Lisa Stendahl Freitag

Option-Based Credit Spreads With Norwegian Market Data

Master's thesis in Financial Economics
Supervisor: Joakim Kvamvold
June 2024

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Abstract

In this thesis, I replicate parts of the methodology proposed in the article by Culp, C. L., Nozawa, Y., and Veronesi, P. (2018), titled “Option-based credit spreads,” published in the *American Economic Review*, volume 108, issue 2. By replicating this methodology, I contribute to filling a literature gap in the field of credit risk by exploring the concept of *pseudo bonds*. The thesis offers an assessment of the replicability of their approach, as well as a contribution to research on the Norwegian fixed-income market.

I create OBX index pseudo bonds using Norwegian market data and study their prices, leverage ratios, yield spreads, and default probabilities during the COVID-19 pandemic and the financial crisis. Moreover, I compare average yield spreads on OBX index pseudo bonds, single-stock pseudo bonds, corporate bonds, and for the Merton model during the COVID-19 pandemic. While Culp et al. performed their analysis for a period covering the financial crisis, I predominantly perform my analyses for the COVID-19 pandemic due to data availability.

Overall, Culp et al.’s method is more productive for the financial crisis than the COVID-19 pandemic for OBX pseudo bonds. The main reason for this is that I achieve a slightly more even distribution of observations among the credit rating categories. In terms of single-stock pseudo bonds, there is insufficient data to get a meaningful result. Furthermore, the resulting default probabilities for single-stock pseudo bonds are highly sensitive to minimal variations in leverage values. I obtain pretty high average spreads from the Merton model, which does not suggest the existence of a *credit spread puzzle* when comparing the average spreads with those for corporate bonds, as discussed in the work by Feldhütter, P. and Schaefer, S. M. (2018), titled “The myth of the credit spread puzzle,” published in *The Review of Financial Studies*, volume 31, issue 8. However, I have to filter away a significant portion of the data to achieve meaningful results from the Merton model.

Sammendrag

I denne masteroppgaven replikerer jeg deler av metodologien i artikkelen av Culp, C. L., Nozawa, Y., og Veronesi, P. (2018), med tittelen "Option-based credit spreads," publisert i *American Economic Review*, volum 108, utgave 2. Ved å replikere deres metodologi bidrar jeg til å fylle et kunnskapshull innen kredittrisiko ved å utforske konseptet *pseudo-obligasjoner*. Masteroppgaven evaluerer replikerbarheten til Culp et al. sin metodologi, og bidrar til forskningen på norske rentepapirer.

Jeg konstruerer pseudo-obligasjoner basert på den norske OBX-indeksen, og studerer deres resulterende priser, gjeldsgrader, kredittmarginer, og misligholdssannsynligheter i løpet av koronapandemien og finanskrisen. Videre konstruerer jeg pseudo-obligasjoner også basert på enkeltaksjer, og sammenlikner gjennomsnittlige kredittmarginer for pseudo-obligasjoner basert på både OBX og enkeltaksjer med empiriske gjennomsnittlige kredittmarginer for selskapsobligasjoner, samt hvordan disse gjennomsnittsverdiene ser ut for selskapsobligasjoner ifølge Mertonmodellen i løpet av koronapandemien. Culp et al. utførte sin analyse på en tidsperiode som dekker finanskrisen, mens jeg hovedsakelig utfører analyser i perioden for koronapandemien grunnet datatilgjengelighet.

Mine funn indikerer at Culp et al. sin metode er bedre egnet til å analysere finanskrisen enn koronapandemien når det gjelder pseudo-obligasjoner basert på OBX-indeksen. Hovedårsaken til dette er at jeg oppnår en litt jevnere fordeling blant kredittvurderingskategoriene. For pseudo-obligasjoner basert på enkeltaksjer har jeg ikke tilstrekkelig data til å få et meningsfylt resultat. Videre er de resulterende misligholdssannsynlighetene for pseudo-obligasjoner basert på enkeltaksjer sensitive for minimale endringer i gjeldsgraden. Jeg finner høye gjennomsnittsverdier for kredittmarginer fra Mertonmodellen, som ikke indikerer at det eksisterer et *credit spread puzzle* i Norge når jeg sammenlikner dem med empriske kredittmarginer for selskapsobligasjoner, som diskutert i arbeidet av Feldhütter, P. and Schaefer, S. M. (2018), med tittelen "The myth of the credit spread puzzle," publisert i

The Review of Financial Studies, volum 31, utgave 8. Imidlertid må jeg filtrere vekk en betydelig mengde data for å få meningsfylte resultater fra Mertonmodellen.

Acknowledgments

This thesis marks the end of my master's degree at the Norwegian University of Science and Technology, where I have spent my last eight years. During these years, I have achieved two bachelor's degrees and soon one master's degree. I have also had the joy of working as a teaching assistant in several courses at NTNU and volunteering in a number of organizations. While staying at NTNU, I had the opportunity to travel abroad to take additional courses and to spend six months on exchange at Sapienza University of Rome.

I want to express my gratitude to my supervisor, Joakim Kvamvold from Folketrygdfondet, for all his support while writing my thesis. I highly appreciate all his feedback and academic contributions, and for allocating time in his schedule for weekly progress meetings.

I am also grateful to my friends and family for their moral support and for always cheering on me. I especially want to thank Tatiana for helping me revise my thesis. Last but not least, I want to thank my boyfriend Gianluca, who has been my biggest supporter and who has a drive that is impossible not to be inspired by.

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Abbreviations and Symbols

Abbreviations

Bps	Basis Points
DD	Distance-to-Default
EDF	Expected Default Frequency
GARCH	Generalized Autoregressive Conditional Heteroskedasticity (model)
HLPB	High-Leverage Pseudo Bond
LIBOR	London Interbank Offered Rate
LLPB	Low-Leverage Pseudo Bond
NIBOR3M	Norwegian Interbank Offered Rate with 3-month Maturity
OECD	The Organization for Economic Co-operation and Development
OIS	Overnight Indexed Swap

Symbols

ϵ_{t+T}	Standardized unexpected asset returns
$\mathcal{N}(\cdot)$	The cumulative distribution function of a normally distributed random variable
P	Put option value

Y_{zc}	Yield of a zero-coupon bond
$\mu_{t,T}$	Expected future growth at time t with maturity T
ϕ	Bond payoff
$\Pr(\cdot)$	Probability mass function
σ_S	Asset volatility
σ_V	Firm volatility
$\sigma_{t,T}$	Volatility scaling parameter at time t with maturity T
A (A_t)	Asset value (Asset value at time t)
B	Bond value
C	Call option value
D	Debt value
E	Equity value
K	Face value of a bond or strike price of an option
$L_{i,t}$	Leverage ratio for leverage K_i at time t
N	Number of observations
$n(\cdot)$	Counting function
P_D	Probability of default of a bond
$p_t(T)$	Ex-ante default probability at time t for maturity T
r	Interest rate
S	Stock price
T	Time to maturity
Z	Risk-free discount factor
YS	Yield Spread

Chapter 1

Introduction

Questions regarding credit risk are critical to understand but can be challenging to answer. Understanding credit risk is essential for policymakers, researchers, and market participants. However, when studying credit risk, empirical methods are less than ideal (Culp et al., 2018). Culp et al. (2018) propose a model-free framework using traded options to analyze credit risk, which offers a controlled environment, fully observable balance sheets, and no corporate frictions. Furthermore, the framework offers the possibility of running what-if experiments (Culp et al., 2024b).

In this thesis, I replicate parts of the methodology put forward by Culp et al. (2018). First, I create index pseudo bonds based on Norwegian data and study their prices, leverage ratios, yield spreads, and default probabilities during the financial crisis and the COVID-19 pandemic. I then compare the results for the two periods. The COVID-19 pandemic disrupted economies across the globe and had a particularly strong impact on the financial markets, which had not seen similar volatilities since the financial crisis (Federal Reserve Bank of St. Louis, 2020).

Furthermore, I compare average yield spreads on OBX index[1] pseudo bonds, single-stock pseudo bonds, corporate bonds, and for the Merton model during the COVID-19 pandemic. Additionally, I create average yield spreads for OBX index pseudo bonds during the financial crisis, for which I have data available. I generally find Culp et al. (2018)'s methodology more productive for the financial crisis than the COVID-19 pandemic for OBX pseudo bonds. In most cases, OBX pseudo bonds have higher average yield spreads than those of empirical corporate bonds during the COVID-19 pandemic. As for single-stock pseudo bonds, there is not enough data to obtain a logical result. I obtain relatively

[1] This index is composed of the twenty-five most traded securities on Oslo Stock Exchange (Euronext, 2023).

high average spreads from the Merton model.

Lastly, to further advance the current research on credit risk, I evaluate the claims suggested by Feldhütter and Schaefer (2018) regarding the existence of the *credit spread puzzle* in the context of the Norwegian market by comparing yield spreads from the Merton model with empirical corporate bond spreads. As a result of such evaluation, their claim appears invalid when formulated on Norwegian market data. That being said, the analysis regarding the Merton model necessitates a filtering of bonds with high debt-to-equity ratios, which removes a significant portion of the data.

1.1 Literature Review

Many papers examine credit risk models and their ability to solve the so-called credit spread puzzle (NBIM, 2011). In the following literature review, I introduce some key statistics on the Norwegian corporate bond market and some research regarding the credit spread puzzle and credit risk.

1.1.1 The Norwegian Corporate Bond Market

According to Rundhaug et al. (2020), "the Norwegian fixed income market is small compared to bond markets in Europe and the United States, and fewer data are available. There are relatively few studies examining Norwegian bonds." The lack of empirical studies on Norwegian bonds was also pointed out by Sæbø (2011). Furthermore, Sæbø (2015) commented on the small amount of historical data for the Norwegian bond market.

The *Organization for Economic Co-operation and Development* (OECD) issued a report in 2022 on corporate bond markets and bondholder rights with Sweden as a focus. However, it also widens up to compare Sweden in a Nordic, European, and international context. Specifically, it compares Swedish data with data from Norway, the U.S., the Netherlands, France, and the U.K., to mention a few (OECD, 2022). This thesis focuses on comparing Norwegian and American data, but it also includes some comparisons of Swedish and Norwegian markets.

Out of the Nordic countries, Norway is the second largest after Sweden in terms of corporate bond market size, measured in outstanding amounts after excluding financial bonds. However, the Nordic markets are relatively small compared to, for instance, the Dutch corporate bond market. The U.S., on the other hand, has large and very active bond markets (OECD, 2022).

Norway had a high degree of industry concentration in its issuance of corporate bonds from 2000 to 2010 and from 2011 to 2021. In both periods, around 55% of the issued bonds were from the energy sector. This result is obtained after excluding financial bonds and real estate. Meanwhile, the U.S. had an industry concentration on about 20% of the issued corporate bonds (OECD, 2022).

Furthermore, the OECD looked at the concentration of the top ten issuers in each of the peer countries. The concentration is measured by the total issuance of shares in each country and the quantity of these shares issued by the top ten issuers. In 2011, the top ten issuers had a 100% fraction of Norway's total issuance. In 2021, however, this fraction decreased to about 45%. This reflects a broadening in the Norwegian market, which became considerably less concentrated this decade. The U.S., in the meantime, had a concentration lower than 20% both in 2011 and 2021 (OECD, 2022).

A considerable share of corporate bonds in Sweden does not have credit ratings. 46% of the corporate bonds issued between 2000 and 2021 lacked ratings. Obtaining a credit rating from a big, international agency is costly, which might explain why a large share of corporate bonds in Sweden are unrated. In the case of Norwegian bonds, about 20% of the non-financial bonds issued between 2000 and 2021 lacked ratings by S&P, Moody's, or Fitch. For the U.S., unrated non-financial bonds issued in the same time interval amount to about 3% (OECD, 2022).

1.1.2 The Credit Spread Puzzle

Generally, spreads are considered as compensation for credit risk from holding corporate bonds, but the yield spreads are often much wider than what expected default losses imply alone. The credit spread puzzle refers to this considerable gap (Amato and Remolona, 2003) and the realization that models like Merton cannot explain the excess return received by corporate bond holders (Merton, 1974; NBIM, 2011).

The efforts in explaining historical credit spreads are typically divided into two approaches: the *structural approach* and the *reduced-form approach* (NBIM, 2011). Merton is one example of a structural model. Another structural model is the Black & Cox model (Black and Cox, 1976). Whether the structural or reduced-form models is preferable is debated, but the structural models are more appropriate for connecting capital structure with credit risk. Merton's model is simple, but we generally do not see much performance improvement when using more complex models (Rundhaug et al., 2020).

Culp et al. (2018) pointed out the challenges of studying credit risk empirically in the context of corporate bonds. Some of these challenges are that corporate bonds are complex and often illiquid, that the market value of firm assets is unobservable, and that leverage is endogenous. Furthermore, empirical methodologies do not allow for counterfactual analyses. In their work, they proposed a new method of studying credit risk with the use of *pseudo firms*. With this method, we obtain simple and observable balance sheets. In their framework, the asset value is based on real securities, while the liabilities are based on equity and zero-coupon bonds. They created a fictitious firm with fully observable asset and debt values.

Feldhütter and Schaefer (2018) challenged the existence of the credit spread puzzle, which they defined as "[. . .] the perceived failure of structural models to explain levels of credit spreads for investment-grade bonds." They pointed to the period during which default probabilities are collected as the reason why researchers find spreads that are much lower than historical spreads. They referred to a typical method of estimating default probabilities at a given maturity and rating, which uses the historical default rate for that maturity and rating. They showed that this method provides a noisy estimate of default probabilities. In their work, they proposed a different method for estimating default probabilities, and using these probabilities with Black & Cox, they found no evidence of a credit spread puzzle for investment grade spreads. Namely, they found that their model matches actual average investment-grade spreads well but underpredicts actual average speculative-grade spreads. They found no such puzzle according to their more specific definition of the credit spread puzzle mentioned above. They found a similar result when applying the Merton model (Feldhütter and Schaefer, 2018).

Sæbø (2015) examined Norwegian corporate bond transactions between 2008 and 2013. He found a credit spread puzzle in the Norwegian market for this period, even after controlling for the biases discussed by Feldhütter and Schaefer in a 2013 working paper.[2] On average, Sæbø (2015) found that the structural model accounted for 28% of the yield spread in the Norwegian data. However, he discussed whether it is correct to call it a *puzzle*. He argued that if investors were only compensated for the expected loss implied by a possible default, they would be risk-neutral. That is, however, not the case, as most investors have a degree of risk aversion. Sæbø (2015) supported his argument using a paper by Agrawal et al. (2004), which built a model consisting of variables measuring default risk and risk aversion. With their model, they claimed to explain 70% of the variation in yield spread. Furthermore, Sæbø (2015) discussed mispricing for different maturities and found that the relative mispricing increases with the remaining time to maturity.

1.1.3 The Sources of Yield Spread

Typically, corporate bonds will offer a higher yield than Treasury bonds at equal maturity. This difference in yield is referred to as the *yield spread* and, in addition to the credit risk of corporate bonds, Huang and Huang (2012) pointed to illiquidity, asymmetric taxes, and

[2] No longer available. The working paper was named "The Credit Spread Puzzle – Myth or Reality," and it likely is an early version of (Feldhütter and Schaefer, 2018) with a similar name.

call and conversion features as sources of the yield spread.

Amato and Remolona (2003) discussed the role of taxes, risk premia, liquidity premia, and the difficulty of diversifying credit risk in this puzzle. In the U.S., corporate bonds are taxed at the state level, but government bonds are not. As returns are compared after taxes, this matters for the yield on a corporate bond, which has to be higher to compensate for the additional tax. Researchers argue that taxes explain a higher fraction of the yield spread on corporate bonds with higher credit ratings. Furthermore, Amato and Remolona (2003) pointed out the volatility of yield spreads as another factor for which investors require a risk premium. The illiquidity of corporate bond markets is another factor, even in the U.S. As the market is less liquid, transactions with corporate bonds are more expensive than transactions with equities and government bonds, for which the investors need compensation. Moreover, Amato and Remolona (2003) discussed the challenge of diversifying credit risk and claimed that this is a neglected explanation for the yield spread size. They argued that diversification difficulties could be a large source of the spread.

Sæbø (2011) found that about one-fourth of the yield spread for Norwegian bonds can be attributed to credit risk for the period 2008-2009. He also found the size and liquidity of the issuer and the sector in which the issuer belongs to be important factors impacting the yield spread. For the U.S., researchers such as Amato and Remolona (2003) discuss the impact of taxes on the yield spread, as government bonds and corporate bonds are taxed differently. However, this is not the case in Norway (Sæbø, 2011).

Culp et al. (2018) found that factors such as bond market illiquidity, overestimation of default risk, and corporate frictions do not explain the credit spread puzzle. They analyzed pseudo bonds based on the SPX index[3] and found results very similar to those for real corporate bonds. However, with pseudo bonds, there are no issues with information asymmetry, managerial frictions, and so on, which other authors have proposed as sources of the credit spread puzzle. They also found pseudo bonds to be more liquid than corporate bonds. Therefore, they argued that these issues cannot be the source of the puzzle. Instead, they observed that the main sources of the excessive yield spread were idiosyncratic asset risk and tail risk.

1.1.4 The Impact of Credit Risk on the Yield Spread

According to Huang and Huang (2012), there has been little consensus on exactly what proportion of the yield spread the credit risk contributes to in studies that have implemented the structural approach, neither for shorter nor longer maturities. Huang and Huang (2012),

[3]This index is composed of the five hundred leading companies in the U.S. (Federal Reserve Bank of St. Louis, 2024).

however, claimed that a consensus could be found within the structural approach on the impact credit risk has on the yield spread. With their approach, they argued that they had found robust conclusions on the impact of credit risk on yield spreads. They concluded that for bonds with a credit rating of Baa and higher, credit risk would account for only 20%-30% of the yield spread. In the case of junk bonds, credit risk was found to be the source of a much larger portion of the yield spread.

1.2 Objectives of the Thesis

The main goal of this thesis is to replicate and validate a set of conclusions obtained from existing literature for the Norwegian market. Specifically, three objectives can be outlined:

O1 Replicating a figure[4] from Culp et al. (2018) with Norwegian market data. This figure plots bond prices, leverage ratios, yield spreads, and default probabilities for SPX index pseudo bonds over time.

O2 Replicating a second figure[5] from Culp et al. (2018) with Norwegian market data. This figure displays average yield spreads across credit ratings for SPX index pseudo bonds, single-stock pseudo bonds, and corporate bonds, as well as those for the Merton model.

O3 Exploring Feldhütter and Schaefer (2018)'s claims regarding the existence of a credit spread puzzle to check if those claims are valid on Norwegian market data. However, I do not replicate their methodology; I only test their results based on my data.

To the best of my knowledge, no peer-reviewed works have evaluated Culp et al. (2018)'s methodology.[6]

All results are produced using codes written in R language (available in the Appendix). The thesis is written in collaboration with *Folketrygdfondet*, which provided the datasets that have made the analyses possible. The datasets contain historical corporate bond data, OBX index prices, and stock data over time.

[4]Figure 1 from Culp et al. (2018).
[5]Figure 2 Panel A from Culp et al. (2018).
[6]However, several papers have applied pseudo bonds in their analyses, such as (Gruenthaler et al., 2022). I found additional works applying pseudo bonds, but do not refer to them here as they are not peer-reviewed.

1.3 Contributions, Limitations, and Structure of the Thesis

By achieving objectives **O1**, **O2**, and **O3** listed in Section 1.2, this thesis makes the following three contributions:

C1 It fills a literature gap in the field of credit risk and contributes to its expansion, which is relevant for policymakers, researchers, and market participants (Culp et al., 2018).

C2 It determines whether a proposed model-free methodology for studying credit risk is portable to other markets and time periods.

C3 It provides a set of findings about the Norwegian fixed income market, primarily during the COVID-19 pandemic, on which there is currently limited research (Rundhaug et al., 2020).

Regarding the limitations of the work, this thesis is limited to replicating two figures from Culp et al. (2018). It should be mentioned that even with an online appendix and some of Culp et al. (2018)'s code provided, the description of their methodology appears insufficient in some instances. According to National Academies of Sciences, Engineering, and Medicine (2019), "replicability is obtaining consistent results across studies aimed at answering the same scientific question, each of which has obtained its own data." One constraint to the replicability of a study is small variations in the steps taken to produce a result (National Academies of Sciences, Engineering, and Medicine, 2019). Due to the frequent lack of clarity of Culp et al. (2018)'s description of their approach, my result may have been produced differently from theirs. This restricts the replicability of their method. Due to this issue, my interpretation of Culp et al. (2018)'s method may differ from someone else's.

Other limitations of this thesis are time and data availability. With more time, the thesis' scope could have been expanded further and built on fewer simplifying assumptions. The limited amount of stock data hindered the possibility of an extended discussion regarding single-stock pseudo bonds. Having all the necessary Norwegian data for the financial crisis would have been beneficial to ensure greater comparability with the work of Culp et al. (2018). As a compromise, due to a lack of data, I primarily considered the period of the COVID-19 pandemic instead of the financial crisis. The COVID-19 pandemic is another period that is not necessarily directly comparable with the financial crisis period. Furthermore, the Merton model in my work did not apply to firms with high debt-to-equity ratios, so I removed the bonds from these firms from my analysis. Chapter 4 presents a deeper analysis of these limitations. These limitations could be addressed in future works.

The structure of the thesis is as follows. Chapter 2 introduces the theoretical foundation of this thesis in addition to a brief historical context. Chapter 3 explains the data used to produce my results and some descriptive statistics. It then explains Culp et al. (2018)'s method and the decisions I made when replicating it. Chapter 4 then presents and discusses my results, along with some of Culp et al. (2018)'s results to facilitate comparison. Finally, Chapter 5 summarizes the thesis and draws conclusions based on my results.

Chapter 2

Background

This chapter presents the theoretical background of the thesis. It describes yield spreads, options, bonds, the models by Black & Scholes and Merton, and pseudo firms. Moreover, it provides a historical context.

2.1 Yield Spreads

Investors who lend funds by purchasing bonds are exposed to *credit risk* (Fabozzi, 2011). Credit risk can be defined as the risk of default or reduction in market value due to reduced credit quality of issuers or counterparties (Duffie and Singleton, 2003). The yield on a bond is composed of a similar default-free bond yield and a premium to compensate for the risks associated with the bond. This premium is called the *yield spread* (Fabozzi, 2011). A part of the yield spread is due to the credit risk in corporate bonds, and the yield spread is therefore often referred to as the *credit spread*. However, other factors also contribute to this premium (Huang and Huang, 2012). Hence, I will mainly use the term yield spread to refer to this risk premium. However, throughout the thesis, I use the terms yield spread, credit spread, and spread interchangeably.

Yield spreads are wider for bonds with lower credit ratings (Berk and DeMarzo, 2016). Furthermore, spreads change systematically with changes in the economy: they widen in a declining economy and tighten during economic expansion. Credit ratings are used to gauge the default risk of bonds. Higher grades indicate lower credit risk; the highest-grade bonds are given the symbol Aaa/AAA. Bonds with a rating of BBB or higher are referred to as *investment-grade bonds*, while bonds with a lower rating are referred to as *high-yield bonds* or *junk bonds* (Fabozzi, 2011). Moody's and Standard & Poor's are two well-known bond rating companies. The best credit rating from Moody's is Aaa, while the best, and equivalent, rating from Standard & Poor's is AAA (Berk and DeMarzo, 2016).

2.2 Theoretical Foundation

This section presents some theoretical elements. It briefly discusses corporate bonds, zero-coupon bonds, financial options, and the interest rate.

