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Probabilistic Planning of Distribution Networks With Optimal DG Placement Under Uncertainties

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***Abstract*—This research paper presents an efficient methodology for distribution network planning under an uncertain environment. As an extension of our previous work presented at the ECCE Asia 2021 conference, here optimal placement and sizing of Renewable Energy Sources (RES)-based Distributed Generations (DGs) are determined considering the generation and load uncertainties. In addition, the optimal tap settings of off-load tap changing transformers present in a network are also determined. Probabilistic non-linear optimization is solved with a sensitivity-based technique to minimize the distribution network losses and improve its voltage stability. The proposed methodology is implemented on standard test systems like the IEEE 69 bus and the Indian 85 bus networks. Further, to determine its real-world functionality, the methodology is tested on a practical radial distribution network of 88 buses present in a remote Froan island of Norway. When compared with existing techniques, the proposed methodology provides much more efficient network planning solutions with lesser power losses. Developed on free and open-source software platforms, it also provides a reliable and cost-effective alternative to network operators to determine their network robustness.**

***Index Terms*—Distributed generation, distribution network planning, network uncertainties, open-source software, optimal DG placement and sizing, probabilistic planning, sensitivity-based approach, wind generation.**

NOMENCLATURE

The important nomenclatures of this paper are listed in this section. Other variables used are described appropriately in their respective sections.

F_{obj}	Objective function of the problem.
P_{Loss}	Network real power loss.
Q_{Loss}	Reactive power absorbed by the network.
A_p, A_q	Associated weights for the different losses.
k, n	Bus indices for the proposed methodology.
N	Number of buses in a network.
l	Number of branches or lines in the network.
τ	Number of transformers in service.
$\mathbb{N}$	Set of all network buses, $\mathbb{N} = [1, \ldots, N]$
Ω	Set of buses except slack, $\Omega = [2, \ldots, N]$
S_{Loss}	Apparent power absorbed by the network.
$\mathbf{S_l}$	Branch power flow vector, $[l \times 1]$
$\boldsymbol{\tau}$	Vector of transformer tap settings, $[\tau \times 1]$
Variables with vector size	$[N \times 1]$
$\mathbf{P_i}, \mathbf{Q_i}$	Real and reactive power injection vectors.
$\mathbf{P_g}, \mathbf{Q_g}$	Corresponding power generation vectors.
$\mathbf{P_d}, \mathbf{Q_d}$	Real and reactive power demand.
$\mathbf{P_{DG}}, \mathbf{Q_{DG}}$	Power generation by the DGs.
$\mathbf{V}$	Bus voltage magnitudes.
$\boldsymbol{\theta}$	Bus voltage phase angles.

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I. INTRODUCTION

GLOBAL warming has been the most important problem demanding immediate attention at present. The electrical power system being the largest producer of greenhouse gases due to the widespread use of fossil-fuel-based power generating units, increased focus has been on using Renewable Energy Sources (RES) for producing power. RES are mostly distributed in nature, with low voltage connections at the Distribution Network (DN) level. Such Distributed Generations (DGs) have stochastic/uncertain power generation profiles owing to the uncertainty of the renewable sources. Additionally, there is a considerable amount of uncertainty in the network load profiles. In such a context, minor imperfections in finding the optimal DG sizing and placement can cause drastic changes in the power flow patterns of the DNs and their operation points.

These lead to substantial increases in network power losses, compromising their overall stability. Increased power losses may also result in an unacceptable voltage profile, mostly at the farthest ends of a radial DN. This in turn requires more investment from the network operators and results in a considerable amount of revenue losses. Hence, for the past few decades, there has been extensive research on finding efficient and suitable methods for the optimal placement and sizing of DGs in a distribution network. This paper, which is an extension of our previous work presented at the ECCE Asia 2021 conference [1], proposes a general and efficient methodology for DN loss minimization with RES-based DG placement and sizing.

A. Literature Review

Although most RES-based DGs have uncertain power generation profiles, earlier research works focused on finding the optimal location and sizing of DGs considering a fully controllable generation. Load uncertainties were also usually not considered. This was done to primarily keep the computational burden low, with the objective function being to minimize the network's real power loss. In [2], an analytical method for finding the optimal DG location and sizing is presented which minimizes the network real power loss. Similarly, the authors in [3] present an analytical method to determine the DG sizes and associated power factors (pfs) at each network bus that results in the greatest reduction in power losses. The optimal DG location, size and pf are then selected which produced the least network losses. In [4], the authors develop three analytical expressions for finding the optimal DG size and their pfs at each network location. The final solution is determined based on the least network losses obtained from all the calculations.

In addition to minimizing the power losses, in some literature, the authors propose methods to improve the voltage stability of radial distribution networks through optimal DG placement. Although network voltage stability is mainly dependent on its dynamic performance, suitable steady-state planning with an objective for improving the same enhances the network performance and reliability during dynamic disturbances. Thus, in [5], the authors propose a voltage stability index-based method to find the weakest network bus which is most prone to a voltage collapse. It is chosen for a DG installation, and the authors show that such an action improves the overall network voltage stability. A methodology is developed in [6] for optimal DG placement based on the voltage stability of a network and to minimize power losses. However, the optimal DG sizes are determined by a search algorithm. In such a context, a comparative study is presented in [7] for the various sensitivity-based indices for optimal DG installation. To reduce the power losses in a network while improving its voltage stability, an analytical method for DG placement and finding its pf is presented by the authors in [8]. The final location and sizing of the DGs are found by a benefit-cost analysis that increases the network loadability limit. Most recently in [9], the authors develop a new sensitivity-based method for the optimal DG placement and sizing problem. The common network loss and voltage sensitivity indices are combined to obtain new indices, which help in achieving more efficient problem-solving.

In recent literature, new methods have also been used to solve the problem of optimal DG placement and sizing. The authors in [10] use a metaheuristic algorithm Particle Swarm Optimization (PSO) to solve the problem with power loss minimization objective. Also, a Mixed Integer Non-linear Programming (MINLP) model-based bi-level formulation is presented by the authors in [11] for solving a similar problem. Here, with the objective of network loss minimization with DGs, the optimization model is split into the Siting Planning Model (SPM) and Capacity Planning Model (CPM). Sensitivity indices are used in SPM to find the possible DG locations in a network, while in CPM, the optimal DG sizes are obtained by Sequential Quadratic Programming (SQP) and Branch and Bound (B&B) techniques. In [12], the authors determine the optimal mix of different types of DGs in a distribution network along with their location and sizing. Here, an analytical method is proposed that effectively minimizes network losses.

As stated earlier, in most of the previous studies, network uncertainties have generally been ignored or considered with much-approximated approaches for the optimal DG placement and sizing problem. However, with the present focus on increasing the power share of RES-based DGs in a power network, recent literature like [13], [14], [15], [16], [17], [18], [19], [20], [21] consider different kinds of network uncertainties in solving the optimal DG placement problem.

