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A stochastic optimization algorithm for the supply vessel planning problem under uncertain demand and uncertain weather conditions

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ABSTRACT

The Supply Vessel Planning Problem (SVPP) with stochastic demands and uncertain weather conditions is a transportation problem occurring in offshore oil and gas logistics. A fleet of supply vessels based at an onshore depot delivers supplies to a set of offshore oil platforms in a weekly sailing schedule. However, the schedules are frequently disrupted due to adverse weather conditions and uncertain demand for cargo from the oil platforms. The two sources of uncertainty are generally addressed in separate, most often through the use of two-phased methods, where simulation is combined with an optimization algorithm. The most common approach to incorporate robustness in the constructed schedules is to use a subjective penalized cost for non-robust voyages, with explicit modelling of recourse actions. In contrast, this paper proposes a two-stage stochastic programming algorithm accounting for both uncertain demand and uncertain weather conditions, allowing for the incorporation of the cost of recourse in the objective function. The cost of each solution is approximated through the use of discrete event simulation within a genetic algorithm. For the tested problem instances, the potential benefit from solving the stochastic program over solving the corresponding deterministic version leads to average relative annual cost savings of approximately 12%.

1. Introduction

The supply vessel planning problem (SVPP) is a maritime transportation problem faced by the offshore oil and gas industry, where a fleet of platform supply vessels (PSVs) is used for the transport of cargo between an onshore depot and a set of offshore installations. Regular delivery of cargo to offshore installations is required for the continuous production of oil. For this, a sailing schedule is constructed and usually repeated weekly. Most commonly, the sailing schedule is changed when there are significant changes in demand from the installations, which usually corresponds to changes in oil production levels. In turn, supply vessels are one of the costliest resources in offshore logistics, with time charter rates for a single vessel reaching tens of thousands of USD. Therefore, offshore oil companies seek to optimize the utilization of such resources (Amiri et al., 2019). It should be noted that additional resources are required for the overall offshore logistics system, including helicopters for the transport of personnel (Santos et al., 2018; Silva and Guedes Soares, 2018), and pipelines or shuttle tankers for the transport of oil (Assis and Camponogara, 2016). In the SVPP, a voyage usually spans several days, and always starts and ends at the depot where the cargo to be distributed among the offshore installations is kept. In each voyage, a vessel may visit more than one installation. Each offshore oil platform may require more than one visit per week and, most often, supply vessel operations can take place only during the day. In situations where the supply vessel arrives at the oil platform when the oil platform is closed for the night, the vessel has to wait until the opening time the next day. Lastly, there is a maximum number of vessels that can be prepared each day at the onshore depot.

A solution to the SVPP is a weekly schedule where voyages are assigned to each vessel in the fleet, and where the costs to be minimized are the sum of voyage costs and charter costs. In a typical scenario, charter costs will be much larger than voyage costs. Fig. 1 presents an example of a solution to the SVPP. In the example, the fleet is composed of two vessels with voyages being assigned so that the two vessels perform the required number of weekly visits for a set of eight installations. Note that all voyages start and end at the onshore depot (labelled as 0 in Fig. 1). Moreover, note that the departures for each installation are evenly spread throughout the week.

However, due to both uncertain weather conditions and uncertain demand, there is often the need to make changes to the planned weekly

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schedules (Aas et al., 2009). Such changes are costly as they usually imply the usage of additional vessels. In turn, deterministic approaches in a setting characterized by high uncertainty will often lead to more frequent rearrangements in the visit schedule. In general, deterministic approaches to supply chain design are unable to increase resilience, thereby risking a significant increase in costs (e.g., costs from lost production) in case of unforeseen disruptive events (Fattahi et al., 2017; Suryawanshi and Dutta, 2022).

If the transport capacity of the fleet is dimensioned according to average demand, in the face of demand fluctuations, frequent reallocations of cargo in time will be required. While excess capacity facilitates the recovery from large demand peaks, which are commonly observed once operations can be resumed after a period of severe weather conditions, excess capacity will also lead to increased charter costs (Aas et al., 2009). Besides demand uncertainty, adverse weather conditions also have a strong negative impact on weekly sailing schedules, affecting both sailing and loading and unloading capabilities. If the significant wave height (i.e., the average wave height of the one-third largest waves, measured trough to crest) exceeds a given threshold, loading and unloading operations cannot take place, and the vessel must wait until favourable weather conditions are met. Adverse weather conditions will also impact sailing speed, a key determinant of fuel costs, which correspond to a significant component of the operating costs. Under adverse weather conditions, if a decision is made to keep within the planned speed, higher fuel consumption will occur, thereby leading to increased costs. If, on the other hand, a choice is made not to compensate for the negative impact of adverse weather conditions on sailing speed, delays in meeting the demand will also occur, leading also to increased costs. However, it should be noted that in the definition of the SVPP speed is usually not included as a decision variable. While the exclusion of speed may, in general, lead to suboptimal solutions (Psaraftis and Kontovas, 2013, 2014), regulations usually demand that at most a given percentage of the machinery power of the vessel is used for sailing or keeping the position alongside the installation, to ensure that the vessel has enough power to accommodate for any unforeseen events. Therefore, speed cannot be freely increased to compensate for the effect of adverse weather conditions.

Mitigation of the consequences of adverse weather conditions is one of the most significant challenges in upstream supply chain logistics (Aas et al., 2009). Costly recourse actions, such as performing additional voyages or chartering additional vessels, will have to be taken in the event of schedule disruption due to either increased demand or adverse weather conditions. There is always a financial trade-off when planning a system to handle uncertainty (Aas et al., 2009). The introduction of robustness in the planning phase will typically increase planned costs. However, such robustness will also lead to less frequent recourse actions, and therefore, decreased recourse costs. To enable the optimization of supply vessel utilization in a realistic setting, where uncertainty plays a major role, this paper considers the stochastic version of the SVPP, with both stochastic demands and uncertain weather conditions, the most important sources of uncertainty faced in practice, being considered.

Published research on the SVPP under uncertainty makes use of two-phased methods combining simulation and optimization or chance-constrained methods at the voyage level to obtain robust schedules (Halvorsen-Weare and Fagerholt, 2017; Kisialiou et al., 2019). Published methods allow arriving at robust schedules, even if the focus is only on uncertain demand as in Kisialiou et al. (2019), or when both uncertain demand and uncertain weather conditions are considered, as in Halvorsen-Weare and Fagerholt (2017). However, the fact that the schedule construction and the assessment of the corresponding expected cost occur in two separate phases, may lead to the obtaining of sub-optimal solutions. In particular, pre-setting a given robustness level to be required for all voyages from which the schedule is to be constructed or requiring the same slack between any two voyages in the constructed schedule may exclude solutions that might perform well under uncertainty (Halvorsen-Weare and Fagerholt, 2017). Additionally, while chance-constrained methods are particularly useful when costs and benefits from decisions at later stages are difficult to assess (Birge and Louveaux, 2011), the cost of recourse in the SVPP corresponds to the same type of cost assessed for first-stage decisions (i.e., charter costs and voyage costs). In contrast, this paper follows the method of Santos et al. (2022), making use of a two-stage stochastic program, with the cost of recourse being considered explicitly in the objective function. Therefore, the expected schedule cost, including the cost of recourse, is minimized in the search procedure, while allowing for a larger solution space to be explored. Therefore, obtained solutions may potentially be less costly than those obtained from the referred two-phased methods, in particular, when compared to alternative approaches proposed to account for both uncertain demand and uncertain weather conditions. However, while in Santos et al. (2022) only uncertain demand is considered, the method proposed in this paper accounts for both uncertain demand and uncertain weather conditions.

Two main characteristics of the SVPP under uncertain demand and

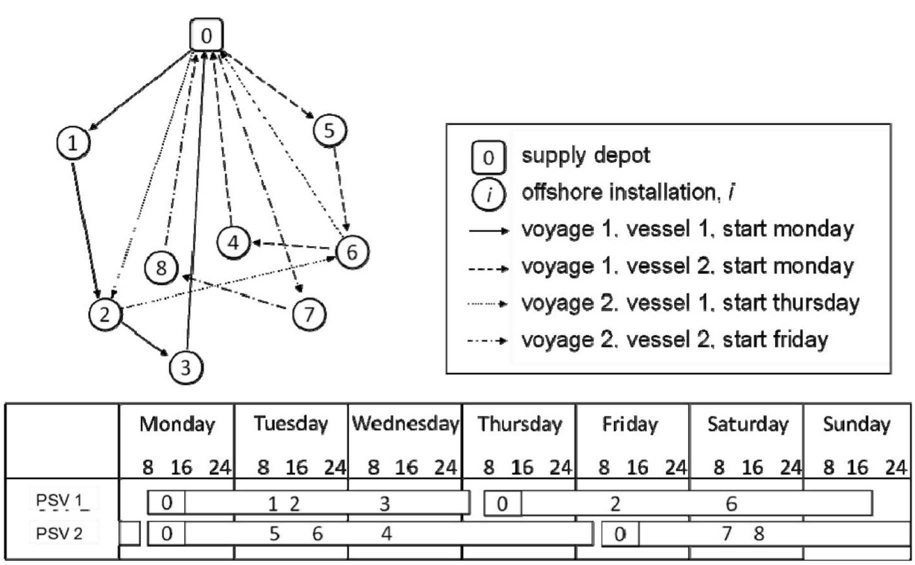


Fig. 1. Example of a solution to the Supply Vessel Planning Problem for an instance with two vessels and eight offshore installations [adapted from Halvorsen-Weare et al. (2012)].

uncertain weather conditions impact the choice of the solution method. First, the SVPP generalizes the Periodic Vehicle Routing Problem (PVRP). Therefore, the SVPP is an NP-hard problem. Second, considering uncertain demand and uncertain weather conditions simultaneously poses a considerable challenge resulting from the intractability of full scenario enumeration. Therefore, as a solution method, a genetic search procedure based on the heuristic originally proposed Vidal et al. (2012) for the PVRP and adapted by Borthen et al. (2018) for the deterministic version of the SVPP is modified through the use of a discrete-event simulation engine to sample scenarios for demand and weather conditions.

