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VBI-2D – Road vehicle-bridge interaction simulation tool and verification framework for Matlab

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ABSTRACT

This document introduces VBI-2D, a user-friendly, efficient, and verified implementation of the vehicle-bridge interaction problem in 2 dimensions for road bridges, implemented in Matlab environment. The software is designed to allow the user to easily simulate the problem with multiple vehicles, road irregularities, and the structural configuration of choice. The theoretical analytical solution is also presented here for verification purposes. This document provides an overview of the software's main features and an introduction on how to use it. Additional guidance is provided in the user guide included in the repository. VBI-2D has been proven to be useful for studies in several research lines and is now accessible to other research groups to explore new ideas in the field of bridge dynamics under traffic loading.

Metadata

Nr	Code metadata description	
C1	Current code version	v1
C2	Permanent link to code/ repository used for this code version	https://github.com/DanielCanteroNTNU/VBI-2D
C3	Permanent link to reproducible capsule	https://codeocean.com/capsule/0,024,181/tree/v1 (Pending publication by CodeOcean. To be done once this paper is accepted)
C4	Legal code license	GNU General Public License v3.0
C5	Code versioning system used	None
C6	Software code languages, tools and services used	Matlab
C7	Compilation requirements, operating environments and dependencies	
C8	If available, link to developer documentation/manual	https://github.com/DanielCanteroNTNU/VBI-2D/tree/main/Documentation
C9	Support email for questions	daniel.cantero@ntnu.no

1. Motivation and significance

Vehicle-bridge interaction (VBI) is the phenomenon that occurs when a moving vehicle traverses a bridge. During that interaction, the responses of the vehicle and the bridge are coupled. The moving vehicle subjects the bridge to loads varying in space and time that deform the bridge. Then again, these displacements change the surface on where the vehicle is travelling, affecting the vehicle's responses. Therefore, the responses of both systems, the vehicle and the bridge, depend on each other. This phenomenon has been extensively studied by many during the last decades, resulting in seminal books by Fryba [1] and Yang et al. [2].

Nowadays, the study of VBI is an active research field, mainly because of its potential applications for structural design, assessment, and monitoring. Generally, every new study either reformulates the VBI problem and develops alternative models or is based on existing numerical implementations privately available to each research group. In addition, the correct simulation of the interaction problem is relatively complex and difficult to verify because of the lack of openly available reference solutions. There is a need of publicly available implementations of the problem and verification examples. Therefore, this document presents an open-source implementation of a VBI simulation tool and corresponding verification model.

The software VBI-2D simulates the vehicle-bridge interaction problem using a 2-dimensional model. It is implemented in Matlab and offers a wide range of modelling possibilities. Fig. 1 presents a schematic overview of the capabilities of VBI-2D software. It simulates the interaction of one or more vehicles crossing a bridge while travelling over an irregularity profile, with possible different properties for each vehicle. The vehicle model can be varied from a simple sprung mass oscillator to

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more complex vehicle configurations with multiple bodies, articulations, and axles. It is possible to generate additional sophisticated vehicle models (fully compatible with VBI-2D) with the tool VEqMon2D [3]. These vehicles can travel over an irregularity profile, which can be generated according to standards, given a specific geometry, or loaded from existing profiles. The bridge is modelled as a beam using a Finite Element formulation with optional number and type of boundary conditions. The user can also specify the properties of individual elements, which permits the representation of complex bridge geometries and conditions. The software performs the simulation of the specified traffic event travelling over the irregularity and bridge. The results are the responses of the vehicle(s) and bridge systems that can easily be retrieved by the user. In addition, the software also offers a range of calculation options and predefined graphical outputs (see Section 3).

The main contribution of VBI-2D is that it provides an open-access verified implementation of the vehicle-bridge interaction problem, that is easy to use and modify. The software democratizes the availability of such simulation tools, which in turn will enable more research groups to work in this field. In fact, it has already been used extensively to produce reliable results for a range of applications.

Arguably, the original focus of VBI studies was (and still is) to evaluate the dynamic responses of bridges under traffic loads. More in particular, it is relevant to know the additional stresses experienced by a bridge during characteristic crossing events in order to facilitate the design or assessment of existing structures. This increase due to the dynamic response of the bridge is generally studied in terms of impact factors [4] or dynamic amplification factors [5]. The original implementation of VBI-2D was used in [6] to evaluate the correctness of these factors, that generally assume that the maximum stresses and bending moments occurs at the mid-span section in simply supported bridges.

