CIVI
MATICS

INTERDISCIPLINARY MATHEMATICAL MODELLING MEETS CIVIC EDUCATION

Co-funded by
the European Union

Erasmus+
Enriching lives, opening minds.

Project information

Project Acronym:	CiviMatics
Project Title:	CiviMatics – Interdisciplinary mathematical modelling meets civic education
Project Number:	2020-1-DE01-KA203-005707
Disclaimer:	This project has been funded with support from the European Commission. This communication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.
National Agency:	Deutscher Akademischer Austauschdienst (DAAD)
Project Website:	https://www.civimatics.eu
Authoring Partner:	Institut für Didaktik der Demokratie, Leibniz University Hannover
Publishing Date:	October 2023
Editors:	Bastian Vajen & Heidi Strømskag
Suggested Citation:	Vajen, B., & Strømskag, H. (Eds.). (2023). *CiviMatics: Interdisciplinary mathematical modelling meets civic education*. The CiviMatics Consortium.
ISBN	978-82-303-6243-3

Consortium

Leibniz University Hannover
Welfengarten 1
30167 Hannover, Germany
Bastian Vajen
b.vajen@ipw.uni-hannover.de

Babeș-Bolyai University
Strada Mihail Kogălniceanu 1
Cluj-Napoca 400000, Romania
Florin Fesnic
fesnic@fspac.ro

Norwegian University of Science and Technology
Høgskoleringen 1
7491 Trondheim, Norway
Heidi Strømskag
heidi.stromskag@ntnu.no

Paderborn University
Warburger Str. 100
33098 Paderborn, Germany
Lara Gildehaus
lara.gildehaus@math.uni-paderborn.de

University of Vienna
Universitätsring 1
1010 Wien, Austria
Nicola Nagy
nicola.nagy@univie.ac.at

TABLE OF CONTENTS

Bastian Vajen | Lara Gildehaus | Heidi Strømskag
Introduction to CiviMatics...5

Heidi Strømskag
Mathematical Modelling Using Study and Research Paths..........................17

Frode Rønning
Mathematical Modelling in a Professional Development Programme......141

Jakob Steinbachner | Nicola Nagy
Is Climate Catastrophe Fair? Fairness, Responsibility and Decolonial Perspectives...152

Bastian Vajen
Interdisciplinary Citizenship Education..208

Lara Gildehaus | Michael Liebendörfer
Implementing Modelling in Teacher Education..264

Florin Fesnic
Stories About Scientific Research..290

Bastian Vajen | Heidi Strømskag | Florin Fesnic
Summary and Outlook...313

CHAPTER 1

Introduction to CiviMatics

Bastian Vajen, Lara Gildehaus, and Heidi Strømskag

with *Yael Fleischmann, Timon Foss-Jähn, Michael Liebendörfer, Nicola Nagy, Frode Rønning, and Jakob Steinbachner*

Aims and Frameworks of CiviMatics

This handbook contains the results of the Erasmus+ project CiviMatics and offers an approach to linking mathematics education and civic education in the field of teacher education and training at the university level. The goal of CiviMatics is to offer educational tools to enhance the competences of future teachers to address complex societal challenges in their classrooms and to combine socio-scientific and mathematical perspectives to help their students understand various aspects connected to these issues. To achieve this, the project focuses on normative modelling, which is the way mathematical modelling and applications of mathematical models shape our reality and influence societal discourses as well as individual and collective behaviour. To exemplify these dimensions of mathematical modelling, various aspects connected to climate change and the human activities furthering it will be used as examples.

The teaching and learning approaches presented in this book as well as the competences connected to them can be linked to the OECD's PISA mathematics framework, aspects of the Sustainable Development Goal 4 and the OECD Learning Framework 2030. The PISA mathematics framework states that mathematical literacy of citizens must take priority over reproducing mathematical techniques or routines (OECD, 2018b, p. 43). In this regard, the PISA mathematics framework defines mathematical literacy as "an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts" (OECD, 2018b, p. 7). In particular, it is important for citizens in the modern world "to reason mathematically and to solve problems and interpret situations in personal, occupational, societal and scientific contexts" as well as "to draw upon certain mathematical knowledge and understanding" (OECD, 2018b, p. 22).

However, part of this mathematical literacy is not to be understood as the subject domain of mathematics, but as a cross-sectional task for education (Weber-Stein & Engel, 2021, p. 166). As mathematical literacy becomes more and more relevant to understand various societal processes due to the progressive mathematisation of society, it becomes more and more connected to other subjects, such as civic education (Mau, 2017, p. 24; Straehler-Pohl, 2017, p. 37; Weber-Stein & Engel, 2021). Consequently, sustainable development goals, such as to ensure "that all learners acquire the knowledge and skills needed to promote sustainable development, including, among others, through education for sustainable development", are intrinsically connected to mathematical competences, as they are essential for describing, understanding, predicting, and communicating various issues connected to climate change (Barwell, 2013, p. 2). Therefore, learning about climate change provides an ideal topic for linking civic education and mathematics, since the topic offers students a field of application for their skills that authentically reaches into the students' lifeworld through the actual effects of mathematical models on political processes (OECD, 2018a, p. 5).

However, before students are able to grasp the connections between the mathematical and societal or political world, it is necessary to also offer teachers insights into these aspects. Thus, this handbook offers various tools to give prospective teachers of mathematics and civic education insight into the workings and principles of the respective subjects and to connect both subjects as part of teacher education. Hence, this handbook will offer an introduction to mathematics education, civic education and mathematical modelling as well as a number of different examples for lectures, seminars or other teaching and learning activities to familiarise future teachers with the links between the two subjects.
In the first chapter, the conceptual principles of the CiviMatics project will be presented. This will start with a brief overview of normative mathematical modelling, which is the cornerstone of this project, negotiating questions about what a model is, what types of models there are, and what relationships exist between classical mathematical modelling and civic education. Based on this, the framework for normative modelling developed in the project and its connection to different subjects is presented. After this theoretical foundation, a detailed description of different examples for a practical implementation as part of teacher education will follow, which include the various approaches in different educational settings from Norway (Chapters 2 & 3), Austria (Chapter 4), Germany (Chapters 5 & 6), and Romania (Chapter 7). These chapters will also provide a didactic commentary with experiences from practice and suggestions for adapting these courses for different educational settings. Additional materials, such as PowerPoint presentations or worksheets used in the seminars can be found in the appendix of the respective chapters as well as on the homepage of the project (https://www.civimatics.eu). The handbook concludes with a summary of its contents and a brief outlook on further research (Chapter 8), which also considers possible applications of the CiviMatics approach outside teacher education at the university level.

Introduction to Mathematical Modelling

Mathematical Modelling

The notion of a model relies on the notion of a system, that is, a reality subject to its own laws. A model is the result of a transformation of a system, usually a simplification, which is supposed to help in generating knowledge about the studied system. In practice, to answer a question relating to a system, one tries to build up a model which is easier, safer, and quicker to study than the system itself. Models are thus used for answering questions or exploring phenomena, possibly guided by research questions. Models always have a descriptive function, but they can also contain statements about what an individual should do, and be used to analyse the system it models. If this is the case, models are referred to as prescriptive, or normative. Depending on the case, a model can thus have descriptive, normative or prescriptive uses. For example, prognostic models (e.g., models of greenhouse gas emissions) are classified as descriptive, but they can be applied in a normative way.

Modelling is understood as the very process of building a model of a system and using it to answer questions about the system at stake. There are different tools for modelling, such as the modelling cycle in the framework of Blum and Leiß (2005) and the Herbartian schema in the Anthropological Theory of the Didactic (ATD, Chevallard, 2019). Insofar as modelling involves valuations, we speak of normative modelling. Models can be prescriptive but non-normative (e.g., a cake recipe), but very often prescriptive models will be normative, because certain actions or outcomes are treated as desirable. It is important to recognise that descriptive models can also be normative if, for example, the descriptive categories used are judgmental ("normal weight") or implicitly value certain actions or outcomes. For example, a model that relates CO_2 emissions in a country to people suggests different consequences than a model that relates CO_2 emissions to economic output. In shorter terms: While a model can be used in a purely descriptive way, that is, as a model of something, as is often the case in physics, for example, it can also be used in a normative way, being a model for something.

Mathematical Modelling in (Higher) Education

Mathematical modelling has become a fixed part of school curricula in numerous countries due to its advantages compared to classical mathematics teaching. Modelling tasks provide an opportunity to connect different types of mathematical knowledge, which classical mathematics instruction often does not accomplish sufficiently, and at the same time combine different competences such as reasoning, modelling, and problem solving that do not have their own place in content-oriented mathematics instruction (Bruder & Krüger, 2018). In particular, modelling tasks also train basic mathematical knowledge and skills that are often not available to students to the desired extent. Furthermore, modelling tasks cannot be solved schematically and can provide a remedy for the widespread problem that students often treat tasks without considering the content, i.e. they only try to extract the numbers and fit them into the currently typical calculation schemes (Bruder & Krüger, 2018).

Despite this great advantage of modelling tasks, modelling processes are mostly given little attention in teacher education and "(basic) competences for teaching mathematical modelling [are] [...] not sufficiently taught" (Borromeo Ferri & Blum, 2018, p. V). This can also be demonstrated in the research of Blum and Leiß (2006), who, when investigating how teachers deal with modelling tasks, found that these involve problems that do not occur in the same way in "classical" mathematics classes. Central to this is the difficulty balancing the independence of students and the intervention of teachers as well as the demanding comparison of results, as often different results to solve a task must be dealt with and different approaches must be discussed and reflected upon (Blum & Leiß, 2006). To be able to successfully incorporate modelling tasks in the mathematics classroom, it is thus important to highlight modelling competences in teacher education and to enable future and current teachers to incorporate relevant tasks in their classrooms.

In the context of structuring the didactic approaches to mathematical modelling, Blum and Borromeo Ferri (2010) described different competences that mathematics teachers need to enable a meaningful handling of modelling tasks in the classroom. In particular, the competence to prepare and conduct reality-based lessons plays a role in the teaching dimension, something that is also demanded outside of modelling processes as a central element of mathematics education to promote the experience of mathematics in everyday life and an opening up of the world through mathematics. Such realistic modelling tasks can also address learner types that are normally less enthusiastic about mathematics (Greefrath et al., 2013). Interdisciplinary approaches offer a useful starting point when seeking a connection to real-world phenomena. Social and economic challenges present an abundance of real-world topics, all of which are well-suited for modelling tasks. These are the kinds of issues typically explored in subjects like civic education or economics.

Link Between Mathematical Modelling and Civic Education

As a central principle, societal problems and their treatment are, as the subject of politics, also the subject of civic education (Goll, 2014, p. 258). Using real problems as the starting point for educational processes enables learners to better understand how democracies deal with challenges and how different viewpoints are negotiated to create possible solutions (Reinhardt, 2018, p. 100). Even if concrete problems and conflicts are constantly changing, their existence represents a fundamental component of democratic societies. Therefore, using societal problems for civic learning processes retains a constant relevance for civic education. However, the increasing complexity of modern societies leads on the one hand to an increasing complexity of problems and possible solutions (Triantafillou, 2020, p. 4). One consequence is a stronger mathematisation of society, which not only leads to new forms of information which influence various decision-making processes, but also reorganises the fundamental conditions of political and societal action (Straehler-Pohl, 2017, p. 37).

One example of this is the problem of climate change. Mathematical processes are central both in describing the problem and in predicting its further development and weighing possible solutions (Barwell, 2013, p. 3). Although the complexity of the problem and the competencies required to understand it exceed the scope of civic education, its analysis and use in the classroom are nevertheless indispensable for civic education. In this respect, it is important for civic education to strive for a stronger interdisciplinary cooperation with other subjects when dealing with complex societal and political problems, such as climate change, and, in this context, to remain open to the contents and didactic principles of other disciplines. An interdisciplinary connection between subjects—such as mathematics and civic education—can be advantageous for both disciplines. For example, mathematical modelling can be used to develop a better understanding of the generation of knowledge about societal problems and the cause-effect relationships of political decisions, while taking a civic perspective on mathematical models can help to better grasp the use of modelling processes and its applicability to the real world.

Normative Modelling and Civic Education

Principles of Civic Education

Although models are used on a regular basis in civic education, didactic discourses about the structure of models as well as their uses for educational processes are lacking. When using models in civic education, the goal is mainly to exemplify political processes and help students to understand their underlying principles and structure. One prevalent model used to exemplify the process of political decision making is the policy cycle. This model describes the policy process as evolving through a sequence of distinct stages. Initially introduced as a normative model in political science aiming to provide an ideal framework for planning and decision making, it has developed into a widely applied framework to organise research on public policy (Jann & Wegrich, 2007). It was also introduced into civic education to help teachers and students grasp real political situations in their complexity, interdependence, and formative elements. The phases of the political cycle, and the categories that influence it, are reformulated into key questions guiding the process of understanding political decision making (Massing, 1995, p. 86). Such categories are for example the division of the political domain into polity (form), policy (process) and politics (content) (Oberle, 2016, p. 25). The policy cycle can be seen as a tool to exemplify the processual structure of political decision-making, which may be influenced by a variety of different variables, but consists of a distinct pattern (Massing, 1995, p. 88). Thus, the use of models such as the policy cycle is twofold: On the one hand, the models should provide students with a simplified, and thus somewhat flawed, picture of reality. On the other hand, they can be used as a tool for analysis, by comparing real political processes with elements of the model and offering a basis for inquiry.

Connected to the use of models such as the policy cycle are often other principles of civic education, such as the problem orientation (Ackermann et al., 2018, pp. 31–33). Problem orientation states that (political) problems and their treatment are, as objects of politics, at the same time objects of civic education (Goll, 2014, p. 258). By taking up political problems (e.g., climate change) and making them the focus of learning about politics, decisions are made about the methodological form of the teaching-learning process. The teaching of problems aims at problem-solving thinking and, if successful, promotes a high degree of judgement competence and political maturity in the learners. To achieve this, Goll (2014) proposes that most approaches have three methodical steps: The analysis of the situation, the discussion of possibilities and the formation of a judgement (p. 263). Such principles of civic education are aimed at helping students acquire the ability to form a political judgement, which represents the core of political education processes (Juchler, 2012, p. 24). The importance of independent political judgement arises from the close connection between the ability to judge and the concept of political maturity. The goal of civic education is to contribute to the development of "political, moral, and ethical autonomy" through political maturity, which as part of self-determination always requires the ability to make independent judgements (Henkenborg, 2012, pp. 28–29). In this context, learners should be enabled to make independent assessments of political, economic, or social issues while weighing different criteria (Reinhardt, 2018, p. 24).

However, the ability to judge cannot be regarded as a stand-alone competency, but has to be integrated into the subject-didactic "triad" of political analysis competence, judgement competence, and action competence (Henkenborg, 2012, pp. 32–34). On the one hand, judgement is therefore dependent on a well-developed analytical competence, since a well-founded judgement appears impossible without penetrating social and political facts and structures; on the other hand, it is also linked to the competence to act, since the rational and independent judgement represents the basis of the political action of democratic citizens. Using societal problems as a topic for civic education processes can help learners understand the causes of such issues and enable them to analyse the political processes that are involved in solving them. Models can be a useful tool to facilitate learning in this context, both regarding the analysis of an issue as well as the political steps that can be taken to solve it. For an issue such as climate change, the understanding of which is dependent on mathematical and political competences, it can be beneficial to combine principles of civic education and mathematics education to enable learners to grasp the issue, analyse possible solutions and act in accordance with their own judgement.

Normative models, which serve as a framework for the interdisciplinary approach to modelling, are approached in two different ways in the context of this project: On the one hand, with the help of a normative modelling cycle developed in the project and, on the other hand, with the help of Study and Research Paths, using the Herbartian schema. These approaches will be briefly presented in the following.

Normative Modelling Cycle

The combination of civic education and mathematics education requires an adaptations of established modelling cycles in order to make political analysis and judgement explicitly visible. Previous representations aim at a mathematical result, which is often checked for its correctness at the end (e.g., by a measurement or an experiment). This unambiguity and verifiability is not given in normative modelling. Our proposal on the methodological level therefore consists of a combination of the steps of political didactic problem-orientation and mathematical modelling, which are made visible in a common model. The basis for this is a modelling cycle that already contains a so-called situation model, that is, a mental representation of the situation (Blum & Leiß, 2005; Borromeo Ferri, 2006, p. 92). Normative modelling, however, requires more, namely a political analysis of the situation (in addition to a mathematical analysis), a discussion of political possibilities, and a judgement formation as subsequent steps. At a minimum, the discussion of political possibilities requires that different possibilities emerge from the mathematical models or that they can be considered from the very beginning. Therefore, the question of selecting models or families of models arises. Policy analysis also requires identification of the interests of involved stakeholders. Neither alternative models nor affected interests emerge on their own. To incorporate these steps, we suggest a new modelling cycle, based on established approaches but adding additional steps for the modelling processes (see Figure 1).

The first step, constructing (1.) does not involve conscious steps, but suggests that in normative modelling we may need to reconcile different conceptions of reality if we are to negotiate solutions in our societies. Simplifying (2.) is one of the most relevant steps. Which parts of the situation model are included at all and how interrelation-ships are simplified essentially determines the result. Here, alternatives have to be considered, their consequences for the model have to be estimated and they have to be classified with regard to political interests. Mathematization (3.) is in itself a technical step, provided that the real model is specified precisely enough. In practice, however, the real model is specified more concretely in this step, so that simplifications similar to those in (2.) are to be expected here as well. The mathematical work (4.) will rarely provide starting points for the political discussion.

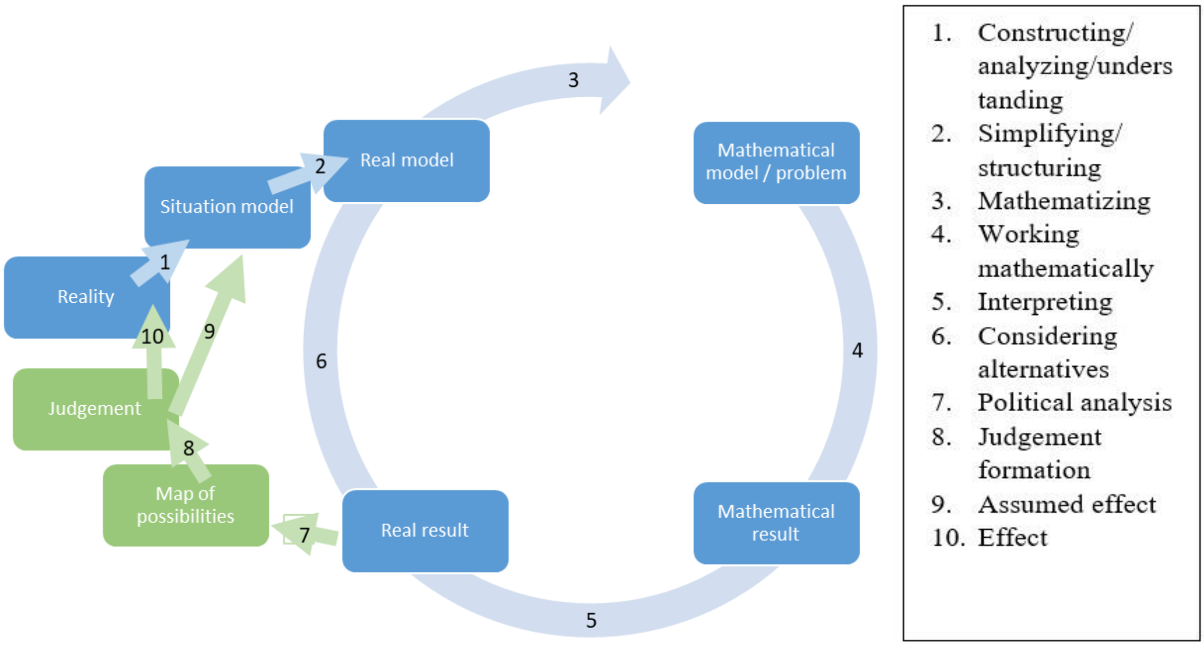
Although alternative actions exist here (e.g., obtaining solutions algebraically or numerically), the differences, if any, should be irrelevant. Interpreting (5.) should also be more of a technical step because it initially involves only the translation of mathematical variables, functions, etc. into reality. However, generalizations could be made at this step, concerning e.g. model assumptions or restrictions of variable ranges, etc. Moreover, the presentation of the results will very often suggest actions, at least implicitly. Such (normative) statements can never be the result of a mathematical calculation and should therefore always be outsourced to the further steps. First, the different possibilities and the different implications related to the stakeholders' interests should be noted through the reflection and critique of the modelling just described. We named that to build a "map of possibilities". After that, everyone

Co-funded by
the European Union

should form their own judgment (8.) by weighing the interests. Finally, it should be acknowledged that decision taken in the classroom might have an impact on the world as we assume it to be at that moment (in terms of our situation model; 9) and as it is (reality; 10).

Figure 1

An interdisciplinary modelling cycle



The presented cycle can be helpful for the creation of models as well as for the analysis of modelling, because it explicitly points out working steps. For example, arguments that one has "recalculated" certain effects are made discussable. Different real results are usually not based on different interpretations (5) or mathematical solutions (4), but partly on mathematisations (3) and especially simplifications (2) as well as perceptions of reality (1), which all have to be discussed explicitly. In the process, it may be possible to identify the interests of the actors concerned, which frame such assumptions.

The Anthropological Theory of the Didactic and Study and Research Paths

The Notion of Praxeology

The *anthropological theory of the didactic* (ATD) postulates that any activity related to the production, diffusion, or acquisition of knowledge should be interpreted as an ordinary human activity, and thus proposes a general model of human activities built on the notion of *praxeology*. This is a key notion in the ATD, explained like this:

> A praxeology is, in some way, the basic unit into which one can analyse human action at large. […] What exactly is a praxeology? We can rely on etymology to guide us here – one can analyse any

> human doing into two main, interrelated components: praxis, i.e. the practical part, on the one hand, and logos, on the other hand. "Logos" is a Greek word which, from pre-Socratic times, has been used steadily to refer to human thinking and reasoning – particularly about the cosmos. […] One fundamental principle of the ATD [states that] no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. […] Of course, a praxeology may be a bad one, with its "praxis" part being made of an inefficient technique – "technique" is here the official word for a "way of doing" – and its "logos" component consisting almost entirely of sheer nonsense – at least from the praxeologist's point of view! (Chevallard, 2006, p. 23).

A praxeology in the ATD is a unit composed of four components (Chevallard, 2019): T, τ, θ and Θ (sometimes referred to as "the four t-s"). T (Latin capital letter t) is a *type* of tasks, τ (Greek tau) is a technique (or a set of techniques) to solve the tasks, θ (Greek theta) is a *technology* (i.e., a discourse) to describe and explain each technique, and Θ (Greek capital theta) is a theory that justifies the technology. T and τ belong to the praxis block of a praxeology, whereas θ and Θ belong to the logos block. A praxeology $\wp$ is thus written: $\wp = [T / \tau / \theta / \Theta]$. *A priori* praxeological analyses are important for classroom experiments, where praxeological models of the knowledge at stake are instrumental in designing interventions to be implemented in the classroom. Praxeological analyses can also be done *a posteriori* to analyse how a praxeology has been built up during the solution of a problem (see e.g., Strømskag, 2021).

From Knowledge "Visits" to Dynamic World Inquiries: A Paradigm Shift

The prevailing didactic paradigm, which we may refer to as the paradigm of *visiting works*, is fundamentally rooted in the notion that there are specific bodies of knowledge, or curricula, which bear significant social relevance (Chevallard, 2015). Within this framework, learners engage primarily with selected praxeologies, and often do so without truly grasping the raisons d'être underpinning them. While this paradigm is not strictly synonymous with a teacher-centric approach, it tends to prioritise the foundational praxeologies over the activities and challenges that might otherwise contextualise them. This is particularly evident in how mathematical knowledge is presented—as a polished, final product. The intrinsic motivations or initial questions that paved the way for its evolution are frequently pushed to the sidelines. Such an approach culminates in what Chevallard (2015) describes as the "monumentalisation" of the curriculum. Here, mathematical entities are held in reverence, almost to the extent of being sacrosanct, with little room for inquiry or contestation. The potential pitfall of this paradigm is that it might render the curriculum as something distant, perhaps even intangible, to students.

On the other hand, the paradigm of *questioning the world* offers a more encompassing pedagogical strategy. The foundational methodological component underpinning the paradigm of questioning the world is the notion of *Study and Research Path* (SRP) based on the so-called *Herbartian schema* (Chevallard, 2019):

$$[S(X, Y, Q) \Rightarrow M] \Rightarrow A^{\heartsuit}$$

Here, the didactic system $S(X, Y, Q)$ is not formed around a given praxeology to be studied, but around a question Q to which X (the students), with the help of Y (the teacher/teachers), has to provide an answer $A^{\heartsuit}$. The study of Q generates an inquiry process involving a didactic milieu M made up of different types of objects or tools for the inquiry:

$$M = \{A_1^{\lozenge}, A_2^{\lozenge}, \ldots, A_m^{\lozenge}, W_1, W_2, \ldots, W_n, Q_1, Q_2, \ldots, Q_p, D_1, D_2, \ldots, D_q\},$$

where the components of M signify the following: $A_i^{\lozenge}$ are existing answers to Q found in the literature and elsewhere; W_j are all types of work that must be used in order to study and understand all the other components of M; Q_k are questions generated by the study of Q and the other components in M; and D_l are datasets that are collected through various types of research during the study of Q.

In the Herbartian schema, the concept of "visiting works" remains but is driven by the need to find productive answers, $A_i^{\lozenge}$, even if it means delving into vast knowledge domains with expert guidance. The motivation behind such visits is not the prestige of $A_i^{\lozenge}$ but its utility in forming $A^{\heartsuit}$. The Herbartian schema outlines the key components of the inquiry process. The dynamics of such a process are articulated through various dialectics, with Bosch (2018) highlighting three as especially significant:

- *Question-Answer Dialectic*: this embodies the iterative essence of research, where answers to initial questions spark further questions, necessitating a deeper exploration into the subject;
- *Media-Milieu Dialectic*: here, a distinction is drawn between the media, which propagates messages, and the milieu, a system devoid of any intention with respect to the question studied. For a message to gain credence, it must endure the scrutiny of the milieu, affirming its authenticity and relevance;
- *Individual and Collectivity Dialectic*: this emphasises the balance between individual contributions and collective aspirations in the research process.

The new paradigm is based on three principles related to curricula (Chevallard, 2018). Firstly, every human community has duties towards its members. An essential duty is that of defining and implementing a community curriculum to ensure that all members of the community are *enabled to think and act appropriately*, in a way beneficial to themselves and to others, in the different social worlds (in particular the worlds of family, profession, and citizenship) in which they are or will be led to live. This aligns with Lange's (2008) emphasis on the competence area of social learning. Similarly, Print (2013) identifies competences essential for democratic citizenship. These competences encompass beliefs in social justice, equality, and the equal treatment of all citizens. Furthermore, they also include the skills necessary for coalition building, cooperation, and the ability to thrive in a multicultural environment.

Secondly, the curriculum within the community should empower its members, either as individuals or in collective groups, to discern, articulate, and address the questions they encounter. This relates to democratic competences like the ability to critically assess information, evaluate stances or decisions, adopt a viewpoint, and substantiate that position, as outlined by Print (2013). Thirdly, to achieve this goal, the community shall define (and revise regularly) a curriculum *core made up of questions* that members of the community "have the right not to be allowed to avoid" (Gagnon, 1995, p. 72).

In summary, the paradigm of visiting works offers a structured approach, prioritising established bodies of knowledge, which can sometimes risk making the curriculum seem fixed or unrelatable to students. On the other hand, the paradigm of questioning the world leans towards fostering inquiry and critical reflection, placing emphasis on the journey of discovery rather than just the destination. This latter approach may present education as more interactive and dynamic.

Practical Applications

The ability to make autonomous political judgments, which is a central component in civic education, is dependent on extensive analytical competence, because without an understanding of social and political issues, a well-founded judgement seems impossible. Complex issues and their solutions, like climate change, require an under-standing of mathematics and civics to properly analyse them and form an informed judgement. Mathematics education and civic education offer a high potential for cooperation in the teaching of complex societal problems due to similar objectives and the reference to comparable concepts. Practical examples of such an interdisciplinary approach will be provided in the next chapter.

References

Ackermann, P., Breit, G., Cremer, W., Massing, P., & Weinbrenner, P. (2018). *Politikdidaktik kurzgefasst: 13 Planungsfragen für den Politikunterricht*. *Wochenschau Politik*. Schwalbach/Ts.: Wochenschau-Verl.

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, *15*(1), 1–16.

Beck, U. (2015). Emancipatory catastrophism: What does it mean to climate change and risk society? *Current Sociology*, *61*(1), 75–88.

Blum, W., & Leiß, D. (2006). Filling up – In the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education* (pp. 1623–1633). Barcelona: Universitat Ramon Llull and ERME.

Blum, W., & Borromeo Ferri, R. (2010). Mathematical modelling in teacher educatio: Experiences from a modelling seminar. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *CERME 6: Proceedings of the sixth congress of the European Society for Research in Mathematics Education, January 28th-February 1st 2009 Lyon (France)* (pp. 2046–2055). Institut national de recherche pédagogique.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM – Mathematics Education*, *38*, 86–95.

Borromeo Ferri, R., & Blum, W. (2018). Vorwort. In R. Borromeo Ferri & W. Blum (Eds.), *Lehrerkompetenzen zum Unterrichten mathematischer Modellierung* (pp. V–IX). Springer Fachmedien Wiesbaden.

Bosch, M. (2018). Study and research paths: A model for inquiry. In B. Sirakov, P. N. de Souza, & M. Viana (Eds.), *International Congress of Mathematicians* (Vol. 3, pp. 4001–4022). Singapore: World Scientific Publishing. https://doi.org/10.1142/9789813272880_0210

Bruder, R., & Krüger, U.-H. (2018). Lerngelegenheiten für Mathematisches Argumentieren, Modellieren und Problemlösen (LEMAMOP). In R. Biehler, T. Lange, T. Leuders, B. Rösken-

Winter, P. Scherer, & C. Selter (Eds.), *Mathematikfortbildungen professionalisieren* (pp. 225–248). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-19028-6_12

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education* (pp. 22–30). Barcelona: Universitat Ramon Llull and ERME.

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Cham: Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Chevallard, Y. (2018). The curricular conundrum in the light of the ATD. In Y. Shimizu & R. Vithal (Eds.), *The Twenty-fourth ICMI Study: School Mathematics Curriculum Reforms: Challenges, Changes and Opportunitie* (pp. 213–220). Tsukuba: University of Tsukuba.

Chevallard, Y. (2019). Introducing the Anthropological Theory of the Didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves%20Chevallard.pdf

Douglas, K. M., & Sutton, R. M. (2015). Climate change: Why the conspiracy theories are dangerous. *Bulletin of the Atomic Scientists*, *71*(2), 98–106.

Gagnon, P. (1995, December). What should children learn? *The Atlantic*. Retrieved from https://www.theatlantic.com/magazine/archive/1995/12/what-should-children-learn/376491/

Goll, T. (2014). Problemorientierung. In W. Sander (Ed.), *Reihe Politik und Bildung: Band 69. Handbuch politische Bildung* (4th ed., pp. 258–266). Schwalbach/Ts.: Wochenschau Verlag.

Greefrath, G., Kaiser, G., Blum, W., & Borromeo Ferri, R. (2013). Mathematisches Modellieren – Eine Einführung in theoretische und didaktische Hintergründe. In R. Borromeo Ferri, G. Greefrath, & G. Kaiser (Eds.), *Mathematisches Modellieren für Schule und Hochschule* (pp. 11–37). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-01580-0_1

Henkenborg, P. (2012). Politische Urteilsfähigkeit als politische Kompetenz in der Demokratie: Der Dreiklang von Analysieren, Urteilen Handeln. *Zeitschrift Für Didaktik Der Gesellschaftswissenschaften*, *3*(2), 28–50.

Jann, W., & Wegrich, K. (2007). Theories of the Policy Cycle. In F. Fischer & G. Miller (Eds.), *Public Administration and Public Policy. Handbook of Public Policy Analysis: Theory, Politics, and Methods* (pp. 43–62). London: Routledge.

Juchler, I. (2012). Politisches Urteilen. *Zeitschrift Für Didaktik Der Gesellschaftswissenschaften*, *3*(2), 10–27.

Kant, I. (2015). *Die drei Kritiken: Kritik der reinen Vernunft (1781/87) ; Kritik der praktischen Vernunft (1788) ; Kritik der Urteilskraft (1790)*. Köln: Anaconda.

Lange, D. (2008). Citizenship Education in Germany. In V. B. Georgi (Ed.), *Schriftenreihe / Bundeszentrale für Politische Bildung: Vol. 666. The making of citizens in Europe: New perspectives on citizenship education* (pp. 89–95). Bonn: Bundeszentrale für Politische Bildung.

Massing, P. (1995). Wege zum Politischen. In P. Massing & G. Weißeno (Eds.), *Schriften zur politischen Didaktik: Vol. 24. Politik als Kern der politischen Bildung: Wege zur Überwindung unpolitischen Politikunterrichts* (pp. 61–98). Wiesbaden: VS Verlag für Sozialwissenschaften.

Mau, S. (2017). *Das metrische Wir: Über die Quantifizierung des Sozialen*. Suhrkamp Verlag.

Michelsen, G., & Overwien, B. (2020). Nachhaltige Entwicklung und Bildung. In P. Bollweg, J. Buchna, T. Coelen, & H.-U. Otto (Eds.), *Handbuch Ganztagsbildung* (pp. 557–574). Wiesbaden: Springer Fachmedien Wiesbaden.

Niss, M. (1983). Considerations and experiences concerning integrated courses in mathematics and other subjects. In M. J. Zweng, T. Green, J. Kilpatrick, H. O. Pollak, & M. Suydam (Eds.), *Proceedings of the Fourth International Congress on Mathematical Education* (pp. 247–249). Boston: Birkhäuser.

Oberle, M. (2016). Politikwissenschaft als Bezugsdisziplin der Politischen Bildung. In M. Oberle & G. Weißeno (Eds.), *Politische Bildung. Politikwissenschaft und Politikdidaktik: Theorie und Empirie* (pp. 17–29). Wiesbaden: Springer Fachmedien Wiesbaden.

OECD. (2018b). *PISA 2022 mathematics framework (draft)*. Paris. https://pisa2022-maths.oecd.org/files/PISA%202022%20Mathematics%20Framework%20Draft.pdf

OECD. (2018a). *The future of education and skills: Education 203*. Paris. https://www.oecd.org/education/2030/E2030%20Position%20Paper%20(05.04.2018).pdf

Ojose, B. (2011). Mathematics Literacy: Are We Able To Put The Mathematics We Learn Into Everyday Use? *Journal of Mathematics Education*, *4*(1), 89–100.

O'Neill, J. (2002). The rhetoric of deliberation: Some problems in Kantian theories of deliberative democracy. *Res Publica*, *8*, 249–268.

Print, M. (2013). Competencies for democratic citizenship in Europe. In M. Print & D. Lange (Eds.), *Civic and political education: Vol. 3. Civic education and competences for engaging citizens in democracies* (pp. 37–51). Rotterdam: Sense Publishers.

Reinhardt, S. (2013). Teaching for democratic learning. In M. Print & D. Lange (Eds.), *Civic and political education: Vol. 3. Civic education and competences for engaging citizens in democracies* (pp. 99–109). Rotterdam: Sense Publishers.

Reinhardt, S. (2018). *Politik Didaktik: Handbuch für die Sekundarstufe I und II*. *Fachdidaktik*. Berlin: Cornelsen.

Straehler-Pohl, H. (2017). De|mathematisation and Ideology in Times of Capitalism: Recovering Critical Distance. In H. Straehler-Pohl, N. Bohlmann, & A. Pais (Eds.), *The Disorder of Mathematics Education* (pp. 35–52). Springer. https://doi.org/10.1007/978-3-319-34006-7_3

Strømskag, H. (2021). Prospective teachers solving a percentage problem: An analysis of the construction of a praxeology. In B. Barquero, I. Florensa, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Extended Abstracts Spring 2019. Trends in Mathematics 13* (pp. 77–86). Cham: Birkhäuser. https://doi.org/10.1007/978-3-030-76413-5_9

Triantafillou, P. (2020). Trapped in the Complexity Bowl? Public Governance and the Liberal Art of Governing. *International Journal of Public Administration*, *43*(14), 1228–1236.

Weber-Stein, F., & Engel, J. (2021). Civic Statistical Literacy und politische Bildung im Informationszeitalter. Kooperative statistik- und politikdidaktische Erkundungen im Feld der Demokratiemessung. In C. Deichmann & M. Partetzke (Eds.), *Demokratie im Stresstest* (pp. 165–192). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-33077-4_10

Zweng, M. J., Green, T., Kilpatrick, J., Pollak, H. O., & Suydam, M. (Eds.). (1983). *Proceedings of the Fourth International Congress on Mathematical Education*. Boston: Birkhäuser. https://doi.org/10.1007/978-1-4684-8223-2

CHAPTER 2

Mathematical Modelling Using Study and Research Paths

Heidi Strømskag

Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, Norway

Introduction

Mathematical Modelling Using Study and Research Paths (MA3001, n.d.) is a 7.5 ECTS master's course in mathematics education at the Norwegian University of Science and Technology in Trondheim (NTNU). It is designed for students enrolled in the master's programme "Natural Science with Teacher Education", geared towards Grades 8–13. Inquiries involving mathematical modelling are the focus of the course, aiming to answer generating questions. The inquiries are shaped in a new didactic paradigm—that of *questioning the world* (Chevallard, 2015)—, which is rooted in the *anthropological theory of the didactic*, the ATD (Chevallard, 2019). This framework introduces a unique methodology for questioning the world, referred to as *Study and Research Paths* (SRPs).

The course involves two inquiries structured as SRPs, both focusing on *climate change*. In addition, the course encompasses several other inquiries that explore various systems, including the thermal insulation capabilities of two brands of thermoses, the likelihood of having a disease in the event of a positive test result, and the braking distance of a vehicle suddenly braking on a horizontal surface, among others. In teacher education, an SRP is a tool with a dual purpose. Firstly, student teachers shall become aware of the raisons d'être of the knowledge they must become sufficiently acquainted with in order to study questions. Secondly, it endeavours to equip them with key capacities to employ SRPs as a didactic tool, particularly when guiding pupils in the study of open modelling questions.

The author of this chapter served as the lecturer of the course in 2022 when it was first launched. Nine students completed the course, all of whom had mathematics as their main subject and either physics (5), computer science (2), biology (1), or chemistry (1) as their second subject in the programme (with the number of students who had each subject shown in parentheses). This chapter provides a broad account of the inaugural run of the course.

Infrastructure of the Course

Knowledge Goals

These are the interrelated knowledge goals of the course:

- Mathematical modelling of systems: the notions of system and model; design of modelling tasks.
- Elementary algebra as a modelling tool: formulas as algebraic models; formulas as equations with parameters.

- The anthropological theory of the didactic: the questioning of the world paradigm; study and research paths; the Herbartian schema.

Upon successful completion of the course, the student will be capable of modelling systems with parameters and have skills in addressing questions about these systems using both elementary algebra and additional modelling tools. Furthermore, the student will be capable of crafting modelling tasks for secondary school, concentrating on the interplay between systems and models. Next, the student will have the competence to explore generating questions through inquiries formatted as SRPs, underpinned by principles and tools derived from the ATD.

Teaching Methods and Study Activities

The course has different teaching and study formats: lectures, exercise classes, inquiries in terms of SRPs, and seminars. The lectures are given to initiate study of theoretical knowledge in mathematics as well as mathematics education, including methods of inquiry and modelling. The exercise classes are structured in a way to facilitate the discussion of tasks or problems, as provided by the lecturer, that the students have already worked on and provided answers to. The SRPs are organised as follows: The lecturer, serving as the supervisor of the class's inquiries, introduces a generating question, Q. Students collaborate in teams to study and answer Q. Regular meetings take place between the class and the lecturer to review the class's progress on Q, and to verify or recalibrate the direction of the inquiries. Seminars are also convened, wherein the teams present interim reports and gain feedback on their developing answers from both other teams and the lecturer. These seminars also serve as a platform to potentially receive suggestions on references they may find useful in their ongoing investigation.

The following activities are mandatory: participation in lectures and exercise classes; implementation and oral presentation of an $\text{SRP}_{\text{pilot}}$; execution of an SRP; composition and oral presentation of a halfway SRP report. Additionally, students are required to provide feedback on another team's halfway SRP report, write a final SRP report, and design a poster based on the conducted SRP.

Requirements, Syllabus, and Assessment

The course requires admission to the study programme "Natural Science With Teacher Education" at NTNU and a minimum of 60 ECTS in mathematics. Mandatory reading applicable to all students is specified at the onset of the semester. In addition to this shared literature, each team will need to utilise resources specifically selected for their teams' inquiries; these are considered obligatory for each respective team. For further details about the common required reading, refer to Appendix A.

The course utilises two types of assessment. Formative assessment is carried out at two stages: first, by fellow students' and the lecturer's feedback on a preliminary (halfway) SRP report; second, by the lecturer's written feedback on a draft of the final SRP report. Summative assessment is carried out in terms of an individual oral examination, where the student brings a poster designed by the team, based on the conducted SRP. The grade scale for the oral examination consists of letters (A–F).

Course Roadmap: Structure and Content with Materials

Unit 1

Title	**A new didactic paradigm and modelling of systems using algebra as a tool**	
Duration[1]	6 hours.	
Organisation	5 × 45 min [Lectures] + 3 × 45 min [Exercise classes].	
Literature	- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer. - Niss, M. (2015). Prescriptive modeling: Challenges and opportunities. In G. Stillman et al. (Eds.), *Mathematical modeling in education research and practice: Cultural, social and cognitive influences* (pp. 67–79). Springer. - Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques, 42*(3), 371–409.	
Topics	- The paradigm of questioning the world - Modelling of systems - Elementary algebra as a modelling tool	
Questions	- What is the thermal insulating capacity of two brands of thermoses? - How can we determine the probability that a person, who has tested positive for a disease, indeed has the disease?	
Materials	PowerPoints	- Introduction (Appendix C1) - Elementary algebra as a modelling tool (Appendix C2) - The ATD and the notions of model and system (Appendix C3) - The ATD and a new didactic paradigm (Appendix C4)
	Tasks	- Modelling of insulating capacity of thermoses (Appendix D1) - Proposed solution to task on insulating capacity of thermoses (Appendix D2) - Modelling of probability (Appendix D3)

Unit 2

Title	**Some tools from the anthropological theory of the didactic**
Duration	3 hours.
Organisation	2 × 45 min [Lecture] + 2 × 45 min [Exercise classes].
Literature	- Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. - Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education, 12*, 71–114.

[1] For each of the five units, the time spent on homework is not counted in the "Duration" column. However, time allocated for team work on the SRPs is included in "Duration", and further details are provided in the "Organisation" column.

	- Markulin, K. et al. (2021). Project-based learning in statistics: A critical analysis. *Caminhos da Educação Matemática em Revista*, *11*(1), 200–220.	
Topics	- Didactic system - Study and research path - Herbartian schema	
Question	How do study and research paths differ from problem- / project-based learning?	
Materials	PowerPoint	- SRPs and the Herbartian schema (Appendix C5)
	Task	- Comparison of SRPs and problem- / project-based learning approaches (Appendix D4)

Unit 3

Title	**Climate change**	
Duration	5.25 hours.	
Organisation	1 × 45 min [Lecture] + 4 × 45 min [$\text{SRP}_{\text{pilot}}$] + 2 × 45 min [Presentation and discussion].	
Literature	Literature selected by each team during their SRP	
Topic	Basic knowledge on climate change	
Generating question	What is climate change, and why is it happening?	
Materials	PowerPoint	- Pilot SRP on climate change by a student group (Appendix C6)
	Task	- A small-scale inquiry into climate change (Appendix D5)

Unit 4

Title	**Carbon capture and storage**	
Duration	30 hours.	
Organisation	6 × 45 min [Lectures and question time[2]] + 30 × 45 min [SRP] + 4 × 45 min [Presentation and discussion].	
Literature	- Strømskag, H. (2022, 21 January). *A note on the Herbartian schema: A dynamic model for a study of a generating question*. Dep. of Mathematical Sciences, NTNU. - Literature selected by each team during their SRP	
Topics	- Models constructed by scientists on CCS - Parameters and their interrelationships	
Generating question	How is carbon capture and storage modelled in the literature? What mathematics is involved in these models? Which parameters are included, and what are the relationships between them?	
Materials	PowerPoints	- Some background to carbon capture and storage (Appendix C7) - Some aspects related to SRPs in teacher education (Appendix C8)
	Tasks	- Inquiry into carbon capture and storage (Appendix D6) - Hydrogeological modelling (Appendix D7)

[2] "Question time" refers to a teaching format wherein the lecturer provides prepared answers to questions that have been submitted in advance.

Co-funded by
the European Union

Unit 5

<table>
<tr><td>Title</td><td colspan="2">The role of modelling in secondary school and design of modelling tasks</td></tr>
<tr><td>Duration</td><td colspan="2">3.75 hours.</td></tr>
<tr><td>Organisation</td><td colspan="2">3 × 45 min [Lectures and question time] + 2 × 45 min [Exercise classes].</td></tr>
<tr><td>Literature</td><td colspan="2">Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. Recherches en Didactique des Mathématiques, 42(3), 371–409.</td></tr>
<tr><td>Topics</td><td colspan="2">- Systems in the natural and the social world
- Instruction of mathematical modelling in secondary school</td></tr>
<tr><td>Question</td><td colspan="2">In which ways can modelling tasks be structured to help students gain knowledge about the systems involved?</td></tr>
<tr><td rowspan="3">Materials</td><td>PowerPoint</td><td>- Modelling of systems and design of modelling tasks (Appendix C9)</td></tr>
<tr><td>Manuscript</td><td>- Note on the dialectic of systems and models (Appendix B)</td></tr>
<tr><td>Tasks</td><td>- Modelling of the Celsius-Fahrenheit relationship (Appendix D8)
- Materials for an answer to the modelling of temperature scales (Appendix D9)
- Modelling of braking distance of a vehicle (Appendix D10)
- Design of modelling tasks for Grades 8–13 (Appendix D11)</td></tr>
</table>

Theoretical Framework

Key Principles of the Anthropological Theory of the Didactic

The course addressed in this chapter is fundamentally rooted in the Anthropological Theory of the Didactic (the ATD), a research framework developed since the 1980s, mainly by Yves Chevallard. This section provides a concise summary of the core concepts of the ATD. The presentation primarily draws upon Chevallard's 2019 article, "Introducing the Anthropological Theory of the Didactic: An attempt at a principled approach". For a more condensed overview of the ATD, one can also refer to Chevallard and Bosch (2019).

The ATD concerns the *didactic*, meaning that the spotlight is on specific actions that individuals or institutions perform or decide, with the potential effect that someone learns something. To study such actions, the concept of *praxeology* has been developed as an analytical tool. It begins with the idea of a *type of tasks*[3] that can be solved using certain *techniques*—task type and technique make up the *praxis* block (knowhow) of a praxeology. Further, every technique requires an explanation or discourse, called *technology*, which in turn has a justifying discourse at a higher level, called *theory*—technology and theory make up the *logos* block (knowledge) of a praxeology. A praxeology is thus a model both of the way we perform, and of the way we explicate and justify, our actions.

The ATD includes a theory of cognition, which proposes that a person has learned something about an object if the person's relation to the object has changed. (Objects are material and non-material components of the world we describe.) Within the framework of the ATD, Chevallard (2015) puts

[3] Solving a quadratic equation, creating a semester plan for a course, or shovelling snow are examples of different types of tasks.

forward a new didactic paradigm, called the “paradigm of questioning the world”. This paradigm presumes a break with the prevailing didactic paradigm, known as the “paradigm of visiting works”, where (in the latter) mathematics teaching is based on the “transmission” of already existing knowledge, followed by students using it to solve some relatively standardised tasks. The knowledge being taught has in this way lost its raison d’être, namely the questions it contributes to answer. The mathematics being studied is then often perceived by the students as decontextualised and purposeless. In the new didactic paradigm, students are given the opportunity to develop genuine knowledge—genuine in the sense that they should also know the purpose and utility of the knowledge. This is the ethos for teaching and learning in the course in question.

Study and Research Paths and the Herbartian Schema

The methodology developed in the ATD to operate in the new paradigm concerns a type of study and research process created by a generating question, where there is a dialectic between research and study (Bosch, 2018). “Research” refers to investigations or problem-solving, while “study” denotes consultation of existing (and available) knowledge that is initiated not only by the teacher but also by the students. The path created in the execution of a study and research process is referred to as a “path”—hence the term *study and research path* (SRP).

A praxeological function that inquiring into a question Q assumes is to lead to studying all sorts of works (including derived questions Q_i). How can we describe what happens in a didactic system $\mathcal{S}$ when a class X studies a question Q under the supervision of teacher(s) Y? The model provided by the ATD is the *reduced Herbartian schema*: $S(X, Y, Q)$ å A (Chevallard, 2019).[4] Here, A is the answer to the question Q that the didactic system is expected to produce. The answer A is usually written with a heart ♥ in superscript: $S(X, Y, Q)$ å $A^{♥}$—the heart hints at the fact that this answer will be “at the heart” of the didactic system, the “authorised” answer to question Q (at least for some time).

In the following, I explain how the reduced Herbartian schema is developed into a (full-fledged) Herbartian schema. The first step involves introducing the didactic milieu, M, which consists of the material and non-material tools that the class collects to conduct its study of the question Q. The reduced Herbartian schema then becomes the semi-reduced Herbartian schema:

[$S(X, Y, Q)$ ➡ M] ➥ $A^{♥}$.[5]

This is understood in such a way that the didactic system creates the milieu M and generates the answer $A^{♥}$ with the help of the milieu denoted by

$$M = \{A_1^{\Diamond}, A_2^{\Diamond}, \ldots, A_m^{\Diamond}, W_1, W_2, \ldots, W_n, Q_1, Q_2, \ldots, Q_p, D_1, D_2, \ldots, D_q\},$$

comprising the following elements:

- $A_i^{\Diamond}$ are existing answers—found in the literature and multimedia resources—that are provided by other persons or institutions. $A_i^{\Diamond}$ is read as “A_i diamond”, where the diamond signifies the “brand” of an institution or person. Therefore, a teacher (in direct instruction), a textbook, or a webpage are institutions that, in reality, “brand” their answers to the questions they tackle.

[4] The adjective *Herbartian* refers to the German philosopher and pedagogue Johann Friedrich Herbart (1776–1841).

[5] The arrow ➡ is read “creates”, and the arrow ➥ is read “generates”.

- W_j are all types of work (theories, experimental plans, historical studies, reports, etc.) that must be used in order to study and understand all the other components of M. To be able to use these works, they must be studied. Studying a work W_j (which is not a question itself) consists of formulating and studying a series of derived questions Q_k to investigate W_j.
- Q_k are questions generated by the study of Q and all the other components in M. Thus, the study of any component in M boils down to studying questions. These derived questions depend on the generating question Q and on the direction the investigation of Q evolves in.
- D_l are datasets that are collected through various types of research during the study of Q.

An SRP will produce partial answers, A_r. These will be part of a "stock taking" that results in the final (though provisional) answer $A^{\heartsuit}$. Therefore, the complete Herbartian scheme, which is a dynamic model of the SRP, is symbolised as follows:

$$[S(X, Y, Q) \Rightarrow \{A_1^{\Diamond}, A_2^{\Diamond}, \ldots, A_m^{\Diamond}, W_1, W_2, \ldots, W_n, Q_1, Q_2, \ldots, Q_p, D_1, D_2, \ldots, D_q\}] \Rightarrow A^{\heartsuit}.$$

Numerous inquiries in the format of SRPs have been carried out and published across various educational levels and countries. Examples include upper secondary schools in Denmark (Jessen, 2017, 2019) and France (Bourgade et al., 2020), teacher education programmes in Mexico and Spain (Barquero et al., 2018; Barquero et al., 2019), as well as Norway (Strømskag, forthcoming). Further examples include engineering education in Spain (Bartolomé et al., 2018, 2021; Amer et al., 2022).

Mathematical Modelling in the Course

The term *model*, as used in the course, is based on the notion of *system*, where a system is anything that has a reality subject to its own laws. A (geometric) sphere is an example of a system; the spread of a virus in a population is another example. Let S be a system. We say that S' is a model of S if, by studying the model S', we can answer certain questions about the system S. One tries to build a model S' of S, which makes it easier, safer, and faster to answer the questions about the system S, by studying the model S' rather than by studying S "directly". Strømskag and Chevallard (2022) have outlined four principles for using elementary algebra when modelling phenomena and objects in the natural and social world:

- Students start from a system S and a question Q raised about it, which appears to require mathematical elements for an adequate treatment.
- They develop a model S' of S in relation to question Q, which is built with elementary algebra and incorporates as many parameters as necessary.
- The students engage with S' to derive an answer considered adequate for Q.
- Concurrently, inspired by this process of inquiring about S, the students uncover the resources of algebra, and study or restudy them to effectively utilise the tools thus acquired.

In the course under consideration, there are two distinct operations that pertain to mathematical modelling: one involves students' own construction of models of systems, while the other involves students' examination of models of systems, created by scientists in various fields. In both instances, the objective is to comprehend systems; however, only in the former case are students engaged in the modelling process themselves. When students model systems, they generate mathematical models that facilitate their ability to respond to inquiries regarding the systems in question. An example of a task of this type is "Modelling of the probability of having a disease given a positive test result". A solution demonstration is presented in the subsequent section.

On the other hand, when students investigate models crafted by scientists, they study relationships between parameters in those models, employing knowledge in mathematics and other fields, either already mastered or essential to acquire during the investigation. Prior to the main SRPs on scientists' models in the field of climate change, pilot SRPs were conducted to achieve two objectives: generating broad knowledge about climate change and gaining experience with a small-scale SRP.[6] The generating question for the main SRPs was about carbon capture and storage (CCS), a technology involving capture of carbon dioxide (CO_2) from industrial sources and power plants, and storage of it in geological formations under the seabed.

The subsequent two sections present digests of inquiries conducted by students in the course, showcasing the two modelling operations.

Inquiry 1: Probability of Having a Disease Given a Positive Test Result

The inquiry presented in this section concerns students' own mathematical modelling.

The System to be Modelled

The starting point is a system, *S*, consisting of a population with an infectious disease and a screening test for the disease with reliability 95%. It is supposed that the test does not produce false negatives (i.e., everyone who has the disease will test positive for it). In the media, it is indicated that screening tests may be completely *illusory* in the sense that a person declared to be infected by the disease may have extremely low risk of actually having the disease.[7] Since the reliability of the test is quite high, this sounds paradoxical, and the question is whether the hint of illusion can really be true. More precisely, the question *Q* to be answered is: How can we ascertain the probability that a person, who has tested positive for a disease, indeed has the disease? (The specific task is detailed in Appendix D3.)

Constructing a model of this system was primarily focused on illustrating the importance of elementary algebra as a modelling tool. Although the students answered *Q* by modelling *S*, the account made here is based on the lecturer's proposed solution. Its purpose in class was to serve as a benchmark, underlining vital attributes of a modelling process and the crucial role played by the parameters of the system under investigation.

Creating a Model and Producing Knowledge about S

These are the parameters of *S* we choose to implement:

- a is the incidence rate (i.e., the relative frequency) of the disease in the population, where $0 \leq a \leq 1$;
- N is the population size;
- p is the probability of having the disease when having tested positive, where $0 \leq p \leq 1$;
- r is the reliability of the screening test, where $0 \leq r \leq 1$.

---

[6] The assignment for the pilot inquiry is detailed in Appendix D5, while one possible result, as determined by one of the student groups, is displayed in Appendix B6.

[7] This has been done, for example, by the French physicist and populariser, Étienne Klein (Klein, 2020).

We have that aN is the number of people infected by the disease, and $(1-a)N$ is the number of non-infected people. Of the non-infected, $(1-r)(1-a)N$ are found positive for the disease. This gives that the total number of positives is equal to $aN + (1-r)(1-a)N$. Therefore, the probability of having the disease when having tested positive is given by:

$$p = \frac{aN}{aN + (1-r)(1-a)N} = \frac{a}{a + (1-r)(1-a)}.$$

Simplification of this expression yields $p = \frac{a}{a + (1-r)(1-a)} = \frac{a}{1-r+ra} = \frac{a}{a\left(\frac{1}{a} - \frac{r}{a} + r\right)} = \frac{1}{\frac{1}{a} - \frac{r}{a} + r} = \frac{1}{r + \frac{1-r}{a}}$. That is, the following equality is a model of the probability sought:

$$p = \frac{1}{r + \frac{1-r}{a}} \qquad (S').$$

S' displays the relationships between the parameters of the system S and provides an answer to the question Q. First, we can observe on S' that p is independent of the population size, N. Second, the following observation explains the apparent paradox in S:

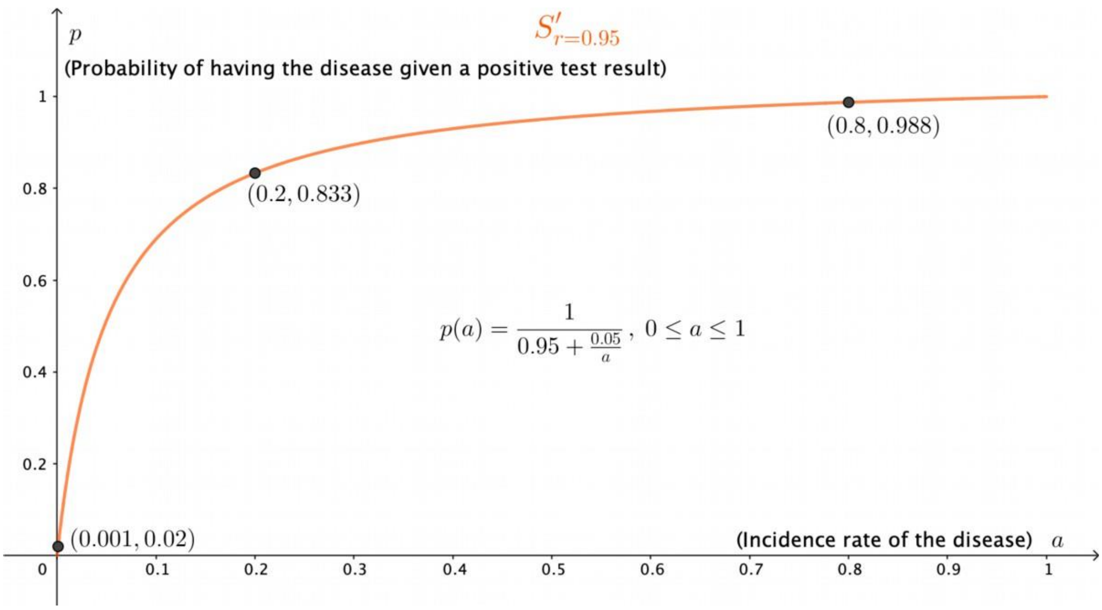
> For a fixed r, we see that when a increases, the fraction $\frac{1-r}{a}$ becomes smaller, and hence the denominator $r + \frac{1-r}{a}$ gets smaller too, so that the probability p increases.

Moreover, this relationship shows that p depends not only on r but also on the relative frequency of the disease, a. Specifically, small values of a yield small values of p. Let us fix $r = 95\% = 0.95$ and do calculations on $S'_{r=0.95}$ to display the relationship between p and a. Our model $S'_{r=0.95}$ is given by the function below, the graph of which is depicted in Figure 1.

$$p(a) = \frac{1}{0{,}95 + \frac{0{,}05}{a}}, \text{ for } 0 \le a \le 1 \qquad (S'_{r=0.95}).$$

Figure 1

A Model of S With r = 0.95



The points plotted on the graph display the following relationships between *a* and *p*:

- $a = 0.1\%$ corresponds to $p \approx 2\%$;
- $a = 20\%$ corresponds to $p \approx 83\%$;
- $a = 80\%$ corresponds to $p \approx 99\%$.

For further reading, the article "Why Every Clinician Should Know Bayes' Rule" by Tiemens et al. (2020) is recommended.

To conclude, probabilistic reasoning and elementary algebra was used to create S', a model that relates the probability of being sick, given a positive test result, to the incidence rate of the disease and the reliability of the test. Algebra was used to explore how changes in one parameter at a time affected the system, S. Further, algebraic transformations were used to derive new formulas based on the original model, S'. ($S'_{r=0.95}$ is one of several formulas developed during the inquiry.) Generally, by using algebra as a tool to model real-world phenomena, we can explore the behaviour of systems in a quantitative and precise way. This can help us to develop new insights into complex systems and to make accurate predictions and sensible decisions.

Inquiry 2: SRPs on Carbon Capture and Storage

The inquiry outlined in this section deals with students' examination of models created by scientists. The generating question for the class's inquiry was this:

> *Q*. "*How is carbon capture and storage modelled in the literature? What mathematics is involved in these models? Which parameters are included, and what are the relationships between them?*"

Guidelines for the SRP report is found in Appendix D5.

Some basic knowledge for investigating models of carbon capture and storage (CCS) was taught in the course as a blend of lectures and seminars. Contextual knowledge for CCS was introduced as a starting point, and additional knowledge was later taught based on needs emerging during the study and research process. While some publications were proposed by the lecturer, most of the resources the students used originated from the need to study derived questions during their inquiry. A summary of the knowledge taught for the CCS inquiries is presented in the following two subsections.

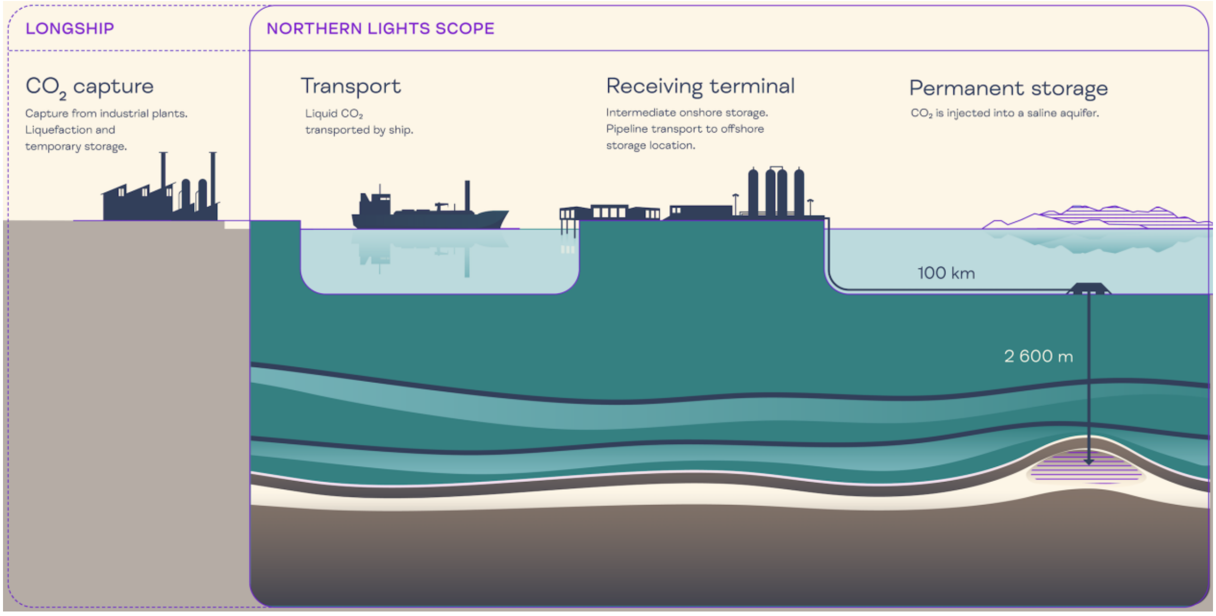
What is Carbon Capture and Storage?

CCS involves two processes: first, the capture of carbon dioxide (CO_2) emissions from industrial processes, such as steel and cement production or from power plants; second, the transport via ship or pipeline and storage of the captured CO_2 in deep underground geological formations. The goal of CCS is to reduce the amount of CO_2 that is released into the atmosphere, to help mitigate climate change.

The Norwegian government decided in 2020 to invest in full-scale CO_2 capture at Norcem's cement factory in Brevik (Norway) and in the transport and storage project Northern Lights (Report to the Norwegian Parliament No. 33, 2019–2020). Norcem's capture facility is the first of its kind in the world, and Northern Lights JV, a collaboration between Equinor, Shell, and TotalEnergies, is the first CO_2 storage facility in Europe that is open to European capture operators. The CO_2 will be transported by ship to a land terminal in Western Norway before it is sent on through a pipeline to an offshore CO_2 storage facility 2,600 meters below the seabed in the North Sea for permanent storage (see Figure 2).

Figure 2

The CCS Process



Note. The figure is taken from "About the Longship Project" (n.d.). Reprinted with permission.

This venture, called the Longship Project, is planned to be fully operative in 2024 (see "Carbon Capture, Utilisation and Storage," 2023).

At an early stage of the class's inquiry into CCS, the lecturer and students decided to delimit the models studied to either carbon capture or carbon storage. Below, I provide a brief background on carbon storage, which will lay the groundwork for a later summary of one team's inquiry into this component of the CCS technology.

Carbon Storage: A Quick Overview

There are four processes related to carbon storage (e.g., Niemi et al., 2017): thermal, hydrological, chemical, and mechanical processes. Team A chose to focus on *hydrological* processes, which involve the movement and behaviour of fluids such as water and CO_2. These processes may include fluid flow through porous rock formations, the displacement of brine and other fluids by injected CO_2, and the effects of hydrostatic pressure on rock properties and geomechanical stability. The next paragraphs briefly describe what dissolution of CO_2 in a background hydrological flow entails.

Normally, when gas is injected into a rock layer, it tends to accumulate in structural traps, such as anticlines. An anticline is a type of geological formation where a rock layer is folded upward into a hill-like shape. Gas can get trapped in these structures, and it doesn't necessarily spread out or dissolve throughout the surrounding rock formation. However, in the case of CO_2 injection into porous rocks, the water present within those rocks is constantly flowing due to background groundwater flow. As the water flows, it continuously dissolves the CO_2 that has been injected into the rock formation, rather than just allowing it to accumulate in a trap. Central in this dissolution process is Darcy's Law, used in geology, hydrology, and civil engineering to model the flow of groundwater, oil, and other fluids through porous media.[8] Darcy's Law states that the rate of flow of a fluid through a porous medium is proportional to the hydraulic gradient and the effective permeability of the medium (Fleurant & Bodin-Fleurant, 2019). Mathematically, this scientific law can be expressed as

$$\dot{V} = \frac{dV}{dt} = -kA\frac{dh}{dl} \qquad (1).$$

The parameters of Equation (1) are: $\dot{V}$ is the volumetric flow rate of the fluid (i.e., the volume of fluid that passes through a given cross-sectional area per unit of time); k is the effective permeability of the porous medium (i.e., its hydraulic conductivity); A is the cross-sectional area of the medium through which the fluid is flowing, $\frac{dh}{dl}$ is the hydraulic gradient, representing the change in hydraulic head (pressure difference) per unit of distance along the direction of flow; and the negative sign indicates that the fluid flows from higher hydraulic head to lower hydraulic head. Appendix D6 contains a task entitled "Modelling in Hydrogeology," which utilises Darcy's Law.

An Inquiry Into Carbon Storage

This section presents a condensed version of Team A's SRP. The system *S* examined is carbon storage in underground saline formations that extend over a relatively large area that can potentially store significant amounts of CO_2. The concepts utilised in their report are the following:[9]

- *Aquifer* denotes a geological formation in the ground where the rocks or sediments have a large content of groundwater.
- *CO_2 plume* in aquifers refers to a volume of CO_2 that has been injected into a deep saline aquifer for the purpose of carbon storage. The CO_2 plume spreads out through the pore spaces in the rock, displacing brine and filling the available pore space.
- *Injection well* is a type of well used to inject fluids or other substances into the ground.
- *Injection array* is a cluster of injection wells used together.
- *Mobility ratio* in geology is a measure of how easily a fluid flows through a porous medium.
- *Permeability* is a measure of how easily a gas or liquid can penetrate a porous medium.
- *Porosity* is the ratio between the volume of voids in a material and the total volume.
- *Saturation* refers to the fraction of the pore volume that is occupied by a specific fluid.
- *Supercritical CO_2* refers to a state of CO_2 above its critical temperature (31.1°C) and above its critical pressure (72.9 atm). Here, CO_2 exhibits both gas-like and liquid-like properties.
- *Viscosity* is a measure of a fluid's resistance to flow.

Team A presented an answer to Q_0 that was made up of a model to calculate the CO_2 storage capacity in deep, saline aquifers at regional level, proposed by MIT environmental engineering experts, Szulczewski and Juanes (2009):

$$C = \frac{2M\Gamma^2(1-S_{WC})}{\Gamma^2+(2-\Gamma)(1-M+M\Gamma)}\rho_{CO_2}\phi HWL_{\text{total}} \qquad (2).$$

[8] "Darcy's Law" originates from the work of Henry Darcy (1803–1858), a French engineer renowned for his significant contributions to hydraulics.

[9] Excerpts from Team A's report, originally written in Norwegian, are translated into English by the author.

Equation (2) is an analytic model represented by an explicit, closed-form expression, which is an

existing answer to Q_0. The model is a rational equation with 9 parameters: C is the mass of trapped CO_2; M is the mobility ratio, measuring fluidity / viscosity of a substance through a porous medium; Γ is the trapping coefficient of CO_2; s_{WC} is the saturation of the connate (i.e., saline, naturally occurring) water in the reservoir; ρ_{CO_2} is the density of CO_2; ϕ is the porosity of the reservoir; H is the net sandstone thickness of the reservoir; W is the length of the injection array in the reservoir; and L_{total} is the total extent of the CO_2 plume after it is trapped (Szulczewski & Juanes, 2009, p. 3309). As for the parameters included, Team A displayed formulas for Γ and M, taken from the same publication. The trapping coefficient, Γ, was defined as the ratio of the residual saturation of CO_2 (s_{rg}), referring to the fraction of pore space that is occupied by trapped CO_2, to the fraction of pore space that is not occupied by trapped connate water ($1 - s_{WC}$):

$$\Gamma = \frac{s_{rg}}{1-s_{WC}} \quad (3).$$

The mobility ratio, M, was formulated in terms of the viscosity of brine (μ_W), the viscosity of CO_2 (μ_g), and the endpoint relative permeability of CO_2 (k_{rg}^*):[10]

$$M = \frac{\frac{1}{\mu_W}}{\frac{k_{rg}^*}{\mu_g}} \quad (4).$$

Interdependence Between Parameters

Identifying parameters involved in S and their interconnectedness was part of Q_0. However, Q_0 did not induce the students to examine the *mathematical aspects* of these relationships, which was neither done in the article they drew on. Therefore, building on insights from Bachu (2015) and Ketzer et al. (2009), the author will apply quantitative reasoning to enrich the understanding of Equations (3) and (4) in the following paragraphs.

The Trapping Coefficient. In Equation (3), Γ represents the fraction of injected CO_2 that is effectively trapped within the subsurface reservoir. The denominator, $1–s_{WC}$, represents the “free” space within the reservoir that is available for CO_2 to be trapped, and the numerator, s_{rg}, represents the fraction of that space that is actually occupied by the trapped CO_2. So, Equation (3) is essentially comparing the amount of trapped CO_2 (s_{rg}) to the amount of available space for trapping ($1–s_{WC}$) to determine the fraction of injected CO_2 that is effectively trapped (Γ). In other words, if the saturation of fossil groundwater s_{WC} is relatively high, then there is less “free” space available for CO_2 trapping, so the trapping coefficient Γ would also be high. A high s_{WC} means less pore space for CO_2 trapping, but also more water displacement by CO_2, which can increase trapping efficiency. Furthermore, connate water may react with CO_2 to form carbonic acid, which can dissolve the minerals in the reservoir rock and create new pore space for CO_2. This is known as mineral trapping, an important mechanism for long-term storage of CO_2 in the formation.

If s_{rg} is low, it means that a smaller fraction of the available space within the underground reservoir is occupied by trapped CO_2. This could be due to factors such as the geological characteristics

[10] The *endpoint relative permeability* of CO_2 is determined experimentally by measuring the flow of CO_2 and groundwater through a sample of the porous medium under fixed pressure and temperature.

of the reservoir, the injection strategy, or the properties of the injected CO_2. If s_{rg} is low, the trapping coefficient Γ would also be low, indicating that a smaller fraction of the injected CO_2 is effectively trapped. This could have implications for the overall effectiveness of CCS as a mitigation strategy for climate change, as a lower trapping coefficient would mean that a larger proportion of the injected CO_2 could potentially leak into the atmosphere over time. In general, a high trapping coefficient is desirable for effective and long-term storage of injected CO_2, as it indicates a higher degree of CO_2 retention within the bedrock formation. Overall, Equation (3) provides a simple way to estimate the trapping coefficient of injected CO_2 in a subsurface reservoir, based on the properties of the reservoir itself.