Performing simulation at each step of the optimization procedure will inevitably lead to large computational times. However, it should be noted that the decisions to be made in the SVPP are not at the operational level where computational time to arrive at a solution would be critical. Instead, decisions in the SVPP are at the strategic/tactical level, with a schedule typically being in use for some months. Therefore, benefits in terms of cost savings from arriving at better solutions should largely surpass the drawbacks from the increased computational effort.

As the main scientific contribution of this paper, a two-stage stochastic programming method is proposed, accommodating the two major sources of uncertainty in offshore logistics: uncertain demand and uncertain weather conditions. By including the cost of recourse in the objective function, the proposed methodology allows for the minimization of the most significant costs faced by offshore oil and gas companies in the operation of offshore supply vessels. To compute the savings from solving the stochastic program (i.e. the value of the stochastic solution), the deterministic version of the problem is also solved, and the corresponding costs under uncertainty are computed. As an additional contribution, the proposed methodology is tested on a set of realistic instances where it is shown to produce high-quality solutions. Note that, as mentioned in Santos et al. (2022), the SVPP under uncertainty corresponds to a sequential (multi-stage) stochastic optimization problem, where a sequence of scenario realizations (e.g., demand and weather condition) is observed, with the required recourse action being implemented only after the observation of each specific scenario outcome. Such a sequence of scenario realizations and implementation of the corresponding required recourse actions is repeated throughout the planning horizon. However, in general, the multi-stage versions of sequential stochastic optimization problems are completely intractable (Shapiro, 2003; Powell, 2014). Therefore, this paper adopts the commonly used approach of using a two-stage approximation, where second-stage decisions are implemented assuming complete knowledge of the uncertainty describing each specific scenario at the start of each week, therefore enabling the construction of a weekly sailing schedule accommodating for any required recourse actions. In turn, the simulation of uncertain demand and weather conditions for a complete year allows the identification of weekly sailing schedules minimizing the expected cost for the complete year.

This paper is structured in the following manner: Section 2 presents the literature; Section 3 presents the mathematical models for the deterministic version of the problem and the two-stage stochastic program with recourse; Section 4 presents the algorithm for the computation of the cost of each solution; Section 5 presents the computational experiments; and Section 6 concludes the paper.

2. Literature review

Research on the SVPP under uncertainty includes papers addressing uncertain weather conditions (Halvorsen-Weare et al., 2012), uncertain demand (Kisialiou et al., 2019), and both uncertain demand and uncertain weather conditions (Halvorsen-Weare and Fagerholt, 2017). While commonly making use of two-phased methods combining simulation and optimization, the referred papers can be broadly classified into two main groups. In the first group, a deterministic approach is used to arrive at robust schedules, with robustness being introduced through the use of probabilistic constraints at the voyage level to accommodate for uncertain demand (Kisialiou et al., 2019), or the use of time slacks to accommodate for stochastic travel and service times resulting from weather uncertainty (Halvorsen-Weare et al., 2012). In the second group, simulation is used to determine the average undelivered demand for each voyage (Halvorsen-Weare and Fagerholt, 2017). Average undelivered demand is then used to set a penalized cost to be applied to each voyage in a deterministic optimization procedure. A method similar to that of Kisialiou et al. (2019) is used by Cruz et al. (2023), who first generate a set of voyages by considering both uncertain demand and uncertain travel times and then use the corresponding voyage reliability level as input to an optimization model to obtain robust weekly schedules. In turn, Ksciuk et al. (2023) review the literature focusing on methods to accommodate for uncertainty in maritime routing and scheduling in general, including supply vessel scheduling.

Adverse weather conditions constitute the most frequent cause for schedule disruption (Aas et al., 2009; Kondratenko and Tarovik, 2020; Rahman et al., 2019). To obtain robust schedules, Halvorsen-Weare et al. (2012) propose a set-covering model where a minimum time slack is required between the arrival at the depot and the departure for the next planned voyage. However, the actual robustness of the constructed schedules is not tested, for instance, through the use of a simulation model. Halvorsen-Weare and Fagerholt (2017) extend the work of Halvorsen-Weare et al. (2012) by using a discrete-event simulation model to determine the average undelivered demand for each voyage. A penalized cost is applied to each voyage in proportion to the average undelivered demand for that voyage and added to the set-covering model of Halvorsen-Weare et al. (2012). The authors use historical weather data for the winter season in the North Sea to model weather conditions as a stochastic process. The expected cost of the obtained solution is computed through the use of discrete event simulation. However, the authors do not consider the possibility of any recourse action. Note that the use of recourse is an intrinsic feature of the real problem faced daily by logistics planners at offshore oil and gas companies.

In turn, methods to handle uncertain demand make use of probabilistic constraints at the voyage level to ensure that the probability of each voyage being capable of accommodating the corresponding demand is above a user-defined threshold (Kisialiou et al., 2019) or upscale demand to allow for some variation (Halvorsen-Weare and Fagerholt, 2017). It should be noted that Halvorsen-Weare and Fagerholt (2017) considers both uncertain demand and uncertain weather conditions through the simultaneous use of upscaled demand and the introduction of time slacks in the constructed schedule.

The referred methods allow for obtaining robust solutions. However, the proposed methods rely on two-phased algorithms, ignoring the cost of recourse, or require the same reliability level for all voyages, irrespective of their duration, number of visits or other individual features. Such a procedure may lead to a significant reduction of the search space and a sharp increase in costs, particularly as robustness is increased essentially through an increase in fleet size. Moreover, the expected schedule cost, which is the cost logistics planners at offshore oil and gas companies seek to minimize and which includes the cost of any recourse action that may be necessary in case of schedule disruption, is ignored in the objective function for all the referred methodologies. Therefore, taking into account that the aim of logistics planners at offshore companies is to minimize the overall schedule cost, including the cost of recourse, rather than maximizing robustness per se, the proposed methodologies can potentially lead to sub-optimal solutions. In contrast, a two-stage stochastic program with recourse allows the minimization of the expected schedule cost, including costs from any recourse actions that may become necessary.

Lastly, Monte-Carlo sampling-based methods have been successfully applied to discrete stochastic optimization problems where explicit enumeration of scenarios becomes impractical. Examples of the application of such methods include vehicle routing (Dalgic et al., 2015;

Kenyon and Morton, 2003; Schrotenboer et al., 2019; Verweij et al., 2003), and supply chain network design (Salehi Sadghiani et al., 2015; Santoso et al., 2005). Monte-Carlo simulation has also been used for the evaluation of solutions within heuristic optimization frameworks for discrete stochastic optimization problems to cope with the difficulties posed by simultaneous NP-hardness and intractability of full scenario enumeration (Alrefaei and Andradóttir, 1999; Gutjahr, 2004). The same general procedure is adopted in this paper through the use of a discrete-event simulation model within a genetic search algorithm, which allows coping with both the NP-hard nature of the SVPP, and the intractability of explicitly enumerating the combined scenarios for demand from the offshore installations and weather conditions.

3. Mathematical models

The set-covering model described in this section is an extension of the voyage-based model proposed by Borthen et al. (2018) for the deterministic version of the SVPP. In turn, Borthen et al. (2018) extend the formulation of Halvorsen-Weare et al. (2012) and Shyshou et al. (2012). A set of constraints to prevent simultaneous visits to an offshore installation are introduced, which are not considered by the referred authors. This is an important characteristic of the real problem, for both operational and safety reasons. The deterministic version of the problem termed the Expected Value Problem, ignores all sources of uncertainty. Afterwards, the deterministic model is extended into a stochastic setting to consider both demand uncertainty and uncertain weather conditions. A two-stage stochastic program is presented to account for the referred uncertainties.

3.1. Expected value problem

Table 1 presents the mathematical notation for the Expected Value Problem (EVP). $\mathcal{R}$ is the set of all pre-generated candidate voyages, $\mathcal{V}$ is the set of time-chartered vessels, and $\mathcal{I}$ is the set of all installations to visit, including the depot. $\mathcal{T}$ is the set of days in the planning horizon (one week), $\mathcal{L}$ is the set of possible voyage durations (in days), and $\mathcal{F}$ is the set of all possible visit frequencies. Additionally, $\mathcal{I}_f \subseteq \mathcal{I}$ is the set of installations with weekly visit frequency f, $\mathcal{R}_v$ is the set of candidate voyages that PSV v can sail, $\mathcal{R}_{vi}$ is the set of candidate voyages that vessel v can sail and that visit platform i and $\mathcal{R}_{vl}$ is the set of candidate voyages of vessel v that have duration l.

The binary decision variable δ_v takes the value one if vessel v is chartered, and zero otherwise. Binary decision variable x_{vrt} takes the value one if vessel v sails voyage r starting on day, and zero otherwise. Parameter C_v^{TC} is the weekly charter cost for vessel v, C_{vr}^{SC} is the sailing cost of vessel v when sailing voyage r. S_i is the number of weekly visits required by offshore installation i; F_v is the number of days vessel v is available during the planning horizon; and B_t is the maximum number of vessels that may be serviced at the depot on day t. A_{vrtid} is a parameter that takes the value one if vessel v visits installation i on day d when starting voyage r on day t, and zero otherwise. The inclusion of a parameter A_{vrtid}, together with the corresponding constraint, constitutes the major extension to the formulation proposed by Borthen et al. (2018) and serves to prevent multiple visits to a given offshore installation from occurring on the same day. Similarly to Shyshou et al. (2012), the spreading of departures is ensured through the use of sub-horizons of length h_f, defined for installations requiring f visits per week, and where $0 \le h_f \le |\mathcal{T}|$. Within each sub-horizon, a minimum of $\underline{P_f}$ and a maximum of $\overline{P_f}$ departures from the depot is required for each installation. To help make the connection between the notation and the problem description, and referring to Fig. 1, note that, for the particular solution shown in Fig. 1, the chosen candidate voyages for vessel 1 are the voyages visiting installations 1, 2, and 3 (voyage 1), and installations 2 and 6 (voyage 2), while the candidate voyages chosen for vessel 2 are the voyages visiting installations 5, 6, and 4 (voyage 1), and installations 7 and 8 (voyage 2). Suppose, for instance, that the set of candidate voyages is available and that the voyage visiting installations 1, 2, and 3 is identified as voyage 6 in the complete set of candidate voyages (before choosing the voyages to be part of the optimal schedule). Then, for the solution shown in Fig. 1, the decision variable $x_{1,6,1}$ will take the value 1, since, according to Fig. 1, the voyage visiting installations 1, 2, and 3 (which would be voyage 6 in the complete set of candidate voyages) is chosen for vessel 1 on day 1 (Monday).