2.2.1 Corporate Bonds

Corporate bonds are bonds issued by corporations (Berk and DeMarzo, 2016). Most corporate bonds, irrespective of country or issuer of origin, are highly illiquid (Goldstein and Namin, 2023). According to Modigliani and Miller's first proposition (MM1), the market value of a firm's assets A is equal to the sum of the market values of the firm's equity E, and debt D (Berk and DeMarzo, 2016):

$$D + E = A \; . \tag{2.1}$$

A typical ratio to evaluate a firm's leverage is the *debt-to-equity ratio* D/E (Berk and DeMarzo, 2016). Fama and French (1992) exclude financial firms from their analysis due to their high leverage-values. A normal leverage value for financial firms might indicate distress for other firms, which can bias their analysis.

2.2.2 Zero-Coupon Bonds

As the name suggests, zero-coupon bonds do not make any coupon payments (Berk and DeMarzo, 2016). Bond prices vary over time and tend to converge toward their face value as they approach maturity. Bond prices are also sensitive to interest rate changes, and the degree of sensitivity is referred to as their duration. Zero-coupon bonds with a longer time to maturity are more sensitive to interest rate changes than bonds with shorter terms, and if the interest rate increases, the bond price typically falls (Berk and DeMarzo, 2016).

2.2.3 Financial Options

Options give the holder the right, but not the obligation, to buy or sell an underlying asset at a fixed price and date. A *put option* is the right to sell underlying at the strike price, and the put payoff is represented in the following equation (Berk and DeMarzo, 2016):

$$\mathrm{P} = \max(K - S, 0) \; , \tag{2.2}$$

where P is the put value, K is the strike price, and S is the value of the underlying stock. Therefore, when holding the price of the underlying constant, the payoff is lower if the strike price is lower. On the other hand, if we consider the strike price constant, a put option's payoff increases if the underlying asset's price decreases. Generally, option values increase as the volatility of the underlying asset increases. Higher volatility yields a higher chance of a positive payoff. Put options are less valuable the lower their strike price is (Berk and DeMarzo, 2016).

2.2.4 Interest Rates

Interest rates depend on the *horizon*, also referred to as the *term* of the investment or the loan. We refer to the relationship between interest rates and investment horizons as the *term structure* of interest rates, while the graphical representation is called the *yield curve*. The gap between long-term and short-term interest rates varies over time. In the U.S., this gap was particularly large in 2008 (Berk and DeMarzo, 2016).

Berk and DeMarzo (2016) looked at interest rates offered for investing in risk-free U.S. Treasuries in November 2006, 2007, and 2008 for different terms. In 2008, the interest rate was below 1% for a one-year term and 5% for a five-year term. In 2007, however, the interest rates were around 3.1% and 3.5%, respectively, resulting in a flatter yield curve. The yield curve was even flatter for the term structure in 2006. If the yield curve is relatively flat, it becomes less critical to differentiate between different interest rates across terms, and we can consider one average interest rate.

The yield curve depends on expectations about future interest rate changes, as stated by Berk and DeMarzo (2016). They considered the case where long-term and short-term interest rates are equal. If the interest rate were expected to increase in the future, there would be no incentive to make long-term investments, as we could make a short-term investment instead and then reinvest with a higher interest rate. Therefore, long-term interest rates are higher than short-term interest rates when interest rates are expected to increase. However, when interest rates are expected to decrease, borrowers will not want to borrow at long-term interest rates. Instead, they can borrow for a shorter term and then take a new loan after the interest rate falls. To attract borrowers, the long-term interest rate has to be lower than the short-term interest rate. This case gives an inverted yield curve. Generally, an inverted yield curve signals an upcoming low economic growth because interest rates fall with economic activity. Before the financial crisis, the yield curve based on U.S. data was inverted. When the economy leaves a recession, the yield curve typically has a positive slope (Berk and DeMarzo, 2016).

2.3 Merton and Black & Scholes

Culp et al. (2018) distinguish between the *Merton insight* and the *Merton model*. The Merton insight is that we can express the value of a corporate bond as the sum of a risk-free bond and a short put option. The Merton model, on the other hand, is used to value risky corporate debt and assumes that the underlying asset value is log-normally distributed. This model utilizes the Black, Scholes, and Merton formula (Culp et al., 2018). Summarized by Rundhaug et al. (2020), "one of the best-known structural models is the Merton (1974) model. This model assumes that a company's equity can be regarded as a call option on its underlying assets with a strike price equal to its liabilities and utilizes the Black and Scholes (1973) option pricing formula."

For a call option with a stock as the underlying asset, the formula for the pricing of a call option C is (Berk and DeMarzo, 2016):

$$C = S \cdot \mathcal{N}(d_1) - e^{-rT} K \cdot \mathcal{N}(d_2) \ , \tag{2.3}$$

with

$$d_1 = \frac{(\ln \frac{S}{K}) + \left(r + \frac{\sigma_S^2}{2}\right)T}{\sigma_S \sqrt{T}} \ , \tag{2.4}$$

$$d_2 = d_1 - \sigma_S \sqrt{T} = \frac{(\ln \frac{S}{K}) + \left(r - \frac{\sigma_S^2}{2}\right)T}{\sigma_S \sqrt{T}} \ , \tag{2.5}$$

where S is the current stock price, r is the risk-free rate, T is the number of years until expiration, K is the strike price, and σ_S is annual volatility of the stock returns. In Equation (2.3) I have replaced the *present value* with e^{-rT}. According to Berk and DeMarzo (2016), "$\mathcal{N}(d)$, the cumulative normal distribution is the probability that a normally distributed random variable will take on a value less than d." Utilizing the *put-call parity*, the formula for the put option value P can be written as:

$$\mathrm{P} = e^{-rT}(1 - \mathcal{N}(d_1)) - S(1 + \mathcal{N}(d_2)) \ . \tag{2.6}$$

As we see from Equations (2.3), (2.4), (2.5), and (2.6), the put option value depends on K, r, T, σ_S, and S. If σ_S increases, the value of any option increases (Berk and DeMarzo, 2016). If r increases, we see from Equations (2.4), (2.5), and (2.6) that the put option value decreases. If S increases, the put option value decreases, and vice versa, as formulated in Equation (2.2).

Merton (1974) developed one of the first models for pricing credit-risky bonds, but Black and Scholes (1973) had already provided the intuition of interpreting capital structure in terms of option contracts (Ammann, 2001). Black and Scholes (1973) consider a company with common stock and zero-coupon bonds outstanding and holding business assets. At maturity, the company pays off the bondholders, if possible, and pays any remaining money to the stockholders. The bondholders own the company assets but have sold the stockholders the option of repurchasing the assets (a call option) (Black and Scholes, 1973). Equivalently, we can say that the stockholders own the company's assets and have purchased a put option from the bondholders (Ammann, 2001). Therefore, we can consider a corporate bond as a default-free bond minus a put option with a strike price of K on the firm's assets. The payoff of the bond ϕ can be written as:

$$\phi = K - \max(K - V, 0) \ , \tag{2.7}$$

where V denotes the total value of the firm's assets. If the firm's asset value is above the face value of the debt, the debt holders are paid off. However, if the face value of debt is higher than the firm's asset value, the firm defaults, bondholders take over and receive remaining assets, and stockholders are left with nothing (Culp et al., 2024d). The payoff of a bond is illustrated in Figure 2.1.

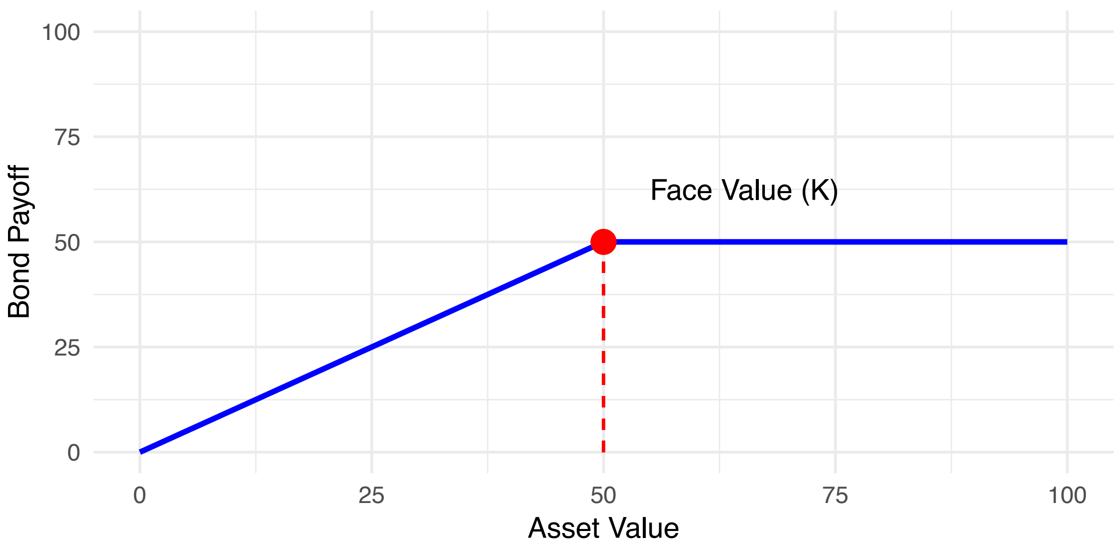


Figure 2.1: Bond payoff for face value $K = 50$, recreated from Culp et al. (2024d).

As stated above, we generally consider a firm bankrupt if its market value is less than the book value of its liabilities. This is, however, from the theoretical point of view. In practice, many firms can continue their activity conditional on them making periodic payments on their debt (Rundhaug et al., 2020). For a corporate bond, the probability of default is considered as the probability that the firm's asset value is lower than the face value of the firm's debt at maturity (Rundhaug et al., 2020). Furthermore, Black and Scholes (1973) state that increasing the proportion of debt, holding the value of the firm constant, increases the firm's probability of default.

2.4 Pseudo Firms

Culp et al. (2018) used the insights from Merton (1974) to study credit risk. They looked at *pseudo firms*, which are hypothetical firms that issue zero-coupon bonds and equity to finance the purchase of tradeable assets. A benefit of looking at pseudo firms is that we do not have to estimate parameters. Instead, we can directly use observable data, such as interest rates and option prices. The balance sheet of a pseudo firm is observable, and the pseudo firm can be used as a laboratory to study the impact of shocks on the firm (Culp et al., 2024c).

The value of the pseudo bond issued by the theoretical firm is equal to the value of a zero-coupon bond minus the value of a put option on the firm's assets. The balance sheet of this firm is observable because we can observe the asset, debt, and equity values. The asset value of the firm is equal to the value of the traded assets, the liabilities are equal to the zero-coupon bond value minus the put option on the firm's assets, and the equity value is equal to asset values minus liabilities (Culp et al., 2024a).

For instance, the asset traded by the pseudo firm can be the SPX index (Culp et al., 2024d). In that case, the value of the pseudo bond $\widehat{B}$ at time t, with strike price K and time to maturity T is:

$$\widehat{B}_t(K,T) = K\widehat{Z}_t(T) - \widehat{\mathrm{P}}_t^{\mathrm{SPX}}(K,T)\ , \tag{2.8}$$

where $\widehat{\mathrm{P}}_t^{\mathrm{SPX}}(K,T)$ denotes the value of a pseudo put option on the SPX index with strike price K and time to maturity T, at time t. $\widehat{Z}_t(T)$ is the risk-free discount factor at time t corresponding to maturity date T. The quantities in the equation with hats indicate that they have prices observable from Treasuries and traded options (Culp et al., 2018). I replicate this approach for the OBX index, providing the following altered equation:

$$\widehat{B}_t(K,T) = K\widehat{Z}_t(T) - \widehat{\mathrm{P}}_t^{\mathrm{OBX}}(K,T)\ . \tag{2.9}$$

As we can see from Equation (2.9), the bond value decreases when the value of the put option increases. There is no need for a model in Culp et al. (2018)'s methodology. It offers a controlled environment where we can choose a capital structure, leverage, riskiness, and more of the pseudo-firm. Issues such as endogenous capital structure and corporate frictions are thus removed (Culp et al., 2018).

2.5 The Financial Crisis and the COVID-19 Pandemic

On March 12th, 2020, Norwegian authorities introduced comprehensive infection control measures to combat the COVID-19 pandemic (Helsedirektoratet, 2020). Internationally, the measures initiated to tackle the pandemic led to significant economic consequences. In Norway, the consequences were reinforced by a decrease in the oil price (Norges Bank, 2020). According to Tjernshaugen et al. (2024), all infection control measures in Norway were repealed in February 2022.

Many consider September 15th, 2008, the beginning of the financial crisis, as this was the day Lehman Brothers filed for bankruptcy (Sæbø, 2011). In 2008, Oslo Børs (Oslo Stock Exchange) fell by 64% in the span of six months. This was the greatest fall since the early 1920s, and the reason was the American mortgage crisis. However, due to high oil revenues, Norway experienced a smaller impact than other countries (Norges Bank, 2024). Culp et al. (2018) analyzed pseudo bonds during the financial crisis and chose to consider an interval from 2007 to the beginning of 2010.

Kozlowski et al. (2021) compared yield spreads during the COVID-19 pandemic and the financial crisis in the U.S. market. They considered the timeline of each crisis and found that there was quite a similar yield spread in the first weeks of the COVID-19 pandemic and the first weeks of the financial crisis. They set the beginning of the COVID-19 pandemic to February 28th, 2020, and the start of the financial crisis to September 15th, 2008. However, four weeks after the beginning of the COVID-19 pandemic, the yield spread rapidly decreased and began returning to its trend around zero. This fast recovery did not occur during the financial crisis, where the yield spread remained between 150 and 300 basis points (bps) for about eight months.

Figure 2.2 displays the *LIBOR-OIS spread* for the period 2002-2024.

LIBOR is the London Interbank Offered Rate, and OIS are overnight index swaps. LIBOR-OIS measures the interbank rate stress and credit concerns and can be considered a health indicator of the banking system (Cui et al., 2016). Figure 2.2 exhibits a spread hovering above 0 bps until the second half of 2007. The spread then fluctuated between 50 bps and 100 bps before rising above 350 bps in the second half of 2008. In the second half of 2009, the spread fluctuated below 50 bps again. In summary, LIBOR-OIS widened considerably during the financial crisis, but the spread was also wide before the official beginning of the financial crisis. In fact, the relatively wide spread lasted for about two years. The same observation regarding the pattern of LIBOR-OIS during the financial crisis was made by Thornton (2009).

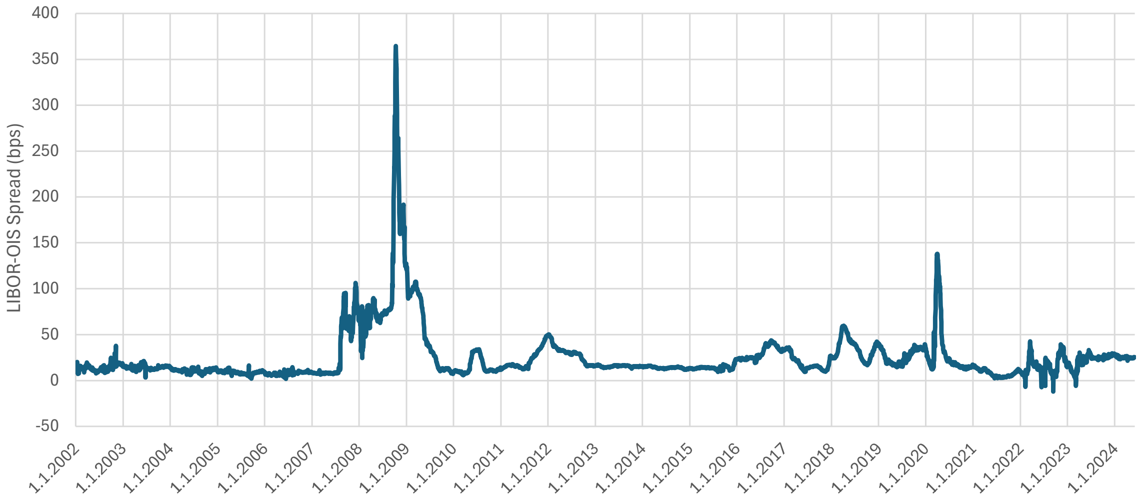


Figure 2.2: LIBOR-OIS spread for the period 2002-2024. Data used to construct the spread are collected from Bloomberg.

Figure 2.2 shows that another significant spike in LIBOR-OIS emerged at the beginning of 2020. The spread was tighter than the financial crisis', with only about 150 bps, and the spike lasted less than half a year. The observation of a significantly tighter spread during the COVID-19 pandemic than the financial crisis does not match the findings by Kozlowski et al. (2021), who may have considered a different indicator to study yield spreads than the LIBOR-OIS. However, both Kozlowski et al. (2021) and Figure 2.2 indicate that the spread lasted longer during the financial crisis than during the pandemic. A tighter spread for the COVID-19 pandemic than the financial crisis signals a smaller degree of distress in the banking system during the pandemic.

Chapter 3

Method

This chapter first describes the data and their preprocessing. Then, it outlines Culp et al. (2018)'s methodology and the choices and assumptions I have made when replicating their results. Culp et al. (2018) proposed using an option-based methodology with observed prices on put options and Treasuries to study credit risk. They construct pseudo firms that issue pseudo bonds and use these to compute credit spreads. To check the external validity of Culp et al. (2018)'s results, I replicate parts of their work on Norwegian data. While Culp et al. (2018) studies the period 2007-2010 and discusses how the financial crisis impacted spreads in the U.S., this work studies a similarly turbulent period from the beginning of 2019 to the end of 2022 with the COVID-19 pandemic in Norway. This is due to data availability. However, I also replicate Culp et al. (2018)'s first figure for the period 2007-2010 for the Norwegian market.

My assumption is that the volatile period of the COVID-19 pandemic will give me similar results to what Culp et al. (2018) obtained for the financial crisis and that I am equipped to test if their method also works on Norwegian data even though I do not have complete data for the same time interval. The methodology for pseudo bonds, including both index pseudo bonds and single-stock pseudo bonds, is explained in further detail in Culp et al. (2014).

3.1 Data Description and Preprocessing

I collect default probabilities and their corresponding credit ratings from Feldhütter and Schaefer (2018), who provide both Moody's historical default probabilities from 1920 to 2012 and default probabilities calculated with their own proposed model. Moody's scheme is referred to as *actual*, while Feldhütter and Schaefer (2018)'s scheme is referred to as *model*.[1] I use these schemes for OBX index pseudo bonds, single-stock pseudo bonds, and default probabilities implied by the Merton model. Feldhütter and Schaefer (2018) provide default probabilities and corresponding credit ratings for maturities of 1-6 years and then for 8, 10, 12, 15, and 20 years.

3.1.1 OBX Index Data

I use daily closing prices for the OBX index, collected from Bloomberg from March 2006 to January 2024. As of December 31st, 2023, the securities with the greatest weights in the OBX index were Equinor with 13.98%, DNB with 10.15%, Aker BP with 8.59%, and Norsk Hydro with 8.35% (Euronext, 2023). I utilize the Norwegian Interbank Offered Rate with a maturity of 3 months, NIBOR3M, as the interest rate, which is a simplification for the maturity-matched interest rate. I employ the implied volatility from historical put options on the index as a volatility measure. Both the implied volatilities and the NIBOR3M rates are collected from Bloomberg.

3.1.2 Corporate Bond Data

Spreads on corporate bonds are calculated by Nordic Bond Pricing. Total liabilities, market capitalization, and credit ratings are gathered from Moody's CreditEdge. The credit ratings are implied from *expected default frequencies* (EDFs) for the specific firms and periods (Moody's Analytics, 2012). I have implied ratings for 1-5 year increments. Issue and maturity dates, collateral type, and interest type are retrieved from Stamdata. The variables mentioned above are collected for the period 2016-2024. The dataset contains forty-three different companies, judging by their MKMV IDs, which are permanent identifiers. Several companies in the dataset have identical MKMV IDs. This occurs because they have changed their names, merged with other companies, or for similar reasons. I categorize the firms after sector, displayed in Figure 3.1. The finance category comprises banks and insurance firms. This composition does not consider that some issuers have issued several bonds while others only issued one.

3.1.3 Stock Data

I have stock data for the companies DNB, Yara, Orkla, Telenor, Equinor, Storebrand, and Norsk Hydro from March 2006 to May 2024. These data include daily stock prices and implied volatilities from put options, which are collected from Bloomberg. As with OBX index pseudo bonds, I apply the NIBOR3M as the interest rate.

[1] For further details on how their rating scheme, *model*, was constructed, the reader can consult Feldhütter and Schaefer (2018).

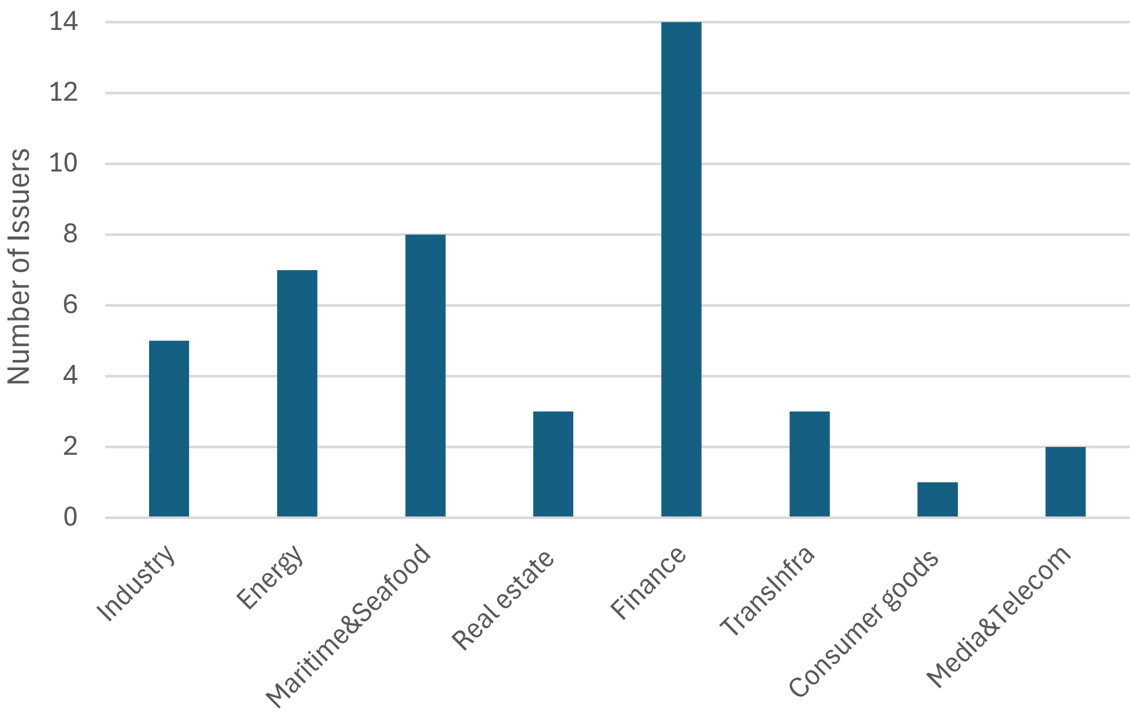


Figure 3.1: Composition of sectors in corporate bond dataset (the shortening TransInfra stands for transportation and infrastructure).

3.1.4 Data Preprocessing

Filtering of the datasets is necessary. After removing NAs, I winsorize the spreads in the corporate bond dataset to remove outliers and avoid disturbances in my results. I do this so that I am left with only non-negative spreads. The winsorizing is also performed to remove the highest 5% of the spreads. Table 3.1 displays descriptive statistics before and after the winsorization for the period January 1st, 2019-December 30th, 2022. In the case of NA's in the implied volatilities from put options on single stocks, I replace the entries with the previous volatility value. The same procedure is used for the OBX index price, NIBOR3M, and the implied volatility from put options on the OBX index.