The Mobility Ratio. In Equation (4), M is a dimensionless quantity that represents the ratio of fluid mobilities between the displaced fluid (brine) and the injected fluid (CO_2), in a subsurface reservoir. It provides insight into how the two fluids interact and move through the reservoir during processes such as carbon storage. The numerator, $\frac{1}{\mu_W}$, represents the inverse of the viscosity of brine (i.e., its fluidity). As the viscosity of brine increases, the mobility ratio decreases, indicating that it is harder for CO_2 to flow through the reservoir. Conversely, as the viscosity of brine decreases, the mobility ratio increases, indicating that it is easier for CO_2 to flow through the reservoir. The denominator of Equation (4), $\frac{k_{rg}^*}{\mu_g}$, represents the ratio of the endpoint relative permeability of CO_2 (k_{rg}^*)—which is a measure of how easily CO_2 can flow through the reservoir relative to brine—, to the viscosity of CO_2. As k_{rg}^* increases, the mobility ratio, M, decreases, indicating that it is easier for CO_2 to flow through the reservoir relative to brine. Conversely, as k_{rg}^* decreases, the mobility ratio M increases, indicating that it is harder for CO_2 to flow through the reservoir relative to brine.

A higher mobility ratio suggests that the injected fluid has greater mobility compared to the displaced fluid. This means that the injected CO_2 can flow more easily through the reservoir, potentially resulting in faster movement and potentially less efficient displacement of brine. Conversely, a lower mobility ratio indicates that the injected fluid has lower mobility relative to the displaced fluid. This suggests that CO_2 faces more resistance or has a harder time flowing through the reservoir, which can result in slower movement and potentially more efficient displacement of the brine.

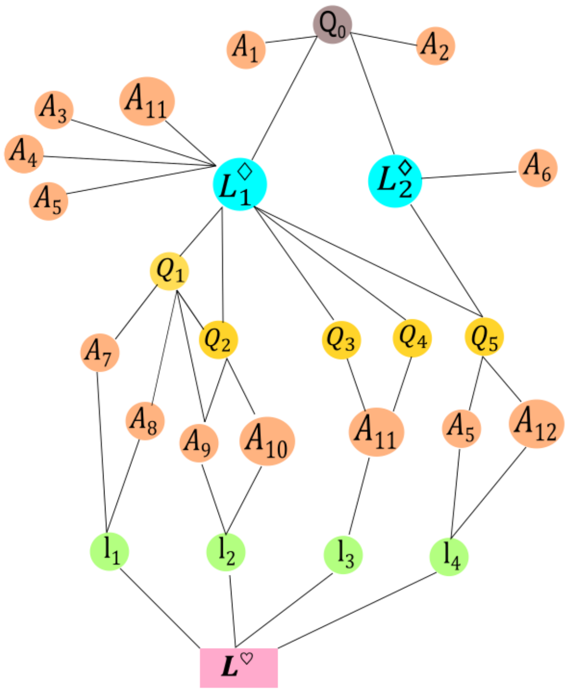
To conclude about M, the relationship between the parameters in Equation (4) is complex and depends on the specific conditions of the reservoir. However, in general, a lower mobility ratio is desirable for effective CO_2 storage, as it indicates that CO_2 is less likely to migrate and potentially leak out of the subsurface storage.

A Pathway Diagram of Team A's Inquiry

Figure 3 is a directed graph illustrating the course of Team A's SRP, accompanied by Table 1, describing the elements of the milieu created and utilised during the inquiry. Note that publications cited in Table 1 are not included in the reference list, since the table is just meant to illustrate the correspondence between the directed graph and the table that describes its nodes.

Figure 3

The SRP on Carbon Storage Carried out by Team A



Note. Q_0 = generating question; Q_j = derived questions; $L_i^{\diamond}$ = existing answers (*L* for "løsning" in Norwegian); A_k = works (*A* for "arbeid" in Norwegian); l_m = partial answers; $L^{\heartsuit}$ = final answer to Q_0. Reproduced with permission.

Table 1

The Milieu Created During the Inquiry Conducted by Team A

Generating question
Q_0: How is carbon capture and storage modelled in the literature? What mathematics is involved in these models? Which parameters are included, and what are the relationships between them?
Existing answers $L_i^\diamond$
$L_1^\diamond$: Calculation of CO_2 storage capacity in deep, saline aquifers (Szulczewski & Juanes, 2009).
$L_2^\diamond$: Estimation of CO_2 storage capacity (Bachu et al., 2007).
Derived questions Q_j
Q_1: What is the relationship between viscosity and permeability in CO_2 storage?
Q_2: What is the mobility ratio about, and why is this an important component in CO_2 storage models?
Q_3: What simplifications of the models have been made, and what are the consequences of them?
Q_4: Are the parameters typically calculated by theory or are they due to empirical work?
Q_5: How important is pressure in calculating storage capacity?
Works A_k
A_1: "This is what you need to know about capture and storage of CO_2" (Sintef, 2019)
A_2: Storage and transport of CO_2 on the continental shelf (Norwegian Ministry of Petroleum and Energy, 2014)
A_3: CO_2 trapping mechanisms (CCP, n.d.)
A_4: Article on capillary capture for geological CO_2 storage (Krevor et al., 2015).
A_5: Video about CO_2 storage by an expert on the topic (Benson, 2021).
A_6: Norwegian website on terminology for the petroleum industry (Petroleumstilsynet, 2022)
A_7: Darcy's law for fluid flow in a porous medium ("Darcy's law", 2022).
A_8: Master's thesis on CO_2 storage in sandstone and limestone (Kvinge, 2012)
A_9: Reservoir engineering (Satter & Iqbal, 2016)
A_{10}: Article on mobility ratios (Bamidele et al., 2009)
A_{11}: Mathematical model of the CO_2 plume footprint in deep, saline aquifers (MacMinn & Juanes, 2009)
A_{12}: How can CO_2 be stored under the Earth? (National Energy Technology Laboratory, 2022)

Note. Partial answers (l_m) are synthesised into $L^\heartsuit$ elsewhere in Team A's report. Reproduced with permission.

Navigating the Interplay of Systems and Models in Task Design

In the preceding section, two distinct cases of mathematical modelling within teacher education were explored. This section transitions into the subsequent phase, which involves instructing student teachers on how to craft modelling activities tailored for secondary education. The capability to develop effective modelling tasks, which enhance understanding of the interplay between systems and models, is a critical skill for student teachers preparing to teach mathematics in Grades 8–13. The task design approach employed in this course is elucidated through a commentary on one of the assignments given to the student teachers. A comprehensive examination of this assignment, including principled explanations and a suggested solution, is given by Strømskag (2023), a manuscript found in Appendix B.[11] What is offered in the following paragraphs is merely a brief summary of the assignment and its solution.

The exercise began with a typical task extracted from a Norwegian textbook (depicted in Figure 4).

Figure 4

Task for Modelling of Stopping Distance

1.130

The stopping distance for a car in motion hinges on both the driver's response time and the braking distance.[12] The table below outlines the stopping distance, denoted by $S(x)$, in metres corresponding to certain speeds in kilometres per hour for a specified car and a specified driver.

x (km/h)	40	60	80	100
$S(x)$ (m)	24	45	73	108

a) Plot the data points from the table in a coordinate system and elucidate why a quadratic function seems to be a suitable fit.
b) Determine the quadratic function, S, that most accurately represents the given data. Sketch the graph incorporating the data points. Ensure that the expression of the function is accurate to three decimal places.
c) Find graphically the speed that would result in a stopping distance of 150 metres.
d) Find graphically the stopping distance corresponding to a speed of 90 km/h.

Note. The task is taken from a Grade 11 textbook, *Sinus 1T*, written by Oldervoll et al. (2020, p. 375). It has been translated into English by the author.

In the textbook task, students were instructed to plot stopping distances against varying vehicle speeds, with the intention of creating a second-degree polynomial model using regression analysis. The specific method for model creation was predetermined by the task's placement under the heading "Polynomial Regression". While the developed model, $S(x) = 0.009x^2 + 0.175x + 3$, did accurately reflect the given data, it fell short in fostering a thorough understanding of the system's underlying mechanics. A clear disconnect existed between the real-world dynamics of the system being studied and the parameters of the model, which subsequently constrained the educational value of the exercise.

In an effort to amplify the educational value of investigating the system in question, the lecturer redirected the task towards principles of inquiry and exploration, with the goal of illuminating the system's defining properties. This involved gaining a nuanced understanding of how system parameters, like the friction coefficient (a critical determinant of braking distance), and speed interact. As part of the refined task, student teachers were asked to model a system where a vehicle, travelling at a specific speed (72 km/h) on a dry, horizontal asphalt surface, needs to make a sudden stop. The question to be answered was about the vehicle's braking distance, which hinged on their ability to understand the

[11] Strømskag (2023) is an extension of a proposed solution originally presented during the course examined here. Its intent is to assist fellow educators who wish to incorporate this assignment into their own course curricula.
[12] Stopping distance = Reaction distance + Braking distance.

Co-funded by
the European Union

implications of friction when brakes are applied. Furthermore, they were prompted to discuss how varying speeds and different road conditions might impact the braking distance.

To calculate the braking distance of a vehicle, the first step involved understanding the dynamics of the forces involved and the vehicle's acceleration. Drawing on Grimenes et al. (2011) and grounded in Newton's laws of motion, the procedure can be outlined as follows: In the vertical direction, the force of gravity (G) and the normal force (N) balance each other out, as per Newton's first law ($N = G$). Meanwhile, the friction force, acting opposite to the vehicle's motion, is proportional to the normal force ($F_{\text{friction}} = \mu N$), where μ is the friction coefficient. Given $N = G = mg$, we derive $F_{\text{friction}} = \mu mg$. In the horizontal direction, the friction force is the sole force working against the movement. Using Newton's second law ($\Sigma F = ma$), where ΣF is equal to $-F_{\text{friction}}$, we find the acceleration a as follows: $-\mu mg = ma$, simplifying to $a = -\mu g$.

With the acceleration known, then the braking distance could be computed, using the equation of motion for uniform acceleration: $v^2 - v_0^2 = 2ad$ (Grimenes et al., 2011, p. 32). The involved parameters were defined as follows:

- a denoted the uniform acceleration of the vehicle (which in the case of braking is negative);
- g denoted the acceleration resulting from gravity (commonly approximated as 9.81 m/s^2 at sea level on Earth);
- m denoted the mass of the vehicle;
- d denoted the distance travelled (which in this case was equal to the braking distance);
- v denoted the velocity of the vehicle in the final state (which in this case was equal to 0);
- v_0 denoted the speed in the initial state (which was the speed at the moment the braking starts);
- μ denoted the coefficient of friction (which depends on the properties of the materials in contact, in this case, the rubber of the tyres and the road surface).

The resultant model was the second-degree polynomial, $d = \frac{v_0{}^2}{2\mu g}$. (For a breakdown of how this formula was derived and further insights into its components, refer to Appendix B.) Besides answering the specific question raised about the system, this model facilitated the generation of general knowledge about the system, such as: the braking distance is independent of the vehicle's mass, and the braking distance is proportional to the square of the initial speed. It was highlighted that this model, as it assumes uniform acceleration throughout braking and ideal conditions, might not yield accurate braking distances in more complex scenarios. However, these very limitations can urge students to reflect on other parameters that could potentially influence braking distance, such as road and tyre conditions, vehicle design, and driver reaction time.

Designed with students having a basic understanding of mechanics in mind, the updated task's final phase focused on tailoring it to students at varying levels. The lecturer proposed a reconfiguration of the task to bypass the need for extensive knowledge of the inherent mechanics of the system. This strategy involved supplying two distinct datasets—one for dry and one for wet road conditions—that correlated the vehicle's speed with its resultant braking distance. Concurrently, students would be introduced to the principle that braking distance is proportional to the square of the speed at the point of braking.

Concluding Comments

The modelling activities implemented in the course provided several advantages for the students. These included enhanced commitment and research capabilities, utilising algebra as a modelling instrument, and cultivating didactic knowledge of task design, particularly focusing on the relationship between systems and models. This chapter demonstrates the integral role that elementary algebra plays in the modelling of systems. Elementary algebra provides the means to express interdependence between parameters, analyse change, optimise performance, solve problems, and make predictions. By using elementary algebraic methods and quantitative reasoning, we can gain a deeper understanding of the world around us and make informed decisions based on quantitative data.

To operate in the paradigm of questioning the world—particularly with open-ended inquiries like the ones on CCS—is challenging for both teachers and students. A new *didactic contract* (Brousseau, 1997) needs to be implemented, because the teacher is generally not an "expert" on the systems studied. The shift away from seeing the teacher as the ultimate authority on knowledge may be difficult for both parties. It calls for students to assume a greater degree of responsibility for their own learning, and, not least, that the teacher allows them to do so. In addition, adoption of the new didactic paradigm entails the introduction of novel tasks for both teachers and students to tackle: in the parlance of the ATD, this translates to a shift in the *topos* of each party. From the teacher's perspective, the ability to pose relevant and thought-provoking generating questions becomes a crucial, albeit demanding, new role. As for the students, they are now expected to focus intensively on one question over a significant duration. This is not just an unfamiliar territory for them, but it can also become a source of frustration, especially if their perseverance wavers.

In conclusion, the course presented in this chapter underlines the critical role of mathematical modelling in questioning and understanding real-world phenomena. This directly aligns with the paradigm of questioning the world (Chevallard, 2015), which encourages not only students, but citizens at large, to interrogate to better understand their surrounding world. The ambition has been to offer useful insights into the effective incorporation of mathematical modelling, with a focus on the dialectic of systems and models, into teacher education. It is hoped that these insights, together with the appendices referenced in the course roadmap, extend their applicability beyond the confines of the course explored in this examination.

References

About the Longship Project. (n.d.). Northern Lights. https://norlights.com/about-the-longship-project/

Amer, M. A., Luque-Corredera, & Bartolomé, E. (2022). Study and Research Paths for learning general chemistry: Analyzing the quality of drinking water. *Journal of Chemistry Education*, *99*(3), 1255–1265. https://doi.org/10.1021/acs.jchemed.1c00971

Bachu, S. (2015). Review of CO_2 storage efficiency in deep saline aquifers. *International Journal of Greenhouse Gas Control*, *40*, 188–202. https://doi.org/10.1016/j.ijggc.2015.01.007

Barquero, B., Bosch, M., & Romo, A. (2018). Mathematical modelling in teacher education: Dealing with institutional constraints. *ZDM Mathematics Education*, *50*, 31–43. https://doi.org/10.1007/s11858-017-0907-z

Barquero, B., Bosch, M., & Wozniak, F. (2019). Modelling praxeologies in teacher education: The cake box. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 1144–1151). Utrecht: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME. https://hal.science/hal-02408705/document

Bartolomé, E, Florensa, I. , & Bosch, M. (2021). Teaching strengths of materials through "Study and Research Paths": Invariants and differences. *International Journal of Engineering Education, 37*(2), 446–460.

Bartolomé, E., Florensa, I., Bosch, M., & Gascón, J. (2018). A 'study and research path' enriching the learning of mechanical engineering. *European Journal of Engineering Education, 44*(3), 330–346. https://doi.org/10.1080/03043797.2018.1490699

Bosch, M. (2018). Study and research paths: A model for inquiry. In B. Sirakov, P. N. de Souza, & M. Viana (Eds.), *International Congress of Mathematicians* (Vol. 3, pp. 4001–4022). Singapore: World Scientific Publishing. https://doi.org/10.1142/9789813272880_0210

Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 67–83). Cham: Springer. https://doi.org/10.1007/978-3-319-05389-9_5

Bourgade, J.-P., Bernad, K., & Matheron, Y. (2020). The need for new teaching praxeologies in the paradigm of questioning the world. In M. Bosch, Y. Chevallard, F. J. Garcia, & J. Monaghan (Eds.), *Working with the Anthropological Theory of the Didactic in mathematics education: A comprehensive casebook* (pp. 139–156). London: Routledge. https://hal.archives-ouvertes.fr/hal-02190795

Brousseau, G. (1997). *The theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Dordrecht: Kluwer. https://doi.org/10.1007/0-306-47211-2

Carbon capture, utilisation and storage. (2023). Equinor.com. https://www.equinor.com/energy/carbon-capture-utilisation-and-storage

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Cham: Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Chevallard, Y. (2019). Introducing the Anthropological Theory of the Didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves%20Chevallard.pdf

Chevallard, Y., & Bosch, M. (2019). Anthropological theory of the didactic (ATD). In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 1–8). Cham: Springer. https://doi.org/10.1007/978-3-319-77487-9_100034-1

Fleurant, C., & Bodin-Fleurant, S. (2019). *Mathematics for Earth science and geography*. Cham: Springer. https://doi.org/10.1007/978-3-319-69242-5

Grimenes, A. A., Jerstad, P., & Sletbak, B. (2011). *Grunnleggende fysikk for universitet og høgskole* [Basic physics for universities and university colleges]. Oslo: Cappelen Damm.

Jessen, B. E. (2017). How to generate autonomous questioning in secondary mathematics teaching? *Recherches en Didactique des Mathématiques, 37*(2–3), 217–246.

Jessen, B. E. (2019). Teachers learning to design and implement mathematical modelling activities through collaboration. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the 11th Congress of the European Society for Research in Mathematics Education* (pp. 1182–1189). Utrecht: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME. https://hal.science/hal-02408819v1/document

Ketzer, J. M., Iglesias, R., Einloft, S., Dullius, J., Ligabue, R., & de Lima, V. (2009). Water–rock–CO_2 interactions in saline aquifers aimed for carbon dioxide storage: Experimental and numerical modeling studies of the Rio Bonito Formation (Permian), southern Brazil. *Applied Geochemistry*, *24*(5), 760–767. https://doi.org/10.1016/j.apgeochem.2009.01.001

Klein, É. (2020). *Je ne suis pas médecin, mais je…* [I am not a medical practitioner, but I…]. Paris: Gallimard. https://lesmardisdelaphilo.com/wp-content/uploads/2020/04/Eienne-Klein-1.pdf

MA3001. (n.d.). *Mathematical modelling using study and research paths*. https://wiki.math.ntnu.no/ma3001/2022v/matematiskmodellering/start

Niemi, A., Bear, J., & Bensabat, J. (2017). *Geological storage of CO_2 in deep saline formations*. Dordrecht: Springer. https://doi.org/10.1007/978-94-024-0996-3

Niss, M. (2015). Prescriptive modeling: Challenges and opportunities. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modeling in education research and practice: Cultural, social and cognitive influences* (pp. 67–79). Cham: Springer. https://doi.org/10.1007/978-3-319-18272-8_5

Oldervoll, T., Svorstøl, O., Vestergaard, B., Gustafsson, E., Osnes, E. R., Jacobsen, R. B., & Pedersen, T. (2020). *Sinus 1T: Matematikk* (4th ed.). Oslo: Cappelen Damm.

Report to the Norwegian Parliament No. 33. (2019–2020). *Longship: Carbon capture and storage* [White Paper]. Norwegian Ministry of Petroleum and Energy. https://www.regjeringen.no/en/dokumenter/meld.-st.-33-20192020/id2765361/?ch=1

Strømskag, H. (2022, 21 January). *A note on the Herbartian schema: A dynamic model for a study of a generating question*. [Unpublished manuscript]. Department of Mathematical Sciences, Norwegian University of Science and Technology.

Strømskag, H. (2023). *On the dialectic of systems and models: The case of braking distance* [Unpublished manuscript]. Department of Mathematical Sciences, Norwegian University of Science and Technology.

Strømskag, H. (forthcoming). Study and research paths on differential calculus in teacher education. In *Proceedings of the 7th International Conference of the Anthropological Theory of the Didactic*. Cham: Birkhäuser.

Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Szulczewski, M., & Juanes, R. (2009). A simple but rigorous model for calculating CO2 storage capacity in deep saline aquifers at the basin scale. *Energy Procedia*, *1*(1), 3307–3314. https://doi.org/10.1016/j.egypro.2009.02.117

Tiemens, B., Wagenwoorde, & Witteman, C. (2020). Why every clinician should know Bayes’ rule. *Health Professions Education*, *6*(3), 320–324. https://doi.org/10.1016/j.hpe.2020.05.002

2.1. Didactic Commentary

Didactic Analysis of MA3001: Insights from the Post-Course Survey

Heidi Strømskag

Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, Norway

Introduction

In Chapter 2 (this volume), I examined the inaugural run of the course MA3001 – "Mathematical Modelling Using Study and Research Paths" (MA3001, n.d.), which took place in 2022 at the Norwegian University of Science and Technology. In this chapter, I conduct a didactic analysis of the course based on student feedback from a post-course survey, using the Anthropological Theory of the Didactic (ATD) (Chevallard, 2015, 2019). Nine students completed the course, and eight of them conducted the survey.

The objectives of the course were diverse. Firstly, it aimed to teach mathematical modelling of systems using elementary algebra as a modelling tool, where parameters played a crucial role. Secondly, it sought to conduct inquiries in the format of study and research paths (SRPs), emphasising the tenets of the paradigm of questioning the world. Finally, it examined models of climate change found in the literature, exploring how system parameters and their interrelationships can elucidate the properties of the modelled systems. The main SRPs in class were about carbon capture and storage (CCS). In each of these areas, meta-level instruction was incorporated, given that the course was integral to a teacher education programme. The bedrock of the course lay in the ATD, whose principles were crucial to the treatment of the course contents. For an account of the scientific elements of the course commented on in this chapter (algebra, ATD, CCS, modelling, SRP), refer to Chapter 2 (this volume).

The subsequent analysis is segmented into two sections: "Mathematical Modelling of Systems Using Elementary Algebra" and "The ATD, Study and Research Paths, and Carbon Capture and Storage". Within each section, the questions and corresponding student responses are displayed. Comments not directly relevant to the topic have been excluded for brevity and clarity. Student feedback is marked with an "S" followed by a numeric identifier (e.g., S1, S2, …, S8). The survey was conducted non-anonymously to evaluate the potential impact of students' prior knowledge. It is noteworthy that S3, S6, S7, and S8 were in their 3rd year of the teacher education programme, while S1, S2, S4, and S5 were in their 4th year. Furthermore, it is relevant that S1, S4, and S5 had previously enrolled in the course MA3061 (n.d.), which introduced the ATD and provided guidance on conducting SRPs focused on the differential calculus present in the Norwegian upper secondary curriculum. In contrast, S2, S3, S6, S7, and S8 had not taken MA3061.

After detailing the students' survey responses, I present my analyses for each of the two sections.

Mathematical Modelling of Systems Using Elementary Algebra

1 How relevant have you found the topics in MA3001 to be for your professional development as a future) mathematics teacher?

	Irrelevant	Not very relevant	Relevant	Very relevant
Subject matter related to modelling	0	0	1	7
Subject matter related to algebra	0	0	2	6

1.1 Comments regarding your answer to the question about the course's relevance to the mathematics teaching profession.

S1. The course has broadened my perspective on what it means to model in mathematics. This has so far been missing (as far as I remember) in other subjects in the programme.

S2. What has been taught about algebra and modelling is particularly relevant because it provides concrete focus areas when teaching these topics. You get a good insight into what is important.

S3. The materials on algebra, modelling and ATD are what I think is most relevant for developing as a mathematics teacher. You learn methods to use within these topics, in addition to seeing the new paradigm from a maths perspective.

2 How suitable do you believe the assignments and ways of working in MA3001 have been in terms of fostering professionalism as a mathematics teacher?

	Unsuitable	Not very suitable	Suitable	Very suitable
Modelling assignments	0	0	2	6
Assignments on algebra	0	0	2	6

2.1 Comments on my answer about assignments and ways of working in the course.

S1. Owing to the fact that "modelling" is one of the core elements of LK20 [National Curriculum for Grades 1–13], the tasks in this course have been very appropriate for developing our expertise in this area. Similarly, algebra is a topic that falls under the core element "Mathematical Knowledge Area". We have also become aware of the importance of algebra as a tool for the core element "Abstraction and Generalisation", emphasising the significance of parameters.

S3. Recognising the value of using parameters within algebra has perhaps been the most pertinent aspect for me. The new curriculum offers less focus on algebra, so it has been beneficial to see how algebra can be used in an exploratory manner where students need to construct, manipulate, and evaluate formulas.

S4. The short tasks we got were very good and useful.

S5. Nice to work on concrete tasks that can be used in teaching in school.

S7. The contents on algebra felt a bit on the side of the main topic [SRP on CCS?], but was very informative.

4 What was particularly interesting or relevant to you in this course (within mathematics, mathematics didactics, physics, geology, economics, chemistry, etc.)?

S2. I particularly found the instruction in algebra and modelling engaging. Several facets of mathematics instruction were revealed that I hadn't previously reflected upon. These insights I will carry forward in my career as a mathematics teacher.

S4. I found it extremely engaging to learn about modelling. It's something I have never been fond of and never truly grasped. This course gave me a clearer understanding of what a model can be, and I was quite taken with the system-model theory we employed. It was succinct and straight to the point.

S5. It was exciting to learn how algebra teaching has evolved over time and how this influences student learning.

S8. It has also been relevant to understand more about how algebra can act as a modelling tool and how one can set the stage for students to attempt construction, manipulation, and evaluation of formulas.

10 Through MA3001, have you developed a new understanding of any bodies of knowledge (different from your previous understanding)? If so, what does this change entail?

S2. Yes, I have developed a new comprehension of algebra, modelling, and parameters. This change can be described as acquiring tools for how students can purposefully work with algebra to find solutions to the questions they wish to answer.

S3. I have gained a greater insight into how to craft algebraic problems where students have to focus on more than just using a formula to find an answer to a problem. I feel that I have a better grasp of how vital it is to use parameters so that students develop skills around using formulas through manipulation and construction in addition to the evaluation aspect.

S4. Yes, refer to my answer to Question 4.

S5. Parameters.

S8. I have learnt a bit more about how algebra can be used as a modelling tool and about different models.

Analysis of Student Survey Responses on Modelling and Algebra

Perspective on Mathematical Modelling and Algebra

The students' responses exhibit a holistic understanding and appreciation of the mathematical modelling and algebra conveyed in the course. As S1 noted, the course filled a significant gap in their prior knowledge, broadening their understanding of mathematical modelling. This hints at the epistemological dimension of the ATD (Florensa et al., 2015), where students recognise a richer set of practices and knowledge related to mathematical modelling.

Student Engagement and New Praxeologies

Responses to Question 4, particularly by S4, shed light on their journey from disinterest or unfamiliarity to a point of clarity and interest. In ATD terms, this transformative experience can be viewed as the result of building new relations to the knowledge at stake (Chevallard & Bosch, 2019). This progression equips students to perform new types of tasks, using different techniques, justified by new technologies, in a new theoretical framework (the ATD). The systematic approach to modelling, as mentioned by S4, might imply the appreciation of a more refined and task-oriented practice.

S5's reflection on the evolution of algebra teaching connects to the historical-epistemological dimension of the ATD. It is imperative to understand not just the "how" (i.e., the functioning) but also the "why" (i.e., the utility) of mathematical practices. By knowing the historical context of algebra, the students can better understand its current pedagogical practices.

Utilising Algebra as a Tool

The responses, especially those of S8, S3, and S2 in various questions, bring forth a nuance in understanding algebra not just as an abstract subject, that is, as a praxeological complex considered from the point of view of its *structure* and (partially) its *functioning*, but as a tool, that is, from a praxeological complex considered from the point of view of its possible *uses*—specifically, as a modelling tool. This resonates with the anthropological notion in the ATD, where algebra becomes a human practice, deeply intertwined with real-world problem solving—and let me stress, this includes mathematical problem solving (Strømskag & Chevallard, 2022).

Modelling Assignments and Assignments on Algebra

The quantitative data gives that 6 of the 8 students found the modelling assignments and assignments on algebra to be "very suitable" and 2 found them suitable for fostering professionalism. This suggests that the students were able to engage in praxeologies of mathematical modelling and algebra. S1's response emphasises the importance of modelling and algebra as core elements in the curriculum. The mention of the core element "Abstraction and Generalisation" and the significance of parameters suggests that the course provided them with the techniques and tools necessary for mathematics teaching in school. S3's comment points to the technique of using parameters in algebra and their usefulness in constructing, manipulating and evaluating formulas, thus emphasising its praxeological value.

Emerging Understandings

The responses to Question 10, especially from S2 and S3, highlight a deepened appreciation of the processes behind the crafting of algebraic problems for Grades 8–13. Their newfound understanding of the role of parameters and the manipulation, construction, and evaluation of formulas underscores the course's effectiveness in altering preconceived notions, which in ATD terminology means that they have developed new relations to objects. This underlines the transformative dimension of the ATD, where prior relations to objects are reshaped through didactic interventions.

In conclusion about modelling and algebra, through the ATD lens, the feedback reveals a deeper institutional and personal transformation in students' engagement with mathematical modelling and algebra. Parameters emerge as a significant component, bridging the gap between algebraic technique and technological depth in modelling. The course seems to have effectively embedded these nuanced elements, fostering a comprehensive understanding of their role in the broader mathematical landscape.

Institutional Dimension: Relevance to the Mathematics Teaching Profession

The institutional dimension refers here to how knowledge is structured, organized, and delivered within an educational institution. The mention of the LK20 National Curriculum for Grades 1–13 in S1's response indicates the alignment of the course content with the institutional demands. By preparing students for the curricular expectations, the course is preparing them for their teaching roles. S3's remark about the "new curriculum" indicates that while institutional priorities might shift (like less focus on algebra), the course was beneficial in showcasing how algebra remains a vital tool. S4's and S5's feedback might suggest that the course's practical and applicable nature, referring to short tasks and concrete tasks, is institutionally relevant, as it prepares them for their roles in classrooms.

S7's remark suggests that algebra seemed somewhat tangential to the primary focus, which I infer to be the SRP on CCS. This implies that the relationship between algebra and the intricate CCS models they delved into might not have been evident to S7. Consequently, there appears to be a need to enhance the teaching of algebra with respect to parameters and their interconnectedness in mathematical models presented in scholarly works (e.g., models developed by scientists). The weakness experienced by S7 is likely related to the limitation discussed in Chapter 2 (this volume). The generating question of the SRPs on CCS was the following:

Q. How is carbon capture and storage modelled in the literature? What mathematics is involved in these models? Which parameters are included, and what are the relationships between them?

However, the students' answers to the last part of the question, about relationships between parameters in the models they studied, were less developed. This indicates that it is necessary to explain what we mean by describing interrelationships between parameters. To exemplify how this may be done, in Chapter 2 (this volume, pp. 29–30), I used quantitative reasoning to explain system properties based on two equations presented in Team A's SRP report—one equation with three and the other with four parameters. Such analyses of models, whether they are constructed by the students themselves or found in the literature, is at the heart of the new didactic paradigm, where the point is to produce knowledge about the systems under investigation, and not just stop when a model is constructed or found. This can be challenging since it requires some knowledge about the system parameters, which in the case of CCS mainly belonged to geology and physics. In the context of teaching modelling, it is important to explain that identifying relationships between parameters means using mathematics (e.g., quantitative or probabilistic reasoning) to explain some of the properties of the system at stake.

The ATD, Study and Research Paths, and Carbon Capture and Storage

1 How relevant have you found the topics in MA3001 to be for your professional development as a (future) mathematics teacher?

	Irrelevant	Not very relevant	Relevant	Very relevant
The subject matter related to the ATD	0	0	1	7
The subject matter related to CCS	0	0	3	5

1.1 Comments regarding your answer to the question about the course's relevance to the mathematics teaching profession.

S1. Would like to have more lectures on the ATD like we had in MA3061 [a course S1 had taken the previous semester]. Comprehensive theory.

S2. ATD and carbon capture and storage are also relevant, but since ATD is not the current didactic paradigm, there are many things that can be difficult to implement in schools.

S3. The materials on algebra, modelling and ATD are what I think is most relevant for developing as a mathematics teacher. You learn methods to use within these topics, in addition to seeing the new paradigm from a maths perspective. The subject matter related to carbon capture and storage is also relevant in that students will develop an ethical view of the environmental problems facing the world. The reason why I see this as slightly less relevant is that the subject matter is very complex, and it can be difficult to understand for upper secondary school students.

S4. In general, I found the course very relevant to my future teaching profession.

S8. Some of the subject matter has been a bit heavy to read since there were long articles in English, but I think the subject matter in the course has generally been relevant for further development of mathematics teachers.

2 How suitable do you believe the assignments and working methods in MA3001 have been in terms of fostering professionalism as a mathematics teacher?

	Unsuitable	Not very suitable	Suitable	Very suitable
Study and research path on CCS	0	0	3	5

2.1 Comments on my answer about assignments and working methods in the course.

S1. I'm keen to try out the SRP working method with my own students, and CCS is particularly relevant due to sustainable development.

S2. The reason I deemed the SRP only "suitable" is that many other subjects are incorporated within CCS than just mathematics, making SRP demanding to conduct as a teacher. Apart from that, I find SRP to be an intriguing form of project-based learning and believe it has been taught very well.

S6. While I appreciate the SRP, it's not exactly my preferred way of working, but it was fun. The tasks were relevant and enjoyable.

S8. I found it very enlightening to learn a new way of working with a problem. Both the work with the SRP and tasks carried out in class are something I will incorporate into my future role as a mathematics teacher.

3 Was there anything you found surprising in MA3001? If yes, please explain.

S2. I was surprised by how involved the students were in the course and how much influence we had. This is something I am not used to from previous courses. The lecturer seemed very genuinely interested in our input, and organised the teaching accordingly.

S4. At the start of the SRP, I thought the topic Heidi [the lecturer] had chosen was surprisingly advanced. Most of us got a little sweaty when we started studying carbon capture and storage. Fortunately, it became more manageable as we realised that we were not supposed to go in depth on everything or "becoming geologists".

4 What was particularly interesting/relevant to you in this course (within mathematics, mathematics didactics, physics, geology, economics, chemistry, etc.)?

S1. ATD as a theory was particularly relevant (especially since I consider writing a master's thesis on it). At the same time, I found it intriguing to explore the phenomenon of CCS and see the significant roles different disciplines play in the entire process.

S2. It was also insightful to partake in group work that demanded such intense collaboration. Throughout our working period, we met various requirements and gave three joint presentations. From prior experiences with group assignments, I'm not accustomed to such high collaborative demands. This was quite positive from my viewpoint, especially since I got on well with my teammate.

S3. Personally, the most captivating part for me was learning about the SRP research method, and how I can utilise it as a teacher. It offered fresh perspectives on, for example, working with interdisciplinarity, which is a primary element in the curriculum. From my practical experiences, many teachers find it challenging to devise schemes of work around this.

S4. I also appreciated the interdisciplinary nature of the SRP, making it highly pertinent for us student teachers.

S7. It was exciting to study both ATD and SRP. It presented a different approach to work that was quite engaging, something I would be keen to delve more into.

S8. I have found it highly engaging to learn and work in a novel way with SRPs.

5 What has been of great importance to your learning in MA3001?

S1. Collaboration has undoubtedly been of great significance for learning. We weren't quite sure where to begin with SRP, but after discussing the task, it all fell into place. Regular seminars have contributed to maintaining the progression of the project.

S2. I would argue that the lecturer and her adaptability to the students have been crucial. I would also highlight my SRP partner. We have had different approaches to the subject and have learned a great deal from one another because of this.

S3. It has been beneficial having seminar-based instruction. Moreover, one doesn't feel overwhelmed with lectures; instead, we have had productive teaching sessions packed with quality content. I think we have been shown numerous exemplary instances of the various topics we have covered, providing us with a clear vision of how we can use modelling, algebra, and SRPs in our future teaching roles. In our work on SRP regarding carbon capture and storage, I feel the lecturer provided excellent feedback and consistent guidance. This has played a role in the motivation for the task. It has also been essential to understand how the task on carbon capture and storage relates to our teaching profession.

S5. Having a passionate lecturer who displays a keen interest in the subject and provides relevant teaching for us as future teachers is invaluable. Fellow students who are also eager to learn and engage enhance the experience.

S6. The opportunity to discuss with others.

S7. The teacher has been incredibly skilled and understanding, recognising that this is new territory for many and genuinely wanting us, the students, to achieve meaningful learning outcomes.

S8. There has been ample opportunity to ask questions about any uncertainties along the way. Collaborative group work has also been advantageous. It's been beneficial to share experiences and pose questions to fellow students.

6 What challenges have you encountered in MA3001?

S1. One of the challenges has been the geological terminology in CCS. Additionally, understanding models with limited information has been tough. Another challenge was adhering to the maximum word count, which, while crucial for maintaining precision in phrasing, proved difficult. The same applied to the exam; 15 minutes pass quickly when there is much to tell.

S2. The most significant challenges stemmed from this being a hectic term. An 8-week placement [in school] concurrent with the subject resulted in limited time for project work at certain junctures. Delving into CCS was also challenging due to its intricate nature and my minimal prior knowledge.

S3. Working with the SRP was notably challenging, as this method was unfamiliar territory for someone in their third year [of the teacher education programme]. It was a different approach compared to what one might be accustomed to, but it became more effective as familiarity grew. The complex nature of the content on carbon capture and storage was also particularly daunting at the onset of the project. There was a lot to grasp, but collaboration with peers and the lecturer made the process more manageable over time. Working on the topic continuously was undoubtedly beneficial, culminating in a feeling of accomplishment when the final report was submitted.

S5. While the topic was unfamiliar, it also meant I learned a great deal about geology and carbon storage.

S7. It was a novel approach, which became a little bit demotivating when progress wasn’t swift.

S8. Initially, it was somewhat challenging to genuinely delve into answering the generating question and to pinpoint where to begin. Furthermore, commitments in other modules meant there wasn’t ample time to immerse oneself in the SRP, making it challenging to get fully acquainted with the task as getting into a good flow took time. Maintaining motivation was also a bit of a hurdle, especially during periods of stagnation.

7 If challenges in the course have been weakened or eliminated during the semester, what has contributed to this?

S1. In terms of geological terminology, it mostly involved reading up on the subject. Regarding the maximum word count, we have been critical of what should be included, simplified our sentences, and received assistance (from you) in eliminating non-essential elements.

S2. My partner’s interest in CCS, or a strong desire to understand the field, has motivated me to spend more time reading. Once I finished my placement, I also had more time for the subject.

S4. It was nice to meet the rest of the class and present to one another.

S5. The lecturer provided valuable feedback on our work, guiding us in a way that made us feel we were on the right track.

S7. The further we delved into the SRP, the easier it became to work with. Receiving feedback from the teacher also helped, reassuring us that our efforts were valuable.

S8. As one got into a good workflow, it became easier. It was also more straightforward to maintain motivation once we had found some answers and began working with the SRP. Being able to ask questions along the way when unsure has also been immensely helpful.

8 Please indicate your level of agreement with the following statements about the didactic paradigm of questioning the world.

	Strongly disagree	Slightly disagree	Somewhat agree	Totally agree
Not knowing the answer to a generating question in advance would be a serious obstacle for me as a future teacher.	0	8	0	0
I will probably try to collaborate with teachers in other subjects to implement SRPs in school.	0	0	4	4
I lack the tools to operate as a teacher in the new didactic paradigm.	2	4	2	0
I liked the fact that in an SRP you come across unexpected facts and ideas that you probably wouldn't have come across otherwise.	0	0	3	5

9 Comments on my answers to the statements in the previous point.

S1. Regarding the first question, about not knowing the answer, it would likely feel different from "usual" teaching. One just has to be confident that students may end up with more knowledge this way.

S2. I wouldn't say that not knowing the answer to a generating question in advance would be a major obstacle, but rather a disadvantage because I want to be in control of what my students get in terms of learning outcomes.

S3. Collaborating with other teachers on SRPs in the school, I believe, is a good way to execute interdisciplinary projects. It doesn't require any extra equipment, but it demands that we, as teachers, embrace a far more open-ended task than we're used to, where together with students we can find answers to, for instance, generating questions about climate challenges.

S6. I plan to get teachers on board who are keen to test out SRP as a method; if not, this can also be applied in other contexts.

S7. I'm not entirely sure how I would specifically set something up in school based on the ATD. It would have been nice to get some concrete examples while we were learning about it.

S8. I would like to use SRPs in teaching, but will probably have some other teachers with me in the beginning. It can be slightly uncomfortable not knowing all the answers in advance, but it will indeed be a good learning experience for me.

10 Through MA3001, have you developed a new understanding of any bodies of knowledge (different from your previous understanding)? If so, what does this change entail?

S1. I have gained a deeper understanding of ATD. I actually thought ATD was a theory resulting from the paradigm of questioning the world, but I realise it's the other way around.

S4. Yes, refer my answer to Question 4.

S5. New understanding within carbon capture and storage, and more knowledge in executing an SRP where we don't know what the final results will be.

S8. Working with SRPs also offers more opportunities for students to think critically and engage in problem-solving.

11 Is there anything you wish were done differently in MA3001? If yes, please explain.

S1. It would have been interesting to know how the ATD differs from the didactics we learn at ILU [Department of teacher education, NTNU]. I can provide the reading list from the didactic course if necessary.

S2. I wish the course had started earlier in the semester, reducing the workload during my placement period.

S4. Regarding the content related to ATD, it might have been insightful to discuss how to introduce it to a secondary school class. The didactic contract for an SRP was touched upon, but a practical discussion on its implementation would have been helpful.

S7. I would have appreciated a concrete example of how one could use ATD and SRPs in schools. Just briefly, as carrying out an SRP [in the course] was very educational, but I'm unsure how one would approach it in a school setting.

12 Free comments.

S2. I am very pleased with this course and found the tasks and themes intriguing. The lecturer has been academically proficient, skilled at adapting, and motivating.

S3. I would say that this has been the didactic course from which I have learned the most. It's the course that has provided the most academic relevance for the job I'll have after completing my studies. It has been educational both in learning how to use modelling and algebra in the exploratory aspect of mathematics, and the SRP has brought new ideas to interdisciplinary work in schools (it can obviously be used within the subject too).

Analysis of Student Survey Responses on the ATD, SRPs, and CCS

The ATD and Working Methods: Relevance to the Mathematics Teaching Profession

The majority (7 of 8 students) found the subject matter related to the ATD to be very relevant. This shows a high appreciation and understanding of the anthropological perspective within mathematics education. The comments emphasize the importance and relevance of the ATD for their development as prospective mathematics teachers. Of the 8 students, 5 found the SRP on CCS to be very suitable as working method in the course. This showcases a general appreciation of the SRP method when it comes to fostering professionalism. The comments on assignment and working methods reflect a range of perspectives on the SRP method: S1 finds SRP relevant especially in the context of sustainable development, which can be seen as an application of the ATD where societal challenges are integrated into mathematical practices. S2 acknowledges the complexity of integrating subjects other than mathematics within CCS, hinting at the intricate balance needed in praxeologies. S6 provides mixed feedback, suggesting that while SRPs might be fun and relevant, it might not align with everyone's "teaching preference."

Surprising Elements in MA3001

S2 and S4 highlighted unexpected aspects of the course that turned out to be positive: S2 was pleasantly surprised by the level of involvement and influence the students had in shaping the course. S4 was initially taken aback by the complexity of the SRP topic, but the apprehension faded as the objectives became clearer.

With respect to interests in and relevance of MA3001, the majority of responses point towards the appreciation of the interdisciplinary approach of the course and the various teaching methods. S1 found the ATD and the multidisciplinary nature of CCS interesting. S2 and S8 appreciated the group work and collaboration in the course. S3 and S4 emphasized the value of the SRP research method and its applicability in teaching. S7 highlighted both the ATD and SRP as engaging different approaches to learning.

Foundational Studying Factors in MA3001

The team-centred approach and the seminar discussions across teams stand out as a recurring theme among the students. S1, S2, S3, and S8 spoke about the significance of collaboration, group work, and the interactions with their peers in enhancing their learning experience. The role of the lecturer was highlighted by several students (S2, S3, S5, and S7), with emphasis on her adaptability, passion, and proficiency in guiding students. S3 and S5 mentioned the seminar-based instruction and quality content as critical contributors to their understanding and engagement. S6 valued discussions with peers, suggesting the course fostered an environment of active engagement and peer learning. S7 praised the instructor's empathetic and skilled teaching approach, acknowledging the novelty of the subject for the students. S8 stressed the importance of open communication and the opportunity to ask questions, reflecting a supportive studying environment.

Overcoming the Hurdles in MA3001

Praxeological Context: The praxeology of the course, involving methodologies like SRP and intricate content like CCS, was a significant source of challenge. However, this challenge was alleviated through various tools, including self-study, feedback from the instructor, and peer collaboration.

Ecosystem of Support: It is evident that the course ecosystem, which encouraged open communication, feedback, and peer support, played a vital role in aiding students to navigate challenges. The lecturer's proactive role and the collaborative spirit of the class were pivotal.

Motivational Aspects: The motivational challenges were countered through peer influence, achieving clarity in the subject, and receiving guidance and reassurance from the lecturer. This indicates that motivation, while internal, can be nurtured by external influences and a supportive learning environment.

In the context of the ATD, understanding these challenges and resolutions helps identify the praxeological obstacles students might face in such courses and offers insights into crafting more effective didactic strategies for similar educational settings in the future.

Beyond Traditional Teaching: Challenges Navigating a New Didactic Paradigm

Praxeological Context: The responses and comments reflect a student body that is adapting to a new teaching paradigm, one that emphasises exploration, team-work, and interdisciplinarity. While there is acknowledgment of its value, there are also concerns about maintaining desired learning outcomes and a clear transition from theoretical understanding to practical application.

Personal and Institutional Adaptation: The personal comfort and confidence of students play a crucial role in determining their readiness to embrace the new didactic paradigm. There is a sense of cautious optimism, coupled with a desire for more tangible examples and support in implementing SRPs as a new method of teaching (supervision) and learning (study) in their future profession as teachers.

Interdisciplinary Collaboration: There is a clear inclination towards interdisciplinary collaboration. This signals a move away from isolated subject teaching towards a more integrated approach that mirrors real-world problem solving.

The students' responses and comments about challenges encountered in the course suggest that while there is a recognition of the value of the paradigm of questioning the world, there is also a need for more robust support systems, practical examples, and collaboration to ease the transition for future teachers.

Strengthened Understanding of Bodies of Knowledge

In their reflections on MA3001, students have demonstrated both a deepening and reshaping of their understanding of certain topics. S1 underwent a pivotal shift in understanding the relationship between the ATD and the paradigm of questioning the world. S4's response, while referring to a previous comment, suggests an evolved grasp of the course's content. S5's insights span a broad spectrum, encompassing both the intricate subject of carbon capture and storage and the intricate pedagogical process of SRPs where outcomes are unpredictable. Lastly, S8 perceives SRPs not just as educational strategies but as instrumental tools to cultivate critical thinking and problem-solving skills in students. This reflection embodies the praxeological dimension of the ATD, highlighting the importance of understanding the types of tasks—mathematical as well as didactic tasks—and the techniques and technologies to solve them.

Students' Desires and Appreciations for MA3001

Through the ATD lens, feedback from MA3001 students highlights key insights into knowledge learnability (partially pertaining to epistemology) and a keenness to translate into the school context acquired ways of studying (pertaining to praxeological analysis and engineering). These insights can be condensed into four main themes, which are briefly outlined below.

Shifting Epistemological Foundations: Student responses underscore their new relations to teaching and learning, marked by a willingness to embrace open-endedness and interdisciplinary teamwork. Several comments indicate students' recognition that engaging in interdisciplinary approaches, like SRPs, can lead to a richer, more integrated perspective on knowledge and problem-solving.

Praxeological Considerations: Feedback points to a desire for practical applications and real-world teaching scenarios, emphasising the praxeological dimension of the ATD. Students are eager to understand how to apply in secondary education the techniques and technologies they have acquired. They seek clarity on transitioning from theory to practice, underlining the importance of tangible examples and techniques for introducing SRPs in schools.

Links With Other Theoretical Frameworks: The comparison between the ATD and the didactics studied at ILU (Department of Teacher Education, NTNU) emerges as a significant theme. One of the students exhibits curiosity about how the ATD complements or contrasts with other theoretical frameworks they have been exposed to. This cross-referencing is open to progress and yet to come.

Affirmation and Satisfaction: Notably, while there are suggestions for improvement, several students communicated profound satisfaction with the course. Students appreciated its academic depth and the versatility it brought to their future teaching careers. The blend of modelling, algebra, and the introduction of SRPs has significantly impacted their perspective on interdisciplinary work in schools. Furthermore, the course's ability to stimulate interest and maintain engagement, backed by supposedly relevant and adaptable lecturing, reinforces its efficacy and resonance with students.

Concluding Comments

Students manifested a comprehensive appreciation of mathematical modelling, with several highlighting how the course has enhanced their understanding in this domain. A transformative learning experience was evident as students navigated from initial unfamiliarity to deeper comprehension, attributed to the ATD framework. Algebra was perceived not merely as an abstract domain but as a vital modelling tool, echoing the anthropological dimension of the ATD. Feedback on modelling assignments suggested the course's efficacy in fostering praxeological practices, with a notable emphasis on parameters. These parameters, vital in bridging algebraic techniques and technological depth in modelling, were seen as crucial in the wider mathematical context. The chapter also explored the course's alignment with institutional expectations, particularly the LK20 National Curriculum. While the course was largely deemed beneficial, some feedback highlighted areas for enhancement, especially regarding the clear integration of algebra and its applicability in broader subjects, like Carbon Capture and Storage (CCS).

As mentioned in the introductory section, S1, S4, and S5 had taken the MA3061 course (n.d.), which introduced the ATD and SRPs, while S2, S3, S6, S7, and S8 had not. Most of the challenges and desires for changes were voiced by S6, S7, and S8. This aligns with the expectation that they might find the course content and methodologies more demanding compared to the 4th year students who had previously taken MA3061. S2, though a 4th year student, had not taken MA3061. However, she faced no significant challenges, possibly due to collaborating with a teammate who had completed MA3061. Similarly, S3, a 3rd year student, experienced few challenges, which might be attributed to his 4th year teammates who had completed MA3061. These observations underscore the potential complexities of teaching a course to a mixed group of students, where some have prior knowledge of the theoretical framework and methodologies, while others do not.

The MA3001 course offered a deep engagement with content and didactic methodologies. Student feedback reveals an overall positive reception but indicates areas for enhancement, particularly concerning translating theoretical concepts into practical applications—which constitutes a research programme to be considered. The blending of modelling, algebra, and SRPs significantly impacted students' views on interdisciplinary teaching, making the course a notable experience in their academic journey.

Finally, I would like to briefly comment on the prerequisites for operating within the paradigm of questioning the world. Regarding supervision and execution of an SRP, neither the teacher nor the students are expected to have studied in advance the fields of knowledge pertinent to the generating question. For the SRPs on CCS, this encompassed geology, physics, and chemistry. The teacher and students study and learn side by side, each embodying what the ATD describes as a Herbartian and procognitive attitude (Chevallard, 2015). Such joint exploration can be challenging, as it redefines traditional roles and expectations. It requires both parties to venture into unfamiliar territories, fostering collaboration and ongoing discovery. This shift necessitates a new didactic contract and a novel *topos* for both teacher and students, topics I have discussed in the concluding section of Chapter 2 (this volume).

References

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Cham: Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Chevallard, Y. (2019). Introducing the Anthropological Theory of the Didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves%20Chevallard.pdf

Chevallard, Y., & Bosch, M. (2019). Anthropological theory of the didactic (ATD). In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 1–8). Cham: Springer. https://doi.org/10.1007/978-3-319-77487-9_100034-1

Florensa, I., Bosch, M., & Gascón, J. (2015). The epistemological dimension in didactics: Two problematic issues. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 2635–2641). Prague: Charles University and ERME. https://hal.science/hal-01289434/document

MA3001. (n.d.). *Mathematical modelling using study and research paths*. https://wiki.math.ntnu.no/ma3001/2022v/matematiskmodellering/start

MA3061. (n.d.). *Design and analysis of mathematics teaching*. https://www.ntnu.edu/studies/courses/MA3061/2021#tab=omEmnet

Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Co-funded by the European Union

Appendix

Appendix A – Reading List

Literature for Mathematical Modelling Using Study and Research Paths

Core texts (for all students)

Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. https://doi.org/10.1007/978-3-319-05389-9_5 [Pages to read: 67–73.]

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Niss, M. (2015). Prescriptive modeling: Challenges and opportunities. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modeling in education research and practice: Cultural, social and cognitive influences* (pp. 67–79). Springer. https://doi.org/10.1007/978-3-319-18272-8_5

Strømskag, H. (2022, 21 January). *Et notat om det herbartiske skjemaet – En dynamisk modell for studiet av et spørsmål* [A note on the Herbartian schema—A dynamic model for the study of a question]. Department of Mathematical Sciences, NTNU.

Strømskag, H. (2022, 23 March). *Et notat om begrepet «didaktisk kontrakt»* [A note on the concept of "didactic contract"]. Department of Mathematical Sciences, NTNU.

Strømskag, H. (2022, 8 April). *Om dialektikken mellom systemer og modeller: Et eksempel med bremselengde* [On the dialectic of systems and models: The case of braking distance]. Department of Mathematical Sciences, NTNU.

Strømskag, H., & Chevallard, Y. (2022a). Conditions for revitalizing the elementary algebra curriculum. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the 12th Congress of the European Society for Research in Mathematics Education* (pp. 620–627). Free University of Bozen-Bolzano and ERME. https://hal.science/hal-03745443v1/document

Supplementary texts (for all students)

Barquero, B., Bosch, M., & Wozniak, F. (2019). Modelling praxeologies in teacher education: the cake box. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the 11th Congress of the European Society for Research in Mathematics Education* (pp. 1144–1151). Utrecht University and ERME. https://hal.archives-ouvertes.fr/CERME11/hal-02408705v1

Bauman, Z. (2007). Forbrukersamfunn [Consumer society] (A. Peretz, Trans.). In G. E. Schjelderup & M. W. Knudsen (Eds.), *Forbrukersosiologi: Makt, tegn og mening i forbrukersamfunnet* (pp. 229–250). Cappelen.

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education, 12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves Chevallard.pdf

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed., pp. 201–204). Routledge.

Doerr, H. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 69–78). Springer. https://doi.org/10.1007/978-0-387-29822-1_5

Education is key to addressing climate change. (n.d.). United Nations. Climate Action. https://www.un.org/en/climatechange/climate-solutions/education-key-addressing-climate-change

Ely, R., & Adams, A. E. (2012). Unknown, placeholder, or variable: What is *x*? *Mathematics Education Research Journal*, *24*, 19–38. https://doi.org/10.1007/s13394-011-0029-9

Markulin, K., Bosch, M., & Florensa, I. (2021). Project-based learning in statistics: A critical analysis. *Caminhos da Educação Matemática em Revista*, *11*(1), 200–220.

Oversby, J. (2015). Teachers' learning about climate change education. *Procedia – Social and Behavioral Sciences*, *167*(8), 23–27. https://doi.org/10.1016/j.sbspro.2014.12.637

Strømskag, H., & Chevallard, Y. (2022b). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Thagaard, T. (2009). *Systematikk og innlevelse: En innføring i kvalitative metoder* [Systematics and empathy: An introduction to qualitative methods] (3rd ed.). Fagbokforlaget. [Pages to read: 62–63.]

Texts for the individual team

In addition to the texts listed above, the required reading for each team includes the literature and other resources utilized in their SRP on carbon capture and storage.

Appendix B

On the Dialectic of Systems and Models: The Case of Braking Distance

Heidi Strømskag

Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, Norway

Introduction

The rapid, abrupt halt of a vehicle in motion: a phenomenon that has saved countless lives and yet, remains shrouded in mystery to many. What really goes into this process? How does the intricate interplay between speed, road surface, and friction determine the braking distance of a vehicle? In this manuscript,[13] I try to unravel this seemingly complex system and delve into its basic mechanics. The ultimate goal is to demonstrate how the design and execution of modelling tasks can enhance students' understanding of the dialectic of systems and models.[14] This interplay between the real-world dynamics of systems being studied on the one hand, and properties of models being constructed, on the other, plays a pivotal role in the didactic paradigm of *questioning the world*, proposed by Chevallard (2015). The principles of this paradigm, foundational to the mathematics education course, "Mathematical Modelling Using Study and Research Paths," taught at the Norwegian University of Science and Technology, are the focus of this discussion.

This manuscript navigates mathematical modelling in teacher education, specifically focusing on the interaction between the system being investigated and the model under construction. The discussion primarily takes its basis from the study titled "Elementary algebra as a modelling tool: A plea for a new curriculum" by Strømskag and Chevallard (2022). Starting with an exploration of a distinctive modelling task on braking distance, sourced from a textbook for Grade 11 (students aged 16–17), it dissects the solutions and critically assesses the interplay between the system and its model within the task. The manuscript further ventures into enhancing the conditions that promote knowledge creation about the investigated system, integrating a concise overview of the phenomenon of *friction* to illuminate the dynamics between the system under investigation and the model under construction. This leads to the unveiling of a revised task about a vehicle's braking distance, meticulously adapted for student teachers, along with a proposed solution. Continuing its trajectory, the manuscript assesses potential adaptations of the revised modelling task to align with varying educational levels, signifying a move towards more sophisticated task design. This advancement is further highlighted in the closing section, proposing a novel task design suitable for Grades 8–13.

[13] This manuscript builds upon a note I developed in 2022, as part of our exploration of braking distance modelling in the course.
[14] In this manuscript, the term "students" is used to encompass both university students and school pupils.

A Modelling Task From a School Textbook

In the Norwegian textbook for Grade 11, *Sinus 1T* (Oldervoll et al., 2020, p. 375)[15], under the headline "Polynomial Regression", the following task is found:

1.130

The stopping distance for a car in motion hinges on both the driver's response time and the braking distance.[16] The table below outlines the stopping distance, denoted by $S(x)$, in metres corresponding to certain speeds in kilometres per hour for a specified car and a specified driver.

x (km/h)	40	60	80	100
$S(x)$ (m)	24	45	73	108

a) Plot the data points from the table in a coordinate system and elucidate why a quadratic function seems to be a suitable fit.

b) Determine the quadratic function, S, that most accurately represents the given data. Sketch the graph incorporating the data points. Ensure that the expression of the function is accurate to three decimal places.

c) Find graphically the speed that would result in a stopping distance of 150 metres.

d) Find graphically the stopping distance corresponding to a speed of 90 km/h.

A Condensed Solution to the Task

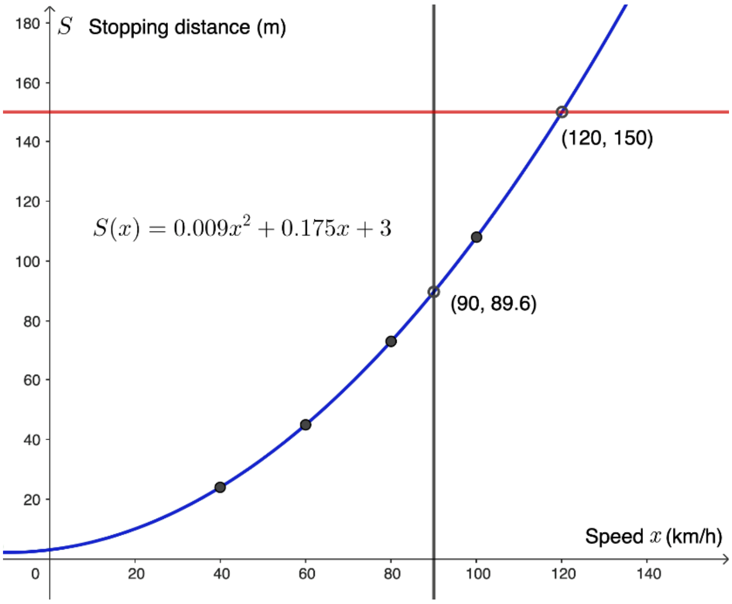
Given that the task is located in the textbook under the headline "Polynomial regression", it is clear that regression analysis is the expected technique for identifying a second-degree polynomial that best fits the given data set. We use GeoGebra and arrive at the following function: $S(x) = 0.009x^2 + 0.175x + 3$. The graph is depicted in Figure 1, where solutions to the final two subtasks are illustrated by the points (90, 89.6) and (120, 150).

[15] The task has been translated into English by the author.

[16] Stopping distance = Reaction distance + Braking distance.

Figure 1

A Graphical Solution to the Textbook Task



Examining Relationships Between the System and the Model in the Task

The model constructed is a second-degree polynomial, $S(x) = ax^2 + bx + c$, with parameters $a = 0.009$, $b = 0.175$, and $c = 3$. While this model fits the given data points, its parameters present limitations as they lack clear physical interpretation. When the speed is zero ($x = 0$), the model counterintuitively suggests a stopping distance of 3 meters ($c = 3$), which opposes real-world scenarios; a stationary vehicle should have no stopping distance.

For positive values of a and b, the quadratic term ax^2 and the linear term bx both suggest that the stopping distance increases with speed, which is generally true. However, the specific coefficients ($a = 0.009$ and $b = 0.175$) have no clear interpretation in the system being studied. They fail to directly correlate with physical factors influencing stopping distance like brake efficiency, tyre grip, and variations in driving conditions, such as road surface. Additionally, these coefficients do not explicitly relate to the complex interplay of psychological perception and physiological response involved in the driver's reaction time.

While the model, shaped by regression analysis, may conform well to the given data, it provides limited insight into the underlying system being modelled. There is a discernible disconnect between the parameters of the model and the real-world dynamics of the system. As a result, the second-degree polynomial model is inapplicable for producing knowledge about the system being investigated, thereby constraining the educational potential of this modelling activity. Might we not then ponder the true essence of this task's significance?

Refined Conditions for Generating Knowledge About the System Studied

In the context of mathematical modelling in education, a fundamental consideration is how we—as mathematics teachers and educators—can amplify the educational value of a modelling activity. One key strategy in this endeavour involves *questioning*. Indeed, asking insightful questions about the system being investigated creates opportunities for exploration and learning by illuminating the defining properties of the system. These properties, or system parameters, and their interrelationships, form the crux of our understanding. Thus, encouraging a study of these parameters aids in unearthing the intrinsic structure and behaviour of the system. The aim is therefore to design modelling tasks—that is, to create conditions—so that students may construct models that *produce knowledge* about the systems under consideration (Chevallard, 1989). This is related to the first stage of the modelling process: the delineation of the system we intend to study, specifying the attributes that are relevant to the study we want to make of this system.

Let us go back to the system where a vehicle in motion brakes abruptly on a horizontal surface. Now, our aim is to create opportunities for students to construct a model that allows them to generate knowledge about the system under investigation. In this scenario, the exclusive focus is placed on the braking distance, as it allows us to isolate and better understand the effects of physical forces at play when the brakes are applied. The reaction distance, although important in real-world situations, introduces additional parameters like human perception and response time, which are outside the scope of our current physics-based exploration.

A key factor in the physics of braking distances is the *friction coefficient*. The friction force between the vehicle's tyres and the road surface is what allows a vehicle to stop. When you apply the brakes, you are essentially using the vehicle's brake system to convert kinetic energy into heat via friction. Changes in this coefficient will greatly affect the system being examined. For instance, on wet or icy roads (where the friction coefficient would be lower), the braking distance will be significantly longer. The next section provides some background knowledge about this phenomenon.

Friction: A Quick Overview

This section provides a concise synopsis of key concepts about friction, largely based on the work of Grimenes et al. (2011). Friction is a force that opposes the movement of one solid object sliding or rolling over another. There are several types of friction, including static friction (friction between objects that are not moving relative to each other), kinetic or sliding friction (friction between objects that are moving relative to each other), and rolling friction (friction that acts on rolling objects).

It is friction that naturally halts various forms of everyday motion unless other forces intervene. We should appreciate the existence of friction, as it is indispensable for many of our daily activities. For instance, without friction, we would be unable to walk, cycle, or drive a vehicle. When we walk, friction provides the necessary grip that allows our feet to push against the ground without slipping. This interaction propels us forward, a consequence of Newton's third law of motion. It is the friction between our feet and the ground that enables us to maintain our balance and manoeuvre in desired directions. In cycling or driving, friction plays a comparable role. The tyres of a bicycle, or a vehicle, grip the surface due to friction, propelling the vehicle forward when power is applied. Additionally, the friction between the bicycle's brake pads and the wheels, or a vehicle's brake system and its wheels, provides the stopping power when required. In the absence of friction, the tyres would slide uncontrollably along the surface, rendering steering and control of the vehicle virtually impossible.

Mathematical Representation of Friction

The force of friction can be represented by the following formula: $F = \mu N$, where F is the force of friction, μ (mu) is the coefficient of friction, which is a dimensionless scalar value that describes the ratio of the force of friction between two bodies to the force pressing them together. N is the normal force, or the perpendicular force with which the surfaces push against each other. In most introductory physics problems, the scenario is something like a box (object) on a flat surface (another object, like a table or the Earth). In these cases, the normal force is equal to the weight of the box, because the surface needs to exert an upward force equal to the weight of the box to prevent it from going through the surface, in accordance with Newton's third law of motion.

Coefficient of Friction

The coefficient of friction (μ) is dependent on the materials involved in the frictional interaction. Typically, the value of μ is established empirically. Table 1 presents a selection of friction coefficient values across different material pairs.

Table 1

Typical Friction Coefficients Between Various Materials

Material 1	Material 2	μ (Friction coefficient)
Steel	Steel	0.6
Steel	Ice	0.05
Steel	Teflon	0.04
Ice	Ice	0.03
Rubber	Dry asphalt	0.7
Rubber	Wet asphalt	0.2
Rubber	Ice	0.02
Wood	Wood	0.3
Hip joint	Hip joint	0.003

Note. The table is adapted from Grimenes et al. (2011, p. 68).

Spotlight on the Interaction Between System and Model

Building upon the analysis of the textbook task, the vision of modelling, and synopsis of friction from the three preceding sections, the following task—inspired by one found in Grimenes et al. (2011, p. 69)—serves as a reformulation of the textbook task. It delves deeper into the theoretical underpinnings of a scenario involving a vehicle braking and sliding on a horizontal surface. This endeavour is geared towards exemplifying a modelling task and its potential solutions, with an emphasis on the intricate interplay between the system under study and the model developed.

Braking Distance of a Vehicle: A Revised Task for Student Teachers

A vehicle is travelling at the speed of 72 km/h on a horizontal road. The vehicle needs to brake suddenly, and the driver presses the brake pedal so hard that the brakes lock. The vehicle then slides along the road until it comes to a stop. We assume that the coefficient of friction between the tyres and the road during sliding is 0.7—that is, the system to be studied involves frictional interaction between rubber and dry asphalt (according to Table 1).

a) Develop a model that allows you to provide answers in response to the following prompts: How long is the braking distance in the described scenario? Provide examples of how different road conditions and varying speeds can impact the braking distance of a vehicle sliding after brake lock.

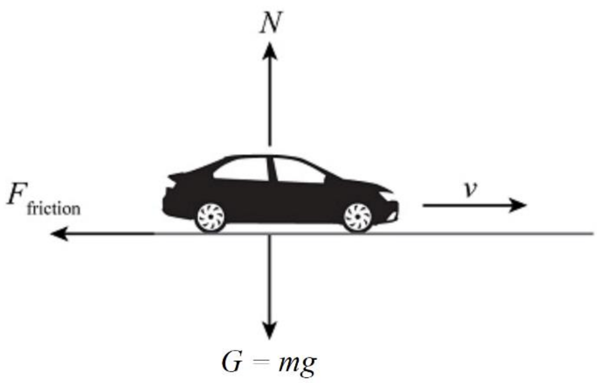
b) Examine the interplay between the system and the model: What insights does the constructed model yield about the system under investigation? Can you describe any specific constraints or limitations associated with this model?

Proposed Solution to the Task

a) In order to determine the braking distance, it is imperative to understand the vehicle's acceleration. This understanding can be gained through Newton's laws of motion. As a first step, we study the forces acting on the vehicle, illustrated in Figure 2. For additional insight, you might consider this tutorial from the website "The Physics Classroom":[17] https://www.physicsclassroom.com/Physics-Video-Tutorial/Newtons-Laws/Force-Of-Friction/Video.

Figure 2

Forces Acting on the Vehicle



In the vertical direction, the force of gravity (G) acts downwards, and the normal force (N) acts upwards. Since there is no acceleration in the vertical direction, Newton's first law states that these forces are equal, $N = G$. As for the direction of the friction force for bodies sliding, it is opposite to the direction of motion. Experiments show that the frictional force acting on a body sliding on a surface has little dependence on speed and on the contact surface area. Experiments have shown that the friction force, $F_{friction}$, is approximately proportional to the normal force on the body from the surface, that is, $F_{friction} = \mu N$, where μ is the friction coefficient for sliding friction. Furthermore, because $N = G = mg$, we have that $F_{friction} = \mu mg$.

[17] "The Physics Classroom" is an online, free to use physics website developed by Tom Henderson, primarily for beginning physics students and their teachers. For more details, see at https://www.physicsclassroom.com/.

In the horizontal direction, only the friction force acts against the movement. We then determine the acceleration, a, using Newton's second law: $\Sigma F = ma$. In this case, $\Sigma F = -F_{\text{friction}}$. Therefore, $-F_{\text{friction}} = ma$, and considering that $F_{\text{friction}} = \mu N$ and $N = mg$, it follows that $-\mu mg = ma$. This can be simplified to: $a = -\mu g$.

We can now find the braking distance using the equation of motion for the constant acceleration case: $v^2 - v_0^2 = 2ad$ (Grimenes et al., 2011, p. 32). The parameters are defined as follows:

- a denotes the constant acceleration of the object (which in the case of braking is negative);
- g denotes the acceleration resulting from gravity (commonly approximated as 9.81 m/s^2 at sea level on Earth);
- m denotes the mass of the object (which in this case is the mass of the vehicle);
- d denotes the distance travelled (which in this case is equal to the braking distance);
- v denotes the velocity of the object in the final state (which in this case is equal to 0);
- v_0 denotes the speed in the initial state (which is the speed at the moment the braking starts);
- μ denotes the coefficient of friction (which depends on the properties of the materials in contact, in this case, the rubber of the tyres and the road surface).

We transform the equation $v^2 - v_0^2 = 2ad$, and get d expressed by the other parameters: $2ad = -v_0^2$ which gives that $d = \frac{-v_0^2}{2a} = \frac{-v_0^2}{2\cdot(-\mu g)} = \frac{v_0^2}{2\mu g}$. This equation is our model for the braking distance of a vehicle travelling on a horizontal road and braking suddenly at speed v_0:

$$d = \frac{v_0^2}{2\mu g}.$$

Using the given values, we can find the braking distance, d. We want to calculate the braking distance in metres and therefore need to convert the speed to m/s. By converting 72 km/h to metres per second, we get 72,000 metres/3,600 seconds, which corresponds to a speed of 20 m/s. We insert this into the equation and get the following result: $d = \frac{(20\text{ m/s})^2}{2\cdot 0.70\cdot 9.81\text{ m/s}^2} \approx 29$ m. This means that at a speed of 72 km/h, the braking distance on dry asphalt ($\mu = 0.7$) will be approximately 29 m.

Different Situations With Varying Parameter Values

We can use the model to calculate the braking distance on wet asphalt at the same speed, 20 m/s. According to Table 1, $\mu = 0.2$ in this case, and we get $d = \frac{(20\text{ m/s})^2}{2\cdot 0.2\cdot 9.81\text{ m/s}^2} \approx 102$ m.

Let us calculate the braking distance on dry asphalt at the speed of 90 km/h. This speed is equivalent to 25 m/s (as 90,000 metres/3,600 seconds = 25 m/s). Substituting into the model, we find that $d = \frac{(25\text{ m/s})^2}{2\cdot 0.7\cdot 9.81\text{ m/s}^2} \approx 45.5$ m. This indicates a braking distance of approximately 45.5 meters.

What if we brake suddenly on wet asphalt at a speed of 90 km/h? This will give a braking distance of $d = \frac{(25\text{ m/s})^2}{2\cdot 0.2\cdot 9.81\text{ m/s}^2} \approx 159$ m. The resulting braking distance of 159 metres is alarmingly long, emphasising the importance of reduced speeds and increased caution when driving on wet or slippery road conditions.

Co-funded by the European Union

b) The model $d = \frac{{v_0}^2}{2\mu g}$ permits generation of the following knowledge:

From our model, we can deduce that the braking distance is independent of the vehicle's mass. This suggests that for any vehicle, given the same speed and friction coefficient, the braking distance will remain constant. Furthermore, the model shows that the braking distance increases with the square of the initial speed (i.e., the speed at which the brakes are applied). To put it another way, if the initial speed changes by a factor of k (from v_0 to kv_0), the braking distance will change by the square of this factor (from d to k^2d).

Let us calculate the braking distance during sudden braking on wet asphalt, when the speed is reduced from 20 m/s to 10 m/s. With a speed of 10 m/s (36 km/h), we get $d = \frac{(10\text{ m/s})^2}{2\cdot 0.2\cdot 9.81\text{ m/s}^2} \approx 25.5$ m. We see here that when the speed is reduced from 20 m/s to $\frac{1}{2}\cdot 20$ m/s, the braking distance is reduced from 102 m to 25.5 m = $\frac{1}{4}\cdot 102$ m. This provides a concrete example of the quadratic proportionality property of the model.

