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Trade-Off Theory for Dual Holders

A dual holder simultaneously owns (private) debt and equity in the same firm. Private debt has a tax advantage, a positive cashflow, which incentivizes its use. This cashflow leads to a lower net cost of debt, which again reduces default risk as well as the cost of external debt. The usual trade-off between tax benefits and bankruptcy costs is altered. Debt priority affects both financing and default decisions. We find that an enterprise-value maximizing firm should issue senior, external debt and junior, private debt, rather than debt with *pari-passu* priority. Our analysis further highlights that tax authorities can effectively curtail the tax-motivated use of private debt through straightforward measures.

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Many medium-sized and larger firms have one or several subsidiaries. Household names such as Amazon and Microsoft have, respectively, more than 40 and 120 subsidiaries. Often, parent companies finance these subsidiaries through both equity investments and intercompany loans. We commonly refer to such loans as private debt. Additionally, subsidiaries may use external loans and credit

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lines. In such scenarios, the parent company is a *dual holder*—it assumes a dual role, simultaneously holding both debt and equity in the same firm. Another example pertains to small businesses, which frequently have only one or a few owners who may also provide loans to their firms. Among publicly listed firms in the United States, Yang (2021) finds that dual holders own more than 3% of the equity. Anton and Lin (2020) document that more than 30% of all U.S. listed firms have at least one dual holder, and they show how dual holdership has increased over time.

The prevalence of dual holdership raises several important economic questions that, to the best of our knowledge, have not been formally addressed in the literature: How does dual holdership affect the firm's default policy? How does it influence the choice of capital structure? And which priority should private debt have relative to external debt in default? We address these questions by extending the traditional trade-off theory to dual holdership.

In the standard trade-off theory (Leland 1994, Miller 1977), tax benefits of debt financing are traded off against bankruptcy costs to determine the optimal leverage. Debt financing yields a tax benefit whenever the combined corporate and dividend tax rate exceeds the investor tax rate on interest payments. Private debt has two direct cash flow effects. First, if coupon payments on private debt are tax deductible for the firm, they reduce the corporate tax. Second, dual holders pay investor tax on private coupon payments from their firm. If there are tax advantages of debt financing, the first effect dominates the second, and private debt reduces the net cost of debt. As such, private debt can be employed for profit shifting, as observed in Saunders-Scott (2015).

We value debt and equity of firms owned by dual holders and derive the optimal default policy, which maximizes the value of equity and private debt. In our model, the firm optimally defaults if earnings fall below an endogenously determined default barrier. The optimal default barrier is proportional to the net cost of debt, which is reduced by the use of private debt. Consequently, dual holdership mitigates default risk.

As is standard in the literature, see, for example, Leland (1994) and Hackbarth, Hennessy, and Leland (2007), we find the optimal leverage by maximizing the enterprise value with respect to the coupon payments. To justify enterprise value as the relevant objective function, consider the following scenario: an entrepreneur sells a firm to the highest bidding investor. The maximum price that the seller can command is the value of the optimally financed firm, representing the highest achievable enterprise value. Therefore, maximizing the enterprise value also maximizes the value of the initial equityholders' investment.

We analyze two types of priorities between external and private creditors. First, the creditors rank *pari-passu*, in which the two types of debt have the same priority in default. The *pari-passu* clause is common in the syndicated loan market and in other types of unsecured debt financing, for instance, in loans both to governments and corporations. It is particularly relevant in the context of dual ownership, as it is widely used in loans to subsidiaries within large conglomerates and small, privately held firms.

Second, many external creditors insist on first priority in default and do not accept *pari-passu* priority. Consequently, we analyze the priority structure with *subordination*, where private debt ranks below external debt. This priority structure is relevant for private loans to small businesses and for larger firms with subsidiaries, and is therefore relevant for dual holders.

Under the *pari-passu* priority structure, also dual holders partly recover their claim in a default, whereas they receive zero under subordination. This difference in recovery amount has implications for both default risk and enterprise value. Compared to the situation with junior, private debt, the positive payout in a default under the *pari-passu* priority structure accelerates the default. This acceleration elevates credit risk for external creditors and reduces the enterprise value. We show that this value is never higher under *pari-passu* than under subordination, but often lower.

Firms can use tax deductible private coupons to replace nondeductible dividend payments. Due to tax benefits of debt, sufficiently large private coupons can lead to a negative net cost of debt, a situation unlikely to be acceptable to tax authorities. This situation is at odds with central economic principles such as scarcity of capital. A negative net cost of debt follows from standard tax principles applied to private debt and not from our choice of modeling framework. This effect underscores the necessity for careful consideration of the regulation and taxation of private debt.

We identify two regions: one characterized by a positive net cost of debt and another by a negative net cost of debt. In the region with a positive cost (risky region), credit risk is present and dual holders rationally use private debt to displace external debt and equity. This model prediction finds empirical support in both Desai, Foley, and Hines Jr (2004) and Buettner et al. (2009), who document that multinational firms employ private debt and external debt as substitutes. This region aligns with the standard scenario in credit risk analyses. In the region with a negative net cost of debt (the no-default region), the tax advantage of private debt is substantial and effectively eliminates the default risk. Here, the firm never defaults, all debt is risk free, and the two types of debt are not substitutes.

The fact that private debt reduces the net cost of debt may explain why tax deductibility of coupon payments is restricted through transfer pricing rules and practice (see, e.g., EY 2019 and OECD 2020). In our model, we introduce an exogenous upper limit on the tax-deductible amount of private coupon payments. We demonstrate how this upper limit determines a firm's optimal use of private debt and, consequently, its optimal capital structure. The debt priority structure can potentially magnify the impact of restricted tax deductibility and lead to qualitatively different default policies. We illustrate by an example a situation where the firm strategically positions itself in the risky region under subordinated priority and in the no-default region under *pari-passu* priority.

Our analysis contributes to the existing literature on financing decisions of firms with multiple debt type. A related study is Hackbarth, Hennessy, and Leland (2007). They analyze capital structure and debt priority structures of a firm with two types of external debt—easy-to-renegotiate bank debt and market debt. In contrast to their study, our study also contributes to our understanding of internal capital markets.

Although empirical evidence regarding dual holders only includes publicly listed firms, where dual holders do not own all stocks, it unanimously suggests that dual holdership mitigates conflicting interests between equityholders and debtholders. Bodnaruk and Rossi (2016) find that target firms in mergers and acquisitions (M&A) with a larger equity share owned by dual holders experience lower M&A equity premia and larger abnormal bond returns. This finding suggests that dual holders consider both their equity and debt investments when they evaluate an offer for a firm in which they own both shares and debt. It is the change in *total* net worth that is of interest for dual holders, rather than the change in equity value alone. Chu (2018) uses mergers between firms that own stocks and firms that own debt in the same firm to analyze how the dividend policy to equityholders changes after a merger. He finds a downward shift in the corporate payout policy so that less funds are paid to equityholders. Yang (2021) provides further evidence supporting the idea that dual holdership reduces these conflicts. He finds that firms owned by dual holders tend to generate fewer patents, but those patents are more profitable. This finding suggests a reduced inclination for asset substitution. Also Chava, Wang, and Zou (2019) address how dual holders can mitigate stockholder-creditor conflicts. They find that loans where (some of) the lenders are dual holders are less likely to have debt covenants that restrict capital expenditures. They also find that such firms are more likely to be granted an unconditional waiver after a covenant violation. Jiang, Li, and Shao (2010) find that credit spreads on loans to firms with dual holders are 18-32 bps lower than the credit spreads on loans to firms without dual holders. Lim, Minton, and Weisbach (2014) find that when hedge funds and private equity funds provide loans to firms in which they hold stocks, they receive a substantial premium on the spread compared to other firm loans. This phenomenon is attributed to the fact that these funds often serve as lenders of last resort. These empirical results are in line with the predictions of our model: dual holdership reduces default risk, and subordinated, private debt carries wider credit spreads than senior, external debt. Chu et al. (2019) find that financially distressed firms with dual holdership are more likely to undergo an out-of-court restructuring, avoiding the costlier route of bankruptcy filing. Francis et al. (2022) find that firms with dual holdership tend to exhibit more aggressive tax behavior than other firms.

In our model, the firm is fully owned by dual holders, an ownership structure commonly found in small businesses and among subsidiaries of larger firms. The findings in the empirical literature suggest that the interests of equityholders and debtholders are better aligned under dual holdership. This empirically documented feature is inherent in our model: the model predictions are based on the assumption that dual holders consider the value of both their equity and debt investments when they determine the firm's optimal default policy. In line with this assumption, Huizinga, Laeven, and Nicodeme (2008) document that different corporate tax rates in different countries affect subsidiaries' optimal debt policies. There are also other factors beside tax rates that determine the use of debt financing and the choice between private debt and external debt. Desai, Foley, and Hines Jr (2004) and Buettner et al. (2009) find

TABLE 1

THE NET AFTER-TAX CASHFLOW TO THE DUAL HOLDERS WHEN TAX DEDUCTIONS ARE LIMITED TO THE AMOUNT C_D

1	EBIT	δ_t
2	– Interest debt	$-C$
3	Earnings before taxes	$\delta_t - C$
4	– Corporate tax	$-\theta_C(\delta_t - C_D)$
5	Earnings before dividend and investor tax	$(1-\theta_C)\delta_t + \theta_C C_D - C$
6	– Dividend tax	$-\theta_{div}((1-\theta_C)\delta_t + \theta_C C_D - C)$
7	After-tax dividend	$(1-\theta_{eff})\delta_t - (1-\theta_{div})(C - \theta_C C_D)$
8	+ After-tax private coupon	$+(1-\theta_i)C_P$
9	After-tax cashflow to dual holders	$(1-\theta_{eff})\delta_t - (1-\theta_{div})(C - \theta_C C_D) + (1-\theta_i)C_P$

that subsidiaries use more private debt when the local, external debt market for the subsidiary is less developed and therefore more expensive to use.

