Doctoral theses at NTNU, 2024:108

Shipra Sachdeva

“If we could have got a realistic situation…”

– Learners’ voices indicating their potential to influence their mathematics learning processes.

Doctoral thesis

NTNU
Norwegian University of Science and Technology
Thesis for the Degree of
Philosophiae Doctor
Faculty of Social and Educational Sciences
Department of Teacher Education

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Norwegian University of
Science and Technology

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Thesis for the Degree of Philosophiae Doctor

Trondheim, April 2024

Norwegian University of Science and Technology
Faculty of Social and Educational Sciences
Department of Teacher Education

NTNU
Norwegian University of
Science and Technology

LOCUMS

NTNU
Norwegian University of Science and Technology

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"शिक्षा ग्रहण अथवा प्रदान की जा सकती है, परंतु विद्या हमें स्वयं प्राप्त करनी पड़ती है। इसीलिए हमें शिक्षार्थी नहीं किन्तु विद्यार्थी कहा जाता है।"

– Unknown

In English (my translation) – "Education can be received or imparted, but we have to acquire knowledge ourselves. That is why we are not called desirous of education, but desirous of knowledge."

Shipra Sachdeva

“If we could have got a realistic situation…”

– Learners’ voices indicating their potential to influence their mathematics learning processes.

Thesis for the degree of *Philosophiae Doctor* (PhD)

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Scientific environment

This thesis is written as a part of studies in Ph.D.-program Educational Sciences – Specialization Teaching Profession and School Research (Ph.D.-programmet: Utdanningsvitenskap – Fagretning lærerprofesjons- og skoleforskning) in the Department of Teacher Education at the Faculty of Social and Educational Sciences (SU), Norwegian University of Science and Technology (NTNU).
The author is employed by Østfold University College at the Department of Natural Sciences, Practical-Aesthetic, Social and Religious Studies, Faculty of Teacher Education and Languages.

The research project LOCUMS

This research is associated with the research project Local Culture for Understanding Mathematics and Science (LOCUMS). LOCUMS was financed by The Norwegian Research Council (Forskningsrådet) in 2015. This project was a co-operation between three universities in Norway, i.e., the Norwegian University of Science and Technology (NTNU), the University in Oslo (UiO) and UiT – The Arctic University of Norway. Therefore, along with the LOCUMS research team at NTNU, I also had the opportunity to interact with the research teams of UiO and UiT – The Arctic University of Norway, including the post-doctoral candidate, Nayla Naoufal at UiO, and the doctoral candidate, Siv Ingrid Nordkild at UiT – The Arctic University of Norway. In addition, the international advisory board for LOCUMS included well-known scholars in the field of mathematics and science education, namely, Glen Aikenhead, Anna Chronaki, Per-Odd Eggen, Halvor Hoveid, Dag Atle Lysne, Anne Birgitte Fyhn, Shaun Nykvist, Rob O'Donoghue, and Marianne Ødegård. The possibility of meeting and interacting with these research colleagues and members of advisory board helped me to broaden my perspective and the field of knowledge.

Other scientific environments:

During my PhD research, national and international conferences in the research field of mathematics education have been the meeting places to know the international mathematics education research community. These conferences include, *Novemberkonferansen* (The November Conference), Conference of the European Research in Mathematics Education (CERME), Mathematics Education and Society (MES), and the Nordic Conference on Mathematics Education (NORMA). These conferences provided me with an arena to get to know and meet both new and well-established mathematics education researchers from all over the world. Among these researchers, I could find a place where my research topic and interests belonged. I am very grateful that I got the opportunity to discuss my research questions with those sharing my research interests. Participating in these conferences to present the papers or posters related to my research and getting constructive feedback has been a learning and knowledge enriching experience for me.

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This PhD thesis could not have been written solely by my efforts. I thank the Almighty God for giving me the strength, determination, and grace to complete this dissertation.

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I wish to express heartfelt gratitude to my supervisors. I thank Associate Professor Dr. Per-Odd Eggen for his enlightening thoughts and constructive comments in our discussions. Your patience to answer my countless questions and suggestions have guided me throughout the process of designing, implementing, data analyses and writing up of this thesis. Professor emerita Dr. Anne Birgitte Fyhn, thank you for answering my questions, providing your knowledgeable feedback and comments on my work. Your guidance has been invaluable. I am extremely grateful to Professor Dr. Toril Eskeland Rangnes for becoming an important driving force in the revision process of writing up this thesis. The demanding discussions and challenging questions asked by you have given me the insight to see and improve the shortcomings of my research. I will always appreciate your kindness to give me time, guidance, and motivation to reflect and grave in-depth, several theoretical and methodological issues concerning this research study. I am also thankful to my supervisors Per-Odd Eggen, Anne Birgitte Fyhn and Toril Eskeland Rangnes for their help with the proof reading of this thesis.

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Finally, I owe a deep debt of gratitude to my family. I cannot express enough gratefulness to my parents, my husband, and my parent-in-laws for loving me unconditionally. Your love has given me personal motivation and reassurance throughout the course of conducting my research, coursework and writing up of this thesis.

Shipra Sachdeva Sharma

Halden,
9th November 2023.

Abstract

The research question explored in this study is: *What can individual learners' voices inform about their critical perspectives towards learning mathematics, and their expressed autonomous involvement in decisions concerning their mathematics learning activities?* Educating young learners to become critical and actively participating future citizens building and sustaining a society based on democratic ideals has been a long-standing concern of education. Research discussing 'Critical Mathematics Education (CME)' and socio-political issues in relation with mathematics education also underline the aims of imparting a virtue of critique, critical thinking, and increasing learners' participation in their own mathematics learning. This thesis can be seen as contributing to the research field expressing the concerns of empowering learners and preparing them for critical citizenship through their mathematics learning process.

The research question contains three research interests addressed in three research papers. The first paper focuses on learners' expressed potential to think critically about their mathematics learning process. Learners were asked questions about *what*, *why* and *how* they learn (in) mathematics, and what would they like to change if they had the opportunity. In the second paper, learners' replies to the questions about *why* they believe learning mathematics will be relevant and important for their personal lives are analysed in-depth. The third paper explores learners' expressed experiences of involvement in decisions concerning their mathematics learning activities and having learner autonomy in their mathematics classrooms.

Individual learner's subjective opinions, perspectives, beliefs, and experiences with learning mathematics are explored in the papers. Simultaneously, the learners were prompted to critically reflect over and evaluate their mathematics learning situations and suggest changes in their mathematics teaching-learning processes. They were also asked to give reasons for their answers and to justify their views about learning mathematics. This combination of a subjective and a critical orientation in data gathering process situates this study in between the social constructivist and critical theoretical paradigms.

A qualitative approach and a hermeneutic phenomenological research design are employed to explore the three research interests outlined above. The data was gathered in two secondary schools in Central Norway by using pre-intervention questionnaires, classroom interventions and post-intervention semi-structured individual interviews with 13-14 years old, lower secondary school learners. 74 learners answered the questionnaires and participated in classroom interventions, and 20 were interviewed. Learners' interview responses became the primary data analysed in the papers and reflexive thematic analysis is employed to conduct the analyses.

The frequency of learners' responses such as, "I do not know" and "I have not thought much about it" give the impression that these learners may not be habitual of or trained in thinking critically about their mathematics learning process. Learners' replies such as, "they have already decided" and "I am used to having it like that" indicate that they may have limited experience with involvement and learner autonomy in their mathematics classrooms. Most of the learners named only elementary calculation skills when asked to mention where they used mathematics they learn (in 8th and 9th grade), but all of them expressed a strong belief in the relevance and importance of learning advanced level mathematics for their lives. The learners grounded their beliefs by voicing their trust in their educational system and the statements they have heard about the relevance and importance of learning mathematics

from their teachers, elders, or other sources of information. However, when asked to reflect critically about their mathematics learning process, some learners demonstrated their potential to critically evaluate *what* they learn, *why* they learn, and *how* they learn the subject content in mathematics. Their capability of assuming learner autonomy to suggest changes in the content and/or the style of their mathematics teaching-learning practices was also identified.

Learners' right of learning to think critically, have the co-responsibility of, and to influence their learning processes is established in the Norwegian Education Act, and underlined as an educational aim in the general part of Norwegian school curriculum. Imparting these competencies among learners through their educational process is argued to prepare them to become responsible learners and critical citizens of a democratic society. However, in mathematics specific curriculum, the words critical thinking and democratic participation signify evolving learners' critical thinking and decision-making abilities as mathematically literate citizens capable of analysing conclusions of mathematical models or statistical data. Learners' responses shown above give reasons to infer that cultivating their critical outlook towards and democratic participation in decisions about their learning activities may not be prioritised as a part of their mathematics classroom routine. Thus, a gap is identified between the concerns of empowering learners and preparing them for critical citizenship through their mathematics education, and their mathematics classroom practices, as expressed by these learners.

This study bears implications for the formulations used in Norwegian mathematics curriculum and for learners' mathematics teaching and learning practices. Inviting learners to develop a critical outlook towards their mathematics education can develop their critical faculties, empowering them to actively participate in the decision-making of their own mathematics learning activities. Listening to learners' voices and incorporating their suggestions in their mathematics classroom practices can be some of initial steps towards achieving the aim of learners' empowerment and critical citizenship through their mathematics education.

Sammendrag

Denne studien tar utgangspunkt i forskningsspørsmålet: *Hva kan vi lære av elevenes egne utsagn om deres kritiske perspektiver om det å lære matematikk, deres autonomi og den medvirkningen de har i beslutninger knyttet til deres matematikkopplæringsaktiviteter?* Det har lenge vært et mål at utdanningen skal bidra til at elever blir aktivt deltakende og kritisk tenkende medborgere som kan ivareta demokratiske verdier i samfunnet. I matematikkdidaktikk, forskningsstudier som setter søkelys på kritiskmatematikkdidaktikk (CME), og sosiopolitiske forhold knyttet til matematikkopplæringen gjenspeiler betydning av å utvikle elevenes kritiske refleksjon gjennom og deres medvirkningsrett i sin egen matematikkopplæring. Avhandlingen er et bidrag til dette forskningsfeltet og tar utgangspunkt i spørsmål knyttet til å myndiggjøre elevene og forberede dem til å bli kritiske medborgere gjennom matematikkopplæringen.

De tre artiklene i avhandlingen belyser forskningsspørsmålet fra ulike vinkler. Den første artikkelen omhandler elevenes potensial til å tenke kritisk om egen matematikkopplæringen. Elevene ble spurt om *hva* de lærer i matematikk, *hvorfor* de lærer dette, *hvordan* opplæringen foregår og *hva de ville endre* dersom de ble gitt mulighet til å gjøre endringer i opplæringen. Den andre artikkelen gjør en dybdeanalyse av elevenes svar knyttet til *relevansen* av det de lærer i matematikk og hvor *viktig* de synes at deres matematikkopplæring er for dere egne liv. Den tredje artikkelen analyserer elevenes utsagn om deres autonomi i klasserommet og hvilke erfaringer de har med å medvirke i avgjørelser knyttet til egen matematikkopplæring.

I artiklene blir individuelle elevers subjektive meninger, perspektiver, antakelser og erfaringer med å lære matematikk utforsket. Elevene ble bedt om å kritisk reflektere over og evaluere deres læringssituasjoner og foreslå endringer i undervisningen i matematikk. De ble også bedt om å begrunne sine svar og synspunkter om å lære matematikk. Denne kombinasjonen av subjektiv og kritisk orientering i datainnsamlingsprosessen plasserer denne studien mellom det sosialkonstruktivistiske og det kritisk-teoretiske paradigmet.

Det er brukt en kvalitativ tilnærming og et hermeneutisk fenomenologisk forskningsdesign for å utforske de tre forskningsinteressene som er skissert ovenfor. Dataene ble samlet inn på to ungdomsskoler i Midt-Norge ved å bruke spørreskjemaer før intervensjon, deretter ble det gjennomført klasseromsintervensjoner og til sist semistrukturerte individuelle intervjuer. 74 elever i alderen 13 – 14 år svarte på spørreskjemaene og deltok i klasseromsintervensjoner, og 20 av disse ble intervjuet. Elevenes intervjusvar er de primære dataene, og disse er analysert ved refleksiv tematisk analyse i artiklene.

Den hyppige frekvensen av svar som «jeg vet ikke» og «jeg har ikke tenkt så mye på det» gir inntrykk av at disse elevene ikke er vante med eller trent i å tenke kritisk om læringsprosessen i matematikk. Andre svar som «det er allerede bestemt» og «Jeg er vant til å ha det slik» indikerer at de kan ha begrenset erfaring med involvering og elevautonomi i matematikkfaget. Når elevene ble spurt om eksempler på bruken av matematikken de lærer (i 8. og 9. klasse), var det stort sett grunnleggende regneferdigheter som ble nevnt, men alle uttrykte en sterk tro på at matematikken de lærte ville få stor betydning og være relevant for dem i framtiden. Elevene grunnga dette ved en tillit til utdanningssystemet og uttalelser fra lærere, foreldre og andre om relevansen og viktigheten av å lære matematikk. Når de ble bedt om å reflektere kritisk over matematikklæringsprosessen, viste noen elever et potensial til å kritisk evaluere hva de lærer, hvorfor de lærer det og hvordan de lærer faginnholdet i matematikk. I disse svarene viste elevene en evne til autonomi og til å foreslå endringer i både faginnhold og undervisningsopplegg i matematikk.

Elevenes rett til å lære kritisk tenkning, deres medansvar og medvirkningsrett er fastsatt i opplæringsloven, og gjentatt i den overordnede læreplanen. Oppøvelsen av slike kompetanser skal forberede elevene til å bli ansvarlige og kritiske borgere i et demokratisk samfunn. I matematikk-læreplanene, kan det virke som om betydningen av ordene kritisk tenkning og demokratisk deltakelse er begrenset til matematikkfaglig innhold som analyser av matematiske modeller eller statistiske data. Elevenes svar gir grunn til å anta at det å utvikle et kritiske syn på og demokratisk deltakelse i deres egne læringsaktiviteter, ikke er prioritert som en del av matematikkopplæringen. Elevenes svar viser at målet om å utvikle og styrke deres kritisk medborgerskap gjennom matematikkundervisningen, ikke gjenspeiles i praksisen disse elevene beskriver fra klasserommet.

Denne studien har betydning for innholdet i matematikklæreplanene, for matematikkundervisningen og for elevenes læringsaktiviteter. Et kritisk syn på matematikkundervisningen kan bidra til å utvikle elevenes evne til kritisk tenkning og gi dem mulighet til å delta aktivt i beslutningsprosesser knyttet til deres egen opplæring. Å lytte til elevene og ta i bruk forslagene deres i opplæringen, kan være viktige skritt mot å nå målet om en opplæring som myndiggjør og utvikler elevenes kritiske medborgerskap gjennom matematikkundervisningen.

List of publications

Paper I:

Sachdeva, S., & Eggen, P.-O. (2021). Learners' Critical Thinking About Learning Mathematics. *International Electronic Journal of Mathematics Education, 16*(3), 1-18. doi: https://doi.org/10.29333/iejme/11003

- **Extended version of:**

Sachdeva, S., & Eggen, P.-O. (2019). Students' Critical Perceptions About Mathematics Education. In J. Subramanian (Ed.), *Proceedings of the Tenth International Mathematics Education and Society Conference (MES10, 28th January - 2nd February, 2019)* (Vol. 10, pp. 761–770). Sri Satya Sai Designing Studio Pvt. Ltd. and International Mathematics Education and Society Conference, MES. https://www.mescommunity.info/proceedings/MES10.pdf

Paper II:

Sachdeva, S., & Eggen, P.-O. (in press). "We learn it [mathematics] at school so one thinks that one will use it …": learners' beliefs about relevance and importance of learning mathematics. *Acta Didactica Norden*.

Paper III:

Sachdeva, S. (2019). Students' experiences of learner autonomy in mathematics classes. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11, February 6 - 10, 2019)* (pp. 1978–1985). Utrecht University and European Society for Research in Mathematics Education, ERME. https://hal.archives-ouvertes.fr/CERME11/hal-02421636

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1 Introduction

In this research, I explore Norwegian lower secondary school learners' voices about, and their experiences with learning mathematics. The aim is to identify learners' potential of reflecting critically over and influencing decisions about their mathematics learning activities. My interest in exploring Norwegian learners' perspectives and my presumptions of how mathematics education in Norway is organised, stem from my own experiences with learning and teaching of mathematics from two countries: India and Norway. Being born and having lived the first 24 years of my life in India, most of what I have learnt in mathematics has been in and from the Indian education system. Therefore, I start writing this thesis by describing my own experiences of having learnt mathematics from two countries as my experiences have conscious or unconscious bearing on me as a person, as a researcher and consequently on this research study. Following this summary, I present the evolvement of the research focus, rationale of the project, and identified gap in (mathematics) education research with respect to the research focus of this study and the research question.

1.1 My experiences with learning and teaching mathematics in India

I attended an English-medium private funded public school until I completed my 12th grade (senior secondary school), and from graduation and onwards (bachelor's and master's degree), I studied in government funded colleges and universities. Since the first day of attending school, the language of instruction for me in all the subjects (except Hindi, my mother tongue) has been English. I remember going to school and sitting in the classroom with other 25 to 30 children every day until my 10th grade. When the timetable of a day included a mathematics class, I would sit as an obedient learner and wait for the mathematics teacher to start the class. I was not particularly interested in, or disliked learning mathematics.

In a usual mathematics class of 30 to 45 minutes, our mathematics teacher would introduce a new or continue the topic we were learning already, as per the curriculum requirements. As learners, we would open our prescribed mathematics textbooks and follow the pages of the textbook referred to by the teacher. The introduction given by the teacher included telling the name of the topic and some example problems from the textbook. The teacher would tell us some rules and solution procedures to solve those example problems followed by a demonstration of how to apply those rules and procedure on selected examples and other similar questions related to that topic in the textbook. Occasionally, these example problems were based on humoristic situations, jokes, or routine life contexts such as shopping, rate of interest at the banks, the use of unknown x's and y's to find discounts and so on.

Following this demonstration by the teacher, we were given similar questions to solve and practice the solution procedure on our own. Practice was considered as the rule of thumb to become better at both doing and learning mathematics. We were often told by our mathematics teachers that, the more a learner practiced solving mathematical problems of different types, the better will s/he learn and perform in mathematics tests and exams. Reflecting on what I was doing while learning mathematics was remembering the rules and procedures demonstrated by my teacher and applying those solution procedures to similar questions. Doing mathematics seemed to me like playing a game in which I had to follow the

prefixed rules and procedures in the prescribed manner and then I could solve the indicated problems. After solving problems at one level of the game (for instance, simplifying algebraic expressions), I would learn the rules and procedures for solving the problems and questions at a higher and a bit difficult level (for instance, solving algebraic equations).

The thrill of getting the right answers to one level of questions and the excitement of reaching the next and more difficult level of questions served as the motivation for me to continue putting efforts into learning mathematics. I continued learning the rules of the game and applying the procedures as demonstrated by the teacher from one grade to another and kept climbing the ladder of learning mathematics by clearing the higher levels of difficulty. When it came to performance, I was an average performing learner in mathematics, scoring anywhere from 50 to 70 out of 100 marks in regular classroom tests and annual exams. Due to my average performance and viewing mathematics learning as a game, I chose to keep learning mathematics in senior secondary school (up to 12th grade) and in the bachelor's degree but I pursued a master's degree in mathematics following my family's desire[1] more than my personal choice. The feeling of and experience with learning mathematics as a *game* sustained through studying bachelor's and master's degree in mathematics. The only change was that the levels of difficulty kept rising and I had to adapt my learning and application strategies to cope up with higher difficulty levels of this game.

After completing the master's degree (M.Sc.) in mathematical science, I did a bachelor's in education (B.Ed.) degree and got qualified for teaching mathematics in schools for learners up to 12th grade (higher secondary level). Our coursework in B.Ed. degree included the cognitive, behaviouristic, and socio-cultural theories of learning in educational psychology and pedagogical knowledge about teaching mathematics, but not much about the interplay between these learning theories and the teaching and learning of mathematics. Following the B.Ed. degree, I got an opportunity to work as a secondary school teacher in a private school before I got selected as an elementary teacher in a government school in India. I have also worked as a junior lecturer in a government college and taught mathematics to undergraduate learners for some time. The experience of teaching mathematics in India was not so different from learning the subject since I followed the same pattern of teaching as I had observed in my own teachers during all the years of my education. I demonstrated the rules and procedures to the learners before giving them similar problems to solve as shown and practice enough that solution method to ensure that they learn and will remember the solution techniques for their exams.

Summing up my experience with learning and teaching of mathematics in India, I realize that I got most exposed to the discipline of mathematics as a commodity rather than to the process of learning mathematics. I experienced the discipline of mathematics as a fixed set of rules, axioms, meaningless symbols, logically deduced theorems, and a bundle of universally valid knowledge which is to be learnt as demonstrated by the teacher and in the textbooks. Little did I know about *formalism* as the philosophy of mathematics education, in which learning mathematics is seen as "a meaningless game played with marks on paper, following rules" (Ernest, 1985, p. 606); or about the instrumental understanding of mathematical knowledge (Mellin-Olsen, 1981; Skemp, 1978).

As a learner of mathematics, I was never asked to take a critical perspective on my own learning activities, nor to decide how I wanted to learn mathematics or was prompted to critically evaluate and give suggestions about any aspect of my own learning processes.

[1] This desire was based on the exchange-value of mathematics which is well established in the Indian society.

However, I had always heard from my teachers and elders that learning and getting good grades in mathematics was very important to secure a financially stable career in the future. Further, the international educational policy documents such as UNESCO (2015), OECD (2019) and research literature for instance Allexsaht-Snider and Hart (2001) and Heymann (2003) also underlines the importance and significance of learning mathematics. As pointed out by Ernest (2004, 2005, 2015) on several occasions, this socio-political status of importance bestowed upon learning mathematics is supported by arguing for the requirement of mathematics to ensure the economic growth of the society, and for the personal, social, and economic growth of mathematics learners. Reflecting on my experiences with learning and teaching mathematics now makes me realize that I shared the view of mathematics as a universally played meaningless but fun game (Sam, 1999), rather than being a meaningful, socially constructed, and negotiated knowledge having a relation to and application in the real world. Perhaps this perception of mathematics was the reason that when asked, I could not explain "how I concluded that $18 + 31$ equals 49" in a parallel session of CERME, 2017, but only said that "I just knew the answer". I did not know how to communicate in or about mathematics.

1.2 My experiences with learning and teaching mathematics in Norway

The experience of learning and teaching mathematics in Norway started with my enrolment to an international master's (M.Sc.) degree in mathematical sciences (with specialization in statistics) at the Norwegian University of Science and Technology (NTNU). The socio-cultural changes and challenges related to moving to a new country were as expected, but a dissimilar approach and style of learning and teaching the universal discipline of mathematics in Norway came as a surprise to me. Though language of instruction was English, and we did have prescribed textbooks, but none of the teachers (except two or three) demonstrated a fixed set of rules and procedures to apply for solving the problems from textbooks. The assignments were not planned in a way that we could just apply the rules and practice solution procedures to arrive at one fixed solution, neither were we supposed to learn theorems by heart and write them in the exams. The mathematical assignments were often planned as a group project or as questions around some social context – a growth or reduction of a population, stochastic processes, predicting behaviour of mathematical models, statistical inferences, etc. To complete the given assignments and projects, I had to discuss mathematical models with my fellow learners, understand the meaning, applicability and outcomes of the models and statistical inferences in relation to the real-life contexts on which these models were built.

It took me a while to get used to this new style of learning and doing mathematics. It was extremely challenging and frustrating to understand and adapt this way of learning mathematics since it was not straightforward. There were universally valid rules, logic, techniques, and solution procedures but it was frustrating that I could not remember the given rules and apply them right away to solve the problems and projects. No wonder I failed badly at first while trying to play the game of learning mathematics in Norway by using the strategies I had learnt in all those years in India. Gradually, by struggling to make sense of mathematics I was learning and its relation, applicability, meaning and bearing upon the prediction and development of real-life contexts provided in the projects and problems given, I managed to adapt to the new style of learning mathematics. This journey of adaptation not only equipped me with a different style of learning mathematics, but also changed my fundamental

understanding of what mathematics is, and what is the meaning of learning mathematical knowledge and skills.

In my understanding, mathematics became a bundle of universal knowledge and skills which were necessary for the world around me to function properly instead of being a meaningless game played by using abstract symbols. The origins and requirement of mathematics as a discipline started to make sense to me. I realized that mathematics is not only necessary and useful to understand the existence and functioning of natural phenomena, but also applicable to comprehend and estimate the development of several real-life situations and social phenomena. Mathematics became a *sensible* and *meaningful* tool to interpret and deal with the world and real-life contexts. Moreover, I discovered that mathematical problems are not always straightforward questions to be solved by using decided procedures, or proving theorems through logical deductions, but they can have a range of correct answers, many possible interpretations, many possible representations of a given situation, etc. Additionally, learning mathematics was not necessarily an all alone individual cognitive project as experienced it in India, but one could have a dialogue and discussion about mathematical problems and understanding. Participation and discussion in group project work was mandatory to get the assignments approved. It was not only prescribed but also required that we discussed different understandings of mathematical problems and different approaches to solve them.

My experience of teaching mathematics in Norway is limited to teaching the courses Mathematical Methods A (MA0001) and Mathematical Methods B (MA0002) at NTNU for students at first degree study/undergraduate level. The language of instruction for these courses was Norwegian so to teach mathematics, I first had to learn mathematical terminology in Norwegian. Since the learners in this course had English textbooks, and I was also a novice in teaching mathematics in Norwegian, so the language of my instruction became a combination of Norwegian and English. These courses focused on introducing functions, differentiation and integration with applications, linear approximation, basic calculus I and II. I got the opportunity to teach in these courses as a part of my work duty included in the work contract for my PhD position in the research project LOCUMS (see 1.3). Being new and unexperienced, I shared the responsibility of this course with experienced mathematics educators. To learn how I can and should teach mathematics in Norway, I attended some lectures of these courses given by experienced lecturers. I observed that teaching mathematics did not mean demonstration of some solution procedures and logical deduction of theorems, but explaining what mathematical concepts mean, how they work and why they can be applied to comprehend and predict real-life contexts, problems, and situations. The practical implications, consequence and meaning of applying a particular mathematical model to a real-life context was also discussed. Following, these observations, I also adopted similar teaching style in independent lectures.

This experience of learning and teaching mathematics in Norway changed my vantage-point of looking at mathematics. The image of mathematics turned from being abstract and meaningless to being a tool of utilization to comprehend the real world and applicable in real-life situations. Though mathematics was still a universally accepted, valid, true, value-neutral, and unquestionable bundle of knowledge for me, but mathematics became a bit *social* in addition to being individual. I learnt that it is possible to have other conversations *about* mathematics with your peers except discussing the solution procedures and mistakes. Any real-life context could be understood, interpreted, and represented in different possible ways using dissimilar mathematical models. Reflecting on my insights now after learning a bit about

the field of mathematics education, I realize that my changed perspective about mathematics resonates with Realistic Mathematics Education (Van den Heuvel-Panhuizen & Drijvers, 2014). Though the image and my view of mathematics had changed but I was still dealing with mathematics as a commercial product having an exchange-value and high status in the society rather than considering the socio-cultural-political process of mathematics education along with the complexities involved in it – an awareness that came by my way through LOCUMS.

1.3 Getting involved in LOCUMS

Due to my interest in teaching, I looked for the opportunities to become a schoolteacher in Norway after completing the master's degree. My pedagogical education (B.Ed.) from India got recognized as one year's pedagogical knowledge in Norway which made me eligible for teaching from grade 1 to 13 in the Norwegian schools. I applied for several teaching positions but could not get through. Therefore, I started looking for research positions in the field of mathematics education as another possible gateway to enter in an academic career and get to know the Norwegian education system.

While looking, I came across the vacancy for a PhD research fellow under the research project Local Culture for Understanding Mathematics and Science (LOCUMS, 2016). The project LOCUMS was a research co-operation between University of Oslo (UiO), Norwegian University of Science and Technology (NTNU) and UiT – The Arctic University of Norway, financed by the Norwegian Research Council (NFR) in 2015 for the period of four years. The aim of this project was to investigate the use of practical activities based on *secondary school* learners' life experiences and cultural backgrounds as a starting point for the learning of concepts and basic skills in mathematics and science. LOCUMS' intention was also to support the move towards a more student-centred education and culturally responsive science and mathematics education for secondary school learners in Oslo, Trondheim and Tromsø.

Two features of this job announcement captured my interest – the connection mentioned between mathematics and learners' life experiences and their culture, and the aim to support learner-centred[2] education. Having perceived mathematics as a value-free, culturally neutral, and universally valid package of knowledge, I was surprised to read about the connection between mathematics and culture. In addition, following my recent experience with learning and teaching mathematics in Norway, I was also excited to be a part of the movement supporting learner-centred education where the concern was to take account of learners' personal interests, real-life experiences, and cultural backgrounds to plan their learning and teaching activities in mathematics. This surprise and excitement motivated me to know more about the connection between mathematics and culture, along with learner-centred approach to mathematics education[3].

Consequently, I decided to apply for this position and started reading mathematics education research concerning these two features. It was this initial reading which turned my head to the world of mathematics education and its research. I see this reading to be my first exposure to the field of mathematics education. Through this reading during the first year of my PhD research period, I got to know what mathematics education as a research field actually

[2] Sometimes the terms student-centred and learner-centred are used interchangeably, but in this thesis a conscious choice is made to use the word learner instead of student to address the subjects of education. This choice is explained in the section 3.1.

[3] I understand the term mathematics education to be the teaching and learning of mathematics.

meant – that both mathematics and mathematics education are influenced by social, cultural and political world of our living; that different educational philosophies and learning theories bear implications for the teaching and learning of mathematics; that secondary school learners' mathematics learning experiences are affected by a number of factors ranging from their own cognition to socio-cultural and political backdrops; and much more. This exposure and awareness served as an eye-opening experience for me. I took my share of time to change my whole outlook towards mathematics education, and to understand and accept mathematics as a value-laden, socio-cultural, context-based and political discipline (Bishop, 1988; Gerdes, 1998; Mellin-Olsen, 1987).

1.3.1 Initiating research under LOCUMS

Following the aims of LOCUMS (for mathematics part), I focused on reading the research concerning the relation between mathematics education, culture, and learners' cultural backgrounds along with the concept of learner-centred education. Reading about the relation between mathematics and culture, I came to know about research concerning ethnomathematics (D'Ambrosio, 1997) and culturally responsive mathematics education (CRME) (Greer et al., 2009; Harding-DeKam, 2014). Ethnomathematical research brings about and acknowledges mathematics embraced in the artefacts of different ethnic and professional cultures, such as carpet and basket weaving in specific ethnic groups (Gerdes, 1988; Masingila, 1994), or in the mathematical terminology used by a specific professional group, such as computer technicians (D'Ambrosio, 1985; D'Ambrósio, 2006). In culturally responsive (mathematics) education, the central idea of is to teach ethnically diverse learners "through their own cultural and experiential filters" (Gay, 2002, p. 106). Culturally responsive teaching aims to address the learning needs of traditionally marginalized learners (Ladson-Billings, 1995; Ladson-Billings, 1994), and use "the cultural knowledge, prior experiences, frames of reference, and performance styles of ethnically diverse students to make learning encounters more relevant and effective for them" (Gay, 2010, p. 31). Implementing CRME would therefore require understanding, recognizing, and acknowledging learners' cultural and experiential frames of reference along with their personal interests in which they use mathematics. CRME recommends organizing the teaching of mathematics in correlation with learners' cultural frames of references, personal experiences, and interests (Parker et al., 2015) instead of presenting mathematics as an abstract, standardized, culture-free and universally true bundle of knowledge.

Since the focus of LOCUMS was not limited to addressing learners' ethnic background or mathematical skills practiced by a specific group, the attention was diverted to culturally responsive mathematics and science education right from the start. Accordingly, focusing on this approach became the starting point for my PhD research as well, but understanding the meaning of "culturally responsiveness" and implementing this approach of mathematics education in the Norwegian context was not a straightforward process. The instances of CRME found in research could not be imitated as it is in Norway. Though cultural diversity is increasing in Norway, yet Norwegian schools are not seen as being challenged to the same extent as for instance France or the UK in terms of cultural differences (LOCUMS, 2016). Further, the cultural diversity in Norway is different due to the presence of both ethnic Norwegian, immigrant, and Sámi (indigenous) people. Therefore, to design and implement any culturally responsive interventions in Norwegian mathematics classrooms, I had to

understand in-depth the nature, characteristics, and requirements of cultural diversity existing in the selected research sites (lower secondary schools in Central Norway).

Reading about the concept of learner-centred education revealed that the notion of *learner-centeredness* is understood in different ways in the research literature, and thus it is difficult to find one all-inclusive definition to this notion (Ding & Li, 2014; McCombs, 2001; Meece, 2003). Researchers such as McCombs and Whisler (1997, p. 11) identify two important features of learner-centred instruction to be:

> "… a focus on individual learners (their heredity, experiences, perspectives, backgrounds, talents, interests, capacities and needs) [and] a focus on learning (the best available knowledge about learning, how it occurs, and what teaching practices are most effective in promoting the highest levels of motivation, learning, and achievement for all learners)".

McCombs and Whisler (1997) further mention the following points as some of the key characteristics of learner-centred classrooms:

- Providing opportunities for students to *choose their own* projects and work at their own pace.
- Including learning activities that are personally and culturally relevant to the students.
- Listening to and respecting *students'* points of view.
- Encouraging *shared decision making* and *student autonomy* and giving students increasing responsibility of their learning.

Therefore, learner-centred education is considered to be a *democratic* educational approach in which decisions regarding learning activities should be made in cooperation with the learners by taking into account their individual interests, backgrounds, experiences, capacities etc. (McCombs, 2001; McCombs & Whisler, 1997; Meece, 2003).

In mathematics education research, discussions about the learner-centred approach of teaching mathematics are often underpinned by the constructivist learning theories where the learners are viewed as the creators of their own mathematical knowledge (Ding & Li, 2014; Zonnefeld, 2015). Learner-centred mathematics classrooms focus on learners' contribution and active participation in their learning activities, where learners can "express their thoughts freely, develop their reflection strategy, and well-connected with daily life" (Ali, 2018, p. 724) so that they become individuals capable of thinking mathematically and solving problems. The feature "listening to and respecting students' points of view" is emphasized in learner-centred mathematical classrooms, and instructional methods such as problem-based, project-based, cooperative, and inquiry-based teaching are employed (Ding & Li, 2014). Most of the information comes from the learners, and they are asked to invent, present, and justify their own solutions to mathematical problems. The teacher acts as a guide who carefully listens to learners' contributions and facilitates their learning in the right direction.

The elements of education as a democratic process, such as, learners' *freedom of choice*, *shared decision making*, *student autonomy* and *listening to learners' points of view* influenced and inspired me the most after reading about the notion of learner-centred education. I also realized that CRME is indeed a learner-centred approach towards teaching and learning of mathematics. I found instances of classroom interventions accounting for learners' observable backgrounds, interests, and experiences after reading mathematics education research literature on CRME (Greer et al., 2009; Rajagopal, 2011). These research studies provide rich information on how CRME interventions can be planed and implemented. However, I missed the instances of exercising these interventions in a more democratic way,

such as, by asking the learners themselves about their interests, by listening to their points of view, or by having a shared decision-making process, etc.

I also noticed an underlying assumption of the research studies about already "knowing" learners' cultural frames of reference while implementing CRME interventions in the classrooms (Parker et al., 2015). What I mean by "knowing" learners' cultural frames of reference is that often it was learners' *observable* ethnicity (African-American, American-Indian, Latino/a learners, etc.) (in several chapters of Greer et al. (2009); (Guha, 2006)), the popular culture (music, technology, sports, food, etc.) they are interested in (for instance in (Leonard et al., 2009); Rajagopal (2011)), or both which were used as the starting points for planning classroom interventions without at first asking the learners themselves about their own interests, experiences, aspirations, etc. Hubert (2014) and Byrd (2016) highlight this shortcoming of the literature, but both these studies investigate learners' experiences with and the effect of having participated in culturally responsive (mathematics) instruction, not the instruction itself being planned in cooperation with and based on learners' input.

Reading the research literature concerning learner-centred education and CRME stimulated my interest even more in LOCUMS. I got particularly drawn towards the democratic approach to education as a two-way process – education being democratic in itself[4], and education for democratic citizenship[5]. The observed gap in the literature concerning lack of attention paid to learners' own viewpoints and inputs stimulated my curiosity to explore learner-centred culturally responsive mathematics education from the perspective of learners themselves (by listening to their own voices[6]). However, I needed to better understand the Norwegian context before planning how this study should be carried out in my research site, which were two multicultural lower secondary schools in Central Norway. It was important to comprehend the meaning of the notion of "culture" and how I could get inputs from learners to plan classroom interventions in a culturally responsive manner.

1.3.2 The LOCUMS meeting at Røros

I got the chance to attend the first international advisory board meeting arranged by LOCUMS in Røros in December 2015. In this meeting, I interacted with several Norwegian members of the international research board for LOCUMS (Marianne Ødegård, Halvor Hoveid, Dag Atle Lysne, Per-Odd Eggen and Anne Birgitte Fyhn) along with its international research members (Glen Aikenhead, Anna Chronaki, Shaun Nykvist and Rob O'Donoghue). Being a cooperation between three Norwegian universities, different sub-projects of LOCUMS were planned for three different locations – north, central, and south of Norway. Located at the Norwegian University of Science and Technology (NTNU), my research was to be conducted in Central Norway. The cultural diversity in these three locations is different in the sense that there the number of learners having multicultural background decreases as one moves from south to north, and many learners in north Norway also belong to the indigenous community,

[4] In the sense that learners are empowered, critical thinkers, become involved, participate in, reflect over, and evaluate the decisions concerning their own learning activities.

[5] In the sense to promote democratic values among learners and to prepare them as critical, empowered, and autonomous citizens who participate in and strive to uphold the democratic values in the society in future.

[6] The term *learners' voices* is used in this thesis against a backdrop of critical pedagogy, wherein the concerns related to learners' empowerment, developing a critical orientation among them, and educating them to become critical, actively participating and transforming citizens in a democratic society stand central.

the Sámi. Consequently, it was a big task to understand the meaning of "culture" and how the culturally responsive interventions should be planned for these different locations.

The discussions in this meeting started by grappling with the concept of *culture* and its meaning in these three locations. A central feature of the conversation was understanding culture in relation to secondary school learners aged between 10 to 15 years. Several themes such as ethnicity, learners' interests, cultural artefacts, daily-life activities, youth sub-culture emerged under the discussions. Suggestion was made to employ the notion of *youth culture* (activities of young learners' interest) for the sub-projects in south and Central Norway since there were many learners in south and Central Norway having different cultural backgrounds (Asian, Middle East, African, European Union countries, etc.). For the research site in north Norway, the suggestion was made on employing *ethnic Sámi culture* (cultural artefacts such as Lávvu, braiding, etc.) as the backdrop since many learners on this site had Sámi background[7]. These suggestions were then left to be explored further and materialized by the PhD and post-doctorate research fellows of respective sub-projects.

Many central ideas of LOCUMS, such as, learner-centred education, different meanings and interpretations of culturally responsive mathematics and science education, teachers' perspectives, research recommendations, planning and implementation of practical classroom interventions inspired by learners' real-life experiences and cultural backgrounds, etc., were discussed in this meeting. However, I felt that a serious consideration of learners' perspectives, their points of view and inputs were missing in these conversations. Analogous to my observations after reading the research literature (Byrd, 2016; Hubert, 2014), I experienced that there was little discussion about asking the learners themselves what they wanted to learn and how. The consideration of questions such as, how these learners could be asked and how they can provide inputs to plan culturally responsive classroom interventions of their own interest were also absent.

I took up these questions and got to know from several group members that one of the reasons of this lack was the range of learners' age which was between 10 to 15 years. The adolescence years of age carry many influences and therefore learners of this age range can be confused, unsure, having doubts about their choice and may not have fully developed thought and self-reflection process. Therefore, a doubt factor was involved in asking learners themselves about their interests, their preferences regarding what they wanted to learn about mathematics and science in school or provide inputs about their *youth culture* for planning classroom projects. Despite this insecurity and being inspired by the democratic features of learner-centred education approach, I proposed that the alternative of asking learners themselves and getting their inputs as our inspiration and starting point for planning classroom interventions should be tried. This suggestion also seemed ethically correct to me since deciding what learners' youth culture is by myself without hearing their inputs seemed like imposing my interpretations onto them, contrary to adopting a democratic approach.

This proposal was seen as uncertain but also interesting by the board members and my supervisor also supported this idea of asking learners themselves, so we decided to adopt this proposal for the sub-project in Central Norway. However, executing this idea required finding answers to some fundamental questions such as, how could we talk to learners about the notion of culture in a way which made sense to them, which age group of learners should we focus on, would the learners be comfortable answering questions related to their interests,

[7] These suggestions got later realized as in indicated in the respective sub-projects.

leisure time activities, personal experiences with learning mathematics[8] at school, express what they want to learn in mathematics, and so on. These questions lead to a meeting with our colleague at NTNU, associate professor Dr. Carla C. Ramirez at the Department of Education and Lifelong Learning, Faculty of Social and Educational Sciences.

In her doctoral thesis, Dr. Ramirez had explored the notion of social inequality among 21 young learners belonging to cultural minorities in three upper secondary schools of Central Norway. Chinga-Ramirez (2015) elaborated on the adjustment strategies adopted by these adolescents to adapt themselves in the Norwegian schools and one of the subject positions brought forward by these adjustment strategies was *the hybrid subject position* (p. 207). In this hybrid subject position, Chinga-Ramirez (2015) illustrated how some of her informants (young learners) having a minority background represented and identified themselves with the values, traditions, norms, etc. from a mixture of many cultures along with their native and Norwegian culture. Hybridity provided the learners with the possibility to blend their subjective positions from different cultures in the homogeneous Norwegian school context and represent themselves as a mixture of several cultures without having to lock their identities within a particular culture and tradition. In doing so, these youngsters create their own hybrid identities through which they navigate in the Norwegian school context.

1.3.3 The youth culture

The notion of hybridity and this perspective of looking at cultural differences among learners made us aware that entering schools with any presumptions about learners' culture may not provide the best ground for an authentic data collection. Therefore, instead of interpreting individual learner's culture based on their ethnicity or nationality, we (the research team in Central Norway) interpreted the concept of culture as *youth culture* which may be hybrid of several cultures (Amit-Talai & Wulff, 1995; Schwartz & Merten, 1967). Youth culture is understood as the "adolescent norms, standards and values which are discussed in a language particularly intelligible to the members of this age-grade" (Schwartz & Merten, 1967, p. 457). A wide range of ideas, beliefs, goals, behaviours, expressions, such as popular music, leisure time activities such as sports, social media, being with friends, and social standards such as clothing, language and vocabulary shared by the youth form the youth culture of the adolescents living in a particular demographic region (Fasick, 1984). Moreover, culture is a dynamic and ever-changing entity (Nieto, 2008), so youth culture would vary with respect to factors like time, place, generational values, technical advances and so on.

Therefore, to listen to the learners' points of view and get to know about their youth culture, we decided to design a questionnaire[9] to ask the learners about practical activities of their interest, their hobbies, their cultural identity, along with questions regarding their interest in learning mathematics and science. The questionnaire contained a combination of close-ended questions, Likert-scale statements, and open-ended questions (see questionnaire attached in appendix number 9.3). Among other themes, these open-ended questions asked the learners to mention what they want to learn more about at school in general, and in the subjects of mathematics and science. The learners studying in 8th or 9th grade (13-14 years old) were chosen as informants under the assumption that teenage learners would be able to

[8] The focus of this thesis is on mathematics only, but the questions had to be planned to cater for both mathematics and science following the research aims of LOCUMS.

[9] For details on designing the questionnaire, see section 4.3.1.

understand the questionnaire statements, and express their interests, experiences with, and expectations from learning mathematics and science in school. The doubt regarding younger learners' understanding of questionnaire statements made us deviate from choosing learners younger than 13 years as informants whereas the schools did not want to use the time of learners studying in 10th grade because they had to prepare for the exams to be conducted on a national level at the end of their academic year. Besides the assumptions of the research team, I also had some prejudices and expectations regarding learners' answers, specifically to the open-ended questions in the questionnaires.

1.3.4 Personal prejudices and expectations

I came to Norway as an Indian immigrant and still identify myself as a first-generation Indian immigrant living abroad. The Indian culture, traditions, values, social ideas, thoughts, popular beliefs, norms, and knowledge bear a strong influence on me. Therefore, it is essential for me to recognize and reflect on my subjective identity as *an Indian (immigrant) non-Norwegian speaking novice researcher*, both in Norway and in the field of mathematics education. When I embarked on my research journey through this PhD position in 2016, my interaction with the Norwegian culture was mainly limited to going to my university and attending mathematics classes. I remember that it was difficult for me to make Norwegian friends due to language and cultural differences. My understanding of how teaching and learning activities in Norwegian educational institutions looked like was also limited to my experiences of attending university courses. I had no experience of and limited knowledge about how teaching and learning activities in Norwegian secondary schools were organized and functioned.

Likewise, I did not have any experience in doing research, planning research activities or of writing and publishing research. This PhD research and all the experiences and knowledge I have got during its course have formed my research journey and developed my understanding of the Norwegian educational and schooling system along with the research field of mathematics education. However, as an Indian immigrant, I was constantly interpreting the experiences I had, and formulating my own understandings and beliefs about Norwegian culture and education. My interpretations and my background from India have formed my worldview and the lenses through which I see and interpret what happens around me and in the society. Each new experience adds more angles to these lenses, continuously enriching and evolving my comprehension of myself as an individual, my worldview, the personal and social situations I experience, and my understanding of the society. Accordingly, despite my little experience with the organization and functioning of Norwegian secondary schools, I had some prejudices and expectations regarding learners' answers to the open-ended questions of the questionnaire mentioned above.

Due to living in a post-colonial society (India being a British colony until 1947) I had heard many popular beliefs about the lifestyle, culture, and structure of the *Western countries*. I had heard that western countries have a better lifestyle, an advanced society where information and technological gadgets have a prominent place pertaining to their status of being developed countries and their economic growth. Western societies enjoy a culture of freedom and a lifestyle where people are free to make their own choices. The rules and laws are very strict, and children are no longer required to live with their parents or family after becoming 18 years old (which was very strange for me, being an Indian). Moreover, even

adolescents could decide to follow their own wishes without being obliged to listen to and follow their family's advice. I am not sure about the exact source of this information (popular beliefs, mass media or social media, traces of colonized thought process or impressions coming from Indians settled abroad) but hearing these popular beliefs created a *well-structured, open, carefree, less sensitive*, and overall, a glossy image of western countries in my mind.

I had a preconception that the features of free will, freedom of choice and following one's own wish will also be reflected in the western (Norwegian) lower secondary schools. I believed that the Norwegian education system would be practicing an inviting and active style of teaching and learning where learners' points of view would get attention, and where young learners would take an active part in their own educational process by voicing their opinions. These preconceptions got strengthened through my personal experience with learning and teaching mathematics in Norway, and I naïvely projected my presumptions on to the learners studying in Norwegian schools. This projection led to my expectations from learners' questionnaire responses concerning their experiences with, and preferences regarding learning mathematics and science at school. I also expected that the learners would be habitual of thinking about their own learning experiences, reflecting over them, and participating in their learning processes by expressing their opinions and sharing them with their teachers.

However, my prejudices and the naivety of my expectations became apparent to me when I got the responses of the 22 learners to whom the questionnaire was first administered. To know about learners' preferences of learning content in mathematics (and science), open-ended questions such as, "Mention what you want to learn more about at the school."; "Is it something in mathematics that you think is quite interesting?", etc. were included in the questionnaires. I was surprised to notice that only a few of the learners had expressed what they found interesting to learn at school (both in general, and in the subjects of mathematics and science). A minority of the learners had mentioned what they want to learn in mathematics, but most of the learners chose to leave the questions unanswered or responded with answers like "I do not know", "Nothing" or "Everything". Since I anticipated that learners would express their learning interests in the questionnaires, their replies made me curious to explore why only a few learners did so. Due to the constraints of time and resources, we decided to design and implement classroom interventions based on interests and activities mentioned by the learners in the questionnaires as our second step of data collection[10].

I understood that direct projection of my prejudices on learners was incorrect, but wondered if my expectations that these learners will be able to *think critically about* their own educational experiences and *participate in the decision-making process* regarding their own learning activities were not coherent with the educational aims drafted for Norwegian secondary school learners. Besides the education policy documents I also wanted to know the perspectives discussed in the research literature regarding learners' position and role in their own learning experiences. Thus, I followed my curiosity to look for the explanations for learners' responses alongside planning further steps of data collection for this research project.

I had the following queries while reading the policy documents as well as the research literature:

[10] Three classroom interventions (each having three steps of data collection) were planned during the whole project. For details about the development and design of the interventions, see section 4.3.

a.) Is it desirable and important that learners express what they want to learn, reflect critically over, evaluate, suggest changes, and take active part in decisions concerning their learning activities, and why; and
b.) In case the competencies mentioned above are desirable, which abilities would individual learners require to attain these competencies?

The Norwegian Education Act (1998) and the Norwegian school curriculum (both general and mathematics specific parts) (Kunnskapsdepartementet, 2006c, 2017; Norwegian Directorate of Education and Training, 2013, 2020) were natural starting points to look for explanations from an education policy's perspective. From the research perspective, I explored what competencies and expectations from learners are listed in (mathematics) education research, when it comes to learners reflecting critically, being involved in, and taking active part in decisions concerning their mathematics learning activities.

In the following text (section 1.4), I present an account of the discussions concerning these two questions above in (mathematics) education research literature. The discussion from an educational policy perspective, is presented as the background of the project in chapter 2. The sections 1.4.2 and 1.5.1 put forward the rationale (research gap) and the research question for my research project respectively.

1.4 Research overview: the critical and participating learner in mathematics

In mathematics education research, I started with exploring how learners' perspectives are presented in mathematics education research, and if they are required to critically think about, and take part in decisions concerning their learning activities in mathematics. I observed that the concerns about bringing forward learners' critical perspectives and their voices are often (but not only) listed under the research literature discussing links between mathematics education and democracy. Specifically, the aims of developing a critical stance among mathematics learners and encouraging their active democratic participation in decisions regarding their mathematics learning processes are often presented under: Critical Mathematics Education (CME) (Skovsmose, 1994a, 2014a), and the socio-political issues related to mathematics education (Gutiérrez, 2013; Valero, 2004b). Here, I present a brief overview of the research fields which have significantly considered imparting a critical stance among mathematics learners, promoting the sense of autonomous involvement (co-responsibility of their own learning), and agency in them.

In adding *critical* before mathematics education, Skovsmose (1994a) opened the possibilities to view mathematics and mathematics education by adopting a critical perspective. By employing CME as a framework, the significance of questioning mathematics and mathematics education got highlighted, rather than accepting mathematics and its education as a universal good endowed upon the humanity and for the society. Critical as a prefix illustrates the possibility of exercising a critique in relation to mathematics and mathematics education (Ernest, 2016). The notion of *mathemacy*, which "may provide educational sense to a notion as 'critical citizenship'" (Skovsmose, 1994b, p. 192, quotes in original), and *reflective knowing* as one of its components, *encourage learners to reflect critically* over mathematical results and consequences these results may have in the society while learning mathematics. Reflective knowing focuses on reflecting critically over the solutions and techniques used to solve mathematical problems, and scrutinizing if using mathematics on a particular problem, model or context is necessary. Further, how applying

mathematics to a particular situation can influence the result and one's understanding of that problem or social context and one's worldview is also emphasized (Skovsmose, 1994b). In addition, the formatting power of mathematics, the role played by mathematics and mathematics education in the society are also highlighted (Skovsmose, 1994b). Mathematics learners are recognized as individual subjects whose *backgrounds*, *foregrounds*, and *intentionality* to learn mathematics influence their participation and efforts they put in learning mathematics (Skovsmose, 2011, 2014b).

Analogous to the CME, research concerning socio-political issues in mathematics education has also focused on development of critical citizens through mathematics education who can live in and thrive to achieve and exercise democratic ideals in the society (Gutiérrez, 2013). However, this research field has devoted special attention to highlight the issues of social justice, power distribution, hegemony of mathematics education, discrimination, inequality, and more in the society for mathematics learners and educators in and through mathematics education (D'Ambrosio, 1990, 2007; Greer & Mukhopadhyay, 2016; Gutstein, 2003, 2006; Valero, 2004b). The concern of educating young mathematics learners to become future critical citizens has been registered in the literature so that they use their mathematical knowledge to understand, be critical about and take actions to mitigate social injustice, discrimination, hegemonical power relationships and so on. Inculcating a critical outlook and awareness among learners is seen as an aim of mathematics education so that they can comprehend the roles mathematics and mathematics education play in these socio-political problems, and how they can be mitigated by using mathematics (see e.g., Gutstein (2006) and Sriraman and Knott (2009)). The aim to provide equal access to learning mathematics for learners is prominent in the socio-political literature to benefit underprivileged or marginalised learners. The literature review conducted by Aguilar and Zavaleta (2012) also demonstrates that the concerns of mathematics education to transform young mathematics learners into independent and actively participating critical citizens, and to build and sustain a democratic society are of international interest.

1.4.1 Presentation of *learners* and their perspectives in mathematics education research

In their review of the research studies exploring the links between mathematics education and democracy, Aguilar and Zavaleta (2012) identified three such links: *Mathematics education as a provider of critical mathematical skills*, *Mathematics education as a social gatekeeper*, and *Mathematics education as a source of values and attitudes*. It is interesting to imagine what role can be projected over to mathematics learners who sit in mathematics classrooms focusing on building these links. When mathematics education acts as a provider of critical mathematical skills, the learners can be seen as the receivers of the skills. They can employ these skills to reflect critically over the results of statistical analyses, the assumptions and limitations of mathematical models forming the basis of political and social decision-making, manipulative strategies used in the advertisements and fake news, and so on. When mathematics education acts as a social gatekeeper, the learners can be seen as being mathematically literate critical citizens who understand the role of mathematics education as a social filter, act against social discrimination and to ensure that everyone has equal access to mathematics education. When mathematics education acts as a source of values and attitudes, the learners can be seen as learning democratic values such as,

"tolerance and respect for diversity, and attitudes about truth that demand the critical analysis of information" (Aguilar & Zavaleta, 2012, p. 6).

According to Vithal (1999), it should be possible for the learners to experience a democratic life in their classrooms to learn democratic values and ideals while learning mathematics. The research studies such as Alrø and Skovsmose (2002), Ernest (2002) and Skovsmose (1994a) also recommend that the interactions should include dialogue, reflection, discussions, conflict of opinions, questioning the content, negotiation of shared goals, challenging of the teacher as an ultimate source of knowledge. Further, Vithal (1999) also suggests that it should be possible to talk back to the authorities (i.e., the teacher) in mathematics classrooms. It can be further observed that these research studies recommend transforming mathematics learners into critical citizens of the society for a collective social good. Moreover, it is also emphasised that learners (as individuals learning mathematics), should experience their mathematics classrooms as a democratic microsociety where they can practice adopting a critical perspective and the value of participation in democracy. Imparting democratic values and critical perspectives among *all* learners through their mathematics education is suggested irrespective of their interest or proficiency in learning mathematics, their socio-economic status, their socio-political, cultural, or historical contexts. By acquiring these competencies, the learners can employ them according to their individual requirements, or collective needs based on their socio-political contexts.

After exploring the competencies to be imparted in learners and the structure of mathematics classrooms promoting democratic values, I examined how mathematics learners are portrayed in mathematics education research literature. Valero (2004a, 2005) highlights *the myth of active learner* by investigating the interpretations of students in mathematics classroom presented in mathematics education research studies adopting cognitive to socio-political perspectives on learning mathematics. In the dominant discourse of (socio) constructivist and socio-cultural mathematics education research, the learners are portrayed as universal *cognitive subjects*. Valero (2005) asserts that, "Mathematics education research talks about this universal, normal child and how he thinks mathematically" (p. 4). It is also portrayed that the learners are interested in learning mathematics and their intentions are to engage in the process of learning. However, from a socio-political point of view, such portrayals of learners "do not allow us to understand the whole complexity of what learning mathematics is […] specially from the perspective of the learners and their perception of their experience" (Valero, 2005, p. 5). A mathematics classroom often has only some learners who like, are really interested, and want to engage in learning mathematics, while other learners as fully grown historic, socio-political beings may have other intentions in learning or not learning mathematics. Valero (2004a) calls for making the representation of learners in mathematics education research more real to *re-humanizing* and *realizing* the learners. In a realized and humanized view of students as whole learners, it is significant to acknowledge that they may have "multiple motives of learning, and who live in a broad context which influences their intentions to participate in school mathematics practices" (Valero, 2004a, p. 48).

Valero (2004a, 2005) has reported the myth of active learner and registered a call to broaden the view of learners under the socio-political perspective of mathematics education research. On the other hand, other scholars have highlighted the need and scarcity of research reporting on learners' critical reflections on and autonomous involvement in learning mathematics. Skovsmose (1994b) emphasises giving learners the opportunity to negotiate and "investigate reasons and goals for suggested teaching-learning processes, and by doing

so, to accentuate their own intentions and incorporate some of them as part of their learning processes" (p. 184). These suggestions are supposed to provide the conditions necessary for mathematics teaching-learning process to be productive. In exemplifying such approaches, Skovsmose (1994b) named the research done by Lindenskov (1993). Under the construct of "Students' curriculum", Lindenskov (1993) illustrated upper secondary school learners' "own criteria for when they themselves think that they learn mathematics meaningfully and based on understanding" (p. 124). Her research findings highlighted that the learners have good reasons for what content they are interested in, and which strategies they adopt while learning mathematics along with their own meta-conceptions, viewpoints and thinking about learning mathematics. Lindenskov (2010) has also urged for involving learners' "voices, intentions and thinking […] in what counts as CME acknowledging the right for all to position them and to be positioned as subjects" (p. 130) and called for more research which heeds to learners' voices.

Skovsmose and Valero (2005) have also questioned "Who has the possibility to participate in decision-making concerning the curriculum?", and suggest that, "With good reasons it can be argued that a bottom-up strategy makes it possible for both students and teachers to be included in curricular decision-making, and that is essential for education to make part of democratic processes in society." (p. 67). Further, Ernest (2004) has also commented that only a few mathematics education research studies have investigated learner' perspectives about learning mathematics and its relevance to them and that, "There is no reason to assume that learners will regard mathematics curricula as "relevant" just because educational and political leaders do so […]" (p. 315).

Several studies in mathematics education research have investigated learners' perspectives on selected aspects of their mathematics learning, for instance,

- the relevance of learning mathematics (e.g., Onion (2004), Sealey and Noyes (2010), Kollosche (2017) and Wiik and Vos (2019));
- their beliefs and attitudes about learning mathematics (e.g., Grootenboer and Marshman (2016), Leder et al. (2002) and Kloosterman (2002));
- the emotional affect of learning mathematics (e.g., Leder and Grootenboer (2005) and Nardi and Steward (2003));
- reflections on doing homework, experiencing learning difficulties and giving tests in mathematics (e.g., Lange (2009), Lange and Meaney (2011), Bagger (2016) and Alrø et al. (2009));
- learners' identities and facilitation of their agency in mathematics classroom (e.g., Bishop (2012), Andersson et al. (2015) and Rangnes and Herheim (2019));
- … and more.

These studies provide important insights into learners' experiences with learning mathematics in the classrooms. However, the research exploring learners' critical reflections on learning mathematics and their potential of evaluating, suggesting changes in, and influencing decisions concerning their mathematics learning activities has been scarce. I did not find many research studies, except Lindenskov (1993, 2010), which have investigated learners' evaluation of their own learning experiences in, expectations from, and potential of decision-making about their mathematics learning processes from their *own* standpoints.

1.4.2 Rationale of the research focus: What and how can the individual learners' voices contribute?

The lack of research considering learners' voices is not only documented in mathematics education research, but in educational research in general as well. Already in 1994, Nieto (1994) put it straight away that, "For the most part, discussions about developing strategies to solve educational problems lack the perspectives of one of the very groups they most affect – students, […]" (p. 392). A further call for investigating more into learners' perspectives is registered by stating that, "Ironically, those who spend the most time in schools and classrooms are often given the least opportunity to talk" (Nieto, 1994, p. 420). However, she also maintains that learners "have important lessons to teach and we need to begin to listen to them more carefully" (Nieto, 1994, p. 420). Giroux (1988, 2016) has also, since 1988, constantly pointed out the insufficiency of attending to learners' perspectives and outlined that learner perspectives provide insight into important elements of teaching and learning processes, which otherwise may not get revealed. The study of Goodlad et al. (1979) supports this assertion by maintaining that learners' experiences of their classroom learning (the *experiential* curricula) may be quite different from the *ideological* (ideal aims of education) or *formal* (officially approved) curricula.

Giroux (1988) writes that listening to learners' perspectives provides "an important starting point for enabling those who have been silenced or marginalised by the schools […] to reclaim the authorship of their own lives" (p. 63). Further, Nieto (1994) adds that "the very act of speaking about their schooling experiences seemed to *act as a catalyst for more critical thinking* about them" (p. 420, italics added). From the mathematics education research overview presented in section 1.4.1 above, it can be concluded that a serious consideration of learners' critical perspectives about, their potential of suggesting changes in and influencing decisions concerning their mathematics learning activities is much wanted in mathematics education research field. Consequently, it can be established that listening to individual learners' perspectives can benefit (mathematics) educational research by:

- Providing access to the experienced domain of the curriculum,
- Acting as an activator of learners' critical thinking about their school and learning experiences,
- Providing significant insights into learners' potential of reflecting critically, suggesting, and taking initiatives to introduce changes in their own learning activities,
- Providing learners with authentic experiences of democratic participation in their classroom microsociety and in educational decisions,
- Assisting in developing democratic values and outlook among learners through practical experiences,
- Supporting the educational aims of empowering learners, and developing them into future critical citizens who are autonomous and take active part in decision-making processes concerning their own lives, or the society, and
- … possibly in more ways.

