

A FIRST BIBLIOGRAPHY ON SET AND VECTOR OPTIMIZATION PROBLEMS WITH RESPECT TO VARIABLE DOMINATION STRUCTURES

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Abstract. This paper lists 107 papers dealing with variable domination structures in vector and set optimization, and its applications and numerics. A classification is given in terms of different sub-branches where variable domination structures occur.

Keywords. variable domination structures; vector optimization; set optimization; vector equilibrium problems.

1. INTRODUCTION

This bibliography lists in alphabetical order by the name of the first author, 107 papers dealing with variable domination structures and its applications and numerics.

Vector and set optimization with variable domination structures (or, if ordering cones are involved, variable ordering structures) is a growing and expanding field of applied mathematics that deals with optimization problems where the domination structure is given by a set-valued map acting between abstract or finite dimensional spaces. The concept of variable domination structures is a generalization of the solution concept with fixed domination structure in multi-objective decision-making problems. A solution concept for problems with general variable domination structures in preference modeling was initiated and first studied by Yu [99] in 1973 (see also Yu [100]). In a practical context, this means that preference depends on the current point in the objective space (so-called decisional wealth in [59]). Since the 2000s, such concepts have been the center of attention in various works. It is our goal in this bibliography to give an overview of the available literature on this and related subjects, and to classify the references according to subfields. We collected the papers to the best of our abilities, but cannot guarantee completeness. The authors will be grateful for any corrections, additions or comments about this bibliography, see the section on how to contribute at the end of this work.

The goal of this bibliography is trifold: First, we wish to provide an introduction for researchers new to this quickly growing (and spreading) field. Secondly, we intend to propose some standardization of notations and concepts to streamline recent efforts in the field. Finally, and most importantly, we mean to initiate a community effort in keeping this review and the work on variable domination structures and their applications updated and thriving.

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2. SOLUTION CONCEPTS FOR VECTOR OPTIMIZATION PROBLEMS WITH VARIABLE DOMINATION STRUCTURE

Now let us briefly recall some definitions related to variable domination structures in optimization.

In the first instance, we bring the classical notion of the Pareto preorder with a fixed ordering cone $C \subset Y$ to mind.

Definition 2.1 (Pareto binary relation). Consider a real linear space Y and a convex cone C in Y . Let $y, v \in Y$. The Pareto binary relation on Y denoted by \leq_C is defined by

$$v \leq_C y : \iff v \in y - C \iff y \in v + C.$$

Preexisting two vectors y and v in a linear outcome space Y , we use the notation $v = y + d$ for some vector $d \in Y$. If y is favored to v by the decision maker, then d can be considered as a *domination factor*. We denote the set of all the domination factors for y together with the zero vector $0 \in Y$ by $D(y)$. Then, the set-valued map $D : Y \rightrightarrows Y$ is referred to as *domination structure*. If $D(y)$ is an *ordering cone* for each $y \in Y$, it is also named a *variable ordering structure* in the literature.

Yu [99] launched the concept of variable domination structures, where the sets $D(y)$ are supposed to be cones, i.e., Yu defined a domination structure as a family of cones $D(y)$, whereas Engau [44] considered it as a set-valued map. Bergstresser, Charnes, and Yu [18] defined domination factors in a finite-dimensional setting with convex domination sets. As distinguished from multi-objective optimization with a fixed ordering cone, we now consider two binary relations in Y with respect to the choice of domination sets.

Definition 2.2 (Binary relations). Let Y be a real linear space and let $D : Y \rightrightarrows Y$ be a set-valued map. We define for $y_1, y_2 \in Y$

$$y_1 \leq_1 y_2 : \iff y_1 \in \{y_2\} - D(y_1) \quad (\text{nondomination binary relation}), \quad (2.1)$$

$$y_1 \leq_2 y_2 : \iff y_1 \in \{y_2\} - D(y_2) \quad (\text{nondomination-like binary relation}). \quad (2.2)$$

Provided $D(\cdot) \equiv C$ for some fixed ordering cone C of Y , both nondomination and nondomination-like binary relations reduce to the classical Pareto binary relation generated by C (see Definition 2.1), i.e., $\leq_1 = \leq_2 = \leq_C$.

Corresponding to both binary relations introduced in Definition 2.2, we consider two solution concepts.

Definition 2.3 (Optimality Notions in Vector Optimization with Variable Domination Structures, [34]). Let Y be a real linear space with zero element 0_Y , let $\emptyset \neq A \subseteq Y$, and let $D : Y \rightrightarrows Y$ be a set-valued map.