The paper is organized as follows: in Section 1, we present the tax system. In Section 2, we present the process for the firm's stochastic earnings. We value the firm's debt and equity and derive the optimal default policies in Section 3. In Section 4, we analyze the optimal capital structure. In Section 5, we analyze the no-default region. Section 6 contains numerical implementations of our results. We conclude the paper in Section 7. Some results and proofs are relegated to the appendices.

1. TAXES

1.1 Two Classes of Debt

We consider a firm that is exclusively owned by dual holders. It is financed by debt and equity. The firm only issues perpetual debt, and the coupons therefore represent interest payments only and do not include down payments on the loans. Let C denote the continuous rate of interest payments on the firm's total debt. It can be decomposed into the coupon rates C_E on external debt and C_P on private debt.

1.2 Tax Rates

Both the firm and the investors are subject to taxation. We denote the tax deductible amount of the coupon payments by $C_D \leq C$. Tax rules dictate the value of C_D, which can be quite intricate and may depend on the firm's total taxable income. For tractability, we assume that C_D is deterministic and time independent.

In our model, the tax system consists of three different types of taxes and a rule for tax deductibility of coupon payments. See Table 1 for an illustration. First, the corporate tax rate is θ_C. At time t, the firm pays corporate tax on the difference between its EBIT—earnings before interest and taxes, denoted δ_t—and the deductible coupon rate C_D. The firm therefore pays taxes at the rate $\theta_C(\delta_t - C_D)$, cf. line 4 of Table 1. For simplicity, we assume that the firm receives full loss offset for any negative profits.[1]

The dual holders and the external creditors are subject to the remaining two types of taxes. The firm's after-tax profits in line 5 of Table 1 are paid to the dual holders as dividends and are taxed at the rate θ_{div}—the second tax rate. From these two tax rates, we get the effective tax rate $\theta_{eff} = (1 - \theta_C)(1 - \theta_{div}) - 1$. The after-tax dividend-rate is given in line 7 of Table 1. Both external creditors and dual holders pay investor taxes at the rate θ_i—the third tax rate—on received coupon payments. Line 8 of Table 1 shows the dualholders' private debt interests after investor tax, and line 9 shows the total after-tax cashflow to the dual holders.

By defining

$$\eta = (1 - \theta_{div})(C - \theta_C C_D) - (1 - \theta_i)C_P, \tag{1}$$

we can write the *after-tax cashflow* in the last line of Table 1 as $(1 - \theta_{eff})\delta_t - \eta$. The first term is the after-tax rate of earnings. The second term η can be interpreted as the *net after-tax cost rate of debt financing*. For simplicity, we refer to this rate as the *net cost of debt*.

1.3 Restrictions on Tax Deductibility

Given our emphasis on dual holders, we make the assumption that any constraints pertaining to the deductibility of coupon payments are linked to the private coupons. More specifically, we assume that

$$C_D = C_E + f(C_P),$$

that is, the tax deductible coupon rate C_D is additively separable in C_E and C_P, and linear in C_E. The function $f(C_P)$ determines the tax deductibility of C_P, so clearly $0 \leq f(C_P) \leq C_P$. This specification of C_D means that there are no restrictions on tax deductibility on external coupons, only on private coupons. The level of external coupons does not influence the deductibility of private coupons.

Under this assumption, η can be written as

$$\eta = (1 - \theta_{eff})C_E - (\theta_{div} - \theta_i)C_P - (\theta_{eff} - \theta_{div})f(C_P). \tag{2}$$

The first term in expression (2) shows that the net cost of debt is increasing in the external coupon C_E. Below we explain the two different effects of private debt, corresponding to each of the two last terms on the right-hand side of expression (2).

1. In certain jurisdictions, carry-forward losses are permissible, which partly justifies this assumption. Negative tax payments are also observed in specific tax regimes, such as the Norwegian petroleum tax regime. Under this regime, companies receive reimbursements from the Norwegian Government in cases where exploration expenses exceed other profits. These companies often operate as subsidiaries of larger international oil corporations and serve as examples of dual ownership, aligning well with our model.

The firm pays coupons on private debt, which reduces the dividend payment to the dual holders. Private debt can therefore be used to convert dividend payments into coupon payments. The second term in expression (2) can be interpreted as the change in the net cost of debt by receiving C_P as coupons (taxed at rate θ_i) instead of dividends (taxed at rate θ_{div}). If $\theta_i > \theta_{div}$, this term is positive, implying that the net cost increases by replacing dividends by private coupons. If $\theta_i < \theta_{div}$, this term is negative, implying that the cost decreases if dividends are replaced by private coupons. In the latter case, private debt reduces the net cost of debt even if private debt is not tax deductible at all, that is, $f(C_P) = 0$.

By using the definition of the effective tax rate θ_{eff}, the third term in expression (2) can be written as $(\theta_{eff} - \theta_{div}) f(C_P) = (1 - \theta_{div})\theta_c f(C_P)$. Because $\theta_{eff} > \theta_{div}$, it shows the dual holders' after-tax value of the firm's reduced tax $\theta_c f(C_P)$ due to private debt.

1.4 A Doubly Restricted Tax System

We illustrate the model with the following tax system. First, we set $f(C_P) = \min(C_P, Q)$, for a constant $Q \geq 0$. Any part of the coupon payment C_P exceeding Q is not tax deductible for the firm. The level Q is determined by tax authorities and is therefore exogenously given. This choice of function $f(C_P)$ is natural to restrict firms' tax deductibility of private coupons. As already discussed regarding the second term in expression (2), if $\theta_{div} > \theta_i$, there are still tax incentives to replace dividends by private debt coupons. To limit also these incentives, we introduce a second modification of the tax system. For $C_P > Q$, the amount Q is taxed at the rate θ_i on the dual holders' hand, while $C_P - Q > 0$ is taxed at the dividend tax-rate θ_{div}. The second modification of the tax system changes the second term of expression (2) to $(-1)(\theta_{div} - \theta_i)\min(C_P, Q)$.

In this doubly restricted tax system, the parameter Q serves two purposes. First, it acts as an upper limit on the tax deductibility of private coupons for the firm. Second, it is the threshold on private coupons for which the investor tax-rate changes from θ_i to θ_{div}. Under this tax system, the net cost of debt changes to

$$\eta_Q = (1 - \theta_{eff})C_E - (\theta_{eff} - \theta_i)\min(C_P, Q). \tag{3}$$

Intuitively, a lax tax system corresponds to a larger value of Q, and a stricter tax systems corresponds to a lower value of Q. Even within the framework of a doubly restricted tax system, a lax tax system has the potential to result in a negative η_Q if C_E is low relative to Q. If $Q = 0$, private debt does not qualify for tax deductions, and there are no tax incentives to substitute dividends with private debt. In this case $\eta_Q = (1 - \theta_{eff})C_E$, an expression of net cost of debt we recognize from models with only external debt.

In the rest of this paper, we use the doubly restricted tax system.

1.5 Debt Structure and Default Risk

Leveraged firms may default on their debt payments. The combination of private and external coupons determines the debt structure. As long as the EBIT flow is positive, an assumption in our model, default can only be optimal if the net cost of debt is strictly positive. The risky region is given by the coupon pairs with a positive η_Q, that is, $\{(C_P, C_E) : \eta_Q > 0\}$. Similarly, the no-default region is given by the coupon pairs $\{(C_P, C_E) : \eta_Q \leq 0\}$.

PROPOSITION 1. *Under the doubly restricted tax system, the risky region is characterized by the inequality*

$$C_E > K \min(C_P, Q), \tag{4}$$

where

$$K = \frac{\theta_{eff} - \theta_i}{1 - \theta_{eff}}. \tag{5}$$

PROOF. The result follows from expression (3) and the condition $\eta_Q > 0$. □

We define *the no-default boundary* as the set of points $\{(C_P, C_E) : C_P, C_E \geq 0\}$, which satisfies

$$C_E = K \min(C_P, Q). \tag{6}$$

This boundary separates the no-default region from the risky region.

The boundary is attainable if the optimal coupon pairs satisfy expression (6). Whether it is attainable or not depends on the tax parameters Q, θ_{eff}, and θ_i. In Sections 3 and 4, we analyze the situation where it is not attainable, that is, inequality (4) is satisfied. We relax this assumption in Section 5.

2. THE EBIT MODEL

We set our analysis in the EBIT framework of Goldstein, Ju, and Leland (2001), commonly referred to as the *classical case*. The time t EBIT-flow δ_t is given as the solution to the stochastic differential equation

$$\mathrm{d}\delta_t = \mu\delta_t \mathrm{d}t + \sigma\delta_t \mathrm{d}B_t, \tag{7}$$

where μ is the constant risk-neutral drift of the EBIT-flow, σ the constant volatility of the EBIT-flow, and $B = \{B_t\}_{t \geq 0}$ a standard Brownian motion defined under the risk-neutral probability (equivalent martingale) measure. The constant $\delta_0 > 0$ represents the time 0 level of the EBIT-flow process, consequently $\delta_t > 0$ for all t. The firm's assets produce a randomly evolving, perpetual, positive EBIT-flow.

Let $r(1-\theta_i) > \mu$ be the after-tax risk-free interest rate. The time t value of the perpetual after-tax earnings flow is given by

$$V_t = \mathbb{E}_t\left[\int_t^{\infty} (1-\theta_{eff})\delta_s e^{-r(1-\theta_i)(s-t)} ds\right] = \frac{(1-\theta_{eff})\delta_t}{r(1-\theta_i)-\mu} = \delta_t U, \tag{8}$$

where $\mathbb{E}_t[\cdot]$ denotes the conditional expectation under the risk-neutral probabilities. The constant

$$U = \frac{1-\theta_{eff}}{r(1-\theta_i)-\mu} \tag{9}$$

can be interpreted as the price/earnings ratio (P/E).