Some Constraints and Limitations

The model $d = \frac{{v_0}^2}{2\mu g}$ is based on the assumption that there is no ABS (Anti-lock Braking System) operating in the system being studied. ABS is a technology that prevents the wheels from locking during braking. Such a mechanism helps to maintain road grip and vehicle stability during braking and can therefore reduce braking distance.

A limitation of the model is that it relies on simplifications and assumptions to describe the braking distance. It assumes uniform acceleration throughout braking and ideal conditions such as a constant coefficient of friction and an even surface. In reality, braking distance can be affected by several factors, including road conditions, tyre condition, vehicle design and driver reaction time. The model therefore provides a simplified description of the system and may have limitations when used to predict accurate braking distances in complex situations.

Lastly, it should be noted that the values of the friction coefficients used in the model are empirically determined and may carry some uncertainty. These coefficients are based on previous observations and experiments under various driving conditions.

Tailoring the Revised Modelling Task to Various Educational Levels

This section delves into the adaptation of the task for different educational levels. The task and its solution, outlined in the previous section, are optimally designed for students already equipped with fundamental knowledge in mechanics. However, its inherent flexibility enables its modification to serve as a learning tool for students just beginning their journey in basic mechanics. This adaptability largely depends on the specific objectives of the modelling task and the time allocation for its execution.

Moreover, the task can be altered in a manner that sidelines the need for an understanding of the mechanics intrinsic to the system being studied. One potential strategy could involve offering two distinct sets of data for system analysis, customised for dry and wet asphalt conditions. These datasets would correlate the vehicle's speed with its respective braking distance under different speed scenarios. For instance, the observations displayed in Table 2 could serve this purpose. (Keep in mind that while students will need to convert speed values from km/h to m/s, it would be beneficial for them to identify this requirement on their own.) Concurrently, students are taught about the phenomenon where a vehicle's braking distance is directly proportional to the square of its speed upon braking.

Table 2

Speed vs Braking Distance on Varying Road Conditions

Speed (km/h)	Braking distance on dry asphalt (m)	Braking distance on wet asphalt (m)
20	2,25	7,87
40	8,99	31,46
60	20,23	70,79
80	35,96	125,85
100	56,18	196,64

This joint presentation of empirical data and underlying principles offers a suitable framework for students to develop the model $d = kv^2$ and calculate the proportionality constant k for both conditions. Such a process not only enhances students' practical application skills in mechanics, but also deepens their understanding of the nature of scientific inquiry and the process of transforming real-world phenomena into mathematical models.

Advancing Task Design

This section introduces a task which centres around the crafting of modelling assignments for secondary school. Mirroring the structure of the braking distance assignment, the student teachers were encouraged to initiate from a textbook task. However, in this instance, the selection of the task was left to their discretion, with the objective to revise and enhance the connection between the system and the model outlined in the original task scenario. The exact wording of the task, including detailed subtasks, is provided in the following section.

Design of Modelling Tasks for Grades 8–13

Identify a modelling task in a secondary school textbook that, in your opinion, inadequately supports the generation of knowledge about the system it aims to model. Your assignment is to design a new task, pertaining to the same system, in a way that strengthens the connection between the system under study and the model to be constructed. Subtasks a–c formulated below are meant to help you in studying the system to be examined, in order to prepare your own task design. Moreover, the note with a proposed solution on braking distance is available for your perusal, and could serve as a useful reference or spark of inspiration in your own work.[18]

a) Delineate the system S that will be the focus of your study. State the initial question Q that you aim to answer regarding S. Define the relevant parameters that are crucial for studying S, along with their respective relationships.

[18] The note referenced herein is elucidated in Footnote 1, found on page 1 of this manuscript.

b) Build up a suitable model S' that can effectively address the initial question Q about the system S. Explain how this model S' can not only answer Q but also produce additional knowledge about the system S. Provide a brief overview of the background knowledge required for constructing the model S'.
c) Discuss the relationships between the system studied (S) and the model constructed (S') including the affordances (strengths and advantages) and constraints (limitations and assumptions) of the chosen model in representing the real-world dynamics of S.
d) Utilising the knowledge generated through the previous subtasks, devise a modelling task suitable for students in Grades 8–13 (select one of the grades). The task should explore the same system (S) and answer a question (Q), concerning S. Q can either be the same or different from the one you addressed previously. Strive to design the task in a way that enables students to develop knowledge about the real-world dynamics of S, while ensuring it is both appropriately challenging and manageable for the chosen grade level.

In closing, it should be recognised that the eventual impact of the proposals and discussions within this manuscript, as they pertain to secondary education, represents an open question, one that awaits further scholarly exploration.

References

Chevallard, Y. (1989). Le passage de l'arithmétique à l'algébrique dans l'enseignement des mathématiques au collège—Deuxième partie. Perspectives curriculaires : la notion de modelisation. *Petit x*, *19*, 45–75. https://publimath.univ-irem.fr/numerisation/PX/IGR89002/IGR89002.pdf

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Grimenes, A. A., Jerstad, P., & Sletbak, B. (2011). *Grunnleggende fysikk for universitet og høgskole* [Basic physics for universities and university colleges]. Oslo: Cappelen Damm.

Oldervoll, T., Svorstøl, O., Vestergaard, B., Gustafsson, E., Osnes, E. R., Jacobsen, R. B., & Pedersen, T. (2020). *Sinus 1T: Matematikk* (4th ed.). Oslo: Cappelen Damm.

Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Co-funded by
the European Union

Appendix C – PowerPoint-Slides

C1 – Introduction

APPENDIX C1

Mathematical Modelling Using Study and Research Paths

INTRODUCTION

Heidi Strømskag
Department of Mathematical Sciences
Norwegian University of Science and Technology

Constituents of the course

Unit 1	Modelling of systems using algebra as a modelling tool A new didactic paradigm
Duration	8 × 45 min.
Organization	5 × 45 min. (lectures) + 3 × 45 min. (exercise classes).
Topics	The paradigm of questioning the world. Modelling of systems. Algebra as a modelling tool.

Unit 2	Some tools from the anthropological theory of the didactic
Duration	2 × 45 min.
Organization	2 × 45 min. (lecture).
Topics	Didactic system; Study and research paths; Herbartian schema.

Unit 3	Climate change
Duration	7 × 45 min.
Organization	1 × 45 min. (lecture) + 4 × 45 min. (SRP_{pilot}) + 2 × 45 min. (presentation and discussion).
Topic	Basic knowledge on climate change.

Unit 4	Carbon capture and storage
Duration	40 × 45 min.
Organization	6 × 45 min. (lectures / question time) + 30 × 45 min. (SRP) + 4 × 45 min. (presentation and discussion).
Topics	Models constructed by scientists on CCS. Parameters and their interrelationships.

Unit 5	The role of modelling in school mathematics Design of modelling tasks
Duration	5 × 45 min.
Organization	3 × 45 min. (lectures) + 2 × 45 min. (exercise classes).
Topics	Systems in the natural and the social world.

Heidi Strømskag Appendix C1 2

Study and research paths (SRPs)

- A type of problem-based learning
- Rooted in the *Anthropological Theory of the Didactic* (ATD)
  - Initiated and mainly developed by Yves Chevallard (Bosch & Gascón, 2014)
- Towards a new didactic paradigm (Chevallard, 2015)
  - Involves asking questions about the world around us (the *Paradigm of Questioning the World*)
  - The current didactic paradigm (which is in decline)?
  - Why do we need a new paradigm?
  - What are the challenges of the new paradigm?
  - What does the national curriculum (LK20) say about mathematics teaching and learning?

Heidi Strømskag Appendix C1 3

Generating question *Q*

Carbon capture and storage is crucial to achieving the Paris Agreement's target of a maximum global temperature increase of 1.5 °C this century.

Q. How is carbon capture and storage modelled in the literature?

What mathematics are included in these models? What parameters are involved, and what is the relationship between them?

Heidi Strømskag Appendix C1 4

An SRP to answer a generating question *Q*

Find existing answers to *Q* (in the literature and multimedia)
To understand these existing answers, one must:
  Study works (study part)
  To understand these works, one must:
    Ask and answer derived questions (research part)
    To answer the derived questions, one must:
      Find existing answers to the derived questions (in the literature and multimedia)
      To understand these existing answers, one must:
        Study works (study part)
        To understand these works, one must:
          Ask and answer new derived questions (research part)
          To answer the new derived questions, one must:
            … etc.

Heidi Strømskag Appendix C1 5

Theoretical tools to tackle mathematical teaching and learning processes

- Didactic system
- Model – system
- Didactic milieu
- Herbartian schema
- Directed graph

Heidi Strømskag Appendix C1 6

Pilot-SRP

Generating question:

Q. What is climate change, and why is it happening?

- Preparation : Read about the Herbartian schema (Strømskag , 2022)
- Work in self-selected groups of two or three

Heidi Strømskag Appendix C1 7

Pilot-SRP (cont.)

Generating question:

Q. What is climate change, and why is it happening?

Procedure for the pilot study

1. Search for sources that can provide *existing answers* to Q.
2. As you study these existing answers, formulate and answer *derived questions*, Q_i, to understand the existing answers. Search for *works* (articles, books, reports, or multimedia resources) that help you understand existing answers and help you answer the Q_i.
3. Write a mini-report from the inquiry, which includes:
   - An answer to Q in terms of a synthesis of the answers to the Q_i.
   - A representation of the pathway of the inquiry, including an overview of the Q_i and works you have studied to arrive at the final answer to Q. This can be done through a directed graph accompanied by a table that explains its nodes.
4. Based on the mini-report, create a PowerPoint that presents the SRP you have conducted. All teams will present their inquiry on Friday 11 February. Time frame for presentation: 5 min. + discussion.

Heidi Strømskag Appendix C1 8

A modelling task

- *Comparison of thermoses*
- Assignment to be found in Blackboard
- To be discussed on 9th February
- Relevant article: Niss (2015)

Heidi Strømskag Appendix C1 9

References

- Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. https://doiorg/10.1007/978-3-319-05389-9_5 [Pages to read: 67–73.]
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer. https://doiorg/10.1007/978-3-319-12688-3_13
- Niss, M. (2015). Prescriptive modeling: Challenges and opportunities In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modeling in education research and practice: Cultural, social and cognitive influences* (pp. 67–79). Springer. https://doiorg/10.1007/978-3-319-18272-8_5

Heidi Strømskag Appendix C1 10

Appendix C – PowerPoint-Slides

C2 – Elementary Algebra as a Modelling Tool

APPENDIX C2

Mathematical Modelling Using Study and Research Paths

ELEMENTARY ALGEBRA AS A MODELLING TOOL

Heidi Strømskag
Department of Mathematical Sciences
Norwegian University of Science and Technology

Elementary algebra (Grade 8–13)

- The role of algebra in school mathematics
- Some history
- Didactic transposition
- What does it take to revitalize school algebra?

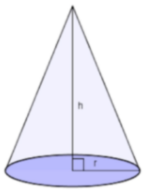
Heidi Strømskag Appendix C2 2

Some history

From the end of the 16th century, it became apparent how algebra was a modelling tool to generate ***formulas***

- via mathematicians like e.g., François Viète, Thomas Harriot, René Descartes

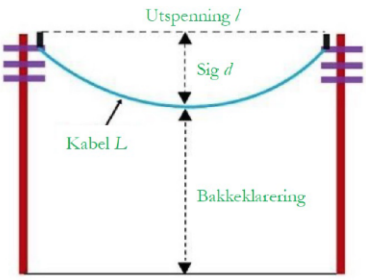
1) First in the capacity of ***formulas as algebraic models***:



Ex 1

$$V = \frac{1}{3}\pi r^2 h$$

(formula for the volume *V* of a coin with radius *r* and height *h*)



Ex 2

$$L = l + \frac{8d^2}{3l}$$

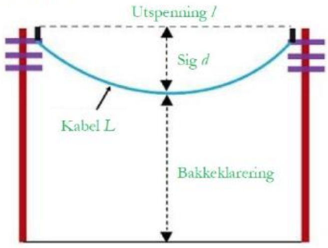
(formula for the length *L* of a cable with span *l* and sag *d*)

Heidi Strømskag Appendix C2 3

Some history (cont.)

2) Then in the capacity of **formulas as equations with parameters:**

- Solving the equation for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$, with respect to r or h.
  - $r = \sqrt{\frac{3V}{\pi h}}$
  - $h = \frac{3V}{\pi r^2}$
- Solving for the cable length, $L = l + \frac{8d^2}{3l}$, with respect to ***d or l***.
  - $d = \sqrt{\frac{3l(L-l)}{8}}$
  - $l^2 - Ll + \frac{8d^2}{3} = 0 \Rightarrow l = \frac{L \pm \sqrt{L^2 - 4\frac{8d^2}{3}}}{2} = \frac{L}{2} \pm \sqrt{\frac{L^2}{4} - \frac{8d^2}{3}}$



Heidi Strømskag Appendix C2 4

The significance of algebra

"One of the most important applications of elementary Algebra is to the use of formulae. In every form of applied science and mathematics … formulae are constantly employed, and their interpretation and manipulation are essential." (Abbott, 1942/1971, s. 69)

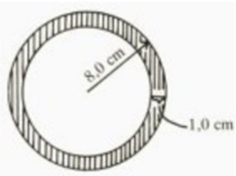
- *Teach yourself algebra* (by Percival Abbott, 1869–1954) in the series "Teach Yourself" (publisher Hodder & Stoughton) was a *bestseller*.
- Percival Abbott was an influential British mathematics teacher (he also wrote *Teach Yourself Calculus*)

Heidi Strømskag Appendix C2 5

Formulas in algebra

- Abbott explains that work on formulas involves 3 operations (Abbott, 1942/1971, p. 69):
  - **Construction**
  - **Manipulation**
  - **Evaluation**

Example (adapted from an exercise inErstad et al., 1984, p. 127)



In this task you will need that a circle with radius r has area πr^2. The figure shows a cross section of a water pipe. The outer radius of the pipe is equal to 8.0 cm and the thickness of the pipe material is equal to 1.0 cm.

a) Calculate the area of the pipe material (shaded in the figure). — Evaluation

Let the outer radius of the pipe be equal to R and the thickness of the pipe be equal to x.

b) Show that the area A of the pipe material is given by $A=2\pi Rx-\pi x^2$. — Construction

c) Find the outer radius R and the thickness x expressed by the other parameters. — Manipulation

d) Calculate the thickness x when $A=88\ \text{cm}^2$ and $R=8.0$ cm. — Evaluation

Heidi Strømskag Appendix C2 6

A key concept: Didactic transposition

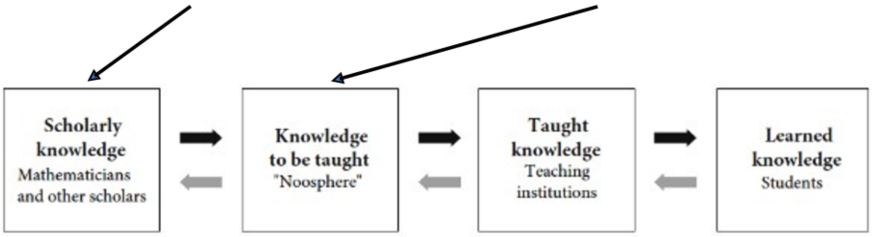
- The theory of ***didactic transposition*** was introduced in 1985 by YvesChevallard (Chevallard, 1985).
- Refers to the ***transformations*** a body of knowledge undergoes from the moment it is produced by scientists, to the moment it is chosen and designed by people from the "noosphere" (curriculum designers, textbook authors, and others) to be taught, until it is actually taught and studied in a given educational institution (Chevallard & Bosch, 2014).

Figure 1. Didactic transposition processes (adapted from Chevallard og Bosch (2014, p. 171)

Heidi Strømskag Appendix C2 7

Analysis of didactic transposition processes

- To study teaching and learning in the classroom, it is not enough to only study what happens in the classroom (e.g., how teachers and students *think* and *do*)
- *The knowledge taught must itself be the object of analysis*!
- The researcher studies transformation processes that knowledge undergoes *between the instances* in the figure below.
  - Anna Karina Kristianslund's master's thesis (NTNU, 2022): Analysis of the didactic transposition processes that differential calculus has undergone from the textbook *Kalkulus* (university) to the textbook *Mønster R1* (Grade 12).



Heidi Strømskag Appendix C2 8

Examples of didactic transformations

- Simplifications
  - Some types of tasks disappear
- Distortions
  - For example, theorems are "reshaped" into definitions
- ...

Purpose?

Heidi Strømskag Appendix C2 9

Result of a *simplification* of algebraic knowledge (Example 1)

Physical law: When resistors R_1, R_2, and R_3 are er connected in parallel, the total resistance, R, is calculated using the equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- To solve this equation with respect to R_i is no longer part of the algebra curriculum for Grade 8–13 → This type of task has disappeared.

Heidi Strømskag Appendix C2 10

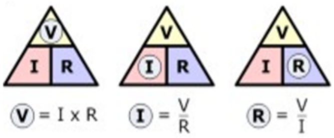
Result of a *distortion* of algebraic knowledge (Example 2a)

Background knowledge:

- Ohm's law says that the electric **voltage** (U) is equal to the product of **resistance** (R) and **amperage** (I). The voltage is the force needed to move the current through a circuit. The resistance tells us how much the material the electricity moves through slows the current.
- $\boldsymbol{U = R \cdot I}$, where U is measured in volts, R is measured in Ohm, and I is measured in amperes.
  - Which of U, R, and I are parameters?

Distortion:

Instead of learning algebraic calculations to solve equations with respect to one of the other parameters in an equation (here the parameters R and I), students in school learn a mnemonic technique (a «triangle trick») – which is a distortion of the knowledge.



Heidi Strømskag Appendix C2 11

Result of a *distortion* of algebraic knowledge (Example 2b)

From the website ***Matematikkens verden*** **[The World of Mathematics]**

https://www.matematikkensverden.no/2015/07/brker-med-symboler.html

Distance, speed, and time

"Sometimes you will come across fractions with letters. The letters are symbols for different values."

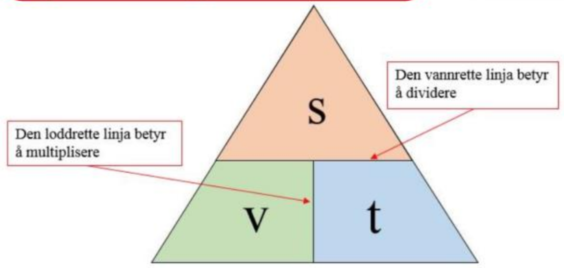
Comment: The triangle trick below involves rote learning of what the horizontal and vertical lines shall mean. But what if we put *v* or *t* on top of the triangle instead of *s*?

$v = \frac{s}{t}$ Denne brøken kan vi snu på, alt ettersom hva vi skal finne ut.

s er strekning og måles i m eller km

t er tid og måles i timer, minutter og sekunder

v er fart og måles i km/t (km/h) eller m/s



Skal vi finne farten blir regnestykket slik:

$v = \frac{s}{t}$

Skal vi finne strekningen blir regnestykket slik:

$s = v \cdot t$

Skal vi finne tiden blir regnestykket slik:

$t = \frac{s}{v}$

Heidi Strømskag Appendix C2 12

Result of a *distortion* of algebraic knowledge (Example 3)

From the textbook *Sinus 1T*:

A formula gives us the value of a variable by help of the value of one or several other variables. The volume V of a sphere is given by

$$V = \frac{4}{3}\pi r^3.$$

In this formula, we find the value of V when we know the value of the variable r, which is the radius. The variable r we call the independent variable and V we call the dependent variable. We choose values for the independent variable and calculate the value of the dependent variable. The formula above also contains the constant $\pi \approx 3.14$.

Sometimes we need values of two variables to calculate the third. The volume V of a cylinder is given by V=πr2h. … In this case we have two independent variables and one dependent variable. In most of the formulas we will work on in this book, we will have one independent and one dependent variable.

$$V = \pi r^2 h.$$

In this case we have two independent variables and one dependent variable. In most of the formulas we will work on in this book, we will have one independent and one dependent variable. (Oldervoll et al., 2020, p. 24)

Parameters have been displaced here: Formulas are fixed, rigid, immobile expressions– they have ceased to be fuel of algebraic work!

Heidi Strømskag Appendix C2 13

Algebra at work

- **Formulas** (in terms of **equations with parameters**) are fundamental in modelling of systems in the world around us.

Construction

Manipulation →

Manipulations are lacking because equations with parameters have disappeared through the didactic transposition of elementary algebra.
This is a serious shortcoming when it comes to modelling of mathematical and extramathematical systems (i.e., systems in nature and in society).

Evaluation

Measure:
To be able to answer questions through modelling of systems in the world around us, we need to *revitalize the algebra curriculum* in school.

Heidi Strømskag Appendix C2 14

Modelling of systems in the world around us

Algebra as an effective *modelling tool* in school:

1. Students start from a system S and a question Q about S
2. Students build a model S' of S, relative to Q, based on elementary algebra—and which will include as many parameters as necessary.
3. They work on S' and arrive at an answer to Q.
4. Through the process of inquiry, they discover the resources of algebra, studying them and then making effective use of them.

(Strømskag & Chevallard, 2022)

Heidi Strømskag Appendix C2 15

Co-funded by
the European Union

Task (temperature scales)

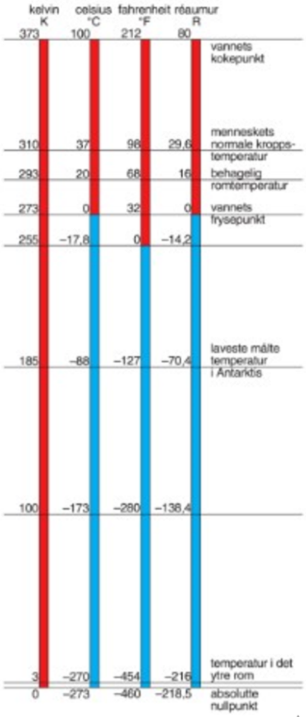


Figure taken from *Store norske leksikon*: https://snl.no/temperaturskala

Heidi Strømskag Appendix C2 16

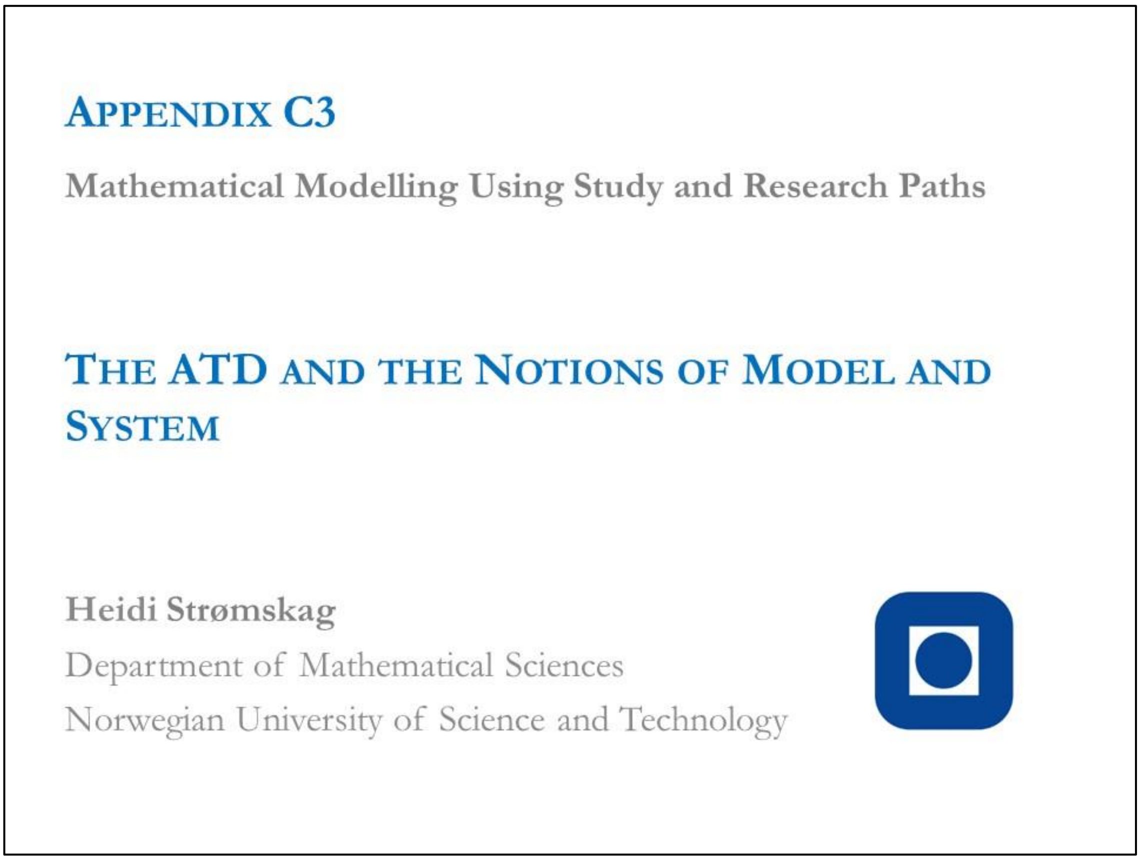
References

- Abbott, P. (1971). *Algebra*. Hodder and Stoughton. (Original work published 1942)
- Chevallard, Y. (1985). *La transposition didactique. Du savoir savant au savoir enseigné*. La pensée sauvage.
- Chevallard, Y., & Bosch, M. (2014). Didactic transposition in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 170–174). Springer. https://doi.org/10.1007/978-94-007-4978-8_48
- Erstad, G., Bjørnsgård, I., & Heir, O. (1984). *Oppgaver matematikk 1MA* [Mathematics tasks Grade 11]. Aschehoug.
- Kristianslund, A. K. (2022). *En prakseologisk analyse av resultatet av didaktiske transposisjonsprosesser innenfor differensialregning i matematikk R1* [Master's thesis, Norwegian University of Science and Technology]. NTNU Open. https://ntnuopen.ntnu.no/ntnu-xmlui/handle/11250/3042505
- Strømskag, H., & Chevallard, Y. (2022b). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Heidi Strømskag Appendix C2 17

Appendix C – PowerPoint-Slides

C3 – The ATD and the Notions of Model and System



Research framework

- The Anthropological Theory of the Didactic, **ATD** (Bosch & Gascón, 2014; Chevallard, 2019)
- **In the ATD, there are four decades of research on the teaching of algebra**

  Selected publications: Chevallard (1984, 1985, 1989, 1990, 1994); Gascón (1993, 1999, 2011); Bosch & Chevallard (1999); Bolea, Bosch & Gascón (1999, 2001, 2004); Ruiz-Munzón, Bosch & Gascón (2007, 2011, 2015, 2020); Ruiz-Munzón (2010); Chevallard & Bosch (2012); Ruiz-Munzón, Matheron, Bosch & Gascón (2012); Bosch (2015); Strømskag & Chevallard (2022a, 2022b, 2023).

Heidi Stromskag Appendix C3 2

Bringing elementary algebra back to full life

Redefining the secondary algebra curriculum in terms of what is *needed* to questioning, understanding and changing the world.

Example 1: Algebra of percentages

For $r > 0$, we have $q\left(1+\frac{r}{100}\right)\left(1-\frac{r}{100}\right) = q\left(1-\frac{r^2}{10\,000}\right) < q$

Heidi Stromskag Appendix C3 3

Bringing elementary algebra back to full life (cont.)

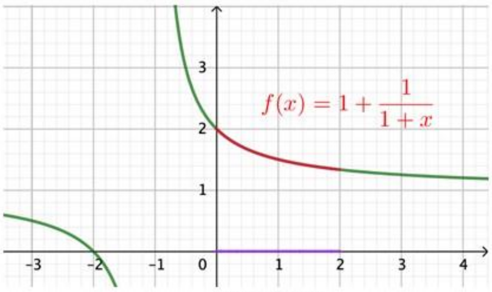
Example 2: An algebraic model of $\sqrt{2}$

$x = \sqrt{2} \Rightarrow x^2 - 1 = 1 \Rightarrow x = 1 + \frac{1}{x+1} = f(x)$, with f a contraction mapping. We have that f is a continuous and decreasing function on the interval [0, 2], and $\sqrt{2}$ is a fixed point of f since $f(\sqrt{2}) = \sqrt{2}$.

Therefore, **if** $a < \sqrt{2} < b$, **then** $f(b) < \sqrt{2} < f(a)$.

$$1 < \sqrt{2} < 2 \Rightarrow 1 + \frac{1}{3} < \sqrt{2} < 1 + \frac{1}{2} \Rightarrow 1 + \frac{2}{5} < \sqrt{2} < 1 + \frac{3}{7} \Rightarrow \cdots$$

$$\Rightarrow 1.4142132 < \sqrt{2} < 1.41421393 \Rightarrow \cdots$$



Heidi Stromskag Appendix C3 4

The notions of *system* and *model* in the ATD

A ***system*** is a fragment of reality that has its own laws. Let us consider a system $\mathcal{S}$:

A system $\mathcal{S}'$ is said to be a ***model*** of $\mathcal{S}$ if, by studying $\mathcal{S}'$, one can produce knowledge about $\mathcal{S}$

- studying *questions* Q about $\mathcal{S}$ by asking $\mathcal{S}'$ about these questions
- choosing models $\mathcal{S}'$ of $\mathcal{S}$ whose study of question Q is easier, safer, quicker than by a "direct" study of $\mathcal{S}$

$\mathcal{S}'$

$\mathcal{S}$

For example, if the radius r of a sphere increases by 20%, the new surface area A' will become $4\pi(1.2r)^2 = 1.44A$, so that the surface area will increase by 44% – a tricky result to obtain experimentally.

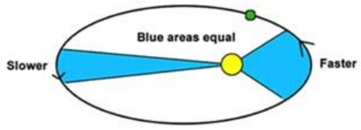
Heidi Stromskag Appendix C3 5

Co-funded by
the European Union

Descriptive versus normative/prescriptive models

- **Descriptive models: models *of* something**
  - Kepler's laws of planetary motion
  - Galileo's laws for bodies in free fall
  - Newton's law of cooling
  - Geometric models of objects

Kepler's 2. law



- **Normative models: models *for* something**
  - Templates in clothing production
  - The pension formula in public pension schemes
  - The formulae for elections and mandates in the distribution of power after elections
  - Gini coefficient for income inequality
  - The BMI formula

$$\text{BMI} = \frac{\text{weight (kg)}}{[\text{height (m)}]^2}$$

(Hilgers, n.d.; Niss, 2015)

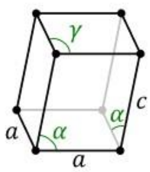
Heidi Stromskag Appendix C3 6

Descriptive versus normative (uses of) models

Given the ordered pair $(\mathcal{S}, \mathcal{S}')$

$\mathcal{S}'$ is a *descriptive* model of $\mathcal{S}$ if $\mathcal{S}'$ is modelled on (or after) $\mathcal{S}$ so that $\mathcal{S}'$ is regarded as a **model *of* $\mathcal{S}$**

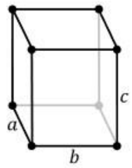
$\mathcal{S}'$



$\mathcal{S}$

$\mathcal{S}'$ is a *normative* model of $\mathcal{S}$ if $\mathcal{S}$ is modelled on (or after) $\mathcal{S}'$, that is, $\mathcal{S}'$ is a **model *for* $\mathcal{S}$**

$\mathcal{S}'$



$\mathcal{S}$

Depending on the *situations studied*, models have descriptive and normative ***uses***

Heidi Stromskag Appendix C3 7

The Herbartian schema – A model of an inquiry

- A **didactic system** is a triplet ***S*(*X*, *Y*, *k*)**
- The **Herbartian schema** is a dynamic model of a didactic system's inquiry into a generating question Q

$$[S(X, Y, Q) \curvearrowright M] \hookrightarrow A^{\heartsuit}$$

$\curvearrowright$ creates

$\hookrightarrow$ generates

(Chevallard, 2019)

Heidi Strømskag | Appendix C3 | 8

The didactic system related to *Q*

$S(X, Y, Q)$

- $X = \{x_1,..., x_9\}$
- $Y = \{\text{lecturer}\}$
- Q: ***How is carbon capture and storage modelled in the literature?***

  What mathematics is involved in these models?

  What are the parameters involved, and what are the relationships between them?
- Students x_i working in 4 teams to answer Q

Heidi Stromskag | Appendix C3 | 9

The milieu *M* for the study of a generating question *Q*

$$M = \{A_1^\lozenge, A_2^\lozenge, \ldots, A_m^\lozenge, W_1, W_2, \ldots, W_n, Q_1, Q_2, \ldots, Q_p, D_1, D_2, \ldots, D_q\}$$

Existing answers found in the literature / on the Internet	Works to study and understand $A_i^\lozenge$	Questions derived from the study of $Q, A_i^\lozenge$, and W_j	Dataset collected through various types of research during the study of Q

Heidi Stromskag Appendix C3 10

References

- Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education, 12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves Chevallard.pdf
- Hilgers, A. (n.d.). *Mit Mathe die Welt verstehen und gestalten*. Friedrich-Verlag. https://www.friedrich-verlag.de/mathematik/modellieren-problemloesen/mathe-und-der-rest-der-welt/
- Niss, M. (2015). Prescriptive modelling: Challenges and opportunities. In G. Stillman, W. Blum, & M. Salett Biembengut (Eds.), *Mathematical modelling in education research and practice: International perspectives on the teaching and learning of mathematical modelling* (pp. 67–79). Springer. https://doi.org/10.1007/978-3-319-18272-8_5

Heidi Stromskag Appendix C3 11

Some ATD research on the teaching of elementary algebra

Bolea, P., Bosch, M., & Gascón, J. (1999). The role of algebraization in the study of a mathematical organization. In I. Schwank (Ed.), *Proceedings of the First Conference of the European Society for Research in Mathematics Education* (Vol. 2, pp. 135–145). Osnabrück: Forschungsinstitut für Mathematikdidaktik and ERME.

Bolea, P., Bosch, M. & Gascón J. (2001). La transposición didáctica de organizaciones matemáticas en proceso de algebrización. *Recherches en Didactique des Mathématiques*, *21*(3), 247–304.

Bolea, P., Bosch, M., & Gascón, J. (2004). Why is modelling not included in the teaching of algebra at secondary school? *Quaderni di Ricerca in Didattica*, *14*, 125–133.

Bosch, M. (2015). Doing research within the anthropological theory of the didactic: The case of school algebra. In S. J. Cho (Ed.), *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp. 51–69). Cham: Springer.

Bosch, M., & Chevallard, Y. (1999). La sensibilité de l'activité mathématique aux ostensifs. *Recherches en Didactique des Mathématiques, 19*(1), 77–124.

Chevallard, Y. (1984). Le passage de l'arithmétique a l'algébrique dans l'enseignement des mathématiques au collège—Première partie. *L'évolution de la transposition didactique, Petit x, 5*, 51–94.

Chevallard, Y. (1985). *La Transposition Didactique. Du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage.

Chevallard, Y. (1989). Le passage de l'arithmétique à l'algébrique dans l'enseignement des mathématiques au collège—Deuxième partie. Perspectives curriculaires : la notion de modelisation. *Petit x, 19*, 45–75.

Some ATD research on the teaching of elementary algebra (cont.)

Chevallard, Y. (1990). Le passage de l'arithmétique à l'algébrique dans l'enseignement des mathématiques au collège—Troisième partie. Perspectives curriculaires: voies d'attaque et problèmes didactiques. *Petit x, 23*, 5–38.

Chevallard, Y. (1994). Enseignement de l'algèbre et transposition didactique. *Rendiconti del seminario matematico Università e Politecnico Torino, 52*(2), 175–234.

Chevallard, Y., & Bosch, M. (2012). L'algèbre entre effacement et réaffirmation. Aspects critiques de l'offre scolaire d'algèbre. *Recherces en Didactique des Mathématiques, Special issue (Enseignement de l'algèbre élémentaire. Bilan et perspectives)*, 13–33.

Gascón, J. (1993). Desarrollo del conocimiento matemático y análisis didáctico: Del patrón análisis-síntesis a la génesis del lenguaje algebraico. *Recherches en Didactique des Mathématiques, 13*(3), 295–332.

Gascón, J. (1999). La naturaleza prealgebraica de la matemática escolar. *Educación Matemática, 11*(1), 77–88.

Gascón, J. (2011). Las tres dimensiones fundamentales de un problema didáctico. El caso del álgebra elemental. *Revista Latinoamericana de Investigación en Matemática Educativa, RELIME, 14*(2), 203–231.

Ruiz-Munzón, N. (2010). *La introducción del álgebra elemental y su desarrollo hacia la modelización functional* [Doctoral dissertation, Universitat Autònoma de Barcelona, Spain]. https://www.educacion.gob.es/teseo/mostrarRef.do?ref=906861

Ruiz-Munzón, N., Bosch, M., & Gascón, J. (2007). The functional algebraic modelling at secondary level. In D. Pitta-Panzati & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2170–2179). Nicosia: University of Cyprus.

Some ATD research on the teaching of elementary algebra (cont.)

Ruiz-Munzón, N., Bosch, M., & Gascón, J. (2011). Un modelo epistemológico de referencia del álgebra como instrumento de modelización. In M. Bosch, J. Gascón, A. Ruiz Olarría, M. Artaud, A. Bronner, Y. Chevallard, G. Cirade, C. Ladage, & M. Larguier (Eds.), *Un panorama de la TAD. An overview on ATD* (CRM Documents, Vol. 10, pp. 743–765). Barcelona: Centre de Recerca Matemàtica.

Ruiz-Munzón, N., Bosch, M., & Gascón, J. (2015). El problema didáctico del álgebra elemental: Un análisis macro-ecológico desde la teoría antropológica de lo didáctico. *REDIMAT*, *4*(2), 106–131.

Ruiz-Munzón, N., Bosch, M., & Gascón, J. (2020). Un modèle épistémologique de référence pour la recherche sur l'algèbre élémentaire. *Nouveaux Cahiers de la Recherche en Éducation, 22*(1), 123–144. https://doi.org/10.7202/1070027ar

Ruiz-Munzón, N., Matheron, Y., Bosch, M., & Gascón, J. (2012). Autour de l'algèbre : les entiers relatifs et la modélisation algébrico-fonctionnelle. *Recherces en Didactique des Mathématiques, Special issue (Enseignement de l'algèbre élémentaire. Bilan et perspectives)*, 81–101.

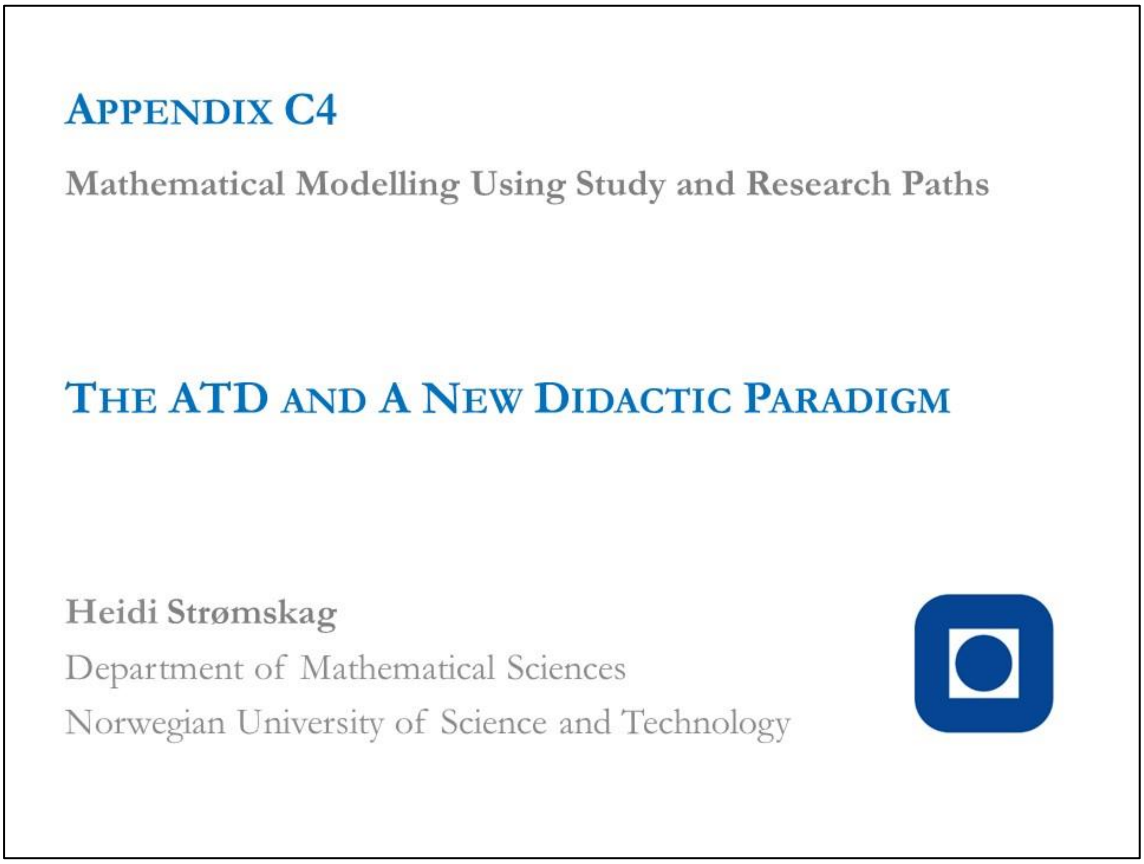
Strømskag, H. & Chevallard, Y. (2022). Conditions for revitalizing the elementary algebra curriculum. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (pp. 620–627)*. Bozen-Bolzano: Free University of Bozen-Bolzano and ERME. https://hal.science/hal-03745443v1/document

Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Strømskag, H., & Chevallard, Y. (2023). Breaches of the didactic contract as a driving force behind learning and non-learning: A story of flaws and wants. *Teaching Mathematics and Its Applications*, *42*(1), 52–64. https://doi.org/10.1093/teamat/hrac003

Appendix C – PowerPoint-Slides

C4 – The ATD and a New Didactic Paradigm



The anthropological theory of the didactic

Founder and main developer of the ATD since the 1980s is Yves Chevallard, professor emeritus at Aix-Marseille Université, France.

Another important contributor is Marianna Bosch, professor at Universitat de Barcelona, Spain.

Heidi Stromskag Appendix C4 2

Briefly on the ATD

ATD is about the ***didactic***

- The focus is on concrete actions that people or institutions perform or decide, which are intended to *teach someone something*.
- ***Praxeology*** is an analytical tool for studying such actions – it is a two-part model of the way we perform and explain human actions:

1) A ***type of tasks*** that can be solved by certain ***techniques*** ➤ task types and techniques make up the *praxis block* of a praxeology (how actions are performed).

2) Any technique needs an explanation called ***technology*** which in turn has a justifying explanation at a higher level, called ***theory*** ➤ technology and theory make up the *logos block* of a praxeology (the knowledge behind actions).

- Examples of different types of tasks: solving a differential equation, designing a task for students, writing a text, making pancake batter, skiing down a hill.

Heidi Stromskag Appendix C4 3

Paradigm

- A paradigm is a kind of *contract* that governs a specific type of human activity
- The "clauses" in such a contract are usually not explicitly formulated
  - Jean-Jacques Rousseau (1762/1988) wrote about the *social contract*:

    "The clauses of this contract is so determined by the nature of the act that the slightest modification would render them null or void, so that, **although they have never perhaps been formally enunciated, they are everywhere the same, everywhere tacitly accepted and recognized**." (p. 92)
- Thomas Kuhn: *The Structure of Scientific Revolutions* (1962)
  - ➢ on scientific paradigms and paradigm shifts

Heidi Stromskag Appendix C4 4

The current didactic paradigm

The paradigm of "*visiting works*" ("knowledge monuments"):

- The reason to study a work *w* is for *w*'s own sake:
  - *w* is considered valuable in its own right—*w* is recognised in the culture and in the curriculum
  - *w* is not studied because of what it allows us to think or do...
- Studying *w* is like visiting a museum
- Any examples you've experienced like this?

Heidi Stromskag Appendix C4 5

Why do we need a new didactic paradigm?

Heidi Stromskag Appendix C4 6

The new didactic paradigm

The paradigm of "questioning the social and natural world around us", rooted in ATD (Chevallard, 2015)

Three principles:

1. Every community has obligations to its members. One of them is to define a curriculum that ensures that community members are able **to think and act appropriately**, in a way that serves themselves and others in the various social settings in which they participate (especially in terms of family, profession and citizenship)
2. The curriculum should enable members to (individually or collectively) **identify, formulate and respond to issues** they face.
3. To achieve Point 2, the community should define (and regularly review) a **core curriculum made up of questions** to which community members have the right and duty to respond.

Heidi Stromskag Appendix C4 7

Co-funded by the European Union

Philosophical inspiration

- German philosopher and founder of pedagogy:
  Johan Friedrich Herbart (1776–1841): a scientist's attitude to the world
  - seeking answers to unanswered questions (for those concerned)

- Investigative approach (enquiry):
  - "Herbartian"
  - "precognitive"
  - "exoteric"

Getty Images/Hulton Archive

Heidi Stromskag Appendix C4 8

The 2020 Norwegian curriculum in mathematics (LK20)?

Homework:

When reading Chevallard (2015), study the curriculums for Mathematics 1P and Mathematics R1 and note down attributes from the two paradigms ("visiting works" and "questioning the world").

Google queries:

- Matematikk P kompetansemål
- Matematikk R kompetansemål

Heidi Stromskag Appendix C4 9

Co-funded by
the European Union

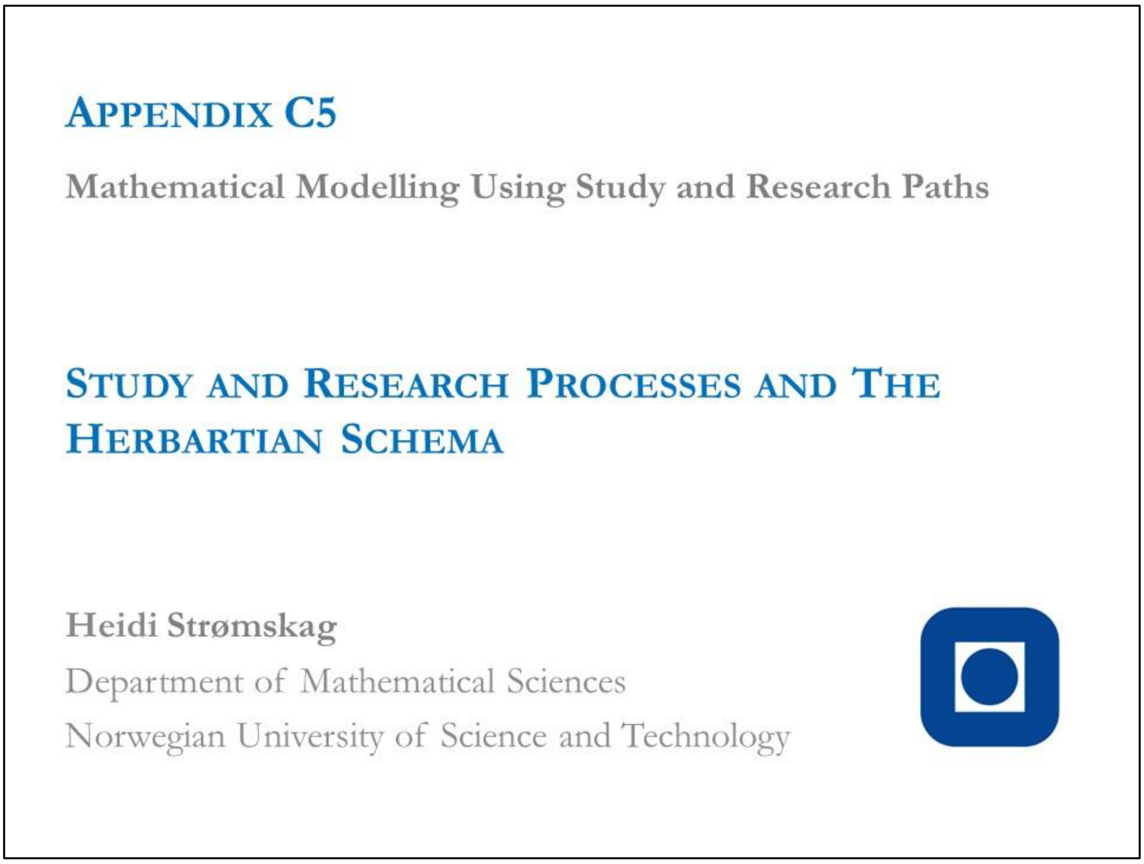
References

- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer. https://doi.org/10.1007/978-3-319-12688-3_13
- Kuhn, T. S. (1962). *The structure of scientific revolutions* (3rd ed.). University of Chicago Press.
- Rousseau, J.-J. (1988). On social contract or principles of political right. In A. Ritter & J. C. Bondanella (Eds.; J. C. Bondanella, Trans.), *Rousseau's philosophical writings* (pp. 84–173). Norton. (Orig. work publ. 1762)

Heidi Stromskag Appendix C4 10

Appendix C – PowerPoint-Slides

C5 – Study and Research Processes and The Herbartian Schema



An SRP to answer a generating question *Q*

Find existing answers to *Q* (in the literature, on the Internet)
To understand these existing answers, one must:
  Study works (study part)
  To understand these works, one must:
    Ask and answer derived questions (research part)
    To answer the derived questions, one must:
      Find existing answers to the derived questions (in the literature, on the Internet)
      To understand these existing answers, one must:
        Study works (study part)
        To understand these works, one must:
          Ask and answer new derived questions (research part)
          To answer the new derived questions, one must:
            … etc.

Heidi Stromskag — Appendix C5 — 2

The Herbartian schema – A model of an inquiry

- A **didactic system** is a triplet $\boldsymbol{S(X, Y, k)}$
- The **Herbartian schema** is a dynamic model of a didactic system's inquiry into a generating question *Q*

$$[S(X, Y, Q) \curvearrowright M] \hookrightarrow A^{\heartsuit}$$

$\curvearrowright$ creates
$\hookrightarrow$ generates

(Chevallard, 2019)

Heidi Stromskag — Appendix C5 — 3

The didactic system related to *Q*

$S(X, Y, Q)$

- $X = \{x_1, \ldots, x_9\}$
- $Y = \{\text{lecturer}\}$
- Q: ***How is carbon capture and storage modelled in the literature?***

  What mathematics is involved in these models?

  What are the parameters involved, and what are the relationships between them?
- Students x_i working in 4 teams to answer Q

Heidi Stromskag Appendix C5 4

The milieu *M* for the study of a generating question *Q*

$$M = \{A_1^\lozenge, A_2^\lozenge, \ldots, A_m^\lozenge, W_1, W_2, \ldots, W_n, Q_1, Q_2, \ldots, Q_p, D_1, D_2, \ldots, D_q\}$$

Existing answers found in the literature and multi-media resources	Works to study and understand $A_i^\lozenge$	Questions derived from the study of $Q, A_i^\lozenge$, and W_j	Dataset collected through various types of research during the study of Q

Heidi Stromskag Appendix C5 5

Co-funded by the European Union

Displaying the path of an inquiry

(example from an SRP on differential calculus, MA3061)

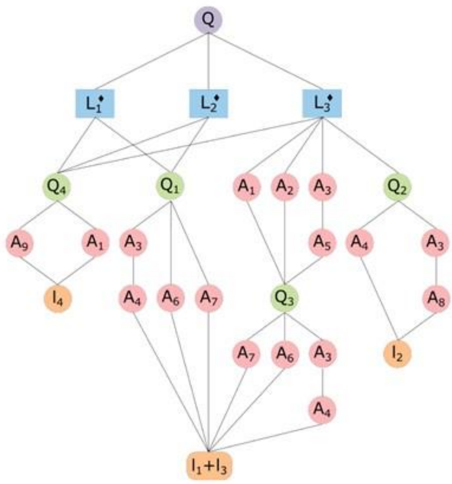


Figure 1. Directed graph displaying the **path of an inquiry** (Team 4, MA3061)

Existing answers, $L_i^\diamond$
$L_1^\diamond$: Sinus 1T and Sinus R1 (publisher Cappelen Damm)
$L_2^\diamond$: Mathematics 1T and Mathematics R1 (publisher Aschehoug)
$L_3^\diamond$: Articles on derivation at National Digital Learning Arena's website for Mathematics 1T
Works, A_i
A_1: Calculus 1 by Adams & Essex (2013)
A_2: Video on limits from NTNU Undervisning (YouTube Channel)
A_3: "Pupils' understanding of the derivative: A case study of R2 pupils' understanding of graphical representations of the derivative." Master's thesis by Fandrem (2016)
A_4: "Students' understanding of differentiation" by Orton (1983)
A_5: "Concept image and concept definition in mathematics with particular reference to limits and continuity" by Tall & Vinner (1981)
A_6: "The historical development of the calculus" by Edwards (1979)
A_7: "The changing concept of change: The derivative from Fermat to Weierstrass" by Grabiner (1983)
A_8: "Introduction to diagnostic teaching in mathematics" by Brekke (1995)
A_9: "Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm" by Chevallard (2015)
Derived questions, Q_i
Q_1. What strength does a graphical approach have and what strength does an algebraic approach have, and how do they supplement each other in an introduction to derivation?
Q_2. Do simplifications appear about derivation in the textbooks $L_1^\diamond$, $L_2^\diamond$, and at $L_3^\diamond$, which can create confusion, and how could they have been avoided?
Q_3. Should pupils first get a proper understanding of limits before they are introduced to derivation?
Q_4. Is "growth speed" an appropriate concept for instantaneous rate of change in a point?

Table 1. **Elements in the milieu** – nodes of the graph in Figure 1 (Team 4, MA3061)

Heidi Stromskag Appendix C5 6

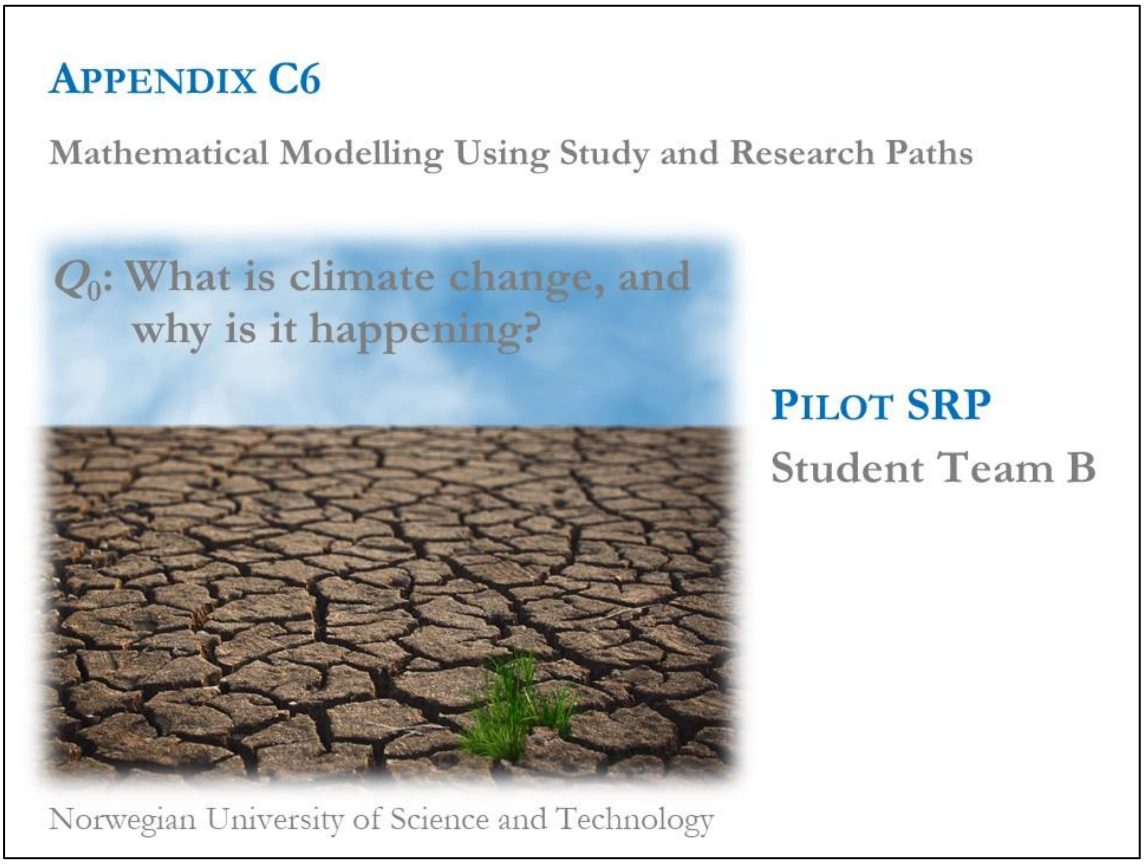
Reference

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education, 12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves Chevallard.pdf

Heidi Stromskag Appendix C5 7

Appendix C – PowerPoint-Slides

C6 – What is climate change, and why is it happening?



Existing answers

- FN-sambandet. (2021). *Klimaendringer* [Climate change]. https://www.fn.no/tema/klima-og-miljoe/klimaendringer
- Miljødirektoratet. (2021). *Klimaendringene skjer her og nå* [Climate change is happening here and now] https://miljostatus.miljodirektoratet.no/tema/klima/

- Earth's temperature is rising
- Ice at the poles is melting
- Sea level rises and becomes more acidic
- More extreme weather

Student Team B Appendix C6 2

Derived questions and answers

Q_1: What is climate, and why does it change?

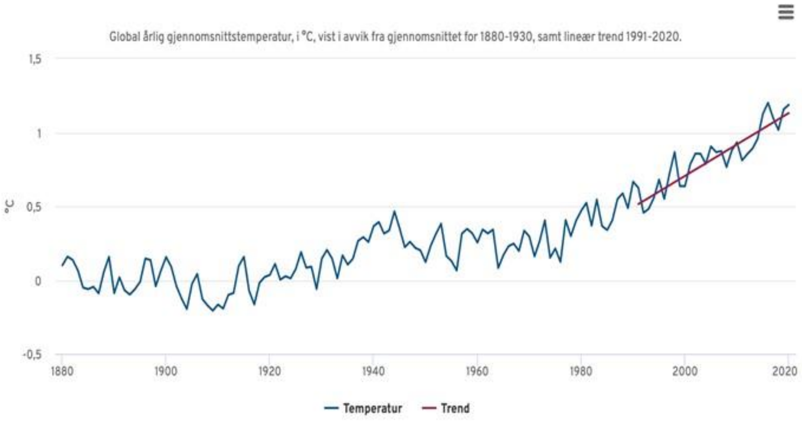
- **W_1:** FN-sambandet. (2021). *Klimaendringer* [Climate change]. https://www.fn.no/tema/klima-og-miljoe/klimaendringer
- **a_1:** Climate is an average of the weather measured over a long period of time. It changes as a result of the Earth emitting more greenhouse gases than it naturally should do. This contributes to an increased greenhouse effect and means that less heat escapes through the atmosphere. Result: increasing temperatures.

Student Team B Appendix C6 3

Derived questions and answers (cont.)

Q_2: How has the global temperature changed over the last 200 years?

- W_2: Energi og klima. (2021). *En varmere klode* [A warmer planet]. https://energiogklima.no/klimavakten/global-temperatur
- a_2:



Student Team B   Appendix C6   4

Derived questions and answers (cont.)

Q_3: Why is there more extreme rainfall than before?

- W_3: Sorteberg et al. (2019). https://www.uib.no/klimaenergi/123978/derfor-f%C3%A5r-vi-ekstremnedb%C3%B8r-knyttet-til-klimaendringene
- a_3: Simplified: A warmer atmosphere can retain more water vapour than a cold atmosphere. The amount of water vapour increases exponentially as a function of temperature, resulting in increased condensation at higher temperatures.

Q_4: How has the coronavirus pandemic affected climate change?

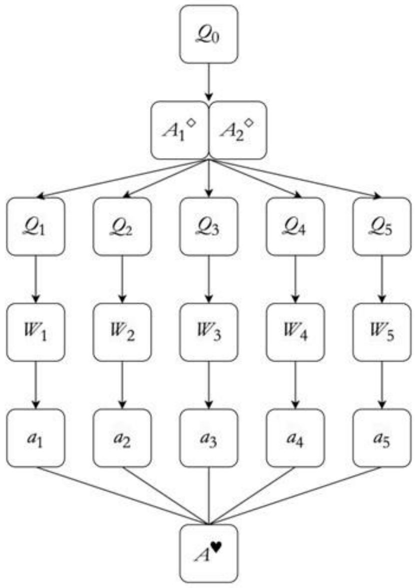
- W_4: Le Quéré et al. (2020). Temporary reduction in daily global CO_2 emissions during the COVID-19 forced confinement. https://www.nature.com/articles/s41558-020-0797-x.pdf
- a_4: Despite reduced CO_2 emissions (up to 17% in April 2020), this has had an equivalent zero effect on climate change.

Q_5: How are changes in sea level measured?

- W_5: Lindsey, R. (2020). *Climate change: Global sea level.* https://www.climate.gov/news-features/understanding-climate/climate-change-global-sea-level
- a_5: Sea level is measured using two main methods: tide gauges and satellite altimeters.

Student Team B   Appendix C6   5

Herbartian schema



Q_0 generating question

$A_i^\diamond$ existing answers

W_j works to be used

a_k partial answers

$A^\heartsuit$ final answer

Student Team B Appendix C6 6

Final (though provisional) answer to Q_0

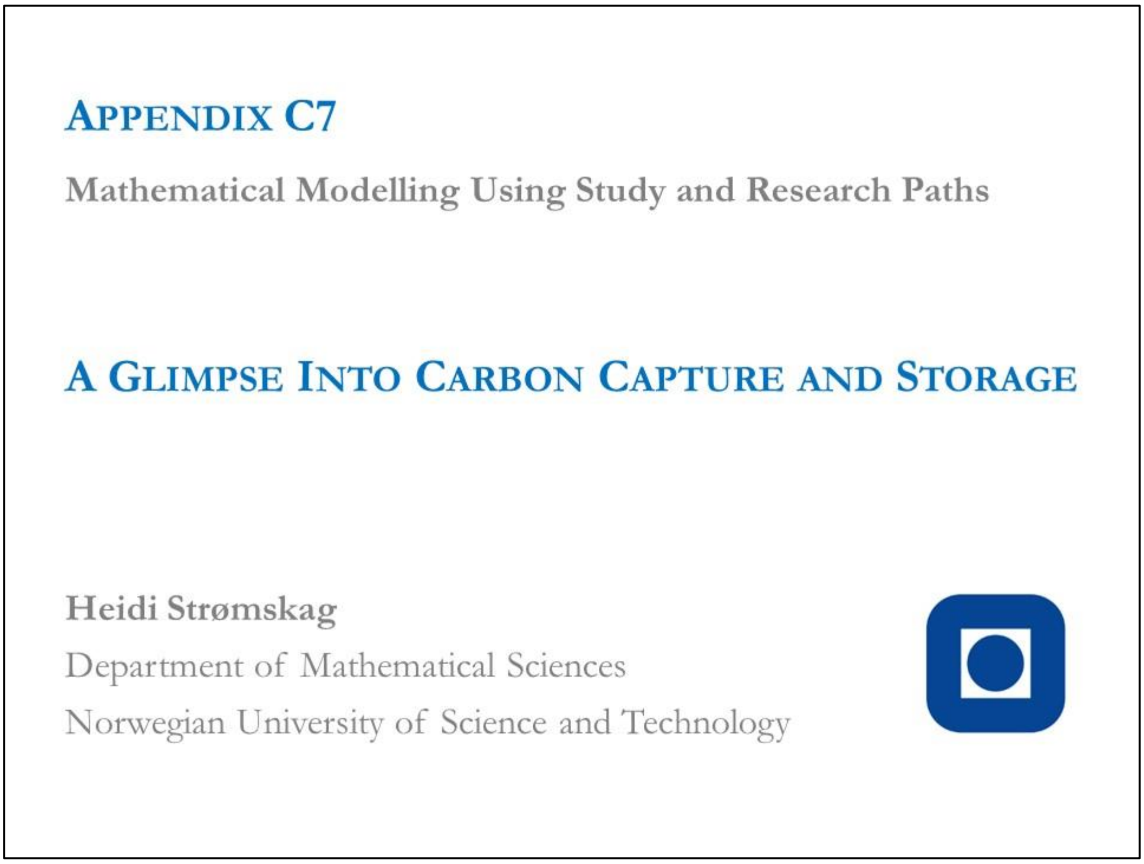
Our final answer $A^\heartsuit$ will be a combination of the existing answers and a synthesis of derived questions:

- **Changes in the average weather over a longer period of time.**
  Unfortunately, a reduction in a short period (like COVID-19) has little effect overall
- **Main cause: Changes in the amount of greenhouse gases in the atmosphere**
  As a result of increased CO_2 in the atmosphere, the pH value of the ocean becomes lower → more acidic (consequences for the ecosystem)
- **Temperature rise causes the ice to melt and the ocean to warm up.**
  The sea is rising: both as a result of "warm" water taking up more space and ice melting
- **More rainfall due to more water vapour in the atmosphere (especially in Norway)**
  This can trigger both floods and landslides

Student Team B Appendix C6 7

Appendix C – PowerPoint-Slides

C7 – A Glimpse Into Carbon Capture and Storage



Some elements on the topic

- What does Carbon Capture and Storage (CCS) involve?
- History of CCS in Norway
- Legislation
- Models

Heidi Stromskag Appendix C7 2

The process towards carbon storage

Phase 1: **Carbon capture**. Here, CO_2 is separated from the rest of the emission gases at the combustion plant.

Phase 2: **Compression** of CO_2 into liquid form.

Phase 3: **Transport** of the compressed CO_2 gas to where it will be stored.

Phase 4: **Carbon storage**. Here, CO_2 is pumped underground in so-called reservoirs. An example could be empty oil reservoirs.

https://northernlightsccs.com/

Karbonfangst og -lagring. (2021, 4 November). Ung Energi. https://ungenergi.no/miljoteknologi/ovrig-miljoteknologi/karbonfangst/

Heidi Stromskag Appendix C7 3

Co-funded by
the European Union

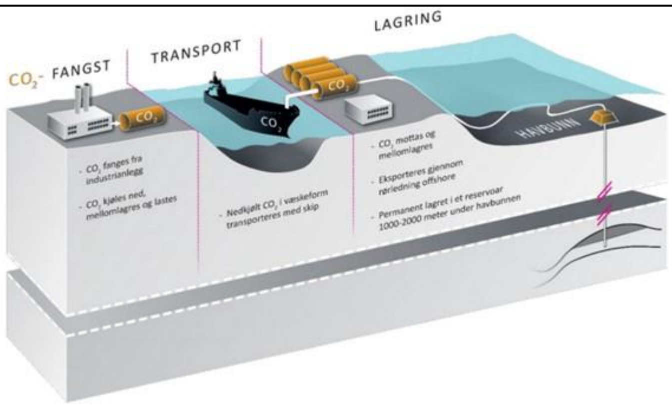


Illustration: Equinor

- Norcem's capture plant is the first of its kind in the world (ca. 900,000 tonnes of CO_2 per year)
- Northern Lights: collaboration between Equinor, Shell and Total
  - The first CO_2 storage facility in Europe that will be opened to European capture operators
  - The CO_2 will be transported by ship to a land terminal in Øygarden in Western Norway, before being piped to an offshore CO_2 storage facility under the seabed in the North Sea for permanent storage.
  - The CO_2 storage facility is located ca. 2500 m below the seabed, south of the Troll field.

https://www.nho.no/tema/energi-miljo-og-klima/artikler/co2-fangst-og-lagring-ccs/

Heidi Stromskag Appendix C7 4

Carbon capture and storage (CCS) – Norway's history

- Podcast from NRK Radio, 9 November 2021 (18 min.):
- https://radio.nrk.no/podkast/oppdatert/l_b463b28d-e9f9-4d15-a3b2-8de9f91d15f2

2007 Mongstad – "Moon landing" project
2012 Realisation of test facility
2013 Testing terminated
2020 New trial - Norcem Brevik

Norcem Brevik cement factory

Heidi Stromskag Appendix C7 5

Co-funded by
the European Union

Regulations on storage and transport of CO2 on the shelf (Legislative Data)

Department	Norwegian Ministry of Petroleum and Energy
Published	In 2014, Booklet 15
Entry into force	5 December 2014
Applies to	Norway
Authorisation	LOV-1963-06-21-12-§3
Announced	8 December 2014
Revised	20 January 2015 (§ 6-2)

§ 1-1. ***Purpose***
The purpose of these regulations is to contribute to sustainable energy and industrial production, by facilitating the utilisation of subsea reservoirs on the continental shelf for environmentally safe storage of CO_2 as a measure to counteract climate change.

§ 1-2. ***The right to subsea reservoirs for storage of CO_2***
The Norwegian state has ownership rights to subsea reservoirs on the continental shelf for utilisation of these for storage of CO_2 and the exclusive right to manage them.

https://lovdata.no/dokument/LTI/forskrift/2014-12-05-1517

Heidi Stromskag Appendix C7 6

Regulations on storage and transport of CO_2 on the continental shelf

Chapter overview:

Chapter 1. Introductory provisions (§§ 1-1–1-11).
Chapter 2. Exploration licence (§§ 2-1–2-6).
Chapter 3. Exploration licence (§§ 3-1–3-5).
Chapter 4. Licence to exploit a subsea reservoir for injection and storage of CO2 (exploitation licence (§§ 4-1–4-15).
Chapter 5. Injection and storage of CO_2 (§§ 5-1–5-13).
Chapter 6. Transport etc. of CO_2 (§§ 6-1–6-3).
Chapter 7. Termination of injection and storage of CO2 (§§ 7-1–7-6).
Chapter 8. Liability for compensation for pollution damage (§§ 8-1–8-8).
Chapter 9. Special rules on compensation to Norwegian fishermen (§§ 9-1–9-6).
Chapter 10. Special requirements for safety (§§ 10-1–10-6).
Chapter 11. General provisions (§§ 11-1–11-24).
Chapter 12. Infringement (§ 12-1).

Appendix I. Criteria for description and assessment of the possible storage site and surrounding area mentioned in § 1-10 of these regulations.

Appendix II. Criteria for establishing and updating the monitoring plan in § 5-4 and the follow-up plan in § 5-7.

Heidi Stromskag Appendix C7 7

What is a model of a system? (Recap from lecture on 11 Feb.)

- A ***system*** is a fragment of reality with its own laws.
- Let S be a given system. We search for an answer to ***questions*** Q about S.
- A system S' is a ***model*** of S if we can produce knowledge of S by studying S'.

We construct/select a model S' of S such that studying Q is easier, safer and faster than studying S "directly".

Remark: There is no such thing as a universal model of a system: a model is *relative* to the question(s) we seek answers to!

Heidi Stromskag Appendix C7 8

Opportunities for studies

- What are the **parameters** included in the categories below, describing the geology of the storage location?
- Search the literature for **mathematical models** that include these parameters.

The static geological model or models shall characterise the complex with respect to:

- Geological structure of the physical trap
- Geomechanical, geochemical and flow properties of the reservoir overburden (cap rocks, seals, porous and permeable layers) and surrounding formations
- Characterisation of fractures and faults and presence of natural and man-made flow paths
- Area and vertical extent of the storage complex
- Pore volume (including distribution of porosity)
- Initial fluid distribution
- Other relevant characteristics

(quoted from Legislative Data)

Heidi Stromskag Appendix C7 9

Some involved concepts

- **Flow** in porous rocks
- Transport
- Dissolution of gas
- Partial differential equation:
  - contains an unknown function of several independent variables and the partial derivatives of the function with respect to these variables
  - used to solve problems involving the spreading of sound or heat, electrodynamics, **fluid mechanics**, elasticity, etc.
- Finite element method:
  - Numerical method for solving partial differential equations in several variables with initial conditions

Figure adapted from "Understanding Porosity and Permeability" (2021)

Understanding porosity and permeability. (2021, 2 June). Earth Resources. Victoria State Government. https://earthresources.vic.gov.au/projects/victorian-gas-program/onshore-conventional-gas/porosity-permeability

Heidi Stromskag   Appendix C7   10

Some background knowledge

- When CO_2 is injected into layers of porous rock, it dissolves into the groundwater.
- The background flow of groundwater causes the gas to continuously dissolve into the water rather than just dissolving vertically into an aquifer (water-bearing rock) or rock layer.
- This is because the gas usually collects in structural traps in rock layers, for example in anticlines (geological term for an uplifted part of a folded rock).
- Using mathematics, partial differential equations describing this movement can be solved so that the concentration and velocity profiles of the aquifer can be predicted.
- This provides information on how much CO_2 can ultimately dissolve in the aquifer, the timescales over which this occurs and the mechanisms that control transport.
- Techniques such as finite element methods allow the analysis to be completed using computers.

Assertion: In a typical aquifer used for CO2 storage, it would take around one million years to dissolve the gas injected into it (Unwin et al., 2016).

Question: What model and mathematical calculations underpin this claim? Which parameters are included?

Unwin, H. J. T., Wells, G. N., & Woods, A. W. (2016). CO2 dissolution in a background hydrological flow. *Journal of Fluid Mechanics*, *789*, 768–784. https://doi.org/10.1017/jfm.2015.752

Heidi Stromskag   Appendix C7   11

Some background knowledge (cont.)

