Master's thesis

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Identifying Structural Changes in the U.S. Business Cycle

Master's thesis in Economics
Supervisor: Leif Anders Thorsrud
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ABSTRACT

This thesis investigates structural changes in the dynamics of the U.S. business cycle, focusing on the period known as the Great Moderation and its aftermath. Utilizing a three-regime Markov Switching Dynamic Factor Model (MSDFM), the thesis expands on Chauvet (1998)'s traditional two-regime approach. By incorporating switching variances, it introduces a third economic regime characterized by moderate growth with lower volatility.

Our empirical analysis uses monthly data up to the end of 2023 retrieved from the Federal Reserve Economic Data (FRED). It confirms that the traditional two-regime model does not adequately capture the complexity of the current economic landscape. By integrating a third regime, our model offers nuanced insights into economic fluctuations and demonstrates improved goodness-of-fit metrics compared to a two-regime model. The model´s classification ability (performance, etc) was validated against historical recession data from the National Bureau of Economic Research (NBER).

Our findings suggest a noticeable shift in economic stability after the Great Moderation. The thesis contributes to macroeconomic modeling by enhancing the empirical framework used to analyze economic business cycles, providing insights beneficial for policymakers and economic stakeholders interested in the dynamics of business cycles.

SAMMENDRAG

Denne avhandlingen undersøker strukturelle endringer i dynamikken i den amerikanske konjunktursyklusen, med fokus på perioden kjent som the Great Moderation og dens ettervirkninger. Ved å benytte en tre-regime Markov Switching Dynamic Factor Model (MSDFM) utvider vi Chauvet (1998) to-regime tilnærmingen ved å inkludere skiftende varians og et tredje økonomisk regime karakterisert av moderat vekst med lavere volatilitet.

Vi bruker månedlige data frem til slutten av 2023 hentet fra Federal Reserve Economic Data (FRED) som grunnlaget for vår empiriske analyse. I vår analyse finner vi at den tradisjonelle to-regime modellen ikke er tilstrekkelig til å fange kompleksiteten i dagens økonomiske landskap. Ved å integrere et tredje regime, karakterisert av moderat vekst og lavere volatilitet, gir vår modell nyanserte innsikter i økonomiske svingninger og viser forbedrede goodness-of-fit-metrikker sammenlignet med en to-regime modell. Modellens prediksjonspresisjon er validert mot historiske resesjonsdata fra National Bureau of Economic Research (NBER).

Våre funn antyder en merkbar endring i økonomisk stabilitet etter Great Moderation. Avhandlingen bidrar til makroøkonomisk modellering ved å forbedre det empiriske rammeverket som brukes til å analysere økonomiske sykluser. Den gir innsikter som er nyttige for beslutningstakere og økonomiske aktører som er interessert i konjunktursyklusens dynamikk.

ACKNOWLEDGMENTS

We want to thank the Institute of Economics at NTNU for supporting us in exploring this field of Economics. We thank our supervisor, Leif Anders Thorsrud for highly valuable insights and feedback during our writing. Additionally, we wish to acknowledge Alex Hubbard, whose computer routines were instrumental in estimating the models presented in this thesis. We also want to thank friends and family for supporting us through the process of writing our master's thesis and our time in Trondheim.

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ABBREVIATIONS

- GDP - Gross Domestic Product
- MS - Markov-Switching
- DFM - Dynamic Factor model
- MSDFM - Markov-Switching Dynamic Factor model
- BFGS algorithm - Broyden–Fletcher–Goldfarb–Shanno algorithm
- ADF - Augmented Dickey-Fuller
- BDS test - Brock-Dechert-Scheinkman test
- PSO - Particle Swarm Optimization
- AR - Auto-regressive
- NBER - National Bureau of Economic Research
- i.i.d. - Independent and Identically Distributed
- ROC - Receiver Operating Characteristic
- AUROC - Area Under Receiver Operating Characteristic

INTRODUCTION

1.1 Thesis Background

In the wake of the "Great Inflation", the volatility of macroeconomic time series has significantly declined and financial crises have become less frequent. This phenomenon, reflecting a broader understanding of economic cycles beyond traditional business cycles, suggests that the economy may have entered a new era of stability. Historically, periods of high and low interest rates have revealed underlying cyclical behaviors that influence economic outcomes.

This is the foundation for understanding the mechanisms that drive the economy and helps explain the possible prevalence of different regimes. The Great Inflation was marked by soaring inflation rates, in contrast to the Great Moderation which was characterized by steady economic growth and consistently low inflation. Stock and Watson (2002) attribute some of the shift from the Great Inflation's turbulent times to the Great Moderation's stability to improved monetary policies. This shift in the economy is key to answering the hypothesis of whether there are more than two economic regimes.

The thesis builds on ideas by Burns and Mitchell (1946) who focused on the analysis of macroeconomic variables to measure the business cycle. Stock and Watson (1988a) later proposed a Dynamic Factor model to classify economic comovements in the business cycle. Hamilton (1989) expanded this line of thought to a non-linear Markov-switching model to capture the asymmetries of the business cycle. The work of Hamilton was integrated into a generalized state space framework by Kim (1994). Both the methods proposed by Hamilton (1989) and the Dynamic Factor models proposed by Stock and Watson (1988a) were combined by Diebold and Rudebusch (1996) into a framework, in which they did not fully estimate. Chauvet (1998) fully estimated the framework proposed by Diebold and Rudebusch (1996) and used Kim (1994)'s general state-space model based on Hamilton (1989)'s work to do so. This thesis expands on the work by Chauvet (1998) to more accurately capture the new dynamics of the business cycle.

This research contributes to the ongoing discussion on economic stability and policy effectiveness, providing valuable insights for policymakers, economists, and stakeholders interested in the dynamics of economic fluctuations. Through rigorous empirical analysis, the thesis aims to illuminate whether an observable change in macroeconomic variables signals a new paradigm in economic stability.

1.2 Research Question

In this thesis, our research question is:

Has there been a shift in the U.S.-business cycle, and is this shift detectable using a Markov Switching Dynamic Factor Model?

The concept of the "Great Moderation" refers to a prolonged period during which major economies experienced low volatility and stable growth, fundamentally altering expectations about economic management and outcomes. The thesis explores whether we are witnessing a reversion to pre-moderation economic regimes or entering a new phase of economic stability. We employ a Markov switching dynamic factor model to empirically investigate this potential structural shift in macroeconomic data. This approach extends the existing two-regime models of economic cycles detailed by Chauvet (1998) and Hamilton (1989) by expanding it to a three-regime model which also incorporates switching variance for the latent dynamic factor. A third economic regime will is hypothesized to be characterized by moderate growth and low volatility. This addition is designed to assess whether economic growth and stability experienced measurable structural changes during the Great Moderation in the U.S. economy."

Coincident indicators in the U.S. economy has traditionally been modeled by the means of the latent dynamic factor. By also modeling the variance of the factor, the probability distributions of the latent dynamic factor will be modeled as a mixture of three normal distributions with differing means and standard deviations. This is as opposed to Chauvet (1998) which models the distributions with unit variance. This means that the latent dynamic factor is sampled from three Gaussian distributions with differing means and standard deviations as opposed to Chauvet (1998) model which models the factor as if it samples from two Gaussian distributions with different means and identical standard deviations.

Our hypothesis is informed by an additional three decades of economic data since Chauvet (1998), which reveals a trend toward reduced economic volatility. This new dataset allows us to robustly test our hypothesis against the evolving economic conditions. The goal is to hypothesize that the specification extension to Chauvet (1998) identifies one recessionary regime characterized by high factor variance and a negative factor mean, one unstable expansionary regime characterized by high factor variance and a positive mean, and one stable moderate expansionary regime characterized by low factor variance and a positive factor mean.

The methods proposed by Kim (1994) are used to estimate the models in this thesis. Kim (1994) expanded Hamilton (1989)'s Markov-switching model into a general state-space format where regime switches occur in both the measurement and transition equations. The observable parameters are linked to a regime variable S_t, which adheres to a Markov process. This will be the foundation for our model estimation methods in detecting the economy's business cycle dynamics.

1.3 Thesis structure

In order to test the hypothesis, we will first go through the relevant economic theory and history related to the Great Inflation and the Great Moderation. Further, we will explain the mechanics of Markov Switching Dynamic Factor models and the applied estimation procedure. The results will be presented from the different models. Finally, the results will be discussed. Special attention will be paid to comparing the two- and three-regime models with switching variance. To conclude this paper we will provide a short summary of our results and discuss further research.

In the context of this study, the term "regime" specifically denotes the Markov regime variable (S_t). This terminological choice is consistent with the conventional usage in this field and is adopted to prevent any ambiguity with the "state variables" typically referenced in state-space models. This distinction is crucial for maintaining clarity in the theoretical framework of our analysis.

1.4 Scope and limitations

Building on the methodological groundwork laid by Chauvet (1998), Kim (1994), and Stock and Watson (1989), this thesis expands upon their frameworks for classifying economic recessions and expansions. The objective is to delve deeper into the structural shifts and variations observed in macroeconomic variables, thereby offering new insights into the effectiveness and impact of monetary policy across different economic contexts. In pursuing this refined analysis, certain methodological simplifications were necessary to maintain the focus and manageability of the research.

The thesis does not consider the implementation of several latent dynamic factors despite it being supported by the model framework. The would diverge with the central thesis on the business cycle as outlined by Burns and Mitchell (1946) who envisioned a single factor capturing the state of entire economy. Incorporating several latent dynamic factors could indeed provide a richer analysis; however, the increased complexity would complicate the model estimation process, and the role and significance of additional latent factors remain ambiguous. for this reason the several factors were omitted from the analysis.

The unprecedented impact of the COVID-19 pandemic has also influenced the results. This thesis addresses the methods used to mitigate the pandemic's effects within the dataset but stops short of categorizing the pandemic as a distinct economic regime or as an outlier. This delineation remains an open question, reflecting the unique and significant disruptions caused by the pandemic, which may require a separate analytical approach or an optimization procedure robust to outlier.

Moreover, this study does not explore the underlying causes of economic moderation. Instead, it focuses on identifying and analyzing the presence of structural changes signaled by a moderation regime within the data. This approach aims to clarify the transitions between economic states and their implications, rather than dissecting the specific drivers of these changes.

THEORY

This chapter will give some context to the essential historical and theoretical aspects. It begins by examining significant economic periods such as the Great Inflation and the Great Moderation, and their impact on current economic policies and theories. Then how the NBER classifies and dates business cycles will be presented. The works of prominent economists such as Burns and Mitchell, Stock and Watson, and Hamilton are discussed to provide foundational insights into the model choice. Furthermore, an explanation for why a Markov switching dynamic factor model is used, highlighting its effectiveness in analyzing economic regimes. This background information is crucial for understanding the empirical analysis and model enhancements presented in the thesis.

2.1 The Great Inflation

Banks (1977)'s paper explores the "Great Inflation", a prolonged period of price increases. Meltzer (2005) marked this as a significant inflationary period originating in the mid-1960s and lasting into the mid-1980s. Banks (1977) suggests that the inflation is largely tied to economic strategies initiated in response to the Great Depression of the 1930s. Key policies, such as deficit spending and transitioning from the gold standard to a fixed currency exchange rate system, played pivotal roles. These were compounded by social and political challenges, further escalating the problem.

Banks (1977) notes that the deep-seated impacts of the Great Depression shaped the 1940s' economic policies. Policymakers implemented expansive fiscal measures, such as significant deficit spending during World War II, aimed at reducing unemployment and boosting economic growth. This pattern extended into the 1960s and 1970s, with the U.S. government routinely running budget deficits during times of economic prosperity. Especially during significant events such as the Vietnam War and the expansion of social welfare programs under President Johnson. The Bretton Woods Agreement established a fixed exchange rate system, linking the US dollar directly to gold. This system facilitated global trade expansion by providing stability in international financial transactions. Additionally, it permitted the United States to sustain a level of spending that exceeded its economic means.

Meltzer (2005) remarks that the end of the Great Inflation in the mid-1980s was attributed to a combination of policy actions by the Federal Reserve and a shift in political will. The inflation, which started in the late 1960s and peaked in the 1970s, was fueled by several factors, including persistent monetary expansion to finance budget deficits, misguided economic policies, and reactions to oil price shocks. Meltzer (2005) references Taylor and National Bureau of Economic Research (2001) and his analysis of the Federal Reserve's policy response to inflation during the period leading up to 1981. According to Taylor, the Federal Reserve did not increase nominal interest rates enough to offset the negative impact of inflation on real interest rates and on expectations for future interest rates. This suggests that the Federal Reserve's policy measures were reactive rather than proactive, lacking the aggressiveness needed to counteract high inflation effectively.

From Bordo and Orphanides (2013) the crucial turning point for inflation occurred when Paul Volcker was appointed Federal Reserve Chairman in 1979. He implemented a series of stringent monetary policies aimed at curbing inflation. These were characterized by significantly higher interest rates which were maintained to squeeze out inflationary expectations from the economy. These policies resulted in a recession in the early 1980s.

This sharp increase in interest rates is known as the Volcker shock of 1980, during which The Federal Reserve implemented an estimation of the Taylor Rule from Taylor and National Bureau of Economic Research (2001). According to this rule, the federal funds rate should increase if inflation is above the target or GDP growth is above its trend, suggesting economic overheating. Conversely, if inflation is below target or GDP growth is sluggish, the rule suggests lowering the rate to stimulate economic activity.

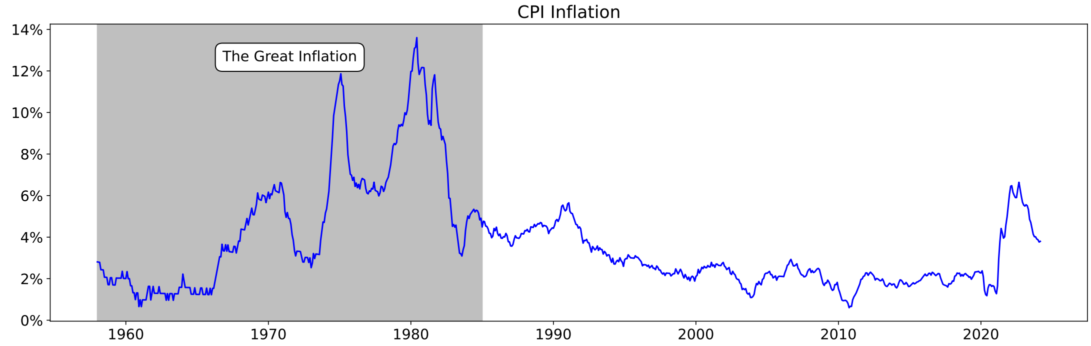

Figure 2.1.1: *Yearly increase in Consumer Price Index for All Urban Consumers. All Items Less Food and Energy in U.S. City Average.* Data retrieved from U.S. Bureau of Labor Statistics (1957).

This change marked a period when central banks adopted new policies. Particularly, the Federal Reserve under Chairman Paul Volcker emphasized controlling inflation over other economic objectives, such as reducing unemployment. The Federal Reserve's efforts led to a significant reduction in inflation by 1983 and the official end of the Great Inflation period by 1984, as observed in the consumer price index (seen in Figure 2.1.1) and other economic indicators.

2.2 The Rise Of The Great Moderation

According to Stock and Watson (2002), the term "The Great Moderation" refers to a substantial decline in the volatility of economic activity, particularly the GDP growth rates among most G7 countries starting in the mid-1980s. They note that in the United States, the standard deviation of GDP growth rates averaged over four quarters was significantly lower in the period from 1984 to 2002 compared to the period from 1960 to 1983. This reduction in volatility is widespread across various sectors within the U.S. economy and is also evident in other G7 nations, albeit with some differences in timing and specific characteristics. Importantly, Stock and Watson also highlight increased international economic integration during this period.