In this class of models, the default event is conceptualized as a situation where the original equityholders transfer control of the firm to the debtholders, who then become the new owners. The debtholders proceed to liquidate the firm, selling its assets to new investors who optimally recapitalize it. The original equityholders optimally let the firm default on debt payments by not paying the creditors' promised coupons the first time τ the value of the EBIT flow hits some threshold level δ_B. Formally,

$$\tau = \inf\{s \geq 0 : \delta_s = \delta_B\} \tag{10}$$

is a stopping time, which we determine endogenously as part of our analysis.

We introduce two basic securities. The present value of receiving one unit of account after tax at the default time τ is given by (see, e.g., Goldstein, Ju, and Leland 2001 and Black and Cox 1976)

$$\pi_1 = \pi_1(\delta) = \left(\frac{\delta}{\delta_B}\right)^{-x}, \tag{11}$$

where

$$x = \frac{1}{\sigma^2}\left[(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r(1-\theta_i)\sigma^2}\right]. \tag{12}$$

Similarly, the present value of an after-tax annuity of one from time 0 to the default time is

$$\pi_2 = \pi_2(\delta) = \frac{1}{r(1-\theta_i)}(1 - \pi_1(\delta)). \tag{13}$$

From the definition of δ_B in expression (10) and from expression (8), it follows that the after-tax value of the future EBIT-flow—and hence the value of an unlevered firm—at the default time τ is

$$V_\tau = \delta_B U.$$

In order to include any additional value due to optimal debt leverage after the first default, we follow the approach by Fischer, Heinkel, and Zechner (1989), Hackbarth, Hennessy, and Leland (2007), Christensen et al. (2014), and others.

Denote the value of a claim on the EBIT process by $H(\delta)$. The valuation function $H(\cdot)$ has a useful scaling property, see Goldstein, Ju, and Leland (2001), Strebulaev (2007), and Christensen et al. (2014); it is homogeneous of degree 1. This fact means that if both the EBIT level δ and the default level δ_B are scaled by the same factor, also the value of the claim is scaled by the same factor. This property also holds for a levered firm if the coupon payment rate is expressed as a proportion of the EBIT level at the date the debt is issued. The economic intuition behind this homogeneity property is quite trivial: if the unit of account changes, say, from USD to Euro for all input variables, then the asset values change accordingly from USD to Euro.

We define the proportional coupon $c = C/\delta$. For conciseness, we temporarily write $H(\delta) = H(\delta, \delta_B, c)$. The homogeneity property means that

$$H(k\delta) = H(k\delta, k\delta_B, c) = kH(\delta, \delta_B, c) = kH(\delta)$$

for any $k > 0$.

The enterprise value W is the sum of after-tax equity and debt values. Let $W^*(\delta)$ be the enterprise value of an optimally levered firm. The original owners of the firm optimally let the firm default at the EBIT level δ_B, at which the liquidation value L is

$$L = W^*(\delta_B) = \delta_B W_1^*, \tag{14}$$

where $W_1^* = W^*(1)$, and we have used the homogeneity property.[2] If there is no value gain from debt leveraging after the first default (as implicitly assumed in some seminal models, for example, Leland (1994) and Goldstein, Ju, and Leland (2001)), $L = V_\tau = \delta_B U$, as explained above. We may also deduce that if there is a gain in value from optimal leverage after the first default, $W_1^* > U$. Note that L is interpretable as the *gross* liquidation value of the firm. This value is the highest price new owners are willing to pay for the assets generating the EBIT flow.

2. We show in Appendix A that W^* is homogeneous of degree 1.

3. DEBT, EQUITY, AND DEFAULT POLICIES

3.1 Debt Valuation

Default is costly for the original debtholders, and we assume that a fraction α of the enterprise value is lost in the liquidation process. The recovery amount for the original debtholders is

$$\begin{aligned}(1-\alpha)L &= (1-\alpha)\delta_B W_1^* \\ &= (1-\gamma)\delta_B U,\end{aligned} \tag{15}$$

where

$$1-\gamma = (1-\alpha)\frac{W_1^*}{U}. \tag{16}$$

By assuming that $\alpha \geq \underline{\alpha} = 1 - U/W_1^*$, $\gamma \geq 0$ and there is a positive cost of bankruptcy. The lower bound $\underline{\alpha}$ can be interpreted as the relative gain in enterprise value for an optimally financed firm compared to an unlevered firm.

At default, creditors divide the recovery amount, which is $(1-\alpha)L$, among themselves based on the priorities outlined in the debt contracts. External creditors receive the payment $\lambda(1-\alpha)L$ at default, and private creditors receive $(1-\lambda)(1-\alpha)L$, where $0 \leq \lambda \leq 1$. The contractual debt priority-structure determines the distribution parameter λ.

The doubly restricted tax system has an impact on the value of debt. The dual holders pay investor tax on private coupons below Q and dividend tax on any private coupons exceeding Q. Private coupons are therefore taxed at the average rate

$$\hat{\theta} = \min\left(1, \frac{Q}{C_P}\right)\theta_i + \max\left(1 - \frac{Q}{C_P}, 0\right)\theta_{div}.$$

PROPOSITION 2. *Under the doubly restricted tax system, the time $t \geq 0$ after-tax value of external debt is*

$$D_E(\delta) = (1-\theta_i)C_E\pi_2 + \lambda(1-\alpha)L\pi_1,$$

and the after-tax value of private debt is

$$D_P(\delta) = (1-\hat{\theta})C_P\pi_2 + (1-\lambda)(1-\alpha)L\pi_1.$$

PROOF. Here, $(1-\theta_i)C_E$ and $(1-\hat{\theta})C_P$ are the after-tax coupons received by the creditors as long as the firm has not defaulted. The time 0 values of these coupons are proportional to π_2. At default, the external creditors and the dual holders are entitled to the recovery amounts $\lambda(1-\alpha)L$ and $(1-\lambda)(1-\alpha)L$, respectively. The time 0 values of these two payments are proportional to π_1. Adding these parts together for each type of debt gives the expressions for $D_E(\delta)$ and $D_P(\delta)$.

□

3.2 *Two Debt Priority Structures*

We analyze two debt priority structures—*pari-passu* and subordination. Under *pari-passu*, any recovery value is distributed proportionally between the private debt holders and the external debt holders. Under subordination, the senior, external creditors have priority over the subordinated, private creditors.

We first analyze the *pari-passu* priority. The only difference from the classical case with only external debt is the presence of two different types of debtholders. The distribution parameter λ is determined by the face values of debt held by the external and private creditors, $D_E(\delta_0)$ and $D_P(\delta_0)$, respectively, and takes the form

$$\lambda = \frac{D_E(\delta_0)}{D_E(\delta_0) + D_P(\delta_0)}. \tag{17}$$

In a competitive market, debt is fairly priced. Here, this fact means that the amount of money a creditor is willing to lend to the firm equals the value of the after-tax payments he receives. Using the bond prices from Proposition 2, the coupon rates C_E and C_P are therefore determined by the system of equations

$$\begin{bmatrix} D_E(\delta_0) = (1-\theta_i)C_E\pi_2(\delta_0) + \lambda(1-\alpha)L\pi_1(\delta_0) \\ D_P(\delta_0) = (1-\hat{\theta})C_P\pi_2(\delta_0) + (1-\lambda)(1-\alpha)L\pi_1(\delta_0) \end{bmatrix}. \tag{18}$$

These coupons secure that the time 0 values of future payments (coupons and recovery amounts) are the same as the amounts lent to the firm at time 0 ($D_E(\delta_0)$ and $D_P(\delta_0)$).

By inserting the expressions for $D_E(\delta_0)$ and $D_P(\delta_0)$ into expression (17) and rearranging, we obtain an explicit expression for λ,

$$\lambda = \frac{(1-\theta_i)C_E}{(1-\theta_i)C_E + (1-\hat{\theta})C_P}. \tag{19}$$

The distribution parameter λ is uniquely determined by the after-tax coupons. If private coupons are taxed at the investor tax-rate θ_i (at all levels of C_P), expression (19) simplifies to $\lambda = C_E/(C_E + C_P)$. That is, the parameter λ is determined by the nominal (pretax) coupons only and is unaffected by tax rates.

Second, we extend our analysis to the priority where external debt is senior to subordinated, private debt. Compared to the classical case, there are two new elements: private debt and subordination.

We focus on the case where external creditors receive the entire liquidation value at default, that is, $\lambda = 1$. To justify this choice of λ, assume first that $\lambda < 1$. Because external creditors have priority over private creditors, a value of $\lambda < 1$ implies that external creditors receive full recourse of their claims. Consequently, external debt is risk free and should yield the risk-free return; only dual holders realize a loss at default. This situation is not common. In Section 3.5, we prove that, under subordination, the implicit assumption in the trade-off theory, that is, $\theta_i \leq \theta_{eff}$, together with a mild restriction on bankruptcy costs, implies that $\lambda = 1$.

To summarize, in the *pari-passu* case, λ is determined from the after-tax coupons. If external debt has priority over private debt, $\lambda = 1$.

Using expression (19) and the above arguments, the debt values under *pari-passu* and subordination follow as corollaries to Proposition 2.

COROLLARY 1. *Under the pari-passu priority structure and the doubly restricted tax system, the value of external debt is given by*

$$D_E(\delta) = (1-\theta_i)C_E\pi_2 + \frac{(1-\theta_i)C_E}{(1-\theta_i)C_E + (1-\hat{\theta})C_P}(1-\alpha)L\pi_1,$$

and the value of private debt is given by

$$D_P(\delta) = (1-\hat{\theta})C_P\pi_2 + \frac{(1-\hat{\theta})C_P}{(1-\theta_i)C_E + (1-\hat{\theta})C_P}(1-\alpha)L\pi_1.$$

COROLLARY 2. *Under subordination and the doubly restricted tax system, the value of external, senior debt is given by*

$$D_E(\delta) = (1-\theta_i)C_E\pi_2 + (1-\alpha)L\pi_1, \tag{20}$$

and the value of private, subordinated debt is given by

$$D_P(\delta) = (1-\hat{\theta})C_P\pi_2. \tag{21}$$

3.3 Equity Value

The time $t \geq 0$ value of equity is given as the value of the after-tax difference between the EBIT flow and the coupon payments from time t until default. We present the value of equity in Proposition 3.

