{x}) = z(x')$ that together with Condition 3 where we have assumed the strict inequality yields:

$$F^i_0(x'^i, z\left(x'\right)) < F^i_0(\tilde{x}^i, z\left(x'\right)) = F^i_0(\tilde{x}^i, z(\tilde{x})).$$

This contradicts with the assumption that x' is a Nash equilibrium because x' and $\tilde{x}$ differ only in the strategies of the ith actor. Thus, $x'^i_j = x'^k_j = x'_j$ for any i, k and fixed j. Therefore for operator $z(x)$ from (13) we have $z(x') = x'^i$ for any i and any Nash equilibrium point x'.

Let us assume that $x'_j > \bar{x}_j = \min_k \hat{x}^k_j$ for some j and select i such that $\bar{x}_j = \hat{x}^i_j$. Taking

$$\tilde{x}^i = \left(x'^i_1, \dots, x'^i_{j-1}, \hat{x}^i_j, x'^i_{j+1}, \dots, x'^i_n\right), \tilde{x} = \left(x'^1, \dots, x'^{i-1}, \tilde{x}^i, x'^{i+1}, \dots, x'^N\right)$$

we obtain that $\tilde{x}^i \in X^i$ due to $\tilde{x}^i \leq x'^i$ and Conditions 1,2. In addition, $z(\tilde{x}) = \tilde{x}^i$ due to $x'_j > \bar{x}_j$ and definition of operator $z(x)$. Observing now that $\hat{x}^i_j = \tilde{x}^i_j < x'^i_j$ and applying the strict version of Condition 4 we obtain:

$$F^i_0(x'^i, z\left(x'\right)) = F^i_0(x'^i, x'^i) < F^i_0(\tilde{x}^i, \tilde{x}^i) = F^i_0(\tilde{x}^i, z(\tilde{x}))$$

which shows again that x' cannot be a Nash equilibrium.

Finally, let us assume that $x'_j < \bar{x}_j$ for some j and select an arbitrary i. In this case we have $x'^i_j < \bar{x}^i_j \leq \hat{x}^i_j$ for any i. Taking $y^i = \left(x'^i_1, \dots, x'^i_{j-1}, \bar{x}^i_j, x'^i_{j+1}, \dots, x'^i_n\right)$ we obtain from the strict version of Condition 4 that $F^i_0(x'^i, x'^i) < F^i_0(y^i, y^i)$. Choosing an arbitrary $k \neq j$ such that $x'_k < \bar{x}_k$ and taking $u^i = \left(y'^i_1, \dots, y'^i_{k-1}, \bar{x}^i_k, y'^i_{k+1}, \dots, y'^i_n\right)$ we get $F^i_0(x'^i, x'^i) < F^i_0(y^i, y^i) < F^i_0(u^i, u^i)$ in a similar way. Continuing with the exchange of x'^i_l by $\bar{x}^i_l$ where $x'_l < \bar{x}_l$ we obtain after the exchange of all such components that $F^i_0(x'^i, x'^i) < F^i_0(\bar{x}^i, \bar{x}^i)$. Recalling that $z\left(x'\right) = x'^i$ and $z\left(\bar{x}\right) = \bar{x}^i$ we obtain

$$F^i_0(x'^i, z\left(x'^i\right)) < F^i_0(\bar{x}^i, z\left(\bar{x}^i\right))$$

which proves that $\bar{x}$ dominates x'.

Proof (*Proposition 2*)**.** The proof consists of checking the conditions of Proposition 1. We take $\left\{Q^C_j, \; j \in \mathcal{P}\right\}$ as x^i from Proposition 1, the objective function from (5) as $F^i_0(x)$ from (13), the difference between left and right hand sides in (6) as $F^i_1\left(x^i\right)$ and the set (7) as V^i.

