**SCOUR AND BURIAL OF SPHERICAL BODIES, SHORT CYLINDERS AND TRUNCATED CONES INDUCED BY BICHROMATIC AND BIDIRECTIONAL WAVES**

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**ABSTRACT**

This article provides a method by which the scour and self-burial of spherical bodies, short cylinders and truncated cones induced by bichromatic and bidirectional waves can be derived. Here empirical scour depth and the self-burial depth formulae for spherical bodies, short cylinders, and truncated cones are used together with a bed shear stress model beneath bichromatic and bidirectional waves. Examples of results are given for unidirectional bichromatic waves and bidirectional monochromatic waves. The results appear to agree qualitatively with physical expectations.

**Keywords**: Scour depth and self-burial; Spherical bodies; Short cylinders; Truncated cones; Shear stress; Bichromatic and bidirectional waves; Examples of results.

**INTRODUCTION**

Small objects located on the seabed are exposed to the boundary layer flow close to the bed. The objects considered here are spherical bodies, short cylinders, and truncated cones, where typical examples are sea mines on the seabed, whilst spherical bodies may also represent stones in scour protection layers of coastal and offshore structures. These objects may experience a range of seabed conditions caused by erodible bed effects. Consequently, the bed may be flat or rippled; an object may be surrounded by a scour hole, and it may be partly/fully buried, due to the interaction between the incoming flow, the object, and the erodible seabed. The equilibrium condition depends on the relative strength between waves and current, the bed form, the bed material, and the ratio between the near-bed oscillatory fluid particle excursion amplitude and the characteristic dimensions of the object. In shallow and intermediate water depths, the flow is commonly a combination of waves and current. In the vicinity of the bed the flow depends on the local flow conditions, i.e. whether it is wave-, tide- (or current-) dominated. Furthermore, waves in the ocean are random including nonlinear and directional spreading effects. More details on the general background of scour in the marine environment are provided by Sumer and Fredsøe (2002).

 Truelsen et al. (2005) provided empirical formulae for the scour depth and self-burial of spherical bodies based on data from laboratory tests for regular waves. For random waves, Myrhaug et al. (2007) presented results for the scour depth and self-burial of spherical bodies using a stochastic method adopting the formulae by Truelsen et al. (2005).

 The scour depth and self-burial of short cylinders have been addressed by Catano-Lopera and Garcia (2006, 2007), who provided results of the burial (2006) and the scour (2007) around short cylinders for combined regular waves and currents based on laboratory experiments. They also presented empirical formulae for the equilibrium burial and scour depths, which were adopted by Myrhaug and Ong (2009) using a stochastic method to give results for burial and scour of short cylinders under combined random waves and currents including second order wave asymmetry effects.

 The scour around and self-burial of truncated cones exposed to currents and regular waves were investigated in laboratory tests by Catano-Lopera et al. (2011), also providing empirical formulae based on data from the experiments. Examples of other works are numerical modelling (Jenkins et al., 2007); field investigations (Guyonic et al. (2007), Mayer et al. (2007), Baeye et al. (2012)); a combined experimental and numerical study of the flow structure around partially buried short cylinders and truncated cones on a deformed seabed in currents (Catano-Lopera et al., 2013); results for the burial and scour of truncated cones due to long-crested and short-crested nonlinear random waves by adopting the empirical formulae by Catano-Lopera et al. (2011) (Myrhaug and Ong, 2014). Rennie et al. (2017) conducted laboratory experiments to examine the initiation of motion and scour burial of objects at the seabed in current flow.

 The purpose of this article is to provide the equilibrium scour depths and self-burial depths of spherical bodies, short cylinders, and truncated cones, for bichromatic and bidirectional waves. The results are based on using the seabed shear stress under bichromatic and bidirectional waves by Myrhaug et al. (2023) combined with the empirical formulae for spherical bodies (Truelsen et al., 2005), short cylinders (Catano-Lopera and Garcia, 2006, 2007), and truncated cones (Catano-Lopera et al., 2011). Examples of results for both unidirectional bichromatic and bidirectional monochromatic waves are given, and the results appear to agree qualitatively with physical expectations.