Table 3.1: Statistics on yield spreads for corporate bonds before and after winsorizing. (N is the number of observations).

N	Mean	Min	Max	1st Quantile	3rd Quantile
153044	115.56	−26313.07	206534.84	32.29	92.70
142911	64.93	0.00	238.91	32.25	85.68

Finally, I filter the corporate bond data by collateral and interest type. I primarily keep bonds with fixed interest and unsecured collateral. Table 3.2 displays statistics for corporate bonds already filtered with respect to the time period and where spreads below zero and in the highest 5% are removed.

Table 3.2: Number of observations N, for corporate bond data remaining after applying filters, and percentages of data remaining relative to starting point (after winsorization and choice of time period).

Filter	N	Remaining Data
None	142911	100%
Unsecured collateral	127809	89.43%
Fixed interest rate	44836	31.37%

3.2 Index Pseudo Bonds

As stated in Section 1.2, objective **O1** deals with the replication of the first figure from Culp et al. (2018) using Norwegian data. This figure displays four plots for SPX pseudo bonds with two different degrees of leverage, K_1 and K_2. The first plot presents the SPX index price alongside the low- and high-leverage SPX pseudo bonds. The second plot shows the two leverage ratios for the SPX pseudo bonds, and the third plot displays their yield spreads. Lastly, the fourth plot shows the default probabilities for the two SPX pseudo bonds (Culp et al., 2018).

I replicate this figure with its four respective plots for both the financial crisis period and the COVID-19 period, and the results can be found in Chapter 4. To replicate the first plot for the financial crisis and the COVID-19 pandemic, I replace the SPX index with the OBX index and plot it alongside the low- and high-leverage pseudo bonds constructed using Equation (2.9). The maturity date for the financial crisis is 18/12/2009, following Culp et al. (2018) for the equivalent period of their study. For the COVID-19 period, I set the maturity date to 30/12/2022. The pseudo bond prices are plotted as percentages of their *principal*, K_i, with $i = 1, 2$ (Culp et al., 2018).

I use historical implied volatilities from put options on the OBX index and choose strike prices with the primary goal of achieving similar leverage ratios to those obtained by Culp et al. (2018). For the financial crisis period, I set the low-leverage strike price at $K_1 = 200$ and the high-leverage strike price at $K_2 = 320$. For the COVID-19 period, the low-leverage strike price is set at $K_1 = 850$, while the high-leverage strike price is set at $K_2 = 1000$. The pseudo bond leverage ratios at time t are defined as $L_{i,t} = K_i/A_t$, where A_t denotes the asset value at time t (Culp et al., 2018). In this thesis, A_t refers to the value of the OBX index over time. For the COVID-19 period, I achieve higher leverage ratios than Culp et al. (2018). This is necessary to get a full range of default probabilities from 0 to 1 for the time period in question.

I replicate the third plot for the financial crisis and the COVID-19 pandemic using the already defined values for Norwegian data. The third plot reports the implied yield spreads YS for the two pseudo bonds, which is found as the difference between the yield of the respective pseudo bond $\mathrm{Y_{zc}}$ and the risk-free yield (this is equivalent to the interest rate r):

$$\mathrm{YS} = \mathrm{Y_{zc}} - r \ . \tag{3.1}$$

The yields of the pseudo bonds are found with the formula for yield to maturity on zero-coupon bonds:

$$\mathrm{Y_{zc}} = \left(\frac{K}{B}\right)^{1/T} - 1 \ , \tag{3.2}$$

where K is the face value, B is the current bond price, and T is years to maturity (Berk and DeMarzo, 2016).

Finally, I replicate the fourth plot, which displays the *ex-ante* default probabilities at time t for a generic value of maturity $T = \tau$:

$$p_t(\tau) = \Pr(A_{t+\tau} < K_i | \mathcal{F}_t) \ , \tag{3.3}$$

where $\Pr(\cdot)$ indicates a probability mass function and $\mathcal{F}_t$ denotes information available at time t (Culp et al., 2018). To find the default probability, I begin by computing the log asset growth following Culp et al. (2014):

$$\ln\left(\frac{A_{t+\tau}}{A_t}\right) = \mu_{t,\tau} + \sigma_{t,\tau}\epsilon_{t+\tau} \ , \tag{3.4}$$

which states that log asset growth consists of an expected component $\mu_{t,\tau}$ and a volatility component $\sigma_{t,\tau}$, which Culp et al. (2018) goes on to estimate with, respectively, a predictive regression and by fitting a GARCH model. The distribution of ϵ_t is unspecified (Culp et al., 2018). For simplicity, I choose rather to find the mean and standard deviation based on the historical data of log asset growth. I can then find the history of shocks:

$$\epsilon_{t+\tau} = \frac{\ln\left(\frac{A_{t+\tau}}{A_t}\right) - \mu_{t,\tau}}{\sigma_{t,\tau}} \ , \tag{3.5}$$

which is used to calculate empirical default probabilities. Equation (3.3) can be rewritten as:

$$p_t(\tau) = \Pr(\epsilon_{t+\tau} < X_{i,t} | \mathcal{F}_t) \ , \tag{3.6}$$

where $X_{i,t}$ is obtained with the following:

$$X_{i,t} = \frac{\ln{(L_{i,t})} - \mu_{i,t}}{\sigma_{t,\tau}} \ , \tag{3.7}$$

where $L_{i,t}$ denotes the leverage ratio. Finally, empirical default probabilities are calculated by dividing the number of events where $\epsilon_{s+\tau} < X_{i,t}$ by the total number of shocks:

$$\widehat{p}_t(\tau) = \frac{n(\epsilon_{s+\tau} < X_{i,t})}{n(\epsilon_{s+t})} \ , \tag{3.8}$$

as stated in (Culp et al., 2014), where $n(\cdot)$, in this instance, is a function that *counts* the number of times the contained statement is true. This procedure is performed on an expanding window in order for the probability at time t to be calculated only with the information available at time t, i.e., *ex-ante* (Culp et al., 2014). We see from Equations (3.7) and (3.8) that the greater the leverage ratio $L_{i,t}$ is, all else constant, the greater will the number of instances be where $\epsilon_{s+\tau} < X_{i,t}$, and the greater will the default probability be.

3.3 Credit Spreads

In the second half of the thesis, I replicate the second figure from Culp et al. (2018), as per objective **O2**, which is a bar chart displaying the spreads in bps for credit ratings ranging from AAA to C. The bars are provided for OBX pseudo bonds, corporate bonds, single-stock pseudo bonds, and the Merton model. To create this bar chart, I use data from the period 2019-2022 instead of 2007-2010 due to data availability for corporate bonds.

3.3.1 OBX Pseudo Bonds

To create the bars for the OBX pseudo bonds, I use their default probabilities found with Equation (3.8). I match each default probability with credit ratings from Feldhütter and Schaefer (2018)'s scheme named *model*. I linearly extrapolate ratings for maturities under one year with increments of 0.1 years. I obtain some negative spreads for the OBX pseudo bonds. To ensure comparability with corporate bonds, I remove negative spreads for the pseudo bonds. After that, I find the average yield spread for each credit rating category. As I have OBX data also for the financial crisis, I create bar charts of average yield spreads for OBX both during the financial crisis and the COVID-19 pandemic. I do this to check if the resulting bar charts become significantly different across periods.

3.3.2 Corporate Bonds

In the case of corporate bonds, Folketrygdfondet has provided credit ratings from Moody's and spreads from Nordic Bond Pricing. I consider data points from January 1st, 2019, to December 30th, 2022. I do not filter the corporate bonds in terms of time to maturity. I utilize credit ratings implied from EDFs for two years. I use a two-year perspective as a middle ground because the bonds have a significant span of maturities, both above and below two years.

I translate the ratings from Moody's scheme to S&P's and then adjust the ratings further to match the categories provided by Feldhütter and Schaefer (2018) in their table with credit ratings and corresponding default probabilities. I do this by removing pluses and minuses, which are included in S&P's scheme. Furthermore, CCC and CC are not included in Feldhütter and Schaefer (2018)'s scheme, so the instances with these ratings are given the rating C instead. The corresponding credit ratings for Moody's, S&P, and Feldhütter and Schaefer (2018) are displayed in Table 3.3.

Table 3.3: Credit rating schemes from Moody's and S&P, and their equivalent rating categories in Feldhütter and Schaefer (2018).

Moody's	S&P	Feldhütter and Schaefer
Aaa	AAA	AAA
Aa1	AA+	AA
Aa2	AA	AA
Aa3	AA-	AA
A1	A+	A
A2	A	A
A3	A-	A
Baa1	BBB+	BBB
Baa2	BBB	BBB
Baa3	BBB-	BBB
Ba1	BB+	BB
Ba2	BB	BB
Ba3	BB-	BB
B1	B+	B
B2	B	B
B3	B-	B
Caa	CCC	C
Ca	CC	C
C	C	C

3.3.3 Single-Stock Pseudo Bonds

The methodology for single-stock pseudo bonds resembles that of the OBX index pseudo bonds. Ex-ante default probabilities are found using Equations (3.5), (3.7), and (3.8). Pseudo bond prices are calculated using Equation (2.9), where the single stock metrics replace the OBX metrics, and yield to maturity is found with Equation (3.2). Yield spreads are then computed by subtracting the risk-free yield from the pseudo bond yield as in Equation (3.1). Finally, each default probability is mapped to a credit rating category using Feldhütter and Schaefer (2018)'s scheme, and the average spread for each credit rating category is calculated. First, I calculate the average yield spread per credit rating for each company, and then I find the average of their averages per credit rating. As with OBX pseudo bonds, I remove negative spreads for the single-stock pseudo bonds to ensure comparability with corporate bonds. For each single-stock pseudo firm, I choose the value of the parameter K to obtain a default probability range between 0 and 1. If forced to choose between a range of 0-0.8 and 0-1.2, I decide to overestimate rather than underestimate default probabilities to secure the whole range.

3.3.4 The Merton Model

The Merton model is used for valuing risky corporate debt, in which the underlying assets are assumed to be log-normally distributed (Culp et al., 2018). I am using the *distance-to-default model* (DD), which is based on the Merton model. Assuming the asset value follows a geometric Brownian motion process, and debt values are constant for the individual firms in the interval $[t, T]$, the Black-Scholes-Merton formula applies (Andersen et al., 2021). This formula is given in Equation (3.9), where E denotes equity value, V asset value, K value of debt, $\mathcal{N}(\cdot)$ is the normal cumulative distribution function, and r is the risk-free interest rate. d_1 and d_2 are defined in Equations (3.10) and (3.11) (Andersen et al., 2021). These equations are equivalent to Equations (2.3), (2.4), and (2.5) in Chapter 2, except that S is replaced with V to denote the firm value:

$$E = V \cdot \mathcal{N}(d_1) - e^{-rT} K \cdot \mathcal{N}(d_2) \,, \tag{3.9}$$

with

$$d_1 = \frac{\ln\left(\frac{V}{K}\right) + \left(r + \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}} \,, \tag{3.10}$$

$$d_2 = d_1 - \sigma_V \sqrt{T} = \frac{\ln\left(\frac{V}{K}\right) + \left(r - \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}} \,, \tag{3.11}$$

where σ_V denotes the firm volatility (Culp et al., 2014). d_1 is the distance of the asset value to the debt value relative to its volatility. The assumption of assets following a geometric Brownian motion process implies that the incremental changes in asset value are normally distributed. Therefore, the probability of default P_{D} can be calculated as follows (Andersen et al., 2021):

$$P_{\text{D}} = \mathcal{N}(-d_2)\ . \tag{3.12}$$

I use corporate bond data to compute default probabilities, where market capitalization is used as equity value and total liabilities as debt value. The historical put implied volatilities on the OBX index are used as volatility measures, and as a risk-free rate, I use the NIBOR3M. Finally, I map each default probability to a credit rating while accounting for time to maturity.

Chapter 4

Results and Discussion

This chapter addresses the objectives **O1**, **O2**, and **O3** presented in Section 1.2.

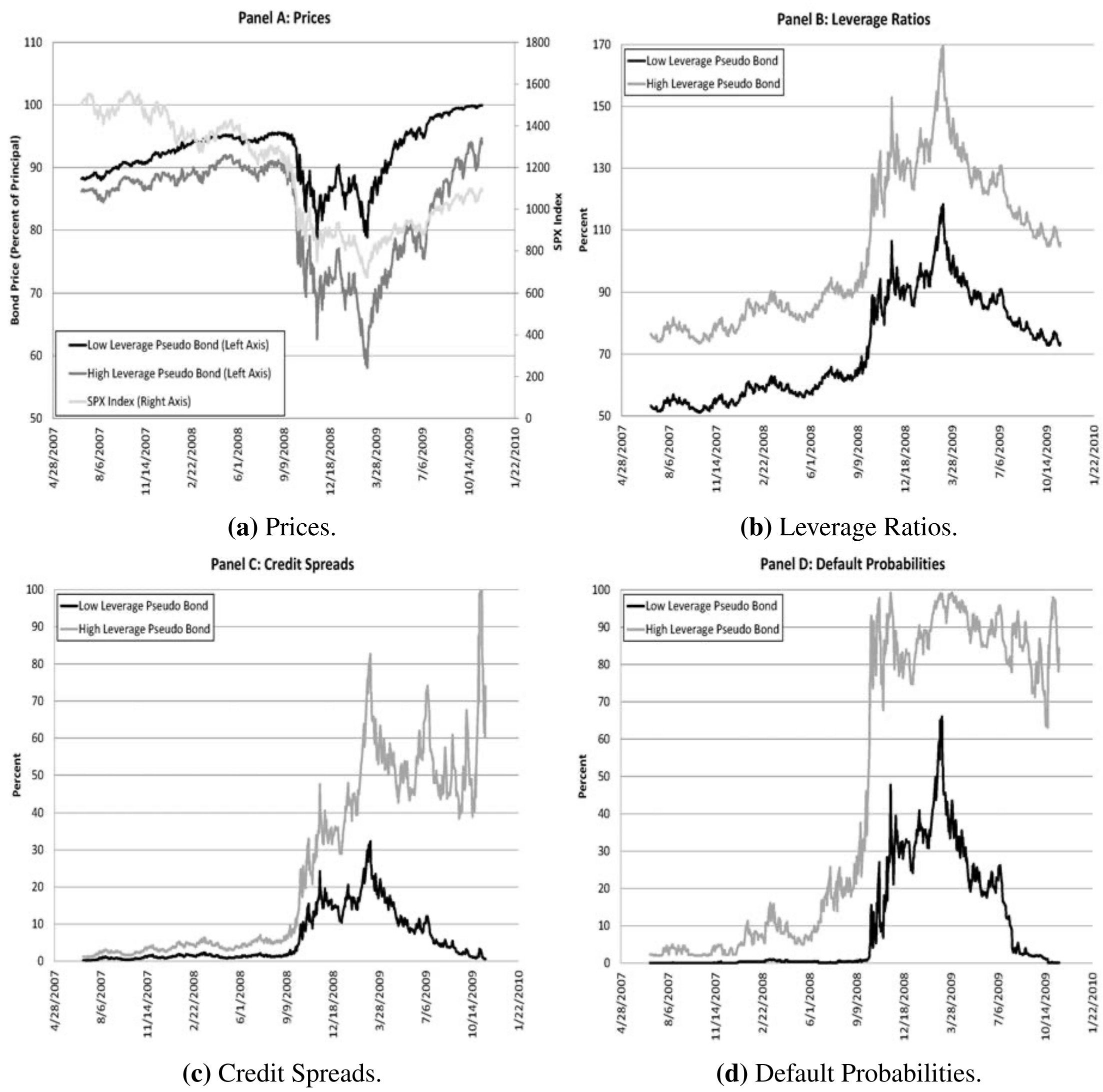
I begin with a presentation of Culp et al. (2018)'s results displayed in Figure 4.1 before I present my own results for the financial crisis and COVID-19 and discuss them in greater detail (objective **O1**). Then, I present Figure 4.6 from Culp et al. (2018) before I discuss my own results instrument by instrument and compare each of the spreads I have obtained (objective **O2**). Moreover, I analyze whether Feldhütter and Schaefer (2018)'s conclusions are transferable to my results for corporate bonds and the Merton model (objective **O3**). Finally, I explore potential sources of error in my results.

4.1 Objective 1: Index Pseudo Bonds

This section's focus is Figure 4.1 from Culp et al. (2018), which displays two SPX pseudo bond prices and their leverage ratios, credit spreads, and default probabilities during the financial crisis. I replicate this figure using Norwegian data for both the financial crisis and the COVID-19 pandemic and compare my results with those of Culp et al. (2018).

4.1.1 Culp et al. (2018)'s results

On the website *The Credit Risk Laboratory* (Culp et al., 2024d), the authors comment further on their results. Figure 4.1 displays four plots with results for the SPX index pseudo bonds. In Figure 4.1(a), the SPX index and the pseudo bond prices are plotted over time. The SPX index is plotted on the right-hand side y-axis, while the pseudo bond prices are plotted as percentage values of their principals on the left-hand side y-axis (Culp et al., 2018). We can observe a significant drop in the value of prices for SPX and the two pseudo bonds during 2008. While the low-leverage pseudo bond eventually recovers and pays 100% of its principal, the high-leverage pseudo bond does not and, therefore, defaults



(a) Prices. **(b)** Leverage Ratios. **(c)** Credit Spreads. **(d)** Default Probabilities.

Figure 4.1: Figure 1 from Culp et al. (2018)'s paper displaying prices, leverage ratios, credit spreads, and default probabilities for the SPX pseudo bonds with two different leverage values for the financial crisis in the U.S. (2007-2010).

(Culp et al., 2024d).

In Figure 4.1(b), we see the leverage ratios of the two pseudo bonds in percentage terms and we can observe a great increase in both ratios during the financial crisis (Culp et al., 2024d). Figure 4.1(c) displays the credit spreads, and Culp et al. (2024d) observe that the spreads are initially low but increase during the financial crisis and that the high-leverage pseudo bond always has a higher spread than that of the low-leverage pseudo bond. While the low-leverage bond spread converges to a negligible number towards maturity, the high-leverage bond spread does not. Finally, in Figure 4.1(d), Culp et al. (2018) displays the ex-ante default probabilities for the two pseudo bonds. The default probabilities increase during the financial crisis. The default probability is always higher for the high-leverage bond than that of the low-leverage bond, and only the low-leverage bond's default probability converges back to a negligible number towards maturity (Culp et al., 2024d).

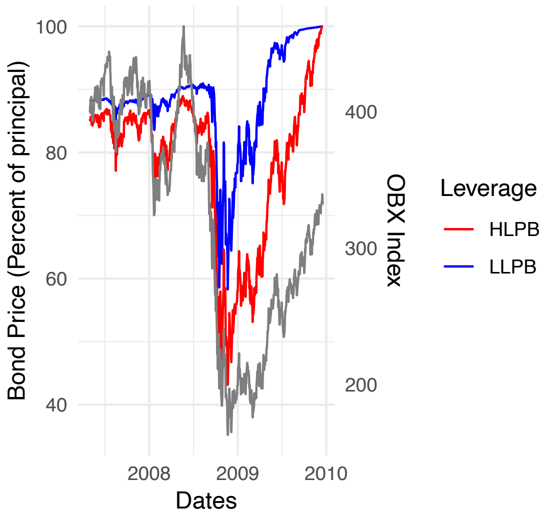
4.1.2 Method Replication for the Financial Crisis in Norway

I create Figure 4.2, which displays the equivalent of Figure 4.1 from Culp et al. (2018) using Norwegian data for 2007-2010. Figure 4.2(a) displays the OBX index and the low- and high-leverage pseudo bond prices. The OBX index falls dramatically in 2008, like the SPX index in Figure 4.1(a). During the 2008 crisis, also both pseudo bond prices drop substantially, similar to what Culp et al. (2018) observed (Culp et al., 2024d). However, unlike their results for the high-leverage bond, my high-leverage bond pays 100% of its principal at maturity, similar to the low-leverage bond. As discussed in Chapter 2, bond prices typically converge to their face value as they approach maturity (Berk and DeMarzo, 2016).

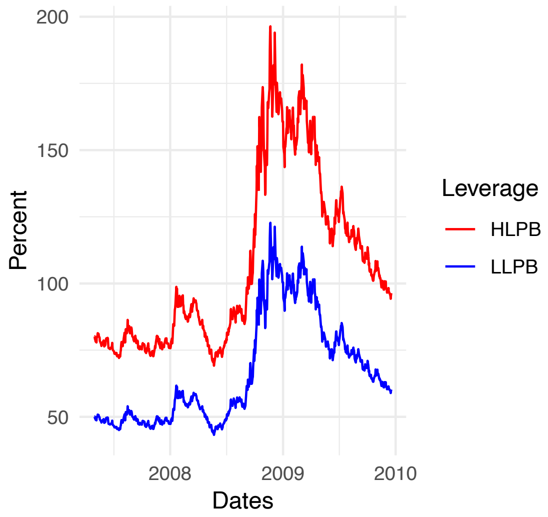
The high-leverage bond has, at times, a more significant price drop than the low-leverage bond during the crisis but does not default, which differs from the observation of SPX by Culp et al. (2024d). In agreement with what is discussed in Chapter 2, put options increase in value when the price of the underlying asset decreases (Berk and DeMarzo, 2016). From Equation (2.9), we see that the bond value falls as the put option value increases. Therefore, the two pseudo bond prices fall in Figure 4.2(a) along with the OBX index.

Figure 4.2(b) displays the leverage ratios for the two OBX pseudo bonds. It shows, similarly to Culp et al. (2024d)'s observations, that the leverage ratios increase considerably during the financial crisis. The leverage ratios are calculated using K_i/A_t, where A_t denotes the value of the OBX index. K_i is constant for its respective pseudo bond (Culp et al., 2018). Since the OBX index falls during the financial crisis, the leverage ratio necessarily has to increase during the same period.

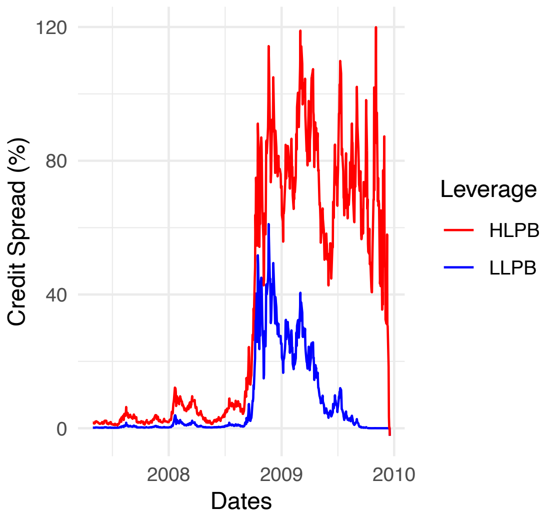
In Figure 4.2(c), the credit spreads for the OBX pseudo bonds are plotted as percentage values. Similarly to Culp et al. (2024d), I observe that the spread for the high-leverage bond almost always is wider than for the low-leverage bond and that both spreads widen during the financial crisis. This can be understood from Equation (3.2). From Figure 4.2(a), we see bond prices falling during the financial crisis. Since the yield to maturity on a zero-coupon bond is found with the bond price as the denominator, the bond yield increases when the bond price falls, and the yield spread, calculated with Equation (3.1), increases. Unlike Culp et al. (2018)'s results, where only the low-leverage pseudo bond spread converges back to a small number (Culp et al., 2024d), both OBX pseudo bond spreads converge to a small number by the end of the sample period.