Table 1
Notation for the expected value problem (EVP).

sets:	
$\mathcal{I}$	set of offshore installations
$\mathcal{V}$	set of vessels
$\mathcal{R}$	set of pre-generated candidate voyages
$\mathcal{T}$	set of days in the planning horizon
$\mathcal{L}$	set of possible voyage durations
$\mathcal{F}$	set of possible visit frequencies
$\mathcal{I}_f$	set of installations with weekly visit frequency $f, f \in \mathcal{F}, \mathcal{I}_f \subseteq \mathcal{I}$
$\mathcal{R}_v$	set of candidate voyages vessel v can sail $, v \in \mathcal{V}, \mathcal{R}_v \subseteq \mathcal{R}$
$\mathcal{R}_{vi}$	set of candidate voyages vessel v can sail and which visit offshore installation $i, i \in \mathcal{I}, v \in \mathcal{V}, \mathcal{R}_{vi} \subseteq \mathcal{R}$
$\mathcal{R}_{vl}$	set of candidate voyages vessel v can sail and which have duration l, $l \in \mathcal{L}, v \in \mathcal{V}, \mathcal{R}_{vi} \subseteq \mathcal{R}$
decision variables:	
$\delta_v \in \{0,1\}$	takes the value one if vessel v is chartered, and zero otherwise, $v \in \mathcal{V}$
$x_{vrt} \in \{0,1\}$	takes the value one if vessel v sails voyage r on day t, and zero otherwise, $v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}$
parameters:	
C_v^{TC}	weekly time charter cost for vessel $v, v \in \mathcal{V}$
C_{vr}^{SC}	sailing cost of vessel v when sailing voyage $r, v \in \mathcal{V}, r \in \mathcal{R}$
S_i	number of weekly visits required by offshore installation $i, i \in \mathcal{I}$
F_v	number of days vessel v is available during the planning horizon, $v \in \mathcal{V}$
B_t	maximum number of vessels that may be loaded at the supply depot on day $t, t \in \mathcal{T}$
$A_{vrtid} \in \{0,1\}$	takes the value one if vessel v visits installation i on day d when starting voyage r on day t, and zero otherwise, $v \in \mathcal{V}, r \in R, i \in \mathcal{I}$, $d \in \mathcal{T}$
h_f	length of sub-horizons for installations requiring f visits per week, $f \in \mathcal{F}$
$\overline{P_f}$	maximum number of departures within each sub-horizon h_f for each considered offshore installation
$\underline{P_f}$	minimum number of departures within each sub-horizon h_f for each considered offshore installation

The mathematical formulation for the deterministic version of the problem is:

EVP:

Minimize:

$$\sum_{v\in\mathcal{V}} C_v^{TC}\delta_v + \sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{t\in\mathcal{T}} C_{vr}^{SC} x_{vrt} \tag{1}$$

subject to:

$$\sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_{vi}}\sum_{t\in\mathcal{T}} x_{vrt} \ge S_i, i \in \mathcal{I} \tag{2}$$

$$\sum_{l\in\mathcal{L}}\sum_{r\in\mathcal{R}_{vl}}\sum_{t\in\mathcal{T}} l x_{vrt} - F_v\delta_v \le 0, v \in \mathcal{V} \tag{3}$$

$$\sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v} x_{vrt} \le B_t, t \in \mathcal{T} \tag{4}$$

$$\sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{t\in\mathcal{T}} A_{vrtid} x_{vrt} \le 1, i \in \mathcal{I}, d \in \mathcal{T} \tag{5}$$

$$\sum_{r\in R_{vl}} x_{vrt} + \sum_{r\in R_v}\sum_{\tau=1}^{l-1} x_{vr,(t+\tau) mod |\mathcal{T}|} \le \delta_v, v \in \mathcal{V}, t \in \mathcal{T}, l \in \mathcal{L} \tag{6}$$

$$\underline{P_f} \leq \sum_{v\in\mathscr{V}}\sum_{r\in R_{vi}}\sum_{\tau=0}^{h_f} x_{vr,(t+\tau) mod|\mathscr{T}|} \leq \overline{P_f}, i\in\mathscr{I}_f, f\in\mathscr{F}, t\in\mathscr{T} \quad (7)$$

$$\delta_v \in \{0,1\}, v\in\mathscr{V} \quad (8)$$

$$x_{vrt} \in \{0,1\}, v\in\mathscr{V}, r\in\mathscr{R}, t\in\mathscr{T} \quad (9)$$

The objective function (1) minimizes the sum of time-charter costs and voyage costs. Constraints (2) ensure that each installation is visited the required number of times. Constraints (3) ensure that each vessel does not sail more days than the number of days for which that vessel is available. Constraints (4) ensure that, on any given day, the number of vessels that are serviced at the depot cannot be greater than the maximum number of vessels that can be serviced at the depot on that same day. Constraints (5) ensure that each installation is visited at most by one vessel on any given day. Note that for constraints (5) to prevent multiple visits from occurring on the same day for a given installation, time windows must also be in place for that installation. Concretely, time windows must be set in such a manner to prevent a visit from being under way at 0 h. For the problem at hand, time windows are assumed to be in place for all installations. Therefore, constraints (5) are sufficient to prevent the occurrence of multiple visits on the same day. To help clarify why constraints (5) can prevent multiple visits from occurring on the same day for a given installation only if time windows are in place, consider Fig. 2. For illustration purposes, a single offshore installation is considered together with two supply vessels. Moreover, two settings are considered: setting A, with no time windows in place, and setting B, with time windows. If no time windows are in place, constraints (5) are not able to prevent a situation such as that illustrated by setting A. In particular, constraints (5) only prevent more than one visit for a given installation from starting on the same day. In the figure, for setting A, the visit for PSV 1 starts on day 1 and the visit for PSV 2 starts on day 2. However, this is not sufficient for the two vessels to have some overlap, i.e. PSV 1 is still servicing the installation at the moment PSV 2 starts the visit. Constraints (5) do not prevent a situation such as that in setting A. However, if time windows are in place, as in setting B, then constraints (5) prevent the existence of overlap since visits must start on different days. Therefore, if time windows are in place, as in setting B, constraints (5) are sufficient to prevent simultaneous visits to an offshore installation (i.e., prevent the existence of visit overlap).

In turn, constraints (6) prevent a vessel from starting a new voyage before returning to the depot from the previous voyage. Lastly, constraints (7) ensure the spreading of departures to a given offshore installation throughout the planning horizon. Note that the use of the modulus operator (mod) in constraints (6) and constraints (7) is required to handle the rolling nature of the planning horizon (i.e., end-of-week effects). Considering that there are seven days in one week (numbered zero to six), for instance, a voyage planned to start on day five and having a duration of three days, would end on day eight (five plus three), therefore outside the duration of one week (seven days). But to be able to model the problem in terms of weekly planning, day eight corresponds to day one (8 mod 7), that is, the second day of the following week, therefore enabling planning to be made for a rolling horizon of one week (i.e., for a schedule that is repeated weekly).

3.2. Two-stage stochastic programming with recourse

Two major changes are introduced in the expected value problem formulation to provide for an extension into a stochastic setting. First, a set of scenarios Ω is introduced, where each scenario ∈ Ω , corresponds to a combination of particular realizations of sea states and demands. Second, recourse actions are introduced to ensure feasibility upon realization of a given ∈ Ω . Similarly to Kisialiou et al. (2019), three types of recourse actions are considered: voyage completions, additional voyages, and emergency voyages. Recourse actions serve to ensure the delivery of demand from failed voyages, where a failed voyage is a planned voyage for which, upon realization of a specific scenario for demand and weather conditions, the vessel originally planned to perform the voyage is unable to transport all the required demand (e.g., the realized demand for the specific realized scenario is larger than that in the originally planned voyage and surpasses the capacity of the vessel originally assigned to perform the voyage).

A *voyage completion* is defined as in Novoa et al. (2006): a vessel that has completed its planned voyage may, in addition, serve one or more installations from failed voyages before returning to the depot. The cost of a voyage completion is the sum of the costs from servicing the additional offshore installations and returning to the depot minus the cost of sailing from the last visited offshore installation in the originally planned voyage to the depot. A planned voyage may have more than one candidate voyage completion. However, at most one voyage completion is undertaken for each planned voyage. Note that a voyage completion corresponds to a sequence of visits to be performed after the visits in the originally planned voyage have been performed and before the vessel returns to the depot. Therefore, for a given voyage, a single vessel can perform at most one sequence of additional visits before returning to the depot, i.e., a vessel cannot perform two voyages in parallel at the same time. So, for each planned voyage, at most one voyage completion is undertaken.

An *additional voyage* is a voyage not included in the planned schedule and which is performed by an idle vessel to service installations from failed voyages. An idle vessel is a vessel in the chartered fleet, i.e., a vessel assigned to perform at least one voyage in the planned schedule, but which is not performing any voyage on the day the additional voyage is started. Moreover, an additional voyage is feasible for a given vessel only if that vessel can return to the depot on time to perform its next planned voyage, so as not to disrupt the remainder of the planned schedule.

Lastly, *emergency voyages* are voyages performed by additional vessels not belonging to the originally chartered fleet and which, therefore, are not assigned to perform any voyage in the originally planned schedule. Each additional vessel is assumed to perform a single

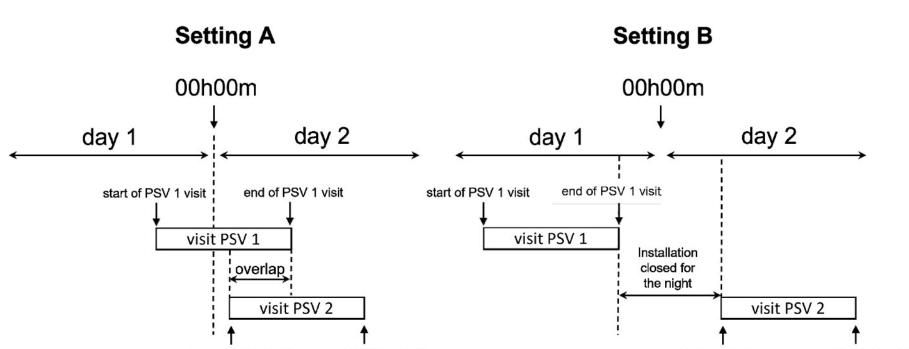


Fig. 2. Illustration of constraints to prevent simultaneous visits to an offshore installation.

additional voyage, with the costs of usage of such vessels corresponding to the sum of chartering costs and voyage costs. Note that additional vessels do not belong to the long-term chartered fleet and are instead chartered to perform single voyages.