- (1) An element $\bar{y} \in A$ is a *nondominated element* of the set A w.r.t. the domination structure D , if $y \not\leq_1 \bar{y}$ for all $y \in A \setminus \{\bar{y}\}$, i.e. if no $y \in A$ exists such that

$$\bar{y} \in \{y\} + D(y) \setminus \{0_Y\},$$

or, equivalently,

$$\bar{y} \notin \bigcup_{y \in A} \{y\} + (D(y) \setminus \{0_Y\}).$$

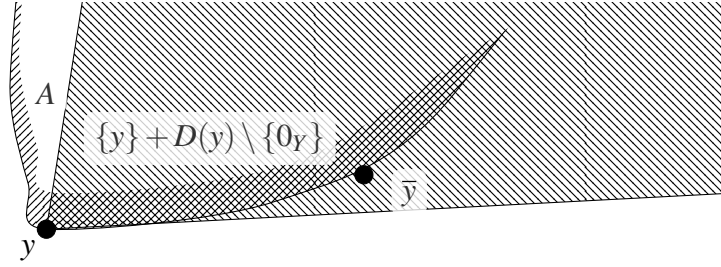


FIGURE 1. $\bar{y} \in A$ is a *not* a nondominated element w.r.t. the domination map D

- (2) An element $\bar{y} \in A$ is a *nondominated-like element* of the set A w.r.t. the domination structure D , if $y \not\prec_2 \bar{y}$ for all $y \in A \setminus \{\bar{y}\}$, i.e. if no $y \in A$ exists such that

$$\bar{y} \in \{y\} + D(\bar{y}) \setminus \{0_Y\},$$

or, equivalently,

$$(\{\bar{y}\} - D(\bar{y})) \cap A = \{\bar{y}\}.$$

Note that in some references, the definition of nondominated/nondominated-like elements differs as the set-valued mapping D maps from a real linear (preimage) space X to the real linear (image) space Y . Within this review, we restrict the representation to the cases where D solely depends on elements of the image space Y . Figures 1 and 2 illustrate the definition of nondominated and nondominated-like elements.

Recollect that the concept of nondominated elements with respect to D in Definition 2.3(1) was launched by Yu in [99, 100] for conic domination structures. The concept of nondominated-like elements with respect to D in Definition 2.3(2) was initiated by Chen, Huang, and Yang in [24, Definition 1.13], compare also Chen and Yang [25, Definition 3.1].

Notice that a nondominated-like element \bar{y} of A w.r.t. D is an element, which is not dominated by another element y with respect to the associated set $D(\bar{y})$ at the nondominated-like element \bar{y} w.r.t. D . On the other hand, given a nondominated element \bar{y} of A w.r.t. D , a domination set $D(y)$ is a set associated with another element y . Characterizations and important properties of nondominated-like elements and nondominated elements can be found in [25, 24, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 99, 100].

Definition 1.2 of *Pareto-type optimal allocations in welfare economics* in [8] also resembles the definition of efficient solutions with variable domination structures, where the variable ordering cone is replaced with a *preference set*.

Note that Eichfelder [34] formulated algorithms based on Jahn-Graef-Younes procedures for solving vector-valued problems with a variable ordering structure. A solution procedure based on multi-objective evolutionary algorithms has been proposed in [50]. Other methods can be found in [15, 16, 19, 21, 80].

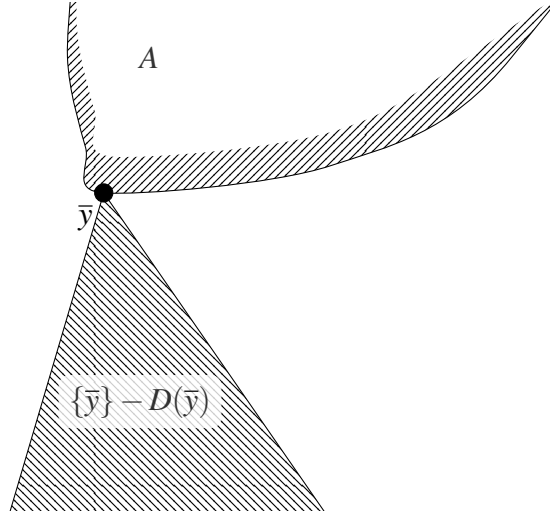


FIGURE 2. $\bar{y} \in A$ is a nondominated-like element w.r.t. the domination map D

3. SET OPTIMIZATION PROBLEMS WITH VARIABLE DOMINATION STRUCTURE

The idea of variable domination structures has also been extended to set optimization problems, where it is assumed that a family \mathcal{A} of sets from the power set of Y is given and minimal elements of \mathcal{A} are to be identified.