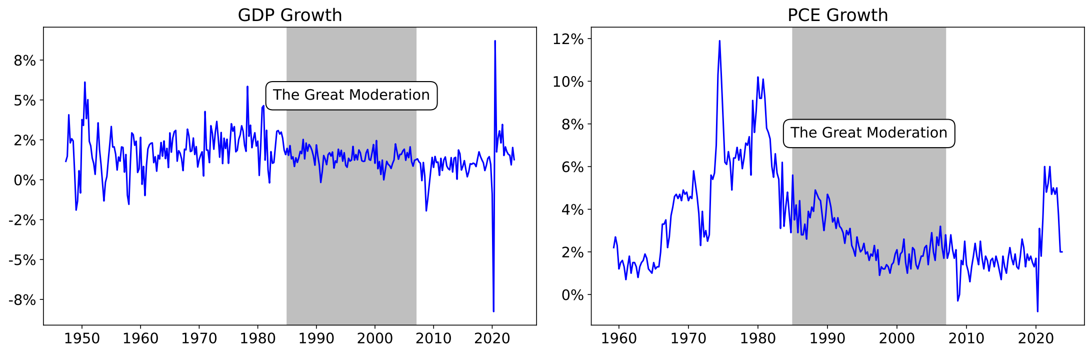

Figure 2.2.1: *Comparison of GDP and PCE Growth.* Data retrieved from U.S. Bureau of Economic Analysis (1946) and U.S. Bureau of Economic Analysis (1959a).

Stock and Watson (2002) explore various explanations for the Great Moderation, focusing particularly on the role of improved monetary policy. They analyze whether policy shifts, especially those aimed at controlling inflation, could have indirectly led to more stable economic growth rates. Their analysis shows that improved monetary policy contributed to controlling inflation, though it accounted for only a small fraction of the reduction in output growth variance. This suggests that other factors may also influence the increased stability.

Other factors such as technological advancements and structural changes in the economy could also have played a significant role in the emergence of the Great Moderation. Stock and Watson (2002) explore the hypothesis that the increasing share of the services sector in the economy, which is typically less volatile than the manufacturing sector, might have contributed to the overall reduction in economic volatility. The structural shift is considered to be part of the broader array of explanations for the decrease in output volatility observed during the Great Moderation. They also explore how the introduction of inventory management methods smoothed production, thereby stabilizing aggregate output. This supports the idea that businesses are now better equipped to handle fluctuations in demand and are more resilient in uncertain times. However, they conclude that while inventory management methods may be advantageous at the individual company level, they do not significantly influence the broader economic stability during business cycles.

The "good luck" hypothesis and its role in the Great Moderation is also discussed by Stock and Watson (2002), with emphasis on the potential impact of smaller macroeconomic shocks during this period. Stock and Watson (2002) explore the idea that the reduction in economic volatility could also be partly attributed to the occurrence of less severe economic shocks, which they refer to as "good luck." They argue that the impact of improved monetary policy might appear more significant due to less volatile productivity shocks and commodity price shocks, in reducing the variance of output growth. This shift makes it seem like policy improvements were more effective than they might have been in isolation. This perspective aligns with the observations of Bean (2009), previously Deputy Governor for Monetary Policy at the Bank of England, who pointed out in his speech that "shocks are not measured directly, only their consequences". Bean suggests that rather than attributing the stability during the Great Moderation to "good luck" or "smaller shocks," it may be more accurate to credit the implementation of effective economic policies and structural changes. These factors collectively have not only reduced the shocks, but also contained their impacts, providing a more nuanced explanation for the observed economic stability.

Clark (2009) suggests that the end of the Great Moderation is considered to be linked to the severe recession that started in late 2007, characterized by significant declines in economic activity that exceeded previous sharp recessions. However, Clark (2009) argues that the occurrence of a severe recession alone does not necessarily imply that the Great Moderation ended permanently. The increase in economic volatility could be temporary, and future volatility is expected to oscillate between high and low levels, with low volatility being more typical. This is also consistent with the findings of Stock and Watson (2012) where they find that the recession of 2007–09 was the result of shocks that were larger versions of shocks previously experienced. The future expected pattern of volatility, according to the Clark (2009), will likely see periodic increases in response to significant shocks. However, it will generally maintain a norm of lower volatility compared to pre-Great Moderation levels. Thus Clark (2009) concludes that the Great Moderation is not necessarily over, but rather it has evolved into a pattern where lower volatility is predominant, punctuated by episodes of higher volatility due to significant shocks.

2.3 National Bureau Of Economic Research And Business Cycle Dating

The Business Cycle Dating Committee of the National Bureau of Economic Research NBER (2024) tracks the progression of U.S. business cycles by marking the chronological peaks and troughs of economic recessions and expansions. The

NBER defines a recession as the period between the peak of economic activity and the following trough. Between each trough and peak, the economy is considered to be in an expansion. NBER (2024) characterizes a recession as a significant and widespread decline in economic activity persisting for several months. This interpretation allows for flexibility in the three criteria of depth, diffusion, and duration, whereby severe conditions in one area can compensate for lesser indicators in another. An example is the sharp and broad decline after the February 2020 peak, which justified classifying the short-lived downturn as a recession. We will use the NBER-defined recession to measure our MSDFM classification ability of recessions. The data series for NBER classified recession are retrieved from Federal Reserve Bank of St. Louis (1854).

The NBER (2024) committee focuses on comprehensive economic indicators rather than isolated sectors to determine the dates of these economic cycles. It bases its assessment on a variety of monthly economic data provided by several federal agencies. Data such as real personal income excluding transfers, nonfarm payroll employment, household employment surveys, real consumer spending, adjusted wholesale-retail sales, and industrial output are all considered. The relative importance of these indicators can vary, with recent emphasis on real personal income and nonfarm payroll employment.

According to NBER (2024), NBER's methodology for determining dates of economic peaks and troughs is deliberately retrospective. It refrains from announcing these turning points until it has enough data to minimize the need for subsequent revisions. The committee waits to confirm a peak until it is certain a recession has taken place, and similarly, it does not declare a trough until it is evident that an expansion has begun. This approach ensures that new economic phases are distinctly recognized and not mistaken for continuations of previous cycles. This results in the NBER's business cycle dating having a considerable delay compared to when the events actually occur.

Chauvet and Piger (2008) highlighted that the MSDFM model presents several key advantages over the traditional NBER method for dating peaks and troughs in business cycles. Its primary strength lies in its real-time performance, which enables more timely and accurate identification of business cycle phases. This enhanced capability allows for more effective monitoring and analysis of economic conditions as they evolve. Chauvet and Piger (2008) show that the MSDFM model is particularly adept at quickly identifying business cycle troughs, achieving this on average about 8 months earlier than NBER announcements. The MSDFM model also demonstrates high precision in real-time applications, consistently matching or closely approximating NBER's turning point dates, typically within one month and never deviating by more than two months. Lastly, the MSDFM model employs a transparent and reproducible statistical algorithm. This approach ensures objectivity and dependability in the dating process and adapts seamlessly to new data.

2.3.1 Early research by Burns and Mitchell 1946

Early research conducted by Burns and Mitchell (1946) focused on the analysis of macroeconomic variables to classify them as leading, lagging, or coincident indicators. Thus providing a descriptive framework for understanding cyclical movements in the economy. However, a significant limitation of their approach is the absence of a mathematical characterization of business cycle measurements. This deficiency results in a model that lacks the information necessary to accurately characterize the business cycles. Additionally, subsequent revisions to these indicators can significantly alter their initial readings.

2.3.2 Stock and Watson's comovement model (1988,1992)

Stock and Watson (1988a) introduced a model that tracks business cycles through the comovements of different economic components, proposing an alternative to the indicators from Burns and Mitchell (1946). Their model is based on negative shocks as an indicator for recessions and positive shocks for expansions. These are characterized within a linear and dynamically stable time series framework. Their approach did not successfully capture the recession of 1990, as evidenced by an analysis using a recession index derived from their non-switching dynamic factor model in Stock and Watson (1992). The inherent linearity of Stock and Watson's framework requires symmetry, resulting in equal scale, duration, and intensity of economic expansions and contractions. Moreover, the model overlooks the possibility of changes in the economy's stochastic structure over time, such as policy shifts.

2.3.3 Hamilton's Markov- switching model

Hamilton (1989) explores the nonlinear characteristics of business cycles by analyzing the growth rate of quarterly GNP through a nonlinear stationary process that includes shifts in its dynamic properties using a Markov switching approach. His research identifies asymmetries between periods of economic expansion and contraction and highlights variations in the behavior of different phases of the business cycle. Despite these insights, his model is univariate and limited in its ability to reflect economic fluctuations and comovements across various aggregate economic variables. Chauvet (1998) explored the limitations of this model and found that when this model is applied to monthly growth rates, it does not adequately capture some historical recessions, which the NBER identified. This suggests that relying on a single coincident variable might not encompass all the relevant information about the business cycle. Individual coincident variables might reflect noise rather than true business cycle trends in the monthly data.

2.3.4 Kim's Dynamic Factor Model

Kim (1994) combines Hamilton (1989)'s nonlinear filter with a nonlinear discrete version of the Kalman filter as described in Kalman (1960). This allows for the estimation of the unobserved state vector and the probabilities associated with the latent Markov regime as in Chauvet (1998). Kim (1994) suggests a solution to the complex computations when combining a Markov-switching and the equations of a state-space model. Each iteration of the Kalman filtering produces an M-fold increase in the number of cases to consider, where M is the total number of regimes at each point in time. This makes estimations of the model virtually intractable. To counteract this problem, approximations introduced by Harrison and Stevens (1976) are used for the estimations. Kim (1994) concludes that the basic filtering, smoothing, and maximum likelihood estimation prove to be adequate approximations. This results in the algorithm giving a significant computation time advantage while being comparable to a model without approximations.

2.3.5 Switching Dynamic Factor of Diebold and Rudebusch (1996)

Diebold and Rudebusch (1996) explored new ways of measuring business cycles. They revise traditional business cycle measurement by integrating both dynamic factor models and regime-switching models into their analysis. They highlighted a shift from the use of linear models, which focused on individual macroeconomic aggregates to models that incorporate complex interactions and non-linear dynamics. They argued that business cycles should be viewed as regime shifts and as comovements across economic variables. They propose a framework that combats the earlier limitations of linear models and aims to capture the complexities and dynamics of the business cycles but do not estimate it.

2.3.6 The framework of Chauvet (1998)

Chauvet (1998) constructs a comprehensive model that synthesizes the frameworks of Stock and Watson and Hamilton, as initially proposed by Diebold and Rudebusch (1996). This integrated approach seeks to encapsulate the concepts of business cycle synchronization and asymmetries as envisioned by Burns and Mitchell, as well as the NBER.

Chauvet (1998) introduces a theoretical model that utilizes a formal probability framework to depict business cycles, producing coincident indicators and probabilities for economic expansions and recessions. The method is designed to be reproducible and allows for the real-time analysis of business cycles. For instance, the likelihood of a recession occurring in a specific month can be determined through these probabilities and coincident indicators, aligned with current macroeconomic signals.

Within this model, business cycles are depicted through a dynamic factor model with regime switching. The dynamic factor, an unobserved variable, encapsulates the common cyclical patterns of various macroeconomic indicators. This factor undergoes discrete shifts to reflect the distinct characteristics of business cycle phases: expansions tend to be prolonged with a longer average duration, whereas recessions are typically sharper and shorter. Thus, Chauvet (1998) models business cycles by representing the synchronized economic activities across different sectors through an unseen dynamic factor. The inherent asymmetry between expansion and contraction phases is modeled by allowing the dynamic factor to switch regimes following a Markov process.

Chauvet (1998) 's model is estimated by optimizing its likelihood function. Empirical evidence indicates that the combined use of the dynamic factor and regime-switching approaches provides an effective representation of the data relative to prior studies. This consistency is maintained across both in-sample and out-of-sample datasets and applies to both revised and real-time data.

2.4 Particle Swarm Optimization

Wang, Tan, and Liu (2018) provides a good overview of the inner workings of the PSO algorithm and is the basis of this entire section.

Particle Swarm Optimization (PSO) is a robust stochastic optimization technique based on the movement and intelligence of swarms. PSO applies the concept of social interaction to problem-solving to enhance the performance of individual solutions, which are represented as particles in the search space. It was developed by Kennedy and Eberhart (1995), inspired by social behaviors of birds and fish. The particles "fly" through the problem space by following the current optimum particles.

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. Each particle updates its position based on its velocity, personal best position, and the global best or local best position found by the swarm. The PSO formula for updating velocity and position are given by:

$$v_{i,d}^{(t+1)} = w \cdot v_{i,d}^{(t)} + c_1 \cdot r_1 \cdot (pbest_{i,d}^{(t)} - x_{i,d}^{(t)}) + c_2 \cdot r_2 \cdot (gbest_d^{(t)} - x_{i,d}^{(t)}), \tag{2.1}$$

$$x_{i,d}^{(t+1)} = x_{i,d}^{(t)} + v_{i,d}^{(t+1)}, \tag{2.2}$$

where $v_{i,d}^{(t)}$ is the velocity of particle i in dimension d at iteration t, $x_{i,d}^{(t)}$ is the current position, $pbest_{i,d}^{(t)}$ is the best previous position of particle i, $gbest_d^{(t)}$ is the best position found by the swarm, c_1 and c_2 are learning factors, r_1 and r_2 are random functions in the range $[0, 1]$ and w is the inertia weight.

The role of the inertia weight w, which controls the impact of the previous velocities on the current one, is crucial in balancing the exploration and exploitation abilities of the swarm. A larger w facilitates exploration while a smaller w promotes exploitation. The acceleration coefficients c_1 and c_2, often called cognitive and social parameters respectively, regulate the personal and collective influence on the movement of particles.

Since its introduction, PSO has been successfully applied to various domains including engineering, economics, and data science, due to its simplicity, efficiency, and ability to converge rapidly to a reasonably good solution. Ongoing research continues to enhance PSO by modifying its inertia weight, learning factors, and introducing hybridization with other metaheuristic techniques to tackle more complex problems.

DATA AND MODEL

In this section, the data sources and the model used for our analysis will be presented. The economic data series used in our models are retrived from the Federal Reserve Economic Data (FRED). Next, the preprocessing steps taken to prepare the data for analysis are described. Then the econometric model used to analyze the data is introduced, based on the methodology outlined by Chauvet (1998). The model is then specified with mathematical equations, explaining the relationships between the observed data and latent factors. Next, the measurement and transition equations are explained while incorporating a Markov switching process to capture regime switching. Additionally, the Kalman and Hamilton filters are used in the estimation process. This model captures the complexity of economic regimes and assesses structural changes in business cycle dynamics. To optimize the parameter estimation Particle Swarm Optimization (PSO) and the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm are used.

3.1 Data

All data utilized in this study were retrieved from the Federal Reserve Economic Data (FRED), a comprehensive database maintained by the Federal Reserve Bank of St. Louis. This database is publicly accessible, allowing for the replication and verification of our analysis. The choice of FRED as a data source aligns with its frequent use in economic research, providing a reliable basis for longitudinal studies.

Following the methodology of the seminal paper by Chauvet (1998), this thesis employs four critical economic series:

- Manufacturing and trade sales (CMRMTSPL). Retrieved from Federal Reserve Bank of St. Louis (1967).
- Total personal income less transfer payments (W875RX1). Retrieved from U.S. Bureau of Economic Analysis (1959b).

