Multi-step Ship Roll Motion Prediction Based on Bi-LSTM and Input Optimization

Shiyang Li, Tongtong Wang, Guoyuan Li, Robert Skulstad, and Houxiang Zhang

***Abstract*— Ship roll is a crucial metric in assessing the vessel's safety in offshore operations. This paper investigates input selection for predicting short-term ship roll motion using the Bidirectional Long Short-Term Memory Network (Bi-LSTM) and the Sobol sensitivity analysis of ship roll based on the predicted models. Considering the complexity of the impact of forces, velocities, and positions with six degrees of freedom on ship roll, a data-driven model is established to represent the relationship adequately. Firstly, one-step prediction models with different time intervals are established based on Bi-LSTM to express the relationship between all input features and output. Afterward, the Sobol sensitivity analysis is carried out to evaluate the impact of input features on the output based on the predicted models. Finally, mathematical statistics are utilized to optimize input selection for multi-step prediction models by analyzing the sensitivity results. The experimental results demonstrate that optimizing the input feature dimensions can improve the accuracy of one-step, five-step, and ten-step prediction models.**

***Index Terms*— Ship Roll Prediction, Sensitivity Analysis, Input Selection, Bi-LSTM**

I. INTRODUCTION

The evolution of intelligent ship technologies has led to an increased demand for advanced ship motion prediction techniques. Such techniques are pivotal in ensuring the safe and efficient operation of ships, achieving navigation accuracy, and mitigating potential risks. Ship motion prediction plays a crucial role in various marine operators, including heavy lifting [1], path planning [2], and trajectory tracking [3]. Numerous offshore operations are carried out under Dynamic Positioning (DP), such as oil and gas drilling, offshore wind turbine installation, and subsea pipeline laying, which require the vessel to maintain stability [4]. Ship roll is directly related to a ship's stability, which is critical in ensuring safe and efficient operations [5]. By building a precise ship roll motion prediction model, operators can better anticipate and respond to potential ship roll events, allowing them to take appropriate actions to avoid unsafe situations.

There are generally two approaches to building ship motion prediction models: the physical model, and the data-driven method. Some researchers employed the physical model to predict ship motion and optimized by parameter identification. A precise physical model is developed and optimized by parameter identification. Wang et al. [6] used the Abkowitz model to predict ship motion and identified hydrodynamic parameters under environmental disturbance based on the support vector machine to establish accurate mathematical models. Meng et al. [7] established the 4 DOF ship whole-ship mathematical model including rolling motion and proposed a parameter identification scheme, support vector regression combined with a modified grey wolf optimizer, to identify the ship motion model. Ship roll motions are influenced by various factors, including the ship's geometry, inertia forces, hydrodynamic forces, and environmental forces, such as wind, waves, and currents [8]. Consequently, It is challenging to describe the roll motion of a ship using the physical model.

Another commonly used method for predicting ship motion is the data-driven model. Considering the ship roll motion is inherently sequential in nature, the recursive neural (RNNs), specifically Long Short-Term Memory Networks (LSTM) are used widely in ship roll motion prediction, due to their ability to capture sequential dependencies and temporal dynamics in time-series data. LSTM offers recurrent connections to memory blocks within the network and introduces gate structures for modulating gradient flow [9]. Bi-LSTM uses two separate RNNs to process the input sequence in opposite directions, which means it can process the input sequence in both forward and backward directions simultaneously. Yin et al. [10] proposed a real-time ensemble prediction model for roll motion by combining the discrete wavelet transform and the variable-structure radial basis function network. Wang et al. [11] proposed a ship roll angle prediction method based on Bi-LSTM and temporal pattern attention mechanism combined deep learning model, to address the issue of low accuracy in predicting ship roll angle with traditional prediction algorithms and single neural network model. Sun et al. [12] proposed a hybrid ship motion prediction model based on LSTM and Gaussian Process Regression, which adopts the idea of two-step prediction. Wei et al. [13] proposed a hybrid three-step prediction model, including adaptive empirical wavelet transform, multi-step forecasting under the multi-input multi-output strategy of Bi-LSTM, and hybrid particle swarm optimization and gravitational search algorithm hyperparameter optimization, which is suitable for different datasets and has strong robustness. Most researchers only work on one-step and at most three-step ship roll motion predictions, however, a multi-step prediction will be useful in practice.

