Optimal class-based storage system with diagonal movements

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Abstract. With the advent of e-commerce activities, warehouse operations require high storage density and high throughput. When the space is limited, high storage density can be attained by puzzle-based storage solutions. Recently a new solution has been introduced by Logistics 4.0 Lab at NTNU and Wheel.me. Storage racks are moved by autonomous wheels in any direction allowing also diagonal movements which might theoretically achieve high throughput. Even though it is identified as most viable method, retrieval time is dependent upon the characteristics of the system. Previous studies investigated retrieval performance largely based on the escort locations (empty slots), Input/Output points and movement types. The purpose of this paper is to apply two-class-based storage policy where high-turnover items are kept nearby the Input/Output point to lower the response time. We conducted a scenario-based analysis by changing the ABC curve, system size, zone A size, and shape factor to examine the impact of ideal system dimensions and zone A boundaries on the performance of the class-based storage system. According to the findings, two-class-based storage system typically outperforms random storage in terms of cycle time of about 10% to 50% depending on the configuration and ABC curve.

Keywords: puzzle-based movable, class-based storage, scenario-based analysis

1 Introduction

As a result of continuous growth in e-commerce operations and online retailing, warehouse technologies in e-commerce contexts have undergone new developments (Boysen et al., 2019). Companies have to manage massive and fluctuating daily order volumes while storing millions of unique items. An allocation of space is needed for storage. Given the scarcity of real estate, prices have gone up in locations near populated cities as a result of urbanization. Companies face challenges which arises from the higher costs related to the needed storage space and the need for quick retrieval concurrently. Compact storage and product accessibility are typically trade-offs in warehouse management (Azadeh et al., 2019). In addition to space available for the storage, warehouse should be operated efficiently as growing inventory turnover rates with shorter storage periods, a large number of orders, and small quantities with faster deliveries always require a flexible automized, and optimized system with higher throughput and higher productivity. Therefore, advancement of material handling is vital in this context. One significant advancement is evolvement of robotics solutions such as autonomous mobile robots (AMRs) which also can be used in narrow-aisle, highly congested warehouse environments (Fragapane et al., 2021).

Puzzle-based storage (PBS) systems are a novel and unique field developed as a result of research on highly compact storage systems. PBS systems are part-to-picker systems that were influenced by the popular 15-puzzle game, in which players must arrange 15 numbered tiles in a 4×4 grid in a sequential pattern by sliding the tiles into only one empty slot. When there is a strong demand for warehouse space, it is not feasible to build up a warehousing design using the area designated for typical aisles. In such cases, "puzzle-based" systems present a possible warehousing option. In puzzle-based systems, unit loads can be moved in one of four directions by shifting escort locations where desirable items can be reach to an input/output location easily (Kota et al., 2015). The puzzle-based storage systems aim to achieve great flexibility while attempting to reduce the number of movements when retrieving the items (Shirazi & Zolghadr, 2021).

In the recent years, through the collaboration between the Logistics 4.0 Lab and the Norwegian company wheel.me, an evolution of PBS has been conceptualized and introduced, exploiting the autonomous wheels developed by wheel.me. Autonomous wheels can replace regular wheels on trolleys, carts or mounted to many other objects like racks and pallets. They consist of computational and mechatronic mechanisms allowing the wheels to sense the environment, avoid collisions and drive autonomously in any direction. Control of the wheels is done through a cloud computing system, which can enable the coordination of multiple objects mounted with autonomous wheels. Mounting autonomous wheels to storage racks removes the need for AMRs to move racks, in this way we can obtain a system with high storage density and high throughput capacity, called Puzzle-Based Movable Rack (PBMR) system.

Built on the recent works done by the authors in modelling PBMR systems (Sgarbossa et al., 2022; Sgarbossa et al., 2023) this paper aims to investigate the impact of class-based storage assignment policy on the throughput and density, and to suggest optimal size of class A. Mathematical modelling and simulation are used as methodology for this study.

