WEATHER OPTIMAL DYNAMIC POSITIONING OF UNDERACTUATED AUVS USING OUTPUT FEEDBACK CONTROL

Jon E. Refsnes [*1] **Asgeir J. Sørensen** [*]
Kristin Y. Pettersen [**]

[*] *Department of Marine Technology*
Norwegian University of Science and Technology (NTNU)
NO-7491 Trondheim, Norway
[**] *Department of Engineering Cybernetics, NTNU*

Abstract: In this paper we present an output feedback control system for dynamic positioning of underactuated AUVs. The control objective is to maintain a desired distance to the target while orienting the vehicle towards the target at all times. The vehicle's location on the sphere circumference around the target can be arbitrary. In fact, this position is determined by current loads acting on the vehicle. Hence, the control scheme presented is a weather-optimal system in the sense that the vehicle automatically will orient itself towards both the target and the current. The complete output feedback system is proven ULAS by Lyapunov theory. Furthermore, by analyzing the inherent dynamics of the controller for the unactuated states, it is shown that the sway and heave velocities converge to a bounded set. A case study on the Minesniper shows satisfactory performance in relatively harsh current conditions.

Keywords: AUV, weather optimal DP, output feedback control, underactuated vehicles

1. INTRODUCTION

Subsea installations entail various demands for survey, inspection and repair work executed by unmanned devices such as AUVs/ROVs. A common method is to use a fully actuated ROV to complete the various tasks. These vehicles are usually energy demanding and they have often poor range and speed capabilities. With a growing number of subsea installations, it is likely to believe that properties such as low cost, fault-tolerant, speed and range capabilities will become more emphasized for survey and inspection missions.

The control problem addressed in this paper is motivated by the Minesniper developed by Kongsberg ASA, Norway. The AUV/ROV is a low cost, torpedo shaped underwater vehicle designed for rapid mine hunting and destruction. There are, however, several additional applications that may be suitable for this device, such as inspection and survey. The vehicles carries a sonar and a video camera in order to detect and visualize the target. Since the vehicle is designed with focus on low cost, the number of on-board sensors and actuators is limited. Nevertheless, by including an observer providing velocity estimates and carefully designing the control system, a high degree of manoeuvrability and tracking accuracy is obtainable.

For industrial related purposes, the control scheme presented in this paper can be useful for several reasons. First, regarding important properties such as reliability and fault-tolerant control. A mission may still be completed even though certain sensors or control actuators malfunction. Second, utilizing this approach may reduce the costs and the design complexity since the amount

[1] This work is sponsored by NTNU and Kongsberg ASA, Norway.

of on-board equipment is limited. Finally, this method minimizes the energy consumption during the dynamic positioning (DP) operation. This increases the endurance, which is an important property for underwater vehicles.

Control and stabilization of underactuated AUVs have been studied by numerous authors over the last decade. Aguiar and Pascoal (2002) have proposed a controller designed for horizontal DP and way-point tracking of the underactuated vehicle SIRENE in the presence of ocean currents. Exponential stabilization of the position and attitude of an underactuated AUV is presented in Pettersen and Egeland (1999). Inspired by the weather optimal control system designed for surface vessels and rigs in Fossen and Strand (2001), we propose an output feedback controller scheme for DP of underactuated AUVs, which is proven locally asymptotically stable. The unactuated states are proven bounded by analyzing the inherent dynamics of the controller, which method is inspired by Fossen *et al.* (2003). Results indicates that the control scheme designed based on the modified control plant model provides satisfactory performance. Furthermore, the target position does not have to be fixed. Simulations show that by moving the target location at low rates, accurate positioning is maintained.

2. THE CONTROL PLANT MODEL

In this section we derive the control plant model, which the observer and controller design will be based on. It is well known that the process plant model of an underwater vehicle, see e.g. Fossen (2002), exhibits highly nonlinear and coupled dynamics increasing the complexity in the controller and observer design. However, for the control objective considered in this paper, there are several valid assumptions that can be applied.

A.1: The velocities are generally small in DP operations. In particular, the angular velocities are sufficiently small such that the nonlinear Coriolis forces and moments become negligible.