PROPOSITION 3. *The time $t \geq 0$ after-tax value of equity is given by*

$$E(\delta) = (\delta - \delta_B\pi_1)U - \big((1-\theta_{eff})C_D + (1-\theta_{div})(C - C_D)\big)\pi_2, \tag{22}$$

where $C_D = C_E + \min(C_P, Q)$.

PROOF. As shown in Table 1, only the deductible coupons reduce the corporate tax. The nondeductible coupons reduce the amount of dividend and therefore also the dividend tax. We calculate

$$E(\delta_t) = \mathbb{E}_t\left[\int_t^{\tau} \big((1-\theta_{eff})(\delta_s - C_D) + (1-\theta_{div})(C - C_D)\big)e^{-r(1-\theta_i)(s-t)}ds\right]$$

$$
\begin{aligned}
=&\mathbb{E}_t\left[\int_t^{\infty}(1-\theta_{eff})\delta_s e^{-r(1-\theta_i)(s-t)}ds\right]\\
&-\mathbb{E}_t\left[e^{-r(1-\theta_i)(\tau-t)}\mathbb{E}_\tau\left[\int_\tau^{\infty}(1-\theta_{eff})\delta_s e^{-r(1-\theta_i)(s-\tau)}ds\right]\right]\\
&-\big((1-\theta_{eff})C_D+(1-\theta_{div})(C-C_D)\big)\mathbb{E}_t\left[\int_t^{\tau}e^{-r(1-\theta_i)(s-t)}ds\right].
\end{aligned}
$$

The result follows from expression (8), the law of iterated expectations, the definitions of the basic securities π_1, π_2, and the default barrier δ_B. □

3.4 Optimal Default Policies

We now present explicit expressions for the optimal default policies for the two priority structures.

3.4.1 Private and external debt rank pari-passu:. In bankruptcy proceedings, if a verdict is reached, all creditors are treated equally under the *pari-passu* priority. They are repaid at the same time and their debt faces the same recovery rate. The only difference, compared to the classical case, is that the recovery amount is split between two types of creditors.

This split in the recovery amount makes the optimal default policy different from the classical default policy. Dual holders will, in addition to the value of equity also rationally include the value of their private debt when they determine the default policy. This change in objective mitigates the conflicting interests between the equityholders and the debtholders.

The recovery amount $(1-\alpha)L$ is the amount to be shared by the debtholders at default. In expression (15), the recovery amount is expressed as a fraction $(1-\gamma)$ of the unlevered firm value $U\delta_B$ at default. Under the *pari-passu* priority structure, the dual holders are entitled to receive the fraction $(1-\lambda) > 0$ of the recovery amount at default. We express the dual holders' recovery rate of the unlevered firm value at default as

$$1-\Lambda=(1-\lambda)(1-\gamma).$$

The parameter Λ is interpreted as the dual holders' loss rate of the unlevered firm value. Thus, the amount ΛL is the firm value, which is lost for the dual holders in default.

From expressions (16) and (A.3), we see that Λ depends on W_1^*, which again depends on the EBIT flow only through its initial value. We therefore use the notation $\Lambda(d)$, where $d=\delta_B/\delta_0$.

PROPOSITION 4. *The optimal default barrier δ_B^{pp} in the pari-passu case is given by the fixed point*

$$\delta_B^{pp} = \frac{J\eta_Q}{\Lambda(\frac{\delta_B^{pp}}{\delta_0})}, \tag{23}$$

where the constant

$$J = \frac{x}{1+x}\frac{1}{r(1-\theta_i)}\frac{1}{U}, \tag{24}$$

x is given in expression (12), η_Q is the net cost of debt from expression (3), and Λ is the dual holders' loss rate of the unlevered firm value at default.

PROOF. See Appendix B. □

First, we observe that solving the fixed-point problem (23) numerically is straightforward. Additionally, Proposition 4 provides a natural extension of the corresponding classical result involving only external debt. Without private debt, $C_P = 0$, the net cost of debt is $(1-\theta_{eff})C_E(1-\theta_{eff})$, and we may interpret the equityholders' loss rate as being one. The classical result $\delta_B = J(1-\theta_{eff})C_E$ therefore follows directly from Proposition 4.

3.4.2 Senior and subordinated debt:. Under subordination, the dual holders get zero payoff at default, that is, the loss rate $\Lambda(d) = \Lambda = 1$. This fact simplifies the analysis of the default policy, allowing us to present an expression for the optimal default barrier as a corollary to Proposition 4.

COROLLARY 3. *The optimal default barrier δ_B^{sub} in the case of senior, external debt and subordinated, private debt is given by*

$$\delta_B^{sub} = J\eta_Q, \tag{25}$$

where J and η_Q are given in Proposition (4) and expression (3), respectively.

We first note that the expression for the default barrier δ_B^{sub} is given in closed form, not as a fixed-point problem as in the *pari-passu* case. Second, we can now write the default barrier in the *pari-passu* case as

$$\delta_B^{pp} = \frac{\delta_B^{sub}}{\Lambda(\frac{\delta_B^{pp}}{\delta_0})}, \tag{26}$$

demonstrating that $\delta_B^{pp} \geq \delta_B^{sub}$, because $0 < \Lambda(d) \leq 1$ for all $d < 1$. In the *pari-passu* case, the default barrier is equal to the corresponding barrier in the subordinated case, scaled up by the reciprocal of the loss rate. A lower loss rate means a higher recovery amount, that is, a higher default payout, incentivizing earlier default. In the subordinated case, where dual holders receive no recovery amount, they delay default until a lower value of the EBIT flow. A lower default barrier signifies a lower default probability.

In the classical case, the net cost of debt is higher than under subordination. *Ceteris paribus*, using private debt, the default barrier δ_B^{sub} under subordination is therefore lower than the default barrier δ_B in the classical case.

3.5 External Creditors Receive the Whole Recovery Amount

In Section 3.1, we argue that the only interesting economic case under subordination occurs when external creditors are entitled to the entire recovery amount upon default. In our model, this situation corresponds to the case where the parameter $\lambda = 1$. Below we prove that $\lambda = 1$, given a mild restriction on bankruptcy costs and the usual assumption that the effective tax rate is higher than the investor tax rate ($\theta_{eff} > \theta_i$).

LEMMA 1. *The face value of external debt* $D_E(\delta_0) > (1-\alpha)L$ *if and only if* $C_E/r > (1-\alpha)L$.

PROOF. From Corollary 2, $D_E(\delta_0) > (1-\alpha)L$ if and only if

$$(1-\theta_i)C_E\pi_2 + (1-\alpha)L\pi_1 > (1-\alpha)L.$$

Rearranging and dividing the latter expression by $r(1-\theta_i)$, we get

$$\frac{(1-\theta_i)C_E}{r(1-\theta_i)}\pi_2 > (1-\alpha)L\frac{1}{r(1-\theta_i)}(1-\pi_1) = (1-\alpha)L\pi_2.$$

The result then follows. □

Lemma 1 shows that the face value of the risky, external debt exceeds the recovery amount if and only if the coupon rate is larger than the risk-free return on the recovery amount, that is, $C_E > (1-\alpha)Lr$.

The restriction on bankruptcy cost is $1-\gamma < (1+x)/x$, or equivalently $\gamma > -\frac{1}{x}$. Observe that the restriction $\alpha \geq \underline{\alpha}$ from Section 3 implies that this new restriction on γ is satisfied.

PROPOSITION 5. *Assume that the investor tax rate* θ_i *is lower than or equal to the effective tax rate* θ_{eff}. *A sufficient condition, which secures that the external creditors receive the whole recovery amount (*$\lambda = 1$*) in the case of subordinated, private debt, is that* $1-\gamma < (1+x)/x$.

PROOF. Under the doubly restricted tax system, $\eta_Q \leq C_E(1-\theta_{eff})$. Because $\theta_{eff} \geq \theta_i$, $C_E(1-\theta_{eff}) \leq C_E(1-\theta_i)$. By applying the bankruptcy cost restriction, $1-\gamma < (1+x)/x$, we can write

$$\frac{x}{1+x}(1-\gamma)\eta_Q < \eta_Q \leq C_E(1-\theta_{eff}) \leq C_E(1-\theta_i). \tag{27}$$

From the above inequalities, we derive an upper bound for the recovery amount. Dividing the inequalities in expression (27) by the after-tax risk-free rate, we obtain the first inequality below. The two subsequent equalities follow by using the optimal default barrier in Corollary 3 and then reformulate it by using expression (15),

$$\frac{C_E}{r} = \frac{C_E(1-\theta_i)}{r(1-\theta_i)} > \frac{x}{1+x}(1-\gamma)\frac{\eta_Q}{r(1-\theta_i)} = (1-\gamma)\delta_B^{sub}U = (1-\alpha)L.$$

We have now proved that $C_E/r > (1-\alpha)L$. The result then follows from Lemma 1. □

4. OPTIMAL CAPITAL STRUCTURE

The enterprise value $W(\delta)$ represents the after-tax value of equity and debt from Propositions 2 and 3, and it can be expressed as

$$W(\delta) = E(\delta) + D_E(\delta) + D_P(\delta). \tag{28}$$

In expression (A.1) in Appendix A, we calculate $W(\delta)$ explicitly. By using expression (15), this expression can be simplified to

$$W(\delta) = \delta U + \pi_2 T - \gamma \delta_B U \pi_1, \tag{29}$$

where

$$T = (\theta_{eff} - \theta_i)(C_E + \min(C_P, Q)). \tag{30}$$

The first term of expression (29) represents the unlevered value of the firm, the second term the value of the tax benefits of debt, and the third term the cost of bankruptcy.