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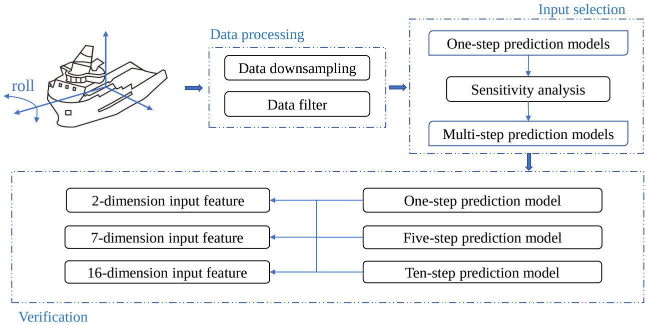

Fig. 1: The result of 10-10 ship model prediction.

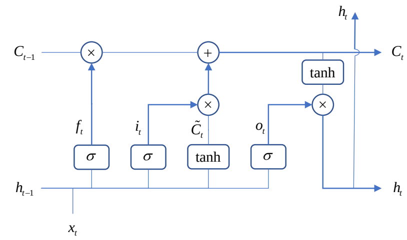

Fig. 2: The structure of the LSTM.

Optimizing the input vector space of the LSTM deep learning model has the potential to enhance the accuracy of ship motion prediction [14]. Forces in the six degrees of freedom are coupled and the degrees of influence on them are different under different degrees of environmental forces [15]. Wang et al. [16] proposed the single-input single-output and the multi-input single-output ship roll prediction methods based on Bi-LSTM, studied the influence of input variables on the ship roll prediction model, and concluded that simply adding input features does not necessarily improve the prediction performance of the model. Zhang et al. [3] employed the wavelet transform to decompose ship motion signals into several frequency scales, which makes LSTM capture the inherent law of ship motion from each frequency scale to establish a multi-scale attention-based LSTM model for the shipboard stabilized platform. Reducing the input dimension of the network has positive effects on computation time as well as network interpretability and generalization ability [17]. To enhance the accuracy of the prediction model, we employ Sobol sensitivity analysis [18] to optimize the input features.

This paper establishes multi-step roll motion prediction models based on Bi-LSTM. Firstly, the data generated by Offshore Simulator Center AS is processed, including downsampling and filtering. Secondly, to enhance the accuracy of the ship prediction model, a Sobol sensitivity analysis is performed to optimize the input features. Finally, the multi-step models are tested using multi-dimension input features. The rest of the paper is organized as follows. Section II elaborated on the methodology, mainly including Bi-LSTM and sensitivity analysis. In Section III, the experiment results are shown. Then, Section IV offers a summary of the paper and presents future work.

II. Framework of Multi-step Ship Roll Motion Prediction

In this section, an effective multi-step prediction scheme is proposed for multi-step ship roll motion prediction when the environmental forces are stable, as shown in Fig. 1. The ship roll motion prediction model is built based on the Bi-LSTM, which is a classical neural network for time-series prediction and can extra time-series information bi-directionally. To improve the accuracy of Bi-LSTM, the Sobol sensitivity analysis is conducted to input selection.

A. *Bidirectional Long Short-term Memory Model*

LSTM is specifically designed to handle long-term dependencies and information retention in time-series data, the structure as shown in Fig. 2. LSTM is a specialized type of RNN that includes a recurrent neural network module in a chain-like architecture for capturing temporal correlations in data. LSTM incorporates three gates, namely the forget gate, input gate, and output gate, for selectively retaining and discarding sequence information and prolonging information storage in the network. Specifically, the forget gate and the input gate are used to regulate forgetting or updating information in the current state, while the output gate controls which information is output during the current state, as shown in Eq. (1)-(6).

