HydroCen
NORWEGIAN RESEARCH CENTRE FOR HYDROPOWER TECHNOLOGY

CENTRE FOR ENVIRONMENT-FRIENDLY ENERGY RESEARCH

HydroCen Report nr. 44

Snow Storage Information in Hydropower Scheduling

With application in the SDDP-based ProdRisk model

Arild Helseth

NTNU SINTEF NINA

HydroCen

The main objective of HydroCen (Norwegian Research Centre for Hydropower Technology) is to enable the Norwegian hydropower sector to meet complex challenges and exploit new opportunities through innovative technological solutions.

The research areas include:

- Hydropower structures
- Turbine and generators
- Market and services
- Environmental design

The Norwegian University of Science and Technology (NTNU) is the host institution and is the main research partner together with SINTEF Energy Research and the Norwegian Institute for Nature Research (NINA).

HydroCen has about 50 national and international partners from industry, R&D institutes and universities.

HydroCen is a Centre for Environment-friendly Energy Research (FME). The FME scheme is established by the Norwegian Research Council.

The objective of the Research Council of Norway FME-scheme is to establish time-limited research centres, which conduct concentrated, focused and long-term research of high international calibre in order to solve specific challenges in the field.

The FME-centres can be established for a maximum period of eight years HydroCen was established in 2016.

Snow Storage Information in Hydropower Scheduling

With application in the SDDP-based ProdRisk model

Arild Helseth

HydroCen

Helseth, A. 2024. Title. HydroCen rapport XX. Norwegian Research Centre for Hydropower Technology

Trondheim, 06 2024

ISSN: 2535-5392
ISBN: 978-82-93602-45-3

KVALITETSSIKRET AV
Birger Mo

NØKKELORD
Driftsplanlegging i vannkraftsystemer

KONTAKTOPPLYSNINGER

HydroCen
Vannkraftlaboratoriet, NTNU
Alfred Getz vei 4
Gløshaugen,
Trondheim

www.HydroCen.no

Abstract

This report concerns the use of snow storage information to improve hydropower scheduling. More specifically, a technique for embedding snow storage information in the stochastic dual dynamic programming (SDDP) is presented and numerically tested in the ProdRisk scheduling model. Snow storage information is applied as an exogenous state variable that can partly explain the inflows to hydropower reservoirs and plants.

Three case studies representing different hydropower systems in Norway are used as test cases. Results show that the use of snow storage improves the scheduling in all cases, leading to better economic performance, more power production, and less spillage.

Sammendrag

Denne rapporten omhandler bruk av informasjon om lagret snø for å forbedre vannkraftplanleggingen. Mer spesifikt er en teknikk for å legge inn snøinformasjon i stokastisk dual dynamisk programmering (SDDP) presentert og numerisk testet i ProdRisk-planleggingsmodellen. Informasjon om snølager brukes som en eksogen tilstandsvariabel som delvis kan forklare tilsigene til vannkraftmagasiner og kraftstasjoner.

Tre casestudier som representerer ulike vannkraftsystemer i Norge brukes som testcase. Resultatene viser at bruk av snøinformasjon forbedrer planleggingen i alle tilfeller, noe som fører til mer økonomisk drift, mer kraftproduksjon og mindre flom.

Content

1 Introduction

1.1 Background

Precipitation to hydropower catchment areas in the Nordic countries comes in the form of rain or snow. In areas with colder climates, precipitation may come solely as snow during the winter period. This snow accumulates, often referred to as *snow storage* (or *snowpack*). This storage cannot be used immediately for hydropower production but holds potential energy that should be accounted for when considering future production. Thus, including information about snow storage in the hydropower scheduling process will likely lead to better informed decision making.

This work was carried out as a part of the HydroCen research center, associated with work package 3.4 on environmental constraints. Previous research in this work package has focused on reservoir constraints [1] [2] [3] and ramping constraints [4] [5] constraints. This work was carried out in the project's final phase and serves to shed light on the representation of snow storage in hydropower scheduling models. In particular, we investigate possibilities for including snow storage information in the stochastic dual dynamic programming algorithm applied in the model ProdRisk.

We will not go into detail about the inflow modeling in ProdRisk here. Ther reader is referred to older [6] and newer [7] documents addressing the inflow model in ProdRisk for further information.

1.2 Context

A key task within hydropower scheduling is to valuate water resources. This is typically carried out in medium-term scheduling models, where ProdRisk [8] and EOPS are examples of such models used in the Nordic countries. Water valuation should be carried out in lieu of the many uncertainties that impact future market prices and availability of water resources.

Inflow to the hydropower system is a key uncertainty that needs to be well represented in the scheduling. By the term *inflow* we refer to water (or energy) entering the reservoir (or power station) to be used for power production.

Existing scheduling tools use a combination of historical inflow statistics and statistical models to represent inflow stochasticity. Inflow prognoses are based on the statistical properties of historical records, possibly adjusted for climatic changes. Snow storage provides additional information about future inflow. Given that information about the snow storage is available, future inflow will consist of a known part (melted snow storage) and an unknown part (future inflow). During the winter an increasing part of the spring flood consists of the known snow storage that accumulates, and a decreasing part from the unknown future inflow. Uncertainty in the spring flood prognosis is therefore continuously reduced as the snow storage builds up during wintertime. This is illustrated in Figure 1.

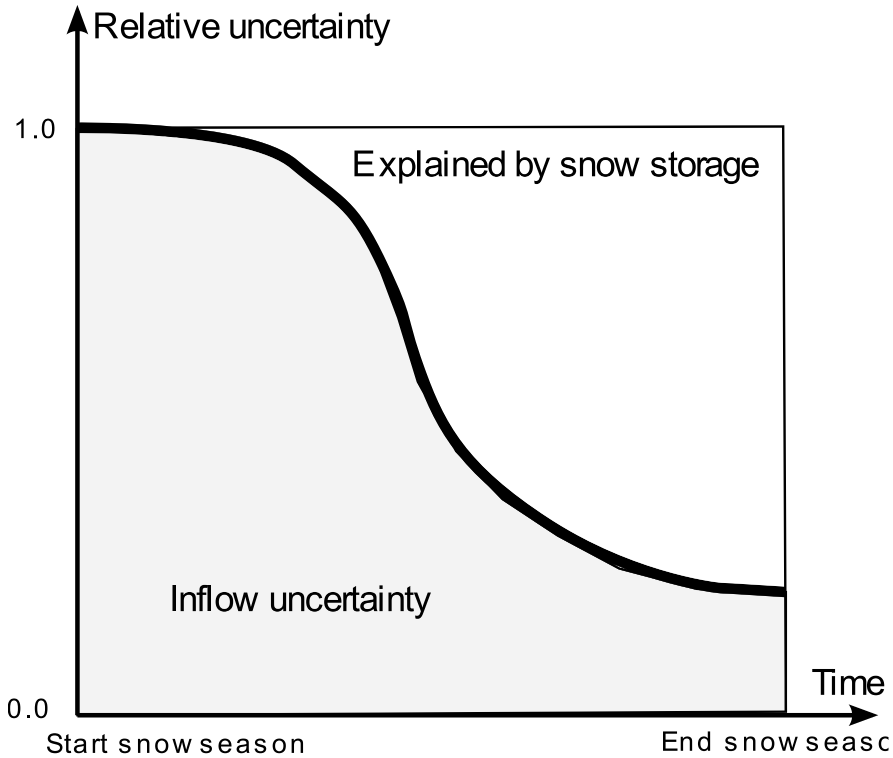

Figure 1 Illustration of how uncertainty in spring flood prognosis is reduced during the snow storage accumulation period. [EOPS manual].

Knowledge about snow storage impacts our expectation about future inflow. For this reason, snow storage, if known, should be considered as part of the system state in hydropower scheduling.

1.2.1 Application of Snow Storage Information

When applying ProdRisk for operational planning, in so-called parallel mode, snow storage information is normally used for spring flood prognosis, covering the first part of the scheduling horizon, as illustrated in Figure 2. The inflow forecast is therefore adjusted according to snow information for the first part of the scheduling horizon, i.e., until the end of the melting season.

For the remaining part of the scheduling horizon, ProdRisk fully relies on the historical inflow scenarios (forward simulation) and stochastic model (backward recursion). Water values are therefore based on the current information of snow storage, but not the long-term correlations between inflow and snow storage. If used for investment studies, in so called serial mode, snow storage information is not used at all in ProdRisk.

This work presents and tests an approach for adding snow storage information as an exogenous explanation to inflow over the entire scheduling horizon shown in Figure 2. Since snow storage must be assumed to be a known system state in the operational planning, it is reasonable to use this information in the longer-term planning, if available. Using this information will serve to reduce uncertainty in the inflow estimates and may thus improve the scheduling.

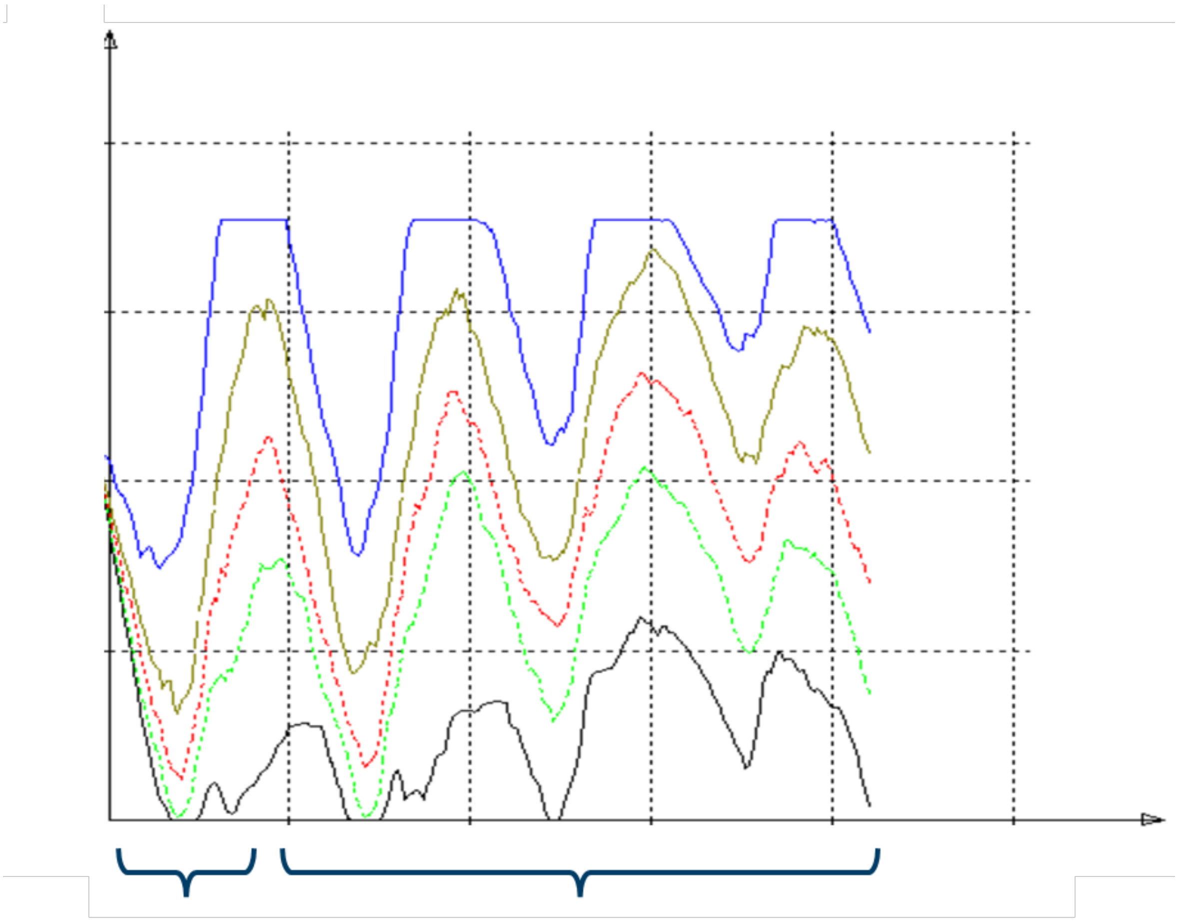

Figure 2 Illustration of the scheduling horizon and the use of inflow/snow forecast. Illustration of reservoir trajectories (percentiles) for a four-year scheduling horizon.