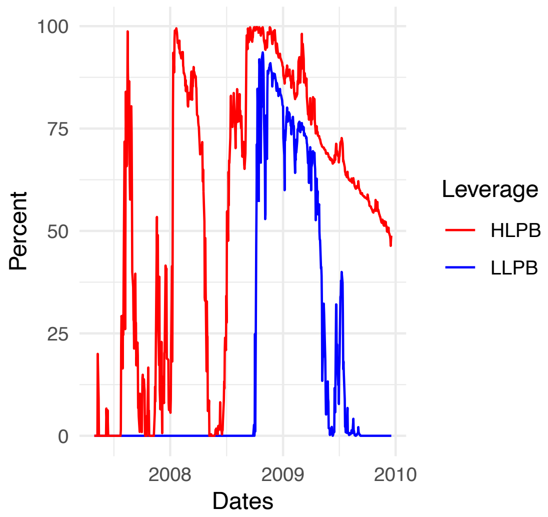
(a) Prices (OBX index in gray).



(b) Leverage Ratios.



(c) Credit Spreads.



(d) Default Probabilities.

Figure 4.2: Results for pseudo bonds during the financial crisis in Norway (2007-2010). LLPB and HLPB refer to the low-leverage and high-leverage pseudo bonds, respectively.

Finally, the default probabilities are displayed in Figure 4.2(d). Both pseudo bonds have higher default probabilities during the financial crisis, but the high-leverage pseudo bond also experiences high default probabilities before the date we typically consider the start of the financial crisis. The default probabilities increase during the financial crisis because of the increased leverage ratio we observe in Figure 4.2(b). This relationship is formulated by Equations (3.7) and (3.8). The high-leverage default probabilities are higher than for the low-leverage pseudo bond exactly because of their different leverage levels.

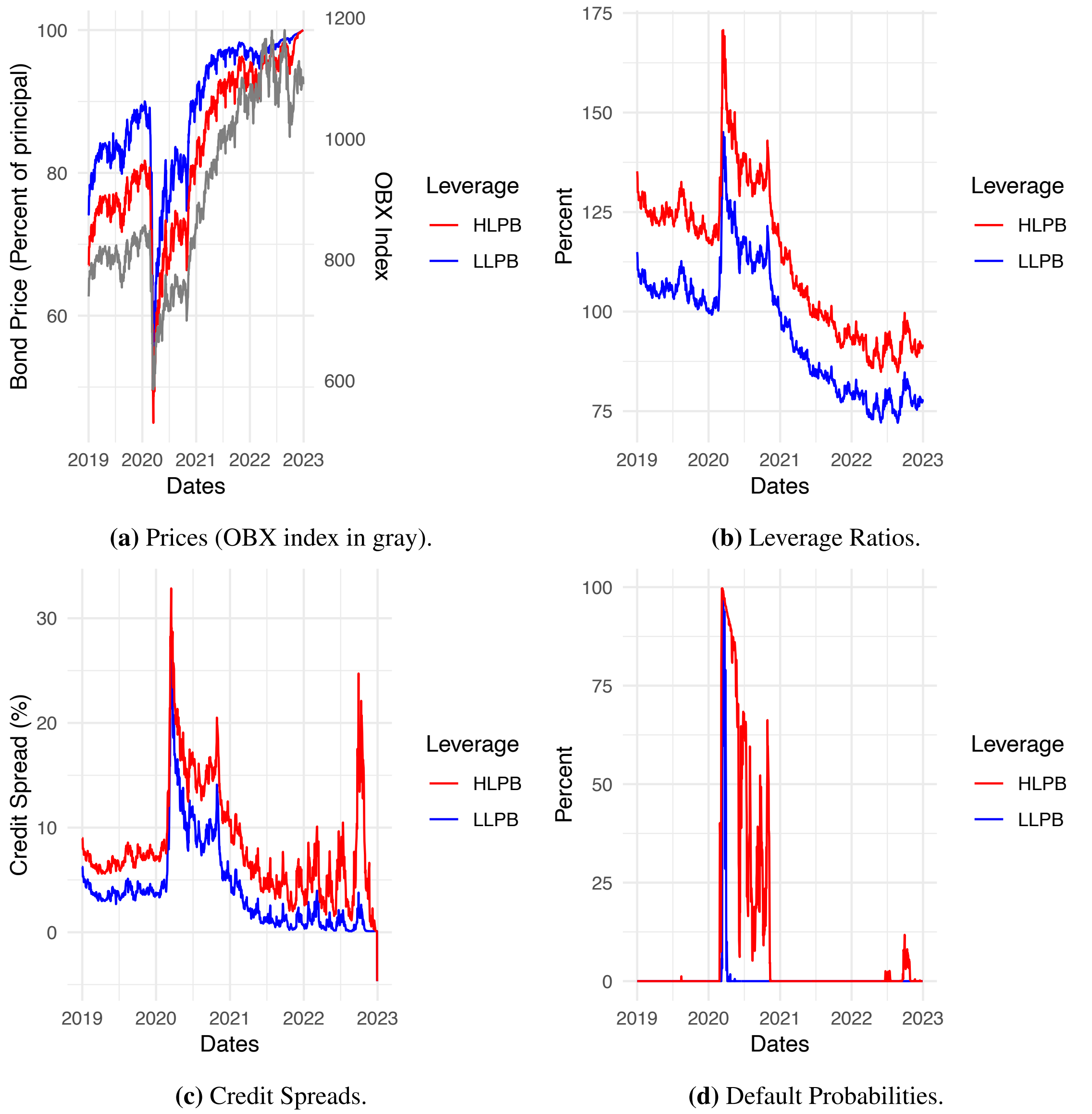
I find that the default probability of the high-leverage pseudo bond is always greater than or equal to, in a few instances, the default probability of the low-leverage pseudo bond. My low-leverage pseudo bond's default probability is similar to that in Culp et al. (2018) but reaches a probability of almost 100% while the maximum for Culp et al. (2018) is around 60%. It does, however, converge back to a negligible value towards maturity. At maturity, my high-leverage pseudo bond has a default probability around 50%, while Culp et al. (2018)'s default probability for the high-leverage pseudo bond still fluctuates around 100%. My resulting default probabilities appear to be more volatile than those of Culp et al. (2018), particularly for the high-leverage pseudo bond.

4.1.3 Method Replication for the COVID-19 Pandemic in Norway

In Figure 4.3, I use Norwegian data for the period 2019-2022 to display the corresponding time series as Figure 4.1 from Culp et al. (2018) and Figure 4.2. In Figure 4.3(a), we see a dramatic drop in the closing price of the OBX index in the first part of 2020. This lower value persists until 2021. Then, the price increases and follows a similar pattern to that before 2020. I observe the same dynamic for the low- and high-leverage pseudo bond prices, which, as discussed previously in this chapter, happens because their underlying asset price falls. As expected, the bond prices converge to their face value when approaching maturity (Berk and DeMarzo, 2016). Just as for the financial crisis, we see in Figure 4.3(b) that the leverage ratios for the pseudo bonds increase during the pandemic. This happens because the OBX index price falls, as previously discussed. Figures 4.2(b) and 4.3(b) show that the leverage ratios increase by more percentage points during the financial crisis for the high-leverage pseudo bond. Furthermore, the figures show that the leverage ratios fall below their starting point after the COVID-19 pandemic. The equivalent did not happen after the financial crisis, due to the OBX index's relatively slower growth in the aftermath of this crisis.

Figure 4.3(c) shows that the credit spreads also widened during the pandemic before tightening again towards 2021. An interesting observation is how low the spreads become compared to those in Figures 4.1(c) and 4.2(c). At their highest, the credit spreads for the financial crisis in the U.S. are 30% for low leverage and 100% for high leverage (Culp et al., 2018). For the financial crisis in Norway, they are 60% and 120%, respectively, while for COVID-19, they are around 20% and 30%. Considering Equation (3.2) for the yield to maturity of zero-coupon bonds, the relationship between the face value and bond price K/B is a determining factor. K is constant for its respective pseudo bond, so the only source of the disparity in Equation (3.2) can be B.

Figure 4.4 shows the ratios K/B for both periods. The ratios are relatively similar before and after their respective crises, and they both experience a spike during their crises.



(a) Prices (OBX index in gray).

(b) Leverage Ratios.

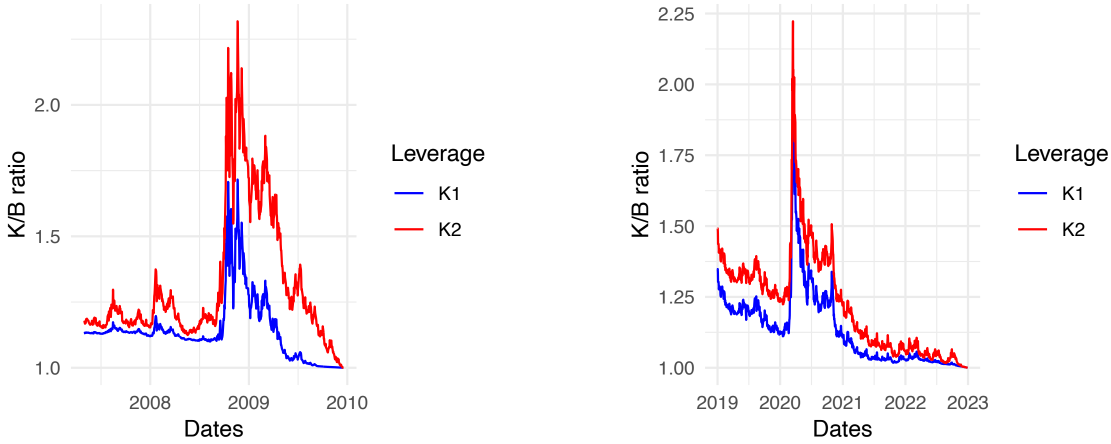
(c) Credit Spreads.

(d) Default Probabilities.

Figure 4.3: Results for pseduo bonds during the COVID-19 pandemic in Norway (2019-2022). LLPB and HLPB refer to the low-leverage and high-leverage pseudo bonds, respectively.

However, the great increase in this ratio lasts longer for the financial crisis than for the COVID-19 pandemic. This results in a higher yield to maturity during the financial crisis and, therefore, also a higher yield spread. This is even though NIBOR3M, used to represent the risk-free interest rate, is considerably higher during the financial crisis than during the pandemic, as seen in Figure 4.5.

Incidentally, another widening happens in Figure 4.3(c) at the beginning of 2023. This is, however, after the COVID-19 pandemic and, therefore, beyond this thesis's scope.



(a) K/B for OBX pseudo bonds during the financial crisis in Norway.

(b) K/B for OBX pseudo bonds during the COVID-19 pandemic in Norway.

Figure 4.4: K/B ratios during the financial crisis (2007-2010) and the COVID-19 pandemic (2019-2022). K_1 refers to the low leverage value, and K_2 to the high leverage value.

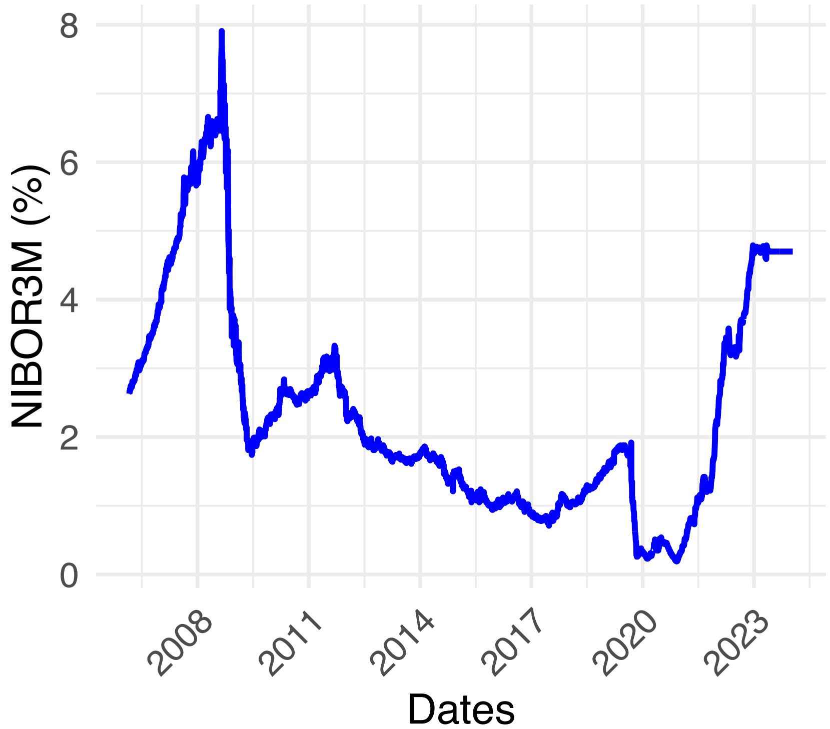


Figure 4.5: NIBOR3M over time.

The default probabilities in Figure 4.3(d) increase dramatically at the beginning of 2020. While the low-leverage pseudo bond's default probability rapidly decreases back to negligible values, the default probability for the high-leverage pseudo bond continues to stay significant until 2021. While the default probabilities are quite bursty for the period

2019-2022 in Figure 4.3(d), especially for the low-leverage pseudo bond, they are more constant for 2007-2010 in Figures 4.2(d) and 4.1(d). Namely, higher default probabilities remain so for a more extended amount of time. A possible reason for this disparity is that I am considering a different period with its own set of characteristics.

Another possible reason is that I am considering a different market, and that there are characteristics in my dataset that differ from the U.S. data. Furthermore, I estimate the mean and the volatility of log asset growth, assuming stationarity, while Culp et al. (2018) ran a predictive regression to estimate the mean and a GARCH model to estimate the volatility (Culp et al., 2018). However, Culp et al. (2018)'s approach for creating Figure 4.1 seems suitable overall also when applied to Norwegian data.

This concludes objective 1 (**O1**) of the thesis, which was to replicate Figure 4.1 from Culp et al. (2018) with Norwegian data. The figure was replicated for two periods, namely the financial crisis and the COVID-19 pandemic, which offered the possibility of comparing Culp et al. (2018)'s method not only with a different market but also with a different time period.

4.2 Objective 2: Credit Spreads

The second objective of this chapter is to replicate Figure 4.6 from Culp et al. (2018) with Norwegian data. The figure displays average credit spreads for corporate bonds, SPX pseudo bonds, single-stock pseudo bonds, and the Merton model during the financial crisis. I replicate this figure using primarily Norwegian data for the COVID-19 pandemic, but I make an additional plot of average spreads for OBX index pseudo bonds during the financial crisis, for which I have sufficient past data to explore potential differences. Culp et al. (2018) use rating categories from Moody's scheme, while I use the rating categories from Feldhütter and Schaefer (2018). In Table 4.1, I have listed the credit rating categories used by Culp et al. (2018) and what I consider equivalent categories based on Table 3.3.

Table 4.1: Credit rating categories by Culp et al. (2018), and their equivalent rating categories according to Feldhütter and Schaefer (2018).

Culp et al.	Feldhütter and Schaefer
Aaa/Aa	AAA-AA
A/Baa	A-BBB
Ba	BB
B	B
Caa-	C

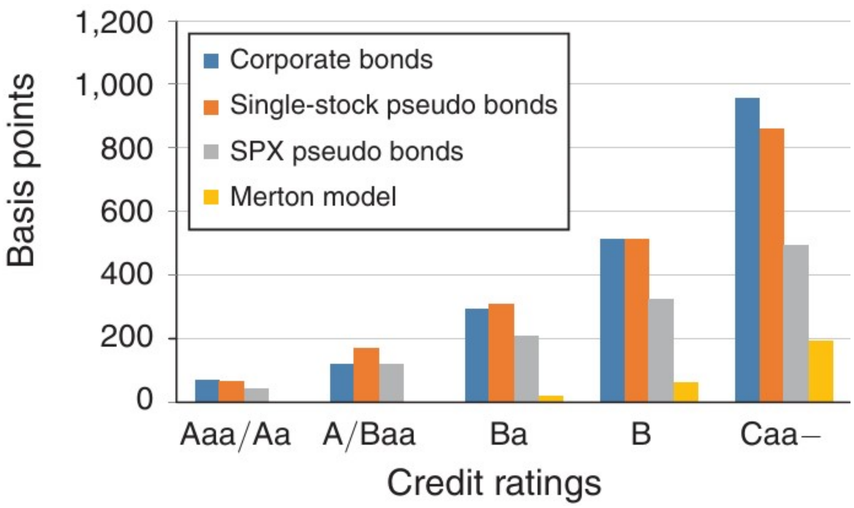


Figure 4.6: Figure 2 Panel A from Culp et al. (2018)'s paper for the financial crisis in the U.S. (2007-2010).

4.2.1 Culp et al. (2018)'s results

Figure 4.6 shows that, according to the log-normal Merton model, credit spreads do not appear before we have a credit rating of Ba or lower. In Table 3.3, Ba1, Ba2, and Ba3 in Moody's rating scheme correspond to BB in Feldhütter and Schaefer (2018)'s scheme, which I consider as a simplified version of S&P's scheme. Furthermore, Figure 4.6 shows that for all credit rating categories, the SPX pseudo bond will have a lower spread than that of corporate bonds and single-stock pseudo bonds and that the distance between them increases with decreasing credit rating. In general, the spreads for the corporate bonds and the single-stock pseudo bonds are at equal levels across credit ratings. Table 4.2 displays the numerical spread values for each rating and asset type found by Culp et al. (2018).

Table 4.2: Resulting yield spreads for different credit ratings and assets by Culp et al. (2018). Reproduction of Table 2 by Culp et al. (2018) for the assets relevant to this thesis. Approximated values for Merton are reported since numerical results were not provided.

Credit Rating	Corporate	Single Stock	SPX	Merton
Aaa/Aa	71	68	42	$\simeq 0$
A/Baa	121	171	119	$\simeq 0$
Ba	293	308	209	< 50
B	512	514	325	< 100
Caa-	956	862	496	$\simeq 200$

4.2.2 Method Replication for the COVID-19 Pandemic in Norway

Table 4.3 summarizes the average spreads for OBX pseudo bonds, empirical corporate bonds, single-stock pseudo bonds, and the Merton model during the COVID-19 pandemic in Norway. The number of credit ratings per category is reported in the column *Count*. In some instances, for OBX pseudo bonds and single-stock pseudo bonds, I obtained floats in the number of observed credit ratings of the different categories. These numbers were rounded to the nearest integer. The reason I obtained floats was that I found the average number of rating categories between different leverages for OBX pseudo bonds and different stocks for single-stock pseudo bonds.

Table 4.3: Resulting yield spreads for different credit ratings and assets, followed by the number of observations of each credit rating category. A hyphen indicates no bond was categorized with the respective credit rating.

Credit Rating	Corporate		Single Stock	
	Spread	Count	Spread	Count
AAA	45	14,396	33.5	959
AA	42.6	18,788	5.88	79
A	55.3	52,287	–	–
BBB	84	47,230	–	–
BB	90.6	8,991	–	–
B	128	1,203	335	2
C	206	16	196	4
Credit Rating	**OBX**		**Merton**	
	Spread	Count	Spread	Count
AAA	49.9	842	76.6	17,214
AA	30.4	79	80.2	2,834
A	136	2	104	4,099
BBB	124	3	105	9,424
BB	143	13	129	6,759
B	164	36	158	1,484
C	204	70	179	302

The statistics presented in Table 4.3 for OBX pseudo bonds, corporate bonds, single-stock pseudo bonds, and the Merton model will be discussed in their respective paragraphs. The statistics for the corporate bonds are for the unfiltered dataset with respect to collateral type and interest type. For Merton, the statistics displayed are found for otherwise unfiltered corporate bond data (after winsorization) with debt-to-equity ratios below two.

OBX Pseudo Bond Spreads

Figure 4.7 displays the average spread implied by the average of the leverages K_1 and K_2 per credit rating for the OBX pseudo bond during the COVID-19 pandemic.

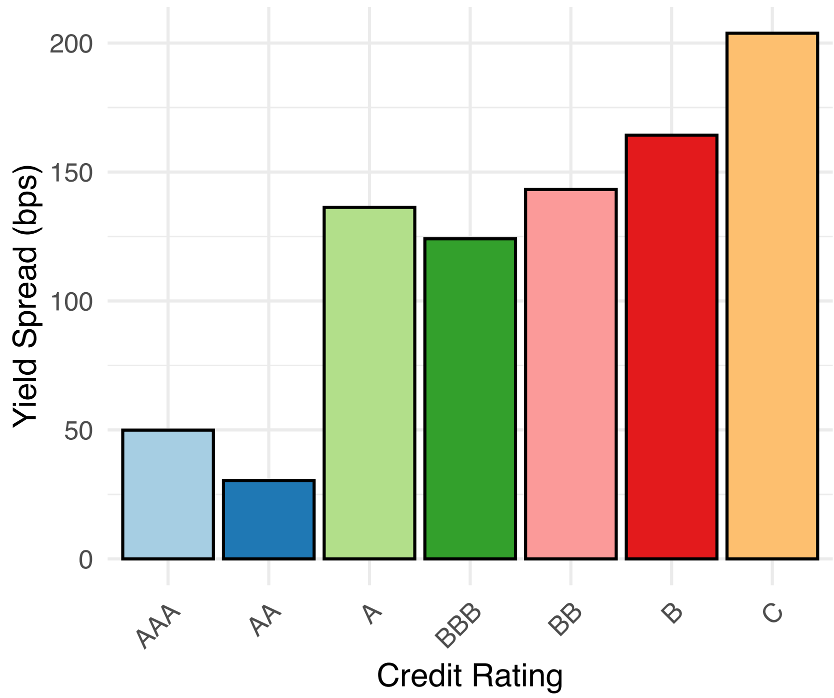


Figure 4.7: Average spread by credit rating following Feldhütter and Schaefer (2018)'s scheme for OBX pseudo bonds during the COVID-19 pandemic in Norway (2019-2022).

From credit rating BBB and lower, the spread increases with decreasing credit quality, like for U.S. data in Figure 4.6. However, I obtain interesting results for credit ratings AAA, AA, and A. Rating AAA has a higher implied spread than AA, and rating A has a higher spread than BBB. A possible reason for this discrepancy is that the number of observations of the credit rating categories fluctuates greatly. Table 4.3 shows that OBX pseudo bonds only have two instances with credit rating A and three instances with rating BBB, while there are 842 instances with AAA rating. Therefore, it is unlikely that my results with ratings A and BBB are representative enough to say something about the average yield spread for their respective categories. It is fair to assume that the same argument holds for the remaining categories as well, namely AA, BB, B, and C, since they also have relatively few observations compared to those of category AAA. Another possible reason for this result is that I have merged several rating categories to obtain Feldhütter and Schaefer (2018)'s rating scheme.

To understand if my results improve when looking at the financial crisis period, where the default probabilities have a less binary behavior, I produce Figure 4.8, which displays

OBX pseudo bond spreads for this period.