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right) \tag{1}$$
$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] + b_i\right) \tag{2}$$
$$\tilde{C}_t = \tanh\left(W_C \cdot [h_{t-1}, x_t] + b_C\right) \tag{3}$$
$$C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t \tag{4}$$
$$o_t = \sigma\left(W_o\, [h_{t-1}, x_t] + b_o\right) \tag{5}$$
$$h_t = o_t * \tanh\left(C_t\right) \tag{6}$$

where h_{t-1} represents the output of the previous cell and x_t represents the input of the current cell. σ represents the $sigmoid$ function. W_f, W_i, and W_o are the weights of the forget gate f_t, the input gate i_t, and the output gate o_t, and W_C is the weight of the state update $\tilde{C}_t$. b_f, b_i, b_c, and b_o are the biases.

Traditional LSTM networks process input sequences in chronological order, only considering past information and ignoring future information. In contrast, bidirectional LSTM networks can process input sequences in both chronological orders, from past to future and from future to past. The hidden states of a bidirectional LSTM are the concatenation of the hidden states from the forward and backward LSTM networks. By considering both past and future information, bidirectional LSTM networks can update the hidden state and cell state more accurately, leading to more accurate predictions at the current time step.

B. Sobol Sensitivity Analysis

The sensitivity analysis can identify and prioritize the most influential inputs based on the relationships between the input variables and the output. The Sobol sensitivity analysis method assesses the impact of input parameters on the output through variance decomposition. The model is represented as $Y = f(X)$, where $X = X_1, X_2, ...X_d$ is an input vector with d uncertain inputs and Y is a univariate model output. Eq. (7) offers the decomposition of $f(X)$.

$$\begin{aligned} Y =& f_0 + \sum_{i=1}^{d} f_i\left(X_i\right) + \sum_{i<j}^{d} f_{ij}\left(X_i, X_j\right) + \cdots \\ &+ f_{1,2,\ldots,d}\left(X_1, X_2, \ldots, X_d\right) \end{aligned} \tag{7}$$

where f_0 is a constant, f_i is a function of X_i, and f_{ij} is a function of X_i and X_j. Assuming that the function $f(X)$ is square-integrable, we can square and integrate the functional decomposition to Eq. (8).

$$\int f^2(\mathbf{X}) d\mathbf{X} - f_0^2 = \sum_{s=1}^{d} \sum_{i_1<\ldots<i_s}^{d} \int f_{i_1\ldots i_s}^2 dX_{i_1} \ldots dX_{i_s} \tag{8}$$

The variance of Y is equal to the expression on the left-hand side of Eq. (8), and the terms on the right-hand side represent the decomposed variance terms with respect to sets of X_i. Therefore, the expression can be transformed into the decomposition of variance, as shown in Eq. (9).

$$\operatorname{Var}(\mathbf{Y}) = \sum_{i=1}^{d} V_i + \sum_{i<j}^{d} V_{ij} + \cdots + V_{12\ldots d} \tag{9}$$

where

$$V_i = \operatorname{Var}_{X_i}\left(E_{\mathbf{X}_{-i}}(Y \mid X_i)\right) \tag{10}$$

$$V_{ij} = \operatorname{Var}_{X_{ij}}\left(E_{\mathbf{X}_{-ij}}(Y \mid X_i, X_j)\right) - V_i - V_j \tag{11}$$

where $\mathbf{X}_{-i}$ denotes the set of all variables, excluding $\mathbf{X}_i$.