Definition 3.1 (Variable Domination Structures in Set Optimization, [62]). Let A, B be nonempty subsets of a real linear space Y , and let $D: Y \times Y \rightrightarrows Y$. Then, we define the *variable upper set less relation* by

$$A \preceq_{D,v}^u B \iff \forall a \in A, \exists b \in B : a \in \{b\} - D(a,b). \quad (3.1)$$

Remark 3.1 ([62, Remark 2.6]). If we replace A by $\{y_1\}$ and B by $\{y_2\}$ in Definition 3.1 and if $D_1: Y \rightrightarrows Y$ is given as a set-valued map that only depends on the first variable such that $D_1(y_1) := D(y_1, y_2)$ is a convex cone for all $y_1, y_2 \in Y$, then we have the following equivalence:

$$\{y_1\} \preceq_{D_1,v}^u \{y_2\} \iff y_1 \in \{y_2\} - D_1(y_1) \iff y_1 \leq_1 y_2,$$

where \leq_1 is given by equivalence (2.1) for $C = D_1$. If, on the other hand, $D_2: Y \rightrightarrows Y$ is given as a set-valued map that only depends on the second variable such that $D_2(y_2) := D(y_1, y_2)$ is a convex cone for all $y_1, y_2 \in Y$, then we have the following equivalence:

$$\{y_1\} \preceq_{D_2,v}^u \{y_2\} \iff y_1 \in \{y_2\} - D_2(y_2) \iff y_1 \leq_2 y_2,$$

where \leq_2 is given by (2.2) for $C = D_2$.

Definition 3.2 (Optimality Notion in Set Optimization with Variable Domination Structures, [63]). Let \mathcal{A} be a family of nonempty subsets of the real linear space Y and let the set relation $\preceq_{D,v}^u$ on the power set of Y be given. Then $\bar{A} \in \mathcal{A}$ is called a *minimal element* of \mathcal{A} w.r.t. $\preceq_{D,v}^u$ if

$$A \preceq_{D,v}^u \bar{A}, A \in \mathcal{A} \implies \bar{A} \preceq_{D,v}^u A.$$

Figure 3 visualizes the optimality notion in set optimization with variable domination structures.

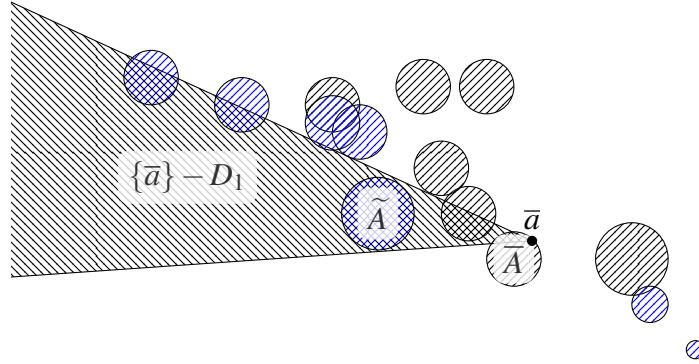


FIGURE 3. Let \mathcal{A} be comprised of several sets, which are closed balls in \mathbb{R}^2 of different radii. $\bar{A} \in \mathcal{A}$ is *not* a minimal element of \mathcal{A} w.r.t. $\preceq_{D,v}^u$, where $\tilde{A} \subset Y$, $\tilde{A} \neq \bar{A}$ and for all $\tilde{a} \in \tilde{A}$, $\bar{a} \in \bar{A}$, $D(\tilde{a}, \bar{a}) = D_1$ and $D(a_1, A_2) = \mathbb{R}_+^2$ for all other $a_1, a_2 \in Y$. The blue sets are all minimal element of \mathcal{A} w.r.t. $\preceq_{D,v}^u$.

Remark 3.2. In the case of a finite family of sets \mathcal{A} , the binary relation (restricted to the elements of \mathcal{A}) can be represented by a directed graph as the domination structure D is a discrete function on a finite domain. In many situations, the task of vector/set optimization with variable domination structures is then equivalent to finding minimal elements in finite graphs and there is a vast literature in the computer science community concerning algorithms for these problems. Typical application cases of variable domination optimization problems in mind (see below), we explicitly refrain from going too much into detail with regards to graph theoretic approaches but want to mention this field of research here for the sake of completeness.

To date and to the best of our knowledge, there exists only one approach for solving set-valued optimization problems with variable domination structures. This can be found in [64], where the authors present a descent method to find approximate minimal solutions of set-valued problems w.r.t. variable domination structures using nonlinear scalarization methods.