**SCOUR AND BURIAL IN REGULAR WAVES**

This section gives a summary of the scour and self-burial in regular waves for spherical bodies, short cylinders, and truncated cones.

**Scour and self-burial of spherical bodies**

The scour and self-burial of a spherical body in regular waves were investigated in laboratory tests by Truelsen et al. (2005). They obtained the following empirical formulae for the equilibrium scour depth *S* (Fig. 1) and the equilibrium self-burial depth (Fig. 2) of a spherical body with diameter *D*:

scour

  (1)

self-burial

  (2)

with the Keulegan-Carpenter number

  (3) where *U* is the undisturbed linear near-bed wave-induced velocity amplitude, and  is the wave period. Here Eqs. (1) and (2) are valid for live-bed scour, for which , and  is the undisturbed Shields parameter defined by

  (4)

Here  is the maximum wave-induced bed shear stress,  is the fluid density, is the acceleration due to gravity, *s* is the sediment density to fluid density ratio,  is the median grain size diameter, and  is the critical Shields parameter for initiation of motion at the bed, i.e. . The scour process attains its equilibrium stage through a transition period.

 The main flow structure causing the scour around spherical bodies placed on the seafloor are the lee-wake vortices governed by , acting as a mechanism transporting the eroded sediments away from the sphere during each half cycle of the wave motion. Moreover, the consequence of the developing scour is that the bearing area of the sphere is reduced causing the load on the soil to increase; the bearing capacity of the soil is exceeded, the soil fails, and the sphere sinks. The whole process continues until the failure of the soil stops, and the sinking ends. More details on the time scale of the scour and on the mechanisms causing the scour and self-burial are provided by Truelsen et al. (2005).

**Scour and self-burial of short cylinders**

Investigations of the scour around and self-burial of short cylinders for combined regular waves and currents were performed in laboratory tests by Catano-Lopera and Garcia (2006, 2007). For regular waves alone, Catano-Lopera and Garcia (2006) proposed the following empirical formulae for the equilibrium scour length of the scour hole around the short cylinder with length , and aspect ratio  in the range 2 to 4 (see Fig. 3):

  (5)

where *L* denotes the downstream length of the scour hole, , and the total length of the scour hole, , for which *L* and the coefficients *p* and *q* are given by, respectively,

  (6)

  (7)

Thus, the upstream length of the scour hole is  (see Fig. 3). Here is defined in Eq. (3), and Eqs. (5) to (7) are valid for . It should be noted that Eqs. (5) to (7) are valid for waves plus current as well, but do not depend explicitly on the current velocity.

 Moreover, the width of the scour hole (Fig. 3) appeared to be in the range 1.3 to 2.5 times the cylinder length , with a mean value of irrespective of .

 For short cylinders Catano-Lopera and Garcia (2006) also proposed the following empirical formula for the equilibrium burial depth *B* (see Fig. 4):

  (8)

 The main mechanism of the scour and self-burial of a short cylinder was observed and discussed by Catano-Lopera and Garcia (2006, 2007) as well as Demir and Garcia (2007), which is summarized as follows. The consequence of the developing scour for a short cylinder placed on the seabed is that the bearing area of the cylinder is reduced causing an increased load on the soil; the soil´s bearing capacity will be exceeded leading to soil failure, and the cylinder will sink. This process continues until the soil failure stops, and the sinking ends (see the three latter references for more details).

**Scour and self-burial of truncated cones**

Investigations of the scour around and self-burial of truncated cones due to regular waves were performed in laboratory tests by Catano-Lopera et al. (2011) leading to the following empirical formulae for the geometric dimensions of the scour hole; the width , the relative downstream length , and the total length  of the scour hole (see Fig. 5):

  (9)

  (10)

where *L* denotes  and , for which *L* and the coefficients *p* and *q* are given by, respectively,

  (11)

  (12)

and Eqs. (9) to (12) are valid for  and .