A.2: When the vehicle is within some radius of the target, the distance and the bearing between the target and vehicle are measured. By the use of sonar and/or video equipment, these measurements can be obtained.

A.3: The pitch angle is limited by $|\theta(t)| < \frac{\pi}{2}$. For most underwater vehicles this is realistic given the inherent restoring moments preventing the vehicle from large pitch angles.

Based on the these assumptions, we consider the following control plant model for DP of an underactuated AUV

$$\dot{\zeta} = R\nu \tag{1a}$$

$$M\dot{\nu} + D(\nu)\nu + g(\Theta) = \tau + b \tag{1b}$$

$$\dot{b} = -T^{-1}b + Bn \tag{1c}$$

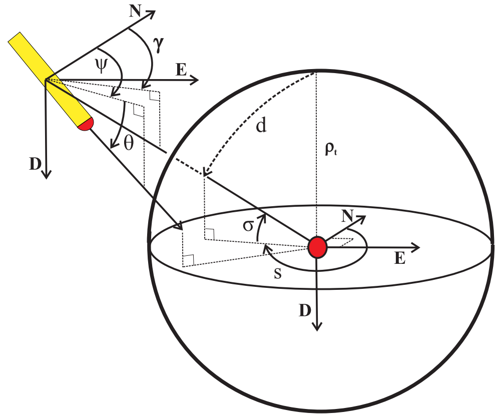

Fig. 1. Geometrical variables used in 3 dimensional DP of AUVs.

where $\zeta = [\rho, s, d, \theta, \psi]^T$. Figure 1 shows the kinematics of the strategy used in this paper, both horizontally and vertically. The variables s [rad] and d [rad] denote the horizontal and vertical location on the desired circumference of the target sphere with radius ρ_t. $M, D(\nu) \in \mathbb{R}^{5\times5}$ are the mass and damping matrices. These are assumed to be diagonal since the diagonal terms are much larger than the off-diagonal. Moreover, the damping is dominated by linear terms. Nevertheless, for slender vehicles, nonlinear damping, in surge in particular, may still be decisive at low speeds. This, however, does not undermine A.1. The gravity vector $g(\Theta)$ is a function of the Euler angles $\Theta = [\theta, \psi]^T$. The bias b is modelled as a Markov process where T is a diagonal matrix of positive time-constants. The bias model is driven by some bounded noise n with a scaling matrix B. It is included to compensate for unmodelled dynamics and environmental disturbances. The velocity vector is given by $\nu = [u, v, w, q, r]^T$, which is defined in the body-frame. Roll is neglected assuming that the vehicle is self-stabilizing. The transformation matrix R yields

$$R = F(\rho_d)\begin{bmatrix} -c\mu c\beta & s\mu & -c\mu s\beta & 0 & 0 \\ -s\mu c\beta & -c\mu & -s\mu c\beta & 0 & 0 \\ -s\beta & 0 & c\beta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{c\theta} \end{bmatrix} \tag{2}$$

$$c(\cdot) = \cos(\cdot), \quad s(\cdot) = \sin(\cdot), \quad c\theta \neq 0$$

$$\mu = \psi - \gamma, \quad \beta = \theta - \sigma$$

$$F(\rho_t) = \mathrm{diag}\left\{1, I_{2\times2}/\rho_t, I_{2\times2}\right\}$$

Note that R is non-singular under A.3, and provided that the desired radius ρ_t is non-zero. In this paper we consider underactuated vehicles with the control vector

$$\tau = [\,\tau_u \;\; 0 \;\; 0 \;\; \tau_q \;\; \tau_r\,]^T \tag{3}$$

2.1 Preliminaries

Defining the following function $d(y) \triangleq D(y)y$, we have by using the mean value theorem that

$D(x)x - D(y)y = \frac{\partial}{\partial z} d(z)\big|_{z=z_0} (x-y)$ where z_0 is on the line segment joining x and y. We apply the following assumption:

A.4: There exists a constant $\delta_m \in \mathbb{R}_+$ such that

$$\|\delta(z)\| \triangleq \left\| \frac{\partial}{\partial z} d(z)\Big|_{z=z_0} \right\| > \delta_m > 0$$

This implies that the hydrodynamic damping includes a linear term, e.g. $D(\nu)\nu = D_l\nu + D_{nl}(\nu)\nu$ where $D_l > 0$ is the linear damping matrix.