- Employees on non-agricultural payrolls (PAYEMS). Retrieved from U.S. Bureau of Labor Statistics (1939).
- Industrial production (INDPRO). Retrieved from Board of Governors of the Federal Reserve System (US) (1919).

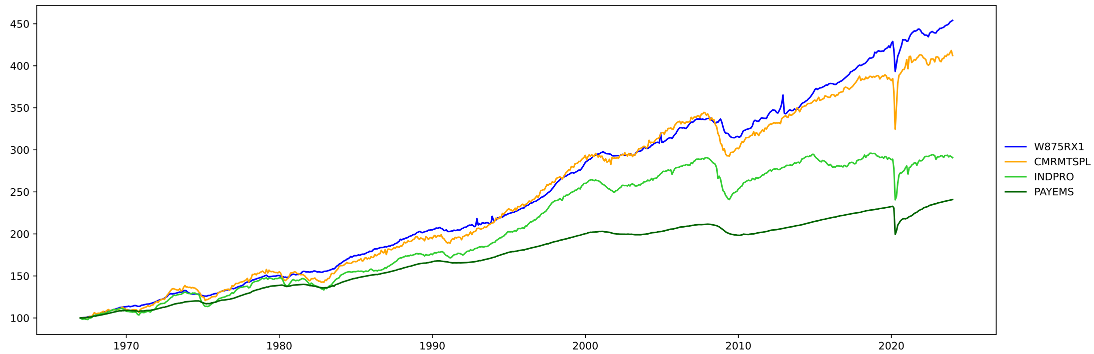

Figure 3.1.1: Base 100 index for our chosen macroeconomic time-series.

These series correspond to the ones used by Chauvet (1998), but have been updated due to the discontinuation of the original series (MTS, PILTP, ENAP and IP). Despite the update, these series cover the same economic indicators and are the ones used by Chauvet and Piger (2008), which suggests that they are suitable replacements for the economic series used by Chauvet (1998).

The dataset spans the period from January 1967 to February 2024. This time frame extends the original analysis by Chauvet (1998), with the additional objective of capturing and analyzing current economic trends. The data preprocessing involved two primary steps: Firstly, the logarithmic differences of the series are computed. Secondly, each series is standardized to achieve a zero mean and unit variance.

Although Chauvet (1998) does not explicitly mention scaling the data to have unit variance, it can be inferred from their results. This inference is supported by the unusually large estimated loadings and standard deviations of errors observed, which suggest that the variance of the dependent variables had been scaled. Further support comes from Chauvet and Piger (2008), who explicitly states that the dependent variables were scaled to unit variance.

Kim and Nelson (2017) supports the practice of scaling to unit variance, noting that it enhances numerical stability and facilitates easier convergence during model estimation.

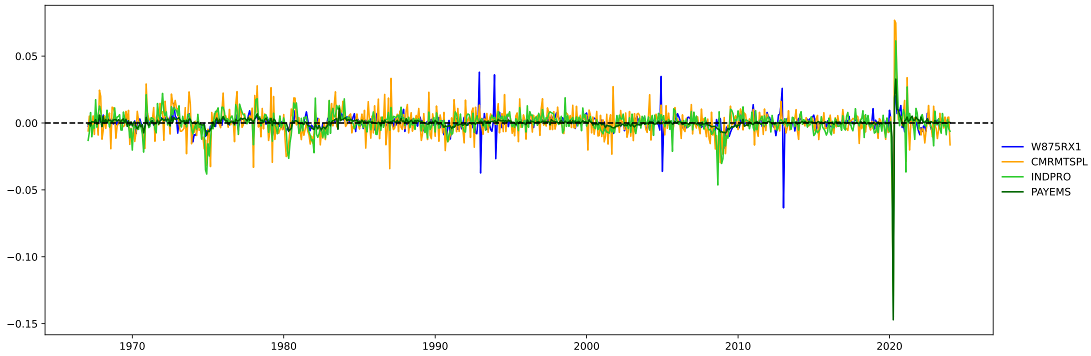

Figure 3.1.2: Pre-processed macroeconomic time-series.

Further, a statistical test for stationarity was conducted, specifically the Augmented Dickey-Fuller test, as detailed by Mushtaq (2011). This test did not provide sufficient evidence to dismiss the hypothesis of non-stationarity in favor of stationarity for each series at the 5% significance level. Moreover, when the cointegration test proposed by Stock and Watson (1988b) was applied to the four coincident economic variables to see whether they exhibited any long-term equilibrium relationships. The test results were unable to refute the null hypothesis, suggesting that the variables are not cointegrated.

	CMRMTSPL	**W875RX1**	**PAYEMS**	**INDPRO**
ADF Statistic	-18.782	-5.604	-19.102	-19.515
P-value	0.000	0.000	0.000	0.000

Table 3.1.1: Evidence against the null hypothesis, reject the null hypothesis. Data has no unit root and is stationary.

Despite these challenges, the chosen dataset from FRED provides a robust foundation for examining the economic indicators of interest. The preprocessing steps ensure the data's suitability for our analysis, adhering to the methodological framework established by Chauvet (1998). Future research may benefit from further clarification of data series generation methods and an exploration of alternative data sources to validate the findings presented herein.

3.2 General Model and Estimation Procedure

The general model closely resembles the one used by Chauvet (1998) and can be expressed as:

$$Y_{it} = \lambda_{ik} * F_{kt} + v_{it} \tag{3.1}$$

$$F_{kt} = \mu_{S_t} + \phi(L_1)F_{kt-1} + \eta_{kt}, \quad \eta_{kt} \sim N(0, \sigma_{1S_t}) \tag{3.2}$$

$$v_{it} = d_i(L_2)v_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, \sigma_2) \tag{3.3}$$

$$\mu_{S_t} = \sum_{m=1}^{M} \mu_m S_{mt}$$

$$\sigma_{1S_t} = \sum_{m=1}^{M} \sigma_m S_{mt}$$

Where 3.1 is the measurement equation and 3.2 and 3.3 together make up the state equation. Y_{it} is a vector of the observed data, F_{kt} is the vector of latent factors, λ_{ik} is the factor loadings and v_{it} is the integrated error. From equation 3.2 one can see that the factors are governed by an $AR(L)$ process where L is a lag operator and ϕ is the estimated parameters with regime-switching mean (μ_{S_t}) aned error term η_{kt} with zero mean and switching variance ($\sigma_1 S_t$). The variance is defined as (σ_{S_t}), where S_{mt} is the switching variable which takes values $\{0, 1\}$ for all time steps where m is the regime and M is the total number of regimes. Equation 3.3 says that the errors of the measurement equation is governed by a $AR(L_2)$ process. The measurement and transition equation can be expressed in its reduced form as seen in Equations 3.4 and 3.5. Where Y_t is a vector of the dependent variables, Z is the observation matrix, ξ_t is the state vector, $\alpha_{\xi S_t}$ is a switching Constant term, T is the transition matrix and u_t is the process noise.

$$Y_t = Z\xi_t \tag{3.4}$$

$$\xi_t = \alpha_{\xi S_t} + T\xi_{t-1} + u_t, \quad u_t \sim N(0, \Sigma_{S_t}). \tag{3.5}$$

This can also be expressed using matrix notation. Equations 3.6 and 3.7 provide an example with $i = 4$, $k = 1$, $L_1 = 2$ and $L_2 = 1$. Which is the same specification used by Chauvet (1998).

Masurement Equations:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} F_{1t} \\ F_{1t-1} \\ v_{1t} \\ v_{2t} \\ v_{3t} \\ v_{4t} \end{bmatrix} \tag{3.6}$$

Transition Equations:

$$\begin{bmatrix} F_{1t} \\ F_{1t-1} \\ v_{1t} \\ v_{2t} \\ v_{3t} \\ v_{4t} \end{bmatrix} = \begin{bmatrix} \alpha_{S_t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_4 \end{bmatrix} * \begin{bmatrix} F_{1t-1} \\ F_{1t-2} \\ v_{1t-1} \\ v_{2t-1} \\ v_{3t-1} \\ v_{4t-1} \end{bmatrix} + \begin{bmatrix} \eta_{S_t} \\ 0 \\ \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{4t} \end{bmatrix} \tag{3.7}$$

The subscript S_t denotes that some of the parameters in the matrices can be time varying depending on the regime S at time t. The number of regimes are denoted with M. The transition probabilities are given Equation 3.2 where each row sums to 1.

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{S1} \\ p_{12} & p_{22} & \cdots & p_{S1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1S} & p_{2S} & \cdots & p_{SS} \end{bmatrix}$$

$$p_{ij} = Prob[S_{t=j} \mid S_{t-1} = i], \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i$$

In a case with no switching, the goal is to forecast the state vector ξ_t based on information up to time $t-1$, denoted as $\xi_{t|t-1}$. However, in the case of Markov switching variables, the goal is to form such a forecast conditional on that S_t being value j and S_{t-1} being value i, which is denoted $\xi^{(i,j)}_{t|t-1}$.

In order to apply the esitimation procedure used in Kim (1994), it is necessary to calculate M^2 forecasts at each time t. Therefore, it is much more computationally demanding in the simple case with no switching. Kim (1994) uses the same forward two-step procedure of the Kalman filter, meaning that it only uses data up to time t to make inference on the unobserved components at time t.

Step 1. The first step is the prediction step. The predictions on the state variables are based on information up to time $t-1$ and all the possible outcomes of S_t denoted by j and the outcome of S_{t-1} denoted by i. This stage is made up of four equations, where the first, Equation 3.8, is the prediction of the state variables based on the model's time series specifications.

$$\xi_{t|t-1}^{(i,j)} = \alpha_j + T\xi_{t-1|t-1}^{i} \tag{3.8}$$

The second, Equation 3.9, is the prediction of the covariance matrix of the state variables.

$$P_{t|t-1}^{(i,j)} = TP_{t-1|t-1}^{i}T' + \Sigma_j \tag{3.9}$$

The third, Equation 3.10, is the prediction error of the time series of interest.

$$\eta_{t|t-1}^{(i,j)} = Y_t - Z\xi_{t|t-1}^{(i,j)} \tag{3.10}$$

Lastly, Equation 3.11, is the variance of the prediction error.

$$f_{t|t-1}^{(i,j)} = ZP_{t|t-1}^{(i,j)}Z' \tag{3.11}$$

Step 2. Once the predictions for the state variables has been made, they are then observed, and the prediction is then updated with information up until time t. This step consists of three equations. The first, Equation 3.12, is the prediction of the state variables on the full information set, where $K_t^{(i,j)}$ is the Kalman gain, which determined the optimal weight to give the new information making prediction about ξ_t. The second, Equation 3.13, is the Kalman gain.

$$\xi_{t|t}^{(i,j)} = \xi_{t|t-1}^{(i,j)} + K_t^{(i,j)}\eta_{t|t-1}^{(i,j)} \tag{3.12}$$

$$K_t^{(i,j)} = P_{t|t-1}^{(i,j)}Z'(f_{t|t-1}^{(i,j)})^{-1} \tag{3.13}$$

The Last, Equation 3.14, is the updating of the covariance matrix of the state variables.

$$P_{t|t}^{(i,j)} = P_{t|t-1}^{(i,j)} - K_t^{(i,j)}Z'P_{t|t-1}^{(i,j)} \tag{3.14}$$

The seven equations above, make up the full Kalman filter routine as described by Kalman (1960) for a given set of regime outcomes $S_t = j$ and $S_{t-1} = i$. If Y_t is missing for any observation the updated variable then becomes:

$$\xi_{t|t}^{(i,j)} = \xi_{t|t-1}^{(i,j)}$$

$$P_{t|t}^{(i,j)} = P_{t|t-1}^{(i,j)}$$

$$K_t^{(i,j)} = 0$$

$$f_{t|t-1}^{(i,j)} = \infty$$

The key contribution of Kim (1994) is to collapse the terms into the best estimate of S_t given by Equations 3.15 and 3.16.

$$\xi_{t|t}^{j} = \frac{\sum_{i=1}^{M} Pr[S_{t-1} = i, S_t = j \mid \psi_t]}{Pr[S_t = j \mid \psi_t]} \xi_{t|t}^{(i,j)} \tag{3.15}$$

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{M} Pr[S_{t-1} = i, S_t = j \mid \psi_t]}{Pr[S_t = j \mid \psi_t]} (P_{t|t}^{(i,j)} + (\xi_{t|t}^{i} - \xi_{t|t}^{(i,j)})(\xi_{t|t}^{i} - \xi_{t|t}^{(i,j)})') \tag{3.16}$$

Note that these collapsed terms involve approximations. This is because ξ_t, conditional on information up to time t, S_t and S_{t-1}, is a mixture of normal distributions. However, the algorithm is still considered to make reasonable inference about ξ_t.

Step 3. After doing the steps for the Kalman filter we will proceed with calculating the marginal- and conditional density of Y_t using the Hamilton filter as described in Hamilton (1989). This step makes inference about the probability terms that show up in the above equations. First, we need to calculate the transition probability given by Equation 3.17.

$$Pr[S_t = j, S_{t-1} = i \mid \psi_{t-1}] = Pr[S_t = j, S_{t-1} = i] Pr[S_t = i \mid \psi_{t-1}] \quad \forall i, j \tag{3.17}$$

Secondly, we calculate the conditional density based on the prediction error decomposition. We first need the joint density of Y_t, S_t and S_{t-1} given by Equation 3.18.

$$\begin{aligned} f(Y_t, S_t = j, S_{t-1} = i \mid \psi_{t-1}) = \\ f(Y_t \mid S_t = j, S_{t-1} = i) Pr[S_t = j, S_{t-1} = i \mid \psi_{t-1}] \quad \forall i, j \end{aligned} \tag{3.18}$$

The marginal density if Y_t is then given by Equation 3.19.

$$f(Y_t \mid \psi_{t-1}) = \sum_{j=1}^{M} \sum_{i=1}^{M} f(Y_t \mid S_t = j, S_{t-1}, \psi_{t-1}) Pr[S_t = j, S_{t-1} = i \mid \psi_{t-1}] \tag{3.19}$$

Where the conditional density of Y_t is defined by Equation 3.20.

$$\begin{aligned} f(Y_t \mid S_t = j, S_{t-1} = i, \psi_{t-1}) = \\ (2\pi)^{-\frac{N}{2}} \left| f_{t|t-1}^{(i,j)} \right|^{-\frac{1}{2}} \exp(-\frac{1}{2} \eta_{t|t-1}^{(i,j)\,\prime} f_{t|t-1}^{(i,j)} \eta_{t|t-1}^{(i,j)}) \quad \forall i, j \end{aligned} \tag{3.20}$$

Where N is the number of variables in Y_t. Finally, we update the probability terms using Equation 3.21 to get Equation 3.22.

$$P[S_t = j, S_{t-1} = i \mid \psi_t] = \frac{f(Y_t \mid S_t = j, S_{t-1} = i, \psi_{t-1})}{f(Y_t \mid \psi_{t-1})} \quad \forall i, j \tag{3.21}$$

$$Pr[s_t = j \mid \psi] = \sum_{i=1}^{M} P[S_t = j, S_{t-1} = i \mid \psi_t] \tag{3.22}$$

Once the Kalman and Hamilton filters have been applied to the data, a backward smoothing procedure can be used to enhance the inference of the state variables at time t. For the basic Kalman filter, this smoothing procedure involves only two equations. The first, Equation 3.23, updates the prediction of the state variables utilizing all available information.

$$\xi_{t|T}^{(j,k)} = \xi_{t|t}^{j} + P_{t|t}^{j} T (P_{t+1|t}^{(j,k)})^{-1} (\xi_{t+1|T}^{k} - \xi_{t+1|t}^{(j,k)}) \tag{3.23}$$