The first-order sensitivity index, also known as the main effect index, is a direct variance-based measure of sensitivity denoted by S_i and expressed as shown in Eq. (12).

$$S_i = \frac{V_i}{\operatorname{Var}(Y)} \tag{12}$$

When dealing with a high number of variables, computing $2d-1$ indices poses a computational challenge. The solution to this problem is the widespread use of the measurement known as the total-order index in the field, S_{Ti}, as shown in Eq. (13).

$$S_{Ti} = 1 - \frac{\operatorname{Var}_{\mathbf{X}_{-i}}\left(E_{X_i}(Y \mid \mathbf{X}_{-i})\right)}{\operatorname{Var}(Y)} \tag{13}$$

The first-order sensitivity index obtained by measuring the effect of changing X_i alone and standardized by the total variance indicates how much the main effect of variable X_i contributes to the variance of the output. The total-effect index measures the overall contribution of X_i to the output variance, including all the variances caused by its interactions with any other input variable of any order. The value range of STi indices is between 0 and 1, where a higher value indicates a higher contribution of the corresponding input parameter to the output variance.

TABLE I: Test ship specifications

Description	Values
Length Between Perpendiculars	82.7 m
Breadth	23.058 m
Draught	7.5 m
Mass	1.0179×10^7 kg

III. Experiment

A. Experiment Setting

The experimental data used in this paper come from a commercial professional simulator developed by the Norwegian company Offshore Simulator Center AS. It features a simulated environment in which users may manipulate the wind, waves, and ocean currents to simulate real-life conditions and offers a library of virtual vessels to choose from. Table I shows the ship specifications we use. In this paper, the ship is in dynamic positioning and the environmental forces are including wind, wave, and swell. The data are generated by the simulator when wind direction, wind velocity, wave direction, and wave height change randomly, wherein the vessel is allowed to return to a balanced state and remain stable before the next change. So in every case, the wind direction, wind velocity, wave direction, and wave height are random constants, since the environmental forces do not change drastically in a short time. During this process, the swell height is kept constant at 0.1 m. During this process, the northern coordinates of the ship are located within the range of (-30.56, 47.15), whereas the eastern coordinates fall within the range of (-20.15, 27.67). Selecting data with constant environmental forces, we obtain 12 groups of data, of which 10 groups are used as a training set and two groups are used as a testing set. Each group contains 16-dimensional features that record the ship's state and environmental conditions for 8 minutes, including roll angle, roll velocity, pitch angle, pitch velocity, yaw angle, yaw velocity, surge, sway, heave, north, east, wind direction, wind velocity, wave direction, and wave height. The data time interval is downsampled to 1 s, so there are 480 data points in every group.

B. Sensitivity Analysis

The one-step prediction models are established for the purpose of conducting sensitivity analysis. This is because sensitivity analysis can only analyze models with scalar output, and it is difficult to describe time series by one scalar value. Furthermore, the LSTM networks capture dynamic information of time series data by propagating the hidden state between time steps, so the input and output time steps

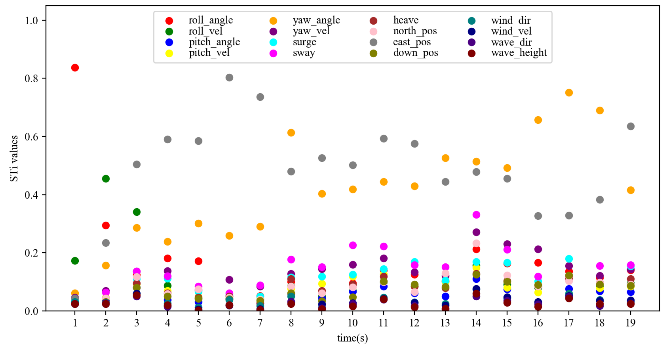

Fig. 3: The result of sensitivity analysis.