The following property yields:

P.1: The transformation matrix satisfies the following

$$\|R\| \leq R_M, \ \|R^{-1}\| \leq \bar{R}_M \tag{4}$$

where $R_M, \bar{R}_M \in \mathbb{R}_+$ under A.3.

We use the following notation in this paper. For any matrix $A(x) = A^T(x) > 0$ for all x, A_m and A_M denote the minimum and maximum eigenvalue of $A(x)$ respectively.

3. OBSERVER DESIGN

Since the velocity vector ν is not measured, an observer is needed to provide velocity estimates for feedback in the controller. Inspired by Celani (2005), we propose the following observe

$$\dot{\hat{\zeta}} = R\hat{\nu} + \lambda L\tilde{\zeta} \tag{5a}$$
$$M\dot{\hat{\nu}} + D(\hat{\nu})\hat{\nu} = \tau + R^{-1}\hat{b} + \lambda^2 MR^{-1}K\tilde{\zeta} \tag{5b}$$
$$\dot{\hat{b}} = -T^{-1}\hat{b} + \kappa\lambda^2 K_b\tilde{\zeta} \tag{5c}$$

where $L, K, K_b \in \mathbb{R}^{5\times5}$ are positive definite and diagonal matrices. The constants $\lambda, \kappa > 0$ are suitable scalars, and the error vectors yield $\tilde{\zeta} = \zeta - \hat{\zeta}, \tilde{\nu} = \nu - \hat{\nu}, \tilde{b} = b - \hat{b}$. To avoid technicalities using both the Earth and body-frame, as in (5), we make the following change of coordinates. Let

$$\nu_\zeta = \dot{\zeta} = R\nu, \quad b_\zeta = Rb$$
$$M^* = R^{-T}MR^{-1}, \quad \delta^*(z) = R^{-T}\delta(z)R^{-1}$$

Then, we define the error variables $x_1 \triangleq \frac{\tilde{\zeta}}{\lambda}$, $x_2 \triangleq \frac{\tilde{\nu}_\zeta}{\lambda^2}$, $x_3 \triangleq \frac{\tilde{b}_\zeta}{\kappa\lambda^2}$, which gives the following error dynamics

$$\dot{x}_1 = \lambda x_2 - \lambda L x_1 \tag{6a}$$
$$\dot{x}_2 = -M^{*-1}\delta^*(z)x_2 + \kappa M^{*-1}x_3 - \lambda K x_1 \tag{6b}$$
$$\dot{x}_3 = -T^{-1}x_3 - \lambda K_b x_1 \tag{6c}$$

It is assumed that the noise n is zero since the bias estimator is driven by estimation errors, (Fossen, 2002).

P.2: The mass matrices M, M^* are positive definite.

Let $x_o \triangleq [x_1^T, x_2^T, x_3^T]^T$ denote the complete estimation error vector and $\chi > 0$ a suitable constant.

Proposition 1. The origin $x_o = 0$ of the observer error dynamics (6) is globally exponentially stable (GES) under A.1-4 and if

$$\chi < \frac{2\lambda K_m L_m}{K_{bM}}, \ \lambda < \frac{2}{T_M K_{bM}}, \ \kappa < \frac{2\delta_m^* M_m^*}{M_M^*} \tag{7}$$

Proof: Consider the following Lyapunov function candidate

$$V_o = \frac{1}{2}x_1^T K x_1 + \frac{1}{2}x_2^T x_2 + \frac{\chi}{2}x_3^T x_3 \tag{8}$$

Differentiating (8) along the state trajectories gives

$$\begin{aligned}\dot{V}_o = &x_1^T K(\lambda x_2 - \lambda L x_1) + \\ &x_2^T(-M^{*-1}\delta^*(z)x_2 + \kappa M^{*-1}x_3 - \lambda K x_1) \\ &+ x_3^T\chi(-T^{-1}x_3 - \lambda K_b x_1)\end{aligned} \tag{9}$$