Condition 1 is satisfied because $F^i_1\left(x^i\right)$ defined in this fashion is componentwise nondecreasing due to $\lambda_{ij} \geq 0$. Condition 2 is satisfied trivially due to (6). Condition 3 is satisfied strictly because the costs c_i and composition coefficients λ_{ij} are positive. Strict satisfaction of Condition 4 is the consequence of concavity of the objective function from (16) , convexity of its feasible set and assumption that its solution is unique.

Proof (*Proposition* 3)**.** Let us select an arbitrary mapping Ξ_C, an arbitrary partition Ξ and an arbitrary collection of positive numbers δ_C, $C \in \mathcal{B}$ that satisfy (20). One such collection exists always: $\delta_C = 1$, $\forall C \in \mathcal{B}$. Since (20) is satisfied for any $i \in \mathcal{I}$, we can write:

$$\begin{aligned} v(\mathcal{I}) &= \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s \min\left\{d_{js}, Q_j^{\mathcal{I}}\right\} - c_i\lambda_{ij}Q_j^{\mathcal{I}}\right)\\ &= \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s \min\left\{\sum_{C\in\mathcal{B}}\delta_C\chi_i(C)d_{js}, Q_j^{\mathcal{I}}\right\} + \right.\\ &\left. -c_i\lambda_{ij}Q_j^{\mathcal{I}}\right). \end{aligned} \tag{A.3}$$

Let us consider

$$\tilde{Q} = \sum_{C\in\mathcal{B}}\delta_C\chi_i(C)Q_j\left(\Xi_C\right)$$

Each of $Q\left(\Xi_C\right)$ is feasible for the profit defining problem of the grand coalition (16) (with C substituted by $\mathcal{I}$ in (16)). Since the weights $\delta_C\chi_i(C)$ are positive and sum up to one and the feasible set of (16) is convex then $\tilde{Q}$ is also feasible for (16). Therefore substituting $Q_j^{\mathcal{I}}$ in (A.3) by $\tilde{Q}$ yields:

$$\begin{aligned} &v(\mathcal{I}) \geq\\ &\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s \min\left\{\sum_{C\in\mathcal{B}}\delta_C\chi_i(C)d_{js},\right.\right.\\ &\left.\left.\sum_{C\in\mathcal{B}}\delta_C\chi_i(C)Q_j\left(\Xi_C\right)\right\} - c_i\lambda_{ij}\sum_{C\in\mathcal{B}}\delta_C\chi_i(C)Q_j\left(\Xi_C\right)\right) \end{aligned} \tag{A.4}$$

Since for any set of numbers $a_i, b_i \in \mathbb{R}$ with i belonging to an arbitrary set of indices we have

$$\min\left\{\sum_i a_i, \sum_i b_i\right\} \geq \sum_i \min\left\{a_i, b_i\right\}$$

then (A.4) can be continued as follows

$$\begin{aligned} v(\mathcal{I}) &\geq \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s\sum_{C\in\mathcal{B}}\delta_C\chi_i(C)\min\left\{d_{js}, Q_j\left(\Xi_C\right)\right\}\right.\\ &\left.-c_i\lambda_{ij}\sum_{C\in\mathcal{B}}\delta_C\chi_i(C)Q_j\left(\Xi_C\right)\right)\\ &= \sum_{C\in\mathcal{B}}\delta_C\sum_{i\in\mathcal{I}}\chi_i(C)\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s\min\left\{d_{js}, Q_j\left(\Xi_C\right)\right\}\right.\\ &\left.-c_i\lambda_{ij}Q_j\left(\Xi_C\right)\right)\\ &= \sum_{C\in\mathcal{B}}\delta_C\sum_{i\in C}\sum_{j\in\mathcal{P}_i}\left(\lambda_{ij}p_j\frac{D_j^{\max}}{\sum_{k\in\mathcal{I}}\lambda_{kj}}\sum_{s\in\Omega}\pi_s\min\left\{d_{js}, Q_j\left(\Xi_C\right)\right\} - c_i\lambda_{ij}Q_j\left(\Xi_C\right)\right)\\ &= \sum_{C\in\mathcal{B}}\delta_C v(C) \end{aligned}$$

Therefore (21) is satisfied and according to Theorem 1 the provision game $(\mathcal{I}, v)$ has nonempty core.