The second, Equation 3.24, updates the covariance matrix of the state variables based on all the available information.

$$P_{t|T}^{(j,k)} = P_{t|t}^{j} + P_{t|t}^{j} T (P_{t+1|t}^{(j,k)})^{-1} (P_{t+1|T}^{k} - P_{t+1|t}^{(j,k)}) (P_{t+1|t}^{(j,k)})^{-1} T P_{t|t}^{j} \tag{3.24}$$

In the case of a Markov-Swicthing model we need two more equations. The first is the joint probability of $S_t = j$ and $S_{t+1} = k$ based on the full information given by Equation 3.25.

$$Pr[S_t = j, S_{t+1} = k \mid \psi_T] = \frac{Pr[S_{t+1} = k \mid \psi_T]Pr[S_t = j \mid \psi_t]Pr[S_{t+1} = k \mid S_t = j]}{Pr[S_{t+1} = k \mid \psi_t]} \tag{3.25}$$

The second is Equation 3.26.

$$Pr[S_t = j \mid \psi_T] = \sum_{k=1}^{M} Pr[S_t = j, S_{t+1} = k \mid \psi_T] \tag{3.26}$$

Finally the filter also requires collapsing the terms in the smoothing algorithm, using the approximation given by Equations 3.23 and 3.24. This gives Equations 3.27 and 3.28.

$$\xi^j_{t|T} = \frac{\sum_{i=1}^{M} Pr[S_t = j, S_{t+1} = k \mid \psi_T]}{Pr[S_t = j \mid \psi_T]} \xi^{(j,k)}_{t|T} \tag{3.27}$$

$$P^j_{t|T} = \frac{\sum_{i=1}^{M} Pr[S_t = j, S_{t+1} = k \mid \psi_T]}{Pr[S_t = j \mid \psi_T]} (P^{(j,k)}_{t|T} + (\xi^j_{t|T} - \xi^{(j,k)}_{t|T})(\xi^j_{t|t} - \xi^{(j,k)}_{t|K})') \tag{3.28}$$

The smoothed value of the state variable is given by Equation 3.29.

$$\xi_{t|T} = \sum_{j=1}^{M} Pr[S_t = j \mid \psi_T] \xi^j_{t|T} \tag{3.29}$$

A by-product of the above filter is that the conditional log-likelihood is obtained from 3.19 where the sample conditional log-likelihood is given by Equation 3.30.

$$LL = \log(f(Y_T, Y_{T-1}, \cdots \mid \psi_0) = \sum_{t=1}^{T} \log(f(Y_t \mid \psi_{t-1})) \tag{3.30}$$

Inserting the marginal density of Y_t given by Equation 3.19, the conditional density of Y_t given by Equation 3.20 and the probability terms given by Equation 3.21 into Equation 3.30. The log-likelihood can be more explicitly written as in Equation 3.31.

$$LL = \sum_{t=1}^{T} \log(\sum_{j=1}^{M} \sum_{i=1}^{M} (2\pi)^{-\frac{N}{2}} \left| f^{(i,j)}_{t|t-1} \right|^{-\frac{1}{2}} \exp(-\frac{1}{2} \eta^{(i,j)\,\prime}_{t|t-1} f^{(i,j)}_{t|t-1} \eta^{(i,j)}_{t|t-1}) \tag{3.31}$$

The filter is derived under the assumption that the parameters of the model is known. To estimate the parameters of the model the log-likelihood function defined in 3.31 can maximized with respect to the unknown parameters using a nonlinear optimization algorithm. Both Chauvet (1998) and Kim (1994) used the

BFGS algorithm for this step.

A notable limitation of the BFGS algorithm is its susceptibility to converge to sub-optimal solutions based on the initial parameters chosen. This highlights the importance of selecting effective initial parameters. Unfortunately, comprehensive studies on identifying optimal initial parameters are lacking. For this reason, a hybrid optimization strategy was adopted. Unlike BFGS, Particle Swarm Optimization (PSO) does not rely on initial parameters, but explores the entire parameter space to find the best solution. These are subsequently used as initial parameters for the BFGS algorithm. Although this hybrid method is more thorough than heuristic selection, it does not assure that the solution is the global maximum of the log-likelihood function. Moreover, this approach is time-intensive and computationally demanding, as the PSO algorithm evaluates many points in the parameter space per iteration.

3.3 Three-regime Dynamic Factor Model Specification

Chauvet (1998) estimated models using both monthly and quarterly data. The model estimated on monthly data will be the main focus of this as it performed the best. Chauvet (1998) used a two-regime Markov Switching Dynamic Factor Model with switching mean and a single dynamic factor. The state variables are governed by an AR(1) process, and can be expressed as such:

$$Y_{it} = \lambda_i * F_t + v_{it} \tag{3.32}$$

$$F_t = \mu_{S_t} + \phi_1 F_{t-1} + \eta_t, \quad \eta_t \sim N(0, \sigma_1) \tag{3.33}$$

$$v_{it} = d_{i1} v_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, \sigma_2) \tag{3.34}$$

$$\mu_{S_t} = \mu_1 S_{1t} + \mu_2 S_{2t}$$

where $S_{mt} = 1$, if $S_t = m$ and $S_{mt} = 0$ otherwise.

Several orders of auto-regressive processes for the loadings, dynamic factor, and integrated errors were tested. Consistent with Diebold and Rudebusch (1996), our findings also suggest that adding additional lags could enhance the approximation quality. Diebold and Rudebusch (1996) notes the possibility that the AR(1) model may induce serial correlation in the error, which could be spuriously detected by regime-switching dynamics. However, higher order AR processes increase the complexity of the model, complicating the estimation process. Consequently, an AR(1) process was chosen for both the integrated errors and the dynamic factor, aligning with the approaches of Chauvet (1998) and Diebold and Rudebusch (1996).

We propose to expand this model to a three regime model with switching variance. The proposed model can be expressed as such:

$$Y_{it} = \lambda_i * F_t + v_{it} \tag{3.35}$$

$$F_t = \mu_{S_t} + \phi_1 F_{t-1} + \eta_t, \quad \eta_t \sim N(0, \sigma_{1_{S_t}}) \tag{3.36}$$

$$v_{it} = d_{i1} v_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, \sigma_2) \tag{3.37}$$

$$\mu_{S_t} = \mu_1 S_{1t} + \mu_2 S_{2t} + \mu_3 S_{3t}$$

$$\sigma_{1_{S_t}} = \sigma_{1_1} S_{1t} + \sigma_{1_2} S_{2t} + \sigma_{1_3} S_{3t}$$

where $S_{mt} = 1$, if $S_t = m$ and $S_{mt} = 0$ otherwise.

3.4 Expected Duration of Regime

Kim (1994) show that the expected duration of any regime can be derived from the diagonal elements of the transition matrix according to Equation 3.38.

$$E[D] = \frac{1}{1 - p_{jj}} \tag{3.38}$$

Here, p_{jj} is the probability of transitioning from regime j to regime j, or put more simply: the probability of staying in the same regime. For instance Chauvet (1998) found a 85.5% probability of staying in a recession the following month given that one is already in a recession. From this, the expected length of a recession would be:

$$\frac{1}{1-0.855} = 6.9$$

This suggests that a recession on average lasts around 7 months. This is the approach used by Hamilton (1989).

EXPERIMENT AND RESULTS

In this section, an evaluation of the performance and adequacy of the proposed three-regime Markov Switching Dynamic Factor Model (MSDFM) will be estimated. Additionally, a comparison with the original two-regime model and other variations will be made. Then the models will be evaluated in order to rank them based on different metrics. The evaluation metrics include goodness-of-fit measures, Receiver Operating Characteristic (ROC) curves, Area Under the ROC Curve (AUROC), and the Brock-Dechert-Scheinkman (BDS) test for independence. Then the estimated models with the evaluation metrics, starting from the simple non-switching dynamic factor model to the full three regime model, will be presented. The purpose of this is to illustrate the effect of increasing the models' complexity, and how that may affect the ability to explain the observed data.

4.1 Model Evaluation And Specification Tests

Conducting statistical tests on the models' switching parameters (μ_{S_t} and σ_{S_t}) and transition probabilities presents unique challenges. Garcia (1998) points out that if the switching parameters are zero, the transition probabilities become unidentified, rendering the score vector for the constrained maximum likelihood estimates (α_{s_t}, σ_{s_t}, p_{ii}) equal to zero. Consequently, traditional tests such as the likelihood ratio, Lagrange multiplier, and Wald tests do not follow standard asymptotic distributions.

Although Garcia (1998) calculated critical values for the asymptotic distribution of the likelihood ratio test comparing a two-regime Markov switching model with switching mean to a model without switching, critical values for other model specifications remain unknown. For this reason, the efficacy of a three-regime model was evaluated by comparing log-likelihood, AIC, and BIC to assess model goodness-of-fit, alongside ROC and AUROC for predictive power on NBER recessions. Additionally, a BDS test was utilized to determine if the estimated models suffer from misspecification. While this method is less definitive than the statistical testing framework provided by Garcia (1998), it is sufficient for a comparative analysis. Especially considering the complexity involved in determining critical values for a test comparing two and three-state models.

To test the significance of the estimated non-switching parameters, the t-statistics were obtained using standard errors calculated from the Hessian matrix, which resulted from BFGS optimization.

4.1.1 Goodness-of-Fit Metrics

To effectively compare different statistical models in terms of their fit to the data, several key metrics were utilized. The Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the log-likelihood ratio (LLR). These metrics provide a robust framework for evaluating the relative quality of each model, taking into account various aspects of model performance and complexity.

$$AIC = 2k - 2\ln \hat{L}, \tag{4.1}$$

The AIC, defined in Equation 4.1, balances the model complexity against the goodness-of-fit, where $\hat{L}$ represents the maximum value of the likelihood function for the model, and k denotes the number of estimated parameters. Lower values of AIC suggest a model with a better fit, given an equivalent level of complexity.

$$BIC = k \ln n - 2\ln \hat{L}, \tag{4.2}$$

Similarly, the BIC is given by Equation 4.2, where n is the number of observations. BIC modifies the penalty for complexity by incorporating the number of observations, typically favoring simpler models compared to AIC, especially as the sample size increases.

$$\lambda_{\text{LR}} = -2\left[\ \ln L(\theta_0) - \ln L(\hat{\theta})\ \right], \tag{4.3}$$

The likelihood ratio, defined in Equation 4.3, compares the log-likelihoods of the estimated model $\hat{\theta}$ and a null model θ_0. This ratio assesses how well the model fits the data relative to the null model. In this thesis a non-switching Dynamic Factor Model is used as the null model. Thus the reported likelihood ratio can be interpreted as the relative improvement of introducing different Markov-Switching specifications to the model.

Collectively, these metrics guide the selection of an optimal model by quantifying the trade-off between complexity and fit, aiding in the identification of models that best capture the underlying patterns of the dataset.

4.1.2 ROC and AUROC

The performance of various model specifications is tested to determine their classification power for recessions. The recessions classified by the NBER serve as the benchmark for a recession indicator. As described in Fawcett (2006) ROC (Receiver Operating Characteristic) and AUROC (Area under the ROC curve) are statistical tools used to evaluate the performance of classification models. The ROC curve illustrates how the true positive rate and false positive rate of a classifier change with different threshold settings. This curve helps visualize the trade-off between catching true positives and avoiding false positives.

The AUROC on the other hand, provides a single metric summarizing the entire ROC curve. It measures the likelihood that the model will rank a randomly chosen positive instance higher than a randomly chosen negative one. An AUROC value ranges from 0.5, indicating a performance no better than random, to 1, indicating perfect classification accuracy. This metric is particularly useful for comparing different model specifications because it condenses the ROC curve information into a single, interpretable number.

ROC and AUROC are relevant in our use case to assess how well the different model specifications correctly identify the recession regime compared to recessions designated by NBER. The data was retrieved from Federal Reserve Bank of St. Louis (1854). ROC curves allow us to visually show the different model specifications on how the true positive rate is relative to the false positive rate in each of the specifications. AUCROC will be used to list and rank the different model specifications, since the higher the area under the ROC curve the more effective a specification is at identifying the NBER recessions. A large AUCROC signifies a higher true positive classification compared to classifying a false positive.

4.1.3 Testing For i.i.d Disturbances

To assess the appropriateness of our model structure, the disturbances in the observable variables will be evaluated. A correctly specified model should exhibit disturbances that are serially independent and exhibit minimal correlation among themselves for each observable variable. Consequently, the sample auto-correlation of the residuals should approximate zero for observations separated by more than one period, and ε_t should resemble white noise. Furthermore, we employ the

Broock et al. (1996) BDS test for non-linear models to verify the assumption taht the distubances are independently and identically distributed (i.i.d.). For the vector $\varepsilon_{tm} = \varepsilon_t, \varepsilon_{t+1}, \varepsilon_{t+2}, \ldots, \varepsilon_{t+m-1}$, we select m values ranging from 2 to 5, and λ equals the standard deviation of ε_t, representing the distance between any two vectors, ε_{tm} and ε_{sm}. This testing approach involves calculating the likelihood that these vectors fall within the distance λ. Although this test was not developed to be a leading indicator it can still help in avoiding misspecification. Therefore, this test gives further evidence into whether the model is correctly specified.

4.2 Estimation Results

In this subsection, the goodness-of-fit, AUROC, and transition probabilities results are presented for multiple model specifications. These metrics lay the foundation for comparing the models to each other. First, the results for a simple non-switching Dynamic Factor model are presented, following this the models gradually increase in complexity.

A non-switching Dynamic Factor model (1-MSDFM) was estimated in order to compare switching and non-switching specifications of the same model. The non-Switching Dynamic Factor model is estimated without a factor mean as a Wald-test suggests that it is not statistically different from zero. This is the reason for omitting it from the model. This is not surprising, as the dependent variables are demeanded in pre-processing so it follows that the dynamic factor should also be zero-mean. The one regime variant of the model shows promise in maximizing the log Likelihood and scoring second to last in AIC and BIC. This specification, however, cannot be measured using ROC curves or AUC due to there being only one regime.

	AIC	BIC	Log-Lik	LR	AUROC
1-DFM	1 485.84	1 544.74	-729.92	-	-
2-MSDFM	441.46	518.49	-203.73	1 052.38	50.59%
2019-2-MSDFM	2 354.63	2 431.66	-1 160.32	-860.79	97.06%
3-MSDFM	320.55	406.63	-141.27	1 177.30	94.69%
4-MSDFM'	305.04	400.19	-131.52	1 196.81	94.63%
4-MSDFM	-129.67	-16.39	89.83	1 639.51	94.33%
Chauvet (1998)	3 587.82	3 659.16	-1 776.91	25.12	99.86%

Table 4.2.1: Likelihood ratios are calculated relative to the 1-DFM model ($\lambda_{\text{LR}} = -2(\ell(\theta_0) - \ell(\hat{\theta}))$). When calculating the AUROC both the COVID-19 regime and the recession regime was regarded as a combined recession regime.

Expanding the single-regime model to a Two-Regime Markov Switching Dynamic Factor Model (2-MSDFM), as specified by Chauvet (1998), yields a significant improvement in the goodness-of-fit metrics over the reference model, as evident from Table 4.2.1. Contrary to the findings of Chauvet (1998) the empirical results indicate that while the solution that maximized the log-likelihood captures the dynamics of the COVID-19 pandemic (as shown in Figure 4.2.1), it fails to capture business cycle dynamics. This observation suggests that the COVID-19 pandemic may be qualitatively different from typical recessions, characterized by its sudden onset, short duration, and greater magnitude.