1.3 Statistics

The energy content of snow in Norway is shown in Figure 3, measured in TWh for the years 1958-2022. Snow storage accumulates until the melting season in spring. In the melting season (approximately week 15-35), snow storage gradually transforms to inflow that can be used for energy production. Although the seasonal pattern is predictable, the annual trajectories shown in Figure 3 show significant uncertainty in snow storage prior to the melting period, as well as the beginning and end of the melting period.

The snow storage information in Figure 3 can be seen together with information about inflow in Norway Figure 4. For each inflow scenario one can find (or estimate) a corresponding snow scenario for a specific catchment area.

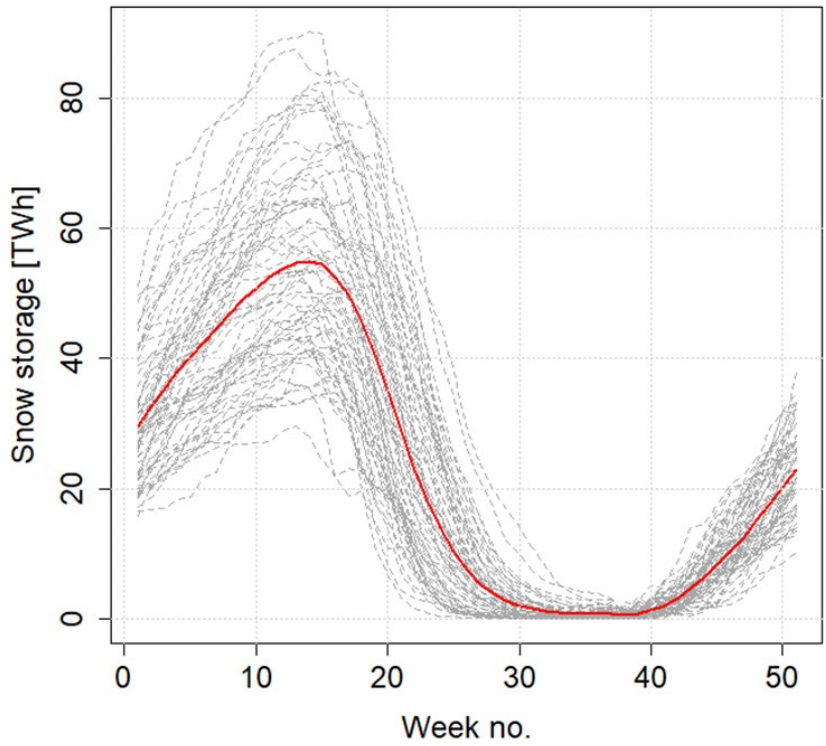

Figure 3 Snow storage per week for years 1958-2022. Scenarios in grey, mean value in red. [https://www.nve.no/energi/analyser-og-statistikk/hydrologiske-data-til-kraftsituasjonsrappor-ten/]

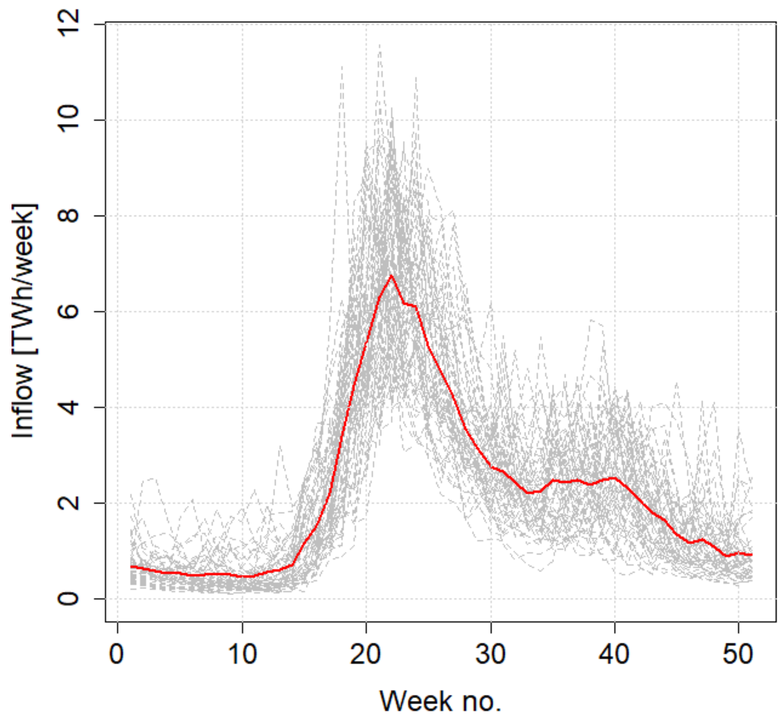

Figure 4 Inflow in Norway, derived from an EMPS dataset. Scenarios in grey, mean value in red.

1.4 Literature Review

This review targets known methodologies that have been applied in the scheduling of hydropower systems and focuses on the use of snow storage information within those.

1.4.1 Stochastic Dynamic Programming

Short Description

The dynamic programming (DP) was formally introduced by [9]. In DP one approaches the overall problem by decomposing the multistage decision-making problem into single-stage problems, which are then solved recursively. DP can accommodate both uncertainty and correlations in time in the hydrologic input by adding a hydrologic variable to the state space vector. The resulting stochastic dynamic programming (SDP) formulation, often referred to as a Markov decision process, may explicitly consider the streamflow lag-1 correlation found in the inflow records. A key challenge with SDP is that all state variables need to be discretized, and consequently the "curse of dimensionality" leads to unacceptable computation times for systems with many state variables. Typically, both reservoir volumes and inflows are considered state variables.

To better capture the complex temporal and spatial correlations of streamflow processes the sampling stochastic dynamic programming algorithm (SSDP) was presented in [10]. Unlike SDP, SSDP follows inflow scenarios when computing the strategy (water values). Similar to SDP, SSDP is also limited by the "curse of dimensionality".

Review

The documents [11] and [12] describe the applied technique for including snow storage as a state in the water value calculation in the EMPS model, which is based on a variant of SDP. The snow storage state is discretized, and the snow storage state is only used in weeks where it provides a significant (correlation larger than 0.5) explanation of future inflow. The algorithm is iterative in the sense that water values without snow information are computed first, followed by a re-computation for weeks where the snow information significantly explains future inflow.

A model based on SDP with a hydrological state variable including snow storage information was presented in [13]. The added state variable is motivated by the concept of temporal persistence. The authors warn that the use of additional hydrological state variables does not always add value, as it affects both the description of hydrological processes and the predictive information available in making release decisions. The importance of including snow storage was found to depend heavily on the objective function, increasing importance with increasing water and firm-power targets.

In [14] it is explained how the inclusion of a hydrologic variable in stochastic and dynamic formulations of the hydropower scheduling problem allows incorporation of information about the state of the hydrologic process. Examples are SDP where inflow from the previous stage is a hydrologic state variable and SSDP where the state variable is a forecast of the total inflow towards the end of the planning horizon. The authors use a combination of snow storage and soil moisture to capture temporal persistence not explained by the traditional inflow model and uses this variable in an SSDP model to find the probabilities of switching from one inflow scenario to another.

In [15] compares an SDP model with three different representations of the hydrological state: inflow in PAR(1), inflow in PAR(5), and snow water equivalent (SWE). The latter proved to be most successful in their case study.

1.4.2 Stochastic Dual Dynamic Programming

Short Description

The uncertainty in a multi-stage stochastic decision-making process can be described by a scenario tree, leading to the formulation of a stochastic optimization problem. Such problems can be solved by decomposition, such as the multi-stage Benders decomposition (MSBD) algorithm [16] for stochastic linear programming problems. In a similar fashion to SDP, the decision-making problems are divided in single-stage problems that are solved iteratively, but unlike SDP the state variables remain continuous, thus circumventing the state of dimensionality. The key challenge with MSBD applied to hydropower scheduling is however the size of the scenario trees needed to cover the panning horizon. To deal with this challenge, a sampling-based variant of MSBD known as stochastic dual dynamic programming (SDDP) was presented in [17]. SDDP efficiently "collapses" the scenario tree by sampling [18] and cut sharing [19].

SDDP inflow uncertainty is usually modeled using statistical time-series models belonging to the family of periodic autoregressive (PAR) models. As the state variables are continuous in SDDP, the stochastic inflow model may be formulated as PAR(p) (of order *p*). In SINTEFs SDDP-based model ProdRisk a PAR(1) model has been found to be appropriate. In contrast, a higher order model is used in the NEWAVE model applied in Brazil [20]. As the inflow model carries information about the state, it should be linear (or at least convex) to meet the convexity requirement of SDDP.

Review

Exogenous hydrological variables were added to the SDDP model in [21]. The endogenously defined reservoir inflows are seen as dependent on exogenous hydrological state variables, and not the other way around, motivating the proposed SDDPX model. X represents the exogenous input to the inflow model, going from MPAR(p) to MPARX(p,b), where b is the order of exogenous variables. In a case study snow storage and winter precipitation were considered exogenous variables.

A different approach was followed in [22] where a periodic autoregressive moving average (PARMA) model was introduced in SDDP. The additional moving average component of PARMA models over PAR allows a more complex correlation structure and thus provides a deeper memory of the inflow process. It is briefly mentioned that the moving average component could potentially capture contributions from snowmelts.

1.4.3 Others

Review

The work in [23] models snow storage explicitly as an additional upstream storage to the reservoir and solves the scheduling problem as a stochastic optimization problem by use of multi-stage Benders decomposition. For this purpose, input data classified as "inflow" is split into "rain" and "snow". However, it is not clear from the presentation how data for "rain" and "snow" were obtained. The author demonstrates that the modeling of snow storage reduces uncertainty in future inflow.

In [24] the benefit for the system of including snow storage information was investigated by use of the fundamental market model FanSi. FanSi is described in detail in [25] and can be characterized as a rolling-horizon simulator, repeatedly solving two-stage stochastic optimization problems by use of Benders decomposition. The experiments are run under the assumption that simultaneous time-series for snow storage and inflow are available. Typically, observed time series for snow storage do not exist, but it is possible to generate such series from, e.g., an HBV-type model based on observed inflows, temperatures, and precipitation. The procedure involves i) estimating the correlation between normalized snow storage and average accumulated inflow until the last week of the snow-melting period, and ii) adjusting inflows in second-stage scenarios according to snow state and the estimated correlation. Historical time series for snow storage were provided by NVE. Results show that generation increases, and spillage decreases with use of snow information.

2 Modeling Approach

In the context of SDDP, the SDDPX modeling approach reviewed in Section 1.4.2 is appealing in the ease of including snow storage information as part of the stochastic inflow model and the state space in SDDP.