Table 2 shows the set notation for the two-stage stochastic program, while Table 3 and

Table 4 show, respectively, the corresponding decision variables and parameters. Note that some of the decision variables in the two-stage stochastic program are in common with the expected value formulation presented in the previous section. Similar to the deterministic formulation, the decision variable δ_v is equal to one if vessel v is chartered, and zero otherwise; and decision variable x_{vrt} is equal to one if vessel v sails voyage r starting on day t, and zero otherwise. Together, decision variables δ_v and x_{vrt} correspond to the set of first stage decisions. Decision variable $y_{vct\omega}$ is equal to one if vessel v performs voyage completion c for a voyage starting on day t in scenario ω, and zero otherwise; $z_{vat\omega}$ is equal to one if vessel v sails additional voyage a on day t in scenario ω, and zero otherwise; $\delta_{u\omega}$ is equal to one if additional vessel u is chartered in scenario ω and zero otherwise; and $w_{uet\omega}$ is equal to one if additional vessel u sails emergency voyage e starting on day t in scenario ω, and zero otherwise. Note that each scenario ω corresponds to a particular realization of demands and weather conditions. Therefore, for a given voyage r visiting some set of installations, under a specific scenario ω_1, the realization of demands may by such that the total demand for voyage r is larger than the capacity of vessel v, and therefore, vessel v cannot sail voyage r under scenario ω_1. In turn, in a different scenario ω_2, the realization of demands for the same voyage r visiting the same set of installations may be such that the total demand for the voyage is smaller than the capacity of vessel v and, therefore, vessel v can sail voyage r under scenario ω_2.

The mathematical formulation for the two-stage stochastic program is:

SPR:

Minimize:

$$\sum_{v\in\mathcal{V}} C_v^{TC}\delta_v + \sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{t\in\mathcal{T}} C_{vr}^{SC}x_{vrt} + \sum_{\omega\in\Omega}\sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}} Pr_\omega \left(\sum_{c\in\mathcal{C}_{v\omega}} C_{vc\omega}^{SC}y_{vct\omega} + \sum_{a\in\mathcal{A}_{v\omega}} C_{va\omega}^{SC}z_{vat\omega}\right) + \sum_{\omega\in\Omega}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}} Pr_\omega \left(\sum_{e\in\mathcal{E}_{u\omega}} C_{ue\omega}^{SC}w_{uet\omega} + C_u^{TC}\delta_{u\omega}\right) \tag{10}$$

subject to:

set covering (EVP): (2),(3),(6),(7),(8),(9) (11)

$$\sum_{v\in\mathcal{V}}\left(\sum_{r\in\mathcal{R}_v} x_{vrt} + \sum_{a\in\mathcal{A}_{v\omega}} z_{vat\omega}\right) + \sum_{u\in\mathcal{U}}\sum_{e\in\mathcal{E}_{u\omega}} w_{uet\omega} \le B_t, t\in\mathcal{T}, \omega\in\Omega \tag{12}$$

$$\sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}}\left(\sum_{r\in\mathcal{R}_{v\omega}} A_{vrtid\omega}x_{vrt} + \sum_{c\in\mathcal{C}_{v\omega}} A_{vctid\omega}y_{vct\omega} + \sum_{a\in\mathcal{A}_{v\omega}} A_{vatid\omega}z_{vat\omega}\right) + \sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}}\sum_{e\in\mathcal{E}_{u\omega}} A_{uetid\omega}w_{uet\omega} \le 1, i\in\mathcal{I}, d\in\mathcal{T}, \omega\in\Omega \tag{13}$$

$$\sum_{e\in\mathcal{E}_{u\omega}}\sum_{t\in\mathcal{T}} w_{uet\omega} \le \delta_{u\omega}, u\in\mathcal{U}, \omega\in\Omega \tag{14}$$

Table 2
Set notation for the two-stage stochastic program with recourse.

sets:	
Ω	set of scenarios
$\mathcal{R}_{v\omega}$	set of planned voyages vessel v can sail in scenario $\omega, v\in\mathcal{V}, \omega\in\Omega$
$\mathcal{R}_{vi\omega}$	set of planned voyages vessel v can sail and which visit installation i in scenario $\omega, v\in\mathcal{V}, \omega\in\Omega, i\in\mathcal{I}$
$\mathcal{R}_{vl\omega}$	set of planned voyages vessel v can sail in scenario ω and which have duration $l, v\in\mathcal{V}, l\in\mathcal{L}, \omega\in\Omega$
C_ω	set of voyage completions in scenario $\omega, \omega\in\Omega$
$C_{r\omega}$	set of voyage completions for planned voyage r in scenario $\omega, r\in\mathcal{R}, \omega\in\Omega$
$C_{rv\omega}$	set of voyage completions for planned voyage r vessel v can sail in scenario ω, $r\in\mathcal{R}, v\in\mathcal{V}, \omega\in\Omega$
$C_{vi\omega}$	set of voyage completions vessel v can sail and which visit installation i in scenario $\omega, v\in\mathcal{V}, i\in\mathcal{I}, \omega\in\Omega$
$C_{rvi\omega}$	set of voyage completions for planned voyage r vessel v can sail and which visit installation i in scenario $\omega, r\in\mathcal{R}, v\in\mathcal{V}, i\in\mathcal{I}, \omega\in\Omega, C_{rvi\omega}\in C_{vi\omega}$
$\mathcal{A}_\omega$	set of additional voyages for vessels in the planned schedule in scenario $\omega, \omega\in\Omega$
$\mathcal{A}_{v\omega}$	set of additional voyages vessel v can sail in scenario $\omega, v\in\mathcal{V}, \omega\in\Omega$
$\mathcal{A}_{vi\omega}$	set of additional voyages vessel v can sail and which visit installation i in scenario $\omega, v\in\mathcal{V}, i\in\mathcal{I}, \omega\in\Omega$
$\mathcal{L}_a$	set of possible additional voyage durations
$\mathcal{U}$	set of additional vessels
$\mathcal{E}_\omega$	set of emergency voyages in scenario $\omega, \omega\in\Omega$
$\mathcal{E}_{u\omega}$	set of emergency voyages additional vessel u can sail in scenario $\omega, u\in\mathcal{U}, \omega\in\Omega$
$\mathcal{E}_{ui\omega}$	set of emergency voyages additional vessel u can sail and which visit installation i in scenario $\omega, u\in\mathcal{U}, i\in\mathcal{I}, \omega\in\Omega$

Table 3
Second stage decision variables for the two-stage stochastic program with recourse.

decision variables:	
$y_{vct\omega}\in\{0,1\}$	takes the value one if vessel v sails voyage completion c for a voyage starting on day t in scenario ω, and zero otherwise, $v\in\mathcal{V}, c\in\mathcal{C}_{v\omega}, t\in\mathcal{T}, \omega\in\Omega$
$z_{vat\omega}\in\{0,1\}$	takes the value one if vessel v sails additional voyage a on day t in scenario ω, and zero otherwise, $v\in\mathcal{V}, a\in\mathcal{A}_{v\omega}, t\in\mathcal{T}, \omega\in\Omega$
$w_{uet\omega}\in\{0,1\}$	takes the value one if additional vessel u sails emergency voyage e starting on day t in scenario ω, and zero otherwise, $u\in\mathcal{U}, e\in\mathcal{E}_{u\omega}, t\in\mathcal{T}, \omega\in\Omega$
$\delta_{u\omega}\in\{0,1\}$	takes the value one if additional vessel u is chartered in scenario ω, and zero otherwise, $u\in\mathcal{U}, \omega\in\Omega$

Table 4
Parameters for the two-stage stochastic program with recourse.

parameters:	
Pr_ω	Probability of scenario $\omega, \omega\in\Omega$
$S_{i\omega}$	number of visits required by installation i in scenario $\omega, i\in\mathcal{I}, \omega\in\Omega$
C_{vr}^{SC}	expected value of the sailing cost of vessel v when sailing voyage $v\in\mathcal{V}, r\in\mathcal{R}_v$
$C_{vc\omega}^{SC}$	sailing cost of vessel v when sailing voyage completion c in scenario $\omega, v\in\mathcal{V}, c\in\mathcal{C}_{v\omega}, \omega\in\Omega$
$C_{va\omega}^{SC}$	sailing cost of vessel v when sailing additional voyage a in scenario $\omega, v\in\mathcal{V}, a\in\mathcal{A}_{v\varpi}, \varpi\in\Omega$
$C_{ue\omega}^{SC}$	sailing cost of additional vessel u when sailing emergency voyage e in scenario $\omega, u\in\mathcal{U}, e\in\mathcal{E}_{u\omega}, \omega\in\Omega$
C_u^{TC}	weekly chartering costs of additional vessel $u, u\in\mathcal{U}$
$A_{vrtid\omega}\in\{0,1\}$	takes the value one if vessel v visits installation i on day d when starting voyage r on day t in scenario ω, and zero otherwise, $v\in\mathcal{V}, r\in\mathcal{R}, i\in\mathcal{I}, t, d\in\mathcal{T}, \omega\in\Omega$
$A_{vctid\omega}\in\{0,1\}$	takes the value one if vessel v visits installation i on day d when sailing voyage completion c for a voyage starting on day t in scenario ω, and zero otherwise, $v\in\mathcal{V}, c\in\mathcal{C}_{v\omega}, i\in\mathcal{I}, t, d\in\mathcal{T}, \omega\in\Omega$
$A_{vatid\omega}\in\{0,1\}$	takes the value one if vessel v visits installation i on day d when starting additional voyage a on day t in scenario ω, and zero otherwise, $v\in\mathcal{V}, a\in\mathcal{A}_{v\omega}, i\in\mathcal{I}, t, d\in\mathcal{T}, \omega\in\Omega$
$A_{uetid\omega}\in\{0,1\}$	takes the value one if vessel v visits installation i on day d when starting emergency voyage e on day t in scenario ω, and zero otherwise, $u\in\mathcal{U}, e\in\mathcal{E}_{u\omega}, i\in\mathcal{I}, t, d\in\mathcal{T}, \omega\in\Omega$