1.5 Shifting the focus from cultural to critical

The research overview presented above answers to the queries put forward in section 1.3.4 from the perspective of (mathematics) education research. The (mathematics)

education research mentioned above establishes the desirability of imparting a critical perspective among learners towards their own (mathematics) learning processes. Likewise, encouraging learners' autonomous involvement in decisions about their (mathematics) learning activities is also suggested. It is also recommended that the learners require abilities such as, adopting a critical outlook, taking initiatives, actively participating in decisions concerning their (mathematics) learning activities to attain the critical outlook and autonomous involvement. Inculcating these abilities in learners can assist in achieving educational goals such as, learners' empowerment, critical citizenship, and democratic participation in the society.

This overview of the research literature mentioned above also made it clear that more research heeding to learners' perspectives about their own learning processes is highly recommended and required. As mentioned earlier, I was drawn towards the democratic approach to education after reading about learner-centred education. I also became interested in knowing more about the reasons behind learners' answers like, "I do not know", "I have not thought about it", "Nothing" or "Everything" in the questionnaires. The combination of my interest in democratic ideas of education, in enquiring learners' perspectives about their own mathematics learning experiences, and the impressions from reading the research literature concerning learners' empowerment caused a shift in my research interests. My research focus shifted away from investigating how practical activities rooted in learners' cultural backgrounds can influence their engagement and motivation to learn mathematics and science. Instead, I became interested in exploring individual learners' critical perspectives towards learning mathematics and their autonomous involvement in decisions concerning their mathematics learning activities.

Thus, the research focus in this study shifted from being on culturally oriented to being critically oriented. Bringing forward learners' perspectives and their voices became central concerns of this research study, and exploring selected learners' critical perspectives towards learning mathematics, and their autonomous involvement in decisions concerning their mathematics learning activities became the objective. This enquiry can illustrate the experiential domain of mathematics curricula from learners' standpoint. Moreover, learners' experiences can highlight if and how the concerns of imparting a critical stance and a sense of autonomous involvement among learners are catered for in their mathematics classrooms. Therefore, this research study can be seen to fill a gap in (mathematics) education research literature by exploring learners' voices, their critical thoughts and their experiences of participation and involvement in decisions concerning their own mathematics learning process.

1.5.1 Research focus and research question

Due to my presumptions about a developed western country (Norway) and a progressive education system, I was surprised by learners' responses in the questionnaires. I became interested in knowing if learners are required to think critically about and have a say in what they want to learn and how their learning activities are designed and planned. I investigated mathematics education research literature concerning CME and socio-political issues to enquire if learners are required to be critical and get involved in decisions concerning

their mathematics learning activities[11]. This literature review highlighted a research gap. The research addressing learners' own experiences and critical reflections regarding their teaching and learning activities in mathematics was limited, and a call for more research highlighting leaners' perspectives was registered. I address this research gap in this research study. Realizing the significance of paying attention to learners' points of view and the important insights their experiences can provide to the field of mathematics education research, I got interested in listening to *learners' voices*. Therefore, in this research project, 8th and 9th grade learners studying in two selected secondary schools in Central Norway are asked to share their critical thoughts about learning mathematics and their experiences with autonomous involvement in decisions concerning their mathematics learning activities. Consequently, the overall research question explored in this thesis is:

> *What can individual learners' voices inform about their critical perspectives towards learning mathematics, and their expressed autonomous involvement in decisions concerning their mathematics learning activities?*

This overall research question is further categorized into three sub-questions which are enquired respectively in the three research papers attached with the thesis. These sub-questions highlight different aspects of young learners' experience with critical thinking and learner autonomy. The research questions formulated in the three papers are as follows:

Reference to the papers	**Research question**
Sachdeva, S., & Eggen, P.-O. (2021). Learners' Critical Thinking About Learning Mathematics. *International Electronic Journal of Mathematics Education, 16*(3), 1-18. doi: https://doi.org/10.29333/iejme/11003 (extended version of Sachdeva and Eggen (2019))	What can learners' expressed mathematics related beliefs reveal regarding their practice with thinking critically about and potential to give suggestions concerning their mathematics learning process? *The scope was narrowed down with the help of three sub-questions, namely – what subject content do learners find interesting/not interesting to learn in mathematics?; why they learn mathematics?; and how their mathematics teaching may be changed?*
Sachdeva, S., & Eggen, P.-O. (Accepted, in press). "We learn it [mathematics] at school so one thinks that one will use it …": learners' beliefs about relevance and importance of learning mathematics. *Acta Didactica Norden (ADNO), xx*(x), xx-xx.	What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the sources of information influencing the formation of their beliefs?
Sachdeva, S. (2019). Students' experiences of learner autonomy in mathematics classes. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11, February 6 - 10, 2019)*	What can young learners' descriptions communicate about their experiences of learner autonomy in their mathematics classes?

[11] I also studied the development of Norwegian educational system, the development of general or overarching parts of different curricula, and specifically mathematics curricula over time. Chapter 2 presents an investigation and discussion of these documents.

(pp. 1978–1985). Utrecht University and European Society for Research in Mathematics Education, ERME. https://hal.archives-ouvertes.fr/CERME11/hal-02421636	
Overall research question of the thesis:	What can individual learners' voices inform about their critical perspectives towards learning mathematics, and their expressed autonomous involvement in decisions concerning their mathematics learning activities?

Table 1 Title and research questions in the three papers, and the overall research question addressed in the thesis.

Table 1 is extended throughout the thesis, gradually adding columns that provide overviews of theory, methodology, methods, analysis, and findings. I want to acknowledge Steffensen (2021) as a source of inspiration for constructing these tables, since she also used similar extended versions of tables to provide an overview of her thesis.

1.6 Organization of the dissertation

This is an article-based thesis which mainly builds upon the findings from the three papers that can be found in chapter 7. This dissertation is written to draw the whole research project together including the findings from different papers. It is organized in the form of chapters discussing the following themes:

The second chapter, Background of the research focus, clarifies the background of this research project by presenting the exploration and discussion of how the words such as, "critical", "critical-thinking", "responsibility", "active participation", "democracy", "citizenship", etc. are used in the educational policy and curriculum documents of Norway. I also present how these words are referred to in the Nordic and international contexts related to education.

The third chapter, Central concepts and theoretical underpinnings, provides a discussion of central concepts employed in this thesis and the theoretical framework forming the basis of this study. The study's theoretical positioning between the paradigms of social constructivism and Critical Theory (critical pedagogy) is also clarified in this chapter.

Next chapter, Methodology, presents the methods of data gathering employed in the research project and an explanation for choosing those techniques. Development process of the questionnaire and interview guides, and the procedure for choosing informants for face-to-face interviews is also clarified. This chapter further explains the choices of adopting a hermeneutic phenomenological research design and reflexive thematic analysis as a method for analysing the data analyses and interpretation. Finally, the trustworthiness, limitations of this research study, and ethical considerations are discussed.

Fifth chapter is Findings in and across the papers. For each paper, the research interest, key findings, and significance of the paper, and what would I have changed if I revised the paper now are described. A discussion about how these papers are different from each other and how they are complementary enough to fit together under one umbrella is also presented.

In chapter six, Discussion, implications, and conclusion, overall findings and conclusions form this research project are presented in a broader context than in the individual papers. The limitations of this study, and how things could have been done differently if I would conduct this research study again are also discussed in this chapter. The implications of this research study and suggestions for future research are also presented, before concluding the chapter.

Chapter seven includes all the *Contributing papers*, right before the *References*. Following *References* are *Appendices,* last chapter of the thesis. It comprises of the written questionnaire, interview guide, project information and consent form given to the learners and their parents, the practical activities, co-author declaration and the ethical clearance certificate from NSD (Norsk senter for forskningsdata, now SIKT) issuing the permission for data collection. The earlier versions of questionnaire and interview guide, all interview transcripts, as well as anonymised version of questionnaire responses can be forwarded, upon request.

2 Background of the research focus

Under section 1.3.4 in chapter 1, I question if, and why, it is desirable that learners reflect critically over, suggest changes in, and take active part in decisions concerning their learning activities. These questions originated when I observed that the questionnaire responses of the first 22 learners to which the questionnaire was administered were not as I anticipated. I wanted to know why most of the learners had replied "I do not know", "Nothing" or "Everything", when asked for instance, to mention what they would like to learn more about in mathematics. In this chapter, I study the Norwegian Education Act, and the general and mathematics specific parts of Norwegian secondary school curriculum[12] with an aim to investigate: If the educational aims drafted for Norwegian lower secondary school learners include the competencies of *thinking critically about* their own educational experiences and *participating in the decision-making process* about their own learning activities. Besides I also enquire how learners' position and role in their own learning experiences are discussed in the Nordic and international education contexts. In the following text, section 2.1 to 2.5 provide an account of my study of policy and curriculum documents, and the Nordic and international contexts with respect to the questions listed above. This exploration will contribute to clarify the background of the research focus adopted in this study.

2.1 The Norwegian Education Act *(Opplæringsloven)*

The Norwegian Education Act (*Opplæringsloven*, in Norwegian) establishes the laws and regulations which govern the functions of educational institutions such as schools in Norway. The Education Act (1998) is the Norwegian law for public and secondary schools, as well as for apprenticeships and adults over the age of upper secondary education (19 years) who have not completed the primary school. Both the Education Act and the regulations to the Education Act specify the structure and organization of compulsory years of education for the Norwegian learners along with their rights and the teachers' and school's duties towards the learners. Children in Norway normally start primary school at the age of six years and have the right to compulsory education for the first 10 years of their schooling. The compulsory years of schooling are distributed as: primary school from grades 1 to 7 (*barneskole*) for learners from six to 13 years of age, followed by the lower secondary school from grades 8 to 10 (*ungdomsskole*) for learners from 13 to 16 years of age. The upper secondary school from grades 11 to 13 (*videregående skole*) for learners from 16 to 19 years of age is their statutory right but is voluntary.

The principles of democracy[13], equality along with human rights and scientific and critical thinking are to be promoted among the Norwegian learners through their educational

[12] Mathematics specific (and some general) education research literature was also examined to investigate same questions. Sections 1.4 and 1.5 in chapter 1 present results of this investigation.

[13] It is important to note that the understanding of the concept of democracy in Norwegian curriculum and educational policy documents does not limit to a style of public administration with having the right to choose the government and the right to freedom. Citizenship and the citizen are central concepts in Norwegian democracy and educational research. Therefore, democracy is understood as a process of decision-making in which the citizens of the society participate equally and actively, where active participation means being involved in, being able to cooperate in, influence, and critically evaluate the decisions so made (see, e.g., Breivega et al. (2019)).

process and learning activities (see, e.g., Briseid (2012) and Breivega et al. (2019)). In their formulations, the regulations to the Education Act (Ministry of Education and Research, 2006) and The Education Act (1998) contain several references to the words such as, democracy, learners' active participation, critical thinking, and learners right to involvement in their own learning processes. These ideas are supposed to lay the foundational cornerstones of the Norwegian educational system and are mentioned as the qualities to be promoted and imparted among the learners through their educational activities. For instance[14], regulations to the Education Act state that, "Alle elevar skal ha moglegheit og tid til å arbeide med saker knytt til elevdemokrati og medverknad[15] i opplæringa". This statement can be translated into English as, "All learners must have the opportunity and time to work on issues related to student democracy and *participation* in education" (my translation and italics).

The objectives of education and training mentioned in The Education Act (1998) also state that, "Education and training are […] to promote democracy, equality and scientific thinking", and that "The pupils and apprentices *must*[16] learn to think critically […] They *must* have joint responsibility and the right to participate" (italics added[17]). These statements mentioned in the regulations to the Education Act and the Norwegian Education Act legally establish learners' right to learn to think critically, to participate, to influence and have a joint responsibility of their own educational processes. These rights seem to have been strengthened more in the proposed and legally approved *new* version of the Norwegian Education Act. In the year 2017, the parliament of Norway appointed a committee to review and evaluate the current Education Act from 1998 and suggested a draft for the new Education Act to meet up the standards of newer times since the current Act is 25 years old. The committee submitted their report in the year 2019 proposing the changes to be incorporated in the new Education Act. These proposals had been under hearing since 2019 and the final draft of the new Education Act was submitted in the parliament in spring of the year 2022. The parliament (Stortinget, 2023) has approved the resolution for the new Education Act in summer 2023, and it will be applicable to the Norwegian schools and educational institutions from August 2024.

The objectives of education and training in the upcoming Education Act also state that, "Education and training are […] to promote democracy, equality and scientific thinking", and that "The pupils and apprentices *must* learn to think critically […] They *must* have joint responsibility and the right to participate" (italics added). Further, learners right to involvement and participation (*medverknad*) are emphasized even more in the new version of the Education Act. Section 10-2 in chapter 10 of the new Education Act states that, "Elevane

[14] I searched for the citations by using the keywords, "kritisk" ("critical" in English), "kritisk tenkning" ("critical thinking" in English), "medverke/medverknad" (NyNorsk) (see footnote 15), "medvirke/medvirkning" (Bokmål) (see footnote 15), and "medansvar" ("co-responsibility" or "joint responsibility" in English) in the online versions of the regulations to the Education Act (Forskrift til opplæringslova) and The Education Act (Opplæringslova) available at lovdata.no.

[15] The Norwegian verb "å medverke" in NyNorsk or "å medvirke" in Norwegian Bokmål does not have an exact substitute in English but it can be translated into verbs such as to participate, to be involved, to cooperate, to influence and to contribute (using Clue online dictionary).

[16] 'Must' and 'shall' are words in English language that have similar meanings. Both indicate the fact that something is mandatory and should be carried out as a duty. However, shall is used more in legal circles while must is used more often by common people. Many believe the word 'shall' to be more formal of the two and fit for use in legal documents and contracts to stress a role or responsibility.

[17] English translations found from the unofficial translation of The Education Act available on the website: https://lovdata.no/dokument/NLE/lov/1998-07-17-61/KAPITTEL_1#KAPITTEL_1.

har rett til medverknad i alt som gjeld dei sjølve etter denne lova, og har rett til å ytre meiningane sine fritt. Elevane skal bli høyrde, og det skal leggjast vekt på meiningane deira etter alder og modning" (Stortinget, 2023, p. 9). Translated into English, this statement submits that, "The pupils have *the right to participate in everything* that applies to them according to this law and have *the right to express their opinions freely*. The pupils *must* be heard, and emphasis must be placed on their opinions according to age and maturity" (my translation, italics added). The new Education Act has further enshrined in law learners' right to self-determination for learners of age 15 years or above. Chapter 25, section 24-5 states that, "Those who have reached the age of 15 *take their own position on questions related to the education*, including consent to individually tailored training, […] and application for admission to further education." (Stortinget, 2023, p. 22, italics added).

The statements mentioned in all the policy documents above[18], emphasize the desirability and importance of providing the opportunities to learners to develop their critical thinking about and active involvement in the decisions concerning their own learning processes depending upon their age and maturity. An underlying assumption which can be deduced from these statements is that, given an opportunity to express, children and young learners can think critically about, evaluate their learning environments, and suggest changes in their learning activities accordingly. The forthcoming version of the Education Act even provides the statutory right to the learners of age 15 (studying in grade 9 or 10) or elder to take their own positions and make decisions when it comes to questions related to their own education. The practical enactment of this right would assume that learners of age 15 and preferably also younger[19] as well should be habitual of being asked to voice their opinions, think critically about, participate in decision-making, cooperate, and take joint responsibility of their learning activities and educational processes. This assumption in turn sets out an expectation that learners, from early years of their education (depending upon their age and maturity), must learn to take their own stand, and have their own opinions about the questions concerning their own learning processes. The culture of critical thinking, participation and involvement in decision-making thus should be a part of learners' classroom practices.

Reading these documents answered the question (a.) above (see 1.3.4) that the educational policy documents emphasize and provide a statutory right to primary and secondary school learners to learn to think critically and get involved in the decisions concerning their own learning processes. The schools have been given the duty to organize the learning activities in a way that learners develop their critical thinking and participate in their own learning processes in a democratic manner. The Education Act and its regulations establish the laws that govern and regulate educational processes to be carried out in the schools, but the objectives outlined in these documents depend on being enacted and realized in the classrooms through a curriculum. In the next section, the common core curriculum guidelines issued in Norway are explored to find out if development of learners' critical

[18] The regulations to the Education Acts, the current Norwegian Education Act, and the forthcoming Norwegian Education Act.

[19] Self-determination cannot be expected to happen or develop instantly. Wehmeyer and Shogren (2016), for instance, suggest that it is a gradually acquired characteristic, emerging in adolescence. Therefore, secondary school learners (between 13 to 16 years of age, who are the informants in this study) should be given the experience and training in thinking critically about and being involved in decisions concerning their own educational activities so that the enactment of self-determination is possible from the age of 15 years.

thinking and their right to participate in decisions concerning their own education process have also been referred to in these guidelines.

2.2 Overarching or general part of the Norwegian school curriculum

The curriculum guidelines (*læreplanen*) in Norway have been issued and amended at different times since the *Normalplanen* from 1939 (N39) (Engelsen, 2020). The Norwegian curriculums and its amendments are usually introduced by an overarching part or the general part which "elaborates on the purpose clause in the Education Act, sets out overall goals for the education and contains the value-based, cultural and knowledge basis for primary school and upper secondary education" (Kunnskapsdepartementet, 2006b, p. 1). The goals, values and principles outlined in the general part of the curriculum apply to, should be catered for, and promoted in the teaching and learning activities of all the subjects taught in the schools, including mathematics.

The curriculum N39 (1939)[20] and the *Læreplan for forsøk på 9-årig skole* (L60, 1960), though did not have any general or overarching part, but were introduced by a short introduction only (Engelsen, 2020). The formulations used in N39 were influenced by the ideas of reform pedagogy and Progressive (education) movement in the USA, so the curricula emphasized that learners' individual activity, and their general and independent work with the subject matter was more important than the acquisition of traditional knowledge (Engelsen, 2020, p. 208). It was only with the School Act of 1959 that "the democratic spirit" became an integral element of the purpose for Norwegian elementary education (Briseid, 2012, p. 51). The brief preface of the curriculum L60 in the year 1960 emphasized imparting the fundamental values of cultural heritage, such as, the democratic ideals, Christian beliefs and morals, scientific methods and thinking, art and poetry among school learners (Engelsen, 2020, p. 208), but the democratic ideals were not elaborated much. The curricula following L60 included a detailed introductory chapter which was named as "the general part" or as "the overarching part" (in the current curriculum). These curricula include: *Mønsterplan for grunnskolen* from the year 1974 (M74), *Møsnterplan for grunnskolen* from the year 1987 (M87), *Læreplanverket for den 10-årige grunnskolen* from 1997 (L97), *Læreplanverket for kunnskapsløftet* from the year 2006 (LK06) (Kunnskapsdepartementet, 2006b), and the currently effective *Læreplanverket for kunnskapsløftet* from the year 2020 (LK20) (Kunnskapsdepartementet, 2017). In the following text, I use the notations M74, M87, L97, LK06 and LK20 to denote the above curricula respectively.

It was with the curriculum M74 that the learners got acknowledged as the *subjects* of learning and as independent participants in the school democracy. Simultaneously, M74 made the call for developing learners' critical thinking and scientific working methodology. Therefore, to better perceive the sense in which the notions "critical" (*kritisk*), "critical thinking" (*kritisk tenkning*) and learners' participation in decision-making process (*medvirkning, medbestemmelse, medinnflytelse* or *medansvar*) are quoted in the curricula

[20] The primary school became mandatory for first seven years in the year 1889 and the N39 was introduced in 1939. In 1960, the curriculum for the primary school for nine years got introduced and the primary school was made mandatory for nine years in the year 1969. In the 1970s and 1980s, a 10-years mandatory primary school system was introduced along with the curriculum M74 in 1974, and its following amendments, M87 in 1987, L97 in 1997, LK06 in 2006 and LK20 in 2020 (the current version).

M74, M87, L97, LK06[21], and LK20[22], I present selected extracts from these curricula in the following sub-sections. The selected extracts are limited to "the general part" (*generell del*) or "the overarching part" (*overordnet del*) of the introductory chapter of the curriculum and are based on the representativeness of their content. Excerpts concerning the use of these notions specifically in "the mathematics curriculum" are presented and discussed in the section 2.3 below.

2.2.1 The notion of "*medvirkning*" (participation, involvement, influence, cooperation, or joint responsibility)

The M74 had a relatively long general part which spanned over 72 pages (from page 9 to page 81). The notion of learners' participation and cooperation in decisions about their own educational processes is mentioned under the heading "Elevenes medvirkning", which when translated to English[23] reads as,

> "The choice of teaching material and the presentation of it can be influenced by the one-sided views and positions of the adult generation. The students have a different experiential background and experience the problems in a different way. They must be allowed to ask their questions based on their own assumptions both in terms of nearby problems and problems of a global nature and be allowed to work with them from different points of view with a consideration to their critical assessment and an independent point of view. The student must gradually get used to being involved in annual and period planning and the choice of teaching material, and to making independent decisions about both teaching material and working methods." (p. 26).

From the citation above, the emphasis on learners' development as partners to be involved in the decisions concerning the material, planning and methods of their own teaching and learning activities becomes clear. Their experiences and perspectives are acknowledged, and importance values has been attributed to their perspectives. The case for their active and independent participation in choosing the material and methods of their learning is made.

Thirteen years later, M74 got replaced by the curriculum M87. The fundamental ideals forming M87 were also progressive, analogous to M74, but Engelsen (2020) highlights that attending to learners' interests and expectations became important to nourish their future prospectives. Providing the learners with opportunities to take a joint responsibility and participate actively in their learning activities was proposed to create in them a sense of belonging towards their school (Engelsen, 2020). While outlining the purpose and duty of the primary school, the general part stated that, "Co-responsibility [*medansvar*] and co-influence [*medinnflytelse*] over one's own work and learning situation provides practical training in democratic ways of thinking and working, and an understanding of the values that characterize a democratic society. The learners should be involved in collaboration on their own working and learning situations right from the first grade" (M87, pp. 19-20). Further,

[21] Curriculum which was applicable when this research study was carried out.

[22] The currently applicable curriculum which became effective in Norwegian schools from August 2023.

[23] All the quotes and citations taken from the Norwegian curriculums from M74 to LK06 are originally in Norwegian, but I have translated them in English to use them in this thesis.

when discussing the learning environment and working methods in the school M87 mentioned that,

> "The students' participation [medvirkning] must include aspects of the teaching and learning activities that have real significance for their working and learning situation. […] This applies to the choice of teaching and learning materials, working methods, and working conditions in general. Participation in these areas must be planned by the teachers, and it must develop over time" (M87, p. 54).

These guidelines mentioned in M87 clarify that learners' cooperation, co-influence and participation in planning and designing their educational activities was not only intended to support learning of the subject matter or influencing their classroom activities but were rooted in the ideal of developing them into participating citizens and bearers of a democratic society. Young learners, as early as from first grade were supposed to be included in and asked questions about the decisions concerning their own learning process.

While L74 and M87 referred to and elaborated on the purpose of the contemporary School or Education Act of Norway, Engelsen (2020) reports that the revised curriculum guidelines of L97 were formed on a detailed analyses of the goals and objectives outlined in the concurrent Education Act and other significant acts concerning upper secondary and vocational education. These guidelines now applied to both elementary, upper secondary, and adult education in Norway and the formulations used in L97 were not normative and at times poetic (Engelsen, 2020). The general part mapped six aspects of being human and education (elementary, secondary, and upper secondary) got assigned the task of building the so called "integrated human" (Engelsen, 2020). Though the importance of learning basic knowledge was highlighted more, yet L97 also demonstrated similar concerns when it came to learners' inclusion in the decisions concerning their learning activities. As a distinctive character of the lower secondary grades, L97 specified the purpose of education as, "the education should contribute to learners gradually taking a greater responsibility for planning their own learning" (L97, p. 74). The general part of L97 were carried forward as the general part of the curriculum guidelines issued in the year 2006, that is, LK06 (though the subject specific curricula were changed, as explained in section 2.3 below).

It is interesting to see the changes introduced to the Norwegian curriculum LK20 (effective from the year 2023) after my research project had concluded. Getting an insight into the direction of these changes, combined with the afterthoughts on this research study can help setting the findings of this study in a perspective and discuss its implications for Norwegian (mathematics) classrooms (see section 6.1.2). The general part of LK06 has been revised and got the name "the overarching part" (*overordnet del*) in LK20. The purpose of the Education Act has been incorporated as a section of this overarching part and it elaborates the purpose of compulsory elementary and upper secondary education as sketched in the Education Act. The core curriculum LK20 has the status as regulations together with the rest of curriculum and comprises of three chapters: 1.) Core values of the education and training, 2.) Principles for education and all-round development, and 3.) Principles for the school's practice.

The values of *democracy and participation* have been identified as the core values of education and training, and it is described that, "The school must be a venue where children and young people experience democracy in practice. *The pupils must experience that they are heard in the day-to-day affairs in school, that they have genuine influence and that they can*

have impact on matters that concern them" (Kunnskapsdepartementet, 2017, italics added). It is further stated that when learners' voices are heard in school, they will experience how they can make their own considered decisions, and such experiences will have a value in present, and will prepare them to become responsible citizens in the society. Not only does the curriculum mention democracy and participation as core values, but also *democracy and citizenship*, as a principle is included in the section of education and all-round development. Additionally, the principles for the school's practice state that learners' "involvement *must* be a part of the school's practice. The pupils must participate and assume co-responsibility in the learning environment which they create together with the teachers every day" (Kunnskapsdepartementet, 2017).

In essence the study of the quotes taken from the curriculums from M74 to LK20 above illustrate the focus placed on learners' involvement and co-influence on decisions concerning their learning activities as a democratic educational practice. Learners are supposed to take co-responsibility of their own learning from a young age, and their involvement, co-operation and participation should have a genuine impact and influence on their learning activities. Thus, it can be interpreted that following the ideals and principles of democracy which form the basis of the Norwegian curriculums, it is desirable and important that learners express what they want to learn, give suggestions, and get autonomously involved in, that is, *"å medvirke"* in the decisions concerning their learning activities. This section has focused on studying the use of the notion of "medvirkning" in the Norwegian curriculum from a historical and developmental perspective. In the following section, I present a similar discussion of the Norwegian curriculums concerning use of the notions "critical", that is, *kritisk* and "critical thinking", that is *kritisk tenkning*.

2.2.2 The notion of "*kritisk*" (critical) or "*kritisk tenkning*" (critical thinking)[24]

The general part of M74 does not refer to the notion of critical thinking but the word "critical" is mentioned in the curriculum on several occasions. The citation taken from M74 in the section above, indicates that developing learners' own independent perspectives and critical awareness along with improving their potential of critical reflection and evaluation formed a strong fundament of their education process. Moreover, the scope of their critical capacity was not limited to participating in planning and designing their learning activities together with their teachers, but also to learn and exercise their ability of critical discernment of the fundaments underlying different social and cultural phenomena. This concern is expressed in M74 as, "but teaching material that exercises criticism against or is in conflict with these basic values also has its justification in school, among other things with a view to developing the ability of independent critical assessment" (p. 27-28). The basic values being referred to in the citation above are, the Christian belief and moral, the democratic ideas, and scientific thinking and methods. Therefore, M74 laid the ground for developing learners' individual critical assessment and judgement abilities.

[24] In my PhD research journey, I became aware that the words *critical* and *critical thinking* are used with different understandings in the education research literature. Research discourses consider an individual *being critical* or developing one's *critical thinking* as a cognitive skill, as a psychological skill (self-reflection), as a socio-political skill (to be an active participating critical citizen of the society, cf. critical pedagogy), as a philosophical virtue to exercise ideological and self-critique, etc. I take up this discussion and my understanding of critical thinking in section 3.3.1 of chapter 3.

The curriculum M87 has also emphasised and employed the word "critical" several times. Development of a "sense of critical assessment and evaluation" among primary and secondary school learners has been outlined as a central concern and purpose of elementary education. It is stated that, "the school must emphasize developing the learners' critical sense […]"(M87, p. 23), and that, "the teaching material must stimulate the [learners'] ability to face new life conditions and situations in society with a healthy critical sense […] and constructive attitude" (M87, p. 43). These citations illustrate that the intent of using the word "critical" was not only to impart a cognitive ability among learners so that they can evaluate arguments and deduce logical conclusions, but also that they embrace "criticality" as a general approach towards living and leading their lives. It was expected that they learn to adopt a critical evaluating approach towards the information they receive and situations they experience so that they can understand the corresponding positive and negative aspects, and consequences before forming their independent opinions, decisions or judgements regarding that information or situation. The learners were encouraged to ask questions and critically evaluate the information they got, also through their learning materials.

It was first in L97 curriculum that both the words "critical" and "critical thinking" are used, with only three mentions of "critical thinking". References to the word "critical" employed a broad sense, that is, to develop the learners' critical judgement in various areas "through experience from assessing performances and expressions against the standards" (L97, p. 24), and that the learners' educational process "must find the difficult balance between respect for established knowledge and the critical attitude that is necessary for the development of new knowledge and for organizing knowledge in new ways" (L97, p. 25). However, the word "critical thinking" was used in a narrower sense under the heading "the creative man" (*det skapende menneske*), which read that, "critical thinking involves testing whether the assumptions for and the individual links in a line of thought hold" (L97, p. 24). A difference that can be observed between how the words "critical" and "critical thinking" are used is that "critical thinking" directs the focus towards judging the logical nature of arguments and conclusions whereas "critical" carries an intent of incorporating a critical perspective in learners' way of looking at the world. These indications remained the same for LK06 as well since the general part for both these curricula was same.

Moving to the currently applicable curriculum LK20, several references to the words "critical" and "critical thinking" can be found. Looking for the keyword "critical" gives 13 hits, which also includes the four references to the keyword "critical thinking". The core values of education and training in the overarching part of LK20 incorporates a section named "critical thinking and ethical awareness". This section repeats the formulation of the Education Act that the learners must learn to think critically and act with ethical awareness, but also elaborates that, "Critical and scientific thinking means applying reason in an inquisitive and systematic way when working with specific practical challenges, phenomena, expressions and forms of knowledge" (Kunnskapsdepartementet, 2017). It further states that, "The pupils must be able to assess different sources of knowledge and think critically *about* how knowledge is developed. […] Critical reflection requires knowledge, but there is also room for uncertainty and unpredictability" (Kunnskapsdepartementet, 2017, italics added). Afterwards, the formulation from L97 that the teaching and training process "must seek a balance between respect for established knowledge and the explorative and creative thinking required to develop new knowledge" (Kunnskapsdepartementet, 2017), is echoed. LK20 also states that *reflection and critical thinking* are parts of acquiring the "competence aims"

(kompetansemål) *in* different subjects[25] and the abilities of reflection and critical thinking are connected to developing perspectives and ethical judgment.

The citations and descriptions using the words "critical" and "critical thinking" in the overarching part of LK20, and earlier curriculums suggest that these words indicate the ability to think and reflect critically *in* and *about* the knowledge the learners gain in different subjects. In other words, if the learners come across a knowledge claim, for instance, "water boils at the temperature of 100°C", then they do not accept this claim without being critical: (a.) *in* science (that is, critically and systematically asking which laws of science make this claim true and how to test if it really holds); and (b.) *about* science (that is, critically and systematically asking why is it important for them to know about this claim and if they could have learnt about this claim in other possible ways, or instead focused on an issue more relevant to their lives). Such an interpretation of the intent to develop an underlying outlook of *criticality* among learners makes it possible to comprehend the difficulty highlighted in the curriculum regarding "seeking a balance between respect for established knowledge and the explorative and creative thinking required to develop new knowledge". The goal of developing such a critical outlook among learners is that they are open but simultaneously sceptical to receiving new knowledge, and that they learn established knowledge but simultaneously are creative and enduring enough to deal with the uncertainty associated with finding new knowledge. This synthesis of the general or overarching parts of the Norwegian curriculums therefore indicates that it is desirable and important that learners reflect over and evaluate their learning activities by adopting a critical outlook. This critical ability can help individual learners to develop their power of discernment, acquire their own independent points of view, make reflected choices, and take decisions regarding important issues in their lives by critically evaluating the positive and negative consequences of those decisions.

2.3 The *mathematics* curriculums in Norway from M74 to LK20

After reading the general or overarching parts of the curriculums from M74 to LK20, it became clear to me that Norwegian learners right from their first grade are supposed to be educated and brought up as critical, independent, and actively participating citizens of a democratic society. These qualities are to be imparted gradually among learners from the day they start their elementary education from 1st grade, and until they become more independent individuals and gain increasing right to self-determination (*selvråderett*) in the age of 15 years when they complete their compulsory education until 10th grade. The aims and objectives suggested in the general or overarching part of the curriculum serve as instructional guidelines and apply to the designing of teaching and learning activities of all the subjects taught in the school. However, to enquire if and in which sense these principles and values were reflected in the contemporary mathematics curriculums, I looked through the formulations of mathematics curriculums from M74 to LK20 referring to the words such as, critical, critical thinking, active participation, cooperation, involvement, independent, decisions, democracy, citizen, and citizenship.

[25] The subject specific curricula in Norway (for instance, mathematics curriculum) outline some competence aims to be achieved in each subject after each grade (for instance, after 2nd grade the learners should have competence in telling the time using a clock and a calendar), and assessments are planned to evaluate learners' successful achievement of these competence aims.

The curriculum M74 from 1974 highlighted the significance of developing learners' capacity of critical evaluation and assessment in general part but the subject specific curriculum of mathematics does not include references to any of the keywords mentioned above. While looking for these keywords in the curriculum of 1987 (M87) three mentions of the word critical can be found under the "statistics" section of mathematics curriculum. These mentions include, "practice in critical assessment of statistical material" as an aim of learning statistical terms, methods, and interpretation of statistical data (M87, p. 201). The necessity of learners' ability to critically evaluate the statistical data material is specified as a competence aim for learners in grades 7 to 9. This requirement is justified based on the need that the learners should be able to comprehend and take an independent stand on the statistical information provided by the authorities when they plan or take decisions on important issues of the society. The concept of involvement and active participation in decision-making process is not mentioned but learners' co-responsibility to understand the decisions about important social issues as democratic citizens of the society can be observed in this justification.

While reading the formulations of L97 (1997), a purpose similar to the call for inclusion in and understanding of the decisions on societal issues voiced in M87 can be observed. The introduction to mathematics curriculum L97 elaborates the place of mathematics in the school and mentions the word critical while mentioning that, "mathematics challenges both ingenuity, critical sense and analytical ability" (p. 153). It is further stated under the same section that the "Knowledge and skills in mathematics are an important basis for active participation in work and leisure and for being able to understand and exert influence on processes in society." (L97, p. 154). Another mention of the word critical comes under the section elaborating on the aims of learning statistics and the need of a critically assessing outlook while interpreting statistical information, graphs, or tables. Consequently, L97 can be seen as repeating the concerns of M87 in terms of employment of the notions critical and active participation of mathematically literate learners in a democratic society.

Further, I explored the curriculum under effect while conducting this research project, that is, LK06 (Kunnskapsdepartementet, 2006a). It can be noticed that the "Purpose" of learning mathematics in school states that, "Active democracy requires citizens who are able to study, understand and critically assess quantitative information, statistical analyses, and economic prognoses. Hence mathematical competence is required to understand and influence processes in society" (Kunnskapsdepartementet, 2006a). Other references to the word critical are related to the subject area of statistics and probability in which critical evaluation, testing the authenticity, trustworthiness and interpretation of data representation, analyses and conclusions are focused. It is interesting to notice the coherence represented in the mathematics curricula from M87 to LK06 regarding the significance of learning mathematics to become a critical and actively participating citizen of a democratic society.

In the currently applicable mathematics curriculum, LK20 (Kunnskapsdepartementet, 2019), I observed that the words critical and critical thinking are mentioned in the 'Relevance and central values' section. The section states that, "Mathematics shall help pupils to develop a precise language for reasoning, critical thinking and communication through abstraction and generalisation" (Kunnskapsdepartementet, 2019). It is further clarified that the capacity of critical thinking in mathematics, "includes critical evaluation of reasonings and arguments and *can arm the pupils to make their own decisions and take a stand on important questions in their own life and in society*" (Kunnskapsdepartementet, 2019, italics added). The curriculum

also mentions that learning mathematics should help in developing learners' capability to work individually and that learners can get aware of their *own* learning in mathematics through collaboration with peers in problem-solving and exploration. Other mentions of the word 'critical' are related to the subject areas of 'Modelling and Application' and 'Statistics'. Through mathematical modelling, the learners are encouraged to critically evaluate the validity and limitations of mathematical models in view of the original situations, and if they can be used in other situations, whereas while learning statistics they are encouraged to "interpret and critically evaluate statistical representations found in media and the local community" (Kunnskapsdepartementet, 2019).

Learners' involvement and participation (*medvirkning*) in their own learning activities is named but as a part of their suggested 'Formative assessment'. It describes that, "The teacher shall facilitate for pupil participation [medvirkning] and stimulate the desire to learn by allowing the pupils to explore mathematics and solve mathematical problems by being creative, modelling and reflecting" (Kunnskapsdepartementet, 2019). Positioning the element of learners' participation as a part of their formative assessment, and not mentioning it as a necessary part of their learning activities in the subject is not explained, so the underlying reason is difficult to understand. Another advancement in mathematics curriculum LK20 is inclusion of the interdisciplinary topic 'Democracy and Citizenship'. The incorporation of this topic in mathematics is explained by stating that, "In mathematics the interdisciplinary topic of democracy and citizenship refers to giving the pupils *the competence to explore and analyse findings from real datasets and data collected from nature, society, working life and everyday life.* […] This type of competence is important in order to formulate one's own arguments and participate in public debate" (Kunnskapsdepartementet, 2019). These formulations call for learners' taking part in their mathematics classroom activities, and as critical mathematically literate citizens in the democratic society, but do *not* include a call for learners' critical thinking about and autonomous involvement in decisions on their own mathematics learning.

The study of Norwegian mathematics curricula from 1987 to 2020 illustrate that the curricula have constantly stressed the importance of a mathematically literate citizen's role in and for democratic society. However, active participation in one's own mathematics teaching and learning activities, and autonomous involvement in decisions concerning mathematics education are not explicitly mentioned. The concerns regarding learners' co-influence, co-operation, and co-responsibility in planning and designing their own learning activities (*medvirkning*), as mentioned in the "general" or "overarching" parts of the respective curricula are not clearly reflected in the subject specific curricula for mathematics. Likewise, the significance of providing learners with the opportunity to think critically about, evaluate and reflect on their own learning and teaching materials and styles in mathematics seems to be left out. It can be summarised that, "the *collective* dimension" of learners being critical, thinking critically, and actively participating in the *society as democratic citizens* is highlighted, but "the *individual* dimension" of learners being critical, thinking critically, and actively participating in decisions of their *own* mathematics learning processes is not equally stressed. Hence, incorporating and employing these concerns mentioned in the general or overarching part depends on teachers' interpretation, incorporation, and operationalisation of such concerns in the subject specific (i.e., mathematics) classroom.

Rønning (2004) explored teachers' interpretation of the general part of the curriculum L97 and suggested that the teachers interpret mathematics as a very important subject having high status and a strict progression. Rønning (2004) further highlighted that considering mathematics as an important subject requiring strict progression makes it difficult to

incorporate interdisciplinary or project work in classroom activities as mentioned in the general part of the curriculum. The teachers suggested that such activities take time from teaching the mathematical content knowledge, which otherwise could have been used to cover the syllabus (Rønning, 2004). Goodlad et al. (1979) also suggest that the curriculum documents and the Education Act are the *formal* understandings of the curriculum, whereas the *perceived* (teachers' interpretations), and the *experienced* (learners' perspectives and experiences) understandings of the curriculum may vary from each other. Therefore, in this thesis I try to gain an insight into the *experienced* domain of mathematics curriculum by listening to secondary learners' voices and experiences with teaching and learning of mathematics. However, to position this study in an international context and to justify and establish its broader significance, it is vital to investigate if the concerns regarding learners' participation and development of their critical orientation represent only the Norwegian style of comprehending the education process or is it more a widespread perspective. For that reason, the sections 2.4 and 2.5 below explore if the agenda of developing learners in to critical and participating individuals and citizens of a democratic society is shared by other nations, and if these concerns are also echoed in international (mathematics) education research.

2.4 The Nordic Model of (mathematics) Education

Taking the broader view, the ideas and philosophies underlying the Norwegian educational policies and curricula are not limited to Norway, but are part of the so-called *Nordic model of education* (see, e.g., Antikainen (2006)), and hence the *Nordic model of (mathematics) education* (see e.g., Dahl and Stedøy (2004)). The Nordic countries comprise of three Scandinavian countries Norway, Sweden and Denmark, along with Finland and Iceland. Antikainen (2006) argues that despite local variations, the values and aims guiding the development of education in the Nordic countries are, "democracy, equality, progressiveness, and pragmatism" (p. 240). Similarly, when describing the educational objectives common to the Nordic countries, Dahl and Stedøy (2004) cite, "equal access to (lifelong) learning, teaching democracy, independence, equality, and the development of critical awareness in pupils" (pp. 4-5). They further mention that the 'Nordic dimension in education' underlines that "the teaching of democratic values is as important as the teaching of knowledge" (Dahl & Stedøy, 2004, p. 5). These research studies also remark that similar underlying values does not mean that the educational structure and school system in the Nordic countries is the same. Nevertheless, this research makes the point that the *Nordic model of (mathematics) education* has underlined the importance of developing and nurturing learners' critical competencies for developing them as future critical citizens of a democratic society so that they can use their mathematical knowledge critically to fight against the social odds, such as, injustice, inequality, discrimination, etc.

Following the common ideology grounding the educational policies, the mathematics education research conducted in these countries also reflect similar concerns in the research studies carried out. For instance in Norway, Steig Mellin-Olsen highlighted the political functioning of mathematics education in the society (see, e.g., (Mellin-Olsen, 1987)) and presented learners' perspectives and rationale for learning mathematics (see, e.g., (Mellin-Olsen, 1984)). In his other publications, he argued that the learners of mathematics are not just receivers of mathematical knowledge but actively participating *subjects* in learning

mathematics, having the ownership of mathematical knowledge they acquire (Mellin-Olsen, 1993b). In several of his works Mellin-Olsen (1987, 1993a, 1993b) also questioned the role, status, and opportunities mathematics learners have and get to gain control over and participation in deciding what knowledge they learn through their mathematics teaching and learning activities. Mellin-Olsen's work was theoretically grounded in Activity Theory, but the issues he raised can be seen as being political and democratic in demanding learners' involvement, participation and consideration of their interests and desires in planning and decisions concerning mathematics classroom activities. Lately, research projects such as, Critical-mathematical argumentation and agency when working with risk in mathematics teaching, and Lived democracy in school have studied how the schools and (mathematics) education processes can imparting democratic values and active participation among learners (see, e.g., Breivega et al. (2019), Herheim et al. (2013), and Rangnes and Herheim (2019)).

Mathematics education research related to adopting a critical orientation and democratic values has also been prominent in Denmark. Ole Skovsmose introduced Critical Theory to the field of mathematics education, opening many possibilities to view mathematics and mathematics education by adopting a critical perspective. He named this critical approach to comprehend mathematics and mathematics education, and the roles played by these two in the socio-cultural-political contexts of a democratic society, as Critical Mathematics Education (CME). In several of his publications Skovsmose (1992, 1994b, 1998) has emphasized the relationship between mathematics education and a democratic society and stressed the need for imparting critical stance among mathematics learners so that they can become critical citizens in a democratic society increasingly dependent on mathematical knowledge. Other scholars such as Paola Valero (2004b, 2017) have also shed a critical light on mathematics education within a socio-political context and the ways in which mathematics learners role is interpreted in their classrooms (Valero, 2005). Analogously, research focusing on the themes such as, learners' agency in mathematics education, their reflections on Swedish national tests, incorporating democratic actions in a mathematics classroom, and their conflict with learners' values is also conducted in Sweden (see, e.g., Andersson and Norén (2011), Bagger (2016) and Andersson and Österling (2019)). In this section I have tried to capture selected studies urging to develop secondary school learners' critical stance towards, and autonomous involvement in their mathematics learning activities. A broader overview of research conducted under the 'Nordic model of mathematics education' can be found in Stedøy (2004).

Summing up the research studies mentioned above, a drive towards developing learners into actively participating, critically reflecting citizens living in, and thriving for a democratic society through learning mathematics can be noticed in the Nordic model of (mathematics) education. However, as Dahl and Stedøy (2004) acknowledge, the commitment of the Nordic model of mathematics education towards democracy, critical awareness, and social equality can be observed in inclusive schools and classrooms, but some classrooms and schools can be perceived as being *un-democratic* if "the pupils do not have influence on the working methods and the content" (p. 7). The authors further state that, "it is one thing to have a curriculum; classroom practice might be very different" (Dahl & Stedøy, 2004, p. 8), where teachers' background, textbooks and examinations play a more important role than the national curriculum and ideas it is built upon. This observation again underlines that including learners in the democratic process of decision-making about and critical evaluation of their educational activities is an aim stated in the *formal* mathematics curriculum, but it cannot be taken for granted as being the *experienced* mathematics curriculum (Goodlad et al., 1979).

It is this *experienced* mathematics curriculum which is the focus of this thesis. In this research study, I have explored "the *individual* dimension" of learners' critical perspectives towards learning mathematics, and their expressed experiences of autonomous involvement (having a partial control and influence[26] over their own learning) in their mathematics classrooms. Having learners studying in two lower secondary schools located in Central Norway as informants in this study, these questions report and bring forward their voices and perspectives concerning their mathematics learning experiences. To further position this study and state its significance in an international context, it is reasonable to look outside the Norwegian and Nordic models of mathematics education. The next section explores if concerns about learners' development as mathematically literate critical citizens can be traced in international mathematics education research as well.

2.5 Outside the Nordic bubble: The study in an international context

Looking outside the Nordic countries, it can be observed that developing critical citizenship skills among learners through their (mathematics) education and preparing them to live in and strive to maintain a democratic society is a concern far from being limited to the Nordic boundaries. The capabilities of critical reflection, taking initiatives, and collaboration (e.g., with peers or group members if working in groups) to analyse and solve problems arising in personal and social contexts are presented as essential skills to be possessed by all citizens of the society (see e.g., OECD (2019), Partnership for 21st Century Skills (P21) (2009), and UNESCO (2013)). Thus, several international educational organizations have stressed developing a critical stance and collaboration skills among future citizens of the society (young learners) as a fundamental aim of the education process worldwide (see e.g., European Commission/EACEA/Eurydice (2017), Leicht et al. (2018) and UNESCO (2015)).

These emphases and discussion however include some tensions. On one hand, soft skills (non-technical, personal attributes), such as, critical reflection, being autonomous (i.e., taking initiatives) and collaboration are underlined in international discussions about children's education. However, simultaneously the international assessment programmes such as, PISA and TIMSS emphasise the measurement of technical skills and content knowledge of school learners around the world. The results of these tests receive international socio-political attention and gain much publicity in printed and social media of many countries. PISA and TIMSS assess learners' knowledge in the subjects of mathematics and science (among other subjects, such as, reading) and thus mathematics is a central place in these assessments. Mathematics is recognised to have played a central role in developing our increasingly technological and digital society, and its economic growth. This recognition underlines the significance of keeping learners up to date with technical mathematical skills (e.g., algebra, statistics, geometry, etc.) (see, e.g., Skovsmose (1998) and D'Ambrosio (1999)). Moreover, research studies such as, D'Ambrosio (1990, 2007) and Skovsmose (1994b) assign an important task to mathematics for developing mathematically literate critical citizens in the democratic society.

The paradox concerning mathematics education in international discussion about learners' education and assessment is that: on one side, the socio-political credit given to technical mathematical skills for economic growth, international assessment programmes,

[26] Through co-responsibility, active participation and involvement in decisions concerning their own mathematics learning processes. See section 3.5.3 for a detailed discussion of the notion *learner autonomy*.

and for developing critical citizenship and democratic participation skills among learners emphasises the importance of teaching and learning of technical skills in mathematics classrooms. Whereas on the other side, developing soft skills of critical reflection, being autonomous, teamwork and collaboration requires learners to adopt a critical stance towards all the information they receive and experiences they gain, which may also include critically perceiving what they learn in mathematics, being involved, suggest changes in decisions about their own learning activities.

This socio-political importance given to technical mathematical knowledge supports teachers' interpretations as reported in Rønning (2004). The teachers asked expressed that they prioritise teaching mathematical content in classrooms rather than using time on developing learners' soft skills of critical reflection, taking initiatives, democratic values, decision-making, etc. Mathematics is interpreted by these teachers as an important subject requiring strict progression in acquiring technical knowledge of mathematical content (cf. Rønning (2004)). These paradoxes are further considered while presenting the discussions and implications of this study in sections 6.1.3, 6.1.4, and 6.2.1 of chapter 6.

2.6 So, what about the research focus?

The investigation of Norwegian Education Act, policy documents and historical development of formal curricula from M74 to LK20 reveal that Norwegian learners are supposed to adopt a critical perspective towards, co-operate in, and influence the decisions concerning their learning activities. It is also emphasised that, through their educational process, the learners develop an underlying outlook of criticality and comprehend their social and personal life contexts, and their worldviews. It was noticed that these competencies are not explicitly listed in mathematics specific curriculum, but the general part of the curriculum applies to teaching-learning of all the subjects in Norwegian schools. Through this exploration, I became aware that development of a critical perspective, a sense of autonomy and democratic participation of secondary school learners in their own educational activities is not only an objective of their educational process, but also a legal right of Norwegian learners. The Nordic model of education and in international discussions concerning learners' educational process also present similar aims.

The exploration of Norwegian education policy and curriculum documents, the Nordic model of education and international concerns about children's education conducted in this chapter support the inference made in section 1.5 after reading mathematics education research discussed in section 1.4. Developing learners' critical perspectives towards, and their autonomous involvement in decisions about their own learning activities is desirable, and these abilities are supposed to contribute achieving the aims of transforming learners into future critical citizens of a democratic society. Considering learners' perspectives, their critical reflections, experiences and heeding their voices is highly recommended. Thus, the exploration of national policy documents and educational aims discussed internationally also support the research focus of this thesis. In this study I explore individual learners' critical perspectives towards learning mathematics, and their expressed autonomous involvement in decisions concerning their mathematics learning activities. This research study can be placed under, *Mathematics as a source of values and attitudes* link between mathematics education and democracy, put forth by Aguilar and Zavaleta (2012).

3 Central concepts and theoretical underpinnings

The aim of this study is to bring forward individual learners' voices informing their critical perspectives towards learning mathematics, and to identify their potential of influencing decisions concerning their mathematics learning activities. Learners' critical perspectives are investigated by asking them to reflect over what, why and how they learn mathematics in their classrooms. In addition, learners' potential of influencing decisions is explored by asking them about their involvement in decisions taken in their mathematics classroom, and to assume learner autonomy to express their choices and suggest changes in their mathematics teaching-learning activities. To position this study theoretically, I explore the theoretical paradigms of cognitive constructivism and Critical Theory in which an individual's capacity of being critical is discussed. Based on the research literature, I present different domains of human functioning in which an individual's capacity of being critical can be exercised and the possibility of moving from one domain of being critical to another. This movement includes transitioning from one theoretical position (i.e., cognitive constructivism) of observing individual's virtue of criticality to another theoretical position (i.e., Critical Theory), and the phases one may experience in this transition. Further, I discuss the role played by the social element (i.e., interactions with others, one's social, cultural, and political contexts) during this transition, and argue for positioning this study on a theoretical pathway going from social constructivism to critical pedagogy (Critical Theory).

The process of positioning this thesis theoretically has not been straight forward or linear. A shift in the focus of the study (from cultural to critical) combined with my limited familiarity with research literature[27] in the beginning of my PhD research period are the reasons of the theoretical route this study has taken. This theoretical journey may seem twisted and incompatible, but it has nevertheless provided significant insights into individual learners' perspectives about their mathematics learning processes. In the following text, I first explain my choice of the word *learners* instead of *students* to address the subjects of (mathematics) education before I discuss the *individual* and *collective* dimensions of learners being critical. Following this discussion, I present the theoretical uncertainty encountered while doing this research and the choice of theoretical positioning of this study between social constructivism and Critical Theory.

3.1 The 'learner' or the 'student'?

Approximately in the middle of my research work, I shifted to using the word *learners* instead of *students* to address the subjects of (mathematics) education, and hence the informants of this study. The first two papers written in the earlier phase of this research (Paper III (5.3) and Sachdeva and Eggen (2019)), address the subjects of education as *students*, whereas a conscious choice to call them as *learners* is made in papers (Paper I (5.1) and Paper II (5.2)) and in this thesis. The discussions about using the right word to address the subjects of education have been on philosophical as well as on pedagogical level (Biesta, 2010; Lieb, 2018; Sanders, 2012). Since the scope of this thesis concerns the field of education, I will elaborate on the pedagogical side of this discussion. To do so, I choose the paper written by

[27] Related to CME, the socio-political issues in mathematics education research, critical pedagogy, and Critical Theory.

Biesta (2010) as the starting point because the paper discusses two most used terms to address the subjects of education, and this thesis also shares its explicit focus on *voice* of the learners and democratic education with this paper.

Biesta (2010) presents and explains three different ways to address the subjects of education, that is, the *student*, the *learner*, and the *speaker*. He explains that the significance of using right word to address subjects of education is not just a matter of language, but "It is […] a matter of […] pathways of meaning and association" tied to these words which may lead easily to some other words (Biesta, 2010, p. 540). In the context of emancipatory education, Biesta (2010) problematizes using the word *learners* to address the subjects of education as he argues that "the learner is constructed in terms of a *lack*. […] The learner is the one who is not yet complete" (p. 541, italics in original). It is further contended that the assumption behind using the word *learner* is that the subject of education is missing something (a skill, knowledge, craft, competence, autonomy, etc. which can be learnt), which he/she must learn from a master or expert (of that knowledge or skill). This lack of knowledge makes the subject of education dependent upon the expert. Thus, it is argued that using the word *learners* does not confer equal status or power to the subjects of education in relation to their educators and establishes a power relationship between them in which the educator's intelligence[28] is more mature than his/her subjects of education. Educator's intelligence can be seen as more mature than those s/he educates in terms of development of a child's brain, or in terms of being able to understand the complexity of subject matter being taught.

In contrast, Biesta (2010) prefers the word *students* to refer to the subjects of education. The justification of this preference is given by using the example from *The ignorant schoolmaster*, the work of Jacques Rancière (1991). In this example schoolteacher Joseph Jacotot is a central figure. Jacotot was an exiled French schoolteacher, who developed an educational approach called the 'universal teaching' while he was teaching French to Flemish students, whose language he did not speak. Jacotot insisted his Flemish students to study the bilingual edition of a novel (written in French and Flemish) to learn French. The factor which made the subjects of education *students* in this example is not that they learnt French without an expert, but that they learnt it without any explanation from the expert who had the knowledge of French. Biesta (2010) highlights that Jacotot did not teach anything to his *students* (since they did not have any shared language), but the *students* still managed to speak and write French because of their own engagement with that bilingual novel. The ignorant schoolmaster had only summoned the learning capacity of his *students*. He considered their intelligence to be equal to his own and demonstrated to the *students* their capacity to learn by themselves.

In this case, the intelligence of the subjects of education was not dependent on their expert's intelligence, and these *students* followed their *own* will. This kind of education, argues Biesta (2010), is an exercise of their liberty, in which the *students* learn by studying on their own, *not without an expert but without having an intelligence that is superior to their own intelligence*. This kind of education is what Biesta (2010) terms as emancipatory education in which the intelligence of the expert and the subjects of education are equal. The task of the educator becomes not to demonstrate that the *learners* are incomplete or lack the knowledge possessed by the educator, but the educator should only ensure that *students*

[28] Biesta (2010) employs the word "intelligence" to indicate *learner's capacity to learn*. The arguments to support the claim that the educator's capacity to learn (or intelligence) is higher than those s/he educates can be developmental (i.e., the learner's brain, frontal lobes, are not mature enough to learn complicated content), or curricular (i.e., the subject matter is too difficult that it needs to be broken down by the educator).

make efforts to learn by themselves. Therefore, "The one who is the subject of education is summoned to *study* […] has become a *student*" (Biesta, 2010, p. 545). It is against the backdrop of democratic and emancipatory education that Biesta (2010) suggests using the term *speakers* to address the subjects of education. When the subjects of education are addressed as *speakers* and are considered to produce *voice*[29], it is only then the process of education can become emancipatory and can liberate its subjects and their intelligence(s).

Considering the backdrop of emancipatory education, the arguments put forward by Biesta (2010) to address the subjects of education as *students* (and so as *speakers*), and not as *learners* are reasonable. However, there are some tensions that can be observed in Biesta's (2010) distinction between the terms *learners* and *students* (and *speakers*). Especially, when considering the curricular argument[30] used to claim that an educator's intelligence (capacity to understand and learn complex subject matter) maybe more mature than those being educated, the claim seems difficult to refuse. Bearing in mind the organisational, institutional, and administrational infrastructure of most national and international educational systems, beginning the process of education by considering the subjects of education as *students* in Biesta's (2010) sense seems very difficult.

The *first question* is if it is reasonable to expect and would it be practicable that the subjects of education right from the start of their educational process (little children) in a school can be the *students* alike the ones taught by Jacotot (i.e., whose intelligence can be summoned to learn what they want or are expected to learn). This question can be better understood by relating it to the prerequisite of *understanding the instructions* of the expert to become a *student*. According to Biesta (2010), "In order for the ignorant one to do the exercises commanded by the master, the ignorant one must already understand what the master says" (p. 550). This requirement of understanding "what the master says", that is, the language of instruction brings up the dilemma inherent in considering the subjects of education as *students* right from the start of their educational process without being *learners* before or parallelly to being *students*. The concerns arising in this case are: how can the intelligence of the subjects of education in schools be summoned to study if they do not understand the language of instruction, and they are not considered as the learners of that language? Can the subjects of education be left on their own to discover the meaning of their language of instruction after an expert has summoned their intelligence to do so? Is it reasonable to expect that little children will acquire and comprehend their language of instruction by themselves (become *students*) without ever lacking an understanding of the meaning, sentence structure, and grammar of that language from an expert (and hence being *learners* of that language)? Is the educational infrastructure supportive of considering the subjects of education as *students* right from the start of their educational processes without being *learners* earlier or simultaneously to being *students*?

The *second question* concerns Biesta's (2010) acknowledgement that the route taken by *students* when summoned to use their intelligence is *unknown*. Pertaining to this unknown route, how can it be avoided that the *students* reinvent the wheel? I do not suggest that the subjects of education should not be let free to explore knowledge on their own by asking this question, but the dilemma arises what an educator should do if he/she detects that *student(s)*

[29] Not in the sense of producing sounds from their mouth, but in the sense of having their own perspectives, points of views, being equally intelligent partners in their own educational processes (Biesta, 2010).

[30] I agree with Biesta's (2010) position that the developmental argument (i.e., the maturity of frontal lobes of children's brain) is not a reason to claim that an expert's intelligence can be superior than the subjects of education he teaches. Thus, the concerns I raise here are related to the curricular argument only.

may reinvent the wheel on the route they have taken? Should the educator let the subjects of education be *students* in this case and continue making the unnecessary effort, or should he/she consider the subjects of education as *learners* in that moment and advise their intelligence by being an expert?

The *third question* in educating *students* in Biesta's (2010) sense relates to the current model of mass education. One may ask how can the model of emancipatory education suggested by Biesta (2010) be adopted, and if it fits well with the model of mass education? How can adopting the emancipatory model of education simultaneously safeguard the concerns of educating children to become *students* in Biesta's (2010) sense and ensure that there is enough workforce to support society's economic growth, and the socio-economic status of discriminated and marginalised groups gets also uplifted? The *fourth question* concerning the distinction between *students* and *learners* relates to the educator's subjective intention, meaning, and understanding he/she has of these words. It is possible that an educator may use the word *learners* to address his/her subjects of education but views them as active subjects and creators of their own knowledge, having their own understandings, points of views and their own *voice*. It is also possible that an educator may use the word *students* to address his/her subjects of education but views them as empty vessels or bank accounts (to use Freire's (1972) words) in which he/she can deposit the knowledge.

Based on the questions and dilemmas highlighted above, I find it difficult to draw clear boundaries between being a *student* and being a *learner*. Understanding the subjects of education as being *students* to be mutually exclusive from them being *learners* is problematic for me. Moreover, it seems unreasonable to expect that the subjects of education in schools can be *students* right from the start of their education process without ever being *learners* of any knowledge or skill earlier or avoid being *learners* and *students* parallelly in all the domains and situations of our life. Based on the discussion above, I do not view the identities of being a *learner*, *student*, and *speaker* as separate domains of the life of the subjects of education, but as a process in which the aim of education can be seen as to develop and evolve from being a *learner* to becoming a *student* and a *speaker* (in Biesta's (2010) sense). It seems difficult to avoid crossing over these identities of being *learners*, *students*, and *speakers*. At different stages of their educational process and in specific contexts, the subjects of education may need to be learners *before* being students and speakers, to be students *before* being learners and speakers, to be speakers *before* being learners and students, or be learners, students, and speakers *parallelly*. In essence the order of being learners, students and/or speakers may not be fixed, and these identities may not be locked up in boxes disjoint from each other.