In [13] and [14], the authors consider uncertainties in network loads and their variations for the optimal DG placement and sizing problem with the inclusion of annual load growth formulations. Several performance indices are used for this purpose. In [15], the authors propose an index-based analytical method to find the optimal location, sizing and pf of wind-based DGs in a DN. Time-dependent load variation and probabilistic wind power generation are considered here. At different load levels in a DN, and under a known power injection from the grid, a similar problem is solved by the authors in [16]. Ref. [17] solves the problem of optimal placement of RES-based DGs in active DNs with improved Harris Hawks Optimizer (HHO) using PSO. The uncertain generation profiles of Solar Photovoltaic (SPV) and Wind Turbine Generators (WTGs) are considered with a probabilistic approach. In [18], the problem objective is to minimize the total energy loss and maximize the profit of renewable DG owners in a DN. A hybrid metaheuristic algorithm is developed in this paper for the optimal DG placement under the stochastic generation profile of the RES-based DGs. Similarly, the authors in [19] use a β-chaotic sequence spotted hyena metaheuristic algorithm to obtain the deterministic and probabilistic locations and sizing of wind turbines in DNs. Objectives for the corresponding planning problem are to reduce network loss and improve its voltage profile and stability. Here, probabilistic planning is obtained with Monte Carlo Simulation (MCS). In [20], the problem of intermittency and uncertainty of RES-based DG units in a DN is overcome by installing controllable and renewable DG units with Reactive Power Sources (RPS). Uncertainties of wind and solar PV-based DGs and load are treated by probabilistic optimization. Probabilistic planning of DGs and Switched Capacitor Banks (SCB) based on various network performance and reliability indices is provided in [21]. The uncertainties of demand, energy price, power generation and equipment availability are considered here in an MINLP formulation, solved by hybrid Krill Herd Optimization (KHO) and Crow Search Algorithm (CSA).

B. Motivation

The above literature review shows that very few of the existing works solve the optimal DG placement and sizing problem with the objectives of minimizing the network losses and improvement of network voltage stability. In most cases, the final solutions are obtained by analytical methods which ignore the vital network operational constraints of line power flow limitations, and limits on bus voltage magnitudes. Such omissions of the essential network constraints may lead to the possible infeasibility of operation, especially for highly loaded distribution networks. The imperfect operation, location, and sizing of DGs may also lead to line overloads and power flow congestions, which in turn can cause segments of the network to trip. In radial systems, this means a loss of all downstream loads. Further, non-optimal DG siting and sizing without consideration of the bus voltage constraints may result in a poor voltage profile of the entire network. In presence of uncertain generation by the RES-based DGs, and uncertain load profiles, the issues of voltage instability in a DN are much more pronounced. The use of average or rated generation level and maximum load demand at each bus for RES-based DG placements may lead to unreliable network operation, especially during large variations of the uncertain variables. In most of the previous works that involve uncertainties in the DG placement problem, load uncertainties are mainly considered with an annual load growth profile. In some cases, generation uncertainties are considered with much simplification such as using variable pfs of the DG units. Stochastic optimization and probabilistic formulation are also used in some literature. However, such planning involves a high computational burden due to the requirement of repeated solutions to the problem for different scenarios.

The literature review also shows that in none of the studies, transformer tap settings are a part of the DG planning problem. Although, these settings play a vital role in maintaining a good voltage profile in a network, reducing the losses, and decreasing the required DG sizes. Transformer tap set points are mainly considered as part of the operational variables rather than planning variables. However, in a DN, the available transformers normally have off-load tap changing provision and they are set at a particular tap for a prolonged time period. Due to such operation, the tap set points may be considered as part of the planning variables and optimized to obtain a better-planned network.

Therefore, to address the shortfalls of the existing literature, in this paper, the optimal DG placement and sizing problem under various generation and load uncertainties, is formulated as a non-linear Optimal Power Flow (OPF) problem and solved by the $2m+1$ Point Estimate Method (PEM) [22]. The proposed methodology is formulated in a generalized form that can handle any probabilistic uncertainty while satisfying all network operational constraints. It obtains optimal DG locations and sizes and assures a reliable network operation with a low computational burden. The optimal transformer tap settings in a DN are also determined, which results in a substantial improvement of the overall network operation.

C. Contributions

The major contributions of this paper and its differences from our previous work [1] may be stated as follows:

1) Various uncertainties of the RES-based DGs and network load demands are considered here for the optimal DG placement and sizing problem by developing an efficient planning methodology based on the $2m+1$ PEM technique. In our previous work, network uncertainties were not considered.
2) Optimal transformer tap settings are determined, which improves the overall network operation. This was also not a part of our previously published conference paper.
3) The optimal DG locations are determined by combined loss sensitivity and voltage stability factor-based approaches, while the DG sizes are obtained by solving non-linear probabilistic AC OPFs, which ensure the satisfaction of all network operational constraints.
4) The methodology is developed on an open and free-to-use software platform and compared with the existing techniques to find its effectiveness.

D. Paper Structure

This paper is organized into the following sections: Section I presents an introduction of the paper, with a detailed overview of the existing literature, motivation for the present work, contributions, and the paper structure. The mathematical formulation of the problem along with a detailed discussion on the various performance indices and the corresponding probabilistic modelling are provided in Section II. The point estimate method is discussed in detail in Section III, followed by the proposed methodology in Section IV. The simulation results and their comparative analyses are discussed in Section V, while the concluding remarks are presented in Section VI.

II. Mathematical Modelling and Indices

In this research work, a methodology is proposed for the optimal placement and sizing of RES-based DGs in a DN. Load uncertainty of the network is also considered. Probabilistic modelling is formulated based on the DN losses and voltage stability indices to minimize the expected losses as well as improve the network voltage stability. All operational constraints are also satisfied to ensure a feasible planning. A mathematical formulation of the problem, the different performance indices used, and the probabilistic modelling of the uncertain variables are described next.