Proof (*Corollary* 1)**.** By Proposition 3, the cooperative provision game described in Section 4.2 has nonempty core because of the validity of the Bondareva-Shapley condition. If the maximum attainable demand does not change when the Platform Operator is not in a Coalition then the game $(\tilde{v}, \mathcal{I})$ has the same structure of the game $(v, \mathcal{I})$. This means that if the demand does not change when the Platform Operator does not join a coalition the value function is given by v and one has

$$\begin{aligned} v(\mathcal{I}) \geq \sum_{C\in\mathcal{B}}\delta_C v(C) \quad &= \delta_{\{1\}}v(\{1\}) + \delta_{\mathcal{I}\setminus\{1\}}v(\mathcal{I}\setminus\{1\}) +\\ &+ \sum_{C\in\mathcal{B}\setminus(\{1\}\cup(\mathcal{I}\setminus\{1\}))}\delta_C v(C). \end{aligned}$$

for every set of δ_C such that

$$\sum_{C\in\mathcal{B}}\delta_C\chi_k(C) = 1,\ \forall k \in N,\ \chi_k(C) = \begin{cases} 1 & \text{if } k \in C\\ 0 & \text{otherwise}\end{cases}$$

Now one has that $\tilde{v}(\{1\}) = 0$ and $\tilde{v}(\mathcal{I}\setminus\{1\}) \leq v(\mathcal{I}\setminus\{1\})$ because of the lower demand, while $\tilde{v}(C) = v(C)$ for all other coalitions C, included the grand coalition $\mathcal{I}$. This implies that

$$\tilde{v}(\mathcal{I}) \geq \sum_{C\in\mathcal{B}}\delta_C\tilde{v}(C).$$

for every set of δ_C such that

$$\sum_{C\in\mathcal{B}}\delta_C\chi_k(C) = 1,\ \forall k \in N,\ \chi_k(C) = \begin{cases} 1 & \text{if } k \in C\\ 0 & \text{otherwise}\end{cases}$$

and this proves the Corollary.

Proof (*Proposition* 4)**.** A game (N, v) is convex if

$$v(S\cup\{i\}) - v(S) \leq v(T\cup\{i\}) - v(T) \qquad \forall S \subseteq T \subseteq N\setminus\{i\}.$$

The small number of agents in the considered game allow us to test the definition for each possible subset. The possible value functions differences for the tests are

$\tilde{v}(\emptyset\cup\{1\}) - \tilde{v}(\emptyset) = \tilde{v}(\{1\}) = 0$ by definition

$\tilde{v}(\emptyset\cup\{2\}) - \tilde{v}(\emptyset) = \tilde{v}(\{2\})$

$\tilde{v}(\{1\}\cup\{2\}) - \tilde{v}(\{1\}) = \tilde{v}(\mathcal{I})$

$\tilde{v}(\{2\}\cup\{1\}) - \tilde{v}(\{2\}) = \tilde{v}(\mathcal{I}) - \tilde{v}(\{2\})$

We can add actor 1 only to coalitions $\emptyset$ and $\{2\}$ and we can add actor 2 only to coalitions $\emptyset$ and $\{1\}$.

1. Condition

   $\tilde{v}(\emptyset\cup\{1\}) - \tilde{v}(\emptyset) \leq \tilde{v}(\{2\}\cup\{1\}) - \tilde{v}(\{2\})$

   coincides with

   $0 \leq \tilde{v}(\mathcal{I}) - \tilde{v}(\{2\}) \implies \tilde{v}(\{2\}) \leq \tilde{v}(\mathcal{I})$

   which must hold since the game has nonempty core
2. Condition

   $\tilde{v}(\emptyset\cup\{2\}) - \tilde{v}(\emptyset) \leq \tilde{v}(\{1\}\cup\{2\}) - \tilde{v}(\{1\})$

   coincides with

   $\tilde{v}(\{2\}) \leq \tilde{v}(\mathcal{I})$

   which is the same expression as the one in the previous point.