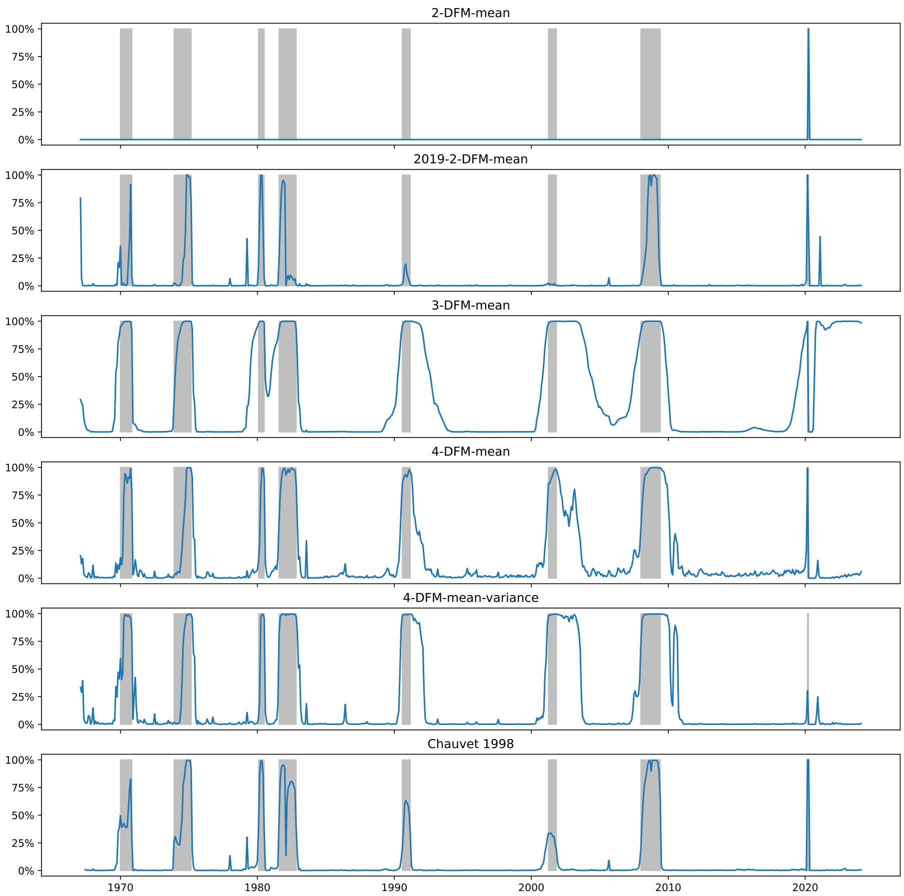

Figure 4.2.1: Estimated Recession Probabilities with NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854). The recession propbabilities of Chauvet (1998)'s model is retrieved from Chauvet, Marcelle and Piger, Jeremy Max (1967).

As depicted in Figure 4.2.1, omitting data prior to the COVID-19 pandemic allows the 2019-2-MSDFM to exhibit business cycle switching. However, evaluating the 2019-2-MSDFM on the entire dataset reveals poorer performance compared to the non-switching factor model in our goodness-of-fit metrics. This necessitates the integration of the COVID-19 pandemic data into the model estimation. Consequently, estimating the three-regime model initially hypothesized to capture a moderate growth regime unexpectedly resulted in identifying a COVID-19 pandemic regime. This shows that the model classifies the COVID-19 recession as quantitatively distinct from the typical recession regime. As a result, the model now includes the traditional expansionary and recessionary regimes, along with a regime specifically to account for the unique characteristics of the COVID-19 pandemic recession.

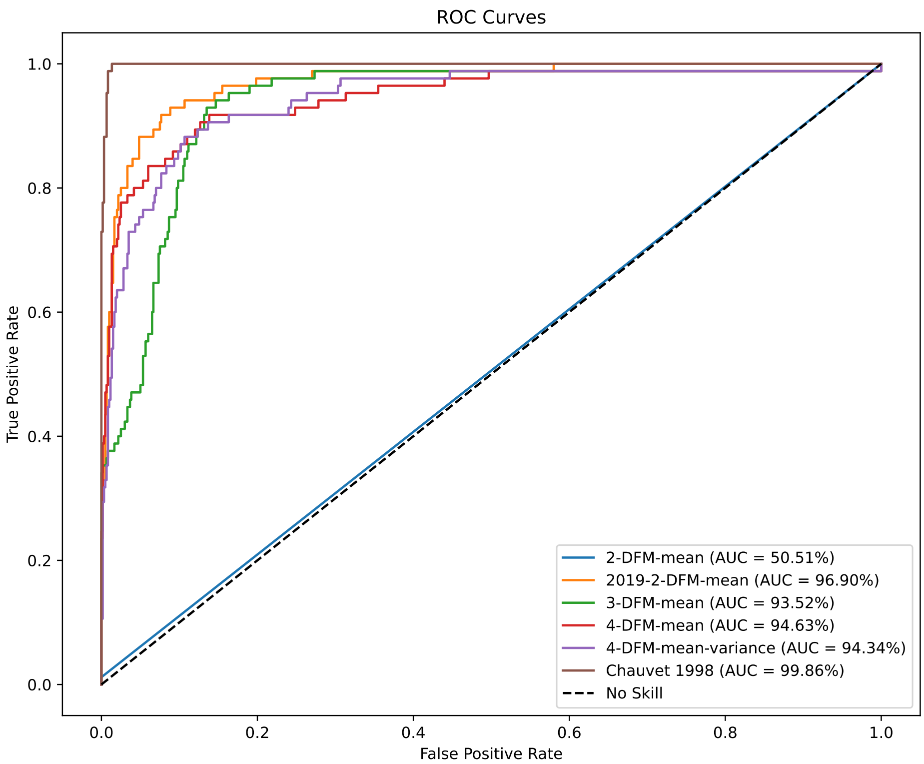

Figure 4.2.2: ROC Curves. Data for Chauvet (1998) was retrieved from Chauvet, Marcelle and Piger, Jeremy Max (1967).

The Three-regime Markov-Switching Dynamic Factor Model (3-MSDFM) exhibits enhanced performance over the two-regime models in terms of goodness-of-fit metrics. This indicates that a third regime is able to capture the complexity of the observed data better than a two-regime model. However, this enhancement is incremental when compared to the substantial improvement observed when transitioning from a non-switching Dynamic Factor model to a two-regime model. Despite its advancements, the 3-MSDFM marginally underperforms in identifying recessions as dated by NBER relative to the 2019-2MSDFM. This is evidenced by the AUROC score and Figure 4.2.2. Conceptually, the 3-MSDFM can be considered a COVID-19 adjusted version of the 2-MSDFM, rendering it comparable to traditional two-regime models.

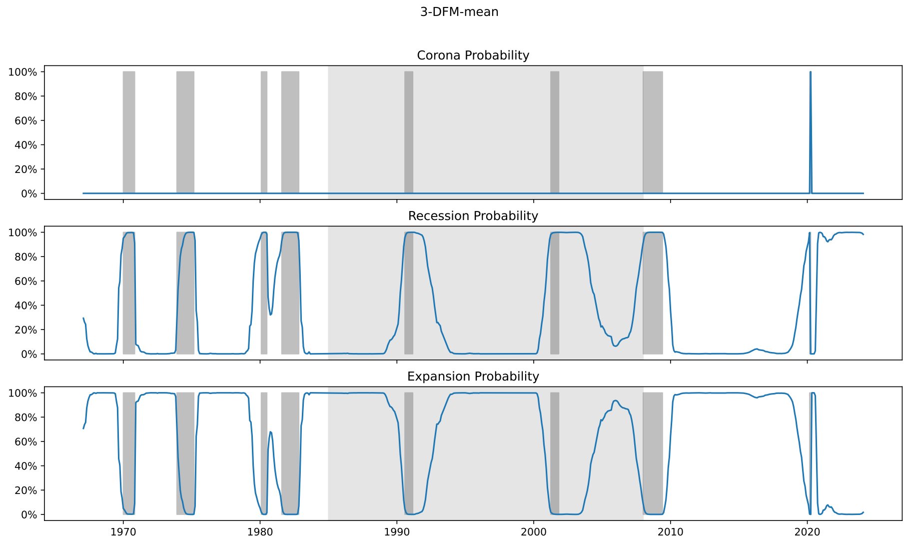

Figure 4.2.3: Estimated transition probabilities for 3-MSDFM with NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

In light of this discovery, a four-regime model was pursued in order to adequately test our hypothesis. The resulting regimes are: Expansion, recession, moderation, and COVID-19. The main focus will be on the three former regimes. The distinct nature of the COVID-19 pandemic is further illustrated in Figure 4.2.4. Here, the regime distributions of the 4-MSDFM model underscore how the pandemic is a significant outlier within the model framework compared to the other regime distributions.

Two variants of the four-regime Markov-Switching Dynamic Factor Model were estimated. One variant with switching variance for the latent dynamic factor, referred to as 4-MSDFM, and another without switching variance, referred to as 4-MSDFM'. Incorporating switching variance enhanced the goodness-of-fit compared to prior models, while exerting minimal impact on the predictive power for NBER recessions. This indicates that the introduction of a fourth regime is better suited to explain the data. By allowing for switching variance the regime distinction was significantly improved. Notably, the introduction of switching variance

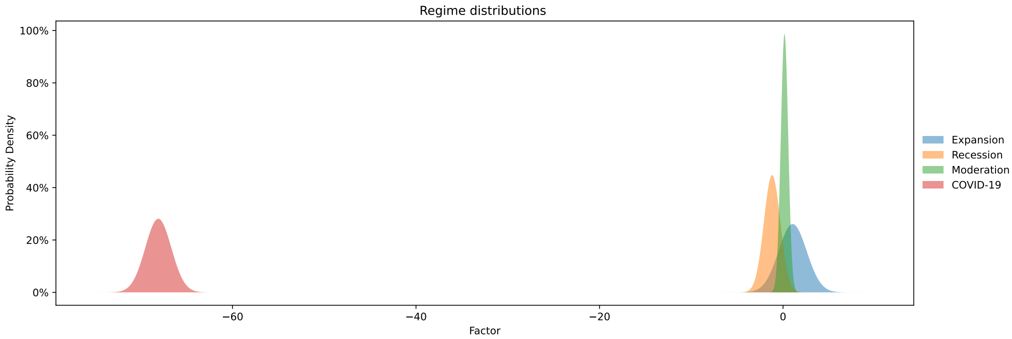

Figure 4.2.4: The probability distributions of the latent dynamic factor for the 4-MSDFM model under the different regimes. This figure illustrates how big an outlier the COVID-19 pandemic is in the model.

in the latent dynamic factor enabled the identification of periods characterized solely by the economic moderation regime. This is clearly demonstrated when comparing the plots in Figure 4.2.7 with those in Figure 4.2.6. Periods of moderation began appearing reliably at the start of the Great Moderation and have been predominant since, except for a brief expansionary period following the COVID-19 lockdowns.

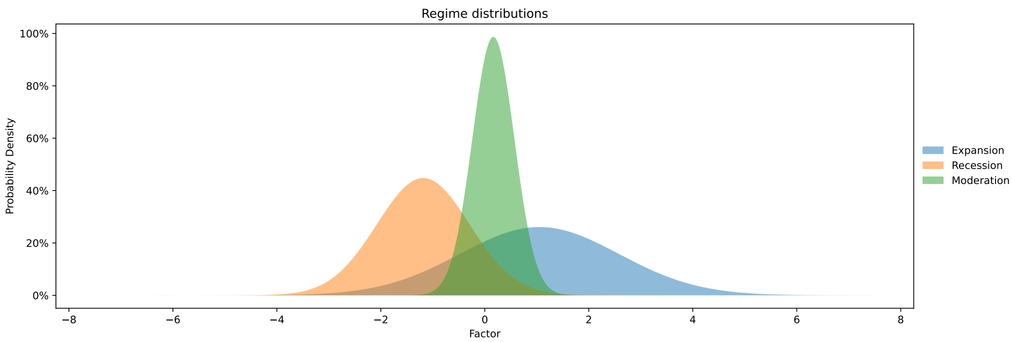

Figure 4.2.5: The probability distributions of the latent dynamic factor for the 4-MSDFM model omitting the COVID-19 regime under the different regimes.

The incorporation of switching variance in the 4-MSDFM model significantly enhanced the characterization of economic phases, particularly periods of moderation and expansion. This enhancement is evident in Figures 4.2.6 and 4.2.7, where the moderate regime is characterized by low volatility and modest growth, and the expansionary regime by high volatility and high growth, as shown in Table 4.2.2. The introduction of a moderate regime allows for a more distinct differentiation between the expansionary and recessionary regimes compared to the 3-MSDFM model. This differentiation is achieved by correctly identifying observations that belong to the moderate regime, which were previously biased towards zero in misspecified models that inaccurately classified these observations into incorrect regimes.

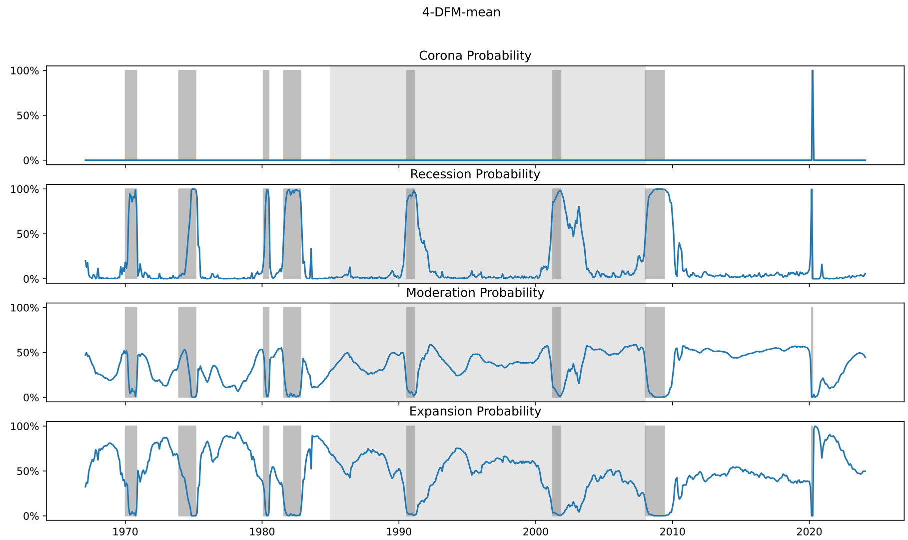

Figure 4.2.6: Estimated transition probabilities for a four-regime Markov-Switching Dynamic Factor Model with switching mean for the latent dynamic factor. NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

Additionally, the estimated variance for the latent dynamic factor in the 4-MSDFM model reveals that the recessionary regime exhibits lower variance than the expansionary regime. This suggests that periods of economic contraction are less volatile than those of expansions. Analysis of the estimated transition probabilities by using Equation 3.38 indicates that the average duration of an expansionary period is 26.9 months, which is significantly longer than that of a recession, at 9 months, and moderation at 6.1 months.

As shown in Table 4.2.2, the probability of remaining within the same regime is lower for the 4-MSDFM compared to the 3-MSDFM. This reduction is expected due to the increased likelihood of switching associated with a higher number of regimes. Additionally, the means of the probability distributions for the 4-MSDFM are more pronounced compared to both the 3-MSDFM and the model described by Chauvet (1998). Specifically, the mean of the recession regime is more negative, and the mean of the expansion regime is more positive. This suggests that models lacking a moderate regime tend to inaccurately classify moderate observations as either expansionary or recessionary. Such misclassification skews the means of the probability distributions of the latent dynamic factors towards zero.