Let $W(C_P, C_E)$ denote the enterprise value as a function of the coupon rates. Denote the optimal external and private coupons by C_E^* and C_P^*, respectively. They can be found by solving the optimization problem

$$W^*(\delta) = \max_{C_P, C_E} W(C_P, C_E) = W(C_P^*, C_E^*). \tag{31}$$

Expression (29) demonstrates that this optimization problem involves a trade-off between the tax benefits of debt and bankruptcy costs. However, there are two novel elements compared to the classical case. First, there are two distinct types of debt, one of which has limited tax deductibility. Second, the use of private debt alters the prices of the basic securities, which are contingent on the default barrier. Under subordination, π_1 decreases as C_P increases, leading to an increase in the value of π_2. Consequently, the value of tax benefits associated with debt increases, while bankruptcy costs decrease when compared to the classical case.

In Appendix A, we show that the optimal enterprise value is proportional to W_1^*, that is,

$$W^*(\delta) = \delta W_1^*,$$

and that the optimal coupons, $C_E^* = \delta c_E^*$ and $C_P^* = \delta c_P^*$. The proportional coupon pair (c_P^*, c_E^*) is such that $W_1(c_P^*, c_E^*) = W_1^*$, cf. expression (A.3).

The use of private debt has the potential to increase the enterprise value, compared to the classical case, which involves only external debt. Let C^* denote the optimal external coupon in the classical case. Let $W^k(C_P, C_E)$ denote the enterprise value for case k, where $k \in \{c, dh\}$ either represents the classical case (c) or the dual holder case (dh). The following proposition establishes that enterprise values in cases with private debt are at least as high as in the classical case.

Proposition 6. *There exists coupon pairs* (C_P^*, C_E^*) *such that the enterprise values*

$$W^c(0, C^*) \leq W^{dh}(C_P^*, C_E^*). \tag{32}$$

Proof. One coupon pair that satisfies Inequality (32) is $(C_P, C_E) = (0, C^*)$. Clearly, maximizing the enterprise value with respect to both C_E and C_P cannot give a lower enterprise value than in the classical case, but possibly a higher value. □

The priority structure between external and private debt not only impacts default risk but also influences the enterprise value. Proposition 7 demonstrates that the maximum enterprise value under subordination is at least as high as the maximum enterprise value under *pari-passu*.

Proposition 7. *The maximum enterprise value in the case with subordinated, private debt is higher than or equal to the maximum enterprise value in the pari-passu case.*

Proof. Let (c_P^{k*}, c_E^{k*}) be a proportional coupon pair, which maximizes W_1^k for priority k, where $k \in \{pp, sub\}$, and pp and sub are abbreviations for *pari-passu* and subordination, respectively. We have that

$$\begin{aligned} W^{pp*}(\delta) &= \delta W_1^{pp*} = \delta W_1^{pp}(c_P^{pp*}, c_E^{pp*}) \\ &\leq \delta W_1^{sub}(c_P^{pp*}, c_E^{pp*}) \leq \delta W_1^{sub}(c_P^{sub*}, c_E^{sub*}) \\ &= \delta W_1^{sub*} = W^{sub*}(\delta). \end{aligned}$$

The first and the last equalities follow from expression (A.2). The second and the third equalities follow from the definition of W_1^*. Proposition 4 establishes that the default barrier $\delta_B^{sub} \leq \delta_B^{pp}$. From expression (A.4), it is easy to show that $\mathrm{d}W_1/\mathrm{d}d < 0$. The first inequality then follows. The second inequality follows from the optimality criterion. □

An important implication of Proposition 7 is that an enterprise-value maximizing firm will always let the dual holders' debt be junior to external debt. *Pari-passu* priority is not the optimal priority structure.

The following propositions characterize the optimal use of private debt.

PROPOSITION 8. *Assume that $C_E < \bar{C}_E$, where the upper bound $\bar{C}_E$ is given in expression (B.2). Under the pari-passu priority structure, the optimal private coupon is $C_P^* = Q$.*

PROOF. See Appendix C. □

PROPOSITION 9. *Under the subordinated priority structure, one optimal private coupon is $C_P^* = Q$.*

PROOF. The enterprise value is given in expression (29). The result follows by observing that the derivative $\mathrm{d}W/\mathrm{d}C_P > 0$ for $C_P < Q$ and $\mathrm{d}W/\mathrm{d}C_P = 0$ for $C_P \geq Q$. □

These propositions show that to maximize enterprise value, the firm should aim for a private coupon as high as the tax restriction allows through the parameter Q. Under *pari-passu*, the optimal private coupon is uniquely given by $C_P^* = Q$ if $C_E < \bar{C}_E$. As we will see in Section 6, the upper bound for external coupon $\bar{C}_E$ is high relative to the optimal external debt coupon. This bound does therefore not seem economically important. Conversely, there is no such upper bound under the subordinated priority structure. Under subordination, private coupons above Q do not impact the enterprise value, creating a continuum of private coupon values that yield the same enterprise value as $C_P = Q$.

Given the optimal private coupon $C_P^* = Q$, we calculate the optimal external coupon numerically.

5. THE NO-DEFAULT REGION

In this section, we use the doubly restricted tax system introduced in Section 1.4 and relax assumption (4) that $C_E > KC_P$. Coupon pairs in the no-default region with $\eta_Q \leq 0$ are now possible. As explained before, negative financing costs are at odds with central economic principles, but may follow from the use and lax taxation of private debt. Within this region, we examine the optimal capital structure and provide conditions under which this capital structure is also globally optimal.

Since there is no default risk in this region, the values of the two basic securities are $\pi_1 = 0$ and $\pi_2 = 1/(r(1-\theta_i))$. The values of debt follow as a corollary to Proposition 2.

COROLLARY 4. *The value of risk-free, external debt is*

$$D_E = \frac{C_E}{r},$$

and the value of risk-free, private debt is

$$D_P = \frac{(1-\hat{\theta})C_P}{r(1-\theta_i)}.$$

We show in Proposition 10 that it is never optimal to let η_Q be strictly negative.

PROPOSITION 10. *It is not optimal to let the net cost of debt η_Q in expression (3) be strictly negative.*

PROOF. To see this claim, assume first that $\eta_Q < 0$. In this case, the dual holders never let the firm default—external debt is risk free and bankruptcy costs are zero. The enterprise value in expression (29) simplifies to

$$W(\delta) = \delta U + \frac{(\theta_{eff} - \theta_i)}{r(1-\theta_i)}(C_E + \min(C_P, Q)).$$

By marginally increasing C_E, bankruptcy costs remain zero, but the firm receives the increased tax benefits from more external debt, increasing the enterprise value. These two effects prevail for increasing values of C_E until $\eta_Q = 0$. □

By combining the result in Proposition 10 and expression (6), we can write the enterprise value on the no-default boundary as

$$W(\delta) = \delta U + \frac{\theta_{eff} - \theta_i}{r(1-\theta_i)}(K+1)\min(C_P, Q), \tag{33}$$

where K is from expression (5).

PROPOSITION 11. *The maximum enterprise value in the no-default region is*

$$W^{\triangle}(\delta) = \delta\Delta,$$

where

$$\Delta = U + \frac{\theta_{eff} - \theta_i}{r(1-\theta_i)}(K+1)\frac{Q}{\delta}.$$

One coupon pair that maximizes the enterprise value in the no-default region is $(C_P^{\triangle}, C_E^{\triangle}) = (Q, KQ)$.

PROOF. From Proposition 10, any optimal coupon pair in the no-default region is on the no-default boundary. By maximizing the enterprise value in expression (33) with respect to C_P, we see that one optimal private coupon is $C_P^{\triangle} = Q$. From Proposition 10 and expression (6), it follows that the corresponding optimal external coupon $C_E^{\triangle} = KQ$. Thus, the coupon pair $(C_P^{\triangle}, C_E^{\triangle}) = (Q, KQ)$ maximizes the enterprise value on the no-default boundary. Inserting this optimal coupon pair into expression (33) gives the maximum enterprise value in the no-default region. □

We note that the enterprise value is linearly increasing in the coupon rates, which in turn are proportional to Q. In the no-default region, the maximum enterprise value is uniquely determined by the exogenously specified tax parameter Q.

The next step is to determine whether any globally optimal coupon pair is situated on the no-default boundary or within the risky region. Recall from previous sections that $W^*(\delta)$ is the maximum enterprise value in the risky region. Denote by W^{**} the maximum enterprise value across both the risky and no-default regions.

PROPOSITION 12. *The globally optimal enterprise value W^{**} is given by*

$$W^{**} = \delta \max(W_1^*, \Delta).$$

If $\Delta > W_1^*$, $W^{**} = W^{\triangle}$ and the optimal coupon pairs are on the no-default boundary. If $\Delta < W_1^*$, the optimal coupon pair is in the risky region. If $\Delta = W_1^*$, the maximum enterprise value is the same in the risky region as on the no-default boundary.

PROOF. If $\Delta > W_1^*$, we have from Proposition 11 and expression (A.2) that

$$W^{**} = W^{\triangle}(\delta) = \delta\Delta > \delta W_1^* = W^*(\delta).$$

If $\Delta < W_1^*$,

$$W^{**} = \delta W_1^* = W^*(\delta) > W^{\triangle}(\delta) = \delta\Delta.$$

If $\Delta = W_1^*$,

$$W^{**} = W^{\triangle}(\delta) = \delta\Delta = \delta W_1^* = W^*(\delta).$$

□

The multiplier Δ is determined by the tax parameters θ_{eff}, θ_i, and Q, as well as the after-tax risk-free interest rate, the price/earnings ratio U, and the time 0 value of δ. In particular, Δ is linearly increasing in Q and is therefore unbounded. Consequently, for sufficiently high values of Q, the optimal coupon pairs are situated on the no-default boundary.

6. NUMERICAL ANALYSIS

We now provide a numerical implementation of our model and illustrate the results using examples. To gain a better understanding of what distinguishes the risky region from the no-default region, we first show how the conditionally optimal coupon pairs $(C_P, C_E^*(C_P))$ transition from the risky region to the no-default boundary as C_P increases. Second, we calculate the values of default barriers to showcase the default policies for the two priority structures. Third, we show how the capital structure decisions, that is, the choice of coupon pairs, impact enterprise values. Given the optimal coupon pairs, it is straightforward to calculate credit spreads.