THKM (thermo-hydro-chemical-mechanical) processes relevant for CCS:

- T – Thermo: thermal simulations (**energy balance equation**) to estimate the change in temperature in the storage and surrounding formations. Temperature change affects the mechanical stresses and how the fluid behaves.
- H – Hydro: flow simulations (**mass balance equation**) to estimate the dispersion of the injected CO_2 and the pore pressure from the injection.
- K – Chemical: chemical effects (**mass balances, chemical equilibria/reactions, kinetics, equations of state**, etc.), there may be geochemical reactions between CO_2 and the formations, dissolution of CO_2 in water, pressure/temperature dependence of CO_2 and formation water, swelling of the formation, etc.
- M – Mechanical: mechanical simulations (**momentum balance equation**) to estimate stress changes due to the injection (which can be compared with fracture limits to check that the bearing is not compromised).

(T. I. Bjornarå, personal communication, 28 February 2022)

Bjornarå, T. I. (2018). *Model development for efficient simulation of CO2 storage* [Doctoral dissertation, University of Bergen, Norway]. https://bora.uib.no/bora-xmlui/handle/1956/17695

Heidi Stromskag Appendix C7 12

Some background knowledge (cont.)

Interaction between the processes THKM on the previous slide means that the mathematical models quickly become complicated:

- For example, if you model hydro flow (H), you calculate the pore pressure, and the pore pressure affects the stress field (M), but the compressibility of the formation (mechanical deformation) also affects the flow so that the two processes are coupled (HM).
- Temperature (T) affects the mechanical stresses (TM) which in turn affect the flow (THM), and they also affect the fluid properties (TH).

Assumptions are often made to eliminate interaction processes to simplify the modelling. For instance, temperature effects are often ignored, and if stresses are not of interest, you "only" need to solve for the flow of CO_2 in the reservoir.

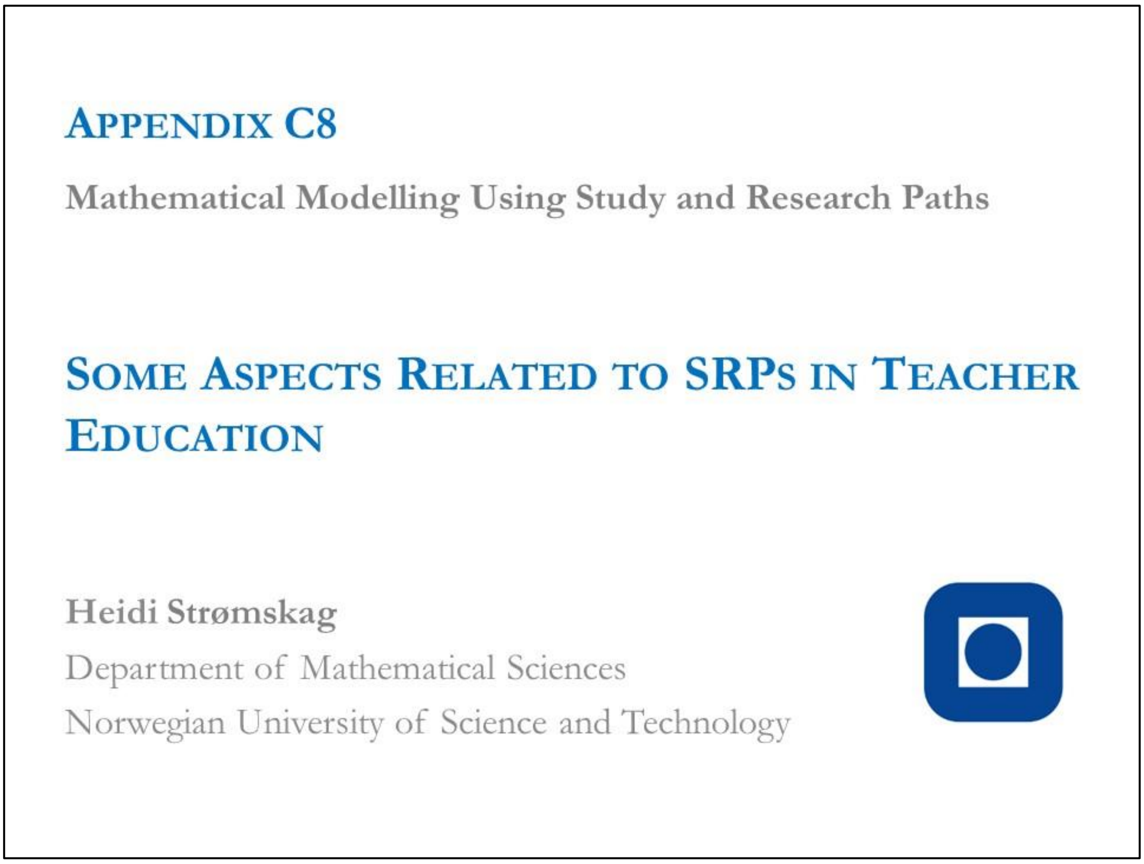
(T. I. Bjørnarå, personal communication, 28 February 2022)

Heidi Stromskag Appendix C7 13

Co-funded by
the European Union

Appendix C – PowerPoint-Slides

C8 – Some Aspects Related to SRPs in Teacher Education



Final report on the SRPs on CCS – some aspects

Information and ideas complementing the guidelines presented earlier

Introduction

- Carbon Capture and Storage: What is it, and why is it important?
- Responsibility of the educational sector: The United Nations Framework Convention on Climate Change (UNFCCC)
- Briefly on SRP as a method to study a question – problem-based teaching/learning format

Theory

- Concepts from the ATD: New didactic paradigm, SRP, Herbartian schema, etc.: Chevallard (2015), Strømskag (2022a)
- System – model: Strømskag and Chevallard (2022)
- Concepts related to the generating question Q (from geology, physics, mathematics, etc.)
- Various kinds of mathematical models

Method

- SRP: Strømskag (2022a)
- Document study (~ content analysis): Cohen et al. (2007), Thagaard (2009)

Heidi Strømskag Appendix C8 2

Final report on the SRP on CCS – some aspects (cont.)

Results

- Mathematical models in the literature on CCS (parameters and relations between them)
- $A_i^{\diamond}$, W_j, Q_k, a_m, $A^{\heartsuit}$ presented with the help of a directed graph and an associated table

Discussion

- On the method used: SRP involves a new *didactic contract* (Brousseau, 1997; Strømskag, 2022b)
- On the topic studied: Challenges related to teaching about climate change (Oversby, 2015)
- What have you learned from conducting this inquiry?

Heidi Strømskag Appendix C8 3

Responsibility of the educational sector

The United Nations Framework Convention on Climate Change, UNFCCC:

- Education is a key stakeholder in addressing the issue of climate change
- The UNFCCC assigns responsibility to parties of the convention to conduct climate change awareness campaigns in education and other public activities to ensure public participation in programmes and access to information on climate change

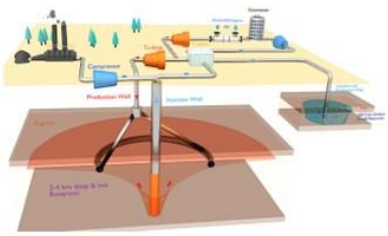
Education can encourage people to change their attitudes and behavior; it also helps them to make informed decisions. In the classroom, young people can be taught the impact of global warming and learn how to adapt to climate change. Education empowers all people, but especially motivates the young to take action. (Education is Key to Addressing Climate Change, n.d.).

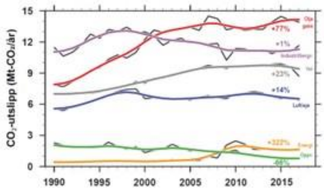
Ethical considerations on the climate issue: human consumption, growth (Bauman, 2007)

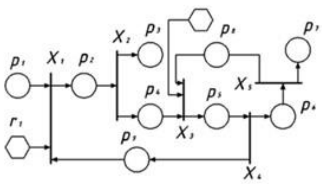
Heidi Stromskag Appendix C8 4

Some types of mathematical models

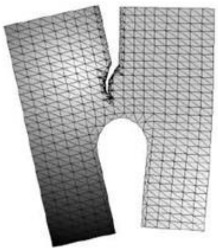
Graphical model (diagram, graph, chart, etc.)







Geometric model (describing shapes)



Heidi Stromskag Appendix C8 5

Co-funded by
the European Union

Some types of mathematical models (cont.)

Analytic model

- Formula
- Equation
- Inequality
- Function
- Differential equation
- Integral
- Linear system
- …

Model with which we can do *calculations*

Analytic model on *closed form*:

A mathematical model that uses a limited number of standard operations. It can contain constants, variables, certain well-known operations (e.g., + − × ÷) and functions (e.g., *n*th root, power, logarithm, trigonometric functions and inverse hyperbolic functions), but no limits, differentials (derivatives) or integrals.

[ELEMENTARY ALGEBRA]

Analytic model on open form (open expression)

A mathematical model involving boundary values, derivation or integration. These types of expressions involve variables or functions that cannot be expressed using a limited number of algebraic operations. Instead, they are defined by a process, such as calculating a boundary value or performing a derivation or integral, that results in a function or expression that depends on one or more variables.

[CALCULUS / ANALYSIS]

Heidi Stromskag Appendix C8 6

"Closed-Form Expression" (2021)

v·t·e	Arithmetic expressions	Polynomial expressions	Algebraic expressions	Closed-form expressions	Analytic expressions	Mathematical expressions
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Elementary arithmetic operation	Yes	Addition, subtraction, and multiplication only	Yes	Yes	Yes	Yes
Finite sum	Yes	Yes	Yes	Yes	Yes	Yes
Finite product	Yes	Yes	Yes	Yes	Yes	Yes
Finite continued fraction	Yes	No	Yes	Yes	Yes	Yes
Variable	No	Yes	Yes	Yes	Yes	Yes
Integer exponent	No	Yes	Yes	Yes	Yes	Yes
Integer nth root	No	No	Yes	Yes	Yes	Yes
Rational exponent	No	No	Yes	Yes	Yes	Yes
Integer factorial	No	No	Yes	Yes	Yes	Yes
Irrational exponent	No	No	No	Yes	Yes	Yes
Logarithm	No	No	No	Yes	Yes	Yes
Trigonometric function	No	No	No	Yes	Yes	Yes
Inverse trigonometric function	No	No	No	Yes	Yes	Yes
Hyperbolic function	No	No	No	Yes	Yes	Yes
Inverse hyperbolic function	No	No	No	Yes	Yes	Yes
Root of a polynomial that is not an algebraic solution	No	No	No	No	Yes	Yes
Gamma function and factorial of a non-integer	No	No	No	No	Yes	Yes
Bessel function	No	No	No	No	Yes	Yes
Special function	No	No	No	No	Yes	Yes
Infinite sum (series) (including power series)	No	No	No	No	Convergent only	Yes
Infinite product	No	No	No	No	Convergent only	Yes
Infinite continued fraction	No	No	No	No	Convergent only	Yes
Limit	No	No	No	No	No	Yes
Derivative	No	No	No	No	No	Yes
Integral	No	No	No	No	No	Yes

Heidi Stromskag Appendix C8 7

Co-funded by the European Union

Example: Szulczewski & Juanes (2009) – model for storage capacity of CO_2 in saline aquifers at large scale

$$C = \left[\frac{2M\Gamma^2(1-S_{wc})}{\Gamma^2 + (2-\Gamma)(1-M+M\Gamma)}\right] \rho_{CO_2} \phi H W L_{total}, \quad (1)$$

$$M = \frac{1/\mu_w}{k^*_{rg}/\mu_g} \quad (2)$$

$$\Gamma = \frac{S_{rg}}{1-S_{wc}} \quad (3)$$

$$L_{max} = \left[\frac{(2-\Gamma)(1-M(1-\Gamma))}{(2-\Gamma)(1-M(1-\Gamma))+\Gamma^2}\right] L_{total} \quad (4)$$

$$L_{inj} = L_{total} - L_{max} \quad (5)$$

Heidi Stromskag Appendix C8 8

Didactic contract (Brousseau, 1997)

- The mutual expectations and obligations that teachers and students have towards each other
- Some rules in the contract are implicit, others are explicit

(see note, Strømskag, 2022b)

Heidi Stromskag Appendix C8 9

Teaching climate change

Complexity related to:

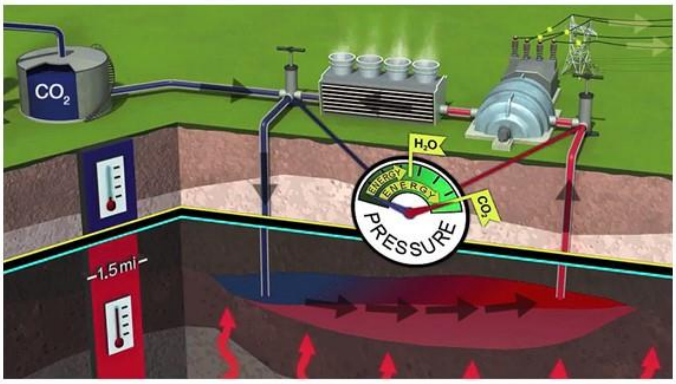
- Claims/knowledge in the field are based on modelling using uncertain and partial data → challenges traditional views of what a science is
- Encompasses expertise in many disciplines (interdisciplinarity)
- Attitudes to environmental issues and consumption are central (see Bauman, 2007)
- Strong links to personal and collective action, often political
- Characterisation of learning within the topic is multifaceted and often beyond the expertise of many teachers

(Oversby, 2015)

Heidi Stromskag Appendix C8 10

Future prospect: Storage of CO_2 for geothermal energy production?

- CO_2 Plume Geothermal (CPG)
- https://www.youtube.com/watch?v=x7fPNLY6h0U&ab_channel=GeothermalEnergyandGeofluidsgroup%2CETHZurich



Heidi Stromskag Appendix C8 11

References

- Bauman, Z. (2007). Forbrukersamfunn [Consumer society] (A. Peretz, Trans.). In G. E. Schjelderup & M. W. Knudsen (Red.), *Forbrukersosiologi: Makt, tegn og mening i forbrukersamfunnet* (pp. 229–250). Cappelen.
- Brousseau, G. (1997). *The theory of didactical situations in mathematics*. (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Kluwer.
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer.
- Closed-form expression. (2021, 13 December). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Closed-form_expression&oldid=1060078029
- Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed., pp. 201–204). Routledge.
- *Education is key to addressing climate change*. (n.d.). United Nations. Climate Action. https://www.un.org/en/climatechange/climate-solutions/education-key-addressing-climate-change
- Oversby, J. (2015). Teachers' learning about climate change education. *Procedia – Social and Behavioral Sciences*, *167*(8), 23–27. https://doi.org/10.1016/j.sbspro.2014.12.637

Heidi Stromskag Appendix C8 12

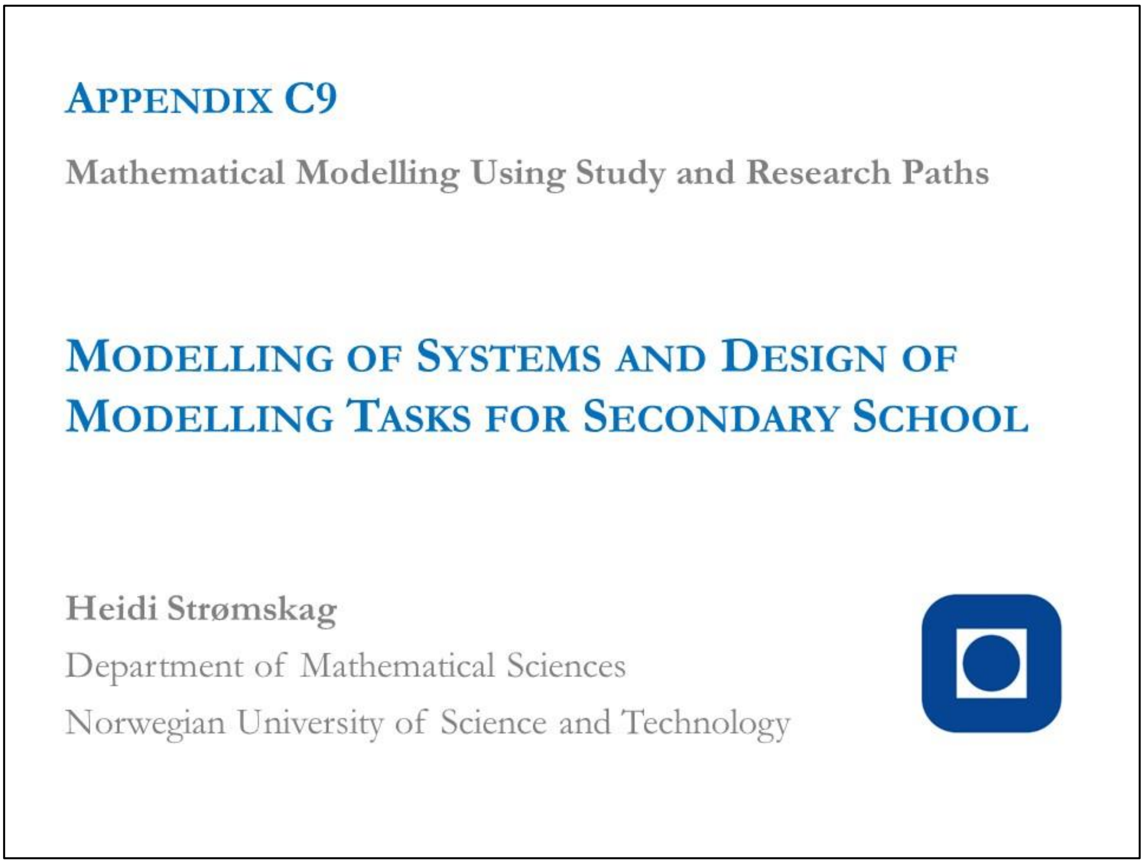
References (cont.)

- Strømskag, H. (2022a, 21 January). *Et notat om det herbartiske skjemaet – En dynamisk modell for studiet av et spørsmål* [A note on the Herbartian schema—A dynamic model for the study of a question]. Department of Mathematical Sciences, NTNU.
- Strømskag, H. (2022b, 23 March). *Et notat om begrepet «didaktisk kontrakt»* [A note on the notion of "didactic contract"]. Department of Mathematical Sciences, NTNU.
- Strømskag, H. & Chevallard, Y. (2022). Conditions for revitalizing the elementary algebra curriculum. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education* (pp. 620–627). Bozen-Bolzano: Free University of Bozen-Bolzano and ERME. https://hal.science/hal-03745443v1/document
- Szulczewski, M., & Juanes, R. (2009). A simple but rigorous model for calculating CO_2 storage capacity in deep saline aquifers at the basin scale. *Energy Procedia*, *1*(1), 3307–3314. https://doi.org/10.1016/j.egypro.2009.02.117
- Thagaard, T. (2009). *Systematikk og innlevelse: En innføring i kvalitative metoder* [Systematics and empathy: An introduction to qualitative methods] (3rd ed.). Fagbokforlaget. [Pages to read: 62–63.]

Heidi Stromskag Appendix C8 13

Appendix C – PowerPoint-Slides

C9 – Modelling of Systems and Design of Modelling Tasks for Secondary School



Inquiry as a pathway: Aligning goals for civilization, society, and education

Goals for the civilization

- Climate conservation: Actively preserving and restoring Earth's climate for future generations.
- Resource redistribution: Facilitating equitable and sustainable sharing of global resources.
- Conflict mitigation: Promoting diplomacy and understanding to prevent wars and other conflicts.

Goals for society

- Citizens being capable of answering questions about the world around them in order to address societal as well as personal needs: Needs relating to technological innovation, social progression, and a comprehensive understanding of natural and social phenomena.

Heidi Stromskag Appendix C9 2

Inquiry as a pathway: Aligning goals for civilization, society, and education (cont.)

Goals for mathematics education

- Educating pupils to tackle questions Q about the intra- and extramathematical world by studying existing answers to Q and posing new, derived questions, Q_i.
- This involves providing opportunities for pupils to become:
  - ***Herbartian*** (i.e., having intellectual curiosity);
  - ***procognitive*** (i.e., being ready to study and learn fields of knowledge new to them);
  - ***exoteric*** (i.e., embracing continuous studying and learning).

(Chevallard, 2015)

Heidi Stromskag Appendix C9 3

Inquiry as a pathway: Aligning goals for civilization, society, and education (cont.)

Goals for mathematics teacher education

- Providing prospective teachers with didactic tools that help them achieve the above goals for compulsory mathematics education. Examples of tools from the ATD (introduced in this course):
  - mathematical modelling based on the notions of a model and system in reference to questions about the system
  - the methodology of inquiries in the format of SRPs
  - the Herbartian schema – a dynamic model for the study of a question
  - directed graph + associated table – a pathway diagram of an inquiry governed by questions
  - formulas and parameters
  - three operations on formulas (construction, transformation, and evaluation)
- How are these tools interrelated? Discuss in groups.

Heidi Stromskag Appendix C9 4

The notions of *system* and *model* in the ATD (recap)

A ***system*** is a fragment of reality that has its own laws. Let us consider a system $\mathcal{S}$.

A system $\mathcal{S}'$ is said to be a ***model*** of $\mathcal{S}$ if, by studying $\mathcal{S}'$, one can produce knowledge about $\mathcal{S}$

- studying *questions* Q about $\mathcal{S}$ by asking $\mathcal{S}'$ about these questions
- choosing models $\mathcal{S}'$ of $\mathcal{S}$ whose study of question Q is easier, safer, quicker than by a "direct" study of $\mathcal{S}$

$\mathcal{S}'$

$\mathcal{S}$

(Strømskag & Chevallard, 2022)

Heidi Stromskag Appendix C9 5

Modelling driven by questions

We use a model to produce knowledge about a system

Main stages in the modelling process:

- Delineation of the system we intend to study, specifying the attributes that are relevant to the study we want to make of this system
- Constructing the model, building on old and new knowledge
- Using the model to produce knowledge about the system under study

(Chevallard, 1989)

Heidi Stromskag Appendix C9 6

Modelling driven by questions

Initial question about a system in the world

AIM: Constructing a model to produce knowledge the system under study

Main stages in the modelling process

- **Delineation of the system** we intend to study, specifying the attributes that are relevant to the study we want to make of this system — Questions about the system
- **Constructing the model**, building on both prior knowledge and knowledge developed as part of the modelling process — Questions about the model
- Using the model to **produce knowledge about the system** under study — Questions about the system-model interplay

Final answer and new questions

Heidi Stromskag Appendix C9 7

A modelling task from a mathematics textbook

A typical task from a Norwegian textbook for Grade 11 (Oldervoll et al., 2020):

1.130
The stopping distance for a car in motion hinges on both the driver's response time and the braking distance. The table below outlines the stopping distance, denoted by $S(x)$, in metres corresponding to certain speeds in kilometres per hour for a specified car and a specified driver.

x (km/h)	40	60	80	100
$S(x)$ (m)	24	45	73	108

a) Plot the data points from the table in a coordinate system and elucidate why a quadratic function seems to be a suitable fit.
b) Determine the quadratic function, S, that most accurately represents the given data. Sketch the graph incorporating the data points. Ensure that the expression of the function is accurate to three decimal places.
c) Find graphically the speed that would result in a stopping distance of 150 metres.
d) Find graphically the stopping distance corresponding to a speed of 90 km/h.

Heidi Stromskag Appendix C9 8

A modelling task from a mathematics textbook (cont.)

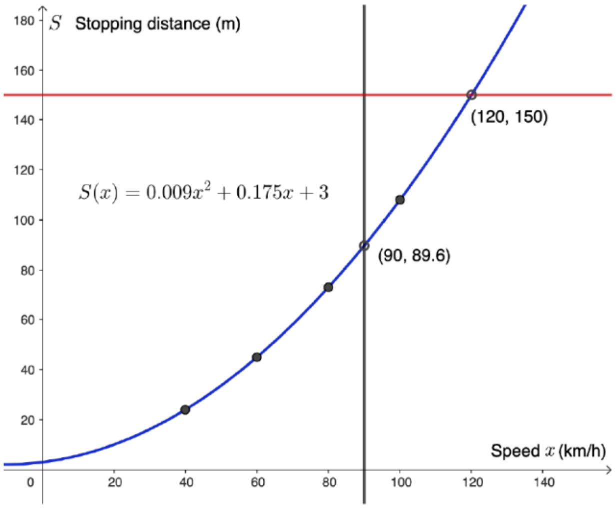
Discuss in groups the following questions about the modelling task from the textbook:

1. What is the *question* asked about the *system* S being studied?
2. Delineate S. Which parameters are relevant to consider?
3. Describe the model S' aimed at in the task.
4. What knowledge can be generated about S with the help of S'?
5. Give an overall comment about the modelling task you have examined.

Heidi Stromskag Appendix C9 9

A modelling task from a mathematics textbook (cont.)

The expected technique: Using regression analysis in GeoGebra to determine a second-degree polynomial that fits the given set of data points: $f(x) = 0.009x^2 + 0.175x + 3$.



Examination of the resulting model

- The model provides a good fit for the given data.
- The model offers minimal understanding of the underlying system being modelled.
- There is no direct interpretation of the parameters in the model related to the real-world system.

Heidi Stromskag Appendix C9 10

Spotlight on the interaction between system and model:
A new task pertaining to the same system

Braking distance of a car: A modelling task

A car is travelling at a speed of 72 km/h on a horizontal road. The car needs to brake suddenly, and the driver presses the brake pedal so hard that the brakes lock. The car then slides along the road until it comes to a stop. We assume that the coefficient of friction between the tyres and the road during sliding is 0.7—that is, the system to be studied involves frictional interaction between rubber and dry asphalt (according to Table 1).

a) Develop a model that allows you to provide answers in response to the following prompts: How long is the braking distance in the described scenario? Provide examples of how different road conditions and varying speeds can impact the braking distance of a vehicle sliding after brake lock.
b) What knowledge does the constructed model allow us to produce about the system under study?
c) Discuss limitations of the model in relation to the system under study.

Material 1	Material 2	μ (Friction coefficient)
Steel	Steel	0.6
Steel	Ice	0.05
Steel	Teflon	0.04
Ice	Ice	0.03
Rubber	Dry asphalt	0.7
Rubber	Wet asphalt	0.2
Rubber	Ice	0.02
Wood	Wood	0.3
Hip joint	Hip joint	0.003

Friction coefficients between various materials. Adapted from Grimenes et al. (2011, p. 68)

Heidi Stromskag Appendix C9 11

Spotlight on the interaction between system and model

Design of modelling tasks for Grades 8–13

Identify a modelling task in a secondary school textbook that, in your opinion, inadequately supports the generation of knowledge about the system it aims to model. Your assignment is to design a new task, pertaining to the same system, in a way that strengthens the connection between the system under study and the model to be constructed. Subtasks a–c formulated below are meant to help you in studying the system to be examined, in order to prepare your own task design. Moreover, the note with a proposed solution on braking distance is available for your perusal, and could serve as a useful reference or spark of inspiration in your own work.

a) Delineate the system S that will be the focus of your study. State the initial question Q that you aim to answer regarding S. Define the relevant parameters that are crucial for studying S, along with their respective relationships.

b) Build up a suitable model S' that can effectively address the initial question Q about the system S. Explain how this model S' can not only answer Q but also produce additional knowledge about the system S. Provide a brief overview of the background knowledge required for constructing the model S'.

c) Discuss the relationships between the system studied (S) and the model constructed (S') including the affordances (strengths and advantages) and constraints (limitations and assumptions) of the chosen model in representing the real-world dynamics of S.

d) Utilising the knowledge generated through the previous subtasks, devise a modelling task suitable for students in Grades 8–13 (select one of the grades). The task should explore the same system (S) and address a question (Q), concerning S. Q can either be the same or different from the one you addressed previously. Strive to design the task in a way that enables students to generate knowledge about the real-world dynamics of S, while ensuring it is both appropriately challenging and manageable for the chosen grade level.

Heidi Strømskag Appendix C9 12

References

- Chevallard, Y. (1989). Le passage de l'arithmétique à l'algébrique dans l'enseignement des mathématiques au collège—Deuxième partie. Perspectives curriculaires : la notion de modelisation. *Petit x, 19*, 45–75.
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Springer
- Grimenes, A. A., Jerstad, P., & Sletbak, B. (2011). *Grunnleggende fysikk for universitet og høgskole* [Basic physics for universities and university colleges]. Cappelen Damm.
- Oldervoll, T., Svorstøl, O., Vestergaard, B., Gustafsson, E., Osnes, E. R., Jacobsen, R. B., & Pedersen, T. (2020). *Sinus 1T: Matematikk* (4th ed.). Cappelen Damm.
- Strømskag, H., & Chevallard, Y. (2022b). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques, 42*(3), 371–409.

Heidi Strømskag Appendix C9 13

Appendix D – Tasks

D1 – Modelling of *thermal insulating capacity* of thermoses

The picture below shows two 1-liter thermoses, with Sarek on the left and Eva Solo on the right.

The question to be answered is:
What are the thermal insulating capacities of the Sarek and Eva Solo thermoses?

Newton's law of cooling will play a role in the modelling of the temperature in the two containers. It states that:

> *"The rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings".*

An experiment has been done where boiling water has been poured into the two containers, and the temperature has been measured at certain points in time after filling. The temperature in the room where the containers were placed during the experiment was 24.2 °C. The temperature of the water just before the lids were screwed on the thermoses was 94.3 °C.

a) For each thermos, build a model that gives the temperature of its water as a function of time after filling of hot water. Use this information: after 3 hours, the water in Sarek measured 88.9 °C, and the water in Eva Solo measured 83.7 °C.

b) Explain whether the models you have built are descriptive or normative.

c) Use the models to designate a ***descriptor*** of the thermal insulating capacity of each thermos. Give this descriptor an explanatory name that may be used by vendors of thermoses.

d) Several observations were made, as shown in the table below. Discuss the validity of your models in light of these measurements.

Time passed after filling (hours)	Sarek	Eva Solo
0	94,3 °C	94,3 °C
1	92,6 °C	89,0 °C
2	90,3 °C	85,8 °C
3	88,9 °C	83,7 °C
5	85,9 °C	78,7 °C
7	82,9 °C	74,6 °C
11	77,8 °C	67,8 °C
12	76,5 °C	66,5 °C
53	47,5 °C	37,0 °C
75	39,9 °C	32,1 °C
96	34,8 °C	28,9 °C

Table 1. Temperatures of the water at points in time after filling of water

Appendix D – Tasks

D2 – Proposed solution: Modelling of thermal insulating capacity of thermoses

a) Let $S(t)$ and $E(t)$ denote the temperature in the Sarek and Eva Solo thermos respectively at time t (in hours). We use Newton's law of cooling to set up a differential equation in each case.

Sarek (k_s is the proportionality factor):

$\frac{dS}{dt} = k_s(S - 24.2)$. Initial condition: $S(0) = 94.3$ og $S(3) = 88.9$.

Eva Solo (k_e is the proportionality factor):

$\frac{dE}{dt} = k_e(E - 24.2)$. Initial condition: $E(0) = 94.3$ og $E(3) = 83.7$.

Solving the differential equation for Sarek:

$$\frac{dS}{dt} = k_s(S - 24.2) \Leftrightarrow \int \frac{dS}{S-24.2} = \int k_s\, dt \Leftrightarrow \ln|S - 24.2| = kt + C$$

$$\Leftrightarrow S - 24.2 = e^{k_s t + C} \Leftrightarrow S(t) = C_1 e^{k_s t} + 24.2.$$

Using the initial conditions to decide C_1 og k_s:
$S(0) = 94.3 \Leftrightarrow C_1 = 70.1$.

That is to say, we have $S(t) = 70.1e^{k_s t} + 24.2$.

Solving the differential equation for Eva Solo:

Because the temperature at $t = 0$ is the same for Eva Solo, we have:
$E(t) = 70.1e^{k_e t} + 24.2$.

We find the proportionality factor in the two cases:

Sarek:
$S(3) = 88{,}9 \Leftrightarrow 70.1e^{3k_s} + 24.2 = 88{,}9 \Leftrightarrow e^{3k_s} = \frac{64.7}{70.1} \Leftrightarrow 3k_s = \ln\left(\frac{564.7}{70.1}\right)$
$\Leftrightarrow k_s \approx -0.0267$.

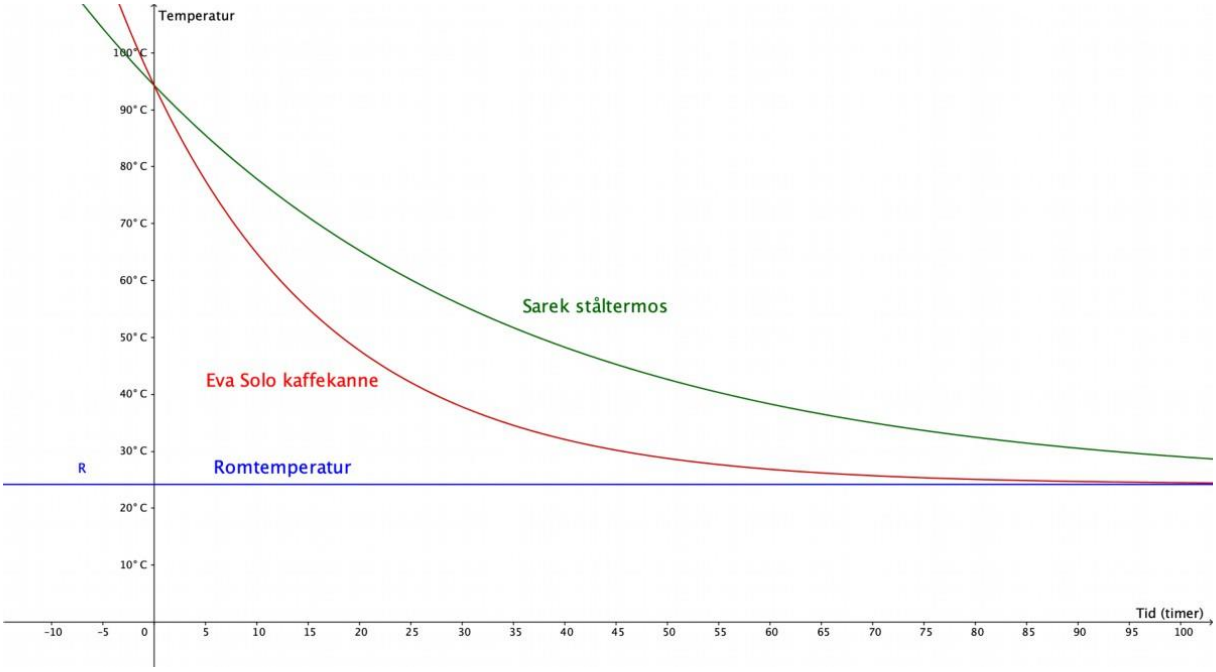
That is to say, we have: $\underline{S(t) = 70.1e^{-0.0267t} + 24.2}$

Eva Solo:
$E(3) = 83.7 \Leftrightarrow 70.1e^{3k_e} + 24.2 = 83.7 \Leftrightarrow e^{3k_e} = \frac{59.5}{70.1}$

$\Leftrightarrow\ 3k_e = \ln\left(\frac{59.5}{70.1}\right) \Leftrightarrow\ k_e \approx -0.0546.$

That is to say, we have: $\underline{E(t) = 70.1e^{-0.0546t} + 24.2}$

The functions $S(t)$ and $E(t)$ are *analytic models*, depending on the choice of initial conditions. Below are *graphical models* for the temperature development in the two thermoses.



The proportionality factor is negative in both cases, and the one with the largest absolute value will give the greatest reduction in temperature per unit of time. We see that the temperature is best maintained in Sarek. This is consistent with the fact that the proportionality factor in the model for Sarek has a smaller absolute value than the model for Eva Solo.

b) These are *descriptive* models. They also have the further property of being *deterministic*, in that they are constructed on the basis of a physical law for temperature development for bodies placed in an environment with a constant temperature (Newton's law of cooling).

c) *Suggestion*. A measure of insulating properties can be called the insulation coefficient, *I*. This will be a ***descriptor*** (see Niss, 2015) and can be calculated according to the equation $I = 100(1 - |k|)$. Note that this equation is an *invention*.

This gives the following insulation coefficients:
Sarek: I = 97,3
Eva Solo: I = 94,5
Furthermore, the insulation coefficient can be linked to intervals that qualitatively describe the insulating properties of thermoses, for example:

$I \in [100, 95)$	very good
$I \in [95, 90)$	good
$I \in [90, 85)$	acceptable
$I \leq 85$	poor

Note: The systems we model differ in terms of how far into the future it is appropriate to consider. For example, a thermos for use in an indoor party does not need to retain heat for as many hours as a thermos for bringing on a longer journey in sub-zero temperatures.

d) Factors contributing to inaccuracy in the data: measurement inaccuracy (the tool used), heat loss when opening-closing, not equal time for measurement in the two thermoses, etc. A *data logger* could have been used to get more accurate data.

Finally, it is possible to use *regression analysis* (e.g., in GeoGebra) to find a model for each thermos. In general, regression models are derived from actual data. They are used to describe the relationship between variables and predict future outcomes. Regression models do not necessarily represent the underlying physics or theoretical principles of a system, but instead provide a statistical relationship between variables.

Reference

Niss, M. (2015). Prescriptive modeling: Challenges and opportunities. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), Mathematical modeling in education research and practice: Cultural, social and cognitive influences (pp. 67–79). Springer. https://doi.org/10.1007/978-3-319-18272-8_5

Appendix D – Tasks

D3 – Modelling of probability of having a disease given a positive test result

The system *S* is a population with an infectious disease, where the relative frequency of the disease is equal to *a*, and the reliability of a screening test for the disease is equal to *r*. Moreover, the screening test is designed in a way that ensures that everyone who has the disease tests positive (i.e., no false negatives).

a) Create a model of the given system that gives you the answer to this question, *Q*: What is the probability, *p*, that a person with a positive test result really has the disease?

b) Explore the model you have created for different values of *p*. Find out, for example, what relative frequency of the disease corresponds to 98% probability of having the disease after receiving a positive test result.

Appendix D – Tasks

D4 – Comparison of study and research paths (SRPs) and problem-/project-based learning approaches

For this investigation, your primary resource will be the article by Markulin et al. (2021). Here are the questions you will study and answer:

a) How does the methodology of SRPs differ from problem-/project-based learning approaches? What are the principles upon which each of these approaches are founded?

b) Imagine a hypothetical question (*Q*) about a system, relevant to study by secondary school students. Explain how *Q* could potentially be tackled differently by the two approaches: SRPs and problem-/project-based learning.

Reference

Markulin, K., Bosch, M., & Florensa, I. (2021). Project-based learning in statistics: A critical analysis. *Caminhos da Educação Matemática em Revista*, *11*(1), 200–220.

Co-funded by
the European Union

Appendix D – Tasks

D5 - Study and research path on climate change: A pilot study

Generating question

Q_{pilot}. *What is climate change, and why is it happening?*

- Preparation: Read about the Herbartian schema (Strømskag, 2022). The note explains the process of an SRP and some tools for presenting such inquiries. Even if you do not need to use all the terminology introduced in this note when presenting the results, the content of the note will still be helpful in the study and research process you will be conducting.
- Work in self-selected groups of two or three.

Procedure for the pilot study

1. Search for sources that can provide *existing answers* to Q.
2. As you study these existing answers, formulate and answer *derived questions*, Q_i, to understand the existing answers. Search for *works* (articles, books, reports, or multimedia resources) that help you understand existing answers and help you answer the Q_i.
3. Write a mini-report from the inquiry, which includes:
    - An answer to Q in terms of a synthesis of the answers to the Q_i.
    - A representation of the pathway of the inquiry, including an overview of the Q_i and works you have studied to arrive at the final answer to Q. This can be done through a directed graph accompanied by a table that explains its nodes.
4. Based on the mini-report, create a PowerPoint that presents the SRP you have conducted. All teams will present their inquiry on Friday 11 February. Time frame for presentation: 5 min. + discussion.

“Hand keep clode”—image downloaded from Pixabay

Appendix D – Tasks

D6 – A study and research process into Carbon Capture and Storage

Generating question

Q_0. *How is carbon capture and storage modelled in the literature?*
What mathematics is involved in these models? Which parameters are included, and what are the relationships between them?

Guidelines for the final SRP report

Length: Max. **6 500 words** (excluding reference list, table of contents, and any appendices).

Note 1: The chapter headings below may well be replaced by more informative headings. Chapters that span several pages should be divided into sub-chapters with suitable headings.

Note 2: APA 7 style should be used for both references in the text itself and in the formatting of the reference list.

The final SRP report shall be given an appropriate **title** and shall contain the following parts:

Table of contents (titles of chapters and sub-chapters)

1. Introduction

- What is this study about? Why is the topic important? Why should it be a topic in education?

2. Didactic framework and theory

- Brief account of the didactic paradigm in which the study is anchored
- Presentation of theoretical tools (e.g., model – system; Herbartian schema)
- Possible glossary of technical terms (briefly explained)

3. Methodology

- Presentation of the research strategy used: SFL and documentary study.
- Presentation of didactic tools in the form of a directed graph with an accompanying table showing the course of the study with the following elements: the generating question, existing solutions, derived questions, works, partial answers.

4. Results

- Presentation of existing answers $A_i^\diamond$ and works W_j that are central to understanding $A_i^\diamond$.
- Presentation of derived questions Q_k and partial answers a_m that are results of studying Q_k, $A_i^\diamond$, and W_j.
- Discussion of the partial answers a_m and presentation of the final (though provisional) answer $A^\heartsuit$.

5. Discussion and concluding remarks

- Comments on the investigation process. Include an overview of derived questions and work that has been omitted (for various reasons that do not need to be explained). What have you learnt from conducting this inquiry? What has been challenging? What are open questions that need further study?

6. References

Milestones/deadlines:

18 March at 23.55:	Submission of a preliminary report to response team and Heidi.
22–23 March:	Presentation of a preliminary report in class. System for feedback.
11 April at 23.55:	Submission of a draft of final report to Heidi for feedback.
13 May at 23.55:	Submission of a final report.

Appendix D – Tasks

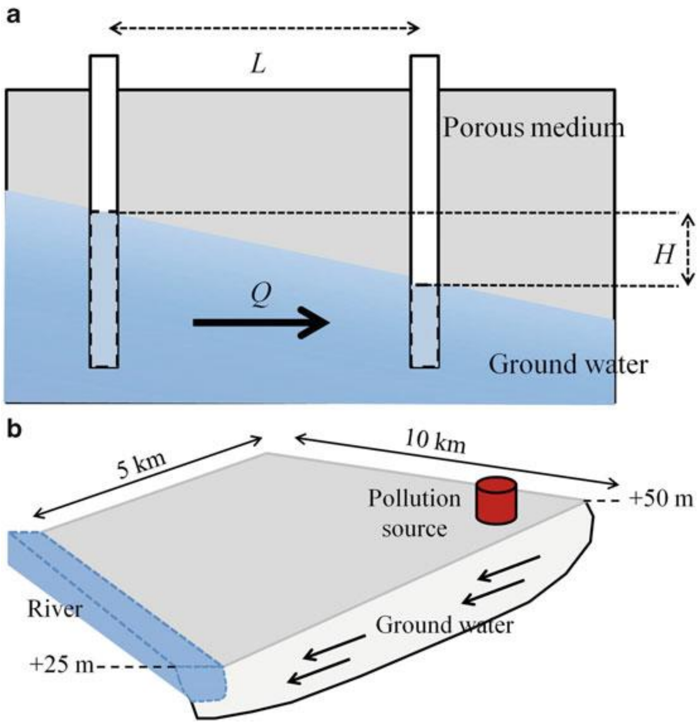
D7 – Hydrogeological modelling: Evaluating aquifer flow and contaminant transport[19]

To determine the flow rate of water in an aquifer, Darcy's law can be used. It gives the relation between the flow through this medium (Q), the hydraulic gradient i, the surface perpendicular to the flow A, and a parameter that characterizes the aquifer k: permeability (expressed in m/s).

Darcy's law is given by the relation $Q = A \times k \times i$. The hydraulic gradient is the ratio between the difference in water depth in two boreholes and the distance between these two boreholes. The hydraulic gradient is $i = \frac{H}{L}$ (see Figure 1).

Figure 1

Principle of flow in an aquifer: The flow rate is proportional to the hydraulic gradient



Note. The figure (1) is taken from Fleurant and Bodin-Fleurant (2019, p. 53) and is reproduced with permission. This figure is not covered by the chapter's CC BY-NC 4.0 license and should not be reproduced without obtaining separate permission from the copyright holder, Springer.

1) What is the unit of measure of the gradient? Of the flow?

2) Justify mathematically and physically that $Q = 0$ m^3/s when $i = 0$.

---

[19] The task is taken from Fleurant and Bodin-Fleurant (2019, p. 52).

3) It is assumed that this aquifer consists of limestone whose estimated permeability is 8.64 m/day, and that the river is 15 m deep. From the graph in Figure 1, determine the flow (in L/s) that goes from the aquifer to the river.

4) By imagining that a soluble contaminant escapes from the tank shown in the figure by a cylinder, calculate the time it will take the contaminant to reach the river.

Reference

Fleurant, C., & Bodin-Fleurant, S. (2019). *Mathematics for Earth science and geography*. Cham: Springer. https://doi.org/10.1007/978-3-319-69242-5

Appendix D – Tasks

D8 – Modelling of temperature scales and designing a Fahrenheit-Celsius conversion task

The modelling task below is motivated by the following exercise, found in the three-volume textbook by Søgaard and Tambs Lyche (1939/1969, p. 104):

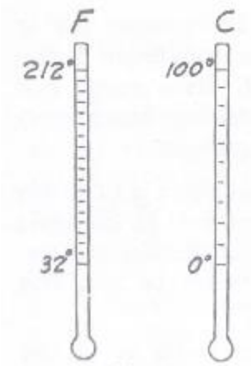


Fig. 17.

Exercise 231. Look at the figure and explain that we will get the Fahrenheit temperature F expressed in terms of the Celsius temperature C by the formula $F=\frac{180}{100}C+32$.

Use this formula and find C in terms of F.

Take $F=100$, 50, 41, 32, 14, 0, and calculate C in each case.

Here is your assignment, the answers to which will be discussed in class in the upcoming exercise class.

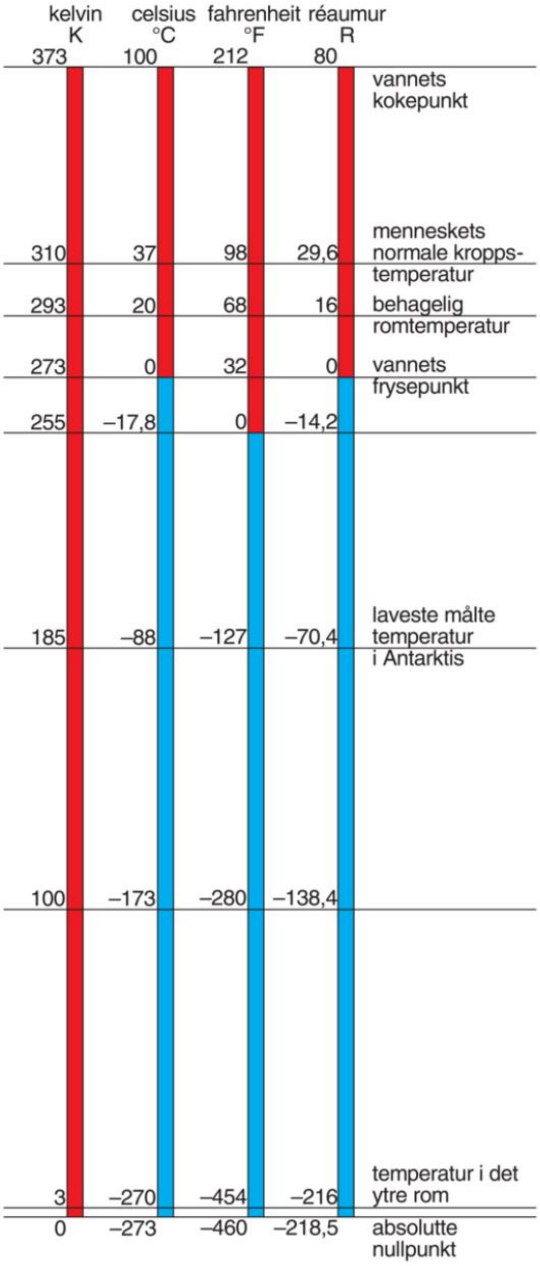
1) Consider Figure 1 on the next page. There is a linear relationship between the temperature scales in Fahrenheit and Celsius. Why is that?
2) What does it mean to have a measurement *scale* or *level*? What types of measurement scales and levels exist, and what are they used for?
3) Design a task intended for upper secondary school, which involves modelling the system with the two temperature scales. You may take the exercise from Søgaard and Tambs Lyche (1939/1969) as a starting point, but ensure that your task explicitly involves incorporating parameters. Moreover, the task should contain elements that makes it necessary for students to carry out construction, transformation, and evaluation of formulas (in the sense of Abbott, 1942/1971, as cited in Strømskag & Chevallard, 2022). Create a proposed solution to the task you have designed.

Reference

Strømskag, H., & Chevallard, Y. (2022). Elementary algebra as a modelling tool: A plea for a new curriculum. *Recherches en Didactique des Mathématiques*, *42*(3), 371–409. https://revue-rdm.com/2022/elementary-algebra-as-a-modelling-tool-a-plea-for-a-new-curriculum/

Figure 1

Temperature scales



Note. The figure is taken from *Store Norske Leksikon*. https://snl.no/temperaturskala. Reproduced with permission.

Appendix D – Tasks

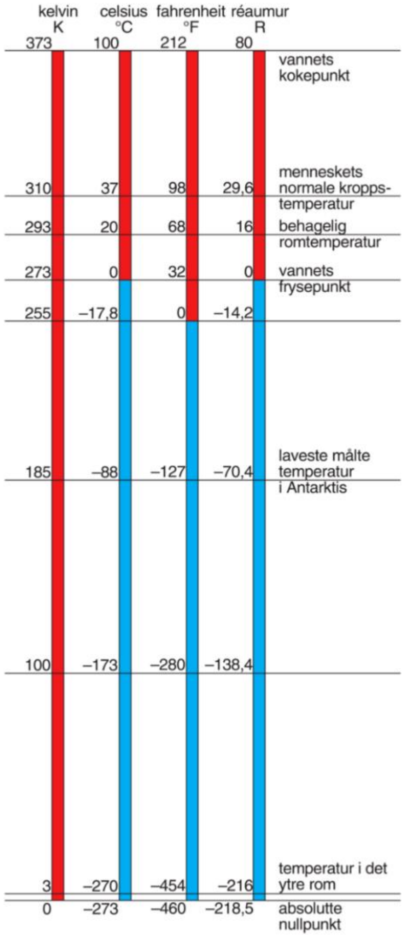
D9 – Materials for an answer to modelling of a relationship between Celsius and Fahrenheit grades

About temperature scales: All temperature scales are calibrated according to the thermal properties of a particular substance or device. Usually this is established by fixing two well-defined temperature points and defining temperature increments via a linear function of the response of the thermometric device. For example, both the Celsius scale and the Fahrenheit scale were originally based on the *linear evolution of a narrow mercury column within a limited temperature range*, each using different reference points and scale increments.

See https://en.wikipedia.org/wiki/Scale_of_temperature

Figure 1

Temperature scales



Note. The figure is taken from *Store Norske Leksikon*. https://snl.no/temperaturskala. Reproduced with permission.

See also https://en.wikipedia.org/wiki/Level_of_measurement. Particularly important here is the following:

Level of measurement, or *scale of measure*, is a classification that describes the *nature of information* within the values assigned to variables. The theory was first developed in 1946 by psychologist Stanley Stevens (1906–1973):

- Nominal level
- Ordinal scale
- Interval scale
- Ratio scale

1) So, we are searching for a linear equation: ***F* = *aC* + *b***, where *F* represents the degree number in Fahrenheit and *C* represents the degree number in Celsius. We have that *F*, *C*, *a*, and *b* are all parameters. We shall determine *a* and *b* from the system given in the exercise.

   Possible contextual reformulation of the textbook exercise:
   Anne is in the US and feels sick. She finds a forehead thermometer that measures temperature in degrees Fahrenheit. She measures her body temperature to be equal to 102 degrees Fahrenheit. Anne doesn't remember exactly how to convert from Fahrenheit to Celsius, but she knows the following: There is a linear relationship between the Celsius and Fahrenheit scales. Furthermore, she knows that 0 degrees Celsius corresponds to 32 degrees Fahrenheit and that 100 degrees Celsius corresponds to 212 degrees Fahrenheit. How can she find out what 102 degrees Fahrenheit is in degrees Celsius?

2) *Construction of a formula.*

   $F = aC + b$

   $32 = a \cdot 0 + b$
   $212 = 100a + b$

   $b = 32$
   $212 = 100a + 32$

   $b = 32$
   $a = \frac{9}{5}$

   That is to say, we have the following formula that is an *equation*: $F = \frac{9}{5}C + 32$ or $\underline{F = 1{,}8C + 32}$.

3) *Transformation.*

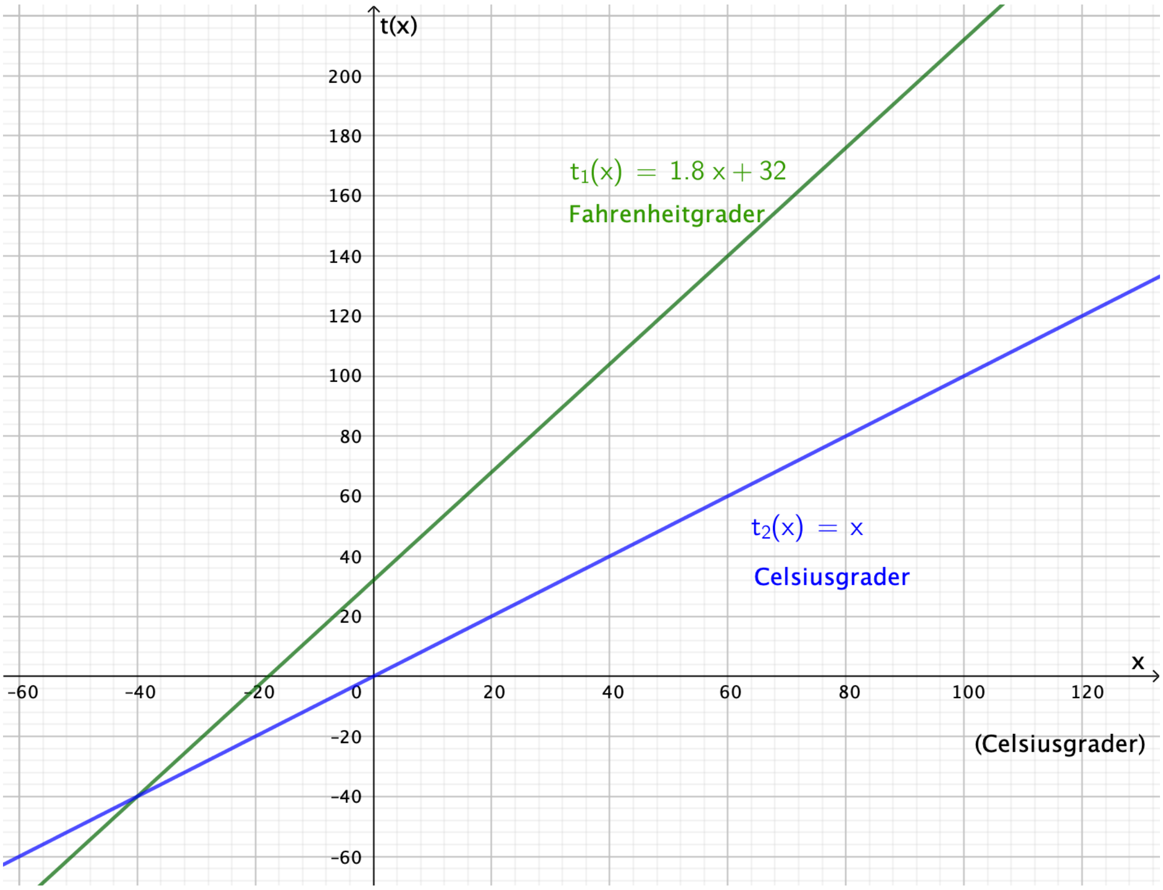
Finding C expressed in terms of F:

$$\frac{9}{5}C = F - 32 \quad \Leftrightarrow \quad \underline{C = \frac{5}{9}(F - 32)}.$$

4) *Evaluation*.

How many degrees Celsius is 100 degrees Fahrenheit?
How many degrees Fahrenheit is 20 degrees Celsius?
Are there any degree numbers where the two temperature scales have the same measure?

Algebraic solution: $F(C) = C \Leftrightarrow 1{,}8\,C + 32 = C \Leftrightarrow \underline{C = -40}$
Graphical solution:



Note. Percival Abbott's (1942/1971) three categories—construction, manipulation, and evaluation—are a *framework of principles* that describe operations on formulae and equations in algebra. It is important to re-establish manipulation of formulae/equations (a neglected area). This is important from a modelling point of view, when working with modelling in school and using algebra as a tool.

Appendix D – Tasks

D10 – Task: Modelling of braking distance

The following task is inspired by Grimenes et al. (2011, p. 69).[20]
A car is travelling at a speed of 72 km/h on a horizontal road. The car needs to brake suddenly, and the driver presses the brake pedal so hard that the brakes lock. The car then slides along the road until it comes to a stop. We assume that the coefficient of friction between the tyres and the road during sliding is 0.7—that is, the system to be studied involves frictional interaction between rubber and dry asphalt, according to Table 1.

Table 1

Typical friction coefficients between various materials

Material 1	Material 2	μ (Friction coefficient)
Steel	Steel	0.6
Steel	Ice	0.05
Steel	Teflon	0.04
Ice	Ice	0.03
Rubber	Dry asphalt	0.7
Rubber	Wet asphalt	0.2
Rubber	Ice	0.02
Wood	Wood	0.3
Hip joint	Hip joint	0.003

Note. The table is adapted from Grimenes et al. (2011, p. 68).

a) What is the system S under investigation, and what is the initial question Q to be answered about S? Delineate S by specifying the parameters that are relevant to the study you will make of it. What knowledge do you need to explain the relationships between the parameters of S?

b) Develop a model S' that can effectively answer the question Q and, further, generate additional knowledge about S. Provide examples of how different road conditions and varying speeds can impact the braking distance of a vehicle sliding after brake lock.

c) What knowledge does the constructed model allow you to produce about the system under investigation? Discuss relationships between the system S and the model S'.

[20] A proposed solution to the task is included in Appendix B. This information was not provided in the original task given to the student teachers. They got a note including a proposed solution after they had worked on the task.

Co-funded by
the European Union

Appendix D – Tasks

D11 – Design of modelling tasks for Grades 8–13

Identify a modelling task in a secondary school textbook that, in your opinion, inadequately supports the generation of knowledge about the system it aims to model. Your assignment is to design a new task, pertaining to the same system, in a way that strengthens the connection between the system under study and the model to be constructed. Subtasks a–c formulated below are meant to help you in studying the system to be examined, in order to prepare your own task design. Moreover, the note with a proposed solution on braking distance is available for your perusal, and could serve as a useful reference or spark of inspiration in your own work.[21]

e) Delineate the system S that will be the focus of your study. State the initial question Q that you aim to answer regarding S. Define the relevant parameters that are crucial for studying S, along with their respective relationships.

f) Build up a suitable model S' that can effectively address the initial question Q about the system S. Explain how this model S' can not only answer Q but also produce additional knowledge about the system S. Provide a brief overview of the background knowledge required for constructing the model S'.

g) Discuss the relationships between the system studied (S) and the model constructed (S') including the affordances (strengths and advantages) and constraints (limitations and assumptions) of the chosen model in representing the real-world dynamics of S.

h) Utilising the knowledge generated through the previous subtasks, devise a modelling task suitable for students in Grades 8–13 (select one of the grades). The task should explore the same system (S) and address a question (Q), concerning S. Q can either be the same or different from the one you addressed previously. Strive to design the task in a way that enables students to develop knowledge about the real-world dynamics of S, while ensuring it is both appropriately challenging and manageable for the chosen grade level.

[21] The note referenced herein is a theoretically substantiated solution proposal that I developed in 2022, as part of our exploration related to modelling of a vehicle's braking distance.

Co-funded by
the European Union

CHAPTER 3

Mathematical Modelling in a Professional Development Programme

Frode Rønning

Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, Norway

Introduction

This report is based on a module called *MA6004 Algebra, Functions and Modelling*, which is part of a professional development programme at NTNU for secondary school teachers (Grades 8–13, with emphasis on Grades 11–13[22]). The target group consists of teachers with insufficient formal background in mathematics to teach at this level, but who do so despite of lacking the formal qualifications. Some also participates in the programme with the aim of getting the possibility to teach mathematics at this level in the future. The study is part-time, and the participants have teaching duties at the same time as they are following the programme. The programme consists of eight modules, each of 7.5 ECTS. Most of the modules are in terms of content identical to the modules taken by regular mathematics students, including future teachers, but the module MA6004 is especially designed for the professional development programme. MA6004 is a module where the emphasis is on learning elementary calculus with differential equations, but with a profile towards teaching in the upper grades of secondary school (11–13). This profile is mainly visible through the emphasis on mathematical modelling as a didactical concept and connects to the Norwegian National Curriculum[23] where modelling is a central topic in mathematics. Modelling and applications is one of six so-called *core elements* in the National Curriculum.

The module MA6004 is placed in the first semester of the programme, alongside with a module with the title *An Introduction to Theories of Knowledge and Learning in Mathematics.* This module contains topics from didactics of mathematics, exemplified through analysis and algebra.

Learning Goals and Structure

The learning goals of the module are formulated in terms of *knowledge*, *skills* and *general competence* as formulated below (see also https://www.ntnu.edu/studies/courses/MA6004/2022/1#tab=omEmnet).

Knowledge

The candidate has

[22] https://www.ntnu.no/videre/matematikk1-8til13 and https://www.ntnu.no/videre/matematikk2-8til13
[23] https://www.udir.no/lk20/mat09-01/om-faget/kjerneelementer?lang=eng

Co-funded by
the European Union

- basic knowledge about modelling as a way of working in mathematics,
- good knowledge about important functions, such as polynomials, rational functions, trigonometric functions, exponential and logarithmic functions, and their properties,
- basic knowledge about differentiation and integration, and applications of these concepts,
- basic knowledge about differential equations and their applications,
- basic knowledge about matrices and their applications,
- basic knowledge about numerical solutions of algebraic equations and differential equations.

Skills

The candidate is able to

- use integration and differentiation to analyse properties of functions,
- model situations from nature and society using mathematical concepts and methods, and assess the validity of these models,
- solve problems using both algebraic and numerical techniques, and use digital tools, including simple programming, in this work,
- assess whether results obtained using digital techniques are reasonable.

General competence

The candidate has acquired

- a good basis for studying more advance mathematical courses,
- good knowledge about mathematical topics and use of digital aids, relevant for teaching mathematics in Grades 8–13.

The module is offered to participants from all over Norway, and the teaching is organised in seminars on campus and additional internet-based support using *Teams.* The module runs from August to December, with two-day seminars in August, September and November. Learning material, printed and in the form of videos, is available on *Teams*, and *Teams* also hosts a chat forum enabling communication among the participants and with the teacher.

The Modelling Aspect

For the module as a whole, the mathematical topics are the most important and take up most of the time spent on the module. These topics are very close to what the participants are expected to teach in upper secondary school. Functions and related topics (differentiation and integration) take up most of the time in upper secondary school mathematics in Norway. The topics are also important prerequisites for modules coming later in the programme, where e.g., analysis is introduced in a more formal way. For this report however, the didactical aspect of mathematical modelling is the most important. In the plans for the meetings (Appendix C) one can see the list of topics covered and where the modelling part was placed. The participants are required to write an assignment of approximately 2500 words that counts 30% towards the final grade. Before handing in the final product, the participants are required to do an oral presentation, and they have the possibility to submit a draft version on which they get feedback from the teacher. The requirements for the assignment are formulated in a two-page document (see Appendix A). Below I have included the introduction to the text (translated to English) containing the requirements.

> In this assignment you are expected to show how to do a modelling process as described in Blum and Ferri (2009). The process should be based on a situation of your own choice, and the mathematics required should be within Chapters 2–7, 9 in the textbook (Gulliksen et al., 2022). You may choose a situation entailing *descriptive modelling* or a situation entailing *prescriptive (normative) modelling* (Niss, 2015). Your work with the model will turn out somewhat differently depending on this choice. One possibility could also be to do a prescriptive modelling process and afterwards reflect on how the situation could be changed to include normative aspects.
> In your work you must show how you make use of digital tools, like e.g., GeoGebra, Excel or programming in a programming language (Python is probably most relevant).

Regarding the modelling aspect, the goal is twofold. The participants should get experience with performing a modelling process themselves, but the intention is also that through this experience they should get sufficient background to orchestrate a modelling project with their own students.

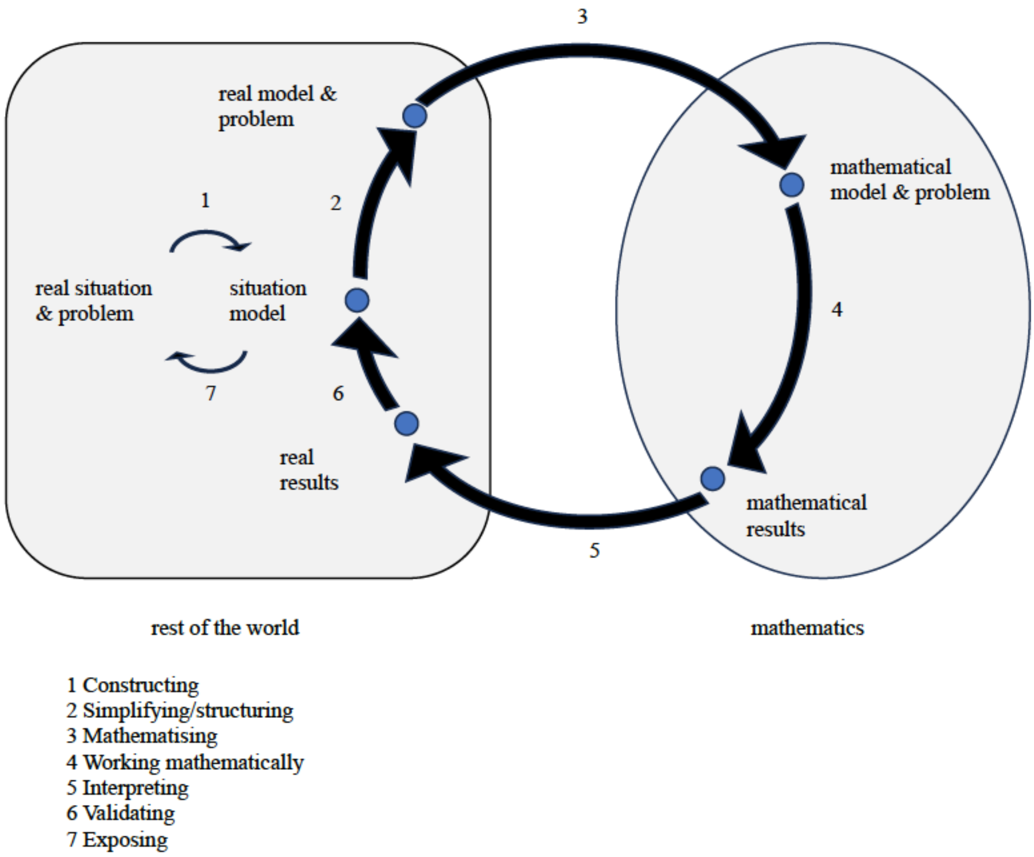
Theory of Mathematical Modelling

The approach to modelling taken in MA6004 is based on the modelling cycle (Blum & Ferri, 2009) as shown in Figure 1. Briefly, this approach entails taking a real situation and problem as a starting point and through a process of simplifying, structuring and mathematising, bringing the problem on a mathematical form so that it becomes a *mathematical* model and problem. From this, the process of working mathematically starts, and mathematical results are obtained. These are interpreted and validated, and through this process, the cycle has brought the problem and its solution/s back to the real world. This approach to modelling is chosen because it is consistent with how modelling is described in the National Curriculum in the core element Modelling and Applications:

> A model in mathematics is a description of reality using mathematical language. The pupils shall gain insight into how mathematical models are used to describe everyday life, working life and society in general. Modelling in mathematics means creating such models. It also means to critically evaluate whether the models are valid and what limitations the models have, evaluate the models in view of the original situations, and evaluate whether they can be used in other situations. Applications in mathematics means giving the pupils insight into how to use mathematics in different situations within and outside of the subject. (Utdanningsdirektoratet, 2020)

Figure 1

The modelling cycle (adapted from Blum & Ferri, 2009, p. 46)



In the quote from the National Curriculum above both the formulation "a description of reality using mathematical language" and the formulation "critically evaluate whether the models are valid and what limitations the models have, evaluate the models in view of the original situations" show that the take on modelling is inspired by the modelling cycle as e.g., described in Blum and Ferri (2009). The approach is also consistent with the definition of *modelling competency* found in Niss and Højgaard (2011):

> Active modelling contains a range of different elements. Firstly, there is the ability to *structure* the real area or situation that is to be modelled. Then comes being able to implement a *mathematisation* of this situation, i.e. translating the objects, relations, problem formulation, etc. into mathematical terms resulting in a mathematical model. Then one has to be able to *work with* the resulting model, including solving the mathematical problems that may arise as well as *validating* the completed model by assessing it both internally (in relation to the model's mathematical properties) and externally (in relation to the area or situation being modelled). Furthermore, there is the ability to *analyse the model critically* both in relation to its own usability and relevance, and in relation to possible alternative models, as well as to *communicate* with others about the model and its results. (Niss & Højgaard, 2011, pp. 58–59)

This approach to modelling is clearly an approach focusing on descriptive modelling. However, prescriptive, or normative, modelling was also discussed in the module. The main examples used in the teaching were taken from Niss (2015). The first example is obviously prescriptive in the sense that it defines how something *should be* instead of describing how something *is*. The example is the A-format (DIN-format) for paper. This format is defined by an algorithm that prescribes both the size and the shape of the paper sheets:

- The sheets should be rectangular
- The largest sheet should have an area of 1 m^2
- By folding a sheet along the longest side, the two pieces obtained should be similar.

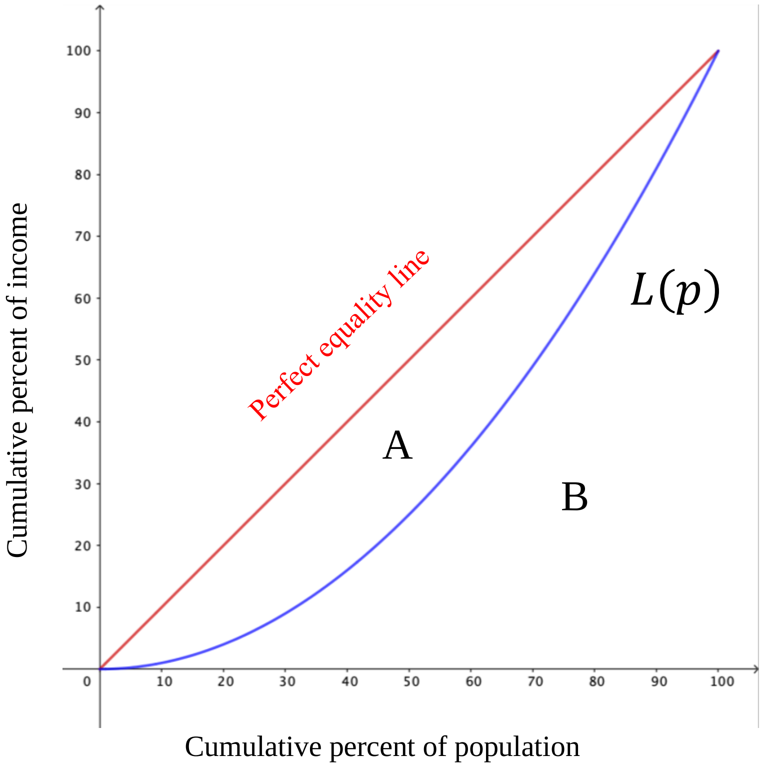
The second example is the so-called Gini index (Sittihyot & Holasut, 2020), as illustrated in Figure 2. In the figure, the blue curve (known as the Lorenz curve) shows the function $L(p)$ defined as the share of the total income in a population earned by the rate p of the population with lowest income. A denotes the area below the red line (the perfect equality line) and the blue curve (the Lorenz curve), and B denotes the area below the Lorenz curve. The Gini-index G is then defined as $G = A/(A + B)$. If $A = 0$ then $G = 0$, and if $B = 0$, then $G = 1$. The following questions were discussed:

- What does it mean if $L(p) = p$?
- What does it mean if $L(p) = 0$ for $p < 1$ and $L(p) = 1$ for $p = 1$?