Then, by completing the squares $\dot{V}_o$ is upper bounded as follows

$$\begin{aligned}\dot{V}_o \leq &-\lambda(K_m L_m - \chi K_{bM}/2)\|x_1\|^2 \\ &-(\delta_m^*/M_M^* - \kappa/(2M_m^*)\|x_2\|^2 \\ &-\chi(1/T_M - \lambda K_{bM}/2)\|x_3\|^2\end{aligned}$$

Hence, if (7) holds, we have that

$$\dot{V}_o \leq -a\|x_o\|^2, \ a > 0 \tag{10}$$

Consequently, it follows by Lyapunov theory that the origin $x_o = 0$ is GES (Khalil, 2002).

□

Remark 1. Note that the globalness is given with respect to the chosen coordinate frame. It is not topologically possible to obtain results that are global in $\mathcal{SO}(3)$ using any coordinate frame of $\mathcal{SO}(3)$ like Euler angles, Euler parameters, Euler-Rodrigues parameters or similar. Due to the topological properties of $\mathcal{SO}(3)$ these representations will either have one singularity or two equilibrium points, something which precludes global results on $\mathcal{SO}(3)$. The results in this section are thus considered global in the chosen coordinate frame.

4. OUTPUT FEEDBACK CONTROL

In this section we design a nonlinear controller utilizing the observer backstepping technique (Krstić *et al.*, 1995). This is carried out in two steps. First, we design a controller assuming that the vehicle is fully actuated. The complete control system, consisting of the observer error dynamics and the tracking error dynamics are then proven stable by Lyapunov theory. Furthermore, since we are considering underactuated vehicles, we analyze the inherent dynamics of the controller which arise because of the unactuated states i.e. sway and heave. It will become clear that due to hydrodynamic damping in all degrees of freedom, the velocities of the unactuated states are bounded.

This approach is inspired by work presented in Fossen *et al.* (2003).

The control objective is stated as

$$\rho(t) \to \rho_d(t),\ \theta(t) \to \theta_d(t),\ \psi(t) \to \psi_d(t) \quad (11)$$

as $t \to \infty$.

4.1 The Guidance System

The desired trajectories are generated by a reference model consisting of a 1st-order low-pass filter cascaded with a mass-damper-spring system, see Fossen (2002, ch.5). The reference input is $[\rho_t, \gamma(t), -\sigma(t)]^T$, and the output are the following continuous and bounded signals

$$\omega_{d1} = [\rho_d, \theta_d, \psi_d]^T \quad (12a)$$

$$\omega_{d2} = [u_d, q_d, r_d]^T = \bar{R}^{-1}\dot{\omega}_{d1} \quad (12b)$$

$$\dot{\omega}_{d2} = [\dot{u}_d, \dot{q}_d, \dot{r}_d]^T \quad (12c)$$

Note that the angles $\gamma(t), \sigma(t)$ are given by the vehicle's position relative to the target.

4.2 The Controller

We define the first error vector according to

$$z_1 \triangleq \hat{\omega}_1 - \omega_{d1} + K_i \int_{t_0}^{t} z_1(h)dh \quad (13)$$

where $\hat{\omega}_1 = [\hat{\rho}, \hat{\theta}, \hat{\psi}]^T$ and $K_i > 0$. Integral action is included to avoid steady state error. Differentiating z_1 with respect to time yields

$$\dot{z}_1 = \bar{R}(\hat{\omega}_2 - \omega_{d2}) + K_i z_1 + \bar{R}_2[\hat{v}, \hat{w}]^T + \lambda h L\tilde{\zeta} \quad (14)$$

where $\hat{\omega}_2 = [\hat{u}, \hat{q}, \hat{r}]^T$ and

$$h = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} s\mu & -c\mu s\beta \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{R} = \begin{bmatrix} -c\mu c\beta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/c\theta \end{bmatrix},\ c\theta \neq 0$$