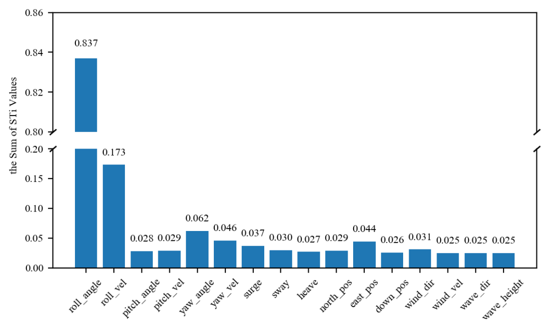

Fig. 4: The statistical result of sensitivity analysis for the one-step model.

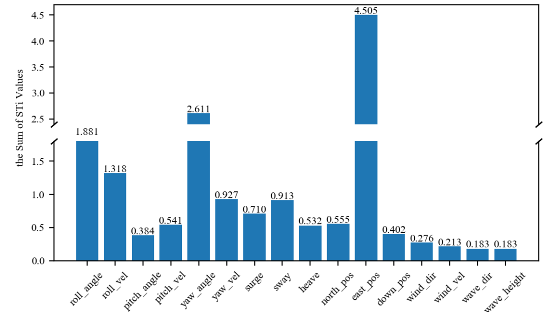

Fig. 5: The statistical result of sensitivity analysis for the five-step model.

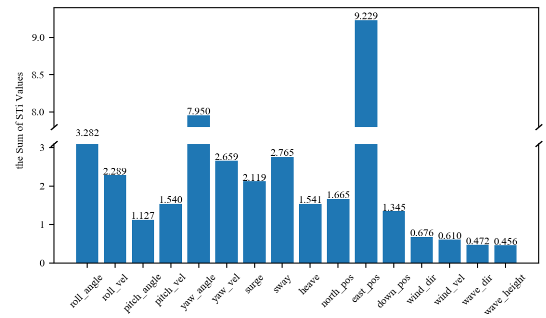

Fig. 6: The statistical result of sensitivity analysis for the ten-step model.

of it must match in length. Thus the multi-step prediction model should be divided into multiple one-step prediction models. The sensitivity analysis of multi-step prediction is transformed into several analyses of the impact of input features on the model output for different time intervals. The analysis needs to determine the degree of influence that input features have on the model output for each time interval. The twenty-one-step models are constructed to predict the ship roll motion for each future nth second ($0 < n \leq 20$) based on the current time state. These models are then used for individual sensitivity analyses, and the sensitivity results are shown in Fig. 3.

The sensitivity analysis results indicate that as the time step changes, the input features have varying impacts on the ship roll angle. Therefore, building up models with different time steps requires different input feature dimensions, which has a significant impact on the output.

C. Input Selection

Statistical analysis is performed to determine the appropriate input selection for establishing a multi-step ship roll motion prediction model based on the results of sensitivity analysis. To build an effective model, the model should incorporate the parameters with higher sensitivity analysis values as they have a more significant impact on the roll.

- **The one-step model** predicts the ship roll motion in the next step, using the current ship states. The results of the sensitivity analysis for the one-step prediction model are presented in Fig. 4 through the total-effect index S_{Ti}. S_i only measures the impact of a single input variable on the model output, while S_{Ti} accounts for the influence of that variable on the model output, as well as their interactions with the variable. Thus, S_{Ti} is preferable for model input selection. The values of S_{Ti} values for the roll angle and roll velocity are higher than those of other variables; accordingly, the one-step model should use roll angle and roll velocity as input features.
- **The five-step model** predicts the ship roll motion in the next five steps, using the current five-step ship states. Consequently, the model must account for the interrelation in time intervals ranging from one to nine. The largest time interval takes place when predicting the next fifth step by the current time state, with the middle interval being nine. Thus, the total-effect index S_{Ti} of variables is summed from the 1s to the 9s time intervals, as shown in Fig. 5. The sensitivity values for the five-step prediction model do not exhibit noteworthy distinctions when compared with those of the one-step prediction model. To remove the effect of the insignificant input features, the first fifty percent of the parameter values would be considered. This decision is based on the finding that the difference between the eighth-largest value, north 0.555, and the