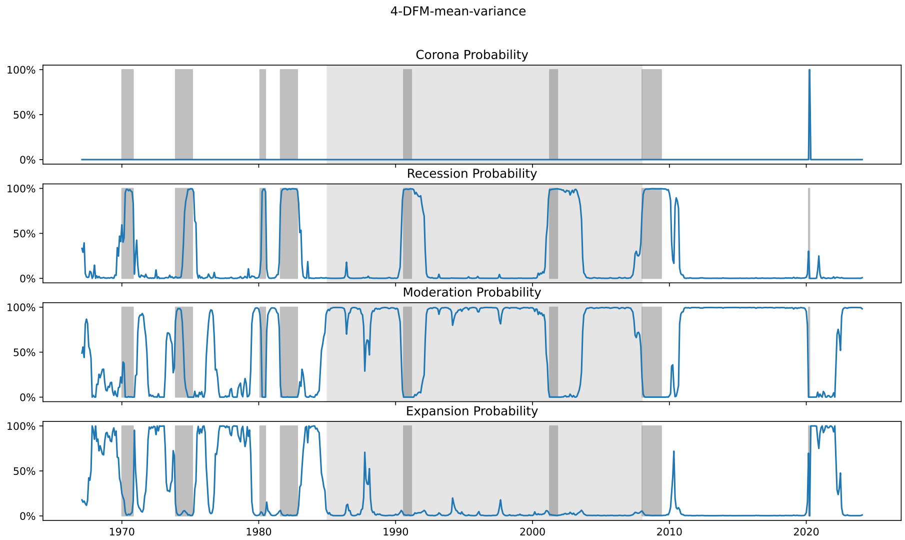

Figure 4.2.7: Estimated transition probabilities for a 4 regime Markov-Switching Dynamic Factor Model with switching mean and variance for the latent dynamic factor. NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

4.2.1 Comparing to Chauvet (1998)

In this section, we will compare the models estimated in this thesis with the model estimated by Chauvet (1998). Special attention will be paid to the 4-MSDFM. From Table 4.2.1, it is evident that models estimated in this thesis exhibit better performance on both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). BIC is particularly suitable for our analysis as it, as mentioned, compensates for variations in sample sizes that tend to inflate the Akaike Information Criterion (AIC). Given that our dataset is larger than the one used by Chauvet (1998), BIC provides a more appropriate metric for comparison. However, these models are less effective in accurately identifying NBER-defined recessions, as illustrated in Figure 4.2.2 and the corresponding Area Under the Receiver Operating Characteristic (AUROC) scores.

Analysis of the estimated transition probabilities reveals close alignment with previous studies, yet with notable differences in recession duration. Specifically, the 4-MSDFM model estimates a longer average recession period of 9 months, compared to 6.9 months as reported by Chauvet (1998), which is confirmed by the recession probabilities depicted in Figure 4.2.1. Here, the 4-MSDFM model consistently predicts longer recession durations relative to those determined by the NBER and Chauvet (1998).

	2-MSDFM	3-MSDFM	4-MSDFM	Chauvet (1998)
p_{dd}	-	0.9785	0.8887	0.855
p_{uu}	0.9971	0.9776	0.9628	0.964
p_{mm}	-	-	0.8349	-
p_{cc}	~ 0	~ 0	~ 0	-
μ_d	-	-1.0847	-1.1848	-0.746
μ_u	0.1166	0.8513	1.0483	0.845
μ_m	-	-	0.1641	-
μ_c	-80.2177	-81.2476	-68.0917	-
$\sigma^2_{F_u}$	-	-	2.3392	-
$\sigma^2_{F_d}$	-	-	0.7934	-
$\sigma^2_{F_m}$	-	-	0.1634	-
$\sigma^2_{F_c}$	-	-	2.0156	-

Table 4.2.2: Switching parameters for the core models. A table containing all of the estimated models can be found in the appendix.

The estimated means for the probability distribution of latent dynamic factor in the 4-MSDFM model are more extreme than those reported by Chauvet (1998), shown in Table 4.2.2. This distinct differentiation arises from the improved classification of observations within the moderate regime, which were previously biased towards zero in misspecified models. These models inaccurately classified the moderate regime observations by assigning them to the general expansionary regime instead.

Table 4.2.3 presents the estimated parameters for our core models , alongside the estimates reported by Chauvet (1998). The estimated auto-regressive (AR) parameter for the latent dynamic factor appears to be smaller and negative in simpler models. Notably, in the 4-MSDFM model, it is not statistically different from zero, contrasting with the findings of Chauvet (1998). The AR-parameters associated with the integrated errors are largely consistent with those estimated by Chauvet (1998). An exception is observed with the employees on non-agricultural payrolls (PAYEMS) parameter, which exhibits a significantly positive value in simpler models and a markedly negative value in the 4-MSDFM.

For the model-specific variance parameters (σ^2), we see a general trend of decreasing variance from the 2-MSDFM to the 4-MSDFM across most variables. This suggests an improvement in model fit with additional dynamic factors, as indicated by lower variance values signifying a better capture of the underlying data structure. The exception is $\sigma^2_{W875RX1}$, which is notably higher in the 4-MSDFM model compared to Chauvet (1998).

4.2.2 Latent Dynamic Factor

In Figure 4.2.8, the latent dynamic factors for various model specifications are depicted. Notably, these factors underwent a significant decline in 2020, due to the COVID-19 pandemic. This figure also incorporates overlays of the different regimes, although the extensive scale required to illustrate the pandemic's impact complicates the visual representation of pre-COVID-19 recessions. To address this, the year 2020 is excluded in Figure 4.2.9, which makes it easier to see movements in the latent dynamic factor. In Figure 4.2.9 there is a clear connection between the different model specifications and when they assign a regime. These figures make it easy to distinguish where the 4-MSDFM model assigns a moderate growth regime compared to where the other models assign an expansionary regime.

4.3 Results of BDS test on the disturbances

In this section, we will present the result of a BDS test on the disturbances of the model. Special attention will be paid to comparing the 2-MSDFM with the other models as it scored poorest on the goodness-of-fit metrics. The BDS test results, summarized in Tables 4.3.1 and 4.3.2, provide insights into the non-linearity and dependence structure of the disturbances. The BDS test statistic is used to test the null hypothesis that the time series data are independently and identically distributed (i.i.d). High values of the BDS statistic and low p-values indicate rejection of the null hypothesis, suggesting that the residuals exhibit significant non-linearity or dependence.

Table 4.3.1 displays the BDS test statistics for various models and economic indicators. Notably, the 2-MSDFM estimated on data before the COVID-19 pandemic shows significantly higher BDS statistics compared to other models. This particulary holds for the employees on non-agricultural payrolls (PAYEMS) indicator, with a value of 22.34, indicating strong evidence against the i.i.d. hypothesis. This suggests that the model, which does not account for the COVID-19 pandemic as a separate regime, has a higher non-linearity or dependence in the disturbances.

Table 4.3.2 presents the p-values associated with the BDS test statistics. Most models show p-values close to zero, reinforcing the rejection of the i.i.d. hypothesis. Again, the 2-MSDFM estimated on data before the COVID-19 pandemic stands out with significantly low p-values. This especially holds for the employees on non-agricultural payrolls (PAYEMS) and industrial production (INDPRO) indicators, highlighting the need for a regime-switching model to account for the COVID-19

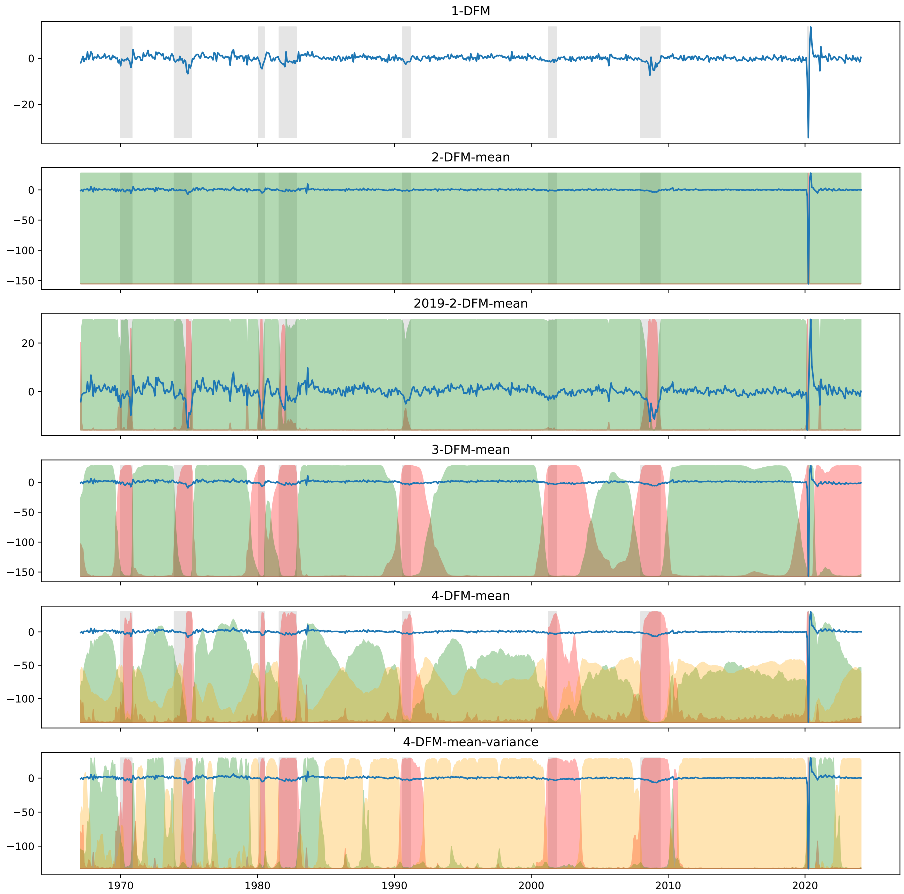

Figure 4.2.8: Estimated latent dynamic factors with probability of regime overlaid. Green is expansionary, red is recessionary, yellow is moderate, and brown is COVID-19. NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

pandemic's effects.

Figures 4.3.1 and 4.3.2 illustrate the disturbances of the models. Figure 4.3.1 shows the disturbances, including data from the COVID-19 period, which indicates larger prediction errors for the 2-MSDFM estimated on data before the COVID-19 pandemic. This supports the modeling of the COVID-19 pandemic as a separate regime due to its substantial impact on prediction accuracy. In contrast, Figure 4.3.2 presents the disturbances omitting data post-2020, which provides a clearer view of model performance under normal conditions. It is evident from these plots that there still are unmodeled structures remaining. Despite this, we see that both the 4-MSDFM and 4-MSDFM' demonstrate smaller magnitude in the

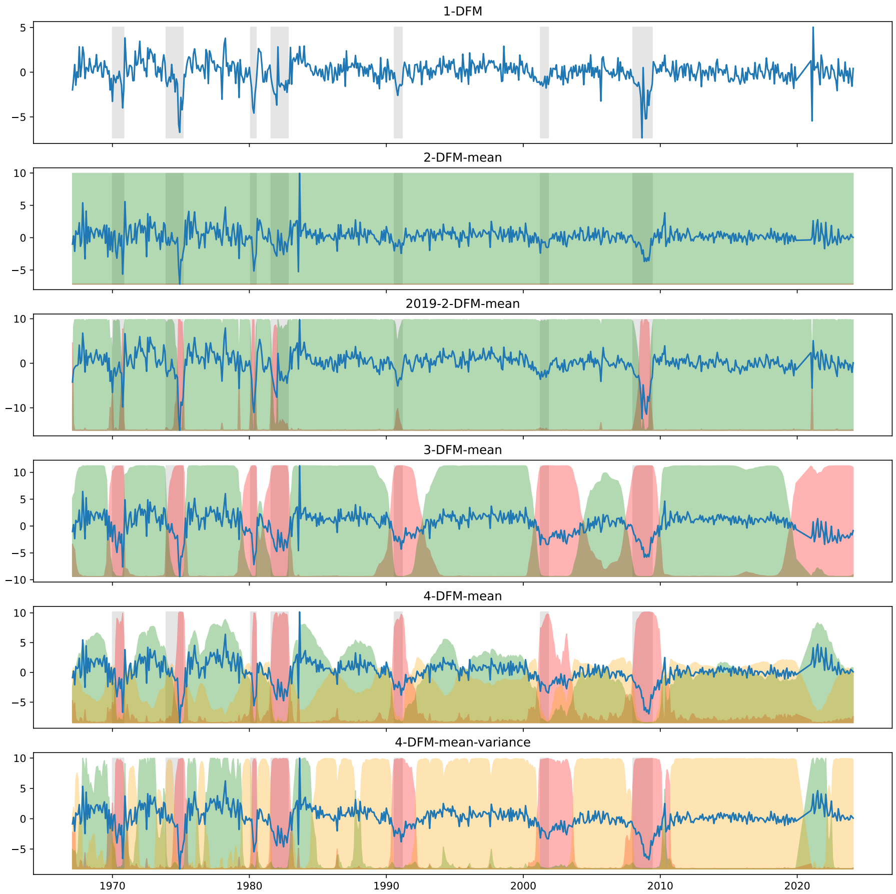

Figure 4.2.9: Estimated latent dynamic factors with probability of regime overlaid (data from is omitted). Green is expansionary, red is recessionary, yellow is moderate, and brown is COVID-19. NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

disturbances compared to the other models.

4.4 Testing Assumptions for Error Terms

This section evaluates the prediction errors from the model by testing the fundamental assumptions required for their validity. These assumptions includes zero mean, homoscedasticity, no autocorrelation, normality, and independence from predictors. Each assumption is assessed using specific diagnostic tools. A Wald test will be used to assess whether the error har zero mean, a Breusch-Pagan test will be used to test for heteroscedasticity, a Ljung-Box test will be used to check for autocorrelation and a Shapiro-Wilk and QQ-plots to assess the normality assumption.

A Wald test was applied to examine whether the residuals exhibit zero mean across all models. The statistical results suggest that the residuals likely conform to the zero mean assumption.

Table 4.4.1 presents the results from the Breusch-Pagan Lagrange Multiplier test, as proposed by Breusch and Pagan (1979), to assess heteroscedasticity. For the manufacturing and trade sales (CMRMTSPL) series, the consistently zero p-values indicate a strong presence of heteroscedasticity, thereby necessitating model specifications that can accommodate varying error variances. Conversely, the total personal income less transfer payments (W875RX1) series shows significant deviations from homoscedasticity in certain models, as evidenced by low p-values, notably in the 4-DFM-mean model, as seen in Table 4.4.1. For non-agricultural payrolls (PAYEMS) and industrial production (INDPRO) series, higher p-values suggest a lesser degree of heteroscedasticity under specific model conditions. The variability in p-values highlights the need for selecting econometric models that appropriately address the unique characteristics of each data series.

Ljung-Box test was employed, as proposed by Ljung and Box (1978), to test for auto-correlation within the error terms. As shown in Table 4.4.2, significant auto-correlation was observed in the residuals. Particularly in the PAYEMS series across all models, indicating that the models may not adequately capture underlying employment data patterns. The INDPRO series also demonstrates substantial auto-correlation, suggesting issues in accurately modeling industrial production. Notably, the CMRMTSPL and W875RX1 series exhibited less consistent auto-correlation, with p-values varying significantly across models. This underscores the complexity of capturing dynamic relationships in macroeconomic data, and highlights the importance of rigorous model validation through residual analysis.

The Shapiro-Wilk test, as delineated by Shapiro and Wilk (1965), was conducted to assess the normality of residuals. The results decisively reject the null hypothesis of normally distributed residuals. This is further corroborated by the QQ-plots in Figures 4.4.1 and 4.4.2, which reveal heavier tails than expected under normality, indicating deviations in the distribution of residuals in both the Non-Switching Dynamic Factor Model and the 4-MSDFM.