A. Mathematical Formulation

The DG placement problem may be formulated as an optimization problem which minimizes the network real power loss and the reactive power consumed, with an improvement of voltage stability [1]. Thus, it can be formulated as:

$$F_{obj} = \min(A_p P_{Loss} + A_q Q_{Loss}) \tag{1}$$

This is subject to the satisfaction of network equality and inequality constraints. Equality constraints are the network power equality constraints to be satisfied at each bus. In compact vector formulation, these are:

$$\begin{aligned} \mathbf{P_i} - \mathbf{P_g} + \mathbf{P_d} - \mathbf{P_{DG}} &= \mathbf{0} \\ \mathbf{Q_i} - \mathbf{Q_g} + \mathbf{Q_d} - \mathbf{Q_{DG}} &= \mathbf{0} \end{aligned} \tag{2}$$

The inequality constraints on the other hand correspond to the physical and operational limits of the various network equipment. Real and reactive power generation limits of the controllable generators and DGs, voltage magnitude and phase limits at the network buses, line power flow limits, and limitations on transformer tap positions constitute the inequality constraints. In vector form these can be stated as follows:

$$\begin{aligned} \mathbf{P_g^{min}} &\le \mathbf{P_g} \le \mathbf{P_g^{max}} \\ \mathbf{Q_g^{min}} &\le \mathbf{Q_g} \le \mathbf{Q_g^{max}} \\ \mathbf{P_{DG}^{min}} &\le \mathbf{P_{DG}} \le \mathbf{P_{DG}^{max}} \\ \mathbf{Q_{DG}^{min}} &\le \mathbf{Q_{DG}} \le \mathbf{Q_{DG}^{max}} \\ \mathbf{V^{min}} &\le \mathbf{V} \le \mathbf{V^{max}} \\ \boldsymbol{\theta}^{\mathbf{min}} &\le \boldsymbol{\theta} \le \boldsymbol{\theta}^{\mathbf{max}} \\ \boldsymbol{\tau}^{\mathbf{min}} &\le \boldsymbol{\tau} \le \boldsymbol{\tau}^{\mathbf{max}} \\ |\mathbf{S_l}| &\le \mathbf{S_l^{max}} \end{aligned} \tag{3}$$

Here, the superscripts min and max denote the minimum and maximum limits of the different variables. Reactive power is absorbed in a power network by its reactances—inductances absorb and capacitances generate. In this paper, Q_{Loss} refers to the absorbed reactive power. Although it should not be typically categorized as a loss, in most existing literature this term has been used to quantify the installation of reactive power support devices for maintaining normal network operation. It may be thought of as, P_{Loss} leading to direct revenue loss, whereas reactive power absorption requires investment in RPS, which is a capital expenditure. To remove any possible ambiguity related to the widely used terminologies, the term Q_{Loss} is used in this paper to denote the total absorbed reactive power. Similarly, power absorbed by the network impedances is denoted as apparent power loss, S_{Loss}.

B. Performance Indices

Usually in most existing literature [5], [7], [9], various sensitivity-based performance indices are developed and used to find the optimal DG placements in a DN. Such an approach provides a quick solution to the problem with much less computational burden compared to the full exhaustive search processes. In this paper, a couple of network power loss and voltage stability indices called Network Loss Sensitivity Factor (NLSF) and Voltage Stability Factor (VSF) [1] are used to find the optimal DG locations, while their optimal sizes are determined by solving non-linear OPFs. Solving the OPFs ensure that the DG placements do not violate any network constraints. In addition, a Tap Sensitivity Factor (TSF) [23] is utilized to find the optimal transformer tap settings. The different sensitivity indices may be defined as follows [1], [23]:

1) Network Loss Sensitivity Factor (NLSF): Different NLSFs with real and reactive power injections at the system buses can be defined as:

$$\begin{aligned} NLSF_{p^k} &= \left|\frac{\partial S_{Loss}}{\partial P_i^k}\right| = \sqrt{\left(\frac{\partial P_{Loss}}{\partial P_i^k}\right)^2 + \left(\frac{\partial Q_{Loss}}{\partial P_i^k}\right)^2} \\ NLSF_{q^k} &= \left|\frac{\partial S_{Loss}}{\partial Q_i^k}\right| = \sqrt{\left(\frac{\partial P_{Loss}}{\partial Q_i^k}\right)^2 + \left(\frac{\partial Q_{Loss}}{\partial Q_i^k}\right)^2} \end{aligned} \tag{4}$$

In the above equation, $NLSF_{p^k}$ and $NLSF_{q^k}$ are the apparent power loss sensitivities with respect to the real and reactive power injections P_i^k and Q_i^k at the k^{th} bus of the network. $\frac{\partial P_{Loss}}{\partial P_i^k}$, $\frac{\partial Q_{Loss}}{\partial P_i^k}$, $\frac{\partial P_{Loss}}{\partial Q_i^k}$ and $\frac{\partial Q_{Loss}}{\partial Q_i^k}$ are the corresponding network real and reactive loss sensitivities with P_i^k and Q_i^k [8].

As power is injected at a bus, the network losses usually decrease, and the associated loss sensitivities have negative values. However, $NLSF_{p^k}$ and $NLSF_{q^k}$ have positive values and buses having higher values of $NLSF$ are the better candidates for a DG placement as they provide a greater reduction in losses per unit of injected power.

2) Voltage Stability Factor (VSF): Good network voltage stability is an important criterion for the reliable operation of a DN. With better voltage stability, the network loadability limit is also increased. Installation of DGs in a DN should not only reduce the power losses but also improve the overall voltage stability of the network. Authors in [5] define Voltage Stability Factor (VSF) at a bus n as follows:

$$\begin{aligned} VSF_n = &\left\{{V_k}^2 - 2\left(r^{kn} P_{d_{eff}} + x^{kn} Q_{d_{eff}}\right)\right\}^2 \\ &- 4\left({r^{kn}}^2 + {x^{kn}}^2\right)\left({P_{d_{eff}}}^2 + {Q_{d_{eff}}}^2\right) \ge 0 \end{aligned} \tag{5}$$

Line resistance and reactance between buses k and n are represented by r^{kn} and x^{kn} respectively, while $P_{d_{eff}}$ and $Q_{d_{eff}}$ are the effective real and reactive power demands at the bus n. Effective demand at a bus is the net demand realized at the bus by the network. It is the sum of all downstream loads connected from the bus till the end of the network and the associated line losses.