Next, we define the following error vector

$$z_2 = \hat{\nu} - \alpha \quad (15)$$

where $\alpha = [\alpha_1, ..., \alpha_5]^T$ is a vector of stabilizing functions and $z_2 = [z_{2,1}, ..., z_{2,5}]^T$. Inserting for $\hat{\nu}$ in (15) into (14) results in

$$\dot{z}_1 = \bar{R}(\alpha_i - \omega_{d2}) + K_i z_1 + hRz_2 + \bar{R}_2[\hat{v}, \hat{w}]^T + \lambda h L\tilde{\zeta} \quad (16)$$

where $\alpha_i = [\alpha_1, \alpha_4, \alpha_5]^T$. To render (16) stable we choose α_i according to

$$\alpha_i = \omega_{d2} - \bar{R}^{-1}((K_i + K_1)z_1 + \bar{R}_2[\alpha_2, \alpha_3]^T) \quad (17)$$

Note that $\bar{R}$ is singular for $|\mu| = \frac{\pi}{2}$, $|\beta| = \frac{\pi}{2}$, i.e. when the vehicle is parallel to the sphere tangent. This is, naturally, because there are no controls in sway and heave. Hence, we can only achieve local stability results when employing this control scheme. The following assumption is applied:

A.5: The initial state of the vehicle satisfies the following:

$$|\mu(t_0)| < \frac{\pi}{2}, |\beta(t_0)| < \frac{\pi}{2} \quad (18)$$

This criteria is easy to satisfy by employing for instance the line-of-sight guidance method prior to the target area.

Consider then the Lyapunov function candidate $V_1 = \frac{\varepsilon}{2} z_1^T z_1$ where $\varepsilon > 0$ is included in order to increase the design freedom of the controller. Differentiating V_1 with respect to time gives

$$\dot{V}_1 = -\varepsilon z_1^T K_1 z_1 + \varepsilon z_1^T (hR + \bar{R}_2 \bar{h}^T) z_2 + \varepsilon z_1^T \lambda h L \tilde{\zeta} \quad (19)$$

where

$$\bar{h} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T$$

Next we choose the second Lyapunov function candidate $V_2 = V_1 + \frac{1}{2} z_2^T M z_2$, which differentiating with respect to time yields

$$\dot{V}_2 = \dot{V}_1 + z_2^T(-D(\hat{\nu})\hat{\nu} - g + \tau + \lambda^2 M R^{-1} K \tilde{\zeta} + \hat{b} - M\dot{\alpha}) \quad (20)$$

We choose the controller as

$$\tau = D(\alpha)\alpha + g - \lambda^2 M R^{-1} K \tilde{\zeta} - \hat{b} + M\dot{\alpha} - K_2 z_2 - \varepsilon(R^T h^T + \bar{h}\bar{R}_2^T) z_1 \quad (21)$$

which results in the following tracking error dynamics

$$\begin{bmatrix} \varepsilon \dot{z}_1 \\ M\dot{z}_2 \end{bmatrix} = \begin{bmatrix} -\varepsilon K_1 & F^T \\ -F & -(\delta(z) + K_2) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} (\varepsilon z_1^T \lambda h L \tilde{\zeta})^T & 0_{5\times1}^T \end{bmatrix}^T \quad (22)$$

where $F = \varepsilon(R^T h^T + \bar{h}\bar{R}_2^T)$. Notice that the right hand side of the system equations (22) includes the variables s and d, while we consider only the $[z_1, z_2]$ tracking error dynamics. We thus consider s and d as general time-varying signals using forward completeness as in Loría *et al.* (2000). To this end, the following preliminary assumption is applied:

P.A.1: The signals s and d exist for all $t > t_0$, i.e. $s, d \in \mathcal{L}_\infty$.

Let $x_t \triangleq [x_o^T, z_1^T, z_2^T]^T$ denote the complete error dynamics.