$$\sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}}\left(\sum_{r\in\mathcal{R}_{vi\omega}} x_{vrt} + \sum_{c\in\mathcal{C}_{vi\omega}} y_{vct\omega} + \sum_{a\in\mathcal{A}_{vi\omega}} z_{vat\omega}\right) + \sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}}\sum_{e\in\mathcal{E}_{ui\omega}} w_{uet\omega} \geq S_{i\omega}, i\in\mathcal{I}, \omega\in\Omega \tag{15}$$

$$x_{vrt} - \sum_{c\in\mathcal{C}_{rv\omega}} y_{vct\omega} \geq 0, v\in\mathcal{V}, r\in\mathcal{R}, t\in\mathcal{T}, \omega\in\Omega \tag{16}$$

$$\sum_{a\in\mathcal{A}_{v\omega l}} z_{vat\omega} + \sum_{\tau=1}^{l_a-1}\left(\sum_{r\in\mathcal{R}_{v\omega}} x_{vr,(t+\tau)mod|\mathcal{T}|} + \sum_{a\in\mathcal{A}_{v\omega}} z_{va,(t+\tau)mod|\mathcal{T}|\omega}\right) \leq \delta_v, v\in\mathcal{V}, t\in\mathcal{T}, l_a\in\mathcal{L}_a, \omega\in\Omega \tag{17}$$

$$\sum_{r\in\mathcal{R}_{vl}} x_{vrt} + \sum_{\tau=1}^{l-1}\left(\sum_{r\in\mathcal{R}_{v\omega}} x_{vr,(t+\tau)mod|\mathcal{T}|} + \sum_{a\in\mathcal{A}_{v\omega}} z_{va,(t+\tau)mod|\mathcal{T}|\omega}\right) \leq \delta_v, v\in\mathcal{V}, t\in\mathcal{T}, l\in\mathcal{L}, \omega\in\Omega \tag{18}$$

$$y_{vct\omega}\in\{0,1\}, v\in\mathcal{V}, c\in\mathcal{C}_{\varpi}, t\in\mathcal{T}, \omega\in\Omega \tag{19}$$

$$z_{vat\omega}\in\{0,1\}, v\in\mathcal{V}, a\in\mathcal{A}_{\varpi}, t\in\mathcal{T}, \omega\in\Omega \tag{20}$$

$$w_{uet\omega}\in\{0,1\}, u\in\mathcal{U}, e\in\mathcal{E}, t\in\mathcal{T}, \omega\in\Omega \tag{21}$$

$$\delta_{u\omega}\in\{0,1\}, u\in\mathcal{U}, \omega\in\Omega \tag{22}$$

The objective function (10) minimizes the sum of chartering costs and voyage costs. Chartering costs include those from vessels in the planned fleet and those from using additional vessels. Voyage costs include costs from planned voyages, voyage completions, additional voyages, and emergency voyages. Constraints (12) ensure that the number of departures from the depot does not exceed the maximum number of vessels that can be serviced at the depot each day; constraints (13) prevent multiple visits to the same offshore installation on the same day, where visits can result from planned voyages, voyage completions, additional voyages, and emergency voyages; constraints (14) ensure that each additional vessel performs a single voyage; constraints (15) ensure that each offshore installation is visited the required number of times under all scenarios; constraints (16) ensure that at most one voyage completion is used for each planned voyage; and constraints (17) ensure that a vessel cannot start a planned voyage or an additional voyage before returning to the depot from an additional voyage, and constraints (18) ensure that a vessel cannot start a planned voyage or an additional voyage before returning to the depot from a planned voyage.

4. Algorithm

The expected value problem and the two-stage stochastic program are solved using a modified version of the hybrid genetic search with adaptive diversity control (HGSADC) proposed by Vidal et al. (2012) for the Periodic Vehicle Routing Problem and adapted by Borthen et al. (2018) for the Supply Vessel Planning Problem. In the HGSADC, the fitness function accounts for the cost of an individual and its contribution to population diversity. Algorithm 1 provides an overview of the HGSADC algorithm. The modifications introduced in the HGSADC when compared to Borthen et al. (2018) correspond essentially to the fitness evaluation phase (line 7 in Algorithm 1). While Borthen et al. (2018) consider only a deterministic setting, here, Monte Carlo simulation is used to simulate each schedule and compute the corresponding expected cost. Additionally, time-windows are considered here, as well as the prevention of simultaneous visits to each offshore installation. Both of these aspects are absent from Borthen et al. (2018).

Algorithm 1. hybrid genetic search with adaptive diversity control

1. read maximum computational time, T_{max}
2. read the maximum number of iterations without improvement, I_{NI}
3. initialize population
4. initialize iteration counter, $i \longleftarrow 0$
5. initialize counter for the number of iterations without improvement, $i_{ni} \longleftarrow 0$
6. while $i_{ni} < I_{NI}$ and computational time $< T_{max}$ do
7.   compute the fitness for each individual in the population (Algorithm 2)
8.   select parent solution s_1 and s_2
9.   generate offspring s_{new} from s_1 and s_2 (crossover)
10.   if s_{new} is feasible then
11.   insert s_{new} into feasible population
12.   else
13.   insert s_{new} into infeasible population
14.   end
15.   if maximum sub-population size $\mu + \lambda$ is reached then
16.   select survivors
17.   end
18.   adjust penalty parameters for violating feasibility constraints
19.   if best solution has not improved for I_{improv} iterations then
20.   diversify population
21.   end
22. return best feasible solution
23. end

Concretely, when adapting the HGSADC for a two-stage stochastic programming algorithm, the cost of each solution is computed through the use of a discrete-event simulation engine. The discrete-event simulation engine considers both uncertain demand and uncertain weather conditions. Recourse actions which are required upon schedule disruption are also accounted for in the simulation procedure. Algorithm 2 provides an overview of the schedule simulation procedure used to compute the fitness function, where U corresponds to total undelivered demand. Note that a schedule s in Algorithm 2 corresponds to a solution s (s_{new}) in Algorithm 1. Moreover, the aim of Algorithm 2 is to compute the fitness for each individual (i.e., the expected cost for each schedule), which is used in the HGSADC to select individuals for the crossover operation.

Algorithm 2. schedule simulation

Input: a schedule s

Output: C_s (cost of schedule s)

$U \leftarrow getUndeliveredDemandFromPlannedVoyages$ (Algorithm)

$U \leftarrow getUndeliveredDemandFromVoyageCompletions(U)$

$U \leftarrow getUndeliveredDemandFromAdditionalVoyages(U)$

$assignRemainingDemandToEmergencyVoyages$

$C_r^{SC} \leftarrow getVoyageCostsForPlannedVoyages$

$C_c^{SC} \leftarrow getVoyageCostsForVoyageCompletions$

$C_a^{SC} \leftarrow getVoyageCostsForAdditionalVoyages$

$C_e^{SC} \leftarrow getVoyageCostsForEmergencyVoyages$

$C_v^{TC} \leftarrow getTimeCharterCostsForVesselsInPlannedSchedule$

$C_{v'}^{TC} \leftarrow getTimeCharterCostsForAdditionalVessels$

$C_s \leftarrow C_r^{SC} + C_c^{SC} + C_a^{SC} + C_e^{SC} + C_v^{TC} + C_{v'}^{TC}$

return C_s

In turn, it should be noted that the simulation procedure described in Algorithm 2 corresponds to sampling of the scenarios described in section 3, i.e., demand and weather conditions are simulated and the cost for each sampled scenario is computed according to Algorithm 2. In particular, if some demand cannot be met by the corresponding planned voyage, an attempt is made to allocate that demand to a voyage completion. Voyage completions are considered for planned voyages departing on the same day as the failed voyage. If no voyage completion is feasible, then an attempt is made to allocate the undelivered demand to an idle vessel, i.e., to an additional voyage for a vessel in the planned schedule but which is not undertaking any voyage on the departure day for the failed planned voyage. Lastly, if additional voyages are also not feasible for the undelivered demand, an additional vessel is used for the transport using an emergency voyage. Note that it is assumed that there is no limit to the number of additional vessels which may be chartered to ensure the delivery of demand. In case there is a limit to the number of additional vessels that may be chartered in a realistic setting, the cost for the demand handled by additional vessels can be interpreted as the cost of unserviced cargo. Nonetheless, and regardless of the interpretation of such costs, when the procedure terminates, all demand will have been assigned to either some planned voyage or delivered through some recourse action.

Recourse actions are attempted in a sequence corresponding to sorting the corresponding costs by ascending order. The least cost recourse action corresponds to reallocating demand from a given failed voyage to another voyage that is already planned to occur, i.e., a voyage completion. The next in order least cost recourse action corresponds to allocating the demand from failed voyages to another vessel that has already been chartered but which is not currently being used, i.e., an additional voyage assigned to an idle vessel. Lastly, the recourse action corresponding to a steeper increase in costs corresponds to the use of an additional vessel, requiring not only voyage costs but also additional charter costs to be incurred in.

4.1. Weather uncertainty

Weather conditions are the most significant source of uncertainty and schedule disruption in the SVPP (Aas et al., 2009). Such uncertainty is accounted for in the simulation engine by modelling sea states as a stochastic process. Similarly to Halvorsen-Weare and Fagerholt (2017), sea states are modelled through the use of a discrete-time Markov chain. However, Halvorsen-Weare & Fagerholt (2017) consider weather conditions only for the winter season as the aim is to directly increase schedule robustness. In contrast, the method presented here aims at minimizing the expected schedule cost while taking into account the variability in weather conditions observed throughout the year, including any seasonality effects. Seasonality effects are captured through the use of a state transition probability matrix specific to each month. The initial state is randomly selected by considering the limiting distribution for the Markov chain for the first month. Once the sea states corresponding to the first month have been determined, the initial state for the second month is set to correspond to the last visited state in the first month. The procedure continues until sea states for a complete year have been obtained through the sequential use of transition probability matrices for the corresponding months. Note that while the procedure is illustrated here by planning for a complete year, a different number of months may be used depending on the specifics of each planning scenario and the foreseen need to make changes to the planned schedule, which are usually the result of changes in demand from the offshore installations.