Variable domination structures have gained increasing interests due their wide range of applications. Variable ordering structures are used in image registration in medical engineering where one aims at merging several medical images gained by different imaging methods as, for instance, computer tomography, magnetic resonance tomography, positron emission tomography, or ultrasound, and one searches for a best transformation map, also called registration (see [34, 36, 94, 95]). They have also been applied to intensity-modulated radiation therapy in [34]. An application of variable domination structures in multi-objective programming within location analysis is presented in [102]. Recently, variable domination structures appeared in the study of variational rationality in behavioral sciences, psychology and some other related areas of behavioral sciences, see [10, 13]. Since set optimization can be interpreted and used as an application of uncertain vector optimization, the variable domination

concept in set optimization has been applied to vector optimization problems with uncertain data in [3, 62, 63, 64, 65, 67, 69].

4. VECTOR VARIATIONAL INEQUALITIES AND EQUILIBRIUM PROBLEMS WITH VARIABLE DOMINATION STRUCTURE

Whether vector or set optimization problems with respect to variable domination structures are studied, the concepts defining the domination are usually based on set inclusions which—in turn—can be re-formulated in various ways. That way, the concepts described above can all be closely linked to separation theory, complementarity problems and, most prominently when screening the literature, variational inequalities/equilibrium problems and their various generalizations. We have therefore decided to include the most tightly connected results in that area also in this review.

In short, for generalized vector variational inequalities with respect to variable domination structures, existence results are studied in the literature (see [22, 43, 49]). Furthermore, applications to vector approximation problems with variable domination structures are discussed.

We also note that the term *variable domination structures* appears frequently in the framework of *vector equilibrium problems*. For example, in [97], a strong vector equilibrium problem with variable domination structure is defined as: Find $\bar{x} \in X$ s. t.

$$\forall y \in X : f(\bar{x}, y) \in D(g(\bar{x})), \quad (4.1)$$

where E and Z are two real Hausdorff topological vector spaces, X is a nonempty subset of E , $D: Z \rightarrow 2^Z$ s. t. for each $x \in X$, $D(x)$ is a proper, closed and convex cone, $g: X \rightarrow Z$, and $f: X \times X \rightarrow Z$. The notation 2^Z indicates the power set of Z . If $D(g(\bar{x})) \equiv C$ (with a fixed cone $C \subset Z$) is constant, then one speaks of a strong vector equilibrium problem (without variable domination). Problems of the form (4.1) appear in a large number of publications, and we restrict ourselves to just mentioning a few in the classification below.

5. CLASSIFICATION

Structuring the literature on the topic can be done by means of several aspects: As the underlying structures are defined by inclusion problems dependent on set-valued mappings, properties of D (such as lower-continuity or monotonicity) and its images (cone-valued, open, connected, absorbant etc.) could be a possibility for classification.

We have, however, decided for a more phenomenological structure and classify this literature study by means of the following seven groups:

- (1) (Set-valued) Vector (quasi-) equilibrium problems with variable domination structures: [1, 2, 14, 45, 47, 51, 53, 70, 71, 73, 74, 96, 97]
- (2) Inclusion problems with moving cones: [77, 93]
- (3) Vector / Multi-objective optimization problems with variable domination structures: [2, 4, 5, 7, 9, 13, 18, 20, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 37, 38, 39, 42, 44, 46, 54, 55, 56, 67, 69, 72, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 98, 99, 100, 101]
- (4) Vector complementarity problems and variational inequalities with variable domination structures: [6, 22, 43, 49, 52]
- (5) Set optimization problems with variable domination structures: [3, 9, 11, 28, 40, 41, 60, 61, 62, 63, 64, 65, 66, 69]

(6) Applications: [8, 9, 10, 12, 13, 27, 36, 42, 48, 59, 63, 67, 68, 69, 75, 76, 87, 88, 89, 90, 91, 94, 95, 102, 103, 104, 105, 106, 107]

(7) Algorithms: [15, 16, 17, 19, 21, 34, 35, 50, 57, 58, 80]

This decision was made to improve the added value for scholars new to the field: An approach based on complex analytical properties would not convey the introductory nature of this overview.

A COMMUNITY EFFORT

We do not claim to be exhaustive within the scope of this short overview paper, neither with respect to the bibliographic entries nor with respect to solution concepts and/or algorithmic ideas. The main purpose is, as indicated in the beginning of this work, to spark a collaborative effort within the community working with variable domination structures in vector and set optimization as well as their applications.

The bibliography in its current form is merely meant to be a starting point. We invite researchers involved in the topic to contact us via

`variable_domination@math.ntnu.no`

if they see relevant work within the field that should be included and wish to increase the scope of this bibliography within an online-forum. It is currently accessible via

https://folk.ntnu.no/markukob/variable_domination

and planned to be published under its own domain soon.

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