Although not explicitly stated in [21], the inclusion of exogenous information can be done in any SDDP model (likely) without further implementation effort, as will be elaborated below. Increasing the state space with exogenous information does not increase computational complexity much. For example, the algorithmic structure and the number of decision variables in the stage-problems are not affected.

The modeling approach presented in the following is very similar to [21], but we will emphasize the integration and use within the ProdRisk model.

2.1 Mathematical Description

The existing inflow model used in ProdRisk follows a PAR(1) formulation:

$$z_t = \phi_p z_{t-1} + \varepsilon_t \tag{1}$$

Where z_t is a vector of normalized inflows for week *t*, ϕ_p the correlation matrix for period *p*, and ε_t is the vector of white noise for week *t*.

Assume that time series for inflow and SWE for the same sites are available. We simply extend the dimensionality of (1) to include both inflow and SWE time series. That is, the PAR(1) estimated parameters (ϕ_p and ε_t) are based on both the inflow and snow time series.

Consider a single-reservoir system with a corresponding inflow (i) and snow (s) time series. The PAR(1) model then becomes:

$$\begin{bmatrix} z_{i,t} \\ z_{s,t} \end{bmatrix} = \begin{bmatrix} \phi_{ii,p} & \phi_{is,p} \\ \phi_{si,p} & \phi_{ss,p} \end{bmatrix} \times \begin{bmatrix} z_{i,t-1} \\ z_{s,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t} \\ \varepsilon_{s,t} \end{bmatrix} \leftarrow \begin{bmatrix} \mu_{i,t} \\ \mu_{s,t} \end{bmatrix} \tag{2}$$

Where μ are the dual values to be used in Benders cuts.

Normalized inflow can be expressed as:

$$z_{\mathrm{i},t} = \phi_{ii,p} z_{i,t-1} + \phi_{is,p} z_{s,t-1} + \varepsilon_{\mathrm{i},t} \tag{3}$$

The second term (red color) shows how the snow storage information ($z_{s,t-1}$) impacts the inflow according to the correlation ($\phi_{is,p}$) between inflow and snow storage in the current week. As we shall see later, this correlation is notable in the melting period.

Normalized snow storage can be expressed as:

$$z_{s,t} = \phi_{si,p} z_{i,t-1} + \phi_{ss,p} z_{s,t-1} + \varepsilon_{\mathrm{i},t} \tag{4}$$

The snow storage has a high autocorrelation ($\phi_{ss,p}$) in the winter season when snow is accumulated.

The Benders cuts comprise state information related to the reservoir volume(v with coefficient π_c), the normalized inflow ($z_{i,t-1}$ with coefficient $\mu_{i,c}$), and the normalized snow storage ($z_{s,t-1}$ with coefficient $\mu_{s,c}$).

$$\alpha_t - \pi_c v - \mu_{i,c}\left(\phi_{ii,p} z_{i,t-1} + \phi_{is,p} z_{s,t-1}\right) - \mu_{s,c}\left(\phi_{si} z_{i,t-1} + \phi_{ss,p} z_{s,t-1}\right) \leq \beta_c \qquad (5)$$

We observe the following from the above expressions:

- [red]: Snow storage information explains the inflow in the current week which in turn enters the reservoir balances of the optimization problem. A high snow storage will increase the inflow in the current week and impact the expectation for the future inflow positively, due to inflow autocorrelation.
- [blue]: Autocorrelation in snow storage provides an important recursive contribution, carrying the value of a high snow storage from the melting season backwards in time towards the winter season.

2.2 Statistical Investigations

From Section 2.1 we realized that the correlation between inflow and snow storage for a specific week and the snow autocorrelation both add to the impact of snow information on hydropower scheduling results. These correlations are studied for a selected set of hydropower sites in the following. These sites are later used as case studies in Section 3.

- **Bergsdalen**, using inflow and snow storage time series for *Bulken*, marked (1) in Figure 5.
- **Lysebotn**, using inflow and snow storage time series for *Jogla*, marked (2) in Figure 5.
- **Rjukan**, using inflow and snow storage time series for *Kvenna*, marked (3) in Figure 5.

Data obtained from NVE covers the years 1958-2022.

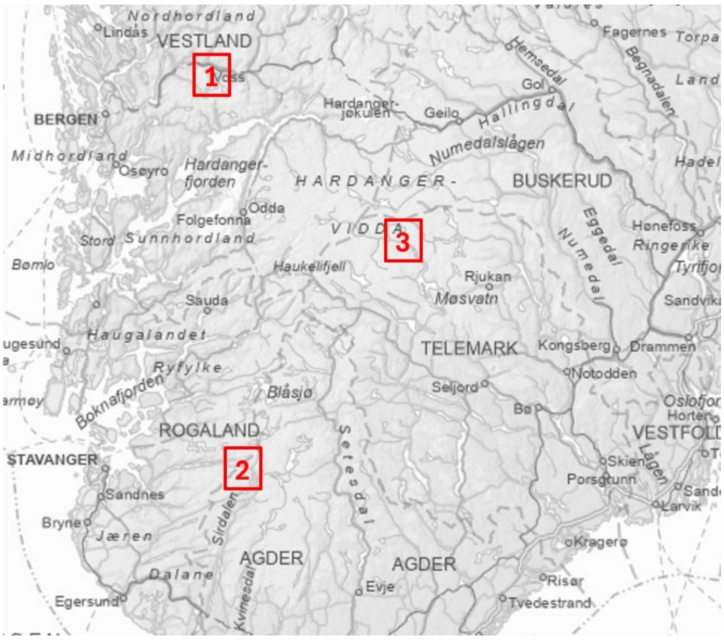

Figure 5 Location of inflow sites (Map obtained from https://atlas.nve.no/).

Time series of inflow and snow storage (snow water equivalent) for the 3 sites are presented in Figure 6, Figure 7, and Figure 8.

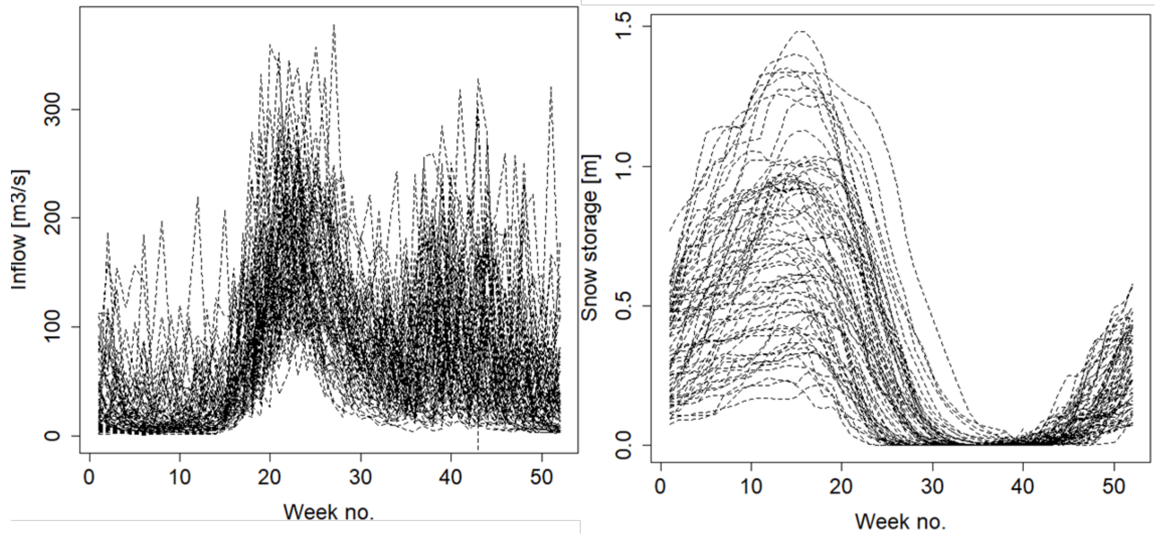

Figure 6 Inflow (left) and snow storage (right) for Bulken.

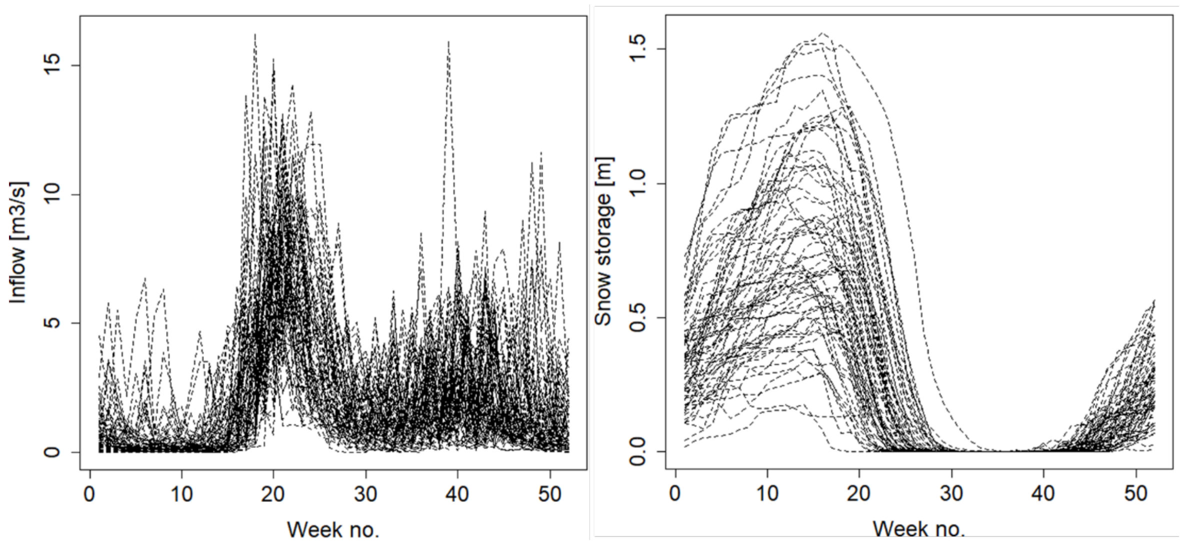

Figure 7 Inflow (left) and snow storage (right) for Jogla.

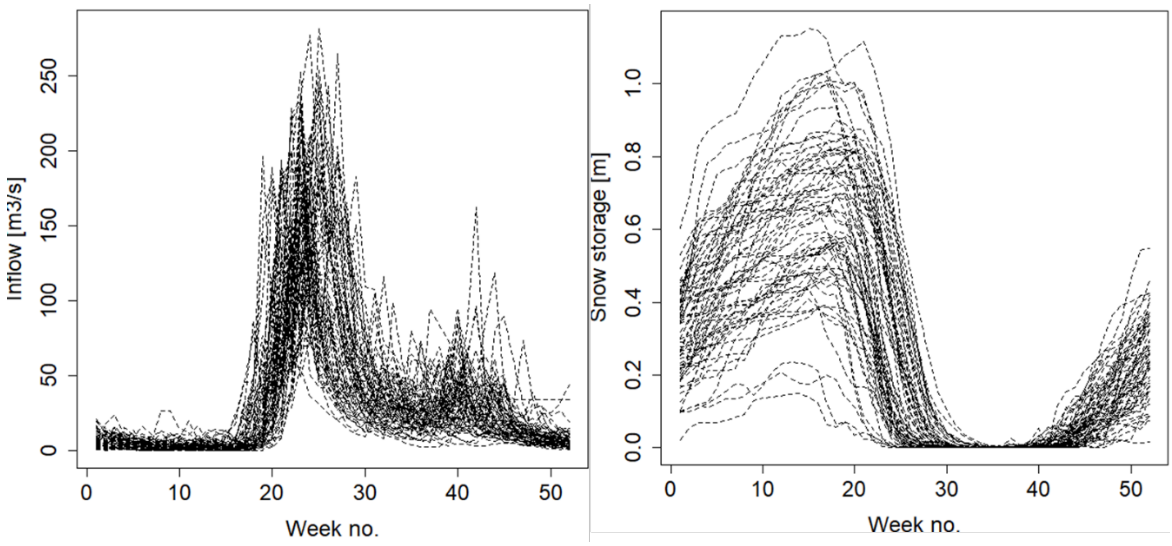

Figure 8 Inflow (left) and snow storage (right) for Kvenna.