VBI-2D has also been used to explore the theoretical implications on the dynamic properties of the coupled vehicle and bridge systems. It is known that the modal properties of the bridge and vehicle(s) change during the crossing event ([1,2]). This evolution of properties was explored numerically with VBI-2D and observed under laboratory conditions in [7]. Then, in [8] this variation in dynamic properties was measured from the responses of two in-service bridges, namely a simply supported steel bridge and a 3-span continuous concrete viaduct. In here, VBI-2D was used to predict and support the analysis of these empirical results.

In recent years, the vehicle-bridge interaction phenomenon has emerged as a possibility to perform structural monitoring based on the analysis of vehicle responses while traversing the target bridge. This idea, that is called indirect, drive-by or vehicle scanning method, is a thriving research field manifested in several recent state-of-the art reviews by multiple groups [9,10,11,12] and a comprehensive book by Yang et al. [13]. Because VBI-2D also calculates the response of traversing vehicles, the software has been recently used to explore the potential of indirect monitoring. For example, in [14] the authors process the responses from vehicles via a deep autoencoder architecture and search for damage sensitive features. Then, [15] explores the potential of using multiple vehicle responses as a dataset to train a probabilistic deep neural network as a tool for damage assessment of bridges. The ideas are evaluated using simulated vehicle traffic events mimicking real operational conditions, which include additional traffic and the perturbing effects of signal noise and temperature variations.

Moreover, the study of bridge behaviour under traffic loading is relevant for bridge weigh-in-motion (BWIM) technology, which uses the responses of a bridge during vehicle crossing to infer the axle weights of the vehicle. This technology has been developed and improved over the last decades, as discussed in [16,17], and is gaining interest because its potential use for vehicle-assisted monitoring applications [18]. Towards that end, VBI-2D was used in [19] to validate the concept of a virtual axle for BWIM systems to track changes in the structural behaviour. Also, in [20], the author proposed the possibility of combining multiple measured bridge responses into an approximation of the loading function. These ideas have been validated numerically with VBI-2D for different bridge configurations, traversed by 5-axle trucks travelling over a road profile.

Therefore, the proven record of past uses of VBI-2D for a variety of topics shows the usefulness and relevance of the software that is openly published here. The following document provides first a review of the theoretical background and analytic verification of the simulated problem. In Section 3, the software architecture and main functionalities are broadly discussed. However, for additional information, the reader is referred to the user guide document included in the repository of the software. Finally, Section 4 showcases one particular example with a sample of default graphical outputs produced by VBI-2D.

2. Modelling background

The vehicle-bridge interaction problem describes the coupled behaviour of a bridge and a moving vehicle, while possibly encountering irregularities on the travelling surface. In its simplest form the problem can be defined as shown in Fig. 2. The vehicle, represented as a single oscillator, consists of a mass m_V moving with variable speed $v(t)$ connected to the road surface $h(t)$ at the contact point by a spring and dashpot system, characterized by stiffness k_V and viscous damping c_V.