 The sinking mechanism of truncated cones was discussed in Catano-Lopera et al. (2011) and is mainly due to the combination of the tunnelling below the cone and the continuous reduction of the span shoulder located underneath the cone center, and it is reduced towards the center. The bearing capacity of the soil will be exceeded as the scour develops causing the soil to fail, and the cone will sink. The whole process continues until the soil failure stops and the sinking ends (more details are given in the latter reference).

 The equilibrium burial depth  of the cone with height , base diameter  and top diameter  with  , is given by the empirical formula:

  (13)

Here  is given in Eq. (3), and  is a representative diameter defined as the average, i.e. , and valid for live-bed scour, where  is defined in Eq. (4). The validity ranges of Eq. (13) are the same as those for Eqs. (9) to (12).

 The main flow structures causing the burial and scour around the truncated cone placed on the seafloor, are the flow patterns surrounding the cone induced by the back and forth motion of the waves determined by . During the first half cycle of the motion, the flow field around the cone is a combination of upstream horseshoe vortices, streamline contractions around and over the cone, as well as vortex shedding in the wake. The vortex shedding leads to sediment suspension, transporting the sediments away from the cone, and farther away by the outer flow. During the second half cycle of the wave motion, the situation is reversed, causing transport of sediments against the wave propagation direction. More details are provided in Catano-Lopera et al. (2011).

 The formulae in Eqs. (9) to (13) are summarized as follows:

  (14)

where  are given in Table 1 for the responses *Y* and the cone dimensions *D*.

**SCOUR IN BICHROMATIC AND BIDIRECTIONAL WAVES**

This section presents results for scour and self-burial beneath unidirectional bichromatic waves, and bidirectional monochromatic waves, for spherical bodies, short cylinders and truncated cones. Examples of results for realistic field conditions are given.

 In bichromatic and bidirectional waves, it is reasonable to assume that it is mainly the highest waves which are responsible for the scour response. However, due to lack of comparative data, it is not known which percentage of the highest waves that contributes to the scour and sinking processes. Thus, as a first approximation, the equivalent monochromatic wave is used. More specifically, the highest waves are represented by the amplitude of the equivalent monochromatic wave for unidirectional bichromatic waves and bidirectional monochromatic waves (more details are given in Appendix A and in the discussion leading to Eqs. (A9) and (A10)). It should be noted, however, that in the case of standing waves, an object located at the node would experience zero bed shear stress potentially leading to no scour. Thus, using the equivalent monochromatic wave amplitude to calculate the Shields parameter will not cover this situation without considering phase angles, but this will not be elaborated further here.

 The semi-analytical model proposed in the present study is established based on the

experimental data reported by Truelsen et al. (2005) for spherical bodies, Catano-Lopera and

Garcia (2006, 2007) for short cylinders, and Catano-Lopera et al. (2011) for truncated cones.

Hence, it is assumed that the flow and structural conditions in the present model follows the

 structural behaviour stated in the experiments. According to these references, there was no

structural dynamics within the given ranges of KC and Shields parameter. This is also

applicable to the present proposed model.

**Scour for unidirectional bichromatic waves**

Spherical bodies and short cylinders

Consider an example with m, corresponding to a pebble (see Soulsby (1997, Fig. 4)), m. Further, the near-bed wave-induced velocity for an equivalent monochromatic wave is given by Eq. (A8) (see Appendix A) for m/s , s , s . This corresponds to , which is within the validity range of the shear stress model (see Eq. (A6) in Appendix A). Thus, the maximum wave-induced velocity amplitude for an equivalent monochromatic wave is m/s with the mean wave frequency rad/s . Then, for a spherical body/short cylinder with m , Eq. (3) gives  (see Eq. (8)). Substitution in Eq. (A9) yields m2/ s2, which from Eq. (4) yields the Shields parameter  for m/s2 and (as for quartz sand), i.e. live bed. The short cylinder aspect ratio is taken as .