Inflow and snow storage data clearly differ for the three metering stations. Whereas Bulken and Jogla are located at modest altitudes where inflow may be significant throughout the year, Kvenna is located at a higher altitude where precipitation primarily comes as snow at wintertime.

Correlations between snow storage and inflow per week is shown in Figure 9. These correlations indicate how much of the current week's inflow can be explained by the prevailing snow storage. It makes sense that these correlations are higher in the melting season. From inspection of the snow storage profiles and the correlations in Figure 9, it is reasonable to define a snow melting season as follows:

Table 1 Definition of snow melting seasons.

SERIES NAME	START WEEK	END WEEK
BULKEN	20	35
JOGLA	18	32
KVENNA	22	35

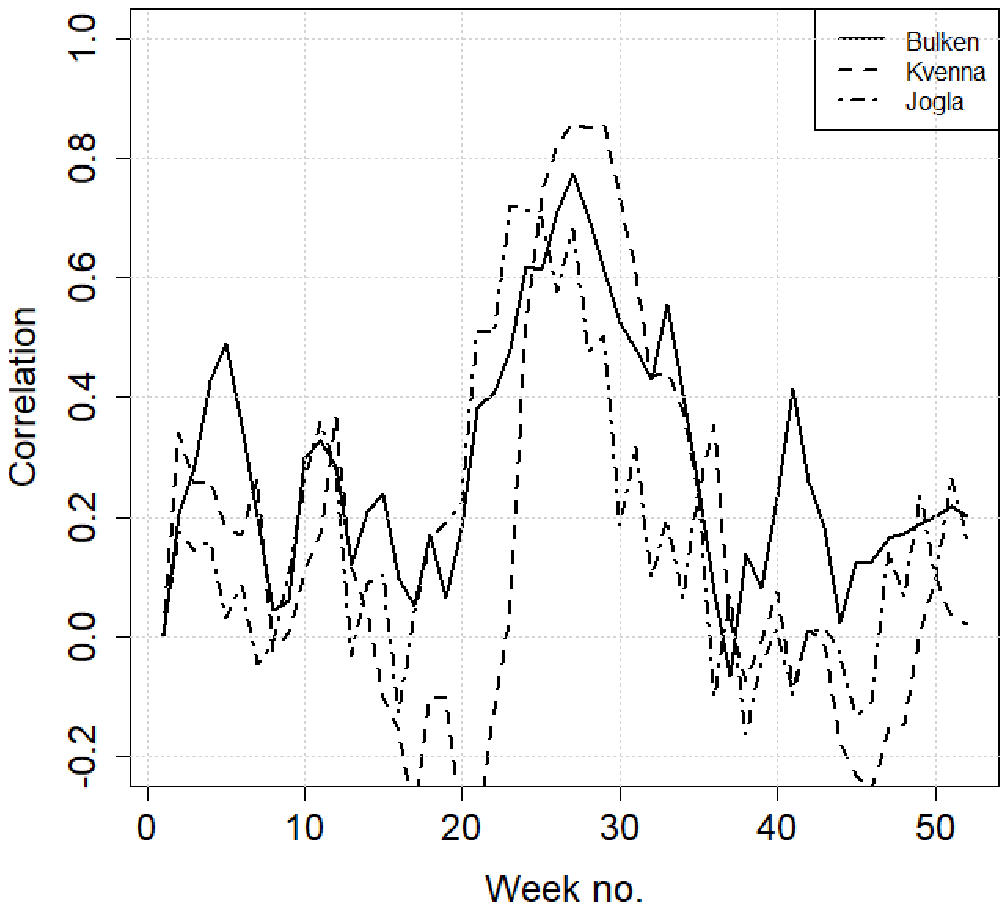

Figure 9 Correlation between snow storage and inflow.

The correlation between snow storage and accumulated inflow towards the end of the melting season is plotted in Figure 10. As one approaches the melting season, a gradually higher share of the accumulated inflow till the end of the melting season is explained by snow storage. This point was also illustrated in Figure 1.

The three sites show similar pattern in that the correlation gradually increases towards the melting season. Since melting starts later for Kvenna, the correlation peaks later and at a higher value than the other two.

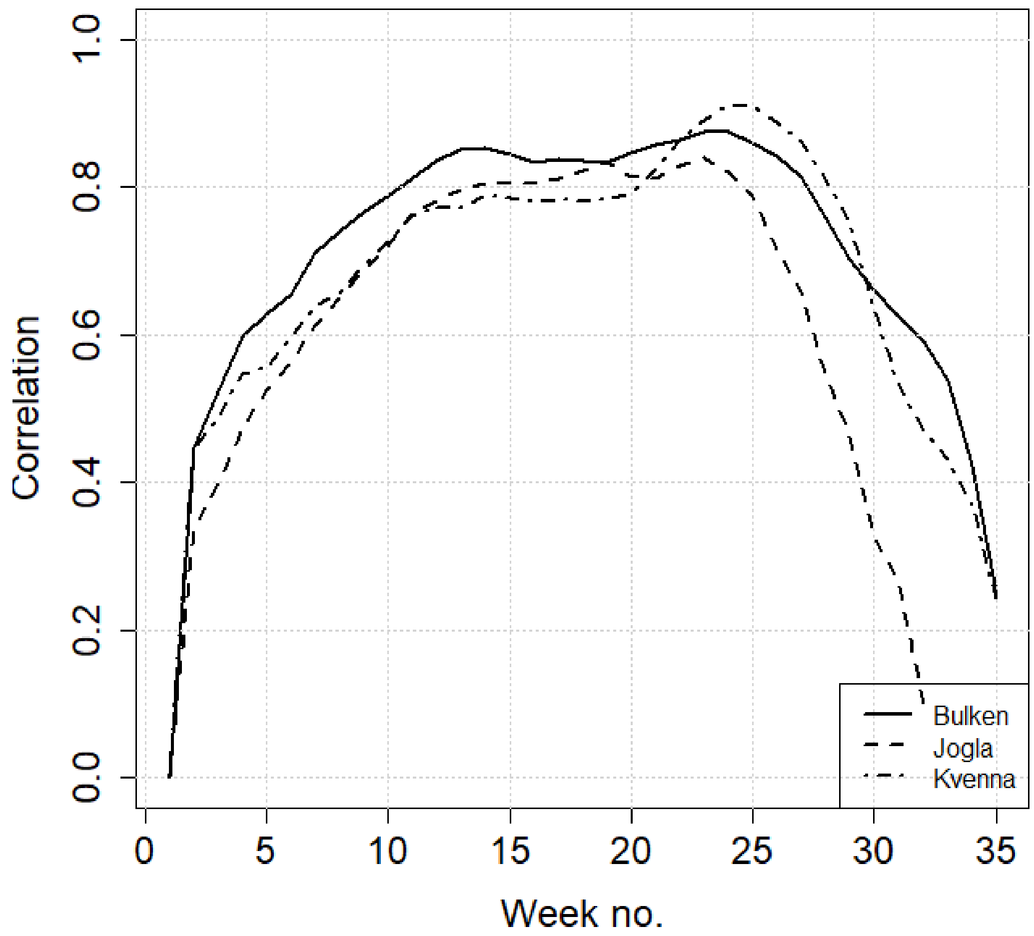

Figure 10 Correlation between snow storage and accumulated inflow towards the end of the melting season.

2.3 Adding Snow Information in ProdRisk

Assume availability of historical records for both inflow and snow storage for the relevant metering stations needed to schedule the operation of a given hydropower system. We seek to use the snow storage information to better predict the inflow, and thus improve the scheduling. Snow storage thus becomes an exogenous variable which serves the sole purpose of explaining inflow. Although snow storage has no direct representation in the optimization problems formulated in the ProdRisk SDDP-algorithm, it can serve as an exogenous state variable with a strong autocorrelation which explains parts of the inflow through melting (shown in Figure 9).

In the following we present a practical way of turning the SDDP model in ProdRisk an SDDPX model, with similar properties to [21]. Although our focus is snow storage as an exogenous state in this work, we underline that this approach is valid for other exogenous states that could explain the inflow.

Consider a hydropower system as illustrated in Figure 11, comprising hydropower modules (reservoir and plant) connected to form a cascade. Each module may have its own regulated (to reservoir) and/or unregulated (to plant) inflow, according to an individual or shared time series of inflow obtained from a metering station. At start-up ProdRisk will identify the number of inflow time series being used and set up the PAR(1) formulation discussed in Section 2.1.

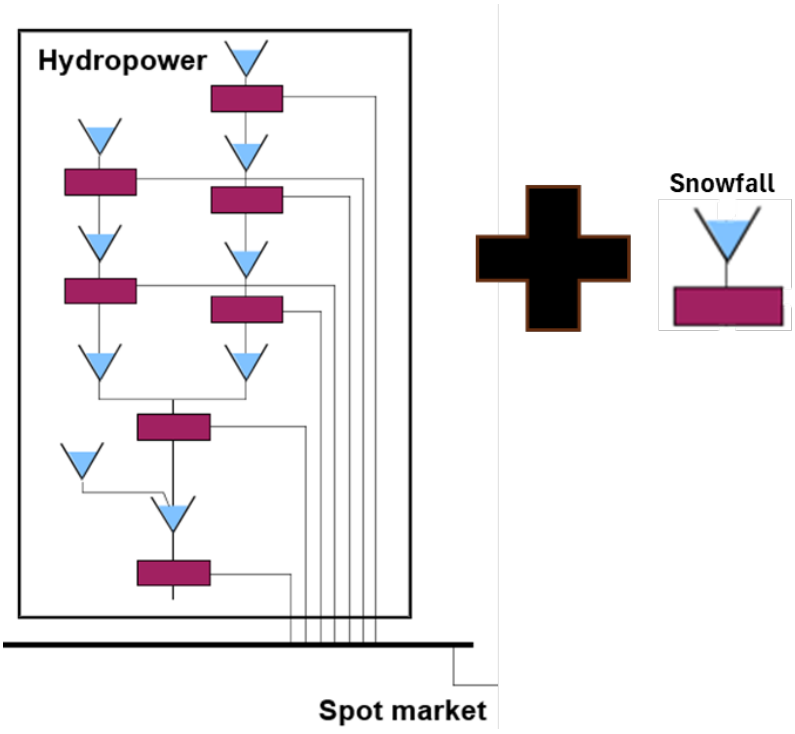

Figure 11 Illustration of modeling technique.