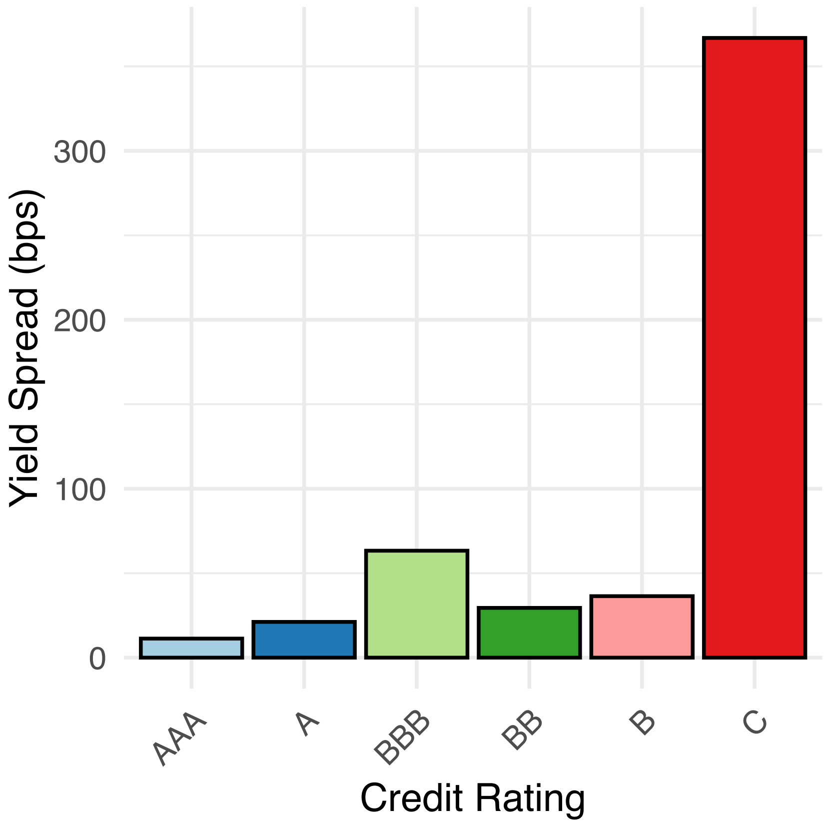


Figure 4.8: Average spread by credit rating following Feldhütter and Schaefer (2018)'s scheme for OBX pseudo bonds during the financial crisis in Norway (2007-2010).

Disregarding the result for category BBB, the spreads follow the expected pattern for decreasing credit quality. For all available rating categories, the result dramatically differs from that in Figure 4.7 for the COVID-19 pandemic. Categories above C have a smaller spread, while category C has more than 100 bps wider spread in Figure 4.8 than in Figure 4.7.

I create Table 4.4 to consider the number of observations of each credit rating reported in Figure 4.8. I can then evaluate the plausibility of my results. As with Table 4.3, I round the average number of observed credit rating categories to the closest integer in Table 4.4. Comparing this table with Table 4.3, I see that the distribution among credit rating categories is slightly more even. This is positive because the average spread per category becomes more reliable. There are significant differences in observations of categories AAA and C between the low-leverage and high-leverage pseudo bonds. This makes sense when considering Figure 4.2(c), where the spread is much wider for the high-leverage pseudo bond during the financial crisis. Most observations are still either

AAA or C, but there is a higher degree of distribution also in the middle, which is expected when looking at Figure 4.2(d). This increases the ability to create reliable bar charts of average yield spreads on rating categories. The relatively high average spread for category BBB comes from the low-leverage pseudo bond, which only has two observations but a very high spread on rating BBB.

Table 4.4: Resulting yield spreads on OBX pseudo bonds during the financial crisis, followed by the number of observations of each credit rating category. K_1 denotes the low-leverage bond, and K_2 the high-leverage bond. A hyphen indicates that no bond was categorized with the respective credit rating.

Credit Rating	K_1		K_2		Average	
	Spread	Count	Spread	Count	Spread	Count
AAA	7.3	464	15.3	83	11.3	274
AA	–	–	–	–	–	–
A	–	–	21.2	6	21.2	6
BBB	104.0	2	22.9	20	63.3	11
BB	38.0	23	20.9	35	29.5	29
B	48.0	16	24.8	39	36.4	28
C	250	184	484.0	506	367.0	345

Empirical Corporate Bond Spreads

Figure 4.9 displays the average yield spread in each credit rating for corporate bonds when I apply filters to remain with bonds with a fixed interest rate and unsecured collateral.

The spreads and credit ratings are, as described in Chapter 2, already provided by Folketrygdfondet, and all I do is convert the credit ratings from Moody's scheme to that of Feldhütter and Schaefer (2018), for then to find the average yield spread for each credit rating category. The average spread is tightest for the best credit rating category and widens with decreasing rating. The results for the Norwegian empirical corporate bonds are similar to the results Culp et al. (2018) find for U.S. data, summarized in Table 4.2, in the sense that the average spreads widen for decreasing credit ratings. However, their corporate yield spreads are wider for all ratings than those for my data. They obtain 71 bps for the category Aaa/Aa, while I obtain below 50 bps for my equivalent range AAA-AA. Another disparity is that I obtain no observations of category C in my most filtered dataset with empirical corporate bonds.

Table 3.2 shows the remaining observations in the corporate bond dataset when imposing different filters. To understand if my result in Figure 4.9 can also be generalized to corporate bonds with floating interest rates and/or secured collateral, I relax the filters and recreate the bar chart for average spreads per credit rating. Figure 4.10 shows the

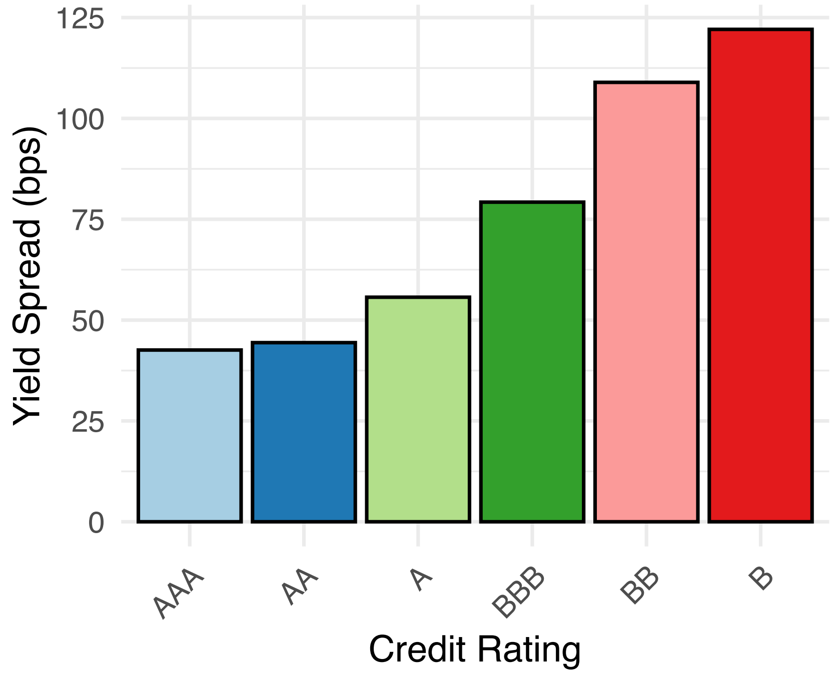
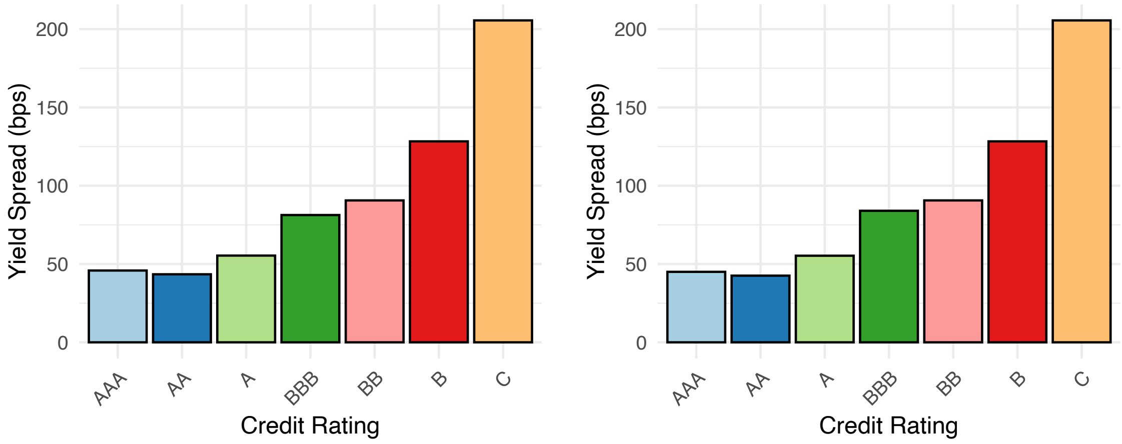


Figure 4.9: Average spread by credit rating following Feldhütter and Schaefer (2018)'s scheme for empirical corporate bond data in Norway during the COVID-19 pandemic (2019-2022). This figure displays results for the most filtered version of the corporate bond dataset.

resulting two bar charts, which appear to be identical. However, there are some minor numerical differences in the average yield spreads between Figures 4.10(a) and 4.10(b). After removing bonds with secured collateral and other types of interest than the fixed interest type, I am only left with 31.37% of the initial observations. Therefore, it is not a surprise that the average spreads look different in Figure 4.10(b), with 100% of the initial observations, with respect to Figure 4.9. When bonds with secured collateral are removed, but all interest types are allowed, shown in Figure 4.10(a), I am left with 89.43% of the observations from the starting point. It appears that removing or adding bonds with secured collateral makes the greatest difference since Figures 4.10(a) and 4.10(b) are almost identical.

Most importantly, the average spread for category C now appears in both Figures 4.10(a) and 4.10(b). It has an average spread of about 200 bps for both. Second, category AAA gets a slightly higher average yield spread than category AA in both figures. The negative spreads are still filtered away, so in case of any negative spreads for instances of ratings AA, they do not pull down the average in my analysis. A possible reason for the discrepancy is that I merge several rating categories from Moody's scheme into one category to obtain ratings within Feldhütter and Schaefer (2018)'s scheme. However, this source of error is also present when I produce Figure 4.9, which does not have a higher spread for AA



(a) Average yield spreads for corporate bonds with unsecured collateral and all interest rate types.

(b) Average yield spreads for corporate bonds with all collateral types and all interest rate types.

Figure 4.10: Average yield spreads per credit rating for empirical corporate bonds with relaxed filters in Norway during the COVID-19 pandemic (2019-2022).

than AAA. That being said, the difference in spreads between categories AAA and AA in Figure 4.9 is less than two percentage points.

Rating category BB has an average spread above 100 bps in Figure 4.9 and below 100 bps in Figure 4.10. The remaining bars are quite similar. Overall, Figure 4.9, which is the most filtered version, is not a representative picture of the less filtered corporate bond spreads. The main reason is that there are no cases of rating category C to use when comparing corporate bond spreads with spreads on category C in other instruments. It is less suitable than Figures 4.10(a) and 4.10(b) to create a comparison with the spreads with the Merton model and the pseudo bonds, such as Culp et al. (2018) did in Figure 4.6. Therefore, I choose to look at the unfiltered corporate bonds in Figure 4.10(b) when comparing corporate spreads to those of other instruments. I do not choose Figure 4.10(a) as it is almost identical to Figure 4.10(b), except for being built on fewer observations. Table 4.3 therefore reports the average spreads for unfiltered corporate bonds. Table 4.3 shows that the empirical corporate bonds most of the time had ratings A or BBB. Compared to OBX pseudo bonds, there are many more observations in total, but there is also a greater distribution between the rating categories.

The disparity between U.S. and Norwegian data increases with decreasing credit quality. Since I only have data from 2016, I cannot check how corporate bond spreads were during the financial crisis in Norway. Perhaps a reason for my lower spreads for corporate bonds is that I am considering a different period than Culp et al. (2018). According to Kozlowski et al. (2021), the yield spreads were higher for a much longer period of time in the U.S. during the financial crisis than for the COVID-19 pandemic. If the same was the case in Norway, it makes sense that my bar chart's average spreads are lower as I have fewer observations with high yield spreads.

As stated in Chapter 1, the Norwegian bond market is smaller than the one in the U.S. (Rundhaug et al., 2020). The industry concentration is higher among the Norwegian corporate bonds than the U.S. ones, and the U.S. market is broader when considering who issues the bonds. Furthermore, Norway has a higher proportion of non-rated non-financial bonds than the U.S. (OECD, 2022). These factors may also explain why the Norwegian corporate bond spreads contrast with the U.S. ones.

Single-Stock Pseudo Bond Spreads

Figure 4.11 displays the average spreads per credit rating for single-stock pseudo bonds.

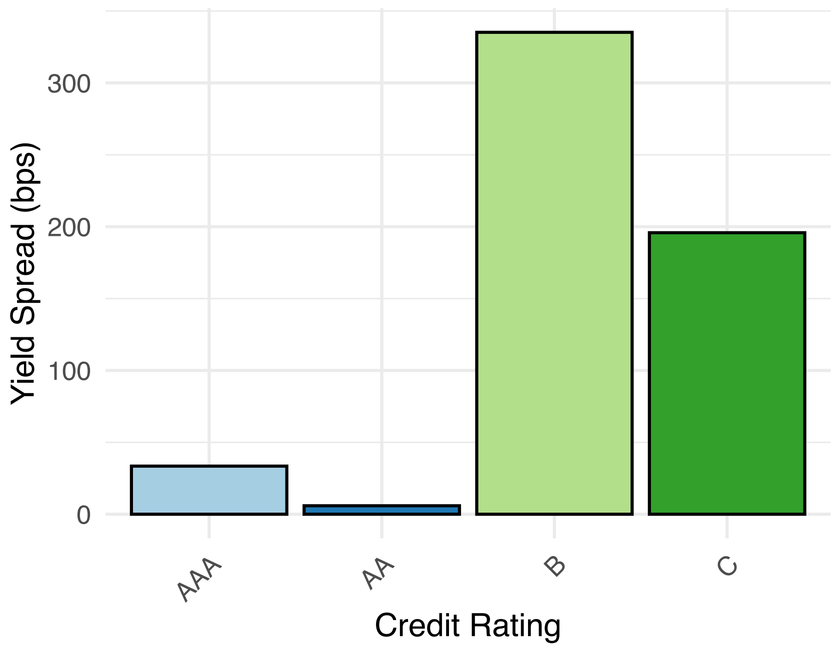
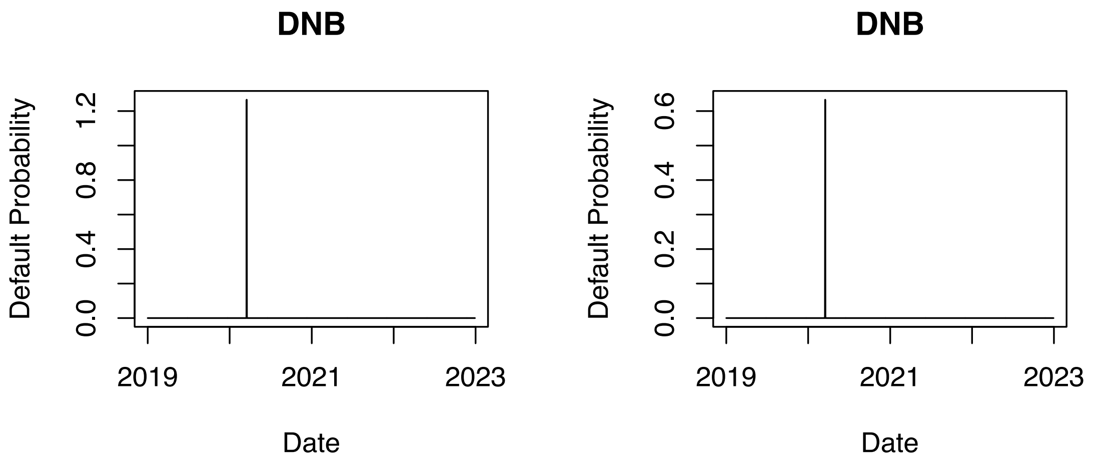


Figure 4.11: Average spread by credit rating following Feldhütter and Schaefer (2018)'s scheme for single-stock pseudo bonds in Norway during the COVID-19 pandemic (2019-2022).

As with OBX pseudo bonds, I get an interesting result for average yield spreads at categories AAA and AA. Furthermore, I get a higher spread for category B than for C. A potential reason for these results is that my dataset contains a small amount of companies. To be specific, only data from seven companies were available for analysis. Therefore, Figure 4.11 is not likely to be comparable with Culp et al. (2018)'s results. Considering the results for single-stock pseudo bonds in Table 4.3, we see that categories B and C only had two and four observations, respectively. Therefore, it is not unexpected that their average spreads provide illogical results. Furthermore, within category C, there is a great difference in average spreads between the different companies. Meanwhile, there are 959 instances of category AAA and 79 of AA. Thus, the average spread for category AAA is the most reliable result, while the result for AA seemingly does not have enough observations to provide the expected result, according to the theory on the relationship between yield spread and credit rating quality (Berk and DeMarzo, 2016).

Minor changes to K greatly impact the resulting default probabilities for single-stock pseudo bonds. I focus on getting a range of default probabilities from 0 to 1, and the choice of leverage as a percentage of the mean stock price of the respective company is dictated by this goal. I illustrate the choice of K in Figure 4.12.



(a) Default probabilities for leverage $K = 64.6\%$ of mean stock price DNB.

(b) Default probabilities for leverage $K = 64.5\%$ of mean stock price DNB.

Figure 4.12: Default probabilities for DNB at different K as percentages of mean stock price during the COVID-19 pandemic (2019-2022).

To achieve more granular changes in values of K, I choose to use a percentage of the mean stock price of the respective company rather than changing K itself. However, minimal alterations are needed to dramatically change the default probabilities even when using percentage values. Figures 4.12(a) and 4.12(b) only have a difference in K of 0.1% but achieve default probability ranges of 0-1.2 and 0-0.6, respectively. Another challenge with these results is their practically binary nature, where there are no default probabilities in between but only extreme values from each side of the range. Furthermore, there are typically just a few instances of very high default probabilities per company in my dataset for single stocks. This explains why I obtain averages of two B-ratings and four C-ratings in Table 4.3. Clearly, Culp et al. (2018)'s method does not produce realistic results for my single-stock pseudo bonds, but the method provides quite probable results for default probabilities of OBX pseudo bonds in Figures 4.2(d) and 4.3(d), even if also Figure 4.3(d) has a more binary nature. The small dataset for single stocks is probably the main cause, as the methods to produce single-stock pseudo bonds and OBX pseudo bonds are similar.

The Merton Model

Figure 4.13 displays the average spreads per credit rating for the Merton model, which I calculated using market capitalization values and debt values for empirical corporate bond data. Filters for collateral type and interest type are not applied to this figure, similar to Figure 4.10(b).

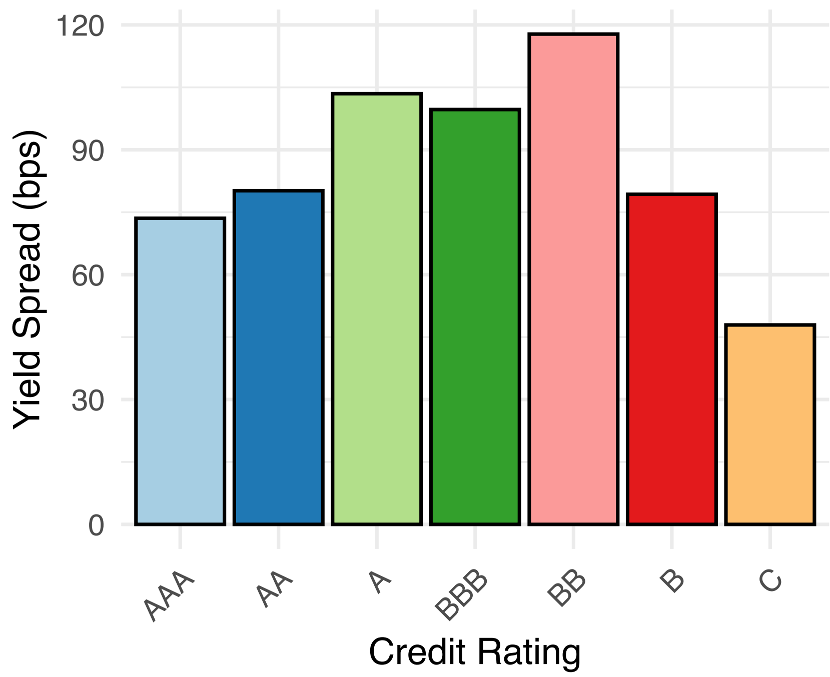


Figure 4.13: Spreads implied by the Merton model for Norwegian data during the COVID-19 pandemic (2019-2022). The data are not filtered with respect to collateral type or interest type.

The resulting spreads in Figure 4.13 are quite different from Culp et al. (2018)'s results for the Merton model in Figure 4.6, which has spreads above zero, at least visibly, for ratings BB, B, and C. Culp et al. (2018) has a yield spread below 50 bps for BB, below 100 bps for B, and about 200 bps for category C. My resulting spreads are clearly much higher than those of Culp et al. (2018) for categories AAA-BBB. For category C, on the other hand, my resulting average yield spread is below 60 bps, which is the lowest of all categories.

When examining each issuer more closely in the corporate bond dataset, I observe tremendously high debt values within the financial sector. This yields very high default probabilities for the respective bonds when applying the Merton model. However, the yield spreads from Nordic Bond Pricing on these bonds are much tighter than expected for such high default probabilities. Therefore, I obtain yield spreads even below 10 bps for C-rated bonds with the current dataset. This can explain why Figure 4.13 yields such a low average spread on category C bonds. Therefore, I choose to filter away the instances with a high degree of total debt compared to market capitalization.

I keep observations where the total debt value is less than two times greater than the market capitalization value. Imposing this filter on the otherwise unfiltered corporate bond dataset reduces the number of observations from 142,911 to 42,116. This reduction follows primarily because the financial firms in the dataset have debt-to-equity ratios greater than two. The reduction is quite dramatic and is logical when looking at Figure 3.1, which shows that the main fraction of issuers in the corporate bond dataset is in the financial sector. As mentioned in Chapter 2, Fama and French (1992) remove financial firms, which might introduce a bias, from their analysis. Figure 4.14 shows the average spreads for this smaller dataset without financial firms, which arguably looks less biased than Figure 4.13.

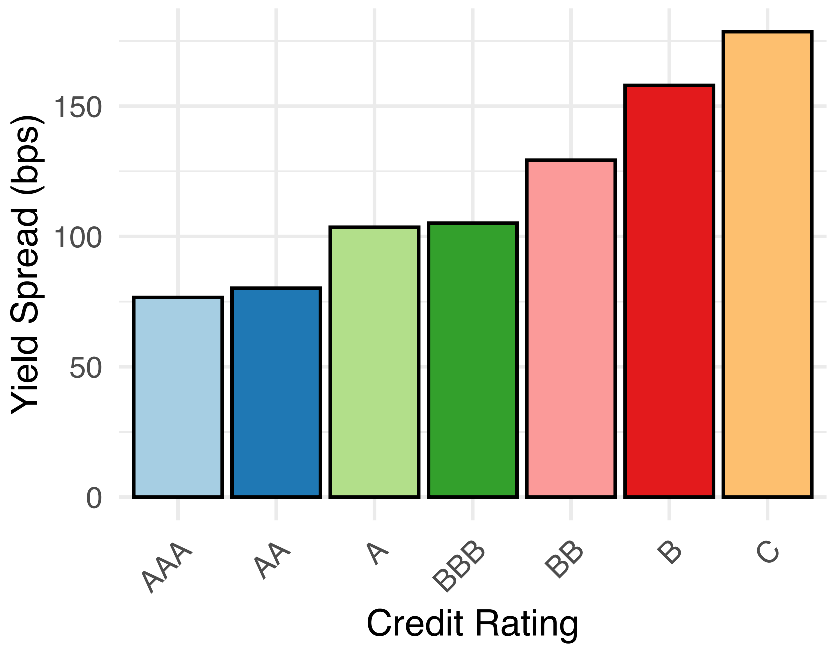


Figure 4.14: Spreads implied by the Merton model for bonds with debt-to-equity ratios below two for Norwegian data during the COVID-19 pandemic (2019-2022). The data are not filtered with respect to collateral type or interest type.