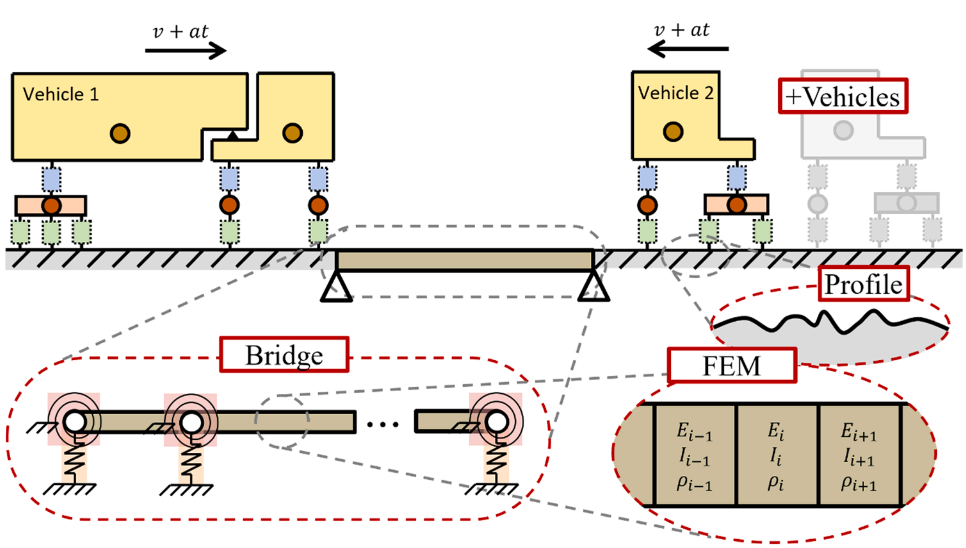

Fig. 1. Graphical description of VBI-2D simulation software.

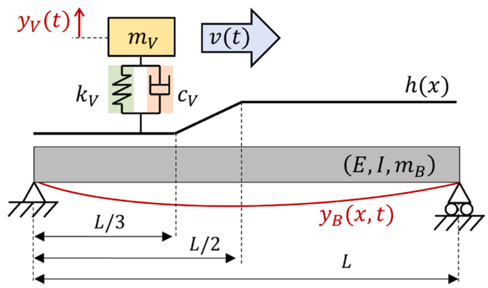

Fig. 2. Description of single moving oscillator travelling over a road profile and traversing a Euler-Bernoulli beam.

The bridge, represented as a simply supported Euler-Bernoulli beam, is defined in terms of its length L, elastic modulus E, second moment of area I, and mass per unit length m_B. The solution to the problem gives the vertical motion of the sprung mass $y_V(t)$ and the deformation of the beam $y_B(x,t)$.

The analytical formulation of the moving oscillator (or sprung mass model) moving over a beam has been presented before, being [2,21] two good examples. Nevertheless, for completeness and verification purposes it is reformulated here for the presented problem statement and adopted variable definitions in Fig. 2.

The vehicle position along the beam is defined in terms of uniformly accelerated forward motion (Eq. (1)), where the instantaneous velocity of the vehicle has been termed v_t.

$$v(t) = v_0 + a\ t = v_t \tag{1}$$

The beam's deformation can be expressed as in Eq. (2), an infinite series of the mode shapes Φ_i multiplied by their corresponding generalized coordinates η_i. This can be approximated by considering the first n modes only, which is deemed accurate if sufficient modes are included.

$$y_B(x,t) = \sum_{i=1}^{\infty} \Phi_i(x)\ \eta_i(t) \approx \sum_{i=1}^{n} \Phi_i(x)\ \eta_i(t) \tag{2}$$

With these definitions it is possible to formulate the equations of motion of the coupled system. These $N = n + 1$ equations can be rearranged into a system of second order differential equations expressed in matrix form (Eq. (3)) for the vector of unknown variables (Eq. (4)). The extended formulation of the coupled system (mass M, damping C, and stiffness K) matrices and force vector F are provided in the Appendix. The step-by-step derivation of the equations of motion and system matrices is provided in a separate 8-page document included in the folder *Documentation* in the software's repository. This analytical formulation of the vehicle-bridge interaction problem has been implemented and solved numerically to act as the reference solution to verify VBI-2D.

$$M\ddot{q} + C\dot{q} + Kq = F \tag{3}$$

$$q = \begin{pmatrix} y_V & \eta_1 & \eta_2 & \dots & \eta_n \end{pmatrix}^T \tag{4}$$

On the other hand, VBI-2D formulates the same problem by describing the beam using a finite element representation. The structure is discretised using beam elements with two degrees of freedom for each node (vertical displacement and rotation). The vehicle(s) equations of motion are combined with the equations for the beam by means of the coupling procedure described in [22] and [23]. The result is another system of second order differential equations with time-varying system matrices that change with the vehicle(s) position. This is solved by direct numerical integration with the Newmark-β numerical scheme as presented in [24].