In addition to these inflow time series, we now introduce one or more snow-storage time series in ProdRisk. This can be done by adding one extra dummy module per times series of snow storage, as illustrated by the module named Snowfall in Figure 11. This module is giving a small reservoir volume, to avoid being treated as a reservoir state variable, and a power station with zero capacity. The sole purpose is to read the snow storage time series and embed it into the PAR(1) inflow model, as shown mathematically in Section 2.1.

The computationally complexity of the computer model will not increase much, since now new variables or constraints are added to the decomposed optimization problems per decision stage. However, a new state variable (the snow storage) is added which may impact overall convergence properties.

3 Computational Experiments

Three hydropower systems were selected as study cases. ProdRisk V10 was run on all cases. The parameterization of the inflow model followed three seasons: before, during, and after the snow melting seasons, according to definitions in Table 1.

For all three cases we used inflow data obtained from the NVE website[1] and snow data obtained in correspondence with NVE.

The same case matrix was used for all cases:

- **Base-1ses** – Similar to default ProdRisk use, e.g., with inflow model parameter estimates per year and without snow storage.
- **Base-3ses** – With inflow model parameter estimates per season, but without snow storage.
- **Snow-1ses** – With inflow model parameter estimates per year and with snow storage.
- **Snow-3ses** – With inflow model parameter estimates per season and with snow storage.

All cases where run in parallel mode (defined initial state) and head optimization was not considered.

[1] https://www.nve.no/vann-og-vassdrag/hydrologiske-data/historiske-data/historiske-vannfoeringsdata-til-produksjonsplanlegging/

3.1 Bergsdalen

The Bergsdalen system was modelled using 4 hydropower modules, comprising two larger reservoirs (Torfinnsvatn and Hamlagrøvatn) and one major downstream power plant (Dale). The inflow to all reservoirs in the system was defined to follow the statistical properties of the Bulken inflow series. In addition, we model a separate snow storage using the Bulken time series for snow storage. An illustration of the system topology with the additional snow storage module is shown in Figure 12.

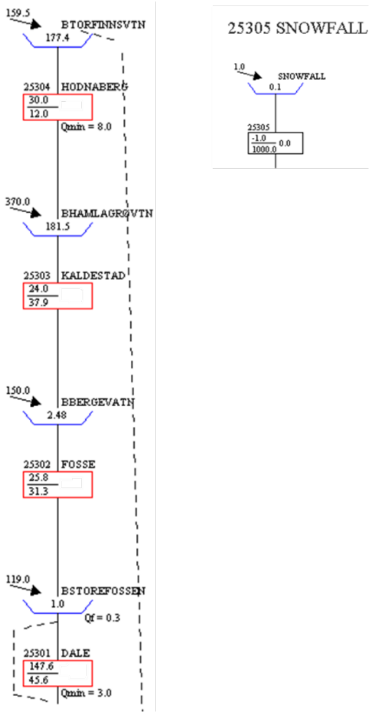

Figure 12 Topology of Bergsdalen system.

3.1.1 Inflow Model

First, we study the parameters estimated for the PAR(1) model constructed by ProdRisk. Correlation matrices per case and per season are presented in Table 2. Some observations that are relevant for the following analyses:

- The inflow autocorrelation shows a non-negligible seasonal variation.
- The inflow autocorrelation in Base-3ses differs significantly from Snow-3ses in the melting season.
- Cross-correlation between snow and inflow (snow explaining inflow in a given week) is significantly higher in the melting period in case Snow-3ses.

Table 2 Correlation matrices for Bergsdalen.

Base-1ses						
	Weeks 1-19		**Weeks 20-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5379	-	0.5379	-	0.5379	-
Snow	-	-	-	-	-	-
Base-3ses						

	Weeks 1-19		Weeks 20-35		Weeks 36-52	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5701	-	0.5814	-	0.4620	-
Snow	-	-	-	-	-	-
Snow-1ses						
	Weeks 1-19		**Weeks 20-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.4851	0.1490	0.4851	0.1490	0.4851	0.1490
Snow	0.0282	0.9325	0.0282	0.9325	0.0282	0.9325
Snow-3ses						
	Weeks 1-19		**Weeks 20-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5409	0.0980	0.4194	0.3220	0.4416	0.0718
Snow	0.0155	0.9882	-0.0343	1.0102	0.0766	0.8199

As a test of the PAR(1) inflow model's ability to capture the correlation between snow storage and accumulated inflow in Bergsdalen/Bulken, we sampled 1000 scenarios of inflow and snow storage using the PAR(1) and compared the computed correlation with that reported using observed data in Figure 10. The result is shown in Figure 13. The results show that correlations found in the sampled series follow the observed patterns, although underestimated.

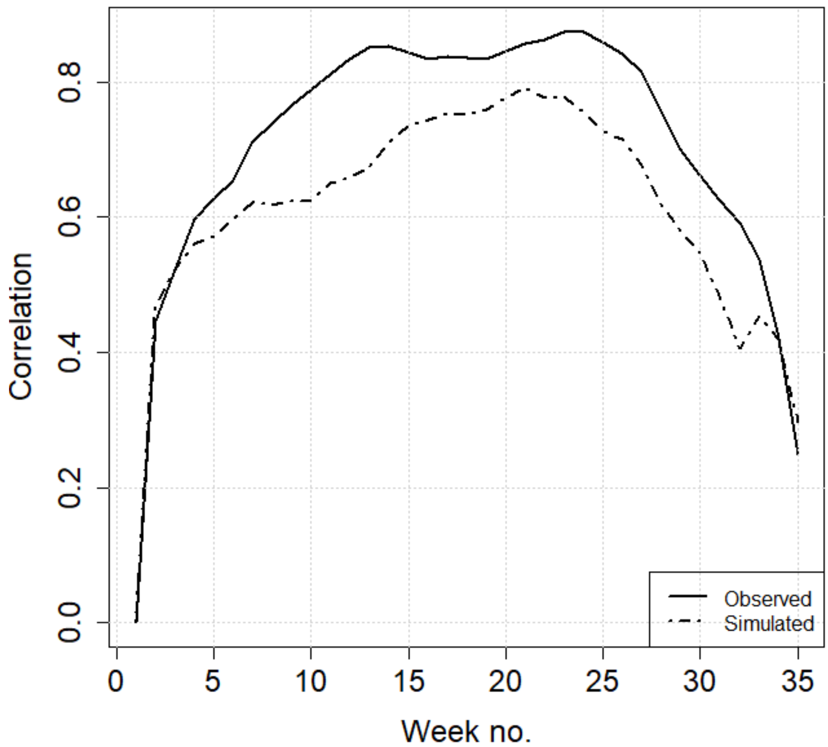

Figure 13 Observed and simulation correlation between snow storage and accumulated inflow for Bergsdalen/Bulken.

3.1.2 Simulation Results

The system was run in parallel mode with a scheduling horizon of 156 weeks, using 53 inflow and snow storage scenarios. The expected values for net profit, spillage and production are presented in Table 3.

Table 3 Performance indicators for Bergsdalen.

	Base-1ses	Base-3ses	Snow-1ses	Snow-3ses
Exp. net profit [MNOK]	66.9	66.8	67.2	67.4
Exp. spillage [GWh]	103.4	103.8	83.3	82.6
Exp. production [GWh]	3124.4	3123.1	3143.2	3137.9

The simulated reservoir trajectories for Hamlagrøvatn and Torfinnsvatn are shown in Figure 14 and Figure 15, respectively, for cases Base-3ses and Snow-3ses. Trajectories differ only marginally, e.g., with Snow-3ses being more careful towards the top of the reservoir for Torfinnsvatn.

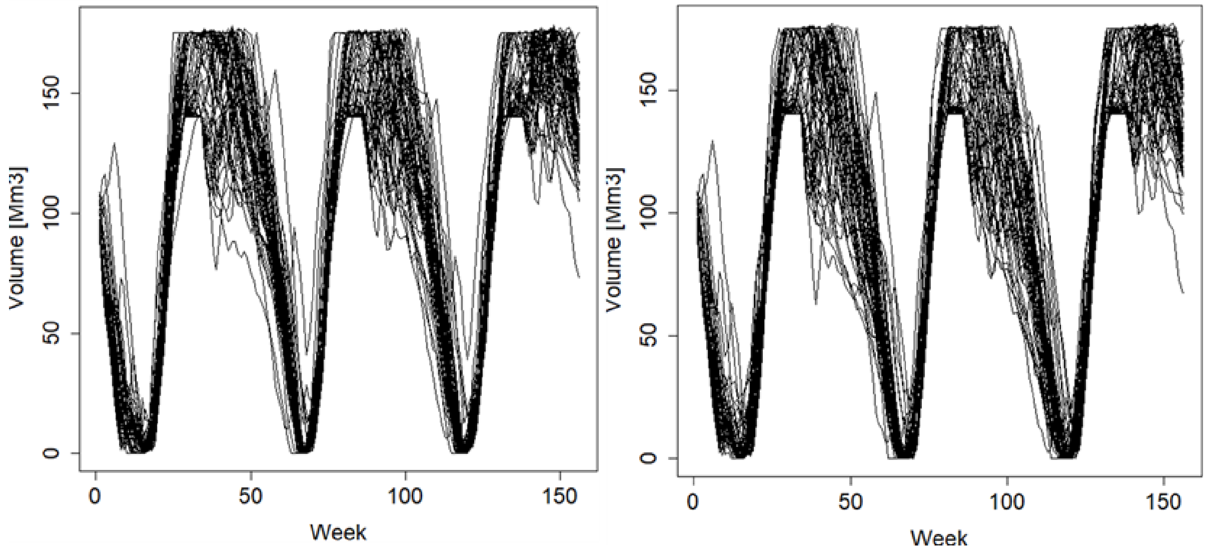

Figure 14 Reservoir trajectories for Hamlagrøvatn, for Base-3ses (left) and Snow-3ses (right).

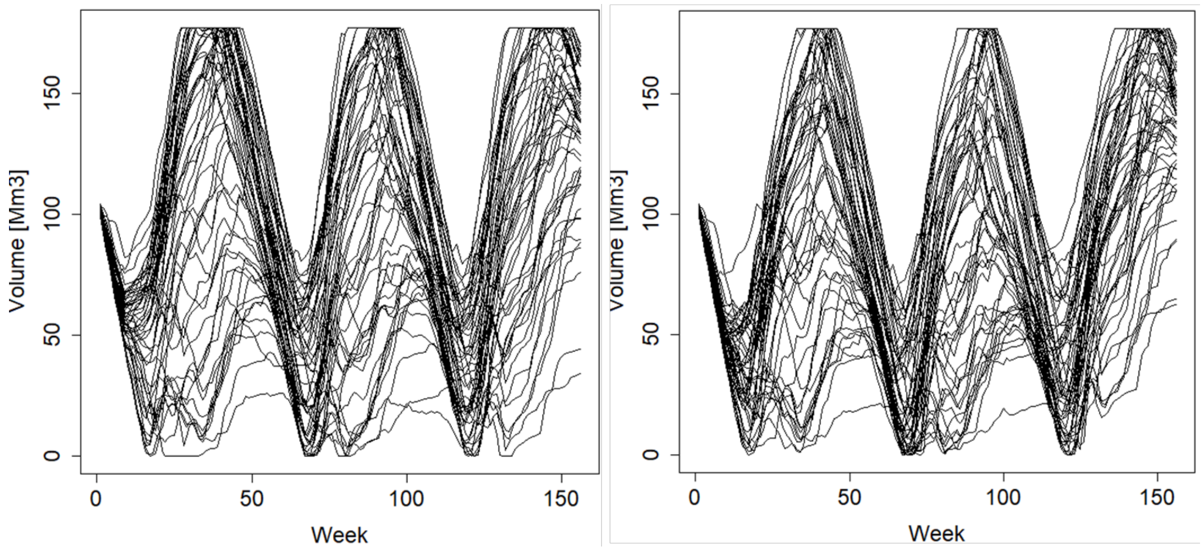

Figure 15 Reservoir trajectories for Storfinnsvatn, for Base-3ses (left) and Snow-3ses (right).