Theorem 1. The origin $x_t = 0$ of the error dynamics (6) and (22) is uniformly locally asymptotically stable (ULAS) under A.1-5, P.A.1 and if the following conditions are satisfied

$$\varepsilon < 2a/(\lambda L_M), \quad \lambda < \min\left\{\frac{2}{T_M K_{bM}}, \frac{2K_{1m}}{L_M}\right\} \quad (23)$$

Proof: Consider the radially unbounded decrescent and positive definite Lyapunov function candidate $V_t = V_o + V_2$. Differentiating with respect to time gives

$$\dot{V}_t = \dot{V}_o - \varepsilon z_1^T K_1 z_1 + \varepsilon z_1^T \lambda h L \tilde{\zeta} - z_2^T(\delta(z) + K_2) z_2 \quad (24)$$

Using (10) and completing the squares results in

$$\dot{V}_t \leq -(a - \varepsilon\lambda L_M/2)\|x_o\|^2 - (\delta_m + K_{2m})\|z_2\|^2 - \varepsilon(K_{1m} - \lambda L_M/2)\|z_1\|^2 \quad (25)$$

Hence, the origin $x_t = 0$ is ULAS if (23) is satisfied.

□

Remark 2. The control scheme presented in this section can be modified for AUVs with control actuators in sway and/or heave by expanding the desired state to e.g. $\omega_{d1} = [\rho_d, s_d, d_d, \theta_d, \psi_d]^T$. Care should be taken for desired vertical attack angles $d_d = \{0, 2\pi\}$ to avoid singularity.

4.3 Stability of the Unactuated States

Inspired by Fossen *et al.* (2003) we now consider the fact that the vehicle is underactuated, recalling the control vector (3). This results in a dynamic constraint in the controller for the unactuated states, i.e. sway and heave. By analyzing the dynamics of the controller we show that the velocities in sway and are bounded.

From (21) we have that

$$\tau = D(\alpha)\alpha + M\dot{\alpha} - f(\cdot) \quad (26)$$

$$f(\cdot) = \lambda^2 M R^{-1} K \tilde{\zeta} + \hat{b} + K_2 z_2 - g + \varepsilon(R^T h^T + \bar{h}\bar{R}_2^T) z_1 \quad (27)$$

where the bounded and converging variables are collected in the function $f(\cdot) = f(\hat{b}, \tilde{\zeta}, z_1, z_2)$. Furthermore, we insert (3) into (26), which leads to the following dynamic constraint on the $\bar{\alpha} \triangleq [\alpha_2, \alpha_3]^T$-dynamics

$$\bar{M}\dot{\bar{\alpha}} + \bar{D}(\bar{\alpha})\bar{\alpha} = \bar{h}^T f(\cdot) \quad (28)$$

where $\bar{M} = \text{diag}\{m_{22}, m_{33}\}$ and $\bar{D} = \text{diag}\{d_{22}, d_{33}\}$. It is clear from the Lyapunov analysis in Section 4.2 that the z_2-dynamics are stable provided that the control vector τ satisfies equation (21). For surge, pitch and heave, the obvious choice is to design the stabilizing function α_i rendering the z_1-dynamics stable since we can assign τ_u, τ_q and τ_r arbitrarily. Then, $[u, q, r]^T \to \alpha_i$ since $z_2 \to 0$ as $t \to \infty$. This is, however, not feasible for the unactuated states. Instead, $\bar{\alpha}$ must satisfy the differential equation (26), which arises since there are no controls in sway or heave. However, the z_2-dynamics are nevertheless ULAS provided that (21) is satisfied. Hence, we use the signals collected in x_t as input to the $\bar{\alpha}$-subsystem.

We proceed by showing that the $\bar{\alpha}$-subsystem is input-to-state stable from f to $\bar{\alpha}$. This is seen by applying for instance $V_{\bar{\alpha}} = \frac{1}{2}\bar{\alpha}^T \bar{M}\bar{\alpha}$ which differentiating along the solutions of $\bar{\alpha}$ gives

$$\dot{V}_{\bar{a}} = -\bar{\alpha}^T \bar{D}(\bar{\alpha})\bar{\alpha} + \bar{\alpha}^T \bar{h}^T f(\cdot) \quad (29)$$