From the definition it is clear that G is a number in the interval $0 \leq G \leq 1$, where $G = 0$ represents perfect equality and $G = 1$ represents maximum inequality. For further details about the teaching sessions on mathematical modelling, see Appendices B1 and B2. The complete list of readings can be found in Appendix D.

Figure 2

The Gini-index



The Gini-index is also an important topic in Civic Education as a model describing socio-economic inequality within a society (Steinbachner & Nagy, 2023). In this paper, both strengths and limitations of the Gini-index are discussed.

The Modelling Project

In the description of the modelling project (Appendix A), the modelling cycle from Blum and Ferri (2009) was actively referred to. Furthermore, the participants were required to find a situation that could be modelled using functions and related concepts such as differentiation, integration and differential equations. This choice was made to link the project to the mathematical topics covered by the module, as well as to relevant topics in upper secondary school mathematics. Apart from these restrictions, the participants were free to choose the situation. It was said that the situation could, but did not have to, be such that it could be used with students in the classes the participants were teaching. The participants were required to formulate a question that the model should help to answer, and they were in particular asked to emphasise steps 2, 3, 4 and 5 in the modelling cycle (Figure 1). They were encouraged, but not required to work with empirical data. However, it was said that in principle it should be possible to find empirical data for the chosen situation.

There were 12 students completing the module, and below is a list of the topics covered in the projects. In a few cases there were more than one student choosing the same topic.

- Modelling the cooling process, comparing different liquids (two projects)

- Modelling/estimating running times (world records) for marathon and half marathon based on historical data or measurements of the student's own running of shorter distances (two projects)
- Modelling the decrease in value for cars
- Modelling energy consumption for refrigerators
- Modelling the cost for buying fuel in different locations
- Modelling the increase in weight in beehives as a function of temperature
- Modelling the stiffness of springs with serial and parallel coupling
- Modelling the development of sunrise/sunset at two different locations in Norway
- Modelling the area of ice in the Barents Sea
- Modelling the reaction kinetics when H_2O_2 is decomposed

The list shows that the chosen topics span over a large spectrum, and they are mostly of a descriptive nature. For some of the projects it is possible to compare measure data with results expected from laws of nature. Examples of this kind are the stiffness of springs (Hooke's law), cooling process (Newton's law of cooling) and decomposing H_2O_2 (Arrhenius' law). Others are purely empirical such as modelling the development of world records in running and the weight of beehives. Some of the projects have, or could have, a normative aspect, and I will go into more details about two of these.

Comparing Price and Energy Consumption of Refrigerators

This project is motivated by the rising prices of electricity and the fact that modern refrigerators are much more energy efficient than old ones. A starting point is the assumption that modern refrigerators have a yearly energy consumption of around 120 kWh whereas old ones may have a consumption of up to 2000 kWh. This could amount to almost 20% of the energy consumption in a small flat. The questions that are formulated in the assignment are:

- By replacing an old refrigerator with a modern one, how long does it take before the investment in the new one is saved, given the current development of energy prices?
- How is this dependent on the age of the old refrigerator?

The model only takes into account the price of the new refrigerator and the energy consumption of an old one versus a new one. This means that the cost of keeping the old refrigerator is set to zero, and no costs for repair and maintenance are taken into account. A Python function was constructed to calculate expenses for a refrigerator over a period of n years with the following parameters as input: Price of refrigerator, energy consumption, price of electricity, and yearly increase in electricity price. Using this function, another function was made to compare the expenses for two different refrigerators. Figure 3 shows a comparison of an old refrigerator (value = 0, energy consumption = 850 kWh, orange curve) with a new one (value = 6995 NOK, energy consumption = 115 kWh, blue curve). The intersection point of the curves gives the number of years when the costs of buying the new refrigerator is "saved".

Figure 3

Total expenses for two refrigerators as a function of time

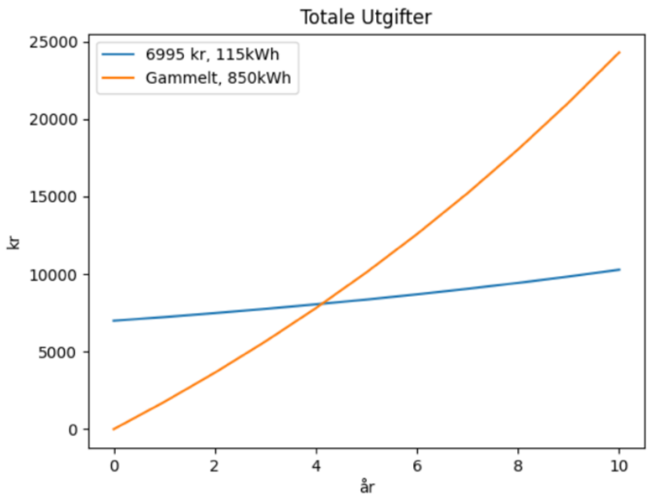
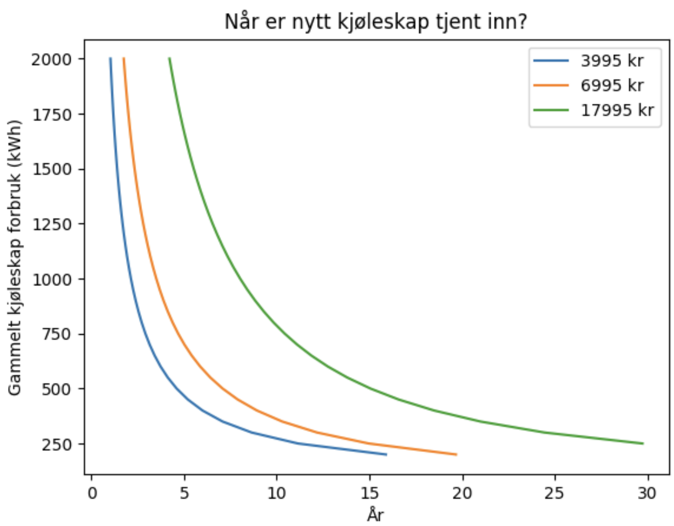


Figure 4 shows a comparison of three refrigerators with different prices, 3995 NOK (blue curve), 6995 NOK (orange curve) and 17995 NOK (green curve). The graph should be read like this: On the vertical axis, choose the energy consumption of the old refrigerator and from this value, draw a line parallel to the horizontal axis. When this line crosses one of the graphs, read off the number of years on the horizontal axis necessary to "save in" the money needed to buy a refrigerator corresponding to the chosen curve. For example, if the old refrigerator uses 1000 kWh, buying a new one at 3995 NOK will be worthwhile after around 2.5 years, whereas buying a new one at 17995 NOK will be worthwhile only after around eight years. It is assumed that all the new ones have the same energy consumption.

Figure 4

Comparing three refrigerators at different prices



As the model was presented in the assignment it did not contain a normative aspect, but it could easily have been modified to also include normative aspects. The important parameters were the price of a new refrigerator and the price of electric energy, and the aim of the modelling process was to minimise the direct and immediate costs for the consumer. Reducing the energy consumption is important also in a global perspective, but to take this into account it would be necessary to look at the consumption of energy to produce a new refrigerator and to transport it from the factory to the retailer and finally to the consumer. The consumer would also need to dispose of the old refrigerator, which again would require energy. Furthermore, in the production process there will be other issues than just energy consumption, e.g., natural resources needed in the components the refrigerator is made of.

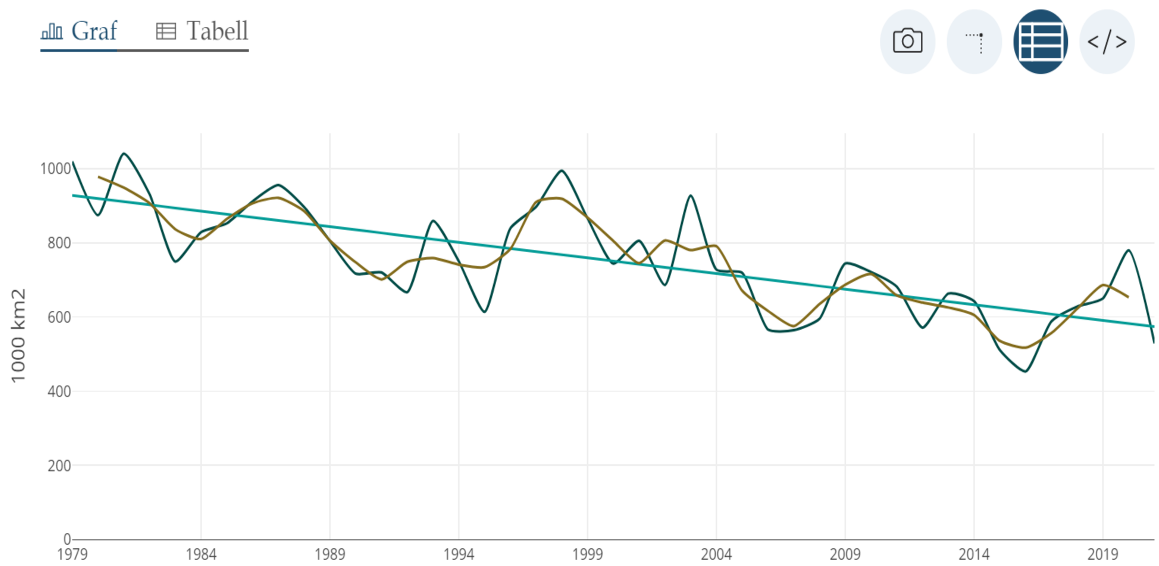
The topic in this assignment can be seen as an example of a topic that could come in many different settings, and that could be well suited for work in school. The main question is targeting one's personal economy: When is it profitable for each individual to choose one solution before another? The project in the list above about cost of fuel could also be placed in the same category. Such questions could easily be given a normative component by looking at them in a more global perspective, taking more parameters into consideration.

Modelling the Development of the Ice in the Barents Sea

This project has a clear normative aspect although the modelling is descriptive. The project starts by quoting the secretary general of the UN and goals set by the Norwegian government about reducing CO_2 emissions by a certain percentage. The project is also connected to learning goals about sustainability in the Norwegian National Curriculum. Global warming is further connected to the area of the ice in the Barents Sea. The project uses available data from 1979 to the present and the aim is to find a function that can predict how large the area of the ice will be in 2032, and to predict when all the ice is gone. The model is purely empirical in the sense that no laws of nature are used to construct the model. The empirical data are illustrated in Figure 5 showing the area of ice as a function of time (black curve). The figure also contains a trend line (green line) as well as a curve showing the mean value taken over three years (brown curve).

Figure 5

Area covered by ice in the Barents Sea as a function of time



The modelling process is based on using the regression tool in GeoGebra. Figure 5 seems to indicate a certain periodic behaviour around a straight line with negative slope (the trend line). Therefore, a reasonable function was proposed to be of the form $f(t) = A - Bx + C\cos(Dt)$, where A, B, C and D are constants.

Summary

The work with modelling in this module showed that the participants were actively engaged in finding situations that could be modelled with functions. All the situations involved descriptive modelling. As I have indicated earlier, the way modelling is described in the National Curriculum, it is seen as descriptive modelling. This could partly explain the choices made, but it is also possible that it takes more effort to find situations that are suited for prescriptive (normative) modelling. In particular, it would be more challenging to find data to include in a normative modelling process. In the example with the refrigerator, it is easy to find data on energy consumption and costs, both of a new refrigerator and of electricity. However, to find data on energy consumption in production, transport or in disposing of the old refrigerator would be more challenging, and these data would probably be much more uncertain. Still, as my two examples show, it could be feasible to change a prescriptive model into containing normative aspects. At least such aspects could, or should, be discussed, if not quantified. Also, for a situation that in itself is connected to normative issues, the example with the ice in the Barents Sea shows that it can be relevant to perform a descriptive modelling process.

It could be raised as a criticism towards the Norwegian National Curriculum in mathematics that the core element Modelling and Applications is solely focusing on descriptive modelling. This criticism is even more appropriate when reading the Core Curriculum which is at a level above the curriculum for the individual subjects. In the Core Curriculum one can find three interdisciplinary topics: Health and life skills, Democracy and citizenship, and Sustainable development. About Sustainable development it is said that "this topic includes issues relating to the environment and climate, poverty and distribution of resources, conflicts, health, equality, demographics and education" (Utdanningsdirektoratet, 2020b). The idea was that all the three interdisciplinary topics should be addressed in all subjects, but it was not really required. Hence, the expert groups designing the curriculum for the individual subjects were free to choose which interdisciplinary topics to include. In mathematics, Sustainable development is not included, which to me seems somewhat surprising. This interdisciplinary topic would have fitted well with the core element Modelling and Applications and made it natural, or even unavoidable, to include normative aspects in the modelling process.

References

Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Gulliksen, T., Hashemi, A. M., & Hole, A. (2022). *Matematikk i praksis* [Mathematics in practice] (7th ed.). Universitetsforlaget.

Niss, M. (2015). Prescriptive modeling: Challenges and opportunities. In G. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modeling in education research and practice: Cultural, social and cognitive influences* (pp. 67–79). Springer. https://doi.org/10.1007/978-3-319-18272-8_5

Niss, M., & Højgaard, T. (2011). *Competencies and mathematical learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark.* Roskilde University.

Sittihyot, T., & Holasut, K. (2020). A simple method for measuring inequality. *Palgrave Communications*, *6*(112), 1–9. https://doi.org/10.1057/s41599-020-0484-6

Steinbachner, J., & Nagy, N. (2023). *Is climate catastrope fair? Fairness, responsibility and decolonial perspectives* (Chapter 4, this volume).

Utdanningsdirektoratet (2020a). *Core elements.* https://www.udir.no/lk20/mat09-01/om-faget/kjerneelementer?lang=eng

Utdanningsdirektoratet (2020b). *Sustainable development.* https://www.udir.no/lk20/overordnet-del/prinsipper-for-laring-utvikling-og-danning/tverrfaglige-temaer/2.5.3-barekraftig-utvikling/?kode=mat01-05&lang=eng

CHAPTER 4

Is Climate Catastrophe Fair? Fairness, Responsibility and Decolonial Perspectives

Jakob Steinbachner & Nicola Nagy

Didaktik der Politischen Bildung, University of Vienna, Vienna, Austria

Political Education and Teachers Education in Austria

Political education is assumed to be a possibility and a necessity for the development of individual skills and society as a whole in an increasingly complex everyday world. Political education should help to deal with individual questions such as one's own identities or also social phenomena such as unequal distributions of power and resources between different genders, and in doing so make an active contribution to the shaping of society and the further development of democracy and human rights. The goal of teaching in political education in Austria is a reflected and reflexive political awareness (BMBWF 2018).

In Austria, political education is carried out in different ways in various educational institutions. Examples include political education in the school curriculum on the one hand and teacher training in history and political education at the University of Vienna on the other.

As already mentioned, the training as well as the subject of political education in Austria is strongly linked to the school subject of history. The hours in school are shared with the subject History (in the general school types) (cf. BMBWF/Zentrum Polis, 2022). Likewise, political education in Austria is one of the "cross-curricular teaching principles", which should mean addressing political issues and democratic action in schools as overall institutions (cf. BMBWF, 2015). In Austria, the training of teachers of political education is also linked to the subject of history. This is exemplified by the curriculum of the subject history and political education for the bachelor's degree in teaching at the University of Vienna. Here, political education is included in a "compulsory module group" with a scope of 10 ECTS (cf. University of Vienna 2022). Of particular interest for this module is the objective to enable students to deal with and teach topics of ecological sustainability and its political implications and the skills for participation at different levels (University of Vienna 2022, pp. 11–12).

Climate *Justice*

The central topic around which the question of political education and possibilities for action develops in this module is that of climate justice. On the one hand, the causes of and possibilities for action against the climate catastrophe should be discussed, and on the other hand, the unjust distribution of cause and effect will be looked at. Different perspectives are taken, and the problem is looked at from different angles.

Fleischmann et al. (2021) highlight that our planet's climate has undergone continuous changes throughout geological time, including significant variations in global average temperatures. They continue that

> the current climatic warming is happening much more rapidly than past warming events. It has become clear that mankind has caused most of the warming of the last century by emitting greenhouse gases to power our modern lives. We do this through burning fossil fuels, agriculture and land use and other activities that drive climate change. The World Meteorological Organization (WMO) reported in 2017 that greenhouse gas emissions in Earth's atmosphere have reached the highest level ever in 800,000 years (Schlein, 2017). […] Climate change involves not only rising temperatures, but also extreme weather events, rising sea levels, changing wildlife populations and habitats, and a number of other impacts. (Fleischmann et al., 2021, p. 6)

In addition to scientific debates, the unjust implications of the climate catastrophe have also come to the attention of journalists and are being discussed, as Chris Mooney shows in his article "Why climate change is really, really unfair". He assumes that "The general rule is, at a global scale, if you're a nation that is going to suffer from climate change, you're very likely not contributing to the problem,". In his article, he shows that the countries that commit the most "environmental bads" often suffer the least from the effects, while those that are already structurally disadvantaged are more affected by the climate catastrophe (Mooney, 2016). This view is widely confirmed by scientific articles. Harlan et al. (2015), for example, assume that climate catastrophe is perceived very differently by rich and poor people a perspective, which will increase in the future because policies aimed to manage climate catastrophe exclude poor and powerless individuals. They even consider climate catastrophe to be mainly caused by inequalities, as the most marginalized countries and societies produce far fewer emissions than wealthy individuals. Given these facts, Harlan et al. identify climate catastrophe as one of the most compelling topics for theorizing injustice and inequality because these inequalities and injustices are systematically produced rather than being a result of individual actions (2015, pp. 127–128).

This perception and increasing popularity and importance has led in recent years to various scientific publications and journalistic elaborations as well as to other comprehensive projects such as the "Global Footprint Network" (https://www.footprintnetwork.org/), international scientific panels such as the IPPC and corresponding relativisation movements such as the NIPCC.

Justice Debates

Firstly we focus on justice debates in a philosophical way to identify our concepts of justice, equality, equity and fairness.

The concept of equality and justice has been explored by Cook & Hegtvedt (1983) and Preisendörfer (2014) in their respective works. Cook & Hegtvedt primarily focus on exchange transactions and their idea of equality is based on the benefit that each actor gains from the exchange. They consider an exchange equal if all actors involved benefit equally, and inequality occurs when the ratio of what is given and received varies among the actors (1983, 218).

Preisendörfer, on the other hand, examines the concept of justice and attempts to distinguish it from inequality, which are often used interchangeably. He seeks to identify factors that indicate a transition from inequality to injustice through empirical research (2014, pp. 34–35). Preisendörfer draws on David Schlosberg's trivalent package, which includes "justice as distribution," "justice as participation," and "justice as recognition." The first component considers justice in terms of distribution, the second includes the question of just procedures and opportunities for participation, while the third addresses minimum standards of human dignity, respect for individual, social, and cultural integrity, and protection from disregard and discrimination (Preisendörfer 2014, p. 33). Preisendörfer's empirical justice research seeks to identify factors that distinguish inequality from injustice, and the outlined factors serve as indicators for such a transition. By analysing these factors, it is possible to identify instances of injustice and to develop strategies to rectify such situations.

Preisendörfer outlines several factors that facilitate the transition from inequality to injustice (2014, pp. 35–40):

1. Clearly pronounced socio-spatial inequalities in environmental burdens
2. Environmental burdens beyond reasonable levels
3. Accumulation of disadvantages
4. Impairments of subjective well-being
5. Disaggregation of causers/beneficiaries and those affected
6. Voluntariness and possibilities of evasion
7. Opportunities for participation and co-determination

Mathematical Modelling in Justice Debates

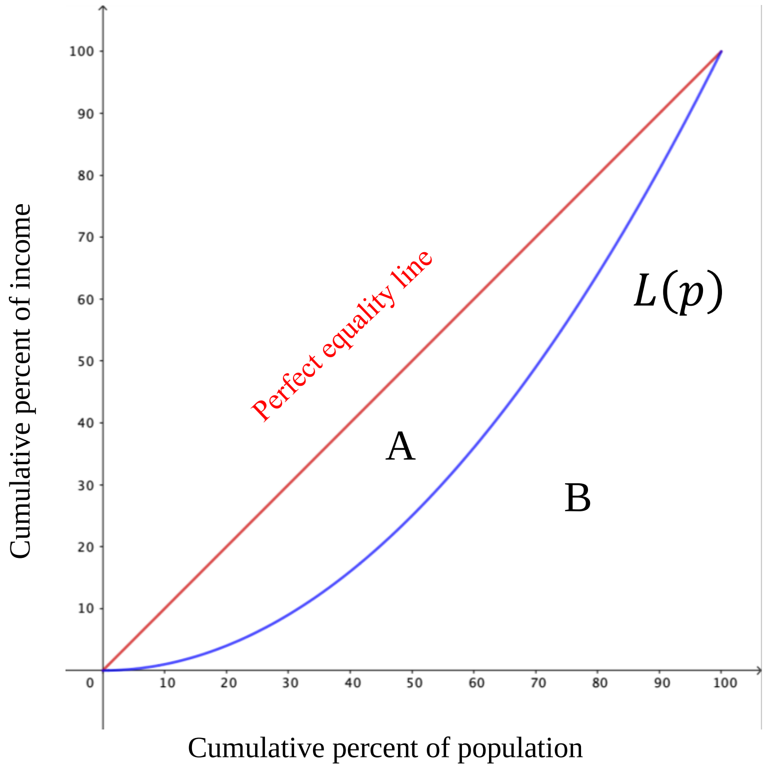
Political issues in the modern world are becoming increasingly complex and refer to various disciplines, including mathematical ones (cf. Gildehaus et al., 2021, pp. 5–6; Lauß, 2022, pp. 116–117). An engagement with mathematical models in political education is thus inevitable in order to enable or understand decision-making processes based on them. This is important, on the one hand, to be able to make decisions oneself or to understand decisions made by others, and on the other hand, to be able to evaluate the validity and reliability of the models on which the decisions are based (Gildehaus et al. 2021, p. 6). A model is referred to here as a transformation of a system, "usually a simplification, which is supposed to help in generating knowledge about the studied system. In practice, to answer a question relating to a system, one tries to build up a model which is easier, safer, and quicker to study than the system itself. Models are thus used for answering questions or exploring facts, guided by research questions. Models always have a descriptive function, but they can also contain statements about what an individual should do. If this is the case, models are referred to as prescriptive, or normative. Depending on the case, a model can thus have descriptive, normative or prescriptive uses." (Gildehaus et al., 2021, p. 7). On the one hand, when dealing with political problems based on mathematical models, it is important to learn and understand the mathematical background. Normative models, however, also require a political analysis in order to be able to recognise political possibilities for action and judgements within this system (Gildehaus et. al., 2021, p. 10). One example for a political problem which also refers to mathematical descriptive and normative models in different perspectives is the climate catastrophe, another one could be justice debates. Descriptive models on the topic of climate catastrophe can be found, for example, on the website of the "Global Footprint Network".

In order to become aware of this model of normative aspects and conclusions for political and social decisions, this model must be analysed for example on the topic of causes, consequences and possible actions for change.

Another topic that can be described by mathematical models is a debate on justice. The Gini coefficient can serve as an example of a mathematical model that describes mostly socioeconomic inequality within a society (but every other size distribution in data-sets with non-negative quantities as well) (Sitthiyot & Holasut 2020, p. 2).

Figure 1

The Gini-index



The Gini index can be derived from the Lorenz curve framework, which plots the cumulative normalized rank of income on the x-axis and the cumulative normalized income on the y-axis. The first 10% on the x-axis represent the 10% of people with the lowest income. The index is calculated by dividing the area between the perfect equality line and the Lorenz curve by the total area under the equality line. A value of 0 indicates perfect equality, while a value of 1 represents maximum inequality (Sitthiyot & Holasut 2020, p. 2).

There are several advantages to using the Gini index. Firstly, it summarizes the entire income distribution with a single statistic, making it relatively easy to interpret. It also allows for easy comparisons between countries and societies with vastly different population sizes. Additionally, the Gini index is updated regularly by various countries and international organizations (Sitthiyot & Holasut 2020, p. 2).

However, there are limitations to the Gini index. A lower Gini index does not always indicate a more equal income distribution, as it does not take into account the social welfare function. Furthermore, the Gini index is more sensitive to changes in the middle of the income distribution and less sensitive to changes at the top or bottom. According to Sitthiyot & Holasut, this means that two or more countries may share the same Gini index but have significantly different income distributions (2020, p. 3).

When analysing this model and debating about equal and inequal distributions and the model's just or unjust implications, normative aspects of justice debates can be made visible.

Different Perspectives in Climate Justice

Gender

The issue of gender inequality has become increasingly crucial in the discourse on climate justice, as has been pointed out by the United Nations (UN) Women (2022) and the Committee on the Elimination of Discrimination Against Women (CEDAW) during their 44th session. In 2009, the OEDAW "expresse[d] its concern about the absence of a gender perspective in […] global and national policies and initiatives on climate change." (OEDAW, 2009, p. 7). They also point out "that climate change does not affect women and men in the same way and has a gender-differentiated impact. [But] […] women are not just helpless victims of climate change—they are powerful agents of change, and their leadership is critical." (OEDAW, 2009, p. 7).

The perspectives presented by Arora-Jonsson (2011) highlight two main issues related to gender and climate catastrophe. Firstly, women in the global south are more likely to be affected by climate catastrophe than men, and secondly, men in the global north are more likely to pollute the environment than women. In both places, women have less representation in decision-making bodies.

Recent literature on gender and climate change emphasises the need to focus on women's perspectives on climate justice since they are the poorest of the poor (Arora-Jonsson 2011, pp. 745–746), more vulnerable to natural disasters (Arora-Jonsson, 2011, pp. 746–747), and more environmentally conscious, thus polluting less (Arora-Jonsson, 2011, pp. 747–748), and yet they have less agency to initiate change (Arora-Jonsson, 2011, p. 749). However, Arora-Jonsson warns against treating women as a homogenous group, as this tends to portray them as suffering due to their marginal social position compared to men. Such a portrayal may deny them agency, construct their vulnerability as a specific problem of their gender, and ignore power imbalances, thereby reinforcing differences between women and men as given and unchangeable. Instead, understanding the configuration of social relations of power and how vulnerability is produced is more critical than generalising the vulnerability of women. In her argument, Arora-Jonsson posits that increasing the efficiency of environmental management involves the active participation of women. The inclusion of women in decision-making processes not only empowers them, but also brings non-male perspectives to the fore, thereby enriching environmental management (Arora-Jonsson, 2011, p. 749).

Arora-Jonsson's conclusions are supported by the statement made by the Committee on the Elimination of Discrimination against Women (CEDAW) at its 44th session, which asserts that "gender equality is essential to the successful initiation, implementation, monitoring and evaluation of climate change policies. The committee calls on states parties to include gender equality as an overarching guiding principle in the United Nations Framework Convention on Climate Change (UNFCCC) agreement (CEDAW, 2009, p. 8).

Decolonial Perspectives

In order to be able to discuss decolonial perspectives, a concept of colonialism must first be clarified. We will use the concept of colonialism from Osterhammel & Jansen (1995, pp. 18–28).
Rachel Harnett (2021, p. 139) assumes that empires and imperialism are a major factor in the emergence of climate catastrophe. It is assumed that modern empires have played a significant role in the rise of fossil fuels. Furthermore, it is argued that the climate catastrophe redefines already existing social, economic and racial inequalities that emerged under colonialism. The climate crisis thus represents a new form of imperialism.

The fact that the climate catastrophe, contrary to the assumption that it is the "single greatest threat to human civilisation" (Harnett, 2021, p. 139), receives far less attention than isolated natural disasters is explained by the "slow-violence theory" (Nixon, 2011, as cited in Harnett, 2021, p. 140). This theory assumes that punctual disasters with a few thousand deaths receive considerably more attention than disasters over longer periods of time with considerably higher numbers of victims due to the less directly noticeable impact.

On average, societies of the Global North have a much higher impact on climate disasters, while people of the Global South often feel their effects first and more strongly. In addition to the effects of climate catastrophe on their immediate living environment (such as more frequent waves of disease, rising sea levels and extreme temperatures), these societies have other disadvantages that often result from their former colonial past. They are lesser industrialized and struggle to keep up with their former colonizers. In addition, climate protection measures often further disadvantage these countries, as their industries often find it more difficult to implement these measures (Harnett, 2021, pp. 139–146). This is what Harnett (2021, p. 146) calls "green neocolonialism".

Generational Perspectives

Changes in consumption patterns have accompanied a shift from precarious survival to growing well-being (Diprose et al., 2021, pp. 103–106), these changes have led to a less sustainable choice of products, creating intergenerational and spatial conflicts (Diprose et al., 2021, p. 107). Despite younger generations' increased awareness of climate change issues, they often make use of short-term or luxury products and are blamed by older generations for not leading a sustainable lifestyle (Dirpose et al., 2021, pp. 109–114). These conflicts are also observed between urban and rural populations, with the latter criticizing urban dwellers for not adopting more responsible and healthier consumption habits (Dirpose et al., 2021, pp. 115–117).

Individual and Systemic Approaches

The phenomenon of climate catastrophe presents us with an unprecedented challenge. Our collective contributions to this complex issue implicate us all, yet attributing responsibility for the resulting deaths is difficult, if not impossible, on an individual level (Dale, 2015, pp. 23–24). In discussing various concepts of responsibility such as causal, moral, and legal responsibility, Dale concludes that, for a variety of reasons, these concepts of responsibility can hardly describe or guide action in the climate catastrophe (Dale, 2015, pp. 23–38). According to the same author, this leads to the conclusion of intervention-responsibility to address climate catastrophe in a globally responsible manner (p. 38). This idea entails a forward-looking conception that directs us towards what different actors in the system, whom Dale refers to as "agents," can do, rather than what has been done so far. These agents operate at various levels of the system and possess varying degrees of power to bring about change, such as an individual acting with different capacities from government actors. Dale identifies four different "families" of "agents" (each individual in one family) with regards to climate catastrophe (Dale, 2015, pp. 38–41): international organizations and regimes, nations and other jurisdictions, individuals, firms. Each agent and each family has the ability to intervene in climate catastrophe (Dale, 2015, p. 38).

Didactic Approaches Teaching Climate Justice

The educational module presented here aligns with the Social Science Issues (SSI) approach developed by Georg Lauß (2022). Contemporary political discourses increasingly rely on scientific findings, necessitating a basic understanding of scientific methods and terminology among participants to engage in meaningful discussions (cf. Lauß, pp. 116–117; Gildehaus et al., 2021, pp. 5–6) what should be taught through this approach. The SSI approach expands the scope of political education by integrating scientific issues and their interconnections with ethical, moral, and political dimensions. Its primary objective is to foster critical awareness based on scientific findings, with actual controversies serving as a launchpad for learning. The approach is grounded in research-based learning, which empowers learners to develop an understanding of complex problems, explore multiple solutions, justify their viewpoints, and evaluate the impact on various stakeholder groups (Lauß, 2022, pp. 117–118).

To operationalize the SSI approach in the realm of political education, Lauß proposes a six-step macro-method (Lauß, 2022, pp. 118–120), including:

1) introducing the case, with the preference for a non-anticipatory approach (for example, through newspaper articles or news sources)
2) posing controversial questions to stimulate critical engagement and encourage learners to examine multiple perspectives
3) introducing relevant scientific theories, concepts, and findings with respect to the issue at hand
4) incorporating inquiry-based learning components, where learners themselves gather data (for example, conducting research on particulate matter limits at specific locations)
5) facilitating discussions on the social and ethical dimensions of the conflict
6) encouraging reflective decision-making processes, where the relevance of scientific arguments is assessed to make informed political judgments

Another important didactic approach to this module is "Historical and Political Learning with Concepts" (Hellmuth & Kühberger, 2016), which emphasizes the importance of using concepts to structure information and facilitate learning of historical and political topics. Students are encouraged to build upon their existing pre-concepts, acquired from their everyday experiences, and adapt them to new situations. This approach allows them to better comprehend and reduce the complexity of abstract historical and political cases.

In line with the SSI approach, the present module aims to cultivate critical awareness among learners. The overarching question, "Is climate catastrophe fair?", requires learners to engage with natural science concepts, especially ones routed in mathematical modelling, related to climate catastrophe and its underlying causes. Furthermore, said question addresses philosophical notions of equality, equity, and justice as they relate to concepts of fairness, as well as political education approaches that thematize the problem of climate catastrophe and its perceived fairness in the global political system. These topics and their corresponding key ideas, serve as the primary focal points of this module, thus combining perspectives of political science, philosophy as well as mathematics.

The module is fully structured by this approach. First, the topic of climate justice is introduced through a journalistic article and thereby first associations to this topic and its concepts are aroused, which are collected and discussed in the class. Subsequently, controversial questions are asked on various subtopics—justice, mathematical models, sustainability, gender and climate catastrophe, etc. (for example, whether they think the climate catastrophe is fair or which problems underlie the unequal distribution of cause and effect of the climate catastrophe)—in order to encourage the learners to adopt perspectives and to engage in a critical debate. As a third step, scientific theory is introduced for various subtopics, which should be applied by the learners themselves to the cases presented as a fourth step, in order to find relevant data independently, for example, by dealing with the website of the "Global Footprint Network" (https://www.footprintnetwork.org/). Afterwards, arguments about the social and ethical dimensions of the problem are discussed, for example using concepts of responsibility, before as a last step, at the very latest at the end of the seminar, usually at the end of the main topics, reflective questions and decisions on the topic are practised and answered with the inclusion of the newly learned scientific theory.

Learning Goals

Guided by the "Reference framework of competences for democratic culture" of the Council of Europe (2018), and aligned with the objectives of this module, successful completion of this module will enable students to achieve the following:

#36: Discusses what can be done to help make the community a better place.
#38: Takes action to stay informed about civic issues.
#62: Can select the most reliable sources of information or advice from the range available.
#69: Can use explicit and specifiable criteria, principles or values to make judgments.
#121: Can assess society's impact on the natural world, for example, in terms of population growth, population development, resource consumption.
#122: Can reflect critically on the risks associated with environmental damage.
#1140: Can use explicit and specifiable criteria, principles or values to make judgments.
#2058: Can reflect critically on the connections between economic, social, political and environ mental processes.
#2059: Can explain the impact that personal choices, political actions and patterns of consumption may have in other parts of the world.

Assessment and ECTS-Scope

This module was created to meet the requirements of the section "UF GP 08 Sozialkunde und Politische Bildung 2" in the curriculum for the teaching profession of the subject History, Political Education in the Bachelor's degree of the University of Vienna. This section corresponds to a scope of 4 ECTS credits.

As a assessment of the course, it is recommended to keep a learning diary parallel to the seminar participation. On the one hand, a learning diary is an instrument for collecting the results of a learning process and can be used for assessment by the teacher, but on the other hand, the focus is on a process of reflection on the increase in knowledge by the learner him/herself. The explicit design can vary (Forum Umweltbildung).

Concluding Comments

In order to understand and assess present political problems, knowledge in various disciplines is required (cf. Gildehaus et al., 2021, pp. 5–6; Lauß, 2022, pp. 116–117). This includes economic, environmental, mathematical and many other disciplines. Climate catastrophe is one such problem that requires political judgement and political commitment in democracies to solve. Due to the current relevance and attention, it is thus a good idea to deal with questions of justice in the climate catastrophe in order to teach students knowledge and skills in different sub-topics. This module is a suggestion to hold a seminar for future teachers to enable them themselves to teach these topics in schools. In addition to comprehensive scientific contributions on various sub-topics, the relevance of this topic in the public sphere is shown on the basis of journalistic and other sources and an analysis of these sources is practised up to decision-making on the basis of these sources. The learning diary can be used to reflect on one's own learning process for one's own future teaching, which should secure the results of the seminar and make it easier for students to re-enter.

Reading List

This section lists texts and other resources students should read or work on in this module.

Texts

Arora-Jonsson, S. (2011). *Virtue and vulnerability: Discourses in woman, gender and climate change.* In: Global Environmental Change 21 (pp. 744–751). https://doi.org/10.1016/j.gloenvcha.2011.01.005

Columbia Climate School (2022). *Colonialism Distorts the Quest to Save Heritage Threatened by Climate Change, Say Researchers.* State of the Planet. https://news.climate.columbia.edu/2022/02/14/colonialism-distorts-the-quest-to-save-heritage-threatened-by-climate-change-say-researchers/

DEUTSCHE PRESSE-AGENTUR – DPA (03.07.2022). *Italy's Verona, Pisa limit water Supplies amid drought.* Daily Sabah. https://www.dailysabah.com/world/europe/italys-verona-pisa-limit-water-supplies-amid-drought

Diprose, K. et. al. (2021). Intergenerational Perspectives on Sustainable Consumption. In: *Climate Change, Consumption and Intergenerational Justice. Lived Experiences in China, Uganda and the UK*. Bristol (pp. 103–127).

Heggie, J. (18.06.2020). *The Leaky Boot: Where is Italy's Water Going?* National Geographic. https://www.nationalgeographic.com/science/article/partner-content-the-leaky-boot-italy

IPCC (2018). *Summary for policymakers*. IPCC: https://www.ipcc.ch/sr15/chapter/spm/

IPCC (2023). *Summary for policymakers*. IPCC. https://www.ipcc.ch/report/ar6/syr/downloads/report/IPCC_AR6_SYR_SPM.pdf

Jamieson, D. (2015). *Responsibility and Climate Change*. In: Global Justice. Theory Practice Rhetoric. Vol. 8 (2) (pp. 23–43). https://doi.org/10.21248/gjn.8.2.86

Middlehurst, M. & T. Eisen (5.5.2021). *Climate justice and gender justice. an essential pairing to get resilience right.* NDI. https://www.ndi.org/our-stories/climate-justice-and-gender-justice-essential-pairing-get-resilience-right

Milken Institute School of Public Health (2020): *Equity vs. Equality: What's the Difference?* Milken Institute School of Public Health. https://onlinepublichealth.gwu.edu/resources/equity-vs-equality/

Mooney, C. (5.2.2015). *Why climate change is really, really unfair*. Washington Post. https://www.washingtonpost.com/news/energy-environment/wp/2016/02/05/why-climate-change-is-really-really-unfair/

NIPCC (2021). *Climate change reconsidered II. Fossil fuels. Summary for policymakers.* http://climatechangereconsidered.org/wp-content/uploads/2018/12/Summary-for-Policymakers-Final.pdf

Perez, C. S. (2020). *Poems.* ETropic: Electronic Journal of Studies in the Tropics, 19(1). https://doi.org/10.25120/etropic.19.1.2020.3676

Shams, E. (2022). *Carbon colonialism: how the Global North are hiding their carbon emissions.* Palatinate. https://www.palatinate.org.uk/carbon-colonialism-how-the-global-north-are-hiding-their-carbon-emissions/

Uzoma, P. (not available). *Colonialism and Climate Change.* University of Washington. https://uw.pressbooks.pub/121climatejustice/chapter/colonialism-and-climate-change/

Other Resources

Chang, A. (2017). *The life cycle of a t-shirt*. TED-Ed. https://www.youtube.com/watch?v=BiSYoeqb_VY

Common Sense Education (10.08.2018). *Rings of Responsibility.* Common Sense Education. https://www.youtube.com/watch?v=fQSnzrB5bso&ab_channel=CommonSenseEducation

Oliver, J. (06.03.2028) *Wake up! (spoken word poetry)*. TEDx Talks. https://www.youtube.com/watch?v=04rfgNvvXz8&ab_channel=TEDxTalks

Preshoff, K. (2018). *What's a smartphone made of?.* TED-Ed. https://www.youtube.com/watch?v=eIdJ22AfsO8

Solli, R. (14.08.2018): *We can be more*. TEDx Talks. https://www.youtube.com/watch?v=lm0r3yFh0zU&ab_channel=TEDxTalks

References

Arora-Jonsson, S. (2011). *Virtue and vulnerability: Discourses in woman, gender and climate change.* In: Global Environmental Change 21. (pp. 744–751). https://doi.org/10.1016/j.gloenvcha.2011.01.005

BMBWF (2015). *Unterrichtsprinzip Politische Bildung, Grundsatzerlass 2015*. https://rundschreiben.bmbwf.gv.at/rundschreiben/?id=700 (accessed on: 23.05.2023)

BMBWF (2018). *Politische Bildung*. https://www.bmbwf.gv.at/Themen/schule/schulpraxis/uek/politbildung.html (accessed on: 23.05.2023)

BMBWF/Zentrum Polis (2015). *Politische Bildung in den Schulen – tabellarische Übersicht*. https://www.politik-lernen.at/dl/qsmmJKJKoMlKnJqx4KJK/Politische_Bildung_in_den_Schulen_tab__bersicht_Stand_J_nner_2022_pdf (accessed on: 23.05.2023)

CEDAW (2009). *Results of the forty-fourth and forty-fifth sessions of the Committee on the Elimination of Discrimination against Women*. https://tbinternet.ohchr.org/_layouts/15/TreatyBodyExternal/SessionDetails1.aspx?SessionID=349&Lang=en (accessed on: 14.03.2023)

Cook, K. S., & Hegtvedt, K. A. (1983). *Distributive Justice, Equity, and Equality*. In: Annual review of sociology. Vol. 9 (1) (pp. 217–241). https://doi.org/10.1146/annurev.so.09.080183.001245

Council of Europe (2018). *Reference framework of compétences for democratic culture*. https://rm.coe.int/prems-008418-gbr-2508-reference-framework-of-competences-vol-2-8573-co/16807bc66d

Diprose, K. et. al. (2021). Intergenerational Perspectives on Sustainable Consumption. In: *Climate Change, Consumption and Intergenerational Justice. Lived Experiences in China, Uganda and the UK*. Bristol. (pp. 103-127.)

Fleischmann, Y., Rønning, F., Strømskag, H., Berger, C., & Mogiani, M. (2021). *Methane emissions causing climate change: An interdisciplinary inquiry*. https://www.idd.uni-hannover.de/fileadmin/idd/Projekte/CiviMatics/Methane_Problem_IO2.pdf (accessed on: 23.11.2021)

Forum Umweltbildung (n.d.). *Das Lerntagebuch*. https://www.umweltbildung.at/praxismaterial/das-lerntagebuch/ (accessed on: 24.05.2023)

Gildehaus, L. et al. (2021). *Framework to develop normative modelling teaching materials.* https://www.idd.uni-hannover.de/fileadmin/idd/Projekte/CiviMatics/CiviMatics_IO1_final.pdf (accessed on: 23.05.2023)

Harlan, S. L./Pellow, D./Roberts J. T./Bell S. E. (2015). Climate Justice and Inequality. In: Riley, Dunlap E. & Robert J. Brulle (Ed.). *Climate change and society. Sociological perspectives*. (pp. 127–163). Oxford. https://doi.org/10.1093/acprof:oso/9780199356102.003.0005

Harnett, R. (2021). *Climate Imperialism: Ecocriticism, Postcolonialism, and Global Climate Change*. ETropic: Electronic Journal of Studies in the Tropics, 20(2). (pp. 138–155). https://doi.org/10.25120/etropic.20.2.2021.3809

Hellmuth, T. & C. Kühberger. *Historisches und politisches Lernen mit Konzepten*. In: Historische Sozialkunde. Geschichte – Fachdidaktik – Politische Bildung. 1(2016). (pp. 3–8).

Jamieson, D. (2015). *Responsibility and Climate Change*. In: Global Justice. Theory Practice Rhetoric. Vol. 8 (2). (pp. 23–43). https://doi.org/10.21248/gjn.8.2.86

Lauss, G. (2022). Politik – Natur – Wissenschaft: Der Social – Science – Issues (SSI) Ansatz in der Politischen Bildung. In: Oberle, M. / Stamer, M. (ed.). *Politische Bildung in Internationaler Perspektive.* Schwalbach. (pp. 116–123).

Mooney, C. (5.2.2015). *Why climate change is really, really unfair*. Washington Post. https://www.washingtonpost.com/news/energy-environment/wp/2016/02/05/why-climate-change-is-really-really-unfair/

Osterhammel, J. & J. Jansen (1995). *Kolonialismus. Geschichte, Formen, Folgen*. München. (translated as: Osterhammel, Jürgen. (2010). Colonialism. A theoretical overview. Princeton, NJ : Markus Wiener Publishers.)

Preisendörfer, P. (2014). *Umweltgerechtigkeit*. In: SozW Soziale Welt. 65/1. (pp. 25-45).

Sitthiyot, T. & K. Holasut (2020): *A simple method for measuring inequality*. In: Palgrave Commun 6. https://doi.org/10.1057/s41599-020-0484-6

University of Vienna (2022). *Teilcurriculum für das Unterrichtsfach Geschichte, und Politische Bildung im Rahmen des Bachelorstudiums zur Erlangung eines Lehramts im Bereich der Sekundarstufe (Allgemeinbildung) im Verbund Nord-Ost*. https://senat.univie.ac.at/fileadmin/user_upload/s_senat/konsolidiert_Lehramt/Teilcurriculum_Geschichte__Sozialkunde_und_Politsche_Bildung_BA_Lehramt.pdf (accessed on: 23.05.2023)

Figures

Figure 1. Biocapacity and Ecological Footprint per capita in Austria. Source: https://data.footprintnetwork.org/?_ga=2.173269795.1067168564.1684845493-1316839659.1684845493#/ (accessed on: 23.05.2023)

Figure 2. The Gini index. In: Sitthiyot, T. & K. Holasut (2020): A simple method for measuring inequality. In: Palgrave Commun 6. (p. 2).

Full Module

Module Title	Is Climate Catastrophe Fair? Fairness, Responsibility and Decolonial Perspectives.
Target Group	The module is intended primarily for undergraduate students pursuing teacher education, but is also suitable for Master's and PhD students. Some activities included in the module are adaptable for use by prospective teachers in their instructional practices, particularly when teaching high school level students.
Module Description Module Description	The educational module presented here aligns with the Social Science Issues (SSI) approach developed by Georg Lauß (2022). The underlying premise of this approach is that contemporary political discourses increasingly rely on scientific findings, necessitating a basic understanding of scientific methods and terminology among participants to engage in meaningful discussions. The SSI approach expands the scope of political education by integrating scientific issues and their interconnections with ethical, moral, and political dimensions. Its primary objective is to foster critical awareness based on scientific findings, with actual controversies serving as a launchpad for learning. The approach is grounded in research-based learning, which empowers learners to develop an understanding of complex problems, explore multiple solutions, justify their viewpoints, and evaluate the impact on various stakeholder groups. In a similar way Gildehaus et. al. (2021) are describing the necessity of making mathematical debates available for political education. They also refer to political debates becoming more complex and need an understanding of various other disciplines to a certain degree to participate in political debates. An engagement with mathematical models in political education is thus inevitable in order to enable or understand decision-making processes based on them. This is important, on the one hand, to be able to make decisions oneself or to understand decisions made by others, and on the other hand, to be able to evaluate the validity and reliability of the models on which the decisions are based (Gildehaus et al. 2021, 5-6). To operationalize the SSI approach in the realm of political education, Lauß proposes a six-step macro-method, including: 1) introducing the case, with the preference for a non-anticipatory approach (for example, through newspaper articles or news sources) 2) posing controversial questions to stimulate critical engagement and encourage learners to examine multiple perspectives 3) introducing relevant scientific theories, concepts, and findings with respect to the issue at hand 4) incorporating inquiry-based learning components, where learners themselves gather data (for example, conducting research on particulate matter limits at specific locations) 5) facilitating discussions on the social and ethical dimensions of the conflict

	6) encouraging reflective decision-making processes, where the relevance of scientific arguments is assessed to make informed political judgments (see Lauß 2022) In line with the SSI approach, the present module aims to cultivate critical awareness among learners. The overarching question, "is climate catastrophe fair?" requires learners to engage with natural science concepts, especially ones routed in mathematical modelling, related to climate change and its underlying causes. Furthermore, said question addresses philosophical notions of equality, equity, and justice as they relate to concepts of fairness, as well as political education approaches that thematise the problem of climate change and its perceived fairness in the global political system. These topics and their corresponding key ideas, serve as the primary focal points of this module, thus combining perspectives of political science, philosophy as well as mathematics.
Explanation of Modules Acronyms and Headings	Acronyms: 1. (Resources) = These are texts, articles, videos, etc. which are used and required for the tasks in the class. Some of them are specially adapted or written for this module and are located directly after the respective session, some are published papers, articles, videos, etc. which are attached as a link. 2. A (Assignments) = These are the tasks for students within the unit. Which assignments are given. Headings: 3. Description = Here is a short didactic explanation of the respective session(s) and a short classification in the macro method of the SSI. 4. Theoretical framework = This short introduction is a summary of some discourses on the topic of the session(s) and is included for the teacher to get a quick overview of the topic but does not replace further study of the topic through the included and referenced texts. 5. In-class activity = This is a schedule of teacher and assignment activities and student activities with an estimate of the time required. 6. Required materials = Here, the required texts and possible further materials such as electronic devices, presentation materials, etc. are briefly mentioned, which are recommended for carrying out the unit. 7. Follow-up = These questions are neither intended as explicit homework nor as a further work step nor as a transition to the next unit. Rather, these questions are possible reflection questions to deal with the teaching and learning success and to reflect on what has been newly learned. 8. References = Here you will find all the texts dealt with in the tasks as well as more in-depth texts on the topic for the teacher to discuss.
Module Prerequisites	Prior knowledge of climate change is a fundamental requirement for this module. Students should possess a basic understanding of the mechanisms involved in climate change, including its underlying causes and impacts. To this end, a helpful resource for familiarizing oneself with recent developments in this field is "What is Climate Change" by NASA Global Climate Change (available online at https://climate.nasa.gov/).

Learning Objectives	Guided by the learning objectives of the CDC, and aligned with the objectives of this module, successful completion of this module will enable students to achieve the following: • engage in informed discussions and identify actionable solutions to address issues affecting their community • demonstrate an ongoing commitment to staying informed about civic issues • assume responsibility for one's actions and choices • express a willingness to consider and challenge one's own ideas and values • demonstrate an ability to learn and acquire new knowledge with minimal guidance • develop analytical and critical thinking skills to evaluate complex issues • use empirical evidence to support one's opinions • apply explicit and defensible criteria, principles, and values to make well-reasoned judgments • cultivate a sense of empathy towards people with different cultural backgrounds • effectively communicate relevant and useful information to group members when working collaboratively • develop a nuanced understanding of global issues and their interconnections, including the impact of human activity on the natural world, such as population development and resource consumption.
Bibliography	Arora-Jonsson, S. (2011). *Virtue and vulnerability: Discourses in woman, gender and climate change.* Global Environmental Change 21. (pp. 744-751) https://doi.org/10.1016/j.gloenvcha.2011.01.005 Chang, A. (05.09.2017). *The life cycle of a t-shirt*. TED-Ed. https://www.youtube.com/watch?v=BiSYoeqb_VY (accessed 22.12.2021) CEDAW (2009). *Results of the forty-fourth and forty-fifth sessions of the Committee on the Elimination of Discrimination against Women*. https://tbinternet.ohchr.org/_layouts/15/TreatyBodyExternal/SessionDetails1.aspx?SessionID=349&Lang=en Columbia Climate School. (14.02.2022). *Colonialism Distorts the Quest to Save Heritage Threatened by Climate Change, Say Researchers*. State of Planet. https://news.climate.columbia.edu/2022/02/14/colonialism-distorts-the-quest-to-save-heritage-threatened-by-climate-change-say-researchers/ (accessed 28.10.2022) Cook, K. S. / Hegtvedt, K. A. (1983). *Distributive Justice, Equity, and Equality*. Annual review of sociology 9 (1) (pp. 217-241) DEUTSCHE PRESSE-AGENTUR – DPA (03.07.2022). *Italy's Verona, Pisa limit water Supplies amid drought.* Daily Sabah. https://www.dailysabah.com/world/europe/italys-verona-pisa-limit-water-supplies-amid-drought (accessed 06.07.2022) Diprose, K. et. al. (2021). *Intergenerational Perspectives on Sustainable Consumption*. In: Diprose, K. et. al. *Climate Change, Consumption and Intergenerational Justice. Lived Experiences in China, Uganda and the UK*. Bristol. (pp. 103-127)

Bibliography	Fiala, N. (2008). *Measuring sustainability: Why the ecological footprint is bad economics and bad environmental science*. Ecological Economies 67. (pp. 519-252) https://doi.org/10.1016/j.ecolecon.2008.07.023 Fleischmann, Y./Rønning, F./Strømskag, H./Berger, C./Mogiani, M. (2021). *Methane emissions causing climate change: An interdisciplinary inquiry.* https://www.idd.uni-hannover.de/fileadmin/idd/Projekte/CiviMatics/Methane_Problem_IO2.pdf Gildehaus, L. et al. (2021). *Framework to develop normative modelling teaching materials.* https://www.idd.uni-hannover.de/fileadmin/idd/Projekte/CiviMatics/CiviMatics_IO1_final.pdf Global Footprint Network. https://data.footprintnetwork.org/?_ga=2.32449276.394242916.1632643265-494322042.1631878242#/ (accessed 21.11.2021) Harlan, S. L./Pellow, D./Roberts J. T./Bell S. E. (2015). *Climate Justice and Inequality*. In: Riley, Dunlap E. & Robert J. Brulle (Ed.). *Climate change and society. Sociological perspectives.* (pp. 127-163). Oxford. https://doi.org/10.1093/acprof:oso/9780199356102.003.0005 Harnett, R. (2021). *Climate Imperialism: Ecocriticism, Postcolonialism, and Global Climate Change.* ETropic: Electronic Journal of Studies in the Tropics, 20(2). (pp. 138–155) https://doi.org/10.25120/etropic.20.2.2021.3809 Heggie, Jon (18.06.2020). *The Leaky Boot: Where is Italy's Water Going? National Geographic*. National Geographic. https://www.nationalgeographic.com/science/article/partner-content-the-leaky-boot-italy (accessed 06.07.2022) Hellmuth, T. & C. Kühberger. *Historisches und politisches Lernen mit Konzepten*. In: Historische Sozialkunde. Geschichte – Fachdidaktik – Politische Bildung 1(2016). (pp. 3-8) Holzleithner, E. (2009). *Globale Gerechtigkeit*. In: Holzleithner, E. (2009). *Gerechtigkeit*. (pp. 99-111). Köln/Weimar/Wien. IPCC (2018). *Summary for policymakers*. https://www.ipcc.ch/site/assets/uploads/sites/2/2019/05/SR15_SPM_version_report_LR.pdf IPCC (2023). *Summary for Policymakers*. https://www.ipcc.ch/report/ar6/syr/downloads/report/IPCC_AR6_SYR_SPM.pdf Jamieson, D. (2015). *Responsibility and Climate Change*. In: Global Justice. Theory Practice Rhetoric. Vol. 8 (2). 2015. (pp. 23-43) Lang, J.: *Disproportionate Impacts of Climate Change*. University of Washington. https://uw.pressbooks.pub/121climatejustice/chapter/disproportionate-impacts-of-climate-change-2/ (accessed 06.11.2022) Lauss, G. (2022). *Politik – Natur – Wissenschaft: Der Social – Science – Issues (SSI) Ansatz in der Politischen Bildung.* In: Oberle, M. / Stamer, M. (ed.). *Politische Bildung in Internationaler Perspektive*. Schwalbach. (pp. 116-123).
Bibliography	

	Middlehurst, M. & T. Eisen (5.5.2021). *Climate justice and gender justice. an essential pairing to get resilience right*. NDI. https://www.ndi.org/our-stories/climate-justice-and-gender-justice-essential-pairing-get-resilience-right (accessed 14.04.2023) Milken Institute School of Public Health (2020): *Equity vs. Equality: What's the Difference?* Milken Institute School of Public Health. https://onlinepublichealth.gwu.edu/resources/equity-vs-equality/ (accessed 21.11.2021) Mooney, C. (5.2.2015): *Why climate change is really, really unfair.* Washington Post. https://www.washingtonpost.com/news/energy-environment/wp/2016/02/05/why-climate-change-is-really-really-unfair/ (accessed 19.11.2021) (accessed 19.11.2021) NIPCC (2021). *Climate change reconsidered II. Fossil fuels. Summary for policymakers*. http://climatecangereconsidered.org/wp-content/uploads/2018/12/Summary-for-Policymakers-Final.pdf Osterhammel, J. & J. Jansen (1995). *Kolonialismus. Geschichte, Formen, Folgen. München.* München. (translated as: Osterhammel, Jürgen. (2010). *Colonialism. A theoretical overview*. Princeton, NJ) Perez, C. S. (2020). *Poems*. ETropic: Electronic Journal of Studies in the Tropics, 19(1). https://doi.org/10.25120/etropic.19.1.2020.3676 • *Rings of fire* (pp. 44-45) • *Halloween in the Anthropocene* (pp. 46) Preisendörfer, P. (2014). *Umweltgerechtigkeit.* In: SozW Soziale Welt. 65/1. (pp. 25-45). Preshoff, K. (01.10.20282018). *What's a smartphone made of?*. TED-Ed. https://www.youtube.com/watch?v=eIdJ22AfsO8 (accessed 22.12.2021) Shams, E. (10.02.2022). *Carbon colonialism: how the Global North are hiding their carbon emissions.* Palatinate. https://www.palatinate.org.uk/carbon-colonialism-how-the-global-north-are-hiding-their-carbon-emissions/ (accessed 28.10.2022) Sitthiyot, T. & K. Holasut (2020): *A simple method for measuring inequality*. In: Palgrave Commun 6 (2020). https://doi.org/10.1057/s41599-020-0484-6 UN Woman (28.02.2022). *Explainer: How gender inequality and climate change are interconnected.* https://www.unwomen.org/en/news-stories/explainer/2022/02/explainer-how-gender-inequality-and-climate-change-are-interconnected Uzoma, P. *Colonialism and Climate Change*. University of Washington. https://uw.pressbooks.pub/121climatejustice/chapter/colonialism-and-climate-change/ (accessed 28.10.2022)

Session 1: Where can I get Information? Newspaper Research

Session Title	Where can I get information? Newspaper research.
Duration	90 minutes
Description	The first step of the SSI framework involves the introduction of the topic, while the second step entails addressing controversial questions to elicit and gather presumptions. The objective of this session is to collate presumptions from students about climate change and its fairness. To achieve this goal, the students will engage with a journalistic source titled "Why climate change is really, really unfair," which challenges their assumptions and provides preliminary insights on the subject matter. In the concluding segment, the students will discuss the fairness of climate change and possible interventions to mitigate its effects. Through this interactive approach, students will obtain a foundational understanding of climate change and have an opportunity to compare and contrast their perspectives with those of their peers.
Theoretical Framework	Fleischmann et al. highlight that our planet's climate has undergone continuous changes throughout geological time, including "significant variations in global average temperatures." They continue that, "the current climatic warming is happening much more rapidly than past warming events. It has become clear that mankind has caused most of the warming of the last century by emitting greenhouse gases to power our modern lives. We do this through burning fossil fuels, agriculture and land use and other activities that drive climate change. The World Meteorological Organization (WMO) reported in 2017 that greenhouse gas emissions in Earth's atmosphere have reached the highest level ever in 800,000 years (Schlein, 2017). […] Climate change involves not only rising temperatures, but also extreme weather events, rising sea levels, changing wildlife populations and habitats, and a number of other impacts." (Fleischmann et al., 6) The Environmental-Equity-Movement is concerned with the problem of environmental burdens created by the relatively privileged that impact underprivileged populations. Although the quality of environmental conditions has improved in wealthy countries in recent years, there is still competition for beneficial environmental conditions, especially in cities. This movement originated in the United States and addressed local environmental inequities that affected socially underprivileged individuals compared to wealthier people. The central thesis of the movement is that less affluent areas, both locally and globally, are more affected by environmental pollution due to various factors, such as the assumption that socially weaker classes are more likely to accept environmental pollution and are expected to offer less resistance. Gordon Walker (2009) highlights the local distance between the place of production and consumption as a central issue in claims of justice. In some cases, it is assumed that the ecological question will replace the social question. The concepts of environmental justice, environmental equity, and environmental equality are used to address conflicts

Theoretical Framework	of equity, justice, and equality, including access to valued resources and recreational areas (Preisendörfer 2014). Harlan et al. (2015) examine the topic from a different perspective, focusing on injustice and inequality instead of theorizing equity. They identify similar issues regarding justice in climate change (Harlan et al. 2015, 127). The impact of climate change is felt differently by rich and poor people and these different impacts will even increase in the future, and policies aimed at managing climate change exclude poor and powerless individuals. They even consider climate change to be mainly caused by inequalities, as the most marginalized countries and societies produce far fewer emissions than wealthy individuals. Nearly 75% of the annual carbon dioxide emissions are produced by the global north, including only 15% of the world's population. Given these facts, Harlan et al. identify climate change as one of the most compelling topics for theorizing injustice and inequality because these inequalities and injustices are systematically produced rather than being a result of individual actions (2015, 128).
In-class Activity	**A 01.01: Step 1 (15 minutes):** At the beginning of the class, the teacher initiates a brainstorming session to gather existing ideas about climate change, including: • its definition • its impact on the country the students currently live in, and other possibly less privileged countries • and potential solutions to stop or reverse it During this stage, the teacher should avoid correcting the students' responses and should only collect their presumptions. **A 01.02: Step 2 (45 minutes):** The teacher distributes a worksheet or provides the link to an article by (R 01.01) Chris Mooney that includes leading questions related to climate change. Students read the article multiple times and may use different colours to mark sentences that correspond to individual questions. Additionally, students can use various sources on the internet to answer these questions. While the teacher may offer assistance, students should try to complete this task independently. **A 01.03: Step 3 (30 Minutes):** Students discuss the article based on leading questions and their notes. Presumptions made in step 1 can also be discussed in terms of their verification or falsification. The teacher moderates the discussion, asks provocative questions (e.g. A01.02) or participates actively if the discussion stalls.
Required Materials	• Chris Mooney's report in the Washington Post (February 5th 2015) titled "Why climate change is really, really unfair" (available online at: https://www.washingtonpost.com/news/energy-environment/wp/2016/02/05/why-climate-change-is-really-really-unfair/) • the worksheet with the leading questions
Follow-up	These are potential follow-up questions:

Co-funded by the European Union

Follow-up	• Why is it important to know the author's biography, or at least parts of it, when reading such articles? • Who is most and least affected by climate change and why? • Who is contributing the most to climate change, and what can be done at the global/national level to hold them responsible? • What measures can we take to stop or reverse climate change?
References	Harlan, S. L./Pellow, D./Roberts J. T./Bell S. E. (2015). *Climate Justice and Inequality*. In: Riley, Dunlap E. & Robert J. Brulle (Ed.). *Climate change and society. Sociological perspectives.* (pp. 127-163). Oxford. https://doi.org/10.1093/acprof:oso/9780199356102.003.0005 Lang, J.: *Disproportionate Impacts of Climate Change*. University of Washington. https://uw.pressbooks.pub/121climatejustice/chapter/disproportionate-impacts-of-climate-change-2/ (accessed 06.11.2022) Mooney, C. (5.2.2015): *Why climate change is really, really unfair.* Washington Post. https://www.washingtonpost.com/news/energy-environment/wp/2016/02/05/why-climate-change-is-really-really-unfair/ (accessed 19.11.2021)

Source: Mooney, C. (5.2.2015): Why climate change is really, really unfair. Washington Post. https://www.washingtonpost.com/news/energy-environment/wp/2016/02/05/why-climate-change-is-really-really-unfair/ (access on 19.11.2021)

A 01.02: Guiding Questions:

(a) What information can be found about the journalist's professional background and credentials, as well as their sources and references for this report?
(b) As per the findings of this report, what is the nature of the correlation between those responsible for polluting the environment and the countries that suffer the greatest impact from the climate crisis?
(c) According to the report, which regions and nations are most affected by the consequences of climate change?
(d) What specific targets have been established to mitigate the effects of the climate crisis to the greatest possible extent?
(e) This report lists a variety of methods to combat the climate catastrophe. Which approaches are mentioned?

Session 2 and 3: Justice and Equality in Perspectives of Mathematical Modelling, Philosophical Terms and Gender

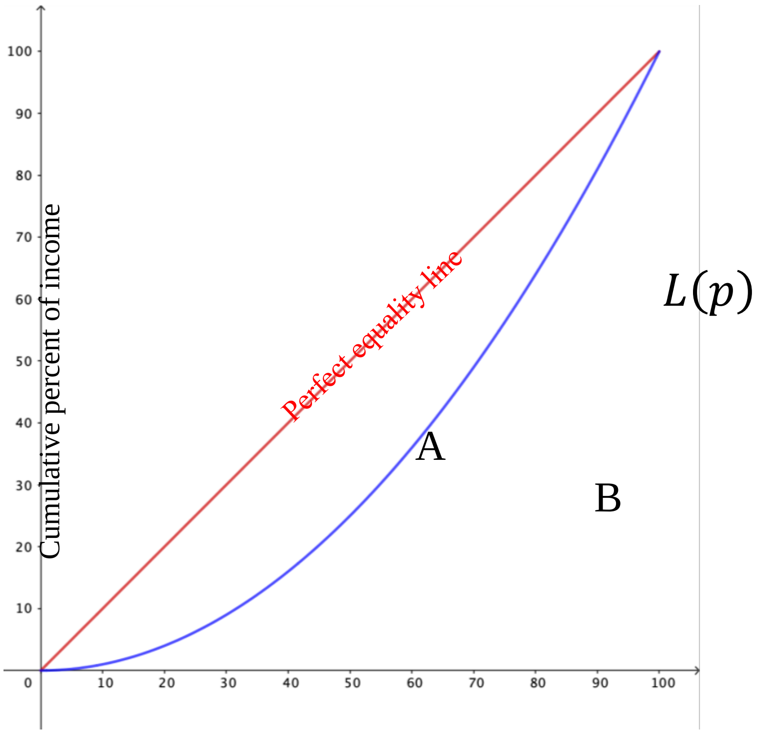
Session Title	Justice and equality in perspectives of mathematical modelling, philosophical terms and gender.
Duration	180 minutes
Description	As per Lauß (2022), it is recommended that educators deliver a theoretical overview through a lecture in accordance with step 3 of the SSI approach, which involves introducing scientific terminology and principles. Nonetheless, for the present module, students are encouraged to undertake self-directed research, with the teacher available to offer guidance and assistance in resolving any potential misunderstandings.

<table>
<tr><td></td><td>During this instructional segment, scientific theories relating to specific concepts should be introduced, with a focus on equality, equity, and justice in the context of mathematical modelling and gender perspectives. Through independent research, students should acquire a deeper understanding of these aforementioned concepts and their relevance to the article on climate change and gender.
By reflecting on these novel ideas, students should be able to formulate their own opinions regarding the topic of climate change and its putative "fairness." The term "fair" has been deliberately included to elicit students' critical thinking and encourage debates on the principles of equality, equity, and justice.
Also these two sessions take some inspiration from the principles of "Historical and Political Learning with Concepts" (Hellmuth & Kühberger: 2016), which emphasize the importance of using concepts to structure information and facilitate learning of historical and political topics. Students are encouraged to build upon their existing pre-concepts, acquired from their everyday experiences, and adapt them to new situations. This approach allows them to better comprehend and reduce the complexity of abstract historical and political cases.</td></tr>
<tr><td>Theoretical Framework</td><td>Equality and Equity
Cook & Hegtvedt (1983) define the term “equality” as a fair exchange of goods, services, or money between parties, whereby the transfer of valued resources is mutually beneficial. The trade is deemed equal if the benefit for all parties is equivalent. In contrast, equity is the ratio of the outcome/input of all parties in the exchange, and any discrepancy between the input and output of one or more parties results in inequity (Cook & Hegtvedt, 1983: 218).
The term “justice” is challenging to standardize or objectively determine, unlike inequality. Preisendörfer (2014) suggests that framing towards injustice is feasible when socially underprivileged classes are disproportionately affected by (environmental) burdens. However, this cannot be taken for granted, and the perception of fairness by the impacted individuals must be considered. Preisendörfer also identifies several factors that could signify a shift from inequality to injustice, including:
1) clearly pronounced socio-areal inequalities in environmental burdens
2) environmental burdens beyond reasonable levels
3) accumulation of disadvantages
4) notable impairments of perceived well-being
5) dislocation of polluters/consumers and affected people
6) insufficient alternative areas for those affected
7) limited opportunities for participation and decision-making
Preisendörfer (2014) anticipates a shift in perception from inequality to injustice, particularly when several of these factors accumulate and exceed reasonable limits. Moreover, inequalities are not considered unjust if they arise from free choice or uncontrollable disasters or accidents.</td></tr>
</table>

The Gini Index: Mathematical Models of (In-)Equality

The Gini index is a mathematical model for measuring socioeconomic inequality, which was first introduced by the Italian statistician Corrado Gini in 1912. It has since become the most widely used method for measuring inequality, not only in socioeconomic contexts, but also in various other fields, such as astrophysics, human geography, and population biology. The Gini index can be applied to any size distribution in data sets with non-negative quantities (Sitthiyot & Holasut, 2020: 2).

The Gini index can be derived from the Lorenz curve framework, which plots the cumulative normalized rank of income on the x-axis and the cumulative normalized income on the y-axis. The first 10% on the x-axis represent the 10% of people with the lowest income. The index is calculated by dividing the area between the perfect equality line and the Lorenz curve by the total area under the equality line. A value of 0 indicates perfect equality, while a value of 1 represents maximum inequality (Sitthiyot & Holasut 2020, 2).



Cumulative percent of population

There are several advantages to using the Gini index. Firstly, it summarizes the entire income distribution with a single statistic, making it relatively easy to interpret. It also allows for easy comparisons between countries and societies with vastly different population sizes. Additionally, the Gini index is updated regularly by various countries and international organizations (Sitthiyot & Holasut, 2020: 2).

However, there are limitations to the Gini index. A lower Gini index does not always indicate a more equal income distribution, as it does not take into account the social welfare function. Furthermore, the Gini index is more sensitive to changes in the middle of the income distribution and less sensitive to changes at the top or bottom.

	According to Sitthiyot & Holasut, this means that two or more countries may share the same Gini index but have significantly different income distributions (2020: 3). For example, Great Britain's and Israel's Gini index in 2015 were nearly identical (0,36). The Gini index of the ration between the share of income of the top 10% and the bottom 10%, on the other hand, were 4,2 in the UK and 5,8 in Israel. As a solution, Sitthiyot and Holasut propose a composite index that considers the income share held by the top and bottom 10% of a population, which may provide a more accurate picture of income inequality. **Global Justice** The concept of global justice is rooted in the understanding that all human beings inhabit the same planet and are inter-connected, despite the presence of national borders. To ensure fairness and equity in the world, it is imperative to consider the question of whether justice can be achieved within national boundaries or if it should be extended to an international level. This fundamental question is at the core of the global justice debate. The United Nations was founded on the principles of global justice, emphasizing the importance of international cooperation to achieve a just world. However, the disintegration of world after the end of the Cold War and the dawn of neoliberal globalization have led to the emergence of new actors in the international arena, including state and non-state actors, transnational corporations, international organizations, and global NGOs. To achieve global justice, it is necessary to establish globally unified standards and minimum working and living standards (Holzleithner 2009). **Gender inequality in climate catastrophe** The issue of gender inequality has become increasingly crucial in the discourse on climate justice, as has been pointed out by the United Nations (UN) Women (2022) and the Committee on the Elimination of Discrimination Against Women (CEDAW) during their 44th session. In 2009, the OEDAW > "expresse[d] its concern about the absence of a gender perspective in the United Nations Framework Convention on Climate Change (UNFCCC) and other global and national policies and initiatives on climate change. From CEDAW's examination of State Parties reports, it is apparent that climate change does not affect women and men in the same way and has a gender-differentiated impact. However, women are not just helpless victims of climate change – they are powerful agents of change and their leadership is critical. All stakeholders should ensure that climate change and disaster risk reduction measures are gender responsive, sensitive to indigenous knowledge systems and respect human rights. Women's right to participate at all levels of decision-making must be guaranteed in climate change policies and programmes" (OEDAW 2009: 7).