 For spherical bodies, substitution in Eqs. (1) and (2) yields m and m, respectively. Similarly, for short cylinders, substitution in Eqs. (5) to (8) yields m, m, m . Thus, m . Here it is noticed that , i.e. the downstream length of the scour hole is larger than the upstream length, which Catano-Lopera and Garcia (2007) attributed to asymmetric wave-induced velocity; examples of this were shown in Catano-Lopera and Garcia (2007, Figs. 2b and 3b). This wave asymmetry effect is not included in the present method of bichromatic and bidirectional waves, but appears here as a result of the inherent features of the empirical scour formulae.

 It is of interest to compare this with the results for one of the wave components, e.g. for m/s ,  s . Then, , m2/ s2, and

< 0.19. For spherical bodies, Eqs. (1) and (2) yield m and m, respectively,

which are smaller than the values for bichromatic waves represented by an equivalent

 monochromatic wave. This is as expected and consistent with representing unidirectional

bichromatic waves with an equivalent monochromatic wave with the wave induced

velocity amplitude m/s. Similarly, for short cylinders, Eqs. (5) to (8) yield

m, m, m , and m , which as expected are

 smaller than those for bichromatic waves. Here it is also noticed that . It should be

 noted that for m/s ,  s, the outcome will be qualitatively similar.

Truncated cones

Consider an example with the same bed conditions, i.e. m, m. Now, the near-bed wave-induced velocity for an equivalent monochromatic wave is given by Eq. (A8) for m/s, s, s. This corresponds to , which is within the validity range of the shear stress model. Thus, the maximum wave-induced velocity amplitude for an equivalent monochromatic wave is m/s with the mean wave frequency rad/s. Then, for a truncated cone with m, m, m, m, Eq.(3) gives , which is in the validity range of Eqs. (11) and (12). Substitution in Eq. (A9) yields m2/ s2, which from Eq. (4) gives the Shields parameter and < 0.34, which is in the validity range of Eqs. (11) and (12).

 For truncated cones, substitution in Eqs. (9) to (13) yields m, m, m, m . Thus, m . Here it is noticed that , which is reasonable to attribute to wave asymmetry effects, although this was not discussed in Catano-Lopera et al. (2011).

 As for spheres and short cylinders, it is of interest to compare this with the results for one of the wave components, e.g. for m/s, s. Then, , m2/ s2, and < 0.34. Thus, Eqs. (9) to (13) yield m, m, m, m and m with , which as expected are smaller than those for bichromatic waves represented by an equivalent monochromatic wave. The outcome will be qualitatively similar for m/s, s.

**Scour for bidirectional monochromatic waves**

Spherical bodies and short cylinders

Consider an example with two monochromatic waves for m with

m/s, s, directional spreading and perpendicular incidence to short cylinders (Fig. 4). Then, based on the discussion of Eq. (A10), this equation is identically the same as using the equivalent monochromatic wave by replacing  in Eq. (A5) with , which substituted in Eq. (A10) gives

  (15)

Furthermore, substitution in Eqs. (3) and (4) gives, respectively,

  (16)

  (17)

where  is substituted in Eq. (17) from Eq. (15).

 The results are given in Table 2 for the directional spreading  ranging from 0 to 146 degrees; i.e. for  it appears that , and consequently no sediment motion. As expected, it appears that and  decrease as increases. Thus, for spherical bodies  and  decrease as  increases. Similarly, for short cylinders  and decrease as  increases. These results are as expected and consistent with representing bidirectional monochromatic waves with an equivalent monochromatic wave with the wave induced velocity amplitude , i.e. reduced by the factor compared with that for an equivalent monochromatic wave for unidirectional bichromatic waves. For short cylinders exposed to bidirectional monochromatic waves it is reasonable to interpret the upstream and downstream scour lengths to represent superpositions of the two directions.

Truncated cones

Consider an example with two monochromatic waves for m with

m/s, s, and directional spreading (Fig. 5). Then, by replacing  in Eq. (A5) with  and substitution in Eq. (A10) give

  (18)

Furthermore, substitution in Eqs. (3) and (4) gives, respectively,

  (19)

  (20)

where  is substituted in Eq. (20) from Eq. (18).

