Bachelor's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
NTNU Business School

Andreas Borhaug

The performance of a Markowitz-based dynamic algorithm compared to the Dow Jones Industrial Average

Bachelor's thesis in Business Administration
Supervisor: Denis Becker
April 2024

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Preface

This analysis does not just mark the completion of the author's undergraduate studies, in Business Administration, with a major in Business Analytics, but also represents a personal journey navigating through the field of financial markets and strategic portfolio management.

The motivation for this research stems from a deep interest in the intersection between the financial markets and data analytics, and how these can be integrated to enhance asset allocation to achieve high returns per unit of risk. Through this work, I aim to contribute to the discussion on the effectiveness of exploiting portfolio optimization compared to investing in renowned benchmarks.

This journey would not have been possible without the guidance and expertise of Denis Becker. His support, insights, and ideas have been invaluable and deeply appreciated.

The content of this paper is the responsibility of the author(s).

Abstract

The Dow Jones Industrial Average, a cornerstone in the financial markets, measures the performance of the most influential entities in the United States. The index exhibits a price-weighted structure, thereby ensuring diversification, making it a suitable alternative for risk-averse investors. However, this structure and diversification come at the cost of potentially higher returns, a trade-off that may not align with investors who are more focused on efficiency rather than a price-weighted structure.

This thesis introduces an algorithm and creates monthly portfolios aiming to maximize the expected Sharpe Ratio, an allocation that optimizes the estimated return per unit of anticipated risk. The algorithm is backtested on the period from 2013 to the end of 2023, where it estimates the expected return and covariance based on data from the previous twelve months. The analysis measures the aggregated return, annual compounded growth rate, volatility, and Beta to determine the risk -/ return profile.

The algorithm nearly outperformed 75% of the assets within the Dow Jones Industrial Average, while also presenting more stable returns than ¾ of the assets. Furthermore, it surpassed the benchmark's level of risk-adjusted return, achieving higher peaks, and a milder decline during the worst market downfall during the period analyzed. This finding illuminates a unique trading strategy for risk-tolerant investors seeking alternatives to traditional indices and suggests that using Modern Portfolio Theory, by means of Sharpe Ratio maximization, provides more efficient returns than an index composed of nearly identical components.

Sammendrag

Formålet med denne oppgaven er å undersøke hvordan en portefølje, som består av aktiva fra Dow Jones Industrial Average, presterer mot indeksen over tid. Indeksen benyttes ofte som et anslag for hvordan større industrivirksomheter presterer i aksjemarkedet. Den følger en prisvektet struktur som gjør at medfører høy grad av diversifisering. Men denne strukturen -og diversifiserings formen går på bekostning av potensielt høyere avkastning, noe som ikke er ønsket for investorer som er mer vekstorientert. Dette leder til oppgavens hensikt og problemstilling:

Kan en porteføljeoptimaliseringsstrategi, som benytter komponentene i Dow Jones Industrial Average, overgå indeksen ved å prioritere risikojustert avkastning?

Dette undersøkes ved å konstruere en algoritme som søker å maksimere Sharpe Ratio, en allokering som optimaliserer forventet avkastning per enhet av estimert risiko. Strategien blir testet på data fra 2013 til slutten av 2023, hvor månedlig allokering baseres på bakgrunn av de tolv foregående månedene. Risiko -/ avkastningsprofil utføres på bakgrunn av volatilitet, Beta, akkumulert -og årlig effektiv avkastning.

Strategien resulterte i avkastning som nesten utkonkurrerte 75% av aksjene i indeksen, og viste mer stabil avkastning enn ¾ av aksjene. Ytterligere overgikk den også referanseindeksens nivå av risikojustert avkastning. Avkastningen nådde høyere topper, og mildere reduksjon under den verste nedgangstiden i perioden som ble analysert. Funnet styrker en påstand om at Markowitz porteføljeteori, med utgangspunkt i maksimal Sharpe Ratio, generer mer effektiv avkastning per enhet risiko enn Dow Jones Industrial Average.

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1 Introduction

The Dow Jones Industrial Average (hereby abbreviated to DJIA) is an index that tracks the performance of thirty major publicly listed industrial corporations listed on the New York Stock Exchange (NYSE) and Nasdaq. It was created by Charles Dow and Edward Dow in 1896 and serves as a benchmark for the overall health of the United States stock market and economy (Ganti, 2024).

To maintain its bearing and accuracy, the index adjusts its value and composition based on the individual assets' pricing and relevance. Furthermore, the index follows a price-weighted structure. The index's value is based on the sum of each corporation's respective share price, divided by a divisor (Ganti, 2024). This approach entails that companies with a higher share price are given more weight and yield a greater impact on the index's value.

Although there are ETFs designed to mimic the DJIA, investors focused on asset growth may miss out on higher returns. This is because the price-weighted nature renders it less sensitive to growth in lower-priced assets. Recognizing this restraint, a new strategy is proposed; a strategy that leverages the assets' growth, regardless of share price, and considers risk-adjusted returns. This leads to the core of the thesis:

Can a portfolio optimization strategy, targeting the components of the DJIA, outperform the index itself in terms of compounded return and risk-adjusted returns, by prioritizing efficiency over a price-weighted structure?

To critically assess the thesis, an algorithm is created that optimizes the expected Sharpe Ratio for each month across the period of 2013-2013. The algorithm does not engage in short selling and uses historical data from the preceding twelve months to estimate the expected portfolio return and standard deviation.

The thesis covers the pertinent theoretical material relevant to the analysis. It then moves on to outline the methodology employed. Subsequently, an analysis is conducted by examining the timeline and specific subperiods. Lastly, the thesis concludes by summarizing the findings and conclusions drawn from the research.

2 Theory

2.1 Return and Expectancy

Expected return refers to the yield an investor expects from an investment over a specified period. In most cases, the expected return is impacted by an influx of factors, such as historical data, theoretical probability of various scenarios, as well as the assessment of future market conditions (Team, 2023). The process of establishing an objective expected return is inherently difficult due to biases and endless possibilities of unforeseen circumstances.

Moreover, this paper exploits two measurements of returns: arithmetic and logarithmic. The arithmetic method involves calculating the change from one date to another. For example, the absolute change between the asset price at the start of the investment compared to the value at the time of sale. This is particularly useful when we want to find the total absolute return or mean return throughout a time series.

$$\text{Arithmetic Return} = \frac{p_t - p_{(t-1)}}{p_{(t-1)}}$$

p = price

t = point in time

Equation 1: Arithmetic Return

Natural returns, however, refer to the mathematical process of converting the prices to logarithmic values and then dividing the price from a specific point in time by the preceding logarithmic price. Unlike the arithmetic returns, the logarithmic returns employ symmetry, entailing that an increase or decrease has the same magnitude. The conversion also captures exponential growth and facilitates aggregation over time. The log returns can be converted back to arithmetic returns at any point in time by applying the sum to the power of *e*.

Natural Logarithmic Return = $\ln \left(\frac{p_t}{p_{(t-1)}}\right)$

p = price

t = time

Conversion to Arithmetic Return = $e^{\sum_1^t (r_1 + r_t)} - 1$

r = Natural logarithmic return at a point in time

Equation 2: Logarithmic Return

In this analysis, the expected return is based on the mean monthly return of the preceding twelve months. While historic performance does not present a guarantee for future returns, it streamlines the process of computing the expected return, as estimating future price is inherently complex given the magnitude of factors that influence stock prices. The total portfolio expected return is calculated by summing the product of each asset's allocation and its respective expected return.