TABLE 2

THE TABLE REPORTS THE PARAMETER VALUES WE USE IN THE NUMERICAL IMPLEMENTATION OF OUR MODEL

$\delta_0 = 30$	$\mu = 0.01$	$\sigma = 0.3$	$\alpha = 0.3$
$r = 0.04$	$\theta_C = 0.25$	$\theta_i = 0.3$	$\theta_{div} = 0.2$

6.1 Choice of Parameter Values

In our analysis, we employ eight parameter values as listed in Table 2.

The initial EBIT value of 30 results in an unlevered asset value of 1,000 in the absence of taxes $(30/(0.04 - 0.01))$. A risk-free interest rate of 4% is in proximity to the yields on 10-year treasury bonds in several industrialized countries.[3] The worldwide average corporate tax rate in 2022, computed across 180 jurisdictions, was 23.37%.[4] Our corporate tax rate of 25% is close to this average. We use a dividend tax-rate of 20%. This rate is the same as the highest tax rate for (qualified) dividends in the United States, and just below the average top dividend tax rate of 24% for European OECD countries.[5,6] These two tax rates give an effective tax rate $\theta_{eff} = 0.4$. We set the investor tax rate $\theta_i = 30\%$, which is below the effective tax rate to secure that there is a tax advantage of debt financing. There seems to be considerable cross-sectional variation in this tax rate across different countries. The after-tax risk-free interest rate is 2.8% $(0.04 \cdot (1 - 0.3))$. The growth rate $\mu = 0.01$, in combination with the risk-free interest rate and the tax rates, gives a P/E-ratio of 35.6, close to the average P/E-ratio of 37 for the S&P-500 constituents in 2021.[7]

6.2 Risky Region and No-Default Region

In Figure 1, we plot the surface for the enterprise value as a function of the coupon pairs (C_P, C_E) for the subordinated priority structure. On the surface, we plot a curve, which shows the conditionally optimal coupon pairs $(C_P, C_E^*(C_P))$. Indeed, the plot demonstrates a discontinuous transition (jump) of the conditionally optimal coupon pairs from the risky region to the no-default boundary as C_P increases to a certain threshold, approximately 19.7, as shown by the dotted curve. This jump in coupon pairs reflects the critical point at which the firm's optimal capital structure shifts from

3. For instance, at the time of writing (April 24, 2023), the yields on 10-year treasuries are about 3.6% in the United States, 2.5% in Germany, and 4.3% in Italy.

4. https://taxfoundation.org/publications/corporate-tax-rates-around-the-world

5. https://www.investopedia.com/terms/q/qualifieddividend.asp

6. https://taxfoundation.org/dividend-tax-rates-europe-2022

7. https://www.currentmarketvaluation.com/models/price-earnings.php

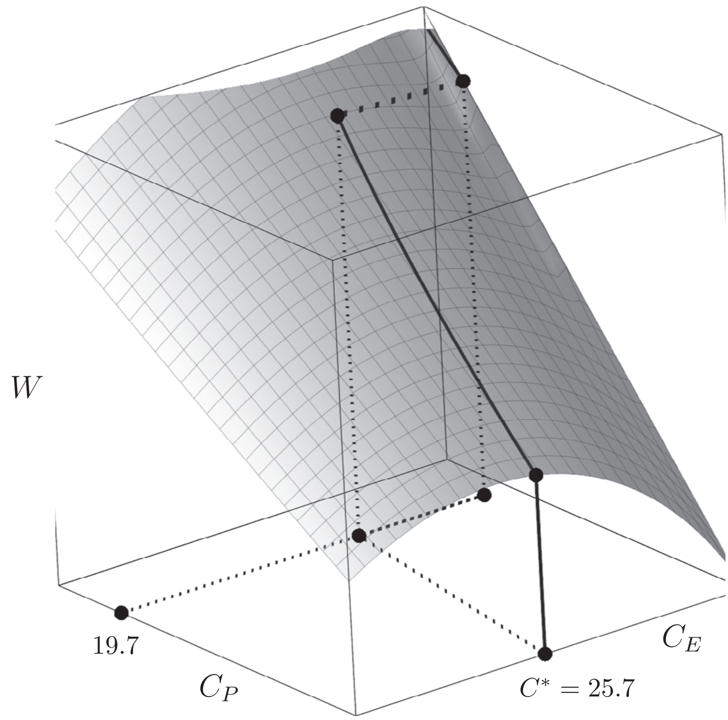


Fig 1. Enterprise Value—Subordinated Priority Structure.

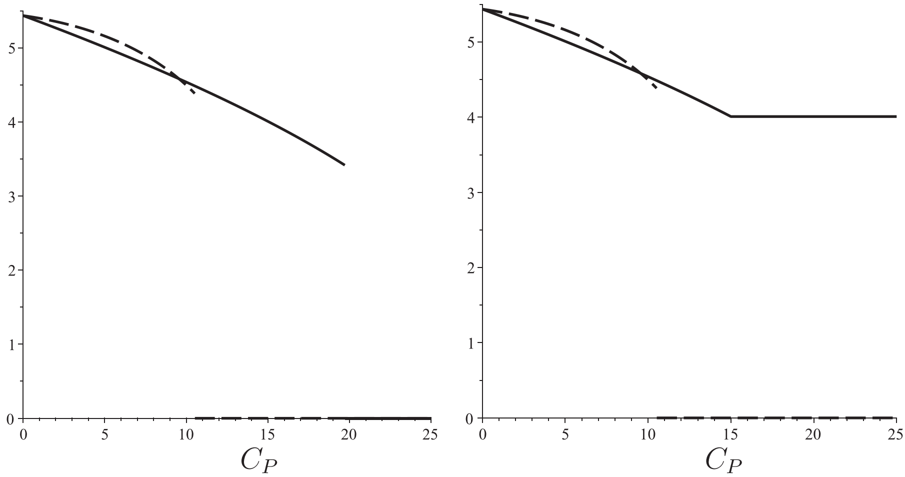
NOTES: The figure plots the enterprise value as a function of C_P and C_E. Other parameter values are given in Table 2.

a risky, leveraged position to a risk-free position with no default risk. It highlights the sensitivity of the capital structure decisions to changes in coupon rates and the trade-off between tax benefits and bankruptcy costs in the model.

6.3 Optimal Default Policy

We numerically illustrate the optimal default policies, that is, the values of the default barriers δ_B for the two priority structures. To highlight the importance of the tax authorities' restrictions, we use "high" and "low" values of Q equal to 30 and 15, respectively.

From panel (a) of Figure 2, we see that under *pari-passu*, there is a discontinuity at $C_P = 10.5$. This discontinuity occurs as the firm's conditionally optimal coupon pairs transition from the risky region to the no-default boundary. For $C_P < 10.5$, the default barrier under *pari-passu* is for most values of C_P higher than the corresponding barrier values under subordination. That we observe a small interval where $\delta_B^{pp} < \delta_B^{sub}$ is not a violation of our earlier claim that the default barrier in the risky region is higher under *pari-passu* than under subordination (see expression (26)). The observation



(a) Default barriers for $Q = 30$.

(b) Default barriers for $Q = 15$.

Fig 2. Default Barriers.

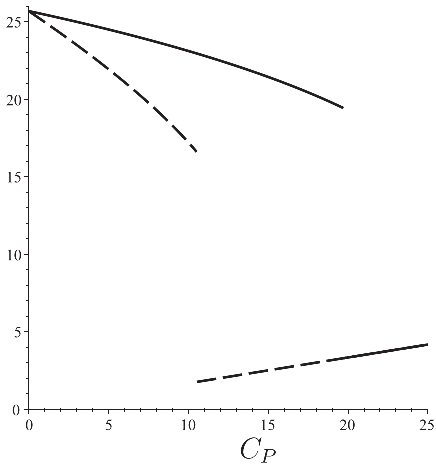
NOTES: The figure plots the optimal default barriers as functions of C_P. The solid lines are for the case with subordinated, private debt and the dashed lines are for the case with *pari-passu* priority. Other parameter values are given in Table 2.

here simply reflects the fact that the conditionally optimal external coupons $C_E^*(C_P)$ differ under the two priority structures. Under subordination, $C_P = 19.7$ is the critical value for which $\eta_Q = 0$ and the coupon pairs $(C_P, C_E^*(C_P))$ transition from the risky region to the no-default boundary. This transition corresponds to the jump we see in Figure 1.

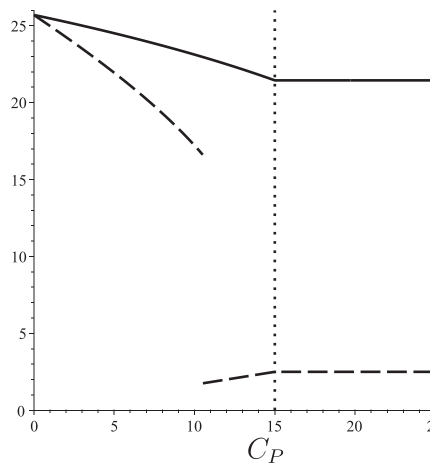
From panel (b) of Figure 2, it is clear that under *pari-passu*, the change in Q from 30 to 15 does not change the conditionally optimal coupon pairs. The transition from the risky region to the no-default boundary takes place at $C_P = 10.5 < Q = 15$. Under subordination, the change in Q has a direct effect on these coupon pairs. The transition would have occurred for a deductible, private coupon of $C_P = 19.7$, which is greater than $Q = 15$. The plot illustrates how the impact of the tax restriction Q varies between the two distinct priority structures.