The strategy computed in Snow-3ses clearly motivates less spillage, as shown from the simulated results for the sum system in Figure 16.

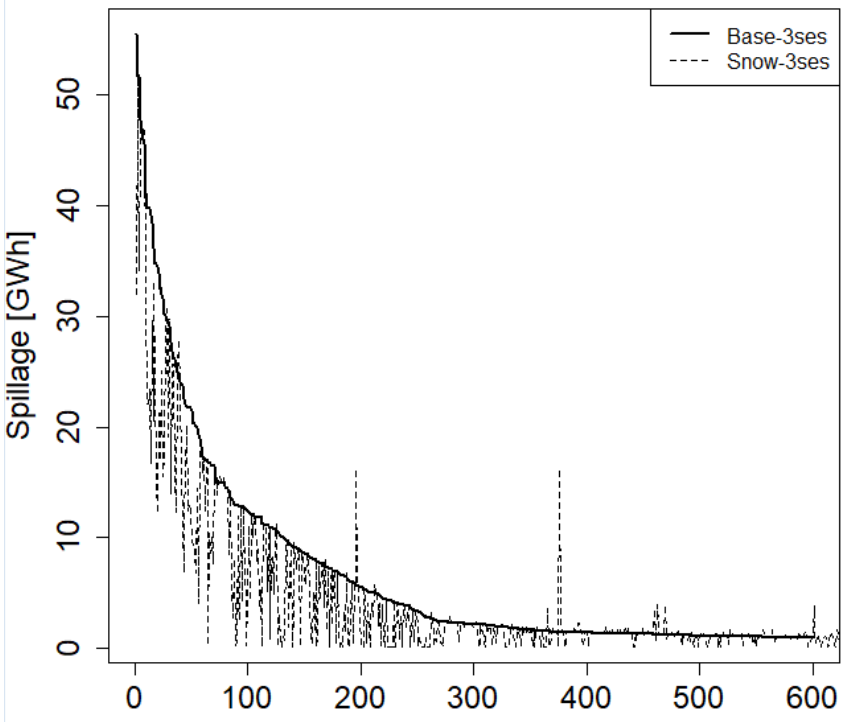

Figure 16 Spillage for the sum system presented as a duration curve sorted according to results from the case Base-3ses.

Cut coefficients for the Hamlagrøvatn reservoir for cases Base-3ses and Snow-3ses are presented in Figure 17. The figure presents the 5% highest and 5% lowest values of the duration curves for all cut coefficients generated during ProdRisk runs for cases Base-3ses (black) and Snow-3ses (red). We observe that coefficients are lower when considering snow storage information in the high end of the duration curve and the opposite in the low end.

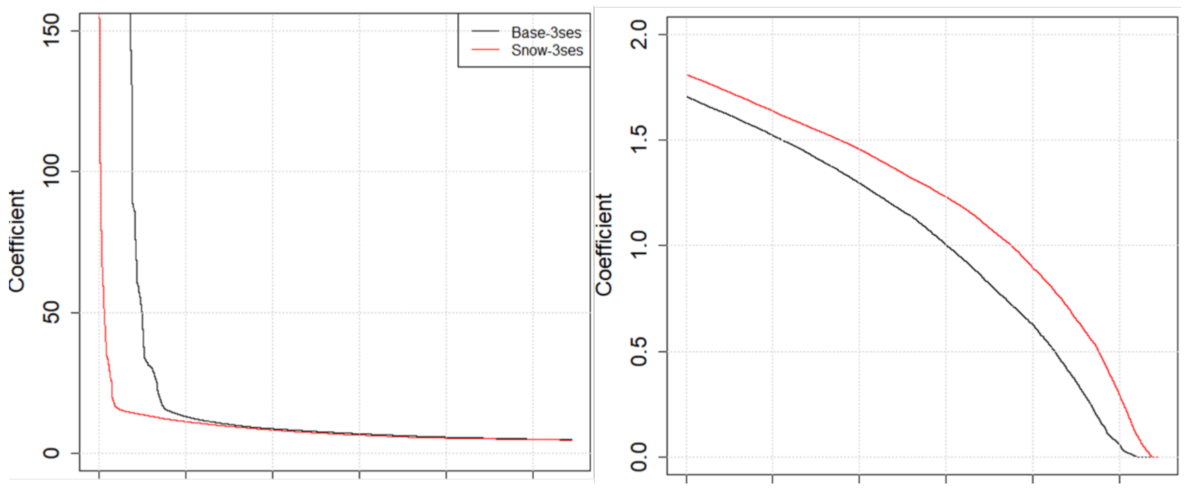

Figure 17 Cut coefficients for Hamlagrøvatn, showing tails of duration curve.

3.2 Lysebotn

The Lysebotn system was modelled using 6 hydropower modules. It comprises one large reservoir (Lyngsvatn) and one major power plant (Lysebotn).

3.2.1 Inflow Model

The inflow to all reservoirs in the system was defined to follow the statistical properties of the Jogla inflow series. In addition, we model a separate snow storage using the Jogla time series for snow storage.

Correlation matrices per case and per season are presented in Table 4. Some observations that are relevant for the following analyses:

- Inflow autocorrelation is significantly higher in the winter/spring season (weeks 1-17) than in autumn/winter (weeks 33-52).

Table 4 Correlation matrices for Lysebotn.

Base-1ses						
	Weeks 1-17		Weeks 18-32		Weeks 33-52	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.4338	-	0.4338	-	0.4338	-
Snow	-	-	-	-	-	-
Base-3ses						
	Weeks 1-17		**Weeks 18-32**		**Weeks 33-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5722	-	0.4824	-	0.2899	-
Snow	-	-	-	-	-	-
Snow-1ses						
	Weeks 1-17		**Weeks 18-32**		**Weeks 33-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.4118	0.1195	0.4118	0.1195	0.4118	0.1195
Snow	-0.0031	0.8873	-0.0031	0.8873	-0.0031	0.8873
Snow-3ses						
	Weeks 1-17		**Weeks 18-32**		**Weeks 33-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5609	0.0843	0.3467	0.3109	0.2886	0.0292
Snow	-0.0190	0.9912	-0.0585	1.0048	0.0033	0.7352

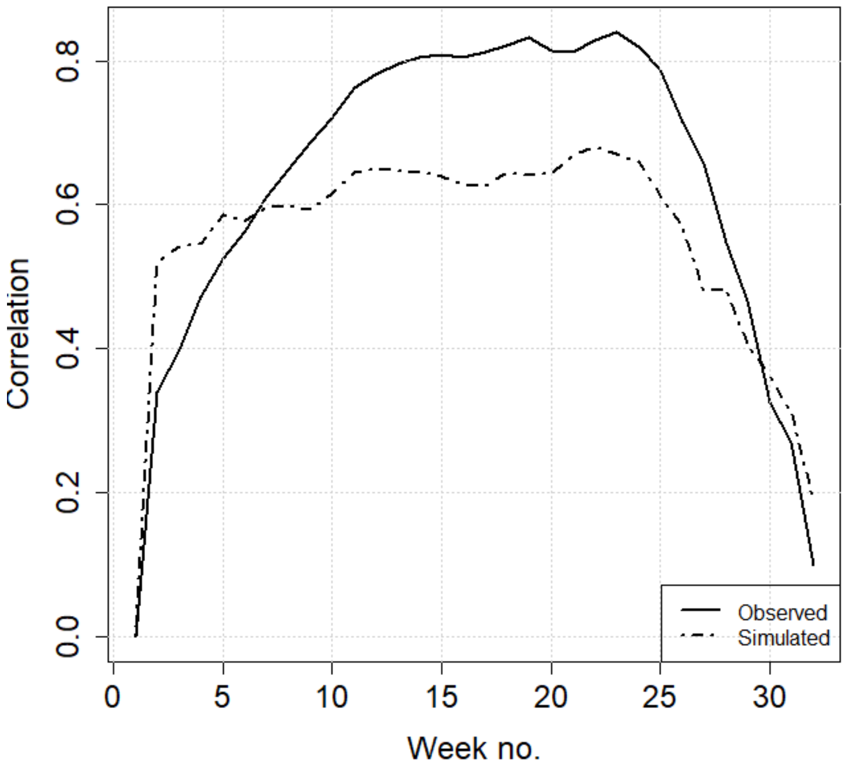

Figure 18 Observed and simulation correlation between snow storage and accumulated inflow for Lysebotn/Jogla.

3.2.2 Simulation Results

The system was run in parallel mode for a scheduling horizon of 260 weeks, using 30 inflow and snow storage scenarios. Expected values for net profit, spillage and production are presented Table 5.

Table 5 Performance indicators for Lysebotn.

	Base-1ses	Base-3ses	Snow-1ses	Snow-3ses
Exp. net profit [MNOK]	4244.5	4250.8	4253.3	4252.0
Exp. spillage [GWh]	393.2	390.8	385.1	388.1
Exp. production [GWh]	7362.9	7368.1	7380.2	7373.9

Reservoir trajectories for Lyngsvatn from ProdRisk runs without and with snow storage information are shown in Figure 19. A maximum reservoir storage constraint was enforced in the middle of the scheduling period. The trajectories are marginally different, where Snow-3ses is more careful towards the top of the reservoir and visits the lower part of the reservoir more frequently than Base-3ses.

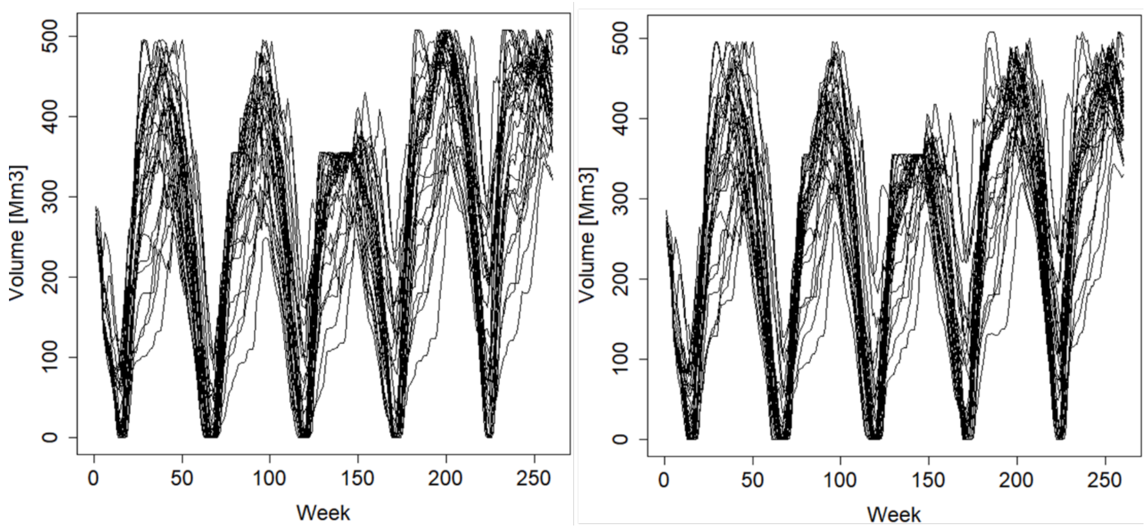

Figure 19 Reservoir trajectories for Lyngsvatn, for Base-3ses (left) and Snow-3ses (right).