The remainder of the paper is as follows. Section 2 gives an exhaustive overview of the literature developed for modelling and designing Puzzle-based storage system. Section 3 presents and analyzes the general mathematical model based on diagonal Movements, while the results are discussed in Section 4 providing some guidelines for practitioners and managers who wants to implement such solution. Finally, Section 5 concludes the papers discussing the limitations of this first study and setting the future research steps.

2 Literature Review

2.1 Puzzle-based storage systems

The first scholars who appear to have studied PBS systems are (Gue & Kim, 2007). They investigated a PBS system where unit loads are sequentially retrieved from one or more empty locations. They contrast standard low-density aisle-based warehouses with puzzle-based systems in terms of storage density and retrieval speed. While traditional warehouses typically outperform puzzle systems in terms of retrieval time, they are less efficient with regard to space. (Kota et al., 2015) use analytical methods to determine the single-load retrieval time expression when several escorts are distributed randomly around the system. For the general case with multiple escorts, they develop an integer program and expand the expression to a system with two escorts. The puzzle-based concept has given rise to a number of compact storage system variations in both practice and literature. Existing research on PBS systems falls into three major categories: system analysis, design optimization, and operations planning and control (He et al., 2023). The literature on system analysis is primarily concerned with the retrieval performance of the PBS system, which includes evaluating the expected retrieval time and comparing it to that of traditional systems, as well as analyzing the effect of escort location on retrieval time (Gue & Kim, 2007). Studies on design optimization prefer to focus on warehouse layout optimization under various storage policies and emerging technologies on advanced systems as well as smarter equipment (Kota et al., 2015). The study done by (Raviv et al., 2023) examined a novel PBS system which use limited number of autonomous mobile robots (AMRs). The retrieval path optimization was mainly discussed through operations planning and control problems (Alfieri et al., 2012; Gue et al., 2014). Recent studies of operational planning in PBS systems show that easing the restrictions imposed by the initial configuration might shorten retrieval times. It is possible to establish virtual aisles that allow requested loads to travel continuously by increasing the number of escorts. Past studies demonstrated that retrieval times might be shortened by using an algorithm that allowed escorts to be moved at the same time (Halseide, 2022). Only recently, thanks to the collaboration between Logistics 4.0 Lab and wheel.me, the authors have introduced a new type of warehouse which evolves the properties and operations of puzzle-based storage systems, called Puzzle-Based Movable Rack (PBMR) system where racks can be moved with autonomous wheels. One additional advantage of such system is that movable racks can move diagonally. The authors modelled and analyzed the system studying different configurations and showing the impact on density and average cycle time/throughput (Sgarbossa et al., 2022; Sgarbossa et al., 2023).

We determine the relevant work for item retrieval based on the desired item number (single or multiple), escort number (single or multiple, fixed location or random location), I/O point number (single or multiple), load movement type (single-load movement, simultaneous movement, and block movement), and objective function (minimize the total completion time of the retrieval process, the total number of moves and the system throughput). Table 1 provides a summary of the pertinent literature.