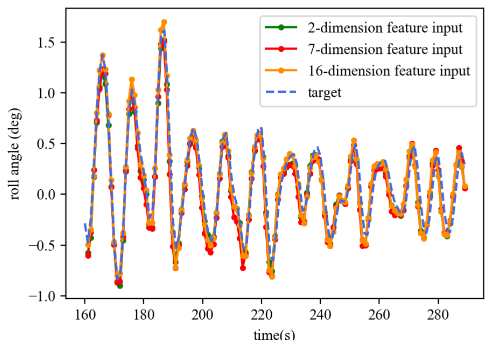

Fig. 7: The result of the one-step ship model prediction.

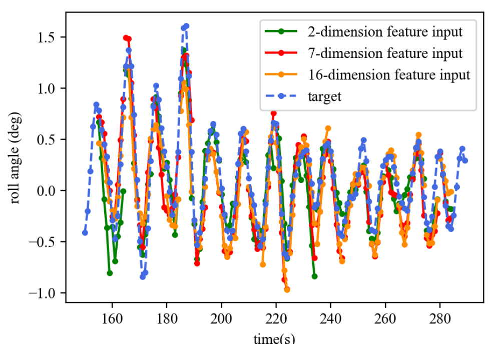

Fig. 8: The result of the five-step ship model prediction.

ninth-largest value, pitch velocity 0.541, is negligible. Thus, all values beyond the eighth are discarded. As a result, the model input includes only the first seven variables, including roll angle, roll velocity, yaw angle, yaw velocity, surge, sway, and east.

- **The ten-step model** predicts the ship roll motion in the ten steps, using the current ten-step ship states, which is similar to the five-step model. So the largest time interval will happen when predicting the next tenth step by the current time state, with the middle interval being nineteen. Thus, the total-effect index S_{Ti} of variables is summed from the 1s to the 19s time intervals, as shown in Fig. 6. It is worth noting that, the S_{Ti} statistical results of the ten-step prediction model has a similar tendency to that of the five-step prediction model. So the chosen model input features include the first seven variables, which are the same as those of the five-step prediction model.

The results of input selection are shown in Table. II. The vessel's dynamic positioning mode and a constant environmental force result in the input to the network being a constant value. The environmental force impacts the network output by influencing changes in other directional forces and moments. It is critical to note that the results of sensitivity analysis obtained under the DP algorithm's influence may not be generalizable to other conditions of predicting roll.

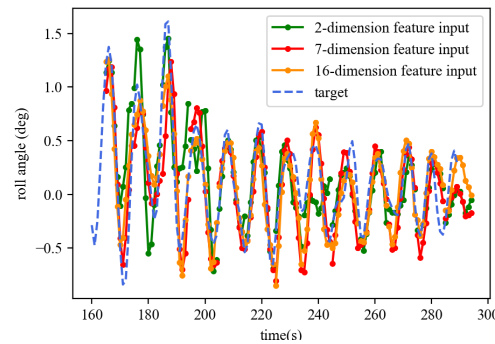

Fig. 9: The result of the ten-step ship model prediction.

TABLE II: The results of input selection

Models	**The Input Features**
The one-step prediction model	roll angle, roll velocity
The five-step prediction model	roll angle, roll velocity, yaw angle, yaw velocity, surge, sway, east
The ten-step prediction model	roll angle, roll velocity, yaw angle, yaw velocity, surge, sway, east

D. *Ship Roll Motion Prediction*

Based on the result of input selection, the one-step, five-step, and ten-step ship roll motion prediction models are established, respectively. The Bi-LSTM model is implemented using Keras deep learning library. The model architecture consists of three layers. The first layer of the model is a Bi-LSTM layer with 128 units, which accepts the input sequence and returns an output sequence of equal length. After the LSTM layer, there is a Dropout layer included, with a dropout rate of 0.2. To predict output for each time step in the sequence, a TimeDistributed Dense layer is added to the model. The model is compiled using the Adam optimizer with the mean squared error loss function. The models with different input feature dimensions are built up to test the effectiveness of input selection.