Expected Return = $E[x]_p = \sum_{x=1}^{n}(\omega_x * E[x]_x)$

ω_x = *Allocation of asset x in the portfolio*

$E[x]_x$ = *Expected Return of asset x*

Equation 3: Expected Portfolio Return

Aggregated returns refer to the accumulated wealth over time compared to the original investment. It is calculated using the formula for the arithmetic return; however, the starting value is held constant. The conversion to aggregated returns is especially useful as it normalizes the prices, making it comparable to other investments. Additionally, it demonstrates the accumulated wealth in percent, effectively illustrating the investment's growth over time.

$$Aggregated\ Return = \frac{p_t - p_0}{p_0}$$

p = price
t = point in time

Equation 4: Aggregated Return

Moreover, another metric used to assess the return of an investment is the compounded annual growth rate (hereby abbreviated to CAGR). This metric measures the annual yield that must be produced to attain the final value, under the assumption that profits are reinvested (Fernando, 2024). As it is a relative measure, it facilitates comparison between different investments.

$$CAGR = \left[\left(\frac{FV}{SV}\right)^{\frac{1}{n}} - 1\right] * 100$$

FV = Final Value

SV = Starting Value

n = years

Equation 5: Compounded Annual Growth Rate (CAGR)

2.3 Risk

Following the concepts of return and expected return, a myriad of factors can lead to a divergence between the projected returns and the actual returns. This divergence is also known as risk – the uncertainty of a financial asset's return compared to the expected return (Chen, 2023).

Unforeseen changes in macroeconomic factors such as geopolitical conditions, financial outlook, or changes in the corporation's performance, can lead to a deviation from the expected value. These factors collectively represent the total risk inherent in an investment and can be classified into two groups: systematic -and idiosyncratic risk factors. Market risk, or systemic risk, pertains to the exposure of risk factors that affect the entire market. As it affects the entire market, it is rigorously challenging to reduce. However, risk associated with a specific company, or idiosyncratic risk, can be moderated through strategic portfolio diversification (Lake, 2022).

2.3.1 Standard Deviation

A key metric for quantifying risk is the standard deviation. It is a statistical measure of the dispersion of data points from the mean in a probability distribution (Institute of Business & Finance, n.d.). Given that the mean monthly return is used as a benchmark for the expected return, it measures the stability of the assets' performance and the total risk exposure. A high standard deviation indicates higher volatility as the returns deviate from the expected return.

$$\sigma_X = \sqrt{{\sigma_X}^2} = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

x = *asset*

σ_X = *standard deviation*

${\sigma_X}^2$ = *variance*

x_i = *asset returns*

$\bar{x}$ = *mean of data points*

n = *number of data points*

Equation 6: Standard Deviation

2.3.2 Covariance and Correlation

Following standard deviation, correlation, and covariance emerge as one the most significant factors for determining the portfolio components and the corresponding allocation. They gauge how variables move together, implying whether they exhibit similarities. These measurements identify assets that do not follow the same return, thereby revealing asset combinations that can mitigate portfolio risk.

Correlation measures the strength of the relationship between two variables (Statista, n.d.). From a financial perspective, it is used to determine whether the assets' returns have a strong linear affiliation, ranging from a -1 to 1. Either side of the scale entails a perfect linear relationship, where a value of 1 suggests that they are moving in an identical linear pattern, whereas a value of -1 portrays an opposite pattern.

Covariance refers to the coherent relationship between two variables where a change in one reflects a change in the other (Surendran, 2022). It can be calculated by summing the product of the deviations from the mean, divided by the number of samples minus one (to account for a sampling).

$$R(X,Y) = \frac{COV(X,Y)}{\sigma_X * \sigma_Y}$$

$$COV(X,Y) = \frac{\sum[(x_i - \bar{x})^2 * (y_i - \bar{y})^2]}{n-1}$$

R =correlation between the assets,

σ_X *and* σ_Y *= standard deviations,*

x_i *and* y_i *= asset returns*

$\bar{x}$ *and* $\bar{y}$ *= mean returns of X and Y*

Equation 7: Correlation and Covariance

2.3.4 Beta

Beta is a measure of the systemic risk of a portfolio compared to the market. It is used in the Capital Asset Pricing Model (CAPM), calculated by the covariance between the portfolio and the benchmark, divided by the benchmark's variance (Team, 2024). A beta above 1 indicates the portfolio has a higher systemic risk compared to the market, whereas a beta below 1 suggests lower systemic risk. A beta of 1 means the portfolio's systemic risk matches the market.

$$\beta = \frac{COV(R_p, R_m)}{Var(R_m)}$$

$COV(R_p, R_m)$ = *Covariance between the portfolio return and the market return*

$Var(R_m)$ = $(\sigma_m)^2$ = *variance of market returns*

Equation 8: Beta

2.3.5 Portfolio Standard Deviation

The total portfolio risk is based on the weighted standard deviation for each asset and the covariances among the assets. The weights refer to the proportion of each asset in the portfolio. The standard deviation for the portfolio is given using the mathematical formula below.

$$\sigma_p = \sqrt{\sum_{x=1}^{N} \omega_x^2 \sigma_x^2 + \sum_{x=1}^{N} \sum_{x \neq y}^{N} \omega_x \omega_y \, Cov(x, y)}$$

σ_p = portfolio standard deviation

x,y = assets in a portfolio consisting of *N* assets

ω = allocation -/ proportion

$Cov(x, y)$ = covariance between the assets

Equation 9: Portfolio Standard Deviation

2.4 Sharpe Ratio

Following returns and standard deviation, the Sharpe ratio measures the balance between the portfolio's excess returns and standard deviation (Samaha, 2023). The excess return refers to the generated return compared to a risk-free alternative. It substantiates the return generated per unit of risk. The Sharpe Ratio can be calculated for both the expected and the actual values, using the formula below.

$$Sharpe\ Ratio = \frac{E[r]_p - rf}{\sigma_p}$$

$E[r]_p$ = expected portfolio return

rf = risk-free rate

σ_p = portfolio standard deviation.

Equation 10: Sharpe Ratio

2.5 Modern Portfolio Theory

From a financial perspective, it is generally denoted that an asset with a high level of potential return carries higher risk. The optimal trade-off level between risk and return depends on the level of risk aversion, however, some portfolios are more efficient than others, as they preserve the expected return, while minimizing risk. As the portfolio's standard deviation factors in the statistical relationship between assets, the total risk can be reduced by investing in assets that follow different patterns of return, thereby minimizing the standard deviation. This process of minimizing the standard deviation is called optimization.

Optimization is a branch of applied mathematics serving to find the optimal solution for an objective function given constraints (Stanford, 2024). In terms of portfolio optimization, it entails minimizing-/maximizing the return, standard deviation, or Sharpe Ratio, by changing the asset allocations.

The economist Henry Markowitz is known for creating the framework for modern portfolio theory, seeking to optimize portfolios through diversification (Investopedia, 2023). This theory applies optimization to mitigate risk by diversifying investments, thus creating efficient portfolios and optimal asset allocations based on specific criteria (University of Washington, 2024). The objective function in this framework can aim to either achieve the minimum variance for a given level of return or vice versa. Consequently, the lowest baseline expected return is the allocation resulting in the lowest portfolio variance, whereas the highest return is the asset allocation providing the highest return. Between these two extremes – minimum variance and maximum return – numerous portfolios can be constructed to achieve the lowest possible level of risk for a given level of return. Together, these portfolios make up the efficient frontier as illustrated in Figure 1.