	2-MSDFM	3-MSDFM	4-MSDFM	Chauvet (1998)
ϕ_1	-0.048	-0.044	0.0145	0.291
	(0.0127)	(0.0123)	(0.0263)	(0.085)
$d_{CRMTSPL}$	-0.0672	-0.0772	-0.1312	-0.240
	(0.0397)	(0.0376)	(0.0378)	(0.052)
$d_{W875RX1}$	-0.0612	-0.0669	-0.1129	-0.087
	(0.0391)	(0.0392)	(0.0384)	(0.057)
d_{PAYEMS}	0.9356	0.9681	-0.9996	-0.171
	(0.0198)	(0.0115)	(0.0013)	(0.052)
d_{INDPRO}	0.22	0.1989	0.1044	0.199
	(0.0428)	(0.0401)	(0.0408)	(0.069)
$\lambda_{CRMTSPL}$	0.171	0.1709	0.1864	0.478
	(0.0147)	(0.009)	(0.012)	(0.040)
$\lambda_{W875RX1}$	0.1455	0.1443	0.1912	0.259
	(0.0141)	(0.0103)	(0.0125)	(0.018)
λ_{PAYEMS}	0.3043	0.3029	0.3569	0.150
	(0.0163)	(0.0067)	(0.0096)	(0.012)
λ_{INDPRO}	0.2003	0.2038	0.2432	0.569
	(0.0154)	(0.0085)	(0.0119)	(0.045)
$\sigma^2_{CRMTSPL}$	0.692	0.6843	0.6442	0.827
	(0.0384)	(0.0373)	(0.0347)	(0.066)
$\sigma^2_{W875RX1}$	0.7823	0.7782	0.7425	0.156
	(0.0428)	(0.0433)	(0.0401)	(0.006)
σ^2_{PAYEMS}	0.007	0.0037	0.0	0.448
	(0.0023)	(0.0011)	(0.0)	(0.049)
σ^2_{INDPRO}	0.5	0.4853	0.4557	0.448
	(0.0279)	(0.0272)	(0.0245)	(0.049)
BIC	518.49	406.63	-16.39	3 659.16
Log Likelihood	-203.73	-141.27	89.83	-1 776.91
Likelihood Ratio	1 052.38	1 177.30	1 639.51	25.12

Table 4.2.3: Note that the metrics reported by Chauvet (1998) are calculated based on different data, and is therefore not directly comparable. A table containing all of the estimated models can be found in the appendix.

	CMRMTSPL	W875RX1	PAYEMS	INDPRO
1-DFM	2.7225	5.5373	8.4076	5.2475
2-DFM-mean	2.6264	5.6444	9.4600	2.6435
2019-2-DFM-mean	17.7518	2.9917	22.3448	11.1822
3-DFM-mean	2.7089	5.4038	9.2659	1.3946
4-DFM-mean	2.7271	5.4679	9.0883	1.6726
4-DFM-mean-variance	2.6740	8.1770	13.2891	5.2654

Table 4.3.1: Test statistics for the BDS test with two embedded dimensions. Higher-order tests also reject the null hypothesis.

	CMRMTSPL	W875RX1	PAYEMS	INDPRO
1-DFM	0.0065	~ 0	~ 0	~ 0
2-DFM-mean	0.0086	~ 0	~ 0	0.0082
2019-2-DFM-mean	~ 0	0.0028	~ 0	~ 0
3-DFM-mean	0.0068	~ 0	~ 0	0.1631
4-DFM-mean	0.0064	~ 0	~ 0	0.0944
4-DFM-mean-variance	0.0075	~ 0	~ 0	0.0000

Table 4.3.2: p-values of BDS test statistics with 2 embedded dimensions. Higher-order tests also reject the null hypothesis.

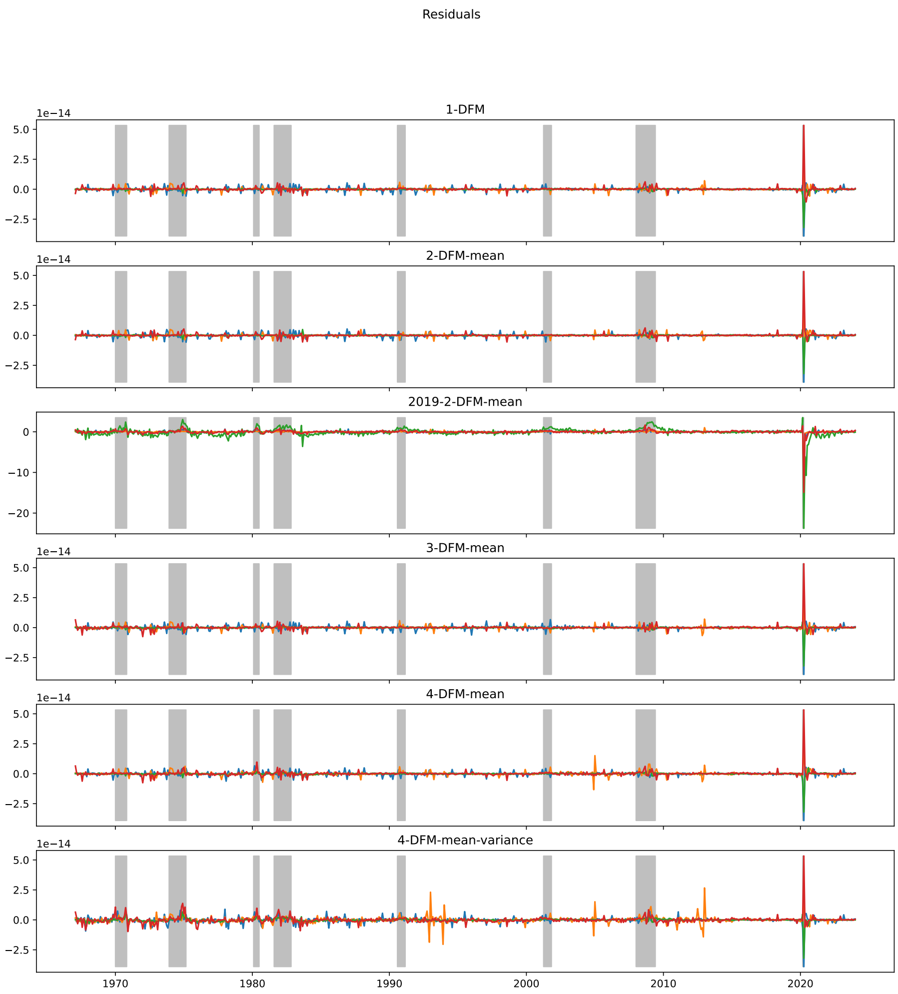

Figure 4.3.1: Model disturbances. Note the scale of the disturbances in the 2019-2-MSDFM. The scale of the prediction error from COVID-19 is greater for the model that does not integrate it. This supports modeling the COVID-19 pandemic as a separate regime. NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

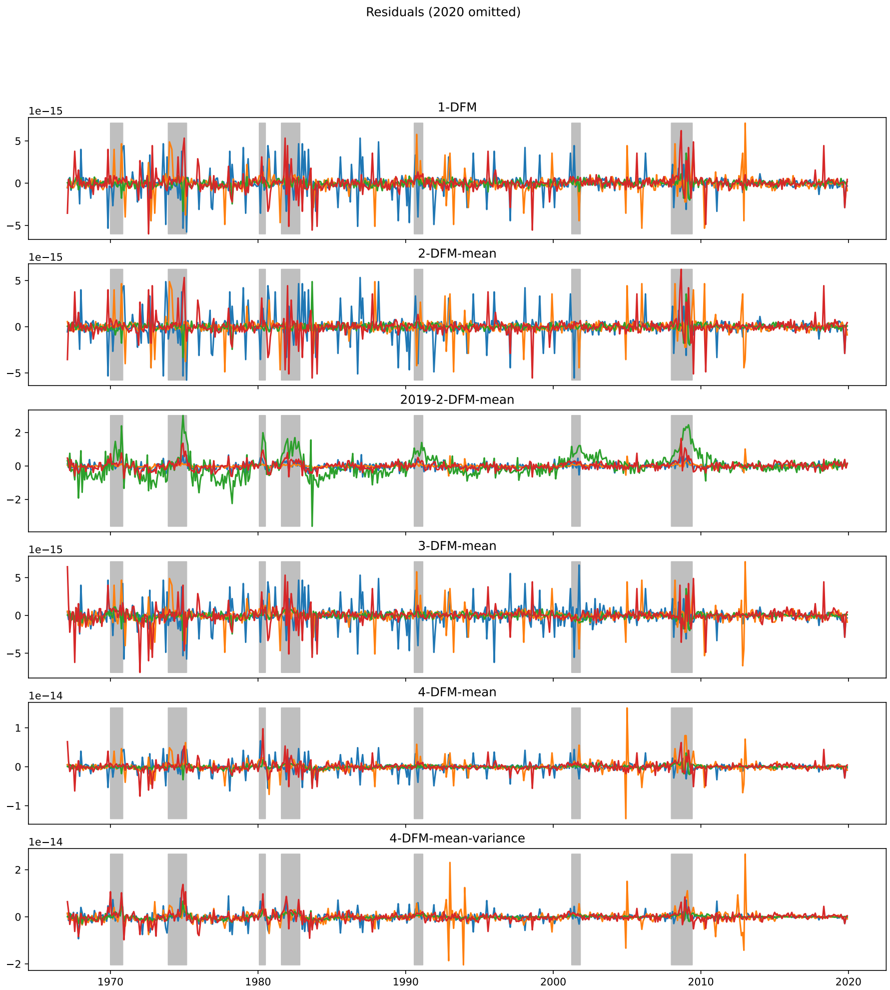

Figure 4.3.2: Model disturbances omitting data after 2020. It is clear that the 2019-2-MSDFM is worse at capturing the underlying structure of the data. NBER classified recessions in grey retrieved from Federal Reserve Bank of St. Louis (1854)

	CMRMTSPL	W875RX1	PAYEMS	INDPRO
1-DFM	0.0	0.0052	0.0150	0.0868
2-DFM-mean	0.0	0.0055	0.0130	0.1297
2019-2-DFM-mean	0.0	0.0019	0.0022	0.1637
3-DFM-mean	0.0	0.0031	0.0131	0.1275
4-DFM-mean	0.0	0.7079	0.0152	0.1415
4-DFM-mean-variance	0.0	0.0000	0.0155	0.2503

Table 4.4.1: p-values for a Breusch-Pagan Lagrange Multiplier test for heteroscedasticity.

	CMRMTSPL	W875RX1	PAYEMS	INDPRO
1-DFM	0.2664	0.0067	0.0000	0.0027
2-DFM-mean	0.0322	0.0215	0.1089	0.5660
2019-2-DFM-mean	0.1195	0.9952	0.0000	0.0024
3-DFM-mean	0.0047	0.0224	0.0900	0.3980
4-DFM-mean	0.1965	0.0322	0.0000	0.1027
4-DFM-mean-variance	0.8179	0.0047	0.0000	0.0048

Table 4.4.2: p-values for Ljung-Box test of auto-correlation in residuals.

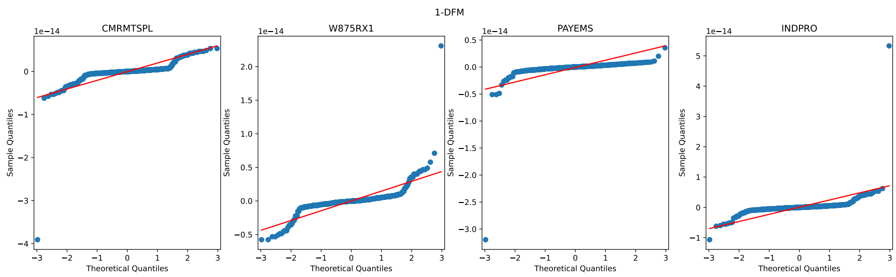

Figure 4.4.1: QQ-plot of the residuals for the Non-Switching Dynamic Factor Model.

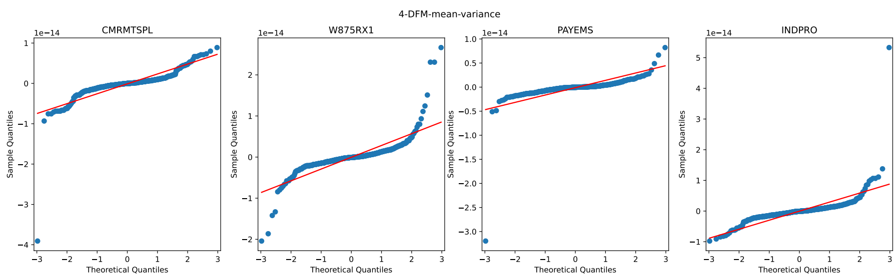

Figure 4.4.2: QQ-plot of the residuals for the 4-MSDFM. QQ-plots for all of the estimated models can be found in the appendix.

COMMENTS AND CONCLUSIONS

5.1 Comments

The empirical findings of this thesis, particularly the identification of three distinct economic regimes through the 4-MSDFM, enhanced our understanding of the U.S. economic stability post the Great Moderation. The 4-MSDFM, which can be interpreted as the COVID-19 adjusted three-regime model expands on the traditional two-regime approach by integrating a third regime with moderate growth and lower volatility. This nuanced approach addresses the limitations of previous models which did not fully capture the complexity of economic fluctuations.

Historically, extensive fiscal measures implemented during periods such as World War II and the Vietnam War parallel the expansive economic policies observed following the COVID-19 pandemic. Similarly, the policies aimed at boosting economic growth and reducing unemployment through deficit spending during the "Great Inflation" resonate with the actions taken by governments after the pandemic. According to Benmelech and Tzur-Ilan (2020), governments and central banks responded to the COVID-19 pandemic by extensively employing both fiscal and monetary tools. The COVID-19 adjusted three-regime model classifies the period shortly after the pandemic-induced recession as an expansionary regime, characterized by high volatility and significant growth. This is depicted in Figure 4.2.7, interestingly this regime has been virtually unobserved since the Great Moderation. This classification indicates that the economic stimulus provided was comparable to that observed during the expansionary phases of the Great Inflation. Notably, this is the first period classified as expansionary since the conclusion of the Great Inflation, signifying a significant shift in economic dynamics towards reduced volatility.

The standard two-regime model with a switching mean has traditionally been adequate for modeling the U.S. business cycle and appears insufficient in the face of the recent COVID-19 pandemic. When data excluding the pandemic period is considered, the 2019-2-MSDFM model more accurately reflects the traditional business cycle dynamics. This suggests that the pandemic introduced anomalies that are not well captured by the simple two-regime approach.

The analysis in this thesis reveals that the COVID-19 pandemic-induced recession deviates significantly from typical business cycle patterns, presenting a substantially greater shock than standard recessions. This finding prompts us to reconsider whether the COVID-19 pandemic should indeed be classified as a recession and whether the National Bureau of Economic Research (NBER) was accurate in its classification.

The 2019-2-MSDFM model exhibits the poorest performance according to goodness-of-fit metrics, indicating it is less effective at accurately representing the observed data. However, it is noteworthy that this model achieves the highest AUROC value among all the models estimated, revealing a complex trade-off. This balance between model fit and recession dating accuracy is also highlighted in Chauvet (1998)'s model. Chauvet (1998)'s model shows poorer goodness-of-fit scores, while more precisely dating recessions as designated by the NBER, compared to all models estimated in this thesis.