	The perspectives presented by Arora-Jonsson (2011) highlight two main issues related to gender and climate change. Firstly, women in the global south are more likely to be affected by climate change than men, and secondly, men in the global north are more likely to pollute the environment than women. In both places, women have less representation in decision-making bodies. The Indian Government's National Action Plan on Climate Change refers to this circumstance as follows: “The impacts of climate change could prove particularly severe for women. With climate change there would be increasing scarcity of water, reductions in yields of forest biomass, and increased risks to human health with children, women and the elderly in a household becoming the most vulnerable. ...special attention should be paid to the aspects of gender” (NAPCC as cited in Arora-Jonsson 2011: 774). Recent literature on gender and climate change emphasises the need to focus on women's perspectives on climate justice since they are the poorest of the poor (Arora-Jonsson 2011: 745-746), more vulnerable to natural disasters (Arora-Jonsson 2011: 746-747), and more environmentally conscious, thus polluting less (Arora-Jonsson 2011: 747-748), and yet they have less agency to initiate change (Arora-Jonsson 2011: 749). However, Arora-Jonsson warns against treating women as a homogenous group, as this tends to portray them as suffering due to their marginal social position compared to men. Such a portrayal may deny them agency, construct their vulnerability as a specific problem of their gender, and ignore power imbalances, thereby reinforcing differences between women and men as given and unchangeable. Instead, understanding the configuration of social relations of power and how vulnerability is produced is more critical than generalising the vulnerability of women. Arora-Jonsson highlights that “[F]irst, that questions of gender and power in environmental management are extremely relevant in a poorer country like India but also very much so in a richer country like Sweden. In the latter, power relations can take forms that make gendered discrimination more difficult to contest. Second, development discourses about equality and empowerment of oppressed third world women bear not only on how gender equality is conceptualised and practiced in the south but also shape the space for gender equality in the North” (Arora-Jonsson 2011: 748). In her argument, Arora-Jonsson posits that increasing the efficiency of environmental management involves the active participation of women. The inclusion of women in decision-making processes not only empowers them, but also brings non-male perspectives to the fore, thereby enriching environmental management. However, she notes that the mere entry of women into existing institutions does not necessarily lead to the dismantling of unequal gender relations. Institutional change and greater flexibility in institutional forms are needed to enable greater participation by diverse groups in decision-making processes (Arora-Jonsson 2011: 749). Arora-Jonsson's conclusions are supported by the statement made by the Committee on the Elimination of Discrimination against Women (CEDAW) at its 44th session, which asserts that "gender equality is essential to the successful initiation,

	implementation, monitoring and evaluation of climate change policies." The committee calls on states parties to include gender equality as an overarching guiding principle in the United Nations Framework Convention on Climate Change (UNFCCC) agreement (CEDAW 2009, 8).
In-class Activity	**A 02.01 (10 minutes):** To get access to pre-concepts of student's understandings of justice a wordcloud[24] on the question "What does justice mean?" is created. Students should collect and shortly discuss the most frequent used topics on this wordcloud for a later reflection on these. **A 02.01: Step 1 (20 minutes):** Students are required to access literature or the internet to learn about the definitions of "equality," "equity," and "justice." The teacher may assist with any uncertainties or misunderstandings. Students should then engage in a plenary discussion to answer the following questions: • What does "justice" mean? • What does "equality" mean? • What does "equity" mean? **A 02.02: Step 2 (25 minutes):** After individual preparation, class discussions should focus on the meaning of each term. The teacher should provide a (R 02.01) thesis paper regarding the texts of (R 02.02) Cook & Hegtvedt (1983) and (R 02.03) Preisendörfer (2014). (If necessary a short lecture by the teacher could be done at this point on the meaning of justice in Preisendörfer (2014)). Students should discuss this thesis paper with partners (2-3 people) regarding new knowledge or confirmation of their believes. They can focus on the wordcloud to identify their new learned additions to their pre-concepts and discuss them. **A 02.03: Step 3 (10 Minutes):** The teacher should deliver a lecture on the Gini index and mathematical modelling based on equality indices. **HW 02.01: Step 4 (Homework):** Students should receive a paper by (R 02.05) Arora-Jonsson (2011) and are asked to identify the most important ideas.

[24] What is a wordcloud? A wordcloud is a visual presentation method for the keywords that are named for a specific recurring word or topic. Often the more frequently mentioned words are printed larger to show greater relevance. There are different providers for this, and a blackboard and chalk are also an option if the digital version cannot be done. We have chosen the provider menti.com for this module.

<table>
<tr><td></td><td>A 03.01: Step 5 (30 minutes):
The class is divided into two groups, with each group focusing on one global area (India and Sweden) as highlighted in Arora-Jonsson's article. Each group prepares short presentations, answering the following questions:
• Which kind of vulnerability regarding climate change is mostly discussed for the area you are presenting?
• What are the most important questions to answer to discuss women’s vulnerability in this area?
• Which solutions are already in use to reduce women’s vulnerability and provide them with agency?
• What does Arora-Jonsson suggest for the future?

A 03.02: Step 6 (20 minutes):
The two groups present their findings and discuss similarities and specifics in both areas.
A 03.03: Step 7 (40 minutes):
Students read an article by (R 02.07) Molly Middlehurst and Tamar Eisen titled "Climate justice and gender justice: an essential pairing to get resilience right." After reading, students should answer the following questions:
• Which factors regarding injustice are named by Preisendörfer (2014) and which factors contribute to a gender justice perspective?
• The article proposes suggestions to handle a gendered unjust climate catastrophe. Which ones? Expand the list by adding a few examples!
• Did the article provide you with new insights on the topic? Which ones?
Students discuss their responses in small groups of two to four people.</td></tr>
<tr><td>Required Materials</td><td>• To facilitate a better understanding of the concepts of "equality," "equity," and "justice," it is recommended that students have access to literature, or preferably the internet. A valuable resource for information about these concepts is the website https://onlinepublichealth.gwu.edu/resources/equity-vs-equality/.
• In addition, it is important for students to have access to the articles related to these concepts either in hard copy or digitally.
• To aid in the presentation, digital devices or other supporting materials should be made available to the students.
• (A presentation for the sessions contents and activities. A suggestion is in Appendix 2.1)</td></tr>
<tr><td>Follow-up</td><td>Potential follow-up activities and questions are:
• Name examples for equality, equity, and justice in other fields (for example in healthcare or education)!
• Do we need to solve issues of inequality, inequity, and injustice globally? If so, how?</td></tr>
<tr><td>References</td><td>Arora-Jonsson, S. (2011). Virtue and vulnerability: Discourses in woman, gender and climate change. Global Environmental Change 21. (pp. 744-751) https://doi.org/10.1016/j.gloenvcha.2011.01.005
CEDAW (2009). Results of the forty-fourth and forty-fifth sessions of the Committee on the Elimination of Discrimination against Women. https://tbinternet.ohchr.org/_layouts/15/TreatyBodyExternal/SessionDetails1.aspx?SessionID=349&Lang=enCook, K. S. & K. A., Hegtvedt (1983). Distributive</td></tr>
</table>

	Justice, Equity, and Equality. In: Annual review of sociology. Vol.9 (1) 1983. p. 217-241. Cook, K. S. / Hegtvedt, K. A. (1983). *Distributive Justice, Equity, and Equality*. Annual review of sociology 9 (1) (pp. 217-241) Holzleithner, E. (2009). *Globale Gerechtigkeit*. In: Holzleithner, E. (2009). *Gerechtigkeit*. (pp. 99-111). Köln/Weimar/Wien. Middlehurst, M. & T. Eisen (5.5.2021). *Climate justice and gender justice. an essential pairing to get resilience right*. NDI. https://www.ndi.org/our-stories/climate-justice-and-gender-justice-essential-pairing-get-resilience-right (accessed 14.04.2023) Milken Institute School of Public Health (2020): *Equity vs. Equality: What's the Difference?* Milken Institute School of Public Health. https://onlinepublichealth.gwu.edu/resources/equity-vs-equality/ (accessed 21.11.2021) Preisendörfer, P. (2014). *Umweltgerechtigkeit.* In: SozW Soziale Welt. 65/1. (pp. 25-45). Sitthiyot, T. & K. Holasut (2020): *A simple method for measuring inequality*. In: Palgrave Commun 6 (2020). https://doi.org/10.1057/s41599-020-0484-6 UN Woman (28.02.2022). *Explainer: How gender inequality and climate change are interconnected.* https://www.unwomen.org/en/news-stories/explainer/2022/02/explainer-how-gender-inequality-and-climate-change-are-interconnected

R 02.01: Equality and justice in Cook & Hegtvedt (1983) und Preisendörfer (2014)

The concept of equality and justice has been explored by Cook and Hegtvedt (1983) and Preisendörfer (2014) in their respective works. Cook and Hegtvedt primarily focus on exchange transactions and their idea of equality is based on the benefit that each actor gains from the exchange. They consider an exchange equal if all actors involved benefit equally, and inequality occurs when the ratio of what is given and received varies among the actors (1983, p. 218).

Preisendörfer, on the other hand, examines the concept of justice and attempts to distinguish it from inequality, which are often used interchangeably. He seeks to identify factors that indicate a transition from inequality to injustice through empirical research (2014: 34-35). Preisendörfer draws on David Schlosberg's trivalent package, which includes "justice as distribution," "justice as participation," and "justice as recognition." The first component considers justice in terms of distribution, the second includes the question of just procedures and opportunities for participation, while the third addresses minimum standards of human dignity, respect for individual, social, and cultural integrity, and protection from disregard and discrimination (Preisendörfer, 2014, p. 33). Preisendörfer's empirical justice research seeks to identify factors that distinguish inequality from injustice, and the outlined factors serve as indicators for such a transition. By analysing these factors, it is possible to identify instances of injustice and to develop strategies to rectify such situations.

Preisendörfer outlines several factors that facilitate the transition from inequality to injustice (2014, pp. 35–40):

1. Clearly pronounced socio-spatial inequalities in environmental burdens
   This criterion considers the assertion of "justice claims" by those affected by environmental burdens. The relative level of comparison groups in the environment is more important than the absolute level.
2. Environmental burdens beyond reasonable levels
   This criterion considers basic human rights and the possibility of leading a dignified life. When the limits of what is reasonable are exceeded, "justice claims" are assumed.
3. Accumulation of disadvantages
   This criterion considers questions of intersectional disadvantages and access to recreational areas and infrastructure, among other things.
4. Impairments of subjective well-being
   This factor focuses on the negative effects of environmental burdens on individuals' subjective well-being.
5. Disaggregation of causers/beneficiaries and those affected
   This criterion is based on the separation of those who cause or benefit from environmental burdens and those who are affected by them.
6. Voluntariness and possibilities of evasion
   This criterion assumes that "justice claims" are more likely to be made when individuals have no way to evade environmental burdens. The costs of finding alternatives are not only theoretical but also monetary.
7. Opportunities for participation and co-determination
   This factor is related to the previous one, as protests and "justice claims" are more likely to occur when those affected have limited or no opportunities to participate and shape the process. If opportunities for participation and co-determination were available, affected individuals would have a chance to change their circumstances in their favour.

Additional literature:

Cook, K. S., & Hegtvedt, K. A. (1983). *Distributive Justice, Equity, and Equality*. Annual review of sociology 9 (1) (pp. 217–241)

Preisendörfer, P. (2014). *Umweltgerechtigkeit.* In: SozW Soziale Welt. 65/1. (pp. 25–45).

Session 4 and 5: Ecological Footprint and Biocapacity. Measuring Sustainability

Session Title	Ecological Footprint and Biocapacity. Measuring Sustainability.
Duration	Two 90-minute sessions including homework
Description	This session mirrors an adaption of steps 3 and 4 of the SSI approach and is based on self-research. Specifically, scientific theory and elements of explorative learning should be incorporated. Lauß (2022) recommends that teachers provide a theoretical introduction to the topic in a lecture. However, in this module, learners are encouraged to obtain information on their own, while the teacher may facilitate their understanding by addressing misunderstandings or guiding them through the material. In line with the

	modification of step 3 and step 4 of the SSI approach, this session is explorative in nature. It begins by introducing basic scientific terminology related to climate change and biodiversity. Subsequently, students conduct comparative research on the effects of climate change in their own country and other countries. As this session involves preparatory research and introductory terminology, it may require some time to be completed. First, students should be familiarized with the problem and their task, followed by group work to complete the task. Finally, the groups should present their findings to the class.
Theoretical Framework	The provided session is based on a series of keywords pertaining to climate change derived from the Ecological Footprint framework proposed by the Global Footprint Network. The most central keywords are presented in the following part: **Biodiversity buffer** refers to the quantity of biocapacity reserved to preserve representative ecosystem types and viable species populations, which depends on biodiversity management practices and the anticipated outcomes. **Biological capacity available per person (or by capita)** is calculated based on the 12.2 billion hectares of biologically productive land and water on Earth in 2019, divided by the number of people living that year (7.7 billion), which amounts to 1.6 global hectares per person. This value also takes into account the wild species competing with humans for biological material and space. **Biological capacity**, or **biocapacity**, represents the capacity of ecosystems to replenish the resources demanded by humans from a given surface. The biocapacity of a surface is determined by its ability to produce biological materials consumed by humans and absorb waste generated by humans. Biocapacity can vary year to year due to climate, management, and the perceived utility of certain inputs to the human economy. The biocapacity of an area is computed by multiplying the actual physical area by the yield factor and the appropriate equivalence factor. Biocapacity is usually expressed in global hectares. **Biologically productive land and water** denotes the land and water area (including marine and inland waters) that supports significant photosynthetic activity and the accumulation of biomass used by humans, excluding non-productive and marginal areas with patchy vegetation. This area also does not include biomass that is not useful to humans. The total biologically productive area on land and water in 2019 was roughly 12.2 billion hectares. The **Carbon Footprint** measures the CO2 emissions linked to fossil fuel use. In the Ecological Footprint accounts, these emissions are transformed into the biologically productive area necessary to absorb the CO2. The Carbon Footprint is added to the Ecological Footprint because it competes with humans for bioproductive space, as increased CO2 concentrations in the atmosphere create ecological debt. Some assessments of Carbon Footprint report results in tonnes

Theoretical Framework	released per year, without translating the amount into the area needed to sequester it. **Consumption** refers to the utilization of goods or services. The term consumption may have different meanings depending on the context. As used in the context of the Ecological Footprint, it refers to the utilization of goods or services which embody all resources necessary to provide them to consumers, including energy. In full life-cycle accounting, the entire production chain, including any losses, is taken into account, such as food lost during processing or harvest, and all the energy required to grow, harvest, process, and transport the food. **Ecological debt** or **biocapacity debt** refers to the accumulation of annual ecological deficits. The Footprint of humanity exceeded global biocapacity for the first time in the early 1970s, and every year since then. As of 2019, this annual overshoot had generated an ecological debt exceeding 17 years of the Earth's total productivity. **Ecological deficit/reserve**, also known as **biocapacity deficit/reserve**, refers to the difference between the biocapacity and Ecological Footprint of a region or country. An ecological deficit occurs when the Footprint of a population exceeds the biocapacity of the area available to that population. Conversely, an ecological reserve exists when the biocapacity of a region exceeds its population's Footprint. In the case of a regional or national ecological deficit, it implies that the region is importing biocapacity through trade, liquidating regional ecological assets, or emitting wastes into global commons such as the atmosphere. The global ecological deficit, unlike the national scale, cannot be compensated for through trade, and thus is equivalent to overshoot. The **Ecological Footprint**, measured in global hectares, quantifies the area of biologically productive land and water an individual, population, or activity requires to produce all the resources it consumes and absorb the waste it generates, utilizing prevailing technology and resource management practices. Because trade is global, an individual or country's Footprint includes land or sea from all over the world. Without further specification, Ecological Footprint generally refers to the Ecological Footprint of consumption. It is important to note that "Ecological Footprint" and "Footprint" are proper nouns and should always be capitalized. **Footprint neutral or negative** refers to human activities or services that result in no increase or a net reduction in humanity's Ecological Footprint. For instance, the activity of insulating an existing house has a Footprint for production and installation of the insulation materials. This insulation, in turn, reduces the energy needed for cooling and heating this existing house. If the Footprint reduction from this energy cutback is equal to or greater than the original Footprint of insulating the house, the latter becomes a Footprint neutral or negative activity. In contrast, making a new house highly energy efficient does not, by itself, make the house Footprint neutral unless, at the same time, it causes reduction in other existing

Theoretical Framework	Footprints, which must be larger than the Footprint of building and occupying the new house. Understanding the **land or area type** is crucial for interpreting the biocapacity trend over time. The Earth's 12.2 billion hectares of biologically productive land and water areas are broadly categorized into five types. These area types for biocapacity support the six Footprint demand types, as described below: **Cropland** is the most bioproductive of all land-use types and is used for producing food and fibre for human consumption, feed for livestock, oil crops, and rubber. However, due to the lack of globally consistent data sets, the current cropland Footprint calculations do not yet account for the extent to which unsustainable agricultural practices may cause long-term degradation of soil. The cropland Footprint includes crop products allocated to livestock and aquaculture feed mixes, as well as those used for fibres and materials. **Forest land** provides two services: the forest product Footprint and the Carbon Footprint. The forest product Footprint is calculated based on the amount of lumber, pulp, timber products, and fuel wood consumed by a country annually. The Carbon Footprint represents the carbon dioxide emissions from burning fossil fuels and embodied carbon in imported goods. The carbon Footprint is the largest portion of humanity's Footprint, and it is represented by the area required to sequester these carbon emissions. The carbon Footprint component of the Ecological Footprint is calculated as the amount of forest land needed to absorb these carbon dioxide emissions. **Grazing land** is used to raise livestock for meat, dairy, hide, and wool products. The grazing land Footprint is calculated by comparing the amount of livestock feed available in a country with the amount of feed required for all livestock in that year, with the remainder of feed demand assumed to come from grazing land. The **fishing grounds** Footprint is calculated based on the maximum sustainable catch estimates for a variety of fish species. These estimates are converted into an equivalent mass of primary production based on the various species' trophic levels. The maximum harvestable primary production is then divided among the continental shelf areas of the world. Fish caught and used in aquaculture feed mixes are also included. The **built-up land** Footprint is calculated based on the area of land covered by human infrastructure, such as transportation, housing, industrial structures, and reservoirs for hydropower. Built-up land may occupy what would previously have been cropland. --- However, despite its extensive use in many publications and public debates, the ecological footprint is also subject to extensive criticism. One example is Nathan Fiala's (2008) critique, which summarises and exemplifies several other publications dealing with the ecological footprint as a measure of sustainability. Fiala concludes that the

	ecological footprint is hardly a measure of the actual sustainability of a country, but rather a measure of the consumption and distribution of resources in relation to population density. As an example of the second of these arguments, he cites the change in the ecological footprint in Canada, which is tied to national borders. While large cities have a much larger footprint than rural areas, this does not indicate more efficient consumption by rural residents, but rather a lower population density and thus lower consumption with a higher biocapacity. The first argument, that the ecological footprint is not so much a measure of sustainability as a measure of consumption, is demonstrated by Fiala (2008) in that while the carbon footprint of an area has a large statistical impact on the ecological footprint of that area, the overuse of agricultural land and thus rendering it unusable has no or even a negative impact on the ecological footprint, as it only measures the area of agricultural land used and not the sustainable management of that land. A third major criticism by Fiala (2008) of the ecological footprint, again in the context of it being a measure of consumption rather than sustainability, is that the ecological footprint uses the average technology level of the world as a measure of the resources required for a product. Technologically more advanced portions could change the footprint strongly in favour of sustainability, as well as technologically very backward portions that hide potentially even more sustainable consumption. Fiala (2008) concludes from this that the ecological footprint, despite several advantages, such as easy comparability and a fairly good representation of the CO2 footprint, should nevertheless be viewed critically and these problems of the model should be taken into account, due to the fact that it is a statistic and can hardly be applied to special cases, many political and social issues with an influence on sustainability.
In-class Activity	**A 04.01: Step 1 (5 minutes):** At the beginning of this activity, the class should be divided into groups of 3-5 individuals. The purpose of these sessions is to promote cooperative work among each group. The teacher should be available to provide advice or research materials if needed and to clarify any misunderstandings that may arise. It is recommended that the teacher notes down the groups for future tasks in the same groups. **A 04.02: Step 2 (10 minutes):** Each group decides on a product from their everyday life and carry out research on its origin and production. Note these groups as teacher and remember the groups participants, these groups are used in later sessions again. **A 04.03: Step 3 (20 Minutes):** Each group should research and understand the following terms: • Ecological Footprint (per person) • Biocapacity

In-class Activity	• Numbers of Earths The Glossary of the website of the Global Footprint Network (available online at: https://www.footprintnetwork.org/resources/glossary/) can be used as a source. Other sources are also acceptable. **A 04.04: Step 4 (25 minutes):** Using the website of the Global Footprint Network, each group should research the biocapacity and ecological footprint of the country where the product they researched is produced and the country they are currently residing in. Specifically, the following website inputs should be used: • "Home" for a graphical overview of the world or each country. • "Explore Data" – "Reserve/Deficit Trends" for a graphical overview of countries or the world's biocapacity reserve/deficit over time. • "Explore Data" – "Analyse by Land-Type" for a separated overview of each land type regarding their biocapacity-reserve in this area. Each group should answer the following questions: • What is the biocapacity of the two countries, and how has it changed in the last decades? • What is the Biological Footprint of an average person in these countries, and how has it changed in the last decades? • Where does the biocapacity of this land result from, and has this result changed over the last decades? • What is the number of earths used for living averagely in this country, and has this number changed over the last decades? **HW 04.01: Step 5 (30 minutes & Homework):** Each group should compare the data they have obtained from their home and from the product-producing country they researched. The following questions should be answered: • What are the differences in each of the asked stats, and how have they changed in each of the countries? • Do strong or weak changes occur in the datasets in one or both countries? • Where might these come from? **A 05.01: Step 6 (30 minutes):** Each group prepares a brief presentation of their findings. **A 05.02: Step 7 (60 minutes):** Each group presents their findings to the class, and the following topics are discussed: • Potential differences between different countries and the findings of the groups about them • The different groups' interpretations of changes in biocapacity, Ecological Footprint, and Numbers of Earths
Required Materials	• Digital devices with internet connectivity, with at least one device available for each group • Materials and hardware/tools that support the presentations (e.g., beamer)

Follow-up	Potential follow-up questions are: • What are the implications of a decrease in biocapacity in a given region? • How does a decline in biocapacity exacerbate the problem of climate catastrophe? • What does it signify if a region has a Number of Earths greater than one, and why is this cause for concern? • What is the significance of a high or low ecological footprint? • What steps can individuals take to reduce their ecological footprint?
References	Fiala, N. (2008). *Measuring sustainability: Why the ecological footprint is bad economics and bad environmental science*. Ecological Economies 67. (pp. 519-252) https://doi.org/10.1016/j.ecolecon.2008.07.023 Global Footprint Network. https://data.footprintnetwork.org/?_ga=2.32449276.394242916.1632643265-494322042.1631878242#/ (accessed 21.11.2021)

Session 6 & 7: Climate Colonialism. Postcolonial and Decolonial Approaches.

Session Title	Climate Colonialism. Postcolonial and Decolonial Approaches.
Duration	Two 90-minute sessions
Description	These two sessions are based on the principles of "Historical and Political Learning with Concepts" (Hellmuth & Kühberger: 2016), which emphasize the importance of using concepts to structure information and facilitate learning of historical and political topics. Students are encouraged to build upon their existing pre-concepts, acquired from their everyday experiences, and adapt them to new situations. This approach allows them to better comprehend and reduce the complexity of abstract historical and political cases. **Session 1:** • A 06.00-06.01: Step 1: Generate a word cloud about "colonialism" • A 06.02: Step 2: Provide an input on colonialism and imperialism • A 06.03: Step 3: Revisit the word cloud and adapt it based on the new input • A 06.04: Step 4: Discuss homework, which includes reading a paper and marking significant phrases that describe the connection between colonialism and climate change • HW 06.01: Step 5: Prepare a picture or poem for a group presentation on climate colonialism **HW 06.01:** Homework: Individually, students familiarize themselves with Rachel Harnetts' paper (2021) "Climate Imperialism: Ecocriticism, Postcolonialism, and Global Climate Change." As a group, continue preparing a picture or poem for the group presentation on climate colonialism.

Co-funded by the European Union

	Session 2: • A 07.01: Step 6: Present the pictures and stories prepared in the previous session • A 07.02: Step 7: Work in groups to analyze and compare two countries, determining if they were colonialists or colonized • A 07.03: Step 8: Discuss findings from step 6 as a class. Also these sessions take parts in the steps 3, 4 and 5 of the SSI. With the introduction of scientific literature regarding the meaning of colonialism and imperialism pre-concepts of the students are challenged and a understanding of these terms in scientific literature is discussed. In several parts of the sessions inquiry-based learning is done, for example as students should do their research on the journalistic and public sources discussing climate colonialism in step A 06.04. With the introduction of Rachel Harnetts paper on climate imperialism (2021) also discussions on moral, ethical and - of cause – responsibility-questions are discussed and introduced.
Theoretical Framework	The discussion of colonialism necessitates an initial definition of the term. A precise definition, as proposed by Osterhammel and Jansens in their seminal work, "Kolonialismus. Geschichte, Formen und Folgen", characterizes colonialism as the imposition of one nation's control over another (1995). Two critical components in Osterhammel and Jansen's definition are the concepts of "ruling" and "the other", both of which are integral to the subjugation inherent in colonialism. The definition espouses three crucial aspects. Firstly, colonialism does not represent the control of rulers over subjects; rather, it encompasses the domination of an entire society by another, thus hindering its natural evolution. The resultant social and cultural changes are intended to serve the interests of the dominant society, not those of the colonized population. Secondly, the degree of otherness assumed by the colonized is a crucial element in colonialism. Colonization involves the imposition of the culture of the colonizing nation, usually European, upon the colonized population. Thirdly, Osterhammel and Jansen contend that colonialism is not merely a structural and historicized domination of one culture over another, but rather a unique way of thinking about such domination. This domination may be legitimized by religious tenets or the civilizing mission, as exemplified by modern development aid initiatives from highly developed nations to less developed ones. The interactions between colonizers and the colonized are marked by a strong hierarchy. Osterhammel and Jansen also discuss the possibility of "colonialism without colonies," which refers to a hierarchy between a strong central part and a much weaker periphery (1995: 21). This situation exists between a nation-state or a connected land empire. In modern times, this strong hierarchy between a (political and economic) strong centre and a less strong periphery is also a good definition of neo-colonial interaction between the global north and the global south, which will be further discussed in Harnett's article on climate colonialism.

Although the climate crisis has been deemed "the single greatest threat to human civilization" (Harnett, 2021: 139) and has garnered significant attention, much of this attention has been focused on the impact of capitalism on this catastrophe. However, Amitav Gosh has emphasized another crucial but often ignored factor: empire and imperialism. Following the ideas of postcolonial literary critics, Gosh argues that modern empires have played a significant role in the rise of fossil fuels. Harnett extends this critique to the contemporary world, calling attention to the concept of petro-colonialism and its implications for U.S. foreign policy. Additionally, she notes that the climate crisis is reinforcing existing economic, social, and racial inequalities that emerged during colonialism, and in her view, the crisis represents a new form of imperialism. Harnett advocates for a dialogue between postcolonial and ecocritical studies, as the former is necessary to unravel the hierarchies that formed during colonialism and amplify the voices of non-speaking or subaltern subjects, while the latter can help us understand the impact of colonialism on geography (Harnett, 2021: 139).

An approach to climate change that is decolonial and anti-colonial is crucial because it recognizes the complexity and diversity of human experience, as different regions, peoples and groups have contributed unequally to the problem and possess differing degrees of political, economic and social power. Chakrabarty terms this the "uneven impact of climate change" (Harnett 2021: 140).

In their paper for this session, Harnett proposes an interpretation of climate change as "slow violence". This concept is defined as "a violence that occurs gradually and out of sight, a violence of delayed destruction that is dispersed across time and space, an attractional violence that is typically not viewed as violence at all" (Nixon 2011, cited in Harnett 2021: 140). By adopting this approach, she highlights the disparity between the visible impact of thousands of deaths after a tsunami and the unnoticed impact of a heat wave. The former receives significant media attention while the latter remains largely unrecognized by the global public (Harnett 2021: 140f).

The uneven impact of climate change is a poignant reality, illustrated by the fact that the tropics have been among the first regions to suffer from its effects. This is exemplified by the drowning of numerous tropical islands as sea levels rise and by the prevalence of disease in these areas, as highlighted in this paper. In contrast, formerly colonized and now post-colonial nations, which were already impoverished by the impact of colonialism, struggle to industrialize and catch up with their western and northern counterparts, who are the main contributors to carbon emissions per capita. The devastation that tropical regions are experiencing due to climate change is what Harnett calls the "environmentalism of the poor." This term refers to countries that were already struggling with poverty and were robbed of their resources without being provided with the infrastructure to extract new ones sustainably. They are now among the most affected by the effects of climate change, without the resources to adequately handle them. Western and northern countries, on the other hand, have the resources

	and infrastructure to manage the effects of climate change to some extent while mostly even being less affected by climate catastrophe yet. Colonizing nations do not feel the same level of impact as the colonized and the later ones lack the same ability and ressources to handle the effects. The effects of Hurricane Maria in 2017 in Puerto Rico and the United States are showing this difference (Harnett 2021: 140-146). Harnett highlights yet another critical point in postcolonial and anti-colonial ecocriticism. As former colonialised nations endeavour to catch up economically, western and northern countries, as the principal contributors, enact laws to decrease the global impact of climate change, primarily impacting the development of poorer countries. Harnett terms these actions "green neocolonialism" by the Global North as perceived by the former colonised countries. They state that "[b]y refusing to acknowledge the economic disadvantage imperialism created for much of the Global South, imperial and former imperial nations turn a blind eye to their past actions and the inequalities that arose because of them" (Harnett 2021: 146). Consequently, many former colonised countries are demanding reparations. Harnett insists that these reparations are necessary if the West and global North expect the South to industrialise without further contributing to climate change significantly. Klein also maintains that the global North is in an "ecological debt." If laws continue to be passed to reduce emissions without assisting the former colonialised to industrialise sustainably, the West and North will reinforce the effects of imperialism on these countries (Harnett 2021: 146-148). In a concluding remark, Harnett highlights the significance of acknowledging and prioritizing the perspectives of indigenous and native populations in the context of climate change management. They provide evidence of environmental advocacy spearheaded by indigenous communities, such as their activism against the construction of the Dakota Access pipeline (Harnett 2021: 149-152).
In-class Activity	**A 06.00: Step 0 (teachers task, preparation before the lesson)** The teacher prepares a wordcloud on the topic of "colonialism" using a suitable tool such as mentimeter.com. **A 06.01: Step 1 (15 minutes):** The code for the wordcloud is shared with students. The teacher asks them to provide insights on their preconceptions about colonialism. The responses will be displayed visually in front of the class, with the most frequent answers appearing the largest. The most commonly cited responses are discussed with the class. **A 06.02: Step 2 (20 minutes):** The teacher offers a presentation on the topic of colonialism and imperialism, drawing on the theoretical framework outlined in Osterhammel and Jansen's (2010) "Colonialism: A Theoretical Overview."

Co-funded by
the European Union

	A 06.03: Step 3 (10 minutes): The class revisits the wordcloud created in step 1 and discusses the most frequently cited responses again, as well as any less commonly cited responses that have risen in importance. It should be considered whether the input from the previous discussion has challenged or changed any preconceptions about colonialism, and which preconceptions still hold true. **A 06.04: Step 4 (10 minutes):** Divide the class into five groups and assign each group one of the following pictures or poems, along with a reference to the article in which it was published: *R 06.01: Source: Shams, E. (2022). Carbon colonialism: how the Global North are hiding their carbon emissions. Palatinate. Online access: https://www.palatinate.org.uk/carbon-colonialism-how-the-global-north-are-hiding-their-carbon-emissions/ (on: 28.10.2022)* *R 06.02: Source: Whose heritage gets saved? In: Columbia Climate School. (2022). Colonialism Distorts the Quest to Save Heritage Threatened by Climate Change, Say Researchers. State of Planet. Online access: https://news.climate.columbia.edu/2022/02/14/colonialism-distorts-the-quest-to-save-heritage-threatened-by-climate-change-say-researchers/ (on: 28.10.2022)* *R 06.03: Source: Uzoma, P. Colonialism and Climate Change. University of Washington. Online access: https://uw.pressbooks.pub/121climatejustice/chapter/colonialism-and-climate-change/ (28.10.2022)* As well as following poems: R 06.04: Perez, C. S. (2020). Rings of fire. In: Poems. ETropic: Electronic Journal of Studies in the Tropics, 19(1). https://doi.org/10.25120/etropic.19.1.2020.3676 (44-45) R 06.05: Perez, C. S. (2020). Halloween in the Antropocene. In: Poems. ETropic: Electronic Journal of Studies in the Tropics, 19(1). https://doi.org/10.25120/etropic.19.1.2020.3676 (46-49) The teacher asks each group to research their assigned picture/poem and discuss it in the context of "climate colonialism." Each group should prepare a short presentation of their findings, using the leading article of their assigned picture/poem and (R 06.06) Rachel Harnett's (2021) "Climate Imperialism: Ecocriticism, Postcolonialism, and Global Climate Change" as theoretical sources. The presentations should not exceed 5-7 minutes each to allow time for discussion. In addition, all students should read

	Harnett's article as homework and mark the most important quotes and potential questions. **HW 06.01: Step 5 (rest of session & Homework):** The groups to conduct initial research and discuss their ideas for the presentation. **A 07.01: Step 6 (50 minutes):** The teacher asks each group to present their findings on the picture/poem they researched and their thoughts on the topic of "climate colonialism." Encourage the class to give feedback or share their own thoughts on the topic, potentially relating to their own picture or article. Each group should present for 5-7 minutes, followed by 3-5 minutes of discussion. **A 07.02: Step 7 (30 minutes):** The groups from Sessions 4 and 5 are brought together on "Ecological Footprint and Biocapacity: Measuring Sustainability." Each group researches their chosen country in terms of the following: • Is the country a former colonized or colonizing country? • How does the country's average income compare to that of your home country? • Describe the development of the country's carbon footprint. • How is the average income developing compared to the carbon footprint? Each group should present their findings in a short presentation. **A 07.03: Step 8 (10 minutes):** Each group presents their findings on their chosen country and discusses how they relate to the topic of climate colonialism. The teacher encourages the class to discuss each presentation in terms of their own findings on "climate colonialism" from the last two sessions, using Harnett's article to support their arguments.
Required Materials	• Access to the internet to share the articles or printed out copies of: Harnett, R. (2021). *Climate Imperialism: Ecocriticism, Postcolonialism, and Global Climate Change.* Perez, C. S. (2020). *Poems*. Shams, E. (10.02.2022). *Carbon colonialism: how the Global North are hiding their carbon emissions.* Columbia Climate School. (2022). *Colonialism Distorts the Quest to Save Heritage Threatened by Climate Change, Say Researchers*. Uzoma, P. *Colonialism and Climate Change*. • Digital devices with internet connectivity to display the above-mentioned poems and pictures
Follow-up	A potential follow-up question is:

	• Harnett (2021: 147): "Since so much of the poverty in these regions is the result of the long history of imperialism, which enriched the colonizing countries and aided in their rise to economic prosperity, it is only fair that these rich imperial nations pay back the developing world for the resources and economic prosperity stolen from them." Is it? The follow article offers further information on the issue: • Lee, Chermaine. (2022). UNDERSTANDING CLIMATE COLONIALISM. Fair Planet. Online access: https://www.fairplanet.org/story/understanding-climate-colonialism/ (on: 28.10.2022)
References	Columbia Climate School. (14.02.2022). *Colonialism Distorts the Quest to Save Heritage Threatened by Climate Change, Say Researchers*. State of Planet. https://news.climate.columbia.edu/2022/02/14/colonialism-distorts-the-quest-to-save-heritage-threatened-by-climate-change-say-researchers/ (accessed 28.10.2022) Harnett, R. (2021). *Climate Imperialism: Ecocriticism, Postcolonialism, and Global Climate Change.* ETropic: Electronic Journal of Studies in the Tropics, 20(2). (pp. 138–155) https://doi.org/10.25120/etropic.20.2.2021.3809 Hellmuth, T. & C. Kühberger. *Historisches und politisches Lernen mit Konzepten*. In: Historische Sozialkunde. Geschichte – Fachdidaktik – Politische Bildung 1(2016). (pp. 3-8) Osterhammel, J. & J. Jansen (1995). *Kolonialismus. Geschichte, Formen, Folgen. München.* München. (translated as: Osterhammel, Jürgen. (2010). *Colonialism. A theoretical overview*. Princeton, NJ) Perez, C. S. (2020). *Poems*. ETropic: Electronic Journal of Studies in the Tropics, 19(1). https://doi.org/10.25120/etropic.19.1.2020.3676 • *Rings of fire* (pp. 44-45) • *Halloween in the Anthropocene* (pp. 46) Shams, E. (10.02.2022). *Carbon colonialism: how the Global North are hiding their carbon emissions.* Palatinate. https://www.palatinate.org.uk/carbon-colonialism-how-the-global-north-are-hiding-their-carbon-emissions/ (accessed 28.10.2022) Uzoma, P. *Colonialism and Climate Change*. University of Washington. https://uw.pressbooks.pub/121climatejustice/chapter/colonialism-and-climate-change/ (accessed 28.10.2022)

Session 8: Sustainable Consumption

Session Title	Sustainable Consumption
Duration	90 minutes
Description	Step 5 of the SSI approach involves an in-depth discussion of the social and ethical conflicts that arise in relation to sustainable consumption. This includes a consideration of the negative impact that consumption can have on producing countries, as well as an exploration of the ways in which the costs of production are often outsourced to

	other regions. As such, this session seeks to raise awareness about the need to consider the wider social and environmental implications of our consumption practices. The discussion will be led by a chapter focused on social and ethical conflicts in sustainable consumption, which will provide a framework for exploring these issues in greater depth. In addition, two Ted-Ed videos will be shown to provide further insight into the environmental impact of unsustainable production practices in the cotton and electronics industries.
Theoretical Framework	The products displayed in supermarkets and shops often conceal the hidden processes of production, distribution, and commercialization. Despite globalization seemingly unifying these processes globally, close examination reveals ever-changing production and consumption patterns that generate diverse social, economic, and political concerns. This session will examine the spatiotemporal variations in consumption patterns and their impact on consumers' daily lives, as well as the production processes of certain consumer goods, thereby rendering their production patterns visible. **Intergenerational Perspectives on Sustainable Consumption** Diprose et al. (2021) present case studies comparing consumption trends and awareness among residents in three different cities across the globe (Jinja, Nanjing, Sheffield). These case studies demonstrate how changes in consumption patterns can be attributed to different factors, including post-wartime expansion (Sheffield), the transition to a market economy (Nanjing), and an increasingly liberalised capitalist economy (Jinja). Although changes in consumption patterns have accompanied a shift from precarious survival to growing well-being, these changes have led to a less sustainable choice of products, creating intergenerational and spatial conflicts. Despite younger generations' increased awareness of climate change issues, they often make use of short-term or luxury products and are blamed by older generations for not leading a sustainable lifestyle. These conflicts are also observed between urban and rural populations, with the latter criticizing urban dwellers for not adopting more responsible and healthier consumption habits. Waste plays a significant role in making consumption patterns more visible and sometimes exacerbates social conflicts. Urban waste is often viewed as a symbol of consumerism and the throwaway society, leading to anxiety about its management and the development of new routines such as recycling. This demonstrates how an increasing availability and awareness of recycling options and needs lead to a larger amount of recycled goods. **The life cycle of a t-shirt** The production of a typical t-shirt involves multiple stages, each with its own environmental impacts. Cotton, the primary raw material, is predominantly grown in the United States, China, and India. However, cotton production is highly water-intensive and involves the use of significant quantities of pesticides, which are harmful to both workers and local ecosystems. In fact, the average t-shirt requires a staggering

Theoretical Framework	2700 litres of water to produce. Although organic cotton that utilizes less water and pesticides exists, it accounts for only a small fraction of all cotton production. After being harvested, cotton is transported to factories in countries such as China, India, and Bangladesh for spinning and manufacturing. However, transportation itself has a high carbon footprint, contributing to climate change. During the manufacturing process, the cotton is treated with various chemicals to achieve desired colours and textures, some of which can be carcinogenic. The chemicals used in the process also lead to toxic wastewater that pollutes rivers, seas, and oceans. The workers involved in processing the cotton into t-shirts often endure poor working conditions and low wages. Once manufactured, the t-shirts are transported around the world to be sold, which again significantly increases their carbon footprint. While some countries produce their clothing locally to reduce shipping-related emissions, clothing is still transported globally multiple times, contributing to approximately 10% of global carbon emissions. The decreasing cost of clothing has led to an increase in consumption, resulting in a 400% escalation in the consumption of clothes from 1994-2014. The environmental impacts of clothing production, from water use and pollution to greenhouse gas emissions, make it the second largest polluter globally. However, there are several ways individuals can work towards sustainable fashion, such as shopping second-hand, purchasing recycled or organic cotton, donating or recycling unwanted clothes, or reusing them for cleaning (Chang: 2017). **What is a smartphone made of?** Every smartphone is composed of a considerable quantity of diverse rare earth elements and precious metals. Rare earth elements are present in numerous regions worldwide in limited concentrations, and their magnetic, phosphorescent, and conductive properties are indispensable for electronic devices like smartphones. Nevertheless, mining these elements is causing significant environmental and social damage, often through open-pit mining, which harms extensive areas, including natural habitats, and leads to air and water pollution. Moreover, nearby people and animals are negatively impacted by this contamination, and in certain cases, displacements of human or animal populations are necessary to open mining areas. In addition to these environmental impacts, substandard working conditions for mining laborers in this process are prevalent, and machines used for mining require significant amounts of oil, exacerbating pollution during the mining of rare earths and precious metals. Additionally, the availability of these elements in our phones is not infinite. As the number of smartphones continues to grow and replacing rare earth elements and precious metals becomes infeasible, the frequent replacement of functioning phones with new ones, leading to the accumulation of electronic waste, necessitates a solution to these electronic device-related issues. Reclaiming the components of our phones is

<table>
<tr><td></td><td>becoming increasingly crucial. In a video, it is recommended to donate old phones, bring them to an e-waste recycling facility, or provide them to a refurbishing company. However, even recycling may cause environmental damage due to the pollution generated by recycling facilities, high energy and oil consumption, and poor working conditions. E-waste is frequently exported to countries with low wages and inadequate working conditions, where laborers (often children or women) are underpaid and lack the knowledge of how to safely disassemble phones, resulting in exposure to toxic elements. Phone waste may also end up in dumps, contaminating nearby soil and water sources (Preshoff 2018).</td></tr>
<tr><td>In-class Activity</td><td>HW 07.01: Step 0:
This step should be performed by the students at home in the last session before this one. A text by (R 08.01) Diprose et. al. (2021) should be handed out to the students. They are informed that they are required to read it at home and take notes or mark quotes in the chapter to answer the following questions:
1) What reasons for increasing consumption impacting the environment with higher production are assumed by people in this chapter?
2) What generation is assumed to live more sustainably, and which traits are assumed to identify this more sustainable lifestyle?
3) What is the most recognized trait of higher consumption in this chapter?
4) Is there a suggestion for more sustainable consumption?
The students should use their notes to create a “panel picture” (one PowerPoint slide, etc.) to present their most important findings concisely.

A 08.01: Step 1 (30 minutes):
The students should utilize their notes taken while reading the chapter to compare their findings and answer the leading questions of HW: 07.01: step 0.
To commence the discussion, two students should be asked to present their “panel picture” and share a few words about their findings and why the arguments/quotes on their slide are the ones chosen to summarize the chapter.
The class should discuss 1) whether these quotes and parts of the chapter were chosen by themselves or if their panels contain other cases and 2) why they chose the same or different ones.

A 08.02: Step 2 (30 minutes):
The leading questions on the videos should be handed out to the students, and they should be given some time to read them.
The life cycle of a t-shirt (R 08.03):
1) Where is cotton mainly produced?
2) What is the impact of cotton production on the environment and the people or animals on cotton fields?
3) Are there further negative environmental impacts in producing t-shirts from cotton after harvesting the raw material? Which ones and where are they mainly caused?</td></tr>
</table>

	4) Where are the produced t-shirts sold afterwards? Are these the same areas of the world where the production and its negative environmental impacts are located? 5) Is there an option to either get cotton with lesser environmental impact or to consume less cotton? What's a smartphone made of? (R 08.02) 1) What is the main problem for our environment with using rare earth elements for our smartphones? 2) Are there also social problems caused in mining rare earth elements and metals for smartphones in near-mining-areas? Which ones? 3) Why is reclaiming resources out of used phones a necessity? 4) What are your individual options to decrease the waste caused by smartphones? 5) Are there problems with these options too? Which ones are assumed in this video? The students watch the two TED-Ed videos by (R 08.02) Kim Preshoff and (R 08.03) Angel Chang in class and take notes according to the leading questions. After the video, the discussion is opened following the leading questions. **A 08.03: Step 4 (30 minutes):** The following questions should be discussed in class. The students should be asked to use their notes from the leading article and the videos to argue with examples: • What impact does high consumption have on our environment? • Which people are mostly affected? • Which ones mainly consume the produced goods? • Which options does every individual have to decrease negative effects of consumption on earth and society?
Required Materials	• Either a beamer or digital devices with internet connectivity and headphones for every student
Follow-up	Potential follow-up questions are: • Consider the social movements of the 1968 era or the contemporary phenomenon of Fridays for Future. A curious observation can be made in the leading article: despite the general assumption that older individuals tend to be more cognizant of environmental issues and exhibit greater sustainability practices, it is the younger generations that are primarily spearheading these movements. Why? • What methods would you recommend to mitigate the harmful effects of consumption on our environment and society?
References	Chang, A. (05.09.2017). *The life cycle of a t-shirt*. TED-Ed. https://www.youtube.com/watch?v=BiSYoeqb_VY (accessed 22.12.2021) Diprose, K. et. al. (2021). *Intergenerational Perspectives on Sustainable Consumption*. In: Diprose, K. et. al. *Climate Change, Consumption and Intergenerational Justice. Lived Experiences in China, Uganda and the UK*. Bristol. (pp. 103-127)

	Preshoff, K. (01.10.2018). *What's a smartphone made of?*. TED-Ed. https://www.youtube.com/watch?v=eIdJ22AfsO8 (accessed 22.12.2021).

Session 9: Counter-Arguing Climate Change Sceptics

Session Title	Counter-arguing Climate Change Sceptics
Duration	90 minutes
Description	This session pertains to steps 5 and 6 of the SSI, wherein social and ethical conflict dimensions will be discussed, and scientific arguments will be evaluated to make political judgements. The topic of climate change will be debated by learners to gain a deeper understanding of various methods and approaches in dealing with the subject. The objective of this session is to encourage a critical analysis of diverse perspectives, distinguishing scientifically valid views from sceptical or scientifically weak ones.
Theoretical Framework	As this session is intended for reflection and debate on previously learned content within the module, no additional theoretical framework is required.
In-class Activity	**A 09.01: Step 1 (30 minutes):** The students will be requested to read the (R 09.02) summary of the NIPCC report "Fossil fuels reconsidered" and identify the main arguments presented by the report. The teacher will encourage students to share their observations during an initial brainstorming session where the teacher will record the arguments on a whiteboard. Moreover, students are also expected to explain the claims or evidence that support the identified arguments. **A 09.02: Step 2 (30 minutes):** Following the brainstorming session, students will be divided into small groups of 2-3 individuals. They will be instructed to devise and document potential counterarguments for three of the statements identified on the whiteboard. During this process, students can use devices with internet connection to conduct research, or they can refer to (R 09.01) the 2018 or 2023 Summary for Policymakers of the IPCC. The students will then affix the sticky notes with their identified counterarguments next to the corresponding arguments on the whiteboard. **A 09.03: Step 3 (30 minutes):** In the final stage, the small groups will be brought together as a plenum. The teacher will present each argument, along with its corresponding counterargument(s). Thereafter, a discussion on the arguments raised and the methods employed by the NIPCC will ensue, and students will critically evaluate the validity of the claims.
Required Materials	• Whiteboard and markers • Sticky notes • Digital devices with internet connectivity, with at least one device available for each group
Follow-up	Potential follow-up questions are:

	• What is the NIPCC? Who are its main representatives? Which are its main donors? • What are NIPCC main arguments? What are their methods? What references do they use? • What is the IPCC? Who are its main representatives? Which are its main donors? • What difference do you notice in organisational terms with the NIPCC?
References	IPCC (2018). *Summary for policymakers*. https://www.ipcc.ch/site/assets/uploads/sites/2/2019/05/SR15_SPM_version_report_LR.pdf IPCC (2023). *Summary for Policymakers*. https://www.ipcc.ch/report/ar6/syr/downloads/report/IPCC_AR6_SYR_SPM.pdf NIPCC (2021). *Climate change reconsidered II. Fossil fuels. Summary for policymakers*. http://climatecangereconsidered.org/wp-content/uploads/2018/12/Summary-for-Policymakers-Final.pdf

Session 10 & 11: Responsibility. Individual and Systemic Approaches

Session Title	Responsibility. Individual and systemic approaches.
Duration	Two 90-minute sessions
Description	These two sessions are based on the text "Historisches und politisches Lernen mit Konzepten" (Hellmuth/Kühberger 2016). This approach to learning historical and political topics involves the identification of concepts as a means of organizing new information. Students can use these learned concepts to structure new information and adapt their concepts to specific cases. The underlying premise of learning with concepts is that individuals already possess pre-concepts that they have learned in their everyday lives. The school's task is to provide students with opportunities, information, and cases to try out, reflect on, and adapt their concepts. These concepts are often highly abstract, and the use of specific historical or political cases and partial concepts may help to reduce their complexity. Session 1: • Step 1: Group work - Creating a poster that depicts their idea of "responsibility" • Step 2: Presenting the posters to the class • Step 3: Watching and discussing videos in groups Homework: Students are asked to read the paper and highlight significant phrases that describe individual and systemic approaches to handling climate change and justice. Session 2: • Step 4: Presenting the results of the group's work with the videos • Step 5: Discussing the paper

	• Step 6: Using new conceptions of "responsibility" in the discussion of water conservation efforts in Italy in 2022 These sessions mainly focus on the step 5 of the SSI although it has a little part related to step 3. Step 3 is part of this sessions as theory regarding responsibility is introduced. Most of the sessions deal with questions regarding moral and ethical takeaways on responsibility debates.
Theoretical Framework	The theoretical framework presented in this context primarily draws upon the work of Jamieson Dale in his article titled "Responsibility and Climate Change" published in the journal *Global Justice.*[25] This article serves as a crucial point of reference to facilitate further discussion in the subsequent parts of the sessions. The phenomenon of climate change presents us with an unprecedented challenge. Our collective contributions to this complex issue implicate us all, yet attributing responsibility for the resulting deaths is difficult, if not impossible, on an individual level (Dale 2015: 23-24). Dale proposes a re-examination of the concept of responsibility in the context of climate change, tracing the evolution of the term from Aristotle to contemporary times (2015: 25-26). He delineates the notions of causal, moral, and legal responsibility, and offers the following example to clarify these concepts: If Jack has a seizure and breaks Jill's model airplane, we may say that Jack was causally but not morally or legally responsible for Jill's loss. If Kelly fails to shovel her sidewalk and Sean slips and falls, we may say that Kelly is morally but not causally or legally responsible for Sean's injury. If Pat is married to a profligate he may be legally but not causally or morally responsible for his partner's debts. (Dale 2015: 25f). Dale posits a nuanced understanding of the concept of responsibility as it pertains to climate change (2015: 26f). He argues that causal responsibility alone is insufficient for moral responsibility and that moral responsibility does not necessarily imply legal responsibility. While we may recognize that our actions contribute to climate change, we may not feel morally responsible for the harm caused, as it does not violate our moral sensibilities. Dale suggests that social change movements aimed at combating climate change must make people feel morally responsible rather than just causally responsible (2015: 28). However, the challenge is that moral responsibility often entails a fear of punishment, which may elicit resistance from individuals. As Dale notes, fear of punishment may make us act against the problem but may also make us act against the person telling us to be responsible (2015: 28). He concludes these thoughts about responsibility by summarizing the terms causal, moral and legal responsibility (2015: 29): Causal responsibility concerns what we bring about and ascriptions are often value-laden. Moral responsibility invokes an action's liability to sanctions. Legal

[25] Theorie Practice Rhetoric, volume 2 (2015), pages 23-43.

	responsibility implies that particular, formal sanctioning practices may be appropriate. Returning to the topic of responsibility in the context of climate change, there are distinct challenges that differ from other issues we encounter in our daily lives. Jamieson Dale's (2015: 29ff) article outlines six main differences: 1) **The power of technology**: "The growth and development of technology, especially in regard to the production and management of energy, is to a great extent responsible for this. While once people had the power to disrupt their local environments, now people have the power to alter the planetary conditions that allowed human life to evolve and that continue to sustain it" (Dale 2015: 30). 2) **Spatial reach of climate change**: The challenge we face is the vast scope of climate change, particularly in terms of the actions that contribute to it. The regions that contribute to climate change are not necessarily the ones where its impacts are felt. Climate change is a global phenomenon, and emissions produced in one area can affect the entire planet. 3) **Temporal reach**: The emissions we produce now will remain in the atmosphere for a considerable time, possibly until the year 3000. Thus, climate change is not only an issue of present harm but also a problem for future generations. 4) **Systematicity of the forces that give rise to it**: The article argues that individual efforts to reduce carbon footprints are commendable, but the manipulation of the carbon cycle is a fundamental issue with the global economic system. Furthermore, Dale notes that climate change damages may be insensitive to individual behaviour and that there are issues of responsibility concerning political acts (2015: 35). 5) **It is the world's largest and most complex collective action problem**: Climate change is a significant issue since every individual plays a part and is affected by it. It is also a complex problem due to the high degree of connectivity in the climate system and the non-linear relationships between variables. 6) **Climate change is world-constituting**: Climate change is a constitutive feature of the modern world, and its causes have shaped the present and the future. It is unknown how many individuals would be alive today if we had never developed cars, trains, or airplanes. In conclusion, it can be argued that the distance between the specific acts of harm and the resulting damages in climate change is considerably greater than in any other problem encountered in our daily lives, both in terms of space and time. Even though we can and will cause harm to many people, no individual will be held solely accountable for the consequences. However, as members of collectives, we do have a responsibility to act in a certain manner (Dale 2015: 32). We already possess an inherent understanding of our obligations in group settings, albeit primarily within small and often homogenous groups. The expansion of these responsibilities can occur in extraordinary circumstances such as during wartime. "Extending the intuition seems to require specific enemies (e.g., Nazis), goals (e.g., winning the war), and means (e.g., resource conservation). Climate change, however, does not share these features. There is no specific enemy, the goal is ill-defined, and the means are (too) many" (Dale 2015: 33).

All these considerations lead to the suggestion that the concept of responsibility is constructed and mobilized for practical purposes, such as when an individual argues that gun violence is not the result of lax gun control but rather the fault of the person who pulled the trigger, thereby denying the need for change in gun control. Dale points out that most arguments regarding responsibility aim to persuade others to adopt one's own perspective, rather than to reveal a fundamental truth about human nature (2015: 36). Bernard Williams rejects the idea of a "correct" conception of responsibility, as individuals require different conceptions of responsibility to navigate the varied circumstances of their lives (as cited in Dale 2015: 36). These differing conceptions of responsibility are inevitable and will often lead to conflicts.

Jamieson Dale proposes the concept of intervention-responsibility to address climate change in a globally responsible manner (2015: 38). This idea entails a forward-looking conception that directs us towards what different actors in the system, whom Dale refers to as "agents," can do, rather than what has been done so far. These agents operate at various levels of the system and possess varying degrees of power to bring about change, such as an individual acting with different capacities from government actors. Dale identifies four different "families" of actors with regards to climate change (Dale 2015: 38ff):

1) **International organizations and regimes**:
   International organizations and regimes have been criticized for their failures in handling climate change, but often lack the power to make the necessary changes. Although they can provide a forum for conscious-raising and sharing of information, and internationalize adaptation, a lot of intervention-responsibility must lie elsewhere. The FCCC, for example, does not have the power to impose sanctions on states not acting towards their ideas.
2) **Nations and other jurisdictions**:
   Dale identifies nations and other jurisdictions as having high capacity for effective action in some areas, but low levels in others. While these agents are often the focus of attention, they have little success in achieving coordinated solutions and fairness in addressing climate change.
3) **Individuals**:
   This category of actors has been extensively discussed in the academic literature. Many texts related to climate change highlight the individual capacity to contribute towards mitigating climate change to "empower" individuals. Nevertheless, the impact of individual actions on climate change remains relatively low. However, it is worth noting that in democratic societies, individual actions may act as a catalyst for collective action and indicate a readiness to comply with laws aimed at reducing greenhouse gas emissions. While individual actions may not have a great effect on reducing emissions, they can facilitate awareness-raising and foster political engagement.

<table>
<tr><td></td><td>4) Firms:
Firms are responsible for increasing emissions every year, and their actions often go unnoticed. They are also spread unequally, with a small number of firms responsible for the majority of carbon emissions. For example, Dale mentions 90 firms from 43 different countries being responsible for 63% of all carbon emissions from 1854-2010. Firms often fund misinformation campaigns and pressure governments to adopt policies in their favour, rather than using their power to lead towards a non-carbon future. Dale notes that these actions go against their intervention-responsibility and should be held responsible for them by other actors. While firms can reduce emissions, there is no guarantee that their actions will result in a fair outcome.</td></tr>
<tr><td>In-class Activity</td><td>A 10.1: Step 1 (20 minutes):
In the first step, the class will be divided into groups of 3 to 4 individuals, and each group will be tasked to create a poster that represents their idea of what "responsibility" means. This could be achieved through drawings, keywords, mind maps, or any other medium of their choice.
A 10.2: Step 2 (30 minutes):
The groups will present their posters to the class, and their peers will be allowed to examine the posters and express their thoughts on the ideas presented. The group will then discuss their thought processes and concepts of responsibility.

A 10.03: Step 3 (40 minutes):
The class will be split into two groups, and each group will watch two videos to discuss later.
Group 1:
This group will watch the following videos:
1) R 10.01: Common Sense Education (10.08.2018): Rings of Responsibility. (online: https://www.youtube.com/watch?v=fQSnzrB5bso&ab_channel=CommonSenseEducation access on 06.07.2022)
2) R 10.02: Solli Raphael (14.08.2018): We can be more. TEDx Talks. (online: https://www.youtube.com/watch?v=lm0r3yFh0zU&ab_channel=TEDxTalks access on 06.07.2022)
Afterwards they engage in a discussion based on the following suggested questions:
• What is the meaning of responsibility, and how do Common Sense Education and Solli Raphael express their ideas of responsibility?
• What are the individual and systemic components of being responsible for the environment according to Solli Raphael and Common Sense Education?
• How are individuals and society interconnected in these videos, and how can each of them impact the other?
• Are there any ideas of responsibility, individual or systemic tasks for being responsible, quoted in these videos that you disagree with or have a different perspective on? If so, which ones, and why?
Group 2:
This group will watch the following videos:</td></tr>
</table>

	1) R 10.03: Common Sense Education (10.08.2018): Rings of responsibility. (online: https://www.youtube.com/watch?v=fQSnzrB5bso&ab_channel=CommonSenseEducation access on 06.07.2022) 2) R 10.04: Jesse Oliver: Wake up! (spoken word poetry). TEDx Talks. (online: https://www.youtube.com/watch?v=04rfgNvvXz8&ab_channel=TEDxTalks access 06.07.2022) Afterwards they engage in a discussion based on the following suggested questions: • What is the meaning of responsibility according to Common Sense Education and Jesse Oliver? • What are the responsibilities of individuals and society in these videos? • Are the responsibilities of ecological problems and anti-racism the same? How are they approached in these videos, and what are the commonalities or differences? • What individual actions or systemic changes are required to act responsibly as human beings? • How are these two approaches (systemic changes or individual actions) connected according to these videos? How can any of these parties influence the others? • Are there any ideas of responsibility, individual or systemic tasks for being responsible, quoted in these videos that you disagree with or have a different perspective on? If so, which ones, and why? **HW 10.01:** Students are required to study the paper (R 10.07) "Responsibility and Climate Change."[26] by Dale Jamieson and select the best quotes concerning systemic or individual approaches and climate change or any questions that may arise. **A 11.01: Step 4 (20 minutes):** In this step, the class will pair up with one member from each of the two groups that watched the videos. Each person will present and discuss their group's findings and discuss similarities and differences in the concepts discussed in both groups. **A 11.02: Step 5 (30 minutes):** Students will discuss the paper by Dale Jamieson (2015) based on their homework, where they will share their thoughts and notable parts of the paper. Similarities in the quotes and thoughts marked will be compared, and potential differences will be discussed. Students will be given time to ask questions, and they can use their phones to find answers as a group. The teacher steps in and provides any necessary information if students are unable to find solutions to their problems. **A 11.03: Step 6 (50 Minutes):** In this final step, students read an article (R 10.05) by Jon Heggie (2020) as well as an article (R 10.06) by the Deutsche Presse Agentur – DPA (2022) (online available at: : https://www.dailysabah.com/world/europe/italys-verona-pisa-limit-water-supplies-

[26] In: *Global* Justice. *Theory, Practice, Rhetoric,* 8/2 (2015), pages 23-43.

	amid-drought) and take notes. The concepts in these articles will be compared to the idea of responsibility discussed in the previous parts of the session(s) and the article by Dale Jamieson (2015). To facilitate the reading process, several questions will be handed out: • Are there any structural issues related to water management in Italy? How and when have these issues been documented? • Are there any possibilities to address this problem at a structural level? Has there been any previous attempt to do so? • What is the underlying cause of the problem? Is it due to structural, individual, or ecological factors? • Who should take responsibility for addressing this problem on a daily basis – individuals or the system? Is there a discussion about the state taking responsibility? • How do these two articles align with Jamieson Dale's concept of responsibility? • Can Jamieson Dale's notion of responsibility be applied to suggest alternative ways to tackle this issue beyond individual responsibility? • What would be your proposed approach to resolving this problem? Should individuals take responsibility, or should the state as a system be responsible?

Required Materials	• Access to the following articles (either online or as hard copies):
	  ○ DEUTSCHE PRESSE-AGENTUR – DPA (03.07.2022). Italy's Verona, Pisa limit water Supplies amid drought.
	  ○ Heggie, Jon (18.06.2020). The Leaky Boot: Where is Italy's Water Going?
	  ○ Jamieson, Dale. Responsibility and Climate Change.
	• Material required for creating a poster (such as pens and paper)
	• Digital devices with internet connectivity, with at least one device available several students

Follow-up	-
References	DEUTSCHE PRESSE-AGENTUR – DPA (03.07.2022). *Italy's Verona, Pisa limit water Supplies amid drought.* Daily Sabah. https://www.dailysabah.com/world/europe/italys-verona-pisa-limit-water-supplies-amid-drought (accessed 06.07.2022) Heggie, Jon (18.06.2020). *The Leaky Boot: Where is Italy's Water Going? National Geographic*. National Geographic. https://www.nationalgeographic.com/science/article/partner-content-the-leaky-boot-italy (accessed 06.07.2022) Hellmuth, T. & C. Kühberger. *Historisches und politisches Lernen mit Konzepten*. In: Historische Sozialkunde. Geschichte – Fachdidaktik – Politische Bildung 1(2016). (pp. 3-8) Jamieson, D. (2015). *Responsibility and Climate Change*. In: Global Justice. Theory Practice Rhetoric. Vol. 8 (2). 2015. (pp. 23-43)

R 10.05: Part of "Heggie, J. (18.06.2020). The Leaky Boot: Where is Italy's Water Going? National Geographic. (online access: https://www.nationalgeographic.com/science/article/partner-content-the-leaky-boot-italy on 06.07.2022):

[…] But if the taps do run dry, nature won't be entirely to blame – man-made factors are making Italy's problems much worse. Two thousand years ago Italy boasted the most advanced aqueducts of the time, carrying water many miles to sate the thirst of its emergent cities. Today, Italy's water infrastructure is struggling, and efforts are being made to repair and replace the outdated and inefficient parts of the network. In the last two years, an additional 20 percent of investment has driven urgent upgrades to Rome's 3,355 miles (5,400 kilometres) of pipes that have been losing some 44 percent of the city's water supply. It's a waste of water that Italy can ill afford because its demand is particularly high. Italy has one of the highest water footprints in Europe, at 3012 cubic yards (2,303 cubic meters) per person per year. That's 25 percent above the European average. The main draw on Italy's water supply is agriculture, as large areas of Italy are not naturally suited for crops and require extensive irrigation. Of the water used for irrigation, 15 percent comes from non-renewable groundwater, twice as much as Spain and seven times that of Greece, draining resources that cannot naturally be replenished. Agriculture also contributes to Italy's problem of water pollution, as fertilizers have joined industrial and domestic contaminants in seeping into the water table. In 2014, the European Court of Justice took legal action against the Italian government for failing to adequately treat water sources inundated with calcium, arsenic, and fluoride.

Session 12: Justice Research Group. Exchange and Reflection of Findings on Responsibility

Session Title	Justice Research Group. Exchange and Reflection of Findings on Responsibility.
Duration	90 minutes
Description	Step 5 of the SSI involves the discussion of the social and ethical dimensions of conflicts. This step is intended to build upon the discussion held at the end of Session 1. The questions raised during this discussion should remain the same, but with the incorporation of new knowledge gained throughout the module. Students are encouraged to reflect on their assumptions and consider how the new information may have changed their thinking and arguments. Step 6 of the SSI, students are expected to use the new theories and assumptions they have learned to compare and contrast their own gain of knowledge with that of their classmates. This step serves as a culmination of the module and provides students with the opportunity to apply the knowledge they have gained to a practical scenario.
Theoretical Framework	As this session is intended for reflection and debate on previously learned content within the module, no additional theoretical framework is required.
In-class Activity	**A 12.01: Step 1 (5 minutes):** The same groups as in Session 3 and 4 should be formed again. The (R 12.01) invitation to a plenary discussion should be distributed to the students.

	A 12.02: Step 2 (55 minutes): In groups, students should prepare for the upcoming discussion. One expert is expected to prepare arguments to represent their group's findings in this module to the class, and possibly present a potential solution to the issue of unfairness in climate catastrophe. The other members of the group should assist the expert in preparation and also prepare themselves for possible counterproposals from other groups' suggestions or for meaningful counter-questions during the discussion. Students should take notes during their preparation. The teacher may assist in clarifying misunderstandings or with phrasing the suggestions of all groups, if needed. **A 12.03: Step 3 (30 minutes):** The groups should convene as a plenum, with the experts seated in front of the class. Each expert will have a chance to present their group's consensus decisions to the public (in this case, the class). The rest of the class should take on the role of a critical audience and question and discuss the proposals. The extent to which these findings match those of the research group should be considered, as well as agreement or disagreement with the proposals of the other groups' experts.
Required Materials	• All notes from the previous parts of this module • Digital devices with internet connectivity, with at least one device available for each group
Follow-up	Potential follow-up questions are: • Do you believe that climate catastrophe is fair? In your understanding, what is the definition of "fair" in the context of climate catastrophe? • What measures can be taken to ensure that climate catastrophe is dealt with in a fair and just manner?
References	-

12.01: Invitation

Invitation You and your group have received an invitation to participate in a plenary discussion on climate justice and equity, and are requested to nominate an expert who can represent your group's views and findings. In preparation for the discussion, please consider the following questions, and use your research, including your findings from Task 3, to support your statements: (a) What should be the primary objective that humanity addresses in addressing global warming: justice, equity, or equality? (b) What factors would contribute to the successful implementation or failure of this goal? (c) What changes would be required in global cooperation to enable the sustainable implementation of these objectives? Please select an expert from your group to speak on behalf of your team and present your group's findings during the plenary discussion.

4.1. Didactic Commentary

Teaching: Is climate catastrophe fair? Fairness, responsibility and decolonial perspectives

Jakob Steinbachner & Nicola Nagy

Department of Teacher Education, University of Vienna, Vienna, Austria

Teaching Experience

The basic idea of the module "Is climate catastrophe fair?" was to show the connection between mathematical and political topics, especially for teachers' education for the subject of history and political education. Climate and justice seemed to be suitable topics to demonstrate the use of mathematical modelling for political debates and possible conclusions from the former for the latter.

The teaching of this module has also shown that the topic of justice in particular is a good illustration of the interconnectedness of mathematics and political decisions. Using the example of debates about injustice and the Gini-Index, debates about what is just and how it can be measured and represented could also be dealt with from the perspective of students of political education. Because of the lack of teaching capabilities of the project-team at the university of Vienna teaching this course was done in several courses of colleagues. This resulted in us not being able to teach the course as a whole in one seminar in one semester and try to discuss every topic with the same group of participants.

At first and partly, this seemed like a disadvantage in the post-processing, reflection and adjustment of the module. Each session should build on each other within the module and the entire module is created in such a way that teaching is intended to be continuous, so that tasks and topics that build on and reference each other can also be treated as such. In the implementation of the teaching, such tasks had to be omitted or the reference to previous tasks—which were not made with the same course—had to be made. However, this showed a strength of the course: individual topic blocks can also be taken out of the overall module with some preparation by the teacher and inserted into other modules in line with seminar topics and seminar processes. This is perhaps a strength of the module that we would not have been able to test in the teaching rehearsal within a seminar running there.

Since the module is very comprehensive and does not fit into every seminar plan, the topics are created as blocks. In most of the topic blocks there are references to other tasks and topic blocks, but these can be omitted or replaced by a short explanation by the teacher, as the teaching samples showed us. In this way, the module can be shortened as desired or only individual topic blocks can be taken from the module without becoming redundant.

Overall, the teaching experience of this course was a positive one. Most of the topics could be dealt with comprehensively by the students (student teachers, bachelor's degree students, history and political education). In particular, the interest of the students was very high, probably also due to the tendency of young people to be more interested in the climate catastrophe. However, some topics may need to be narrowed down further. For example, in the teaching of Sessions 2 and 3, especially in the discussion of justice and equity terms, due to a very open approach to the term through all the sources that the students themselves found in their research, a very broad discussion broke out about the meaning of these terms, which could have been clarified around the scope of one unit. Here we recommend the use of suggestions of sources for the students' research, which we added later and also used in the course of this unit. Overall, however, the students were able to work on the contents of the courses without major problems and to continue thinking independently; no major problems arose anywhere. The greatest difficulty was understanding different mathematical models of students from the History and Political Education teacher training programme. Here, a reference to the ideas of the didactic considerations of this module (cf. Gildehaus et al., 2021, pp. 5–6; Lauß, 2022, pp. 116–117) is recommended and had a motivating effect.

Other regularly occurring problems are often more due to the size of subject blocks. Since many assignments encourage discussion and this, if productive, does not need to be interrupted by the teacher, not every assignment can be completed in some seminars. Here, however, the assignments are designed in such a way that the omission of short reflection lectures or the discussion of another summary text can be given up as homework or omitted.

References

Amnesty International (2019, 10 December). *Amnesty Umfrage zu Menschenrechten: Klimawandel wichtigstes Thema unserer Zeit für „Generation Z“*. https://www.amnesty.at/presse/amnesty-umfrage-zu-menschenrechten-klimawandel-wichtigstes-thema-unserer-zeit-fuer-generation-z/ (accessed 6 June 2023)

Gildehaus, L. et al. (2021). *Framework to develop normative modelling teaching materials.* https://www.idd.uni-hannover.de/fileadmin/idd/Projekte/CiviMatics/CiviMatics_IO1_final.pdf (accessed 23 May 2023)

Lauß, G. (2022). Politik – Natur – Wissenschaft: Der Social – Science – Issues (SSI) Ansatz in der Politischen Bildung. In M. Oberle & M. Stamer (Eds.). *Politische Bildung in internationaler Perspektive* (pp. 116–123). Schwalbach.

CHAPTER 5

Interdisciplinary Citizenship Education

Bastian Vajen

Institute of Didactics of Democracy, Leibniz Universität Hannover, Hannover, Germany

Introduction

The goal of this course is to introduce students to the principles of and interdisciplinary approaches to citizenship education. To achieve this, students will first receive a short introduction to citizenship education as a subject and a whole-school approach. Following this, the students will work on approaches which connect citizenship education with other subjects, such as geography, history, physics or mathematics. Due to the focus of this project, mathematics education and its potential for interdisciplinary learning processes with citizenship education and vice versa will be highlighted. Finally, students are expected to work on their own ideas and design lesson plans which connect competences of citizenship education with competences of other subjects.

The target group of this course are students who, in the best-case scenario, have already attended an introduction to citizenship education or a similar course. However, it is also possible for students without prior knowledge to attend this course since it offers a brief introduction to citizenship education in its first three sessions. Thus, this course is suitable for teacher training in general. Each session is designed to last 90 minutes, as this is a common duration for a seminar session at German universities.

At the Leibniz University Hannover, this course was designed for students studying politics at a bachelor level with the aim of becoming citizenship education teachers after finishing their master's degree. It was incorporated into the introductory module for citizenship education, consisting of two seminars. The first course offers a general introduction to citizenship education, usually taken during the third semester, while the second functions as an advanced course, offering a detailed insight into one specific element of citizenship education. Students were free to choose the second course from a variety of seminars, for example interdisciplinary citizen-ship education, education for sustainable development or nonformal citizenship education. To pass the module, students were required to attend both seminars and write one term paper at the end of either the first or the second course, which earned them ten ECTS-points. Teaching this course as a standalone course would equate to approximately five to seven ECTS-points, de-pending on the workload required for the final exam.

The objectives of this course are the following: The students should (a) understand the fundamentals of citizenship education, (b) understand citizenship education as a whole-school-approach, (c) gain insight into interdisciplinary citizenship education opportunities, (d) gain an understanding of the foundations of mathematics education and mathematical modelling, (e) gain insight into practical implementation possibilities of interdisciplinary citizenship education.

Teaching Methods and Study Activities

Although this course uses different teaching and study methods, every session is structured similarly by starting with a discussion of the required reading, a (group-)work phase and a comparison and discussion of its results. As part of this course, students are usually asked to prepare one or more texts for each session by reading them and adding comments, questions or feedback during the reading process. To encourage this, the lecturer can provide certain mandatory questions or tasks the students have to finish before each lesson and thus ensure that every student is familiar with the contents of the text to a certain degree. Building on this preparation, every session starts with the discussion of the respective required reading, whereby either the students or the lecturer can start the process by asking questions or stating comments about certain elements of the texts.