A low bus VSF value indicates that the bus is nearer to its voltage collapse point. Therefore, to have good voltage stability, the value of its VSF should be as high as possible. As the installation of a DG injects real and reactive power at a bus and improves its VSF value, the buses with low values of VSF are ideal locations to install a DG [1]. Now, the stability of a network is determined by its weakest bus, thus, Network Voltage Stability Factor (NVSF) is the VSF of its weakest bus. It is defined as follows:

$$NVSF = \min_{n\in\Omega} [VSF_n] \tag{6}$$

3) Tap Sensitivity Factor (TSF): As discussed earlier, the transformer off-nominal tap settings in a DN offer a versatile control parameter for the network operators to optimize its operation. Optimal transformer tap settings provide a good network voltage profile and also minimize power losses. In most of the DNs, taps of off-load tap changing transformers are fixed at a specified position for prolonged periods. Therefore, when such tap settings are integrated with the optimal DG planning problem, a better-optimized network is obtained. After the DG locations and sizes are determined, the tap settings can be obtained by a gradient search method. In a DN, the reduction of real power losses is more important than reducing the reactive power absorption. Thus in this respect, TSF can be defined as the sensitivity of network real power losses with a change in the transformer tap positions [23]:

$$TSF = \frac{\partial P_{Loss}}{\partial \tau^{kn}} = \frac{\partial P_{Loss}}{\partial P_i^n} \times \frac{\partial P_i^n}{\partial \tau^{kn}} \tag{7}$$

$\frac{\partial P_i^n}{\partial \tau^{kn}}$ is defined as [23]:

$$\frac{\partial P_i^n}{\partial \tau^{kn}} = \frac{V^k V^n}{{\tau^{kn}}^2} \left[\mathbb{b}^{kn} \sin\theta^{kn} - \mathbb{g}^{kn} \cos\theta^{kn}\right] \tag{8}$$

Here, $\mathbb{b}^{kn}$ and $\mathbb{g}^{kn}$ denote the line susceptance and conductance values between buses k and n; and θ^{kn} denote the voltage phase angle difference between the same buses. As a tap set point is lowered, the voltage at the transformer secondary increases, thereby reducing the network power losses. Therefore, the search direction for the optimal tap position is the negative of the gradient. Tap t^{kn} is adjusted at each iteration of the gradient search method as follows:

$$\text{At each iteration } \iota,\ {\tau^{kn}}_{\iota} = {\tau^{kn}}_{\iota-1} - TSF^{-1} \times \phi_{step} \tag{9}$$

In (9), ϕ_{step} is the step size for change in P_{Loss}. This should be selected judiciously to obtain an accurate tap setting with a reasonable computational burden. A smaller step size helps to obtain a better result at the expense of a greatly increased computational burden. Again, larger step sizes are much more susceptible to overlooking a potentially better solution. In a practical distribution transformer, it is not possible to set the tap ratios so precisely. Thus, a very accurate estimation of the step size and its precise tuning for the proposed algorithm is not required. However, a good, guessed value found by running the simulation a few times with various step sizes allows reaching the final solution within a justified time frame.

C. Probabilistic Modelling

When network generation uncertainties due to RES-based DG units and load uncertainties are considered, deterministic placement and sizing of DGs do not provide optimal planning. In many cases, these uncertainties may even cause violations of the various network variables. Probabilistic realization of the DG locations and sizes obtained by the PEM result in a feasible final planning solution with much less computational burden than the stochastic methods. The first step for probabilistic planning is to determine the corresponding probabilistic models of the uncertainties. Such models of the different uncertainties considered in this paper can be obtained as follows:

1) Probabilistic Modelling of Wind Turbine Generator (WTG): A WTG shows non-linear power output characteristics to the wind speed variations. The intermittency and uncertainty of the wind flow pattern also cause the output power from a WTG to be intermittent. Usually, it is difficult to accurately forecast the wind speed at a specific place and time. However, probabilistic modelling of the wind flow pattern at a location is much more readily available, and it can be used to predict the expected value of a WTG power output. Wind flow patterns can be approximated to the Weibull's distribution, with the Probability Density Function (pdf) as:

$$f_{pdf}(u) = \frac{\beta}{\alpha}\,(u/\alpha)^{\beta-1} \exp\left(-\left(\frac{u}{\alpha}\right)^{\beta}\right) \tag{10}$$

Here, $f_{pdf}(u)$ is the pdf of wind speed variation, u, with $(\alpha > 0)$ and $(\beta > 0)$ being the scale and shape parameters of the Weibull's pdf. These parameters can be obtained from the mean and standard deviation of the historical wind speed data. The power output of a WTG with wind speed variations is defined as per the following:

$$P_{gW} = \begin{cases} 0 & 0 \le u < u_{ci};\ \ u_{co} \le u \\ P_{gW_{rt}} \times \frac{u-u_{ci}}{u_r-u_{ci}} & u_{ci} \le u < u_{rt} \\ P_{gW_{rt}} & u_{rt} \le u < u_{co} \end{cases} \tag{11}$$

Here, P_{gW} is the WTG power output at wind speed u and $P_{gW_{rt}}$ is the rated power of the WTG; whereas u_{ci}, u_{co} and u_{rt} are the cut-in, cut-out and rated wind speeds respectively. For a variety of wind speeds as per Weibull's distribution, a corresponding distribution of a WTG power output is also achievable.

2) Probabilistic Modelling of System Loads: The uncertain load variations at each network bus are considered to follow the normal distribution. Load pdf is expressed as follows:

$$f_{pdf}(d) = \frac{1}{\sqrt{2\pi\sigma_d^2}} \exp\left(-\frac{(d-\mu_d)^2}{2\sigma_d^2}\right) \tag{12}$$

In the above equation, $f_{pdf}(d)$ is the pdf of bus demand/load, d, while $(\sigma_d > 0)$ is the standard deviation and $(\mu_d > 0)$ is the mean value.

III. Point Estimate Method

In 1998, Hong [22] proposed a point estimate method to probabilistically estimate the expected output of a function of random variables from only a few exact evaluations. It provides an efficient calculation of the expected function outcomes with high accuracy [24], [25]. While there are many variations of this method, the most common ones used are the two and three-PEMs. These methods considerably reduce the computational burden of finding the expected outcome of such functions compared to the MCS method, where a large number of exact function evaluations at different realizations of the random variables are required. Thus, the accuracy of the MCS method is dependent on the number of samples collected and their variation. On the contrary, in PEM, the sample numbers and their variations do not affect the accuracy of the final expected value.

Three-PEM provides more accuracy than two-PEM, and therefore, the former is used in this paper to probabilistically determine the DG locations and their sizes in a DN. For completeness, a brief overview of the three-PEM for a function with a single random variable is provided as follows [22]:

Let there be a single discrete random variable g_b which has Y distinct points. Therefore, the pdf of g_b is $f_{pdf}(g_b)$ which is the same as its probability distribution function. $Z = R(g_b)$ be a function of g_b. Thus, the mean or expected value μ_{g_b}, standard deviation σ_{g_b} and coefficient of variation ε_{g_b} of g_b can be expressed as follows:

$$\mu_{g_b} = E(g_b) = \sum_{y=1}^{Y} [g_{b,y} f_{pdf}(g_{b,y})] \tag{13}$$

$$\sigma_{g_b}{}^2 = E\left(g_b{}^2\right) - \mu_{g_b}^2 \tag{14}$$

$$\varepsilon_{g_b} = \frac{\mu_{g_b}}{\sigma_{g_b}} \tag{15}$$

In the above expressions, $g_{b,y}$ denotes the y^{th} location of the variable g_b. Similarly, the mean and standard deviation of Z can be obtained by replacing g_b by $R(g_b)$. Now, by three-PEM, the objective is to obtain the mean value of $R(g_b)$ from its three actual evaluations. Thus,

$$p_{b,1}R(g_{b,1}) + p_{b,2}R(g_{b,2}) + p_{b,3}R(g_{b,3}) = E(R(g_1)) \tag{16}$$

Here, $p_{b,1}$, $p_{b,2}$ and $p_{b,3}$ are the weights associated with the random variable locations $g_{b,1}$, $g_{b,2}$ and $g_{b,3}$ respectively. The values of these weights are required to be found out from the three-PEM. A detailed description of this is provided in the Appendix.