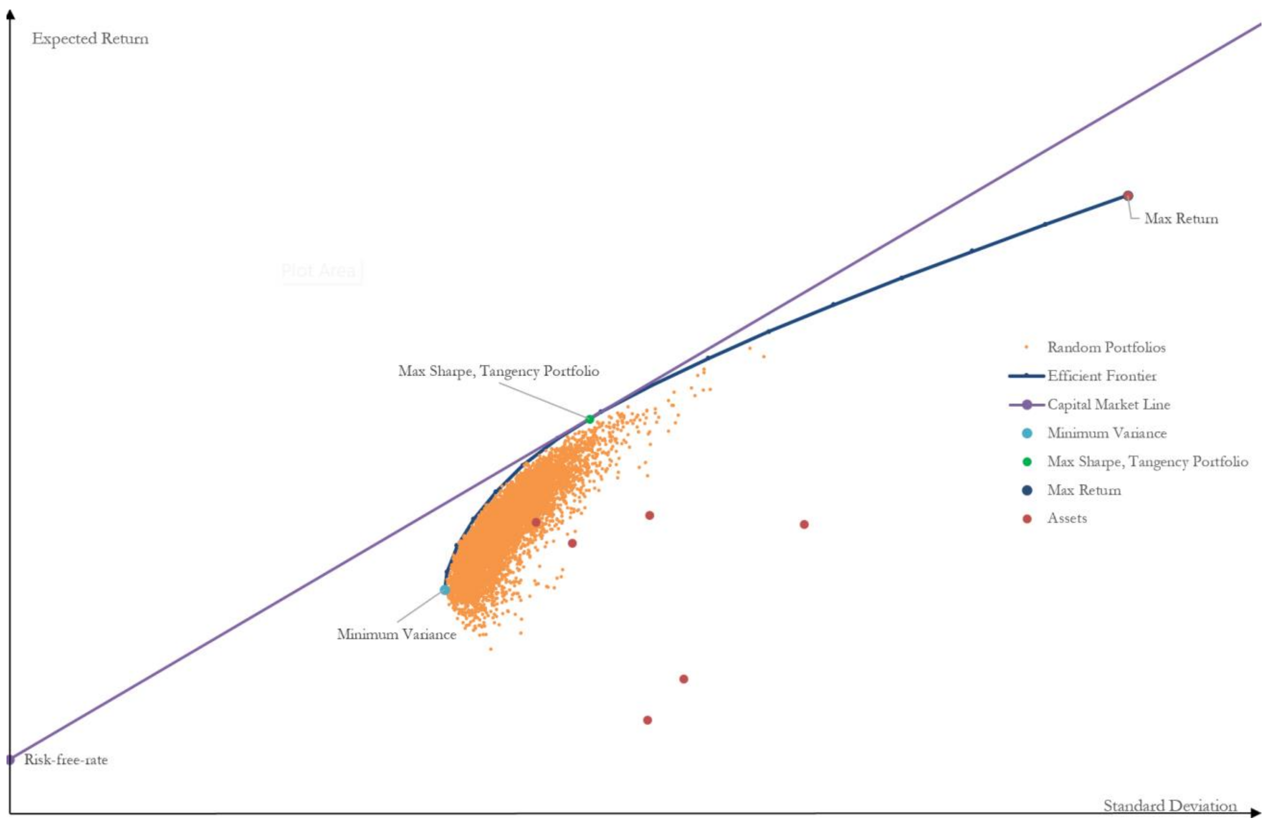

Figure 1: Illustration of Efficient Frontier

3 Methodology

This project aims to investigate whether a portfolio composed of stocks from an established index, specifically the Dow Jones Industrial Average (DJIA) can be optimized to maximize the Sharpe Ratio, to achieve higher risk-adjusted returns compared to the overall index. It evaluates the effectiveness by exploiting a rolling window technique. The operation is performed by an algorithm that backtests the strategy from 2013 to the end of 2023.

3.1 Trading Strategy

The trading strategy employs a rolling window technique that seeks to maximize the expected Sharpe Ratio for each month throughout the entire period. To navigate through the unknown future movements, the trading strategy is based on historic performance. The monthly allocations are based on the logarithmic returns from the preceding twelve months. Given that the strategy looks back at historical data, instead of assessing current market conditions, it is expected that the algorithm will experience a delay in terms of optimal asset allocations. This means that the invests after growth has taken place, in anticipation that the asset will continue to grow in the future. Moreover, the intention behind the rolling window is to capture the general trajectory among the assets in the portfolio. Instead of using the previous month to establish the holdings for the upcoming month, a twelve-month window is used to mitigate anomalies that occur each month. This methodology entails that each month will have an impact of $^{1}/_{12}$ on the overall assessment.

3.2 Data

The dataset used in this analysis was derived from the Dow Jones Industrial Average Index (DJIA) as of January 1st, 2024, with the exception of Dow Inc. This asset was excluded from the dataset due to missing data prior to 1st of April 2019. General Electric was instead used as a substitution given its long historic standing in the index (Williams, 2017). Moreover, during the period in question, several stocks have been introduced and removed from the index. Table 1 illustrates the assets used in the portfolio, whereas Table 2 illustrates the changes in DJIA.

Portfolio used for optimization

Sector	Ticker	Distribution
Technology	AAPL, CSCO, INTC, MSFT, CRM, IBM	*20.0%*
Financial Services	AXP, GS, JPM, TRV, V	*16.7%*
Industrial Goods	BA, CAT, MMM, GE, HON	*16.7%*
Healthcare	AMGN, JNJ, MRK, UNH	*13.3%*
Consumer Goods	KO, MCD, PG, NKE	*13.3%*
Retail	WBA, WMT, HD	*10.0%*
Energy	CVX	*3.3%*
Telecommunications	VZ	*3.3%*
Entertainment	DIS	*3.3%*

Table 1: Assets Used in The Analysis

Changes in the Dow Jones Industrial Average Index (DJIA), 2013-2023

Date	Deleted	Added
September 20, 2013	AA	NKE
September 20, 2013	BAC	GS
September 20, 2013	HPQ	V
March 18, 2015	T	AAPL
June 26, 2018	GE	WBA
April 2, 2019	DD (due to separation from DOW)[1]	(DOW)[2]
August 31, 2020	XOM	CRM
August 31, 2020	RTX	HON
August 31, 2020	PFE	AMGN

Table 2: Changes in the DJIA 2023-2023

(Dogs of the Dow, 2024)

[1] DD (DowDuPont) separated into three individual publicly traded companies, with the following tickers: DOW, DD, and CTVA.
[2] DD Following the separation, DOW was the only company to remain in DJIA.
(SEC, 2019)

The monthly holdings are based on the previous 252 trading days – the average number of trading days in a twelve-month period. As the time series stems from 2013 to the end of 2023, there are 132 look-back periods, entailing a vast number of data points. To automate the process of data retrieval, the prices are fetched using the *yfinance* library in Python, an open-source tool that fetches data from Yahoo Finance through their public APIs (PyPi, 2024). To address inconsistencies identified in the data from Yahoo Finance, the prices are rounded to five decimal places, which is the lowest observed discrepancy.

Upon collecting the data, the daily prices are converted to logarithmic returns. Subsequently, the covariances are calculated and converted to monthly figures. This is achieved by the multiplication of 21, the average number of trading days per month. The prices are then resampled to monthly values and converted to monthly returns. Consequently, the mean from the preceding monthly returns serves as the expected return.

3.3 Optimization

Having established the formulas for the expected return and covariance for each asset, the portfolio metrics are calculated, facilitating the maximization of Sharpe Ratio. The maximization is based on two constraints, no short selling and the sum of the portfolio must be equal to 100%, entailing that all available capital is invested. It should be noted that this investment strategy does not impose any bounds in terms of variety or number of asset classes, entailing that the algorithm is allowed to concentrate on a particular sector or asset. This can severely affect the level of diversification and result in increased idiosyncratic risk.

Moreover, the optimization is solved in Python using the library *SciPy* which provides several minimization tools. Given the task at hand to maximize the Sharpe Ratio with minimization tools, the objective function is inversed to a negative form, thereby maximizing by means of minimization.

The optimization problem is formulated as the following

Minimize:

$$-\left(\frac{E[x]_p - rf}{\sigma_p}\right)$$

Given constraints:

$$0 \leq \omega_x \leq 1 \; for \; all \; x$$
$$\sum_{i=1}^{n} \omega_x = 1$$

Where:

$$E[x]_p = \sum_{x=1}^{n} \omega_x * E[x]_x$$
$$rf = 0$$

p= portfolio,
x = asset
ω = allocation -/ proportion
E[x] = expected return
rf = risk-free rate

Equation 11: Optimization of expected Sharpe Ratio

4 Results and Analysis

4.1 Entire Window

4.1.1 Correlation

All of the assets in the portfolio exhibited a positive correlation with each other during the period period in question, 2013-2023. As illustrated in the frequency table below, most of the assets had a moderate degree of correlation, with a minimum of 0.21 and a maximum of 0.81. These findings imply that the assets have a moderate degree of linear similarities in their levels of return. Furthermore, this is an indication that the portfolios constructed present limited diversification, as the potential for reducing portfolio risk is limited.

Correlation, summary

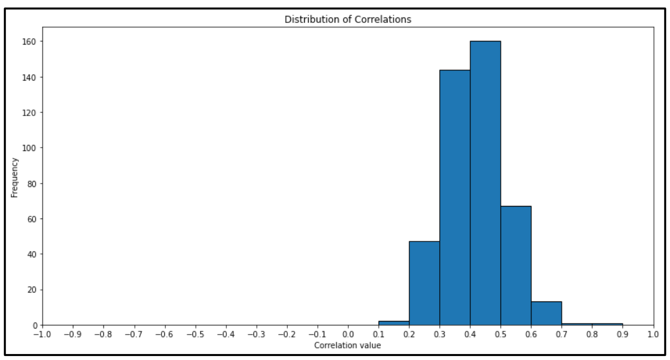

Figure 2: Correlation Distribution

Correlation value	*Frequency*	*Distribution*
−1 ↔ 0	**0**	0.0 %
0.1 ↔ 0.2	2	0.5%
0.2 ↔ 0.3	47	10.8 %
0.3 ↔ 0.4	144	33.1 %
0.4 ↔ 0.5	160	36.8 %
0.5 ↔ 0.6	67	15.4 %
0.6 ↔ 0.7	13	3.0 %
0.7 ↔ 0.8	1	0.2 %
0.8 ↔ 0.9	1	0.2%
0.9 ↔ 1.0	0	0.0%
Total	***435***	***100%***

Table 3: Correlation Frequency

Moreover, it should be noted that a strong correlation is expected for comparable assets operating in the same industry, as they often are impacted by the same variables. For instance, both Morgan Stanley (MS) and Goldman Sachs (GS) operate as financial institutions and generate income from investment banking, wealth management, trading, and other financial services (McClay, 2023). They exhibited a correlation of 0.82. The lowest correlation observed was between Walmart (WMT) and General Electric (GE) with a value of 0.18.

One noteworthy topic following the positive correlation values is that most industry leaders tend to move in the same direction over time, depending on the perceived market outlook. For instance, during bullish periods, investors are fueled by optimism, causing the general stock market to rise (Gallagher, 2024). Furthermore, renowned indexes and corporations with significant market capitalization are also deeply correlated with broad macroeconomic indicators and conditions (Danso, 2020). Since the DJIA is an indicator of the financial market, it is natural the assets in the portfolio are impacted by the same variables, thus producing a positively correlated relationship.

Nevertheless, as illustrated in Table 3, the majority exhibited a correlation between 0.3 and 0.6. These levels of correlation are evident when examining the assets' compounded return over time.

4.1.2 Compounded Returns

Figure 3 illustrates the aggregated return over time, compared to their initial price at the start of the timeline. Table 4 serves as a statistical summary of the figure. Almost all of the assets exhibited positive growth during the period. In extension to the correlation analysis, this positive growth innately contributes to an underlying positive correlation, as the majority of the assets move in the same direction over time.

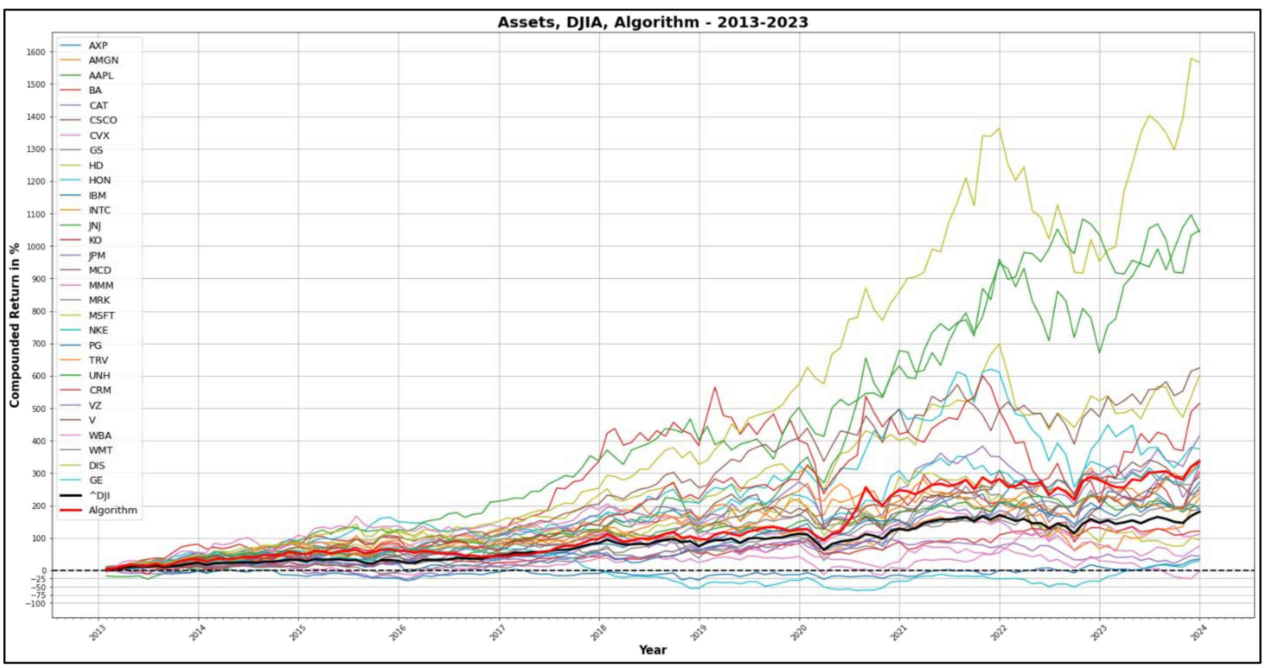

Figure 3: Aggregated Returns – Assets, DJIA, Algorithm

Measurement	*CAGR%*	*Total Return*	*Monthly Mean Return*	*Monthly Standard Deviation*
Mean	12,47	341,60	1,18	6,57
Min	-0,26	-2,80	0,28	4,27
25th percentile	9,27	166,12	0,84	5,33
50th percentile	12,50	265,01	1,20	6,25
75th percentile	14,67	350,42	1,42	7,81
Max	29,16	1565,77	2,36	10,65

Table 4: Descriptive Statistics based on aggregated returns for DJIA, Assets, and Algorithm

The lowest observed return was -2.80% which is attributable to the performance of WBA. This was the only asset analyzed that provided a negative yield. Moreover, GE is also notable for its poor performance, totaling a return of 23% over an eleven-year period. This was far below the general tendency. However, poor performance leading up to mid-2018 was expected given its removal from the index. Nevertheless, it experienced significant growth in 2023, which enabled it to recapture its standing and provide an overall positive yield. The best-performing asset during the timeframe was MSFT. It showcased stable growth maintaining a standard deviation below the 50th percentile and yielded a fifteenfold return, corresponding to a compounded annual growth rate of 29.16%.

In extension to the assets' performance, the DJIA itself delivered a positive return alongside the lowest standard deviation among the assets considered. The low level of fluctuations can be attributed to the level of diversification inherent in the index's price-weighted structure. Furthermore, the returns generated from the DJIA placed it in the lower quartile, with a total return of 181% and a CAGR of 9.86%. This entails that 75 percent of the assets analyzed yielded a higher return than the index itself.

The algorithm achieved a notable total return of 334%, corresponding to a compounded annual growth rate of 14.29%. This positioned it near the 75th percentile of returns. It also presented a standard deviation below the 25th percentile, with a value of 5.29%. In comparison to the DJIA, the algorithm surpassed the DJIA compounded returns in 125 out of 132 months. DJIA had a higher aggregated return between the end of 2016 to mid-2017.

4.1.3 Monthly Performance

Aggregated returns are particularly sensitive to outliers; a brief period of substantial returns can offset a long period of subpar performance. Consequently, this section focuses on the monthly returns. Figure 4 presents the monthly change of both the DJIA and the algorithm. Notably, they shared many of the same characteristics, moving in unison on many occasions.

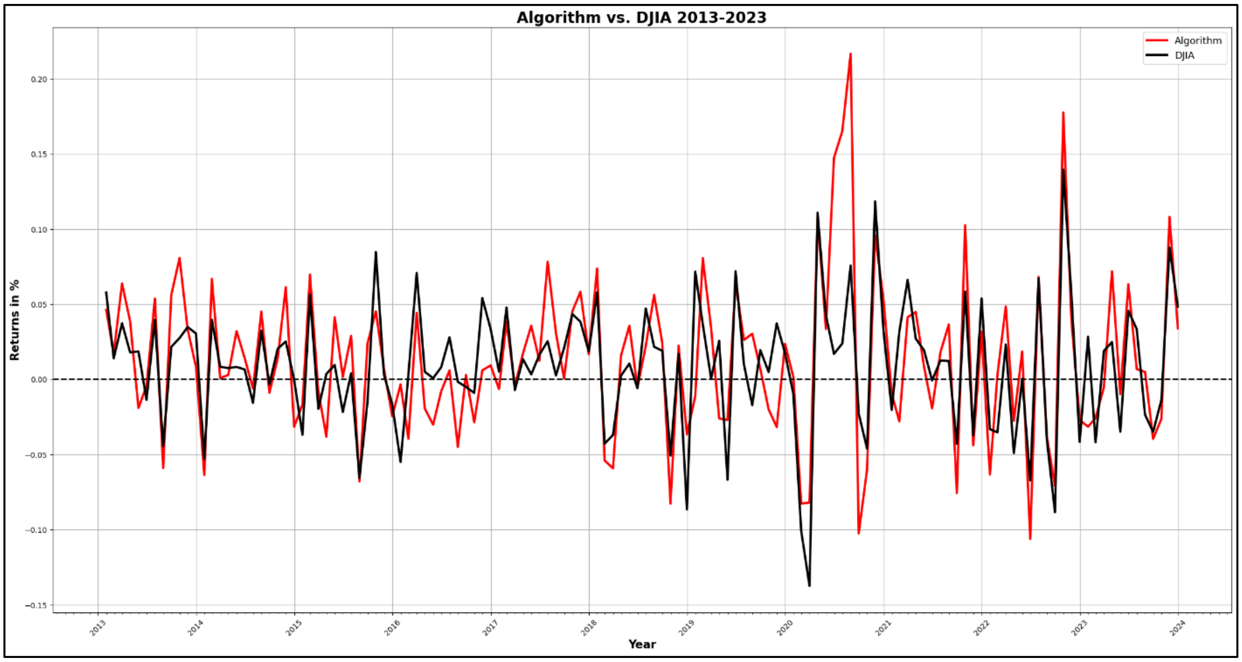

Figure 4: Monthly Return – Algorithm vs. DJIA

The summer months of 2020 stood out as the algorithm generated substantial returns. June, July, and August recorded returns of 14.74%, 16.51%, and 21.66%. These months also entailed high growth for the DJIA, but with less intensity. More detailed descriptive statistics are provided in Table 5, highlighting the percentiles, mean, standard deviation, and monthly wins.

Measurement	Mean	*Standard Deviation*	*Min*	*25th percentile*	*50th percentile*	*75th percentile*	*Max*	*Monthly wins*
Algorithm	1.25%	5.31%	-10.62	-2.49%	0.84%	4.13%	21.66%	71
DJIA	0.89%	4.27%	-13.74%	-1.50%	1.13%	3.28%	13.95%	61

Table 5: Descriptive Statistics Based on Monthly Returns for DJIA – Algorithm

The 25th percentile indicates that the algorithm experienced declines more often than the benchmark. Furthermore, the median, illustrated by the 50th percentile, reveals that the DJIA provided more stable returns. However, the upper quartile signifies that the algorithm is better equipped to provide higher returns than the DJIA. This is more clearly depicted in Figure 5 showcasing the quartiles and outliers.