To evaluate the correctness of VBI-2D, its numerical results are compared to the solution based on the analytical formulation. As indicated in [21] the case of a moving oscillator includes the main characteristics of the VBI problem and is therefore deemed appropriate to verify VBI-2D. The same problem solved via two different tools should render the same solution. More precisely, the case presented in Fig. 2 is simulated for a sprung mass model (with properties taken from [2]) moving at 25 m/s traversing a 25 m simply supported beam (with numerical properties taken from [25]), while travelling over a ramp of height 1 cm. In order to progressively verify different aspects of the problem, four separate verification cases are studied, directly comparing VBI-2D results to the analytical formulation solution in Fig. 3. The first case corresponds to the sprung mass (without damper) traversing the beam over a smooth profile, which verifies the coupling procedure. Next, the simulation is repeated including the ramp, confirming the correct inclusion of the road profile irregularities in the simulation software. Then, the effect of vehicle damping was included, ensuring that VBI-2D correctly accounts for the additional coupling terms that appear in the problem. Finally, the software is verified for the case of the vehicle moving with accelerated forward motion (a=10 m/s^2). All four verification cases in Fig. 3 show matching results between the simulated and the reference solution, concluding that the VBI-2D provides correct solutions. Note, for the sake of reproducibility, the user can find in the software's repository the implementations to: obtain the reference solutions from the analytical expression (script *V01*), run VBI-2D with the particular properties for each of the verification cases (script *V02*), and to plot Fig. 3 (scrip *V03*).

For the simulation of the vehicle-bridge interaction, there exist several options to effectively couple the behaviour of both systems. Broadly speaking, the alternatives can be categorized into coupled or iterative. The coupled solution updates the system matrices with the position of the vehicle for every time step in the numerical integration procedure. On the other hand, iterative methods calculate the response of the vehicle and the bridge multiple times, while updating the compatibility conditions in every iteration until certain convergence criteria has been reached. However, note that the modelling of VBI is an active research field, and there exist a variety of methods and variations to achieve the interaction, as discussed in recent reviews [26] and [27]. Nevertheless, any solution procedure should tend to the correct solution. Eventual discrepancies might be attributed to the choice of time step or spurious effects of the numerical calculations. VBI-2D has implementations for both solution paradigms, i.e., coupled and iterative, delivering the same results. Extended explanations of the alternative interaction procedures in VBI-2D can be seen in Section 3.2.

3. Software description

3.1. Software architecture

VBI-2D simulates the vehicle-bridge interaction for traffic events with multiple vehicles traveling over a road profile and traversing a bridge. The user has the possibility to configure the simulation to

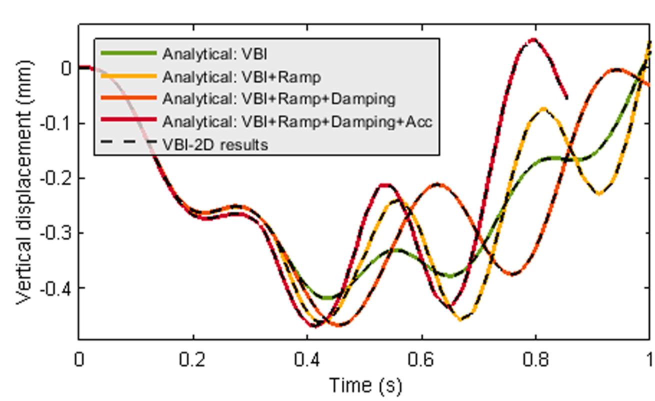

Fig. 3. Comparison of results between VBI-2D and the reference solution.

represent a wide variety of cases. The traffic events can be composed of several vehicles, each with different models, properties, and travelling patterns. The bridge can be defined with the desired mechanical properties supported by multiple types of boundary conditions.

The software has been designed to be easy to use and modify. It has been implemented for Matlab environment and has a straightforward workflow as indicated in Fig. 4. To run a user-defined simulation it is only necessary to define the appropriate values and options at the start of *A01_VBI_input_and_run.m*. In particular, the user can specify several vehicles, where the parameters and options for vehicle *i* are saved in the indexed structure variable *Veh(i)*. The software includes a selection of already implemented vehicle models (e.g. sprung mass, quarter-car, 2-axle vehicle, 5-axle truck with articulation). The user can obtain additional vehicle models by generating the Matlab functions that generate the appropriate vehicle system matrices using VEqMon2D tool [3]. Then the profile can be defined using a selection of already implemented shapes and irregularities or load it from an existing file. Next, the user can specify the bridge properties, boundary conditions, number of elements and condition of the bridge. Finally, there exist a range of calculation options that allow the user to specify the time step to use, solution procedure, desired graphical output, and other relevant options for the simulation.