In Fig. 7, 8, and 9, the Bi-LSTM model is established to predict the ship roll motion in the next one, five, and ten steps, respectively, based on input features in the previous. The green line shows the predicted roll angle of the Bi-LSTM model with two-dimensional input features (roll angle and roll velocity), the input selection results of the one-step ship roll motion prediction model. The red line presents the predicted roll angle of the model with seven-dimensional input features, including roll angle, roll velocity, yaw angle, yaw velocity, surge, sway, and east, the input selection results of the five-step and ten-step ship roll motion prediction model. Additionally, the orange line represents the predicted roll angle of the 16-dimensional input features model, including roll angle, roll velocity, pitch angle, pitch velocity, yaw angle, yaw velocity, surge, sway, heave, north, east, down, wind direction, wind velocity, wave direction, and wave height, which are all the input features of the data.

To evaluate the validity of the model prediction, mean

TABLE III: The error of the prediction models

Error	One-Step Prediction			Five-Step Prediction			Ten-Step Prediction		
	16-dimension	*2-dimension*	*7-dimension*	*16-dimension*	*2-dimension*	*7-dimension*	*16-dimension*	*2-dimension*	*7-dimension*
MAE	0.0512	**0.0459**	0.0473	0.1625	0.1673	**0.1513**	0.1903	0.2156	**0.1806**
MSE	0.0039	**0.0034**	0.0040	0.0454	0.0624	**0.0465**	0.0619	0.1021	**0.0626**
RMSE	0.0623	**0.0587**	0.0629	0.2130	0.2492	**0.2156**	0.2489	0.3196	**0.2502**

absolute error (MAE), mean square error (MSE), and root-mean-square error (RMSE) are adopted as the evaluation criteria. The error results are shown in Table III. It can be found that the performance of the predicted model is superior upon input feature selection. Input features required for different steps predicted networks exhibit variability. Since the five-step prediction and the ten-step prediction produce the same input feature result, one could speculate that the input feature would remain constant as the number of steps increases.

IV. Conclusion

The multi-step ship roll motion prediction model is established based on the Bi-LSTM, including one, five, and ten steps. To improve the accuracy of the prediction model, the input selection is conducted through Sobol sensitivity analysis. Because the sensitivity analysis can be only conducted on the model with the scalar output, the sensitivity analysis of multi-step prediction is separated into that of prediction with different time intervals. The results show the efficiency of the multi-step prediction and input selection. The ability to predict roll motion with a high degree of accuracy can help to enhance the safety and efficiency of maritime operations, reduce the risk of injury and damage, and improve the comfort and well-being of crew and passengers. Efforts for future work will be made to improve the LSTM network to make it more suitable for roll prediction, enhance the accuracy of the short-term roll motion prediction and extend the predicted horizon.

Acknowledgment

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References

[1] Y. Chu, G. Li, L. I. Hatledal, F. T. Holmeset, and H. Zhang, "Coupling of dynamic reaction forces of a heavy load crane and ship motion responses in waves," *Ships and Offshore structures*, vol. 16, no. sup1, pp. 58–67, 2021.

[2] G. Li, H. P. Hildre, and H. Zhang, "Toward time-optimal trajectory planning for autonomous ship maneuvering in close-range encounters," *IEEE Journal of Oceanic Engineering*, vol. 45, no. 4, pp. 1219–1234, 2019.

[3] T. Zhang, X.-Q. Zheng, and M.-X. Liu, "Multiscale attention-based LSTM for ship motion prediction," *Ocean Engineering*, vol. 230, p. 109066, 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0029801821005011

[4] M. V. Clavijo, A. M. Schleder, E. L. Droguett, and M. R. Martins, "RAM analysis of dynamic positioning system: An approach taking into account uncertainties and criticality equipment ratings," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 236, no. 6, pp. 1104–1134, 2022.