The goal of this first phase is to clear up misunderstandings, explain connections to certain theories the students may not be familiar with, to point out the central message of the text and to discuss differing opinions about it and its merits. This initial phase is followed by a task the students should work on during the session, which aims at a deeper understanding of the session topics and the texts. These tasks are usually worked on in groups and can involve different methods, such as concept mapping, priority games or lesson planning. After the task is finished, the students are asked to compare and discuss their results, connect them to the topic of the session, the texts and – if possible – the discipline as whole.

The goal of this process is to offer a deeper understanding of the literature and with it the topics of this course while also encouraging all students to actively participate.

Requirements, Syllabus and Assessment

The course was designed to be taught as part of the Bachelor of Arts program for students of political science who want to become teachers after finishing their Master of Education degree. Because the Leibniz University Hannover offers different types of teacher education programmes, this course was open to prospective teachers of general education, vocational education, and special education. The course utilizes two types of assessment. To pass the course, students have to answer a weekly question connected to the required reading for each session. Passing this course is necessary to earn the 10 ECTS points connected to the introductory mod-ule for citizenship education described in the introduction.

For grading, the students are asked to write a term paper (10–12 pages), which is due by the end of the semester. Students are free to choose their topic and research question as long as it is related to the topics covered by this course. Choosing the method of examination is usually the responsibility of the lecturer and other types of exams, like an oral or a written exam, would also be suitable for this type of module.

Structure of the Module and Literature

<u>01 Introduction and organizational matters</u>

- -

<u>02 Citizenship education – Goals and competences</u>

Himmelmann, G. (2013). Competences for Teaching, Learning and Living Democratic Citizenship. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 3–8). Sense Publishers.

Sander, W. (2004). Incitement to freedom: competencies of political education in a world of difference. *The Development Education Journal*, *11*(1), 9–11.

03 Citizenship education – Competences and educational practices

Print, M. (2013). Competencies for Democratic Citizenship in Europe. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 37–50). Sense Publishers.

Reinhardt, S. (2013). Teaching for Democratic Learning. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 99–110). Sense Publishers.

04 Citizenship education – Democratic schools

Edelstein, W. (2011). Education for Democracy: reasons and strategies. *European Journal of Education*, 46(1), 127–137. https://doi.org/10.1111/j.1465-3435.2010.01463.x

Solhaug, T. (2018). Democratic Schools – Analytical Perspectives. *JSSE - Journal of Social Science Education*, 17(1), 2–12. https://doi.org/10.4119/jsse-858

Additional Literature

Schutz, A. (2001). John Dewey's Conundrum: Can Democratic Schools Empower? *Teachers College Record*, 103(2), 267–302. https://doi.org/10.1111/0161-4681.00116

05 Interdisciplinary citizenship education – Possibilities and limitations

Syed, G. K. (2013). How Appropriate is it to Teach Citizenship through Main Curriculum Subjects? *Citizenship, Social and Economics Education*, 12(2), 136–142. https://doi.org/10.2304/csee.2013.12.2.136

06 Interdisciplinary citizenship education – Social studies

Barton, K. C. (2017). Shared Principles in History and Social Science Education. In M. Carretero, S. Berger, & M. Grever (Eds.), *Palgrave Handbook of Research in Historical Culture and Education* (pp. 449–467). Palgrave Macmillan UK. https://doi.org/10.1057/978-1-137-52908-4_24

Thornton, S. J. (2018). Geography as a Social Study: Its Significance for Civic Competence: Its Significance for Civic Competence. In E. E. Shin (Ed.), *Spatial Citizenship Education: Citizenship Through Geography* (pp. 10–21). Routledge.

07 Interdisciplinary citizenship education – Other subjects

Davies, I. (2004). Science and citizenship education. *International Journal of Science Education*, 26(14), 1751–1763. https://doi.org/10.1080/0950069042000230785

Porto, M. (2018). Intercultural Citizenship Education in the Language Classroom. In I. Davies, L.-C. Ho, D. Kiwan, C. L. Peck, A. Peterson, E. Sant, & Y. Waghid (Eds.), *The Palgrave Handbook of*

Co-funded by the European Union

Global Citizenship and Education (pp. 489–506). Palgrave Macmillan UK. https://doi.org/10.1057/978-1-137-59733-5_31

08 Mathematics education – Philosophy

Ernest, P. (2018). The Philosophy of Mathematics Education: An Overview. In P. Ernest (Ed.), *The Philosophy of Mathematics Education Today* (pp. 13–38). Springer International Publishing.

09 Mathematics education – Mathematical Modelling

Blum, W., & Leiß, D. (2007). How do Students and Teachers Deal with Modelling Problems. In P. Galbraith, W. Blum, S. Khan, C. Haines, & C. R. Haines (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics; proceedings from the twelfth international conference on the teaching of mathematical modelling and applications* (pp. 222–231). WP Woodhead Publ; Horwood.

Maass, K., Artigue, M., Burkhardt, H., Doorman, M., English, L. D., Geiger, V., Krainer, K., Potari, D., & Schoenfeld, A. (2022). Mathematical modelling – a key to citizenship education. In N. Buchholtz, B. Schwarz, & K. Vorhölter (Eds.), *Initiationen mathematikdidaktischer Forschung* (pp. 31–50). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-36766-4_2

10 Mathematics education – Descriptive and normative models

Niss, M. (2015). Prescriptive Modelling – Challenges and Opportunities. In G. A. Stillman, W. Blum, & M. Salett Biembengut (Eds.), *International Perspectives on the Teaching and Learning of Mathematical Modelling. Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences* (pp. 67–79). Springer International Publishing. https://doi.org/10.1007/978-3-319-18272-8_5

Pohlkamp, S., & Heitzer, J. (2021). Normative modelling as a paradigm of the formatting power of mathematics: Educational value and learning environments. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (pp. 799–808). Tredition.

Additional Literature

Gildehaus, L. & Liebendörfer M. (2021). "CiviMatics - Mathematical Modelling meets Civic Education", In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (pp. 167-171). Tredition

11 Practical examples – Lesson plans I

Additional Literature

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16. https://doi.org/10.1080/14794802.2012.756633

12 Practical examples – Lesson plans II

Additional Literature

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16. https://doi.org/10.1080/14794802.2012.756633

13 Conclusion and evaluation

-

Contents and Contexts

The underlying concept of this course is to teach students about areas in which citizenship education can be combined with other subjects – especially mathematics – and to facilitate a discussion about possible benefits and shortcomings of such an approach. As with all interdisciplinary learning processes, besides possible benefit of expanding the understanding of connections between different scientific disciplines and enhancing the ability to comprehend the way various domains perceive, analyse and solve certain problems, there is also the challenge to properly address each individual discipline and combine similar levels of complexity for each subject when designing curricula or individual lessons. As these issues are still part of on an on-going academic debate, they cannot be solved by this course. However, by broadening students' perspectives about their own subject and the possibility of combining it with other subjects, it can lay the foundation to address these issues throughout within the course and offer a first step to encourage students to incorporate interdisciplinary learning processes when becoming school teachers.

To achieve this, this course is structured in a sequential manner, first presenting the principles of citizenship education and subsequently introducing other subjects and the different ways in which they can add to or overlap with citizenship education. Mathematical education takes a prominent role here, in that the fundamentals of the subject are also presented and discussed in detail. In the end, the students are tasked with converting what they learned throughout the course into an interdisciplinary lesson plan by finding a relevant topic and designing a lesson around it which combines learning goals of at least two different subjects.

After the introductory session, the course starts with an introduction to the central goals of citizenship education. To help students conceptualize the principles of citizenship education, the required reading consist of two texts by Himmelmann (2013) and Sander (2003) which highlight the goals of citizenship education (Sander, 2003) and its contribution to a democratic society (Himmelmann, 2013). The main task of this session is the creation of a concept map, which should help students understand the different elements of citizenship education stated in the required reading by visualizing them. The results can also be used as a reference point for later sessions when discussing the principles and goals of other subjects.

The following session again focuses on citizenship education. The required reading by Print (2013) and Reinhardt (2013) discuss different competences and behaviours of democratic citizens, which should be facilitated by citizenship education. As this issue is part of a controversial discussion and it often appears that citizenship education should prepare students for all different kinds of issues, the task of this session involves a priority game, in which the students are asked to prioritize statements about the central competences of democratic citizens and use the one they deem most important to create their own definition of what citizenship education should achieve. The results can be used to facilitate a discourse about the contribution of other subjects and can serve as a resource in upcoming sessions.

The fourth session is the last one which focuses only on citizenship education and starts broadening the perspective on it by working with two texts by Edelstein (2011) and Solhaug (2018) which describe citizenship education not as subject, but a general task for the whole school. This whole school approach aims at facilitating democratic participation and democratic practices in schools and the texts suggest different criteria to identify democratic schools. Building on this, the task for this session is to outline a lesson or a unit that incorporates essential elements of a democratic schools. The subject is not limited to citizenship education, or any subject for that matter, and can also involve extracurricular projects. The idea is to think about ways in which certain methods or practices can be used to teach students democratic skills and behaviour, irrespective of a certain subject. The goal of this session is thus to decouple citizenship education from a subject to a certain degree and use this as a starting point to introduce ways in which other subjects can be connected to citizenship education.

This process is also the subject of the fifth session, for which the students are asked to read a text by Syed (2013) which argues that it is more appropriate to teach citizenship through other subjects than to limit it to one statutory subject. Especially for German students, where citizenship education as a subject has an important tradition, this is a controversial statement. This session should thus facilitate a discourse about the necessity of the subject and the ability and inability of other subjects to teach topics usually addressed as part of citizenship education. Building on this, the students are tasked to work with school curricula and identify topics that are suitable for other subjects and topics that are not. Connected the last session, the goal of this session is for students to discuss the standalone value of citizenship education and its connection to other subjects.

This connection is expanded in the sixth session with the introduction of history and geography as two subjects that share certain elements with citizenship education. The required reading for this session include texts by Baron (2017) and Thornton (2018), which discuss this from the viewpoint of their respective subject. The task for this session is to use one principle of citizenship education – problem orientation – and to choose a problem analysed from a political and historical and/or geographical perspective.

Similarly, the next session introduces science and language education as subjects that can be connected to citizenship education and, based on the texts by Davies (2004) and Porto (2018), should again facilitate a discussion about the criteria for a successful integration of citizenship education with other subjects. Again, the students are asked to find suitable topics and learning goals that satisfy the principles of citizenship education and discussed subjects.

Building on this, the next session introduces the principles of mathematics education. The idea here was to progress further away from subjects that seem closely related to citizenship education, like history, and through discussing language learning and science education arrive at mathematics, a subject not often associated with citizenship education. During this process, students should continuously reflect on what central elements of citizenship education are and if or how they can be included into other subjects. The goal to this session is to see if and where the goals of mathematics education and citizenship education connect or even overlap. For this, a text by Ernest (2018) was chosen, which outlines principles and controversies of mathematics education. Other texts, for example by Ole

Skovsmose, would also be suitable, however the text by Ernest was chosen because it contains a fairly condensed overview of the field of mathematics education. The main task of this session is again to create a concept map and, if time permits, also compare it to the one created for citizenship education during the second session. The main goal of this task is again to help students better understand the required reading by visualising central elements and to create a reference point for future sessions. With regards to the connection to citizenship education, the relations of mathematics to society discussed by Ernest are especially fruitful.

Connected to this, the next session introduces modelling as a key element of mathematics education and again asks students to find suitable topics for interdisciplinary learning processes that cover both political and mathematical issues. The texts by Blum & Leiß (2007) and Maass et al. (2022) outline both the role mathematical modelling play in mathematics education and the benefit it can offer to citizenship education. The latter aspect will be further addressed in the next session, which introduces the process of normative modelling created in connection with this project. Here, texts by Niss (2015) and Pohlkmap and Heitzer (2021) highlight the societal relevance of mathematical modelling, the way normative assumptions are connected to mathematical modelling and the implication for teaching modelling. These texts can also be supplemented by materials provided within the CiviMatics-handbook or the text by Gildehaus and Liebendörfer (2021).

The final two sessions should combine all texts and materials discussed previously and give students the time and space to design a detailed lesson plan for a topic that connects elements of citizenship education with another subject. The choices can be limited by the lecturer, for example by specifically asking students to use citizenship education and mathematics, but this should depend on the academic background of your students. However, offering the students the opportunity to dive into topics and connections they find most interesting will most likely lead more engagement by the students.

Detailed Lesson Plans

Session 1

Session title	Introduction and organizational matters
Duration	45-60 Minutes
Goal of the session	This session should introduce the structure of the module and the requirements expected from the students to pass it. These requirements can of course be adapted, depending on the guidelines of your university. To pass the seminar, the students are required to hand in a short response (1/2 page) to a question or task handed every week. The relevant document will be uploaded one week before the session and will be due two hours beforehand. The content will always be related to the texts the students are expected to read as preparation or any given session. For Example, the next session will take place on a Monday (17.04.2023). After this session, a document will be uploaded to a learning platform or send out to the students via mail. This document will include the following task: *Describe the goal of citizenship education based on the texts by Himmelmann (2013) and Sander (2004).* The students will hand in an answer to this question one week later, two hours before the session (24.04.2023). To successfully pass this seminar, the students have to hand in a response to such questions every week on time. For their grades, students will have to write a term paper (12-15 pages) at the end of the course. The paper could be a theoretical paper comparing different didactic approaches, a practical paper aimed at developing learning materials or a meta-analysis of research connected to a certain research question. The topics should be related to the content of this course. The paper will be due at the end of the semester (in Germany usually two months after the course ended).
Theoretical framework	-
Activity in class	**Step 1 (20-30 minutes)** Open the class detailing the requirements described above. It can be beneficial to also roughly outline possible topics and the structure of the term paper as well as the requirements connected to the weekly tasks. The better the students know what will be expected from them, the better they will be able to use the contents of the following sessions to develop their research question and structure their paper. If you like you can also present the learning goals you set out for your students. **Step 2 (25-30 minutes)**

	After every question connected to the requirements are answered, also present the lesson plan and the required reading for every session. Try to highlight the common thread connecting the seminar session.
Required materials	You can use the PowerPoint-Presentation provided with the course materials or develop your own. There are no additional required materials.
Homework	Task for the next session: *Describe the goal of citizenship education based on the texts by Himmelmann (2013) and Sander (2004).*
References	Required reading for the next session: Himmelmann, G. (2013). Competences for Teaching, Learning and Living Democratic Citizenship. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 3–8). Sense Publishers. Sander, W. (2004). Incitement to freedom: competencies of political education in a world of difference. *The Development Education Journal*, *11*(1), 9–11.

Session 2

Session title	Citizenship education – Goals and competences		
Duration	90 minutes		
Goal of the session	This session should help students conceptualize the goals of citizenship education and guidelines connected to it.		
Suggested timetable	Time	Content	Materials
	10 min.	Introduction Use the task as a starting point for the session. The task was: *Describe the goal of citizenship education based on the texts by Himmelmann (2013) and Sander (2004).* Ask your students to summarize their findings and to point to the relevant sections of the text. If you want to facilitate a discussion, you can prepare additional questions for the students. Such as: *How does the goal you describe relate to the Beutelsbach Consensus?* (Sander, 2004, p. 9)	-Required reading

		How does Himmelmann relate his goals for citizenship education to his theory of democracy? (Himmelmann, 2013, p. 6) These questions should encourage students to relate certain aspects of citizenship education described by these texts, if they haven't done this already on their own in preparation for this session.	
	10 min.	<u>Concept-Mapping – How to do it</u> The next task adds to this by asking the students to create a concept map related to citizenship education. If your students are not familiar with concept mapping, you can give them a short introduction You can find suggestion for it in the appendix. The task for the students will be: *Based on the two texts you have read, prepare a concept map on the key elements of citizenship education*	PowerPoint 02
	35 min.	<u>Concept-Mapping – Working with the texts</u> The students should gather in small groups (3-5 students, depending on the size of your class) and start working on their concept map. Be sure to bring enough materials for everyone.	Flipchart paper Moderation cards Felt tip pens
	30 min.	<u>Comparing results and discussion</u> The concept maps are distributed in the seminar room. There are different ways to compare the results: • You could do a group puzzle connected to a gallery walk. The groups are rearranged so that every group has one student from each concept map in it. The groups walk throughout the classroom and describe their concept maps to each other • You could to individual presentations, where the students present their concept maps to the whole class Afterwards, there are again two different ways to use the results: • You can let the students discuss the differences of their maps, the emphasis	Pin board Pins Scotch tape

		they put on various issues and give additional insight based on the texts or your understanding of citizenship education • You can let the students reflect on the use of concept maps to visualize aspects of texts and gallery walks to compare results for their own practice as future teachers. Questions to facilitate such a discussion could be: ○ For what kind of content are concept maps suitable? ○ How can it be implemented in the classroom?	
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-
Theoretical framework	**Incitement to freedom** Sander (2004) frames the goal of citizenship education, or political education as he calls is, as an incitement to freedom. He states: "Normatively, political education is an *incitement to freedom*. It encourages people to live political freedom – for the success of political actions in independent societies does not only depend on abilities but also on the *courage* to put one's own mind and common sense to public use. In modern democracies, where human rights are granted, this appears to be an easy task – one that is incomparable to the courage it takes to resist in systems of dictatorship. But those who move beyond their immediate social surroundings and open themselves up to the complex public structure of modern societies will soon have experiences that require quite some learning, to be productively dealt with. Plurality and cultural difference confront the individual with the relativity of their own worldview. Higher authorities defining obligatory truths no longer exist – political decisions are to be made in conditions of incomplete and insecure knowledge. Therefore, difference also implies insecurity, and where the courage to manage this is lacking, counter-modernity might gain a foothold." Political education not only *encourages* people to live their political freedom, it also enables them to do so, mediating *competencies* that enable people to seize their rights in a self-assured way and with a chance to succeed. These		

<table>
<tr><td></td><td>
<blockquote>competencies are based upon the normative presumption of an orientation toward the principle of individual political autonomy (and therefore political freedom). Yet the competencies themselves are not to be seen as expectations of a certain behaviour, but as a set of abilities that individuals have free access to, that become visible through the progress of learning, and that are open to technical evaluation.” (p. 9)</blockquote>
Education for democratic citizenship
Himmelmann (2013) focuses on education for democratic citizenship and broad understanding of politics and democracy. He states:
<blockquote>“The changing use of terminology in the field of citizenship education correspond to the changes in concerns and concepts of citizenship – as conceived to meet actual and future challenges of democratic societies. Over and beyond the different vocabulary and approaches the focus of modern citizenship education has shifted from mere state-centered, nation-centered or even narrow political “instruction” to a broader “citizenship education,” more specially, to a new “education for democratic citizenship.
(…)
Instead of fostering passive and affirmative learning we find emphasis on active, social, cooperative and critical learning. Instead of call for more obedience and loyalty to the ruling powers, new concepts strive for experimental and practical, for social, moral and responsible self-government and participation of the learners in the society they belong to. Instead of nationalistic, patriotic, ethnic, tribal, racial or even mono-religious learning we find the call for intercultural and environmental education, of peace education, moral and social as well as media learning. Instead of accumulation and testing of mere knowledge, future citizenship education should stress equal efforts on (1) democratic knowledge and understanding, (2) democratic values, attitudes and common awareness. These competencies should be accompanied by (3) practical skills like problem solving, conflict solution, service learning, entrepreneurial or project learning and civic engagement.” (p.4) </blockquote>
</td></tr>
<tr><td>Required materials</td><td>-Laptop, projector or smart board, Power-Point
-Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape</td></tr>
<tr><td>Homework</td><td>Task for the next session:
Describe two important competencies of democratic citizens described in the texts by Print (2013) and Reinhardt (2013)</td></tr>
<tr><td>References</td><td><u>This session:</u></td></tr>
</table>

Co-funded by
the European Union

	Himmelmann, G. (2013). Competences for Teaching, Learning and Living Democratic Citizenship. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 3–8). Sense Publishers. Sander, W. (2004). Incitement to freedom: competencies of political education in a world of difference. *The Development Education Journal*, *11*(1), 9–11. <u>Next session:</u> Print, M. (2013). Competencies for Democratic Citizenship in Europe. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 37–50). Sense Publishers. Reinhardt, S. (2013). Teaching for Democratic Learning. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 99–110). Sense Publishers.

Session 3

Session title	The goal of this session is to familiarize the students with competences being discussed in connection with citizenship education.		
Duration	90 minutes		
Goal of the session	The students learn about what competences are and different dimensions of civic competences or democratic competences being discussed in the literature.		
Suggested timetable	Time	Content	Materials
	20 min.	<u>Introduction</u> You can use the task for this session as a starting point for a discussion. The task was: *Describe two important competencies of democratic citizens described in the texts by Print (2013) and Reinhardt (2013).* Again, use the first 15 minutes to ask your students to summarize their findings and to point to the relevant sections of the text. If you want to facilitate a discussion, you can prepare additional questions for the students. Such as: *Print discusses four key groups of active behaviours, while Reinhardt describes five central competences of democratic citizens. Which ones would you see as most important*	-Required reading

		for a "good" citizen? (Print, 2013, p. 40; Reinhardt, 2013, p. 100) *How can the teaching of citizenship education discussed by Reinhardt be related to these competences?* (Reinhardt, 2013, p. 104) These questions should encourage students to discuss the relevance of certain competences for active citizens in democracy.	
	40 min.	Priority game The next task builds on this by asking students to get together in groups and prioritize competences of democratic citizens. It is a decision-making game in which learners have to select a certain number of key statements, explanations, solutions, policies, etc. First, they select 10 out of a list of 30 on their own, then they narrow them down in a small group (3-5 students) to 5 essential statements. The tasks are the following: 1. *Prioritize what you consider to be the ten most important competencies of democratic citizens.* 2. *Discuss your priorities with your group and agree on five competencies of democratic citizens.* 3. *Based on the final five key statements, create your own definition a competent democratic citizen*	-PowerPoint 03 -Priority game worksheet (01_A)
	25 min.	Comparing results and discussion The students will visualize their priorities on a flipchart and present their results. Afterwards you use their different priorities to relate them to different types of citizenship (Print, 2013, 39 f.) or discuss with your students how to develop these competences through different educational settings and tasks. Connected to this, you can also use different ways of teaching citizenship education	-Flipchart paper -Felt tip pens

		(Reinhardt, 2013, p. 104) to create a rough unit addressing certain priorities with your students.	
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-
Theoretical framework	**Civic competence and active citizenship** Print (2004) describes competences, civic competences and active citizenship the following way: "A competence is a capacity, a potential and the ability or means to engage in a phenomenon. It refers to a complex combination of knowledge, skills, understandings, values, attitudes and desire which lead to effective, embodied human action in a particular domain." (...) The term "*civic competence*" refers to the knowledge, attitudes, values and skills needed for a participation in civic and political life, that is, necessary to be able to play the role of a citizen and that enables a person to become an active citizen. For Audigier (2000) "… the core competences associated with democratic citizenship are those called for by the construction of a free and autonomous person, aware of his rights and duties in a society where the power to establish the law, i.e. the rules of community life which define the framework in which the freedom of each is exercised, and where the appointment and control of the people who exercise this power are under the supervision of all the citizens" (p. 37 f.). What then might we see as being a 'good' citizen? Another way of addressing this is to identify key groups of 'active' behaviours: 1. Engage and participate in traditional political activities such as voting, joining political parties and being a candidate for election. 2. Engagement in the form of voluntary community activities. This might be working with welfare agencies such as a homeless shelter, collecting for charities or contributing to your local community clean-up. 3. Participating in activities and movements that seek to make changes to social and political directions. Mostly these are seen in		

	a positive sense such as signing petitions or joining a legal demonstration on a social issue. Some may be 'negative' or illegal such as illegal demonstrations or damaging property. 4. Participating in self-directing, beneficial behaviours such as financial self-sufficiency and creative problem- solving such as saving water in one's home or being energy efficient (p. 40) **Competencies of a democratic citizen** Reinhardt (2013) describes the following competences of a democratic citizen: "The competent democratic citizen is able to: 1) *Take others' perspective roles*: The views and expectations of others, also of the generalized other, are seen and integrated. 2) *Handle conflicts*: Conflicting interests, values and identities are approached with tolerance and "resolved" responsibly. 3) *Use social sciences*: Institutions, structural frameworks and individual actions in society (e.g. in politics, economy, law and other partial systems) are analyzed by employing social sciences. 4) *Use moral and political reasons*: Judgements on political issues need two sorts of criteria, those referring to the functioning of the political system and those referring to individual and / or collective terms of morals/ethics. 5) *Participate in democracy*: Everyday face-to-face life, work life, civil society and the over-all democratic state give the opportunity for and are dependent on the participation of citizens. These competencies break the idea of the competent democratic citizen down and render a more precise and concrete notion to our thinking about teaching and learning democracy." (p.100)
Required materials	• Laptop, projector or smart board, Power-Point • Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape
Homework	Task for the next session: *According to Edelstein (2011) and Solhaug (2018), what are central democratic practices "democratic schools" should implement?*
References	<u>This session:</u> Print, M. (2013). Competencies for Democratic Citizenship in Europe. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 37–50). Sense Publishers. Reinhardt, S. (2013). Teaching for Democratic Learning. In M. Print & D. Lange (Eds.), *Civic education and competences for engaging citizens in democracies* (pp. 99–110). Sense Publishers.

	Next session: Edelstein, W. (2011). Education for Democracy: reasons and strategies. *European Journal of Education*, 46(1), 127–137. https://doi.org/10.1111/j.1465-3435.2010.01463.x Solhaug, T. (2018). Democratic Schools – Analytical Perspectives. *JSSE - Journal of Social Science Education*, 17(1), 2–12. https://doi.org/10.4119/jsse-858

Session 4

Session title	In this session, the students should learn about the whole school approach of citizenship education, which focuses on democratic participation and democratic practices in schools.		
Duration	90 minutes		
Goal of the session	The students get know approaches that highlight the necessity of a whole school approach when it comes to citizenship education and focus on democratic participation and democratic practices in schools.		
Suggested timetable	Time	Content	Materials

	30 min.	Introduction
		Again, the question given to the students at the end of the last session can be used as an introduction to this session:
		According to Edelstein (2011) and Solhaug (2018), what are central democratic practices "democratic schools" should implement?
		
		However, this time it may be beneficial to start with a more in-depth discussion of the texts and the elements of democratic schools they are presenting. Questions for a moderated discussion could be:
		
		• What is Edelstein's (2011, p. 127 f.) diagnosis of society?
		    ○ Do you agree with it?
		• What are the practices Edelstein (2011, p. 131 f.) connects to democratic schools?
		    ○ Do you agree with them?
		    ○ Do you see potential issues?
		        ▪ E.g. students taking regressive political stances
			-Required reading
			-PowerPoint 04

		as part of their civic engagement • Do you agree with Solhaug's (2018, p. 8) table detailing elements of democratic schools? 　○ Which ones are most important for you? 　○ Is something missing? These questions can also be integrated into separate tasks and/or methods like: 　○ A priority game 　○ Think/pair/share approaches The goal of these questions is to facilitate a discussion about the merits of democratic-school-approaches and help students articulate their (implicit) priorities connected to educating "democratic" citizens.	
	35 min.	<u>Democratic schools – theory and practice</u> The next task builds on this by again assigning the students to small groups (3-5) and ask them to design a lesson or a teaching unit that incorporates essential elements of a democratic schools. You can either design a lesson in a subject related to citizenship education or of any other subject you are studying to become a teacher (e.g. how could a unit in mathematics incorporate elements of democratic school culture?). Students can address content, social forms as well as teacher behaviour.	-Flipchart paper -Felt tip pens
	20 min.	<u>Comparing results and discussion</u> The students will visualize their lessens and present their results. Again, you can use different social forms for the comparison, such a quick presentation or a gallery walk. During or after the presentation phase, you can discuss the following questions with your students: • Are certain subjects better suited then others to incorporate elements of a democratic school culture (e.g. service learning)	-Pinboard -Pins -Scotch tape

<table>
<tr>
<td rowspan="2"></td>
<td></td>
<td>Which elements go beyond the level of individual lessons and have to be addressed by the whole school
Can you identify shortcomings or do you take issues with the democratic-schools-approach?</td>
<td></td>
</tr>
<tr>
<td>5 min.</td>
<td><u>Open questions</u>
Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven’t been addressed during the session.</td>
<td>-</td>
</tr>
<tr>
<td>Theoretical framework</td>
<td colspan="3">Democratic schools as an answer to current crises
Edelstein (2011) describes current crisis situation as a reason to focus more on democratic schools. He writes:

<blockquote>“The importance of learning democracy in school is linked to the present crisis, a crisis that presents the system in which we live with perilous challenges and risks, for which both governments and citizens in general are ill prepared. Beyond the recent crisis of the financial system that determines the present social and economic experience and the political disillusionment of millions worldwide, political scientists such as Herfried Münkler in Germany (Münkler & Wassermann 2008) and Colin Crouch in England (2004) have identified serious threats to the very foundations and basic components of democratic systems: the corrosion, as Münkler calls it, of the sociomoral resources of democracy.” (p. 127.)</blockquote>
The main three tasks of democratic schools connected to this are as follows:

<blockquote>When schools are called upon to provide the socio-moral resources required to develop the democratic habits that serve to maintain a democratic system of government in the future, they are also called upon to design the kind of educational lifeworld that is conducive to the democratic empowerment of the children who attend them in the present. Schools are called upon to provide for a democratic form of life. In fact, ‘learning democracy’ is not a single task with a well-defined outcome. Rather, it consists of a variety of different yet interconnected tasks:
(a) learning about democracy in order to become a knowing and conscious democratic actor in (future) situations of social and political choice and decision (Rawls, 1971);
(b) learning through democracy by participating in a democratic school community, and thus acquire sustainable democratic habits (Dewey, 1963; 2004);</blockquote></td>
</tr>
</table>

> (c) learning *for* democracy, including the construction and ongoing development of democratic forms of life, based on cooperation and participation in local, national and transnational contexts (Himmelmann, 2007). (p. 130)

He also identifies three main categories for social-educational practices of a democratic school:

> The seemingly unlimited variety of socio-educational practices can be subsumed under three major headings: *(a) democratic self-government; (b) social projects; and (c) civic engagement*. Several have been identified and illustrated by case reports in the publication on citizenship education in Europe ('Schools for society. Learning democracy in Europe') by Susanne Frank and Ted Huddlestone — a project promoted by ILDE (Initiative for Learning Democracy in Europe) of the Network of European Foundations and supported by the Freudenberg Foundation in Germany and the Citizenship Foundation in London. I shall briefly describe the types of democracy-enhancing action indicated by the three categories above: classroom council as a prototype of democratic self-government; service learning as a prototype of the democracy enhancing social project; and volunteering as a prototype of civic engagement. (p. 131)

Competencies of a democratic citizen

Sollhaug (2018, p. 6) lists summarizes analytical aspects of a democratic school:

Short name	**Elaborations**
Democratic participation	Democratic participation in schools may vary according to its breadth (the number and relevant participants, its depth (the qualities of participation) and its range (the subject matters which is to be decided on).
Schools as institutions	Institutions have their formal regulation, the norms governing practice, and culture which reflect a degree of democratic practice. Democratic schools have a supportive regulatory legal framework, norms which support school democracy, and a culture which support an inclusive democratic practice.
Knowledge	Democratic schools provide teaching and learning processes which support students'

		knowledge construction, social learning, and citizenship practices. Democratic schools acknowledge that a diversity of students have diverse life experiences which need to be present in the process of learning.
	Student council	Democratic schools have effective student councils, which provide opportunities for student participation and being critical of issues of importance to their schooling
	Democratic values and virtues- Freedom	It is being argued that the more freedom students are given participate in school, the more democratic the school is.
	Equity	Building on the concept of equity, I argue that the more students experience equity in their schooling, the more democratic the school is.
	Tolerance	It is argued that political tolerance is a necessity for democratic practice, and consequently, schools where students and teachers show great tolerance for diverse views and behaviour are more democratic than schools which have less tolerant students and teachers.
	Solidarity	Schools with teachers and students who practice a culture of solidarity will experience more support for their school work and will be more democratic than schools with a less solidarity.
	Protection	Schools, which provide effective protection of its students, are more democratic than schools, which provide less effective protection.
	Inclusiveness	Schools, which have a practice of inclusiveness in schooling along with the criteria for inclusiveness mentioned above, will be more
Required materials	• Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape	
Homework	Task for the next session: *Summarize the reasons Syed (2013) gives for why citizenship education should be taught through other subjects.*	
References	This session:	

	Edelstein, W. (2011). Education for Democracy: reasons and strategies. *European Journal of Education*, 46(1), 127–137. https://doi.org/10.1111/j.1465-3435.2010.01463.x Solhaug, T. (2018). Democratic Schools – Analytical Perspectives. *JSSE - Journal of Social Science Education*, 17(1), 2–12. https://doi.org/10.4119/jsse-858 Next session: Syed, G. K. (2013). How Appropriate is it to Teach Citizenship through Main Curriculum Subjects? *Citizenship, Social and Economics Education*, 12(2), 136–142. https://doi.org/10.2304/csee.2013.12.2.136

Session 5

Session title	Interdisciplinary citizenship education – Possibilities and limitations		
Duration	90 minutes		
Goal of the session	Based on the content of the past sessions, the students should discuss the necessities of a separate subject for citizenship education (e.g. relating to the competences of a democratic citizen covered two sessions ago) and start connecting content of citizenship education to other subjects		
Suggested timetable	Time	Content	Materials
	20 min.	Introduction Again, the question given to the students at the end of the last session can be used as an introduction to this session: *Summarize the reasons Syed (2013) gives for why citizenship education should be taught through other subjects.* Also, since this text comes to a controversial conclusion ("that it is more appropriate to teach citizenship through other subjects than to limit it to one statutory subject"), it may again be beneficial to start with a more in-depth discussion of the text and relate it to the texts covered in the previous sessions. Questions for a moderated discussion could be: *Syed argues that "is more appropriate to teach citizenship through other subjects". Do you think that competences of democratic citizen's outlined by Print (2013) and Reinhardt (2013) can also be attained in other subjects?*	-Required reading -PowerPoint 05

Co-funded by the European Union

		◦ *If yes, which are appropriate for other subjects and which aren't?* ◦ *Can the whole school approach of democratic schools and the approach suggest by Syed cover all necessary competences of democratic citizens?* *Syed warns that "limiting citizenship to one subject" could lead to "indoctrination".* (Syed, 2013, p. 140) ◦ *Do you feel like such a concern is valid?* ▪ (You could use the Beutelsbach Consensus as a reference) *Are you in favour of citizenship education as a separate subject or as a topic which is taught through other subjects?* ◦ *Which advantages or disadvantages do you see?* ◦ *Can there be a best of both worlds? If so, how?* The last question can be used as a transition to the next task.	
	45 min.	<u>Working with the curriculum</u> This task will depend on your national context and the way citizenship education is taught. In Germany, there are different subjects connected to citizenship education, varying between state and types of school, as well as an effort to additionally establish a whole-school-approach. Thus, there are curricula of a standalone subject of citizenship education which can be used for this task. If the situation in your country is different, you can use related curricula and assign covering certain subjects or topics beforehand. One example could to assign (2-5) groups, depending on the various types of citizenship education in your county and the type of teaching degree your students aspire to, and hand out curricula detailing the topics and competences students should	-School curricula Possibly: -Flipchart paper -Felt tip pens

		attain after finishing a certain type of school. The students are then asked to study the curricula and find links to other subjects (in the best case the one they are also studying as part of their teaching degree). The task is to look for possibilities to teach certain topics in other subjects and come up with one example for a lesson. The tasks are the following: ○ Study the curriculum and find at least • a) one topic which you could also teach in another subject and • b) one topic (if possible) that you would classify as impossible to cover as part of another subject. ○ Come up with an idea for a lesson connected to a) which would combine competences and learning goals from both the original subject and citizenship education. ○ Give an explanation for why you think the topic of b) needs its own subject to address it properly.	
	20 min.	Comparing results and discussion Compare the results of the group work. You can again ask the students to visualize it, however this may make the comparison a bit time consuming. Ask the other groups to give feedback after one group presented their idea. Also, encourage critical remarks regarding both a) and b). Can the idea really address competences from both subjects? Is the topic really not suitable for another subject?	Possibly: -Pinboard -Pins -Scotch tape
	5 min.	Open questions Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session. If the discussion if fruitful, feel free to skip this part as you see fit.	-
Theoretical framework	**Citizenship education through other subjects** The main aim of Syed's (2013) article is the following: "In this article, the author argues that it is more appropriate to teach citizenship through other subjects than to limit it to one statutory subject.		

> The author's argument is based on three important issues. The first issue is that citizenship, being an aim of education rather than a means, deserves to be part of the whole curriculum. This is to say that citizenship is one of the most important purposes that education should serve, and thus this purpose of education should not just be limited to one way of achieving it, like an ordinary subject. The second argument is that citizenship should not be confined to the limits of one subject, as it is much more beneficial for the nature of the subject to be taught in collaboration with different subjects, such as history, English, science and geography." (p. 136)

She further describes here reasons the following way:

> These young citizens need to be equipped with the proper insight to carry on the citizenry. For this purpose, all education that they acquire should be infused with the complex issues of citizenship. Brett (2005) defines 'education for citizenship' to be 'equipping students with a set of tools which will enable them to participate effectively, actively and responsibly within their adult life', while education is meant 'to help prepare people to live a better individual life and also to contribute to the improvement of wider society (locally, nationally and globally)' (Davies et al, 2002, p. 113)." (p. 137).

Her arguments are based on studies outlining the benefits of incorporating topics of citizenship education in other subjects:

> There is ample literature to suggest links between the main curriculum subjects and citizenship. History, as Crick suggests, has the biggest role to play in teaching citizenship (foreword, in Arthur et al, 2001). The Crick report noted that the 'topical and contemporary issues' which may be termed the 'lifeblood' of citizenship can be dealt with in reference to evidence from history (Qualifications and Curriculum Authority, 1998). Concepts such as power, freedom, equality, democracy and racism, which are central to the debates of citizenship, can be explored explicitly through all subjects, especially history (Brett, 2005). Links can be found between teaching citizenship and geography (Lambert & Machon, 2004). Pykett (2012) argues that the issue of identity can be taught and explained best through geography, as citizenship is determined usually through geographical boundaries. Explicit interrelated teaching will help towards a better understanding of such issues on the part of learners. English as a subject has always been used for social purposes (Spurgeon, 1995). In the 1990s in England, racism emerged as a radical issue, and the use of George Orwell's Animal Farm, a novel against racism, in the English curriculum ensured the necessary education regarding this clearly citizenship debate

	(Spurgeon, 1994). Moreover, there is great potential for possible collaboration between citizenship education and science education (Davies, 2004; Levinson, 2010). 'The role of nature and technology as agents in democratic processes' (Leach et al, 2005, p. 12) cannot be ignored and has to be made evident to learners through unequivocal education for citizenship in alliance with science. (p. 138) However, she also acknowledges possible challenges (p. 139 f.), such as: The first challenge to teaching citizenship through all subjects is a misunderstood conception of the cross-curricular thematic approach to citizenship. It is not the mere similarity of its content with any main subjects (Counsell, 2002, cited in Brett, 2005), as it is commonly perceived to be, but rather an infusion that provides room for the exploration of complex ideas relevant to citizenship debates (see also Brett, 2005). A 2003 Ofsted report shows a great misunderstanding on the part of a history department, taking work on suffragettes to be an example of work on democracy without a proper exploration of democratic concepts. This can be resolved with proper professional training (see Foster et al, 2005). (p. 139)
Required materials	• School curricula • Flipchart paper, felt tip pens
Homework	Task for the next session: *What areas of overlap between citizenship education and history or geography do the texts by Barton (2017) and Thornton (2018) identify?*
References	<u>This session:</u> Syed, G. K. (2013). How Appropriate is it to Teach Citizenship through Main Curriculum Subjects? *Citizenship, Social and Economics Education*, 12(2), 136–142. https://doi.org/10.2304/csee.2013.12.2.136 <u>Next session:</u> Barton, K. C. (2017). Shared Principles in History and Social Science Education. In M. Carretero, S. Berger, & M. Grever (Eds.), *Palgrave Handbook of Research in Historical Culture and Education* (pp. 449–467). Palgrave Macmillan UK. https://doi.org/10.1057/978-1-137-52908-4_24 Thornton, S. J. (2018). Geography as a Social Study: Its Significance for Civic Competence: Its Significance for Civic Competence. In E. E. Shin (Ed.), *Spatial Citizenship Education: Citizenship Through Geography* (pp. 10–21). Routledge.

Session 6

Session title	Interdisciplinary citizenship education – Social studies		
Duration	90 minutes		
Goal of the session	The goal of this session is for the students to familiarize themselves with approaches and concepts from other subjects (history and geography) and how these approaches can be related to concepts of citizenship education.		
Suggested timetable	Time	Content	Materials
	20 min.	<u>Introduction</u> You can again use the question the students were asked to answer as preparation fort this session as a starting point. The task was: *What areas of overlap between citizenship education and history or geography do the texts by Barton (2017) and Thornton (2018) identify?* You can also use the different areas mentioned, especially the ones described by Barton, to facilitate a discussion about the overlap between their understanding of central areas and competences of citizenship education (e.g. the ones discussed by Print (2013) and Reinhardt (2013)). You can also connect this topic to the last session and ask the question if these areas can replace citizenship education as a subject to a degree or if they could solely be seen as an addition. This also works the other way around: How can they be implemented into regular citizenship education lessons. These questions should encourage students to discuss overlaps between citizenship education and other subjects, which can be used as an introduction for the next task.	-Required reading -PowerPoint 06
	40 min.	<u>Problem orientation (see Reinhardt, 2013, S. 104f.)</u> The task for students for this session is to come up with a societal or political problem that can be analysed from a political, historical or geographical perspective. This problem can be used to design a lesson connected to certain competences of citizenship education and geography/history or all three combined. The students should be encouraged	Possibly: -Flipchart paper -Felt tip pens

		to think about a problem which they could use to design a lesson that incorporates learning process attributed to citizenship education and/or history (e.g. with regards to knowledge, analytical competences or judgements). Such an example could be: • Political disputes between Germany and Poland about further reparations • Refugee routes and European refugee policies • Influence of colonialism on contemporary border conflicts and international peacekeeping possibilities This task is again best suited for working in small groups. The tasks are the following: • *Find a political or societal problem that can be used for a lesson which combines citizenship education with geography and/or history* • *Outline the aspects that are suited to convey historical/political/geographical knowledge or would help students attain relevant competences.*	
	25 min.	<u>Comparing results and discussion</u> Compare the results of the group work. You can again ask the students to visualize it as part of the previous task, however this may make the comparison a bit time consuming. Ask the other groups to give feedback after one group presented their idea and encourage a discussion about its feasibility.	Possibly: -Pinboard -Pins -Scotch tape
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-

Theoretical framework	**Shared principles in history and social science education** Barton (2017) describes identities a number of areas overlap between history and social science education. According to him, these areas are "perspective", "causation", "agency", "evidence" and "concepts" Although he acknowledges that subjects compete for curricular space and interdisciplinary approaches can be seen as something which takes away from one subject and benefiting another, he also highlights the relevance connecting different subjects: "A In the context of general education at the primary and secondary levels, however, emphasizing these differences may have the unintended consequence of impeding students' understanding of each subject by failing to capitalize on areas of overlap and similarity (Thornton & Barton, 2010)." (p. 450) As an example for one area, he describes the relevance of "perspective" the following way: "Perspective, also known as empathy, is one of the most widely discussed elements of historical thinking (e.g. Barton & Levstik, 2004; Endacott & Brooks, 2013; Knight, 1989; Lee & Ashby, 2001), even though it may not receive as much classroom attention as many educators believe that it should. In order to understand past social structures, as well as the actions of people in history, students must understand how people at the time saw the world; they need to recognize the values, attitudes and beliefs that motivated people in a given period, rather than thinking that they shared the same perspectives as people today. (p. 450) (…) Students must come to understand, then, that in studying any society—past or present—they must attend to both the existence and influence of societal perspectives, as well as to be aware of how those perspectives are characterized by diversity, change and power relations. This idea is not unique to any one discipline, and systematic and coordinated attention to this topic across sub jects would help students better recognize the need to take perspective into account." (p. 453) **Geography as a social study** Thornton (2018) connects citizenship education and geography by using broad definitions of "citizenship" and "civic competences" (p. 11). He refers to the approaches of Lucy Sprague Mitchell as an example for a practical connection: "No educator has better exemplified how this "enhancement" might be accomplished than Lucy Sprague Mitchell (1991). Here I will focus on geography in elementary education although these methods are readily

	adaptable for older students. Mitchell was concerned that schoolchildren live in a world of "end-products . . . removed from the forces which determine the functioning" of the environment they inhabit (p. 6). Mitchell saw active geographic inquiry as the remedy. By "endproducts" she meant, for example, that with the food they consumed schoolchildren lacked understanding of how food moved farm to table. The educational explorations Mitchell recommended might include visits to a local food distribution warehouse so children could "develop the habits of firsthand observation and experimentation and the attendant 'relationship thinking'" (p. 7)." (p. 14).
Required materials	• Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape
Homework	Task for the next session: *What criteria Davies (2004) and Porto (20218) reference for successful integration of citizenship education with other subjects?*
References	This session: Barton, K. C. (2017). Shared Principles in History and Social Science Education. In M. Carretero, S. Berger, & M. Grever (Eds.), *Palgrave Handbook of Research in Historical Culture and Education* (pp. 449–467). Palgrave Macmillan UK. https://doi.org/10.1057/978-1-137-52908-4_24 Thornton, S. J. (2018). Geography as a Social Study: Its Significance for Civic Competence: Its Significance for Civic Competence. In E. E. Shin (Ed.), *Spatial Citizenship Education: Citizenship Through Geography* (pp. 10–21). Routledge. Next session: Davies, I. (2004). Science and citizenship education. *International Journal of Science Education*, 26(14), 1751–1763. https://doi.org/10.1080/0950069042000230785 Porto, M. (2018). Intercultural Citizenship Education in the Language Classroom. In I. Davies, L.-C. Ho, D. Kiwan, C. L. Peck, A. Peterson, E. Sant, & Y. Waghid (Eds.), *The Palgrave Handbook of Global Citizenship and Education* (pp. 489–506). Palgrave Macmillan UK. https://doi.org/10.1057/978-1-137-59733-5_31

Session 7

Session title	Citizenship education – Other subjects		
Duration	90 minutes		
Goal of the session	In this session, the students should learn about the connection between citizenship education and other subjects, such as science or language education.		
Suggested timetable	Time	Content	Materials
	20 min.	<u>Introduction</u> Use the task as a starting point for the session. The task was: *What criteria Davies (2004) and Porto (2018) reference for successful integration of citizenship education with other subjects?* Similar to last session, you can use the list of criteria mentioned in both text (Davies, 2004, p. 1760; Porto, 2018, p. 493 f.) as a starting point and compare them to central areas and competences of citizenship education (e.g. the ones discussed by Print (2013) and Reinhardt (2013)). Similarly, you can use these lists, or parts of them, to start a discussion about topics or learning goals for citizenship education as well as issues the students take with them. For example, you can connect the third point of an intercultural citizenship project mentioned by Porto (p. 498) to discuss issues with facilitating community engagement of students discussed during the session covering democratic schools. Similarly, students can think about how to incorporate activities asking students to "explain their views, their understandings and their arguments" (Davies, 2004, p. 1760) in a lesson about a science-related topic. These questions should again encourage students to discuss overlaps between citizenship education and the subjects covered in these texts, which can be used as a transition to the next task.	-Required reading -PowerPoint 07
	40 min.	<u>Lesson plan – Finding suitable topics and learning goals</u> The task for students for this session is similar to last one and can again be related to the didactic principles	Possibly: -Flipchart paper -Felt tip pens

		and teaching methods outlined by Reinhardt (2013, p. 104). For example, you could let your students focus on case orientation and let them design a case study or on political-moral orientation with the development of a dilemma method, and let them develop ideas for interdisciplinary learning goals and lesson plans. Since the subjects in this session seem to be "further removed" from citizenship education and learning goals are not as easy to align as with history or geography, the focus of this session should be on encouraging the students to find suitable topics and develop learning goals for students that would be appropriate for citizenship education as well as language learning or science education. This task is again well suited for group work and can be visualised in various ways. If you plan to include a presentation phase or a gallery walk, you may need to adjust the time reserved for the presentation phase accordingly. A task for the students could look like this: • *Get together in small groups and come up with a topic which could cover a lesson in citizenship education and science education or language learning* • *Formulate learning goals for the students which would be appropriate for both subjects*	
	25 min.	<u>Comparing results and discussion</u> Compare the results of the group work. Again, ask the other groups to give feedback after one group presented their idea and encourage a discussion about its appropriateness and the achievability of the learning goals.	Possibly: -Pinboard -Pins -Scotch tape
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-
Theoretical framework	**Science and citizenship education**		

Davies (2004) describes the necessity for connecting science and citizenship education the following way:

> "First, the issues that face citizens today often involve, or appear to involve, some scientific knowledge. Genetically modified food, sources of energy, and genetic engineering all have a high profile. There is widespread recognition that the issues reported in the popular and 'quality' media (for example, Pringle 2003) provide a need and, perhaps, opportunities to develop a valuable form of scientific understanding." (p. 1751)

He also identifies a number of challenges that have to be overcome to achieve this. One example is the following:

> "My second point about the need to overcome challenges between science and citizenship relates to those who seem to claim, even if only by omission, that science is a rather narrow academic pursuit with little need for elaboration about the connections with the social and political." (p. 1755 f.).

He also emphasizes that lessons connecting both subjects have to address central aspects of citizenship education:

> It is, of course, necessary to ensure that these goals are actually related to key aspects of citizenship. I would not want to pretend that as soon as students begin to use evidence, for example, that they are automatically 'doing' citizenship (they could simply be participating in a science experiment). Any classroom materials would probably need to be based around a key concept (inequality or justice or identity or others that related directly to the nature of society) and linked explicitly to a contemporary issue. These materials and activities would need to be designed to encourage students:
>
> - to explain their views, their understandings and their arguments;
> - to tolerate, accommodate, include and reflect upon opinions and views that may be different from their own; and to participate in the consideration and debate of these ideas in the classroom
> - and (ideally) use this experience and understanding in their life outside school. (p. 1760)

Intercultural citizenship education in the language classroom

	Porto (2018) presents an approach highlighting the integration of certain aspects of citizenship education into the language learning classroom: "In this view, language learning is pushed beyond the linguistic and the communicative, and also beyond the instrumental (learning a language for communication, employment, etc.), toward an intercultural citizenship perspective that connects the foreign language classroom with the community, whether local, national, regional, and/or global. Inter cultural citizenship becomes the content of foreign language education." (p.490) Referring to Byram et al. (2017), she highlights three characteristics of an intercultural citizenship project in the language classroom: Pedagogically, Byram et al. (2017) outline the following characteristics of an intercultural citizenship project in the language classroom: (1) citizenship is the content of language lessons through the inclusion of themes of social relevance (the environment, ecology, languages, peace and conflict, diversity, linguistic and other rights, sustainability, poverty, hunger, etc.); (2) students who speak different native languages engage with others in intercultural communication in a transnational project and develop a sense of community called "transnational identification"; (3) students engage in criticality and reflexivity not only at the level of thought but also by taking action in the community (at local, national, regional or global levels).
Required materials	• Laptop, projector or smart board, Power-Point • Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape
Homework	Task for the next session: *Describe the goal of mathematics education based on the text by Ernest (2018).*
References	This session: Davies, I. (2004). Science and citizenship education. *International Journal of Science Education*, 26(14), 1751–1763. https://doi.org/10.1080/0950069042000230785 Porto, M. (2018). Intercultural Citizenship Education in the Language Classroom. In I. Davies, L.-C. Ho, D. Kiwan, C. L. Peck, A. Peterson, E. Sant, & Y. Waghid (Eds.), *The Palgrave Handbook of Global Citizenship and Education* (pp. 489–506). Palgrave Macmillan UK. https://doi.org/10.1057/978-1-137-59733-5_31

	Next session: Ernest, P. (2018). The Philosophy of Mathematics Education: An Overview. In P. Ernest (Ed.), *The Philosophy of Mathematics Education Today* (pp. 13–38). Springer International Publishing.

Session 8

Session title	Mathematics education – Philosophy		
Duration	90 minutes		
Goal of the session	Moving on to another subject, which will be covered more in depth than the ones beforehand, the students will get to know the principles and the philosophy of mathematics education as a basis for the following sessions.		
Suggested timetable	Time	Content	Materials
	10 min.	Introduction Use the task as a starting point for the session. The task was: *Describe the goal of mathematics education based on the text by Ernest (2018).* Since the text by Ernest is quite dense and it is likely that a number of your students are not familiar with mathematics education, it may be beneficial to either start with a quick discussion of the task for this session or with just answering any questions that came up for your students while reading the text. It may be better to discuss further issues during the concept mapping phase or afterwards.	-Required reading -PowerPoint 08
	50 min.	Concept-Mapping – Working with the text The next task is similar to the one given in the second session, since your students should also create a concept map covering the philosophy of mathematics education. Your students should be familiar with the concept, but you can again give them a reminder of what they are supposed to do by using the PowerPoint-Slides from the second session. The task for the students will be: *Prepare a concept map on the philosophy of mathematics education based on the text by Ernest (2018)*	-Flipchart paper -Moderation cards -Felt tip pens

	25 min.	<u>Comparing results and discussion</u> The concept maps are distributed in the seminar room. This time, there are three different ways to use the results. Similar to last time: • You could do a group puzzle connected to a gallery walk. The groups are rearranged so that every group has one student from each concept map in it. The groups walk throughout the classroom and describe their concept maps to each other • You could to individual presentations, where the students present their concept maps to the whole class But you could also use the concept maps from the third session and ask the students to compare their maps of citizenship education with the maps of mathematics education. Ask them point out similarities and discuss possible connection when it comes to classroom practice. If you chose one of the first two options, there are again two different ways to use the results: • You can let the students discuss the differences of their maps, the emphasis they put on various issues and give additional insight based on the texts or your understanding of citizenship education • You can let the students reflect on the use of concept maps to visualize aspects of texts and gallery walks to compare results for their own practice as future teachers. Questions to facilitate such a discussion could be: o For what kind of content are concept maps suitable?	-Pin board -Pins -Scotch tape
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-

Theoretical framework	**Philosophy of mathematics education** Ernest (2018) understands a philosophy of mathematics education the following way: "Understood in its simplest sense mathematics education is about the practice or activity of teaching mathematics. The philosophy of some activity or domain is its aim, rationale or underlying purpose. So the simplest sense of 'philosophy of mathematics education' concerns the aim or rationale behind the practice of teaching mathematics. This issue is a vitally important one, central to the philosophy of mathematics education, as well as to mathematics education as a whole. The purpose of teaching mathematics also implicates the aim of learning mathematics, because learning is inseparable from teaching, although they can be conceived of separately. In practice an active teacher presupposes one or more learners, and only in pathological situations can one have teaching without learning. However, the converse does not hold, for informal learning is often self-directed and takes place without explicit teaching." (p. 14) (…) "So the philosophy of mathematics education should also be understood to include the application of philosophical concepts and methods, such as a critical attitude to claims as well as detailed conceptual analyses of the concepts, theories, methodologies or results of mathematics education research, and mathematics itself (Ernest, 1998; Skovsmose, 1994)." (p. 15) To clarify these statements, Ernest introduces different topic areas and with it five questions related to the philosophy of mathematics education: • Question 1: What Is Mathematics? • Question 2. How Does Mathematics Relate to Society? • Question 3: What is Learning and Learning Mathematics, in Particular? • Question 4. What is Teaching and Teaching Mathematics, in Particular? • Question 5: What is the (Philosophical) Status of Mathematics Education as Knowledge Field? Each of these questions comes with a description of essential controversies. For example, controversy number five relates to the association of mathematics education with mathematical and educational science: "**Controversy 5:** Should mathematics education, as a university discipline, be accommodated within education departments or mathematics departments? Different countries answer this in different ways, and are not

	always fully consistent within themselves. This question matters, for location in a mathematics department within a scientific faculty often brings significantly better resourcing than housing in an education department, within a social science faculty. However, in some traditional mathematics departments mathematics specialists are looked down upon as not being 'real mathematicians', whereas in many education departments mathematics educationists are on a par with their education colleagues." (p. 23) Furthermore, Ernest (2018) also provides and analysis of research and theories in mathematics education: "The questions listed above interrogate and problematise the practices of teaching and learning mathematics and related issues from a low- or non-theoretical perspective. Starting from questions in this way represents a 'bottom up' introduction to the scope and nature of the philosophy of mathematics education. Simply speaking, this is putting practice before theory. In contrast, a 'top down' approach can use the branches of philosophy to provide conceptual frameworks for analysing philosophical concerns in research in mathematics education. In what follows, research and theories in mathematics education are analysed according to the branches of philosophy they draw upon, including metaphysics and ontology, epistemology, social and political philosophy, ethics, methodology, and aesthetics. (p. 25)" These constitute a top-down perspective on the philosophy of mathematics education, while the first part represents a bottom-up perspective. Ernest (2018) summarizes this the following way: From the bottom-up perspective one can characterize the area in terms of questions such as: What are the aims and purposes of teaching and learning mathematics? What is mathematics? How does mathematics relate to society? What is learning mathematics? What is mathematics teaching? What is the status of mathematics education as knowledge field? Using a 'top down' perspective the field can be characterised based on the branches of philosophy involved. Looking briefly into the contributions of ontology and metaphysics, aesthetics, epistemology and learning theory, social philosophy, ethics, and the research methodology of mathematics education reveals both how rich and deep the contributions of philosophy are to the theoretical foundations of our field of study. (p. 33)

Required materials	• Laptop, projector or smart board, Power-Point • Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape
Homework	Task for the next session: *What are the steps of a modelling cycle and how can modelling be related to citizenship education?*
References	This session: Ernest, P. (2018). The Philosophy of Mathematics Education: An Overview. In P. Ernest (Ed.), *The Philosophy of Mathematics Education Today* (pp. 13–38). Springer International Publishing. Next session: Blum, W., & Leiß, D. (2007). How do Students and Teachers Deal with Modelling Problems. In P. Galbraith, W. Blum, S. Khan, C. Haines, & C. R. Haines (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics; proceedings from the twelfth international conference on the teaching of mathematical modelling and applications* (pp. 222–231). WP Woodhead Publ; Horwood. Maass, K., Artigue, M., Burkhardt, H., Doorman, M., English, L. D., Geiger, V., Krainer, K., Potari, D., & Schoenfeld, A. (2022). Mathematical modelling – a key to citizenship education. In N. Buchholtz, B. Schwarz, & K. Vorhölter (Eds.), *Initiationen mathematikdidaktischer Forschung* (pp. 31–50). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-36766-4_2.

Session 9

Session title	Mathematics education – Mathematical Modelling		
Duration	90 minutes		
Goal of the session	Connected to the last session, the students will get to know one key element of mathematics education, mathematical modelling, and how it can be related to citizenship education.		
Suggested timetable	Time	Content	Materials
	25 min.	Introduction You can use the question for this session as a starting point. The task was: *What are the steps of a modelling cycle and how can modelling be related to citizenship education?*	-Required reading -PowerPoint 09

		However, it may also be beneficial to use a short PowerPoint presentation to explain the steps and the logic of a mathematical modelling cycle to the students. Based on this, you can discuss the second part of the question, the relation to citizenship education. This introduction should set the stage for the main task of this session, which will ask the students to find a problem from the field of citizenship education which involves mathematical models and think about ways both sides of the problem, the political side and the mathematical side, can be connected in an interdisciplinary lesson. These ideas can be used for the next session, where the students are asked to design a lesson based on the normative modelling cycle.	
	35 min.	<u>Topics suitable for mathematical modelling and citizenship education</u> The task for this session is to find suitable topics for interdisciplinary learning, which cover both political and mathematical issues. Again, it may be beneficial to ask the students to work in small groups. As the text by Maass et al. (2022) suggest, COVID-19 has shown the connection between mathematical models and political decisions quite well. Climate change and climate models or modelling of emissions as something that informs policy decisions are areas which can connect mathematical and political issues as well. Another topic may be economic or social policies, like the minimum wage or government subsidies. The task for the students in this session is not to map out a whole lesson, but to think about suitable topics and how to connect their inherent mathematical and political elements. Depending on your class and the number of students studying mathematics, you can ask your students to go into more detail regarding the mathematic elements of the topic and the mathematical competences. You can also hand out curricula from citizenship education or mathematics to guide the students by reminding them	Possibly: -Flipchart paper -Felt tip pens Possibly: -Citizenship education curricula -Mathematics curricula

		which topics are covered in school and what the intended learning outcomes are. The tasks for the students could be the following: • *Find a topic which includes aspect that are suitable for a mathematics lesson and a lesson in citizenship education at the same time.* • *Describe the goals you would set out for your students if you would use this topic for a lesson of mathematics and citizenship education.* • *Relate the topic to the respective curricula*	
	25 min.	<u>Comparing results and discussion</u> Again, let your students present the results of their group work and ask them to give each other feedback and discuss their ideas. You can also lead the discussion more towards the central competences of citizenship education covered in earlier lessons and facilitate a discussion about the necessity of mathematical skills for a democratic citizen.	Possibly: -Pinboard -Pins -Scotch tape
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-
Theoretical framework	**Mathematical modelling – a key to citizenship education** Maass et al. (2022) highlight the connection between mathematics education and citizenship education by referring to the COVID-19 pandemic: "The story of COVID-19 demonstrates that citizens around the world need to understand how mathematics contributes to understanding global challenges and ways of overcoming them. Citizens, and their leaders, need to understand that predictions—such as the number of likely casualties—are based on models that make use of assumptions and the best inputs available. They also need to learn to critically evaluate reports based on the predictions of models that affect decisions and to deal with the inherent uncertainty in an appropriate way. These capabilities make it clear that mathematical modelling is a key element of citizenship education." (p. 33) They also give a summary of the steps of modelling processes and its connection to inquiry-based learning in the mathematics classroom:		

> "These modelling processes have been discussed since the late 1960s. The modelling cycle from the Shell Centre (Burkhardt, 1981) conceptualizes the following steps of the modelling process: (1) understanding the real situation (situation); (2) making assumptions and representations, (3) simplifying the situation and mathematizing it (model); (4) analyzing the model and solving it (mathematical solution); (5) interpreting the solution; (6) validating the interpreted solution; and (7) reporting on the solution found." (p. 35).
>
> (…)
>
> "Particularly, we can say that modelling and inquiry-based approaches are linked, if not at the level of definitions, then at least at the level of practices. The questions that serve as a starting point for the inquiries developed by pupils and students often have their source in extra-mathematical situations linked to their daily lives, their extra-curricular concerns and, increasingly, to environmental and societal issues." (p. 37)

The purpose of mathematical modelling, according to them, is:

> "Mathematical modelling serves various purposes. First, mathematical modelling has been shown to foster mathematical reasoning and thinking and to develop the ability to solve complex problems in real-world contexts (cf. Maass, 2004). Alternatively, mathematical modelling can be an approach to learning mathematics itself, the core of the Realistic Mathematics Education tradition, where it is referred to as emergent modelling (Doorman, 2019; Gravemeijer, 1999). Modelling also provides an avenue for learning by inquiry." (p. 37)

Summarizing the connection between mathematical modelling and societal needs, they state:

> "With the massive 2020 global pandemic disruptions, mathematical modelling played a major part in COVID-19 infection control, yet the projections from modelling became a source of social controversy (Rhodes & Lancaster, 2020). An appreciation of the critical nature of mathematical models in society (Barbosa, 2006) and how assumptions made in the modelling process can impact on societal decisions need greater attention in the classroom. Mathematical problem solving across the grades needs to incorporate not only modelling but also critiquing what a model yields, particularly where what is learned is important in society (Barbosa, 2006). To engage in modelling includes developing conceptual innovations in response to society's needs; such innovations require changes from the conventional ways of thinking that are applied to typical school word problems (Lesh et al., 2013)." (p. 38)

	Educational approaches to mathematical modelling Blum & Leiß (2007) offer insights into the way students and teachers deal with mathematical modelling problems. Their paper offers insights into a typical modelling task and the way students and teachers respond to it (p. 224 ff.)
Required materials	• Laptop, projector or smart board, Power-Point • Flipchart paper, felt tip pens, pin board, pins, scotch tape
Homework	Task for the next session: *According to the text from Niss (2015) and Pohlkamp & Heitzer (2021), what types of modelling exist and what are real life examples for them?*
References	<u>This session:</u> Blum, W., & Leiß, D. (2007). How do Students and Teachers Deal with Modelling Problems. In P. Galbraith, W. Blum, S. Khan, C. Haines, & C. R. Haines (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics; proceedings from the twelfth international conference on the teaching of mathematical modelling and applications* (pp. 222–231). WP Woodhead Publ; Horwood. Maass, K., Artigue, M., Burkhardt, H., Doorman, M., English, L. D., Geiger, V., Krainer, K., Potari, D., & Schoenfeld, A. (2022). Mathematical modelling – a key to citizenship education. In N. Buchholtz, B. Schwarz, & K. Vorhölter (Eds.), *Initiationen mathematikdidaktischer Forschung* (pp. 31–50). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-36766-4_2 <u>Next session:</u> Niss, M. (2015). Prescriptive Modelling – Challenges and Opportunities. In G. A. Stillman, W. Blum, & M. Salett Biembengut (Eds.), *International Perspectives on the Teaching and Learning of Mathematical Modelling. Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences* (pp. 67–79). Springer International Publishing. https://doi.org/10.1007/978-3-319-18272-8_5 Pohlkamp, S., & Heitzer, J. (2021). Normative modelling as a paradigm of the formatting power of mathematics: Educational value and learning environments. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference: 3 Volumes* (pp. 799–808). Tredition <u>Additional Literature</u> Gildehaus, L. & Liebendörfer M. (2021). "CiviMatics - Mathematical Modelling meets Civic Education", In D. Kollosche (Ed.), *Exploring new ways to*

	connect: Proceedings of the Eleventh International Mathematics Education and Society Conference (Vol. 1, pp. 167-171). Tredition.

Session 10)

Session title	Mathematics education – Descriptive and normative models		
Duration	90 minutes		
Goal of the session	Connected to the previous session, the students will get to know approaches which directly connect mathematical modelling to citizenship education and how to use normative modelling for interdisciplinary learning processes.		
Suggested timetable	Time	Content	Materials
	15 min.	<u>Introduction</u> You can use the question for this session as a starting point: *According to the text from Niss (2015) and Pohlkamp & Heitzer (2021), what types of modelling exist and what are real life examples for them?* However, it may again be beneficial to use a short PowerPoint presentation to explain the differences between the types of models and connect them to real world examples if you feel like your students did not grasp this aspect of the text accordingly. As a second step, you may want to take the three learning targets proposed by Pohlkamp & Heitzer (2021) for normative modelling tasks and ask your students to connect them to learning goals and competences of citizenship education. Are there overlaps? What are the differences? You can also use a PowerPoint-Slide summarizing the descriptions of Print (2013) and Reinhardt (2013) as a basis for this discussion	-Required reading -PowerPoint 10
	50 min.	<u>Normative modelling lessons</u> The task this session builds on the tasks of the previous session and asks the students design a lesson incorporating normative models while using the normative modelling cycle as a guideline. It may be beneficial to let them visualize their ideas, since	Possibly: -Flipchart paper -Felt tip pens

Co-funded by
the European Union

		this may make it easier to see the references to the modelling cycle and help them organize the steps they want to take during their lesson. The tasks for the students could be the following: • *Design a lesson which addresses the learning targets for normative models proposed by Pohlkamp & Heitzer (2021) while also referencing competences of citizenship education (e.g. Reinhardt, 2013).* o *Use the modeling cycle as a guideline* o *You can build on the topics and learning goals you discussed during last session.*	
	20 min.	<u>Comparing results and discussion</u> Again, let your students present the results of their group work and ask them to give each other feedback and discuss their ideas. You can also lead the discussion more towards the central competences of citizenship education covered and possible curricular connections you covered during the last session. If your students cannot finish their tasks during the allotted time, feel free to compare the current state of their lesson. They can build on their progress during the next two sessions.	Possibly: -Pinboard -Pins -Scotch tape
	5 min.	<u>Open questions</u> Give the students the chance to ask any questions related to the content of this session or the course as a whole that haven't been addressed during the session.	-
Theoretical framework	**Normative modelling and social reality** Pohlkamp & Heitzer (2021) start their text by highlighting the relevance the societal relevance of mathematical modelling: "Furthermore, it is indispensable from the perspective of promoting citizen empowerment that they discover how and wherefore mathematics is used and which impacts has this modelling for reality. More than being able to calculate their income tax, students should know the main determining		

	decisions behind this tax model and not confuse this arbitrary variable setting with physical constants." (p. 799) They identify different types of modelling processes, which they characterize the following way: "Davis and Hersh (1986, p. 115) enumerate three functions of mathematics: descriptive, predictive and prescriptive. This enumeration can be complemented and transferred to models: There are models that describe, that predict, that prescribe and that explain (Henn, 2002, p. 6)" (p. 800) (…) Nevertheless, the first three can be summarised as the result of descriptive modelling as they have a certain approach in common: 'Descriptive' relates to the fact that all three models are based on a mathematisation of a real-life phenomenon (Fig. 1). The apple had fallen before Newton formulated his law of gravity. The Arctic Ice would have melted and would continue to melt even without a mathematical description or prediction. Freudenthal (1978) speaks of models as "after images" or "plaster cast" (p. 131). The mathematical modelling serves as a reproduction for a better processing, analysis and understanding of the original phenomenon. (p. 800) (…) In the case of normative modelling, mathematics is however used to design standards and rules in order to shape or even create reality (Fig. 2). Hence, the reference to formatting norms explains the denomination although the term prescriptive modelling would be a synonym (Greefrath & Vorhölter, 2016, p. 9). For further illustration of this type of modelling, Freudenthal (1978) uses the metaphor of the "knitting pattern" (p. 131), the comparison with playing rules is by experience particularly suitable for students. One example of the formatting power of normative modelling is the income taxes: You can only pay taxes, once a tax rate has been defined. (p. 801) However, the authors also highlight that these distinctions refer to the process of modelling, not models itself: "Nevertheless, such an example could contribute to misconceptions: Of course, a given tax rate also describes the amount due and a citizen can predict one's tax debt with help of the mathematical function. The categorisation 'descriptive vs. normative' refers more to the intention of the original modelling process than to the handling of the resulted model. That is why we prefer to attribute these adjectives only to 'modelling' and normally not to 'model'. Furthermore, this dichotomy is based on idealised prototypes." (p. 801).