Upon execution of the main script, the software runs *A02_Calculations.m*, which the user does not need to edit. This script performs all calculations necessary to obtain the vehicle(s) and bridge responses. Essentially, it processes all the inputs, defines relevant auxiliary quantities, creates the corresponding vehicle and bridge models, performs the modal analysis of the systems, executes the interaction procedure, solves the problem numerically by direct integration, and post-processes the results to obtain relevant quantities for the user. All this information is conveniently stored in the structure variable *Sol* in the workspace. In addition, VBI-2D also executes the script *A03_Results_plotting.m* that produces all the graphical outputs requested by the user.

3.2. Software functionalities

In VBI-2D the user can simulate user-defined models via adaptable inputs that enable the definition of a wide range of varied and complex configurations. The main options available to the user are summarized graphically in Fig. 5. Each of these functionalities are further presented next. However, note the user is referred to the *User guide* included in the repository for extended guidance.

Vehicle: Multiple vehicles of different types with different properties and travel patterns can be defined. One possible configuration is called a

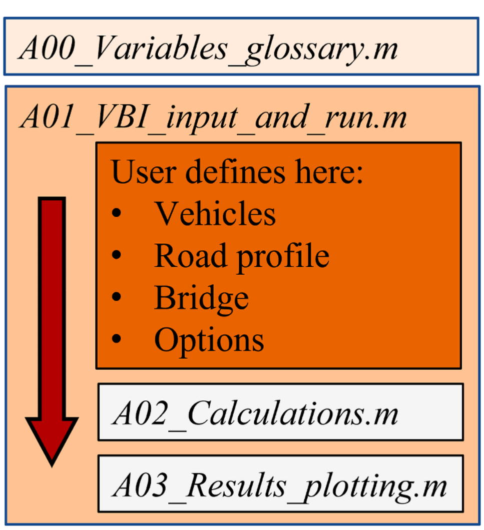

Fig. 4. VBI-2D workflow and corresponding script names.

traffic event, which might consist of vehicles travelling in opposite directions meeting on the bridge. The folder *Vehicle_equations* contains a sample of vehicle models which are readily available, that describe them in terms of their components' degrees of freedom (vertical and rotational displacements). Additional vehicle models could be generated with VEqMon2D [3], which is a framework to generate Matlab functions for the equations of motion of 2D vehicles with multiple bodies, articulations, and axle configurations. For example, the 5-axle truck model shown in Fig. 5 corresponds to the vehicle model *Vehicle_2A3_G_1_1_3.m* (Body 1 with 2 axles, articulation, and Body 2 with one axle group with 3 axles). Therefore, with all these options it is possible to define any desired traffic loading on the bridge. For that, the user needs to specify the model type, mechanical properties, geometry, and travel pattern (initial position, direction, speed, and acceleration) for each vehicle. The reference system for the vehicle position definition is centred at the left end of the bridge, where negative values of the x-coordinate correspond to a location to the left of the bridge.

Profile: In addition to the case without any irregularity on the road (called smooth), there exist several types of profile shapes available to the user. The user can easily define a road profile with specific shapes such as a step, a ramp, or a sinusoidal geometry, with the desired location and magnitude. However, arguably the most relevant profile type for VBI simulations is one that resembles actual road irregularities. That can be generated according to the ISO-8608 [28] specification. VBI-2D generates instances of random profiles based on the power spectral densities as in the standard, which are categorized from Class A to Class H depending on the level of roughness (being Class A the smoothest). In addition, the user can also load profiles from existing files, such as measured road irregularities. VBI-2D models the contact interface between wheels and road (and bridge) as single contact points, while in realty the interface is a small surface due to the tyre's deformation. In an attempt to roughly approximate the effect of an actual tyre contact patch, VBI-2D has an option to apply a moving average filter to the profile, as suggested in [29]. Finally, note that the profile might be longer than the bridge, to include the approach distances of the vehicles before they reach the bridge.