Table 1: Literature summary of PBS systems

Author & Year	Prime Objective				Configurations & policies															
		Throught put	Retrieval time	No. of Moves	Escorts		Retrieval		I/O		Movements								Storage	
	Density				Single	Multiple	Single	Multiple	Single	Multiple	Load	Block	Simultaneous	Sequential	Fixed aisle	Virtual aisles	Diagonal	Rectilinear	Random	Class-based
Gue and B. Kim (2007).			√	√	√	√	√		√		√	√		√				√	√	
Taylor & Gue (2008)			√	√	√	√	√		√		√		√	√				√	√	
Rohit, Kota V, G Don Taylor and K.R. Gue (2010)			√	√	√	√			√		√		√	√				√	√	
Alfieri et al. (2012)	√		√			√	√		√					√				√	√	
Gue, K.R., K. Furmans et al. (2014).	√	√						√					√			√		√	√	
Rohit, Kota V, G Don Taylor and K.R. Gue (2015)			√	√	√	√			√		√		√	√				√	√	
Zaerpour, N. , Yu, Y. , & De Koster, R. (2017)			√			√	√		√					√		√		√		√
Mirzaei, M. , De Koster, R. , & Zaerpour, N. (2017)			√			√		√			√			√				√	√	
Yalcin, A. , Koberstein, A. , & Schocke, K. O. (2019)			√	√		√	√				√	√	√	√				√	√	
Bukchin, Y. , & Raviv, T. (2020)			√	√		√	√			√		√	√					√	√	
Guo et al. (2021)			√			√		√		√	√	√	√					√	√	
Ma, Y., Chen, H., & Yu, Y. (2021)				√		√	√				√		√	√				√	√	
Shirazi, E and Zolghadr, M. (2021)			√	√	√	√		√		√			√					√	√	
Rosenfield A. (2021)			√	√		√	√			√	√		√	√				√	√	
Yu, H. , Yu, Y. , & de Koster, R. (2021)			√			√	√		√			√	√					√	√	
Bukchin, Y. , & Raviv, T. (2022)			√	√		√	√		√	√	√	√	√					√	√	
Wu H. & Chen Z. (2022)	√			√	√	√	√		√			√						√	√	
He, J et al. (2023)			√			√		√			√		√					√	√	
Raviv, T., Bukchin, Y. , & de Koster, R. (2023)			√	√			√		√	√	√	√	√					√	√	
Sgarbossa, F., Halseide, M., Timenes, A. (2022)			√			√	√		√		√				√	√	√	√	√	
Sgarbossa, F., Halseide, M., Timenes, A. (2023)			√			√	√		√		√				√	√	√	√	√	
This paper			√			√	√		√		√		√		√	√	√	√		√

2.2 Class-based storage systems

In practice, storage system classifies inventory items into A, B, or C classes based on how much of the overall annual demand they account for. A class items are those with the highest percentages, while C class items are those with the lowest percentages. The rest are goods in the B class. ABC curves can be created based on the three classes to represent the relationship between the percentage of inventory items and the corresponding percentage of total annual demand. This ratio has a practical relationship with the well-known ABC curve skewness (Zhang et al., 2013).

Despite the fact that class-based storage is more effective, in reality, random storage is frequently employed. Two-class-based storage has been demonstrated to dramatically decrease a storage system's reaction time. Classifying unit loads into high-turnover and low-turnover classes makes it easy to put two-class-based storage into practice. The first zone, which includes a collection of locations nearer the I/O point, is designated for the high-turnover unit loads. The response time of the system can be further slashed by creating a two-class-based live-cube system with ideal system dimensions and first zone border boundaries (Yu & Koster, 2009). Additionally, it is necessary to provide answers to issues regarding the effectiveness of an ideal first zone boundary and ideal system dimensions for varied COI curve skewness, system size, first zone size, and shape factor. In (Zaerpour et al., 2017) had addressed these issues significantly through their study. They studied a live-cube compact storage system that can generate virtual aisles for any required load and proposed a strategy to minimize the response time by optimizing the system's dimensions and the first zone border of a two-class system. In an effort to simplify system design, work done by (Zhang et al., 2013) tackles zone boundary optimization for puzzle-based compact storage systems when a three class-based storage policy is used.

2.3 Research gap and contributions of the study

It is clear from the literature that the study of PBMR system is still at the beginning and the investigation of the impact of class-based storage assignment policy has not been carried out yet. In this paper, we study two class-based storage system in compact storage which use puzzle based movable rack solutions with autonomous wheels. The main contributions of this study can be listed as follows.

1. As far as we are aware, this work is the first to suggest a configuration that allows for diagonal movement in a two-class based PBS system, which can reduce travel distance compared to rectilinear movements.
2. We discussed how the performance of a two-class storage system with an optimal first zone boundary and optimal dimensions is affected by the skewness of the COI curve and how does the performance of an optimal two-class based system depend on the size of the whole system?
3. We integrated mathematical model with simulation model to understand the retrieval performance.
4. We examined the impact of skewness of the COI curve, system size, shape of the first zone on the performance of class-based system as well as comparison

with randomized and two class based policy through a scenario-based analysis.