6.4 Enterprise Value

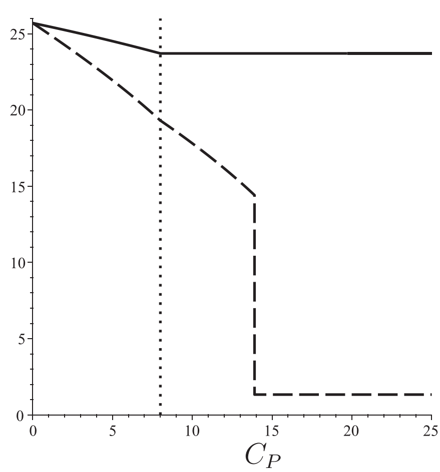
In panel (a) of Figure 3, we observe that in the risky region, the two types of coupons act as substitutes. As private coupons increase, the conditionally optimal external coupons decrease. This substitution effect is more pronounced under the *pari-passu* structure than under subordination. Additionally, the panel illustrates that the coupons are complements on the no-default boundary. However, this complementarity ceases to exist when $C_P = Q = 30$, and beyond this point, the coupons are neither complements nor substitutes (not shown in the plot). Furthermore, it highlights



(a) Coupon combinations for $Q = 30$.



(b) Coupon combinations for $Q = 15$.



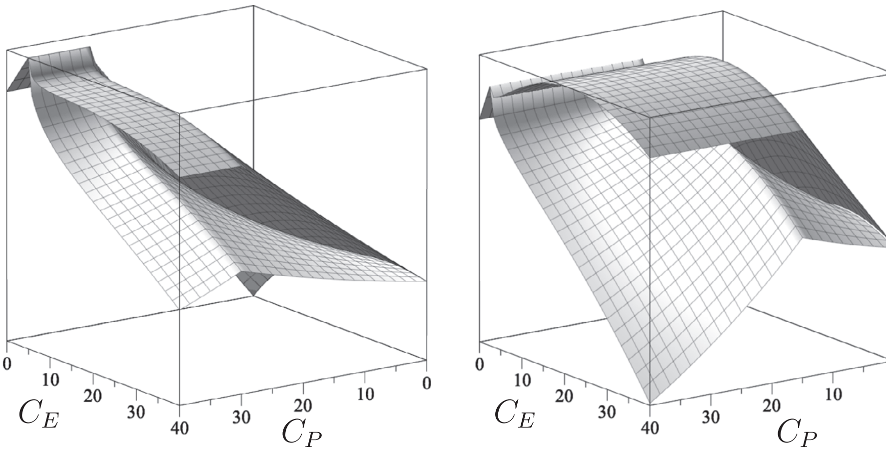
(c) Coupon combinations for $Q = 8$.

Fig 3. Conditionally Optimal Coupon Pairs.

NOTES: The figure plots conditionally optimal coupon pairs $(C_P, C_E^*(C_P))$. The solid lines are for the case with subordinated, private debt and the dashed lines are for the case with *pari-passu* priority. The dotted, vertical lines represent the values of Q. Parameter values are given in Table 2.

that the conditionally optimal coupon pairs converge on the no-default boundary for both priority structures, rendering the priority structure irrelevant when there is no default risk.

In panel (b), we depict the same graphs as in panel (a), but with a reduced Q value from 30 to 15. In both priority structures, the plots reveal that coupons are neither



(a) Enterprise values for $Q = 30$. (b) Enterprise values for $Q = 15$.

Fig 4. Enterprise Values.

NOTES: The figure plots the enterprise values as functions of C_P and C_E for $Q = 30$ and $Q = 15$. In each plot, the upper surface is for the case with subordinated debt and the lower surface is for the *pari-passu* case. The value of the upper bound $\bar{C}_E = 47.03$. Other parameter values are given in Table 2.

complements nor substitutes for $C_P \geq Q$. As C_P exceeds Q, the conditionally optimal external coupons remain constant. This effect is observed in both the risky region under subordination and in the no-default region under *pari-passu*.

The situation is more intricate under *pari-passu*. Consider an even lower value of Q, say 8, which is below the value of $C_P = 10.5$ where the coupon pairs would transition from the risky region to the no-default boundary if Q was greater than 10.5. By increasing C_P in the interval $[Q, 10.5)$, the firm still reduces C_E, but at a slightly lower rate than when $C_P < Q$, see panel (c) of Figure 3. The reason for this different behavior, compared to under subordination where C_E does not change, is that the dual holders receive a larger fraction of the liquidation value in case of default when C_P is high and C_E is low. In fact, with the given parameters and $Q = 8$, this reduction in C_E for increases in C_P continues until $C_P = 13.9$, at which point $\eta_Q = 0$. However, the optimal $C_P^* = Q$ here and the no-default boundary is therefore not attainable.

In Figure 4, we plot the surfaces for the enterprise values for two different levels of Q. The upper surfaces correspond to the subordinated priority structure, whereas the lower surfaces pertain to *pari-passu*, showcasing the findings from Proposition 7. Panel (a) highlights how, when $Q = 30$, the maximum enterprise values for both priority structures are achieved on the no-default boundary. Panel (b) illustrates the impact of reducing Q to the lower value of 15. Under subordination, the maximum value lies within the risky region, whereas under *pari-passu*, it shifts to the no-default boundary.

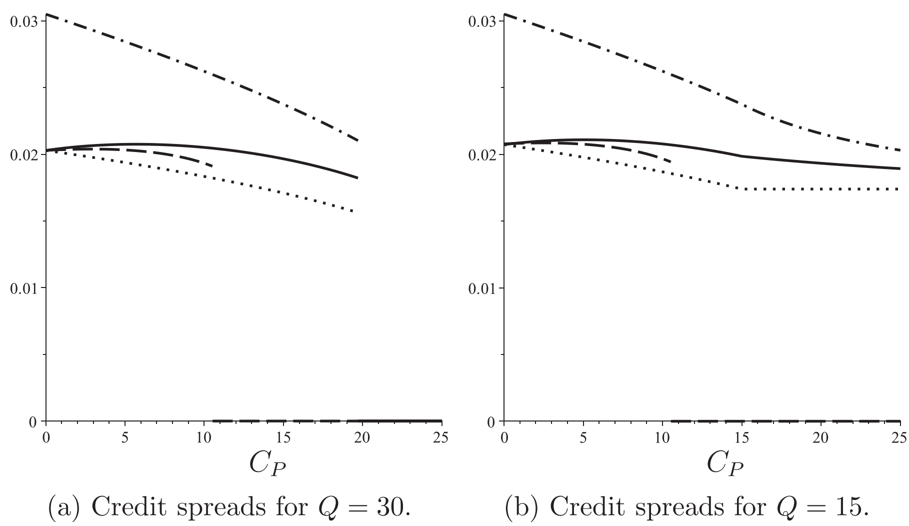


Fig 5. Credit Spreads.

NOTES: The figure plots credit spreads for conditionally optimal coupon pairs $(C_P, C_E^*(C_P))$ on $Q = 30$ and $Q = 15$. The solid lines show spreads on total debt in the case with subordinated, private debt. The dash-dotted lines show spreads for subordinated, private debt. The dotted lines show spreads on senior, external debt. The dashed lines show spreads for both private and external debt with *pari-passu* priority. Other parameter values are given in Table 2.

6.5 Credit Spreads

Our model allows us to compute credit spreads for various coupon combinations. We use the results in Corollaries 1 and 2 and compute the credit spreads for the two debt types as

$$s_j = \frac{C_j}{D_j(\delta)} - r,$$

where $j = E, P$, and for the total debt as

$$s = \frac{C_P + C_E}{D_P(\delta) + D_E(\delta)} - r.$$

Under *pari-passu*, both types of debt possess identical recovery rates, resulting in equivalent credit spreads. The plots in Figure 5 illustrate the credit spreads. Compared to the case without private debt, the spreads are wider for small values of C_P, but tighter thereafter. In panel (a), there is an interval with zero credit spreads for both priority structures. This zero spread is due to the coupon pairs residing on the no-default boundary, eliminating credit risk. In panel (b), this observation applies only to the *pari-passu* case.

Under subordination, the entire recovery amount goes to the external creditors in the event of a default. Consequently, external debt has tighter credit spreads than the more risky private debt. The plots illustrate that the spreads on both external and private debt tighten as C_P increases. The spreads on private debt tighten at a faster rate than the spreads on external debt. As external and private debt are substitutes in the risky region, the spread on total debt is first widening in C_P and subsequently tightening. For lower values of C_P, the total cost of debt increases when cheap external debt is substituted by more expensive private debt.

7. CONCLUSIONS

By using trade-off theory, we analyze financial decision making by a firm with two types of debt—debt to external creditors and debt to the equityholders. In this situation, the equityholders are dual holders. Private debt offers a tax advantage, which we mitigate by introducing a constraint into the tax system. We explore two distinct priority structures: *pari-passu* and subordination. Under the *pari-passu* priority structure, all creditors are on equal footing in default. Under the second priority structure with subordination, the external creditors are senior to the junior, private creditors, which also own the firm's equity.

We focus on three distinct financial decisions. First, we determine the optimal default policy under both priority structures. All else being equal, a firm with subordinated, private debt exhibits lower default risk compared to a firm financed only by external debt. Similarly, a firm with subordinated, private debt has lower default risk than a firm where private debt ranks *pari-passu* with external debt.

Second, we analyze the firm's optimal capital structure, that is, the optimal choice of coupons on external debt and private debt. We identify two distinct regions, which are separated by combinations of external and private coupons. In the first region, with relatively more external debt, credit risk is present and both types of creditors take this risk into account when pricing the debt—credit spreads are strictly positive. With relatively more private debt, the coupon pairs transition from the risky region to the no-default region. In this region, credit risk vanishes, and both the default barrier and credit spreads are equal to zero.

Third, we demonstrate that subordination never gives lower enterprise value than *pari-passu*.