Niss (2015) uses slightly different terminology to describe the differences between modelling processes. He summarizes the difference between descriptive and prescriptive modelling the following way:

> "On the basis of these examples – and hosts of other examples – it seems fair to conclude that prescriptive modelling, even though it shares significant features with descriptive modelling, differs from it in characteristic ways, which become visible when considering the modelling cycle. Whilst the preparation of the extra mathematical domain for modelling by way of specification and idealisation can be rather similar in descriptive and prescriptive modelling, the mathematisation part may – but need not – be very different, since in prescriptive modelling there may not be any clue whatsoever concerning how to come up with a sensible model to meet the aim of the modelling. The de-mathematisation and the validation parts are normally very different as they are largely absent in prescriptive modelling. Mathematical treatment sometimes is similar and sometimes different in the two sorts of modelling, depending on the context and situation. This suggests that the sub-processes of the modelling cycle may not be able to fully capture what happens in evaluating models arising from prescriptive modelling. Instead, meta-validation becomes crucial." (p. 76)

For educational process connected to normative modelling, Pohlkamp & Heitzer (2021) identify three learning targets:

> Doing justice to the normative type when teaching modelling is tied to complementary learning targets. The general objective of explaining the formatting power of mathematics by comprehends amongst others the following targets (Pohlkamp, 2020):
>
> 1. Exemplifying prototypic characteristics of normative modelling: Discovering the abovementioned characteristics as ambiguity and subjectivity facilitates to deconstruct the myths about Mathematics that Hersh (1991) identified. This initiates a reflection of mathematics, its use and role in society as another positive impulse.
> 2. Outlining premises, decisions and alternatives: The task is to encourage critical minds. Confronted with supposedly completed models, a responsible citizen should be qualified to detect other options and especially be careful if they are being told that there is not an alternative.
> 3. Judging normative modellings as well as the possibility of judgment: While comparing different outcomes given the same cause for the normative modelling, students encounter

	the same challenges as during the modelling. As there cannot be a benchmark as exactness of reproduction, the choice of quality criteria is subjective and arbitrary as well. Whether a tax model is fair depends on your understanding of fairness.
Required materials	• Laptop, projector or smart board, Power-Point • Flipchart paper, felt tip pens, pin board, pins, scotch tape
Homework	Task for the next session: *No task*
References	<u>This session:</u> Niss, M. (2015). Prescriptive Modelling – Challenges and Opportunities. In G. A. Stillman, W. Blum, & M. Salett Biembengut (Eds.), *International Perspectives on the Teaching and Learning of Mathematical Modelling. Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences* (pp. 67–79). Springer International Publishing. https://doi.org/10.1007/978-3-319-18272-8_5 Pohlkamp, S., & Heitzer, J. (2021). Normative modelling as a paradigm of the formatting power of mathematics: Educational value and learning environments. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference: 3 Volumes* (pp. 799–808). Tredition <u>Next session:</u> Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16. https://doi.org/10.1080/14794802.2012.756633

Session 11 & 12

Session title	Practical examples – Lesson plans I + II		
Duration	2x90 minutes		
Goal of the session	During these two sessions, the students should develop a more in-depth lesson plan which connects citizenship education with at least another subject		
Suggested timetable	Time	Content	Materials
	10 min.	<u>Introduction</u> During this session, the students are tasked with creating a detailed lesson plan for a topic which connects elements of citizenship education with another subject. Again, the students can work on the task in small groups (3-5 students).	-PowerPoint 11

		The students are free to choose the topics and the subjects they want to cover, but you can narrow both down if you like. The students can choose if they want to build on their results from the previous lesson, be it mathematics, history, geography, science or languages, but they are free to pick a totally different subject (like sports, arts or music). Since the previous two sessions covered mathematics education, a number of students will likely decide to further build on their partially developed ideas. If the students have time, you can also ask them to start developing learning materials, for example an introduction to the lesson or a task combining the two subjects. Alternatively, you can ask your students to evaluate existing materials you have access to. The tasks for the first approach are the following: • *Building on your previous idea, design a lesson plan addressing competences of citizenship education and another subject* • *If time permits it, also start working on materials for the students (e.g. an introduction to the lesson or a worksheet covering one competence or a number of competences)* • *Visualize your lesson. Include a timetable and connect the learning goals to each task.* During the second session, you can use the introduction to address issues the students raised in the previous session or skip in and let them work on their lessons,	
	80 min. (I) 30min (II)	<u>Developing the lesson</u> During this phase, the students work together within their groups. You can provide a number of materials, both to visualize their ideas and to generate ideas. The text for this lesson serves ads to the latter point by detailing the areas mathematics is involved with climate change research.	-Flipchart paper -Felt tip pens -moderation cards <u>Possibly:</u>

			-Laptop, projector or smart board, Power-Point Possibly: -Curricula
	55 min.	Comparing results and discussion Let the groups present their results and ask the other students to provide feedback. Again, there are different ways to do this: You could do a group puzzle connected to a gallery walk. The groups are rearranged so that every group has one student from each concept map in it. The groups walk throughout the classroom and describe their concept maps to each other. You could do a group gallery walk, where the results are hung up throughout the room and the whole class visits them, while the respective group explains their ideas You could to individual presentations, where the students present their concept maps to the whole class. The goal is for the students to share their results and get an impression about different ideas to approach interdisciplinary citizenship education. You can steer the conversation to certain aspect, like competences, or you can let the students chose the areas they want to highlight.	- Pinboard -Pins -Scotch tape Possibly: Laptop, projector or smart board, Power-Point
Theoretical framework	**Climate change and mathematics** Barwell (2013, p. 3 f.) describes three areas where mathematics is involved in climate change and climate change research: description, prediction and communication: The *description* of climate change is mostly based on relatively simple statistics concepts and methods. Without mathematics, we would have little awareness of climate change as a system-wide phenomenon. (…) The *prediction* of the likely future course of climate change is based on more advanced mathematics. Developing predictions about future global, regional or local effects of climate change draws on a range of advanced mathematical methods, including mathematical modelling, differential equations, non-linear systems and stochastic processes (McKenzie 2007,		

	22-3). Several different climate models have been developed to relate greenhouse gas emissions to changes in the Earth's climate. (…) The *communicatio*n of climate change also entails the use of mathematics and, more particularly, mathematical or statistical literacy. Climate change is now explained or discussed in a wide range of non-scientific contexts, including public media, official websites, blogs, official publications, reports and so on. Interpreting and, in some cases, participating in the production of these texts entails some level of engagement with the mathematics used to describe and predict climate change. Additionally, a degree of statistical literacy is also necessary, in relation to the use and interpretation of data, graphs and accounts of the mathematics involved." (p. 3) He also highlights the relevance for mathematics education the following way: "A critical mathematics education approach to the issue of climate change would, therefore, start with data, but would include a political dimension. Students could, for example, examine different graphs for a selection of countries showing national emissions per annum, cumulative national emissions per annum since the industrial revolution, and emissions per capita, in order to consider who is responsible and who should bear the greatest burden in reducing global greenhouse gas emissions. Such data relates directly to current international debates about the successor treaty to Kyoto. Discussion could also examine the challenge of climate uncertainty and, in relation to this uncertainty, explore how mathematical analysis is set against individual experience, values and social relations, perhaps through simulations similar to that developed by Pratt et al. (2011). Indeed, the use of simulations would be a viable way to introduce students to some of the more challenging aspects of the mathematics of climate change, such as the modelling processes and non-linearity I have discussed. Through the use of simulations, students can come to understand the significance of these kinds of ideas, without needing to learn the advanced mathematics behind them. Such an approach would also allow students to engage with the contradictions inherent in post-normal science, such as the apparent contradiction between maintaining a standard of living and taking action on climate change. To take this kind of approach further, students need to develop and engage with a wider peer community, through, for example, communication with climate scientists, politicians or community representatives." (p. 14)

Required materials	• Laptop, projector or smart board, Power-Point • Flipchart paper, moderation cards, felt tip pens, pin board, pins, scotch tape • School curricula
Homework	Task for the next session: *No task*
References	This session: Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16. https://doi.org/10.1080/14794802.2012.756633 Next session: -

Session 13

Session title	Conclusion and evaluation
Duration	60-90 minutes
Goal of the session	The goal of this session is either to summarize the contents of this session and/or the give the students a chance to evaluate the course. Thus, there is no strict timetable and you can also use this session as a joker if you need additional time to address certain topics during the previous session of the course.
Follow-up	Task for the next session: -
References	This session: -

5.1. Didactic Commentary

Interdisciplinary Citizenship Education – Insights from practice

Bastian Vajen

Institute of Didactics of Democracy, Leibniz Universität Hannover, Hannover, Germany

Introduction and Structure of the Module

The underlying idea of this course is to illustrate different approaches to combine citizenship education with other subjects - especially mathematics -, to discuss the possible advantages and disadvantages of these approaches are to encourage the students to think about ways interdisciplinary learning can be implemented as part of regular school education. Accordingly, the course is structured in a way to allow students to gradually get to know different perspectives on (interdisciplinary) citizenship education. The first two sessions focus on central aspects of citizenship education, which are then gradually expanded in the following session through whole-school-approaches to citizenship education and other subjects discussing political elements of their teachings. Mathematics education takes a prominent position here, which is why this course also includes a brief and selective introduction to the principles of mathematics education before concepts that combine mathematics education with citizenship education are discussed. At the end of the seminar, the contents of the previous sessions are applied in the development of lesson plans, allowing the students to put their theoretical ideas into practice.

Looking back, it can be said that the overall structure of the seminar worked well when teaching citizenship education students. The seminar was mainly attended by students studying politics in combination with another subject and the aim of becoming a teacher after finishing their master's degree. For these students, who in the vast majority of cases had already attended an introduction to political education, the two introductory sessions proved helpful, but not too extensive to appear redundant. As different introductory courses are offered at this department, the texts differed from the literature a number of students had already read in their introductory course and provided appropriate added value. They also provided a good basis for the following sessions. The students participating in this course were able to references the literature and central context connected to it of earlier sessions when discussing aspects of the current session (for example in Session 7).

The subsequent sessions of citizenship education as a whole-school approach and the potentials and limits of citizenship education in other subjects seemed to be a good connection to the previous sessions in terms of content. Especially with regards to the goals and central competences of citizenship education discussed in the first two sessions, the following sessions were well suited as a theoretical background for the discussion of the approaches presented in the require readings. The subsequent sessions dealing with educational approaches connecting citizenship education with other subjects, such as geography, history, language teaching and science education, were also facilitated by the heterogeneous group students, who were in part studying the subjects discussed here. However, it was noticeable in both sessions that not all students were equally committed to actively participating and that for them, the relevance of the respective topics for their own later professional future was not entirely apparent.

This tendency became even more pronounced in the sessions dealing with the basics of mathematics educations. Even if the underlying idea of the course remained transparent for most students, the engagement with the principles of mathematics education seemed to be difficult for some. During these seminar sessions, some students reported their complicated relationship with mathematics, which dated back to their school experience and how it seemed to be somewhat inaccessible to them. However, the approaches combining citizenship education and mathematics education, which were discussed during the tenth session, appeared to be more relevant for the students and helped them engage better with both subjects. The subsequent task of developing their own lesson plans was also particularly motivating for the students, as was the case with practical examples throughout the seminar.

Overall, the basic structure of the seminar appears to be fundamentally suitable for the purposes described at the beginning of this chapter. Especially for students studying to become citizenship education teachers and who do not study this subject in combination with mathematics, the successive steps towards other subjects taken throughout the course seemed to be helpful.

Tasks and Sessions

In retrospect, the task the students had to complete and hand in before each session can be seen as a double-edged sword. On the one hand, the additional extrinsic motivation meant that almost all students were adequately prepared for each session. The students had read the texts for each session at lease superficially and could either summarise the central statements of each text and discuss them with each other or name certain aspects that they had not understood, and which could then be discussed and contextualised during the seminar. In this respect, it was often possible to keep the discussion of the texts relatively short and enable the students to deal with the respective session topics in greater depth. The demands associated with the assignments also meant that the students also had to attentively read texts for sessions with topics they were not particularly interested in - such as the principles of mathematics education. Accordingly, it was possible have productive activities and discussion in all sessions.

On the other hand, the students perceived the workload for this seminar as particularly high. The preparation of several texts, some of them form disciplines they were not familiar with, was perceived by some students as very time-consuming. In particular the text by Ernest (2018) on the philosophy of mathematics education was described by many students as very demanding. Accordingly, the most frequently voiced suggestion for improvement by students was to limit their homework in one way or another, for example by reducing the number of tasks and giving them freedom to choose the sessions for which to prepare them. Of course, these suggestions somewhat undermine the basic idea that all students are encouraged by the assignments to deal with the required reading at least superficially, which creates a good foundation for the seminar sessions. Here, depending on the students' motivation and the legal requirements of each respective university, it could be possible to change these tasks or to contemplate if successful preparation for and participation in each session can also be ensured through other activities

The design of the sessions, on the other hand, was evaluated positively by almost all students. Here, the absence of student presentations, which are also used as a mandatory task by some lecturers, as well as the regular practical focus of the sessions were highlighted as positive. The students appreciated being able to deal with practical aspects, such as concepts for school lessons or school curricula, during their group work phases. It should be noted here that these sessions could not always adhere entirely to the recommended schedule. In some sessions, for example the fifth session, the discussion about the text and the added value of citizenship education was so pronounced that the group work phase had to be shortened. The activities involving concept maps during the second and the eighth session took longer than expected, so the discussion of the results had to be shortened. In the second session, this was due to the complexity of the results, for which the students asked to have more time to develop them. During the eighth session, to the complexity of the text became an issue for the student, which had to be discussed for a longer period of time led to ongoing questions during the group work phase. The last session, which was not filled with any other content apart from the evaluation of the course and the students' questions, gave some leeway to address certain aspects that had not been dealt with in sufficient detail in the respective sessions. In conclusion however, the design of the sessions seems to be appropriate for the content of the seminar, but may require spontaneous adaptation, which can vary from learning group to learning group.

Conceptions and Content

Overall, many students seemed to get relevant impulses for linking citizenship education with other subjects, and here especially with mathematics, from this course. Two students described the learning effect of the seminar as follows:

> Well, before the course I had a bit of a hard time connecting mathematics and politics myself. But now, one is aware that mathematics takes place from time to time in probably all subject areas, and especially in citizenship education with statistics and such, but with the materials we used, as I said, current examples, it was also very good for me to see again how present mathematics actually is. And that definitely helped me and I also found some of the topic drafts or lesson drafts very cool, so that I say, okay, I could also imagine maybe trying that out later, and so from my side I think the feedback regarding the whole project was consistently positive. (S04)

> I had also considered beforehand, in the first session of the entire seminar, which subjects we could imagine combining with citizenship education, and almost all the subjects that we had ever had at school came up. Only with mathematics did we find it very, very difficult. We hadn't really found an idea, and we had a lot of mathematical things in our heads and thought: How do I bring that into citizenship education? But I also think that this had changed a lot for me [throughout the course]. (S01).

Overall, the idea of citizenship education as a component of other subjects, both from the perspective of citizenship education itself as well as from the perspective of other subjects, was seen as a great interesting idea and a benefit by the students. Especially for the subject of mathematics, the perspective of some students has changed significantly. In this respect, this course seemed to offer a good opportunity for students to reflect on the educational content of their own subject as well as the opportunities and limits of connecting it to of citizenship education on the one hand and to broaden their perspective for initiating interdisciplinary learning processes in their later practice as teachers on the other.

CHAPTER 6

Implementing Modelling in Teacher Education

Lara Gildehaus & Michael Liebendörfer

Institute of Mathematics, Paderborn University, Germany

General Introduction

In the following, we present three modules, each of which was conducted as part of a course: an introductory course in mathematical thinking, a mathematical content course, and a mathematics didactics course. This covers all types of lectures offered at the University of Paderborn for mathematics teaching students. The central, common content is normative modelling, which is described in more detail below. After normative modelling has been introduced, there is a separate description for each of the modules, which includes the embedding in the course.

The integration of normative modelling into existing courses offers the great advantage that teaching can be directly enriched without having to change the formal rules of the courses. Thus, however, normative modelling always remains only a small part of the course, which retains its central learning objectives according to the study regulations. Therefore, the three modules presented here are rather short. The connection to the further contents of the module succeeds with varying degrees of smoothness. In the introductory course to mathematical thinking and the mathematics didactics lecture it succeeds well, in the subject course one can see the different character of the contents.

The design of the modules is based on the assumption that students have had little or no previous exposure to normative modelling. This corresponds to the state of the literature (Blum & Ferri, 2009) and also to our practical experience. From a didactic point of view, students' own activities in the field of normative modelling are at least as important as getting explained the theory about it. Therefore, and due to the limited learning time allocated to the modules, the theory of normative modelling is only partially addressed in the three modules. Nevertheless, we consider it very useful to present it here in more detail than it will be found later in the materials. The theoretical foundations were developed in the CiviMatics project precisely for the purpose of providing a basis for teaching.

Background: Normative Modelling

Mathematical modelling is firmly anchored in the educational standards for the German school system (Bildungsstandards, 2015) and is either explicitly located in the teacher training programme or implicitly via applied mathematics. As stated in Chapter 1 (this volume), the type of model can be theoretically differentiated, between descriptive and normative modelling.

Modelling used in school lessons is rarely normative, but almost always descriptive. Common examples of tasks refer at most to two different real models that are predominantly descriptive, and which are then compared in terms of their desirability (Besser et al., 2020). This may also have to do with the fact that normative modelling assigns teachers demanding tasks beyond ordinary modelling. Based on our theoretical thoughts we present three implementations in the following, where preservice teachers may learn how to teach about normative modelling as well. Those implementations are oriented at the introduced (normative) modelling cycles (Vos & Frejd, 2022) in Chapter 1 (this volume).

Module 1: Introduction to Mathematical Thinking

This module comprises of two sessions on normative modelling within a general course introducing mathematical thinking. The course was a 6 credits course designated to the first semester in the bachelor programme for preservice mathematics teachers for lower secondary school (Grades 5–10). The general goal of this course was that students acquire profound knowledge about mathematics in its historical and cultural context. The most prominent element is the idea of proof. The principles of mathematical proof are to be introduced to the students, as well as the verification of arguments when conjecturing and proving mathematical statements. Furthermore, the course addresses how mathematical theories are built up (axioms, definitions, theorems). According to the study regulations, the course should also include mathematical modelling as another fundamental inspiration, use and way of mathematical thinking. Unlike in previous years, we included normative modelling in this final part of the lecture. We only report the part on modelling but not the parts on proof and mathematical theories.

Learning Goals and Structure

The learning goals of this module are as follows:

- Students know how to use a modelling cycle for different purposes, e. g. analysing real problems, analysing student solutions, and designing exercises. In particular, students know what is meant by the terms of the inner-mathematical and outer-mathematical world.
- Students know the difference between descriptive and normative modelling and can discuss this difference from a didactical perspective.
- Students can evaluate and discuss the relevance of (normative) mathematical modelling in school and reflect on the necessity of normative mathematical modelling in terms of citizenship education.

The general course is taught in lectures and tutorials on campus. Both lecture and tutorial run for 90 minutes once a week over the complete semester (14 weeks in total). While in the lecture new contents are presented and introduced, they are usually applied in exercises during the tutorials. Additionally, students are handing in weekly obligatory homework with exercises similar to those from the tutorial. The topic of (normative) modelling is usually taught at the end of the course. We addressed the topic of normative modelling in two lectures, two tutorials and two homework assignments. In the following, we present these two sessions of one lecture, tutorial and homework at a time.

Overview

Title	Normative mathematical modelling
Duration	4 x 90 min. + 1 x homework
Organization	2 x 90 min. (lecture) + 2 x 90 min. (tutorial) + 1 x homework

Co-funded by
the European Union

Relevant Literature	**Introduction to mathematical modelling:** Borromeo Ferri, R., Greefrath, G., & Kaiser, G. (Eds.). (2013). *Mathematisches Modellieren für Schule und Hochschule: Theoretische und didaktische Hintergründe*. Springer Spektrum. Kaiser, G., Blum, W., Ferri, R. B., & Stillman, G. (2011). Trends in teaching and learning of mathematical modelling. Springer Netherlands. **Modelling cycles:** Blum, W., & Leiss, D. (2005, February). „Filling Up "-the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In *CERME 4–Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (Vol. 1623). Sant Feliu de Guíxois: FUNDEMI IQS–Universitat. Gildehaus, L., & Liebendörfer, M. (2021). CiviMatics - Mathematical modelling meets civic education. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 167–171). Tredition. **Educational standards for mathematics (Germany):** Kultusministerkonferenz. (2012). Bildungsstandards im Fach Mathematik für die allgemeine Hochschulreife. Kultusministerkonferenz. http://www.kmk.org/fileadmin/Dateien/veroeffentlichungen_beschluesse/2012/2012_10_18-Bildungsstandards-Mathe-Abi.pdf 7 **General example tasks on (normative) modelling:** Marxer, M., Prediger, S., & Schnell, S. (2010). Wie verteilen wir die Müllgebühren? – Bildungswirksame Erfahrungen beim Entwickeln und Diskutieren normativer Modellierungen. Praxis der Mathematik in der Schule, 52(36), 19–25. Rellensmann, J. (2019). Mathematisches Modellieren [die Feuerwehraufgabe]. *Selbst erstellte Skizzen beim mathematischen Modellieren: Ergebnisse einer empirischen Untersuchung*, 5-30, Springer Spektrum. Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt?. *Journal of mathematical modelling and application*, *1*(1), 45–58. **Example on CO_2-Emissions:** VDI (2021). *Deutschlands Verantwortung für die globalen CO2-Emissionen.* From: https://www.vdi.de/news/detail/deutschlands-anteil-an-den-globalen-co2-emissionen (Status: 07/07/2023) Appunn, K., Eriksen, F., Wettengel, J. (2023). Germany's greenhouse gas emissions and energy transition targets. From:

	https://www.cleanenergywire.org/factsheets/germanys-greenhouse-gas-emissions-and-climate-targets (Status: 07/07/2023) **Example on teacher shortage:** Wiarda-Blog (2022). *Schluss mit der Lückenstopferei!* From: https://www.jmwiarda.de/2022/01/31/schluss-mit-der-lückenstopferei/ (Status: 07/07/2023) Fokken, S. (2022). Bildungsforscher hält Berechnungen der Kultusminister teilweise für »unseriös«. From: https://www.spiegel.de/panorama/bildung/lehrermangel-an-schulen-bildungsforscher-haelt-kmk-berechnungen-teilweise-fuer-unserioes-a-27193b0a-7537-46ee-b06b-be6bb6094c4d (Status: 07/07/2023) Nehra, W. (2022.). Study reveals Germany could be in need of over 80.000 teachers by 2030. From: https://www.iamexpat.de/education/education-news/study-reveals-germany-could-be-need-over-80000-teachers-2030 (Status: 07/07/2023)
Topics	Introduction to (normative) mathematical modelling Modelling cycles Developing normative modelling

Session 1

The aim of the first lecture is to give a basic overview on mathematical modelling. To this end, the lecture starts with activating students' prerequisites on mathematical modelling and then tries to raise an awareness towards political dimensions of mathematical modelling in the further process. Hence, the modelling cycle (Blum & Leiß, 2005) is introduced based on step-by-step examples. This is followed by an example, where students apply the modelling cycle to a specific task "the fire brigade" (Rellensmann, 2019). Building on a discussion on that "fire brigade" example, normative mathematical modelling is introduced. For this purpose, the inner-mathematical and extra-mathematical world is briefly discussed, as well as the goals of modelling. Along the way, normative modelling is defined, identified and compared with descriptive modelling. In a short exercise, normative and descriptive models are named and assigned by the students. The lecture finally starts a discussion on the question "How do we distribute the rubbish fees?" (Marxer et al., 2010) as an example of a normative modelling task in mathematics education and ends with the introduction of the normative modelling cycle (Gildehaus & Liebendörfer, 2021).

In the first tutorial, the contents of the lecture are repeated and deepened. Modelling has to be identified as normative or descriptive in task 1 and students are asked to create further examples. Subsequently, the students start their own normative modelling by developing a suitable question. Some examples in the context of climate change are provided but are not mandatory to choose. The students complete their modelling process in the homework.

Background

Modelling is mainly considered here from a formal, but also from a didactic perspective. The essential core idea is the mediation or translation between the mathematical and the real world. To this end the lecture is based on the modelling cycles according to Blum & Leiß (2005).

Concrete learning activities

Lecture 1:

- Introduction to the lecture: Warm-up by collecting students' thoughts on the questions: "What role does mathematics play in political decision-making processes?
- What is the role of mathematics education in the critical literacy of students?"
- Presenting mathematical modelling as part of the educational standards.
- Collecting students previous experiences with mathematical modelling
- Introducing the modelling cycle based on an example (Blum & Leiß, 2005)
- Discussing the modelling cycle with the students
- Joint development and discussion of another example task with the students
- Explanation of the inner-mathematical and outer-mathematical world
- Introduction to normative modelling by explaining normative and descriptive statements
- Asking students to assign examples to an either normative or descriptive modelling
- Introducing the example task "How do we distribute the rubbish fees?"
- Plenary discussion
- Summarising the differences of descriptive and normative modelling

Tutorial 1:

- Task 1: Students assign given examples to either normative or descriptive modellings
- Task 2: Students develop their own normative modelling example. They either choose out of two open topics (modelling the climate impact of certain foods or modelling the climate impact of electric cars) or choose their own topic. They are then developing a concrete question and investigating the question with the help of the normative modelling cycle.

Homework 1:

- Students are supposed to finish their normative modelling examples and to prepare them for presentation in the next week.

Materials

You can find the lecture slides for this session as well as the exercise sheets for the tutorial and homework in the Appendix.

Session 2

The aim of the second lecture is to deepen and strengthen the idea of normative modelling and to get students to know how they could use it in their future mathematics lessons. The lecture starts with a short introduction and repetition of the normative modelling cycle using a given example. An article on the topic of CO2 budgets is presented. The students are asked to name important assumptions that underly the CO2 budget. With the help of another article, the students shall discuss and apply the further steps of the normative modelling cycle. Reflecting on modelling, other possible types of models are discussed. Finally, the normative use of prognostic models is discussed using the example on the estimated numbers for teachers needed in future years. At the very end of the lecture, modelling is summarised and students are asked about their individual attitudes towards the use of (normative) modelling in future mathematics lessons.

In the tutorial, the homework of the last week is presented and discussed. In particular, the students present different results of their modelling considering different assumptions, followed by a general discussion with all students.

Background

Modelling is predominantly considered here from a subject-specific perspective, but also from a didactic perspective. The essential core idea is to reflect on different assumptions and their consequences in the normative modelling process. Political dimensions of mathematical modelling should be made visible. To this end, a normative modelling cycle (Gildehaus & Liebendörfer, 2021) is being introduced, expanding the cycle of Blum & Leiß (2005) from the previous lecture.

Concrete learning activities

Lecture 2:

- Introduction to lecture: refreshing the idea of normative modelling with a few examples
- Joint development of assumptions for the CO2 budget with the help of an article and the normative modelling cycle
- further discussion in plenary regarding the further steps in the modelling cycle and decisions for the CO2 budget model
- Discussion on possible modelling besides descriptive and normative
- Developing the normative use of predictive models using the example of teachers' needs
- Discussing the questions "What components does a model need, to predict whether we will have enough (or too many / too few) teachers in Germany in 2030" and "What would you bring in if you want the demand to be particularly high / low?" in the plenary
- Asking students about their attitude towards the importance of (normative) modelling and discussing if the last two lectures have changed their view .

Tutorial 2:

- Presenting the results of the homework in the exercise
- Discussing together the normative modelling presented

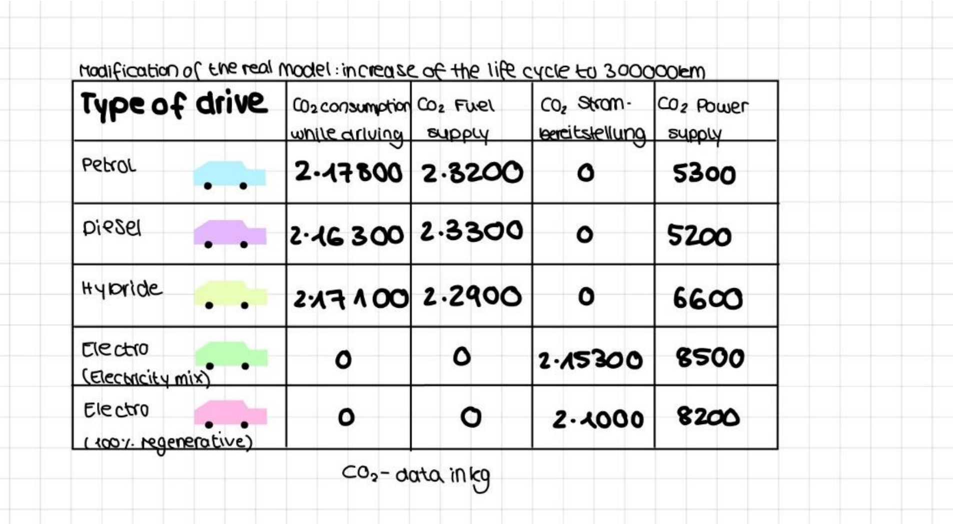
Materials

You can find the lecture slides for this session as well as the exercise sheets for the tutorial and homework in the Appendix.

Evaluation and comments

We evaluated the modules with two different cohorts of students. Although the students were theoretically expected to have basic knowledge of mathematical modelling, most of them reported that they had very little experience of modelling during their school years. Thus, mathematical modelling might be underrepresented in schools at the current moment and the step-by-step introduction on modelling with the help of the modelling cycle was helpful. After the first lecture, students discussed descriptive and normative modelling quite well in the tutorial and showed little to no problems to identify normative or descriptive modelling in five examples. However, building their own models was rather challenging for most of the students.

We identified two main challenges in their modelling processes: First, they had problems to identify a concrete question or aim for their modelling, in particular when they did not refer to the provided examples. Secondly, they were challenged to use and apply demanding mathematical concepts in their models. Accordingly, all the model's students developed had very basic mathematical content such as proportional calculations and descriptive statistics (see figure 1 for an example).


Modification of the real model: increase of the life cycle to 300000km

Type of drive	CO_2 consumption while driving	CO_2 Fuel supply	CO_2 Strom-bereitstellung	CO_2 Power supply
Petrol	2·17800	2·3200	0	5300
Diesel	2·16300	2·3300	0	5200
Hybride	2·17100	2·2900	0	6600
Electro (Electricity mix)	0	0	2·15300	8500
Electro (100% regenerative)	0	0	2·1000	8200

CO_2-data in kg

Figure 1: Student work on the carbon footprint of different cars

Thus, when introducing normative modelling we recommend giving students guidelines in terms of possible questions and aims to model, even though this limits the open character of the modelling task. Furthermore, it should not be underestimated that applying mathematical content on an open and self-selected problem seems to be challenging for students, specifically for first-year students like we had in our implementation. Concrete examples that also include advanced mathematics may be helpful. In general, students' feedback on the modelling tasks was very positive, they enjoyed bringing in their own interests and were very engaged in the discussion and presentations of their work.

Module 2: Elements of Mathematics

Module 2 includes a short, homework-based implementation on normative modelling within a general course on formal mathematics. At Paderborn University, this course provides 6 credits points for master students. It is designated to third semester preservice mathematics teachers for primary school (grades 1 to 4). The goal of this course is that students acquire profound knowledge on working mathematically. The focus is rather on strategies and thinking than specific contents. Questions like: “How are mathematical concepts formed? How is mathematical knowledge legitimised? What questions does mathematics pursue?” are addressed. Framed by those questions, generic proofs, symbolic representations and formal proofs (e.g. direct and indirect proofs in basic number theory and algebra) are taught. In addition to those contents, a short excurse on modelling is provided.

As the (normative) modelling implementation developed here, mainly stands for its own, broad applications to different courses seem possible. Learning prerequisites are general contents from school mathematics lessons as well as a basic understanding about mathematical modelling. Students should also be familiar with didactic concepts in mathematics and be able to create and reflect on tasks according to subject-specific and didactical aspects.

Learning Goals and Structure

The learning goals of this module are as follows:

- Students know the difference between descriptive and normative modelling and can discuss this difference from a didactical perspective.
- Students can evaluate and discuss the relevance of (normative) mathematical modelling in school and reflect on the necessity of normative mathematical modelling in terms of citizenship education.
- Students are able to create normative modelling tasks and reflect how they can be applied in the classroom.

The general course is taught in lectures and tutorials on campus. Both lecture and tutorial run for 90 minutes once a week over the complete semester (14 weeks in total). While in the lecture new contents are presented and introduced, they are usually applied in exercises during the tutorials. Additionally, students are handing in a weekly obligatory homework with similar exercises to that of the tutorial. In this course, the topic of modelling is comprised into one homework session, where students work through the content independently and complete an exercise sheet for this purpose. Thus, this session can easily be integrated, for example as “bonus homework” or teaching-related excursion into different mathematics courses.

Overview

Title	Brief introduction to normative modelling
Duration	1 x homework
Organization	Independently from the course
Relevant Literature	Marxer, M., Prediger, S., & Schnell, S. (o. J.). *Wie verteilen wir die Müllgebühren? – Bildungswirksame Erfahrungen beim Entwickeln und Diskutieren normativer Modellierungen*. Tu-dortmund.de. Retrived at 13. März 2023, from http://www.mathematik.tu-dortmund.de/~prediger/veroeff/10-Marxer_Prediger_Schnell_PM-H36-Webversion.pdf

Topics	Normative modelling in primary education classrooms

Session

The aim of the homework is to reactivate the idea of modelling in mathematics lessons and to extend it with regard to normative modelling. Based on this, students are supposed to reflect on how to integrate normative modelling into mathematics lessons. For this purpose, an introductory overview text is provided and shall be summarised by the students. In the next step, a sample task on normative modelling shall be analysed. Based on this, the students finally develop their own task for normative modelling and explain the didactic potential of such a task.

Background

Mathematical modelling is introduced here in a very condensed form. Nevertheless, building on the prior knowledge of the students, it can succeed here in giving a first insight into normative modelling. For this purpose, the introduction is closely oriented to Marxer et al. (2010) and the students' independent reflection on the potential of such normative modelling tasks.

Concrete learning activities

Homework:

- Read the text by Marxer et al.(2010).
- Summarise it in your own words and take a look at...
    - → ... what distinguishes normative and descriptive modelling
- → ... why normative modelling should play a role in mathematics education.
- Discuss whether normative modelling is also relevant in the teaching of mathematics in primary school.
- Develop a task for normative modelling that you could imagine using in your later lessons.
- Justify the didactic potential of your task, for example, by giving possible solutions of students and the resulting occasions for discussion.

Materials

You can find the exercise sheet for the homework in the Appendix.

Evaluation and comments

We implemented and evaluated this module with one cohort of students. Although the module stands more or less on its own, our students showed almost no problems to work out the content independently and wrote great summaries. We should keep in mind, however, that as second year master students, they were quite experienced. All students described the second task (to develop their own normative modelling and to reflect how to apply this into their future lessons) as very motivating. However, when analysing their developed modelling tasks, we found that almost all students struggled to provide an open character in their modelling tasks. Instead, the developed modelling tasks were loaded with implicit or explicit information to shape students' assumptions. One example is presented in Figure 3. The task starts with the question how to distribute the water cost in a given household. A family consisting of four members (mom, dad, child, baby) is named as well as the overall cost of 400 €. Additional information is given on who is showering how often, indicating a rather "clear" distribution of the water cost, that limits the normative potential of the task. When introducing the development of normative modelling tasks to students, we should thus keep in mind that mathematical tasks often come with "one right solution" that is promoted and preservice mathematics teachers may not be used to generating open tasks.

Water costs of a family

This is how the four family members deal with water:

Father: showers every day
Daughter: showers every other day
Mother: showers every day, cooks with the water, cleans
Baby son: is bathed twice a week

The water costs in the household amount to 400 Euros in one year, how should the payment be divided among the family members?

Task:
1) In *individual work*, think about possible distributions and draw up terms for them.
2) Get together *in groups* and discuss your approaches. Agree on a solution.
3) Think about how the distribution would look if there were two annual costs?

Figure 2: Normative modelling task developed by a student

Module 3: Normative Modelling within teaching Stochastics

In the following, we present a two-session implementation on normative modelling within a general course on teaching stochastics. The general goal of this course is that students acquire profound knowledge about stochastics from a didactical perspective. More concretely, the students learn about the development and aspects of the concept of relative frequency and describe typical difficulties of understanding when calculating with ratios and when dealing with the concept of chance. They know relevant "Grundvorstellungen" and "fundamental ideas" around stochastics and how to apply these into learning environments. Students analyse and evaluate educational standards and curricula on stochastics. They are able to reflect on didactical principles for lesson design.

The target group of this course are preservice mathematics teachers in the last year of their bachelor programme (fifth semester). Learning prerequisites are contents from stochastics lessons at school as well as a basic understanding of mathematical modelling. Ideally, the students already attended a related course, at Paderborn the "Elements of Stochastics", where the same contents are addressed from a formal, mathematical perspective, to provide students with some general understanding of subject-specific contents in stochastics.

At Paderborn University, this course is part of a 15 credits points module for bachelor programs (7.5 points are designated to the "Elements of Stochastics", 7.5 points to the "Teaching of Stochastics").

It is designated to fifth semester preservice mathematics teachers for lower secondary school (grades 5 to 10).

Learning Goals and Structure

The learning goals of this module are as follows:

- Students know how to use the (normative) modelling cycle and they are able to work with them from different perspectives, e. g. analysing existing taks or students' solutions as well as designing exercises.
- Students know the difference between descriptive and normative modelling and can discuss this difference from a didactical perspective.
- Students can evaluate and discuss the relevance of (normative) mathematical modelling in school, specifically in stochastics lessons and reflect on the necessity of normative mathematical modelling in terms of citizenship education.
- Students internalise how (descriptive) data analysis and randomness in real data can be connected with decisions in mathematical modelling.

The general course is taught in lectures and tutorials on campus. Both lecture and tutorial run for 90 minutes once a week over the complete semester (14 weeks in total). While in the lecture, new contents are presented and introduced, these contents are usually applied in exercises during the tutorials. Additionally, students hand in weekly obligatory homework with exercises similar to those of the tutorial.

We implemented the topic of (normative) modelling into the middle of the course, when it comes to data analysis and different ways of descriptive statistics, as well as statistical literacy. We addressed the topic of normative modelling within two lectures, two tutorials and two homework. We present those two sessions in the following.

Overview

Title	(Normative) modelling and data
Duration	4 x 90 min. + 2 x homework
Organization	2 x 90 min. (lecture) + 2 x 90 min. (tutorial) + 2 x homework
Common Literature	**Reality in modelling** Kaiser, G. (1995). *Realitätsbezüge im Mathematikunterricht – Ein Überblick über die aktuelle und historische Diskussion*. In: G. Graumann, T. Jahnke, G. Kaiser & J. Meyer (Hrsg.), Materialien für einen realitätsbezogenen Mathematikunterricht. Band 2 (ISTRON) (S. 66–84). Hildesheim: Franzbecker. Kaiser, G., Stender, P. (2013). *Complex modelling problems in co-operative, self-directed learning environments.* In: Stillmann, G., Kaiser, G. , Blum, Brown, J.(Hrsg.): Teaching mathematical modelling. Connecting to research and practice. Dordrecht u.a.: Springer (2013) S. 277-293

	Kaiser, G., & Maaß, K. (2006). Vorstellungen über Mathematik und ihre Bedeutung für die Behandlung von Realitätsbezügen. *Realitätsnaher Mathematikunterricht vom Fach aus und für die Praxis*, 83-94. Kaiser, G., Stender, P. (2013). Complex modelling problems in co-operative, self-directed learning environments. In: Stillmann, G., Kaiser, G. , Blum, Brown, J.(Hrsg.): *Teaching mathematical modelling. Connecting to research and practice*. Dordrecht u.a.: Springer (2013) S. 277-293 **Modelling with data/stochastic modelling/levels of data analysis:** Blum, W., & Leiss, D. (2005). „Filling Up "-the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In *CERME 4–Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (Vol. 1623). Sant Feliu de Guíxois: FUNDEMI IQS–Universitat. Eichler, A., Vogel, M. (2013). *Leitidee Daten und Zufall*. Wiesbaden: Springer, S. 132-140. Gildehaus, L., & Liebendörfer, M. (2021). CiviMatics - Mathematical modelling meets civic education. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 1, pp. 167-171). Tredition. **Example tasks modelling around "Oil":** Statista (2023).Weltweiter Erdölverbrauch in den Jahren 1970 bis 2022. From: http://de.statista.com/statistik/daten/studie/40384/umfrage/welt-insgesamt---erdoelverbrauch-in-tausend-barrel-pro-tag/ (Status: 07/07/2023) Statista (2023). *Oil consumption worldwide from 1970 to 2021*. https://www.statista.com/statistics/265261/global-oil-consumption-in-million-metric-tons/ (Status: 07/07/2023) Südkurier (2022). *Gute Frage: Wie lange reicht uns eigentlich das Öl noch?*. From: https://www.suedkurier.de/ueberregional/wirtschaft/energiekrise-wie-lange-reicht-das-oel-noch;art416,11291735# (Status: 07/07/2023) Lorenz, A. (2023). When will we run out of fossil fuels? From: https://www.fairplanet.org/story/when-will-we-run-out-of-fossil fuels/#:~:text=Global%20consumption%20of%20oil%20is,reserves%20in%20about %2047%20years. (Status: 07/07/2023). **ESD Orientation Framework** Bildung für nachhaltige Entwicklung (BNE): Pädagogische Ansätze, politische Rahmenbedingun- gen und praktische Unterstützungsmöglichkeiten. (2022). Hamburg. Alsina, Á. (2022). On Integrating Mathematics Education and Sustainability in Teacher Training: Why, to What End and How?. In Controversial Issues and Social Problems

for an Integrated Disciplinary Teaching (pp. 9-21). Cham: Springer International Publishing.

ESD and modelling

Reiss, K., Ufer, S., Ulm, V., & Wienholtz (2015). Orientierungsrahmen für den Lernbereich Globale Entwicklung - G. 4.4. 1 Mathematik fachdidaktischer Teil., S. 316.

Baykal, I. & Semiz, G. K. (2020). Middle school pre-service mathematics teachers' opinions related to mathematics education for sustainability. Eurasian Journal of Educational Research, 20(89), 111-136.

Critical statistics education

Gal, I. (2002). Adults' statistical literacy: Meanings, components, responsibil-ities. *Revue Internationale de Statistique [International Statistical Re-view], 70*(1), 1–25.

Gal, I. (2019). Understanding statistical literacy: About knowledge of contexts and models. In J. M. Contreras, M. M. Gea, M. M. López-Martín y E. Molina-Portillo (Eds.), *Actas del Tercer Congreso Internacional Virtual de Educación Estadística* (pp. 1-15), Haifa.

Heymann, H. W. (1996). *Allgemeinbildung und Mathematik* (1. Aufl.). Julius Beltz.

Gal, I. (2019). Understanding statistical literacy: About knowledge of contexts and models.

Krüger, K. (2016): Statistische Grundbildung fördern. *mathematik lehren 197*, 2-7.

Example 1: Corona Pandemic

Maier-Borst, H., Martin, A. (2021). *Wie unterschiedlich die vierte Welle Geimpfte und Ungeimpfte trifft.* From: https://www.rbb24.de/panorama/thema/corona/beitraege/2021/11/unterschied-geimpft-ungeimpft-inzidenzen-krankenhaus.html (Status: 07/07/2023)

Nordström, P., Ballin, M., & Nordström, A. (2022). Risk of infection, hospitalisation, and death up to 9 months after a second dose of COVID-19 vaccine: a retrospective, total population cohort study in Sweden. The Lancet, 399(10327), 814-823.

Example 2: Age structure

Destatis.de. Bevölkerungspyramide: Altersstruktur Deutschlands von 1950 - 2060. From: https://service.destatis.de/bevoelkerungspyramide/index.html (Status: 07/07/2023)

Destatis.de. Population pyramid: Germany's age structure from 1950 - 2060 From: https://service.destatis.de/bevoelkerungspyramide/index.html#!l=en (Status: 07/07/2023)

Example 3: Employment data

Arbeitsagentur.de. Entgeltatlas - Bundesagentur für Arbeit. From: https://web.arbeitsagentur.de/entgeltatlas/beruf/128366 (Status: 07/07/2023)

	Arbeitsagentur.de. Remuneration of full-time employees again on the upswing. From: https://www.arbeitsagentur.de/en/press/en-2022-34-remuneration-of-full-time-employees-again-on-the-upswing (Status: 07/ß7/2023) **Networking** Eichler, A., Vogel, M. (2015), *Leitidee Daten und Zufall*. Wiesbaden: Springer, S.141ff. **Computers and data analysis** Eichler, A., Vogel, M. (2015), *Leitidee Daten und Zufall*. Wiesbaden: Springer, S.144ff. Cobb, P. (1998, July). Analyzing the mathematical learning of the classroom community: The case of statistical data analysis. In NOTE 328p.; For volumes 2-4, see SE 062 272-274; for the 1998 conference see ED 416 082-085. (p. 66).
Topics	(Normative) modelling Data analysis and mathematical modelling Critical statistics education and normative modelling

Session 1

The aim of the lecture is to provide a deeper understanding of the methods of descriptive statistics and how randomness in data can be handled. As part of this, mathematical modelling is introduced and connected to the concepts of data analysis. To this end, the lecture starts with the question how to describe, analyse and model reality. It refers to three possible types of reality references (Kaiser & Maaß, 2006) that are connected to mathematical modelling in general and specifically mathematical modelling cycles. Based on an example task, relevant steps of the search processes for data and methods and models to analyse those data are presented and discussed. In a second example about yeast growth, considerations are made together with the students how growth can be modelled, based on real yeast growth data. Afterwards, the introduced contents are discussed from a didactical perspective: didactic considerations on the topic of modelling with data are set up and reconnected to the ideas of data analysis and reality references. Finally, the possibilities of implementing ESD (Education for Sustainable Development) into mathematics classrooms dealing with data analysis and modelling are discussed in a short excursus.

In the tutorial, the modelling cycle according to Blum & Leiß (2005), as well as the normative modelling cycle (Gildehaus & Liebendörfer, 2021) are deepened, and students analyse and discuss given tasks with regard to their potential for normative modelling. The aim is to foster students' reflections on normative modelling by making normative components in modelling more visible and discussable. In the first exercise of the tutorial, students practice and discuss modelling in the context of growth of Covid infections using GeoGebra. Based on different growth models, different predictions on the development are to be made and reflected. Furthermore, they analyse and discuss the didactic potential for modelling with those different growth models, as well as (dis)advantages of using GeoGebra in this context.

In the second task, students assign data to the three stages of data analysis (Eichler & Vogel, 2013) and explain how they would deal with the situation with the help of the modelling cycle. Finally, the potential of the task in relation to “Teaching for sustainability / ESD” is discussed.
Both homework tasks are similar, they repeat normative modelling, here based on data of the extinction of insects, as well as based on data about carbon footprints of different food. Both exercises are then analysed and reflected in terms of their didactical potential. Finally, the students reflect on whether they would use normative modelling tasks such as the presented ones in their own future lessons. The complete session plan including literature, materials and concrete activities can be found in the appendix in English.

Background

Modelling is introduced here fundamentally from a didactic perspective and connected to data collection and analysis. Based on examples of normative modelling, the students work out and discuss the didactic potential and the relevance of a reflected handling of normative modelling in mathematics lessons. They also reflect how they themselves could introduce normative modelling into their future teaching practices.

Concrete learning activities

Lecture 1:

- Introduction to the lecture by explaining the types of reality references (Kaiser, 1995) and a quote on modelling:
    - → Defining modelling: "Modelling is about understanding a reality-related situation through the use of mathematical means, structuring it and leading it to a solution, as well as recognising and assessing mathematics in reality." (Blum & Leiß, 2009, p. 40f)
- Illustration of mathematical modelling based on a cycle by Kaiser/ Stender, 2015 and by Eichler/ Vogel, 2013
- Concluding processes of “searching relevant data, applying methods and models for analysis with the example task on oil stocks
- Students are asked questions about the example: “Consider how long the oil stock will last under a given assumption. Create further scenarios for the development of global oil consumption. Create a mathematically justified forecast of how long the reserves will last for each of the other scenarios.”
- Students should analyse the oil task: "Which elements of descriptive statistics are used“? → alternative example as supplement (further source for the question how long the oil will last)
- Further task example for clarification: this is about modelling data by means of graphs→ introduction to the example with a fictitious class discussion (initial question after the discussion: why are we still modelling then?)
- concrete application: Example of modelling population growth→ Question to students: What could be a “good” model for the given situation?
- Question: What else could be considered besides a linear function?
- Another example: Yeast growth
    - → Introduction: Advantages of yeast (Kohorst & Portscheller 1999): Task for the students: Modelling growth by graphical representation, making hypotheses, determining characteristic values and further representation of data in other ways.
- Introducing explanation of didactic considerations for modelling with data, the stochastic modelling cycle and stochastic modelling
- Introducing the normative modelling cycle

- Explanation model Data analysis and reality reference: Three stages of data analysis by Eicher/ Vogel, 2015
- Excursus ESD (Education for Sustainable Development): Explanation and mathematics as an opportunity for ESD, orientation framework for the learning area, core competences of ESD, ESD and modelling, ESD and descriptive statistics.

Tutorial 1:
- Task 1: Modelling the growth of Covid infections with GeoGebra (checking by changing parameters, adapting graph to given graph); explanation of the didactic potential for modelling with data of the task, naming didactic advantages and disadvantages for the use of GeoGebra at this point.
- Task 2: Interpretation of statistics by assigning the situation to the three stages of data analysis (orientation to the normative modelling cycle), discussion of the potential of the task with regard to "Teaching for sustainability"

Homework 1:
- Task 1: Modelling the extinction of insects with GeoGebra (adjusting the parameters so that the resulting function runs through the points as well as considering the long-term significance of the different function possibilities); explaining the didactic potential for modelling with data from the task, naming advantages and disadvantages of using GeoGebra at this point.
- Task 2: Interpretation of different statistics for the same initial question and explanation of how to deal with the situation (orientation to the normative modelling cycle), discussion of the potential of the task in relation to "Teaching for sustainability", reflection of using normative modelling tasks in lessons

Materials

You can find the lecture slides for this session as well as the exercise sheets for the tutorial and homework in the Appendix.

Session 2

The aim of the second lecture is to get to know basics of critical statistics education, as well as networking those basics with the ones on (normative) modelling from the previous lecture. Additionally, digital tools to teach and learn about critical statistics education and normative modelling are discussed.

Thus, at first, the importance of critical statistics education/statistical literacy is discussed. Components for statistical literacy are explained and a checklist for the critical examination of information is presented. Based on three different examples it is presented how statistical literacy can be promoted within the process of data analysis in mathematics classrooms and in general.

In the tutorial, students start with a short repeat from a previous lecture: they work with correlations as part of data analysis and critically reflect them for possible conclusions and statements. This connects to the actual lecture on statistical literacy and normative modelling. Accordingly, students analyse data from a newspaper article and relate and reflect their process to the components of critical statistics education from the lecture in the second task.

The homework repeats this exercise in a similar way, where another newspaper article has to be analysed and reflected based on the given checklist for statistical literacy. Finally, students are asked to create a mind map where they connect previous lecture contents, specifically on descriptive statistics towards the ideas of normative modelling and critical statistics education.

Background

Modelling is taken up here in the context of statistical literacy. In particular, the underlying assumptions of data processing are being discussed and critically reflected.

Concrete learning activites

Lecture 2:

- Introduction to the lecture, providing examples on the importance of statistical literacy.
- Presenting the components of statistical literacy according to Gal 2002
- Presenting a checklist for the critical examination of information to enhance statistical literacy.
- Conveying the goal of critical statistics education as part of maturity and democratic participation
  → linking critical statistics education with general education by naming areas of application
- Showing examples of critical statistics education with the involvement of the plenary:
- Example 1: Corona pandemic: with the help of given data, the students should assign the components.
- Example 2: Data and forecasts on population statistics - How does demographic change have an impact?: With the help of given data, the students should allocate the components.
- Example 3: Exploring interactive data representations and interpreting them in three given situations: with the help of given data, students should match the components.

Tutorial 2:

- Task 1: 4 statements are given, and the students have to determine their correlations and then assign a statement to a situation.
- Task 2: Newspaper article: Doubt about unemployment figures is to be analysed with regard to the formulated statement and the diagram presented, questions are to be answered with the help of another statistic, reference of the components to the processing carried out

Homework 2:

- Task 1: A diagram on the number of photos posted to selected hashtags on Instagram is being presented: Students check the diagram for statistical information using the checklist and explain how the diagram can be used to promote statistical literacy.
- Task 2: Create a MindMap with given terms for descriptive statistics and (normative) modelling

Materials

You can find the lecture slides for this session as well as the exercise sheets for the tutorial and homework in the Appendix.

Evaluation and comments

We implemented and evaluated this course with two cohorts of students. The embedment of mathematical modelling into the stochastics, as well as the fact that most of the students had prior knowledge on mathematical modelling from their previous studies, facilitated the integration of modelling into the course. Students quickly adapted the contents and showed no problems to understand the contents. They reported to really enjoy working with the GeoGebra environments and realistic problems and real data.

However, while our main focus would have been on the political dimension of modelling, with respect to normative modelling, the students evaluated different aspects as more important: When they reflected if they would use normative modelling tasks in their future mathematics lessons, they raised many concerns about time and effort. As advantages they named the possibly motivating effect of real-world applications as their main reason, but not the potential to discuss political dimensions. Thus, like in many cases, students understanding of the content, was not directly connected to students valuing the content. We may keep in mind, that preservice mathematics teachers' attitude towards political dimensions of mathematics and more specifically normative modelling, is not necessarily open.

Summary

Concluding, the modelling activities implemented in the different courses here provided several benefits for students, including increased commitment and motivation and critical thinking skills. However, it became also visible that normative modelling comes with several challenges for mathematics (preservice) teachers, since open tasks and discussions are sometimes in contrast to common mathematical classroom norms. Within the limited implementation sessions we had, we could not always address those challenges completely and convince preservice teachers of the need and benefits of implementing normative modelling. Still, we provided very important steps towards this direction. Our presented modules allow an easy and uncomplicated implementation into very different courses of teacher education. They all showed great comprehensiveness and provide a good opportunity to introduce students to normative modelling and to encourage them to reflect about political dimensions of mathematics.

References (those not already given in the overviews)

Besser, M., Kleickmann, T., & Vergin, J. (2020). Fachdidaktische Bewertung von Unterrichtsqualität am Beispiel mathematischen Modellierens. In H.-S. Siller, W. Weigel, & J. F. Wörler (Eds.), *Beiträge zum Mathematikunterricht 2020* (pp. 121–124). WTM.

Blum, W., & Leiss, D. (2005). „Filling Up "-the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In *CERME 4–Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (Vol. 1623). Sant Feliu de Guíxois: FUNDEMI IQS–Universitat.

Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Kaiser, G., Blum, W., Ferri, R. B., & Stillman, G. (2011). *Trends in teaching and learning of mathematical modelling.* Springer Netherlands.

Kaiser, G., Stender, P. (2013). Complex modelling problems in co-operative, self-directed learning environments. In G. Stillmann, G. Kaiser, W. Blum, & J. Brown (Eds.), *Teaching mathematical modelling. Connecting to research and practice* (pp. 277–293). Dordrecht: Springer.

Sekretariat der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland. (2015). *Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife: Beschluss der Kultusministerkonferenz von 18.10.2012*. Wolters Kluwer.

Vos, P., & Frejd, P. (2022). The modelling cycle as analytic research tool and how it can be enriched beyond the cognitive dimension. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education* (pp. 1185–1192). Bozen-Bolzano: Free University of Bozen-Bolzano and ERME. https://hal.science/hal-03759063

6.1. Didactic Commentary

Didactic reflections on “Implementing Modelling in Teacher Education”

Lara Gildehaus, and Michael Liebendörfer

Institute of Mathematics, Paderborn University, Germany

Introduction

In the previous chapter, we presented three modules, each of which was conducted as part of a course: an introductory course in mathematical thinking, a mathematical content course and a mathematics didactics course. These cover all types of lectures offered at the University of Paderborn for mathematics teaching students. While we already presented short comments on the evaluation of the modules, in the following we are providing a broader analysis about our experiences and challenges.

The central common content of our modules is normative modelling. Since this is often underrepresented in mathematics classrooms (in university as well as in school), little is known about teaching potentials and challenges. In general, teaching about mathematical modelling can be challenging for teachers as well as students (Blum & Borromeo-Ferri, 2009). Specifically, open modelling tasks with different possibilities of solutions may be challenging for teachers to handle in the classroom (Besser et al., 2020). On the other hand, mathematical modelling tasks may be advantageous as they may increase students’ motivation (Greefrath et al., 2013).

In the following, we evaluate if these general findings are also true for normative modelling and what other observations can be found.

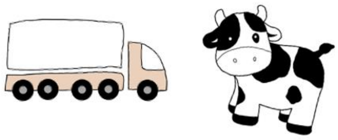
Module 1: Introduction to Mathematical Thinking

This module comprises two sessions on normative modelling within a general course introducing mathematical thinking. The course was a 6 credits course designated to the first semester in the bachelor programme for preservice mathematics teachers for lower secondary school (Grades 5–10). At the end of the course we implemented two weeks about (normative) modelling, including two lectures, two tutorials and one homework. We collected student’s homework on their developed normative modelling and several anonymous surveys during the lecture, that asked them about their experiences and reflections around mathematical modelling.

Challenges

As we mentioned in the previous chapter already, the main challenges we observed in our courses were that of choosing an adequate topic and that of applying mathematics accordingly. Altogether, 12 students worked on the homework sheet, herby 4 students decided to work on normative modelling about the climate impact of different food, 6 decided to work on the climate impact of electric cars and 2 students worked on their own topic (distributing state money to different sports). Thus, it was very helpful for them to have some suggested topics available. Those two students who opted for their own topic were also known as high-achieving students in the course. However, from a mathematical perspective none of the students used more than elementary algebra when modelling. More concretely, they used proportional calculations based on average means they had researched (see Figure 1 for an example). Thus, they were not able to apply their mathematical expertise, they should have gained during the semester, for their normative modelling. When introducing normative modelling, we recommend giving students guidelines in terms of possible questions and aims to model, even though this limits the open character of the modelling task. Furthermore, it should not be underestimated that applying mathematical content on an open and self-selected problem seems to be challenging for students, specifically for first-year students like we had in our implementation. Concrete examples that also include advanced mathematics may be helpful.

Mathematical model and mathematical results



- a cow produces around 191 liter of methane per day: 5,348 kg CO2
- Soya for feed = 4,500,000t soya per year in Germany: 2,500.00kg CO2 per year
- Transport distances = around 20,400 km (rule of thumb 130.35 g CO2/km): 2659.14 kg CO2
- Methane production of a cow: around 219 kg of pure meat→ $Value\ of\ CO2/219$kg = 24,42 kg CO2 per day→ multiplied by 365 days: **8913.33 kg CO2 per year per kilo**
- soya fed: $value\ of\ CO2/\ 80\ million$ (data refer to the whole of Germany) → **0,003125 kg emissions** per person by feeding soya per year
- Transport: $2659{,}14\ kg\ CO2/219\ kg\ meat$ → **12,14 kg CO2 emissions per kilo** due to transport
- All results must be multiplied by 60

Figure 1: Student work on the carbon footprint of vegetarians and non-vegetarians

Potentials

During and at the end of the second lecture, we asked students the following questions in an anonymous online survey:

1. Which role do you think mathematics play for politics?
2. Which role do you think mathematics lessons play for a critical literacy ("Allgemeinbildung") of students?
3. Did your attitude about the relevance of mathematical modelling change during the last two weeks? If so, how did it change?

In total, 13 students answered those open questions. In question 1, most students answered with "it is (very) relevant" or similar. Only a few of them provided examples or argumentations, such as "mathematics is relevant for political decision making", "mathematics calculates probabilities", "mathematics promotes rational thinking", "mathematics provides statistics". For Question 2, however, the answers were similarly vague. Most of the students stated, that mathematics plays an important role, "because it is an essential competence, e.g., for later jobs, life in general", only some students described, that mathematics would promote specific "thinking, that would be relevant to understand and reflect many topics". Regarding Question 3, all students agreed that their attitude about the relevance of modelling had changed. Given explanations were that they "gained a more critical perspective", that they "find modelling more relevant now in general", that they "reflected on the role of assumptions in modelling" and that they "reflected on the presentation of modelling results".

In general, our implementation showed high potential to let students reflect about modelling, normative modelling and the role of mathematics for political decision making in general, even though some of their reflections were still vague, what may have been based on the rather short implementation. We may note though, that even though the survey was conducted anonymous, it may have had an impact, that it took part during the lecture and right after the modelling implementation.

Moreover, we collected informal oral feedback from the students during lectures and tutorials. Within this they reported that the modelling tasks were motivating, and they enjoyed bringing in their own interests.

Module 2: Elements of Mathematics

Module 2 includes a short, homework-based implementation on normative modelling within a general course on formal mathematics. At Paderborn University, this course provides 6 credits points for master students. It is designated to third semester preservice mathematics teachers for primary school (Grades 1–4).

We collected and analysed student's homework where they developed their own modelling tasks for teaching in primary school and also reflected on the didactic potential of their task. Furthermore they discussed the relevance of normative modelling for primary school.

Challenges

In total, 17 students handed in their homework with their own developed modelling task. However, when analysing their developed modelling tasks, we found that almost all students struggled to provide an open character in their modelling tasks. Instead, they developed modelling tasks that were loaded with implicit or explicit information to shape students' assumptions. One example is presented in Figure 2. Here, it is clearly indicated who contributed what to the eating and living costs, for example the number of rooms used, and the average amount of eaten food is presented. This may indicate an exact solution, that covers all of the given information.

Task

A holiday cottage in Denmark is rented by Lena's family and other relatives. They all travel from different cities in Germany and are always very much looking forward to the holiday. At the end of the holiday, the 10-year-old Lena and her twin brother Matz want to calculate fairly who has to pay how much, because they have a lot to think about in the process.

Spending

Cost of the house for 7 days: 1300 euros

Cost for all meals for all 10 people for 7 days: 450 euros

In total 10 people took part in the holiday

- Family 1: Lena, Matz, Mum, Dad
  - all were in the holiday cottage the whole time
  - they had two double rooms
- Family 2: Aunt Kathrin and Uncle Hugo
  - they arrived one day later
  - they had a double room
  - Uncle Hugo usually eats twice as big portions as everyone else
- Family 3: Grandma Martha
  - was in the holiday cottage all the time
  - she had a single room
  - Grandma Martha usually eats very little because otherwise she gets stomachache
- Family 4: Aunt Heike, Uncle William and Baby Lilian
  - they were in the holiday cottage the whole time
  - they had a double room with a baby cot

1) Homework: Think about how Lena and Matz can distribute the costs. Which family has to pay how much? Give reasons!

Figure 2: Normative modelling task developed by a student

In the reflection about the implementation, more than half of the students provided exactly one “obvious” solution to their developed modelling tasks, instead of reflecting on the openness and potential to discuss different solutions. While they technically did provide different “possible solutions” it was often the case, that these “possible solutions” seemed very unlikely, given the provided task and information. For example, with regard to the task in figure 2 it was written that “one solution could be to equally distribute the costs among all people”. Even though the students tried to provide “open modelling tasks” in theory, most of them had practically “one obvious solution” in mind, hindering a truly open discussion. When introducing the development of normative modelling tasks to students, we should thus keep in mind that mathematical tasks often come with “one right solution” that is promoted, and preservice mathematics teachers may not be used to generating open tasks.

Potentials

Within the homework as well, we asked students to discuss the following questions:

1. Briefly summarise in your own words why normative modelling should (or should not) play a role in mathematics education.
2. Discuss whether normative modelling is also relevant in mathematics education in primary school. What is the argument in favour? What are the arguments against?

Overall 26 students answered those questions. The first question was often answered very general again, with "It should play a role, because it is a relevant for a critical citizenship/to understand the world" or similar. However, students also mentioned "It should play a role because students can learn about different solutions to one problem" and "students can learn how to discuss with each other". Thus, our students seemed to value normative modelling in specific and a political dimension of mathematics in general, even though some of them also discussed further advantages.

Regarding the second question, the arguments in favour were very similar to those given in question 1, some students also added "real world applications as motivating" and "gaining problem solving skills" as further advantages. Arguments against normative modelling in primary school were that "students may not have enough knowledge" or "tasks may be too complex" and thus "students could be overloaded", "students could lose focus", "students could become frustrated". Several times it was also mentioned that there would not be enough time for normative modelling during the general lessons. Specifically the value of "learning about different solutions" and "learning how to discuss with each other" may seem contradictory, given the fact that our students did not truly develop open modelling tasks. Still, they seemed to reflect honestly about normative modelling and did try to provide open tasks. We discuss this problematic of "openness is not always openness" further in the general section.

Again, within informal oral communication many students described developing their own normative modelling task and reflect how to apply this into their future lessons as very motivating and interesting.

Module 3: Normative Modelling Within Teaching Stochastics

In the course "Teaching Stochastics", we implemented two sessions on (normative) modelling. The general goal of this course is that students acquire profound knowledge about stochastics from a didactical perspective. At Paderborn University, this course is part of a 15 credits points module for bachelor programs (7.5 points are designated to the "Elements of Stochastics", and 7.5 points to the "Teaching of Stochastics"). It is designated to fifth semester preservice mathematics teachers for lower secondary school (Grades 5 –10). Students attended two lectures, two tutorials and handed in two homework on data analysis, mathematical (normative) modelling and critical statistics education.

Challenges

Within their second homework, at the end of the (normative) modelling implementation, students answered the following question:

1. Explain whether, you could later use such tasks on normative modelling that you have learned about in this lecture and the exercises in your teaching. What speaks in favour of this from your personal perspective? What is against it?

In total, 36 students discussed this question. While they named several arguments in favour of using normative modelling task, many of them also mentioned several challenges and disadvantages. Those were for example general concerns that there would “not be enough time” and that normative modelling is “not explicitly part of the curricula”. One may “lose the focus on mathematics”, specifically, when “researching data in the internet”. Furthermore students also mentioned that normative modelling may require a lot of skills, that students not necessarily bring, such as “handling big data sources”, “researching on the internet”, “understanding complex situations” and “discussing”.

While these all seem like reasonable concerns, we also found some students to be rather critical about a political dimension of mathematics. They mentioned that mathematics and more concretely the mathematics teacher may “lose their neutrality” when teaching about normative modelling tasks. Specifically in the context of climate change they stated that “every voice should be heard” and one should be aware “not to blame specific groups, e.g. non-electrical car drivers”. Thus, those students seemed to distance themselves from a political dimension in mathematics (education).

Potentials

Students also named several advantages they saw in normative modelling tasks. Most of them mentioned the “real-world connection” and “authenticity”, as well as a general “motivation” and the possibility to foster specific competences. In general, most students concluded their argumentation that they could imagine implementing at least some normative modelling tasks into their future teaching. Still, we have to note, that the potential of discussing a political dimension of mathematics based on normative modelling was not mentioned as an advantage (but disadvantage as seen above). We discuss this further in the following section.

General Comments

After reviewing those different experiences and evaluations we may conclude some general comments on an abstract level.

Normative Modelling Requires Knowledge and Attitude on Different Levels

Our first observation is, that learning in the context of normative modelling requires not only to gain knowledge, but also to reflect one’s own attitudes. Thus, if students do either not have enough knowledge or an open enough attitude, it seems to be very hard to discuss normative modelling with them.

Moreover, this duality of knowledge and attitude can be observed on two different levels. The first level is the individual level, where in our case preservice teachers learn about normative modelling. The second level is a more collective level, where teachers teach about normative modelling. Here again, they do not only need specific knowledge, but an attitude that values a political dimension in mathematics.

Normative Modelling Cannot (yet) be Assessed

The identified duality and different levels above may explain, why it seems very hard to actually assess or rate normative modelling “competencies”. Even though most of our students reported to value a political dimension of mathematics after our implementations, for example of Module 1, it remains unclear to what extent. Their given answers indicated different levels, because some of them were rather vague, e.g. “mathematics is important in life”, while others showed concreter and deeper level of reflection, e.g. “normative mathematical modelling may unreflectively be taken as objective results that play an important role in political decision-making processes, thus it is important to identify and discuss assumptions in normative modelling”.

Similarly, in the second module, where students were requested to develop their own normative modelling tasks, we argued that they did not truly develop open tasks, while they themselves described these as open. Thus, assessing normative modelling is obviously very normative in such a way that we can hardly measure to what extent students gained knowledge or changed their attitudes.

Normative modelling challenges existing classroom norms in mathematics

In line with our observation about “openness” in mathematical modelling tasks, normative modelling tasks seem to challenge given mathematical classroom norms. This is not only the case for the norm that “mathematical tasks have one concrete answer”, but also for the objectivity and unambiguity. This became apparent, for example, when some students in Module 3 mentioned that mathematics teachers shall not lose their objectivity in the classroom.

We thus finally conclude: Taking normative modelling seriously often requires open discussions in classrooms that are not used to open discussion (at least, in mathematics).

References

Besser, M., Kleickmann, T., & Vergin, J. (2020). Fachdidaktische Bewertung von Unterrichtsqualität am Beispiel mathematischen Modellierens. In H.-S. Siller, W. Weigel, & J. F. Wörler (Eds.), *Beiträge zum Mathematikunterricht 2020* (pp. 121–124). WTM.

Blum, W., & Borromeo-Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Greefrath, G., Kaiser, G., Blum, W., & Borromeo Ferri, R. (2013). Mathematisches Modellieren – Eine Einführung in theoretische und didaktische Hintergründe. In R. Borromeo Ferri, G. Greefrath, & G. Kaiser (Eds.), *Mathematisches Modellieren für Schule und Hochschule* (pp. 11–38). Wiesbaden: Springer Fachmedien.

CHAPTER 7

Stories About Scientific Research

Florin Fesnic

Center for the Study of Democracy, Department of Political Science, Babeş-Bolyai University Cluj, Romania

Introduction

Civic and scientific education are the cornerstones of a successful democratic society. Both play a pivotal role in fostering social and economic wellbeing. By empowering individuals with the knowledge to actively participate in their democracy, make informed decisions, promote innovation, embrace diversity, and contribute to economic prosperity, these forms of education create a virtuous cycle that sustains a thriving nation. Governments, educators, and communities must recognize and invest in the importance of civic and scientific education to build a brighter and more prosperous future for all. Only through an educated and engaged citizenry can we overcome challenges, uphold democratic values, and foster a society that prioritizes the collective wellbeing of its members.

Even though the focus in this course is on offering examples of scientific research to illustrate the logic of scientific inquiry, the normative (i.e., civic) aspects are also present, implicitly or, in some cases, even explicitly. That is to say, an important criterion to select the examples discussed in this class, in addition to their scientific value, was to connect them to the social, economic, political and/or medical wellbeing of citizens and societies. One of the most telling examples in this respect will be the discussion of how societies and governments have responded to the Covid-19 epidemic and the crucial role played in this context by scientific and civic education.

The potential public for this class (or, rather, for the forthcoming book based on it) is wide-ranging. The initial public for the course, intended to be offered as a semester-long elective, are undergraduate students in social sciences (e.g., political science or sociology). It can be either a standalone class or an addition to more traditional introductory courses in research methods. Nonetheless, the public can be much broader than this, such as students from various fields (e.g., medicine), high school students wanting to continue their education in universities, or the general public. One of the most important messages of this class is that, while the examples discussed are very diverse, the logic of scientific research is very similar across scientific fields.

Infrastructure of the Course

Knowledge Goals

These are the interrelated knowledge goals of the course:

The main goal of the course is to improve the scientific literacy as well as the civic knowledge and consciousness of students

1) A closely related goal is to make students aware of the unifying logic of scientific inquiry across very diverse fields (e.g., political science and medicine)
2) Depending on the context and the students' degree of previous exposure to research methods, and additional benefit can be making them aware of the basic concepts (e.g., independent and dependent variables) or methods (e.g., linear regression) in scientific research

After completing the course, the students should be better equipped to understand and critically assess, from both an empirical (i.e., scientific), as well as normative (civic) perspective some of the important current debates in the public space (e.g., the reality, the causes and consequences of causes climate change, or the benefits of vaccination).

Teaching, Infrastructure, Requirements, and Assessment

As of right now, this course was never taught as a formal class. It is taught either as an elective class or as a supplement to a more formal introduction to research methods class. In this latter case, the material for/from this class can be used during seminars, to illustrate how the concepts and methods learned during lectures (e.g., the experimental method, or linear regression) can be used to analyse real-world examples. Obviously, depending on the specific situation, teaching, the requirements and the assessment can, and should, be adjusted accordingly. Below is just an example of how I would do these if I was teaching this as a standalone class.

The class is designed in a seminar-like format. Students will be evaluated based on (1) active class participation (20% of the final grade)—including, but not limited to, their answers to questions provided in advance, and (2) a final exam (80% of the final grade)—this will consist of three short essays based on a selection from the list of question provided in advance (see below). All the required readings will be provided in advance, either as pdf files (via email, as attachments) or, when feasible/available, via links to web. Additionally, after each class, I will provide a PowerPoint with the lecture/main points discussed in class, as well as a YouTube video with the recording of the discussion. On this latter point, I must note that (1) the class (in this specific case) will be an elective, and therefore students will not have to take the class, (2) only my face will be visible in the recording. Finally, all the datasets used throughout the semester will also be available to students.

Course Roadmap: Structure and Content of Learning Units (weeks)

Week 1

Title	**Introduction. What is the scientific method?**
Duration	Two hours
Organization	Lecture and discussion
Literature	Eric J. Johnson, Daniel Goldstein. 2003 (Nov 21). "Do Defaults Save Lives?" *Science* 302, 5649, pp. 1338-1339. DOI: 10.1126/science.1091721. Wynn, Charles M., and Arthur W. Wiggins. 2017. *Quantum Leaps in the Wrong Direction: Where Real Science Ends... and Pseudoscience Begins*, 2nd ed. Oxford University Press (Chapter 1, "The Road to Reality: Scientific Method," pp. 1-10).

Topics	The scientific method (versus "common sense")
Questions	What is the scientific method? How is it different from "common sense"? Why is it important that the public has at least a basic understanding of science and the scientific method?

Weeks 2 & 3

Title	**Brief introduction to basic research methods and methodological concepts**
Duration	Two to six hours (as needed) **Note:** Depending on how much previous exposure to research methods students had, this section can be extended, shortened, or skipped altogether
Organization	Lecture and discussion
Literature	Hague, Rod, Martin Harrop, and John McCormick. 2019. *Comparative Government and Politics: An Introduction*, 11th ed. Red Globe Press (Chapter 3, "Comparative Methods: An Overview".
Topics	Basic concepts (e.g., dependent and independent variable) and methods (e.g., linear regression)
Questions	What are the most basic concepts and methods in scientific research? Why and how are they useful?

Week 4

Title	**Blind Retrospection: Why Shark Attacks Are Bad for Democracy**
Duration	Two hours
Organization	Lecture and discussion
Literature	Achen, Christopher H., and Larry M. Bartels. 2003. "Blind Retrospection: Why Shark Attacks Are Bad For Democracy." