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Co-rotational Reduced Order Kalman Filter Finite Element Method for the Real-time Implementation of Large-scale Structural Models

Bismi Rasheed [*,**] **Morten Dinhoff Pedersen** [***] **Hans Georg Schaathun** [*]

[*] *Cyber-Physical Systems Laboratory, Department of ICT and Natural Sciences, NTNU - Norwegian University of Science and Technology, Ålesund, Norway (e-mail: bismi.rasheed@ntnu.no, hasc@ntnu.no)*

[**] *Ålesund Biomechanics Lab, Ålesund General Hospital, Møre and Romsdal Hospital Trust, Ålesund, Norway*

[***] *Department of Engineering Cybernetics, NTNU - Norwegian University of Science and Technology, Trondheim, Norway (e-mail: morten.pedersen@ntnu.no)*

Abstract: The real-time implementation of large-scale structural models without sacrificing their physical realism is challenging. This paper introduces a conceptual framework named the Co-rotational Reduced Order Kalman Filter Finite Element Method (CROKF-FEM) for the realistic linear elastic analysis of large-scale structures in real-time. The standard linear finite element method (FEM) is employed for high-precision deformation modeling, and element matrices are pre-computed in the space domain. To avoid the inflated elements of standard linear FEM due to large rotation, a co-rotational formulation is devised. Then, the system is discretized in the time domain using the Wilson- θ implicit integration scheme and a further state-space model is developed. Subsequently, a reduced-order model is constructed using the balanced truncation method. Finally, a Kalman filter algorithm is created on the reduced model for online estimation. This approach preserves all the benefits of traditional linear FEM with a similar deformation accuracy while achieving significant real-time performance.

Keywords: Kalman Filtering, Model Order Reduction, Corotational Formulation, Finite Element Method, Real-time Modeling, Soft Tissue Deformation

1. INTRODUCTION

Large-scale structures exist in wider fields of biology and engineering. The accurate modeling of these structures is difficult due to the complex underlying physical mechanisms and high degrees of freedom (DOF) present in the domain of interest. The minimal requirement of a visual feedback rate of $30Hz$ renders it more difficult to execute in real-time. The common methods for modeling these large-scale structures can be classified into two categories. One category prioritizes faster computing speed over simulation precision. Mass-Spring Modeling (MSM) (Tan et al., 2020) and Position-Based Dynamics (PBD) (Wang et al., 2021) fall under this group. The other class focuses on accurate modeling based on continuum mechanics. The Finite Element Method (FEM) (Bathe , 2006) is a well-known numerical technique from this group in which the mechanical behavior of the structure is characterized based on stress-strain relations by discretizing the continuum into a finite number of elements connected with nodes, called a finite element mesh. Modeling using traditional FEM is computationally expensive, even for linear elastic problems.

Numerous studies have focused on improving the computational efficiency of FEM. Pre-computation of the constant matrices helps to reduce FE computation time. The appropriate choice of numerical schemes also accelerates the FE computation, for instance, Total Lagrangian Explicit Dynamics (TLED) (Meister et al., 2020). However, the requirement of the small time step in the explicit scheme makes this method not suitable for real-time implementation. The co-rotational formulation incorporates large rotations to produce precise deformation (Marinkovic et al., 2018), but simulation performance suffers substantially because the system matrix needs to be constructed at each time step. Although the use of a Graphical Processing Unit (GPU) accelerates the computational performance (Pellicer-Valero et al., 2020; Wang et al., 2021), the large-scale structural models often lead to unmanageable demands on computational resources (Lu et al., 2021). Real-time performance was attainable with FEM-based machine learning (ML) (Pellicer-Valero et al., 2020; Meister et al., 2020), albeit at the expense of higher computing demands made during model training. The computational head can be diminished by reducing the set of variables to a lower dimension using Model Order Reduction (MOR) techniques (Gao and Shang., 2020) such as Proper Or-

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thogonal Decomposition (POD) and Proper Generalized Decomposition (PGD). MOR also requires compromising the precision of the simulation. Online estimation of the deformation using a Kalman filter (KF) is another option for minimizing the expensive computational burden (Xie et al., 2020). Nevertheless, for large-scale models, this approach needs to manage certain computational loads. Song et al. (2022) enhanced the performance of the non-linear FEM by combining extended KF and POD. However, the stiffness matrices have to be recomputed, and a nonlinear system of equations has to be solved to compute the forces at every time step. Generally, shifting from linear to nonlinear analysis for precise modeling would incur higher computational costs, but staying in the linear regime for real-time requirements results in inaccurate deformation modeling. Hence, we aim to solve large rotations in large-scale structural models in real time while retaining all the benefits of standard linear FEM.

The motivation for developing this framework is to achieve a higher visual and haptic feedback rate in soft tissue deformation for the realization of a digital twin model for arthroscopic knee surgery (Bjelland et al., 2022). It is crucial that the soft tissue reacts to an external stimulus with physical realism and in real-time when designing and training surgical procedures (Xie et al., 2020). The proposed framework could also be employed in the linear elastic analysis of large and complex structures in aviation, aerospace, and shipping because of the difficulty of balancing the conflicting criterion of real-time versus physical realism.

This paper presents a novel framework, named the Co-rotational Reduced Order Kalman Filter Finite Element Method (CROKF-FEM), to accelerate the computational performance of multidimensional physical models. In CROKF-FEM, the continuum is discretized in space based on a linear elastic theory using FEM and pre-computes the constant matrices. The co-rotational formulation is then used to account for the large rotations. Subsequently, the model is discretized in the time domain using the Wilson-θ implicit integration method and further transformed to the state-space formulation. Following this, the state-space equation is reduced using the balanced truncation technique. Finally, the Kalman filter is employed for the online estimation of deformation. This method is suitable for real-world scenarios because the system state equation is constructed by considering the process noise of the system.

2. METHODOLOGY

In this section, the proposed framework of the Co-rotational Reduced Order Kalman Filter Finite Element Method (CROKF-FEM) is explained.

2.1 Linear Elasticity

The mechanical deformation of large-scale structures can be described using infinitesimal strain theory and linear elastic theory (Sadd , 2009). Consider the deformation of a domain from undeformed state ${}^0\Omega$ to deformed state Ω with nodal positions ${}^0X = [{}^0x, {}^0y, {}^0z]^T$ and $X = [x, y, z]^T$ in ${}^0\Omega$ and Ω respectively. The right superscript T throughout the formulation denotes the transpose of the associated vector or matrix. The strain energy (E) of the continuum is defined as

$$E_\Omega = \frac{1}{2}\int_\Omega \sigma_{ij}\,\varepsilon_{ij}\,dX. \tag{1}$$

The relationship between stress (σ_{ij}) and strain (ε_{kl}) tensors is represented using generalized Hooke's law

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

where the matrix C_{ijkl} is fourth order isotropic tensor; $C_{ijkl} = \lambda\,\delta_{ij}\,\delta_{kl} + 2\mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ with $\lambda = \frac{E}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$. Here, E and ν are Young's modulus and Poisson's ratio of the material, respectively.

In matrix form, $C_{ijkl} = D$ is rewritten as

$$D = \lambda\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}. \tag{3}$$

The strain-displacement relation connects the strain tensor ε_{ij} to displacement gradient tensor $u_{i,j}$ is given by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{4}$$

2.2 Spatial Discretization

The considered elastic continuum is discretized into the finite number of four-noded tetrahedral elements. The shape functions, N_1, N_2, N_3, and N_4, are used to describe the distribution of displacement in terms of nodal values of displacement $u = [u_x\, u_y\, u_z]^T$;

$$u = \sum_{i=1}^{4} N_{ie}(X)u_{ie} \tag{5}$$

where subscript e represents the element.

Using the derivatives of shape functions B, the strain-displacement relations can be rewritten as $\varepsilon = Bu$,

where

$$B = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & 0 & \dots & \frac{\partial N_4}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \dots & 0 & \frac{\partial N_4}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_1}{\partial z} & \dots & 0 & 0 & \frac{\partial N_4}{\partial z} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & \dots & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial z} & \frac{\partial N_1}{\partial y} & \dots & 0 & \frac{\partial N_4}{\partial z} & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial z} & 0 & \frac{\partial N_1}{\partial x} & \dots & \frac{\partial N_4}{\partial z} & 0 & \frac{\partial N_4}{\partial x} \end{bmatrix}. \tag{6}$$

Then the strain energy for an element e is

$$E_e = \frac{1}{2}\int_{\Omega_e} {u_e}^T {B_e}^T D B_e u_e dX. \tag{7}$$

The equilibrium equation gives element force as

$$f_e^{int} = \frac{1}{2}\int_{\Omega_e} {B_e}^T D B_e u_e dX. \tag{8}$$

Substituting (8) in (7) yields, $f_e^{int} = K_e u_e$, where the K_e is the element stiffness matrix,

$$K_e = {B_e}^T D B_e V_e. \tag{9}$$

2.3 Co-rotational Formulation

The deformation of massive structures is usually accompanied by arbitrarily large rotations, thus invalidating the assumptions inherent in infinitesimal strain theory. These large rotations cannot be handled by linear FEM, resulting in an artificial enlargement of the model (Marinkovic et al., 2018). Therefore, a suitable formalization that takes into account geometric non-linearity due to large rotations must be devised. The co-rotational formulation presented here is adapted from Marinkovic et al. (2018).

For each element, e, the coordinate transformation of the undeformed configuration in the local reference frame at time $t = 0$, ${}^0Xe$, to the deformed configuration in the current frame at time t, ${}^tX_e$, is expressed as ${}^tX_e = A_e\,{}^0X_e$, where A_e is the transformation matrix. The rotational part at time t, ${}^tR_e$, of the transformation matrix, describes the rigid body rotation between the initial and current configuration. Then, the quantity ${}^tR_e{}^T\,{}^tX_e - {}^0X_e$ gives the displacement free of any rotations. Premultiplying this with the pre-computed element stiffness matrix gives the internal forces of each element in the reference frame. The internal forces in the current frame can be computed by

$${}^t f_e^{int} = {}^tR_e\,{}^0K_e\,{}^tR_e{}^T\,{}^tX_e - {}^tR_e\,{}^0K_e\,{}^0X_e, \tag{10}$$

where ${}^tR_e\,{}^0K_e\,{}^tR_e{}^T$ is defined as an element warped stiffness matrix (${}^tK_e$), and the quantity ${}^0K_e\,{}^0X_e$ can be precomputed.

Then, the global warped stiffness matrix (${}^tK$) and global internal force vector (f^{int}) can be represented as

$$\begin{aligned} {}^tK &= \sum {}^tK_e\,, \\ f^{int} &= \sum {}^t f_e^{int}. \end{aligned} \tag{11}$$

2.4 Dynamic Equilibrium Equation

The dynamic equilibrium equation to represent the deformation of the system can be written as

$$M\ddot{U} + C\dot{U} = f^{ext} - f^{int} \tag{12}$$

where the mass matrix M is a function of material density ρ; $M = \int_V \rho N^T N dV$, and $f^{int} = KU$. In (12), $\ddot{U}$ is the acceleration vector and $\dot{U}$ is the velocity vector with U being the nodal displacement vector.

The damping matrix is determined by $C = \alpha M + \beta K$ with small values of damping coefficient α and β. f^{ext} is the external force.

2.5 Temporal Discretization

Wilson - θ implicit time integration scheme is employed for the time discretization (Xie et al., 2020). The acceleration ($\ddot{U}_t$), velocity ($\dot{U}_t$), and displacement (U_t) at time $t + \Delta t$ can be written as

$$\begin{aligned} \ddot{U}_{t+\Delta t} &= \frac{6}{\theta^3\,\Delta t^2}(U_{t+\theta\Delta t} - U_t) - \frac{6}{\theta^2\,\Delta t}\dot{U}_t + (1 - \frac{3}{\theta})\ddot{U}_t\,, \\ \dot{U}_{t+\Delta t} &= \dot{U}_t + \ddot{U}_t\,\Delta t + \frac{\Delta t}{2}(\ddot{U}_{t+\Delta t} - \ddot{U}_t)\,, \\ U_{t+\Delta t} &= U_t + \Delta t\,\dot{U}_t + \frac{1}{2}\ddot{U}_t\,\Delta t^2 + \frac{\Delta t^2}{6}(\ddot{U}_{t+\Delta t} - \ddot{U}_t). \end{aligned} \tag{13}$$

$U_{t+\theta\Delta t}$ is required for calculating (13). Applying the integration scheme to dynamic equilibrium equation (12) results

$$\begin{aligned} &(\frac{6}{\theta^2\,\Delta t^2}M + \frac{3}{\theta\Delta t}C + K)\,U_{t+\theta\Delta t} = f_t + (2M + \frac{\theta\Delta t}{2}C)\,\ddot{U}_t \\ &+(\frac{6}{\theta\,\Delta t}M + 2C)\,\dot{U}_t + (\frac{6}{\theta^2\,\Delta t^2}M + \frac{3}{\theta\Delta t}C)\,U_t \end{aligned} \tag{14}$$

where the integration constant should be taken as $\theta \geq 1.37$ for unconditional stability.

2.6 State Space Representation

The displacement, velocity, and acceleration are considered unknown variables. The state space formulation presented here is adapted from Xie et al. (2020). Rewriting (14) gives

$$U_{t+\theta\Delta t} = J^{-1}AU_t + J^{-1}Y\ddot{U}_t + J^{-1}B\dot{U}_t + J^{-1}If_t \tag{15}$$

where I is identity matrix, and A, B, Y and J are

$$\begin{aligned} A &= \frac{6}{\theta^2\Delta t^2}M + \frac{3}{\theta\Delta t}C, \\ B &= \frac{6}{\theta\,\Delta t}M + 2C, \\ Y &= 2M + \frac{\theta\Delta t}{2}C, \\ J &= \frac{6}{\theta^2\,\Delta t^2}M + \frac{3}{\theta\Delta t}C + K. \end{aligned} \tag{16}$$

We need the displacements at each time step Δt instead of $\theta\Delta t$. For this, substitute (15) in equation (13) for $\ddot{U}_{t+\Delta t}$ yields

$$\ddot{U}_{t+\Delta t} = A^{aa}\ddot{U}_t + A^{va}\dot{U}_t + A^{xa}U_t + A^{fa}f_t. \tag{17}$$

Further substituting (17) in equation (13) for $U_{t+\Delta t}$ yields

$$U_{t+\Delta t} = A^a\ddot{U}_t + A^v\dot{U}_t + A^xU_t + A^f f_t \tag{18}$$

where

$$\begin{aligned} A^a &= \frac{\Delta t^2\theta^3 I - \Delta t^2\theta^2 I + 2J^{-1}Y}{2\theta^3}, \\ A^v &= \frac{\Delta t\theta^3 I - \Delta t\theta I + J^{-1}B}{\theta^3}, \\ A^x &= \frac{\theta^3 I + J^{-1}A - I}{\theta^3}, \\ A^f &= \frac{J^{-1}}{\theta^3}. \end{aligned} \tag{19}$$

Finally rearranging (18) gives system state equation

$$U_{t+\Delta t} = FU_t + Gz_t \tag{20}$$

with

$$\begin{aligned} F &= A_x;\; G = [A_a\;\; A_v\;\; A_f]\,, \\ z_t &= \left[\ddot{U}_t\,\dot{U}_t\,U_t\right]^T. \end{aligned} \tag{21}$$

where U_t is the system state vector, z_t is the control input, F is state transmission matrix, and G is control input matrix.

2.7 Model Order Reduction

Lower-order approximations of complicated systems are derived using model order reduction techniques. Balanced truncation is one of the most common model reduction schemes. This formulation aims to construct a reduced-order model by eliminating the states which are least observable and controllable.

Balanced Truncation: The formation described here is based on (Arif et al., 2014). Consider the state-space representation of the dynamic system equation (12)

$$\begin{aligned} U_{t+\Delta t} &= FU_t + Gz_t \\ y_t &= HU_t \end{aligned} \tag{22}$$

where y_t is the measurement vector at time t, and H is the measurement matrix.

The controllability gramian (W_c) and observability gramian (W_o), which are symmetric positive definite matrices, are obtained from the unique solution of the Lyapunov equation.

$$\begin{aligned} FW_cF^T + GG^t - W_c = 0 \\ F^TW_oF + H^TH - W_o = 0 \end{aligned} \tag{23}$$

The system (22) is balanced only if W_c and W_o are equal and diagonal

$$W_c = W_o = diag(\sigma_1, \sigma_2, ..., \sigma_n) = \Sigma \tag{24}$$

where σ_i's are positive real numbers; $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_n > 0$. These are ordered Hankel singular values of the system (22). The balanced gramian Σ can be partitioned based on its Hankel singular value.

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{25}$$

$\Sigma_1 = diag(\sigma_1, \sigma_2, ..., \sigma_r)$ and $\Sigma_2 = diag(\sigma_{r+1}, \sigma_{r+2}, ..., \sigma_n)$ with $\sigma_r >> \sigma_{r+1}$.

Subsequently, we can generate the balanced system

$$\begin{aligned} \tilde{U}_{t+\Delta t} &= \tilde{F}\tilde{U}_t + \tilde{G}z_t \\ \tilde{y}_t &= \tilde{H}\tilde{U}_t \end{aligned} \tag{26}$$

where $\tilde{F}$, $\tilde{G}$, and $\tilde{H}$ are matrices of the balanced system with appropriate dimensions. The system (26) can be partitioned based on (25).

$$\Psi = \left(\begin{array}{cc|c} \tilde{F}_{11} & \tilde{F}_{12} & \tilde{G}_1 \\ \tilde{F}_{21} & \tilde{F}_{22} & \tilde{G}_2 \\ \hline \tilde{H}_1 & \tilde{H}_2 & 0 \end{array}\right) \tag{27}$$

with

$$\begin{aligned} \tilde{U}_t &= \begin{pmatrix} \tilde{U}_{1_k} \\ \tilde{U}_{2_k} \end{pmatrix}; \tilde{F} = \begin{pmatrix} \tilde{F}_{11} & \tilde{F}_{12} \\ \tilde{F}_{21} & \tilde{F}_{22} \end{pmatrix} \\ \tilde{G} &= \begin{pmatrix} \tilde{G}_1 \\ \tilde{G}_2 \end{pmatrix}; \tilde{H} = \begin{pmatrix} \tilde{H}_1 & \tilde{H}_2 \end{pmatrix} \end{aligned} \tag{28}$$

where $\tilde{U}_{1_k}$ and $\tilde{U}_{2_k}$ are state variables corresponding to the large and small Hankel singular values, respectively. Finally, the reduced order model for the system can be obtained by discarding the state variables in the system (22) corresponding to small Hankel singular values.

$$\begin{aligned} \tilde{U}_{r_{(t+\Delta t)}} &= \tilde{F}_r\tilde{U}_{r_{(t)}} + \tilde{G}_r z_t \\ \tilde{y}_{r_{(t)}} &= \tilde{H}_r\tilde{U}_{r_{(t)}} \end{aligned} \tag{29}$$

To avoid complications with the symbols, the (29) is rewritten as

$$\begin{aligned} \mathcal{U}_{t+\Delta t} &= \mathcal{F}\mathcal{U}_t + \mathcal{G}\mathcal{Z}_t, \\ \mathcal{Y}_t &= \mathcal{H}\mathcal{U}_t. \end{aligned} \tag{30}$$

2.8 Kalman Filter Implementation

The Kalman filter system state equation for deformation analysis is described as

$$\begin{aligned} \mathcal{U}_{t+\Delta t} &= \mathcal{F}\mathcal{U}_t + \mathcal{G}\mathcal{Z}_t + \mathrm{w}_t \\ \mathcal{Y}_t &= \mathcal{H}\mathcal{U}_t + \mathrm{n}_t \end{aligned} \tag{31}$$

where w_t is the process noise and n_t is the measurement noise, they can be specified as $\mathrm{w}_t \approx \mathcal{N}(0, \mathcal{Q})$ and $\mathrm{n}_t \approx \mathcal{N}(0, \mathcal{R})$. $\mathcal{Q}$ and $\mathcal{R}$ are process noise covariance and measurement noise covariance, which are assumed to be uncorrelated and to have a Gaussian distribution with a zero mean.

The prediction stage of KF method is given by

$$\begin{aligned} \bar{\mathcal{U}}_{t+\Delta t} &= \mathcal{F}\mathcal{U}_t + \mathcal{G}\mathcal{Z}_t \\ \bar{\mathcal{P}}_{t+\Delta t} &= \mathcal{F}\mathcal{P}_t\mathcal{F}^T + \mathcal{Q} \end{aligned} \tag{32}$$

where $\bar{\mathcal{U}}_{t+\Delta t}$ and $\bar{\mathcal{P}}_{t+\Delta t}$ are priori estimated displacement and its associated error covariance.

The measurement stage is represented as

$$\begin{aligned} \mathcal{K}_{t+\Delta t} &= \bar{\mathcal{P}}_{t+\Delta t}\mathcal{H}^T(\mathcal{H}\bar{\mathcal{P}}_{t+\Delta t}\mathcal{H}^T + \mathcal{R})^{-1} \\ \mathcal{U}_{t+\Delta t} &= \bar{\mathcal{U}}_{t+\Delta t} + \mathcal{K}_{t+\Delta t}(\mathcal{Y}_t - \mathcal{H}\bar{\mathcal{U}}_{t+\Delta t}) \\ \mathcal{P}_{t+\Delta t} &= \bar{\mathcal{P}}_{t+\Delta t} - \mathcal{K}_{t+\Delta t}\mathcal{H}\bar{\mathcal{P}}_{t+\Delta t} \end{aligned} \tag{33}$$

where $\mathcal{K}_{t+\Delta t}$ represents the kalman gain.

In the CROKF-FEM algorithm, the values of M, ${}^0K_e$, ${}^0K_e^0X_e$, $\bar{\mathcal{P}}_{t+\Delta t}$, $\mathcal{K}_{t+\Delta t}$, and $\mathcal{P}_{t+\Delta t}$ can be pre-computed before the online simulation.

3. RESULTS AND DISCUSSION

Precise simulation of micro- to nano-scale deformation in real-time is crucial when dealing with large-scale structures in the fields of aerospace, aviation, defense, medical and biological systems, and so on. In contrast to MSM approaches, space discretization by linear FEM in accordance with infinitesimal strain theory and linear elastic theory lays the foundation for accurate simulation in our framework. The linear elastic theory takes into account the linear stress-strain behavior of the structure, whereas the infinitesimal strain theory describes solid behavior for deformations considerably smaller than the body dimensions. Obviously, these assumptions fail to include the material and geometric nonlinearities of the considered continuum. This, however, allows the stiffness matrix to be pre-computed (section 2.2), which saves time spent online.

The aforementioned structures typically undergo large rotations during mechanical deformation. The assumption of Hookean elastic forces fails because Cauchy's stress tensor

is not rotationally invariant in this situation. Simulation using traditional linear FEM results in inflated elements (Marinkovic et al., 2018); one way to avoid this is to depart from linear to non-linear analysis, which is undesirable due to the increased computational load. This dilemma can be tackled by employing a co-rotational formulation, which solves the problem of large rotations while retaining all the benefits of traditional linear FEM. Because of this, the proposed framework can incorporate geometric non-linearity (section 2.3).

Our framework avoided the use of an explicit integration scheme, which is only stable in small time steps. The Wilson- θ implicit integration scheme employed (section 2.5) for the time domain discretization guarantees stability in all circumstances. To achieve exceptional numerical stability with rapid convergence, this system employs an updated Lagrangian formulation (Ozkul , 2004). Furthermore, the use of larger time steps improves the computational efficiency of the simulation.

Although the pre-computation of element stiffness matrices assists in improving computing performance, the further implementation of the co-rotational formulation negates this gain because of the warped element stiffness matrix calculation at each time step. Additionally, the high number of state variables in (22) increases the computational time significantly. Therefore, we proposed MOR techniques to reduce the order in the system. The balanced truncation method is used to create a reduced model (section 2.7.1) that preserves the stability, controllability, and observability of the dynamic system under consideration.

On the basis of local measurement data, the recommended Kalman filter technique on the reduced model estimates deformation online (section 2.8). Specifically, this performs state estimation $\mathcal{U}_t$ via feedback control. The CROKF-FEM effectively filters out measurement noises and achieves accuracy comparable to that of the traditional FEM. This enables the framework to achieve real-time performance without compromising physical realism.

This paper only focuses on the construction of CROKF-FEM without any quantitative analysis. The consideration of the linear elastic, isotropic material model would impose limitations on the algorithm. This leads to the deviation of mechanical deformation from the real-world scenario. However, compared to linear FEM with an isotropic material model, the suggested framework is superior; this is due to the inclusion of geometric non-linearity and real-time performance.

4. CONCLUSION

This paper outlines a conceptual framework for the real-time implementation of precise analysis of large-scale structural models. In CROKF-FEM, the continuum is discretized in space by FEM, and a co-rotational formulation is employed. The Wilson- θ implicit integration scheme was then used to develop system-state equations in the time domain. Furthermore, state variable estimation of a discrete dynamic system was applied to the reduced model using a Kalman filter. The introduction of a co-rotational formulation handles the geometric non-linearities due to large rotations. Pre-computation of element matrices, reduction of the order of the system, and online estimation using the Kalman filter make the framework ideal for real-time or near-real-time applications. The proposed CROKF-FEM also enhances simulation effectiveness in dynamic modeling with the use of larger time steps.

Future research work includes the implementation and comparative analysis of the proposed framework with traditional linear FEM using a primitive geometry. The framework will be improved based on the performance analysis. Subsequently, it will be used to simulate soft tissue deformation in real-time during knee arthroscopic surgery.

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