This results in the following upper bound

$$\dot{V}_{\bar{a}} \leq -(\bar{\delta}_m - e)\|\bar{\alpha}\|^2 - e\|\bar{\alpha}\|^2 + \|\bar{\alpha}\|\|f(\cdot)\| \quad (30)$$

where $e > 0$ is scalar. Hence, we have that $\dot{V}_{\bar{a}} \leq -(\bar{\delta}_m - e)\|\bar{\alpha}\|^2$, $\forall \|\bar{\alpha}\| \geq \frac{1}{e}\|f(\cdot)\|$. Therefore, $\bar{\alpha}$ converges to the bounded set $\left\{\bar{\alpha} : \|\bar{\alpha}\| \leq \frac{1}{e}\|f(\cdot)\|\right\}$. Moreover, since $z_2(t) \to 0$ and $x_o(t) \to 0$ as $t \to \infty$, it follows that $\bar{\alpha}(t) \to [v(t), w(t)]^T$ as $t \to \infty$.

4.4 Forward Completeness of the Overall System

In this section, we prove that the preliminary assumption P.A.1 holds. This is carried out by showing forward completeness of the overall system.

Proposition 2. The time-varying vector $\Gamma(t) \triangleq [x_t^T, \bar{\alpha}^T]^T$ exists for all $t \geq t_0$.

Proof: Consider the following Lyapunov function $V_{fc} = V_t + V_{\bar{\alpha}} + \frac{1}{2}b^T b$. Differentiating V_{fc} using (24) and (29) gives $V_{fc} \leq \varepsilon z_1^T \lambda h L \tilde{\zeta} + \bar{\alpha}^T \bar{h}^T f(\cdot)$. Furthermore, by employing Young's inequality, P.1 and using that $\left\|\tilde{\zeta}\right\| \leq \|x_o\|$ and $\left\|\tilde{b}\right\| \leq \|x_o\|$, it can easily be verified that $\dot{V}_{fc} \leq cV_{fc}$ is satisfied, where $c > 0$ is a constant. Hence, the overall system is forward complete and the vector $\Gamma(t)$ exists and can be continued for all $t \geq t_0$.

□

5. CASE STUDY: THE MINESNIPER MKII

The simulations are carried out on the Minesniper MkII with length 2m, width 0.17m and weight 40kg. Note that the vehicle model used to test the derived controller is the highly nonlinear and coupled process plant model (Refsnes *et al.*, 2006). The current velocity is set to 0.5m/s from South East (300deg.). White noise is added to the state measurements ζ. Figure 2 shows the horizontal position of the Minesniper MkII performing DP on a target at 5m depth and with desired radius $\rho_t = 5$m. The green square [10,0]m denotes the initial aiming target whereas the blue diamonds represent a moving target with final location [25,5]m, denoted by the red star. It is seen from Fig. 2, that due to the current, the vehicle orients itself towards both the target and the current, illustrating the weather optimal property. Figure 3 presents the tracking trajectories and tracking errors. The

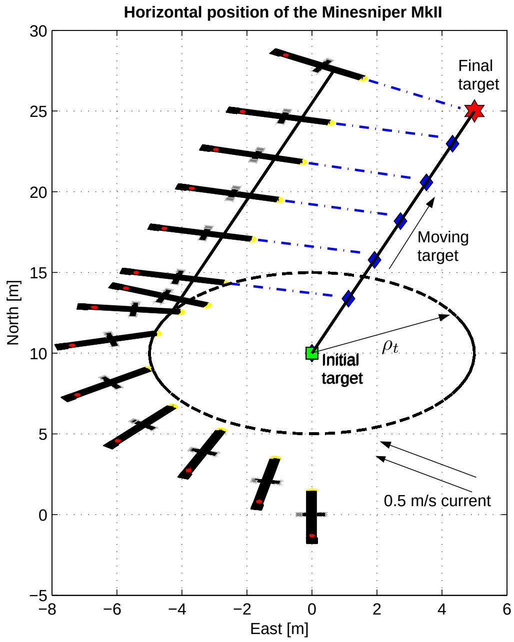

Fig. 2. Horizontal position of the Minesniper MkII in DP.