Once a particular realization of sea states has been obtained for a complete year, the values of such sea states are used for the computation of both voyage and service times. In particular, sea states are considered to have the same impacts on sailing speed and service time as those considered in Halvorsen-Weare and Fagerholt (2017) shown in Table 5.

4.2. Simulation of planned voyages, voyage completions and emergency voyages

The simulation procedure for planned voyages is detailed in Algorithm 3 and Algorithm 4. All vessels in the planned schedule are required to return to the depot on time for the next planned voyage. Therefore, planned voyages may be simulated for a complete year to determine the need for any recourse action to be implemented. Furthermore, a procedure is used to ensure the feasibility of planned voyages in terms of vessel capacity, where the smallest simulated demands are removed until the total remaining cargo for the planned voyage is smaller than or equal to the capacity of the vessel assigned to perform that voyage. In Algorithm 3, as in Algorithm 2, U corresponds to total undelivered

Table 5
Impact of sea states on sailing speed and service time [adapted from Halvorsen-Weare and Fagerholt (2017)].

sea state	significant wave height [m]	change in sailing speed [kn]	change in service time [%]
1	≤ 2.5	0	0%
2	(2.5, 3.5]	0	+20%
3	(3.5, 4.5]	−2	+30%
4	> 4.5	−3	waiting on weather

demand. To keep track of the corresponding installation and day for which there is undelivered demand (and allow a subsequent application of some recourse action) information on the installation *i* and the day *t* for which there is undelivered demand is kept in *U*.

Algorithm 3. simulation of planned voyages

Input: a schedule s; a list of sea states for a time horizon of one year

Output: U (undelivered demand for a complete year)

1. initialize an empty map of undelivered demand for a complete year, $U \leftarrow \emptyset$
2. **foreach** day of year $t \in 1, \ldots, 365$ **do**
3.   **foreach** planned voyage r in schedule s starting on day t **do**
4. $u_{ti} \leftarrow getUndeliveredDemandForPlannedVoyage(r)$, where i is the installation (Algorithm 4)
5. $U(t,i) \leftarrow U(t,i) \cup u_{ti}$
6.   **end**
7. **end**
8. **return** U

Note that Algorithm 3 is embedded in Algorithm 2 and serves only to simulate the planned voyages and obtain the corresponding undelivered demand, as signalled in Algorithm 2. No recourse actions are mentioned or outlined in Algorithm 3. All recourse actions are outlined in Algorithm 2.

For additional visits inserted into planned voyages through voyage completions, the least insertion cost criteria are adopted, followed by the application of a best feasible 2-OPT exchange heuristic. Note that the referred heuristic is applied only for additional visits inserted into voyage completions, and not to the visits in the originally planned voyage. Visits in the originally planned voyage are kept in the same order as in the planned schedule, as schedule regularity is favoured over cost minimization. Moreover, the same heuristic procedure is applied to both additional voyages and emergency voyages. The use of the greedy procedure allows for a decrease in the computational effort when compared to an exact solution approach, enabling the simulation procedure to be embedded within the genetic search algorithm. However, taking into account that, on the one hand, in a typical setting, chartering costs are significantly higher than voyage costs, and on the other hand, recourse actions are attempted in order of increasing costs, the adopted general greedy procedure should provide for a good approximation to an exact solution approach. Lastly, to approximate the expected cost of each solution, 30 simulation runs are performed for each schedule, with each run corresponding to a complete year. While in Algorithms 2 and 3, *U* corresponds to the set of all undelivered demands, in Algorithm 4 *u* is the set of undelivered demands for a single planned voyage (i.e., the set of all *u* is the set *U*). In line 4, the value of the smallest simulated demand for a given voyage is assigned to u_i. where the subscript *i* refers to the installation having the demand u_i. While the criterion in the iteration is always choosing the smallest demand when total demand is still larger than the capacity of the vessel, keeping track of the information on the corresponding installation (*i*) is required to enable subsequent recourse actions to be applied, i.e., to have information on which installations to visit through voyage completions, additional voyages, or emergency voyages. If the installation information *i* is not recorded, then it would not be possible to know which installations require recourse and which do not. Additionally, Algorithm 4 simulates each planned voyage in two steps. First (steps 1 to 7 in Algorithm 4), installations are removed from the voyage until feasibility is achieved in terms of the capacity of the vessel. Afterwards (steps 8 to 16 in Algorithm 4), and considering the impact of weather conditions on sailing speed, feasibility in terms of travel time is also ensured by removing each installation if performing the corresponding visit would prevent the vessel from returning to the depot on time for the next planned voyage.

Algorithm 4. simulation of a single planned voyage

Input: a vessel assigned to perform the planned voyage, v; a list of installations to be visited, $\mathcal{I}$; an ordered list of sea states for a time horizon of one year, S

Output: u, a map of undelivered demand for the planned voyage

1. initialize an empty map of undelivered demand $u \leftarrow \emptyset$
2. $D \leftarrow getListOfSimulatedDemandsForInstallations$
3. **while** *total simulated demand exceeds capacity of the vessel assigned to perform voyage* **do**
4. $u_i \leftarrow$ remove smallest simulated demand from D
5.   remove installation i from list of installations to visit, $\mathcal{I} \leftarrow \mathcal{I} \backslash \{i\}$
6. $u \leftarrow u + u_i$
7. **end**
8. $i \leftarrow 0$
9. initialize next installation to visit, $nextInstallation \leftarrow \mathcal{I}(i)$
10. **while** vessel v is able to visit $nextInstallation$ and return on time for next planed visit **do**
11.   compute costs and travel times to reach $nextInstallation$
12. $D \leftarrow D \backslash D(i)$
13. $i \leftarrow i + 1$
14. $nextInstallation \leftarrow \mathcal{I}(i)$
15. **end**
16. $u \leftarrow u + D$
17. **return** U

---

5. Computational experiments

The proposed two-stage stochastic programming algorithm was tested on problem instances constructed from data for the Santos Basin in Brazil. In particular, problem instances with five, 10, 15, 20, and 25 offshore installations were generated, with the average distance from the onshore depot being 250 km. In a realistic setting for Santos Basin, in Brazil, a fleet of PSVs harboured at a single port is planned to serve approximately 20 offshore installations. The majority of the offshore installations require two weekly visits, while the smaller installations require only one visit per week. Moreover, historical data was used for both the modelling of weather conditions and for the construction of probabilistic models for the simulation of demand. Demand for each visit is modelled as a random variable following a negative binomial distribution (*r*,*p*), where *r* is the number of successes and *p* is the probability of success, with $r = 3$ and $p = 0.05$.

Two types of costs are computed for each solution: the planned cost, and the expected cost. Planned schedule cost is computed by assuming all random variables to take a value equal to the corresponding expected value, i.e., by solving the expected value problem presented in Section 3.1. Planned cost includes both voyage costs and charter costs for vessels in the planned schedule. In turn, the expected schedule cost is computed by considering the probabilistic models for each random variable. Expected schedule cost includes costs computed from the simulation of voyages as well as charter costs. In particular, voyage costs included in the expected schedule cost are costs computed from the simulation of planned voyages, voyage completions, additional voyages, and emergency voyages. In turn, charter costs included in the expected schedule cost are charter costs for vessels in the planned schedule and charter costs for additional vessels used for emergency voyages. To compute the expected schedule cost for the solution from the deterministic approach, simulation is applied as a last step in the algorithm for the Expected Value Problem, while for the two-stage stochastic program, simulation is applied at each iteration of the algorithm. While in Brazil no functioning spot market exists allowing for the hiring of vessels on short notice, the computed recourse cost should be interpreted as an approximation to the investment cost that would be required for chartering vessels through long-term charters to be able to fulfil the demand from the required emergency voyages.

The algorithms were coded in Java programming language and compiled with JavaSE-1.8.0_231. The computational experiments were conducted on a computer with 8 GB of RAM and a 1.80 GHz processor.

Both Table 6 and Fig. 3 show that the two-stage stochastic program with recourse provides for an expected cost which is lower than that obtained through the use of the deterministic approach commonly used by logistics planners at offshore oil and gas companies. In particular, savings from the application of the stochastic program range from approximately 55 thousand USD per week for the problem instance with five installations, thereby leading to annual savings of approximately three million USD, to 154 thousand USD per week for the problem instance with 25 installations, thereby leading to annual savings of approximately 8 million USD. In relative terms, savings from using the stochastic program over the deterministic algorithm correspond to an average decrease in costs of approximately 12%. Table 6 also shows that there is a steep increase in the computational effort required for the stochastic programming approach. Note that each method was run 20 times, and the reported results correspond to an average of such runs. In particular, note that the fractional result for the size of the chartered fleet reported in Table 6 for the problem instance with 25 installations (EVP) results from the fact that for some runs of the HGSADC (i.e., with some particular seeds for the genetic algorithm), the output fleet size was 5, whereas for others the output fleet size was 4, and averaging over the 20 runs resulted in an average fleet size of 4.67 vessels.

In turn, such a steep increase in computational effort may seem to defeat one of the main purposes of using a heuristic approach, which is to provide a good solution in a relatively short computational time. However, the magnitude of the savings obtained from using the stochastic programming approach should largely compensate for the increase in required computational time. Moreover, since the SVPP is not an operational planning problem, but rather a problem in which decisions pertaining to the long term, i.e. the SVPP is a strategic planning problem, computational times shown in Table 6 should not prevent the practical application of the proposed methodology given the magnitude of the obtained costs savings.