 The results are given in Table 3 for the directional spreading  ranging from 0 to 150 degrees; i.e. for  it appears that , and consequently no sediment motion. As expected, it appears that and  decrease as increases. Thus, for truncated cones  and  decrease as  increases. As for spherical bodies and short cylinders, these results are as expected. For truncated cones exposed to bidirectional monochromatic waves it is also reasonable to interpret the upstream and downstream scour lengths to represent superpositions of the two directions.

 To the best knowledge of the authors, no comparative data exist from physical experiments, numerical simulations or field observations on scour around and self-burial of spherical bodies, short cylinders and truncated cones in the open literature. However, comparison with data in the validity range of the present method for these objects are required to make conclusions regarding the validity of the method.

**SUMMARY AND CONCLUSIONS**

A method for predicting the scour and self-burial of spherical bodies, short cylinders and truncated cones induced by bichromatic and bidirectional waves is provided. Results are illustrated by three examples of applications representing realistic field conditions, which agree qualitatively with physical expectations.

 For bidirectional monochromatic waves the scour depth and the self-burial depth around spherical bodies, the self-burial depth and the total/upstream/downstream length of the scour hole around short cylinders, as well as the burial depth/width and the total/upstream/downstream length of the scour hole around truncated cones decrease as the directional spreading increases.

 Comparison with data in the validity range of the method is required in order to conclude regarding its validity.

**APPENDIX A. BED SHEAR STRESS BENEATH BICHROMATIC AND BIDIRECTIONAL WAVES**

The present approach is based on utilizing the recent results in Myrhaug et al. (2023) who presented the seabed shear stress beneath bichromatic and birectional waves for large bed roughness. Here a summary of the formulae of the bed shear stress is provided.

 Consider two monochromatic waves with wave frequencies , , phase angles , , propagating in directions , , horizontal free stream wave-induced velocity vectors

  (A1)

and bed shear stress vectors

  (A2)

Here  denotes the horizontal free stream velocity amplitude vector,  is the bed shear stress amplitude vector,  is the bed shear stress phase, is the phase lag between bed shear stress and velocity, *t* is the time, and i= is the complex unit. Beneath bichromatic and bidirectional waves the combined bed shear stress is

  (A3)

The maximum bed shear stress for , is

  (A4)

where  denotes the magnitude of . For , it should be noticed that  is independent of the phase angles  and . However, apart from a special case provided in a subsequent example, bichromatic waves (forwill be discussed.

 Here the Christoffersen and Jonsson´s (1985) time-independent eddy viscosity for fully

 rough turbulent boundary layer flow beneath harmonic waves for large roughness is adopted,

where the bed shear stress amplitude associated with a sinusoidal wave with wave frequency  and free stream velocity amplitude  is obtained using the laminar solution . Then, the constant (in time and space) eddy viscosity replaces the kinematic viscosity , where ,  is the Nikuradse equivalent sand roughness, and  is the maximum bed friction velocity. For a sinusoidal wave, substitution of  for  yields , and consequently

  (A5)

 Moreover, the bed shear stress amplitude for each harmonic wave component is , which substituted in Eq. (A4) and solved for  , yields

  (A6)

This result is valid for  where  is the free stream excursion amplitude. Thus, Eq. (A6) gives the bed shear stress explicitly being a consequence of replacing the kinematic viscosity with the eddy viscosity for large bed roughness. Otherwise, by using e.g. a common wave friction factor would generally involve iteration to determine the bed shear stress.