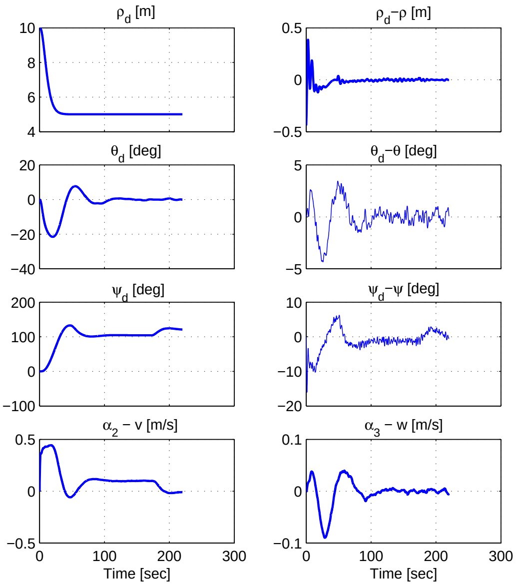

Fig. 3. Left: Desired tracking trajectories. Right: Tracking error. Bottom: $\bar{\alpha} - [v, w]^T$.

figure shows satisfactory tracking performance. Moreover, the two bottom plots present the error between the $\bar{\alpha}$ and the actual velocities in sway and heave, respectively.

6. CONCLUSION

An output feedback controller has been developed for DP of an underactuated AUV. It consisted of a nonlinear Luenberger observer and a controller that was designed using the observer backstepping technique. The control objective was to orient the vehicle towards the target while keeping a fixed distance to the target at all times. By employing conventional Lyapunov theory, the total error dynamics consisting of the estimation errors and the tracking errors, were proven ULAS. Moreover, it was shown that the velocities in sway and heave converged to a bounded set by analyzing the inherent dynamics of the controller that arose for the unactuated states. Simulation results showed that the method provided accurate positioning, and that it was weather optimal in the sense that the vehicle automatically converged to the stable equilibrium position oriented towards the target and the current.

7. ACKNOWLEDGEMENTS

The authors would like to thank Kongsberg ASA for their support and contribution to the project.

REFERENCES

Aguiar, A. P. and A. M. Pascoal (2002). Dynamic positioning and way-point tracking of underactuated AUVs in the presence of ocean currents. In: *Proc. 41st IEEE Conf. Decision & Control*. Las Vegas, Nevada, USA. pp. 2105–2110.

Celani, F. (2005). An asymptotic observer for robot manipulators with position measurements. arXiv math.OC/0507072.

Fossen, T. I. (2002). *Marine Control Systems: Guidance, Navigation and Control of Ships, Rigs and Underwater Vehicles*. Marine Cybernetics. Trondheim, Norway. http://www.marinecybernetics.com.

Fossen, T. I. and J. P. Strand (2001). Nonlinear passive weather optimal positioning control (WOPC) system for ships and rigs: experimental results. *Automatica* **37**, 701–715.

Fossen, T. I., M. Breivik and R. Skjetne (2003). Line-of-sight path following of underactuated marine craft. In: *Proc. IFAC Manoeuvring and Control of Marine Craft (MCMC)*. Girona, Spain.

Khalil, H. K. (2002). *Nonlinear Systems*. 3rd ed.. Prentice-Hall. New York.

Krstić, M., I. Kanellakopoulos and P. V. Kokotović (1995). *Nonlinear and Adaptive Control Design*. John Wiley & Sons Ltd. New York.

Loría, A., T. I. Fossen and E. Panteley (2000). A separation principle for dynamic positioning of ships: Theoretical and experimental results. *IEEE Transactions on Control Systems Technology* **8**(2), 332–343.

Pettersen, K. Y. and O. Egeland (1999). Time-varying exponential stabilization of the position and attitude of an underactuated autonomous underwater vehicle. *IEEE Transactions on Automatic Control* **44**(1), 112–115.

Refsnes, J. E., A. J. Sørensen and K. Y. Pettersen (2006). Robust observer design for underwater vehicles. Munich, Germany. To appear in Proc. Conference on Control Applications (CCA).