Following the argumentation above, using the right word to address the subjects of education matters less than the educator's understanding and intent of using that word. As Biesta (2010) and Freire (1972) emphasise, educators should consider the subjects of education as independent individuals having their own points of view and their own *voices* rather than being the objects of education (i.e., empty bank accounts) in which the deposit of knowledge can and should be made. Further, striking a balance between the requirement of mass education and education for emancipation also requires consideration. Therefore, the subjects of education can be viewed as independent individuals and accepted in their roles as both *learners*[31], *students*[32], and *speakers*[33] at different stages of their education process. The

[31] When they require explanations.

[32] When they learn something on their own after their intelligence is summoned.

[33] Who can already *voice* their opinions, critique, disagreements, rejections, etc.

aim of emancipatory educational process could be to ensure that these independent *speakers* start the school as *learners* and leave the school as *students*.

I prefer using the word *learners* to address the subjects of education as I have argued above that being a student straight away seems problematic to me, and since the current educational system can better facilitate the journey from being a *learner* to a *student* than the other way round. In addition, the reason for preferring the word *learner* is the active connotation that can be attached to the term *learner* as described by Sanders (2012). Lieb (2018), cites Sanders (2012), and suggests that the distinctions between the student and learner identities carry with them existential undertones and point to self-identification as "an objectified follower (student) or as a subjective leader of one's own learning processes. Essentially, the *student* is passive while the *learner* has agency and is self-empowered" (p. 21, italics in original). There is no consensus on the difference between these two terms, and both are preferred or criticized, but I understand the term *learner* to mean an active, self-motivated, and self-empowered subject of education. Therefore, I address the informants of my study as *learners* voicing their own mathematics learning experiences.

3.2 *Individual* and *collective* dimensions: uncertain theoretical position

I mentioned the *individual* and *collective* dimensions of learners being critical, thinking critically, and actively participating in decisions concerning them personally or socially. I identified these dimensions after investigating the use of words 'critical' and 'critical thinking' in the formulations of general (/overarching) and mathematics specific curriculum guidelines of Norway (see chapter 2, section 2.3). These formulations emphasise developing a critical stance among learners, and their involvement in decisions affecting them, to become an individual critical thinker (i.e., using one's logical reasoning and argumentation skills), and to become a critical citizen (i.e., actively participate and assist in maintaining social justice and equality) in a democratic society.

When I started searching literature to understand learners' responses, analyse the data, and write about the findings, the first keyword I used was 'critical thinking'. The reason of using this keyword was that it was employed in the Norwegian Education Act, the general part, and the mathematics specific part of the Norwegian curriculum LK06. Another reason was the possibility to understand and further examine individual learners' responses such as, "I do not know", "I have not thought about it", "Nothing" or "Everything" when asked about their experiences and aspirations regarding their mathematics learning. The meaning and understanding in which the word 'critical thinking' was employed in the Education Act was, however, broader than the sense in which it was used in the mathematics specific curriculum of LK06. The word 'critical' in the general part denoted the concern to impart a critical outlook among learners to understand and view the world around them and avoid accepting common truisms uncritically without questioning. Whereas the word 'critical thinking' (both in general and mathematics specific parts of the curriculum) was used in a narrower sense to direct the focus towards judging the logical nature of arguments, facts and conclusions (see chapter 2, section 2.3 for more details).

At the early stage of my PhD study, I had limited knowledge about the (mathematics) education research literature or literature related to the philosophy of science due to my background being from applied mathematics. Therefore, I understood the broad usage of the terms 'critical' and 'critical thinking' in the general part of LK06 as referring to both, the

collective (questioning one's personal and socio-political contexts) and the individual dimensions (using one's logical reasoning and argumentation skills to reach an objective conclusion) of being critical and participating in the decision-making processes. Whereas the narrow usage in mathematics specific part of LK06 was understood as referring only to the individual dimension of being critical and participating in the decision-making processes.

Due to time constraints related to LOCUMS, I moved forward in my research with my own ambiguous interpretations of the words 'critical' and 'critical thinking' and interviewed selected individual learners to know more about their experiences, reflections, evaluations, and suggestions to influence their mathematics learning activities. Personally, I understood the term 'critical thinking' in a broader sense as individual learners' potential of reflecting over and evaluating their mathematics learning experiences critically; and participating in decision-making processes to improve their mathematics learning activities. However, I struggled with the theoretical positioning of this study at that time.

Gradually, I got aware of the different possible interpretations of the words 'critical' and 'critical thinking', distinct theoretical underpinnings and educational research traditions focusing on the individual and collective competencies of being critical and participating in decision-making processes. In educational research literature, these competencies are often placed under *two educational approaches* namely: the *critical thinking* approach and the *critical pedagogy* approach. "Each invokes the term "critical" as a values educational goal: urging teachers to help students become more sceptical toward commonly accepted truisms. Each says, in its own way, "Do not let yourself be deceived."" (Burbules & Berk, 1999, p. 45 , quotes in original). The critical thinking approach suggested for instance by, (Ennis, 1964), (Siegel, 1980) and (Facione, 1990, 1992) emphasises developing the *individual's* competence of logical argumentation, evaluate unsubstantiated truth claims, seek evidence and information, make inferences, critical self-reflection, etc. among learners. Whereas the critical pedagogy approach proposed for instance by, (Freire, 1972), (Giroux, 1993) and (McLaren, 1994) emphasises developing a *collective* critical consciousness among learners to promote critical citizenship and active democratic participation through their educational processes.

I also experienced this distinction while reading mathematics education research studies. In mathematics education research, several studies (see, e.g., Agoestanto et al. (2017), Aizikovitsh-Udi and Cheng (2015) and Applebaum and Leikin (2007)) have investigated the individual dimension of learners being critical and decision-making (to judge conclusions of mathematical models, statistical inferences, etc.) in correlation to critical thinking approach. Further, the collective dimension of learners being critical and decision-making (to reflect over social injustice, social discrimination, etc.) is discussed in correlation to critical pedagogy approach in other research studies (see, e.g., (D'Ambrosio, 1990), Skovsmose (1992, 1998), and Skovsmose and Valero (2001, 2005)). After reading this literature, my understanding of the individual dimension of being critical, participating and being involved in decision-making processes resonated with the critical thinking approach, and that of the collective dimension of being critical, participating and being involved in decision-making processes resonated with the critical pedagogy approach. Despite this good fit, my confusion regarding the theoretical positioning of this research study increased after getting to know that the critical thinking (i.e., my individual dimension) and critical pedagogical approach (i.e., my collective dimension) are placed under two distinct theoretical paradigms, often considered as incompatible (see section 3.3.1).

The reason of this uncertainty was my interest in highlighting *individual* learners' *critical* perspective towards learning mathematics as a *social* process, and their expressed

experiences of participation and involvement in decisions concerning their mathematics learning activities. The data I collected involved individual learners' perspectives and their voices about their mathematics learning experiences. Though their experiences were *individual* and their thoughts potentially *critical*, but the contexts of their learning, the interviews and the tool used during the interviews to understand their voices (that is, the 'language') were *social* and cultural in nature. Therefore, my perspective of looking at the data I had did not fit entirely well either with the critical thinking approach or with the critical pedagogical approach. Moreover, I struggled to find suitable data analyses tools to interpret individual learners' interview responses focusing on the *individual* (but not cognitive or collective) dimension of being critical.

This theoretical uncertainty, struggle with finding suitable data analyses tools and having data highlighting individual learners' voices and experiences of phenomena (both learning and interviewing) taking place in social contexts made it difficult for me to position this study in the theoretical landscape. My theoretical struggle is also visible in the concepts and notions used in this thesis and the papers attached to it. These writings employ the terms indicating both the individual (e.g., critical thinking, beliefs, and learner autonomy), and the collective (e.g., critical citizenship, autonomous involvement, and democratic participation) dimensions of learners being critical and participating in decision-making process. These processes may influence the learners personally as individuals as well as socially as citizens. A greater emphasis on individual dimension however can be observed throughout the thesis pertaining to having individual learners' interview responses as units of data analyses, and to understand and interpret individual learners' voices and experiences of learning mathematics (for instance, using critical thinking skills framework for data analysis in Paper I). Nonetheless, using concepts and terms from both critical thinking and critical pedagogical approaches, and having the *social* element as well made it challenging (but worth the struggle) to position this study in the theoretical landscape.

3.3 The theoretical positioning of the study

In this section, the differences, and similarities between the theoretical underpinnings of the critical thinking and critical pedagogical approaches are elaborated based on the research literature, Barnett (1997), Burbules and Berk (1999), Davies (2015) and Johnson and Morris (2010). Both Barnett (1997) and Davies (2015) have written in the context of higher education, but pertaining to the educational aims of imparting critical stance and democratic participation in secondary school learners, this literature is also relevant to be discussed in the context of secondary school education. The theoretical incompatibility between the philosophy of science traditions underlying critical thinking and critical pedagogy approaches as highlighted in this literature is also presented. This discussion is continued by introducing the *social* element of this research study and positioning this social element in correlation with the theoretical traditions underlying the critical thinking and critical pedagogical approaches. Conclusively, the theoretical positioning of this study in relation to these different theoretical traditions is presented and clarified.

3.3.1 Theoretical incompatibility? – Critical thinking and critical pedagogy traditions

The individual and collective dimensions of being critical and participating in decision-making interpreted by me can be placed under two theoretical paradigms, that is, *cognitive constructivist* (critical thinking) and *Critical Theory* (critical pedagogy) respectively. Burbules and Berk (1999) suggest that the idea of being critical in the critical thinking (cognitive constructivist) tradition is seen as an individual being "more discerning in recognising faulty arguments, hasty generalizations, assertions lacking evidence, truth claims based on unreliable authority, ambiguous or obscure concepts, and so forth" (p. 46). The critical pedagogy tradition however differs in the conception of being critical. In critical pedagogy, specific belief claims are not considered "as propositions to be assessed for their truth content, but as parts of systems of belief and action that have aggregate effects within the power structures of society. It asks first about these systems of belief and action, *who benefits*?" (Burbules & Berk, 1999, p. 47). In other words, the capability of thinking critically in a critical theoretical tradition concerns one's ability to be critical about accepting the ways of society, take into consideration the social context around a problem, addressing issues of social injustice, power, and acting against power, etc. Johnson and Morris (2010) and Davies (2015) also note the same differences in the understanding of being critical, and what it entails in literature concerning critical thinking and critical pedagogy as Burbules and Berk (1999).

Burbules and Berk (1999) further indicate that the term 'critical thinking' is employed in the research studies investigating the cognitive and psychological domain of human behaviour stemming from the cognitive constructivist paradigm. Whereas the term 'critical citizenship' is often observed in the research literature concerning critical pedagogy stemming from the Critical Theory[34] paradigm. Johnson and Morris (2010) support the same by suggesting that critical thinking literature emphasises an *individual's* trait of applying logic and being able to reach *objective*, nonpartisan, and sound conclusions (i.e., in the cognitive sphere of human activity). However, critical pedagogy literature emphasises an individual's trait of living as a critical citizen in the society (i.e., in the socio-political sphere of human activity). The critical pedagogical approach aims at fostering a *collective* critical capacity in citizens to take sides of the *subjects* facing discrimination and injustice, and enabling them to stand against such injustice, unfair power structures and discrimination. These theoretical paradigms are thus seen as being relatively incompatible in the literature.

3.3.1.1 The core virtue of criticality

The critical thinking and critical pedagogical traditions direct their attention to being critical in different domains of human functioning in the world (i.e., cognitive, and socio-political), but one can question if the *core virtue of criticality*, (i.e., an individual's core capacity of *being critical*) is also different in different domains of human functioning in the world. In his work, Davies (2015) refers to the term *criticality* as "a term of fairly recent origin" (p. 64), and employs *criticality* deliberately as a neutral word implying "no particular account of critical thinking or theoretical emphasis" (p. 63). Barnett (1997) puts forward the notions of 'criticality' and 'critical being' and defines an individual's capability of thinking critically in terms of taking a critical stance. He claims that "Critical persons are more than just critical thinkers. They are able to critically engage with the world and with themselves as with

[34] "The roots of critical pedagogy lie in the critical theories of the Frankfurt School […]" (Johnson & Morris, 2010, p. 79).

knowledge" (Barnett, 1997, p. 1). Davies (2015) cites Barnett (1997) in elaborating that the term "criticality" […]:

> "extends beyond the individual to the individual's participation in the world, i.e., in the form of responsible citizenship. This is a concept of critical thinking involving students reflection on their knowledge and simultaneously developing powers of critical thinking, critical self-reflection and critical action—and thereby developing (as a result) critical *being*" (p. 65, italics in original).

An individual's criticality or the "critical spirit" therefore comprises of three elements, *critical thinking*, *critical reflecting* and *critical acting*. Barnett (1997) identified three domains of an individual's life in which s/he may exercise his/her criticality, namely, critical reason/thinking (examining knowledge and ideas), critical self-reflection (examining experiences of self) and critical action (examining action in the world), but the "critical spirit" of the person who is *being critical* in these domains is the same. This relationship is also visualised as a Venn diagram involving three interlocking circles (see Figure 1), in which the "critical person" is located at its core.

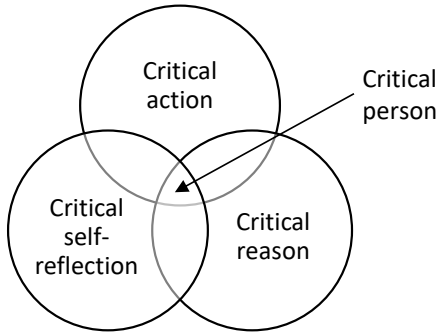

Figure 1 Critical being as the integration of the three forms of criticality (critical reason, critical self-reflection, and critical action, figure reconstructed with permission from Barnett (1997), p. 105).

Burbules and Berk (1999) also describe that though critical thinking and critical pedagogy traditions suggest some different ways and domains in which critical beings can exercise their "criticality", but at broader level, both share some common concerns.

> "[…] both critical thinking and critical pedagogy authors would argue that by helping to make people more critical in thought and action, […] educators can help to free learners to see the world as it is and to act accordingly; critical education can increase freedom and enlarge the scope of human possibilities" (p. 46).

Likewise, though Johnson and Morris (2010) acknowledge that a strong socio-political element encouraging the learners to engage with issues of power, inequality, and a call for critical action to transform the society differentiate the theoretical positions of critical thinking (apolitical) and critical pedagogy (socio-political). However, referring to Barnett (1997), Johnson and Morris (2010) also assert that "the boundaries between critical thinking and critical pedagogy have thus become blurred" (p. 80). They have also identified three elements

which would seem common in literature concerning both critical thinking and critical pedagogy (see Figure 2), i.e., dialogue or argument, logic, and discovering 'new' knowledge.

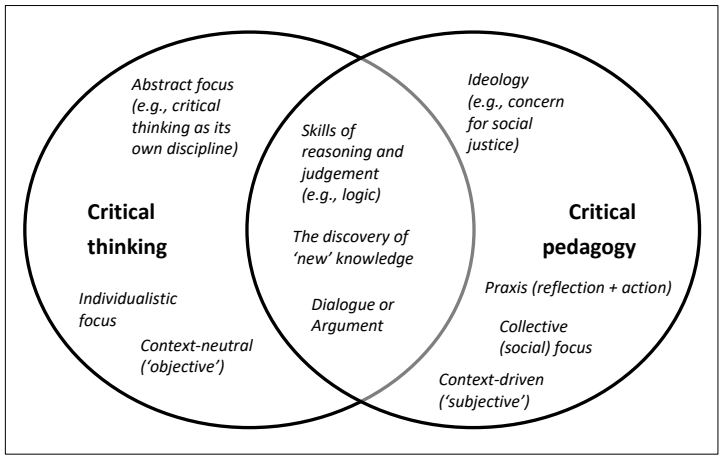

Figure 2 Intersections between critical thinking and critical pedagogy (reconstructed with permission from Johnson & Morris (2010), p. 80).

Consequently, the research literature cited above discussing critical thinking (i.e., cognitive constructivism) and critical pedagogy (i.e., Critical Theory) traditions suggest that an individual's *core virtue of being critical* and the aim to make people more critical in thought and action through critical education is the same and equally emphasised in both these traditions. The situations and contexts towards which the critical person diverts his/her criticality may be different, but imparting a stance of criticality among learners and transforming them into critical beings are the common objectives of both these positions.

It can therefore be argued that the theoretical paradigms of cognitive constructivism (underlying critical thinking approach) and Critical Theory (underlying critical pedagogy approach) are not two mutually exclusive positions locked in separate boxes when considering individual learners' capacity of being and participating critically in the world. The research studies discussed above demonstrate that the boundaries between these theoretical underpinnings are not solid. These boundaries have several openings and crossing through them is possible. In other words, it is difficult to state and differentiate clearly that when does one stop being critical thinker and start being a critical citizen, or vice-versa. The common concern of critical thinking and critical pedagogy traditions to educate *critical beings* make the boundaries between them flexible and permeable.

My research interest in this thesis is to explore individual learners' capacity of being *critical* about learning mathematics, their expressed involvement in decision-making processes, and their potential of suggesting changes in their mathematics learning activities. Following the reasoning above, this research interest as well as the data collected in this study do not fit entirely well either with the individual focus of critical thinking approach, or with the collective focus of the critical pedagogy approach. As a result, I want to position this research study in this "common and shared space" (visualised in Figure 2) between these theoretical positions. I argue that the *social* element present in my research study can be positioned in this common space since the 'individualistic focus' starts turning into a 'collective

(social) focus' when one crosses the boundary of critical thinking (cognitive constructivism) and enters in the domain of critical pedagogy (Critical Theory), and vice versa. I also argue that this transition passes through the theoretical paradigm of social constructivism if a theoretical pathway is stretched out from the cognitive constructivist paradigm to the Critical Theory paradigm. In the next section, I discuss the possibility of stretching out such a pathway between cognitive constructivism and Critical Theory passing through social constructivism and positioning this study theoretically between social constructivism and Critical Theory.

3.3.2 Extension and transition of the virtue of criticality

The cognitive constructivist and Critical Theory traditions emphasise being critical in the intellectual and socio-political domains of human functioning respectively, but I have reasoned that an individual's core virtue of criticality is the same. Based on this argument, one can further ask that how this 'same' virtue of criticality is then exercised in these distinct domains of human functioning. It is this question of *extension* and *transition* of the virtue of criticality which is dealt with in this section. There are other questions[35] and concerns related to individual's capacity of being critical which can be discussed with respect to the cognitive constructivist and Critical Theory paradigms of philosophy of science. However, due to my research interest in exploring individual learners' potential of being critical towards and being involved in decisions regarding their own mathematics learning activities, I discuss if it is possible to extend the domain of exercising one's criticality from cognitive to critical. In case such an extension is possible, I examine: (a.) which phase(s) of transition may the virtue criticality pass through while travelling from the cognitive to the socio-political domain of being critical, and (b.) what is the role played by the *social* element in this phase of transition.

This query is relevant for my research interest since I have not asked the informants of my study to be critical in an entirely cognitive sense (e.g., logical reasoning while solving a mathematical task), nor in a sense where they reflect over the role mathematics plays in their socio-political contexts (e.g., in relation to power relations, or to study social inequalities). Instead, the informants are asked to critically reflect over what, why and how they learn mathematics, and their expressed experiences of having autonomous involvement in decisions concerning their learning activities. This research focus, thus, involves exploring learners' critical self-reflection where the object of their reflection is the social process of their mathematics learning and the socio-political contexts associated with this process. Consequently, though learners' reflections will be individual (and cognitive), but these reflections are critical and about the social and socio-political contexts with which the learners interact.

To discuss the possibility extending the domain of the virtue of criticality, I put forward a hypothesis: A critical citizen ⇔ a critical thinker? This hypothesis can be elaborated as: (a.) Does being a critical thinker imply that one is also a critical citizen? and (b.) Does being a critical citizen imply that one is also a critical thinker? These questions can be asked on the background of individual (cognitive) and collective (socio-political) dimensions of developing learners' criticality which are focused by critical thinking and critical pedagogical traditions

[35] For instance, the scope and limitations of *being critical* in critical thinking and critical pedagogy traditions, transferability, and transition of the virtue of criticality from one domain to another, the direction of such transferability and transition, etc.

respectively. Researchers such as Barnett (1997), Burbules and Berk (1999), Johnson and Morris (2010) and Davies (2015) demonstrate that despite unlike focus, the boundaries between the cognitive and socio-political domains of exercising criticality are *not* seen as being strict and rigid by *all* the experts who belong to these traditions.

Barnett (1997) suggests that critical thinking seen as using the cognitive skills by individuals without moving towards critique is not sufficient, it is "thinking without a critical edge" (p. 17). In reviewing the critical thinking and critical pedagogy traditions, Burbules and Berk (1999) also maintain that,

> "Both the skills-based view and the skills-plus-dispositions view [of critical thinking] are still focused on the individual person. But it is only in the context of social relations that these dispositions […] can be formed or expressed, and for this reason the practices of critical thinking *inherently* involve bringing about certain social conditions. Part of what it is to be a critical thinker is to be engaged in certain kinds of conversations and relations with others […]" (p. 49, italics in original).

On the other hand, viewing the socio-political element to be inherent in the critical pedagogy tradition, they state that, "For critical pedagogy, […] self-emancipation is contingent upon social emancipation. […] individual criticality is intimately linked to social criticality […]" (Burbules & Berk, 1999, p. 55). These statements clarify that being a critical thinker *inherently* involves engaging in *social* interactions, relations, circumstances, and an examination of such social circumstances is not considered a part of the critical thinking tradition. Whereas the critical pedagogy tradition does not perceive an individual being separate from one's social, especially political circumstances. In critical pedagogy tradition, being a critical thinker (individual criticality) is intimately linked to acquiring critical citizenship competency (social criticality).

3.3.2.1 From critical thinker to critical citizen: a transit through social constructivism?

Both the critical thinking and critical pedagogy traditions involve and include the *social* element, but the emphasis laid on the social is almost negligible in the critical thinking tradition, whereas the critical pedagogy tradition mostly attends to the (socio-)political perspective rather than the correlation between the social and the individual. The quote, "Critical thinking's claim is, at heart, to teach how to think critically, not politically; for critical pedagogy, this is a false distinction", form Burbules and Berk (1999, p. 55) also confirms the explicit focus that critical pedagogy places on the (socio-)political element. The correlation between the individual and social element in relation to individual's capacity of being critical in the world is highlighted in the research of Johnson and Morris (2010), and Davies (2015). These studies suggest extended models of critical thinking (Davies, 2015), and of critical citizenship education (Johnson & Morris, 2010) in which the domains of exercising individual's virtue of criticality are expanded, from individual (cognitive) to collective (socio-political). This expansion happens by including and passing through the *social* sphere of human existence and exercising individual's virtue of criticality in this social domain of one's functioning.

Davies (2015) presents the different movements of criticality (see Figure 3) with the help of several axis diagrams, including an individual axis and a socio-cultural axis of criticality as the axes of this diagram. Starting from what he terms as "the critical thinking movement", he continues to include both individual and socio-cultural elements which have been stressed

in literature discussing individual's capacity of being critical in the world and presents the following figure as his final version of the axis diagram:

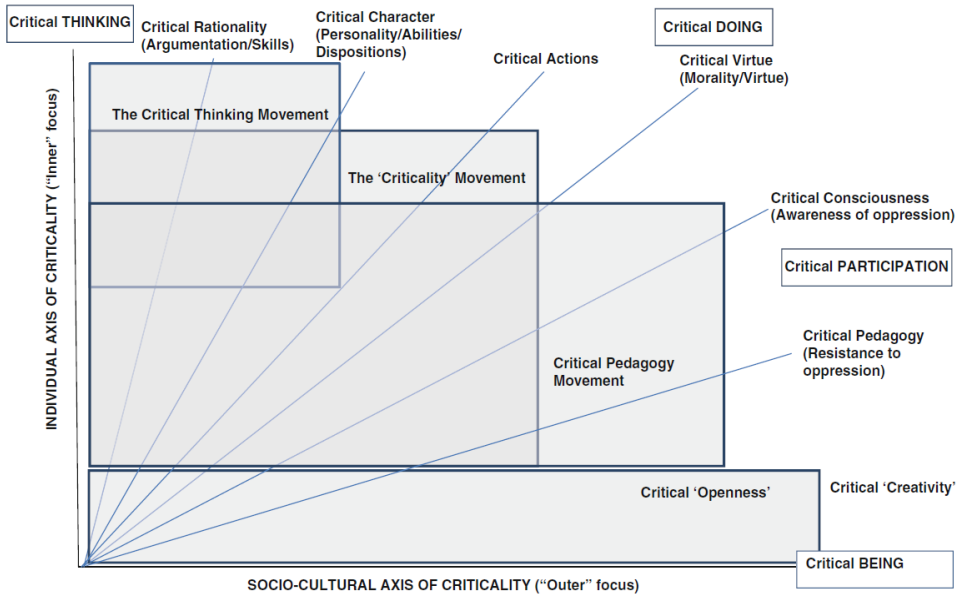

Figure 3 The axis diagram including the individual and socio-cultural elements of critical thinking.

Note. The extended version of axis diagram showing different "movements" of criticality reviewed by Davies (2015). From *Higher Education: Handbook of Theory and Research: Volume 30* (p. 82), by M. Davies, 2015, Springer International Publishing. Copyright 2015 by Springer International Publishing Switzerland. Reprinted with permission.

Including the individual axis of criticality (the "inner" focus), and the socio-cultural axis of criticality (the "outer" focus), this axis diagram (Figure 3) shows an expansion of domains of exercising individual's criticality in research literature discussed in Davies (2015). The extension of individual's domain of 'being critical' in this diagram starts from one's cognitive skills in the critical thinking movement to include more and more elements from one's socio-cultural world of existence up to the critical pedagogy movement and critical 'openness'. Likewise, in their framework for critical citizenship education, Johnson and Morris (2010, p. 90, Table 2) emphasise developing learners' *individual* independent critical thinking skills, as well as their *social* skills in dialogue, critical interpretation of others' viewpoints, active participation, and their capacity to reflect critically on one's 'status' in communities and society, and speaking with one's own voice.

The final model for critical thinking as proposed by Davies (2015) (see Figure 4) can be seen as expanding the spheres of exercising an individual's criticality from the individual (cognitive constructivism), to the social (social constructivism), and the socio-political (Critical Theory) domains of their functioning. The innermost circle in these concentric circles comprises of individual cognitive skills such as argumentation, and as it expands outwards the individual starts interacting with others, encountering his/her social contexts, and experiences his/her social and societal relationships – also, those of power, discrimination, and injustice. This model suggests that the development of learners' capacity of being critical can start from

an individual dimension and continue to reach the collective dimension (or the other way round), and during this development social interactions will take place.

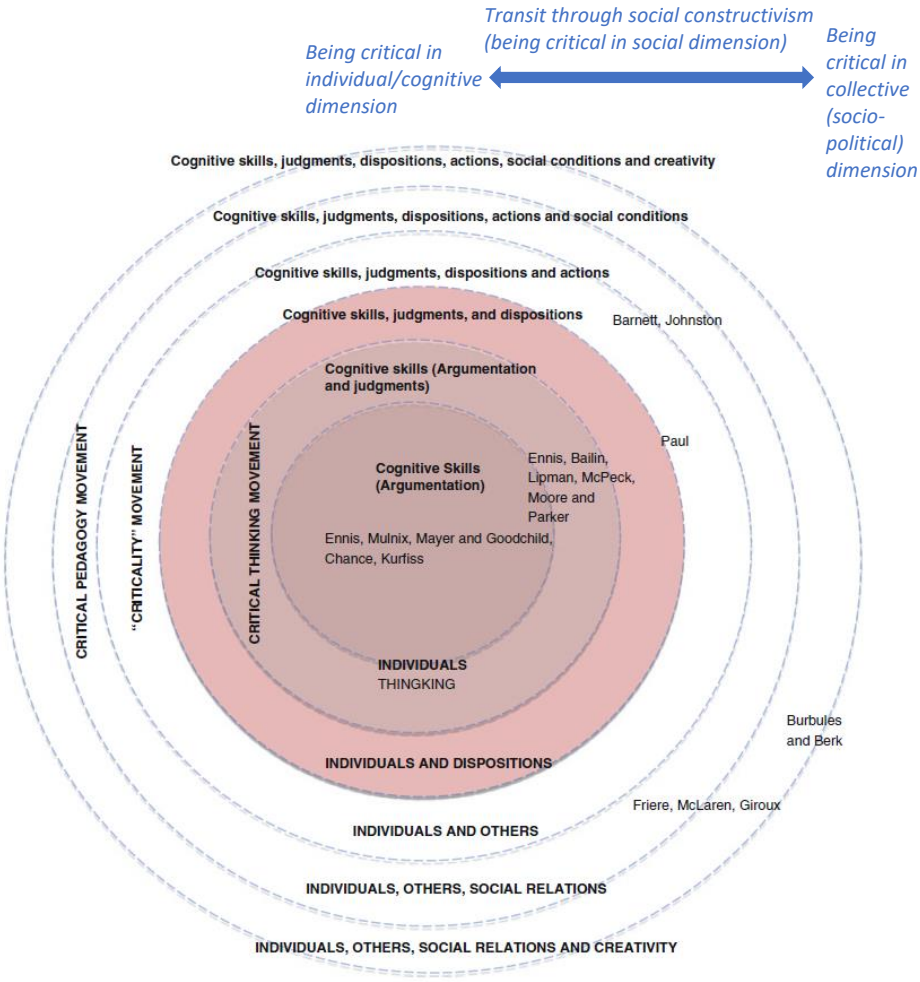

Figure 4 The proposed model of critical thinking in higher education.

Note. The extended version of axis diagram showing different "movements" of criticality reviewed by Davies (2015). From *Higher Education: Handbook of Theory and Research: Volume 30* (p. 82), by M. Davies, 2015, Springer International Publishing. Copyright 2015 by Springer International Publishing Switzerland. Reprinted with permission.

This phase of transition, where individual learners encounter and confront the *social* element in form of interactions with others or their social contexts is what I perceive as the transit of the virtue of criticality through *social constructivist* theoretical position. I have tried to represent this phase of transition with the help of a blue left-right arrow drawn in top right corner of Figure 4. This social constructivist position can therefore be seen as located between the cognitive constructivist and the critical pedagogical (Critical Theory) positions. In and through this transition, the learners interact and cooperate with others in social settings, and

learn to critically observe, analyse, and reflect over their own knowledge, thoughts, experiences, situations and status in their communities and society.

Social constructivism is a philosophical position accepting that both social interactions and individual meaning making play pivotal and crucial parts in an individual's knowledge construction process (Ernest, 1991, 1998). In reviewing the background traditions that emerged into social constructivism as a philosophical position, Ernest (1994) summarises that, "social constructivism originated in sociology and philosophy, with inputs also from symbolic interactionism and Soviet psychology, and subsequently it influenced modern developments in social psychology and educational studies […]" (p. 306). Pertaining to diverse starting points and encompassing different perspectives, social constructivism can refer to widely divergent viewpoints.

The works of Piaget and Vygotsky are often linked with social constructivism since both emphasise the importance of social interactions in learners' knowledge acquisition process. The Piagetian version positions individual's learning in the centre of the learning process, whereas social domain and social interaction is more valued in the Vygotskian perspective (Ernest, 1994; Jaworski, 2002). However, the common stance of these viewpoints is that "the social domain impacts on the developing individual in some crucially formative way, and the individual constructs (or appropriates) her meanings in response to her experiences in social contexts" (Ernest, 1994, pp. 306-307). In addition, "Social constructivism links subjective and objective knowledge in a cycle in which each contributes to the renewal of the other. […] Objective knowledge is internalized and reconstructed by individuals, […] to become the individual's subjective knowledge" (Ernest, 1991, p. 43), which then is used by the individuals to create new knowledge, and thereby completing the cycle. Following the framework of critical citizenship presented by Johnson and Morris (2010) and the model of critical thinking proposed by Davies (2015), it can be argued that the extension of individual's virtue of criticality (i.e., from being a critical thinker to a critical citizen, or vice-versa) transitions through the theoretical position of social constructivism (see Figure 4). I position this research study in this transitional pathway.

3.3.2.2 Positioning this research study and some reflections

The inclusion of and transition through the *social* element in Davies' (2015) model of critical thinking suggests that a theoretical pathway can be stretched out from the cognitive constructivist theoretical position to the critical pedagogy (Critical Theory) theoretical position. I represent such a pathway in Figure 5 below (the blue left-right arrow taken from Figure 4). This pathway transitions through the theoretical position of *social constructivism* with a *critical* element, represented by the white oval with permeable boundaries in Figure 5 below. This transition passing through social constructivism can expand individual learners' virtue of criticality from the left side (cognitive constructivism) of the blue arrow (i.e., being critical in individual dimension) to the right side (Critical Theory) of the blue arrow (i.e., being critical in collective dimension) in Figure 5 below. The transition can also happen from the right side (collective dimension) of the arrow to the left side (individual) of the arrow. Likewise, this transition will also pass through social constructivism, the social domain of learners' functioning and learning to be critical in their social interactions in the world. The white oval in Figure 5 below with permeable boundaries represents the theoretical position of social constructivism with a critical element.

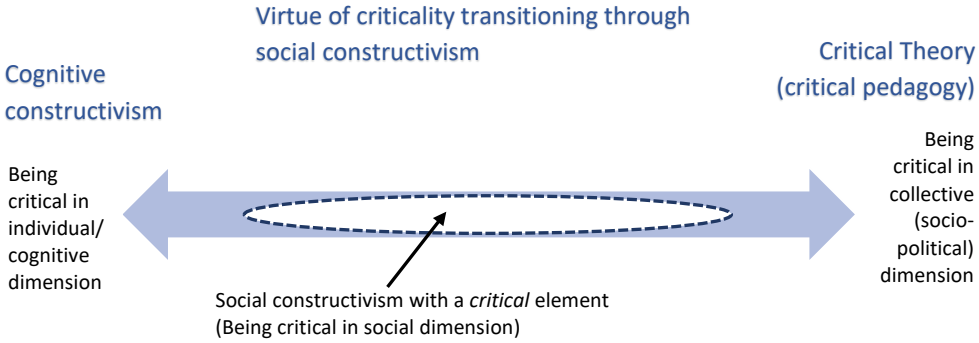

Figure 5 The virtue of criticality transitioning through theoretical position of social constructivism.

The research interest of this study lies in exploring *individual* learners' *critical* perspectives towards, and their involvement in decisions concerning their mathematics learning activities. The contexts in which learners acquire their learning experiences in mathematics are *social*. Considering this *individualistic* focus, the *social* contexts in which mathematics learning experiences are acquired, and an interest in knowing learners' *critical* self-reflections, I position this research study in the grey oval as drawn in Figure 6 below. This grey oval is placed within the theoretical pathway going from social constructivism to Critical Theory (critical pedagogy). The grey oval denotes that the theoretical position of this study is not a fixed point, but a "domain, in movement". This "domain, in movement" can oscillate between social constructivism and Critical Theory. My theoretical journey in this research project started from social constructivism but gradually moved towards the socio-political dimension more, as the underlying reasons of learners' responses unfolded (see 4.4). In moving back and forth, this "domain, in movement" (the grey oval) provided me with a conceptual framework including notions and literature stemming from both the research fields (e.g., critical thinking, self-reflection, beliefs, autonomy, agency, empowerment, critical citizenship, democracy, etc.) which I have employed to explore my research focus.

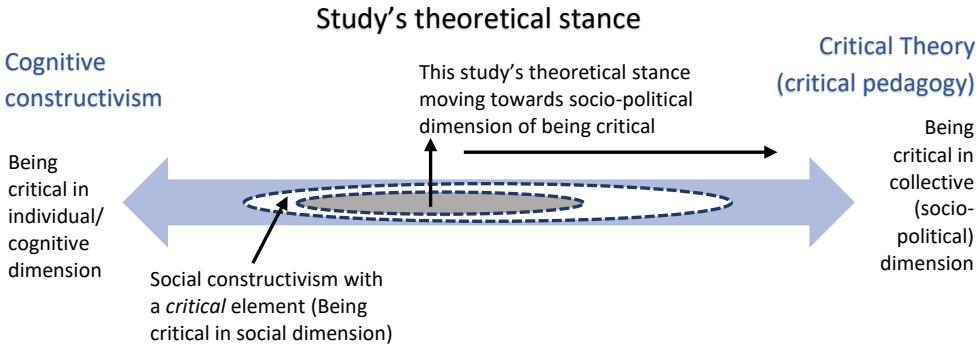

Figure 6 Theoretical positioning of this research study between social constructivism and Critical Theory.

This theoretical position (see Figure 6) resonates well with the focus of data collection on individual learners, the understanding of mathematics learning activities as social processes and with the interest to explore individual learners' critical thoughts and involvement in decision making in their mathematics classroom. It is also coherent with the research literature cited above in suggesting that individual learner's capacity of being critical can expand from one domain of human functioning to another (individual, social, and socio-political) and is beyond the boundaries of different theoretical paradigms (openness and creativity). This capacity is flexible and can be exercised in being critical individually (logical argumentation), socially (in one's social experiences and interactions with others), and collectively (critical engagement in interpreting and transforming one's socio-political contexts). Therefore, to understand and analyse this capacity, it was important for me to see the possibilities of transitioning between different theoretical paradigms despite the incompatibility and dissimilarities.

In positioning my study in this manner, I do not suggest that the theoretical positions of cognitive constructivism, social constructivism, and Critical Theory (critical pedagogy) are reconcilable or compatible. Rather my attempt is to visualise a spectrum of possibilities (the blue arrow, the white oval, and the grey oval in Figure 5 and Figure 6) which is available between these theoretical positions. Further, I suggest that an individual's virtue of criticality is not locked up in definite boundaries but can oscillate freely on the spectrum of possibilities. It can be placed on a specific position or may be a "domain, in movement" moving between possible positions available on this spectrum.

The theoretical journey made in this thesis may seem twisted and incompatible. Rethinking the choice of terminology now after getting to know the research literature related to critical thinking, social constructivism and critical pedagogy and realising the incompatibility of cognitive constructivist and Critical Theory paradigms, I recognize the potential of improvement in the theoretical underpinnings of this thesis (discussed in section 6.3). However, despite this realization and recognition, this research study is not void of a theoretical stance and its findings bear significant implications for mathematics education research (see section 6.2). This study brings forward individual learners' voices and critical reflections about their mathematics learning processes taking place in social contexts which provide significant insights into individual learners' perspectives about their own mathematics learning activities. In eliciting learners' viewpoints and critical reflections about learning mathematics, this study contributes to the research gap of exploring learners' voices, intentions and thinking in CME as highlighted by Lindenskov (2010).

3.4 Social constructivism and Critical Mathematics Education (CME)

This study aims to elicit individual learners' voices informing about their critical perspective towards learning mathematics, and their expressed experiences of autonomous involvement in decisions concerning their own mathematics learning activities. Theoretically, I position this research study as a "domain, in movement" (grey oval in Figure 6) between social constructivism that to critical pedagogy (Critical Theory). These two paradigms are placed on a theoretical pathway stretched between cognitive constructivism and critical pedagogy (Critical Theory). In the following text, I consider the connection between social constructivism and the concerns related to critical pedagogy voiced in mathematics education research in relation to the research focus and theoretical positioning of this study.

The research literature concerning CME and socio-political role of mathematics education (see, e.g., D'Ambrosio (1990), Aguilar and Zavaleta (2012), Skovsmose and Valero (2002) and Gutstein (2006)) promotes imparting critical citizenship, active democratic participation, action against social inequality, learners' empowerment, etc. among learners through their mathematics education. The concerns of developing a critical citizenship and active democratic participation among learners through their mathematics education are encouraged, but there are also accompanying contradictions. Skovsmose and Valero (2002) discuss the paradoxes of inclusion and citizenship in relation to the call for democratic participation and critical citizenship. According to Skovsmose and Valero (2002), "The paradox of inclusion refers to the fact that the current globalization, which proclaims universal access and inclusion as a stated principle, can also be associated with processes of exclusion" (p. 401), for instance, in creating a global and universal curriculum for school mathematics, many local socio-political and cultural contexts are overlooked. Further, the paradox of citizenship "refers to the fact that, on the one hand, education seems ready to prepare for active citizenship, but, on the other hand, it seems to ensure adaptation of the individual to the given social order" (Skovsmose & Valero, 2002, p. 386). I acknowledge these paradoxes and will discuss them along with the findings of this study in section 6.1.3 of chapter 6, but first I elaborate on some theoretical questions related to the calls for critical citizenship and democratic participation.

I discuss some central concepts introduced and elaborated in CME and socio-political mathematics education research to: (a.) explore the relation of social constructivism with CME and the socio-political research in mathematics education, and (b.) to consider how social constructivism can contribute to illuminate the individual dimension of learners' being critical. The elaborations of these concepts highlight the roles, responsibilities and opportunities mathematics learners may have in learning mathematics with a 'critical' element. These concepts include, for instance, *mathemacy*, reflective knowing, critical citizenship (Skovsmose, 1992, 1994b), learners' empowerment (Ernest, 2002, 2016), and their active democratic participation in their mathematics classroom (Lindenskov, 2010; Mellin-Olsen, 1987; Skovsmose, 1992; Skovsmose & Valero, 2005). The discussions regarding learners' roles and positioning in CME and socio-political research stand central to this research study due to its goal of highlighting learners' voices, experiences, and their critical reflections about learning mathematics.

In CME, Skovsmose (1992, 1994b) proposes and discusses the notion of 'mathemacy'. He clarifies that the intention of introducing mathemacy in the context of mathematics education is to explore the possibility of providing a *critical* dimension to the teaching and learning of mathematics. Skovsmose (1992) examines if mathemacy in mathematics education can be a competence equivalent to *literacy* as suggested by Freire (1972). Following Freire (1972), the aim of literacy is not only to impart the competences of reading and writing among learners but also to promote a sense of *critical consciousness* among them. The objective of promoting peoples' critical consciousness through their educational processes is to make them aware of their own socio-political positions and empower them to act for gaining collective emancipation (rather than individual knowledge gaining) in society. The learners are encouraged to reflect and act critically in their socio-political contexts so that they can understand and transform their own position and condition of living in the society. They are supposed to create a collective critical citizenry and achieve social and cultural empowerment and emancipation from social inequality, hegemonic power relations and injustice (see, e.g., Freire (1972), Giroux (1989) and Skovsmose (1992)).

In conceiving mathemacy as the mathematical counterpart of Freire's literacy, Skovsmose (1992) imagines the possibility of understanding mathemacy as a radical construct "rooted in a spirit of critique and project of possibilities that enables people to participate in the understanding and transformation of their society and, therefore, mathemacy becomes a precondition for social and cultural emancipation" (p. 2). In this conception of mathemacy, Skovsmose (1992, 1994b), like Freire (1972) and Giroux (1989), suggests mathematics education for learners' empowerment, critical citizenship and democratic participation. Skovsmose (1994a, p. 48) maintains that "*mathemacy must be seen as composed of different competencies: a mathematical, a technological and a reflective. And especially: reflective knowing has to be developed to provide mathemacy with a critical dimension*" (original italics). *Mathematical knowing* includes the knowledge of traditional mathematical skills such as reproducing theorems and proofs and mastering a variety of algorithms. *Technological knowing* includes the abilities in applying the knowledge of mathematical skills and algorithms to build mathematical models and solve practical problems. Finally, *reflective knowing* "refers to the competence in reflecting upon and evaluating the use of mathematics" (Skovsmose, 1994a, p. 47).

Since reflective knowing is mathemacy's element with a critical dimension, it has been discussed in different ways. Skovsmose (1992, 1994b) mentions reflective knowing in contexts of learners reflecting critically on the application and consequences of mathematical models employed in the society. Similar interpretation is also found in the work of Gutstein (2003, 2006), which underlines the significance of adopting a critical stance towards the contexts and results of mathematical modelling and data adopted from real-life contexts to reflect and act against the socio-political issues of social injustice, discrimination, inequality, and more. These interpretations emphasise the collective dimension of mathematics learners being and thinking critically and developing critical and democratic citizenship through mathematics education. However, a collective critical and democratic citizenry requires individual citizens having the capacity to be critical, exercise their criticality and take autonomy to participate.

The collective ideals of democracy, critical citizenship and active democratic participation require individuals having a trained intellect. This requirement is clearly stated in a quote by Jay Bryan Nash (1953, p. 37), "Democracy assumes freedom; freedom assumes choice. But to be able to choose, man must have a trained intellect and be disciplined in choices pertinent not only to the good of himself but to the good of all" (cited in Kaufman (1989, p. 169)). Further, Norén and Valero (2022) elaborate that, "Democracy requires people who can think rationally and sensibly, who can adopt a critical and independent stance against tendentious influences and who can analyse, compare and compile" (p. 169, my translation, original in Swedish). Freire (1972) has also stressed the idea of *self-awareness* along with the urge of developing learners' collective critical consciousness since a collective critical consciousness emerges from a collection of individuals who are critically self-conscious about their own socio-political contexts.

In this thesis, I aim to explore and understand the individual dimension of mathematics learners being critical and reflecting over the ways and experiences of learning mathematics in the classroom. It is in relation to this individual dimension of being critical that I discuss the relation of social constructivism with CME and the socio-political research in mathematics education. Skovsmose (1994b, 2011) considers this individual dimension of mathematics learners being critical *towards* (not *in* and *through*) their mathematics learning experiences. Elaborating on the notions of reflections, Skovsmose (1994b) states that, "Reflections can address not only the social role of mathematics but also the actual teaching–learning

situation; and from a vantage point the students may make their own learning process an object for reflection" (p. 175). The notions such as learners' *intentionality* in learning mathematics, *sharing,* and *negotiating* these intentions with their teachers are also introduced and discussed theoretically to clarify this individual (or *subjective* as used by Skovsmose (1994b)) aspect of learners' critical reflections on their own mathematics learning process. However, this discourse does not exemplify how these notions can be practically implemented in the classroom.

Nevertheless, the question of interest in this section is which theoretical position can be employed to explore the *individual* dimension of mathematics learners being *critical* towards their mathematics learning processes which are inherently of a *social* character? On the one hand the contexts in which individual learners learn mathematics, the essential element of their reflections such as interacting with others (their teachers), the use of language to communicate, the cultural and political frames of references forming and influencing their learning experiences are inherently social in nature. However, on the other hand, their personal experiences are inherently subjective and individual. Further, the focus is on the critical element. Similar questions can be asked in discussing the notions of learners' empowerment and their active democratic participation in their mathematics classrooms.

Both Skovsmose (1994b), Ernest (2002) and Skovsmose and Valero (2005) emphasise that learners' ability to negotiate with authorities, being involved in and influencing the decision-making processes concerning their own mathematics education process are central tenets of learners' empowerment and active democratic participation in their mathematics learning activities. Skovsmose (1994b) highlights that "Learning for democracy could also mean learning how to interact with authorities, and in this case, 'learning by doing' makes sense" (p. 149). Likewise, Ernest (2002) suggests adopting a "questioning and decision making learning style in the classroom" including "the questioning of content and the negotiation of shared goals" (p. 8) to achieve the aims of CME. Skovsmose and Valero (2005) also question "Who has the possibility to participate in decision-making concerning the curriculum?" and argue that "a bottom-up strategy makes it possible for both students and teachers to be included in curricula decision-making, and that this is essential for education to make part of democratic processes in society" (p. 67). However, concerning learners' participation in these decision-making and negotiation processes, one can question that which theoretical paradigm can cater for enquiring individual learners' voices, experiences, perceptions, perspectives, and critical reflections on being a part of these democratic practices?

I see the white oval presented in Figure 5 as a possible answer to this question. Placed on the theoretical pathway stretched between cognitive constructivism and Critical Theory, this oval demonstrates that – the transition from being a critical thinker (cognitively) to becoming a critical citizen (socio-politically, cf. Critical Theory) passes through social constructivism (one's social interactions). One can adopt social constructivism as a theoretical position in case one's research aims are closer to enquiring individualistic aspects of learners' criticality (e.g., the grey oval is closer to social constructivism in Figure 6). This theoretical position can also move towards the other end of the pathway (e.g., the grey oval in Figure 6 can move closer to Critical Theory (critical pedagogy)) as one's research moves away from focusing on individualistic, and closer to enquiring collective aspects of learners' criticality.

Social constructivism, as explained by Ernest (1994), attends to both individual and social factors that influence formation of individual's experiences and knowledge. Hence, it provides a suitable theoretical position to explore individual learners' critical reflections about and experiences of their mathematics learning processes, which are inherently social in nature

(Ernest, 1998). This suitability of social constructivism can be employed in exploring the individual dimension of learners' criticality towards their (social) learning experiences in mathematics, whereas the Critical Theoretical position is used in CME and socio-political research to discuss the collective (socio-political) sphere of learners' criticality through learning mathematics. Thus, I envisage *social constructivism with a critical element* (see Figure 6) as a theoretical position which can be adopted to conduct research studies exploring learners' individualistic perspectives (e.g., their thoughts, experiences, critical reflections, beliefs, autonomy, agency, etc.) in CME and socio-political mathematics education research.

3.5 Social constructivism and central concepts in the papers

The papers attached to this thesis employ some concepts and terminology which are often placed under and can be seen as belonging to the cognitive constructivist tradition (e.g., learners' critical thinking, beliefs, and learner autonomy). These concepts are defined in detail in the papers attached to the thesis, but in this section, I clarify my understanding and operationalization of these concepts to justify the theoretical positioning of this study as shown in Figure 6. Further, Table 2 presents an overview of the social constructivist and critical features of these central concepts.

3.5.1 Social constructivism and learners' critical thinking about learning mathematics

As mentioned before the keyword critical thinking guided my literature search and influenced my choices made early during the PhD period. The concept of critical thinking is usually understood in terms of cognitive abilities such as, argumentation, logical reasoning, evaluating evidence, deducing conclusions, and the like. In this research, the aim was not to assess learners' cognitive abilities of argumentation, logical reasoning, etc. but to explore their critical thoughts *about* learning mathematics. Thus, the prefix 'critical' is used in a sense of being opposite to 'uncritical', and to pursue a *critique* of the social process of their mathematics learning. Ernest (2016) explains different meanings of the word critical and critique, being one of those meanings, means "to analyse the merits and faults of something, typically a cultural product, possibly to uncover and evaluate its hidden dimensions of meaning, and social and cultural significance" (pp. 100-101).

Accordingly, the concept of critical thinking was operationalised as learners' ability of analyse the merits and potential of improvement in their mathematics learning processes by asking them questions, such as, what content they like/dislike to learn in mathematics and why, and eventually what they want to learn in mathematics given a free choice and how would they like its teaching to be. The operationalisation of critical thinking in this way brings forward its connection with social constructivism. Firstly, the critical thoughts of learners which I wanted to enquire were their subjective understandings about the social process of their mathematics learning, instead of being cognitive (i.e., not analysing the logic of a truth claims or any arguments). Secondly, the process of communication between me and my informants was social, and the socio-cultural tool of 'language' was our medium of interaction. Thirdly, the interview was also conducted in a social context (i.e., in learners' school environment) which may have influenced learners' perspectives, experiences, and voices.

3.5.2 Social constructivism and learners' beliefs about relevance and importance of learning mathematics

Mathematics education research studies such as Leder et al. (2002) and Leder and Grootenboer (2005) demonstrate that investigating learners' mathematics related beliefs have received much attention in the research field. Learners' mathematics related beliefs to explore the affective (emotional and psychological) side of their mathematics learning experiences are investigated in many research studies. Instances of such research include studies exploring learners' motivation (Kloosterman, 2002), attitudes (Grootenboer & Marshman, 2016), mathematical identities (Andersson et al., 2015; Bishop, 2012), and affection (Leder & Grootenboer, 2005; Nardi & Steward, 2003) towards learning mathematics. Moreover, learners' beliefs about the relevance (usefulness) of studying mathematics are also well-investigated (see for instance, Nyabanyaba (1999), Onion (2004), Wedege (2007), Sealey and Noyes (2010), Kollosche (2017) and Wiik and Vos (2019)). These studies, however, do not differentiate between the concepts of relevance and importance, nor do they explore the sources of information forming learners' beliefs about the relevance and importance of learning mathematics.

Bar-Tal (1990) defines beliefs as "[…] units of cognition" (p. 12), but the formation of beliefs is not entirely cognitive, that is an individual's beliefs are formed based on different sources of information. Bar-Tal (1990) describes three categories of beliefs, namely, descriptive beliefs (based in individual's direct experiences), inferential beliefs (based on individual's inference of a situation based on rules of logic), and informational beliefs (based on information received by others). These sources of information form the connection between an individual's beliefs and social constructivism. Though beliefs are subjective units of cognition, yet the formation of these units of cognition are considerably influenced by the individual's social contexts and interactions. Bouvier (2004) has explored the influence of collective and socialised beliefs on individual beliefs by proposing the notions of *plural* subject and *polyphonic* subject. In acting as a plural subject, an individual may personally disagree with a shared belief formed in a group but fail to state the disagreement due to social commitment towards that group. The concept of a polyphonic subject, on the other hand, relies on the idea that, "individuals are to some extent the reflection of other people" (Bouvier, 2004, p. 388) and that a belief is social or socialised (might have emerged through various discussions with other people) – even if it is deeply personal (Bouvier, 2004, p. 389). This discussion indicates that one's personal beliefs are distinct from social beliefs, but the two sets of beliefs influence each other to a varying extent. Rydgren (2009) further highlights that identifying to what extent a particular belief is individual's own, formed under social (others') influence, or as a combination of the two, can be more challenging. This contribution of one's subjectivity and social contexts and interactions in forming one's beliefs connects learners' beliefs and social constructivism.

3.5.3 Social constructivism and learners' expressed experiences of autonomous involvement (learner autonomy) in mathematics classroom

Analogous to critical thinking, employing the notions of autonomous involvement (learner autonomy) in this study started with looking for a keyword to address the concern of promoting learners' joint responsibility and their right to participate in decisions regarding

their own educational processes (as stated in The Education Act (1998)). Similar concern to encourage learners' active democratic participation in their mathematics learning process is also listed in CME and socio-political mathematics education research. The policy documents and mathematics education research contributed to form my understanding that "learners' participation" in decisions about, and their right "to influence" their educational processes entail their *active* involvement in their learning processes. I derived the meaning of learners' *active* involvement from the descriptions used in policy documents and research literature, such as, (a.) learners should make a choice of teaching and learning materials, working methods, and working conditions (e.g., in M87), and (b.) that they should negotiate their learning goals with their teachers, can have discussions or conflict of opinions with them (see e.g., Ernest (2002)), etc. Thus, to *actively* participate in making choices, negotiating goals, or suggesting alternatives influencing their educational processes, the learners are required to reflect over the choices they have, and evaluate their consequences for their learning activities. Such reflections, as Mellin-Olsen (1993b) suggests, would require that learners' take *initiative* and *ownership* of their own learning processes.

While looking for research literature which may contribute to address this concern, I came across the concept of *autonomy* in literature discussing the learning of English as a foreign language. In the context of education and learning, Holec (1981) defines autonomy as, "the ability to take charge of one's learning" (p. 3). This definition of autonomy captured my interest to indicate the elements of learners taking initiative and ownership of their own learning processes. Therefore, I chose the keyword *autonomous involvement*[36] in the research question, rather than involvement only. Learners can be involved in a decision by asking then to answer mere "yes" or "no" to a question[37] or by giving them a choice from a selection of alternatives available[38]. However, learners would require taking initiative and ownership of their learning activities, so that they take an independent stance on available choices, evaluate their consequences, come up with their own suggestions, etc., to be *autonomously* involved in their learning processes and influence them. Thus, Holec's (1981) idea of autonomy resonated partially with my understanding of learners' active participation in their learning processes (taking charge seen in coherence with taking initiative and ownership). However, I wanted to address learners' ability to take *co-responsibility*, *cooperate* with their teachers and peers, and participate actively in the decisions concerning their learning processes, instead of taking a complete charge of their own learning process as Holec (1981) suggests.

Looking for alternative concepts or ways in which autonomy is defined, I found the concept of *learner autonomy* in Little's (1991, 2003) research work. Little (1991, p. 4) defines learner autonomy[39] as, "a *capacity* – for detachment, critical reflection, decision-making and independent action. It presupposes, but also entails, that the learner will develop a particular kind of psychological relation to the *process* and *content* of his learning" (italics added). In mentioning the qualities of autonomous learners, Little (2003) asserts that they "understand the purpose of their learning program, explicitly accept responsibility for their learning, share

[36] I did not observe these learners' regular mathematics lessons. The data I have analysed is their interview responses, providing access to their *expressed* experiences of autonomous involvement in their mathematics classroom.

[37] For instance, a teacher may ask learners if they want to play a mathematical game or not in their mathematics lesson.

[38] For instance, a teacher may give the learners choice between working individually or in groups to solve mathematical problems.

[39] Learner autonomy, as a *capacity* of an individual is different from the *actions* autonomous learners take. *Autonomy* and *agency* entail a similar distinction, but capacity of autonomy would precede autonomous actions.

in the setting of learning goals, take initiatives in planning and execution of learning activities, and regularly review their learning and evaluate its effectiveness" (p. 1). This definition of learner autonomy and qualities of autonomous learners seemed coherent to the abilities listed in the policy documents and mathematics education research requiring learners to co-operate in, take the co-responsibility of, participate in decision-making processes, and influence their own learning activities.

Little (1991, pp. 4-5) also clarifies that though the notion of learner autonomy "implies that the learner enjoys a high degree of freedom. But it is important to insist that the freedoms conferred by autonomy are never absolute, always conditional and constrained". In other words, learner autonomy is not synonymous to independent self-instruction. Rather, learner autonomy, focuses on learners sharing the responsibility and negotiating the goals and content of their learning processes in cooperation with their teachers. Therefore, the keyword learner autonomy and its understanding corresponded to what I was looking for to address the goals of promoting learners' joint responsibility and their right to participate in decisions regarding their own learning processes. Thus, learner autonomy in this study is understood as learners' capacity to become a partner, not only a participant in their mathematics learning process so that they can influence and take co-responsibility of their own mathematics learning together with their teacher. The notion of learners' autonomous involvement in their learning activities is hence addressed in this study by using the concept of learner autonomy.

In mathematics education research, such descriptions of learners' involvement and control of their mathematics learning activities are made in the work of Mellin-Olsen (1993b). He voiced the concern of considering learners as active learning subjects and giving them the opportunity to partially control and get involved in decisions concerning their own mathematics learning activities. Yackel and Cobb (1996) and Ben-Zvi and Sfard (2007)[40] use the terms intellectual autonomy and learner autonomy respectively in their research, but these notions are discussed with respect to the cognitive context of learning mathematics (i.e., freely choosing solution methods to solve mathematical tasks) rather than being an active democratic participant of, and influencing one's own mathematics learning activities. It is in the latter sense that this thesis aims to explore learners' expressed experiences of learner autonomy in their mathematics classrooms.

Being a capacity of an individual, learner autonomy is not directly visible but according to Deci and Ryan (1987), it can be observed in one's behaviour, descriptions of one's own experiences and actions. Further, despite being a subjective sense of freedom, choice and volition, an individual's experiences of learner autonomy depend upon if one understands his/her social contexts, external events, interpersonal interactions, along with intrapersonal interactions to be autonomy supportive or controlling (Deci & Ryan, 1987). Thus, gaining a sense of learner autonomy would not only depend upon a learner's personality and character, but the classroom structure, interaction, one's relation with the teacher, and one's peers also play an important role in building learners' perception of having learner autonomy. It is this role which one's social contexts and interactions play in his/her comprehension of a situation as being autonomy supportive or controlling which links the theoretical position of social constructivism to learners' individual capacity of learner autonomy.

[40] In their study, Ben-Zvi and Sfard (2007) also suggest that learner autonomy should not be understood as giving learners complete freedom to learn what they want on their own. The authors emphasise that learners should have expert guidance under which they develop their own rational and critical thinking.

In sections, 3.5.1, 3.5.2, and 3.5.3 above, I have clarified the connection between the theoretical position of social constructivism and central concepts employed in different papers attached to this thesis. These connections are summed up in Table 2 below.

Title of the papers	Research question	Social constructivist feature	Critical theoretical feature
Learners' critical thinking about learning mathematics (extended version of Sachdeva and Eggen (2019))	What can learners' expressed mathematics related beliefs reveal regarding their practice with thinking critically about and potential to give suggestions concerning their mathematics learning process?	Learners as individuals participating in the socio-political context of their mathematics learning process. **Learners' critical thinking** about their mathematics learning process is explored.	Learners asked to exercise the **critique** of one's mathematics learning process.
"We learn it [mathematics] at school so one thinks that one will use it …": learners' beliefs about relevance and importance of learning mathematics	What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the sources of information influencing the formation of their beliefs?	**Learners' subjective beliefs** about the relevance and importance of learning mathematics are explored. The influence of socio-political contexts and interactions on learners' beliefs.	Learners asked to **reflect critically over and justify** their relevance and importance beliefs about learning mathematics for themselves, and to **consider the socio-political influence** on one's subjective beliefs critically.
Students' experiences of learner autonomy in mathematics classes	What can young learners' descriptions communicate about their experiences of learner autonomy in their mathematics classes?	Individual **learners' expressed subjective experiences** of having learner autonomy in social contexts and interactions of their mathematics classrooms are explored.	Learners asked to **assume freedom of choice and learner autonomy** to suggest alternatives and changes in one's mathematics teaching-learning processes.

Table 2 An overview of each paper's title, research questions, and the theoretical considerations.

4 Methodology

In this study, learners' potential of influencing their mathematics learning processes is explored by studying their capacities of being critical about learning mathematics, and by assuming the autonomy to take part in decision-making and suggest changes in their mathematics teaching-learning activities. Investigating specifically these two capacities became the focus because they are frequently discussed in educational policy documents and in socio-political mathematics education research. In doing so, I intend to position learners' voices and identify their potential to influence their mathematics learning processes under (mathematics) educational research that emphasises the development of learners' critical citizenship and active democratic participation skills through learning mathematics. Consequently, this research study fills a gap and contributes to the call for investigating learners' perspectives in research concerning CME and socio-political issues in mathematics education (see for instance Vithal (1999) and Lindenskov (2010)).