[5] T. Manderbacka, N. Themelis, I. Bačkalov, E. Boulougouris, E. Eliopoulou, H. Hashimoto, D. Konovessis, J.-F. Leguen, M. M. González, and C. A. Rodríguez, "An overview of the current research on stability of ships and ocean vehicles: The STAB2018 perspective," *Ocean Engineering*, vol. 186, p. 106090, 2019.

[6] T. Wang, G. Li, B. Wu, V. Æsøy, and H. Zhang, "Parameter identification of ship manoeuvring model under disturbance using support vector machine method," *Ships and Offshore Structures*, vol. 16, no. S1, pp. 13–21, 2021.

[7] Y. Meng, X. Zhang, and J. Zhu, "Parameter identification of ship motion mathematical model based on full-scale trial data," *International Journal of Naval Architecture and Ocean Engineering*, vol. 14, p. 100437, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2092678222000036

[8] N. Patil and S. Rajendran, "A time-domain method for analyzing the ship roll stabilization based on active fin control," *Ocean Systems Engineering*, vol. 11, no. 3, pp. 275–299, 2021.

[9] L. Alzubaidi, J. Zhang, A. J. Humaidi, A. Al-Dujaili, Y. Duan, O. Al-Shamma, J. Santamaría, M. A. Fadhel, M. Al-Amidie, and L. Farhan, "Review of deep learning: Concepts, CNN architectures, challenges, applications, future directions," *Journal of big Data*, vol. 8, pp. 1–74, 2021.

[10] J.-C. Yin, A. N. Perakis, and N. Wang, "A real-time ship roll motion prediction using wavelet transform and variable RBF network," *Ocean Engineering*, vol. 160, pp. 10–19, 2018. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0029801818305523

[11] Y. Wang, H. Wang, D. Zou, and H. Fu, "Ship roll prediction algorithm based on Bi-LSTM-TPA combined model," 2021.

[12] Q. Sun, Z. Tang, J. Gao, and G. Zhang, "Short-term ship motion attitude prediction based on LSTM and GPR," *Applied Ocean Research*, vol. 118, p. 102927, 2022.

[13] Y. Wei, Z. Chen, C. Zhao, Y. Tu, X. Chen, and R. Yang, "A BiLSTM hybrid model for ship roll multi-step forecasting based on decomposition and hyperparameter optimization," *Ocean Engineering*, vol. 242, p. 110138, 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0029801821014591

[14] Y. Liu, W. Duan, L. Huang, S. Duan, and X. Ma, "The input vector space optimization for LSTM deep learning model in real-time prediction of ship motions," *Ocean Engineering*, vol. 213, p. 107681, 2020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0029801820306739

[15] W. Lyu, O. el Moctar, and T. E. Schellin, "Ship motion-sloshing interaction with forward speed in oblique waves," *Ocean Engineering*, vol. 250, p. 110999, 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0029801822004231

[16] Y. Wang, H. Wang, B. Zhou, and H. Fu, "Multi-dimensional prediction method based on Bi-LSTMC for ship roll," *Ocean Engineering*, vol. 242, p. 110106, 2021.

[17] R. Skulstad, G. Li, T. I. Fossen, B. Vik, and H. Zhang, "Dead reckoning of dynamically positioned ships: Using an efficient recurrent neural network," *IEEE Robotics and Automation Magazine*, vol. 26, no. 3, pp. 39–51, 2019.

[18] X. Cheng, G. Li, R. Skulstad, P. Major, S. Chen, H. P. Hildre, and H. Zhang, "Data-driven uncertainty and sensitivity analysis for ship motion modeling in offshore operations," *Ocean Engineering*, vol. 179, pp. 261–272, 2019.