Appendix B. Appendix II

From the results of the case study, we observe two types of behavior: either the revenue shares among developers stabilize at a fixed value, or they converge to the same level for each developer. To ensure the robustness of our results we consider whether these conclusions hold for different configurations of the key parameters by conducting a sensitivity analysis. This analysis explores how changes in development hours, development costs, pricing strategies, and development capacity influence the revenue sharing scheme among the participants considered in the case study. Namely, we defined bounds for a selection of critical parameters as follows:

- Development hours for video component ($\lambda_{2,1}$): between 500 and 2000 h.
- Cost per hour for video development (c_2): between \$80 and \$300.
- Price point for the app (p_1): between \$2 and \$10.
- Capacity for audio development (W_3): between 1500 and 4000 person-hours.

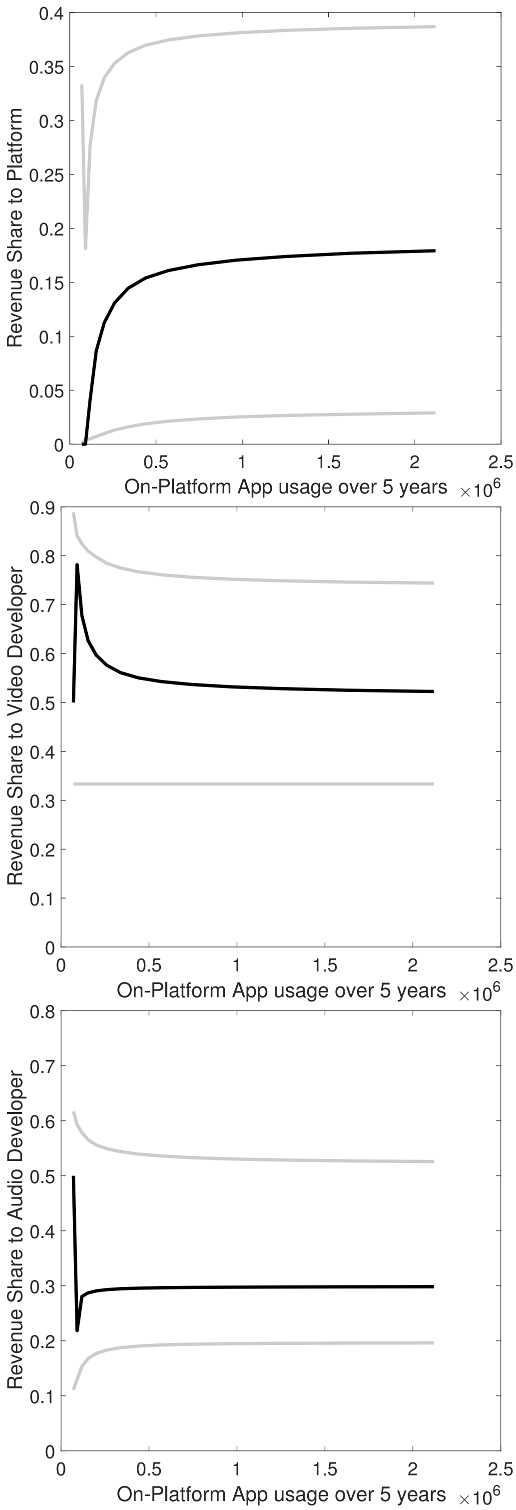

Fig. 5. Sensitivity on the effect of the cumulated demand on the revenue sharing scheme.