Learners' perspectives could only be accessed by attending to their point of views and experiences of learning mathematics which could be achieved by asking them questions, by listening to (or reading) their responses to those questions, and by observing them. Therefore, this study adopts *qualitative* research methods such as (partially open-ended) questionnaires and semi-structured interviews to gain insight into learners' perspectives and experiences of learning mathematics. While gathering and especially while beginning to look for ways to analyse the data, and interpret learners' responses, I became aware that the choice of a qualitative research design entails choosing specific methodological, theoretical, epistemological, and ontological perspectives. These perspectives highlight a researcher's assumptions regarding what kind of knowledge one believes to attain by doing research in a chosen way, how that knowledge can be attained, and what characteristics the produced knowledge claims will have. With this realisation, I started my journey to explore the methodological underpinnings of this research study which is presented in the following text. In sections 4.1 and 4.2, I describe the methodological, theoretical, epistemological, and ontological stances adopted in this study, and relate them to my research interest. Section 4.3 presents the methods of data gathering, followed by section 4.4 describing the data analyses process. The trustworthiness of this study is discussed in section 4.5 and section 4.6 presents the ethical considerations catered for in this research project.

4.1 Methodological and theoretical stances

The focus of my research study became to gain insight into learners' critical perspectives towards learning mathematics and listening to their voices indicating their involvement in and potential of influencing their own mathematics learning activities. To gain such insights and listen to learners' voice, I wanted to reach the learners themselves and get the opportunity to ask them questions about their *experiences of being* in their mathematics classrooms. However, to understand the potential inherent in learners' voices, I interpreted and reinterpreted their responses to my questions constantly in relation to the context of their experiences of learning mathematics and the research literature I was reading during my research journey. Due to the focus on exploring learners' lived experiences, this study adopts a *methodological* stance of *phenomenological research*, and since understanding relative

meanings of their voices is aimed, this study adopts the *theoretical perspective* of *interpretivism* based on Crotty's (1998, p. 5) classification.

Crotty (1998) writes that the interpretivist approach "*looks for culturally derived and historically situated interpretations of the social life-world*" (p. 67, original italics). Further, Creswell (2007) asserts that a phenomenological study "describes the meaning for several individuals of their *lived experiences* of a concept or a phenomenon" (p. 57, original italics), though phenomenologists can focus on different features of those experiences while describing the meaning. Wojnar and Swanson (2007) and Creswell (2007) discuss two such features, namely, the *universal essence* of a lived experience, and the context dependent *interpreted meaning* of a lived experience. Wojnar and Swanson (2007) describe that the *descriptive phenomenology*[41] approach focuses on finding the universal essence of a lived experience, whereas the *hermeneutic phenomenology*[42], in alignment with the interpretivist approach, focuses on understanding and interpreting the context dependent meaning of a lived experience. Connelly (2010) clarify that the difference between these two approaches exists in how they handle the bracketing of researcher's presumptions or biases about the phenomenon as

> *"Descriptive phenomenologists try to bracket or put aside these presuppositions or biases so they do not affect the study. Interpretive phenomenologists do not believe these ideas can be put aside because they are a part of the person; the researcher only can be aware of them and any effect they have on the study"* (p. 127).

I place this study under the hermeneutic[43] phenomenological approach since the focus here is not to find a universal or common essence in learners' experiences of learning mathematics, but to interpret contextual meanings of their experiences. These meanings could be relative and different for individual learners. Moreover, discussing the bracketing of researcher's presumptions, Wojnar and Swanson (2007) report that "Heideggerian phenomenology is based on the perspective that the understanding of individuals cannot occur in isolation of their culture, social context, or historical period in which they live" (p. 174). However, Wojnar and Swanson (2007) also underline that the researcher should be conscious of one's own preconceptions and biases so that s/he is cautious of his/her interpretations and does not override informants' experiences with his/hers. Therefore, the hermeneutic phenomenological approach also acknowledges that I have my own situated preconceptions of social lifeworld as a person, and I should be aware of these preunderstandings while cautiously interpreting learners' experiences in relation to the social context and data obtained from other relevant sources.

4.2 Epistemological and ontological stances

Focusing on the phenomenological research methodologically, and adopting the theoretical perspective of interpretivism, this study can be located under the *epistemological*

[41] Also known as Husserlian or transcendental phenomenology since it was proposed by Edmund Husserl.
[42] Also known as Heideggerian or interpretive phenomenology since it was proposed by Martin Heidegger.
[43] Hermeneutics was, and is, the science of interpreting religious texts, but nowadays 'texts' are understood to include written or verbal communication, arts, music, etc. also, and hermeneutics is employed to interpret these unwritten sources (Crotty, 1998). A detailed description of hermeneutics is given by Crotty (1998).

paradigm of *social constructivism*[44]. Social constructivism is often described as a strand or branch of the *constructionist* epistemology, which is often combined with interpretivism and manifests itself in phenomenological studies (see for instance, Crotty (1998), Creswell (2007) and Burr (2015)). Crotty (1998) explains that in constructionist view meaning (and hence knowledge) is not discovered objectively, but "meanings are *constructed* by human beings as they engage with the world they are interpreting" (p. 43, italics added) and "constructivism is primarily an individualistic understanding of the constructionist position" (p. 58). Therefore, constructivism focuses exclusively on "the meaning-making activity of the individual mind" (Crotty, 1998, p. 58) as these meanings are subjectively constructed when an individual human subject engages with objects in the world and makes sense of them (Crotty, 1998, p. 79).

While constructivism focuses on an individual's subjective meaning-making, social constructivism emphasises that "these subjective meanings are negotiated socially and historically" (Creswell, 2007, p. 21). The subjective meanings of lived experiences "are not simply imprinted on individuals but are formed through interaction with others (hence social constructivism) and through historical and cultural norms that operate in individuals' lives" (Creswell, 2007, p. 21). My research interest was to explore and interpret individual learners' lived experiences of learning mathematics and how they understand and express these experiences in their questionnaire responses and interview conversations with me. Learning mathematics in schools is inherently a social process involving interactions and discussions with their peers and teachers in their mathematics classroom. The social context of learning mathematics and their interactions with others can influence individual learners' lived experiences of learning mathematics. Therefore, adopting social constructivism as an epistemological stance allows me to cater for both subjective meaning which individual learners construct while engaging with their mathematics learning activities, and the social influences on their lived experiences of learning mathematics.

The choice of phenomenological research clarifies my methodological assumption that knowledge about a *phenomenon* (i.e., learning mathematics) can be gained by studying learners' lived experiences of learning mathematics and being in their mathematics classrooms. The choice of interpretivism as the theoretical perspective clarifies my assumption that the meaning which learners ascribe to their mathematics learning experiences can be understood by interpreting their responses to questionnaire and interview questions. The choice of social constructivism as an epistemological stance clarifies my assumption that individual learners construct subjective meanings (and knowledge) of their mathematics learning experiences while engaging in their mathematics learning activities and interpreting those activities in its social context. As a researcher, adopting social constructivist perspective as in this study would imply that knowledge about learners' experiencing the phenomenon of learning mathematics can be gained in two steps: (1.) interacting with them about these experiences; and (2.) cautiously interpreting their responses in relation to the social contexts, my own understandings, and information obtained from other relevant sources about this phenomenon.

[44] I am aware of constructionism as another strand of this epistemological paradigm as mentioned and defined by Crotty (1998) and Burr (2015). However, I choose social constructivism since my focus is to explore the meaning construction activity of individual learner's minds and how these meanings are affected by social interactions. Whereas constructionism and social constructionism focus on the collective construction of meaning and the extent to which these constructions are the product of social forces shaped by language and other cultural or historical processes (Burr, 2015; Crotty, 1998).

This implication clarifies my *ontological* position in this study as a researcher, that is, meaning (knowledge or truth) cannot be described simply as objective or as subjective, but "objectivity and subjectivity need to be brought together and held together indissolubly" to construct the meaning (knowledge or truth) (Crotty, 1998, p. 44). *Meaning* is constructed when human consciousness is directed towards the object and simultaneously the object is shaped by human consciousness. Crotty (1998) further argues that objective reality may exist without human consciousness engaging with and interpreting it, but *meaningful reality* cannot and therefore, the constructionist viewpoint is at once realist and relativist (p. 63).

From this ontological perspective, I highlight learners' potential of influencing their mathematics learning activities inherent in their voices. This potential emerged as knowledge when I interacted with, and interpreted learners' written and verbal responses. Hence, the knowledge of the phenomenon (learning mathematics) is constructed through an interaction between: (a.) the object (i.e., learners' lived experiences of learning mathematics), (b.) the subject (i.e., learners' own interpretations of their experiences), and (c.) the researcher (my own understandings), and all partners contribute equally to this knowledge construction process.

4.3 Methods of data gathering

To know about and to make sense of leaners' experiences of learning mathematics and being in mathematics classroom, data gathering methods allowing reflective interaction with the informants were required. Postholm (2005) asserts that qualitative research implies understanding participant's perspective. The qualitative researcher directs his/her focus on informants' everyday activities in their natural context, but the research is influenced by his/her theoretical standpoint (Postholm, 2005, p. 17). Therefore, the data is gathered using the qualitative inquiry methods such as, partially open-ended questionnaires, classroom interventions and individual face-to-face semi-structured interviews.

The first step to gather the data was to get in touch with schools and learners who could be our informants. The criteria to choose the schools for data collection was inspired from the research focus of LOCUMS (2016), and therefore *we*[45] (the team of LOCUMS) wished to contact schools having a large number of multicultural learners so that we can achieve a variation in the cultural backdrop of learners. The idea was to ask learners themselves about their experiences of learning mathematics and science, and real-life application of advanced-level knowledge of these subjects. Therefore, it was reasonable to get in touch with learners who had experienced learning advanced-level mathematics and have learnt mathematics for quite some years (e.g., for 8-10 years). We chose secondary school learners in 8th and 9th grades as the informants of this study. After making this choice, information e-mails were sent to principals, and in some cases to secondary mathematics and science teachers in these schools to inform and invite them for participating in this research study. Approximately 45-50 schools in and around Central Norway were contacted, however the response over the e-mails was negligible, so we decided to reach the principals of some schools over the telephone.

[45] "We" as used in this chapter hereafter indicates the team members of LOCUMS research group associated with NTNU. My supervisor Dr. Per-Odd Eggen has participated actively with me in all the sessions of data gathering process while Dr. Dag Atle Lysne participated with us in the last data gathering session.

Telephonic conversation resulted in getting an opportunity for the research team to visit these schools personally and explain the aim of this project so that the school principal and the teachers could decide if they are interested to participate. These school visits gained a positive response, and the research team got access to two secondary schools located in Central Norway, willing to participate in our study. In the first school, learners studying in 8th and 9th grades became our informants, whereas the interaction was made with 9th graders in the other school. These three classes had two to three learners each coming from a different cultural background than Norwegian. This cultural variation involved a blend of learners coming from Norwegian, Sámi, Eritrean, Afghani, and Iranian cultural backdrops respectively. However, due to the shift in my research interest towards a socio-political direction, this cultural variance was not focused on while analysing the data.

A total of 74 learners studying in 8th and 9th grade (that is, 13-14 years old), contributed to the data gathering process. The data was collected twice in the 8th grade and once in the 9th grade of the first school, followed by one iteration in the 9th grade of the second school. Altogether, four iterations of the same research design were conducted where every phase except the questionnaire part was repeated each time the research team got into a new classroom with different learners. Figure 7 below illustrates the various phases of the research design and data collection in a chronological order.

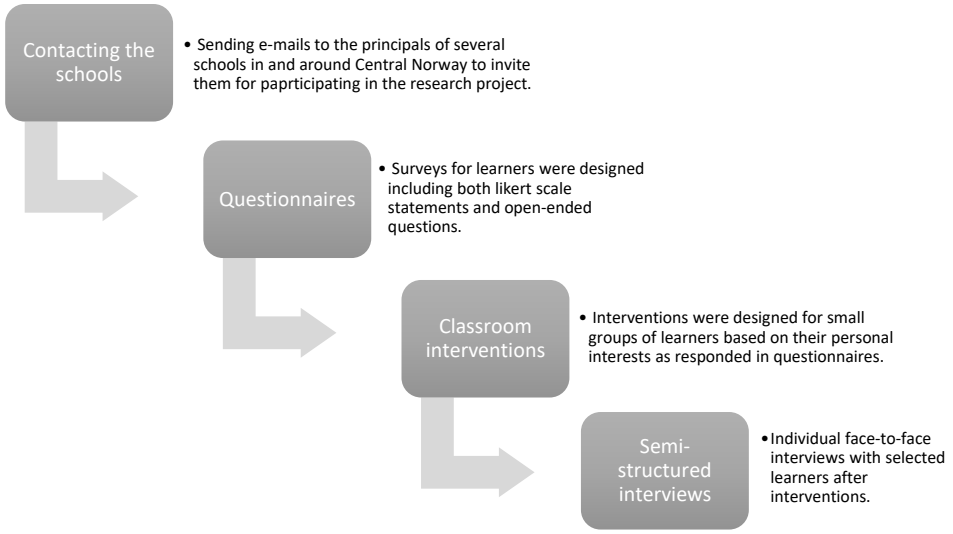

Figure 7 The different phases of data collection.

After finalizing the schools and classrooms, we conducted a few meetings with learners' mathematics and science[46] teachers to clarify the research design and plan how the classroom interventions along with number of iterations will be executed. Three research members from the team of LOCUMS located in Trondheim were involved in planning group activities and practical tasks for learners and the teachers later participated in executing classroom interventions. During this planning phase, several e-mails were exchanged between the researcher and these teachers so that enough information is sent out to the learners and

[46] The project LOCUMS focused on both mathematics and science education, but in this thesis, I focus on learners' responses to questions concerning their mathematics learning experiences only.

their parents/guardians about this sub-study of LOCUMS (conducted in Central Norway, on which this thesis is based). An information letter was written and sent to learners' parents/guardians through their teachers. This letter informed the learners and their parents/guardians about this sub-study of LOCUMS, its research objectives, and the three stages of data collection process to obtain their consent to voluntary and anonymous participation in the data collection process. These stages of data collection consisted of pre-intervention questionnaires, classroom interventions entailing practical group tasks for learners to be solved using their knowledge in mathematics and science, and the final stage involving face-to-face individual semi-structured interviews with selected learners. The first three classroom interventions took place in school number 1 and the last classroom intervention took place in school number 2. The process of data collection is illustrated in Table 3 below and the sections following the table provide further details about the design of questionnaires, classroom interventions and interviews along with the ethical issues concerning data collection process.

School	Standard	Number of participating learners	Number of responded questionnaires	Number of classroom interventions	Number of interviews
School 1	9th	22	22	1	5
School 1	8th	21	20	2	10
School 2	9th	32	32	1	5
Total		75	74	4	20

Table 3 An overview of sample size and collected data material.

4.3.1 Questionnaires

The reason of choosing questionnaires as our first step to get in touch with learners was two-fold: (a.) to gain insight into learners' interests and *youth culture* so that we could design relevant practical tasks for them to solve; and (b.) to get preliminary idea about their experiences of learning mathematics and science to plan interview questions. Pitura (2023) states that questionnaires have been a traditional data gathering method employed in quantitative studies, but recently questionnaires including open-ended questions[47] which can elicit informants' beliefs and opinions regarding the issue in question have also gained popularity in qualitative research approaches such as phenomenology, case study, etc. In qualitative studies, these questionnaires can supplement other data gathering methods such as interviews, observations, etc. Questionnaires including both open and closed-ended (Likert-scale statements) questions are employed in this study to get a larger overview of learners' interests, thoughts about culture, their learning experiences, etc. so that we could gather learners' input to design classroom interventions and an in-depth interview guide to be used later in data gathering process.

Initial inspiration of which questions to include in the questionnaire and how to formulate them was derived from the questionnaire designed for the ROSE project (see Schreiner and Sjøberg (2004)). A five-point scale was used for these Likert-statements moving from 'strongly agree' – 'agree' – 'disagree' – 'strongly disagree' and 'do not know', whereas

[47] Open-ended questions alone or in combination with closed-ended questions.

the open-ended questions were formulated as short questions with an open space under to write a response. The questionnaire was distributed into four main sections, namely, personal information; leisure time interests and cultural identity; relation with learning mathematics and science; and future career aspirations; and the school environment, friends, and social participation. Each of these sections further focused on different themes for instance, their leisure time interests, thoughts about culture and being in a multicultural classroom, interests in, experiences of, and wishes about learning mathematics and science, future career expectations, school, and classroom environment, and more (see 9.3). Selected Likert-scale statements were designed to gain information regarding learners' leisure time interests to design practical group tasks for them, and some open-ended questions were designed to get a preliminary insight into learners' thoughts and views about their learning experiences of mathematics and science at school.

The questionnaires were designed in Norwegian language therefore, we carried out a quality check of the language used, and questions formulated in the questionnaires before administering questionnaires to the learners. We gave the questionnaire to a young school going learner of same age as our informants and asked her if the language used was suitable and the questions were adapted to young learners. Minor changes in the language of some of the questions were made after getting feedback from her so that the questionnaire is adapted to young learners. Additionally, the questionnaire was also quality assured by one of our colleagues, Berit Bungum, having rich experience in conducting qualitative research. The questionnaires were administered to learners by their respective class teachers without presence of the research team.

After the questionnaire got administered for the first time to a class consisting of 22 learners, their answers like, "nothing", "I don't know", "everything", or blank space to the question, "is there something in mathematics which you think is especially interesting?", stimulated my interest in knowing more about individual learners' personal experiences of learning mathematics rather than the cultural perspective which was central in LOCUMS. Learners' questionnaire responses served as entry points for me to better understand my research interest in exploring learners' perspectives. Therefore, I revised the questionnaire for the forthcoming rounds and added more open-ended questions in it, such as, "what do you think is useful to learn in mathematics?", "what do you think is useful to learn in science?". The final revised version of the questionnaire is attached here as appendix number 9.3.

4.3.2 Classroom interventions

After administering questionnaires in a classroom, the next phase of data gathering was to design and conduct classroom interventions rooted in activities of learners' interests which they had mentioned in their questionnaire responses. The idea of conducting classroom interventions was grounded in the 'original' research interest of LOCUMS to investigate the use of practical activities based on learners' life experiences and cultural backgrounds as a starting point for the learning of concepts and basic skills in mathematics and science (LOCUMS, 2016). However, due to the shift in my research interest in exploring learners' perspectives and experiences of learning mathematics in-depth, I chose to analyse the data gathered through questionnaires and individual semi-structured interviews rather than the classroom interventions. These classroom interventions are (among other things) a part of the background for conducting interviews but they do not form a part of data set which is analysed

in the papers. These interventions have had significance for the quality of semi-structured interviews conducted later. I was present as a participating observer in all the classroom interventions which were planned and got the opportunity to interact with the learners in an informal setting before the interviews themselves. This informal meeting contributed to achieve a more comfortable interview situation afterwards. Moreover, learners' reflections on their experiences of participating in these interventions were also a theme of discussion in the interviews, so that the conversation could be more relatable for the leaners. Thus, these interventions provided me with a background and common frames of reference for having interview conversations with the learners later.

To design the classroom interventions, we were inspired by a design-based research approach as described by Anderson and Shattuck (2012) and started by noting learners' interests reported in the questionnaires. Their interest areas were then clustered under different themes such as, cooking, carpentry, first-aid, space and astronomy, biology, playing football and so on. We finalised four interest areas for each intervention class and the learners were then given the choice to opt for which group they wanted to be placed in. Each group consisted of four to five learners. The tasks were designed so that in addition to being related to learners' activities of interest, the knowledge of mathematics and science was required to solve them completely. The classroom interventions presenting all the practical tasks given to the learners are attached to the thesis (see appendix 9.4.1 to 9.4.4). As an example, the learners who chose cooking as their interest area got the task to cook a nutritious meal for an athlete of their own age. In addition, they were asked to mention which nutrients will the athlete get from the meal they cooked and to make a nutrition value chart to calculate the amount of energy, fat, protein etc. the meal will provide. Another important feature of these tasks was that each group of learners had to construct an artefact or a product at the end of their group project. Learners in the cooking group for instance prepared a one-time meal considering an athlete of their own age which was shared by the group itself and their classmates. The classroom interventions and selected groups of learners working on the tasks were audio and video recorded.

4.3.3 Semi-structured individual face-to-face interviews

Individual face-to-face semi-structured interviews with learners were used to get an insight into their perspectives, beliefs, and experiences of learning mathematics at school. Interviews are known to be the method of data collection when the intention is to get an insight into informants' perspectives, thoughts, beliefs, understandings, and experiences of a certain situation, phenomenon, or context (Postholm, 2005). The semi-structured interviews lie in-between the continuum of unstructured and structured interviews (Bryman, 2016; Kvale & Brinkmann, 2015). The conversations in semi-structured interviews are loosely structured around central themes of research interest which are to be covered during the interview, mostly including open-ended questions. This loose structure of interviews allows the interviewer to steer and follow-up the conversations and interviewee's responses (Magaldi & Berler, 2020). Semi-structured interviews also contribute to having a balanced sense of freedom between the interviewer and the interviewee to create a safe space for sharing one's views, experiences, and opinions. All the interviews were conducted and audio-recorded by me, with no involvement of other members of LOCUMS research team. The interviews were

conducted in Norwegian. The following sections explain the process of selecting informants for the interviews, designing interview guide and my role as an interviewer.

4.3.3.1 Selecting interview informants

The individual semi-structured interviews conducted face-to-face with selected learners became the primary source of data which is analysed and presented in the papers attached to this thesis. After conducting the questionnaires and classroom interventions, some learners from each classroom were selected for conducting the in-depth semi-structured interviews. One learner from each group working on different group tasks was selected. A total of 20 semi-structured interviews were conducted and each interview lasted between 45-90 minutes. The selection of interviewees was done on the basis of principle of maximum variation (Bryman, 2016; Denzin & Lincoln, 2018). Therefore, an attempt was made to choose 3 to 4 learners among those who were observed to be more, average, and less engaged while participating in the classroom interventions. Likewise, 3 to 4 learners among those who wrote in the questionnaire that they are highly, moderately, or not interested in learning mathematics [and science]. In selecting learners based on this principle, it was ensured that the interview data would represent the different learners' experiences with learning mathematics. Hence, the interview data presents views and opinions of a representative sample of learners of a usual secondary school classroom.

In addition to selecting the learners having varying interest and motivation to learn mathematics, the principal of variation was also applied to select learners having different cultural backgrounds. The cultural variation was considered to cater for the aims of LOCUMS. Among these 20 interviewees (10 girls, 10 boys), a majority (15) of learners were Norwegian, whereas there were two Sámi, one Eritrean, one Arabic and one learner from Afghanistan. This cultural variation, however, was not focused while analysing the data pertaining to the research interest in exploring learners' mathematics experiences in-depth. All the learners having other cultural backgrounds than Norwegian (with one exception, a learner from Afghanistan) had been in Norway for more than three years and understood Norwegian well enough to carry out the interviews in Norwegian. Interview with the learner from Afghanistan was carried out in Pashto with the help of a translator. He was still learning Norwegian for most of the time in school and therefore he attended a different class. Thus, this interview was not included in the interview data which was analysed while writing the papers.

4.3.3.2 Designing the interview guide

For semi-structured interviews, an interview guide is planned to keep the track of themes which should be talked about in the interview, but this structure is not so strict. Most of the questions are open-ended with a minority of questions with closed answers such as, name or age of the informant. The interview guide, analogous to the questionnaires, included several themes of enquiry. These themes included learners' reflections about their general learning process in the school, their views about mathematics and science education, their social participation in the classroom, friends and school environment, their views about culture and integration in the classrooms, cultural identity, future career aspirations, and their experiences of participating in the classroom interventions. This blend of several themes present in the interview guide can be explained in terms of broad research objectives of LOCUMS.

However, since my research interest shifted towards exploring and understanding learners' perspectives on learning mathematics in-depth, I included more themes in the interview guide focusing on their experiences of and their viewpoints about learning mathematics. A few instances of themes included in later versions of interview guide concern asking learners about the use and importance of learning mathematics, the reasons of holding certain beliefs about learning mathematics, changes which they want to incorporate in their mathematics learning activities if given a chance, and more. The final revised version of interview guide is attached to this thesis as appendix number 9.5. The interview guide and the questions were also quality checked by other members of LOCUMS research team at NTNU.

4.3.3.3 My role as an interviewer

In this section I reflect over my role as an interviewer and discuss the issue of authority and power relation in the interviews I conducted. Kvale and Brinkmann (2015) write about the asymmetric power relation between an interviewer and the informants in the context of research interviews and emphasise that the researcher should acknowledge the existence of these power relations. They further underline that this asymmetric distribution of power is unavoidable in an interview and can influence the knowledge produced under interviews. A researcher should therefore reflect over and handle this relationship of power responsibly. Each interview was stared with establishing a rapport with informants. While establishing the rapport, I informed the learners about my duty of confidentiality, that there were no right or wrong reasons of the questions I was asking, and I was interested in their honest responses and knowing their experiences and viewpoints. They were also told that their responses will not have any influence on their grades, and they can deny replying to a question if they do not want to answer it. Considering learners' classroom situation, I did not have any teaching duty towards these learners and in that sense, I did not have authority over them in their classroom.

Norwegian language was the mode and medium of communication in the interviews, so having an immigrant background and being a non-native speaker of Norwegian language made it challenging for me to conduct interviews in Norwegian. Though I was fluent in speaking Norwegian when I started to gather data, but my language skills were not developed to the extent that I could understand different dialects, match the pace of the learners, or to understand the slang these youngsters may employ to communicate with me. Therefore, I had to stop several times while interacting with learners and ask them to repeat and clarify what they were saying. There were also instances in which learners mentioned some contexts, such as a game (*førstemann til 1000*) they played or a particular topic (*søyle diagram*) they learnt in mathematics classroom which I could not understand and had to ask the learners for explanations. The issues with language made me pause and repeat the conversation many times under the interviews, but in my view, the contexts and situations in which learners took the authority to explain things to me contributed to mitigate my authority as an interviewer to some extent. Consequently, my language proficiency and immigrant background have also had some positive influences in the interview contexts.

I also reflect over the interview techniques which I used under the interviews. I adopted a discussion-based interview style and asked open-ended questions. The discussion revolved around a theme, for instance, why does the learner think that learning mathematics is important for him/her. The learner was then given time (sometimes pauses of 5-10 seconds) to think for him/herself and respond to this enquiry. Learners' responses were usually

followed-up by asking them to confirm what they replied, by repeating their responses so that they could confirm or by making a claim before them which they could accept or reject. In some instances, I also repeated what learners had said earlier to ask a follow-up question. I was cautious of not asking leading questions or to pressurise learners but on listening to the interview conversations later and reflecting critically over them, I realised that there were some instances of asking leading questions or where learners may have felt some pressure to reply in a few parts of the conversations. I take self-criticism for such instances which occurred during the interviews, but also underline that these were the instances in which learners were prompted to set aside what seemed to be their preconceptions while answering the questions.

Addressing the phenomenological spirit in data gathering process, learners were asked to reflect critically and justify their answers to certain questions. For instance, while answering why do they think learning mathematics is important for them, many learners replied that they think so because their teachers and/or elders have told them so. In this case, I insisted them to set aside what they are told by others and communicate what they personally believe or have experienced about the importance of learning mathematics. Rethinking these interview instances in which they were insisted to return to their own thoughts, beliefs and lived experiences of learning mathematics, I realise that some learners may have felt pressurised while reflect over them critically and justify their answers. These parts of the interviews entail scope of improvement, but the overall context, discussion-based conversations, instances in which learners took the charge and explained things to me, and having no classroom authority over learners are the characteristics contributing to reduced influence of asymmetrical power relation between the researcher (me) and the informants in these interviews.

4.3.4 The *critical* outlook in gathering data

In chapter 3, section 3.3.2.2, I argued for positioning this study between the social constructivism and critical theory. In this section, I explain how the critical outlook influenced data gathering process by drawing on the literature presenting critical pedagogy and phenomenology as a research methodology. The critical outlook is reflected in the data gathering process when the learners were prompted to reflect critically and critique their mathematics learning activities. The learners were asked to justify and reason for their beliefs about learning mathematics. In addition, learners were also asked to reflect critically over their experiences of learning mathematics. For instance, learners were asked about why learning mathematics is useful and important for them personally, if they have questioned their teachers or elders about the applicability of the subject content like equations in their everyday life, what would they like to learn in mathematics if they had the freedom to choose, etc. In these conversations, the learners were asked to think critically about learning mathematics, assume learner autonomy to choose, take decisions and suggest changes in their mathematics teaching-learning activities. Consequently, the critical outlook from a critical pedagogical perspective is adopted in gathering data for this study by encouraging learners to voice their personal opinions and critique their mathematics learning experiences; and by providing them with the opportunities to assume autonomy and suggest changes in their teaching-learning activities of mathematics.

Another feature of data gathering process in which the critical outlook is reflected is in the use of phenomenology as a research methodology itself. Crotty (1998) writes that phenomenology "is an exercise in *critique*. It calls into question what we take for granted" (p. 83). It is further stated that taking a fresh look at phenomena would call into question the current meanings we attribute to phenomena. Crotty (1998) explains that people are encouraged to exercise the *critical* spirit of phenomenology by reflecting over one's lived experiences by setting aside one's preconceptions and taken for granted meanings attributed to these experiences to find the *universal essence* of these experiences. However, in this study, my focus was not to find a universal essence in learners' experiences of learning mathematics, but to explore the *subjective* (not influenced by information received from other sources) meanings, and beliefs which learners themselves attribute to their experiences of learning mathematics. For instance, when I asked them about why they think learning mathematics is important for them, I asked the learners to set aside what their teachers or elders have told them about the importance of learning mathematics, and answer what they think themselves. By doing so, I wanted to gain an insight into individual learner's subjective meanings, experiences, and interpretations of their lived experiences of learning mathematics rather than the taken for granted interpretations of what others had told them about the importance of learning mathematics. Encouraging learners to critically reflect over their established understandings about learning mathematics can be seen in connection to the critical and reflective spirit of phenomenology, as phenomenology is described as "a reflective enterprise, and in its reflection is critical" (Larrabee, 1990, p. 201).

This call to set aside one's preconceptions while reflecting over a phenomenon may remind of the line of thought followed by *descriptive* phenomenologists, but the intention of the call made to learners in this study was aimed at hearing their subjective voices about their *own* experiences of learning mathematics instead of hearing others' voices in their answers. One can ask if a researcher adopting a hermeneutic/interpretive approach to phenomenological enquiry can make such a call to his/her informants to set aside one's preunderstandings while answering interview questions since the aim of this approach precisely is to find different meanings inherent in individual's subjective experiences of a phenomenon. One can further ask if this call for setting aside one's preconceptions is different in its style or aim when made by adopting the descriptive or hermeneutic approach in conducting a phenomenological enquiry. Based on the literature cited above, I can point out that the former approach aims at finding a common essence of a phenomenon experienced by different individuals, and the latter aims to interpret different meanings which individuals experiencing that phenomenon attribute to it. However, the questions asked above demand further contemplation.

4.4 Data analysis

To interpret and understand the meanings learners attribute to their experiences of learning mathematics, I adopted a hermeneutic/interpretivist approach to analyse the data. The hermeneutic approach attempts to understand the world of social realities by studying lived experiences of the participants of the study. These social realities do not exist objectively but are constructed as a result of social interactions, interpretations and actions of human beings who are constantly engaged in interpreting, acting in and constructing this world of social realities. Benner (1985) describes hermeneutics as a "systematic approach to

interpreting a text" in which "interview material and observations are turned into text through transcription" (p. 9). The *text* which I interpreted in this study was learners' questionnaire and interview responses. The questionnaire responses were already written text whereas the interviews were transcribed by me to turn them into text. In hermeneutics, being systematic and critically reflexive while interpreting the texts is emphasised (see for instance, Wojnar and Swanson (2007)). The concepts of *double hermeneutic* and *hermeneutic circle* elaborated by Crotty (1998) can contribute to understanding how one can be systematic and adopt a reflexive approach while interpreting text.

Referring to the task of interpretation as faced by social scientists, Giddens (1976, 1979) suggested that social science researchers need to engage in the process of "double hermeneutic" to understand the meaning of social realities as experienced by the informants of their study. Double hermeneutic implies that social science researchers engage in two-levels of interpretation while interpreting a text or phenomenon (Crotty, 1998). In the first level, the researcher enters in and interprets the frames of meaning (social, cultural, and historical contexts) in which his/her informants (common man) make sense of their lived experiences, and in the second level he/she reinterprets these meanings in relation to the frames of reference (technical concepts and schemes) shared by the community of social scientists. In other words, social science researchers are interpreting the interpretations of their informants and making sense of their informants' sense-making activity. In understanding the concept of double hermeneutic, I recognised the importance of being aware of the social, cultural, and historical contexts in which learners gain their lived experiences of learning mathematics, and in which I interviewed them. Further, I also recognised the significance of interpreting learners' descriptions in relation to relevant literature (policy documents and mathematics education research) so that the findings are relevant for the research community emphasising development of critical citizenship and active democratic participation skills among learners through their mathematics education.

Another concept of importance in an interpretive process is that of hermeneutic circle as suggested by Heidegger (1962). Wojnar and Swanson (2007) elaborate the concept of hermeneutic circle as follows,

> "According to Heidegger, the interpretive process is circular, moving back-and-forth between the whole and its parts and between the investigator's forestructure of understanding [interpreter's preunderstandings] and what was learned through the investigation. Heidegger (1962) referred to this process as entering into a hermeneutic circle of understanding that reveals a blending of meanings as articulated by the researcher and the participants […]. The goal of hermeneutic inquiry is to identify the participants' meanings from the blend of the researcher's understanding of the phenomenon, participant-generated information, and data obtained from other relevant sources" (p. 175).

Understanding the concept of hermeneutic circle made me aware of my own sociocultural background, preconceptions and presuppositions which may play a role in my interpretations. In interpreting the data gathered during this study, I also engaged with the concept of hermeneutic circle by moving back and forth between my own preunderstandings, the information generated by my informants and the data I obtained from other relevant sources (such as reading the literature and communicating with members of LOCUMS research team). These movements of going back-and-forth between transcripts, own understandings and data

obtained from other sources were carried out in a "dynamic, non-linear style of thinking" as Smith et al. (2009, p. 28) point out. For instance, sometimes my back-and-forth movements started from reading the transcripts and research literature and then interpreting them in relation to educational policy documents and discussing these interpretations with LOCUMS team members. At other times, I would start with reading policy documents and with my own preunderstandings and then move to the research literature, discussion with others and the transcripts.

To organise and structure my interpretations in a better way, I found some steps which I could follow in carrying out a hermeneutic phenomenological analysis of the data gathered in this study. Taking inspiration from the interpretation theory of Paul Ricoeur, Lindseth and Norberg (2004) propose three methodological steps to enter in a hermeneutical circle. These steps include: (a.) formulating a naïve understanding of the text from an initial reading, (b.) dividing the text into meaning units which are then condensed to form sub-themes, themes, and main themes and comparing these themes with naïve understanding; and (c.) reading the text as a whole, and reflecting upon the naïve understanding and the themes in relation to literature to formulate a comprehensive understanding of the meaning of lived experience.

These three steps for entering in the hermeneutic circle gave me an approach, but I found a detailed toolkit to start analysing the data when I came across thematic analysis (TA) (Braun & Clarke, 2006, 2019; Braun et al., 2018) as a method of data analysis. After reading more about TA, I realised that in its approach to analysing data, TA is quite coherent with the three steps mentioned by Lindseth and Norberg (2004). Both the approaches recommend starting with gaining an initial understanding of the data, followed by dividing the data into meaningful chunks of information, and then to identify themes which emerge by comparing and synthesising the data with other relevant sources (constant reflection of one's understandings in relation to transcripts, reading literature, personal assumptions, discussions with other researchers, etc.).

Braun and Clarke (2006) mention six-phases in which TA is carried out, namely, (a.) initial familiarization with the data, (b.) generating initial codes, (c.) searching for themes, (d.) reviewing themes, (e.) defining and naming themes, and (f.) producing the report. The first phase of TA can be related to the first step of gaining a naïve understanding of data as suggested by Ricoeur. The second and third phases can be related to condensing text into themes, sub-themes, and main themes. Finally, the fourth and fifth steps can be related to the last step of Ricoeur's theory of reading the text as a whole and reflecting upon the naïve understanding and the themes in relation to literature and formulating a comprehensive understanding of the meaning of lived experience. However, an advantage of adopting TA as an approach to data analysis for me was that it was more concrete in detailing the process of coding and finding themes, and entailed a stepwise procedure which I could easily operationalise. Thus, to practicalize these three steps of interpretive analysis as suggested by Lindseth and Norberg (2004) (inspired by Paul Ricoeur) in my data analysis process, I decided to employ TA as a toolkit and stepwise guide.

4.4.1 The interpretation process – from questionnaires and interviews to learners' voices

Learners' responses to questions concerning their mathematics learning process in questionnaires and interviews were the *units of data analysis* in this study. I started my data analysis journey with a naïve reading of learners' responses to the questionnaire statements

and queries concerning their mathematics learning. In this reading, I discovered that many learners had not answered what they wanted to learn in mathematics when asked to suggest their choices. Further, some of the learners' responses to the Likert-scale statements concerning their relation to learning mathematics also seemed inconsistent to me. For instance, there were several learners who agreed or strongly agreed to the statement, "I am interested in what I learn in mathematics and science" and at the same time agreed or strongly agreed to the statement, "I think that what I learn in mathematics and science is waste of time". Another example is that learners agreed or strongly agreed to the statement, "I like to learn mathematics", and at the same time agreeing or strongly agreeing to the statement, "I think that what I learn in mathematics and science is waste of time". This initial reading of learners' questionnaire responses inspired me to design the themes and questions which I wanted to ask from learners in the interviews. Therefore, I view my familiarization with questionnaire responses as a forerunner to my actual data analysis process, and as a source of data triangulation whereas interview data as principal source of data which is analysed.

In each iteration of our data gathering process, I went through learners' questionnaire responses, followed by designing and participating in classroom interventions before I designed an interview guide and finally interviewed the learners. After each iteration, I revised both questionnaires and interview guide to incorporate my learnings from the last round and to improve the techniques of data gathering. Consequently, the transcription of interviews was done in between two iterations of the research design. I transcribed the first ten interviews directly in English language. Though it was more time consuming for me, but it was more practical with respect to writing about the findings of the study. However, the last ten interviews were transcribed in Norwegian. I also started with transcribing whole interviews, but gradually I decided to transcribe those parts of interviews which were concerned with learner's mathematics education process.

Learners' interview responses are the primary units of data analysed in all the three papers attached to this thesis. The preliminary readings of questionnaires are presented in the first paper only to complement the data from interviews. It is in the phase of analysing interview transcripts that I started employing TA. Braun et al. (2018) outline three *schools* (possible approaches) to data analysis associated with TA namely, the *coding reliability*, *codebook*, and *reflexive*. I position the approach which I used to TA under the *reflexive* school of TA. Braun et al. (2018) emphasise the researcher's active role in knowledge production process and researcher's subjectivity is considered as a resource in the process of producing knowledge. The authors also write about other distinctive features of reflexive TA such as, coding is a reflective and iterative process which keeps evolving, coding does not start from any prefixed code books, and that the aim of coding and theme generation in reflexive TA is not to "accurately" summarise the data, but to provide a coherent and compelling *interpretation* of the data, grounded in the data (Braun et al., 2018, p. 848). These features align well with my choice of a hermeneutic phenomenological interpretive approach due to the acceptance of researcher's subjectivity in the interpretation process.

I underwent the phase of *familiarisation* with interview data when I was transcribing the interviews to turn them into texts. Simultaneously I noted my initial ideas (for instance, the incoherence in learners' responses, their answers such as 'I do not know' indicating that they may not be habitual of thinking critically about learning mathematics, etc.) and entered the hermeneutic circle of interpretation. While I was transcribing and making my initial notes, I was also reading research literature, planning for the next round of iterating the research design in another class, and discussing my ideas with the members of local research team of

LOCUMS at the same time. In between these phases of reading, planning and discussions, I would again go back to transcribing the rest of the interviews. This cycle of moving back-and-forth between transcribing, reading, planning, and discussing, is what I understand as my involvement with hermeneutic circle of interpretation. The context of these interpretations and the scope of this circle kept on evolving and increasing as I started reflecting over my initial understandings in relation to the transcriptions of all interviews, learners' socio-cultural and historical context, my own presumptions and experiences of learning mathematics, educational policy documents, research literature and discussions with other researchers. It is this dynamic, and non-linear but reflective and recursive style of thinking, which helped me to go through the phases of *generating initial codes inductively* from the data and *identifying potential themes* and sub-themes at both *semantic* and *latent* levels. For instance, some of the initial codes I generated were – inconsistent, incoherent, relevance, importance, participation, co-responsibility, cooperation etc. Further, some of the potential themes were – learners' thoughts about what to learn in mathematics, why to learn mathematics, how to learn mathematics, learners' critical thinking, learners' beliefs, democracy, decision-making etc.

Byrne (2022) explains *inductive coding* as a 'data-driven' approach to coding which starts from the data and is free from any preconceived theory or conceptual framework (p. 1396). He further clarifies that coding at a semantic level entails that codes are "identified through the explicit or surface meanings of the data" whereas coding at a latent level entails going "beyond the descriptive level of the data" and attempting "to identify hidden meanings or underlying assumptions […] that may shape […] the descriptive content of the data" (Byrne, 2022, p. 1397). Consequently, when coding style is latent, the analysis becomes much more interpretive, requiring researcher to be more creative and active in creating the codes. Since my approach to generative initial codes and identifying themes and sub-themes was 'data-driven' and the analysis frameworks I ended up using in the papers (see Table 4) resulted from my engagement with the data first and then with the literature, I understand it as an inductive coding. Analogously, the generation of codes and themes do not focus only on the descriptive or explicit meaning of the content but also the hidden meanings and underlying assumptions, therefore, I understand that the codes and themes were generated by emphasising both semantic and latent approaches.

After generating initial codes and identifying potential themes, I entered in the phases of *reviewing*, and *defining and naming* the themes. In these phases, generated themes were reviewed with respect to coded interview extracts, compared with my naïve understandings, and revised in relation to other sources such as research literature, discussions, socio-cultural contexts, etc. so that an overall comprehensive understanding of learners' lived experiences of learning mathematics can be formulated. The *main* themes which resulted from this synthesis were, *learners' critical thinking about learning mathematics*, *learners' beliefs about the relevance and importance of learning mathematics*, and *learners' experience of learner autonomy in mathematics classrooms*. Finally, the *written reports* illuminating learners' experiences and presenting their voices about learning mathematics were produced in the form of three papers (representing three main themes) in which compelling extract examples were exhibited and the analysis was related to the research question and literature. Table 4 presents an overview of the written reports including the data material, the socio-cultural context, analysis frameworks, themes and sub-themes which were generated during data analyses.

An overview of the data material and interpretation procedure employed for data analysis in each of the papers is presented in Table 4 below.

Title of the papers	**Research question**	**Social constructivist feature**	**Critical theoretical feature**	**Data material and approach to data analyses**	**Data analyses frameworks**	**Themes and sub-themes**
Learners' critical thinking about learning mathematics (extended version of Sachdeva and Eggen (2019))	What can learners' expressed mathematics related beliefs reveal regarding their practice with thinking critically about and potential to give suggestions concerning their mathematics learning process?	Learners as individuals participating in the socio-political context of their mathematics learning process. **Learners' critical thinking** about their mathematics learning process is explored.	Learners asked to exercise the **critique** of one's mathematics learning process.	74 learners' responses to selected statements in the questionnaires and interviews with 19 learners. Hermeneutic or interpretivist phenomenology.	Counting learners' responses to selected Likert-scale statements in the questionnaire by using MS Excel and reflexive TA of interview excerpts using **Critical Thinking skills** framework.	**Main theme:** Learners' critical thinking *about* learning mathematics **Sub-themes:** what, why and how to learn mathematics.
"We learn it [mathematics] at school so one thinks that one will use it …": learners' beliefs about relevance and importance of learning mathematics	What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the sources of information influencing the formation of their beliefs?	**Learners' subjective beliefs** about the relevance and importance of learning mathematics are explored. The influence of socio-political contexts and interactions on learners' beliefs.	Learners asked to **reflect critically over and justify** their relevance and importance beliefs about learning mathematics for themselves, and to **consider the socio-political influence** on one's subjective beliefs critically.	Interviews with 19 learners. Hermeneutic or interpretivist phenomenology.	Reflexive TA of interview excerpts using three categories of beliefs, **descriptive**, **inferential**, and **informational**.	**Main theme:** Learners' beliefs about learning mathematics **Sub-themes:** relevance beliefs, importance beliefs, descriptive beliefs, inferential beliefs, informational beliefs.

Students' experiences of learner autonomy in mathematics classes	What can young learners' descriptions communicate about their experiences of learner autonomy in their mathematics classes?	Individual **learners' expressed subjective experiences** of having learner autonomy in social contexts and interactions of their mathematics classrooms are explored.	Learners asked to **assume freedom of choice and learner autonomy** to suggest alternatives and changes in one's mathematics teaching-learning processes.	Interviews with first 10 learners. Hermeneutic or interpretivist phenomenology.	Preliminary inductive analyses of leaners' experiences of having learner autonomy in mathematics classrooms.	**Main theme:** Learner autonomy in mathematics classroom

Table 4 An overview of each paper's title, research questions, theoretical considerations, and the data analyses process.

4.4.2 The *critical* outlook in analysing data

Like discussing how the critical outlook was integrated in data gathering process (see section 4.3.4), in this section I describe how the critical outlook was integrated in data analyses by drawing on the literature concerning CME, critical pedagogy and hermeneutics/interpretivism as an analytic paradigm. This discussion will support the theoretical underpinnings of this thesis, which are grounded between social constructivism and Critical Theory.

The integration of a critical outlook in the phase of interpreting the data occurred when the data was analysed in relation to the research literature concerning CME, socio-political issues concerning mathematics education, policy documents and critical pedagogy. The concerns such as, developing learners' critical reflection towards their mathematics education, growing them into future critical citizens, democratic participation in educational decision-making and learners' empowerment, were focused on while interpreting learners' interview responses in relation to the context around their mathematics education. This constant cycle of going back-and-forth between the hermeneutic circle of *text (data)* and *context* (literature, discussions, my own subjectivity, policy documents, social context, etc.) marks the presence of a critical edge in the process of interpreting data and to position and present learners' voices in relation to critical pedagogy's concerns with empowering learners and developing their critical citizenship and active democratic participation skills.

Another means by which the critical outlook was incorporated in data interpretation and analysis process is the use of hermeneutic (or interpretive) phenomenology and reflexive TA as a way of doing analyses itself. Wojnar and Swanson (2007) emphasise that though the contextual features of a lived experience in interpretive (or hermeneutic) phenomenology can be "generated from a blend of meanings and understandings articulated by the researcher and participants" (p. 177), but the researcher must be self-reflective in the process of interpretation. Highlighting Ricoueur's theory of interpretation, Lindseth and Norberg (2004) emphasise that in attempting to gain a comprehensive understanding of a lived experience, we (the researchers) may not free ourselves from our preunderstandings, and we are only aware of some features of it. Therefore, "through *critical* reflection, we can revise, broaden and deepen our awareness" (Lindseth & Norberg, 2004, p. 150, italics added). Similarly, Braun et al. (2018) explain that to determine themes as conceptually founded patters "requires depth of (close and *critical*) engagement to move beyond the surface or obvious content of the data and to identify implicitly or unexpected unifying patterns of meaning" (p. 848, italics added). Consequently, both hermeneutic (or interpretive) phenomenology and reflexive TA encourage the researcher to adopt a critical approach and be cautious that one's preunderstandings can influence data analyses process. Assuming such critical orientation between the cycles of interpretation and reinterpretation of data in relation to one's own bias, research literature, and in discussions with others can make the researcher watchful of not overinterpreting or putting in one's own presuppositions in the interpretations. This critical orientation was incorporated in data analyses process of this study by being critical and self-reflective in repeated and recursive cycles of going back-and-forth between the *text (data)* and *context* (literature, discussions, my own subjectivity, policy documents, social context, etc.).

4.5 Trustworthiness of the study

Unlike quantitative studies, the reliability and validity of a qualitative study is considered based on its *trustworthiness* (Bryman, 2016; Guba & Lincoln, 1989; Lincoln & Guba, 1985). The objective of this qualitative study is not to establish any statistical tests or to make any generalizable or universal statements about learners' experiences of learning mathematics (that is, the phenomenon enquired). Therefore, the trustworthiness of this qualitative study can be presented by its credibility, transferability, dependability and confirmability, rather than its statistical validity (Bryman, 2016; Nowell et al., 2017). To establish the trustworthiness of a qualitative research study, it is significant that the research design and the data analyses processes are transparently described along with its findings and conclusions. The sub-sections below discuss this study's trustworthiness.

4.5.1 Credibility

The first criteria to establish the trustworthiness of a qualitative research study is to ensure its credibility. The accuracy and believability of the research results are questioned in order to establish and judge the credibility of a qualitative study (Creswell, 2014). Nowell et al. (2017) suggest that credibility addresses the correspondence between respondents' views and the researchers' representation of them, and can be addressed through a range of techniques such as data triangulation, researcher triangulation, peer briefing, prolonged engagement, member checking (conforming interpretations with participants), etc. In this study the credibility was achieved by using data triangulation, researcher triangulation and peer briefing. Gathering learners' perspectives by employing both questionnaires and individual semi-structured interviews is used as a mode of data triangulation, and data from both the instruments conform to observations for instance, that learners were not habitual of thinking critically about learning mathematics. Moreover, adopting a self-critical orientation while interpreting and analysing raw data in relation to personal presumptions and research literature, researcher triangulation and peer briefing techniques were also used to check preliminary findings against the raw data, strengthening the credibility of findings. The coherence of research findings presented in the papers with similar previous research literature also adds to the credibility of findings.

4.5.2 Transferability

Bryman (2016) suggests that the transferability of a qualitative research entails the applicability of research findings to other contexts. The transferability of research findings in a qualitative study cannot be established by the researcher but is often decided by the readers. However, the researcher's task is to assist the reader's consideration regarding transferability by providing a rich and thick description of the context of research, the research design, data gathering and analyses strategies. In this thesis, a detailed description of the research context and analysis tools is provided so that the transferability of research findings in other contexts like this study can be examined. Moreover, elaborated information of the context of learners' mathematics education such as, a description of Norwegian educational

policy, insights about the socio-cultural settings of their classrooms, details of data gathering and analyses techniques, etc. is provided to further ease the evaluation of transferability.

4.5.3 Dependability

The third criteria of evaluating trustworthiness of a qualitative study is named as dependability (Miles et al., 2014). Dependability entails that the research findings are consistent and can be replicated. To attain dependability, "the researchers can ensure the research process is logical, traceable and clearly documented" (Nowell et al., 2017) (p. 3). Therefore, the preceding sections (see section 4.3 to 4.4) present a clear account of the choices made and steps taken throughout the process of gathering and analysing data to make the process more traceable and open to evaluate researcher's reasoning. To further support dependability of the study, the questionnaire, designed classroom interventions and interview guide for semi-structured interviews are also attached as appendices to this thesis.

4.5.4 Confirmability

While establishing the fourth criteria of trustworthiness, namely, the confirmability for a qualitative study, it is important to demonstrate that researcher's interpretations and findings are clearly derived from the data (Nowell et al., 2017). The researcher is required to make clear that how conclusion and interpretations have been reached. Guba and Lincoln (1989) maintain that establishing confirmability is contingent upon the attainment of the study's credibility, transferability, and dependability. Therefore, researchers are encouraged to reason for their "theoretical, methodological, and analytical choices throughout the entire study so that others can understand how and why decisions were made" (Nowell et al., 2017, p. 3). Considering the confirmability of this study in the previous sections, I have explained my theoretical, methodological, and analytical choices that were taken throughout the study along with the reasons motivating these choices.

4.5.5 Limitations of the study

Along with discussing the trustworthiness of a qualitative research study, it is also significant to specify its limitations. The first limitation of this study lies in its scope. This sub-project of LOCUMS was carried out in two lower secondary schools located in Central Norway. The participants for questionnaires and classroom interventions were 74 8^{th} and 9^{th} grade learners and the interviews were conducted only with 20 learners. Consequently, the sample size of this study is small and therefore the findings are relative and contextual. Therefore, the findings of these study cannot be generalised. The second factor that can be considered as a limitation of the study is the delay in clarification of its research interest. The research interest took a turn from focusing on cultural aspects related to learners' mathematics and science learning activities to investigating learners' perspectives and their experiences of learning mathematics under a critical lens after getting the first questionnaire responses. Accordingly, the questionnaire and interview guides were revised, and more questions were added asking learners to reflect critically over their mathematics learning experiences. Introducing these changes earlier could have contributed to increased goal-orientation of the study from the

beginning. The third limitation of this study is lacking member checking. The trustworthiness of the study could be increased if we could have asked the learners' (that is our informants) to check and confirm if the interpretations and findings of this study are consistent with their perspectives and views. However, we could not achieve member checking due to the constraints of time and resources available for the project.

Reflecting on the methodological journey I took while gathering and analysing data in this research study and while writing up its findings, I realise that my awareness of the choices and decisions made during the project have evolved. I have become conscious of acknowledging my presumptions and preconceptions I had when I embarked on this methodological journey and how these presumptions may have influenced my interpretation and analysis of learners' responses. This process of evolvement and my development as a researcher surfaces in, and can be observed by noticing the difference in the style of writing the *data analyses* and *finding* sections of Paper I and Paper II. Considering the limitations of this study, I have developed a recognition of the potential of improvement in writing the papers and in conducting this research study. These potentials of improvement are discussed in sections 5.1.3, 5.2.3, 5.3.3, and 6.3 respectively.

4.6 Ethical considerations

Tangen (2014) signifies the importance of using ethical considerations of a study to reflect over the protection and benefit of research participants, on the one side, and the internal and external quality of research on the other. The ethical reflections can be divided into four main categories including: reflections in the design phase discussing the justification and relevance for the field and practice; reflections during the data gathering and analysis; ethical considerations while reporting and publishing a project and finally, the reflections concerning role of the researcher bringing ethical challenges in the research study.

Considering the first domain of reflection, the justification and relevance of this research study is elaborated and established in the first chapter of this thesis by explaining the requirement of mathematics education research studies which highlight learners' perspectives and bring forward their voices about learning mathematics. These ethical reflections, however, are not only limited to justifying the need of research for the sake of filling a research gap in existing literature, but it also entails clarifying the agenda of research to the research participants and the potential informants of the study. Therefore, it was significant to provide enough information to the research participants so that they are aware of the agenda and rationale of this study. Consequently, a document providing written information to the participating schools and the learners as informants was distributed (attached in appendix 9.2) along with informing them about my duty to deal with the data confidentially. However, since the informants in this study are minors, their parents and guardians were given written information about the project and their consent was attained to take audio and video recordings of the classroom activities and the semi-structured interviews. In addition, it was also ensured during the rapport of each interview that learners are aware of my duty of confidentiality and their right to withdraw their consent without providing any reason for this withdrawal.

The second domain of reflection includes the data gathering and analysis process. To get permission for gathering sensitive and personally identifiable data, I applied to the Norwegian Center for Research Data (NSD, now SIKT). I received the permission (attached in

appendix 9.1) from NSD to collect the data with instructions regarding how to handle and store sensitive information. Further reflections concerning data gathering and analysis methods are presented earlier in this chapter.

According to Tangen (2014) the ethical responsibility of a researchers does not end after taking informed consent from the participants and ensuring the confidential treatment of sensitive data collected as per the demand of research project, but it is also essential that the researcher maintains the promise of anonymity for the protection and benefit of the participants. Keeping the agreement of confidentiality, no names of the schools, teachers or the learners are revealed in this thesis. The location of the schools is also not disclosed. This anonymity was also ensured in the research papers published as a part of this thesis where no names are mentioned. These papers only mention the Norwegian school context, the information about age and gender of the informants and their cultural backgrounds. This information does not give access to any personally identifiable attributes of the informants.

Finally, in the fourth domain of ethical reflection, Tangen (2014) maintains that participation of the researcher in the context of study may bring challenges for the conduct of research. I have reflected over and discussed my role as a participant in the research process earlier in this chapter where I illustrated how my presence under the data gathering process may or may not have influenced the quality of data that was accumulated. Following these domains of ethical reflections throughout the data gathering and analyses process have allowed me to stay transparent in dealing with the challenges and opportunities provided by this research project and simultaneously assure the protection of rights and benefits of the participants.

5 Findings in and across the papers

The quest of this research study started from reflections I had made after reading the research literature discussing learner-centred educational approaches. The interest to know learners' perspectives about their own mathematics learning processes increased after reading the concerns of empowering learners and developing their critical thinking faculties and active democratic participation skills registered in educational policy documents and in research literature. Imparting these abilities among learners is listed as an aim of learners' mathematics education process. Further, learners' questionnaire responses contrary to my assumptions from Norwegian learners having the right to co-operate in decisions concerning their own educational activities contributed to my curiosity to know more about the learners' opinions, beliefs, and experiences of learning mathematics. To cater for these research interests, I explored individual learners' voices informing about their critical perspective towards learning mathematics and investigated their expressed experiences of autonomous involvement in decisions concerning their mathematics learning activities.

The three papers attached to this thesis inquire into different aspects of these research interests in depth, which collectively contribute to answer the overall research question of the project. In Paper I (an extended version of Sachdeva and Eggen (2019)), learners' potential of thinking critically *about* their personal mathematics learning process is explored and analysed using the Critical Thinking skills framework presented by Facione (1990). Paper II explores learners' relevance and importance beliefs about learning mathematics for their own lives using the categories of beliefs, i.e., descriptive, inferential, and informational beliefs presented by Bar-Tal (1990). Paper III presents preliminary analyses of first ten learners' experience with exercising learner autonomy in their mathematics classrooms. A summary of the research focus and findings of these three papers is presented in the following text along with discussing the red thread going through these papers. I also write about planned changes to be made in the third paper to develop it into a journal article.

5.1 Paper I: Learners' critical thinking about learning mathematics

Sachdeva and Eggen (2021). Learners' Critical Thinking About Learning Mathematics. *International Electronic Journal of Mathematics Education, 16*(3), 1-18. doi: https://doi.org/10.29333/iejme/11003.

5.1.1 Research interest

It is an extension of the conference paper *"Students' critical perceptions about mathematics education"* (Sachdeva & Eggen, 2019) presented at the 9th international Mathematics Education and Society (MES) conference held in India in 2019. The conference paper presented a preliminary analyses of interview responses from first ten learners who were interviewed. This paper extends the analyses process to include all the 74 learners' responses to selected statements in the questionnaire and all the 19 learners' interview responses to selected questions. Further, the critical thinking skills framework submitted by Facione (1990) was employed for analysing the data. The following research question is enquired in the paper:

What can learners' expressed mathematics related beliefs reveal regarding their practice with thinking critically about and potential to give suggestions concerning their mathematics learning process?

The scope of the research question was narrowed down with the help of three sub-questions: *what* subject content do learners find interesting/not interesting to learn in mathematics?; *why* they learn mathematics?; and *how* their mathematics teaching may be changed?

5.1.2 Key findings and significance of the paper

Based on the literature review regarding critical thinking in mathematics education research field, I observed that learners' critical thinking *in* mathematics is often investigated whereas their critical reflections *about* their personal mathematics learning process are seldom investigated. Learners' responses such as, "I do not know" and "I have not thought much about it" indicate that they are not habitual of thinking critically about what content in mathematics they like to learn. Moreover, they seemed to struggle justifying their own beliefs about why learning mathematics is relevant and important for their personal lives and hesitate in suggesting improvements to make learning mathematics meaningful for them. Consequently, learners' mathematics teaching and learning processes and their personal beliefs concerning the same does not seem to be the objects of their critique. They also demonstrate trust in their education system and seldom question any decisions made in their classrooms. Therefore, a meta perspective of their learning contexts does not seem to be developing among learners. However, if they are prompted and encouraged to think critically about mathematics education, a few learners display the potential of contributing to improve their mathematics learning process by suggesting *what* content they could learn in mathematics and *how*.

On a conceptual level, the significance of this paper lies in the discussion of different ways in which critical thinking has been addressed in mathematics education research field to help learners to gain a meta-perspective of their mathematics learning. The limitations of different ways in which critical thinking is interpreted in mathematics education research are presented, and the potential of using critical thinking to observe one's beliefs and personal mathematics learning process is discussed. On the level of data analysis, the self-regulation skill and its sub-skills, self-examination, and self-correction, from critical thinking skills framework are used to analyse learners qualitative interview responses which is a novice approach. The critical thinking skills framework is broadly used to analyse quantitative data to explore learners' critical thinking skills in mathematical problem-solving, but this framework has seldom been employed in a qualitative study. In this study, learners' critical faculties regarding their *personal* thoughts and beliefs about their mathematics learning are examined by using this framework.

5.1.3 If written now, what I would have changed in the paper?

Reflecting on how Paper I is written and structured, I recognize several changes which could have improved the paper. The choice of words used to describe the informants is one

thing. In expressing that the learners were not habitual of thinking critically about their mathematics learning, the words *unawareness, uncritical* and *inexperienced* are used. If I had rewritten the paper now, I would have used other words like, *learners' replies indicate that they may not be used to* or *habitual of*, to indicate that learners express that they do not usually think critically over the questions of *what*, *why* and *how*, related to their mathematics learning process. The paper is long, so I also recognise the scope of shortening it along with structuring it better to write short sections and avoid repetitions. The data analyses section has another shortcoming that the analyses is less elaborated. This limitation may give the impression that some of the claims are unsubstantiated, such as, the learners lack training, experience, and practice in thinking critically about learning mathematics.