Spillage for the sum system is lower when considering snow information, as shown in Figure 20.

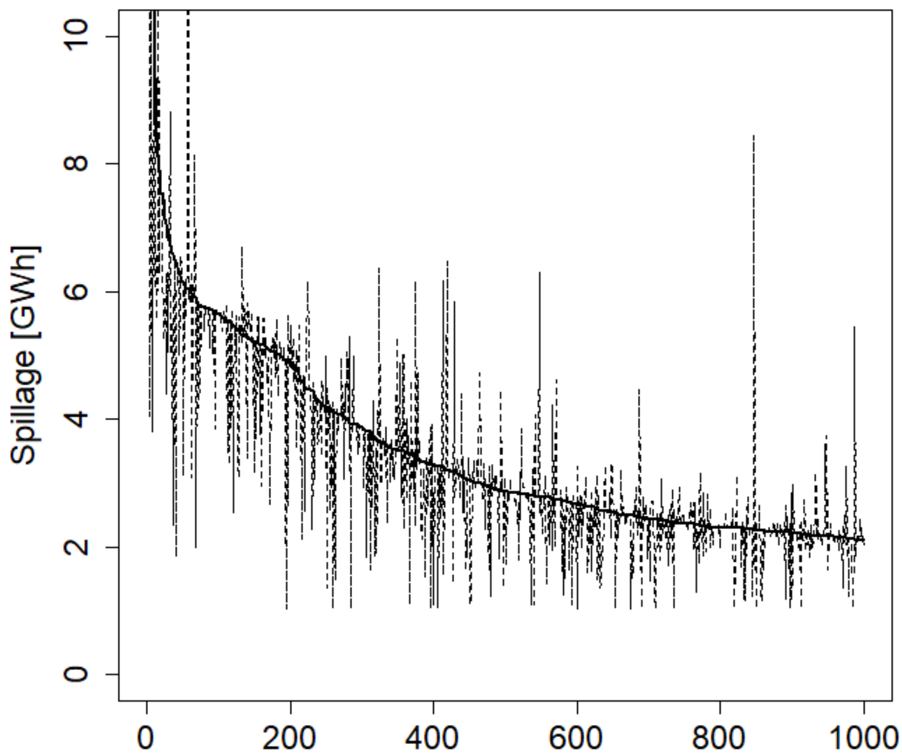

Figure 20 Spillage for the Lysebotn system presented as a duration curve sorted according to results from the case Base-3ses.

Cut coefficients for the Lyngsvatn reservoir for cases Base-3ses and Snow-3ses are presented in Figure 21. The figure presents the 5% highest and 5% lowest values of the duration curves for all cut coefficients generated during ProdRisk runs for cases Base-3ses (black) and Snow-3ses (red). We observe that coefficients are higher when considering snow storage information in both ends of the duration curve.

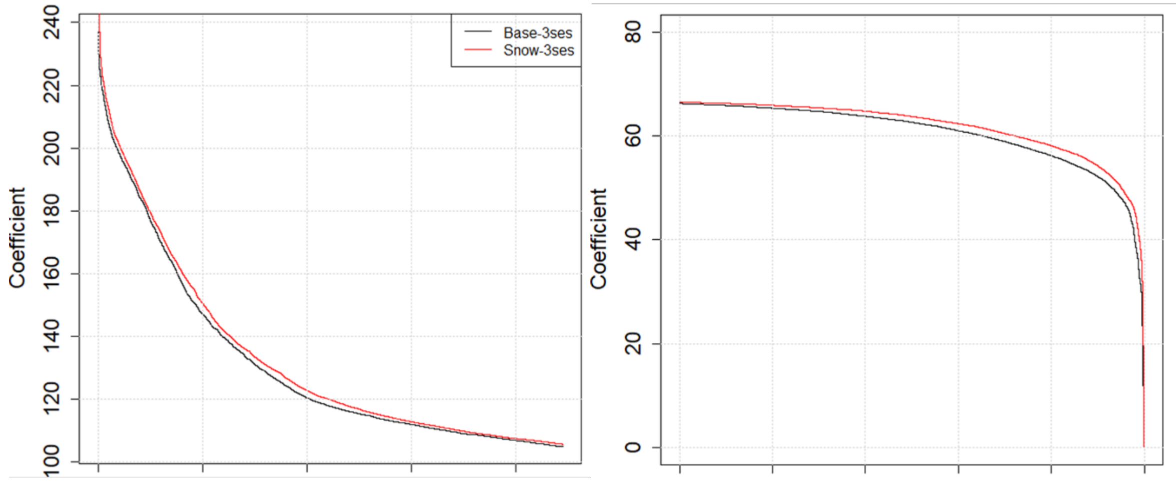

Figure 21 Cut coefficients for Lyngsvatn, showing tails of duration curve.

3.3 Rjukan

The Rjukan system is modelled using 10 hydropower modules. It comprises one large (Møsvatn) and two medium-sized (Mårvatn and Kalhovd) reservoirs, and has several power stations with significant capacity downstream.

3.3.1 Inflow Model

The inflow to all reservoirs in the system was defined to follow the statistical properties of the Kvenna inflow series. In addition, we model a separate snow storage using the Kvenna time series for snow storage.

Correlation matrices per case and per season are presented in Table 6. Some observations that are relevant for the following analyses:

- Variations in inflow autocorrelation across seasons are less than for the previous cases.

Table 6 Correlation matrices for Rjukan.

Base-1ses						
	Weeks 1-21		**Weeks 22-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.6352		0.6352		0.6352	
Snow						
Base-3ses						
	Weeks 1-21		**Weeks 22-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5883		0.6661		0.6668	
Snow						
Snow-1ses						
	Weeks 1-21		**Weeks 22-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.6233	0.0581	0.6233	0.0581	0.6233	0.0581
Snow	0.0294	0.8872	0.0294	0.8872	0.0294	0.8872
Snow-3ses						
	Weeks 1-21		**Weeks 22-35**		**Weeks 36-52**	
	Inflow	*Snow*	*Inflow*	*Snow*	*Inflow*	*Snow*
Inflow	0.5879	0.0027	0.5034	0.3290	0.6686	-0.0276
Snow	0.0194	0.9847	-0.0385	1.0010	0.0401	0.7034

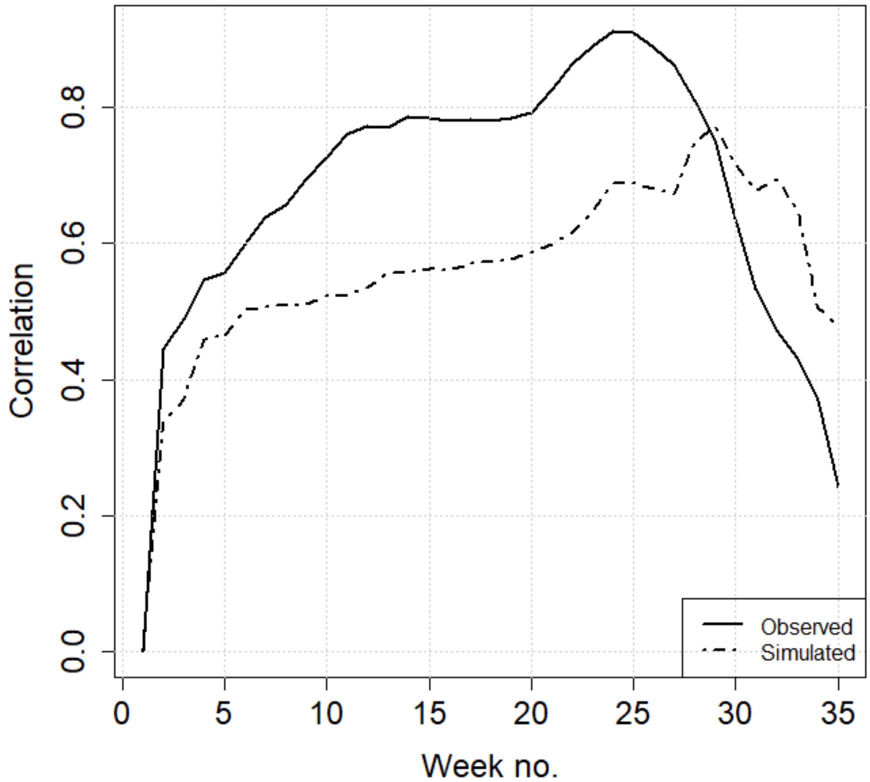

Figure 22 Observed and simulation correlation between snow storage and accumulated inflow for Rjukan/Kvenna.

3.3.2 Simulation Results

The system was run in parallel mode from week 36 to 260 using 54 inflow and snow storage scenarios.

Table 7 Performance indicators for Rjukan.

	Base-1ses	Base-3ses	Snow-1ses	Snow-3ses
Exp. net profit [MNOK]	3080.8	3078.7	3080.3	3084.5
Exp. spillage [GWh]	582.3	605.4	411.8	346.8
Exp. production [GWh]	14821.3	14794.7	15033.2	15092.5

Reservoir trajectories from ProdRisk runs without and with snow storage information are shown in Figure 23. The trajectories are marginally different, where Snow-3ses is more careful towards the lower part of the reservoir than Base-3ses, indicating higher water values.

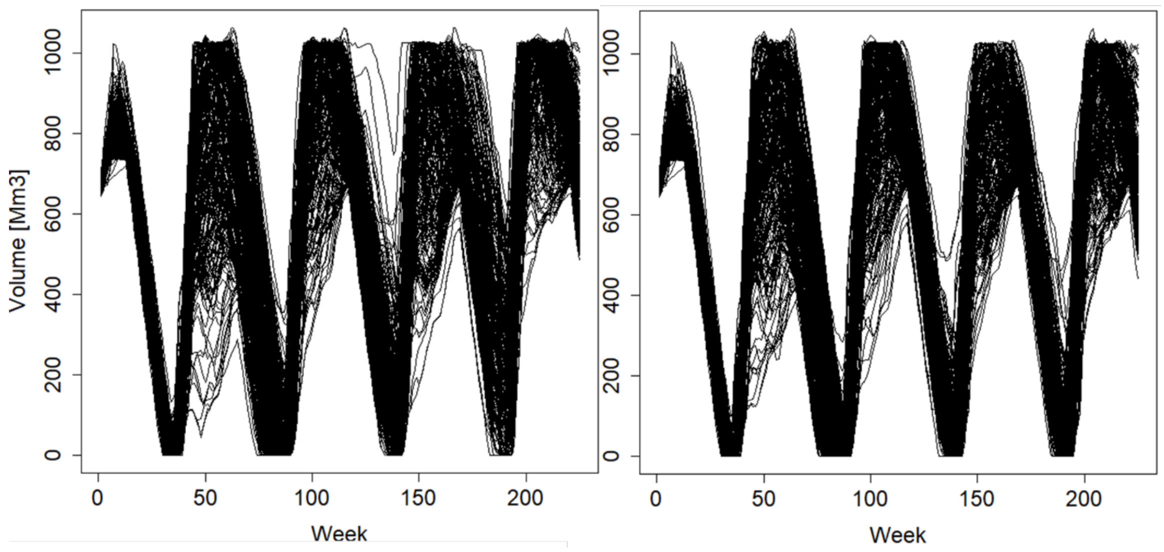

Figure 23 Reservoir trajectories for Møsvatn, for Base-3ses (left) and Snow-3ses (right).