 First, for unidirectional bichromatic waves with  and , Eq. (A6) yields

  (A7)

This corresponds to the maximum bed shear stress beneath two waves with the following wave-induced velocity near the bed

  (A8)

where  and . That is, corresponding to the equivalent monochromatic wave with the wave-induced velocity having an amplitude  (i.e. two times that for monochromatic waves) and mean wave frequency , which substituted in Eq. (A5) yields

  (A9)

 Second, for bidirectional monochromatic waves with ,  where  is the wave period, and , Eq. (A6) yields

  (A10)

One should notice that although  in this case, Eq. (A6) is valid since (see Eqs. (A1) and (A2), as well as Myrhaug et al. (2023, Eq. (A4)) for and ). This result corresponds to the maximum bed shear stress beneath two waves with frequency  propagating perpendicular to the short cylinder with directional spreading , i.e. corresponding to that for the equivalent monochromatic wave . Thus, Eq. (A10) is obtained by replacing  in Eq. (A5) with  (i.e. reduced by the factor compared with that for an equivalent monochromatic wave for unidirectional bichromatic waves).

**DATA AVAILABILITY STATEMENT**

All data and models used during the study appear in the published article.

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Table 1. Summary of scour responses , object dimensions  and coefficients  in

Eq. (14).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  | 0.0125 | 0.8  | 0.1  |
|  |  | 1.03  | 0.1  | 0.05  |
|  |  | 0.42 | 0.46 | 0  |
|  |  | 0.80 | 0.34  | 0  |

Table 2. Example of results for spherical bodies/short cylinders for bidirectional monochromatic waves; m , m , m/s , s , m.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  (deg) |  |  | Sphericalbodies (m) | Sphericalbodies(m) | Shortcylinder(m) | Shortcylinder(m) | Shortcylinder(m) | Shortcylinder(m) | Shortcylinder-(m) |
| 0 | 44 | 0.264 | 0.102 | 0.242 | 0.32  | 3.12  | 5.51 | 2.39 | 0.73 |
| 30 | 42.4  | 0.252  | 0.101  | 0.241  | 0.31  | 3.06 | 5.39 | 2.33  | 0.73 |
| 60 | 38.2 | 0.219 | 0.100  | 0.237 | 0.28  | 2.88 | 5.06  | 2.18  | 0.70 |
| 90 | 31.2 | 0.166 | 0.097 | 0.227 | 0.23  | 2.58 | 4.48  | 1.90 | 0.68 |
| 120 | 22 | 0.105 | 0.091 | 0.202 | 0.17  | 2.12 | 3.63  | 1.51 | 0.61 |
| 146 | 13 | 0.0511 | 0.081 | 0.151 | 0.10  | 1.58 | 2.65  | 1.07 | 0.51 |

Table 3. Example of results for truncated cones for bidirectional monochromatic waves; m , m , m/s , s , m.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  (deg) |  |  | Cones (m) | Cones(m) | Cones(m) | Cones(m) | Cones(m) | Cones-(m) |
| 0 | 13.9 | 0.314 | 0.46 | 0.95 | 1.08  | 1.47 | 0.39 | 0.69 |
| 30 | 13.4  | 0.300  | 0.44  | 0.94  | 1.06  | 1.45 | 0.39 | 0.67 |
| 60 | 12 | 0.259 | 0.40  | 0.93 | 1.01  | 1.40 | 0.39  | 0.62 |
| 90 | 9.8 | 0.198 | 0.33 | 0.90 | 0.92  | 1.30 | 0.38  | 0.54 |
| 120 | 7 | 0.125 | 0.24 | 0.85 | 0.79  | 1.16 | 0.37  | 0.42 |
| 150 | 3.6 | 0.0518 | 0.13 | 0.76 | 0.58  | 0.93 | 0.35  | 0.23 |

Figure 4. Definition sketch of the self-burial depth (*B*) of a short cylinder.

Figure 5. Definition sketch of the self-burial depth (*Bd*), the scour width (*Ws*), and the lengths

 of the scour hole () around a truncated cone.



Figure 1. Definition sketch of scour depth (*D*) around a spherical body.



Figure 2. Definition sketch of self-burial depth (*e*) around a spherical body.



Figure 3. Definition sketch of the lengths () and the width (*W*) of the scour hole

 around a short cylinder.



Figure 4. Definition sketch of the self-burial depth (*B*) of a short cylinder.



Figure 5. Definition sketch of the self-burial depth (*Bd*), the scour width (*Ws*), and the lengths

 of the scour hole () around a truncated cone.