1) Probabilistic Solution by PEM: A particular advantage of the PEM is that it can be suitably generalized for functions of more than one random variable. Let, $\mathbf{G} = [g_1\ g_2\ g_3 \ldots\ldots .g_m]$ be a set of m distinct random variables, for a function $Z = R(\mathbf{G})$. Then, the process to obtain the mean $\mu_Z = E(R(\mathbf{G}))$ can be generalized as follows:

1) The mean and variances of all the random variables g_b are computed. $b = 1, 2, \ldots, m$
2) Calculate $p_{b,a}$, $\lambda_{g_b,3}$, $\lambda_{g_b,4}$, and $\zeta_{b,a}$ for each g_b; $a = 1, 2$. $p_{b,3} = (\frac{1}{m}) - p_{b,1} - p_{b,2}$. $\zeta_{b,3} = 0$.
3) Set *E(Z)* = 0.
4) Choose a single random variable g_b, $b = 1, 2, \ldots, m$.
5) $\forall a$, calculate the point estimates of g_b, i.e. $g_{b,a}$.
6) Evaluate $Z = R(\mathbf{G})$, with $g_{b,a}$, while setting all the other random variables at their mean positions i.e., $Z = R(\mu_{g_1}, \mu_{g_2}, \mu_{g_3}, \ldots, g_{b,a}, \ldots, \mu_{g_m})$, $a = 1, 2, 3$.
7) Update the function mean: $E(Z) = E(Z) + p_{b,a} \times R(\mu_{g_1}, \mu_{g_2}, \mu_{g_3}, \ldots, g_{b,a}, \ldots, \mu_{g_m})$.
8) Exit the process if all the random variables are considered for the calculation. Otherwise repeat steps 4 to 8.

Here, $\zeta_{b,a}$ are the constants to be determined for b^{th} random variable at a^{th} location, so that, $g_{b,a} = \mu_{g_b} + \zeta_{b,a}\sigma_{g_b}$, $a = 1, 2, 3$. $\lambda_{g_b,t}$ denote the ratio of $M'_{ct}(g_b)$ to $\sigma_{g_b}{}^t$, where, $M'_{ct}(g_b)$ is the t^{th} order central moment of g_b. $\lambda_{g_b,3}$ and $\lambda_{g_b,4}$ are known as coefficient of skewness and kurtosis of g_b respectively.

Now, for obtaining a fair estimation of μ_Z, the deterministic solution of Z must be solved for $3m$ number of times for m random variables. However, it can be observed that $\forall b$, $\zeta_{b,3} = 0$ i.e. for m number of times all variables are at their mean values, $\boldsymbol{\mu}_{\boldsymbol{\mu}_\mathbf{G}} = [\mu_{g_1}\ \mu_{g_2}\ \mu_{g_3}\ \ldots\ \mu_{g^m}]$. Therefore, it is sufficient to solve the deterministic solution of Z only once at this point, with the value of associated weight suitably augmented as:

$$p_{b,0} = \sum_{b=1}^{m} p_{b,3} \tag{17}$$

Therefore, for a function of m random variables, there involves $2\,m + 1$ repeated evaluations of the function Z, and so, the method is also known as $2\,m + 1$ PEM. The solution of any probabilistic function can be conveniently obtained if Z is replaced by that function in the above formulation.

IV. Proposed Methodology

In this section, the proposed solution methodology for optimal DG placement and sizing will be discussed in detail. The performance indices described in Section II-B can be directly used to find the optimal DG locations and transformer tap positions when power generation from the DGs are completely controllable. However, with RES-based DGs having uncertain, uncontrollable generation, and with network load uncertainties, expected values of the performance indices must be used to find the potential DG locations. Their expected values are obtained by applying the $2m + 1$ PEM as discussed in Section III.

The optimal DG placement and sizing problem is a nonconvex MINLP problem, in which the global optimality of the results cannot be guaranteed. However, achieving a good local optimum solution and ascertaining its quality can be done from a comparative analysis with existing methodologies. To obtain the global optimum location and sizing, the only way is to run OPF for all possible combinations of network locations and select the location and DG size that produces the lowest objective function value. Such exhaustive searching may be possible for a small system, but as the network size and the number of DG installations increase, there is a combinatorial explosion, making the process prohibitively complex. It only results in a minimal improvement of the objective function at the expense of a tremendous increase in computational burden [1]. Hence, narrowing down the search area with suitable network indicators results in achieving the proper balance between the quality of the final solution and the computational complexity of the methodology.

From the expected values of NLSF and VSF for each network bus, the corresponding bus rankings are computed. The buses are ranked according to their NLSF values in descending order, and as per their VSF values in ascending order. Next, a percentage of the best buses from each ranked set are chosen as per the user's preference and a superset of potential buses for DG installation is formed. In this paper, the best $\frac{1}{4}^{\text{th}}$ buses from each set is selected for the superset [1]. For each of these bus locations, the expected values of the objective function shown in (1) are computed by PEM. This operation also provides the expected size of the DGs to be installed at the respective bus locations. The final installation location and size of DGs are selected which provides the minimum expected value of the objective function F_{obj}. After the first DG installation site is determined, the next DG location is found by repeating the entire process. The number of DGs to be installed is determined by the user. This process of sequential addition of DGs in a network help to obtain good final planning with much less computational burden [1]. For the first DG installation in the network, the performance indices are computed considering only the load uncertainties. However, both load and generation uncertainties are considered for computing the NLSF and VSF values for the next DG installations. After all the DGs are installed in the network, the transformers' optimal tap settings are determined by the iterative gradient descent method. During each iteration, estimated sizes of the DGs are also determined through the probabilistic solution of F_{obj} to obtain an appropriately optimized network planning. A small tolerance value (υ) is selected as the convergence criteria of the gradient descent method. The method also terminates when the probabilistic solution of F_{obj} fails to converge. A detailed flow chart is depicted in Fig. 1 [23] to illustrate the complete proposed methodology.

The DG planning methodology developed in this research work is based on open-source and free-to-use software. The simulation codes are written in GNU Octave [26], while the objective function F_{obj} is solved with MATPOWER [27], [28] using the MATPOWER Interior Point Solver (MIPS) [29].