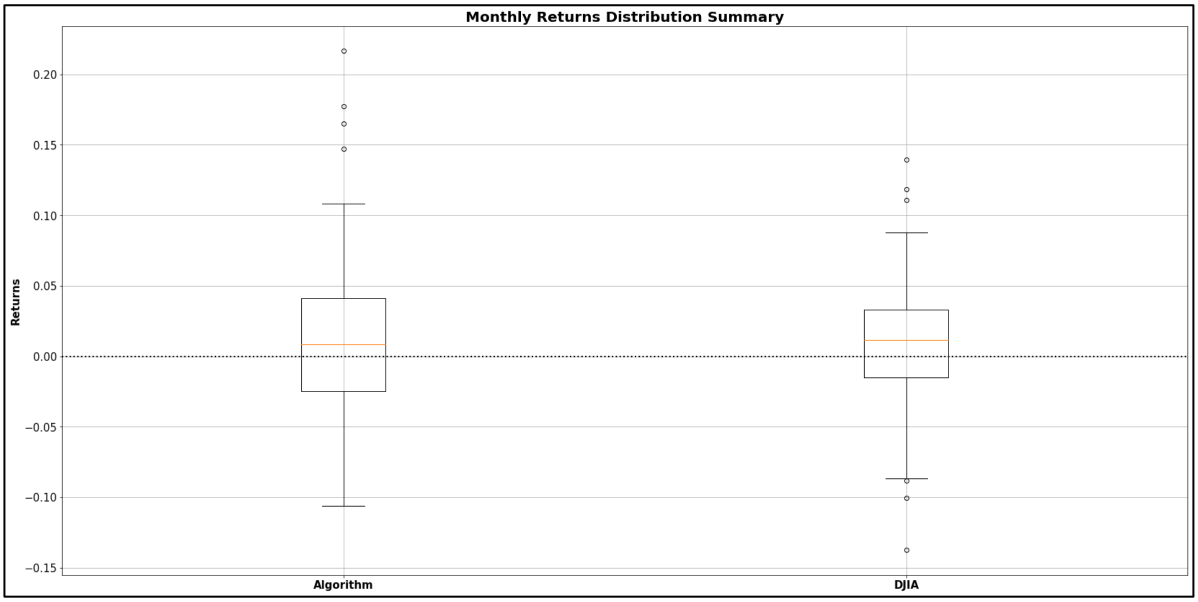

Figure 5: Boxplot of DJIA and algorithm's monthly distribution

The potential for greater returns is highlighted by a higher 75th percentile and a greater number of positive outliers. Nevertheless, the increased opportunity for superior returns is accompanied by increased volatility, as evidenced by a differential of 1.04% in the standard deviations. Furthermore, the algorithm also portrayed a wider range of returns, spanning a range from -10.62% to 21.66%. This outcome was anticipated as the algorithm's architecture enables it to invest in fewer assets with substantial positions, making it more prone to volatility. However, the DJIA recorded the bottommost return with a nadir of -13.74 %. This was not anticipated given that the index held a lower standard deviation and a higher median return.

Nevertheless, this occurrence was an outlier and does not reflect the broad trend of negative returns. The DJIA encountered a negative monthly return in 34.09% of the observed periods with a median of -3.50%. In contrast, the algorithm encountered a higher frequency of negative returns, occurring in 39.39% of the months, and a less severe median of -2.83%.

The algorithm achieved the highest monthly yield 71 times out of a total of 132 possible, equaling a win percentage of 53.79%. Despite having a relatively similar frequency of monthly wins as the DJIA, the algorithm provided markedly higher total yield, thus endorsing the assertion that the outperformance was due to a higher 75^{th} percentile and positive anomalies.

4.1.4 Sharpe and Beta

Measurement	*Algorithm*	*DJIA*
Correlation	0.754	0.754
Monthly Sharpe Ratio	0.236	0.209
Annualized Sharpe Ratio	0.818	0.725
Beta (DJIA as the benchmark)	0.938	1.000

Table 6: Sharpe and Beta – Algorithm and DJIA

Following the affirmation that the algorithm's performance is caused by few, but high returns, the algorithm still generated a superior Sharpe Ratio, as presented in Table 6. The algorithm's mean monthly return of 1.25% and standard deviation of 5.31% generated a score of 0.236, thereby beating the index's monthly Sharpe Ratio of 0.209. In an annualized context, the product of the monthly Sharpe Ratio and the square root of twelve, the algorithm attained a ratio of 0.818, whereas the DJIA achieved a ratio of 0.725. This entails that, despite higher volatility, the algorithm was slightly more efficient at delivering returns per unit of risk than the DJIA.

The assessment of market sensitivity revealed a Beta of 0.938 against the DJIA. Although the assets in the portfolio are an appropriation of the index, the algorithm is set to maximize the Sharpe Ratio, leading to a different weight structure. These disparities abate the correlation between the portfolio and the DJIA, resulting in a correlation of 0.754. Since the Beta is derived from the strength of the relationship between the market and the portfolio, the covariance between them was below the market's variance, leading to a Beta below 1.

The Beta indicates that the portfolio's performance was less volatile to changes in the overall stock market, in this case measured by the DJIA. In more detail, with a lower Beta, it substantiates that the algorithm experienced less systematic risk. Given that the algorithm presented a higher standard deviation, it signifies that the algorithm was more concentrated on individual assets, leading to more exposure to idiosyncratic risk and individual risk factors.

4.2 Timeline Subset Analysis

4.2.1 Bull market

Market Growth measured by well-known indices

Index	Ticker	Compounded Average Growth Rate (CAGR) 2009-2019	Historic Compounded Average Growth Rate
DJIA	*^DJI*	11.03%	8.16%
Nasdaq 100	*^NDX*	19.22%	13.56%
S&P500	*^GSPC*	11.97%	5,83%
NYSE Composite	*^NYA*	8.09%	6.24%
Nasdaq Composite	*^IXIC*	16.76%	9.63%
Vanguard Total Stock Market Index Fund	*VTI*	14.46%	7.99%

Table 7: Market Growth, Bull Window

During the period leading up to 2020, a majority of the assets experienced significant returns, showcasing the dynamics of a bullish market. This phase is marked by robust growth across the overall stock market. After the financial crisis in 2008, prominent indices such as the NYSE Composite, Nasdaq Composite, and Vanguard Total Stock Market Index Fund experienced considerable returns. These indices present an overview of the performance of publicly traded companies listed on both the NYSE and Nasdaq. Furthermore, indexes such as the Nasdaq 100 and the S&P500 measure the companies with the highest market capitalization. As seen in Table 7, all of them outperformed their historic CAGR, denoting a bullish period from 2009-2019. This growth is also illustrated in Figure 6. As the bull period overlaps with the window in the analysis, it entails that the returns generated by the algorithm are not just specific for the algorithm, but rather the stock market in general.

Although the algorithm yielded a higher total return in the bull window than the DJIA, it does not mean that it is a better investment during a bull window. Several other indexes outperformed both the DJIA and the algorithm's returns.

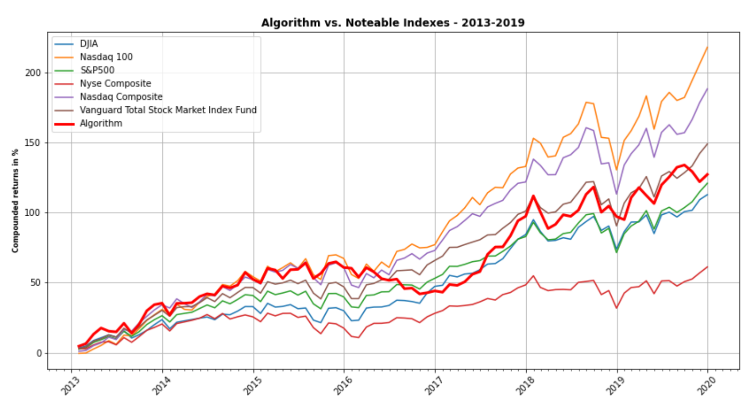

Figure 6: Algorithm vs. market Indexes during bull window

The Beta value stood at 1.037, indicating that the algorithm was more responsive to fluctuations in the DJIA during the time bull window, meaning more exposure to systemic risk. Moreover, during the downfall in late 2018, the compounded return across the indices declined ranging from -24% at its lowest to -38% at the most. The DJIA's decline was recorded at -24%. The algorithm, however, only experienced a downfall of -18%, beating both the DJIA and the other indices.