Bridge: The finite element model of the bridge is defined in terms of geometrical and mechanical properties of the beam and mesh (see Section 2), which can be specified in the input. For the user's convenience, VBI-2D also includes predefined properties taken from [25], deemed to represent generic concrete bridges with 9 m to 43 m long spans. Then, the supports can be defined by several boundary conditions with vertical and/or rotational springs. These are designated in the model with arrays indicating their location, vertical stiffness, and rotational stiffness, where entries with *Inf* value are possible and correspond to fixed boundary conditions. This allows the user to define a wide range of structural configurations, for example multi-span continuous bridges. In addition, the user has the possibility to modify the properties of particular beam elements and boundary conditions. This option has been called 'Damage' in the software because it was originally designed to represent damage in a bridge, for example, by reducing the stiffness of one single element. Nevertheless, the possibility of easily defining individual properties of elements and boundary conditions enables the representation of a wide range of structural configurations. For example, as shown in [30], it is possible to model a cable stayed bridge representing the cables as vertical sprung supports, or a cantilever bridge with variable cross-sections by using different properties for each beam element.

Solution procedure: The vehicle(s) and beam models interact during a crossing event. This interaction is achieved by either directly coupling the equations of motion or solved iteratively. VBI-2D offers three alternative but equivalent procedures to achieve this interaction, as shown conceptually in Fig. 5. (1) The coupled procedure updates the system matrices in each time step [23]. (2) Iterations of full simulations, where the vehicles are simulated first for a given profile to obtain the contact forces, which in turn are applied to the bridge [31]. The road

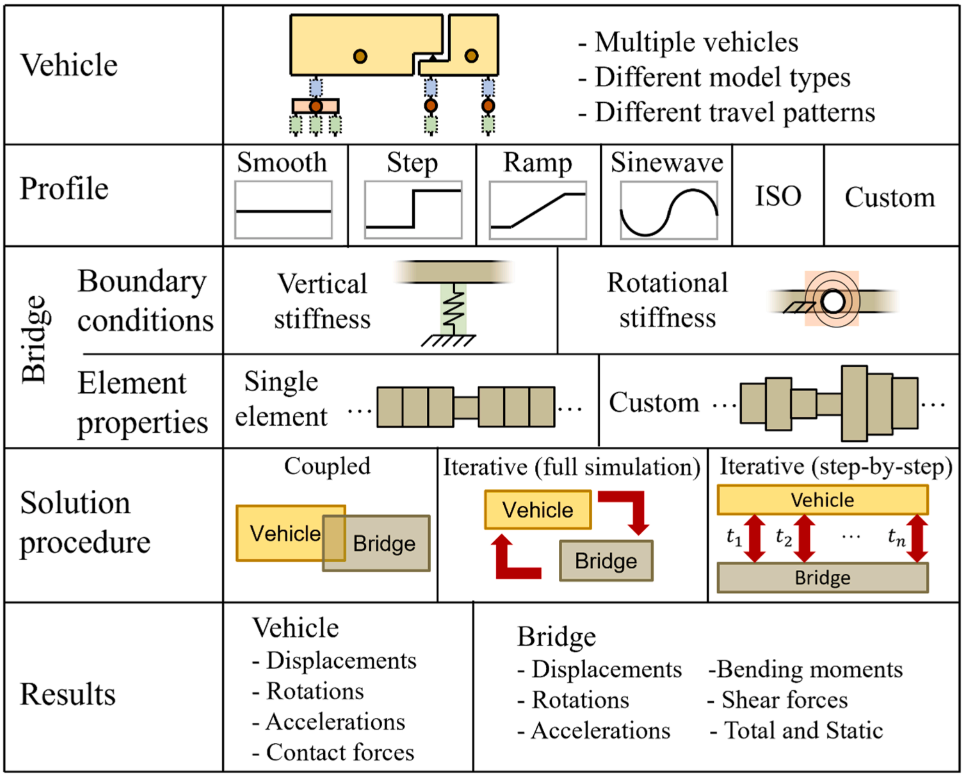

Fig. 5. Main features of VBI-2D.

profile and contact forces are updated in each iteration and repeated until convergence is reached. (3) Similarly, the same iterative procedure is performed but repeated within every time step of the numerical solver. All three procedures give the same result. However, note that VBI-2D has been optimized to solve the interaction via the coupled procedure (1), which is also set as the default.