V. Results

The proposed methodology is tested for applicability on a couple of test networks and a practical DN network present in Norway. The test systems are the standard IEEE 69 bus [30] and Indian 85 bus [31] radial distribution networks. The practical system is an 88-bus network present in a remote Froan island in Norway [23]. Testing the proposed methodology on these systems establish its efficacy. Voltage magnitudes at the buses for the first two test systems are allowed to vary within $\pm 5\%$ of their base values. Whereas, for the Froan network, voltage magnitude at the slack bus is restricted within $\pm 5\%$, and at the other buses these are allowed to vary between $\pm 10\%$ of their base values.

WTG-based DGs are assumed to be installed at each network, with the corresponding wind flow patterns to be following that used in [19]. All WTG characteristics are the same as that used in [32] for 3 MW turbines. The base loads for the networks at each of the buses are considered as the mean load values (μ_d) to account for the load uncertainty, and corresponding σ_d values are considered as 25% of the mean values. Such high values of μ_d and σ_d ensure that there is enough load variation to cover most of the unusual situations. For finding the optimal size of

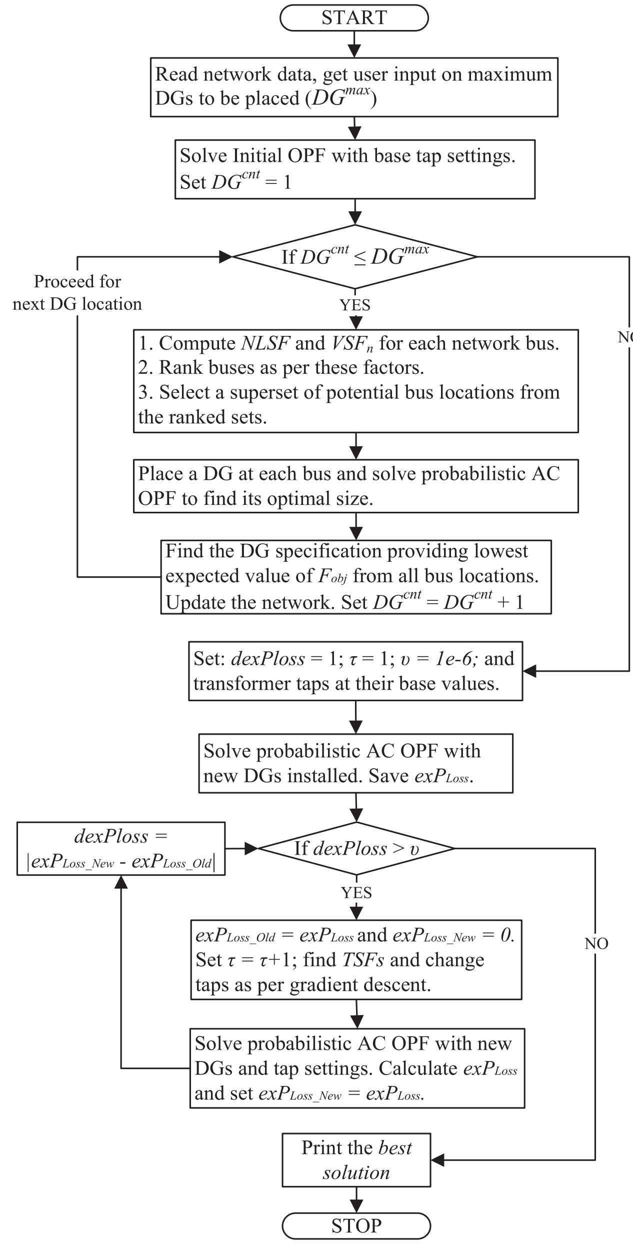

Fig. 1. Algorithm flow chart of the proposed methodology [23].

the WTGs, expected real power generations from the WTGs are calculated by PEM, and their rated power capacity $P_{gW_{rt}}$ are assumed as twice the expected real power generations, rounded to the next MW. However, the maximum rated power of the WTGs are limited to 3MW for the IEEE 69 bus and the Indian 85 bus systems.

In this paper, the research work aims to reduce both real and reactive power losses by the same degree. Hence in (1), values of the corresponding weights are set as 1, i.e. $A_p = A_q = 1$. The program simulations are run on a personal computer with Intel(R) Core(TM) i7-8650 CPU @ 1.90 GHz (base frequency) and 16 GB of RAM. For obtaining the WTG generation pattern and load variations in the PEM, 100,000 random data are generated based on their respective pdfs.

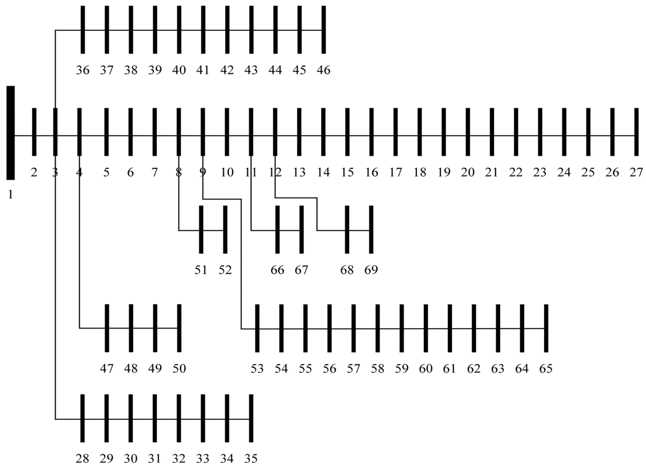

Fig. 2. Single line diagram of the IEEE 69 bus network [33].

TABLE I
OPTIMAL DG PLACEMENT RESULTS FOR THE IEEE 69 BUS NETWORK

No. of DGs	Bus No.	Exp. DG Power Gen. Real (kW)	Exp. DG Power Gen. Reactive (kVAR)	$P_{gW_{rt}}$ (kW)	pf (lag)	Exp. Network Losses Real (kW)	Exp. Network Losses Reactive (kVAR)	NVSF
1	61	1834.86	1307.54	3000	0.814	31.408	18.098	1.084
2	61	1834.86	1208.45	3000	0.835	13.737	10.027	1.126
	12	953.77	547.19	2000	0.867			
3	61	1834.86	1000.65	3000	0.878	7.841	7.443	1.165
	12	953.77	546.76	2000	0.867			
	64	394.38	205.26	1000	0.887			

A. IEEE 69 Bus Network

The IEEE 69 bus network is a radial distribution system with 69 buses and 68 branches. The base real and reactive power demands are 3802.19 kW and 2694.60 kVAR respectively. Without any DG installation, the real and reactive power losses are 200.640 kW and 91.193 kVAR, with an initial NVSF value of 0.834. A single-line diagram of the network is shown in Fig. 2 [33]. Three WTGs are assumed to be installed on the network. Considering the uncertainty of loads and generation by WTGs, results for the optimal DG placement problem obtained by the proposed methodology are provided in Table I.