To investigate the relative impact of uncertain weather conditions and uncertain demand, Table 7 shows the results from optimizing against demand uncertainty only, against weather uncertainty, and against both demand uncertainty and weather uncertainty. For the three approaches, in the last iteration of the algorithm, the simulation accounts for both demand uncertainty and weather uncertainty. For instance, when assessing the expected cost from optimizing against demand uncertainty only (referred to in Table 7 as SPR, demand), at every iteration of the algorithm, simulation accounts for only demand uncertainty, whereas weather is modelled to take a value equal to the corresponding expected value. However, in the last iteration, the simulation also incorporates weather uncertainty. Therefore, for the three tested approaches, simulation is performed in the same manner in the last iteration, i.e., including both demand uncertainty and weather uncertainty.

Table 7 shows that, for the tested problem instances, weather uncertainty plays a minor role. Solutions obtained from solving the stochastic program accounting only for weather uncertainty approximate those obtained from solving the expected value problem, which is seen when comparing results from Table 7 to those in Table 6. Therefore, demand accounts, by far, for most of the uncertainty in the tested problem instances. This falls in line with empirical observation of offshore logistics in Brazil, where a considerable variation in demand is observed between offshore installations and also from week to week for a single offshore installation (Leite, 2012). In contrast, weather conditions in Santos Basin are seldom adverse. Table 8 shows the observed frequency for each sea state, where sea states are defined as in Table 5, from historical data for Santos Basin in Brazil.

From Table 8 it can be seen that sea states 1 and 2 account for approximately 96% of the observed sea states throughout the year. Recall from Table 5 that sea states 1 and 2 have no impact on the sailing speed of the vessel, and that sea state 1 has no impact on service time. Furthermore, even when considering the worse month, where worse is defined as the month having the highest frequency for the most severe sea states, i.e., the month of May, sea states 1 and 2 still account for approximately 92% of the observed sea states. Therefore, the impact of simulated sea states will, for the vast majority of cases, be essentially equivalent to that observed if instead a sea state equal to the expected value is used, i.e., sea state 1. Therefore optimizing against weather uncertainty only leads to solutions which are similar to those obtained from solving the expected value problem. This can be seen from the fact that costs for the expected value problem (shown in Table 6) are of the same order of magnitude as those shown in Table 7 for the stochastic program accounting for weather uncertainty only. As a result, optimizing against weather uncertainty only essentially adds no robustness when compared to that observed when using the expected value approach. In contrast, optimizing against uncertain demand provides, by far, most of the added robustness when compared to that from the expected value problem, a situation which agrees with previous empirical analysis of offshore logistics in Brazil. Note that this situation is in sharp contrast with that observed, for instance, in the North Sea,

Table 6
Summary results from solving the expected value problem (EVP), and the two-stage stochastic program with recourse (SPR) (results averaged over 20 runs for each methodology).

# installations/# weekly visits	solution	planned cost [10^3 USD]	expected cost [10^3 USD]	size of chartered fleet	expected number of additional PSVs	CPU time [secs]
5/9	EVP	381.17	442.98	2	0.33	56.88
5/9	SPR	382.58	388.08	2	0.03	233.19
10/18	EVP	388.70	665.53	2	1.52	287.94
10/18	SPR	392.08	594.53	2	1.10	1532.23
15/26	EVP	580.06	911.14	3	1.80	613.41
15/26	SPR	585.82	824.62	3	1.28	3529.17
20/36	EVP	772.76	1205.49	4	2.34	1146.15
20/36	SPR	779.85	1075.97	4	1.57	6721.02
25/45	EVP	909.37	1437.68	4.67	2.83	1893.28
25/45	SPR	971.80	1283.36	5	1.57	12415.26

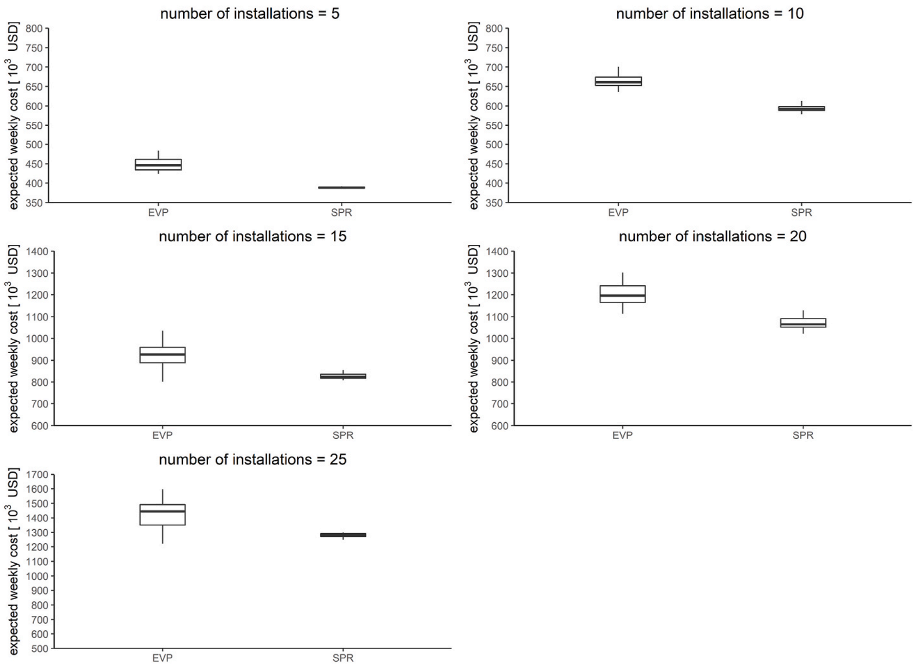


Fig. 3. Expected costs for the deterministic approach and the stochastic programming methodology for problem instances with five, 10, 15, 20, and 25 installations. EVP = expected value problem; SPR = two-stage stochastic programming with recourse.

Table 7
Summary results from solving the two-stage stochastic program with recourse (SPR) accommodating for only demand uncertainty, only weather uncertainty, and both demand and weather uncertainty simultaneously.

# installations/# weekly visits	solution	planned cost [10^3 USD]	expected cost [10^3 USD]	size of chartered fleet	expected number of additional PSVs	CPU time [secs]
5/9	SPR, demand	382.89	390.88	2	0.04	196.58
5/9	SPR, weather	381.65	434.51	2	0.13	192.05
5/9	SPR, demand + weather	382.58	388.08	2	0.03	233.19
10/18	SPR, demand	393.13	604.14	2	1.14	854.73
10/18	SPR, weather	389.42	661.48	2	1.49	932.67
10/18	SPR, demand + weather	392.08	594.53	2	1.10	1532.23
15/26	SPR, demand	586.01	837.03	3	1.35	1876.54
15/26	SPR, weather	582.17	902.81	3	1.77	1989.24
20/36	SPR, demand + weather	585.82	824.62	3	1.28	3529.17
20/36	SPR, demand	779.14	1093.46	4	1.66	4376.92
20/36	SPR, weather	775.64	1194.16	4	2.29	4494.29
20/36	SPR, demand + weather	779.85	1075.97	4	1.57	6721.02
25/45	SPR, demand	962.06	1291.17	4.88	1.69	7389.24
25/45	SPR, weather	910.29	1424.23	4.57	2.98	8425.02
25/45	SPR, demand + weather	971.80	1283.36	5	1.57	12415.26

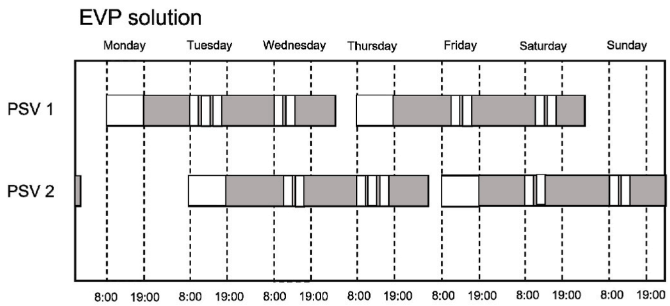
Table 8
Observed frequency for each sea state from historical data for Santos Basin in Brazil, considering all year and only for the month showing the highest frequency for sea states three and four (month of May) (weather data collected from NOAA [n.d.]).

state	frequency (all year)	frequency (month of May)
1	0.727	0.680
2	0.232	0.240
3	0.043	0.062
4	0.008	0.019

where weather conditions account for most of the uncertainty in offshore logistics.

Nonetheless, it can be seen from Tables 6 and 7 that, while the benefits from optimizing against uncertain demand greatly outweigh those from optimizing against uncertain weather conditions for the tested problem instances, there is still some benefits from optimizing against uncertain weather conditions. This is seen from the fact that the expected cost obtained from optimizing against uncertain weather conditions is smaller than that obtained from solving the expected value problem.

Lastly, Fig. 4 shows two solutions with one being obtained through the use of the deterministic approach (EVP), and the other through the use of the two-stage stochastic programming method. In Fig. 4, the sailing legs for each vessel are coloured in grey, while visits to



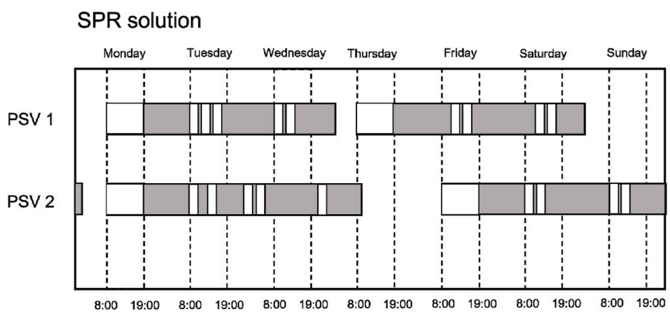


Fig. 4. Comparison of two solutions obtained from the deterministic approach and the two-stage stochastic program for the problem instance with 10 installations.

installations are in white. All voyages start from the depot, where the loading operations start at 8h00 and end at 19h00.

It can be seen that the solutions are similar. However, the first voyage for PSV 2 has a shorter duration in the deterministic approach, which would contribute to a smaller planned cost. However, between the arrival of PSV 2 from its first voyage and the departure for the second planned voyage, there is relatively little slack, which makes the solution less robust. In contrast, the first weekly voyage for PSV 2 in the stochastic programming method has a longer duration, which contributes to the planning cost for the stochastic solution being larger than that for the deterministic approach. However, in the solution for the stochastic method, there is also slacker between the arrival of PSV 2 from the first voyage and the departure for the second voyage, which makes the schedule more robust, and therefore, it is likely to need fewer recourse actions to compensate for any failed voyages.