3 General Model with Diagonal Movements and Analysis

We propose square grid system (m×m) with two class-based storage. The shape of the class A is also having square shape. The movement policy for retrieving the items have both rectilinear and diagonal movements. We followed the previous work that was attempted to employ diagonal movement for this study (Halseide, 2022). For this system there is only one I/O point is available.

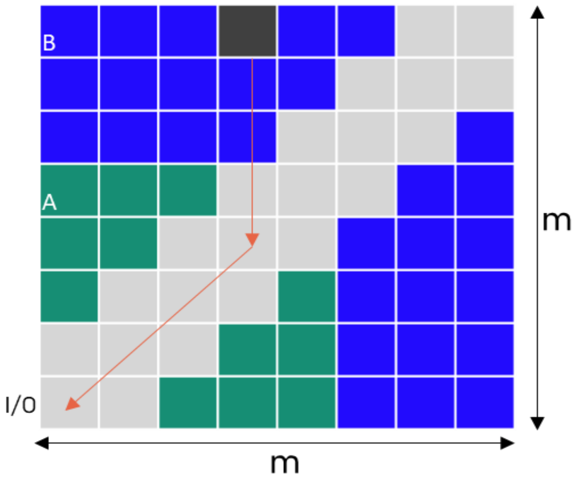

Figure 1: Puzzle based storage system with m×m dimensions

3.1 Storage capacity & density

Size of the square grid is m×m. To create a diagonal aisle in the square grid, 3m-2 escorts are required. We only locate the minimum number of escorts for the system. Therefore, total storage capacity is;

$$C = m^2 - 3m + 2 \tag{1}$$

Density for an m×m grid is modeled as the capacity divided by the grid size. Grid size is the total number of cells in the grid, so the density is expressed as;

$$d = \frac{m^2 - 3m + 2}{m^2} \tag{2}$$

3.2 Average travel distance and cycle time

In this study we define the average travel distance when two class based storage system is implemented. Travel distance is the total route length when item is traveling from its initial location to the I/O location including both rectilinear and diagonal movements. First, average travel distance for the whole grid is calculated using below equation and simulated the results for different m values using Python.

$$Avg\ D = \frac{1}{m^2-3m+2}\left(\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\max(i,J) + (\sqrt{2}-1)\min(i,j) - \frac{(m-1)m}{\sqrt{2}} - \sqrt{2}m^2 + 3\sqrt{2}m - 2m - 2\sqrt{2} + 2\right) \quad (3)$$

We calculated the dimensions of the class A, using percentage of picks in class A.

a: *dimension of the side of class A*
rA: *percentage of racks of class A on the total number of racks in the grid*

$$a = \frac{3+\sqrt{9-4(2-r_A(m^2-3m+2))}}{2} \quad (4)$$

When considering class A dimensions, average distance for class A and class B can be denoted from below equations.

d_A: *average travel distance for class A*
d_B: *average travel distance for class B*

$$Avg\ d_A = \pi r^2\left(\sum_{i=0}^{a-1}\sum_{j=0}^{a-1}\max(i,J) + (\sqrt{2}-1)\min(i,j) - \frac{(a-1)a}{\sqrt{2}} - \sqrt{2}a^2 + 3\sqrt{2}a - 2a - 2\sqrt{2} + 2\right) \quad (5)$$

$$Avg\ d_B = \pi r^2\left(\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\max(i,J) + (\sqrt{2}-1)\min(i,j) - \frac{(m-1)m}{\sqrt{2}} - \sqrt{2}m^2 + 3\sqrt{2}m - 2m - 2\sqrt{2} + 2\right) - Avg\ D_A \quad (6)$$

It is possible to indicate the proportion of loads in the grid that need to be moved in order to open a rectilinear aisle as a fraction β. All loads that are not adjacent to the diagonal aisle necessitate an extra step.

β_A: *fraction of racks that need the additional step in class A*
β_B: *fraction of racks that need the additional step in class B*

$$\beta_A = \frac{a^2-5a+6}{a^2-3a+2} \quad (7)$$

$$\beta_B = \frac{m^2-5m+6-(a^2-5a+6)}{a^2-3a+2-(a^2-3a+2)} \quad (8)$$

Finally, we calculated the average cycle time for the system using below equations. In this work, cycle time is defined as the total time needed to finish all of the procedures that allow a load to be moved from its starting point. The requested load must travel to the IO-location during these phases, hence cycle time rather than retrieval time is used to describe how long it takes. We assumed the velocity (v) is equal to 1 for this study.

$T\bar{A}$: $average\ cycle\ time\ for\ class\ A$
$T\bar{B}$:$average\ cycle\ time\ for\ class\ B$
p_A: $percentage\ of\ picks\ in\ class\ A\ on\ the\ total\ number\ of\ picks$
$\bar{T}$: $average\ cycle\ time$

$$\mathrm{T\bar{A}} = (\mathrm{\beta A} + \mathrm{d\bar{A}}) * \mathrm{v} \quad (9)$$
$$\mathrm{T\bar{B}} = (\mathrm{\beta B} + \mathrm{d\bar{B}}) * \mathrm{v} \quad (10)$$
$$\bar{\mathrm{T}} = p_A * \mathrm{T\bar{A}} + (1 - p_A) * \mathrm{T\bar{B}} \quad (11)$$

In order to calculate total average time, p_A is required. In general, it is known as a cumulative percent demand. The demand frequency curve represents the relationship between ranked cumulative percent demand (p_A) and the proportion of goods in inventory (r_A). The demand frequency curve is represented by the model in (Bender, 1981) work.

$$p_A = \frac{(1+S)r_A}{S+r_A} \quad (12)$$

It is common to find that a tiny portion of the items account for a sizable portion of the total demand. The demand curve's skewness is determined by the shape parameter (S). It is shown by the notation "100x=100F(x)" that 100x% of the items correspond to 100F(x)% of the overall demand. In this study, we employ the 20/40, 20/50, 20/60, 20/70, 20/80 and 20/90 curves, with S values of 0.6, 0.332, 0.2, 0.12, 0.0667, and 0.0285 respectively.

3.3 Scenario-based Analysis

Scenario based study was done by using different sub cases varying the percentage of class A racks and COI value.
Each sub case, m value changes from 1 to 100. Therefore, 6600 (66×100) scenarios were examined. Impact for the average travel time was investigated varying the m value, r_A and COI value. r_A=1 is the random scenario.

As the fixed parameters, shape of the system; Square, number of input/ Output locations; 1 and minimum number of escorts required for a system is used. Size (m), class A percentage (r_A) and demand curve (COI value) are the system variables. The main performance metrics are the density and average cycle time. Density depends only on the value of m. It is identifiable that even though number of escorts increased with the m value, still the density is increasing.

Zone A dimensions, density, average distance for class A and B, Average cycle time for class A and B were calculated which will depend on m, r_A and COI values in class-based system. Through the study, we need to highlight the average cycle time for whole system when class-based storage was applied. Therefore, in results section, we illustrate the impact of different parameters to the performance metrics (cycle time).

Table 2. Total subcases

COI / r_A	20-40%	20-50%	20-60%	20-70%	20-80%	20-90%
0.05	Case 1	12	23	34	45	56
0.1	2	13	24	35	46	57
0.15	3	14	25	36	47	58
0.2	4	15	26	37	48	59
0.25	5	16	27	38	49	60
0.3	6	17	28	39	50	61
0.35	7	18	29	40	51	62
0.4	8	19	30	41	52	63
0.45	9	20	31	42	53	64
0.5	10	21	32	43	54	65
1	11	22	33	44	55	66

4 Results & Discussion

4.1 Impact of class A percentage to the average travel time

The charts developed through the scenario-based analysis results illustrate the performance of the retrievals when changing the different parameters.

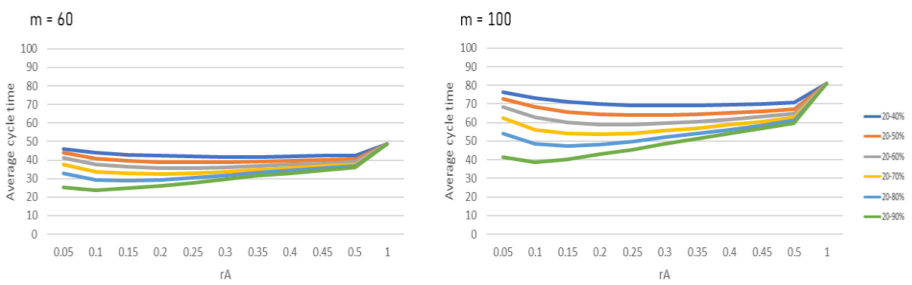

Fig. 2. Impact of r_A on the average cycle time for selected m values

Fig. *2* interprets how average cycle time deviate with the percentage of A. This is shown for the for different COI values and selected m values (m=60 and m=100). Average travel has reduced until $r_A = 0.1$ and again increases after that point for both m values. Therefore, lowest value can be obtained when r_A= 0.1 and COI=20-90% in both cases. When compare m=60 and m=100, average cycle time is considerably lower in m=60 for most cases.

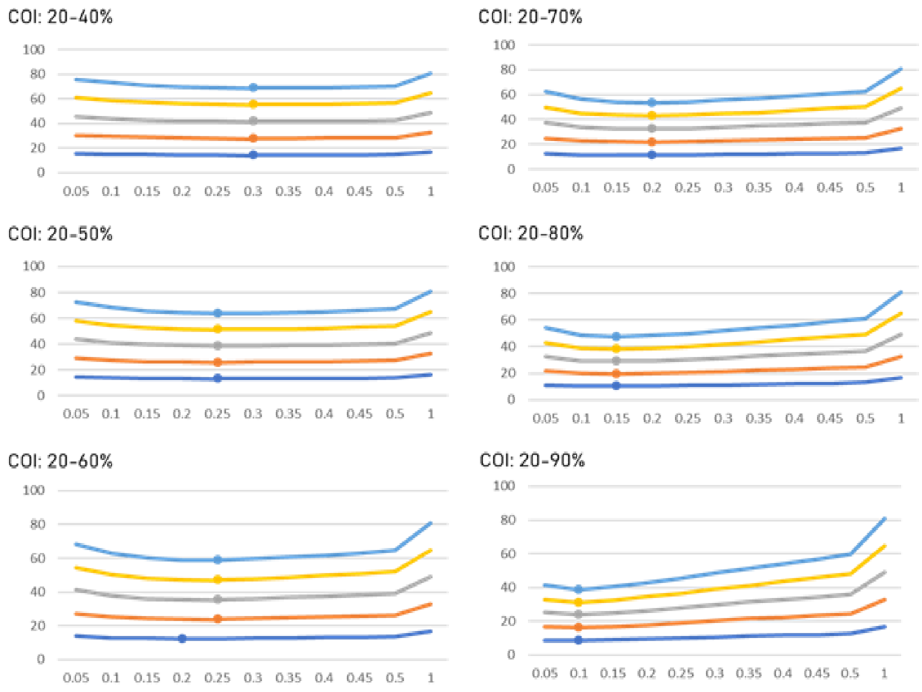

Fig. 3. Impact of r_A on the average cycle time for different COI curves

For different m values, variation of average travel time with the percentage of A is depicted in Fig. 3. Different charts were generated for each COI and for each chart, minimum travel time can be identified as pointed in dot. When m=20-40% minimum average time value can be found in r_A=0.3. When m=20-50%, r_A=0.25 and in COI= 20-60%, r_A=0.25 or 0.2 and in COI=20-70%, r_A=0.2… Overall, it can be identified that when the COI percentage is increasing, optimal r_A will move towards the 0. These identified optimal r_A values for each COI value are used compare each with random scenario.