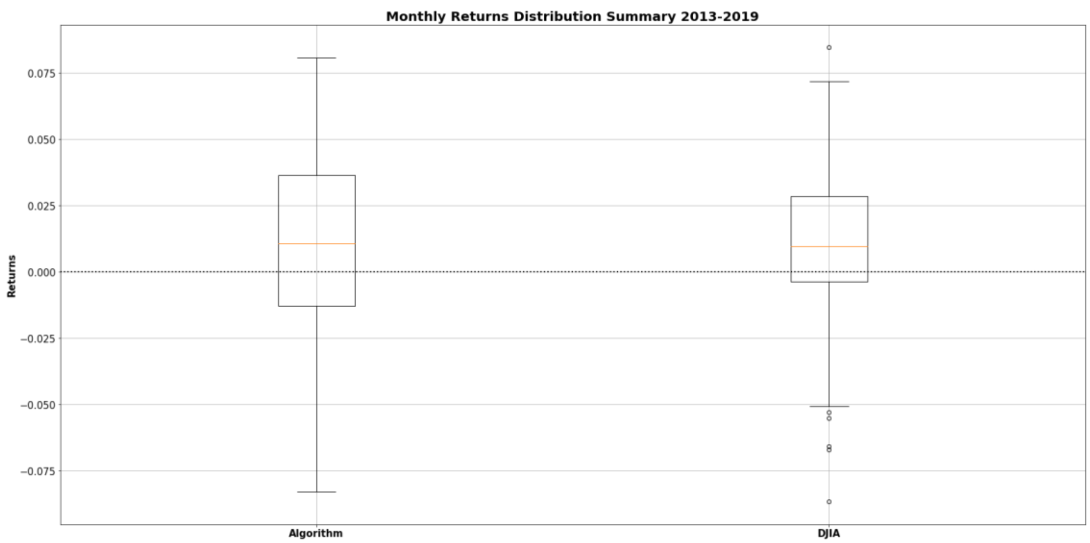

Figure 7: Boxplot of DJIA and algorithm's monthly distribution 2013-2019

The boxplot of the bull window shares several similarities with the one observed over the total period. Once again, the algorithm presented a wider interquartile range and registered a lower 25th percentile. The 75th percentile was also higher. The median was also higher for the algorithm, a disparity from the total window, indicating that the algorithm normally provided a higher yield, as half of the months were higher than those of the DJIA. During the total window, the high mean return was caused by extreme values north of the median. This was not the case during the bull window. In short, the algorithm provided more positive returns, leading to a general outperformance of the DJIA.

4.2.2 COVID-19 pandemic

The year 2020 was marked by uncertainty, lockdowns, and insecurity regarding the future financial outlook. As previously illustrated in Figure 3, the assets in the portfolio experienced a decline until reaching a bottom in March 2020, innately causing both the DJIA and the portfolio to produce a negative result. Consequently, the financial markets experienced substantial turbulence, causing the algorithm and DJIA to produce a negative yield for the first few months. This is illustrated in Figure 8.

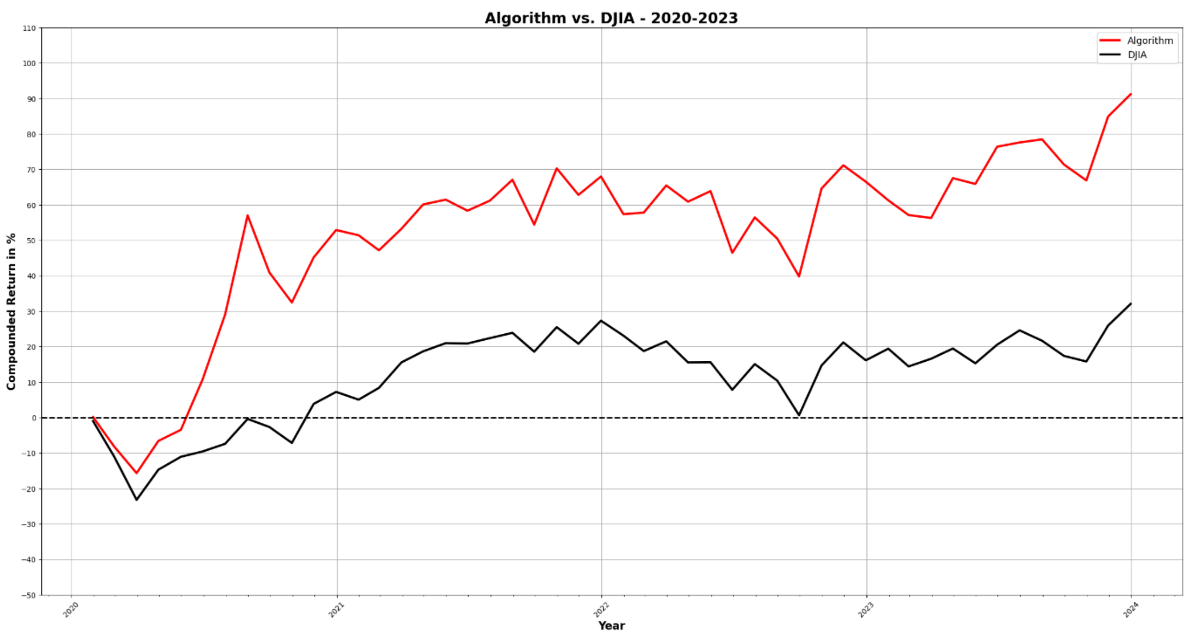

Figure 8: Compounded Returns– Algorithm vs DJIA 2020-2023

The DJIA experienced the heaviest downfall, with a negative aggregated return at -23.20% at the end of March, compared to the algorithm at -15.67%. The period after March 2020 was followed by a phase of recovery and growth. Shortly after experiencing the nadir, the algorithm rapidly experienced superior growth. This was caused by abnormal levels of high returns. As stated earlier, June, July, and August recorded figures of 14.74%, 16.51%, and 21.66%. These values are attributed to Apple's (AAPL) growth, as the algorithm exclusively invested in this single asset during that period. The returns from this asset exceeded the DJIA, causing the algorithm to generate a higher compounded return throughout the period. The specific monthly returns are provided in Figure 9, showcasing the monthly change throughout the period marked by COVID-19 to 2023.