Spillage for the sum system is lower when considering snow information, as shown in Figure 24.

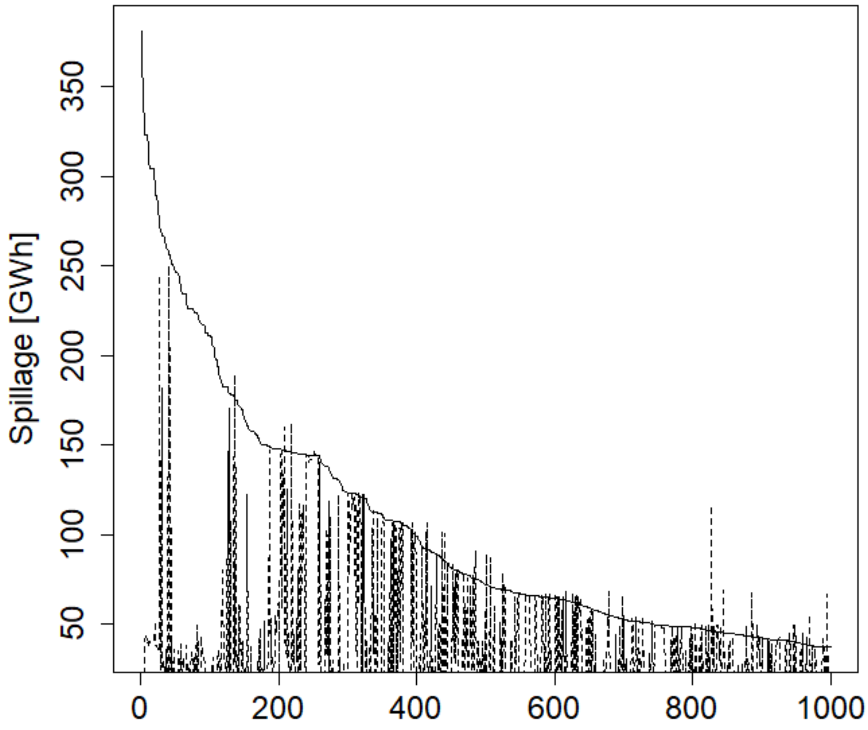

Figure 24 Spillage for the Rjukan system presented as a duration curve sorted according to results from the case Base-3ses.

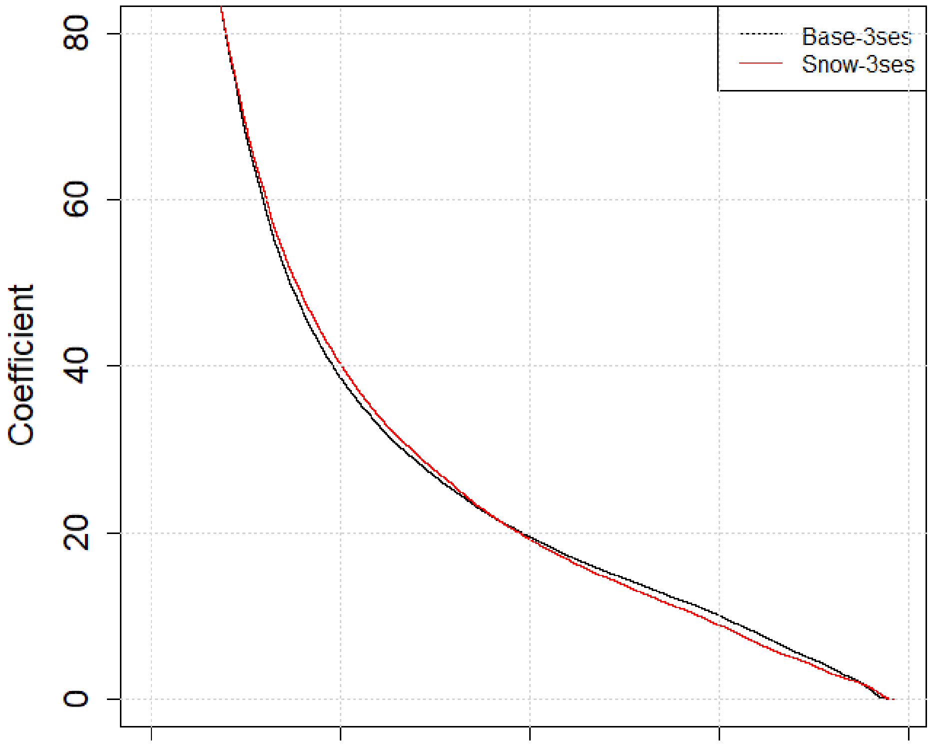

Figure 25 Cut coefficients for Møsvatn, duration curve.

4 Conclusions

For hydropower systems in col cold climate zones, snow storage information may serve as an exogenous state variable that can partly explain the inflows to hydropower reservoirs and plants. In this work we tested a technique for embedding snow storage information as an exogenous state variable in the ProdRisk hydropower scheduling model which is based on the SDDP-algorithm.

A review detected that a similar approach, known as SDDPX in [21], has been presented earlier in the scientific literature. Despite the lack of scientific novelty, the presented approach introduces an easy integration of snow storage information in the widely used SDDP-based scheduling model ProdRisk. It can be seen as a practical advice for ProdRisk users but may also be useful for similar type of scheduling models. By associating time series with snow storage information with a so-called “dummy module”, the estimated PAR(1) model jointly represents snow storage and inflow as state variables in SDDP.

Three case studies representing different hydropower systems in Norway were used as test cases. Results showed that the use of snow storage improves the scheduling in all cases, leading to better economic performance, more power production, and less spillage.

5 References

[1] A. Helseth, B. Mo and H. O. Hågenvik, "Nonconvex Environmental Constraints in Hydropower Scheduling," in *In Proc. International Conference on Probabilistic Methods Applied to Power Systems*, Liege, 2020.

[2] A. Helseth, B. Mo, H. O. Hågenvik and L. E. Schäffer, "Hydropower Scheduling with State-dependent Discharge Constraints – an SDDP approach," *Journal of Water Resources Planning and Management,* vol. 148, 2022.

[3] L. E. Schäffer, A. Helseth and M. Korpås, "A stochastic dynamic programming model for hydropower scheduling with state-dependent maximum discharge constraints," *Renewable Energy,* vol. 194, pp. 571-581, 2022.

[4] A. Helseth, S. and B. Mo, "Approximating Ramping Constraints in Hydropower Scheduling," in *19th International Conference on the European Energy Market*, Lappeenranta, 2023.

[5] A. Helseth and S. Rex, "Ramping Constraints in Hydropower Scheduling - With application in the SDDP-based ProdRisk model," 2023.

[6] A. Gjelsvik, "Studiar i tilsigsmodellering for SDDP," 2005.

[7] S. Mathisen, "Improved Inflow Modelling For the SDDP Algorithm," 2022.

[8] SINTEF Energi, "https://www.sintef.no/programvare/prodrisk/".

[9] R. Bellman, Dynamic Programming, 1 ed., Princeton, NJ: Princeton University Press, 1957.

[10] J. Kelman, J. R. Stedinger, L. A. Cooper, E. Hsu and S. Q. Yuan, "Sampling stochastic dynamic programming applied to reservoir operation," *Water Resources Research,* vol. 26, pp. 447-454, 1990.

[11] A. Haugstad, "Vannverdiberegning med infromasjon om snømagasinet," 1993.

[12] B. Mo and A. Haugstad, "Snømagasin i Samkjøringsmodellen," 1997.

[13] J. A. Tejada-Guibert, S. A. Johnson and J. R. Stedinger, "The Value of Hydrologic Information in Stochastic Dynamic Programming Models of a Multireservoir System," *Water Resources Research,* vol. 31, pp. 2571-2579, 1995.

[14] P. Côté, D. Haguma, R. Leconte and S. Krau, "Stochastic optimisation of Hydro-Quebec hydropower installations: a statistical comparison between SDP and SSDP methods," *Canadian Journal of Civil Engineering,* vol. 38, pp. 1427-1434, 2011.

[15] Q. Desreumaux, P. Côté and R. Leconte, "Role of hydrologic information in stochastic dynamic programming: a case study of the Kemano hydropower system in British Columbia," *Canadian Journal of Civil Engineering,* vol. 41, pp. 839-844, 2014.

[16] J. R. Birge, "Decomposition and partitioning methods for multi-stage stochastic linear programs," *Operations Research,* vol. 33, pp. 989-1007, 1985.

[17] M. V. F. Pereira and L. M. V. G. Pinto, "Multi-stage stochastic optimization applied to energy planning," *Mathematical Programming,* vol. 52, pp. 359-375, 1991.

[18] A. Shapiro, "Analysis of stochastic dual dynamic programming method," *European Journal of Operational Research,* vol. 209, pp. 63-72, 2011.

[19] G. Infanger and D. P. Morton, "Cut sharing for multistage stochastic linear programs with interstage dependency," *Mathematical Programming,* vol. 75, pp. 241-256, 1996.

[20] M. E. P. Maceira, A. C. G. Melo, J. F. M. Pessanha, C. B. Cruz, V. A. Almeida and T. C. Justino, "Combining monthly wind and inflow uncertainties in the stochastic dual dynamic programming," *Energy Systems,* 2023.

[21] J. Pina, A. Tilmant and P. Côté, "Optimizing Multireservoir System Operating Policies Using Exogenous Hydrologic Variables," *Water Resources Research,* vol. 53, pp. 9845-9859, 2017.

[22] Y. Mbeutcha, M. Gendreau and G. Emiel, "Benefit of PARMA Modeling for Long-Term Hydroelectric Scheduling Using Stochastic Dual Dynamic Programming," *Journal of Water Resources Planning and Management,* vol. 147, p. 05021002, 2021.

[23] M. Hindsberger, "Modeling a Hydrothermal System with Hydro- and Snow Reservoirs," *Journal of Energy Engineering,* vol. 131, pp. 98-117, 2005.

[24] B. Mo, A. Helseth and G. Warland, "Modeling and Quantifying the Importance of SnowStorage Information for the Nordic Power System," in *15th International Conference on the European Energy Market*, Lodz, 2018.

[25] A. Helseth, B. Mo, A. L. Henden and G. Warland, "Detailed Long-Term Hydro-Thermal Scheduling for Expansion Planning in the Nordic Power System," *IET Generation, Transmission and Distribution,* vol. 12, pp. 441-447, 2018.

[26] L. E. Schäffer, A. Adeva-Bustos, T. H. Bakken, A. Helseth and M. Korpås, "Modelling of Environmental Constraints for Hydropower Optimization Problems – a Review," in *International Conference on the European Energy Market (EEM)*, Stockholm, 2020.

[27] A. Helseth, A. C. G. Melo, Q. M. Ploussard, B. Mo, M. E. P. Maceira, A. Botterud and N. Voisin, "Hydropower Scheduling Toolchains: Comparing Experiences in Brazil, Norway, and USA and Implications for Synergistic Research," *Journal of Water Resources Planning and Management,* vol. 149, 2023.

www.hydrocen.no

ISSN: 2532-5392
ISBN: 978-82-93602-45-3

HydroCen
v/ Vannkraftlaboratoriet, NTNU
Alfred Getz vei 4,
Gløshaugen, Trondheim

www.hydrocen.no
HydroCen
@FMEHydroCen