A close observation of the Table I reveals that with an increased number of DG installations, power losses of the network decrease steadily. For each DG installation, network voltage stability improves considerably than the base case with no DGs. The best possible bus locations for the DG placements are 61, 12 and 64, with the rated sizes being 3 MW, 2 MW and 1 MW respectively. A 100% reliable operation of the network has been observed in the PEM. Therefore, with the rated WTGs in place, operating under the estimated wind flow pattern, the network functions appropriately, satisfying all the operational constraints.

A comparative study with the results in [19] is presented in Table II. However, for uncertain planning in [19], only 85% of the base load value at each bus is considered as the corresponding mean load. This results in 17.65% more mean load value being considered by the proposed methodology in this paper. It is observed from Tables I and II that, with even a considerably increased mean load value, the proposed methodology provided a 7.92% and 53.21% reduction in the expected network real power losses for one and two WTG placements respectively. Also, for the latter case, the DG installation locations found by the proposed methodology are buses 61 and 12, whereas, the optimal locations stated in [19] are buses 61 and 62. This proves the effectiveness of the proposed technique in finding the optimal DG sizes and their placement.

TABLE II
COMPARATIVE STUDY OF THE RESULTS OF THE IEEE 69 BUS NETWORK

No. of DGs	Methodology in ref. [19] Bus No.	Methodology in ref. [19] Exp. Real Loss (kW)	Prop. Methodology Bus No.	Prop. Methodology Exp. Real Loss (kW)	Loss red. by the Prop. Methodology
1	61	34.110	61	31.408	7.92%
2	61, 62	29.360	61, 12	13.737	53.21%

TABLE III
OPTIMAL DG PLACEMENT RESULTS FOR THE INDIAN 85 BUS NETWORK

No. of DGs	Bus No.	Exp. DG Power Gen. Real (kW)	Exp. DG Power Gen. Reactive (kVAR)	$P_{gW_{rt}}$ (kW)	pf (lag)	Exp. Network Losses Real (kW)	Exp. Network Losses Reactive (kVAR)	NVSF
1	28	2503.5	2502.77	3000	0.707	195.200	107.752	0.859
2	28	2503.5	1625.98	3000	0.839	70.558	34.033	1.016
	60	1526.52	1326.99	3000	0.755			
3	28	2503.5	902.169	3000	0.941	44.473	21.418	1.107
	60	1526.52	1323.14	3000	0.756			
	48	455.496	609.195	1000	0.599			

B. Indian 85 Bus Network

This radial system is part of a much larger practical distribution network present in India, with 85 buses and 84 branches. The system has a base real power demand of 2570.28 kW and reactive power demand of 2622.21 kVAR. For this system, a consideration for annual load/demand growth is included in addition to the usual load uncertainty. A planning horizon of 5 years is considered with a yearly load growth rate of 7.5%. Thus, base loads at each bus after 5 years become:

$$\forall k \in \mathbb{N}, \quad P_d^{k^{new}} = P_d^{k^{old}} \times (1 + 0.075)^5$$
$$Q_d^{k^{new}} = Q_d^{k^{old}} \times (1 + 0.075)^5 \qquad (18)$$

These increased base load values at each bus are used for solving the optimal DG placement problem with the usual uncertainties of generation and load demands. Before any DG installation, the network real power loss is 530.923 kW, the reactive loss is 333.884 kVAR, and the NVSF value is 0.704.

Similar to the IEEE 69 bus case, here also three DG installations are considered. The gradual reduction of the network losses and improvements in NVSF values are shown in Table III. It is observed from Table III that the optimal locations for the three DG installations are bus numbers 28, 60 and 48, which

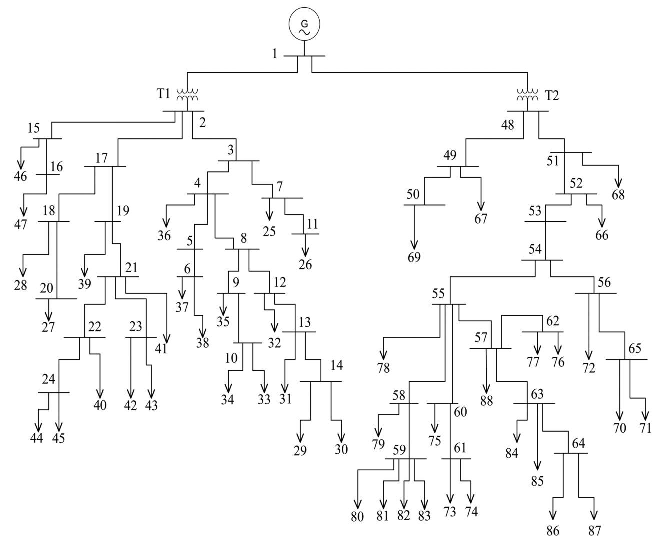

Fig. 3. Single line diagram of the Froan network [23].

resulted in a 57.24% increase in the NVSF value, with a 91.62% reduction in the real power loss, and 93.58% reduction in the reactive power loss with respect to the base case having no DG installation.

C. Froan Network

The Froan distribution network is a real DN present in a remote Froan island in Norway. This island is located almost a hundred kilometres into the sea from the Trondheim coast. Its distribution network is connected to the power grid of mainland Norway with an old undersea cable, which requires replacement in the near future to ensure a reliable power supply. However, replacement of the cable is extremely costly, which prompted the network operator [34] to opt for RES-based DGs to be installed locally. This network is a radial system with 88 buses, and 87 branches with a maximum demand of 198.992 kW and 27.154 kVAR. In the network, there are two main feeders which are connected to the single grid power supply through two step-down off-load tap changing transformers. The transformer taps are considered to have an operational range between 0.8 and 1. The maximum flow limit for each branch and transformer is considered to be 200 kVA. A detailed single-line diagram of the Froan network is shown in Fig. 3 [23].

This network has heavy loads connected at its farthest ends, hence with maximum base demand, even deterministic AC OPFs fail to converge without appropriate DG installations as discussed in [23]. At only 40% of the bus loading, the initial network losses are 20.326 kW and 12.008 kVAR. The initial value of NVSF as 0.243 shows that there exists a very poor voltage profile. For installation of the WTG-based DGs, it is assumed that their rated maximum capacities are 300 kW, while the wind flow characteristics and other DG parameters are the same as that used in the previous case studies. The network operator required a plan for the installation of two DGs and the detailed results for two DG installations on this network are provided in Table IV. The final NVSF value after the installation of 2 DGs increases to 0.766 (improvement of 215.23% compared to the 40% loading case), indicating a substantial betterment of the voltage profile and network reliability. However, compared to the deterministic solution presented in [23], the expected power losses for the uncertain case increase, with a 20.04% decrease in the NVSF value. This is a normal condition, where the overall network operational stability decreases compared to the deterministic case to accommodate the generation and load uncertainties.