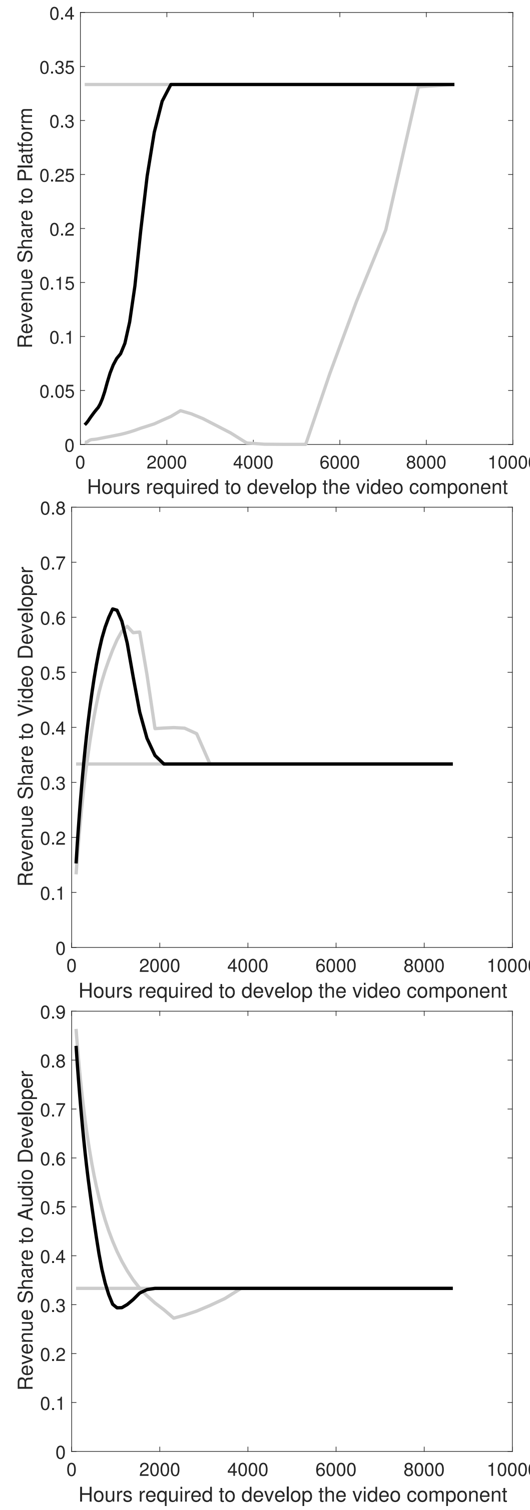

Fig. 6. Sensitivity on the effect of the complexity of the video component development on the revenue sharing scheme.

Then, we explored how combinations of extreme values for these key parameters influence revenue distribution with growing on-platform user bases and increased complexity in video component development. Our aim was to see if revenue shares stabilize when user bases expand and if they tend to become more uniform among participants as development complexity rises, beyond just the scenario examined in our case study.

To assess the sensitivity of revenue impacts from growing customer bases, we have considered parameters $c_2, \lambda_{2,1}$, and p_1, testing all combinations of their bounds. By solving the revenue sharing problem for these combinations, we identified the highest and lowest value final outcomes to compare with our case study results. Fig. 5 illustrates these findings: the case study solution is in black, while the sensitivity analysis extremes are in light grey. It is important to note that these grey curves may not stem from the same test and thus might not sum to one. In every simulation instance, the behavior mirrored the case study, showing a stable revenue share for each participant as on-platform demand increased.

To explore how increased complexity in developing the video component affects outcomes, we examined parameters c_2, W_3, and p_1, utilizing all combinations of their bounds. From these simulations, we charted the cases with the highest and lowest value initial outcomes for each participant in light grey, comparing them to the case study's revenue shares. These comparisons are shown in Fig. 6. Across all scenarios, revenue shares for participants converged to a uniform value, though the paths to this convergence varied, sometimes non-monotonically, as exemplified by the Platform's revenue share in the upper chart of Fig. 6. Despite these variations, revenue shares for all participants eventually stabilized at a common value, driven by the coalition's revenues tapering off to zero.

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