One implication arising from our analysis is the expectation of observing significant amounts of private debt, restrained primarily by regulatory limitations if they exist. However, there are several reasons why this implication may not hold. Our model was designed to analyze private debt in a dual holder setting, and it does not constitute a model of optimal portfolio choices for dual holders. Dual holders may find more attractive investment opportunities elsewhere, either for risk diversification or to achieve a superior risk-to-reward ratio. Constraints stemming from capital scarcity and liquidity could also curtail the use of private debt by dual holders.

Our model has several empirically testable implications. First, it predicts reduced default probabilities for firms with dual holdership. Second, it suggests that firms owned by dual holders should consistently incorporate both private and external debt into their capital structure. Last, it posits that firms with dual holdership should employ subordinated, private debt.

Our results highlight that a firm's inclination to default on its debt payments is contingent upon the ownership of the debt. These findings could potentially extend to default incentives for firm owners who do not fall under the category of dual holders. For example, the default incentives of sovereign entities may vary based on whether their debt is held by foreign or domestic investors. Likewise, for other firms, these incentives may be influenced by whether their debt is held by influential banks or passive investors.

APPENDIX A: ENTERPRISE VALUE

From Propositions 2 and 3, we have that the enterprise value is given by

$$\begin{aligned} W(\delta) =& (\delta - \delta_B \pi_1)U - \pi_2\big((1-\theta_{eff})C_D + (1-\theta_{div})(C - C_D)\big) \\ &+ \pi_2\big((1-\theta_i)C_E + (1-\hat{\theta})C_P)\big) + (1-\alpha)L\pi_1. \end{aligned}$$

To rewrite the expression for the enterprise value $W(\delta)$, we use that $C_D = C_E + \min(C_P, Q)$ and the definition of $\hat{\theta}$. We also use that $C - C_D = \max(C_P - Q, 0)$ and that $\min(C_P, Q) + \max(C_P - Q, 0) = C_P$, so

$$\begin{aligned} W(\delta) =& (\delta - \delta_B \pi_1)U - \pi_2\big(\theta_{eff} \min(C_P, Q) - \theta_{div} \max(C_P - Q, 0)\big) \\ &+ \pi_2((1-\theta_i)C_E + C_P - \theta_i \min(C_P, Q) - \theta_{div} \max(C_P - Q, 0)) \\ &+ (1-\alpha)L\pi_1. \end{aligned}$$

Collecting like terms, the enterprise value simplifies to

$$W(\delta) = \delta U + \pi_2\big((\theta_{eff} - \theta_i)(C_E + \min(C_P, Q)\big) - \pi_1((1-\alpha)L - \delta_B U), \tag{A.1}$$

where the first term represents the unlevered value of the firm, the second term the value of the tax benefits of debt, and the third term the bankruptcy cost. Let the proportional tax-deductible coupon

$$c_D = \frac{C_D}{\delta} = \frac{C_E + \min(C_P, Q)}{\delta}.$$

We now use that $L = \delta_B W_1^* = W^*(\delta)d$, where $d = \delta_B/\delta$, cf. expression (15), and the homogeneity property of $W^*(\delta)$. We write the enterprise value as

$$W(\delta) = \delta U(1 - \pi_1 d) + \pi_2 \delta((\theta_{eff} - \theta_i)c_D) + W^*(\delta)d(1-\alpha)\pi_1.$$

This equation also holds for the optimal enterprise value $W^*(\delta)$. Thus, inserting $W^*(\delta)$ on the left-hand side, we can solve for $W^*(\delta)$ to get

$$W^*(\delta) = \delta W_1^*, \tag{A.2}$$

where

$$W_1^* = \max_{c_P, c_E} W_1 = W(c_P^*, c_E^*), \tag{A.3}$$

and

$$W_1 = \frac{(1 - d\pi_1)U + \pi_2(\theta_{eff} - \theta_i)c_D}{1 - d(1-\alpha)\pi_1}. \tag{A.4}$$

Here, $W_1 = W_1(c_P, c_E)$ is a function of the proportional coupons and the prices of the two basic securities, π_1 and π_2, which are functions of d. Therefore, the expression for W_1 only depends on the ratio d, not on the level of δ. We therefore also write $W_1 = W_1(d)$. It is easy to verify from expression (A.2) that $W^*(\delta)$ satisfies the homogeneity property explained in Section 2. In numerical implementations, we solve the optimization problem (A.3) numerically.

APPENDIX B: PROOF OF PROPOSITION 4

For notational simplicity, we let $\delta_B = \delta_B^{pp}$, and we denote the loss rate by $\Lambda(\delta)$. We first derive an expression for the dual holder's private value, that is, the sum of the values of equity and private debt. This expression follows from expression (22) and Corollary 1, where we use expression (15) and similar arguments as in Appendix A. We find that

$$E(\delta) + D_P(\delta) = (\delta - \delta_B\pi_1(\delta))U - \eta_Q\pi_2(\delta) + (1 - \Lambda(\delta))\delta_B\pi_1(\delta)U$$

$$= (\delta - \Lambda(\delta)\delta_B\pi_1(\delta))U - \eta_Q\pi_2(\delta).$$

We calculate the default policy that maximizes the value of this expression. To this end, we use the smooth-pasting condition (see, e.g., Brekke and Øksendal 1991, Dixit and Pindyck 1993, or Øksendal 1995)

$$\left.\frac{d(E(\delta) + D_P(\delta))}{d\delta}\right|_{\delta=\delta_B} = \left.\frac{\mathrm{d}\Omega(\delta)}{\mathrm{d}\delta}\right|_{\delta=\delta_B}, \tag{B.1}$$

where $\Omega(\delta) = (1-\lambda)(1-\alpha)L(\delta) = (1 - \Lambda(\delta))\delta U$ is the dual holders' recovery amount at default for $\delta = \delta_B$. We calculate the left- and the right-hand sides of this equation as

$$\left.\frac{\mathrm{d}(E(\delta) + D_P(\delta))}{\mathrm{d}\delta}\right|_{\delta=\delta_B} = U(1 + x\Lambda(\delta) - \delta_B\Lambda'(\delta)) - \frac{x\eta}{r(1-\theta_i)\delta_B},$$

and

$$\left.\frac{\mathrm{d}\Omega(\delta)}{\mathrm{d}\delta}\right|_{\delta=\delta_B} = U(1 - \Lambda(\delta) - \delta_B\Lambda'(\delta)),$$

respectively. Observe that $\Lambda(\delta)|_{\delta=\delta_B} = \Lambda(\delta)$. The explanation for this result is that $\Lambda(\delta)$ depends only on the initial value of δ, which determines the constant d, the ratio of the default barrier to the initial δ value. The same explanation holds for $\Lambda'(\delta)$. The result of the proposition follows by equating the two latter expressions and solving for δ_B. Note, in particular, that the solution does not depend on $\Lambda'(\delta)$.

APPENDIX C: PROOF OF PROPOSITION 8

We now prove that $C_P^* = Q$ for $C_E < \bar{C}_E$ under *pari-passu*.

From expression (29), if follows that

$$\frac{\mathrm{d}W}{\mathrm{d}C_P} = \begin{cases} (\theta_{eff} - \theta_i)\pi_2 - \frac{\mathrm{d}\delta_B}{\mathrm{d}C_P}A, & \text{if } C_P < Q, \\ -\frac{\mathrm{d}\delta_B}{\mathrm{d}C_P}A, & \text{if } C_P \geq Q, \end{cases} \tag{B.1}$$

where $A = \gamma U \frac{\mathrm{d}(d\pi_1)}{\mathrm{d}d} - T\frac{\mathrm{d}\pi_2}{\mathrm{d}d} > 0$.

From Proposition 4, we see that increasing the private coupon has two effects on the default barrier. First, increasing C_P reduces the net cost of debt if $C_P < Q$, which reduces δ_B. Second, increasing C_P reduces the loss rate Λ, which increases δ_B. For $C_P < Q$, there are two opposing effects.

After implicit differentiation of expression (23), it follows that

$$\frac{\mathrm{d}\delta_B}{\mathrm{d}C_P} = \begin{cases} \frac{J(C_P+C_E)}{C_E+\gamma C_P}\left(\frac{\eta_Q(1-\gamma)C_E}{(C_P+C_E)(C_E+\gamma C_P)} - \theta_{eff} + \theta_i\right) \gtrless 0 \text{ if } C_P < Q, \\ \frac{J\eta_Q(1-\gamma)\beta C_E}{(C_E+\beta\gamma C_P)^2} > 0 \text{ if } C_P \geq Q, \end{cases}$$

where $\beta = (1-\hat{\theta})/(1-\theta_i)$. Due to the sign of the derivative, we can conclude that $\mathrm{d}W/\mathrm{d}C_P < 0$ for $C_P \geq Q$, but it may be positive or negative for $C_P < Q$.

For $C_P < Q$, a few standard manipulations yield that

$$\frac{\mathrm{d}W}{\mathrm{d}C_P} > 0 \iff B_1 - B_2 + kB_3 > 0,$$

where

$$k = \frac{(\theta_{eff} - \theta_i)(C_E + \gamma C_P)}{\gamma\eta_Q + (\theta_{eff} - \theta_i)(C_E + \gamma C_P)},$$

$$B_1 = (\theta_{eff} - \theta_i)(C_E + \gamma C_P)(C_E + C_P),$$

$$B_2 = \eta_Q(1-\gamma)C_E,$$

and

$$B_3 = \frac{\pi_2(C_E + \gamma C_P)r(1-\theta_i)\eta_Q}{\pi_1 x},$$

where x is from expression (12). To determine an upper bound $\bar{C}_E$ for the external coupon, we assume that this bound is decreasing in C_P (this is reasonable and also easy to verify numerically). By inserting $C_P = 0$ into the condition $B_1 - B_2 + kB_3 = 0$, solving for C_E, and denoting the solution $\bar{C}_E$, we find that

$$\bar{C}_E = \frac{(1 + (1 - \gamma - K)x(\frac{\gamma}{K} + 1))^{-\frac{1}{x}}}{J(1 - \theta_{eff})}\delta_0, \tag{B.2}$$

where K and J are from expressions (5) and (24), respectively.

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