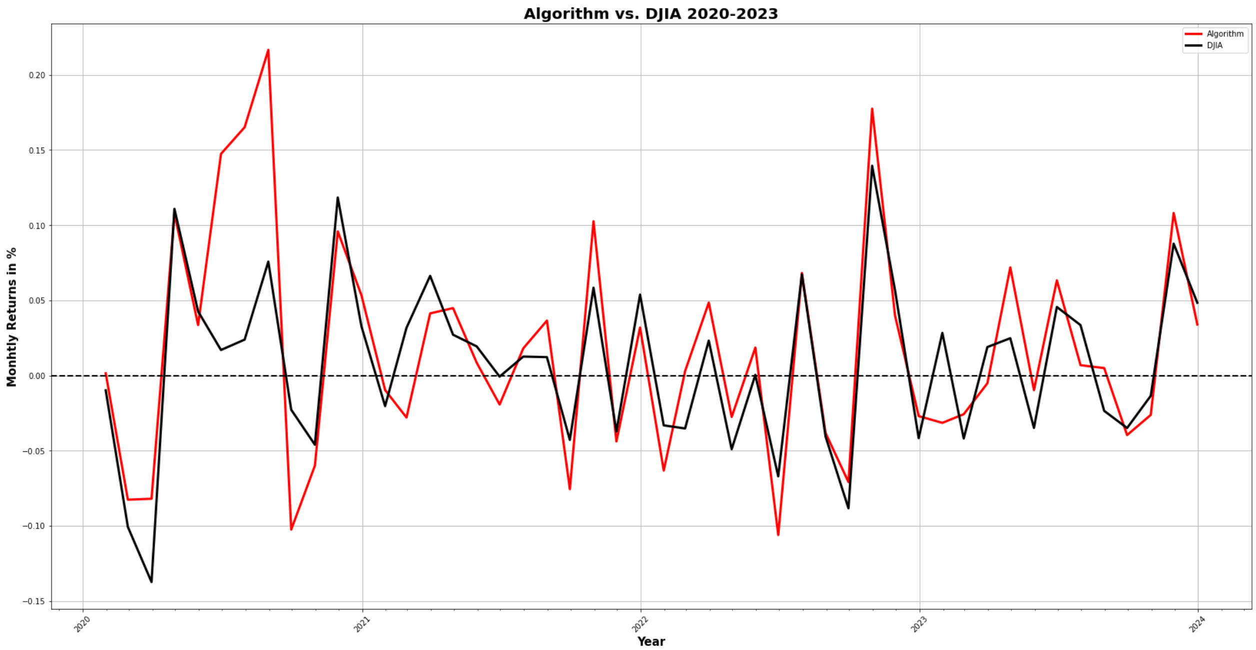

Figure 9: Monthly Return – Algorithm vs. DJIA 2013-2023

After the extensive period of growth, the algorithm and the DJIA exhibited many similarities. Notably, the algorithm experienced more volatility, marked by a higher degree of fluctuations in highs and lows. Similarly to the total window, the algorithm produced a higher 75th percentile and a lesser median, as marked by the boxplot in Figure 10. Contrary to the previous analysis of the bull window, the DJIA experienced a lower 25th percentile, entailing that the bottom 25% of returns were lower than the algorithm. Furthermore, the mean monthly returns were also higher for the algorithm, much attributed to the high returns generated and an elevated bottom quartile.

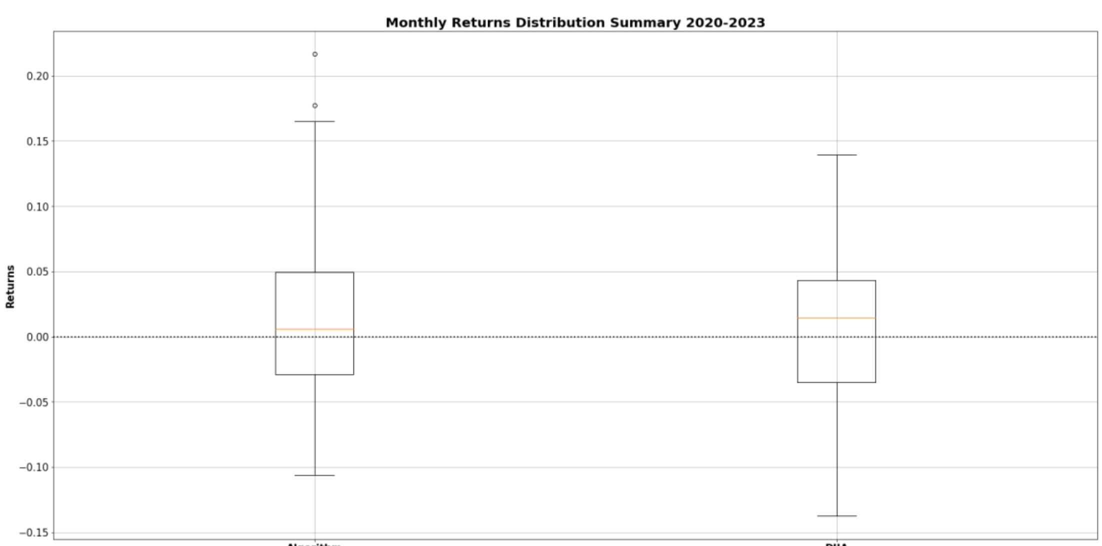

Figure 10: Boxplot of DJIA and the Algorithm's Monthly Distribution 2020-2023

5 Summary and conclusion

During the bull window leading up to 2020, the assets and broader indices experienced significant growth and returns. During this subperiod, the algorithm experienced a higher monthly median return than the DJIA. However, the trajectory was affected by a wider range of interquartile returns and a higher standard deviation. Although the algorithm yielded higher returns, there were other indices, such as the Nasdaq Composite and Nasdaq100, that outperformed the algorithm,

Moreover, during the subperiod dominated by the pandemic, the algorithm demonstrated resilience to low extreme values, dampening the effect of the market turndown. Nevertheless, it still experienced a decline in value, but less profound than the DJIA. The diminished effect was due to more concentration on individual assets, generating more exposure to individual risk factors. This proved to be more beneficial as the holdings were less affected by the pandemic and thrived during the recovery period. It capitalized on Apple's (AAPL) growth, resulting in a solid yield throughout the summer of 2021. After 2021, both the algorithm and the DJIA experienced many of the same highs and lows. However, given the substantial return from Apple, the algorithm was better positioned to yield a higher accumulated return, and thereby a higher CAGR.

Across the entire timeline, all the assets displayed a correlation, with the majority exhibiting moderate levels. The parallel movements entailed limited opportunities to offset risk and hence limited diversification. Consequently, the algorithm exhibited a deeper concentration on fewer assets. This concentrated strategy proved to be more beneficial in terms of risk-adjusted returns.

The strategy to maximize the Sharpe Ratio using a twelve-month rolling window proved to outperform the benchmark in terms of compounded annual growth rate. It nearly generated returns beating 75% of the assets, while simultaneously presenting more stable returns than ¾ of the assets. The algorithm achieved higher peaks and a milder decline during the worst market downfall during the period analyzed. This finding illuminates a unique trading strategy for risk-tolerant investors seeking alternatives to traditional indices and suggests that using Modern Portfolio Theory, by means of Sharpe Ratio maximization, provides more efficient returns than an index composed of nearly identical components.

Given the conclusion and the findings in the paper, an analysis was conducted by using the monthly weights one month ahead. It should be noted that this is not feasible, except for instances of backtesting, as it involves using future data. Nevertheless, it yielded some very interesting results, where it dominated both the DJIA and the algorithm. Several attempts were made to construct a Deep Learning algorithm to predict the values one month ahead in time, by using an LSTM architecture. This entailed using historic returns to determine the next month's returns. Unfortunately, due to the inherent complexity of the stock market, it did not provide sufficient results. Furthermore, other attempts involved predictions of covariance and monthly returns. Due to poor model statistics, these findings were not presented. The topic of portfolio optimization combined with artificial intelligence is nevertheless an interesting subject. Further study is strongly endorsed as it may yield significant enhancements in portfolio management.

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