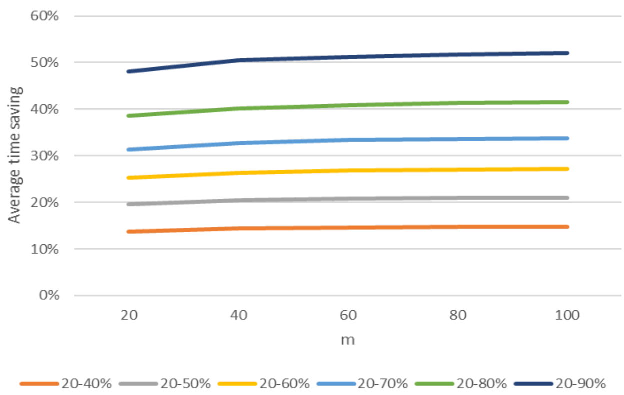

Fig. 4. Travel time savings as a percentage when optimal rA value used for each COI value

The deviation of average cycle time when optimal r_A applied is shown as a savings percentage to random scenario. The percentage savings is shown for different m values for each COI. Plotted the chart based on the saving percentage and m values. COI 20-90% shows higher percentage of savings for all the m values.

As **Fig.** *2* and Fig. *3* interprets, average cycle time is significantly lower for different r_A than random storage in all cases. The percentage of cycle time savings for the optimal r_A values is between 10% -55%.

4.2 Impact of skewness of COI curve to the average travel time

Even though the skewness of demand curve changed, average cycle time lower in class-based storage compared to random storage. However, significant difference can be identified for various demand curves.

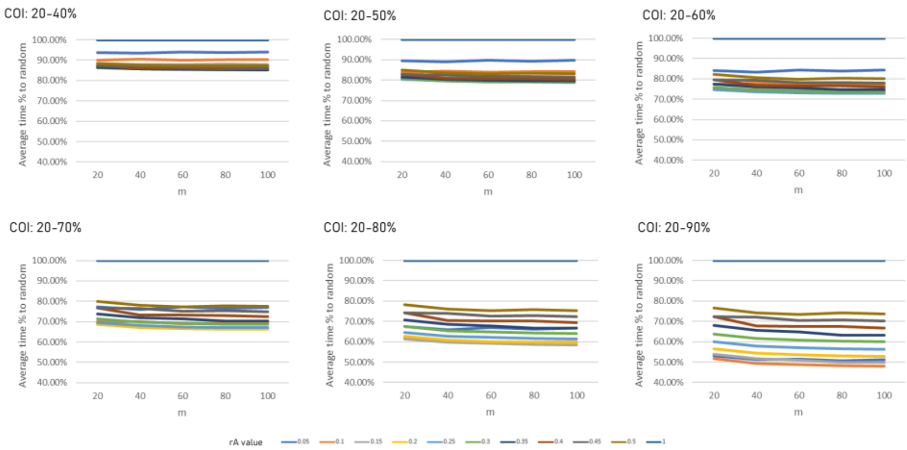

Fig. 5. Percentage of average travel time compared to random scenario

For each COI value, average travel time was plotted as a percentage to the random situation using different charts as in Fig. *5*. In each chart random scenario is illustrated in blue color line which is falling along the 100% gridline. When the COI curve is changing from 20-40% situation to 20-90% the gap between random situation and other r_A situations is increasing. Still the average travel time is lower in most cases.