TABLE IV
OPTIMAL DG PLACEMENT RESULTS FOR THE FROAN NETWORK

No. of DGs	Bus No.	Exp. DG Power Gen.		$P_{gW_{rt}}$ (kW)	pf (lag)	Transformer Tap Settings		NVSF
		Real (kW)	Reactive (kVAR)					
1	55	132.596	13.286	300	0.995	τ^{1-2}	0.9488	0.766
2	51	40.1701	4.195	100	0.994	τ^{1-48}	0.9574	
Expected Network Losses								
Expected Real Losses (kW)					9.675			
Expected Reactive Losses (kVAR)					4.637			

VI. Conclusion

This paper proposes an efficient planning methodology for finding the optimal location and sizes of RES-based DGs in a DN. The generation uncertainty of wind turbine-based DGs and the network demand uncertainties are appropriately considered in the proposed formulation with a probabilistic realization through three-point PEM. Expected values of loads and generations are used along with different sensitivity-based performance indices to obtain the potential DG locations, their rated sizes and optimal transformer tap set points. The final planning results are obtained by solving non-linear probabilistic AC OPF problems for each potential DG location and selecting the location which provides the least power losses. The solution of probabilistic AC OPF for each location ensures that all network operational constraints are satisfied under generation and load uncertainties. When applied on a number of standard and practical test systems, the proposed methodology minimizes the total power losses with a substantial increase in network voltage stability as depicted in Section V. Comparative studies also establish the advantages and benefits of the method.

A gradient search is also conducted to find the optimal tap settings of the off-load tap-changing transformers present in the network. This helps to improve the system voltage stability with simultaneous reduction of power losses. The proposed methodology is developed on free-to-use software platforms which shall provide the network operators with a cost-effective tool to examine their network stability in presence of RES-based DGs and demand uncertainty.

Appendix

This section describes in detail the three-point PEM. The t^{th} order central moment of g_b is defined as:

$$M'_{ct}(g_b) = \sum_{y=1}^{Y} \left[(g_{b,y} - \mu_{g_b})^t \, f_{pdf}(g_{b,y}) \right] \tag{19}$$

By definition, $\lambda_{g_b,1} = 0$, $\lambda_{g_b,2} = 1$. Now, for the discrete random variable g_b, the Taylor series expansion of $R(g_{b,y})$ around the mean μ_{g_b} can be expressed as:

$$R(g_{b,y}) = R(\mu_{g_b}) + \sum_{t=1}^{\infty}\left[\frac{1}{t!}R^{t'}(\mu_{g_b})(g_{b,y} - \mu_{g_b})^t\right] \quad (20)$$

Here, $R^{t'}(-)$ denotes the t^{th} derivative of $R(-)$ with respect to g_b. The mean of Z can be calculated as:

$$\begin{aligned}\mu_Z = E(R(g_b)) &= \sum_{y=1}^{Y} R(g_{b,y}) f_{pdf}(g_{b,y}) \\ &= R(\mu_{g_b}) + \sum_{y=1}^{Y}\left[\sum_{t=1}^{\infty}\frac{1}{t!}R^{t'}(\mu_{g_b})(g_{b,y} - \mu_{g_b})^t\right] f_{pdf}(g_{b,y}) \\ &= R(\mu_{g_b}) + \sum_{t=1}^{\infty}\left[\frac{1}{t!}R^{t'}(\mu_{g_b})\sum_{y=1}^{Y}(g_{b,y} - \mu_{g_b})^t f_{pdf}(g_{b,y})\right] \\ &= R(\mu_{g_b}) + \sum_{t=1}^{\infty}\left[\frac{1}{t!}R^{t'}(\mu_{g_b})\lambda_{g_b,t}{\sigma_{g_b}}^t\right] \quad (21)\end{aligned}$$

Multiplying (20) by $p_{b,a}$ with g_b replaced by $g_{b,a}$ and summing up the three expressions results in:

$$\begin{aligned}&p_{b,1}R(g_{b,1}) + p_{b,2}R(g_{b,2}) + p_{b,3}R(g_{b,3}) \\ &\quad = R(\mu_{g_b})(p_{b,1} + p_{b,2} + p_{b,3}) \\ &\qquad + \sum_{t=1}^{\infty}\left[\frac{1}{t!}R^{t'}(\mu_{g_b})\left(p_{b,1}{\zeta_{b,1}}^t + p_{b,2}{\zeta_{b,2}}^t\right){\sigma_{g_b}}^t\right] \quad (22)\end{aligned}$$

In three-PEM, the expected value of a function is determined from only three points of actual realization of the function. Therefore, we must have:

$$p_{b,1}R(g_{b,1}) + p_{b,2}R(g_{b,2}) + p_{b,3}R(g_{b,3}) = E(R(g_b)) \quad (23)$$

By comparing the first five terms of the RHS of (22) with RHS of (21), we get:

$$\begin{aligned}&p_{b,1} + p_{b,2} + p_{b,3} = 1;\ \ p_{b,1}\zeta_{b,1} + p_{b,1}\zeta_{b,1} = 0; \\ &p_{b,1}{\zeta_{b,1}}^2 + p_{b,2}{\zeta_{b,2}}^2 = 1;\ \ p_{b,1}{\zeta_{b,2}}^3 + p_{b,2}{\zeta_{b,2}}^3 = \lambda_{g_b,1}; \\ &p_{b,1}{\zeta_{b,2}}^4 + p_{b,2}{\zeta_{b,2}}^4 = \lambda_{g^b,4} \quad (24)\end{aligned}$$

Finally solving (24), we get:

$$\zeta_{b,2} = \frac{\lambda_{g_b,3}}{2} + \sqrt{\left[\lambda_{g_b,4} - 3\left(\frac{\lambda_{g_b,3}}{2}\right)^2\right]} \quad (25)$$

$$\zeta_{b,1} = \frac{\lambda_{g_b,3}}{2} - \sqrt{\left[\lambda_{g_b,4} - 3\left(\frac{\lambda_{g_b,3}}{2}\right)^2\right]} \quad (26)$$

$$\begin{aligned}&p_{b,1} = \frac{1}{\zeta_{b,1}(\zeta_{b,1} - \zeta_{b,2})};\ \ p_{b,2} = \frac{-1}{\zeta_{b,2}(\zeta_{b,1} - \zeta_{b,2})}; \\ &p_{b,3} = 1 - p_{b,1} - p_{b,2} \quad (27)\end{aligned}$$

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