The trade-off between recession categorization and model fit presents a compelling dilemma, prompting us to consider whether a model's complexity contributes to over-fitting despite scoring better on both AIC and BIC. Interestingly, it appears that models with poorer performance in explaining observed data may actually offer clearer distinctions between economic regimes. This indicates that goodness-of-fit is a poor indicator of inference in the business cycle. Grasping this trade-off is vital for achieving a balance between model accuracy and practical usefulness in economic regime classification.

The log-likelihood function of the Markov-Switching Dynamic Factor Model is non-convex. This implies that the certainty of attaining the global maximum in parameter estimation is compromised. Despite employing a rigorous hybrid optimization approach that combines Particle Swarm Optimization and BFGS, it remains uncertain whether the parameters estimated in this study indeed correspond to those that maximize the likelihood function. Furthermore, the apparent trade-off between goodness-of-fit and the performance of NBER recession classification calls for a reconsideration. It raises the question of whether maximizing the likelihood function is the most effective method. This method is used for estimating parameters in models designed to provide robust inference on structural changes in the U.S. business cycle.

Contrary to the findings of Chauvet (1998), this analysis reveals evidence of serial dependence in the disturbances. Additionally, auto-correlated and heteroskedastic residuals were observed, as indicated by the Ljung-Box test for auto-correlation and the Breusch-Pagan test for heteroskedasticity for all estimated models. This indicates that there is some structure not captured by the model. However, this may neither be unexpected nor significant. While the latent dynamic factor aims to capture commonalities across individual series, it inevitably neglects idiosyncratic structures such as individual seasonality and shocks, which are attempted modeled by the integrated error term. The presence of structured disturbances in the model indicates that the integrated errors fail to adequately capture said idiosyncratic seasonalities and shocks. Assuming that the process governing the latent dynamic factor is correctly specified, the misspecification is isolated to the idiosyncratic part of the series and should not affect the inference of the commonalities in the series and by extension the business cycle.

However, if the processes determining the business cycle are misspecified the above argument does not hold. If it is the case that the modes should be estimated with higher-order auto-regressive processes for the latent dynamic factor, there could still be some common structure in the disturbances. Along with Diebold and Rudebusch (1996), this thesis also finds evidence that this may be the case. However, Diebold and Rudebusch (1996) is more concerned with the possibility that the AR(1) model may induce serial correlation in error, which could be spuriously detected by regime-switching dynamics, not the validity of the error term. Another explanation for the structured disturbances is that, despite the rigorous optimization scheme, it was unsuccessful in maximizing the likelihood function. Consequently, it appears that the optimization procedure may not have successfully identified the optimal solution for the models.

As mentioned in the introduction the thesis explores whether we are witnessing a reversion to pre-moderation economic regimes or entering a new phase of economic stability. From our findings, we can infer that following the Great Moderation we can quantitatively detect the existence of a moderate growth regime, both before and after the recessions of 2007-08 and 2020. Thus reaffirming the arguments of Clark (2009), where he concludes that over time the macroeconomic volatility will undergo occasional shifts between high and low levels in volatility, with the norm being low volatility. Thus disproving the revision back to pre-moderation economic regimes, even though recessions still appear.

5.2 Limitations and Future Research

The statistical significance of a three-regime model was not tested within this thesis. While Garcia (1998) provides a method for computing critical values for statistical tests, extending this methodology to a three-regime model was considered beyond the scope of this study due to its complexity. Garcia (1998) determined critical values for the asymptotic distribution of the likelihood ratio test in a two-regime Markov switching model, but similar values for a three-regime model have yet to be established. As such, the effectiveness of the three-regime model was evaluated using goodness-of-fit metrics rather than the more definitive statistical testing framework suggested by Garcia (1998). Developing critical values for comparing two and three-regime models would enhance the rigor of the evaluation and strengthen the findings of this thesis.

This thesis utilized Maximum Likelihood Estimation, which assumes a normal distribution. Future studies should consider robust optimization techniques that do not assume normality, but rather an elliptical distribution allowing for the heavy tails observed in this paper. By modeling with an elliptical distribution that accounts for heavier tails instead of a normal distribution, the COVID-19 data can appear less anomalous. Additionally, the rejection of the Shapiro-Wilk test, which indicates that residuals are not normally distributed, supports the argument for using an alternative distribution in the maximum likelihood estimation. This is visually depicted in Figure 4.4.2, where the red line represents a normal distribution and where the residuals visibly diverge from this line.

5.3 Conclusions

This thesis set out to explore the applicability of a Markov-Switching Dynamic Factor model for analyzing structural changes in the business cycle dynamics of the U.S. economy, particularly in light of the Great Moderation. This thesis introduced modifications to the original model, incorporating three Markov-switching regimes and introducing switching variance in the latent factor, aimed at better reflecting the evolving macroeconomic dynamics, especially in regard to volatility.

The main focus has been on whether these shifts have necessitated adjustments to the model's configuration to enhance its descriptive and predictive accuracy. This involves modeling the variance of the dynamic latent factor and introducing a moderation regime characterized by low growth and volatility. The empirical analysis, employing monthly data up to the end of 2023, provides evidence suggesting that the traditional two-regime model does not adequately capture the complexity of the current economic landscape. Integrating a third regime, characterized by moderate growth and lower volatility, the model provided deeper insight into economic fluctuations and demonstrated improved goodness-of-fit metrics.

The results support the theoretical framework proposed by Chauvet (1998), validating the utility of Markov switching models in identifying distinct economic phases. However, unlike Chauvet (1998), which modeled two regimes with switching means, our findings suggest that an additional regime as well as modeling the volatility is necessary to adequately describe the evolving dynamics of the economy, particularly in light of new economic data post-2000.

Despite the enhancements, the models did not pass the Brock-Dechert-Scheinkman (BDS) test and several assumptions about the error terms were violated. These results indicate potential misspecifications in the model that could affect its reliability and accuracy. These limitations suggest the need for further refinement of the model and its assumptions, which could involve exploring alternative specifications or estimation methods.

In essence, this thesis not only supports the foundational theories of Markov switching dynamics, but also encourages a reevaluation of how these models are constructed in the face of evolving business cycles. The empirical validity of this approach is reinforced by its alignment with observed data trends and increased explanatory power.

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APPENDICES

Additional Results

	1-DFM	2-MSDFM	2019-2-MSDFM	3-MSDFM	4-MSDFM'	4-MSDFM
ϕ_1	0.2518	-0.048	0.4101	-0.044	-0.0419	0.0145
	(0.0416)	(0.0127)	(0.0587)	(0.0123)	(0.0136)	(0.0263)
$d_{CRMTSPL}$	-0.2294	-0.0672	-0.3624	-0.0772	-0.1302	-0.1312
	(0.0413)	(0.0397)	(0.0398)	(0.0376)	(0.0379)	(0.0378)
$d_{W875RX1}$	-0.1461	-0.0612	-0.1542	-0.0669	-0.1134	-0.1129
	(0.0398)	(0.0391)	(0.0402)	(0.0392)	(0.0385)	(0.0384)
d_{PAYEMS}	-0.1123	0.9356	0.5343	0.9681	-0.9964	-0.9996
	(0.0479)	(0.0198)	(0.0571)	(0.0115)	(0.01)	(0.0013)
d_{INDPRO}	0.0331	0.22	-0.2475	0.1989	0.1036	0.1044
	(0.0924)	(0.0428)	(0.0604)	(0.0401)	(0.0412)	(0.0408)
$\lambda_{CRMTSPL}$	0.691	0.171	0.3975	0.1709	0.2066	0.1864
	(0.0316)	(0.0147)	(0.028)	(0.009)	(0.012)	(0.012)
$\lambda_{W875RX1}$	0.5318	0.1455	0.2618	0.1443	0.1798	0.1912
	(0.0353)	(0.0141)	(0.0285)	(0.0103)	(0.0125)	(0.018)
λ_{PAYEMS}	0.7592	0.3043	0.515	0.3029	0.3496	0.3569
	(0.0325)	(0.0163)	(0.0371)	(0.0067)	(0.0096)	(0.0096)
λ_{INDPRO}	0.8955	0.2003	0.5889	0.2038	0.247	0.2432
	(0.0303)	(0.0154)	(0.0339)	(0.0085)	(0.0119)	(0.0119)
$\sigma^2_{CRMTSPL}$	0.4373	0.692	0.5872	0.6843	0.645	0.6442
	(0.0277)	(0.0384)	(0.0372)	(0.0373)	(0.0349)	(0.0347)
$\sigma^2_{W875RX1}$	0.6908	0.7823	0.8497	0.7782	0.7417	0.7425
	(0.0401)	(0.0428)	(0.0491)	(0.0433)	(0.0401)	(0.0401)
σ^2_{PAYEMS}	0.3674	0.007	0.3048	0.0037	~ 0	~ 0
	(0.0266)	(0.0023)	(0.0396)	(0.0011)	(~ 0)	(~ 0)
σ^2_{INDPRO}	0.1393	0.5	0.3293	0.4853	0.4575	0.4557
	(0.0243)	(0.0279)	(0.0382)	(0.0272)	(0.0247)	(0.0245)

Table .0.1: Estimated non-switching model parameters. In general we find that estimated parameters are statistically significant dispite having minor differences between the models.

	2-MSDFM	2019-2-MSDFM	3-MSDFM	4-MSDFM	4-MSDFM
p_{dd}	-	0.7583	0.9785	0.8357	0.8887
p_{uu}	0.9971	0.9868	0.9776	0.8529	0.9628
p_{cc}	~ 0.0	-	~ 0.0	~ 0.0	~ 0.0
p_{mm}	-	-	-	0.8903	0.8349
μ_d	-	-2.5214	-1.0847	-1.3368	-1.1848
μ_u	0.1166	0.1218	0.8513	0.4074	1.0483
μ_c	-80.2177	-	-81.2476	-68.1088	-68.0917
μ_m	-	-	-	0.115	0.1641
$\sigma^2_{F_u}$	-	-	-	-	2.3392
$\sigma^2_{F_d}$	-	-	-	-	0.7934
$\sigma^2_{F_c}$	-	-	-	-	2.0156
$\sigma^2_{F_m}$	-	-	-	-	0.1634

Table .0.2: Estimated model parameters. Subscript u denotes expansionary regime, d denotes recessionary regime, m denotes moderate regime and c denotes the Covid-19 regime. p_{jj} is the probability of transitioning from regime j to regime j, or put simply: The probability of staying in the same regime.

QQ-Plots of Residuals

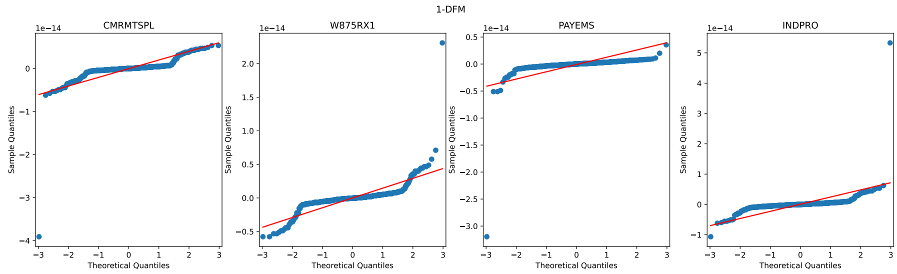

Figure .0.1: QQ-plot of the residuals for the Non-Switching Dynamic Factor Model.

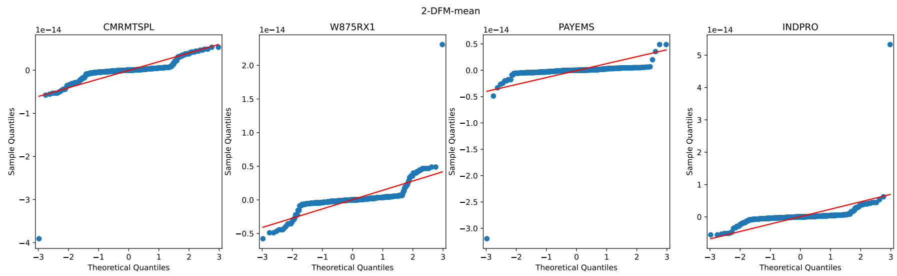

Figure .0.2: QQ-plot of the residuals for the 2-MSDFM.

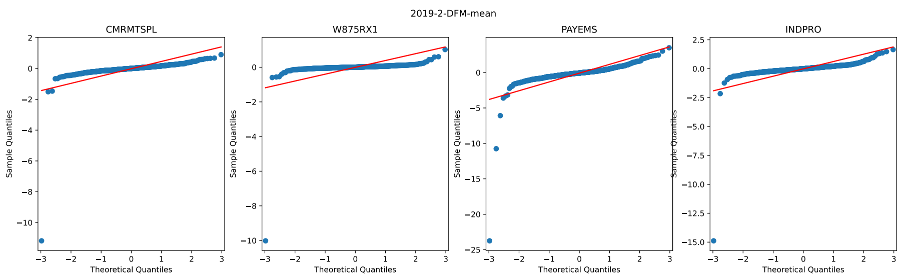

Figure .0.3: QQ-plot of the residuals for the 2019-2-MSDFM.

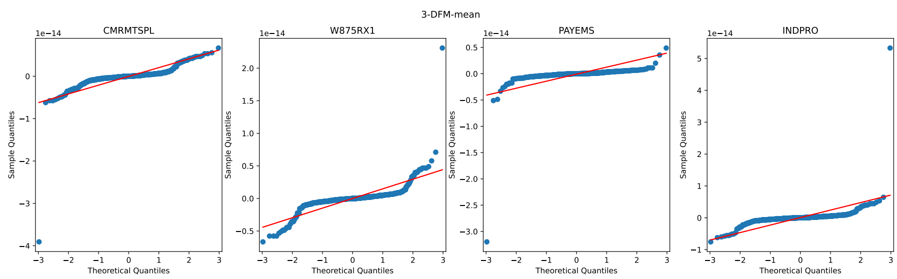

Figure .0.4: QQ-plot of the residuals for the 3-MSDFM.

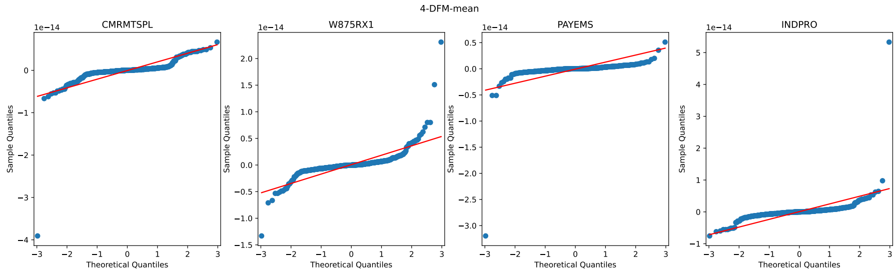

Figure .0.5: QQ-plot of the residuals for the 4-MSDFM'.

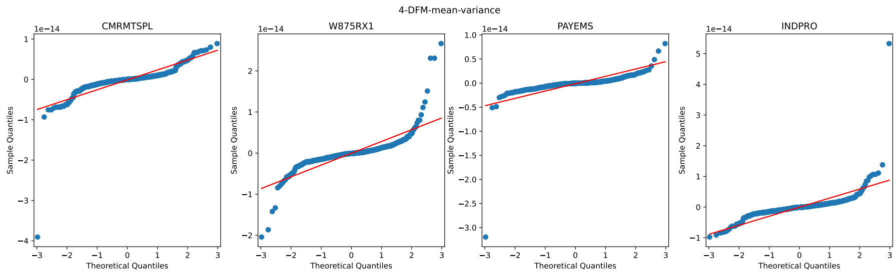

Figure .0.6: QQ-plot of the residuals for the 4-MSDFM.

Code

All the code used can be found on Github in the link below.

`https://tinyurl.com/Master-MSDFM`

You can also scan the QR code below.

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