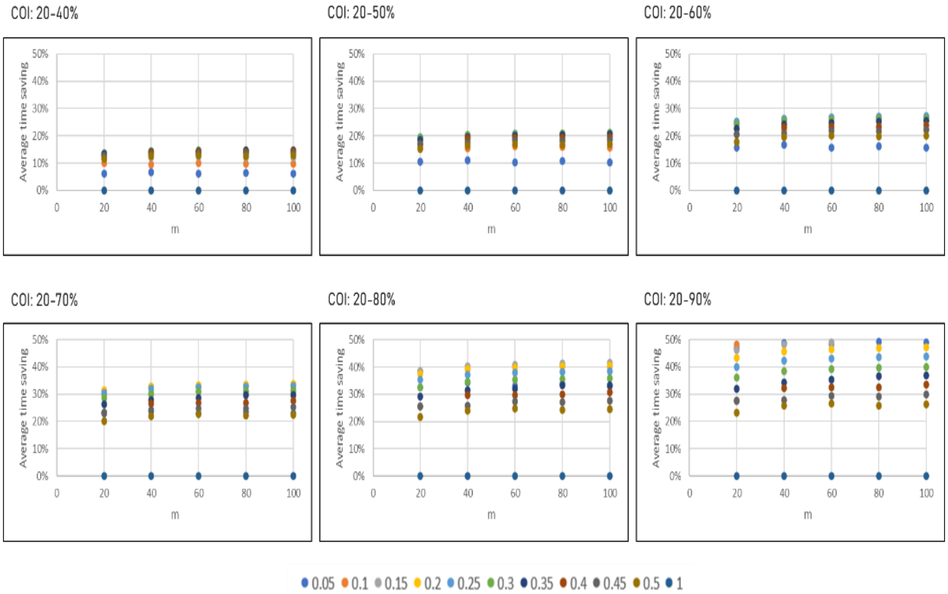

Fig. 6. Average travel time savings range for each COI

Percentage of average travel time saving compared to the random scenario was plotted in the next slide. Saving percentage was plotted to understand the range of savings pertaining to different m values for each COI. Therefore, COI = 20-90% situation has a wide savings range than other situations.

5 Conclusion and Recommendations for Future Research

Even though, retrieval of single load per grid instance is possible through the proposed model, our system with diagonal movement and storage racks moved by autonomous wheels may reach a comparably high throughput. When the storage area grows, single load retrieval would have a negative impact on throughput capacity. On the other hand, PBS systems can use robotics applications to boost throughput. Creating algorithms that extend the movement policy to allow for the simultaneous retrieval of numerous loads.

In this study, we mainly investigate two-class storage for puzzle-based movable racks system. Through the proposed mathematical model, we calculate the cycle time varying the system dimensions, class A boundaries and demand curve. For any COI curve, the ideal first zone boundary and the dimensions of a live-cube system can be calculated analytically. The findings demonstrate that, when compared to a random storage policy, an optimal two-class-based storage policy can greatly decrease the cycle time in the storage system. For instance, approximately 50% decrease in cycle time can be achieved for any system size with a COI curve of 20–90%. The key findings through this study can be listed as below.

1. The gaps between the optimal two-class-based storage and the random storage can be increased by increasing the skewness of the ABC curve (i.e., decreasing S).
2. The average cycle time for the random case is lower in two-class-based storage for different zone A sizes. However, after identified optimal zone A percentages, cycle time increases with zone A size more steeply. This is due to the likelihood that low-turnover items will occupy the grids closest to the I/O point as the size of the zone A increases.
3. It can be observed that for lower system size (m), average cycle time is lower. However, the gap between class-based situation and random storage is almost similar. This was already discussed in (Zaerpour et al., 2017) in their work. Thus, it can be equally gaining advantages for both small and large sized systems while adopting optimal r_A and demand curve at the time.

As wells as adopting class-based storage for PBS is effective method, various design and operational improvements to the system can multiply the performance. If the loads in PBS system can be arranged in a way which travel diagonally to shorten trip distances, there should be opportunities to increase throughput capacity while decreasing mobility. Investigation of how we can use dynamic strategies to allocate escorts in storage classes while increasing number of escorts can be examined in future works. Further, considering different shapes (triangular, rectangular, hexagon) and utilization of the unit loads of class A to retrieve more efficiently would be a promising area to discover. Therefore, our work implies that further research on PBS systems with diagonal movement and class-based concept has more room for improvement.

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