The relationship between spreads and credit ratings is now as expected from Berk and

DeMarzo (2016). This result is used further in the final comparison of the average spreads between instruments and corresponds to the statistics reported for the Merton model in Table 4.3. However, the spreads in Figure 4.14 are not directly comparable to the empirical corporate bond spreads as the dataset used to generate this figure is much smaller. The Merton model appears unsuitable for firms with a high debt-to-equity ratio, at least when matching default probabilities implied by the Merton model with empirical spreads. This reveals a weakness of the Merton model, at least in the form that I applied in this thesis.

4.2.3 Spreads Compared

Figure 4.15 displays average spreads per credit rating category for corporate bonds, pseudo bonds, and the Merton model side by side. It facilitates a comparison with Culp et al. (2018)'s average spreads in Figure 4.6.

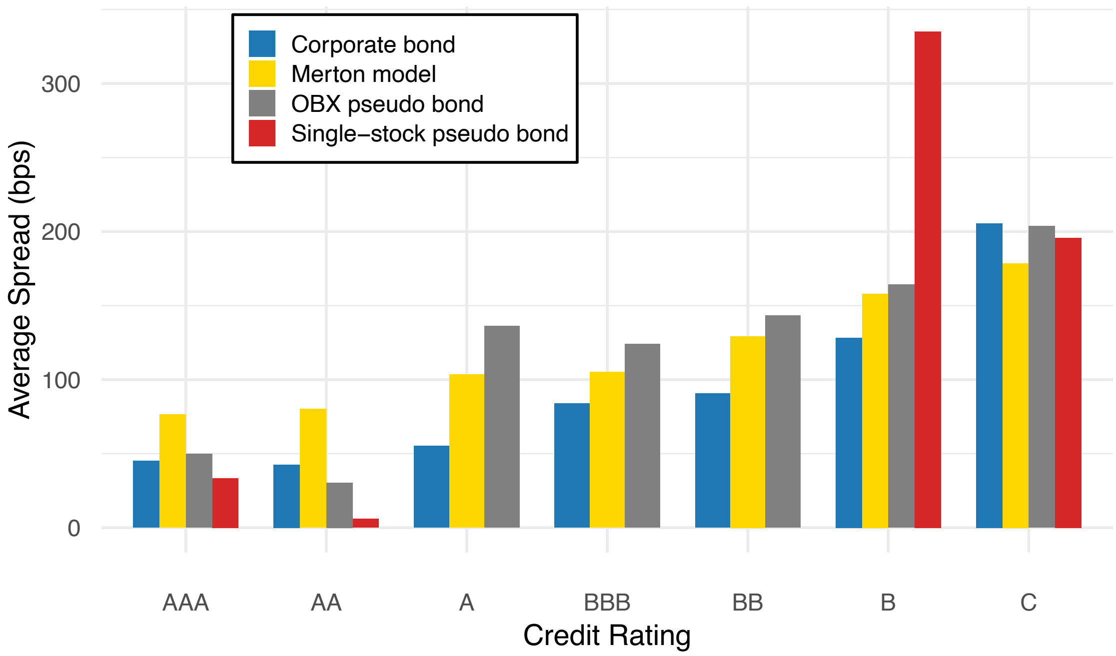


Figure 4.15: Figure 2 Panel A in Culp et al. (2018) replicated with Norwegian data for the COVID-19 pandemic (2019-2022).

As previously stated, Culp et al. (2018) find that the SPX pseudo bonds always have lower average spreads than the single-stock pseudo bonds and the corporate bonds and that the spreads for corporate bonds and single-stock pseudo bonds are at quite similar levels. They also find that, in all instances, the spread implied by the Merton model is much lower than the yield spreads found for the other instruments.

It must be said that it is difficult to provide a meaningful comparison of the single-stock pseudo bonds in Figure 4.15. My result is limited by a small amount of data and few observations on categories between the endpoints of the credit rating scale. However, the remaining results will be discussed in detail. I obtain average spreads that are usually higher on OBX pseudo bonds than corporate bonds. However, they are pretty similar in ratings AAA, AA, and C.

According to Figure 4.15, the Merton model seems to overestimate the default probability for all ratings except C when comparing it with empirical corporate bonds. Nevertheless, a comparison of the spreads from Merton and the corporate bonds is likely to be a comparison of apples and oranges. Neither the data for corporate bonds nor the Merton model are filtered by collateral type or interest type, but the data used in the Merton model is filtered by the debt-to-equity ratio. This yields a much smaller dataset lacking financial firms, which contributes to the largest fraction of the dataset. When attempting to filter the corporate bond dataset on the same debt-to-equity ratio necessary to obtain the meaningful average spreads from Merton, I lose several rating categories completely in the bar chart for average spreads on corporate bonds. Therefore, such a bar chart is not included in the thesis. This illustrates further how ill-suited it is to compare the results for corporate bonds and the Merton model.

This concludes objective 2 (**O2**), which was to replicate Culp et al. (2018)'s figure displaying average yield spreads per credit rating category for pseudo bonds, corporate bonds, and the Merton model. Due to a shortage of stock data and a substantial loss of data when applying the debt-to-equity ratio filter before using the Merton model, objective **O2** could not be achieved fully. Corporate bond data was only available for the COVID-19 pandemic period, which hindered a comparison between U.S. and Norwegian results during the same period.

4.3 Objective 3: A Discussion on Feldhütter and Schaefer (2018)'s Results

In this section, I address the third and last objective of the thesis. As discussed in Chapter 1, Feldhütter and Schaefer (2018) analyze whether the Black & Cox model is able to explain the yield spreads on corporate bonds or not. They do so by comparing the actual and model-implied yield spreads on corporate bonds. As inputs in the Black & Cox model, they use asset volatility, leverage ratio, payout ratio, and recovery ratio of the bond of the issuing firm (Feldhütter and Schaefer, 2018). While Feldhütter and Schaefer (2018) compute spreads on corporate bonds, I already have these provided. Therefore, my approach is not identical to theirs. As they do, I compute default probabilities with a structural model, but instead of Black & Cox, I use the Merton model discussed in Chapter 3. This also means that my input variables are different from theirs.

Nevertheless, I compare the actual spreads for my corporate bonds with those implied by the Merton model. I explain in Chapter 3 how I use Feldhütter and Schaefer (2018)'s rating scheme called *model* to translate the default probabilities implied by the Merton model to credit ratings. I do this to see if their model removes the so-called credit spread puzzle, which Feldhütter and Schaefer (2018) claim not to exist.

In the context of this thesis, I compare Figures 4.10(b) and 4.14, which display the average corporate bond spreads from the unfiltered dataset and the average spreads implied by the Merton model when filtered for high debt-to-equity ratios, respectively. These average spreads correspond to the, respectively, blue and yellow bars in Figure 4.15.

As presented in Chapter 1, the credit spread puzzle refers to how actual yield spreads are much higher than what would be expected based on models such as Merton (Amato and Remolona, 2003; NBIM, 2011). For all ratings above C, the Merton model yields a wider average spread than the observed average spreads on corporate bonds in Figure 4.15. Judging from this alone, the credit spread puzzle does not exist in Norwegian data. On the contrary, the Merton model appears to overestimate the average spreads on all credit ratings except C. However, a large portion of the corporate data had to be filtered away to construct the result for Merton. Therefore, it is unlikely that these results are transferable to the Norwegian market as a whole.

Table 4.5 offers a comparison of average spreads per credit rating between the rating schemes *model* and *actual* when applied to default probabilities from the Merton model. For some rating categories, *actual* is closer to the average spreads observed for corporate bonds. However, it performs slightly worse than *model* for category C. Overall, *model* and *actual* provide quite similar results for average spreads in the Merton model. Therefore, it is unlikely that Feldhütter and Schaefer (2018)'s scheme, *model*, is the reason why I achieve such high average spreads in the Merton model. It must be reiterated that I do not follow the exact steps of Feldhütter and Schaefer (2018), which can be another reason why I obtain different results. However, the main source of error in the Merton analysis is the fact that I have to exclude such a great part of my corporate bond data to get meaningful results.

This concludes objective 3 (**O3**), which was to understand if the credit spread puzzle is present when applying Feldhütter and Schaefer (2018)'s scheme in mapping default probabilities to credit ratings. When comparing average spreads for the Merton model and the empirical corporate bonds, I find the Merton model to overestimate yield spreads for all credit rating categories, but C. This alone would indicate that there is no credit spread puzzle in Norwegian markets. However, as Table 4.5 showed, there is little difference between Moody's and Feldhütter and Schaefer (2018)'s average spreads, which indicates that their rating scheme alone was not the deciding factor in the analysis.

Instead, the analysis was somewhat hampered by the necessity of constructing results for Merton on a different dataset than the full corporate bond dataset. Financial firms were excluded when calculating default probabilities to achieve meaningful results for the Merton model. However, these firms were not removed when finding the average spreads for corporate bonds due to the following loss of several credit rating categories.

Table 4.5: Average spreads per credit rating from the Merton model, with the schemes named actual and model compared.

Credit Rating	*Model*	*Actual*
AAA	76.6	76.3
AA	80.2	99.1
A	104	105
BBB	105	103
BB	129	120
B	158	141
C	179	177

4.4 Potential Sources of Error

4.4.1 Different Markets and Different Periods

In Chapter 3, I have assumed that I would obtain similar results to those of Culp et al. (2018) as I replicate their study on another volatile period. However, my results are different from theirs in several ways. This might be because the COVID-19 pandemic and the financial crisis had different dynamics. As presented in Chapter 2, the increased yield spread lasted much longer during the financial crisis than during COVID-19 in the U.S. (Kozlowski et al., 2021). Assuming the same was the case in Norway, this might explain the more *extreme* behavior I observe in my results for COVID-19, with default probabilities above zero lasting for only one day at a time, and therefore very few observations with credit ratings other than AAA or C. Based on my results, it seems that Culp et al. (2018)'s method is unsuitable for studying pseudo-bonds for any given period.

Another potential reason is that I am considering a different country with its own characteristics. As stated in Chapter 1, there are fewer bonds with ratings in Norway than in the U.S. (OECD, 2022), which implies that a smaller amount of Norwegian bonds will be available to compare with U.S. bonds. Moreover, the Norwegian bond market is smaller than the U.S. market (Rundhaug et al., 2020). Also, OECD (2022) found the Norwegian bond market to be different from U.S. market in terms of industry concentration and issuer composition.

4.4.2 The Interest Rate

As discussed in Chapter 2, interest rates can differ depending on the horizon (Berk and DeMarzo, 2016). Therefore, using NIBOR3M as an interest rate measure can cause inaccuracies in the resulting yield spreads as it only considers a horizon of three months. Most of the time, the bonds in this thesis have a longer time to maturity than three months. Before recessions, yield curves tend to be inverted, giving a higher interest rate for short-term rates than long-term rates (Berk and DeMarzo, 2016). This indicates that NIBOR3M might have been higher than the real interest rates for my instruments. Conversely, NIBOR3M might be too low during and after a recession compared to the maturity-matched interest rate for my instruments. My choice of interest rate in this thesis does not affect the yield spreads for corporate bonds in Figure 4.9, which are provided by Nordic Bond Pricing. The same spreads are applied when finding average yield spreads for the Merton model.

4.4.3 Other Assumptions

As stated in Chapter 3, I use the standard deviation and mean of log asset growth instead of forecasting and using a GARCH model. It is difficult to say how different my results are due to this simplification, but my results are likely less precise because of this choice. Even though many of the corporate bonds have a maturity above two years, I used ratings implied by EDFs for two years. This can also lead to imprecise categorizations of the corporate bonds. Furthermore, I only keep bonds with a debt-to-equity ratio below two to avoid a bias when producing average spreads with the Merton model. I achieve less biased results with this exclusion, but it leaves me with results based on a much smaller dataset, which are unlikely to be comparable with the results for corporate bonds where no debt-to-equity filter is imposed. Modifying Merton's model could possibly mitigate this issue. As previously mentioned, another source of error can be the merging I did of several credit rating categories to match the scheme of Feldhütter and Schaefer (2018).

Chapter 5

Conclusions and Further Works

In this thesis, I replicated parts of Culp et al. (2018)'s approach in their paper named "Option-based credit spreads." I produced OBX pseudo bonds and single-stock pseudo bonds according to their method and compared their spreads with those according to the Merton model and those of empirical corporate bonds. However, I compared the spreads in a period different from the financial crisis due to data availability.

I found that Culp et al. (2018)'s approach for producing index pseudo bonds is quite transferable to Norwegian data, especially when conducted during the financial crisis. However, the results became somewhat less fruitful when the method was applied for the COVID-19 period. Moreover, I found that the average spreads on pseudo bonds were impacted by very few observations on some credit rating categories. This was the case for both OBX pseudo bonds and single-stock pseudo bonds and resulted from the default probabilities' dichotomous movement with spikes usually lasting only one day.

In addition to the COVID-19 pandemic, I considered OBX pseudo bond spreads during the financial crisis. When doing this, I obtained a more even distribution between the different credit rating categories. This improved the reliability of the resulting average spreads. Therefore, it appears that the main source of atypical results for single-stock pseudo bonds was the lack of data, while for the OBX index pseudo bonds, it was the time period in question.

Furthermore, the choice of leverage values proved to be challenging when studying single-stock pseudo bonds, as the resulting default probability was surprisingly sensitive to minimal changes in leverage. I excluded bonds with a high debt-to-equity ratio to obtain meaningful average spreads for the Merton model. Considering that the main portion of bond issuers in the dataset are in the financial sector, I lost a significant portion of the data. Therefore, it is unlikely that the results for Merton are directly comparable with the average corporate bond spreads, which are based on the whole dataset. That being said, it is still interesting to observe how the Merton model yields higher average yield spreads than those for corporate bonds, which does not suggest the existence of a credit spread puzzle as suggested by Feldhütter and Schaefer (2018) in their paper named “The myth of the credit spread puzzle.”

As presented in Chapter 1, one of the contributions of this thesis is filling a literature gap in the field of credit risk. More specifically, it furthers a discussion started by Culp et al. (2018) on pseudo bonds and pseudo firms. Moreover, the thesis offers a critical review of the replicability of Culp et al. (2018)’s methodology. Additionally, it provides a set of findings about the Norwegian fixed income market, on which there is currently little research (Rundhaug et al., 2020).

Further works might include: (a) repeating this study on U.S. data for the COVID-19 pandemic to see if default probabilities become equally bursty as for Norwegian data. Using U.S. data becomes especially necessary when studying single-stock pseudo bonds, as the availability of single-stock options in Norway is limited; (b) creating a more accurate measurement of volatility and risk-free interest rate appears necessary as it is likely to be a source of inaccuracy in my analysis; and (c) creating a modified version of the Merton model that can create meaningful results for financial firms with high debt-to-equity ratios.

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Appendix

This appendix contains the code in R language used to obtain the results presented in the thesis.

```
############# IMPORTING PACKAGES #############

library(readxl)
library(tidyverse)
library(dplyr)
library(zoo)
library(ggplot2)
library(tidyr)
library(data.table)
library(lubridate)
library(foreach)
library(doParallel)
library(iterators)

############# DATA TREATMENT #############

# Importing dataset OBX
impliedVols <- read_xlsx("impliedVols.xlsx")

# Convert 'Dates' column to Date type
impliedVols$Dates <- as.Date(impliedVols$Dates)

# Convert 'PX_LAST' column to numeric type
impliedVols$PX_LAST <- as.numeric(impliedVols$PX_LAST)
# NAs get replaced with the previous value
impliedVols$PX_LAST <- na.locf(impliedVols$PX_LAST, na.rm = FALSE)

# Convert 'HIST_PUT_IMP_VOL' column to numeric type
```

```r
impliedVols$HIST_PUT_IMP_VOL <- as.numeric(impliedVols$HIST_PUT_IMP_VOL)
# NAs get replaced with the previous value
impliedVols$HIST_PUT_IMP_VOL <- na.locf(impliedVols$HIST_PUT_IMP_VOL, na.rm = FALSE)

# Convert 'NIBOR3M' column to numeric type
impliedVols$NIBOR3M <- as.numeric(impliedVols$NIBOR3M)
# NAs get replaced with the previous value
impliedVols$NIBOR3M <- na.locf(impliedVols$NIBOR3M, na.rm = FALSE)

# NAs DNB
impliedVols$DNB_HIST_PUT_IMP_VOL <- na.locf(impliedVols$DNB_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$DNB_PX_LAST <- na.locf(impliedVols$DNB_PX_LAST, na.rm = FALSE)

# NAs EQNR
impliedVols$EQNR_HIST_PUT_IMP_VOL <- na.locf(impliedVols$EQNR_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$EQNR_PX_LAST <- na.locf(impliedVols$EQNR_PX_LAST, na.rm = FALSE)

# NAs NHY
impliedVols$NHY_HIST_PUT_IMP_VOL <- na.locf(impliedVols$NHY_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$NHY_PX_LAST <- na.locf(impliedVols$NHY_PX_LAST, na.rm = FALSE)

# NAs ORK
impliedVols$ORK_HIST_PUT_IMP_VOL <- na.locf(impliedVols$ORK_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$ORK_PX_LAST <- na.locf(impliedVols$ORK_PX_LAST, na.rm = FALSE)

# NAs STB
impliedVols$STB_HIST_PUT_IMP_VOL <- na.locf(impliedVols$STB_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$STB_PX_LAST <- na.locf(impliedVols$STB_PX_LAST, na.rm = FALSE)
```

```
# NAs TEL
impliedVols$TEL_HIST_PUT_IMP_VOL <- na.locf(impliedVols$TEL_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$TEL_PX_LAST <- na.locf(impliedVols$TEL_PX_LAST, na.rm = FALSE)

# NAs YAR
impliedVols$YAR_HIST_PUT_IMP_VOL <- na.locf(impliedVols$YAR_HIST_PUT_IMP_VOL, na.rm = FALSE)
# NAs get replaced with the previous value
impliedVols$YAR_PX_LAST <- na.locf(impliedVols$YAR_PX_LAST, na.rm = FALSE)

############# DEFINING VARIABLES AND CONSTANTS #############

# The time interval for plots is 28/4/2007 - 18/12/2009.
# BUT: YYYY-MM-DD
# For financial crisis
#start_date <- as.Date("2007-04-28")
#end_date <- as.Date("2009-12-18") # maturity date

# For covid pandemic
start_date <- as.Date("2019-01-01")
end_date <- as.Date("2022-12-30")

# We get our subset time period
subset <- impliedVols[start_date <= impliedVols$Dates & impliedVols$Dates <= end_date, ]

# Parameters financial crisis.
#K1 <- 200 # LOW LEVERAGE
#K2 <- 320 # HIGH LEVERAGE

# Parameters covid pandemic.
K1 <- 850
K2 <- 1000

############# FUNCTION FOR FINDING PUT PRICE #############
BSPutPrice <- function(S, K, T, r, sigma) {
  # Ensure all inputs are numeric to avoid type issues
  S <- as.numeric(S)
  K <- as.numeric(K)
  T <- as.numeric(T)
```

```r
  r <- as.numeric(r)
  sigma <- as.numeric(sigma)

  # Calculate d1 and d2 using vector operations
  d1 <- (log(S / K) + (r + sigma^2 / 2) * T) / (sigma * sqrt(T))
  d2 <- d1 - sigma * sqrt(T)

  # Calculate put prices using vector operations
  put_prices <- K * exp(-r * T) * pnorm(-d2) - S * pnorm(-d1)

  # Return the computed put prices
  return(put_prices)
}

############# FUNCTION YTM ZERO-COUPON BOND #############
calculate_ytm_zero_coupon <- function(price, face_value, years_to_maturity) {
  ytm <- (face_value / price)^(1 / years_to_maturity) - 1
  return(ytm)
}

############# FUNCTION EXTRAPOLATING CREDIT RATINGS ############
extrapolateCreditRatingsBelowOneYear <- function(df) {
  one_year_probs <- df[df$maturity == 1, -1]  # Extract 1-year probabilities
  two_year_probs <- df[df$maturity == 2, -1]  # Extract 2-year probabilities

  slopes <- (two_year_probs - one_year_probs) / (2 - 1)  # Calculate slopes for extrapolation

  # Include 0 in the sequence of extrapolated maturities
  extrapolated_maturities <- seq(0.0, 0.9, by = 0.1)
  extrapolated_df <- data.frame(maturity = extrapolated_maturities)

  for (rating in colnames(one_year_probs)) {
    intercept <- as.numeric(one_year_probs[[rating]]) - as.numeric(slopes[[rating]]) * 1
    # Calculate extrapolated probabilities
    extrapolated_probs <- as.numeric(slopes[[rating]]) * extrapolated_maturities + intercept

    # Ensure no negative probabilities: replace negatives with zero
    extrapolated_probs <- ifelse(extrapolated_probs < 0, 0, extrapolated_probs)
```

```
      extrapolated_df[[rating]] <- extrapolated_probs
    }

    return(extrapolated_df)
}

############# Plot NIBOR3M ########
ggplot(impliedVols, aes(x = Dates, y = NIBOR3M)) +
  geom_line(color = "blue", size = 0.8) +
  labs(x = "Dates",
       y = "NIBOR3M (%)") +
  theme_minimal() +
  theme(
    plot.title = element_text(hjust = 0.5, size = 16),
    axis.text.x = element_text(angle = 45, hjust = 1, size = 10),
    axis.text.y = element_text(size = 10),
    axis.title = element_text(size = 12)
  ) +
  scale_x_date(date_breaks = "3 year", date_labels = "%Y")

############# Plot HIST_PUT_IMP_VOL ######
ggplot(impliedVols, aes(x = Dates, y = HIST_PUT_IMP_VOL)) +
  geom_line(color = "blue", size = 0.8) +
  labs(x = "Dates",
       y = "Implied Volatility (%)") +
  theme_minimal() +
  theme(
    plot.title = element_text(hjust = 0.5, size = 16),
    axis.text.x = element_text(angle = 45, hjust = 1, size = 10),
    axis.text.y = element_text(size = 10),
    axis.title = element_text(size = 12)
  ) +
  scale_x_date(date_breaks = "3 year", date_labels = "%Y")

############# FIG 1 PANEL A: PRICES #############

# WE FIND THE PUT OPTION VALUES OF THE INDEX

# Apply BSPutPrice function for K1 and K2 over the subset time period
put_prices_K1 <- BSPutPrice(subset$PX_LAST, K1, (end_date - subset$
    Dates)/365, subset$NIBOR3M /100, subset$HIST_PUT_IMP_VOL /100)
put_prices_K2 <- BSPutPrice(subset$PX_LAST, K2, (end_date - subset$
    Dates)/365, subset$NIBOR3M /100, subset$HIST_PUT_IMP_VOL /100)

```

```r
# Create time series data frames for these two
ppK1 <- data.frame(Dates = subset$Dates, PutPrice_K1 = put_prices_K1)
ppK2 <- data.frame(Dates = subset$Dates, PutPrice_K2 = put_prices_K2)

# Calculate K * exp(-r * t) for K1 and K2
discounted_prices_K1 <- K1 * exp(-subset$NIBOR3M /100 * as.numeric((end_date - subset$Dates)/365))
discounted_prices_K2 <- K2 * exp(-subset$NIBOR3M /100 * as.numeric((end_date - subset$Dates)/365))

# Create time series data frames for these two
dpK1 <- data.frame(Dates = subset$Dates, DiscPrice_K1 = discounted_prices_K1)
dpK2 <- data.frame(Dates = subset$Dates, DiscPrice_K2 = discounted_prices_K2)

# Create new time series based on the formula K * exp(-r * t) - put price
# Low leverage bond value
bond_value_K1 <- data.frame(Dates = subset$Dates, LowLevBond_K1 = (discounted_prices_K1 - put_prices_K1))
# High leverage bond value
bond_value_K2 <- data.frame(Dates = subset$Dates, HighLevBond_K2 = (discounted_prices_K2 - put_prices_K2))

# Low leverage bond value percent of principal
percent_principal_K1 <- data.frame(Dates = subset$Dates, Percent_of_principal_K1 = (bond_value_K1$LowLevBond_K1 / K1)*100)
# High leverage bond value percent of principal
percent_principal_K2 <- data.frame(Dates = subset$Dates, Percent_of_principal_K2 = (bond_value_K2$HighLevBond_K2 / K2)*100)

# Make a new dataframe for dates and OBX index values for our subset time period
OBX_df <- subset[,1:2]

# Setter inn forrige verdi for NA values
OBX_df <- OBX_df %>%
  mutate(PX_LAST = ifelse(PX_LAST == "#N/A N/A", NA, PX_LAST),
         PX_LAST = na.locf(PX_LAST, na.rm = FALSE))
OBX_df$PX_LAST <- as.numeric(OBX_df$PX_LAST)

percent_principal_K1 <- percent_principal_K1 %>%
  mutate(Percent_of_principal_K1 = na.locf(Percent_of_principal_K1, na.rm = FALSE))
```

```

percent_principal_K2 <- percent_principal_K2 %>%
  mutate(Percent_of_principal_K2 = na.locf(Percent_of_principal_K2, na.rm = FALSE))

bond_value_K1 <- bond_value_K1 %>%
  mutate(LowLevBond_K1 = na.locf(LowLevBond_K1, na.rm = FALSE))

bond_value_K2 <- bond_value_K2 %>%
  mutate(HighLevBond_K2 = na.locf(HighLevBond_K2, na.rm = FALSE))

# Collecting all data in one data frame
dfs <- list(OBX_df, percent_principal_K1, percent_principal_K2, bond_value_K1, bond_value_K2)

# Merging multiple data frames on "Dates" column
merged_df <- Reduce(function(x, y) merge(x, y, by = "Dates", all = TRUE), dfs)

df <- merged_df

# Assuming df is your dataframe
df$Dates <- as.Date(df$Dates)  # Ensure Dates column is Date type

# Determine the scale factor for the secondary axis
primary_max <- max(df$Percent_of_principal_K1, df$Percent_of_principal_K2, na.rm = TRUE)
secondary_max <- max(df$PX_LAST, na.rm = TRUE)
scale_factor <- primary_max / secondary_max

# Plot
ggplot(df, aes(x = Dates)) +
  geom_line(aes(y = Percent_of_principal_K1, color = "LLPB")) +
  geom_line(aes(y = Percent_of_principal_K2, color = "HLPB")) +
  geom_line(aes(y = PX_LAST * scale_factor, color = "PX_LAST")) +
  scale_color_manual(values = c("LLPB" = "blue", "HLPB" = "red")) +
  scale_y_continuous(name = "Bond Price (Percent of principal)",
                     sec.axis = sec_axis(~ . / scale_factor, name = "OBX Index")) +
  labs(y = "Percent", color = "Leverage") +  # Adjusted label for legend
  theme_minimal()

#Test
df$KoverB_K1 <- K1 / df$LowLevBond_K1
```

```
df$KoverB_K2 <- K2 / df$HighLevBond_K2

# Plot
ggplot(df, aes(x = Dates)) +
  geom_line(aes(y = KoverB_K1, color = "K1")) +
  geom_line(aes(y = KoverB_K2, color = "K2")) +
  scale_color_manual(values = c("K1" = "blue", "K2" = "red")) +
  labs(y = "K/B ratio", color = "Leverage") +  # Adjusted label for legend
  theme_minimal()

############# FIG 1 PANEL B: LEVERAGE RATIOS #############
# Panel B reports market leverage of the two pseudo firms, L = K/A, in percentage terms.
# Where B = leverage value and A = OBX
# Make two new dataframes, and then merge them with the df dataframe

# Low leverage
market_leverage_K1 <- data.frame(Dates = subset$Dates, Market_Leverage_K1 = ((K1 / df$PX_LAST)*100))

# High leverage
market_leverage_K2 <- data.frame(Dates = subset$Dates, Market_Leverage_K2 = ((K2 / df$PX_LAST)*100))

# MERGE
# Collecting all data in one data frame
dfs <- list(OBX_df, bond_value_K1, bond_value_K2, market_leverage_K1, market_leverage_K2)

# Merging multiple data frames on "Dates" column
merged_df <- Reduce(function(x, y) merge(x, y, by = "Dates", all = TRUE), dfs)

df <- merged_df

# Plot

# Create the plot with Market_Leverage_K1 and Market_Leverage_K2 on the y-axis
ggplot(df, aes(x = Dates)) +
  geom_line(aes(y = Market_Leverage_K1, color = "LLPB")) +
  geom_line(aes(y = Market_Leverage_K2, color = "HLPB")) +
  scale_color_manual(values = c("LLPB" = "blue", "HLPB" = "red")) +
  labs(y = "Percent", color = "Leverage") +
```

```r
  theme_minimal()

############# FIG 1 PANEL C: CREDIT SPREADS #####################

df$ytm_K1 <- calculate_ytm_zero_coupon(df$LowLevBond_K1, K1, as.numeric(end_date - subset$Dates)/365)
df$ytm_K2 <- calculate_ytm_zero_coupon(df$HighLevBond_K2, K2, as.numeric(end_date - subset$Dates)/365)

df$spread_K1 <- (df$ytm_K1 - (subset$NIBOR3M / 100))*100
df$spread_K2 <- (df$ytm_K2 - (subset$NIBOR3M / 100))*100

# Plot the credit spreads
ggplot(df, aes(x = Dates)) +
  geom_line(aes(y = spread_K1, color = "LLPB")) +
  geom_line(aes(y = spread_K2, color = "HLPB")) +
  scale_color_manual(values = c("LLPB" = "blue", "HLPB" = "red")) +
  labs(y = "Credit Spread (%)", color = "Leverage") +
  theme_minimal()

############# FIG 1 PANEL D: EX ANTE DEFAULT PROBABILITIES #############

# FINAL VALUE, A_T+TAU
final_obx_value <- df[df$Dates == end_date, "PX_LAST"]

# LOG ASSET GROWTH = LOG(A_T+TAU / A_T)
df$log_asset_growth <- log(final_obx_value/ df$PX_LAST)

# HISTORICAL MEAN OF LOG ASSET GROWTH
mean_log_asset_growth <- mean(df$log_asset_growth, na.rm = TRUE)  # na.rm = TRUE removes NA values before calculation

# HISTORICAL STANDARD DEVIATION OF LOG ASSET GROWTH
stdev_log_asset_growth <- sd(df$log_asset_growth, na.rm = TRUE)

# HISTORY OF SHOCKS
df$shocks <- (df$log_asset_growth - mean_log_asset_growth) / stdev_log_asset_growth

# FROM EQUATION 10
# LEVERAGE RATIO LOW LEVERAGE K1
df$leverage_ratio_K1 <- K1 / df$PX_LAST

# LEVERAGE RATIO HIGH LEVERAGE K2
```

```r
df$leverage_ratio_K2 <- K2 / df$PX_LAST

# EQUATION 11
df$XK1 <- (log(df$leverage_ratio_K1) - mean_log_asset_growth) / stdev_log_asset_growth
df$XK2 <- (log(df$leverage_ratio_K2) - mean_log_asset_growth) / stdev_log_asset_growth

# ESTIMATE EMPIRICAL DEFAULT PROBABILITIES

# Initialize vectors to store the default probabilities for each time point
df$default_prob_K1_expanding <- rep(NA, nrow(df))
df$default_prob_K2_expanding <- rep(NA, nrow(df))

# Loop through each row in the dataframe to calculate default probabilities using only data up to that point
for(i in 1:nrow(df)) {
  # For each time t, calculate the proportion of shocks up to time t that are less than XK1 and XK2 at time t
  if (i > 1) { # Ensure there is at least one prior data point to calculate the mean
    df$default_prob_K1_expanding[i] <- mean(df$shocks[1:i] < df$XK1[i], na.rm = TRUE)
    df$default_prob_K2_expanding[i] <- mean(df$shocks[1:i] < df$XK2[i], na.rm = TRUE)
  } else {
    # For the first row, default probabilities can't be calculated due to lack of prior data
    df$default_prob_K1_expanding[i] <- NA
    df$default_prob_K2_expanding[i] <- NA
  }
}

# Replace NAs with the next value
df$default_prob_K1_expanding <- na.locf(df$default_prob_K1_expanding, fromLast = TRUE, na.rm = FALSE)
df$default_prob_K2_expanding <- na.locf(df$default_prob_K2_expanding, fromLast = TRUE, na.rm = FALSE)

# Multiply by 100 to get percentages
df$default_prob_K1_expanding_percent <- df$default_prob_K1_expanding * 100
df$default_prob_K2_expanding_percent <- df$default_prob_K2_expanding * 100
```

```

# Plot
ggplot(df, aes(x = Dates)) +
  geom_line(aes(y = default_prob_K1_expanding_percent, color = "LLPB")) +
  geom_line(aes(y = default_prob_K2_expanding_percent, color = "HLPB")) +
  scale_color_manual(values = c("LLPB" = "blue", "HLPB" = "red")) +
  labs(y = "Percent", color = "Leverage") +
  theme_minimal()

############# Figure 2 Panel A #############################
############# CREDIT RATINGS FROM TABLE 8 FELDHÜTTER & SCHAEFER #####################

# Define the specific maturities
maturity <- c(1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20)

# Model Default Rates for "AAA"
model_AAA <- c(0.0001, 0.0004, 0.0010, 0.0016, 0.0022, 0.0028, 0.0041, 0.0054, 0.0067, 0.0087, 0.0118)

# Actual Default Rates for "AAA"
actual_AAA <- c(0.00, 0.0001, 0.0003, 0.0009, 0.0017, 0.0025, 0.0052, 0.0087, 0.0116, 0.0138, 0.0171)

# Model Default Rates for "AA"
model_AA <- c(0.0001, 0.0008, 0.0019, 0.0031, 0.0045, 0.0059, 0.0087, 0.0116, 0.0145, 0.0189, 0.0258)

# Actual Default Rates for "AA"
actual_AA <- c(0.0007, 0.0022, 0.0035, 0.0054, 0.0083, 0.0117, 0.0183, 0.0250, 0.0334, 0.0452, 0.0558)

# Model Default Rates for "A"
model_A <- c(0.0002, 0.0014, 0.0034, 0.0060, 0.0091, 0.0126, 0.0200, 0.0277, 0.0355, 0.0467, 0.0637)

# Actual Default Rates for "A"
actual_A <- c(0.0010, 0.0031, 0.0064, 0.0099, 0.0138, 0.0178, 0.0266, 0.0362, 0.0461, 0.0599, 0.0793)

# Model Default Rates for "BBB"
model_BBB <- c(0.0030, 0.0095, 0.0178, 0.0270, 0.0364, 0.0457, 0.0637, 0.0804, 0.0954, 0.1153, 0.1424)
```

```
# Actual Default Rates for "BBB"
actual_BBB <- c(0.0029, 0.0086, 0.0154, 0.0229, 0.0310, 0.0390, 0.0546, 0.0711, 0.0872, 0.1087, 0.1376)

# Model Default Rates for "BB"
model_BB <- c(0.0223, 0.0561, 0.0876, 0.1154, 0.1399, 0.1615, 0.1978, 0.2272, 0.2515, 0.2809, 0.3174)

# Actual Default Rates for "BB"
actual_BB <- c(0.0135, 0.0327, 0.0546, 0.0775, 0.0992, 0.1197, 0.1571, 0.1927, 0.2247, 0.2665, 0.3206)

# Model Default Rates for "B"
model_B <- c(0.0843, 0.1640, 0.2239, 0.2704, 0.3078, 0.3386, 0.3865, 0.4224, 0.4505, 0.4831, 0.5217)

# Actual Default Rates for "B"
actual_B <- c(0.0380, 0.0871, 0.1372, 0.1816, 0.2206, 0.2554, 0.3141, 0.3589, 0.3958, 0.4422, 0.4914)

# Model Default Rates for "C"
model_C <- c(0.2332, 0.3611, 0.4379, 0.4907, 0.5299, 0.5605, 0.6057, 0.6380, 0.6624, 0.6900, 0.7217)

# Actual Default Rates for "C"
actual_C <- c(0.1402, 0.2381, 0.3121, 0.3686, 0.4140, 0.4478, 0.4963, 0.5388, 0.5802, 0.6376, 0.7134)

# Create DataFrames
model_df <- data.frame(maturity, AAA = model_AAA, AA = model_AA, A = model_A, BBB = model_BBB,
                       BB = model_BB, B = model_B, C = model_C)
actual_df <- data.frame(maturity, AAA = actual_AAA, AA = actual_AA, A = actual_A,
                        BBB = actual_BBB, BB = actual_BB, B = actual_B, C = actual_C)

############# EXTRAPOLATED FOR LESS THAN 1 YEAR #########
# Extrapolate ratings for maturity less than 1 year
extrapolated_model_ratings <- extrapolateCreditRatingsBelowOneYear(model_df)
extrapolated_actual_ratings <- extrapolateCreditRatingsBelowOneYear(actual_df)

```

```
# Merge the dataframes so we are left with only two dataframes again
# Append extrapolated values to the original dataframes
model_df_with_extrapolation <- rbind(model_df, extrapolated_model_
    ratings)
actual_df_with_extrapolation <- rbind(actual_df, extrapolated_actual_
    ratings)

# Order the combined dataframe by maturity
model_df_with_extrapolation <- model_df_with_extrapolation[order(model_
    df_with_extrapolation$maturity), ]
actual_df_with_extrapolation <- actual_df_with_extrapolation[order(
    actual_df_with_extrapolation$maturity), ]

################### OBX MAPPING #################################

# FOR K1
df1 <- subset(df, spread_K1 > 0)
test1 <- df1
test1$years_to_mat <- as.numeric((end_date - df1$Dates)/365)
test1 <- test1[c("Dates", "spread_K1","default_prob_K1_expanding","
    years_to_mat")]

# Function to find the closest maturity index
find_closest_maturity <- function(maturity, maturities) {

  which.min(abs(maturities - maturity))

}

# Function to find the closest default frequency and return the
    corresponding column name
find_closest_frequency <- function(frequency, frequencies) {

  names(frequencies)[which.min(abs(frequencies - frequency))]

}

# Map the values
map_values_K1 <- function(external_df, model_df) {

  results <- vector("character", nrow(external_df))

  for (i in seq_len(nrow(external_df))) {

    row <- external_df[i, ]
```

```

    closest_idx <- find_closest_maturity(row$years_to_mat, model_df$maturity)

    # Extract the row in model_df for the closest maturity

    maturity_row <- model_df[closest_idx, -1]  # Exclude the 'maturity' column

    # Find the closest frequency column

    results[i] <- find_closest_frequency(row$default_prob_K1_expanding, maturity_row)

  }

  external_df$Mapped_Column <- results

  return(external_df)

}

# Run the mapping
mapped_df1 <- map_values_K1(test1, model_df_with_extrapolation)

# Calculate the average spread for each Mapped_Column, including count of each group
average_spread_obx_K1 <- mapped_df1 %>%
  group_by(Mapped_Column) %>%
  dplyr::summarise(Average_Spread = mean(spread_K1 * 10, na.rm = TRUE))

# Convert Mapped_Column to a factor with levels in the specified order
average_spread_obx_K1$Mapped_Column <- factor(average_spread_obx_K1$Mapped_Column,
                                        levels = c("AAA", "AA", "A", "BBB", "BB", "B", "C"))

# Create a bar chart with ggplot2, now with the ordered factor
ggplot(average_spread_obx_K1, aes(x = Mapped_Column, y = Average_Spread, fill = Mapped_Column)) +
  geom_bar(stat = "identity", position = "dodge", color = "black", show.legend = FALSE) +  # Adding the black outline
  theme_minimal() +
  labs(x = "Credit Rating", y = "Yield Spread (bps)") +
  scale_fill_brewer(palette = "Paired") +
```

```r
  theme(axis.text.x = element_text(angle = 45, hjust = 1))

# FOR K2
df2 <- subset(df, spread_K2 > 0)
test2 <- df2
test2$years_to_mat <- as.numeric((end_date - df2$Dates)/365)
test2 <- test2[c("Dates", "spread_K2","default_prob_K2_expanding","years_to_mat")]

# Map the values
map_values_K2 <- function(external_df, model_df) {
  results <- vector("character", nrow(external_df))
  for (i in seq_len(nrow(external_df))) {
    row <- external_df[i, ]
    closest_idx <- find_closest_maturity(row$years_to_mat, model_df$maturity)
    # Extract the row in model_df for the closest maturity
    maturity_row <- model_df[closest_idx, -1]  # Exclude the 'maturity' column
    # Find the closest frequency column
    results[i] <- find_closest_frequency(row$default_prob_K2_expanding, maturity_row)
  }
  external_df$Mapped_Column <- results
  return(external_df)
}

# Run the mapping
mapped_df2 <- map_values_K2(test2, model_df_with_extrapolation)

# Calculate the average spread for each Mapped_Column, including count of each group
average_spread_obx_K2 <- mapped_df2 %>%
  group_by(Mapped_Column) %>%
  dplyr::summarise(Average_Spread = mean(spread_K2 * 10, na.rm = TRUE))

average_spread_obx_K2$Mapped_Column <- factor(average_spread_obx_K2$Mapped_Column,
                                                                levels = c("AAA", "AA", "A", "BBB", "BB", "B", "C"))

# Create a bar chart with ggplot2, now with the ordered factor
ggplot(average_spread_obx_K2, aes(x = Mapped_Column, y = Average_Spread, fill = Mapped_Column)) +
```

```
  geom_bar(stat = "identity", position = "dodge", color = "black", show.legend = FALSE) +  # Adding the black outline
  theme_minimal() +
  labs(x = "Credit Rating", y = "Yield Spread (bps)") +
  scale_fill_brewer(palette = "Paired") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))

# Finally looking at the average between K1 and K2
together <- bind_rows(average_spread_obx_K2, average_spread_obx_K1)

grouped <- together %>%
  group_by(Mapped_Column) %>%
  summarise(Average_Spread = mean(Average_Spread, na.rm = TRUE))

ggplot(grouped, aes(x = Mapped_Column, y = Average_Spread, fill = Mapped_Column)) +
  geom_bar(stat = "identity", position = "dodge", color = "black", show.legend = FALSE) +  # Adding the black outline
  theme_minimal() +
  labs(x = "Credit Rating", y = "Yield Spread (bps)") +
  scale_fill_brewer(palette = "Paired") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))

# counting the number of observed credit ratings per category
# low-leverage
rating_counts_K1 <- mapped_df1 %>%
  group_by(Mapped_Column) %>%
  summarise(Count = n())
# high-leverage
rating_counts_K2 <- mapped_df2 %>%
  group_by(Mapped_Column) %>%
  summarise(Count = n())
#Average
combined_counts <- full_join(rating_counts_K1, rating_counts_K2, by = "Mapped_Column", suffix = c("_mapped_df1", "_mapped_df2")) %>%
  mutate(AvgCount = rowMeans(select(., starts_with("Count_")), na.rm = TRUE))

############# CORPORATE BONDS ###################

############# Importing data, data treatment, filtering ######

# Whole bond data
BondData <- read_delim("bond_data_full.csv", delim = ";", escape_double = FALSE, trim_ws = TRUE)
```

```r

# Removing unused columns
BondData <- BondData %>%
  select(-c(TKR, ISSUERISIN, PRICE_DATE, EDF1, ISSUED_VOLUME, EDFDATE, LCT, DLTT, DLC, EDF3, EDF4, EDF5, CURRENCY,
            SHORT_NAME, DURATION, SPREADDURATION, MODIFIEDDURATION, SEC_NAME,
            CCGMMIS1, CCGMMIS3, CCGMMIS4, CCGMMIS5, COMPANY_NAME))  # the '-' sign indicates you want to drop these columns

# Make SPREAD column numerical values
BondData$SPREAD <- as.numeric(BondData$SPREAD)

# Remove any NA
BondData <- na.omit(BondData)

# Make the dates into date type
BondData$MAPDATE <- as.character(BondData$MAPDATE)
BondData$MAPDATE <- as.Date(BondData$MAPDATE, format="%Y%m%d")

# Ordered
BondData <- arrange(BondData, MAPDATE)

start_date <- as.Date("2019-01-01")
end_date <- as.Date("2022-12-30")

# Correct time period
BondData <- BondData[start_date <= BondData$MAPDATE & BondData$MAPDATE <= end_date, ]

# Same for issue date and maturity date.
# The gsub functio replaces any occurrences of "." or "/" with "-" in the date columns
BondData$MATURITY_DT <- gsub("[./]", "-", BondData$MATURITY_DT)
BondData$ISSUE_DT <- gsub("[./]", "-", BondData$ISSUE_DT)

# Make the dates in date format with yyyy-mm-dd format
BondData$MATURITY_DT <- mdy(BondData$MATURITY_DT)
BondData$ISSUE_DT <- mdy(BondData$ISSUE_DT)

# Find extreme values for SPREAD
spread_quantiles <- quantile(BondData$SPREAD,c(0.05,0.95), na.rm = TRUE)
bottom_quantile_spread <- spread_quantiles[1]
top_quantile_spread <- spread_quantiles[2]
```

```r

# winsorize the right tail at 95% and the bottom tail at zero
BondData <- subset(BondData, SPREAD < top_quantile_spread & SPREAD > 0)

# Add r and sigma into my other dataset as well
BondData <- BondData %>%
  left_join(impliedVols %>% select(Dates, NIBOR3M, HIST_PUT_IMP_VOL), by = c("MAPDATE" = "Dates"))

BondData$Maturity <- (BondData$MATURITY_DT - BondData$MAPDATE)/365
BondData$Maturity <- as.numeric(BondData$Maturity)

#BondDataTimeFilter <- BondData %>%
#  filter(Maturity >= 2)

# Time difference
# BondData$time_difference_days <- BondData$MATURITY_DT - BondData$ISSUE_DT

# Filter the dataframe for maturities 2 years or longer
#BondDataTimeFilter <- BondData %>%
#  filter(time_difference_days >= 2 * 365.25)

# Apply filter for collateral_type equal to 9
BondDataCollateralFilter <- BondData %>%
  filter(FK_COLLATERAL_TYPE == 9)

# Apply additional filter for interest type = fixed
BondDataCollateralInterestFilter <- BondDataCollateralFilter %>%
  filter(INTEREST_TYPE == "Fixed")

############# Translating from Moody's to S&P #################

moodys_to_sp <- c(Aaa = "AAA", Aa1 = "AA+", Aa2 = "AA", Aa3 = "AA-",
               A1 = "A+", A2 = "A", A3 = "A-", Baa1 = "BBB+", Baa2 = "BBB",
               Baa3 = "BBB-", Ba1 = "BB+", Ba2 = "BB", Ba3 = "BB-",
               B1 = "B+", B2 = "B", B3 = "B-", Caa1 = "CCC+", Caa2 = "CCC",
               Caa3 = "CCC-", Ca = "CC", C = "C")

# Whole dataset
BondData$sp_rating <- moodys_to_sp[BondData$CCGMMIS2]

# With fixed and floating interest
```

```
BondDataCollateralFilter$sp_rating <- moodys_to_sp[BondDataCollateralFilter$CCGMMIS2]

# With fixed interest only
BondDataCollateralInterestFilter$sp_rating <- moodys_to_sp[BondDataCollateralInterestFilter$CCGMMIS2]

##### Translating from S&P to Feldhütters range of ratings  #####
# I do this by giving the equivalent rating without the + or - sign.
# CCC and CC falls down to C

sp_to_feldhutter <- c("AAA" = "AAA", "AA+" = "AA", "AA-" = "AA", "AA" = "AA", "A+" = "A",
                      "A-" = "A", "A" = "A", "BBB" = "BBB", "BBB+" = "BBB", "BBB-" = "BBB",
                      "BB+" = "BB","BB-" = "BB", "BB" = "BB", "B" = "B","B+" = "B","B-" = "B",
                      "CCC+" = "C","CCC" = "C", "CCC-" = "C", "CC" = "C", "C" = "C")

# Whole dataset
BondData$feldhutter_rating <- sp_to_feldhutter[BondData$sp_rating]

# With fixed and floating interest
BondDataCollateralFilter$feldhutter_rating <- sp_to_feldhutter[BondDataCollateralFilter$sp_rating]

# With fixed interest only
BondDataCollateralInterestFilter$feldhutter_rating <- sp_to_feldhutter[BondDataCollateralInterestFilter$sp_rating]

############# Separating each rating, finding average spread, plotting moody rating and spread #########

# Find the average spreads for each rating category

# With fixed and floating & with all collateral types
average_spreads <- BondData %>%
  group_by(feldhutter_rating) %>%
  summarize(AverageSpread = mean(SPREAD, na.rm = TRUE))

# Fixed and floating interest
#average_spreads <- BondDataCollateralFilter %>%
#  group_by(feldhutter_rating) %>%
#  summarize(AverageSpread = mean(SPREAD, na.rm = TRUE))
```

```

# Fixed interest only
#average_spreads <- BondDataCollateralInterestFilter %>%
#  group_by(feldhutter_rating) %>%
#  summarize(AverageSpread = mean(SPREAD, na.rm = TRUE))

# Order from highest to lowest rating
average_spreads$feldhutter_rating <- factor(average_spreads$feldhutter_rating, levels = c("AAA", "AA", "A", "BBB", "BB", "B", "CCC", "CC", "C"))

# What i plot here is the Moody's ratings translated to the categories that Feldhutter use
# I do not use Feldhutter's scheme. I am simply providing the same categories for comparison reasons.
# Plot
ggplot(average_spreads, aes(x = feldhutter_rating, y = AverageSpread, fill = feldhutter_rating)) +
  geom_bar(stat = "identity", color = "black", show.legend = FALSE) + # Prevents the legend from being shown
  theme_minimal() +
  labs(x = "Credit Rating", y = "Yield Spread (bps)") +
  scale_fill_brewer(palette = "Paired") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))

# counting the number of observed credit ratings per category
rating_counts_corp <- BondData %>%
  group_by(feldhutter_rating) %>%
  summarise(Count = n())

############# LOG-NORMAL MERTON SPREADS ####################

# Removing too high debt instances
BondDataMerton <- subset(BondData, (LT / MKTCAP) < 2)

# d1 and d2
BondDataMerton$d1 <- (log((BondDataMerton$MKTCAP + BondDataMerton$LT)/BondDataMerton$LT) + ((BondDataMerton$NIBOR3M / 100) + 0.5 * (BondDataMerton$HIST_PUT_IMP_VOL /100 )^2 ) * BondDataMerton$Maturity) / ((BondDataMerton$HIST_PUT_IMP_VOL /100 ) * sqrt(BondDataMerton$Maturity))
BondDataMerton$d2 <- BondDataMerton$d1 - (BondDataMerton$HIST_PUT_IMP_VOL / 100) * sqrt(BondDataMerton$Maturity)

# Default probability
```

```r
BondDataMerton$def_prob <- pnorm(-BondDataMerton$d2)

# Ensure column names match expected names in functions
test <- BondDataMerton %>%
  select(MAPDATE, SPREAD, def_prob, Maturity, CONAME)

# Function to find the closest maturity index
find_closest_maturity <- function(maturity, maturities) {
  which.min(abs(maturities - maturity))
}

# Function to find the closest default frequency and return the corresponding column name
find_closest_frequency <- function(frequency, frequencies) {
  if (length(frequencies) == 0) {
    return(NULL)
  }
  column_index <- which.min(abs(frequencies - frequency))
  if (length(column_index) == 0) {
    return(NULL)
  }
  return(names(frequencies)[column_index])
}

# Map the values
map_values_new <- function(my_df, credit_ratings_df) {
  # Ensure necessary columns are present
  if (!("Maturity" %in% colnames(my_df)) || !("def_prob" %in% colnames(my_df))) {
    stop("The input dataframe must contain 'Maturity' and 'def_prob' columns.")
  }

  results <- vector("character", nrow(my_df))
  for (i in seq_len(nrow(my_df))) {
    row <- my_df[i, ]
    closest_idx <- find_closest_maturity(row$Maturity, credit_ratings_df$maturity)

    # Extract the row in credit_ratings_df for the closest maturity
    maturity_row <- credit_ratings_df[closest_idx, -1]  # Exclude the 'maturity' column

    # Debug prints
    print(paste("Processing row:", i))
```

```
    print(paste("Closest maturity index:", closest_idx))
    print("Maturity row contents:")
    print(maturity_row)

    closest_frequency <- find_closest_frequency(row$def_prob, maturity_row)

    # Debug print
    print(paste("Closest frequency column name:", closest_frequency))

    if (is.null(closest_frequency)) {
      results[i] <- NA  # or handle the error appropriately
    } else {
      results[i] <- closest_frequency
    }
  }
  my_df$Mapped_Column <- results
  return(my_df)
}

# Perform mapping
test <- map_values_new(test, actual_df_with_extrapolation)

# Calculate average spreads by rating model
avg_spreads_merton <- test %>%
  group_by(Mapped_Column) %>%
  summarise(AvgSpread = mean(SPREAD, na.rm = TRUE)) %>%
  ungroup()

ratings_order <- c(’AAA’, ’AA’, ’A’, ’BBB’, ’BB’, ’B’, ’C’)

avg_spreads_merton$Mapped_Column <- factor(avg_spreads_merton$Mapped_Column, levels = ratings_order)

# PLOT
ggplot(avg_spreads_merton, aes(x = Mapped_Column, y = AvgSpread, fill = Mapped_Column)) +
  geom_bar(stat = "identity", color = "black", position = "dodge") +
  scale_fill_brewer(palette = "Paired") +
  theme_minimal() +
  labs(title = "",   # Remove title
       x = "Credit Rating",
       y = "Yield Spread (bps)",
       fill = "Credit Rating") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1),
```

```
      legend.position = "none")  # Remove legend

# Count number of each credit rating
rating_counts_merton <- test %>%
  group_by(Mapped_Column) %>%
  summarise(Count = n())

############# SINGLE-STOCK PSEUDO BONDS #####################

# Function to calculate financial metrics for a company
financial_analysis <- function(data, price_col, vol_col, rate_col, percentage_of_avg_price) {
  library(zoo)  # Ensure this library is loaded for na.locf

  # Convert dates to Date object
  data$Dates <- as.Date(data$Dates)

  # Check if Dates column exists and is converted
  if (!"Dates" %in% colnames(data)) {
    stop("The Dates column is missing.")
  }

  # Calculate average stock price
  avg_stock_price <- mean(data[[price_col]], na.rm = TRUE)

  # Calculate K as a percentage of the average stock price
  K <- avg_stock_price * (percentage_of_avg_price / 100)

  # Calculate maturity
  end_date <- max(data$Dates)
  data$Maturity <- as.numeric((end_date - data$Dates) / 365)

  # Calculate put prices
  put_prices <- BSPutPrice(data[[price_col]], K, data$Maturity, data[[rate_col]] / 100, data[[vol_col]] / 100)

  # Create data frames for put prices and discounted prices
  pp <- data.frame(Dates = data$Dates, PutPrice = put_prices)
  discounted_prices <- K * exp(-data[[rate_col]] / 100 * data$Maturity)
  dp <- data.frame(Dates = data$Dates, DiscPrice = discounted_prices)

  # Calculate bond values
  bond_value <- data.frame(Dates = data$Dates, Bond = (discounted_prices - put_prices))

```

```r
# Prepare data frame for merging
main_df <- data.frame(Dates = data$Dates, Price = data[[price_col]], Maturity = data$Maturity)

# Merge all data frames
merged_data <- Reduce(function(x, y) merge(x, y, by = "Dates", all = TRUE), list(main_df, pp, dp, bond_value))

# Calculate additional metrics
merged_data$ytm <- calculate_ytm_zero_coupon(merged_data$Bond, K, merged_data$Maturity)
merged_data$spread <- (merged_data$ytm - (data[[rate_col]] / 100)) * 100 * 10

# Calculate log asset growth
final_value <- merged_data[merged_data$Dates == end_date, "Price"]
merged_data$log_asset_growth <- log(final_value / merged_data$Price)
mean_log_asset_growth <- mean(merged_data$log_asset_growth, na.rm = TRUE)
stdev_log_asset_growth <- sd(merged_data$log_asset_growth, na.rm = TRUE)

# Calculate shocks, leverage ratio, and X for default probabilities
merged_data$shocks <- (merged_data$log_asset_growth - mean_log_asset_growth) / stdev_log_asset_growth
merged_data$leverage_ratio <- K / merged_data$Price
merged_data$X <- (log(merged_data$leverage_ratio) - mean_log_asset_growth) / stdev_log_asset_growth

# Estimate empirical default probabilities
merged_data$default_prob_expanding <- rep(NA, nrow(merged_data))
for (i in 1:nrow(merged_data)) {
  if (i > 1) {
    merged_data$default_prob_expanding[i] <- mean(merged_data$shocks[1:i] < merged_data$X[i], na.rm = TRUE)
  } else {
    merged_data$default_prob_expanding[i] <- NA
  }
}
merged_data$default_prob_expanding <- na.locf(merged_data$default_prob_expanding, fromLast = TRUE, na.rm = FALSE) * 100

# Plots
plot(merged_data$Dates, merged_data$Bond, type = "l", xlab = "Date", ylab = "Bond Value", main = "Bond Value Over Time")
```

```
  plot(merged_data$Dates, merged_data$spread, type = "l", xlab = "Date", ylab = "Yield Spread", main = "Yield Spread Over Time")
  plot(merged_data$Dates, merged_data$default_prob_expanding, type = "l", xlab = "Date", ylab = "Default Probability", main = "Default Probability Over Time")

  return(merged_data)
}

# Apply function to each single-stock bond [Range 0 - 0.3]
results_DNB <- financial_analysis(subset, "DNB_PX_LAST", "DNB_HIST_PUT_IMP_VOL", "NIBOR3M", 64.3)
results_EQNR <- financial_analysis(subset, "EQNR_PX_LAST", "EQNR_HIST_PUT_IMP_VOL", "NIBOR3M", 82.5)
results_NHY <- financial_analysis(subset, "NHY_PX_LAST", "NHY_HIST_PUT_IMP_VOL", "NIBOR3M", 77.8)
results_ORK <- financial_analysis(subset, "ORK_PX_LAST", "ORK_HIST_PUT_IMP_VOL", "NIBOR3M", 61.4)
results_STB <- financial_analysis(subset, "STB_PX_LAST", "STB_HIST_PUT_IMP_VOL", "NIBOR3M", 56.9)
results_TEL <- financial_analysis(subset, "TEL_PX_LAST", "TEL_HIST_PUT_IMP_VOL", "NIBOR3M", 28.9)
results_YAR <- financial_analysis(subset, "YAR_PX_LAST", "YAR_HIST_PUT_IMP_VOL", "NIBOR3M", 71.3)

# Apply function to each single-stock bond [Range 0 - 0.3 - 0.6]
results_DNB <- financial_analysis(subset, "DNB_PX_LAST", "DNB_HIST_PUT_IMP_VOL", "NIBOR3M", 64.5)
results_EQNR <- financial_analysis(subset, "EQNR_PX_LAST", "EQNR_HIST_PUT_IMP_VOL", "NIBOR3M", 82.8)
results_NHY <- financial_analysis(subset, "NHY_PX_LAST", "NHY_HIST_PUT_IMP_VOL", "NIBOR3M", 77.9)
results_ORK <- financial_analysis(subset, "ORK_PX_LAST", "ORK_HIST_PUT_IMP_VOL", "NIBOR3M", 61.7)
results_STB <- financial_analysis(subset, "STB_PX_LAST", "STB_HIST_PUT_IMP_VOL", "NIBOR3M", 57)
results_TEL <- financial_analysis(subset, "TEL_PX_LAST", "TEL_HIST_PUT_IMP_VOL", "NIBOR3M", 28.9)
results_YAR <- financial_analysis(subset, "YAR_PX_LAST", "YAR_HIST_PUT_IMP_VOL", "NIBOR3M", 71.5)

# Apply function to each single-stock bond [Range 0 - 0.8 - 1]
results_DNB <- financial_analysis(subset, "DNB_PX_LAST", "DNB_HIST_PUT_IMP_VOL", "NIBOR3M", 64.55)
```

```r
results_EQNR <- financial_analysis(subset, "EQNR_PX_LAST", "EQNR_HIST_PUT_IMP_VOL", "NIBOR3M", 83)
results_NHY <- financial_analysis(subset, "NHY_PX_LAST", "NHY_HIST_PUT_IMP_VOL", "NIBOR3M", 78.055)
results_ORK <- financial_analysis(subset, "ORK_PX_LAST", "ORK_HIST_PUT_IMP_VOL", "NIBOR3M", 62)
results_STB <- financial_analysis(subset, "STB_PX_LAST", "STB_HIST_PUT_IMP_VOL", "NIBOR3M", 57.3)
results_TEL <- financial_analysis(subset, "TEL_PX_LAST", "TEL_HIST_PUT_IMP_VOL", "NIBOR3M", 29)
results_YAR <- financial_analysis(subset, "YAR_PX_LAST", "YAR_HIST_PUT_IMP_VOL", "NIBOR3M", 71.572)

# Apply function to each single-stock bond [Range 0 - 1.2]
results_DNB <- financial_analysis(subset, "DNB_PX_LAST", "DNB_HIST_PUT_IMP_VOL", "NIBOR3M", 64.6)
results_EQNR <- financial_analysis(subset, "EQNR_PX_LAST", "EQNR_HIST_PUT_IMP_VOL", "NIBOR3M", 83.3)
results_NHY <- financial_analysis(subset, "NHY_PX_LAST", "NHY_HIST_PUT_IMP_VOL", "NIBOR3M", 78.1)
results_ORK <- financial_analysis(subset, "ORK_PX_LAST", "ORK_HIST_PUT_IMP_VOL", "NIBOR3M", 62.4)
results_STB <- financial_analysis(subset, "STB_PX_LAST", "STB_HIST_PUT_IMP_VOL", "NIBOR3M", 57.5)
results_TEL <- financial_analysis(subset, "TEL_PX_LAST", "TEL_HIST_PUT_IMP_VOL", "NIBOR3M", 29.1)
results_YAR <- financial_analysis(subset, "YAR_PX_LAST", "YAR_HIST_PUT_IMP_VOL", "NIBOR3M", 71.58)

# Rename the columns for consistency
colnames(results_DNB)[colnames(results_DNB) == "default_prob_expanding"] <- "def_prob"
colnames(results_EQNR)[colnames(results_EQNR) == "default_prob_expanding"] <- "def_prob"
colnames(results_NHY)[colnames(results_NHY) == "default_prob_expanding"] <- "def_prob"
colnames(results_ORK)[colnames(results_ORK) == "default_prob_expanding"] <- "def_prob"
colnames(results_STB)[colnames(results_STB) == "default_prob_expanding"] <- "def_prob"
colnames(results_TEL)[colnames(results_TEL) == "default_prob_expanding"] <- "def_prob"
colnames(results_YAR)[colnames(results_YAR) == "default_prob_expanding"] <- "def_prob"

```

```
# WINSORIZE NEGATIVE SPREADS
results_DNB <- subset(results_DNB, spread > 0)
results_EQNR <- subset(results_EQNR, spread > 0)
results_NHY <- subset(results_NHY, spread > 0)
results_ORK <- subset(results_ORK, spread > 0)
results_STB <- subset(results_STB, spread > 0)
results_TEL <- subset(results_TEL, spread > 0)
results_YAR <- subset(results_YAR, spread > 0)

# Map credit ratings to default probabilities and maturities
results_DNB$rating <- map_values_new(results_DNB, model_df_with_
    extrapolation)
results_EQNR$rating <- map_values_new(results_EQNR, model_df_with_
    extrapolation)
results_NHY$rating <- map_values_new(results_NHY, model_df_with_
    extrapolation)
results_ORK$rating <- map_values_new(results_ORK, model_df_with_
    extrapolation)
results_STB$rating <- map_values_new(results_STB, model_df_with_
    extrapolation)
results_TEL$rating <- map_values_new(results_TEL, model_df_with_
    extrapolation)
results_YAR$rating <- map_values_new(results_YAR, model_df_with_
    extrapolation)

# Function to find average spread per credit rating
calculate_average_spread <- function(df) {
  df %>%
    group_by(rating$Mapped_Column) %>%
    summarise(AverageSpread = mean(spread, na.rm = TRUE))
}

# Find the average spread per credit rating for each dataframe
avg_spread_DNB <- calculate_average_spread(results_DNB)
avg_spread_EQNR <- calculate_average_spread(results_EQNR)
avg_spread_NHY <- calculate_average_spread(results_NHY)
avg_spread_ORK <- calculate_average_spread(results_ORK)
avg_spread_STB <- calculate_average_spread(results_STB)
avg_spread_TEL <- calculate_average_spread(results_TEL)
avg_spread_YAR <- calculate_average_spread(results_YAR)

# Then find the "average of the averages"
combined_averages <- bind_rows(avg_spread_DNB, avg_spread_EQNR, avg_
    spread_NHY,
```

```
                               avg_spread_ORK, avg_spread_STB, avg_spread_TEL,
                               avg_spread_YAR)

colnames(combined_averages) <- c("Rating", "AverageSpread")

# Calculate the overall average of these averages for each credit rating
overall_average_spreads <- combined_averages %>%
  group_by(Rating) %>%
  summarise(OverallAverageSpread = mean(AverageSpread, na.rm = TRUE))

ratings_order <- c('AAA', 'AA', 'A', 'BBB', 'BB', 'B', 'C')

overall_average_spreads$Rating <- factor(overall_average_spreads$Rating, levels = ratings_order)

# Plot
ggplot(overall_average_spreads, aes(x = Rating, y = OverallAverageSpread, fill = Rating)) +
  geom_bar(stat = "identity", color = "black", position = "dodge") +
  scale_fill_brewer(palette = "Paired") +
  theme_minimal() +
  labs(title = "",   # Remove title
       x = "Credit Rating",
       y = "Yield Spread (bps)",
       fill = "Credit Rating") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1),
        legend.position = "none")  # Remove legend

# DNB
rating_counts_DNB <- results_DNB  %>%
  group_by(rating$Mapped_Column) %>%
  summarise(Count = n())
# EQNR
rating_counts_EQNR <- results_EQNR  %>%
  group_by(rating$Mapped_Column) %>%
  summarise(Count = n())
# NHY
rating_counts_NHY <- results_NHY  %>%
  group_by(rating$Mapped_Column) %>%
  summarise(Count = n())
# ORK
rating_counts_ORK <- results_ORK  %>%
  group_by(rating$Mapped_Column) %>%
```

```
  summarise(Count = n())
# STB
rating_counts_STB <- results_STB  %>%
  group_by(rating$Mapped_Column) %>%
  summarise(Count = n())
# TEL
rating_counts_TEL <- results_TEL  %>%
  group_by(rating$Mapped_Column) %>%
  summarise(Count = n())
# YAR
rating_counts_YAR <- results_YAR  %>%
  group_by(rating$Mapped_Column) %>%
  summarise(Count = n())
# Combine the counts
combined_counts_single <- bind_rows(rating_counts_DNB, rating_counts_EQNR, rating_counts_NHY,
                              rating_counts_ORK, rating_counts_STB, rating_counts_TEL,
                              rating_counts_YAR)
# Find average amount of each rating
colnames(combined_counts_single) <- c("Rating", "Count")

# Calculate the overall average of these averages for each credit rating
average_number_ratings <- combined_counts_single %>%
  group_by(Rating) %>%
  summarise(AverageCount = mean(Count, na.rm = TRUE))

# ILLUSTRATION CHOICE OF K FOR DNB
plot(results_DNB$Dates, results_DNB$Bond, type = "l", xlab = "Date", ylab = "Bond Value", main = "Bond Value Over Time")
plot(results_DNB$Dates, results_DNB$spread, type = "l", xlab = "Date", ylab = "Yield Spread", main = "Yield Spread Over Time")
plot(results_DNB$Dates, results_DNB$default_prob_expanding, type = "l", xlab = "Date", ylab = "Default Probability", main = "DNB")

####### Plot bond payoff ####

# Define parameters
K <- 50  # Face value of the bond
asset_values <- seq(0, 100, by = 1)  # Range of asset values from 50 to 150

# Calculate bond payoff
bond_payoff <- pmin(asset_values, K)
```

```r

# Create a dataframe
dataphrame <- data.frame(AssetValue = asset_values, BondPayoff = bond_payoff)

ggplot(dataphrame, aes(x = AssetValue, y = BondPayoff)) +
  geom_line(color = "blue", size = 1) +  # Plot bond payoff as a line
  annotate("point", x = K, y = K, color = "red", size = 4) +  # Highlight the point where A(t) = K
  labs(x = "Asset Value", y = "Bond Payoff") +
  theme_minimal() +
  geom_segment(x = K, y = 0, xend = K, yend = K, linetype = "dashed", color = "red") +  # Dashed line stopping at the payoff
  annotate("text", x = K + 5, y = K + 10, label = "Face Value (K)", hjust = 0, vjust = 0) +
  ylim(0, 100)  # Adjusted y-limit to add space above

############# PLOT ALL SPREADS TOGETHER #####

# single-stock
overall_average_spreads$Instrument <- "Single-stock pseudo bond"
# corporate bond
average_spreads$Instrument <- "Corporate bond"
# merton
avg_spreads_merton$Instrumet <- "Merton model"
# obx pseudo bond
grouped$Instrument <- "OBX pseudo bond"

# GIVE ALL INSTRUMENTS THE SAME COLUMN NAMES
colnames(overall_average_spreads) <- c("CreditRating", "Spread", "Instrument")
colnames(average_spreads) <- c("CreditRating", "Spread", "Instrument")
colnames(avg_spreads_merton) <- c("CreditRating", "Spread", "Instrument")
colnames(grouped) <- c("CreditRating", "Spread", "Instrument")

# Combine them
all_instruments <- bind_rows(overall_average_spreads, average_spreads, avg_spreads_merton, grouped)

# Define custom colors for each instrument
custom_colors <- c("Corporate bond" = "#1f77b4", # blue
                   "OBX pseudo bond" = "#808080", # grey
                   "Merton model" = "#FFD700", # yellow
                   "Single-stock pseudo bond" = "#d62728") # red
```

```

# Plot
ggplot(all_instruments, aes(x = CreditRating, y = Spread, fill = Instrument)) +
  geom_bar(stat = "identity", position = position_dodge(width = 0.75), width = 0.75) +
  scale_fill_manual(values = custom_colors) +
  labs(x = "Credit Rating",
       y = "Average Spread (bps)",
       fill = NULL) +
  theme_minimal() +
  theme(legend.position = c(0.3, 0.85),
    legend.key.size = unit(0.4, "cm"),
    legend.background = element_rect(fill = "white", color = "black", size = 0.5),
    legend.title = element_blank(),
    axis.text.x = element_text(margin = margin(t = 10)),
    axis.ticks.length = unit(0.2, "cm")
)
```

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