6. Conclusion

This paper presents a two-stage stochastic programming algorithm with recourse for the supply vessel planning problem under uncertain demand and uncertain weather conditions. Both uncertain demand and uncertain weather conditions are frequent causes of schedule disruption in offshore logistics. Such disruption makes it necessary to implement costly recourse actions to ensure the delivery of cargo from failed voyages. Recourse actions include the relocation of visits to other planned voyages, performing additional voyages, and the use of additional vessels chartered specifically to ensure the delivery of demand from failed voyages. Alternative methodologies accommodating both demand and weather uncertainty, rely on two-phased methods combining simulation and optimization. In the first phase, simulation is applied at the voyage level to compute the average undelivered demand. In the second phase, a deterministic optimization algorithm is used to arrive at robust schedules, with a penalty being applied to each voyage, where such penalty is computed from the average undelivered demand obtained from the simulation model in the first phase. While the referred method allows obtaining schedules which are robust against both weather and demand uncertainty simultaneously, the penalty to be applied to each voyage is essentially subjective rather than corresponding to actual costs faced by the offshore oil and gas companies. Moreover, schedules obtained without the incorporation of the referred penalties may also perform well under uncertainty. In contrast, a two-stage stochastic program with recourse allows for the direct incorporation of the real costs faced by offshore companies in the face of schedule disruption. Therefore, a two-stage stochastic programming algorithm provides for the minimization of the actual costs faced by offshore oil and gas companies. While the use of simulation to approximate the expected schedule cost leads to an increase in computational time, the magnitude of the savings obtained should largely compensate for such drawbacks. Moreover, taking into account that the Supply Vessel Planning Problem is a strategic planning problem, with a given fleet and the corresponding sailing schedule typically being in use for several months, the proposed methodology allows obtaining solutions which are robust under weather and demand uncertainty with the minimized cost within a reasonable computation time, allowing for its practical application. For the tested problem instances, when compared to the deterministic approach commonly used by logistics planners at offshore oil and gas companies, the application of the proposed methodology leads to annual savings ranging from approximately three million USD, for the test instance with five installations, to eight million USD, with 25 installations. A comparison is also made of the relative importance of demand uncertainty and weather uncertainty in offshore logistics. For the tested problem instances, results suggest that uncertain demand plays a much larger role in offshore logistics in Brazil when compared to uncertain weather conditions, which is in agreement with previous empirical observations and is in sharp contrast with the situation observed, for instance, in the North Sea, where uncertain weather conditions play a much larger role in offshore logistics, as observed, for instance, in Halvorsen-Weare et al. (2012).

CRediT authorship contribution statement

A.M.P. Santos: Methodology, Formal analysis, Visualization, Writing – original draft. **K. Fagerholt:** Writing – review & editing. **C. Guedes Soares:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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References

Aas, B., Sr, Ø., Wallace, S., 2009. The role of supply vessels in offshore logistics. Marit. Econ. Logist. 11, 302–325.
Alrefaei, M.H., Andradóttir, S., 1999. Simulated annealing algorithm with constant temperature for discrete stochastic optimization. Manag. Sci. 45 (5).
Amiri, M., Amin, S.H., Tavakkoli-Moghaddam, R., 2019. A Lagrangean decomposition approach for a novel two-echelon node-based location-routing problem in an offshore oil and gas supply chain. Transport. Res. E Logist. Transport. Rev. 128, 96–114.

Assis, L.S., Camponogara, E., 2016. A MILP model for planning the trips of dynamic positioned tankers with variable travel time. Transport. Res. E Logist. Transport. Rev. 93, 372–388.
Birge, J.R., Louveaux, F.v., 2011. Introduction to Stochastic Programming. Springer, New York.
Borthen, T., Loennechen, H., Wang, X., Fagerholt, K., Vidal, T., 2018. A genetic search-based heuristic for a fleet size and periodic routing problem with application to offshore supply planning. EURO Journal on Transportation and Logistics 7 (2), 121–150.
Cruz, R., Mendes, A.B., Bahiense, L., 2023. Schedule robustness in the periodic supply vessels planning problem with stochastic demand and travel time. Int. Trans. Oper. Res. https://doi.org/10.1111/itor.13241.
Dalgic, Y., Lazakis, I., Dinwoodie, I., McMillan, D., Revie, M., 2015. Advanced logistics planning for offshore wind farm operation and maintenance activities. Ocean. Eng. 101, 211–226.
Fattahi, M., Govindan, K., Keyvanshokooh, E., 2017. Responsive and resilient supply chain network design under operational and disruption risks with delivery lead-time sensitive customers. Transport. Res. E Logist. Transport. Rev. 101, 176–200.
Gutjahr, W.J., 2004. S-ACO: an ant-based approach to combinatorial optimization under uncertainty. In: Dorigo, M., Birattari, M., Blum, C., Gambardella, L.M., Mondada, F., Stützle, T. (Eds.), Ant Colony Optimization and Swarm Intelligence. Springer Berlin Heidelberg, pp. 238–249.
Halvorsen-Weare, E., Fagerholt, K., 2017. Optimization in offshore supply vessel planning. Optim. Eng. 18, 317–341.
Halvorsen-Weare, E., Fagerholt, K., Nonås, L., Asbjørnslett, B., 2012. Optimal fleet composition and periodic routing of offshore supply vessels. Eur. J. Oper. Res. 223, 508–517.
Kenyon, A.S., Morton, D.P., 2003. Stochastic vehicle routing with random travel times. Transport. Sci. 37 (1).
Kisialiou, Y., Gribkovskaia, I., Laporte, G., 2019. Supply vessel routing and scheduling under uncertain demand. Transport. Res. C Emerg. Technol. 104, 305–316.
Kondratenko, Tarovik, V., 2020. Analysis of the impact of arctic-related factors on offshore support vessels design and fleet composition performance. Ocean. Eng. 203.
Ksciuk, J., Kuhlemann, S., Tierney, K., Koberstein, A., 2023. Uncertainty in maritime ship routing and scheduling: a Literature review. Eur. J. Oper. Res. 308 (2), 499–524.
Leite, R.P., 2012. Maritime Transport of Deck Cargo to Petrobras Fields in Campos Basin: an Empirical Analysis, Identification and Quantification of Improvement Points. Departamento de Engenharia Industrial, Pontificia Universidade Catolica do Rio de Janeiro, MSc Thesis, Rio de Janeiro, Brazil.
NOAA https://polar.ncep.noaa.gov/waves/CFSR_hindcast.shtml, accessed on16/3/2018.
Novoa, C., Berger, R., Linderoth, J., Storer, R., 2006. A Set-Partitioning-Based Model for the Stochastic Vehicle Routing Problem. Lehigh University Technical report (Issue January).
Powell, W.B., 2014. Clearing the jungle of stochastic optimization. In*Bridging data and decisions*, Informs. 109–137.
Psaraftis, H., Kontovas, C., 2013. Speed models for energy-efficient maritime transportation: a taxonomy and survey. Transport. Res. C Emerg. Technol. 26, 331–351.
Psaraftis, H., Kontovas, C., 2014. Ship speed optimization: concepts, models and combined speed-routing scenarios. Transport. Res. C Emerg. Technol. 44, 52–69.
Rahman, M.S., Khan, F., Shaikh, A., Ahmed, S., Imtiaz, S., 2019. Development of risk model for marine logistics support to offshore oil and gas operations in remote and harsh environments. Ocean. Eng. 174, 125–134.
Salehi Sadghiani, N., Torabi, S.A., Sahebjamnia, N., 2015. Retail supply chain network design under operational and disruption risks. Transport. Res. E Logist. Transport. Rev. 75.
Santos, A.M.P., Fagerholt, K., Laporte, G., Guedes Soares, C., 2022. A stochastic optimization approach for the supply vessel planning problem under uncertain demand. Transp. Res. Part B Methodol. 162, 209–228.
Santos, A.M.P., Silva, L.M.R., Santos, T.A., Guedes Soares, C., 2018. A simulation approach applied to the optimization of offshore crew transportation problems. In: Guedes Soares, C., P, T.A. (Eds.), Maritime Transportation and Harvesting of Sea Resources, 2. Taylor & Francis, pp. 1133–1140.
Santoso, T., Ahmed, S., Goetschalckx, M., Shapiro, A., 2005. A stochastic programming approach for supply chain network design under uncertainty. Eur. J. Oper. Res. 167 (1).
Shapiro, A., 2003. Monte Carlo sampling methods. Handb. Oper. Res. Manag. Sci. 10, 353–425.
Schrotenboer, A., Ursavas, E., Vis, I., 2019. Mixed Integer Programming Models for Maintenance Planning at Offshore Wind Farms under Uncertainty, 112. Transportation Research Part C Emerging Technologies.
Shyshou, A., Gribkovskaia, I., Laporte, G., Fagerholt, K., 2012. A large neighbourhood search heuristic for a periodic supply vessel planning problem arising in offshore oil and gas operations. INFOR Inf. Syst. Oper. Res. 50 (4), 195–204.
Silva, L.M.R., Guedes Soares, C., 2018. Helicopter fleet size and mixed vehicle routing problem for crew exchange on an offshore oil and gas field. In: Guedes Soares, C., Teixeira, A.P. (Eds.), Maritime Transportation and Harvesting of Sea Resources. Taylor & Francis, pp. 1157–1164.
Suryawanshi, P., Dutta, P., 2022. Optimization models for supply chains under risk, uncertainty, and resilience: a state-of-the-art review and future research directions. Transport. Res. E Logist. Transport. Rev. 157.
Verweij, B., Ahmed, S., Kleywegt, A.J., Nemhauser, G., Shapiro, A., 2003. The sample average approximation method applied to stochastic routing problems: a computational study. Comput. Optim. Appl. 24 (2–3).
Vidal, T., Crainic, T.G., Gendreau, M., Lahrichi, N., Rei, W., 2012. A hybrid genetic algorithm for multidepot and periodic vehicle routing problems. Oper. Res. 60, 611–624.