Results: All the results of a simulation are conveniently stored in the workspace. This information is organized into structure variables, with the results for the vehicle *i* and the beam in *Sol.Veh(i)* and *Sol.Beam* respectively. For example, the user can retrieve the time history of the beam's bending moment at every node from *Sol.Beam.BM.value_xt*.

In addition to the features highlighted above, VBI-2D has more interesting content and options:

- A 28-page user-guide for first time users of the software that explains in greater detail how to use the software, define the inputs, work with the outputs, and configure the various available options.
- A glossary listing all the variables used in the software, either as input, output, option, or auxiliary variable, followed by short descriptions (in *A00_Variables_glossary.m*).
- A document with the step-by-step derivation of the analytical expressions of the VBI problem for a sprung mass model (shown in Fig. 2).
- Verification scripts to compare the software's solution to the analytical results
- The software includes a set of reasonable default values for the various settings and options, to minimize the number of definitions required by novice users.
- Option to switch on/off the interaction between vehicle(s) and the bridge, which enables the simulation of the moving load problem.
- Graphical outputs of the simulation results
- Plus, other minor features and options available, reported in the user guide.

It is worth noting that VBI-2D has been developed as a research tool. The implementations have been optimized for Matlab environment to the best of the author's capabilities. The result is a computationally efficient tool, capable of simulating single vehicle events in seconds. Moreover, the implementation has been designed to keep the script accessible and understandable, which should allow the research community to adopt, improve, and modify it according to their research needs. On the other hand, the software has some inherent limitations, which arguably could be listed as: it represents the problem in 2 dimensions, no horizontal forces or deformations are included in the model, and it is not possible to model a moving mass in direct contact with the bridge (This is more relevant for railway bridges and that has been implemented in [32]).

4. Illustrative examples

To illustrate the capabilities of VBI-2D, some simulation results of a traffic event crossing a bridge are shown in Fig. 6. This event is composed of 2 vehicles travelling in opposing directions, being the first vehicle a 5-axle articulated truck moving at constant speed (90 km/h), while the other is a 2-axle vehicle with accelerated forward motion (initial speed 70 km/h, acceleration 10 m/s^2). The mechanical properties of both vehicles are taken from [15] and are travelling over a Class A road profile. The bridge is a 25 m long simply supported beam. Fig. 6 shows a sample of possible graphical outputs that can be easily requested after the simulation is completed. It shows time history responses of the vehicle and bridge, with superimposed information like axle position in time (see Beam deformation plot) and corresponding static responses (see Beam BM plot). The reader can reproduce this simulation and generate the results in Fig. 6 with the script *E01_Example.m* included in the repository.

5. Impact

The software VBI-2D is a robust, efficient, and verified implementation of the VBI problem in 2 dimensions that has been a useful

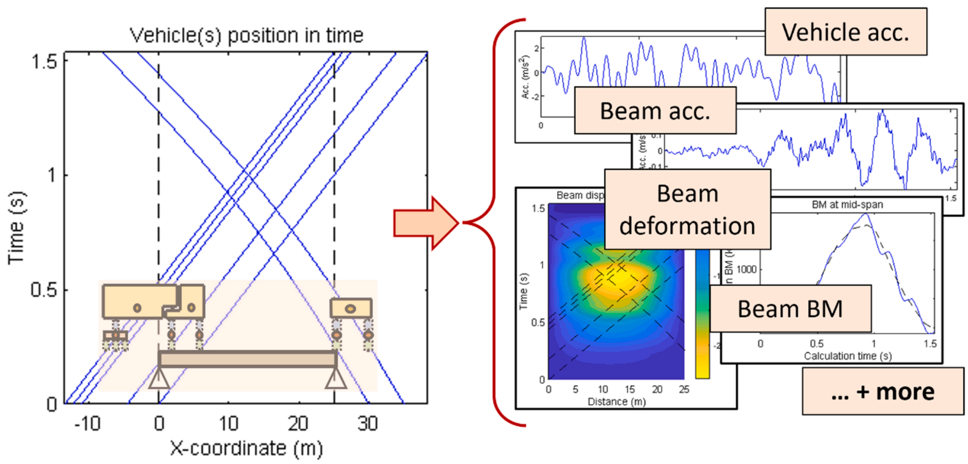

Fig. 6. Illustrative example of VBI-2D simulation of crossing event with two vehicles and a sample of graphical outputs.