The interview transcripts include instances in which learners expressed that they are not asked to suggest alternatives or changes in their mathematics teaching-learning activities. In rewriting the paper, I would have included such excerpts so that the claims could be better supported. I have subsequently become aware of differentiating clearly between what the learners expressed in their interview responses and what is my interpretation (influenced by my worldview) of learners' interview responses. I also realise the significance of substantiating my interpretations with: (a.) learners' statements (the words they used) in their replies, and (b.) the theoretical and methodological lenses I chose to analyse their statements.

5.2 Paper II: Learners' beliefs about relevance and importance of learning mathematics

Sachdeva, S., & Eggen, P.-O. (in press). "We learn it [mathematics] at school so one thinks that one will use it …": learners' beliefs about relevance and importance of learning mathematics. *Acta Didactica Norden*.

5.2.1 Research interest

This article is accepted for publication in the journal. In this paper, learners' responses to the question of *why* they learn mathematics (a part of Paper I) are analysed in further detail. While analysing the data for Paper I, I observed that most of the learners' responses to why they think learning mathematics is relevant included references to elementary calculation skills which they used in their daily lives instead of referring to the secondary-level mathematical knowledge they were learning in 8^{th} and 9^{th} grades. In addition, their responses to the question about why they consider learning mathematics to be important for their personal lives (present and future) included reference to the statements about importance of learning mathematics they had heard from teachers, elders, or other information sources. These observations stimulated my interest in exploring learners' beliefs about the relevance and importance of learning mathematics and the sources of information influencing the formation of their beliefs. The research question investigated in this paper is:

> *What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the sources of information influencing the formation of their beliefs?*

5.2.2 Key findings and significance of the paper

The data analyses illustrate that learners seldom reflect over the relevance and importance of learning mathematics for their own lives, and express trust in their school system that the content chosen for them by the school will be relevant for them. They also express strong belief in the relevance and importance of mathematics education. Their relevance beliefs are grounded in *extrinsic* value (use-value) of learning mathematics. On the other hand, their importance beliefs are grounded only in the exchange-value of learning mathematics, with no reference to the *intrinsic* value of gaining mathematical knowledge and skills. Despite this strong belief, majority of learners refer only to the basic arithmetic operations as an example of useful mathematical knowledge. Further, learners refer to the statement heard from others as reasons for their strong belief in the importance of learning mathematics. Nevertheless, except a few, the learners did not seem to question the relevance or importance of secondary-level mathematical subject content they were learning.

Categorising learners' beliefs according to the belief categories suggested by Bar-Tal (1990) reveals that learners' relevance beliefs seem to be formed on the bases of their direct experiences, inferences, and information received from others. Further, their importance beliefs seem to be formed on the bases of their inferences and information received from others, and not on their direct experiences with intrinsic value of learning mathematics. The significance of this paper lies in its suggestion to treat the relevance and importance of learning mathematics as two concepts distinct from each other in mathematics education research and the difference in learners' responses to questions concerning these two concepts. In addition, the discussion is related to the concerns of learners' empowerment and developing their capability to critically question and participate in decisions concerning their mathematics learning activities registered in research literature writing about CME and socio-political issues related to mathematics education.

5.2.3 If written now, what would I have changed in the paper?

The limited number of words allowed in a journal article restrict the scope of details which can be included in data analysis part, and the discussion of possible implications in the paper. These are two sections of the paper I would have elaborated more on if I were to revise it. To give a detailed account of how the inductive reflexive TA was carried out by going back and forth between the categories of beliefs and learners' responses would have contributed to clarify the nuances of the data analyses and assist replication of the study. Additionally, a tension was observed between the formulations of educational policy documents and mathematics education research literature emphasising learners' empowerment and the Norwegian mathematics curriculum not stating these concerns explicitly. I would elaborate this issue if I had the scope to do so in the paper.

5.3 Paper III: Students' experiences of learner autonomy in mathematics classes

Sachdeva, S. (2019). Students' experiences of learner autonomy in mathematics classes. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*

(CERME11, February 6 - 10, 2019) (pp. 1978–1985). Utrecht University and European Society for Research in Mathematics Education, ERME. https://hal.archives-ouvertes.fr/CERME11/hal-02421636

5.3.1 Research interest

In this paper, learners' responses to the questions concerning *how* they learn mathematics (a part of Paper I), and how they would like to learn mathematics if given the chance to change, are analysed in further detail. While analysing the data for Paper I, I observed that most of the learners first expressed that they were satisfied with their mathematics teaching, but when prompted to reflect again, assume freedom and learner autonomy to choose and decide, they came up with suggestions to change their style of mathematics teaching. Therefore, the focus of this paper is to illuminate learners' experiences with being involved in decision-making activities and cooperate (i.e., experience of learner autonomy) in designing their mathematics learning activities in their mathematics classrooms. The paper is written during an early phase of data interpretation process and is based upon the preliminary analysis of interview responses of the first 10 learners. It is a conference paper which was limited in its scope and discusses the preliminary findings from the early phase of data interpretation and analyses. The research question explored in this paper is:

> *What can young learners' descriptions communicate about their experiences of learner autonomy in their mathematics classes?*

5.3.2 Key findings and significance of the paper

Learners' responses concerning involvement in decisions concerning and having learner autonomy in their mathematics classroom exhibit that they may have limited experience with self-control and self-decision. The learners expressed that by and large it is their teacher who has the authority in mathematics classroom, and the decisions regarding what will happen in their mathematics classroom are usually made in advance by school authorities and the teachers. They also expressed their trust in the decisions made for them by their school and teachers. Therefore, the learners' autonomy may not get much attention to emerge and get practiced. Learners' responses also suggested that they subsequently may get used to accepting their mathematics learning activities as they are practiced in routine. They may also not claim their right to be involved in the decision-making process, or to influence their mathematics teaching-learning practices. Their descriptions of their routine classroom practices gave the impression that their experiences of learning mathematics may be of a traditional[48] style. On being asked to suggest changes in their classroom practices, they were hesitant of proposing alternatives. However, when encouraged, some learners' potential to become the discussion partners of their teachers and suggest, design, co-operate and improve the teaching-learning practices in mathematics manifested itself.

[48] Demonstration of a method to solve a mathematical problem by the teacher followed by learners practicing the same method by solving similar problems in their notebooks.

In mathematics education research, the concept of learners' autonomy is often discussed in relation with learners' intellectual capacities to find out new strategies and methods to solve mathematical problems (see e.g., Yackel and Cobb (1996) and Ben-Zvi and Sfard (2007)). In this paper, the concept of learner autonomy has been discussed in relation to learners' taking charge of their own learning in a sense that they understand the purpose of their learning program, can take initiatives, and collaborate with their teachers to provide a form to and get engaged in their own learning process. Such an understanding correlated with how learners' active democratic participation in decisions concerning their mathematics learning activities is described by scholars such as, Mellin-Olsen (1993b), Ernest (2002), Skovsmose and Valero (2005) and Lindenskov (2010). This study, hence, contributes to the research concerning learner autonomy in mathematics education by providing the learners with an opportunity to express their experiences regarding learner autonomy in mathematics classes, not only in an intellectual sense, but also to assume an active autonomous participation in decisions about their mathematics learning process.

5.3.3 How I plan to revise and rewrite this paper?

This paper was written in the early phase of data analyses and was limited in its scope due to being a conference paper. Consequently, it has shortcomings and should be rewritten to communicate its scientific contribution clearly to the research field. Therefore, in this section I discuss how I plan to revise and rewrite this paper. Taking a general overview, the background and context of the research should be elaborated more with the help of Norwegian educational policy and research literature discussing learners' autonomy in their educational processes. However, there are two specific issues I want to improve: (a.) clarifying the meaning and understanding of the concept of learner autonomy in relation to mathematics education, and (b.) the data analysis.

Learner autonomy is discussed in mathematics education literature on different forms. For instance, Yackel and Cobb (1996) and Ben-Zvi and Sfard (2007) discuss learners' autonomy in an intellectual sense that learners can work independently, choose strategies for solving a mathematical problem-solving on their own, decide their flow of task themselves, have free communication in the class, etc. On the other hand, Mellin-Olsen (1987, 1993b), wrote about learners' having the possibility to take control and ownership of the learning activities taking place in their mathematics classroom in a broader sense, and sharing this control and ownership with their teachers. The latter form of discussing learners' control and ownership is coherent with my understanding of learners' autonomous and active democratic participation in the decisions concerning their own mathematics learning. Therefore, I will elaborate my understanding of the concept of learner autonomy in close association with learners' possibility to take control and ownership of parts of their own mathematics learning activities.

Mellin-Olsen (1993b) mentioned three levels of control related to the didactical activity of learning mathematics: control on the *goal-level*, on the *choice-level*, and on the *use-level*. Control on the goal-level entails having authority to choose the goal of a didactical activity (e.g., choosing which tasks to work with). Control on the choice-level entails having the authority to select (choose) the tools or instruments for a didactical activity (e.g., selecting which knowledge and skills to employ). Finally, the control on the use-level entails having the authority to choose how to use the selected instrument or tool in a didactical activity (e.g.,

choosing in which way the knowledge or skill should be used to solve the tasks) (see e.g., Mellin-Olsen (1993b, p. 66)). The intellectual sense of understanding learner autonomy (i.e., Yackel and Cobb (1996) and Ben-Zvi and Sfard (2007)) can be seen as being coherent to the control at the use-level as described by Mellin-Olsen (1993b). In the revised version of this paper, I aim to discuss this and possibly other correlations between the two ways of looking at learners' authority and ownership in their own mathematics learning activities.

The understanding of taking the charge of one's own learning (cf. Mellin-Olsen (1993b)) also correlates with how Little (1991) defines the notion of *learner autonomy*. Little (1991) considers individual's autonomy as his/her "*capacity* – for […] critical reflection, decision-making, and independent action" (p. 4, original italics). Little (1991) further maintains that having learner autonomy "presupposes, but also entails, that the learner will develop a particular kind of psychological relation to the process and content of his learning. The capacity of autonomy will be displayed both in the way the learner learns and […] transfers what has been learned to wider contexts" (p. 4). Later, by referring to Holec (1981), Little (2003) also clarified that "**autonomous learners** understand the purpose of their learning […], explicitly accept responsibility for their learning, share in the setting of learning goals, take initiatives in planning and executing learning activities, and regularly review their learning and evaluate its effectiveness" (p. 1, bold in original). However, these literary sources also emphasise that learner autonomy is not equal to independent or teacher-free learning. In the revised version of this paper, I will explain the notion of learner autonomy in mathematics learning around the understandings of this concept put forward by Mellin-Olsen (1987, 1993b) and Little (1991).

To revise the data analysis part, the idea is to employ the three levels of "kontroll av kunnskaper" (*control of knowledge)* described by Mellin-Olsen (1993b). In the interviews, I asked learners question such as, "if you had the chance, what would you change in your mathematics classroom?", "who takes decisions in mathematics classrooms?", "do you get to choose how you want to learn mathematics, if yes, what do you get to choose and decide?", "would you like to get responsibility of your own learning activities", etc. Mellin-Olsen (1993b) explains that *a person (a learner)* can be said to have *control over knowledge* if s/he can: (a.) apply the knowledge when required without the teacher's supervision; and (b.) has the metalevel insight that s/he masters that knowledge, has the right of disposal, and can grab control over that knowledge on his/her own (p. 65). Further, he mentions three levels of control which can be derived in a situation where knowledge is mediated:

N1: control over the goal-oriented didactical activity, i.e., *goal-level control*;
N2: control over the selection and choice of tools which should be used in that didactical activity, i.e., *choice-level control*; and
N3: control over the use of chosen tools in the activity, i.e., *use-level control* (p. 66).

Taking the didactical activity of learning any subject as an example, Mellin-Olsen (1993b) elaborates that the distribution of control between the teacher and the learner varies from one subject to another and also from one teacher to another. Further, the three levels of control, according to Mellin-Olsen (1993b), create eight possibilities of distributing control of the learning activity between the teacher and the student, as shown in the Table 5 below. The possibility 2 listed in Table 5, i.e., $\mathrm{control}(N1, N2, N3) = (cT, cT, cL)$, indicates that the teacher (**T**) has control (**c**) over the *goal-level* (**N1**) and *choice-level* (**N2**), whereas the learner (**L**) has control over the *use-level* (**N3**) of the didactical activity.

Levels of control →	N1	N2	N3
Possible distribution of control ↓			
1	cT	cT	cT
2	cT	cT	cL
3	cT	cL	cT
4	cT	cL	cL
5	cL	cT	cT
6	cL	cT	cL
7	cL	cL	cT
8	cL	cL	cL

Table 5 Possibilities of distribution of control (c) between the teacher (T) and the learner (L). Adapted from Mellin-Olsen (1993b, p. 67).

The possibilities of distribution (2 – 8) in which learners assume the control of any level of their learning activity can be related to Little's (2003) description of autonomous learners in which the learners take initiatives, charge and responsibility of either the goal, the tools or the use of tools involved in their mathematics learning activity. Therefore, both learners and teachers can have autonomy to control different levels of the didactical activity of mathematics learning. I will use these levels and possibilities of control distribution to analyse learners' interview responses to the interview questions mentioned above. The idea is to analyse learners' responses to explore on which levels of control do they experience that their teacher assumes the autonomy and identify on which levels of control they themselves express the desire to assume autonomy in their mathematics classroom if given a free choice. I will also explore if they express any levels of control on which they would not like to assume autonomy in relation to their mathematics learning activity. Thus, the revised version of this paper will unfold learners' expressed experiences of having learner autonomy in their classrooms (the current version) to identify their potential of assuming learner autonomy and influence the decisions taken about their mathematics learning activities (in revised version).

5.4 Tying a *red thread* across the papers

The papers mentioned above can be seen as diverting from each other in certain aspects, yet they relate to each other in a fundamental sense. In this section I draw the *red thread* which ties these papers together while maintaining each of the papers' independent contribution. In the following text, the differences between these three papers are highlighted before illustrating their commonalities and points of intersection.

Each paper presented above explores a different aspect of young mathematics learners' voices about their own mathematics learning processes. In Paper I, learners' responses are analysed to identify their potential of critically perceiving and suggesting changes in the content (*what*), the relevance and importance (*why*), the way (*how*) of their own mathematics teaching-learning processes. In other words, this paper presents an overview of learners' critical thinking about *what*, *why* and *how* of their mathematics learning. In the second paper, learners' responses to the question of *why* learn mathematics, i.e., why they consider learning mathematics to be relevant and important for their lives are addressed in-depth. Learners' responses to the question of *how* they participate in their mathematics learning activities are thoroughly analysed in the third paper. Their participation is identified in their expressed

experiences of having learner autonomy in mathematics classroom, being involved in decisions concerning their learning activities, and suggesting changes in the teaching-learning processes of mathematics. Therefore, these papers pay attention to different concepts by using dissimilar frameworks for data analysis and bring forward distinct findings about the questions investigated.

Despite the differences, these papers have several points of intersection which draw the red thread through them. A central and common aspect in all these papers is their relationship to the concepts of critical thinking and active democratic participation of learners in decisions concerning their mathematics learning processes. These three papers can be considered as a collective work illustrating learners' voices about their own mathematics learning processes, as an overview of the findings in each of the papers demonstrates in Table 6 below. They bring forward learners' potential of not only critically considering, but also suggesting changes in their mathematics teaching-learning activities to make them more meaningful for themselves. Another common characteristic in these papers is that they derive their motivation from the fundamental aim of the educational process, i.e., to develop learners' critical orientation so that they become critically aware, active, and autonomously participating citizens of the society based on and sustaining democratic principles. These concerns correlate with the issues of learners' empowerment and developing their critical citizenship skills in and through their mathematics learning process, as discussed in the research literature.

Title of the papers	Research question	Social constructivist feature	Critical theoretical feature	Data material and approach to data analyses	Data analyses frameworks	Themes and sub-themes	Findings
Learners' critical thinking about learning mathematics (extended version of Sachdeva and Eggen (2019))	What can learners' expressed mathematics related beliefs reveal regarding their practice with thinking critically about and potential to give suggestions concerning their mathematics learning process?	Learners as individuals participating in the socio-political context of their mathematics learning process. **Learners' critical thinking** about their mathematics learning process is explored	Learners asked to exercise the **critique** of one's mathematics learning process	74 learners' responses to selected statements in the questionnaires and interviews with 19 learners Hermeneutic or interpretivist phenomenology	Counting learners' responses to selected Likert-scale statements in the questionnaire and reflexive TA of interview excerpts using **Critical Thinking skills** framework	**Main theme:** Learners' critical thinking *about* learning mathematics **Sub-themes:** what, why and how to learn mathematics	Learners' responses such as, "I don´t know" or "I have not thought about it" indicate that they are not habitual of thinking critically about their mathematics learning processes. They express their trust in the socio-political institution of school and accept school's requirement of learning mathematics. When asked to think critically, their potential of contributing to improve their mathematics learning processes becomes visible.
"We learn it [mathematics] at school so one thinks that one will use it …": learners' beliefs about relevance and	What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the	**Learners' subjective beliefs** about the relevance and importance of learning mathematics are explored. The influence of socio-political	Learners asked to **reflect critically over and justify** their relevance and importance beliefs about	Interviews with 19 learners. Hermeneutic or interpretivist phenomenology	Reflexive TA of interview excerpts using three categories of beliefs, **descriptive**, **inferential**, and **informational**	**Main theme:** Learners' beliefs about learning mathematics **Sub-themes:** relevance beliefs, importance beliefs,	The learners' answers gave the reason to differentiate between relevance and importance of learning mathematics. Learners expressed a strong belief in the relevance and importance of learning mathematics. They

importance of learning mathematics	sources of information influencing the formation of their beliefs?	contexts and interactions on learners' beliefs	learning mathematics for themselves, and to **consider the socio-political influence** on one's subjective beliefs critically			descriptive beliefs, inferential beliefs, informational beliefs	seldom questioned these beliefs. I find that their relevance beliefs are formed on the bases of use-value of mathematics, and their importance beliefs are formed on the bases of exchange-value of mathematics instead of the intrinsic value of mathematical knowledge itself.
Students' experiences of learner autonomy in mathematics classes	What can young learners' descriptions communicate about their experiences of learner autonomy in their mathematics classes?	Individual **learners' expressed subjective experiences** of having learner autonomy in social contexts and interactions of their mathematics classrooms are explored	Learners asked to **assume freedom of choice and learner autonomy** to suggest alternatives and changes in one's mathematics teaching-learning processes	Interviews with first 10 learners. Hermeneutic or interpretivist phenomenology	Preliminary inductive analyses of leaners' experiences of having learner autonomy in mathematics classrooms	**Main theme:** Learner autonomy in mathematics classroom	Learners' expressed experiences of learner autonomy in mathematics classroom are limited to the opportunities provided by their teachers. A few learners exhibited a desire to attain autonomy and demonstrate the potential to suggest changes influencing their mathematics learning processes but without taking complete responsibility.

Table 6 An overview of each paper's title, research questions, theoretical considerations, data analyses, and findings.

6 Discussion, implications, and conclusion

The journey of this thesis started with an interest in exploring learners' voices and their experiences of learning mathematics from a critical perspective after reading their questionnaire responses. Their interview responses provided insight in their potential to adopt a critical outlook towards learning mathematics, being involved in the decisions concerning and suggesting changes in their mathematics learning activities. In this chapter, I discuss the findings as presented in the last chapter considering previous research studies and educational policy documents explored in the chapters: 'Introduction' and 'Background of the research focus'. After discussing the findings, the implications and conclusion of this study are also outlined.

6.1 Discussing findings of the study

I started this thesis with writing about my own experiences of learning and teaching mathematics from Norway and India and expressed interest in knowing Norwegian secondary school learners' voices about their experiences of learning mathematics. Beginning from the same point, I start this section by reflecting over Norwegian learners' expressed voices and experiences, the primary findings in this study, in relation to mine. Further, I discuss the secondary findings of the study, that is, the tension between the formal and experienced curriculum, the dilemma of preparing learners for their future lives and potential conflicting political interests highlighted in learners' educational policy documents.

6.1.1 Seeing myself in learners

Being a mathematics learner in the school, I had not reflected upon the questions such as what I am learning in mathematics, why I am learning mathematics, and how I am being taught mathematics. It was one of the subjects that was a part of my timetable at school, it was chosen by my school as a socio-political institution, and I was supposed to learn it to obey the decision of my school and instructions of my teachers. I followed what I was told and supposed to follow and did not reflect or questioned anything critically. I noticed similar features in Norwegian secondary school mathematics learners' responses in questionnaires and interviews. Therefore, I highlight three characteristics of learners' responses in which I could reflect my own experiences of learning mathematics in school. The first characteristic is most of the learners' expression that they had not reflected over the questions of what, why and how concerning their mathematics learning processes. They associated their mathematics lessons with solving problems in their textbooks. They described their classroom routine starting with the teachers' demonstration of a solution method to solve mathematical problems and learners solving similar problems to practice the solution method. These descriptions were like my own classroom experience of learning mathematics.

The second characteristic is learners' expressed trust in the socio-political institution of school and in their educational system. They stated that they learn secondary-level mathematical subject content at school and even if they have not yet experienced situations in which they could use the knowledge of mathematics learnt in 8th and 9th grades, yet gaining

that knowledge is important for them since it is taught in their school. Though a few learners expressed that it would have been better to learn how they could handle their finances (making budgets or paying bills, etc.) in their adult life instead of cramming formulas in mathematics, yet they expressed their trust that they would learn such stuff later (in grade 10 or above) in their school. The underlying reason and factors influencing learners' expressed trust in their school and educational system is not clear, but their responses did not seem much different than how I perceived my school and educational system as a secondary school learner.

The third characteristic is their acceptance of the socio-political rhetoric of importance of learning mathematics as indicated by the research (see e.g., Ernest (2004)) and emphasis on the importance of learning mathematics as stated in the national curriculum and international educational policy documents (see e.g., UNESCO (2015), OECD (2019) and Kunnskapsdepartementet (2019)). Learners repeated the statements heard from their teachers, elders, and other informational sources to justify their belief in the relevance and importance of learning mathematics. Similar findings have been indicated by Onion (2004), Kollosche (2017), and Ernest (2004) for instance. The reasons supporting the importance of learning mathematics were often the desire of getting a stable career, future financial security, and personal economic growth through by a good job with a handsome salary (see, e.g., Wiik and Vos (2019)). These statements reminded me of the choice I had made to pursue a master's degree in mathematics despite my interest to continue studying psychology after bachelors. Reflecting over these statements by learners, I recognise the echo of my family's argumentation for pursuing a higher degree in mathematics since this choice could increase my chances of getting a good job and build a better career than in psychology.

Besides noticing these similar features and analogous to the findings of Lindenskov (2010), I also identified the potential visible in some of the learners' voices to adopt a critical perspective towards and assume learner autonomy to suggest changes in their mathematics learning processes. Though most of the learners' responses indicated that they were not habitual of thinking critically over their mathematics learning processes, but when encouraged to reflect critically, a few of them suggested alternate subject content or teaching methods to change their classroom practices and to make learning mathematics more meaningful and relevant for themselves. Their potential of taking some charge and negotiating shared goals of their own mathematics learning activities in cooperation with their teachers became visible. It is difficult for me to predict if I would have been able to do the same since I never got asked to do so. The reason for why only some of the learners could adopt that outlook is not clear from the findings of this study. However, it may be speculated that the general part of curriculum's emphasis laid on imparting a comprehensive critical orientation among learners towards what they learn, experience, and the information they get can be a reason underlying such outlook.

6.1.2 The discrepancy between the formal and experiential curricula

Learners' statements and responses illustrate the discrepancy and a gap of understanding between the formal and experiential domains of their curriculum. The formal curricula, as suggested by Goodlad et al. (1979) are officially approved by the state and local school boards, whereas the experiential curricula is how learners experience their curricula in their teaching-learning activities. Considering only the formal part first, the formulations

adopted in the general part and subject (mathematics) specific part of the Norwegian curriculum raising concerns about empowering learners and developing their critical thinking, citizenship skills, active and autonomous democratic participation in their educational process differ from each other. While the general part of the curriculum underlines imparting a critical and sceptical orientation among learners towards their own learning processes and life situations in general, the mathematics specific part of curriculum (LK20) stresses inculcating critical thinking among learners with respect to data quantification and analysis. Thus, the notions of democracy and citizenship are mentioned in the current mathematics curriculum LK20, but encouraging learners to critically evaluate their own content or situation of learning mathematics is not stated explicitly (Kunnskapsdepartementet, 2019). This difference of contexts in which the concepts of critical thinking, citizenship and democratic participation are employed in the formal curricula itself can cause a gap of understanding among teachers about what these skills entail and how these competencies can be taught in a subject specific context (cf. Rønning (2004)).

Looking at learners' responses from this gap of understanding, it can be understood that the experiential curricula (i.e., their expressed classroom experiences) diverge from the formulations and objectives indicated in the general part of the formal curricula. This gap can explain learners' responses such as "I do not know", "I have not thought much about it", or "I do not think more beyond that", etc. when asked to adopt a critical perspective towards their teaching-learning processes of mathematics. Further, the indication of their limited experiences with being involved in the decisions concerning their learning activities and having learner autonomy in mathematics classroom also seem fair since imparting these competencies among learners is not stated as an explicit aim of their learning process in Norwegian mathematics curriculum.

However, it is not only the gap of understanding due to different contexts in which the words critical, democracy, participation, citizenship, etc. are used in curriculum formulations which may underlie the discrepancy between the formal and experiential curricula. Viewing these curriculum formulations from an analytical perspective, they reveal a set of conflicting political interests that surface as one tries to interpret the meaning of these formulations, as discussed below.

6.1.3 Conflicting socio-political interests?

The formal curriculum can be viewed as a socio-political statement approved by the nation's government alone or in co-operation with the members of education department, school boards, etc. Being a socio-political document, the formal curriculum reflects the aims and purposes of learners' educational process as envisioned in the society and influenced by the developments in the global job market and requirement of workers in different fields. The formal curriculum reciprocates the competencies to be imparted among learners through their educational processes catering for the contemporary social and economic requirements of the society, and for learners' individual needs. In attempting to cater for both individual and collective needs through learners' (mathematics) education process, the formal curricula may involve some conflicting political interests.

The statements employed in the formal Norwegian mathematics curriculum exemplify a possible instance of such conflicting political interest. On one hand the mathematics curriculum emphasises the importance and relevance of learning mathematics for individual

learner's personal and social growth and future financial stability. The statements like, "[M]athematics is important subject […]", and "[M]athematics shall prepare pupils for a society and working life […]" mentioned in the current mathematics curriculum, LK20 (Kunnskapsdepartementet, 2019) illustrate the emphases laid on learning mathematical knowledge and skills to get a job in the future. On the other hand, the general part of the curriculum underlines developing learners' critical thinking, citizenship, and active democratic participation skills through their (mathematics) education so that they can critically evaluate, take the co-responsibility of, influence their own learning processes, and can take decisions concerning their own lives and society (Kunnskapsdepartementet, 2017; The Education Act, 1998).

At first sight, these purpose statements indicating socio-political interests in the formal curricula may not seem conflicting but considering the *paradox of citizenship* presented by Skovsmose and Valero (2005) the contradiction inherent in these statements can be observed. Skovsmose and Valero (2005) express the paradox of citizenship in mathematics education as, "On the one hand, mathematics education seems ready to prepare students for active citizenship, but, on the other hand, it seems to ensure adaption of the individual to a given social order" (p. 61). From learners' perspective this paradox can be interpreted as learning the skills and knowledge of mathematics to secure a job in the future on one hand, and learning to adopt a critical perspective towards, being involved in decision-making processes and assuming learner autonomy in one's own mathematics learning activities on the other. The latter of which is argued to include questioning of the content, significance, and negotiating goals for one's own mathematics learning (cf. Ernest (2002)).

Learning mathematical knowledge and skills required by the school and being critical towards learning the same competencies simultaneously may be challenging for the learners pertaining to the trust they exhibited in their educational and school system under the interviews. Learners' responses like, "I do not think one can find any such jobs [not requiring the knowledge of mathematics and science]", and "… we learn it [mathematics] at school so one thinks one will use it …" demonstrate the conflict faced by them when asked to adopt a critical perspective towards the significance of learning mathematics for their lives. Thus, I find it legitimate to ask if learners' trust and unquestioned acceptance of the school's requirement of learning mathematics is: (a.) only a matter of discrepancy between the formal and experiential domains of the curricula, or (b.) does it reveal the socio-political documents' (educational policy, formal curricula, etc.) attempt to maintain that the objectives of educating learners can simultaneously serve the interests underlying two conflicting views of education, i.e., capitalist and critical (see e.g., Giroux (2016) and Valero (2017)).

6.1.4 Preparing learners for tests versus critical and active autonomous citizenship

Until now I have discussed the study's findings from learners' perspectives, but considering the teachers' perspectives is equally important. Teachers play a significant role in how mathematics teaching-learning processes take place and how the formal curriculum is implemented in mathematics classrooms. Goodlad et al. (1979) pointed out that, "What teachers perceive the curriculum of their classrooms to be and what they actually are teaching may be quite different things" (p. 62). In other words, teachers' perceptions, and operationalisation of the formal curriculum in mathematics classroom can have a considerable impact on if and how the aims expressed in the curriculum are realised in the classrooms.

It can be seen as one of the limitations of this study that no data was gathered to illustrate teachers' perspectives and opinions about teaching mathematics. Teachers' perspectives and understandings about critical thinking, citizenship, learners' active and autonomous democratic participation employed in general, and mathematics specific parts of the curriculum can provide important insight into how teachers structure and implement the formal mathematics curriculum in their classroom. Teachers are bestowed upon the responsibility of constructing classroom activities by keeping in mind the goals of education listed in the formal curriculum. Consequently, they also face the difficult choice of catering for both capitalist and critical interests mentioned in the curriculum documents. In other words, they face the challenge of balancing between: (a.) imparting mathematical knowledge and skills among learners so that they get a job and become financial independent in the future, and (b.) imparting a general sceptical outlook among them so that they can perceive their mathematics learning activities critically, get involved in decisions and assume learner autonomy in relation to their learning processes in mathematics[49].

I present the balancing above as difficult and challenging due to the focus placed on testing and assessment in mathematics nationally and internationally. The international tests such as TIMSS and PISA assess learners' mathematical knowledge from different countries over the world. These countries are then ranged on a high to low performing scale according to the performance and scores achieved by the learners of respective countries. Analogously, the national assessments in Norway (i.e., oral, and written exams) conducted at the end of 10th grade often evaluate learners' mathematical knowledge on the bases of their capability to solve mathematical problems, extract relevant quantitative information out of the questions given and apply procedures and techniques to find the correct answer. Since the assessment of procedural and technical skills in mathematics play a significant role in positioning not only learners and schools, but also different countries on a performance scale ranging from low to high performance, preparing learners for these exams may receive most attention in mathematics classrooms. Learners' interview responses such as, "I have not experienced a situation where I would need it [mathematics] except for the tests", and "I just think that in our lessons […] now we are going to solve problems in our book" provide indications of that solving textbook problems may be prioritised in their mathematics classroom. Therefore, it can be asked if the socio-political focus placed on evaluation of technical mathematical knowledge and achieving a high position on the performance scale play a role in selecting which competencies (technical or critical) are prioritised by teachers when they plan their mathematics lessons. The teachers thus may face a difficult choice of constantly choosing between preparing learners for the tests versus preparing them to become critical and active autonomous citizens through their mathematics education.

6.2 Implications of findings of the study

The discussion over the findings of this research study bear implications for the practice and research in the field of mathematics education. These implications are presented in the following sub-sections.

[49] That is, to attain critical citizenship and active democratic participation through their mathematics education.

6.2.1 Empirical implications: mathematics education policy and classroom practices

The *first* area of concern for which the findings of this study bear implications is the formulations in the educational policy documents and formal curriculum which guide the teaching-learning practices of mathematics education. Learning to think critically [through their educational process], taking co-responsibility and exercising their right to cooperate in and influence [their educational processes] are among the goals enlisted in the formal curriculum (The Education Act, 1998) and also in mathematics specific curriculum (Kunnskapsdepartementet, 2019). The general part of the curriculum emphasises imparting a general critical orientation and involving learners to have co-responsibility and cooperate in decisions concerning their educational processes and evolve them as critical autonomous citizens of the society. Whereas the mathematics specific part of curriculum emphasises imparting critical thinking among learners to provide them with the capabilities to judge arguments, reason, analyse data and evaluate conclusions of mathematical problems involving data. These differences in the meaning, scope, and definition of these concepts may result in interpretations which can be broad providing much scope of involving learners to critically evaluate and assume learner autonomy of their mathematics learning processes, or narrow, i.e., limiting the scope and aspects in which learners can be critical towards or have an influence on their own learning activities. In addition, these interpretations may vary from one school or institution to another and from one teacher to another, possibly causing the discrepancies between formal and experiential domains of curriculum. Therefore, the references made to concepts such as, critical thinking, democratic participation, citizenship, etc. in mathematics curriculum should be broad and explicit enough to include imparting a critical outlook and learner autonomy among learners as an explicit goal of mathematics teaching and learning. Accordingly, the curriculum formulations can invite and encourage mathematics educators to incorporate a critical orientation and learner autonomy not only *in* learning mathematics[50], but also *about* learning mathematics[51].

The *second* area of concern for which this study's findings bear implications is classroom practices. Ernest (2002) emphasises the role of classroom practices in mathematics to empower learners, and maintains that, "Teaching approaches should include discussions, permitted conflict of opinions […] but with justification offered, the challenging of the teacher as and ultimate source of knowledge […], the questioning of content and the negotiation of shared goals" (p. 8). This quotation suggests the significance of planning classroom activities to provide learners with the experience, training, and practice in acquiring a critical perspective towards learning mathematics[52]. Mellin-Olsen (1993b) and Valero (2005) also suggest that learners cannot be expected to assume control, ownership, involvement, and empowerment with respect to their mathematics learning on their own. To be able to think of alternatives, the learners should get the opportunities to participate in their school mathematics practices and to activate themselves with respect to their teachers and knowledge so that they learn to take initiatives and take control. Gaining experiences that are different than the regular way of organizing their teaching-learning practices of mathematics

[50] That is, to reason and argument while solving mathematical problems.

[51] That is, to critically evaluate, be involved in decisions of, and suggest changes in their mathematics learning activities.

[52] Since learners' ability of thinking critically may not transfer from one sphere of life where they apply it (e.g., in solving mathematical problems) to another sphere of their life (e.g., critically evaluating their learning situations) on its own.

can stimulate their thought process and activate their potential to influence their own mathematics learning.

Learners' interview and questionnaire responses such as, "I am used to having it like this so … I have done it all these years, I think it's a fine way to learn …", indicate that they may not be used to considering their mathematics teaching practices critically. Their replies further suggest that they trust their education and school system and decision made by these authorities concerning their learning processes. A combination of this trust in the socio-political system along with learners' not reflecting critically and participating in decisions about their own learning activities may make them disclaim their right of having a co-responsibility of and cooperating in their learning processes. Further, this unquestioned acceptance and trust in socio-political rhetoric about mathematics education, traditional and dominant ways of teaching and learning mathematics may continue to be self-reinforcing through the statements which learners hear from others. Developing learners' critical outlook towards their mathematics education can allow them to observe the inconsistencies inherent in their own and socio-political perspectives about mathematics education so that they can critically review the role of mathematics education in their own lives and in society.

Another aspect related to these 8th and 9th grade learners' practice and training in taking decisions and making choices about their own mathematics learning processes concern their decision to continue learning mathematics after 10th grade. Norwegian learners, after their 10th grade are required to reflect thoroughly and decide if they will continue to learn mathematics in their higher secondary education. While taking such decisions, it is important for them to critically evaluate the options available for them, the long-term consequences of each available option and to take informed decisions about their own mathematics education. Thus, if learners are to critically evaluate and voice their opinions about their mathematics learning, then their classroom activities need to include practices such as, discussions, conflict of opinions, questioning the content, reflective knowing about mathematics, negotiating shared goals, taking initiatives, suggesting changes in the content and teaching style of mathematics, etc. Learners should be involved as *real actors* (to use Valero's (2005) words) and as a part of their mathematics education process from a socio-political perspective. Therefore, this study implies that learners receive training and experience of voicing their opinions and exhibiting their potential to influence their own mathematics learning processes. By doing so they can learn to question and evaluate their mathematics education and practices critically, get involved in talking decisions, assume autonomy to suggest changes and cooperate with their teachers to influence their learning activities. Gaining these competencies through their mathematics education can assist in realising the aims of learners' empowerment and critical citizenship through their mathematics education. Moreover, mathematics classrooms can become miniature democratic societies using learner-centred educational approaches in which decisions about learning activities are made in cooperation with learners, by taking into account their individual interests, backgrounds, experiences, etc.

However, these implications do not suggest that such classroom practices can be seamlessly adopted and incorporated straightaway into mathematics classrooms, and they will generate desired results easily after being implemented. Adopting such practices would be time consuming and require efforts on the part of both learners and teachers along with adjusting and organising the classroom environment to achieve these goals. Moreover, there are several factors influencing the adoption of such teaching-learning practices in mathematics classrooms. Teachers' beliefs about mathematics teaching, their perceptions of formal curricula, their interpretations of the concepts employed in the formulations, and their

classroom culture are instances of one such factor (i.e., the teachers) which can have significant role in adopting and transforming mathematics classroom practices. Other examples of such factors can be creating mathematical tasks, structuring classroom collaborations and interactions between learners and between teachers and learners, the priorities and obligations of the school as a socio-political institution, the culture and interests of learners' parents and the society in general, teacher education, socio-political interests and requirements of national and global job markets, etc. The sections, 6.1.3 and 6.1.4 above also present some of the paradoxes and dilemmas inherent in these factors which may influence the adoption of such teaching-learning practices in mathematics classrooms.

6.2.2 Research implications: openings for future research

This study responds to the calls put forward by for instance, Nieto (1994) and Giroux (1988, 2016) for more research illustrating learners' perspectives about their own educational processes. Concerning the field of mathematics education research specifically, this study contributes to the similar call of authors for instance, Mellin-Olsen (1993b), Vithal (1999), Skovsmose and Valero (2002), Ernest (2004), and Lindenskov (2010) to bring forward individual learners' voices and experiences with learning mathematics. Research studies such as, Gutstein et al. (1997) and Gutstein (2003) have explored learners' critical perspectives while learning mathematics from a socio-political point of view, but these studies aimed at developing a collective awareness among learners about the issues of justice and equality through learning mathematics. Individual learner's perspectives are not focused except for in some selected studies (see, e.g., Lindenskov (2010)). Norén and Valero (2022) suggest that democracy requires people who have a trained intellect, can adopt a critical and independent stance, and think rationally and sensibly to analyse tendentious influences.

Therefore, this study illustrates the significance of exploring individual learners' voices about, and their experiences with learning mathematics. In enquiring so, their potential of perceiving different sides of their mathematics learning processes critically and influencing their mathematics learning activities is identified. However, this research contribution has also highlighted some other queries related to individual learners that require further exploration and opens new research opportunities. Some of these queries, for instance include: (a.) how can *individual* learners be encouraged to perceive their mathematics learning critically, (b.) what dilemmas do learners get exposed to and experience while considering their mathematics education process in a critical sense, (c.) how individual learners can acquire reflective knowing about learning mathematics, (d.) which classroom practices can encourage learners to assume control and autonomy of their own learning processes, etc. These issues can also be explored with respect to a group of learners, thus considering the development of a *collective* critical orientation of that group towards learning mathematics.

Analogous to exploring learners' perspectives it is reasonable to investigate teachers' perspectives and interpretations of the formal mathematics curriculum and how they design and execute the curriculum to develop learners' critical orientation towards and their active autonomous participation in mathematics learning activities. The classroom practices, collaboration and interactions constitute another facet which needs to be explored under the socio-political and critical lens. Vithal (1999) highlights that much of the research concerning the CME and socio-political issues related to mathematics education has been theoretical, but the "question is what happens when an attempt is made to deliberately realise such a link in

mathematics classroom" (p. 27). Researching classroom practices from a critical perspective can respond to this call.

Another area of future research includes exploring the conflicting socio-political interests highlighted in the formal curriculum, educational policy documents, and the underlying reasons of these paradoxes. Further, it can also be investigated that how these conflicting interests influence teachers' interpretation of curriculum, how they balance between the priorities of preparing learners for tests, critical citizenship, and active autonomous participation, and how they design and plan mathematics classroom practices according to these interests. In addition, the role and influence of parents' perspectives, cultural aspects, the obligations of schools as socio-political institutions, etc. can be examined.

Considering the theoretical position and data analysis frameworks chosen in this study, I also argue that future research studies may benefit by exploring: (a.) if the frameworks reserved to study cognitive or psychological aspects (e.g., beliefs, motivation, autonomy, agency, critical thoughts, experiences, reflections, etc.) of an individual's worldview also include some social aspects, and (b.) if these frameworks can be operationalised to explore individual learner's perspectives about their (mathematics) educational processes. The interaction between and intersection of the concepts utilised in cognitive, social, and critical paradigms have provided me with insights into leaners' voices and experiences, and further research may also take advantage of this combination. Another possibility is to develop new frameworks to interpret and analyse individual learners' perspectives with a socio-political and critical orientation.

Therefore, more research is welcome and needed to get a better understanding of empirical, theoretical, and methodological aspects related to empowering learners, and developing their critical citizenship and active autonomous participation skills in their mathematics learning processes. Further, the dilemmas inherent in these contexts of exploration should also be illustrated and enquired.

6.3 What could be done differently?

After finishing this research study and reflecting on the choices made in the process, I realise that some changes could have contributed to increase the quality and trustworthiness of this research. One such change could be made in the *theoretical choices* made in the project. In this study the research interest shifted from a cultural focus to exploring individual learners' critical perspectives towards their mathematics learning processes. This shift of interest also caused a shift in the choice of theoretical lenses employed in the study. On one hand, the insights from cognitive and social constructivism have helped me to understand the notions of an individual's thought, opinion, thinking, reflection, criticality, beliefs, autonomy, etc. On the other hand, the insights from critical pedagogy and Critical Theory have helped me to analyse the meaning of learners' perspectives shaped by their socio-political contexts and interpret them into their voices indicating their potential to influence their mathematics learning processes. I have argued for my theoretical positioning between social constructivism and critical pedagogy in chapter 3, but I faced challenges concerning the compatibility of theoretical paradigms while conducting and writing about this study. These challenges cannot be avoided completely but choosing a critical orientation from the beginning of this study could have contributed to positioning it thoroughly in Critical Theory. An awareness of this

theoretical conflict earlier could have helped me to explore *critical constructivism* (Kincheloe, 2005) as the theoretical paradigm for positioning this study.

I got introduced to critical constructivism as a theoretical paradigm short time before I started to write this thesis. After reading a bit about it, I realised that my arguments for theoretical positioning of this study between social constructivism and critical pedagogy (Critical Theory) may align with the theoretical underpinnings of critical constructivism. However, I chose not to employ critical constructivism as a theoretical paradigm to position this study in this thesis because of two reasons. Firstly, my current understanding and knowledge of this theoretical stance is limited. Secondly, I wanted to present the development of research project and its theoretical underpinnings as per my own understandings and struggles I experienced in making theoretical choices while I was conducting this research and analysing the data. I wanted to give an account of my own evolvement as a researcher instead of giving the impression that the 'critical' perspective was a part of this research project right from its onset. While conducting this research, I had worked with the theoretical concepts and ideas of cognitive constructivism, social constructivism, and critical pedagogy (Critical Theory), so I sticked to these concepts to clarify theoretical underpinnings of this project. It can be considered that choosing critical constructivism as a paradigm to position this study theoretically may have contributed to increase its quality by enhancing its theoretical compatibility, but I need to delve deeper into how my understanding of this research project fits with critical constructivism as a theoretical paradigm to be sure of its contribution.

Another change I envision that may have improved the thesis is the process of *data analyses*. With an early focus on concepts from critical pedagogy and socio-political aspects in mathematics education research, I could have analysed the data by using critical discourse analyses. This method of analysing data could shed light on learners' experiences with power relations in their mathematics classrooms, their trust in educational and school system, their instrumental and social rationale of learning mathematics, the influence of socio-political contexts on their perspectives, etc. to find different narratives of mathematics learners and the discourses they participate in. Critical discourse analysis could be combined with critical constructivism as the study's theoretical positioning. Accordingly, I may have ended up in using different notions such as, critical consciousness, agency, praxis, intentions, foregrounds, backgrounds, etc. from CME and socio-political research in mathematics education to explore *individual* learners' perspectives towards and experiences with learning mathematics.

Though there is a potential of improving the theoretical and methodological choices made in this study, yet I contend that these choices and decisions have served as a strength of this project in providing me with the insights I have got into learners' perspectives. The choices have been challenging but that does not mean that they were wrong. On the contrary, a combination of social constructivism and a critical element has provided me with a unique angle to understand and interpret learners' perspectives which may not have been possible for me to acquire if I had chosen a definite theoretical stance from the start. The findings of this study can take CME and socio-political mathematics education research in the direction of enquiring individual learners' perspectives and hearing to their voices by combining the conceptual apparatus of different theoretical positions. Valero's (2004b) conclusion that:

> "Adopting a socio-political approach is not only a matter of choosing a particular set of theories and methodologies. It is an 'attitude' that seeks for consistency between the former and our activity as researcher. This attitude also shows that the researcher is in search of appropriate ways for communicating the interpretations of her or his activity" (p. 20),

aptly summarises my position as a researcher adopting the socio-political approach to mathematics education research.

6.4 Conclusion – what do the learners' voices communicate?

The overall research question of this thesis is:

> *What can individual learners' voices inform about their critical perspectives towards learning mathematics, and their expressed autonomous involvement in decisions concerning their mathematics learning activities?*

Learners' responses indicate that most of them are not habitual of adopting a critical perspective towards and questioning their learning activities in mathematics. The learners' replies further suggest that their involvement in decisions concerning their mathematics learning and their experiences of having learner autonomy in mathematics classrooms are limited to the opportunities provided by their teachers. There can be several reasons underlying learners' replies, for example, my authority as an interviewer, they have not been asked about it before, their understanding of their role as learners in school, etc., but their responses suggest two possible explanations: (a.) such practices are not a part of their routine mathematics classroom; and (b.) they trust the socio-political institution of school and mediated significance of learning mathematics to succeed in the future job market. Moreover, learners' perspectives seem to be formed based on the combination of impressions they subjectively (thus cognitively) construct, the information they receive from their social interactions and their societal (including political, cultural, and historical) contexts. Thus, the two possible reasons outlined above may make them accept the requirements of learning mathematics at school without questioning.

Learners' voices reveal that some of them adopted a critical stance towards learning mathematics and assume learner autonomy in their mathematics classroom when asked to do so. These few learners exhibit the potential to suggest changes in and influence their mathematics learning process to improve it. They also expressed the desire to assume the co-responsibility, be the discussion partners and co-operate with their teachers in transforming the teaching-learning practices of mathematics. However, their potential of influencing their mathematics learning processes may remain hidden and unpolished if they trust their educational and school system without exercising a critique of their learning and if their classroom practices do not include: (a.) adopting a critical stance towards learning mathematics; and (b.) provide them with the experience of assuming learner autonomy and taking partial control of their own mathematics learning to improve it. Learners' unquestioned trust may impede the process of their empowerment and their development into future critical citizens and active autonomous participants and actors of a democratic society, taking decisions concerning their own (mathematics) learning processes, their lives, and the society.

It seems apt to quote Valero (2004a) here in saying that "Empowerment needs to be defined in terms of the potentialities for students to participate in school mathematics practices. They get empowered when, through that participation, they position themselves in ways that are significant for the development of the practice" (p. 49). The analyses of learners' responses in this study seem to support Nieto's (1994) claim that speaking to learners about their schooling experiences can act as a catalyst for stimulating more critical thinking about those experiences. The experiences of being empowered and having autonomy of their own

learning can in-turn motivate learners to seek more empowerment and autonomy. Learners, who are critical thinkers, are empowered and active autonomous participators, can in turn contribute to uphold the values and principles of critical citizenship and active autonomous participation in a future democratic society. Thus, the processes of learners' empowerment, their active autonomous participation in the society, and development of their critical citizenship competencies can be envisioned to be in a cyclic relation (see Figure 8), each element motivating and supporting the growth of the other.

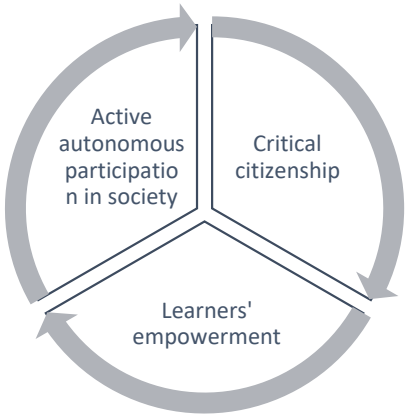

Figure 8 Cyclic relation between learners' empowerment, autonomous participation and critical citizenship skills.

Finally, I conclude this thesis with reflecting on my personal perspectives and presumptions I held about Norwegian secondary school learners, their education and school system, and about the process of conducting research. My presumption about Norwegian learners was that they would be actively involved in decisions concerning their mathematics learning practices and critically aware about what, why and how they want to learn mathematics. These presumptions proved to be wrong for selected learners who were my informants. Another assumption concerned the Norwegian Education Act, general part of the curriculum, and mathematics curriculum and these documents' operationalisation in learners' mathematics classroom. I assumed that these documents would be coherent in their formulations and the educational aims listed in the formal curricula would be seamlessly incorporated in the operationalised and aligned with the experiential curricula. However, conducting this research revealed the discrepancies between different domains of curricula, conflicting socio-political interests, and teachers' dilemma of preparing learners for tests versus the educational aim of preparing them for critical and autonomous citizenship.

Before entering the field of research, I thought that one can find an answer to the general questions related to mathematics education, such as, how mathematics should be taught, how it can be learnt, what are the right ways to teach and learn mathematics, etc. However, this research journey made me aware of: (a.) many ways of teaching and learning mathematics which cannot be judged as being completely right or wrong: (b.) the nitty-gritty nuances of employing different teaching and learning approaches, and (c.) knowledge of a wide range of cognitive to critical, and individual to socio-political contexts and situations influencing the mathematics education process. Getting this huge variety of insights into the research field, I realise how little I can say about a specific research question I have explored

in this thesis and that this study contributes a tiny yet significant part of knowledge to the vast field of mathematics education research.

What I can say is that some learners' voices indicate an opportunity to improve their mathematics teaching-learning practices, and their potential to make suitable choices and take well-informed and reflected decisions regarding their own mathematics learning. One such improvement in learners' mathematics learning practices could be to widen the scope of interpreting educational goals concerning learners' empowerment, their active autonomous participation in decision-making and their development as future critical citizens in their mathematics curriculum. Another improvement could be shifting the focus of socio-political discourse establishing the significance of learning mathematics. The socio-political discourses may avoid focusing highly on increasing learners' academic achievements in mathematics. Rather these discourses may equally encourage developing learners into critical and active autonomous citizens of the society. Introducing these changes in mathematics education practices does not require adding more subject content in mathematics curriculum, but it requires that learners are provided with the possibility to participate and be involved in decisions concerning their own learning activities.

The purpose is to adopt a critical outlook towards learning mathematics, which must not be mistaken as a negative outlook. Neither can the learners, teachers, parents, or the socio-political institution of school be blamed for learners' unquestioned acceptance of the requirement of learning mathematics. What I seek to make a case for is learners' right to think critically about, cooperate in, and take co-responsibility of their own mathematics learning process so that they can experience having learner autonomy, increased ownership, and feel empowered in relation to learning mathematics. These experiences can bring forward learners' potential to influence and improve their mathematics learning processes and can help them to develop their critical citizenship and democratic participation competencies.

Providing such experiences to the learner in mathematics classroom can also benefit the practices of teaching and learning mathematics. Being open to critique from learners can provide opportunities to constantly evolve the scope, significance and meaning of mathematics education for learners' empowerment, critical citizenship, autonomy, and active democratic participation. What Skovsmose (2003) expressed about mathematics, that, "Mathematics is neither good nor bad—But far from neutral" (p. 229) can probably be also expressed about mathematics education, that, "*Mathematics education* is neither good nor bad—But far from neutral". Therefore, it can be argued that the objective of mathematics education cannot be to scare the learners to believe that they may fail in their lives if they are not good at mathematics, nor to force them to learn mathematics even if they do not see it as being relevant and important for their own lives. Neither can mathematics education promise a high salary job, rich, successful, and problem-free lives to its learners because of their good achievement in mathematics. However, one can consider the positive and negative consequences of mathematics education for its learners. In doing so, the disaffection, disinterest, mathematics anxiety, etc. may be some negative consequences of mathematics education. Whereas evolving learners as critically aware, reflective, autonomous, and actively participating citizens conscious of the positive and negative aspects of learning and applying mathematics in their lives and in society may be the positive consequences of mathematics education. This underlying critical stance can help the learners to understand the possibilities, limitations, and consequences of learning mathematics to attain cognitive, social, and socio-political competencies. In addition, the learners may also understand the value of having learner autonomy and being involved in decisions concerning their learning activities, so that

they can make suitable choices and reflected decisions about learning and applying mathematics in different domains of their own lives and in society. By being autonomously involved in their learning process, the learners can learn to adopt a critical stance towards learning mathematics to choose and decide if learning mathematics is beneficial for them or not, and how they want to learn it. Learners' responses analysed in this study exhibit their potential of undertaking thorough critical reflection and providing reasonable arguments to support their suggestions to influence and improve their mathematics learning experiences.

Hence, the call for and the requirement of listening to learners' voices informing their potential of influencing their own mathematics learning processes is appropriate. Though I have more questions (than answers) in my mind after finishing this research study than I had when I started it, yet through my journey I have understood that teaching-learning practices of mathematics are not perfect or rigid. These practices can assist in evolving learners into critically aware citizens and can empower them to become autonomous and get involved in decisions concerning their lives as individuals or citizens. Further, if learners' expressed experiences of their mathematics classroom indicate that they may not get the opportunities to develop, train and practice their critical stance towards, or having autonomous involvement in decisions concerning their learning activities — then such possibilities may not become a part of their classroom routine only by theoretically criticizing this routine or by accepting it uncritically. These opportunities can only be integrated in their mathematics classroom routine by encouraging the learners to adopt a critical outlook *towards* and allowing them to exercise a critique of their own mathematics learning experiences. Heeding to their expressed critique, listening to their voices such as, "If we could have got a realistic situation …", and incorporating their suggestions in their classroom practices can provide learners with the opportunities to acquire a critical perspective towards, and autonomous involvement in their mathematics learning processes. Providing such opportunities to learners in their mathematics classrooms may further contribute to realise the aims of mathematics education to empower the learners, and to develop them into critical and active autonomous citizens living in, building, and striving to maintain a society based on democratic values.

7 Contributing papers

7.1 Paper I: Learners' critical thinking about learning mathematics

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Learners' Critical Thinking About Learning Mathematics

Shipra Sachdeva [1,2*], Per-Odd Eggen [2]

[1] Østfold University College, NORWAY
[2] Norwegian University of Science and Technology, NORWAY
***Corresponding Author:** shipra.sachdeva@hiof.no

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ABSTRACT

Developing critical thinking practices among young learners through mathematics education is a topic of attention for mathematics education research community. Learners' critical thinking concerning cognitive and social aspects of their mathematics education have been explored in several research studies. However, learners' critical thinking concerning their personal beliefs about their mathematics learning process have not received much consideration. In this paper, learners' practice with thinking critically *about* and their potential to suggest changes in their mathematics learning process is explored based on their expressed beliefs *about* learning mathematics. Learners of eighth and ninth grade in two Norwegian schools responded to a questionnaire and were interviewed to gather their opinions concerning their mathematics learning process. Data analysis indicates that learners seldom think critically, and hold inconsistent beliefs about mathematics and its learning process. Moreover, they struggle to observe their own beliefs critically, and hesitate in suggesting alternatives to make learning mathematics meaningful for them. Consequently, learners' critical attitude towards their mathematics learning process and their personal beliefs in order to gain a meta-perspective of their learning contexts does not seem to be evolving effectively. However, if learners are encouraged to think critically about mathematics education, their potential of contributing to improve their mathematics learning process becomes visible. We recommend that young learners get training in and are encouraged to think critically *about* their mathematics learning process so that they are equipped to make reflected choices related to learning mathematics in their personal lives.

Keywords: critical thinking, mathematics education, learners' perspectives, learners' beliefs

INTRODUCTION

The word 'critical' can have different meanings depending on the frame of reference of its use (Ernest, 2016). A situation can be 'critical' or at a point of *crisis*, when an action can dramatically improve or deteriorate the conditions. Secondly, *criticism* or *criticizing* can be a form of conveying disapproval, disagreement or negative comments about an argument, situation, decision etc. Additionally, 'critical' can be understood in terms of *critique* (being critical), as contrary to being 'uncritical'. In this sense, being critical includes analyzing the merits, demerits and consequences of any belief, judgement, choice, opinion, product, context etc., be it socio-cultural, political or personal (Ernest, 2016). In this paper, the word 'critical' is used to mean the opposite of being 'uncritical', and refers to the learners' knowledge of evaluating their personal beliefs, inferences, choices, etc. *critically* in order to take informed decisions and action(s) for their personal and societal betterment.

Adopting a critical stance while making choices in life is essential for learners. They should be aware and capable of critically analyzing their viewpoints and situations in cognitive, social and personal spheres of their life to survive and succeed in a complex society (Facione, 1990). In addition, learners are envisioned as future critical citizens of the society, using their critical thinking potential to promote justice and democracy (Skovsmose, 1998). Consequently, teaching and learning of critical thinking practices is often recommended in the education research literature (Bybee & Fuchs, 2006; Saavedra & Opfer, 2012). A number of reports issued by elected commissions (Ludvigsen et al., 2015; Partnership for 21st Century Skills (P21), 2009) and the educational policy documents of some countries also mention development of critical thinking competences as one of the fundamental aims of education. For example, the Norwegian Education Act states that, "[Through education] students and apprentices *must learn to think critically* and, act ethically and environmentally consciously. They *must have co-responsibility* and *the right to co-operation*" (Opplæringsloven, 1998) (translated and added italics). An interpretation of this statement can be that learners should learn to *think critically* through and about their education, take *responsibility* and have the *right to co-operate* in decisions regarding their education. Moreover, learning critical thinking abilities is also mentioned as an educational ideal (Siegel, 1980), and learners' moral right "because in the end students must choose for themselves; there is no escaping this truth" (Norris, 1985, p. 40).

Therefore, the requirement and importance of acquiring critical thinking competence for learners to achieve both individual and societal good is well established.

Mathematics is strongly represented as a fundamental school subject in countries across the globe. Mathematics education, therefore, has a vital role in educating children to become critically thoughtful, responsible and co-operative beings acting in the society. Accordingly, developing critical thinking competence among mathematics learners has been a concern for mathematics education since decades (Jansson, 1986; Kuntze et al., 2017). Evolving "critical thinking skills [through mathematics education] is an implicitly hoped-for outcome of using the NCTM's *Standards*" (Gutstein, 2003, p. 66). However, neither NCTM Standards nor mathematics education research literature explicitly specify or limit the aspect(s) of learning mathematics, concerning which, learners' critical thinking should be developed. Consequently, mathematics education researchers have understood and used the term *critical thinking* diversely in different contexts to discuss development of learners' critical competences.

Learners' *critical thinking* in mathematics education research literature have been discussed mainly as – a set of cognitive skills used to draw logical conclusions and take informed decisions while solving mathematical problems (Aizikovitsh-Udi & Cheng, 2015; Kuntze et al., 2017; O'Daffer & Thornquist, 1993); and secondly as – an attitude to understand and reflect over the role(s) of mathematics and mathematics education in socio-political and cultural contexts to promote justice and democratic concerns in the society (Gutstein et al., 1997; Skovsmose, 1994b, 1998). The first point emphasizes learning critical thinking *in* mathematics to acquire mathematical procedures for problem-solving and finding unbiased logical results. The second point roots critical thinking in the spirit of Critical Mathematics Education (CME) (inspired by critical pedagogy) Freire (1972); (Skovsmose, 1994a) to consider mathematics and mathematics education as objects of reflection and critique in society. This classification highlights the attention directed to imparting critical thinking among mathematics learners regarding the *cognitive* and *social* aspects of their mathematics learning process respectively, whereas the *personal* aspect seems to be missing. Therefore, in this paper we focus on learners' practice with critical thinking regarding their *personal* beliefs *about* their mathematics learning process, and their potential to participate in this process.

Thinking critically in social and cognitive aspects of one's life cannot be complete without, or compensate for being critical in one's *personal* life. Moreover, applying critical thinking cognitively, *in* the process of solving mathematical tasks and understanding social complexities through learning mathematics, cannot be taken to be the same as thinking critically *about* learning mathematics in personal life. Young mathematics learners worldwide need to make a personal choice concerning their mathematics learning early in their educational pathway. In Norway, as in many other countries, 14-15-year-old learners decide if, and what specific direction (vocational or theoretical) of mathematics they want to pursue in their upper secondary school by the end of their compulsory school years (i.e., after tenth grade). Developing learners' critical thinking faculties through mathematics education can be helpful to make a well-reflected choice of this kind. Thus, developing learners' critical abilities through mathematics education is mentioned as a central aim in the recently revised Norwegian mathematics curriculum (specified under '*fagrelevans og sentrale verdier*' [subject relevance and central values]). In the Norwegian mathematics curriculum, it is stated that, "Critical thinking in mathematics involves critical evaluation of reasoning and argumentation, and can equip the learners [with the competence] to make their own choices and to address important questions concerning their own [personal] lives and the society" (Norwegian Directorate of Education and Training, 2020) (translated). Therefore, the scope of learning critical thinking through mathematics education can be seen as directly related to learners' *personal* lives and choices. The Norwegian mathematics curriculum emphasizes that mathematics education should evolve learners' critical attitude towards the decisions they make in their *personal* lives; and gives a personal dimension to learning critical thinking through mathematics education in addition to the cognitive and social dimensions. Consequently, developing a tendency to think critically *about* their personal beliefs concerning mathematics and its learning process may help learners to attain a meta-perspective of learning mathematics and to make informed personal choices about their mathematics education.

Statement of Problem and Research Question(s)

Learners' practice with thinking critically *about* their *personal* beliefs concerning their mathematics learning process are seldom investigated in mathematics education research literature. This research paper addresses this research gap. By practice with thinking critically, we mean learners' tendency to critically observe their *personal* beliefs *about* mathematics and its learning process to gain a *meta-perspective* of it in their own lives. Therefore, in this paper learners' practice with thinking critically and give suggestions *about* their own mathematics learning process is explored based on their expressed beliefs regarding this process. The research question, *"What can learners' expressed mathematics related beliefs reveal regarding their practice with thinking critically about and potential to give suggestions concerning their mathematics learning process?"* is investigated in this paper. The scope of learners' mathematics related beliefs was narrowed down with the help of three sub-questions, namely – *what* subject content do learners find interesting to learn in mathematics?; *why* they learn mathematics in their opinion?; and *how* their mathematics teaching may be improved?

CONCEPTUAL FRAMEWORK

Defining Beliefs

Learners' first-hand experiences, opinions and beliefs are used to explore their *critical* orientation while they talk about their mathematics learning process. Learners' beliefs about mathematics and mathematics education play an important role in learning of the subject and are well-researched (Leder, Pehkonen, & Törner, 2002; Maaß & Schlöglmann, 2009). Furinghetti and Pehkonen (2002) mention that it is difficult to point out a single universal definition of beliefs because of different goal-orientations

and contexts in which this term is used. However, Leder et al. (2002) cite Bar-Tal (1990), describing *beliefs* to be viewed as, "… units of cognition. They constitute the totality of an individual's knowledge including what people consider as facts, opinions, hypotheses, as well as faith. Accordingly, any content can be the subject of a belief." (p. 12). Beliefs can be formed through a variety of sources, be it direct personal experiences, inferences about a context or information provided by outside sources etc. (Bar-Tal, 1990). Various types, classifications, structures, and assessment methods of beliefs discussed in the literature are beyond the scope of this paper. Therefore, adapting Bar-Tal's definition, learners' mathematics and mathematics education related beliefs are understood as their conscious or unconscious opinions, thoughts, ideas, perceptions or hypotheses about their mathematics learning process which they consider to be true.

Defining Critical Thinking

Analogous to the word *critical, critical thinking* is also defined differently in educational, socio-political and psychological research contexts. Critical thinking has acquired several definitions[1], over its research history. In 1988, the American Philosophical Association (APA) founded a panel of 46 experts (including educationalists, philosophers and psychologists) to develop a consensus *definition* and *fundamental skills* comprising critical thinking for educational instruction and assessment purposes. The panel's consensus presented a total of six *core critical thinking cognitive skills* with 16 sub-skills (**Figure 1**), and *affective dispositions* (not focused in this paper) including habits of mind of a good critical thinker in their conclusive report. The consensus defined critical thinking as, "…[a] purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or contextual considerations upon which that judgment is based" Facione (1990, p. 3). Therefore, critical thinking evolved as an ability comprising *core cognitive critical thinking skills* and *affective critical thinking dispositions*.

Interpretation		
Categorization	Decoding Significance	Clarifying Meaning
Analysis		
Examining Ideas	Identifying Arguments	Analyzing Arguments
Evaluation		
Assessing Claims	Assessing Arguments	
Inference		
Querying Evidence	Conjecturing Alternatives	Drawing Conclusions
Explanation		
Stating Results	Justifying Procedures	Presenting Arguments
Self-regulation		
Self-examination	Self-correction	

Figure 1. Cognitive critical thinking skills and sub-skills adapted from (Facione, 1990, p. 12)

Facione (1992) simplified APA's consensus definition of critical thinking as a process to "make a purposeful, reflective judgement about what to *believe* or what to do – precisely the kind of judgement which is the focus of critical thinking" (p.17, italics added). Analogously in this paper, *critical thinking* is understood as an ability to reflect over one's beliefs, circumstances and actions for making purposeful, reflective judgements and choices about what to believe and how to act responsibly for improving one's life, without harming others. In mathematics education context, besides being the ability to solve mathematical problems logically and reflecting over mathematics' role in society, critical thinking can be comprehended as a process tool for learners to consciously reflect upon and gain a meta-perspective *about* their own mathematics teaching-learning process. Therefore, learners' critical thinking *about* their own mathematics learning process was analyzed by using the *self-examination* and *self-correction* sub-skills of the sixth core cognitive critical thinking, *self-regulation* (**Figure 1**). The APA consensus' definitions of sub-skills *self-examination* and *self-correction* were adapted to analyze learners' interview responses.

[1] Mentioning critical thinking as "the propensity and skill to engage in an activity with reflective skepticism" (McPeck, 1981, p. 8) or as "disciplined self-directed thinking which exemplifies the perfections of thinking appropriate to a particular mode or domain of thinking" (Paul, 1993, p. 33), see Beyer (1985) for a literature review on defining critical thinking.

LITERATURE REVIEW

Learners' Beliefs and Mathematics Education Research

In mathematics education, *beliefs* have emerged from being a hidden variable in the mathematics learning process to being a thoroughly investigated topic of research (Goldin et al., 2009; Leder et al., 2002). The research constituting beliefs in mathematics education is rich and diverse, particularly if one also includes the studies using terms like ideas, conceptions, perceptions, attitudes, values etc. in place of the term *beliefs*. Research regarding beliefs about mathematics and mathematics learning process have involved both learners and teachers (both in-service and pre-service) as informants, and may concern either *cognitive* domain (for example, a polygon having three sides is a triangle), or the *affective* domain (for example, mathematics is difficult) (McDonough & Sullivan, 2014). However, most of the studies concerning beliefs are placed under the umbrella of *affective issues* in mathematics education research (Leder & Grootenboer, 2005; Zan et al., 2006). In this study, learners' beliefs about mathematics are explored to get an insight into their practice with thinking critically about and suggesting changes in their mathematics learning process. Consequently, this study takes learners' perspective in focus within the affective domain of enquiry related to their mathematics learning process.

Op't Eynde, De Corte, and Verschaffel (2002) define learners' mathematics-related beliefs as, "… the implicitly or explicitly held subjective conceptions students hold to be true, that influence their mathematical learning and problem solving" (p. 16). Research studies enlightening learners' mathematics-related beliefs conducted within the affective domain may be classified as focusing on either positive or negative affect towards mathematics (Hannula et al., 2016; Zan et al., 2006). Taking the positive affect on one hand, learners' beliefs have paved the way to explore their interest, enjoyment, liking etc. towards mathematics, which is usually seen to be correlated with increased motivation, engagement and achievement (Hannula et al., 2016). Whereas, considering the negative affect on the other hand, learners' beliefs have helped to explore their mathematics anxiety, stress, fear etc. which are seen to be correlated with the feelings of disaffection, failure and loss of interest in learning mathematics (Zan et al., 2006). Therefore, the literature regarding learners' mathematics-related beliefs includes a large spectrum of studies exploring their beliefs about topics such as – the nature of mathematics (Young-Loveridge et al., 2006), attitude towards learning mathematics (Grootenboer & Marshman, 2016), self-efficacy (Kele & Sharma, 2014), mathematics teaching (Mapolelo, 2009), the use-value of mathematics (Kollosche, 2017; Onion, 2004; Pais, 2013), their mathematical identities (Andersson et al., 2015; Bishop, 2012), emotional disaffection (Nardi & Steward, 2003), anxiety (Young et al., 2012) towards mathematics; and more.

Learners' beliefs regarding the nature of mathematics report that they seldom think about this topic (Kloosterman, 2002), they view mathematics as a useful and important (Grootenboer & Marshman, 2016), but also as a difficult and boring subject (Nardi & Steward, 2003). They usually hold a positive attitude towards learning the subject, however, only a few wish to become mathematicians (Grootenboer & Marshman, 2016). When it comes to the specific questions of *what, why* and *how* in mathematics learning process, more studies have examined learners' beliefs concerning the *why* of learning mathematics, rather than *what* to learn in mathematics or *how*. In our literature search, Lindenskov (2010) came up as the only study that took learners' perspective over *what* content learners' themselves are interested to learn in mathematics. The results suggest that if given an opportunity, learners clearly mention what they understand or not while learning mathematics and devote their attention towards related curriculum accordingly. Thus, Lindenskov (2010) suggests that if given the opportunity learners' can critically evaluate their mathematics curriculum and also draws implications of her study for the role of learners in CME research. Concerning the question of *how*, Mapolelo (2009) and Nardi and Steward (2003) reported that learners experience mathematics classes to be tiring, lecture-oriented and the use of mathematical language seemed to be a barrier in learning the subject. However, the learners not only see the problems but, if allowed, can also point out effective instruction strategies to improve the learning outcome of their mathematics classes (Clare & Sue, 2013). Moving to the question of *why*, the investigations Onion (2004), Sealey and Noyes (2010), Pais (2013), and Kollosche (2017) uncover that though learners believe mathematics to be an important subject to learn, yet they struggle to respond when asked about where they use and *why* they need to learn advanced mathematics at school.

These studies indicate that the learners are not used to think over what it takes to learn or why knowing mathematics is important (Kloosterman, 2002), and find it difficult to answer the questions about *what, why* and *how* of their mathematics learning process. In addition, they hold inconsistent beliefs about mathematics and mathematics education, indicating that thinking critically about personal mathematics learning experiences is not a part of a regular mathematics class and despite being potential, their voices are often not encouraged or heard (Clare & Sue, 2013). Though evident, the reasons for this inconsistence in learners' beliefs have not been the focus of research. Learners' beliefs opened a gateway to study their motivation, engagement, affection etc. towards mathematics but their contrary beliefs about mathematics and possible explanation for them are not explored in the research literature. In this paper, learners' beliefs are therefore studied in light of their *critical thinking* skills in order to look for the possible reasons of these logical inconsistencies existing in learners' beliefs about mathematics. The sub-skills *self-examination* and *self-correction* of the sixth critical thinking skill *self-regulation* from critical thinking skills framework (**Figure 1**) are used to analyze learners' expressed beliefs about mathematics (details in *Data Analysis Framework* section).

Learners' Critical Thinking and Mathematics Education Research

Analogous to the research concerning beliefs in mathematics education, the research related to learners' critical thinking is also wide and diverse. Critical thinking is discussed from being a cognitive toolkit in order to solve mathematical problems in a logical and deductive manner (Jablonka, 2014; O'Daffer & Thornquist, 1993), to an attitude which learners as future citizens of mathematical society should be able to adopt in order to strive for a just and equal social structure (Gutstein, 2003; Skovsmose, 1998). This section elaborates these variations concerning critical thinking research in relation to mathematics education and suggests the connection between learners' beliefs and critical thinking in light of the research question asked in this study.

Critical thinking for mathematical problem-solving and its limitations

The "first wave" (McLaren, 1994, p. ix) of psychological and educational research concerning critical thinking presented it as the cognitive processes resulting in analyses of information objectively, argumentation, drawing conclusions, deductive and logical reasoning etc. (Ennis, 1964). The requirement of being objective, logical and rational while thinking critically had similarities with the *absolutist* view of mathematical knowledge (Ernest, 1985, 1991). This view of mathematics education in which being objective was essential for the knowledge to be seen as mathematical, provided convenient grounds for the partnership of critical thinking and mathematical problem-solving. Gutstein et al. (1997) also highlight this correlation by drawing on NCTM Standard (National Council of Teachers of Mathematics [NCTM], 1989) using *mathematics as reasoning* by citing "… students understanding and applying reasoning processes, creating and judging mathematical arguments, and validating their own thinking and answers" (p. 712).

This partnership leads to testing learners' critical thinking skills quantitatively (using mathematical tasks as test items), and usually in experimental settings (pre-and post-tests, control and treatment groups). Standardized tests, such as, Ennis-Weir Critical Thinking Essay Test (E-W), California Critical Thinking Skills Test (CCTST) and Cornell Level X Critical Thinking Test (CL-X)[2] got developed for the purpose. Likewise, CCTST-N[3] and James Madison University's Quantitative Reasoning (QR) Test[4] were developed to assess learners' critical thinking skills (named as QR, numeracy or quantitative literacy skills), specifically in mathematics. Since these tests measure cognitive critical thinking skills such as, proposing hypotheses, analysis, evaluation, drawing inferences and conclusions, most of the questions (usually multiple-choice) posed included graphs, tables, numbers and data to reach a decision. Mathematics education researchers also employed these tests (Aizikovitsh-Udi & Cheng, 2015) and contextual mathematical problems (Firdaus et al., 2015; Palinussa, 2013) to assess learners' critical thinking skills.

Using critical thinking to solve tailored (contextual) mathematical problems (in classroom/critical thinking tests) can indicate learners' successful implementation of critical thinking within a *cognitive* domain, usually divorced from their *social, political, cultural* contexts and *personal* life-choices. Jablonka (2014, p. 122) mentions that, even *cognitive* critical thinking "does not automatically emerge as a by-product of any specific mathematics curriculum …". Hence, learners' critical thinking ability may not transfer from a particular area of the discipline to social and personal spheres of their lives by itself. To gain a meta-perspective of their mathematics learning process, understand and participate in improving it, learners should be acquainted with applying critical thinking to reflect the role of mathematics and mathematics education in the *social, political, cultural* aspects of their lives, and in making their *personal-life* choices.

Critical thinking for understanding mathematics and mathematics education in socio-political and cultural contexts and its limitations

By introducing the "second wave" of *critical thinking*, critical pedagogy researchers reminded that all *critical thinking* is carried out by *someone* in *some social context* (McLaren, 1994, p. x). They insisted that if one is to use critical thinking in their lives, a complete de-subjectification and de-contextualization of any daily life situation cannot be possible. Hence, achieving full objectivity while thinking critically by discarding thinker's social, political, cultural contexts, and personal beliefs and/or prejudices is illusionary. Moreover, creativity and imagination may seem like opposites to *critical thinking,* and any creative solutions to a problem would be inacceptable if critical thinking should only propose logically deducible solutions.

Critical pedagogy, through education, aims to make learners conscious of *thinking critically* to reflect over their socio-political and cultural settings, and their personal beliefs to understand and rectify hidden hegemonic ideas, power, privilege, injustices and inequalities in society (Freire, 1972). Thus, 'second wave' critical thinkers propose an alliance of critical thinking and critical pedagogy, in which *critical thinking* should enable learners to critically observe, reflect, understand and analyze their social contexts and *personal* beliefs from a distance; and take planned actions to promote social justice, equity and peace in the society. This alliance also influenced mathematics education research.

CME (Skovsmose, 1994a) and the socio-political turn (Gutiérrez, 2013) in mathematics education make it clear that imparting *critical thinking competence* among mathematics learners is important to promote social justice, equity and critical citizenship through mathematics education (Gutstein et al., 1997). The need of making mathematics learners critically reflect and critique mathematics in the spirit of critical pedagogy is argued, though not particularly using the term *critical thinking*. Gutstein (2003, 2006) and Sriraman and Knott (2009) provide examples of using mathematics education with critical pedagogy to promote learners' socio-political and cultural consciousness about mathematics and mathematics education's role in society; and how it can help promoting social justice, critical citizenship and equity. This way learners can be made aware of the role mathematics and mathematics education play in their socio-political and cultural surroundings and how it indeed shapes their thoughts, lives and society. Also, Skovsmose (1994b), seemingly supports the 'second wave' of critical thinking, rejecting the possibility to "… reduce 'reflection' and 'critical thinking' to 'logical awareness' ... informal logic and to criticism 'inside the disciplines' …" (p. 217). He recommends CME to develop *mathemacy* in learners – a competence parallel to *literacy* (Freire, 1972) gained by critical education, as advocated in critical pedagogy. Evolving learners' critical reflective skills for attaining mathematical, technological and *reflective* knowledge[5] are favored to develop *mathemacy* (Skovsmose, 1994b). Learners are expected to critically reflect the

[2] See Hatcher (2011) and Liu, Frankel, and Roohr (2014, pp. 5-7) for detailed overview of available Critical Thinking tests.
[3] California Critical Thinking Skills Test with Numeracy (CCTST-N).
[4] See https://www.madisonassessment.com/view-demo/ for sample test items and Grawe (2011) for a detailed review of QR assessment tests.
[5] *Mathematical* knowledge includes learning mathematical content – numbers, algebra, methods, theorems, proofs etc. used in mathematical modelling and problem-solving. *Technological* knowledge enables learners to model a given situation mathematically and applying mathematical knowledge to analyze or solve the situation. *Reflective* knowledge, however uses the *critical reflection* component to reflect over the legitimacy and role of applying mathematical and technological knowledge to a situation and the influence mathematics exerts on those contexts.

use of mathematics in society using their *reflective* knowledge about mathematics gained through CME. Similar to *literacy*, the aim of *mathemacy* is to empower learners to be critical citizens through mathematics education.

However, critical socio-political awareness alone is not sufficient to bring social justice and equality, since to act consciously for bringing change in personal lives or society, people need *self-awareness* prior to socio-political awareness (Freire, 1972). In mathematics education context, Gutstein (2003) says that being critical about use of mathematics in socio-political contexts such as statistical data and results can help learners only *read* the world using mathematics. This reading can promote equity and justice *theoretically*, whereas to *write* (to *act* for changing) the world using mathematics, learners need to be active carriers of that change. Therefore, though learners may understand the role of mathematics and mathematics education in their socio-political and cultural contexts using critical thinking, it is not obvious that they can think critically about their own mathematics learning process for taking reflected decisions in their *personal* lives.

Critical thinking for gaining a meta-perspective of learning mathematics in personal life and its potential

Previous sections describe the *first* and *second* waves of research concerning *critical thinking.* McLaren (1994, p. xii), however, describes a potential "third wave" of *critical thinking* aimed at making critical thinkers aware of the importance of being critical to their *personal beliefs.* Such critical thinkers observe their own beliefs, choices, decisions and actions in their personal lives and societal circumstances critically to reflect on how they themselves influence and are affected by inequalities and injustice prevalent in society. Simplified, the 'third wave' of *critical thinking* can be seen as being a *critical thinker* in *personal* sphere of one's life.

In mathematics education, research regarding *critical thinking* is not precise. Different perspectives highlight learning and applying *critical thinking* in *cognitive* and *social* spheres of one's life through mathematics education (see *Introduction*). However, learners' potential of *critical thinking* to make reflected choices and take informed, well-reasoned decisions about learning mathematics in their *personal* lives does not seem to be carefully researched. This focus on researching learners' application of critical thinking skills only in the cognitive and social spheres of their life while learning mathematics has failed to consider the opportunity to find out if and how they adopt a critical attitude towards learning mathematics in their *personal* lives. Consequently, the potential and practice of learners in applying critical thinking skills to their *personal* mathematics learning process remains undiscovered. Learners are expected to make choices in mathematics education, especially at the transition points (at the end of their compulsory school years) of their educational pathway where they take important decisions regarding education for their future lives. Such choices should not be made uncritically. Therefore, McLaren's third wave of critical thinking applies to mathematics education as well. Learners should use critical thinking to gain a *meta-perspective* of learning mathematics in their personal life, so that they not only decide what to *believe*, but also can figure out how to *act* to improve their mathematics learning process. These *meta-perspectives* about their mathematics learning process can be different. A call for learners to achieve such *meta-perspectives* is also scattered around in the literature concerning CME. We outline three types of non-exhaustive and overlapping *meta-perspectives* below.

Firstly, learners' *critical thinking* about personal *beliefs, choices* and *decisions* concerning mathematics learning in their lives may involve *critically* understanding *what* content they are interested/not interested in learning, *how* mathematics is being taught to them, *why* they are/are *not* learning mathematics, and the purpose it serves in their present and future lives. Learners choose if they want to study mathematics or *not* and eventually the direction(s) they wish to pursue in further mathematics education. Such decisions involve *critical thinking* about personal reasons, desires and ways of learning mathematics or *not*. This *meta-perspective* would allow them to not only "grasp a meaning but also to have the possibility of *negotiating* a meaning about the content of their [mathematics] education" (Skovsmose, 1994b, p. 93) (parentheses and italics added). Acquiring such a meta-perspective can give learners the possibility to co-operate with teachers to influence and evaluate their teaching strategies and take decisions to make mathematics classes meaningful for themselves. This way learner cans develop a meta-language to have a say, take *charge*, start *acting* and *participating* (Skovsmose, 1994b, 2001) in their mathematics classrooms. Similar concerns are found in previous studies encouraging to extend the focus of CME to involve students more profoundly (Clare & Sue, 2013; Powell & Brantlinger, 2008), give them freedom to decide their own curriculum in mathematics (Lindenskov, 2010) and know its relevance in their *personal* lives (Kollosche, 2017).

The second type of *meta-perspective,* involves learners' *thinking critically* and understanding their strategies of learning mathematics and the ways to improve them. For instance, while doing mathematics, they can *critically reflect* upon their patterns of study, learning strategies, and regulate their personal practicing methods, studying structures, schedules etc. that work for them. By gaining such a *meta-perspective*, learners can improve their *learning* strategies leading to feel success in learning mathematics and enhance their mathematics learning experience on their own (Malmivuori, 2006).

In the third type of *meta-perspective*, learners become aware of *critically* understanding their personal beliefs about mathematics and mathematics education and the reasons for holding them. First and second *meta-perspectives* allow learners to understand how they can influence their mathematics learning, while the third *meta-perspective* encourages learners to *think critically* about how learning mathematics influences them and the society. Several papers forward concerns about learners acquiring an inferiority complex while struggling to learn and succeed in academic mathematics under CME research. Greer and Mukhopadhyay (2012) raise the issue of hegemony of mathematics education in society and "intellectual violence" (p. 236) exerted on learners by the dominant societal status and teaching structure of mathematics. D'Ambrosio (2010) also pointed out the responsibility of mathematics and mathematics educators to address the problem of survival with dignity for all individuals in the society. The need to *critically* consider the interplay between their personal beliefs about mathematics, and how these beliefs in-turn influence and reflect in the dominant structures of society through mathematics education is addressed for both struggling

	Strongly disagree	Disagree	Agree	Strongly Agree	Do not know
To learn mathematics and science is important for me because it will improve my career opportunities	☐	☐	☐	☐	☐
I am interested in what I learn in mathematics and science	☐	☐	☐	☐	☐
Mathematics and science are important subjects for me because I need them for my future studies	☐	☐	☐	☐	☐

Figure 2. An excerpt of statements from the questionnaire

and successful learners of academic mathematics. This self-awareness can make learners *conscious to reflect* over their role[6] in hegemonic structures and *able to act* for changing the status quo of dominant ideologies and traditions of mathematics education in society in and respect people holding different views.

Gaining these meta-perspectives through critical thinking can provide learners with a *potential* to improve their mathematics learning process for the betterment of their personal lives and the society. They can figure out better ways to learn mathematics, co-operate with their teachers to improve their mathematics teaching, and prioritize their choices while learning mathematics in their present and future lives. In addition, aim of the third wave of *critical thinking* to make the thinker aware of his/her own ideological situatedness in the society and work for emancipation (not adaption) can be realized through this *potential*. However, in this study, learners' meta-perspective of *first* type is explored since their expressed beliefs regarding *what, why* and *how* of their mathematics learning process are analyzed to identify their practice with *thinking critically* about this process.

METHOD

A qualitative approach is adopted to investigate learners' *personal* beliefs and reasons for learning mathematics (Creswell, 2014). Presented data was collected as part of the research project *Local Culture for Understanding Mathematics and Science* (LOCUMS, 2016); aiming to explore the role of *practical tasks rooted in students' own interests and local culture* to learn mathematics and science. Here, we analyze parts of the data collected under sub-project of LOCUMS carried out in middle Norway. Two schools which included learners from diverse cultural backgrounds were chosen as the sites of data collection in accordance with the research aims of LOCUMS. Four classroom interventions were planned which took place in 8th (two interventions) and 9th (two interventions) grades of two schools located in middle-Norway. Each intervention included three steps of data collection. In first step, learners responded to a paper-pen questionnaire, second step included learners working to solve practical group tasks (4-5 learners in each group) and the final step included face-to-face individual interviews with selected learners. This paper is based upon the data collected in questionnaires and interviews. In total, 74 learners from these two schools participated in the questionnaires and interventions, and 20 learners were selected for conducting semi-structured interviews. Both the questionnaires and interviews were conducted in Norwegian, audio-recorded, and later transcribed for analyses.

The questionnaire was designed by deriving the inspiration from the ROSE survey (Schreiner & Sjøberg, 2004). Different parts of the questionnaire were devoted to different themes which LOCUMS focused on. The questionnaire started for example with asking for the introduction of the learner and moving ahead to their personal life interests, leisure time activities, thoughts about culture, their multicultural classrooms, connection with mathematics and science, future education and job perspectives, school environment, social participation and a particular section for the immigrant learners. Each of these parts included both five-point Likert-scale statements and some open-ended questions where learners could write down their opinions freely. Since LOCUMS was directed to both mathematics and science subjects, the statements in the questionnaire include both these subjects. This paper discusses the results from learners' responses to the section "your connection with mathematics and science" in the questionnaire part. This section included 24 Likert-scale statements concerning learner's relation with mathematics and science subjects. Out of these 24 statements, 12 statements were about both mathematics and science, eight concerned only science, and four only mathematics. Learners had to respond to the extent they agreed with each given statement on a scale from *strongly disagree* to – *strongly agree* (see **Figure 2**).

Due to the focus of LOCUMS on both mathematics and science education, the statements in this section of questionnaire were about both these subjects. In addition, the statements carried both positive and negative connotations, for example, some statements dealing with mathematics were, *'I like to learn mathematics'; 'I think what I learn in mathematics is a waste of time'; 'I am simply not good at doing mathematics'* etc. Since statements dealing with both mathematics and science made it difficult to separate learners' beliefs about mathematics from those about science, their questionnaire responses were used as a starting point to design the interviews. Any incoherence noticed in their questionnaire responses laid the foundations for preparing interview questions directly related to their beliefs regarding mathematics, and its learning process. Consequently, the

[6] Either by contributing to authoritative ideologies, by adapting and adjusting to these hierarchical structures or by contributing to injustice and inequality in any other sense.

questionnaire responses are presented here only as a precursor to the interview analyses, which is the main source of information for this study.

An interview guide was developed in accordance with selected learner's responses on the questionnaire statements about mathematics and science, and their performance and engagement levels in the intervention. Each interview lasted for about one to one and a half hour, where the questions were divided into different sections. These sections started with an individual introduction and moved to enquire into the learner's general views about education, school environment, culture and cultural differences[7]. Further, learners' beliefs in-depth about their learning process in mathematics and science subjects separately were explored; and the concluding section enquired their experiences with the interventions. Since the research question deals with learners' practice with thinking critically *about* and give suggestions concerning their personal mathematics learning experience, the interview questions focused on their choices, decisions and personal experiences in mathematics classroom are presented here. Specific issues such as, *what* they are/are not interested in learning, *why* they learn and *how* they are taught mathematics, were discussed with learners. In this way, learners were made to *self-examine* their opinions and suggest ways, which can enrich their mathematics learning experience.

Our informants include learners studying in 8th and 9th grades (13-14 years old) who will soon reach a transition stage in their educational pathway (i.e., in 10th grade) where they prioritize and take important decisions concerning their further mathematics education. The whole data set consists of 74 responded paper-pen surveys, video recordings of learners solving group tasks and audio recordings from 20 individual (10 boys, 10 girls) interviews with selected learners. All the 74 learners got and answered the questionnaires (no sampling), whereas the 20 informants for in-depth semi-structured interviews were selected by following the principle of maximum variation. Based on a learner's questionnaire responses and his/her performance and participation level in the classroom intervention, a representative selection of five learners (one from each group in a classroom) after each intervention was made. The selected learners included students who liked/did not like, were highly, moderately or not much interested in learning mathematics, and were highly, moderately or not so enthusiastic while participating in the intervention. Attention was also paid to interview the learners from different cultural backgrounds. Majority (15) of the interviewed learners were Norwegian but some of them had different mother tongue (Sámi (2), Eritrean (1), Arabic (1) and Pashto (1)) and diverse cultural backgrounds. All except one (interviewed in Pashto) of them had been in Norway for more than three years and understood Norwegian well during the interviews despite having a different mother tongue. However, to investigate their critical thinking in-depth, this study focuses on learners' responses, not the variation in their culture and language.

DATA ANALYSIS FRAMEWORK

Both questionnaire and interview data were analyzed to explore learners' *critical thinking* about their personal choices and decisions concerning learning mathematics in their lives. Preliminary analysis of questionnaires highlighted learners' inconsistent responses to statements concerning their mathematics learning. For instance, a learner *strongly agreeing* that she/he likes learning mathematics but strongly disagreeing that she/he is interested in what she/he learns in mathematics was interpreted as an incoherent response. These conflicting responses were followed up in the interviews. Both questionnaire statements revealing learners' incoherent responses and selected interview excerpts are presented here. Questionnaire responses are analyzed in MS Excel, and interview excerpts are analyzed descriptively using the sub-skills, *self-examination* and *self-correction* of the sixth core cognitive critical thinking, *self-regulation* (**Figure 1**).

While holding certain beliefs, making specific choices or decisions in lives, people often have personal reasons and justifications for doing so while being unconscious of these reasons. Therefore, when asked to explain their beliefs, choices and decisions, the process of clarifying can make them conscious of those underlying reasons, assumptions, bias etc. *Critical thinking*, as defined, presumes existence of beliefs, hypotheses or opinions, but being *critical* involves questioning those beliefs and ideas logically. Beliefs are the objects of reflection for critical thinking and *self-regulation* critical thinking skill provides the opportunity to analyze our personal beliefs rationally (Facione, 1990). *Self-regulation* focuses on the process of self-consciously questioning and evaluating reasons for the judgements and decisions made by oneself (i.e., *self-examination*), and correcting one's reasoning or beliefs in case it reveals any bias or erroneous assumptions (i.e., *self-correction*). Since the incoherence of learners' beliefs concerning their mathematics learning process is focused in this study, the skills of *self-examination* and *self-correction* are employed to analyze learners' interview responses. Learners' beliefs are enquired along with the reasons and justifications for holding those beliefs to explore if this inconsistence may be related to their practice with *thinking critically*. The definitions of *self-examination* and *self-correction* were operationalized as follows:

- *self-examination* is understood as "… an objective and thoughtful meta-cognitive self-assessment of one's opinions and reasons for holding them'; 'judging the extent to which one's thinking is influenced by deficiencies in one's knowledge, or by stereotypes, prejudices, emotions or any other factors constraining one's objectivity or rationality." (Facione, 1990, p. 19), and
- *self-correction*, is understood as, " where self-examination reveals errors or deficiencies, to design reasonable procedures to remedy or correct, if possible, those mistakes in one's opinions and their causes" (ibid.).

[7] The questions concerning learners' views about education, mathematics, science, culture and cultural differences in their own class etc. intended to meet the objectives of LOCUMS.

Our aim in these interviews was to explore learners' practice with critical thinking *about* learning mathematics. These definitions allowed us to explore learners' beliefs, assumptions, thoughts, opinions etc. presented in the interview conversations to highlight their practice with critical thinking skills of *self-examination* and *self-correction* about learning mathematics.

Ethical Considerations

This project was reported to the Norwegian Centre for Research Data (*Norsk senter for forskningsdata*, NSD) and permission to collect data was obtained with a clearance number 50556/3/AMS. Both young learners and their parents and/or guardians were informed in written about the project and their written consent was obtained to collect data involving their children. They were made aware that all data will be treated confidentially. It was also conveyed that their participation is voluntary and they can withdraw their consent whenever they wish without giving any reason.

DATA INTERPRETATION AND DISCUSSION

Questionnaire Analyses

This section presents analyses of learners' responses to the statements of the section "your connection with mathematics and science" in the questionnaire. There were 24 statements in this section, out of which eight statements dealt only with science and 16 with both mathematics and science. In 11 of these 16 statements learners were asked about their achievement levels in mathematics and science, and the remaining five statements enquired about their interest, liking and importance of learning these subjects for further studies and career opportunities. Since learners' achievement level is not the priority of this paper, some analyses of learners' responses to only these five statements are presented here. These statements being – '**I** like to learn mathematics'; '**I** am interested in what **I** learn in mathematics and science'; 'To learn mathematics and science is important for me because it will improve my career opportunities', 'Mathematics and science are important subjects for me because **I** need them when **I** will study further'; and '**I** think that what **I** learn in mathematics is waste of time'.

On comparing learners' responses to these statements, we observed an inconsistence in their answers. A comparison analysis of opinions of several learners appeared to be incoherent and unsure. For example, if a learner strongly agreed/agreed to both the statements, *'I like to learn mathematics'* and, *'I think what I learn in mathematics is waste of time'* or vice-versa[8], it was interpreted as an incoherent response. Another example is a learner strongly agreeing/agreeing that, *'I like to learn mathematics'*, but strongly disagreeing/disagreeing that, *'I am interested in what I learn in mathematics and science'*; or the opposite[9]. Therefore, we collected learners' responses to these five statements from all 74 questionnaires and compared them with each other to find out the proportion of incoherent answers for each pair of the statements. **Table 1** shows these statement pairs and the number of incoherent responses from the learners. This analysis promoted our interest in enquiring if practice with thinking critically can be a possible explanation of the 'mismatch' between learners' responses, which formed the bases of our research question for this paper. **Table 1** is presented as a forerunner leading towards the interview process and research question.

As depicted in **Table 1**, 18% (13 out of 74) learners stated that they *like* to learn mathematics, but are *not interested* in learning the same, or vice-versa. 32% hold divergent views about *liking* to learn mathematics, and the *importance* of learning mathematics and science when it comes to future career opportunities. Simultaneously, responses of 31% of learners are incoherent when it comes to *liking* to learn mathematics and the *importance* of learning mathematics and science for further studies. 28% responded that what they learn in mathematics is *waste of time* despite they *like* to learn mathematics, or they do not think that learning mathematics is a waste of time, yet they do not *like* it. Further, 22% give disconnected responses to being *interested* in learning mathematics and science in relation to its *importance* for future careers. 23% have paradoxical views regarding their *interest* in learning mathematics and science, and the *need* for learning these subjects for further studies, whereas, 20% reported that though they are *interested*, what they learn in mathematics seems to be *waste of time*. The results for the last three pairs of statements in **Table 1** seem to be more intelligible. Only 5% of students have diverging views regarding *importance* of learning mathematics and science for *future careers*, and the need of these disciplines for *further studies*. Just 11% state that learning mathematics is *waste of time,* which seems to be correlated with that learning mathematics and science is *important* for their *future career*. Learning mathematics seemed to be a *waste of time* for only 5% of learners as they report that they need to learn mathematics and science to *study further*.

This preliminary analysis of questionnaire responses brought the confusion apparent in learners' beliefs to our notice which provoked our curiosity to know if learners consciously and critically examine and correct their personal beliefs about learning mathematics. Though majority of the learners (over 50% for all the statement pairs) provide coherent answers, but the lack of harmony and indecisiveness seems evident for about 30-40% of learners. The questionnaire responses, however, did not help us to understand why learners were indecisive or held divergent opinions. Therefore, semi-structured interviews were conducted to delve deeper into learners' application of critical thinking while interacting *about* their mathematics learning process. In interviews, learners were asked to follow-up and justify any incoherent answers in-depth. Their responses were then analyzed using *self-examination* and *self-correction* to explore their use of critical thinking for making choices and decisions about learning mathematics in personal lives.

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[8] That is, strongly disagreeing/disagreeing to both the statements, *'I think what I learn in mathematics is waste of time'* and, *'I like to learn mathematics'*.

[9] That is, strongly disagreeing/disagreeing that, *'I like to learn mathematics'*, but strongly agreeing/agreeing that, *'I am interested in what I learn in mathematics and science'*.

Table 1. Overview over learners' incoherent responses to pairs of the five statements in questionnaire

Pair number	Statement number 1	Statement number 2	No. of learners (out of 74) giving incoherent responses to statement number 1 and 2
1	I like to learn mathematics	I am interested in what I learn in mathematics and science	Agree or disagree to both the statements (Coherent responses): 46 Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Incoherent responses): 13 Do not know/other answers: 15
2	I like to learn mathematics	To learn mathematics and science is important for me because it will improve my career opportunities	Agree or disagree to both the statements (Coherent responses): 41 Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Incoherent responses): 24 Do not know/other answers: 9
3	I like to learn mathematics	Mathematics and science are important subjects for me because I need them when I will study further	Agree or disagree to both the statements (Coherent responses): 40 Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Incoherent responses): 23 Do not know/other answers: 11
4	I like to learn mathematics	I think that what I learn in mathematics is waste of time	Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Coherent responses): 43 Agree or disagree to both the statements (Incoherent responses): 21 Do not know/other answers: 10
5	I am interested in what I learn in mathematics and science	To learn mathematics and science is important for me because it will improve my career opportunities	Agree or disagree to both the statements (Coherent responses): 43 Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Incoherent responses): 16 Do not know/other answers: 15
6	I am interested in what I learn in mathematics and science	Mathematics and science are important subjects for me because I need them when I will study further	Agree or disagree to both the statements (Coherent responses): 41 Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Incoherent responses): 17 Do not know/other answers: 16
7	I am interested in what I learn in mathematics and science	I think that what I learn in mathematics is waste of time	Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Coherent responses): 41 Agree or disagree to both the statements (Incoherent responses): 15 Do not know/other answers: 18
8	To learn mathematics and science is important for me because it will improve my career opportunities	Mathematics and science are important subjects for me as I need them for further studies	Agree or disagree to both the statements (Coherent responses): 60 Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Incoherent responses): 4 Do not know/other answers: 10
9	To learn mathematics and science is important for me because it will improve my career opportunities	I think that what I learn in mathematics is waste of time	Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Coherent responses): 54 Agree or disagree to both the statements (Incoherent responses): 8 Do not know/other answers: 12
10	Mathematics and science are important subjects for me because I need them when I will study further	I think that what I learn in mathematics is waste of time	Agree to statement no. 1 and disagree to statement no. 2 or vice-versa (Coherent responses): 56 Agree or disagree to both the statements (Incoherent responses): 4 Do not know/other answers: 14

Learners' Interview Responses about What, Why and How of Learning Mathematics

This section presents selected interview excerpts in which learners were asked about *what* they think is interesting to learn in mathematics, *why* they think it is important to learn mathematics and *how* to improve mathematics teaching. Being semi-structured, the interviews were conducted like informal conversations and the questions asked from different learners were similar but not identical. In order to explore learners critical thinking in terms of self-examination and self-correction, they were often asked to provide reasons for their responses. In the following text, different learner's response (indicated by using a number as a subscript under letter 'L' in the transcripts below) to same question is used as the unit of analysis. A descriptive analysis of learners' interview responses is presented below. Learners' responses starting with the phrases *"I" think*, *view*, *believe, use* etc. have been chosen to evaluate their application of self-examination and self-correction skills. First, representative interview responses of learners concerning *what* is interesting and *what* seems to be waste of time for them to learn in mathematics are presented and discussed. Later, learners' responses concerning *why* they learn and *how* they suggest to improve mathematics teaching are explored. An interview excerpt concerning the interest in learning mathematics follows:

I (Interviewer): … is there something in mathematics you are especially interested in learning?

L_4 ($Learner_{Number}$): I like to calculate the areas and … construct

I: is there something in mathematics which you think is useless to learn?

L_4: no…

I: but do you think then that you need to learn everything in mathematics that you learn in the class?

L4: maybe not everything but …it is smart to be able to do that …

Here, the learner quickly expresses what content of mathematics she likes to learn; however, it seems difficult to answer if there is any content in mathematics which seems useless for her to learn. This was common for most of the interviewees. 16 out of 20 learners promptly mentioned what specific content in mathematics they are interested in learning. Moreover, when justifying their choices 12 out of 20 were able to provide reasons for their interest in learning that specific content, may it be just for fun, further studies or future job. These numbers give an impression coherent to Lindenskov (2010) interpretation that leaners usually *self-examine* and can justify their personal interests in learning specific mathematics content. However, learners seemed to be confused and unsure when asked to name the topics which may not be useful for them. In the interview excerpt presented above, the learner quickly concludes that she does not find any topic in mathematics to be useless to learn. The paradox, however, is that though the learner thinks that not everything is useful to learn, yet she could not mention any topic in mathematics that is useless to learn. This unawareness of critically evaluating both perspectives indicates a negligence of self-examining one's personal opinions in-depth. This dilemma was also clear in other interviews, for instance:

I: … is it something in mathematics which you think is especially interesting?

L6: I think it's like the fractions and the forms [figures] and such things… I think that it's fun to do that…

I: is there something in mathematics you think is waste of time?

L6: eh… no…it's well maybe… or I don't know actually

I: but eh… maybe there is something that you feel that you have no need to learn or … like you cannot use it for something? Or is it just that … like okay it's nice to know about everything then?

L6: yea… it's maybe like this or like… I don't know actually. I can't think of any words [topics] like which could be felt like waste of time…

I: yea but it's no hurry like you don't have to answer quickly… you can take your time to think and if you think it would help?

L6: it's maybe the same [histograms, as mentioned earlier in the interview] …

…

I: will you say that it's difficult to learn or you just can't find any use of it?

L6: I don't just find any use of it… I don't know…

In this extract, learner easily picks up what is especially interesting for her to learn in mathematics, but when asked about the content she does not find useful, the words *'I don't know'* appear frequently. Only on being urged and given time and to reflect, she came up with a topic which did not seem useful for herself. Only eight out of 20 could name topics they think are waste of time to learn for them and just seven could give a reason for why learning that specific topic in mathematics is in vain for them.

Learners' responses and frequent use of the words *'I don't know'* indicate their self-doubt, unfamiliarity and lack of *critically self-examining* their personal interests in learning mathematics. Even if learning everything they are taught does not appear to be correct, possible or useful, but not trying to do so is experienced to be negative since mastering mathematics is experienced to signify being clever and smart. Therefore, an unconscious resistance to gain an overall critical meta-perspective of one's own mathematics learning also surfaces. These contrasting responses reveal that learners are not used to think critically about the questions related to the mathematical content they *do not* think is interesting to learn. However, when asked to be critical and clarify their own opinions mentioned earlier in the interview, they do mention the content which does not seem personally beneficial for them. Considering both, *what* content *is* and *is not* interesting for them, is important for learners to make reflected personal choice regarding what field of mathematics they want to pursue in the future, and opt out of the content which is boring for them.

Being able to use and apply acquired knowledge in practical lives and professions legitimates to a great extent *why* that knowledge is gained. Therefore, learners were asked to justify *why* they learn mathematics by providing examples of using mathematics in their lives. While mentioning these uses, 17 of 20 learners named only the application of elementary mathematics in real-life (i.e. basic arithmetic calculations while shopping, cooking etc.), whereas only three of them presented examples of using mathematics they learn in their current grade. For instance:

I: but like in general? …, is it something then you can use mathematics for?

L8: eh… not for a lot of things but maybe … or at least little bit… if you have to calculate something so…eh…mass density or you should find out … or if you have to make a bit like makeshift or MacGyver [a TV hero]… it's like to make something out of the things you find and then it becomes something…you can calculate how things work out like if you had to for example make an electric cycle from only the things you find in a garage… so you need to calculate things and know how the things work out … such that it could work…

L_8's argument of being able to do machine makeovers represents a situation where one may need 8th and 9th grade mathematics in a real-life situation, however, the learner imagines a fictional character (i.e., MacGyver), not himself using mathematics in own real-life. Two more learners provided such examples including: to read geometrical and numerical data using maps and a compass; and applying statistics to any data collected, though specifying that he uses statistics only at school. Though specifying that they have not used mathematics in this way yet in their own everyday life, these three learners display some potential to *self-examine* their personal beliefs about *why* they learn mathematics. Uses of elementary mathematics provided by the remaining 17 learners include examples such as:

> *I: but can you give examples on where you use mathematics? Like the mathematics you learn at the school? Do you use it anytime in your daily life or…?*
>
> *L_6: yes… I use it like when I am at a shop so I use to pay by cash and to find out how much I'm going to get back and different things like this and then I use it when I make food …then one needs to know like liters and deciliters and such things then…*

L_8's response associates "the relevance of mathematics with learning skills *in elementary mathematics*" (Kollosche, 2017, p. 637) (italics in original), without considering why and where this learner needs advanced mathematics such as algebra or geometry in his/her life. Estimation of price using basic arithmetic operations is curriculum of primary school, not what they learn in 8th and 9th grade mathematics. Lack of *self-examining* and *self-correcting* one's beliefs about *why* one learns mathematics is also visible in the following response:

> *I: yes… I am listening to what you are saying but actually it is a bit interesting that… algebra is a very… ok it's right what you are saying that it's [algebra is] just setting in a letter instead of a number but why are you going to need it…like? You are saying that it is useful but can you give an example about where you will need it?*
>
> *L_5: I've just heard that one needs it…*
>
> *I: heard…? Where?*
>
> *L_5: from teachers and other adults…*
>
> *I: ok, but have you sometime asked them where they use it?*
>
> *L_5: no…*
>
> *I: no… have you sometime asked them 'where can I use an equation?'*
>
> *L_5: no … I haven't asked him [the teacher] no…*
>
> *I: has he [the teacher] said anything about where you will use an equation?*
>
> *L_5: no…*
>
> *I: ok… well …but you still think that though you don't know that where one needs or can use it …so you think that it is useful to learn?*
>
> *L_5: yes… because we learn it…*
>
> *I: and important?*
>
> *L_5: it's not the most important thing but …like… eh… it is no…I just mean that people should know about it and be able to do it but I don't know why …we learn it at the school so one thinks that one will use it…*

L_5 is evidently influenced by the widespread belief that mathematics is a useful subject to learn and everyone is going to need it later in their lives. The phrase '*we learn it at the school so one thinks that one will use it…*' illustrates learner's trust in mathematics curriculum adopted in their classroom to an extent that she does not feel the need to critically examine the information received by the teachers or other adults. Consequently, she is not conscious of demanding reasons, justifications and real-life applications of mathematics learnt in school. Moreover, though she could not mention any uses of advanced mathematics in her life, yet she denied that learning it may not be so useful for her. This instance not only illustrates the lack of *self-examination* of one's personal beliefs about learning mathematics, but also the denial to *self-correct* those beliefs if found unjustified, which again can hinder her in developing a meta-perspective of first type about learning mathematics. Similar seems to be the case for the other 17 learners who could only provide elementary uses of mathematics despite being in 8th or 9th grade, but still believed that learning advanced mathematics is and will be useful for them. This unconsciousness to *self-examine* and *self-correct* (if needed) their beliefs can inhibit evolving *self-regulatory critical thinking* skills among learners and sustain the incoherence of their beliefs regarding their own mathematics learning process.

Finally, the interview questions focused on *how* learners are taught and what changes they suggest improvement in their mathematics lessons. Several learners mentioned that mathematics lessons at times are either difficult, boring or obligatory for

them. This general expression of learner dissatisfaction encouraged questioning the learners about their personal experiences of and suggest changes in mathematics teaching to improve it. Learners were not acquainted with answering such questions and it was not easy for them to spontaneously suggest changes in their mathematics lessons. However, when persuaded, 15 out of 20 learners proposed changes in their mathematics teaching. For example:

> *I: hmm .... have you sometime thought that there is something I would like to be changed in [mathematics] teaching?*
>
> *L_{12}: no ... I have not thought about it ...*
>
> *I: are you satisfied with the way teachers teach you?*
>
> *L_4: yes…*
>
> *I: why do you think that this way to be taught is alright?*
>
> *L_4: because… eh… I don't know maybe I'm used to having it like this so…I have done it all these years so, I think it's a fine way to learn…*
>
> *I: but if you had a chance…would you change the teaching in mathematics and science?*
>
> *L_4: no…*
>
> *I: nothing?*
>
> *L_4: maybe a bit more [practical] activities in mathematics but otherwise I don't think so of anything…*

Learner L_{12} replied that she has *never thought* about bringing any changes in their mathematics teaching, symbolizing unawareness to think critically and *self-examining* their mathematics teaching. Similarly, L_4 gave an expression of being *'used to having taught like this'*, thus thinking some other approach than the *usual* way seems to be difficult for her. However, when asked to reconsider, she could recommend more practical mathematics lessons since much sitting and getting instructions from the teachers was mentioned as being boring (Nardi & Steward, 2003). Changes proposed by these 15 learners ranged from including more practical activities to teachers using more real-life based tasks, alternative explanations and giving more time for slow learners to catch-up in the lessons. However, lack of practical activities in mathematics lessons was a recurring concern and also frustrating for some learners as illustrated:

> *I: had you involved other activities like more practical activities… [in mathematics lessons]?*
>
> *L_8: eh… I would have done it because what we've done in the whole 8th grade is just to write… write and write and solve the tasks and then it becomes quite boring and you lose the interest and you sit there just to write and when the class is over so you think 'oh yes, finally finished…'*

Here, L_8 suggests including more practical activities in order to get over the monotony of mathematics lessons. The changes mentioned above include general suggestions, but one of the learners even managed to sketch a complete lesson plan for teaching a topic she suggested in the interview. She expressed that learning to make budget for a family in an Excel worksheet can be useful instead of using one to two weeks to cram formulas for calculating volumes and areas of geometrical figures, which are easily accessible on their smartphones. Her reply follows:

> *I: hmmm…if you have a suggestion about that you could have learnt to set up a budget…have you also thought of how would you have liked to learn it? In what way?*
>
> *L_5: eh… if we could have got a realistic situation… and then we could have got a task about it so it would have… for example set up budget for a whole family for a month and you get different expenses and the teacher and you get the income and you have to pay the tax and you have to pay different and you should have a bit sum as saving if you sometimes get into a trouble and such things … like which are important to learn like you don't need to take loan and you don't have a lot of debt because you just got into a trouble which you never expected in your budget… so we learn how one should use his money because I mean that mathematics…we use a lot of mathematics in society like money and much is controlled by money and money gives power so… because people should not use up…because of people should have better … because like now we have got confirmed… like I don't have any concept of… like I don't know what 1 million kroner is… it's a lot and lot but you can't manage to buy a house for one million kroners… so we should learn more about the value like how much 1000 kroner [NOK] is worth … and such…*

L_5's reply presents an apt instance of using critical thinking to evaluate and examine her beliefs and in both personal and social domains of learning mathematics in her life. This critical outlook indicates learner's potential to acquire a meta-perspective of *first type* about her own mathematics learning not only to make choices, decisions and suggest improvements in her mathematics

learning experiences, but also to critically observe mathematics' role in the society. These extracts show that though learners possess the capability to think critically and suggest changes in mathematics lessons, but their potential is hidden and they are unsure about doing so due to lack of training and experience. Further, it seems unlikely that learners themselves will initiate any such discussion and communicate their ideas to mathematics teachers unless asked. These examples indicate that learners are likely to gain a meta-perspective regarding *what, why* and *how* to improve their mathematics lessons by evaluating and suggesting improvements in these lessons, provided their *self-examination* and *self-correction* critical thinking skills are encouraged and facilitated. Our data however does not indicate that learners' views are different or appear to be influenced by variations in their cultural backgrounds.

Lack of adopting such a critical attitude towards learning mathematics in their personal lives makes it difficult for learners to gain the meta-perspective of *first type* so that they can make personal life choices and decisions concerning their mathematics education for further studies or career. This inexperience in decision-making indicates learners' *uncritical* obedience to the pre-decided mathematics curriculum and the traditional or dominant ways of teaching and learning mathematics at school. These speculations about learners' unconsciousness can be grounded in their contrasting questionnaire responses, combined with interview answers such as, *'I don't know'* and *'I'm used to having it like this so…I have done it all these years so, I think it's a fine way to learn…'*. However, though being inexperienced in thinking critically, the learners do not lack the potential to do so. When given time, encouragement to clarify their responses, and asked to reflect over their choices, they seem willing to explain their thoughts in depth and can also investigate and understand the reasons of their beliefs. Some of them have an advanced potential to contribute in improving their mathematics teaching-learning process, but they are not likely to initiate or communicate these thoughts to their teachers if not encouraged to do so.

Learners investigated in this study are young so it is not surprising that they are unsure and *'don't know'* their choices exactly, but it can be problematic if they have *'never thought about it'* until now. They will soon reach a transition stage in their educational pathway (i.e., starting higher secondary after 10th grade), where choosing the right direction (theoretical or vocational) for further mathematics learning will be important for their lives. Our study suggests that these learners lack the practice in thinking critically *about* their learning process of mathematics and struggle to mention their personal interests, reasons and favorable strategies to learn mathematics. These findings indicate a mismatch between what is expected of the learners at this age in terms of deciding the direction of their future mathematics education, and what prerequisites they have acquired in terms of critical thinking to assess the options available to make a well informed and reasoned choice concerning mathematics education in their personal life. Regardless of their achievement levels in mathematics, and whether or not they decide to study mathematics further, mathematics education should be experienced as meaningful, not causing inferiority complex or a feeling of unworthiness among these learners. Therefore, more awareness and attention needs to be diverted for inspiring them to be critical towards their mathematics related beliefs so that they can suspect the widespread claims prevalent in the society about mathematics instead of just accepting or leaving them unquestioned.

TRUSTWORTHINESS AND LIMITATIONS

The trustworthiness of this study is addressed using the criteria *credibility*, *transferability*, *dependability* and *confirmability* (Bryman, 2016). The *credibility* is achieved by triangulating the two research methods – questionnaires and semi-structured interviews. The paradox in learners' responses is evident in both questionnaires and the interviews. Moreover, the findings of the study are in coherence with the previous studies reviewed in the paper. The criterion of *transferability* is taken care by providing a thick description of the context of this study, the details of informants and an elaborated account of data analyses procedure. *Dependability* is assured by keeping the log and regular meetings of the research team through all phases of the research process. A language and quality check of questionnaire, intervention design and semi-structure interview guide was done by fellow researcher having experience in conducting similar studies. In addition, the analyses of questionnaire and interview responses were also discussed with the research team to review the findings. Furthermore, though complete objectification cannot be obtained but *conformability* of the study is ensured by keeping an objective outlook during the phases of data collection and analyses. The researchers neither had any involvement in learners' mathematics and science lessons on daily bases, nor any control over their achievement in these disciplines, before or after this project. In this way, it was avoided to exert any influence on the learners to provide falsified or have hidden any information while answering the questionnaire and interview questions. Researchers' subjective values or preferences therefore, can be assumed to have little influence on the conduct of the research and the findings derived from it.

Despite adopting the measures to establish the trustworthiness of the findings, this study has limitations such as, lack of similar previous research, and the constraints on time and resources to well-design instruments for exploring other possible reasons of the incoherence in learners' beliefs. Employing the lens of critical thinking faculties, *self-examination* and *self-correction* does not seem to be a familiar approach to visualize the incoherence observed in learners' mathematics related beliefs. Therefore, scarcity of similar previous research can be considered as a limitation for the way this study was designed and conducted. In addition, the overarching focus of LOCUMS on both mathematics and science subjects restricted the amount of time and resources available. Consequently, it became difficult to develop professionally advised questionnaires to access learners' self-examination and self-correction critical thinking skills, specifically for their mathematics learning experiences. Though investigating mathematics and science beliefs simultaneously can be considered as a multidisciplinary approach, the study's accountability and reliability within mathematics education might be increased by adopting such measures. Nevertheless, these limitations open up the possibilities for further research on developing such instruments and interview guides to advance the concerning field of investigation.

SUMMARY AND CONCLUSION

Learners' mathematics related beliefs are investigated under the domain of affect research in mathematics education. Most studies concerning learners' beliefs explore their motivation, engagement, anxiety, stress etc. towards learning mathematics through their beliefs, but a few studies are found enquiring their beliefs regarding the *what, why* and *how* of their mathematics learning process. These investigations illustrate that the learners' hold incoherent beliefs about mathematics and mathematics education, however, this incoherence has not been the center of attention for further research in mathematics education. We take this incoherence in learners' mathematics related beliefs as a starting point to explore their practice with thinking critically about and their potential to suggest changes in their mathematics learning process. Specifically, the critical thinking skills of *self-examination* and *self-correction* are analyzed. Therefore, along with their beliefs learners were also asked to provide the justifications and reasons for holding them.

Our literature review highlighted that research concerning critical thinking in mathematics education mainly discusses it as a tool to learn *cognitive* practices of inference, drawing conclusions etc. within a discipline; and sometimes to highlight *socio-political* power imbalances. However, APA's consensus definition of critical thinking[10], critical pedagogy research's argument for *second* and *third wave* of critical thinking; and recent mathematics education research challenge the limited view of critical thinking. The importance of developing and exploring learners' application of critical thinking in *personal* domain of their lives, along with the *cognitive* and *socio-political* is recognized in this paper. We propose that learners should acquire habit of critical thinking to gain a meta-perspective *about* their *personal* mathematics learning process so that they can make choices and decisions about learning mathematics in their future lives. In this way, they may become aware to participate and take actions for making mathematics learning process meaningful for them in accordance with the central values of the Norwegian mathematics curriculum.

Consequently, in this study learners' practice with thinking critically and their potential to suggest changes in their mathematics learning process is explored by analyzing their expressed mathematics related beliefs. The incoherence evident in the questionnaire responses of 8th and 9th grade learners is presented and *self-examination* and *self-correction* (**Figure 1**) skills are used to interpret their interview responses to explore their application of critical thinking with respect to their *personal* mathematics learning process. Interview excerpts including incidents where learners apply, and do not apply *self-examination* and *self-correction* skills to their mathematics related beliefs are represented. Based on our interpretations of the questionnaire and interview data, we conclude that the inconsistence evident in learners' mathematics related beliefs can be related to the lack of practice in thinking critically *about* their mathematics learning process. Both questionnaire and interview responses indicate that these learners have limited or no experience in thinking critically about learning mathematics, and specifically in applying the skills of *self-examining* and *self-correcting* on their mathematics related beliefs. This leaves an impression that learners' critical thinking faculties *in* learning mathematics seems to be common in classrooms whereas, critical thinking *about* learning mathematics does not seem to get much attention. Moreover, learners do not seem aware of their right to observe their mathematics learning process critically in order to take a co-responsibility and co-operate in improving their own mathematics learning experience as per the Norwegian Act of Education. Thus, learners' potential to cooperate with their teachers to influence their mathematics education is intimidated, hence, being hardly visible and utilized.

The focus on critical thinking in mathematics concerning just the *cognitive* and *social* aspects do not provide a holistic picture of interaction between mathematics education, the learner, and the society. Therefore, it is significant to make mathematics learners aware from young age of being critical to their *personal* beliefs *about* mathematics, their mathematics learning process, and the role mathematics education plays in the society to make them reflect and consciously take decisions regarding mathematics in their personal and social lives. Therefore, we encourage that learners should learn to argue for and justify the legitimacy of their personal choices about their own mathematics learning process in mathematics classrooms while learning mathematics. Such critical thinking can stop learners from *uncritically* feeling obliged to follow a pre-decided mathematics curriculum, and use their potential to influence their mathematics learning process for the best of themselves and the society. We recommend that young mathematics learners are encouraged to and get training in thinking critically *about* their mathematics learning process along with gaining critical thinking skills for problem-solving in mathematics. Such training can equip them with a meta-perspective of their mathematics education in order to gain a holistic view of it and make reflected choices concerning mathematics in their personal lives. More research on this area is welcome and needed.

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[10] That is, making a reflective judgement about what to believe and what to do in situations one faces in their real-life.

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7.2 Paper II: Learners' beliefs about relevance and importance of learning mathematics

"We learn it [mathematics] at school so one thinks that one will use it …": Learners' beliefs about relevance and importance of learning mathematics.

Shipra Sachdeva[1] and Per-Odd Eggen

Abstract

In this study, we explore Norwegian learners' beliefs about the relevance and importance of learning mathematics. The data material consists of semi-structured interviews with nineteen lower secondary school learners (13-14 years old). The analyses indicate that the learners believe in the relevance of learning secondary level mathematics though they struggle to give examples of this relevance in their personal lives. Further, learning mathematics is also believed to be important for their lives but the sources of these beliefs are often statements heard from others. The analyses also suggest that the learners seldom reflect, evaluate, or question the relevance and importance of learning mathematics critically. A scarcity of such reflections indicates a tension between learners' beliefs and the curriculum's aim to empower learners to think critically and contribute to the decisions concerning their own mathematics learning process. We discuss the implications of this tension for learners' empowerment and development as critical citizens, who can participate and voice their opinions in discourses about the significance and roles of mathematics education in their personal lives and society.

Keywords: relevance, importance, mathematics education, learners' beliefs, socio-political discourse, learners' empowerment.

Introduction

Learners' beliefs about learning and teaching of mathematics can influence their attitude and motivation to learn mathematics (Grootenboer & Marshman, 2016a; Leder et al., 2002).

[1] Corresponding author.

Therefore, their beliefs about the relevance and importance of learning secondary level mathematics in their present and future lives are well investigated (see e.g., Grootenboer and Marshman (2016b); Kollosche (2017); Onion (2004) and Pais (2013)). Findings of these studies reveal that learners believe school mathematics to be relevant and important for their lives (Kollosche, 2017; Onion, 2004; Pais, 2013), and have a positive attitude towards learning it (Grootenboer & Marshman, 2016b). However, simultaneously, Onion (2004), Nardi and Steward (2003) and Kollosche (2017) report that learners consider mathematics to be a difficult and boring subject. Further, learners cite using elementary calculation skills (Kollosche, 2017), assessment (Onion, 2004), qualifying for a well-paid job (Wiik & Vos, 2019), or "to do homework" (Alrø et al., 2009, p. 15), rather than the knowledge of mathematics, as reasons for believing that learning *secondary level* mathematics is relevant and important for them.

Kloosterman (2002) and Grootenboer and Marshman (2016b) illuminate this paradox by reporting that secondary school learners are seldom conscious of, or used to reflectively considering or questioning if and for what they need to learn mathematics, and how it is relevant and important for their lives. Based on this observation, one may ask how secondary school learners formulate their beliefs about the relevance and importance of learning mathematics for themselves. In this paper we address this research gap by exploring learners' beliefs about the relevance and importance of learning mathematics and the sources of information forming these beliefs. The research question we investigate is:

> *What are Norwegian secondary school learners' beliefs about the relevance and importance of learning mathematics, and what are the sources of information influencing the formation of their beliefs?*

Background and context

Prior to exploring the question above, it is reasonable to enquire if secondary school learners are expected, encouraged, or trained to consider and critically reflect over their beliefs about the relevance and importance of learning mathematics for themselves. In this section, we explore this enquiry based on the socio-political discourse prevailing in the research literature and educational policy documents about the relevance and importance of learning mathematics.

Socio-political discourse on the relevance and importance of learning mathematics

On analysing learners' beliefs about the relevance of mathematics' curriculum for themselves, Sealey and Noyes (2010) noted that learners consider mathematics as "a power subject, giving access to higher paid careers and economic security" and, this "discourse (of the middle classes and their teachers) seems to go largely unchallenged" (p. 250). Grootenboer and Marshman (2016b) also pointed out that, "saying that mathematics is important is a generally unquestioned view" (p. 57). Studies such as, Ernest (2004), Pais (2013), Valero (2017) and Kollosche (2017) speculate the dominant socio-political discourse on the relevance and importance of learning mathematics to be a reason underlying these unquestioned beliefs.

The international educational policy documents issued by UNESCO (2015) and OECD (2019), and the research literature for instance, Heymann (2003) and Ernest (2005) also underpin this discourse by emphasising the usefulness of learning mathematics for learners' personal, economic, and societal development. The universal accept of this usefulness and importance of learning mathematics is further underlined in educational policies of many countries (e.g., the "Mathematics for all" policy (Allexsaht-Snider & Hart, 2001); Kunnskapsdepartementet (2019)).

The Norwegian secondary school curriculum for mathematics also reflects this socio-political discourse explicitly in its current (LK20) and former (LK06) versions. Statements like "Both boys and girls must have the opportunity to gain rich experiences from the subject of mathematics that create positive attitudes to […] the subject" (Kunnskapsdepartementet, 2006); and "Mathematics is an important subject […]" (Kunnskapsdepartementet, 2019) promote the relevance and importance of learning mathematics and aim to develop a positive attitude among learners towards learning mathematics.

However, Ernest (2004) points out that, 'There is no reason to assume that learners will regard mathematics curricula as "relevant" just because educational and political leaders do so […]' (p. 315). It is further claimed that learners' own views of mathematics and its relevance to their *personal* goals are missing from the discussion about relevancy of mathematics education, and their "beliefs about the relevance or utility of mathematics often reflects the prevailing rhetoric about the importance and high valuation of mathematics in society" (ibid., p. 316). Nonetheless, some recent studies have critically assessed this prevalent socio-political rhetoric and the roles mathematics education plays in society. These studies also outline

possible outcomes these rhetoric and social roles of mathematics education may have for learners' views about the relevance and importance of learning mathematics.

Questioning the unquestioned socio-political discourse

Lundin (2012), Pais (2013), Valero (2017) and Kollosche (2018) critically evaluate the prestigious status attributed to secondary school mathematics and problematize its socio-political roles. These studies highlight the roles played by mathematics in society such as, being a discipline training learners to be obedient, to follow rules and experience boredom in the classroom (Kollosche, 2018; Valero, 2017). Their research points at mathematics operating as a discipline to groom learners as future citizen-workers of a bureaucratic society rather than being a set of skills or knowledge relevant and important for all learners, who may have different personal goals. In addition to critically questioning this dominant socio-political discourse, a call to empower learners to negotiate the subject content they learn in mathematics and its relevance to them has also been voiced in the research.

Mellin-Olsen (1987, 1993) emphasised that the learners should *own*[2] the aims of their education but also noted that, "the pupil has usually been considered as one who reflects on the mathematical content of the situation, and not *about* the learning situation" (1987, p. 20). Critical Mathematics Education (CME) research further addresses this concern. Alrø and Skovsmose (2002) assert the need of dialogue with the learners to, "the extent that they are able to recognise the intentions and to identify with them, they can be joint owners of their education" (p. 43). Ernest (2001) also acknowledged that empowering learners as individuals and citizens in accordance with the aims of CME will, "require the use of a questioning and decision-making learning style in the classroom. Teaching approaches should include discussions, […], the *questioning of content* and the *negotiation of shared goals*". (p. 288, italics added). This appeal underpins the need of empowering learners (mathematically, socially, and epistemologically) in their mathematics learning processes by taking a joint ownership, questioning the relevance of the subject content, participating in decisions, negotiating shared goals, and critically evaluating their learning situation (see Ernest (2002) for details). These concerns correlate with CME's aims to empower the learners and impart democratic values and critical citizenship skills[3] in them through mathematics education.

[2] That learners get the opportunities to pose and solve problems relevant and interesting for themselves, participate in the decisions about the content of their own mathematics learning, and evaluate their learning situations.

[3] Promoting democratic values can empower learners to participate, negotiate, co-operate, and influence the decisions concerning their own mathematics learning. Promoting critical citizenship skills can empower learners to apply their own critical judgement to the claims or declarations of oneself, any

Empowering learners to influence their learning processes along with developing their critical thinking are also the aims of learners' educational process as per the Norwegian Education Act (NEA): "The pupils […] must learn to think critically […]. They must have joint responsibility and the right to participate" (The Education Act, 1998). An understanding of this statement can be that education should empower the learners to take a joint responsibility of, think critically about, and exercise their right to co-operate and contribute to decisions concerning their education. These objectives are also echoed in the general part of the core curriculum of Norway (applicable to all the subjects taught at primary or secondary level). Not only the Norwegian, but also the 'Nordic model' of education underlines the goals of learners' empowerment, democracy, critical citizenship through learners' education process (Andersson & Österling, 2019; Dahl & Stedøy, 2004). Thus, empowering learners to question the subject content's relevance for themselves, to negotiate shared goals, to participate in the decision-making, to reflect about and evaluate their learning situation in mathematics also become the aims of mathematics education. The statement, "Mathematics shall help pupils to […] become more aware of their own learning" listed in the Norwegian mathematics curriculum LK20 (Kunnskapsdepartementet, 2019, see section "Relevance and central values") also indicates similar concerns.

Based on the discussion above, it is reasonable to infer that secondary school learners are expected and encouraged to critically reflect over and question the relevance and importance of the subject content they learn in mathematics and get the opportunity to negotiate shared learning goals for themselves. Learners should also be aspired to participate, take ownership of, influence and co-operate in decisions concerning their mathematics learning process. Such practice and training are seldom observed as a tradition in school mathematics (Kloosterman, 2002) despite the expectations of research and curriculum guidelines to heed learners' voices. The learners are rarely asked to think critically *about* learning mathematics (Sachdeva & Eggen, 2021) or have a say in the socio-political discourse regarding their mathematics education (Sealey & Noyes, 2010). Little research has focused on how learners' beliefs about the relevance and importance of learning mathematics are formed (Grootenboer & Marshman, 2016b), or how can they contribute to their own mathematics learning process by critically evaluating this relevance and importance against their personal goals (Ernest, 2004). Therefore,

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other person, interest groups or authorities. In mathematics classroom context, learners' can be empowered by involving them to participate and critically evaluate the decisions concerning their own mathematics learning. Empowered learners can in-turn promote democracy and critical citizenship in the society.

in this study we enquire learners' beliefs about the relevance and importance of learning mathematics for them personally, and the sources of information influencing the formation of these beliefs. Further, we discuss implications of learners' expressed beliefs for empowering them to contribute to their own mathematics learning process.

Conceptual framework

Beliefs

There exists a plenty of literature illustrating the nature, structure, definition, qualities, content, influence and classifications of beliefs presented by different researchers (see Green (1971); Bar-Tal (1990) and Leder et al. (2002) for instance). The significance of learners', teachers', and educators' beliefs for the teaching and learning of mathematics is explored in research literature (e.g., Goldin et al. (2009) and Maaß and Schlöglmann (2009)) examining the affective dimension of teaching or learning mathematics. However, the research is not unanimous upon a universal definition of beliefs (ibid.). Furinghetti and Pehkonen (2002) present a thorough review of research concerning definitions and characterizations of beliefs, but a discussion of all these is beyond the scope of this paper. In the following text, we present selected definitions, and classifications which can assist the aim of exploring learners' beliefs about the relevance and importance of learning mathematics and the sources of information forming them.

Definition of beliefs

Bar-Tal (1990), defines beliefs as "[…] units of cognition. They constitute the totality of an individual's knowledge including what people consider as facts, opinions, or hypotheses, as well as faith. Accordingly, any content can be the subject of a belief." (p. 12). Goldin (2002) defines beliefs as, "multiply-encoded, internal cognitive/affective configurations, to which holder attributes truth value of some kind (e.g., empirical truth, validity, or applicability)" (p. 59). Despite the difference between these two definitions, the commonalities such as beliefs are 'highly subjective cognitive entities about some aspect of an individual's world which are considered to be true by the individual (the subject) holding those entities', constitute an understanding of beliefs in the research community. An instance of such a cognitive entity can be a learner's belief that, "learning is to reproduce the knowledge in an exam". In this example, the learner's subjective opinion (cognitive entity/belief) is about what learning (an aspect of learner's world) is for, and this belief is held true by the learner.

Beliefs are also clearly distinguished from other concepts discussed under the affective domain such as, attitudes, values, and emotions. McLeod (1989) suggested that "beliefs, attitudes and emotions differ in the ways that cognition is involved in the affective response" (p. 246). While "beliefs are mainly cognitive in nature" and are formed "slowly over a relatively long period of time", the emotions usually "have a much stronger affective component" (ibid.). Goldin (2002) defined attitudes as, "moderately stable predispositions toward ways of feeling in classes of situations", and values as, "deeply-held preferences, possibly characterized as "personal truths", stable" (p. 61). Therefore, the notions of beliefs, values, attitudes, and emotions can be listed in order of increasing affective involvement and intensity (emotions being most affective and intense, beliefs least) and decreasing cognitive involvement and stability (beliefs being most cognitive, cf. 'units of cognition', and stable, emotions least).

In this study, we follow the same distinction between beliefs, attitudes, emotions, and values as presented above. Adapting Bar-Tal's definition of beliefs, we understand learners' beliefs as their subjective opinions, hypotheses, as well as faith about the relevance and importance of learning mathematics for their lives.

Sources of information influencing learners' beliefs

Bar-Tal (1990), building on the work of Bem (1970) and Fishbein and Ajzen (1975), classified beliefs into three categories based on the sources of information influencing their formation. These three categories, ***descriptive***, ***inferential***, and ***informational*** beliefs, indicate one's beliefs formed on the information gathered through direct subjective experience, rules of logical inferences, and gained by other sources, respectively. Descriptive beliefs are derived from the perceptions acquired through one's senses and direct experiences. Inferential beliefs are formed based upon the rules of logic that allow inferences, made by thinking over already stored beliefs (personal and/or socialized) collected in the past. Lastly, informational beliefs are formed on the basis of information provided by outside sources such as other individuals, books, television, newspapers, etc. (Bar-Tal, 1990, p. 13).

Hernandez-Martinez and Vos (2018) acknowledge the formation of learners' relevance beliefs based on their personal motives or interest in mathematics, and/or based on a general relevance mediated to them by others[4]. Likewise, learners' importance beliefs regarding learning mathematics may be formed based on their subjective perception of values inherent to mathematics, and/or based on the importance mediated to them by others. These sources of

[4] Social and/or political sources of information such as, teachers, elders, researchers, governmental authorities and institutions, educational policies, mass media, etc.

information influencing learners' beliefs correlate with Bar-Tal's (1990) categorisation of beliefs (cf. subjective motives to descriptive beliefs and information mediated by others to informative beliefs). Therefore, we adopt Bar-Tal's (1990) classification of beliefs described above to categorise the sources of information influencing the formation of learners' beliefs (see Analyses and findings section).

Relevance

Wedege (2007) suggests that the question of relevance of mathematics education is always grounded in a context involving questions such as *what* (in mathematics) is relevant and *why*. These questions are further elaborated by Nyabanyaba (1999) and Jablonka (2007) as relevance *of what*, relevance *to whom*, relevant *according to whom*, relevance *for what end* or *purpose* with the reference to both individual and general (socio-political) objectives of learning mathematics as pointed out by Wedege (2007). The term *relevance* has also been discussed with reference to words such as utility (Ernest, 2004), current or future usefulness (Sealey & Noyes, 2010), need and demand (Wedege, 2007). Therefore, the concept of relevance is not precisely defined, but is related to the notions of *usefulness* and *meaningfulness* (Hernandez-Martinez & Vos, 2018; Priniski et al., 2018). Hernandez-Martinez and Vos (2018) suggest that "usefulness is a property of the topic being learnt, […] while relevance is a *connection* between the topic being learnt, its usefulness and a learner" (p. 246). Priniski et al. (2018) consider *relevance* to be "a personally meaningful connection to the individual" (p. 12).

Learning mathematics can be useful for a learner to achieve the aim of becoming a mathematician. For another learner, learning mathematics maybe meaningful because of the joy experienced in solving mathematical tasks. In this paper, the relevance of mathematics education is understood as its property of being useful or meaningful for the learners. Further, the connections between learning mathematics and its usefulness or meaningfulness established by learners are understood as their *relevance beliefs*.

Importance

The notions of *importance* and relevance are often treated as equivalents and used synonymously or together. This synonymous relation is visible in research and different official policy documents (for instance, see Allexsaht-Snider and Hart (2001), NCTM (2000), OECD (2019) or Kunnskapsdepartementet (2019)).

Research studies in mathematics education do not explicitly differentiate relevance from importance, but this distinction is made elsewise. Solomon and Heller (1982) assert that the difference between importance and relevance is critical, "for that which is relevant is not

necessarily important, and that which is important is not necessarily relevant"[5] (p. 165). The authors explain that the importance of anything is considered based on its *intrinsic* characteristics such as meaning, import, consequences, prominence, and value, whereas the relevance of anything is considered based on its *extrinsic* characteristics such as technique, application, usefulness, and service.

In mathematics education research, the notion of relevance is much discussed (see Relevance section), but the importance of learning mathematics seems to be treated implicitly under its relevance. Learners' beliefs particularly about the importance of learning mathematics (different from relevance) are not much explored. In this study, the concepts of relevance and importance are considered as being supplementary to, but different from each other. We understand the importance of mathematics education as the import and values that are intrinsic to learning mathematics (such as power of abstraction, imagination, estimation, simulation, etc.). However, learners' beliefs about the importance of learning mathematics might diverge from this understanding of import values. For instance, research studies such as Pais (2013) and Wiik and Vos (2019) have found that learners believe learning mathematics to be important for their future because of its exchange-value in the job-market. Here, we consider learners' beliefs about the importance of learning mathematics as their *importance beliefs*.

Method

This study is part of a larger research project called Local Culture for Understanding Mathematics and Science (LOCUMS, 2016-21), aimed at exploring the role of practical tasks (rooted in learners' own interests and local culture) in the learning of mathematics and science. The data was collected under the sub-project of LOCUMS carried out in middle Norway. Two schools including learners from diverse cultural backgrounds were chosen as the sites of data collection in accordance with the research aims of LOCUMS and four classroom interventions were planned. The interventions took place in 8th and 9th grades of these two schools with learners 13-14 years of age. Each intervention included three steps of data collection. In first step, 74 learners responded to a paper-pen questionnaire, second step included learners working to solve practical group tasks (4-5 learners in each group) and the final step included face-to-face individual semi-structured interviews with 19 selected learners. The questionnaire responses exhibited that many learners wrote no suggestions about what they wanted to learn

[5] As an example of this difference, learning mathematics may be relevant for a learner because of the joy experienced in solving mathematical problems, but he/she may not consider learning mathematics to be important because of the intrinsic values (e.g., power of abstraction) it promotes; or vice-versa.

in mathematics and science. This scarcity of suggestions was then enquired in-depth under the interviews along with learners' relevance and importance beliefs about learning mathematics.

After classroom interventions, a representative sample (learners having high, average, and low interest in learning mathematics) of four or five learners from each class (one from each group) were interviewed, making a total of nineteen interviews (11 girls, 8 boys). The interviews, each lasting for 45–90 minutes, were conducted in Norwegian, audio-recorded, and later transcribed for analyses. This paper is based upon the analyses of learners' responses to the interview questions exploring their beliefs about the relevance and importance of learning mathematics for themselves.

The interviewer was present with the interviewed learners under the interventions to assist, answer their questions and collect data while they were working on practical group tasks. Learners were therefore acquainted to and had an informal interaction with the interviewer before the interviews. Interview techniques such as waiting, conforming, non-academic language, comforting the learners in case they did not answer a question, etc. were used to reduce unfortunate authority of the interviewer, and to ensure learners' honest responses. The interviewer had no teaching duties or personal relation with the learners. Written information about the research project was provided to learners' parents/guardians and their consent to interview the learners was obtained. The interviews started by informing every learner about interviewer's duty of confidentiality and their anonymity. Learners were assured that the information provided by them will not influence their teachers, education, or grades in any way.

The choice of words in the interview questions was made to avoid a gap of understanding between the interviewer and the learners. Words such as *useful*, *need* and *use* were employed instead of the words *relevance* or *meaningful* to enquire learners' beliefs about the relevance of learning mathematics, and the words *important* and *importance* were used to enquire their beliefs about the importance of learning mathematics. Learners were asked *if* and *why* they believe learning mathematics to be important and relevant for themselves. The questions asked were for instance, what mathematical content they find useful to learn, where they use/will use mathematics, is mathematics important for them to learn and why do they believe so, etc. They were also asked to give reasons for their beliefs to identify the sources of information underlying and forming their beliefs. The interview excerpts presented in this paper are selected due to the representativeness and clarity in learners' responses. The unit of analysis is learners' responses to the interview questions.

Exploring learners' beliefs is seen as difficult since their beliefs are subjectively valid but not necessarily static or explicit for themselves. Lester (2002) observed this difficulty and

doubted if "[the data] accurately indicate what the students really believe. I do not think most students really think much about what they believe about mathematics and as a result are not very aware of their beliefs" (p. 353). Therefore, capturing learners' beliefs accurately is difficult, which results in the limitation that the findings of this study cannot be generalized and will only represent 'there and then' beliefs of selected Norwegian learners. We dealt with this limitation to some extent by asking the learners to not only express their beliefs, but also to mention the reasons for holding them. The credibility of the findings of this study is further established by discussing these findings with previous research results (see Discussion).

Analyses and findings

The analyses were initially driven by the data where learners' answers including words such as *useful*, *fun*, *need* and *use* were taken as indications of their relevance beliefs, and their use of words *important* and *importance* were taken as indications of their importance beliefs. Later, Bar-Tal's (1990) categories of beliefs were used to classify learners' beliefs as descriptive, inferential or informational based on the sources of information forming these beliefs. The abductive process of going back and forth between the data and conceptual framework including the notions of relevance, importance and categories of beliefs resulted in the following schematic overview table (Table 1) used to categorise learners' beliefs.

Beliefs' category ⟶	Descriptive	Inferential	Informational
Relevance and Importance beliefs about learning mathematics ↓			
Relevance beliefs *[Extrinsic]*			
Importance beliefs *[Intrinsic]*			

Table 1 Relevance and importance beliefs table (before data analysis)

After the data analyses process, the table above was filled with learners' beliefs about the relevance and importance of learning mathematics expressed during the interviews (see Table 1 in attachment). In the next two sections, we present and analyse learners' interview responses on relevance (useful/meaningful) and importance of learning mathematics for themselves.

The *relevant* mathematics

Fifteen out of nineteen learners mentioned elementary arithmetic as most relevant mathematical subject matter because of its usefulness in their everyday life, for instance in

shopping and budget estimation. Whereas four learners gave examples of secondary-level mathematical subject content they use in their lives such as using maps and compass for spatial orientation, calculating mass, density,[6] and knowledge of geometrical construction in case of becoming a carpenter. Some replies also included references to assessment in mathematics, as shown in the following excerpt:

I[7]: What is useful for you to learn in mathematics?

L_{15}[8]: I find most of it quite useful … eh … well, useful … almost everything, especially just plus and minus and division and … multiplication … these are things I find very useful …

I: But what about other stuff you learn now, like algebra and x, y and z or …

L_{15}: Ok … I do not know how useful it will be to me, but I think it is fun … [laughing]

I: [Also laughing] Yes, you find it fun, but …

L_{15}: I do not know how useful it will be in the future … I have not experienced a situation where I would need it except for the tests … so …

The learner first mentions *most of* the mathematical subject content as quite useful. Saying the word useful once again, "eh … well, useful …", s/he goes on to include *almost everything*. However, to give specific examples s/he mentions basic arithmetic skills *especially* to be *very useful*.

When the learner is specifically asked about the usefulness of learning algebra s/he replies that though s/he finds learning algebra to be *fun*, s/he *does not know* how useful it will be in the future and specifies that s/he has *not experienced a situation* where s/he would need algebra *except for the tests*. The learner's reply indicates that though s/he does not know the relevance of learning algebra for future life, yet assessment and evaluation criteria in mathematics (an institutional requirement) serve as reasons for believing that it is relevant.

This belief in the usefulness of basic arithmetic seems to be based on direct experiences of needing these skills to do everyday calculations and is categorised as a *descriptive* belief. Whereas the reply regarding algebra indicates that learning algebra is experienced to be meaningful (*fun*), and the belief about its usefulness is grounded in the inference that s/he will need it for passing the tests. Therefore, it is categorised as an *inferential* belief (see Table 1 in attachment).

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[6] This learner (L_8) mentioned that one can for instance make an electric bicycle out of things one has in the garage if one can calculate mass, density etc.

[7] *I* stands for the *interviewer*.

[8] L_n specifies the *learner's* reply in n^{th} interview.

Using the phrase, "I do not know …" twice in the answer indicates that the learner may not have thought much about the relevance of learning mathematics for her/himself. Some other learners also expressed the same:

I: What do you yourself believe to be useful to learn in mathematics?

L_{12}: To calculate …

I: OK … and?

L_{12}: Ah … [long pause] … I do not know …

I: Have you ever thought about it?

L_{12}: No … I just think that in our lessons … that, OK, now we are going to calculate and solve problems in our book … I do not think more beyond that …

The learner mentions that learning *to calculate* in mathematics is useful. On being asked to mention more examples, s/he did not come up with other useful subject matter to learn in mathematics (saying "I do not know" after a long pause). When asked if s/he has *ever thought about* what is useful to learn in mathematics, the learner answered that s/he has *not* done so. S/he added that in mathematics lesson s/he thinks that they will *calculate and solve problems in their book*, and s/he does *not think more beyond that*. This remark indicates that learners are not used to consciously reflect over the relevance of learning mathematics for their lives and such reflections may not be a part of their "routine" (*calculate and solve problems in our book*) mathematics classroom expectations.

While some learners seem not being used to thinking over the relevance of learning mathematics for themselves, other learners said that they have heard from *others* that learning mathematics is relevant for them. The following extract exemplifies this:

I: Why are you going to need it [algebra] …?

L_5: I have just heard that one needs it …

I: Heard? Where?

L_5: From teachers and other adults …

I: But have you ever asked them where they use it?

L_5: No …

[…]

I: Has he [the teacher] said anything about where you will use an equation?

L_5: No …

I: Ok … well … but you still think that though you do not know where one needs or can use it … yet you think that it is useful to learn?

L_5: Yes … because we learn it …

On being asked *why* s/he is going to need algebra and equations; the reply was that s/he has *just heard that one needs it* from her/his *teachers and other adults*. This reply indicates that the learner accepts the stated requirement of learning mathematics. The source of learner's belief in the relevance of learning mathematics is the information received by others and therefore this belief is categorised as an *informational* belief. When asked if s/he has enquired the teacher and other adults about where they use algebra or where s/he can use equations, the reply was that s/he has not questioned this claim. The reasons of not questioning such claims are not clear but the last line in the interview segment, "yes … because we learn it" indicates that this learner trusts his/her school system and that the curricula chosen for her/him must be relevant. This trust is also reflected in the following example:

I: But can you use equations and algebra somewhere [later in your life] …?

L_{11}: Eh … yeah, not that I know where one will use it but I know that one can use most of what we learn in mathematics somewhere but there are more important things that we should have really learnt first such as … how one pays a bill and how one … eh … everything like that … eh … I have not learnt much of that … though we will definitely learn such stuff later …

This learner did *not know where one will use* the algebra and equations they learn, but s/he knows that *one can use most of what they learn in mathematics somewhere*. However, s/he also reflects critically and admits that there are *more important things* that they *should have really learnt first such as…how one pays a bill* … and which they *have not learnt much of* (e.g., using mathematical knowledge in their adult life), yet simultaneously expresses her/his trust in the education system that they will ***definitely*** learn that stuff later. This trust is more visible in leaners' replies about the importance of learning mathematics, presented in the following section.

The *important* mathematics

Despite struggling (responses like, *I do not know*, long pauses, *I have just heard that one needs it*, etc.) to find the usefulness of learning secondary-level mathematics in their lives, *all* the learners answered that learning mathematics is important for their lives. This importance of learning mathematics surfaced in several interviews (in form of various reasons) and inspired us to explore *why* the learners considered learning 8th and 9th grade mathematics to be important. The following interview segments present some responses:

I: As you said that it is difficult to learn but do you think it will be important for your life later?

L_1: Much of it is important … yes … like … [long pause]

[…]

I: So, why do you think that really? What can you use it for later in your life then?

L_1: It is about what you will work with … like … yeah so, if one will work in a shop or be a hairdresser and for example if one has to stand at the cash counter, so, it's a must that one has enough knowledge of mathematics, one should be able to do that … and much like that for many different jobs one must have a knowledge of mathematics …

[…]

I: But can't one find any jobs where one does not need to know much about mathematics and science?

L_1: There are no such jobs…or I do not think one can find any such jobs…

L_1 expressed earlier in the interview that s/he finds learning mathematics and science to be difficult since these subjects require thinking hard (much concentration), yet on being asked about the importance of learning mathematics s/he answered, "much of it is important". When s/he tried to come up with instances of this important subject content s/he did not mention any (tries to recall while saying "yes … like …" but takes a long pause). On being asked about the reason of thinking mathematics to be important, s/he mentioned future job opportunities (though being a hairdresser or handling a cash counter rarely require more than basic arithmetic skills). The learner even inferred that there do not exist any jobs not requiring the knowledge of school mathematics and science.

Another reason for considering mathematics learning as important for future, included helping one's own children in their mathematics homework when it is their turn:

I: Do you think that what you learn in mathematics and science will be important for you, later in your life?

L_2: Yeah … they say that at least …

I: Who?

L_2: The teachers … they say that at least …

I: Yeah … but do you think so yourself? You should answer for yourself now …

L_2: Yeah … I maybe think that too … like if you will have children then you can help them with their homework …

I: Yeah … but like in everyday life like … in real situations … where are you going to see mathematics and science actually?

L_2: Eh … it is difficult! [pause]

I: Where are you going to use it in your life?

L_2: Where I would use … no … I do not know …

[…]

I: You do not know …

L_2: No … I have not thought much about it …

Being asked about the importance of learning mathematics, L_2 instantly replied, "yeah…they say that at least", and when asked to specify who, s/he quickly says that it is "the teachers". The influence of what teachers (classroom's authority) inform about the importance of learning mathematics becomes clear from this instance. On insisting to answer the question according to her/his personal thoughts and experiences, L_2 replied, "I maybe think that too", and supports her/his answer by imagining a future situation where s/he can use her/his knowledge in mathematics to help her/his children in their homework. L_2 found it *difficult* to find instances where s/he would use advanced mathematics in her/his daily life. The statement, "No … I have not thought much about it", indicates that the learner does not often reflect over the reasons of believing in the importance of learning mathematics. L_2's replies for believing in the *importance* of learning advanced mathematics indicate that the socio-political importance of learning school mathematics gets reinforced among learners since they 'hear' about it from their teachers and do not reflect over or critically question such claims.

In several interviews, learners stated that they do not think over the importance of learning mathematics since they are informed about its importance by their teachers, parents, popular media etc. Interview segment with L_5 illustrates this case even more:

I: And important?

L_5: It is not the most important thing but … like … eh … I just mean that people should know about it and be able to do it, but I do not know why … we learn it at school so one thinks that one will use it …

This learner explicitly expresses her/his trust in the schooling system that though learning mathematics may not be *the most important thing*, yet *people should know* the mathematics taught in the school (though admitting, *but I do not know why* ...) since they *learn it at school* (a socio-political institution). However, on the other hand, the same trust may make the learner disclaim her/his right (as per NEA) to critically question the importance and relevance of what they are taught.

Learner's replies in the interview segments presented above indicate their *inferential* (e.g., inferring that they will need mathematics to teach their own children, or in their future jobs, etc.) and *informational* (e.g., the teacher/other has told them about the importance of

learning mathematics) importance beliefs about learning mathematics. However, none of the learners' responses included their *descriptive* (directly experienced) beliefs about the importance of learning mathematics, based on the intrinsic values (e.g., the power of abstraction, estimation, etc.) of learning mathematics (see Table 1 in attachment).

Discussion

The interview responses of learners indicate that a distinction between the notions of relevance and importance, suggested by Solomon and Heller (1982), can also be made in the context of learning and teaching mathematics. This distinction may not be clear for the learners, but it can be noticed in learners' replies expressing their relevance beliefs and importance beliefs about learning mathematics. Their responses to the questions regarding *relevance* of learning mathematics are based on the extrinsic application or usefulness (*use-value*) of mathematics. However, their answers about the *importance* of learning mathematics point towards the inferred or informed prominence or consequences (*exchange-value*, in terms of job, degree, own children's education, etc.) of learning mathematics, rather than the intrinsic meaning or import of mathematical knowledge itself. The choice of words of the interview questions can be a partial reason, but this dissimilarity was consistently noticed in responses of many learners.

The relevance of learning mathematics surfaced in learners' responses both as its usefulness for calculations (L_{12} and L_{15}) and meaningfulness for enjoyment (*fun*, L_{15}). Learners' responses as presented in 'The *relevant* mathematics' section, exhibit their strong belief in the relevance of learning 8th and 9th grade school mathematics despite referring mostly to elementary arithmetic skills as the subject content they use in their daily lives. A majority of secondary school learners interviewed by Kollosche (2017) also associated the relevance of learning mathematics to the mastery of basic calculation skills. L_{15}'s response about needing mathematics in the tests is coherent with the findings of Onion (2004) and Alrø et al. (2009) where learners reported that the mathematics they learn is useful only in their mathematics lessons, to do homework, and for exams. Learners' relevance beliefs about learning mathematics further seem to be formed on the bases of all the three sources of information suggested by Bar-Tal (1990), that is, their descriptive, inferential and informational beliefs.

The importance of learning mathematics becomes apparent in learners' reference to future situations in which they expect to require mathematics, such as to get a job (L_1) or to teach mathematics to their own children (L_2), which can be seen as a circular argument for

learning mathematics. This finding correlates with Wiik and Vos (2019), reporting that learners choose to learn mathematics in anticipation of getting high-paid jobs in future. The analyses of interview extracts presented in 'The *important* mathematics' section reveal learners' strong belief in the importance of learning mathematics for their lives despite not being able to justify this belief. Such importance beliefs seem to support the suspicion of Kollosche (2018) and Valero (2017) that mathematics as a school discipline is required to train learners to become future citizen-workers instead of critical citizens. Further, learners' importance beliefs about learning mathematics seem to be formed on the bases of inferences they make or the information they receive from others, instead of their own direct experiences.

The number of learners replying, "I do not know" and "I have not thought about it" (15 of 19 learners), and the frequency of such replies indicate that reflecting consciously and critically over the relevance or the importance of learning mathematics for their lives is not usual for them, as Kloosterman (2002) also pointed out. Such reflections do not seem to be a part of their mathematics classroom routine or expectations (L_{12}) either. They express their trust in the educational system, and state that the content chosen by the authorities (teachers, other adults, school) and policy makers for them to learn should be relevant and important for them. Teachers' and other adults' statements also seem to influence the formation of their beliefs about the relevance and importance of learning secondary-level mathematics. The socio-political status and value attributed to mathematical knowledge for succeeding in life plays a significant role in forming learners' beliefs. Similar to the findings of Sealey and Noyes (2010) and Grootenboer and Marshman (2016b), none of the 19 interviewed learners questioned or critically evaluated this dominant socio-political rhetoric of high valuation or requirement of learning mathematics in the society.

This unquestioned acceptance contrasts with NEA's and CME's intention to empower learners to think critically, co-operate in decisions concerning their own (mathematics) educational processes and develop critical citizenship skills through learning mathematics (Ernest, 2001; Mellin-Olsen, 1987). Consequently, a *tension* emerges between the intentions of NEA and CME, and the expressed realization of these aims in learners' interview responses, bearing consequences for empowering learners to contribute to their own mathematics learning process by thinking critically.

Learners (like L_{11}) exhibit the potential of critical consideration and suggesting practical mathematical content (paying bills) that could be taught before algebra or equations, still such suggestions may not be further conveyed to their teachers or higher authorities under the presumption that such content will be chosen and taught later. L_{11}'s answer indicates that given

the opportunity to participate and cooperate in the decision-making process, learners can act as discussion partners suggesting alternative mathematical subject matter personally relevant and important for them. However, in these interviews, only a few learners make such suggestions. It is also unknown if their proposals get forwarded or have an influence on their mathematics learning content or activities. Consequently, their potential of participating in and critically questioning the dominant socio-political rhetoric of relevance and importance of learning mathematics may remain hidden, and their empowerment and critical citizenship skills unpolished.

The data analysed in this paper does not uncover the cause of this tension, but a possible reason may be a gap in the understanding of becoming an *empowered* learner through learning mathematics. By this gap, we mean closely linking the notion of learners' empowerment to becoming what Ernest (2002) terms as a *mathematically*[9] empowered learner, but *not* a *socially* and *epistemologically* empowered learner[10]. The formulations using the terms 'critical', 'citizenship' and 'democracy' in LK06 can be seen as exemplifying this gap of understanding[11]. We argue that adopting a broader understanding of becoming an empowered learner through learning mathematics suggested by Ernest (2002) may contribute to realize NEA's and CME's aim of learners' empowerment and critical citizenship, as envisioned in the 'Nordic model' of (mathematics) education (Dahl & Stedøy, 2004).

Implications

The distinction in learners' replies concerning the relevance and importance of learning mathematics implies that the concepts of *relevance* and *importance* deserve to be treated distinctively, rather than synonymously, in mathematics education research. This distinction can contribute to differentiate and communicate the relevance (extrinsic values) and importance (intrinsic values) of teaching and learning mathematics in general. Specifically, this difference can contribute to understand nuances underlying learners' (also educators') beliefs about the relevance and importance of teaching and learning mathematics. These nuances can assist in:

[9] Ernest (2002) suggests that a mathematically empowered learner can use his/her mathematical knowledge to pose, solve, evaluate, and discuss mathematical problems and models critically.
[10] A socially and epistemologically empowered learner, as per Ernest (2002) can critically analyse and challenge the underlying socio-political assumptions, authority, power structures, roles, uses, abuses, purpose, etc. associated with learning mathematics.
[11] In the current and revised version of the Norwegian mathematics curriculum, LK20, the scope of reference to the terms critical, democracy and citizenship is expanded. However, democracy and citizenship are still closely connected to data quantification and its critical evaluation rather than empowering learners to evaluating one's own learning situation while learning mathematics critically.

(a.) exploring the motivation and reasons of learners' (also educators') involvement in the learning and teaching of mathematics more precisely; and (b.) posing and answering further research questions (e.g., how can the learners directly experience the intrinsic values of learning mathematics, how does an understanding of intrinsic values of learning mathematics affect learners' mathematics learning process, etc.).

The 8th and 9th grade learners interviewed in this study will be making crucial decisions regarding their own future mathematics education and career during their tenth grade. However, the analyses indicate that these learners are not used to making reflected choices and critically evaluating the decisions about their own mathematics learning process. Involving learners in decisions regarding their own mathematics learning process can be a step towards mathematics education for learners' empowerment and critical citizenship. Therefore, mathematics classroom practices can reflect the aim of promoting learners' empowerment, for instance by incorporating a questioning learning style, negotiating shared goals, critically evaluating the subject content in mathematics, etc. to empower the learners to take informed decisions and apply critical judgements to their own mathematics education.

The findings of this study also bear implications for the formulations employed in the mathematics curriculum of Norway. The references made to the notions of critical thinking, democracy and citizenship can invite and encourage mathematics educators to incorporate not only mathematical, but also the social and epistemological empowerment in mathematics teaching and learning practices.

Conclusion

This study contributes to the research field by establishing that there exist nuanced differences in learners' beliefs concerning the relevance and importance of learning mathematics, and this distinction should be acknowledged in mathematics education research. Mapping of sources influencing learners' belief formation as descriptive, inferential, and informational, contribute to an increased understanding about the sources of information that influence the formation of learner's beliefs about the relevance and importance of learning mathematics for their own lives.

We also maintain that though the interviewed learners are not used to reflect over the relevance and importance of learning mathematics, yet some of them exhibit a potential to suggest constructive alternatives to improve their own mathematics learning process. Giving learners time and inviting them to critically evaluate and co-operate in decisions concerning

their own mathematics learning activities can encourage them to voice their suggestions. Making these changes in mathematics learning and teaching can assist in realizing NEA's and CME's aim of mathematics education for learners' empowerment and critical citizenship.

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7.3 Paper III: Students' experiences of learner autonomy in mathematics classes

Students' experiences of learner autonomy in mathematics classes

Shipra Sachdeva

Students' experiences of learner autonomy in mathematics classes

Shipra Sachdeva

Norwegian University of Science and Technology (NTNU), Norway; shipra.sachdeva@ntnu.no

This paper presents representative excerpts from post-project interviews conducted with 8th and 9th grade students from a Norwegian countryside school. Interviews attempt to explore students' experiences of learner autonomy in mathematics classes. These students participated in a larger study where they responded to a pre-project questionnaire followed by solving practical group projects using mathematics and science knowledge and post-project interviews. Preliminary interview analysis reveals that students' experiences with learner autonomy in mathematics classes are limited to opportunities provided by their teachers together with an insecurity in perceiving themselves as responsible autonomous learners. However, these learners clearly exhibit a desire to acquire autonomy, the potential to suggest changes, and participate in discussions and decisions concerning their mathematics teaching-learning process, together with their teachers.

Keywords: Student experience, learner autonomy, classroom environment and mathematics instruction.

Introduction

Developing autonomy in learners has been emphasized as a goal of mathematics education (Ben-Zvi & Sfard, 2007; Yackel & Cobb, 1996). Learner-centered teaching strategies such as mathematics teaching based on real-life contexts, inquiry-based and problem-centered learning (Wheatley, 1992; Yackel & Cobb, 1996) have been discussed to increase learners' autonomy in learning mathematics. However, discussions related to autonomy in mathematics education literature have mostly depicted it as an *intellectual* attribute to be acquired while working with mathematical problems in order to identify, enquire and develop alternative solutions, and/or as the ability to scaffold mathematical algorithms and procedures in a better way (McConney & Perry, 2011; Mueller, Yankelewitz, & Maher, 2014; Wood, 2016; Yackel & Cobb, 1996). Specifically, intellectual autonomy has been defined as, "students' awareness of and willingness to draw on their own intellectual capabilities when making mathematical decisions and judgements" (Cobb & Yackel, 1998, p. 170). However, less researched aspects of autonomy in mathematics classes are the ones in which learners themselves "develop a particular kind of psychological relation to the process and content of [their mathematics] learning" (Little, 1991, p. 4). Developing such a *psychological* relationship with their mathematics learning can help students better understand, intervene and improve the ways *they* learn mathematics.

The first section of chapter one of the Norwegian Education Act (NEA), states that "[through education] students and apprentices must learn to think critically, and act in an ethically and environmentally conscious way. They *must have co-responsibility* and *the right to influence* [their education]" (Opplæringsloven, 1998/2018, italics added). For pupils to have the responsibility and right to influence their mathematics education, they need to take charge of their own learning in mathematics, that is become autonomous, at least partially. Further, if learners are supposed to apply their mathematical knowledge toolkit effectively in order to solve real-life problems after finishing formal education, they should be autonomous learners. Not only to solve their own real-

life problems using mathematics, but also to participate as critical, responsible and active future citizens of society, learners need to have experience and training in understanding and thinking over their current situations along with taking right decisions and actions to reach desired outcomes. This would require learners to acquire and practice both, the *intellectual* (i.e., the capability to take reasoned decisions), and the *psychological* aspects (i.e., a perception and an experience in exercising) of their autonomy. Although each learner is autonomous and possesses some decision-making skills, these should be nourished by getting an experience of exercising them within their immediate community and own peer-group (in the mathematics class). By learning how to balance power, authority, freedom and co-responsibility among themselves, students would effectively use their autonomy to take charge of their own lives, decisions and their consequences, and benefit society. Consequently, researching psychological aspect of *learner autonomy* in mathematics classes, in addition to its intellectual aspect, is equally significant if learners are to be motivated to succeed in mathematics (George, 2012).

By *learner autonomy* in this paper, I mean learners' ability to develop a psychological relation with, and freedom to take *responsibility* of their own mathematics learning process. This *responsibility* includes learners taking initiatives, participating in discussions, planning and executing self-beneficial mathematics learning activities, making decisions about what one wants to learn, how one likes to learn, at what pace and why together with their teachers, and simultaneously reflecting on these choices. Learner autonomy is a widely researched concept and is considered to be an essential attribute for learners of any second/foreign language because of its positive correlation with language learning (Little, 2003). However, the positive influence of acquiring leaner autonomy may not be limited only to second/foreign language learning. Therefore, in this paper, I explore the research question "What can young learners' descriptions communicate about their experiences of learner autonomy in their mathematics classes?" in order to explore young learners' experiences with learner autonomy in mathematics classes as per the aim of NEA.

Background of the study

This study is a part of a bigger research project called Local Culture for Understanding Mathematics and Science (LOCUMS, 2016). This project explores if using practical activities rooted in *learners' own culture* can benefit them in learning mathematics and science in lower secondary classes. Building on experiences from a former research project where solving practical tasks rooted in students' local culture in discipline design and technology promoted their school engagement (Lysne & Hoveid, 2013), LOCUMS intends to research if similar approach can increase students' interests in learning mathematics and science. While reviewing the literature, I observed that learner-centered intervention studies carried out in mathematics classrooms are often planned either by the researchers or the teachers, and learners' choices remain un(der)stated. For designing practical activities rooted in students' local culture for learning mathematics and science, it was important to get informed about learners' interests. Therefore, I designed a pre-intervention questionnaire consisting open and closed ended questions about themes such as: learner's general views about education, perceptions about mathematics and science education, their activities of interest, what they desire to learn about at the school, their thoughts on culture etc. The purpose of these questionnaires was two-fold – to design a learner-centered teaching approach voicing learners' opinions, choices and interests which acted as an input for designing practical activities;

and to explore learners' potential of taking reflected decisions and responsibility of learning mathematics. It was while analyzing students' questionnaire responses such as "I don't know what I want to learn in mathematics", "I do not know what [content] is useful to learn in mathematics", "I am not interested in learning mathematics, yet learning mathematics is important", to open-ended questions concerning mathematics that the notion of *learner autonomy* in mathematics classes emerged as my research interest. Therefore, learners' autonomy was investigated further through semi-structured interviews. Data gathered for LOCUMS was directed towards both mathematics and science, and consisted of a cycle of pre-project student questionnaires, practical group tasks, followed by individual semi-structured interviews with selected students. Group tasks required knowledge of mathematics and science to be solved. Semi-structured interviews were focused to probe learners' experiences of autonomy in mathematics classrooms among broader themes such as getting a vision of learners' experience on project day, their general outlook towards learning mathematics and science, the extent of activities used in the mathematics and science teaching and its relevance etc. Learners' responses about experiences of learner autonomy in mathematics classes collected in the first ten semi-structured interviews are presented in this paper.

Theoretical framework

Learner autonomy was first defined as "the ability to take charge of one's own learning" (Holec, 1981). The rationale of fostering autonomy in learners "… is quite simply that a teacher may not always be available to assist. Learners need to be able to learn on their own because they do not always have access to the kind or amount of individual instruction they need …" (Cotterall, 1995, p. 220). The first definition of learner autonomy was later elaborated by Little (1991) as, "a capacity – for detachment, critical reflection, decision-making, and independent action" (p. 4). Further, Little (2003) mentions that, "… *autonomous learners* understand the purpose of their learning program …, take initiatives in planning and executing learning activities, and regularly review their learning and evaluate its effectiveness" (p. 4, italics added). Therefore, learner autonomy entails more than intellectual autonomy, meaning the learners can "act freely with a sense of volition and choice" (Deci & Flaste, 1996, p. 89) and involves an activating psychological process in order to attain autonomy in relation to one's learning processes. Little (1991) moreover reminds of the difference between an autonomous learner and an independent learner. Being an autonomous learner does not mean that one is an independent learner and is able to learn without any assistance of the teacher, but it means taking a partial control of one's own learning process. By having a partial control, the learner should her/himself understand and reflect upon one's learning curve in order to figure out what strategies work best for her/himself. Resultantly, a learner can comprehend how one learns better and design, plan, execute and analyze self-beneficial mathematics learning strategies.

Students' intellectual autonomy has been discussed, but, I found fewer studies devoted to promoting learners' autonomy among students in mathematics education research literature. Research initiatives in directions such as *critical mathematics education* (Skovsmose, 2014), *culturally-responsive mathematics education* (Greer, Mukhopadhyay, Nelson-Barber, & Powell, 2009), *mathematics education for social justice* (Gutstein, 2003), and *sociocultural and sociopolitical awareness* (Sriraman & Knott, 2009) have illuminated social implications of learning mathematics for the learners and our future society. These research areas address concerns to acknowledge learners' interests, promoting pupils' critical awareness towards sociocultural and sociopolitical

issues involving mathematical calculations, and engage students in struggle for social justice through mathematics education. For learners to become mathematically literate critical citizens of society, they should be able to understand their responsibility of learning mathematics, comprehend the role mathematics plays in their lives and society, and make decisions involving mathematical calculation. These capabilities require students to understand and take charge of their own learning in mathematics. Skovsmose (2014) mentions that, "It is a preoccupation of critical mathematics education … to develop a mathematics education that might provide new possibilities for the students" (p. 117). This study tries to enhance research concerning learner autonomy in mathematics education by providing the learners with an opportunity to express their experiences regarding learner autonomy in mathematics classes. The focus is to explore if learners are aware of their responsibility and can suggest changes to improve the quality of mathematics classes, provided they can assume more control of their mathematics teaching-learning process, as NEA expects from them.

Method and study participants

Qualitative research design using semi-structured interviews with students was adopted as a method to learn about students' experiences. Since I wanted to know individual opinions, perceptions and thoughts of the learners participating in mathematics classes, interviewing seemed as an optimal way to proceed. Semi-structured interviews provided me with the opportunity to engage the learners in a free conversation with occasional follow-ups, without the restrictions of time limitations and a strict structure. In this way, participants could also, to some extent, control the direction of the interview, so that I avoided being the steering authority in the interview, and could gather authentic and trustworthy information.

Learners of age 13 to 14 years, studying in 8th and 9th grade in a countryside school located in central Norway were informants of this study. One learner per group was chosen from 4 or 5 groups in each class, based on the level of their activity on the project day. Keeping in mind the principal of representativeness, an attempt was made to select students with different interests, level of activity (high, medium and low), achievement in mathematics (high, moderate and low achievers) etc. so that various experiences could be gathered. Here, I present representative excerpts from interviews with 5 girls (one of them with Sámi background) and 5 boys (one of them with Eritrean background). Most of these students' parents were working or driving farms, holding an average socio-economic status.

Results and discussion

The interviews were conducted in Norwegian, translated to English and interview transcripts were analyzed in order to identify learners' experiences regarding learner autonomy in mathematics classes. Interview questions were designed so that learners had to reflect on their mathematics teaching, make choices, take decisions and suggest changes in it; and were analyzed to find learner responses involving words such as *control, decision, responsibility* etc. concerning their mathematics education. This section presents selected interview excerpts followed by the *descriptive analysis* of learners' responses from the first ten informants. In the following transcripts, *I* indicates the interviewer and L_{number} indicates which of the ten learners is responding. Interviews were conducted after practical group activities and hence in the first extract presented below, the

interviewer is asking how in the learners' experience the learning situation on the day of project was different from the learning situation in their usual mathematics classroom. The following snippet illustrates the learners' experience:

I: eh… do you think this way to repeat mathematics and science content was different from the usual teaching?

L_3: yes…

I: how would you describe that why was it different from usual teaching? … … how would you describe that the situation was different then? what was the difference?

L_3: … we did not sit in the classroom and raised our hands and talked like… we do not calculate like we discuss so much and find it together…

I: but is it then different form usual teaching or would you say that it's also you who controls there as well?

L_3: no… there it's the teacher who has more control

I: but do you get it [the responsibility] usually like in mathematics and science classes?

L_6: or… like we don't get to decide everything on our own because then it's like they have already decided what we should do from beforehand but also…

Both of the learners' responded that the learning situation they experienced on the day of project was different in terms of having control and responsibility. These experiences of learners about usual mathematics classroom exhibit limited experience with self-control and self-decision where the teacher is usually the one who manages the class and everything that is to happen in the classroom is decided beforehand. These classroom experiences, where learners are not exposed to the responsibility of their own learning, may inhibit learners' potential to understand, reflect, analyze and make decisions to improve their own learning processes for their self-benefit. Therefore, only when provoked a bit to assume autonomy, most of the learners suggested probable improvements in mathematics classroom:

I: are you satisfied with the way teachers teach you?

L_4: yes…

I: why do you think that this way to be taught is alright?

L_4: because… eh… I don't know maybe I'm used to having it like that like this so…I have done it all these years so, I think it's a fine way to learn…

I: but if you had a chance… would you change the teaching in mathematics …?

L_4: no…

I: nothing?

L_4: maybe a bit more activities in mathematics but otherwise I don't think so of anything…

I: would you have included any other activities like some practical activities in mathematics teaching?

L_8: eh… I would have done it because what we've done in the whole 8th grade is just to write, write and write and solve the task and then it becomes quite boring and you lose the interest and you sit there just to write and when the class is over so you think "oh yes, finally finished…".

In the first snippet, learner L_4 expresses being *used to have been taught like this* as the reason of being satisfied with the teaching. Consequently, thinking about some other way of being taught or suggesting a change to the *usual* way sounds like a difficult task to him/her. However, when asked again in a way where she/he could choose, she/he suggested an improvement in the mode of mathematics instruction. Similarly, the frustration of mathematics classes lacking practical activities and a suggestion to include the same if given the opportunity is evident in L_8's response. The next excerpt shows that given the possibility to suggest changes and design their mathematics teaching themselves, learners can acquire autonomy in mathematics learning.

I: hmmm… now when you have a suggestion about that you could have learnt to set up a budget… have you also thought of how would you have liked to learn it? In what way?

L_5: eh… if we could have got a realistic situation… and then we could have got a task about it so it would have… for example set up a budget for a whole family for a month and you get different expenses and the teacher and you get the income and you have to pay the tax and you have to pay different and you should have a bit sum as saving if you sometimes get into a trouble and such things… like which are important to learn like you don't need to take a loan and you don't have a lot of debt because you just got into a trouble which you never expected in your budget… so we learn how one should use his money because I mean that mathematics… we use a lot of mathematics in society like money and much is controlled by money and money gives power so… because people should not use up… because of people should have better … … like I don't have any concept of… like I don't know what 1 million kroner is… it's a lot and lot but you can't manage to buy a house for one million kroners… so we should learn more about the value like how much 1000 kroner is worth … and such…

The learner (with Sámi background) not only mentions what he/she desires to learn but also suggests how the lesson could be planned and what kind of tasks they could have worked on. This segment shows that learners have a potential of developing autonomy and co-responsibility in mathematics learning. Further, the extract below presents learners' response to the question of sharing responsibility of their learning with the teacher:

I: what is the difference between if it's only me [who, as a teacher] decided everything and if you are also with me when I decide?

L_9: it becomes more fun if both decide… then… I think it could have happened that people would have liked to come to school more often...

I: when they are asked…?

L_9: yes… when they get to decide a bit what they do at the school…

As indicated in the response of L_9 (Eritrean immigrant, in Norway since three years), the thought of gaining a little bit charge over one's own learning can not only encourage their autonomy but also that they would have liked to come to school more often, exhibiting the importance of listening to student voices.

Conclusion

Learners' responses concerning autonomy in their mathematics classroom exhibit limited experience with self-control and self-decision. This indicates that decisions regarding what will happen in mathematics classroom are usually made in advance by school authorities and the teachers, which can act as a constraint for learners' autonomy to emerge and being practiced. Consequently, learners *get used to* mathematics classes, trust decisions of their school and teachers, and consequently see the *traditional way* to be the only way of mathematics education. This can leave learners unfamiliar with assuming autonomy to comprehend their learning processes in depth and limit their creative potential to experiment with better and self-beneficial ways of learning mathematics. Moreover, if concerns for social justice, adopting a critical stance towards mathematics' role in society and learners' personal lives, and raising consciousness towards sociopolitical and socio-economic issues through mathematics education are to be fulfilled; learners need to experience autonomy in their mathematics education. This way they can feel confident to put forward their argument, discuss and design better learning opportunities in mathematics with their teachers and, moreover, get engaged in, discuss and debate about social issues and initiate social changes in wider society.

Moreover, enthusiasm of learners to experience a partnership and co-responsibility of their own mathematics learning is the same regardless of their ethnic background. Interview excerpts from learners L_3 and L_9 depict similar experiences in mathematics classrooms and they desire similar changes (i.e., *a culture of promoting learner autonomy*) in mathematics learning as their Norwegian classmates. Similar responses from learners having diverse backgrounds complemented my focus on *youth culture* instead of ethnic backgrounds in this study.

Observing learners' potential to suggest, design, co-operate and improve teaching-learning strategies in mathematics, I conclude that learners should be *heard*, encouraged to be critical, take responsibility and decisions regarding their mathematics education to make them autonomous, cooperative and responsible mathematics learners fulfilling the aim of NEA, and become mathematically literate critically aware citizens to deal with challenges of our future society. For encouraging learners to be autonomous, they should be provided with time and space to plan and execute their own learning strategies together with their teachers, and approaches promoting autonomous behavior (Deci & Ryan, 1987) should be adopted. Moreover, to make learners think critically, decide, have more control and co-responsibility of their learning with teachers, just asking them simple questions (Croninger & Croninger, 2016) can be the first step of an interaction leading them to assume autonomy. Learner autonomy can make learners aware of their right to social justice by balancing the power relations between students and teachers. Autonomous learners can build a personal relationship with mathematics and gain experience of participation and authority of

understanding mathematics' role in his/her personal (i.e., their home, classroom, school etc.) and wider society (as visible in L_5's response).

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9 Appendices

9.1 The ethical clearance certificate from NSD – approval for data collection

Shipra Sachdeva
Program for lærerutdanning NTNU

7491 TRONDHEIM

Vår dato: 24.11.2016 Vår ref: 50556 / 3 / AMS Deres dato: Deres ref:

TILBAKEMELDING PÅ MELDING OM BEHANDLING AV PERSONOPPLYSNINGER

Vi viser til melding om behandling av personopplysninger, mottatt 13.10.2016. Meldingen gjelder prosjektet:

50556	*Local Culture for Understanding Mathematics and Science (LOCUMS)*
Behandlingsansvarlig	*NTNU, ved institusjonens øverste leder*
Daglig ansvarlig	*Shipra Sachdeva*

Personvernombudet har vurdert prosjektet, og finner at behandlingen av personopplysninger vil være regulert av § 7-27 i personopplysningsforskriften. Personvernombudet tilrår at prosjektet gjennomføres.

Personvernombudets tilråding forutsetter at prosjektet gjennomføres i tråd med opplysningene gitt i meldeskjemaet, korrespondanse med ombudet, ombudets kommentarer samt personopplysningsloven og helseregisterloven med forskrifter. Behandlingen av personopplysninger kan settes i gang.

Det gjøres oppmerksom på at det skal gis ny melding dersom behandlingen endres i forhold til de opplysninger som ligger til grunn for personvernombudets vurdering. Endringsmeldinger gis via et eget skjema, http://www.nsd.uib.no/personvern/meldeplikt/skjema.html. Det skal også gis melding etter tre år dersom prosjektet fortsatt pågår. Meldinger skal skje skriftlig til ombudet.

Personvernombudet har lagt ut opplysninger om prosjektet i en offentlig database, http://pvo.nsd.no/prosjekt.

Personvernombudet vil ved prosjektets avslutning, 31.12.2019, rette en henvendelse angående status for behandlingen av personopplysninger.

Vennlig hilsen

Katrine Utaaker Segadal

Anne-Mette Somby

Kontaktperson: Anne-Mette Somby tlf: 55 58 24 10
Vedlegg: Prosjektvurdering

Dokumentet er elektronisk produsert og godkjent ved NSDs rutiner for elektronisk godkjenning.

NSD – Norsk senter for forskningsdata AS Harald Hårfagres gate 29 Tel: +47-55 58 21 17 nsd@nsd.no Org.nr. 985 321 884
NSD – Norwegian Centre for Research Data NO-5007 Bergen, NORWAY Faks: +47-55 58 96 50 www.nsd.no

9.2 Information given to schoolteachers, learners and their parents/guardians

Forespørsel om deltakelse i forskningsprosjekt

I prosjektet vil aktiviteter knyttet til elevenes egen kultur være utgangspunkt for undervisning og læring i matematikk og naturfag.
"Prosjekts originalnavn: LOCUMS – Local Culture for Understanding Mathematics and Science"

Bakgrunn og formål
Kultur er grunnleggende i vår livsstil, væremåte, tankemåte og arbeid har en viktig rolle i hvert barns læringsprosess. I prosjektet LOCUMS vil vi undersøke effektene av å la elevene bruke sin egen kultur som utgangspunkt for opplæringen. Vi vil se om kultur kan fungere som en inngangsport for å skape tilknytning mellom skolefag og elevens liv utenfor skolen. Prosjektet vil ta utgangspunkt i praktiske aktiviteter knyttet til elevenes egen kulturelle identitet, og vi vil prøve å bruke disse aktivitetene som en del av undervisningen.

Kultur betyr i denne sammenhengen ikke etnisitet, men elevenes egen ungdomskultur, og reflekterer flere sider av livet som for eksempel kunst, mat, dans, musikk, idrett – det vil si praktiske aktiviteter som hører til elevenes interesse og dagligliv. Elever vil ikke bli bedt om å framme sin etnisitet i de praktiske aktivitetene, men det vil være opptil hver enkelt elev om han/hun ønsker å utnytte sin etniske bakgrunn i prosjektet.

Formålet med forskningsprosjektet er å observere forandringen i læringsprosessen når elevene får økt innflytelse på utgangspunktet for undervisningen. Langsiktige mål er å forbedre elevenes interesse for matematikk og naturfag, å motivere elevene til å lære og å redusere frafallet. Student-prosjektene kan også hjelpe elever til å reflektere over sammenhengen mellom matematikk og naturfag i skolen og praktiske aktiviteter knyttet til deres dagligliv og identitet. Idéen er å gi elevene eierskap til hvordan de ønsker å lære og å gi forskere innblikk i hvordan kulturell og personlig involvering påvirker læring.

Dette PhD-prosjektet er en del av LOCUMS (Local Culture for Understanding in Mathematics and Science), finansiert av Norges Forskningsråd. Det er et samarbeidsprosjekt mellom 3 norske universiteter, Norges teknisk-og naturvitenskapelige universitet, Universitet i Oslo og Universitet i Tromsø. To stipendiater og én post-doktor vil jobbe med det samme prosjektet. Shipra Sachdeva er PhD-stipendiat ved NTNU og vil utføre forskning i skoler med barn fra flere ulike kulturer.

I dette forskningsprosjektet ønsker vi å jobbe i skoler som har elever med variert kulturell bakgrunn. Derfor har vi valgt ****** skole som samarbeidsparter i skoleåret 2016-17. Primært ønsker vi å arbeide med elever fra 8. og 9. trinn og deres lærere. Et vanlig elev-prosjekt vil vare ca. 5 timer per uke i tre uker. I tillegg til naturfag og matematikk kan det være aktuell å inkludere fag som musikk og kunst og håndverk, noe vi vil avtale med den enkelte samarbeidsskolen. I løpet av prosjektet vil elever bli delt i grupper basert på egne interesser og hobbyer.

Hva innebærer det å delta i prosjektet?
Siden dette er et forskningsprosjekt vil det skje en datainnsamling. Det er viktig å observere og dokumentere hva som skjer i undervisningsoppleggene og å undersøke hvordan lærere og

elever opplever de endrede undervisnings- og læringsforholdene. Vi planlegger å ta lyd- og videoopptak av klasseaktiviteter og gruppeaktiviteter. Vi vil be noen elever om å bære hodekamera i tillegg til at det kan bli tatt lyd- og bildeopptak av hele klassen. Det vil bli gjennomført intervjuer av elever. I tillegg vil vi be elevene fylle ut spørreskjemaer for å få oversikt over hobbyer, interesser, skolemiljø og familiebakgrunn. Elevene kan velge selv om de vil svare på de enkelte spørsmålene.

Foresatte kan de be om å få se både intervjuguider og spørreskjema på forhånd.

Hva vil det skje med personlig informasjon under prosjektet og når det blir avsluttet?
Alle personlige opplysninger vil bli behandlet konfidensielt. Ingen andre enn PhD-kandidaten (Shipra Sachdeva) og hennes veiledere på NTNU vil få tilgang til personlige data som blir samlet inn i løpet av prosjektet. Dersom det blir leid inn hjelp for å transkribere (lage utskrift fra) lydopptakene vil alle personlige opplysninger bli slettet på forhånd. Alle lyd- og videoopptak vil bli lagret på sikker måte og uten tilkobling til internett.
Prosjektet er planlagt avsluttet i 2019. Resultater fra prosjektet vil bli publisert i en doktorgradsavhandling, men verken avhandlingen eller publiserte artikler vil inneholde noen referanse som kan brukes til å identifisere enkeltelever. Etter at prosjektet er avsluttet vil alle data bli anonymisert. Dette vil senest skje i løpet av 2019. Dersom det blir aktuelt med intervjuer i etterkant av prosjektet vil deltakerne få en ny forespørsel og informasjonsbrev om dette.

Frivillig deltakelse
Det er frivillig å delta i prosjektet og en elev kan når som helst velge å trekke seg uten å begrunne dette nærmere. I et sånt tilfelle vil all personlig informasjon om eleven bli anonymisert. Det vil ikke ha konsekvenser for elevenes karaterer om de velger å ikke delta i prosjektet.
Prosjektet er utformet i samarbeid med personvernombudet for forskning, Norsk samfunnsvitenskapelig datatjeneste (NSD).

Har noen spørsmål i forbindelse med denne henvendelsen, eller ønsker å bli informert om resultatene fra undersøkelsen når de foreligger, så ta gjerne kontakt med personene som er angitt under.

Med vennlig hilsen,
Shipra Sachdeva
Stipendiat
NTNU, Program for lærerutdanning
Skolelaboratoriet, Realfagsbygget/ A4-125
Høgskoleringen 5
7491 TRONDHEIM
E-post: shipra.sachdeva@ntnu.no

Prosjektleder:
Per-Odd Eggen
Førsteamanuensis
NTNU, Program for lærerutdanning
Skolelaboratoriet, Realfagsbygget/ A4-141
Høgskoleringen 5
7491 TRONDHEIM
E-post: perodde@plu.ntnu.no

Samtykkeerklæring for deltakelse i forskningsprosjektet LOCUMS ved **** skole for skoleår ****-****.**

Samtykkeerklæring fra elev og foreldre/foresatt til at
_____(elev) deltar i forskningsprosjektet LOCUMS med fokus på kulturinspirert undervisning/læring i matematikk og naturfag.

Vi har mottatt informasjon om prosjektet og er villig til å delta i studiet.

Dato og sted:

Signatur (foreldre/foresatte): _____ Tlf. nr. _____

Signatur (elev): _____

9.3 Pre-project questionnaire distributed to learners

Spørreskjema for elevene på *. trinn ved **** skole, **** (semester), ****(år)**

"LOCUMS, Local Culture for Understanding Mathematics and Science"

Innledning:

Dette skoleåret vil vi be dere om å bli med i et forskningsprosjekt som heter LOCUMS. I prosjektet vil vi ta utgangspunkt i elevenes egne interesser og jobbe med praktiske aktiviteter. Vi ønsker å møte skoleklasser der det er elever fra mange ulike kulturer, og at dere som deltar skal jobbe med aktiviteter som oppleves meningsfulle. I neste omgang er målet å knytte skolefag til disse aktivitetene. Vi vil alltid prøve å koble aktivitetene til matematikk og naturfag, men det kan også være andre fag, som for eksempel kunst og håndverk, musikk eller mat og helse.

LOCUMS ønsker å undersøke sammenhengen mellom elevenes kultur og deres interesse og motivasjon for læring i matematikk og naturfag. For å forske på undervisningen trenger vi mange opplysninger, og vi ønsker å bruke både spørreundersøkelser, intervju, lyd- og filmopptak.

Dette spørreskjemaet er laget som et utgangspunkt for intervju med elever, der vi ønsker å finne ut mer om

- personlige opplysninger
- familiebakgrunn
- hva du er interessert i å lære mer om på skolen
- hvilke fritidsinteresser du har og hvilken kultur kjenner du deg hjemme i
- hvilken interesse du har for å lære matematikk og naturfag
- hvilke praktiske aktiviteter du deltar i
- hva du tenker om framtidsjobben din
- hvordan du liker deg på skolen og i klasserommet
- ditt forhold til de andre elevene og andre venner
- selvsikkerhet, selvbilde, trivsel, tilhørighet og din rolle som medelev i klassen
- skolearbeidet ditt hjemme

Vi ønsker at du fyller ut spørreskjemaet og leverer det inn til meg (Shipra) i løpet av to til tre dager. Det er ingen andre enn jeg (Shipra Sachdeva) som får vite hva du har svart på disse spørsmålene. Jeg vil gjerne vite din mening, men du trenger ikke svare på alle spørsmålene om du ikke vil. Dersom det kommer fram personlige opplysninger i svarene dine kan du være helt trygg på at de ikke vil komme til noen andre. Alle data som blir brukt fra denne spørreundersøkelsen vil bli anonymisert. Dersom du er usikker på noe, så kan du spørre meg (Shipra) eller en lærer som er med i prosjektet.

Jeg takker dere på forhånd!!

Spørreskjema for elevene på *. trinn ved **** skole, **** (semester), ****(år)**

"LOCUMS, Local Culture for Understanding Mathematics and Science"

Les dette nøye før du svarer på spørreskjemaet:

1. Jeg vil gjerne at du svarer ærlig på spørsmålene.
2. Mange av spørsmålene er avkrysningsspørsmål. Du skal krysse av det alternativet/de alternativene som du synes passer best for deg og din situasjon for hvert spørsmål.
3. Noen av spørsmålene har et kommentarfelt der du kan skrive åpne svar, forklaringer og/eller tilleggsinformasjon.
4. Ingen svar er rett eller feil her – vi vil bare høre din ærlige mening.
5. Dersom det kommer fram personlige opplysninger i spørreskjemaet vil ingen andre enn Shipra få tilgang til opplysningene.
6. Svarene du gir på spørreskjemaet vil ikke ha noen innvirkning på karakterene dine.

Opplysninger om deg og din familie:

Navn (frivillig):

Alder:

Gutt/Jente:

Morsmål:

Ble du født i Norge?

a) Ja ☐
b) Nei ☐

Hvis du ikke er født i Norge, hvor gammel var du da du kom til Norge?
(Skriv null (0) hvis du var yngre enn 12 måneder)

_____ år.

Har du gått i barnehage/førskole i Norge?

a) Ja ☐
b) Nei ☐

I hvilket land ble foreldrene dine født?

Mor: _____
Far: _____

Hvilket språk snakker du hjemme det meste av tiden?

_____

Hvor godt kan du norsk?
(Kryss av bare én boks i hver linje)

	Veldig Bra	Bra	OK	Ikke så bra	Dårlig
Snakking	☐	☐	☐	☐	☐
Skriving	☐	☐	☐	☐	☐
Lesing	☐	☐	☐	☐	☐
Å forstå når andre snakker	☐	☐	☐	☐	☐

Fritidsinteresser/hobbyer, praktiske aktiviteter og kulturell identitet:

Hvilke fritidsinteresser/hobbyer har du?
(Kryss av på en skala fra *ikke interessant* til *svært interessant* for hvert punkt nedenfor)

	Ikke interessert	Lite interessert	Middels interessert	Svært interessert	Vet ikke
Se på TV/film	☐	☐	☐	☐	☐
Spille data/video spill	☐	☐	☐	☐	☐
Gå på tur/trening	☐	☐	☐	☐	☐
Å være sammen med venner	☐	☐	☐	☐	☐
Spille/høre på musikk	☐	☐	☐	☐	☐
Skrive dikt, musikk, historier osv.	☐	☐	☐	☐	☐

	Ikke interessert	Lite interessert	Middels interessert	Svært interessert	Vet ikke
Danse	☐	☐	☐	☐	☐
Klær/Mote	☐	☐	☐	☐	☐
Lese aviser, blader, nettaviser, nyheter osv.	☐	☐	☐	☐	☐
Å følge med på sosiale media	☐	☐	☐	☐	☐
Jobbe med/utforske teknologi, smarttelefoner, datamaskiner osv.	☐	☐	☐	☐	☐
Hagearbeid, natur, blomster, klima, miljø osv.	☐	☐	☐	☐	☐
Dyr, kjæledyr, dyreliv osv.	☐	☐	☐	☐	☐
Idrett / trening	☐	☐	☐	☐	☐
Utendørsleker	☐	☐	☐	☐	☐
Innendørsleker	☐	☐	☐	☐	☐
Håndverk og kunst (skulptur, keramikk, matlaging, maling, sying, strikking, broderi, snekkeri, fotografering osv.)	☐	☐	☐	☐	☐
Være sammen med familie	☐	☐	☐	☐	☐
Lese, skrive og studere faglige tekster	☐	☐	☐	☐	☐

Har du andre hobbyer/interesser enn de som er nevnt ovenfor? Hvilke?

_____
_____
_____

Hvilke praktiske aktiviteter/håndverk liker du best å holde på med?

_____
_____
_____

Hva betyr ordet *kultur* for deg? Hvilke andre ord dukker først opp i hodet ditt når du hører ordet *kultur*?

_____
_____
_____

Hvordan vil du beskrive din personlige *kultur*?

_____
_____
_____

Føler du at det er forskjell mellom norsk kultur og andre etniske (hjemlandets/samisk/annen) kulturer representert i klassen? Hva er forskjellig?

_____
_____
_____

Når alle elevene er sammen i klassen, tror du da at det er noen kulturelle forskjeller mellom dere?

_____
_____
_____

Ditt forhold til matematikk og naturfag (realfag), framtidsjobb:

Hvor enig er du i disse utsagnene?
(Kryss av i bare én boks i hver linje)

	Svært uenig (Strongly disagree)	Uenig (Disagree)	Enig (Agree)	Svært enig (Strongly Agree)	Vet ikke (Do not know)
Jeg liker realfagbøker	☐	☐	☐	☐	☐
Å gjøre en innsats i realfag er viktig fordi det vil hjelpe meg i det arbeidet jeg vil gjøre senere	☐	☐	☐	☐	☐
Jeg gleder meg til realfagtimene	☐	☐	☐	☐	☐
Jeg jobber med realfag fordi jeg liker det	☐	☐	☐	☐	☐
Å lære matematikk og naturfag er viktig for meg fordi det vil bedre mine yrkesmuligheter	☐	☐	☐	☐	☐

	Svært uenig (Strongly disagree)	Uenig (Disagree)	Enig (Agree)	Svært enig (Strongly Agree)	Vet ikke (Do not know)
Jeg er interessert i det jeg lærer i matematikk og naturfag	☐	☐	☐	☐	☐
Matematikk og naturfag er viktige fag for meg fordi jeg trenger det når jeg skal studere videre	☐	☐	☐	☐	☐
Mye av det jeg lærer i realfag, vil hjelpe meg til å få jobb	☐	☐	☐	☐	☐
Jeg er ofte bekymret at realfag-timene blir vanskelig for meg	☐	☐	☐	☐	☐
Jeg er rett og slett ikke flink i matematikk	☐	☐	☐	☐	☐
Jeg er rett og slett ikke flink i naturfag	☐	☐	☐	☐	☐
Jeg tror at det jeg lærer i matematikk er bortkastet tid	☐	☐	☐	☐	☐
Jeg tror at det jeg lærer i naturfag er bortkastet tid	☐	☐	☐	☐	☐
Jeg liker å lære matematikk	☐	☐	☐	☐	☐
Jeg liker å lære fysikk (atomer, radioaktivitet)	☐	☐	☐	☐	☐
Jeg er interessert i å lære om verdensrommet (stjerner, planeter og universet)	☐	☐	☐	☐	☐
Jeg liker biologi (medisin, kirurgi, menneskekropp osv.)	☐	☐	☐	☐	☐
Jeg er interessert i forskning	☐	☐	☐	☐	☐
Jeg liker å lære kjemi (kjemikalier, reaksjoner, molekyler)	☐	☐	☐	☐	☐
Jeg lærer realfag raskt	☐	☐	☐	☐	☐
Jeg blir veldig stresset når jeg må gjøre lekser i matematikk-og naturfag	☐	☐	☐	☐	☐
Jeg forstår selv det vanskeligste i matematikk og naturfag	☐	☐	☐	☐	☐
Jeg er redd for at jeg vil få dårlig karakter i matematikk	☐	☐	☐	☐	☐
Jeg er redd for at jeg vil få dårlig karakter i naturfag	☐	☐	☐	☐	☐

Er det noe i naturfaget som du synes er spesielt interessant, for eksempel, biologi, fysikk og/eller kjemi?

_____
_____
_____

Er det noe i matematikkfaget som du synes er spesielt interessant, for eksempel, regning, geometri og/eller trigonometri?

_____
_____
_____

Beskriv hva du ønsker at du kunne lære mer om på skolen.

_____
_____
_____

Hva synes du det er *nyttig å lære*?

_____
_____
_____

Hva vil du si at *nyttig læring* i matematikk er for deg?

_____
_____
_____

Hva vil du si at *nyttig læring* i naturfag er for deg?

_____
_____
_____

Hva av dette har du tenkt å fullføre?
(Kryss av i de boksene som passer)

a) Ungdomsskole ☐
b) Videregående skole på yrkesfaglig studieretning ☐
c) Videregående skole med allmenne, økonomiske, og administrative fag ☐
d) En kort utdanning med varighet fra 1 til 2 år ☐
e) En utdanning på universitet/høgskole som varer i minst 3 år (f. eks. sykepleier, ingeniør, fysioterapeut) ☐
f) En utdanning på universitet/høgskole som varer i minst 5 år (f. eks. medisinstudiet, jusstudiet, studium med hovedfag) ☐

Hvor viktig vil dette være for deg i din framtidige jobb?

(Kryss av i bare én boks i hver linje)

	Ikke viktig	Lite viktig	Viktig	Svært viktig	Vet ikke
Sosialt arbeid – hjelpe andre og jobbe med mennesker	☐	☐	☐	☐	☐
Dyr – forske på dem	☐	☐	☐	☐	☐
Klima og miljø	☐	☐	☐	☐	☐
Jobbe med teknologi og data	☐	☐	☐	☐	☐
Håndverk	☐	☐	☐	☐	☐
Jobbe med noe som er lett og enkelt	☐	☐	☐	☐	☐
Skape nye idéer	☐	☐	☐	☐	☐
Jobbe slik at jeg har mye tid til venner	☐	☐	☐	☐	☐
Jobbe med noe kreativt og nytt	☐	☐	☐	☐	☐
Jobbe med maskiner og verktøy	☐	☐	☐	☐	☐
Jobbe slik at jeg får mye tid sammen med familien min	☐	☐	☐	☐	☐
Være berømt	☐	☐	☐	☐	☐
Tjene mye penger	☐	☐	☐	☐	☐
Jobbe selvstendig og bestemme over meg selv	☐	☐	☐	☐	☐
En jobb som involverer mye reise og møter med nye folk	☐	☐	☐	☐	☐
Være sjefen på jobben og kontrollere andre	☐	☐	☐	☐	☐
Jobbe sammen med andre i et lag/team	☐	☐	☐	☐	☐
Utvikle kunnskap, evner og holdninger	☐	☐	☐	☐	☐

Har du andre forventninger til framtidsjobben din?

_____

_____

_____

Skolemiljø, venner og sosial deltakelse:

Tenk på disse setningene om skolen din: **Hvor mye er du enig i disse utsagnene?**
(Kryss av i bare én boks for hver linje)

	Svært uenig	Uenig	Enig	Svært enig	Vet ikke
Skolen har gjort mye for å forberede meg på voksenlivet og jobbmarkedet	☐	☐	☐	☐	☐
Å gå på skole er bortkastet tid	☐	☐	☐	☐	☐
Skolen har hjulpet meg til å bli trygg på å ta beslutninger	☐	☐	☐	☐	☐
Elevene kommer godt overens med de fleste lærerne	☐	☐	☐	☐	☐
Lærerne er virkelig interessert i hva jeg sier og gjør	☐	☐	☐	☐	☐
Lærerne hjelper meg når jeg trenger hjelp	☐	☐	☐	☐	☐
Jeg har mange venner i klassen	☐	☐	☐	☐	☐
Jeg liker å jobbe sammen med vennene mine med faglige oppgaver	☐	☐	☐	☐	☐
Lærere og andre elever liker meg	☐	☐	☐	☐	☐
Jeg føler at jeg ikke passer inn	☐	☐	☐	☐	☐
Jeg føler meg ensom og annerledes	☐	☐	☐	☐	☐
Jeg liker å hjelpe andre med oppgaver og lekser	☐	☐	☐	☐	☐
Jeg får hjelp til å løse oppgaver	☐	☐	☐	☐	☐
Jeg deltar i aktiviteter som foregår på skolen eller i klassen	☐	☐	☐	☐	☐
Jeg liker å arbeide sammen med elever som er fra andre land	☐	☐	☐	☐	☐
Lærere planlegger matematikk- og naturfagstimene slik at alle elever blir interessert i å lære	☐	☐	☐	☐	☐
Læreren forstår mine sterke og svake sider og tilpasser opplæring slik at den passer for meg	☐	☐	☐	☐	☐

	Svært uenig	Uenig	Enig	Svært enig	Vet ikke
Utenlandske elever er fornøyd med klassemiljø og læring	☐	☐	☐	☐	☐

Hvis det er mer informasjon som du ønsker å gi om deg, dine venner, din klasse eller skolen, så bruk skrivefeltet nedenfor.

_____

_____

_____

(Bare for utenlandske elever) Hvor enig er du i disse utsagnene?
(Kryss av i bare én boks i hver linje)

	Svært uenig	Uenig	Enig	Svært enig	Vet ikke
Jeg liker meg i Norge	☐	☐	☐	☐	☐
Jeg savner hjemlandet mitt	☐	☐	☐	☐	☐
Hjemmemiljøet mitt er ikke mye norsk	☐	☐	☐	☐	☐
Foreldrene mine snakker norsk sammen	☐	☐	☐	☐	☐
Foreldrene mine snakker et annet språk enn norsk sammen	☐	☐	☐	☐	☐
Jeg savner vennene mine fra hjemlandet mitt	☐	☐	☐	☐	☐
Jeg savner kultur fra hjemlandet mitt	☐	☐	☐	☐	☐
Jeg savner tradisjoner fra hjemlandet mitt	☐	☐	☐	☐	☐
Jeg liker norske tradisjoner	☐	☐	☐	☐	☐
Jeg liker norsk tenke- og væremåte	☐	☐	☐	☐	☐
Jeg savner å være sammen med norske venner	☐	☐	☐	☐	☐
Jeg ønsker å bli integrert i det norske samfunnet så fort som mulig	☐	☐	☐	☐	☐
Jeg er/ønsker å bli vant til norsk kultur og rutiner	☐	☐	☐	☐	☐
Jeg liker meg i norsk skole	☐	☐	☐	☐	☐

	Svært uenig	Uenig	Enig	Svært enig	Vet ikke
Det er behagelig å gå på skole her	☐	☐	☐	☐	☐
Jeg ønsker ikke å ha kontakt med norske klassekamerater	☐	☐	☐	☐	☐
Jeg liker norsk mat	☐	☐	☐	☐	☐
Jeg har det bedre i Norge enn i hjemlandet mitt	☐	☐	☐	☐	☐

Er det noe annet du har lyst til å fortelle om ditt hjemland og kultur eller om deg selv?

_____
_____
_____

Hva engasjerer deg? Generelt, i forhold til matematikk, i forhold til naturfag.

_____
_____
_____

Savner du noe spesifikt fra det du gjorde i hjemlandet ditt, en del av din kultur eller om deg selv?

_____
_____
_____

Jeg finner at det er vanskelig å bli involvert/inkludert/integrert med andre i klassen. Jeg har problemer i å forstå språk, kontekst og hva andre i klassen driver med på og snakker om?

_____
_____
_____

9.4 Designed practical activities

9.4.1 Classroom Intervention 1 – plan for project-work

Pilotstudie LOCUMS, **** januar 2017

Her er en foreløpig plan for aktivitetene til de fire elevgruppene som skal delta i pilotstudien for LOCUMS-prosjektet i januar 2017. Vi håper at dere kan bidra med innspill, kommentarer og spørsmål slik at vi får en endelig plan.

I tillegg til oppgavene har vi lagt noen krevende utfordringer for hver gruppe som de kan jobbe med hvis de får tid til overs. Det å gjøre oppgavene ferdig vil være førsteprioritet men hvis elever i en gruppe får tid kan de jobbe med oppgavene merket "Hvis dere får tid:" samme dag. I tilfelle det ikke er tid, kan disse utfordringene eventuelt løses senere.

Vi vil lage en liste over kompetansemål fra de ulike læreplanene som kan bli berørt i disse fire prosjektene, og i etterkant vil vi analysere hvilke kompetansemål som reelt ble berørt og der vi kan anta at elevene fikk et faglig utbytte.

Målet vårt med prosjektet er å svare på forskningsspørsmålene i LOCUMS-prosjektet. Vi er ikke ute etter å sammenligne våre aktiviteter med ordinær undervisning, men vi vil organisere aktiviteter som gir oss mulighet til å få fram data om effekten av å gi elevene innflytelse på sine egne læringsaktiviteter. Aktivitetene vil fungere som en arena for datainnsamling, i hovedsak film- og lydopptakene fra aktivitetene.

Sted og Dato: Trondheim, 08. desember, 2016

Hilsen Shipra Sachdeva og Per-Odd Eggen

Gruppe 1: Snekring

Prioritet no. 1

Dere skal lage en vedkasse av finér. Kassen skal være 60 cm bred og ha plass for innholdet i en 80-liters vedsekk. Lokket skal være hengslet slik at det står stødig når det er åpent, selv om kassen står helt inntil veggen. Kassen skal være fin å se på.

1) Planlegg hvordan kassen skal være og lag en arbeidstegning med målestokk.
2) Fordel arbeidet mellom dere.
3) Lag kassen ferdig.

Forutsetninger: De får utlevert finérplater og treverk, skruer, lim og bormaskin.

Tid til disposisjon:

Hva ønsker vi å se?

1) Planleggingen – om dere har laget en god plan som alle forstår og kunne bruke i arbeidet.
2) Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av.
3) Produktet – kan kassen brukes til vedkasse? Ble den fin og nøyaktig laget? Var målene innenfor kravet?

Hvis dere får tid:

Hvilke andre muligheter hadde dere for utforming når kassen skal være 60 cm bred og 80 liter? Diskuter oppgaven slik at alle i gruppa forstår både spørsmål og løsninger. Bruk gjerne hjelpemidler som f. eks. PC, telefon, illustrasjoner osv.

Gruppe 2: Matlaging

Prioritet no. 1

Dere skal lage sunn og næringsrik middagsmat for idrettsutøvere, for eksempel for 9.klassinger som trener mye. Det skal være nok mat for minst 4 elever, og i dag skal alle elevene i klassen dele det dere lager slik at det blir en "smakbit" til hver. Maten skal inneholde alle viktige næringsstoffer i riktig mengde som trengs for å ha nok energi til trening. Lag noe som dere tror de fleste liker godt. Begrunn også hvorfor er den mengde nok for ham/hun og hva bruker kroppen de forskjellige næringsstoffene til?

1) Planlegg hva dere vil lage og lag en plan for arbeidet. Det skal være mulig for dere som gruppe å lage ferdig maten på to timer.
2) Lag en oversikt til journalistgruppen om hva maten inneholder og hvorfor dette er riktig mat for en idrettsutøver.
3) Fordel arbeidet mellom dere.
4) Lag maten ferdig.

Forutsetninger: De får penger til å handle råstoff og vil bli kjørt til og fra butikken.

Tid til disposisjon:

Hva ønsker vi å se?

1) Planleggingen – om dere har laget en god plan som alle forstår og kunne bruke i arbeidet.
2) Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av.
3) Produktet – planla dere et måltid som inneholdt mange viktige næringsstoffer i en god sammensetning for en idrettsutøver? Greide dere å lage et bra måltid?

Hvis dere får tid:

Dersom det er ønskelig med en annen sammensetning av næringsstoffer, for eksempel ha mer protein, men mindre karbohydrater, hvordan kan dere oppnå det og samtidig ha kontroll på næringsinnholdet? Diskuter oppgaven slik at alle i gruppa forstår både spørsmål og løsninger. Bruk gjerne hjelpemidler som f. eks. PC, telefon, illustrasjoner osv.

Gruppe 3: Journalistgruppe

Prioritet no. 1

Dere skal dokumentere og rapportere det arbeidet blir gjort av de andre gruppene i klassen. I Norge i dag trenger vi journalister som ikke bare tar bilder og rapporterer hendelser men har også kunnskap om hvordan ting skjer og fungerer. Derfor skal dere lage en avisreportasje om de andre gruppene i klassen der dere viser om de har funnet fram til løsninger på oppgavene de fikk og hvordan de løste dem. For hver gruppe skal dere lage et bilde (foto eller tegning) med en bildetekst som illustrerer utfordringene hver av gruppene jobbet med. Hvorfor var dette en krevende utfordring? Hvilke valg måtte elevene i hver gruppe ta? Sørg for at dette kommer tydelig fram i reportasjen. Du kan ikke bruke mer enn 45 minutter på å snakke med hver gruppe, men dere kan være til stede og ta bilder så mye dere vil.

1) 1 Planlegg hva vil du spørre de gruppene om og hvordan vil dere lage et innlegg om arbeidet de gjør.
2) 2 Fordel arbeidet mellom dere.
3) 3 Lag rapportene ferdig.

Forutsetninger: De får bruke deres mobiltelefoner for å ta bilder og lydopptak. Dere får papir, blyanter, farger osv. hvis dere trenger og/eller kan jobbe på PC hvis dere vil lage en digital rapport.

Tid til disposisjon:

Hva ønsker vi å se?

1) Planleggingen – om dere har laget en god plan som alle forstår og kunne bruke i arbeidet.
2) Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av.
3) Produktet – beskrev reportasjen de ulike prosjektene på en god måte? Bidro bildet med bildetekst til dette? Ville den vært egnet som avisreportasje?

Hvis dere får tid:

Hvordan kan dere illustrere utfordringene som de andre gruppene har fått på en sånn måte at det er lett å skjønne for en som leser reportasjen? Diskuter oppgaven slik at alle i gruppa forstår både spørsmål og løsninger. Bruk gjerne hjelpemidler som f. eks. PC, telefon, illustrasjoner osv.

Gruppe 4: Verdensrommet

Prioritet no. 1

Dere skal lage modell av solsystemet i plastelina (byggeleire). Modellen skal være i målestokk og få plass inne i rommet og kunne brukes til å undervise om solsystemet. Målestokk betyr her at de ulike planetene og sola skal ha riktig størrelse i forhold til hverandre og at avstanden mellom dem skal være riktig i forhold til størrelsen på planetene. Dere skal også beregne hvor langt unna modellen av sola dere måtte ha plassert den nærmeste stjerna (Proxima Centauri) for at denne avstanden skulle bli en del av modellen.

Bruk programmet Stellarium til å bestemme hvilken retning de ulike planetene skal ha i forhold til sola.

1) 1 Planlegg modellen og lag en arbeidstegning.
2) 2 Fordel arbeidet mellom dere.
3) 3 Lag modellen + en liten plakat for hver planet der dere skriver noen sentrale opplysninger.

Forutsetninger: De får utlevert byggeleire og 4 iPader med Stellarium installert.

Tid til disposisjon:

Hva ønsker vi å se?

1) Planleggingen – om dere har laget en god plan som alle forstår og kunne bruke i arbeidet.
2) Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av.
3) Produktet - Ble modellen fin og nøyaktig laget? Var det enkelt for en besøkende å forstå modellen?

Hvis dere får tid:

Hvor lang tid brukte romskipet New Horizons på å passere dvergplaneten Pluto da 13. juli 2015? Hvor lang tid tok signalene fra romskipet og til jorda og tilbake da New Horizons passerte Pluto? Hvordan kunne folk på jorda styre for eksempel skal styre kameraene som tok bildene? Diskuter oppgaven slik at alle i gruppa forstår både spørsmål og løsninger. Bruk gjerne hjelpemidler som f. eks. PC, telefon, illustrasjoner osv.

9.4.2 Classroom Intervention 2 – plan for project-work

Pilotstudie LOCUMS, ***** 4. april 2017

Her er en plan for aktivitetene til de fire elevgruppene som skal delta i pilotstudien for LOCUMS-prosjektet i 4. april 2017. Kom gjerne med innspill, kommentarer og spørsmål.

Vi vil lage en liste over kompetansemål fra de ulike læreplanene som kan bli berørt i disse fire prosjektene, og i etterkant vil vi analysere hvilke kompetansemål som reelt ble berørt og der vi kan anta at elevene fikk et faglig utbytte.

Målet vårt med prosjektet er å svare på forskningsspørsmålene i LOCUMS-prosjektet. Vi er ikke ute etter å sammenligne våre aktiviteter med ordinær undervisning, men vi vil organisere aktiviteter som gir oss mulighet til å få fram data om effekten av å gi elevene innflytelse på sine egne læringsaktiviteter. Aktivitetene vil fungere som en arena for datainnsamling, i hovedsak film- og lydopptakene fra aktivitetene.

Sted og Dato: Trondheim, 29. mars, 2017

Hilsen Shipra Sachdeva og Per-Odd Eggen

Gruppe 1: Koking på bål

Oppdrag

Dere skal lage to bål som dere skal sammenligne. Det ene bålet skal lages med tørr ved, det andre med trevirke som dere finner ute i naturen. Dere skal koke en liter vann på hvert av bålene og finne ut hvor lang tid det tar før vannet koker. Mål temperaturen på vannet på forhånd. Deretter skal dere koke en liter vann på en kokeplate der effekten er kjent.

1. Prøv å finne ut hvor stor effekt de to bålet dere laget hadde.
2. Var det forskjell på de to bålene?
3. Hvor stor effekt har bålet sammenlignet med ei kokeplate?
4. Vis ulike beregningsmåter for effekten av bålet og av kokeplata.

Som utgangspunkt kan dere bruke at 1 kWh (en kilowatt i en time) i teorien er nok til å varme opp 8,6 liter vann fra 0 – 100 ^{o}C eller 10,7 liter vann fra 20 – 100 ^{o}C

Forutsetninger: De får utlevert tørr ved og fyrstikker + et kokekar, vann og et termometer.

Tid til disposisjon: Hele dagen. Vi starter ute og fortsetter på klasserommet.

Hva ønsker vi å se?

1. Om dere kan fordele arbeidet godt, lage et godt bål, henge opp kokekaret og koke vannet uten at det skjer uhell eller blir for mye søl.
2. Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av?
3. Om dere kan beregne hvor stor effekt de to bålene hadde og om dere kan forklare forskjellen på de to bålene og kokeplata.

Tilleggsoppdrag hvis dere får tid:

Kan dere finne andre måter for å beregne hvor stor effekt det er av en kokeplate eller et bål?

Gruppe 2: Matlaging

Oppdrag

Dere skal lage Granola for turbruk og lage en brosjyre for produktet.

1. Vei alle ingrediensene hver for seg slik at dere har de opplysningene dere trenger når dere skal lage brosjyren.
2. Fordel arbeidet og lag Granola i to varianter – som frokostblanding eller som en blokk. Hva var mest vellykket?
3. Lag en brosjyre som består av ett A4-ark som skal brettes slik at det blir en firesiders trykksak. Den skal inneholde en omtale av produktet og en varedeklarasjon med alle næringsstoffer og energiinnhold.
4. Alle i klassen bør få smake på produktet.

Forutsetninger: De får utdelt råvarene dere trenger.

Hva ønsker vi å se?

1. Planleggingen – om dere har laget en god plan som alle forstår og at dere fordeler arbeidet på en god måte.
2. Brosjyren skal være beregnet for folk som går lange turer i skog og mark og som trenger lett, næringsrik mat. Den skal være fin å se på og inneholde alle de opplysningene som en friluftsperson trenger om produktet.
3. At produktet faller i smak i klassen.

Tilleggsoppdrag hvis dere får tid:

Finn gjerne ut hvor mye næring som er anbefalt for en skiløper som skal gå lange turer (over flere dager) og trekke pulk eller bære med seg all maten. Hvor mye Granola trengs det per dag på en sånn tur?

Oppskrift Granola: (dette er et eksempel som kan varieres etter ønske!)
250 gram havregryn, 50 gram solsikkekjerner, 50 gram sesamfrø, 5o gram mandler (skivet), 50 gram valnøtter, grovt hakket, 50 gram hasselnøtter, grovt hakket,25 gram brunt sukker, 2 klyper salt, 1 dl jus , 1 ss honning, 1 ss smør (eventuelt olje) 75-100 gram tørket frukt og bær (feks eple, aprikos, tranebær, fiken)
Rør sammen alt det tørre med unntak av sukker og tørket frukt/bær. Ha eplejuice, smør, sukker og honning i en kasserolle, og varm opp til alt er oppløst. Hell blandingen oppi det tørre og rør godt sammen. Hell blandingen over på et bakepapirkledt stekebrett og fordel den jevnt utover. Stek granolaen midt i ovnen ved 140 grader (på vanlig over- og undervarme) i 50-60 minutter, til blandingen har en jevn brunfarge. Rør i blandingen cirka hvert 10-15 minutt for å få et jevnt stekt resultat.
La blandingen avkjøles, tilsett tørket frukt og bær, og hell deretter blandingen på et stort glass eller i en boks. Frokostblandingen er holdbar i ca 1 måned. Prøv også å blande inn frukt og bær før det stivner og trykk det hele sammen til en form som et knekkebrød.

Gruppe 3: Biologi

Oppdrag

Dere skal ut i skogen og finne minst ti levende organismer som dere kan ta med inn i klasserommet. Bruk et termokamera for å sjekke om de er varmere enn omgivelsene. Det kan være insekter, små planter eller andre organismer. (Dere får låne et termokamera til dette.) Prøv å finne både planter og dyr.

1. Samle planter og dyr og ta dem med inn i klasserommet.
2. Se hva som skjer etter hvert som temperaturen stiger. Ta bilder!
3. Sammenlign generelt at hvordan greier planter og smådyr å overleve gjennom vinteren?

Velg en eller to elever i klassen som kan skrive en tekst noenlunde raskt på pc. Mål tida som trengs for å skrive de linjene som er skrevet med blått i denne teksten. La de samme elevene holde hånda i isvann så lenge de greier. Prøv deretter å skrive den samme teksten og mål tida. Var det noen forskjell? Skriv ned fire tilpasninger som mennesket har for å tåle kulde og fire for å tåle varme. Hvorfor må vi mennesker holde jevn temperatur?

Hva ønsker vi å se?

1. At dere har samlet inn ulike organismer og at dere kan prøve å bestemme hva de er.
2. Samarbeidet – at alle fikk bidra med en del av arbeidet.
3. At dere gjennomførte kuldeforsøket på en god måte.
4. Skriv en oversikt over forskjeller på mennesker og andre dyr når det gjelder tilpasninger til varme og kulde.

Tilleggsoppdrag hvis dere får tid:

Ta bilde med termo-kamera av en elev med lite klær, gjerne utendørs. Hvor på kroppen er varmetapet størst?

Gruppe 4: Verdensrommet

Oppdrag

Dere skal lage modell av jorda og månen som kan brukes til å forklare hvorfor månen hele tida skifter mellom nymåne, halvmåne og fullmåne.

1. Planlegg modellen og lag en arbeidstegning der størrelsene på jorda og månen og avstanden mellom dem er riktig i forhold til hverandre.
2. a.) Hvorfor er månen synlig bare deler av døgnet?
   b.) Hvorfor skifter månen mellom nymåne, halvmåne og fullmåne?
   c.) Hvilken betydning har månen for livet på jorda?
   d.) Har månen innvirkning på samfunnet, tradisjonene våre eller dagliglivet?

Forutsetninger: Dere får utlevert byggeleire og en iPad med Stellarium installert.

Hva ønsker vi å se?

1. Planleggingen – om dere har laget en god plan som alle forstår og kunne bruke i arbeidet.
2. Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av.
3. Produktet - Ble modellen fin og nøyaktig laget? Har dere laget forklaringer som er lette å forstå?

Tilleggsoppdrag hvis dere får tid:

Den 11. april blir det fullmåne. Når kommer den første fullmånen etter det igjen? Er fullmånen synlig på himmelen på samme sted (det vil si i samme himmelretning) hver gang?

9.4.3 Classroom Intervention 3 – plan for project-work

Pilotstudie LOCUMS, **** 13. juni 2017

Her er et forslag for aktivitetene til de fire elevgruppene som skal delta i pilotstudien for LOCUMS-prosjektet den 13. juni 2017. Denne gangen ønsker vi innspill, kommentarer og spørsmål fra både lærere og elever.

Vi vil lage en liste over kompetansemål fra de ulike læreplanene som kan bli berørt i disse fire prosjektene, og i etterkant vil vi analysere hvilke kompetansemål som reelt ble berørt og der vi kan anta at elevene fikk et faglig utbytte.

Målet vårt med prosjektet er å svare på forskningsspørsmålene i LOCUMS-prosjektet. Vi er ikke ute etter å sammenligne våre aktiviteter med ordinær undervisning, men vi vil organisere aktiviteter som gir oss mulighet til å få fram data om effekten av å gi elevene innflytelse på sine egne læringsaktiviteter. Aktivitetene vil fungere som en arena for datainnsamling, i hovedsak film- og lydopptakene fra aktivitetene.

Sted og Dato: Trondheim, 29. mai, 2017

Hilsen Shipra Sachdeva og Per-Odd Eggen.

Plan for dagen:

1) Dere får 3 og 3½ timer til å gjøre praktiske forberedelser og for å lage en presentasjon. Fordel tida slik det passer.
2) Til slutt skal vi se presentasjonene sammen. Vi går ut fra at presentasjonene vil ta mellom 1 og 1½ time.

Denne gangen vil vi legge spesielt merke til om dere fordeler arbeidet slik at alle får best mulig utbytte.
NB! Gi oss gjerne en tilbakemelding om disse planene. Har dere gode forslag til forbedringer? Vi hører gjerne fra dere på e-post: shipra.sachdeva@ntnu.no eller per.eggen@ntnu.no

Gruppe 1: Biologi/Menneskekroppen/Førstehjelp

Oppdrag

Dere skal lage en oversikt over symptomer og hva som skal gjøres i fem ulike førstehjelpssituasjoner, for eksempel beinbrudd eller et hjerneslag. Dere skal ha ansvar for å presentere en førstehjelpssituasjon hver, men det er selvsagt lov å samarbeide. Tenk deg at du er den første som kommer til en person som har en skade eller sykdom, og lag en kort beskrivelse av hvilke problemer som må løses og hva som er det første du skal gjøre. Dere bør prøve mest mulig i praksis, for eksempel hva dere gjør med et beinbrudd hvis det blør, hvis en person har epilepsi eller lignende. På slutten av dagen får dere et kvarter til å presentere det dere har kommet fram til for de andre i klassen. Dette kan være et rollespill, en PowerPoint eller lignende. Ta gjerne bilder eller filmer underveis slik at dere kan bruke dem hvis dere skal lage en presentasjon.

For hver av de ulike skadene/sykdommene

1. Prøv å finne fram hvilke organer i kroppen som blir rammet først ved den skaden/sykdommen dere jobber med. Kan det være mer enn et organ som blir rammet? Kan det variere fra person til person? Hvis ja, i hvilke nødssituasjoner kan det variere?
2. Lag en oversikt over symptomer som dere trenger å kjenne til ved hver nødssituasjon.
3. Hva skal man gjøre hvis man er den første som kommer og kan hjelpe i hver av disse situasjonene?
4. Finn i lag at hvor i Snåsa dere kan få vite mer om førstehjelp og om det er mulig for dere å ta førstehjelpskurs.
5. Finnes det noen tiltak som kan brukes i alle situasjoner uansett symptomer?

Forutsetninger: De får i-pad og PC med tilgang til internett for å finne informasjon.

Hva ønsker vi å se?

1. Om dere kan fordele arbeidet godt, lage et godt rollespill/presentasjon som kan være nyttig i en gitt situasjon.
2. Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av?
3. Om dere kan beskrive hva en førstehjelper kan bidra med i nødssituasjon og om dere kjenner viktige forhold ved de nevnte skadene/sykdommene.

Gruppe 2: Astronomi, tid og bevegelse

Vi er mest vant til 10-tall-systemet, men når vi regner med tid eller himmelretninger bruker vi andre tallsystemer. En sirkel (eller et kompass) er delt inn i 360 grader, ikke hundre. Et år har 365 dager, en måned har fra 28 til 31 dager, et døgn har 24 timer, en time har seksti minutter som igjen består av seksti sekunder. Sekunder deles ikke opp i sektsideler, men etter ti-talls-systemet i tideler, hunderdeler og tusendeler. Hva er årsaken til alle disse forskjellige tallsystemene, og kan vi finne forklaringene i solsystemet, vår egen kropp eller andre steder?

Oppdrag

Dere skal presentere en presentasjon for å vise jordas rotasjon rundt sola og rundt sin egen akse og om det er en sammenheng med klokka vi bruker i dag og sirkelgeometri i matematikk. Dere kan bruke bilder eller tegninger dere lager for å lage presentasjon. Hver av dere bør svare på en av de spørsmålene nevnt under og lag presentasjon sammen. Ta gjerne utgangspunkt i videoen "Why are there 360 degrees in a circle?" og hent informasjon for å lage deres egen presentasjon.

1. Hvorfor har vi 360 grader i en sirkel og 365 dager i året? Er det noe sammenheng mellom disse to? Anta at jorda bruker 360 dager å gå rundt sola. Kan du vise jordas bevegelse i sin bane rundt sola ved hjelp av en sirkel? Har det noe å si for antall dager vi har i måneden?
2. Anta at jorda bruker 360 dager å gå rundt sola. Hvor mange måneder ville det blitt dersom hver måned skal ha like mange dager? Finnes det land som har en annen kalender enn oss? (Tips: sjekk Etiopia, eller forskjellen på gregoriansk og juliansk kalender.)
3. Bruk en 360^{o} sirkel for å vise hvor mange grader jorda beveger seg i sin bane rundt sola per måned.
4. Kan du beskrive jordas bevegelse rundt sin egen akse ved bruk av en sirkel? Går det an å plassere 24 timer i den sirkelen? Hvor mange grader vil det ta for at jorda kommer tilbake akkurat på samme plass etter å ha fullført en rotasjon? Hvor mange grader vi jorda gå i en time? Hvor langt tid vil det ta for jorda å rotere en grad?
5. Hvorfor har vi 60 minutter i en time og 60 sekunder i ett minutt? Kanskje det finnes flere mulige forklaringer?

Forutsetninger: Dere får utdelt PC (-er) for å finne ut informasjon og tegnesaker, passer osv.

Hva ønsker vi å se?

1. Planleggingen – om dere har laget en god plan og at dere fordeler arbeidet på en god måte.
2. At presentasjon forklarer tydelig hva dere har jobbet med og fant ut. Det skal være forklarende for andre elever i klassen.

Gruppe 3: Ballspill/Sannsynlighet

Oppdrag

Dere skal ut på fotballbanen/idrettsbanen og prøve å treffe mål/kurv fra tre forskjellige avstander i banen. Velg én ballidrett (fotball, eller håndball eller kurvball) Hver av dere skal kaste/sparke ballen minst 10 ganger mot målet/kurven fra disse tre ståstedene. Det må være så langt hold at det er en utfordring å treffe målet. Skriv ned hvor mange ganger hver av dere treffer mål. Når en av dere sparker/kaster ball, kan en annen observere og skrive ned resultat i en tabell.

Hvis dere velger fotball: Gjenta forsøket med keeper i målet.

1. Hvor mange ganger har dere sparket/kastet ball totalt?
2. Hva er sannsynlighet for å treffe mål fra de tre avstandene for hele gruppa og for hver enkelt av dere? (Beregn eventuelt både for tomt mål og for mål med keeper.)
3. Lag figurer eller tabeller som viser hvordan sannsynlighet for å treffe mål endres med avstanden for hele gruppa og for hver enkelt av dere.
4. Kan dere knytte disse resultatene til forskjellige taktikker i lagidrett (fotball eller håndball eller kurvball) og presentere resultatet for de andre i klassen?

Forutsetninger: De får skrivesaker og PC for å lage en presentasjon. De får hjelp fra læreren for å regne ut sannsynlighet.

Hva ønsker vi å se?

1. At alle har sparket/kastet ball og bidratt til statistikken og at alle fikk notert resultatene fra sine egne prøver.
2. Dere har laget en tabell som presenterer oversikt over resultater både for dere alle og for hver enkelt.
3. Samarbeidet – at alle fikk bidra med en del av arbeidet.
4. At dere kan bestemme sannsynligheten for å treffe mål fra tre forskjellige steder.

Gruppe 4: Regnskap/Budsjett

Oppdrag

Sett opp et regnskap for din familie i en måned. Du kan gjerne ta utgangspunkt i gjennomsnittslønn for en person i Norge, som er 43.300 kr per mnd. før skatt.

Prøv å beregne alle utgifter og inntekter som for eks. mat, klær, strøm, barn, reise til jobb og skole, skatt, barnetrygd osv. Lag en oversikt som viser fordelingen av utgifter og inntekter, gjerne som sirkeldiagram. Eksempel på ekstra kostnader kan være sykdom, ferieplanlegging, selskap, bursdagsfeiring osv. Sett opp hvert deres budsjett og vis hvordan dere vil prioritere å bruke de pengene som eventuelt blir til overs i løpet av måneden eller i løpet av et år. Det er fint om det kommer fram ulike ønsker og prioriteringer.

Forutsetninger: De får bruke kalkulator, PC og skrivesaker. Det er lov å spørre eller søke etter alle slags opplysninger, men ikke sjekk inntekten til enkeltpersoner – det er ikke nødvendig i denne oppgaven.

Tid til disposisjon: Tre timer til å sette opp forslag til budsjett slik at det kan presenteres for klassen.

Hva ønsker vi å se?

1. At dere kan lage et realistisk budsjett og at dere kan prioritere – det vil si at dere kan velge hva dere kan spare på og hva dere dermed kan bruke mer til.
2. Samarbeidet – fikk alle bidra med en del av arbeidet, ble arbeidet utført effektivt og at alle gjorde viktige deler av arbeidet som de kunne lære av.
3. Produktet - Har dere laget en god presentasjon som er lett å forstå?

Pilotstudie LOCUMS, ******** februar 2018

Her er en oversikt over NTNU-prosjektet LOCUMS sine aktiviteter på ******** skole 2. og 9. februar 2018. Det er planlagt å bruke fire skoletimer (to timer hver dag) der dere skal brygge en modell av et drømmeprosjekt (rom, lekeanlegg eller lignende). Prosjektet skal dokumenteres av en av gruppedeltakerne som tar bilder og lager en presentasjon.

Oppdrag: Mitt drømme – (hus/ kjøkken/ rom/ lekeplass/ treningsplass/ idrettsplass eller lignende).

Dere skal planlegge, designe og til slutt bygge en fysisk modell av deres drømme – (hus/ kjøkken/ rom/ lekeplass/ treningsplass/ idrettsplass) – velg en av disse alternativene. Anta at dere har 150m^2 til disposisjon. Dere står fritt fram til å velge form, størrelse på gjenstander osv. til deres drømmeprosjekt. Dersom dere får tid kan dere også lage en oversikt over kostnadene for hele byggeprosjektet.

Plan for timene som blir brukt i prosjektet:

1. Dere får 4 timer totalt for å jobbe med dette prosjektet. Vi har foreslått en tidsfordeling mellom deloppgavene (se punkt 3 og 4), men fordel tida slik det passer for gruppa deres.
2. Dere skal jobbe i grupper der én har hovedansvaret for å lage en presentasjon som dokumenterer arbeidet og som sendes (se e-postadresser nedenfor) på slutten av prosjektet. Ta gjerne bilder av måling, av notater, av samarbeid og praktiske utfordringer. Lag gjerne presentasjonen som en PowerPoint, video eller på en annen måte. De andre i gruppa har hovedansvaret for planlegging og bygging av modellen.
3. Dere kan bruke de den første timen til å planlegge og designe bygget eller anlegget dere velger. Dere kan lage en skisse på papir eller bruke SketchUp eller et annet tegneprogram på PC som dere er kjent med.
4. De neste 3 timene kan brukes for å lage en realistisk modell av drømmebygget eller anlegget. Dere får utlevert byggemateriale for å bygge modellen. Dere må angi hva som ville være størrelsesforholdet dersom dere skulle lage et tilsvarende bygg i virkeligheten.
5. Til slutt kan den presentasjonsansvarlige bruke ½-1 time for å lage ferdig presentasjonen.

Hva ønsker vi å se?

Dere må passe særlig på:

1. Om dere kan lage en god modell der dere får fram størrelsen på byggverket og en oversikt over hva det vil koste å bygge det.
2. Hvilke problemer dere må løse for å lage modellen.
3. Samarbeidet –ble arbeidet utført effektivt og fikk alle gjøre viktige deler av arbeidet som de kunne lære av?
4. Om dere har laget en god presentasjon av ideene og det ferdige prosjektet.

Vi hører gjerne fra dere på e-post: shipra.sachdeva@ntnu.no eller per.eggen@ntnu.no

9.5 Interview guide for semi-structured interviews

Intervjuguiden til bruk for ** (semester), **** (år) – Elevene**
" Kulturell (/Elev)-inspirert undervisning-læring"

Navn: Alder:

Gruppe:

Innledning:

Dette skoleåret har dere vært med i et forskningsprosjekt som heter LOCUMS. Formålet med prosjektet er å observere hvordan bruk av praktiske aktiviteter innenfor elevenes egen interesse og kultur påvirker undervisning -og læringsprosessen.

Vi samler inn data gjennom spørreundersøkelser, intervju av elever og lyd -og filmopptak av klassen.

Jeg (Shipra Sachdeva) vil gjøre intervjuet etter elevprosjektet for å samle inn informasjon som jeg tror har betydning for læring i realfag.

Tema for de ulike delene av intervjuet:

1. Har du det bra på skolen og i klasserommet?
2. Interesse for å lære matematikk- og naturfag
3. Hobbyer, kulturell identitet, og praktiske aktiviteter du er glad i
4. Sosial deltakelse og kommunikasjon
5. Inkludering og aktivitet i klassen – for utenlandske elever i klassen
6. Interesse for praktiske og teoretisk rettede aktiviteter i undervisningen, synes du at de praktiske aktivitetene dere gjør i naturfag -og matematikkundervisningen er interessante og relevante (gjenspeiler realfagsbruk i ditt liv) for deg
7. Dine forventninger fra lærere, fagstoff, læringsstil og hvordan fagstoffet bør bli undervist
8. Erfaringer fra den delen av prosjektet dere selv deltok i.

Jeg (Shipra Sachdeva) er interessert i å vite dine ærlige meninger og du kan ta den tida du vil før du svarer. Det er ikke nødvendig å svare på alle spørsmålene. Alle personlige opplysninger vil bli anonymisert. Det er ingen riktige eller feil svar og de svarene du gir har ingen innflytelse på karakterene dine. Lærerne får ikke vite noe fra samtalen vi har i intervjuet.

Spørsmål etter tema 1

Skole- og klassemiljø:

- Liker du deg på skolen? Hvorfor/hvorfor ikke?
- Har du mange venner i klassen? Er det behagelig og føler du deg trygt i klasserommet? Hvis ikke, hvorfor?
- Hva tenker du at er viktig for voksenlivet og tror du at skolen hjelper til med å forberede deg på voksenlivet? Hvordan?
- Har du tillit til at det skolen velger å lære deg om er riktig for deg og du kommer til å trenge det senere i livet?
- Hvilket fag liker du best? Hvorfor?
- Hva ønsker du å lære mer om på skolen? Hvor kommer den interessen fra?

Spørsmål etter tema 2

Interesse for fagene matematikk og naturfag:

- Hva tror du er matematikk?
- Etter din mening, hva er nyttig å lære om matematikk og naturfag? Lærer du noe matematikk og naturfag på skolen som du har nytte av i reelle situasjoner? Kan du gi et eksempel?
- Når tror du at du har lært noe i matematikk? Kan du gi noen eksempel på når du følte at du har lært noen ting i matematikk? Hva kommer den følelse av?
- Er du interessert i å lære matematikk? Hvorfor/hvorfor ikke?
- Er det noe i matematikkfaget som du synes er spesielt interessant? Hvor kommer den interessen fra og hvorfor er du interessert i å lære akkurat dette?
- Er det noe i matematikkfaget som du synes er bortkastet tid? Hvis ja, hva? Hvorfor tror du at det er bortkastet? Har du noe alternativ å foreslå isteden for det temaet? Hvordan ville du ha lært foreslåtte temaer på skolen?
- Er du interessert i å lære naturfag? Hvorfor/hvorfor ikke? Hvor kommer den interessen fra?
- Er det noe i naturfaget som du synes er spesielt interessant? Hvor kommer den interessen fra og hvorfor er du interessert i å lære akkurat dette?
- Er det noe i naturfaget som du synes er bortkastet tid? Hvis ja, hva? Hvorfor tror du at det er bortkastet? Har du noe alternativ å foreslå isteden for det temaet? Hvordan ville du ha lært foreslåtte temaer på skolen?
- Får du noen problemer når du studerer matematikk? ... og naturfag? Hvilke problemer er det i tilfelle?
- Får du hjelp fra foreldre/foresatte og lærere til å forstå fagstoffet i matematikk og naturfag?
- Tror du at det du lærer om matematikk og naturfag vil være viktig for deg senere i livet? Hvis ja, hvordan og hva kan du bruke det til i livet ditt? Hvis nei, hvorfor ikke?

Spørsmål etter tema 3

Individuelt- og felles interesser, hobbyer, praktiske aktiviteter og kulturell identitet:

- Hva gjør du i fritida di? Hvilke hobbyer og interesser har du? Hvilke praktiske aktiviteter er du glad i?
- Er du godt fornøyd med sosiale livet ditt? Tror du at du klarer å kommunisere det du vil på skolen, når du omgås med familie og med vennegjengen din?
- Er du ofte sammen med vennene dine? Gjør dere lekser, studier, prosjekter sammen?
- Har du noen felles interesser med vennene dine? Hva slags aktiviteter gjør dere når dere er sammen?
- Liker og eventuelt deltar dere i hverandres individuelle interesser? Hvordan?
- Hva betyr begrepet kultur for deg?
- Hvordan vil du definere din identitet (hvem vil du si at du er/hvordan vil du beskrive deg selv)?
- Samiske/Utenlandske elever – Tror dere at deres kultur er annerledes enn de andre elever i klassen? Hvordan?

Spørsmål etter tema 4

Inkludering og aktivitet i klassen

- Synes du at du er godt inkludert i klassen? Blir du tatt imot på en god måte?
- Har du mange venner? Utenlandske eller norske?
- Hvordan blir utenlandske elevene inkludert i klassen? Har de egen gruppe?
- Hvor ofte snakker du sammen med utenlandske og/eller norske klassekameratene dine?
- Er det gjensidig interesse når norske og utenlandske elever snakker sammen?

Spørsmål etter tema 6

Praktiske aktiviteter i undervisning og dets relevans i hverdagsliv:

- Er det ofte praktiske aktiviteter i matematikk- og naturfagundervisningen?
- Er disse aktivitetene interessante for deg?
- Liker du slike praktiske aktiviteter og synes du de hjelper deg å forstå matematikk- og naturfag?
- Tror du at praktiske aktiviteter hjelper deg å forstå fagstoffet og tror du at du kan ha nytte av dem i livet utenom skolen?

Spørsmål etter tema 7

Forventninger fra lærere og fagstoffet:

- Liker du matte –og naturfagslærerne dine? Tror du at elevene kommer godt overens med lærerne? Hvorfor/hvorfor ikke?
- Får du hjelp av lærerne når du trenger det?
- Tror du at lærerne virkelig er interesserte i hva du mener og gjør? Hva gjør at du tror det du tror?
- Hva tror du er forskjellen mellom å hjelpe og å være interessert i (lærerne hjelper dem men læreren er ikke interessert i det dem gjør – kan spørre det igjen i intervju)?
- Prøver lærerne å knytte læring i klasserommet til noe som du trenger utenom skolen? Gir læreren eksemplarer på slike tilknyttinger?
- Er du fornøyd med måten lærerne underviser på? Hvorfor/hvorfor ikke? Hva vil du ha annerledes?

Spørsmål etter tema 8

- **Spørsmål relatert gruppe og aktivitetsvalg:**
  - Hvorfor var du med på ..... gruppe?
  - Hva slags forventinger hadde du fra den gruppe du valgte/ble plassert på?
  - Hvorfor er du interessert i for eks. å lage mat eller andre aktiviteter?
  - Hva er din hensikt (motiv/purpose)/din motivasjon for å lage mat eller andre aktiviteter?
  - Fikk du hjelp av noen ekspert/erfart person for å gjøre oppgaver den dagen? Hvor mye hjelp fikk du av de som veiledet dere? Var den hjelpen nyttig og du lærte noe fra det som du ikke visste fra før av? Eks. Fortalte eksperten noe mer om fordelene av disse næringsstoffene enn det som sto i boka?

- **Dine erfaringer, refleksjoner knyttet til at dere blir spurt om å velge tema dere selv er interesserte i.:**
  - Hva synes du om oppgavene dere fikk? Hvordan hadde oppgavene vært om dere fikk lage dem selv? Hvor vanskelig eller lett var de for deg? Hadde du forandret noe i oppgaven om du fikk mulighet? Hva hadde du forandret?
  - Du ga en interesse for å lære om ..... Kan du se noen sammenheng mellom din egen interesse og det dere gjorde på prosjektet dere fikk?
  - Var det noe i oppgavene dere fikk som du selv hadde interesse av å lære? (som beskrevet i spørreskjema)?
  - Var det noe i oppgavene som dere fikk som du ikke har noen interesse av?
  - Var den praktiske aktiviteten du gjorde i dette prosjekt forskjellig fra de du gjør i vanlig matte og naturfag undervisning? Hva var forskjellen?
  - Følte du at du trenger matte og naturfagkunnskap for å gjennomføre prosjektet?
  - Lærte du noe om matematikk og/eller naturfag gjennom prosjektet? Hvis ja, hva og hvis ikke, hvorfor tror du det – altså når vil du synes at du har lært noe i matte? Kan du gi ett eksempel på når du trodde at du lærte noe om matte og/eller naturfag?

- Var denne måten å lære matematikk og naturfag forskjellig fra vanlig undervisning? Hvordan?
- Fikk du noen erfaringer fra prosjektet som du selv tror var nyttige? I tilfelle, hvordan var de nyttige?
- Kan du knytte kunnskapen du fikk i prosjektet til det du er interessert i å lære/vite om? Hva slags nytte fikk du av den kunnskapen? Tror du at du kommer til å bruke den kunnskapen i virkelige livssituasjoner?
- Hvordan fungerte det for deg å arbeide i en gruppe? (opplevelser når du snakket og jobbet sammen med kameratene dine)
- Hvordan synes du samarbeidet fungerte mellom lagmedlemmene? Var alle med og hadde like mye ansvar når dere gjorde oppgaven?
- Hvis det var noen som ikke tok så mye ansvar eller ikke var like interessert og aktiv som andre, hva gjorde du og de andre i gruppen med det?
- Hvis du ikke tok kontroll på dette, tror du at det var et godt lag og lagarbeid? Hvorfor valgte du å ikke gjør noe med det?
- Tror du at du kunne bestemme over (ha autonomi, ansvar og eierskap) din egen læring og hva du ønsket å lære ut fra dine egne behov/ din egen kulturelle identitet?

- **Når de fikk gjøre ferdig oppgavene i etterkant av prosjektet:**
  - Hva var følelsen og hvordan var motivasjon hos elevene da?
  - Fikk du råd/hjelp av noen erfaren/ekspert person den dagen også?
  - Hva gjorde de andre gruppene som ble ferdig med prosjektet – (for eks. til matlagingsgruppe) – gjorde de noen beregninger etterpå – hvorfor eller hvorfor ikke?

Spørsmål etter tema 5

Intervju for innvandrere elever**:**

- Tell me about how you are doing in Norway?
- What do you miss most from your home country? Do you miss something special from your school in your homeland? Way of teaching or so?
- Do you like Norway?
- Do you like to come to school?
- Do you like to learn Norwegian? Do you feel that you learn enough to communicate with other children?
- Do you think you are different from Norwegian students or similar?
- How are you different or similar? If similar, what you think about the way of clothing, people's behavior, traditions, customs, religion and rules of the society and youth interests etc. here and in your homeland? Do you think these things are similar or different?
- Do these differences show up and influence you when you are in the class? Does it limit your possibilities to make friends, be together with other Norwegian students?
- Do these differences show up and influence the way you are taught and the way you learn when you work in groups?

- If yes, do you then want to change anything in the way you are taught/classroom settings? What would you like to change and why?
- Where is it easier to make friends?
- Which subjects do you like?
- Have you been to school in your homeland?
- If you What would you like to learn at school?
- What would you say is different at the school here and in your home country?
- What do you think about the subject mathematics? Do you like it or not?
- What within mathematics does interest you? Something at all?
- If you are interested in learning science and mathematics, how would you like to learn them at this school?

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