resource in several investigations in the past. It offers the possibility to simulate a wide range of situations considering different traffic events, road conditions, and bridge configurations. VBI-2D is publicly available in the repository, together with additional supportive resources and documentation, for its readily adoption by the research community. Ultimately, the goal of VBI-2D is to facilitate the study of the VBI problem to a broader community. It is expected that it will foster advances in related fields, such as bridge dynamics due to traffic loading, structural monitoring, and damage detection, among others.

6. Conclusions

This document has presented the software VBI-2D, which is a Matlab implementation to simulate the interaction between moving vehicles and a bridge. The simulation results provided by the software are verified against the theoretical analytical solution, which has been introduced here and is further elaborated and implemented in the software's repository. Furthermore, the user is presented with the software's architecture and main functionalities. It was highlighted that VBI-2D offers the possibility to easily define user-defined loading events with multiple vehicles, include the effect of road irregularities, and model a wide range of structural configurations to represent the bridge. The software includes additional configurations and options that have been introduced here and are discussed further in the user guide. The provided software constitutes and accessible implementation of the VBI problem that can readily be used to explore new ideas in the field of bridge dynamics under traffic loading.

7. Appendix

System matrices in Eq. (3) considering the first n modes of vibration of the beam.

$$M = \begin{pmatrix} m_V & 0 & 0 & \dots & 0 \\ 0 & m_B & 0 & \dots & 0 \\ 0 & 0 & m_B & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & m_B \end{pmatrix} \tag{5}$$

$$C = c_V \begin{pmatrix} 1 & -\Phi_1 & -\Phi_2 & \dots & -\Phi_n \\ -\Phi_1 & \Phi_1^2 & \Phi_1\Phi_2 & \dots & \Phi_1\Phi_n \\ -\Phi_2 & \Phi_2\Phi_1 & \Phi_2^2 & \dots & \Phi_2\Phi_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\Phi_n & \Phi_n\Phi_1 & \Phi_n\Phi_2 & \dots & \Phi_n^2 \end{pmatrix} \tag{6}$$

$$K = K_k + K_c \tag{7}$$

$$K_k = k_V \begin{pmatrix} 1 & -\Phi_1 & -\Phi_2 & \dots & -\Phi_n \\ -\Phi_1 & m_B\omega_{B,1}^2 + \Phi_1^2 & \Phi_1\Phi_2 & \dots & \Phi_1\Phi_n \\ -\Phi_2 & \Phi_2\Phi_1 & m_B\omega_{B,2}^2 + \Phi_2^2 & \dots & \Phi_2\Phi_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\Phi_n & \Phi_n\Phi_1 & \Phi_n\Phi_2 & \dots & m_B\omega_{B,n}^2 + \Phi_n^2 \end{pmatrix} \tag{8}$$

$$K_c = v_t c_V \begin{pmatrix} 0 & -\Phi_1' & -\Phi_2' & \dots & -\Phi_n' \\ 0 & \phi_1\phi_1' & \phi_1\phi_2' & \dots & \phi_1\phi_n' \\ 0 & \phi_2\phi_1' & \phi_2\phi_2' & \dots & \phi_2\phi_n' \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \phi_n\phi_1' & \phi_n\phi_2' & \dots & \phi_n\phi_n' \end{pmatrix} \tag{9}$$

$$F = \begin{pmatrix} k_V h + c_V \dot{h} \\ (m_V g - k_V h - c_V \dot{h})\Phi_1 \\ (m_V g - k_V h - c_V \dot{h})\Phi_2 \\ \vdots \\ (m_V g - k_V h - c_V \dot{h})\Phi_n \end{pmatrix} \tag{10}$$

Where $\Phi_i = \Phi_i(x_V, t)$ is the i th mode of vibration value at the location of the vehicle x_V and Φ_i' its first spatial derivative. Also, g is the gravity acceleration and $\dot{h}$ the first time-derivative of the road profile.

CRediT authorship contribution statement

Daniel Cantero: Conceptualization, Software, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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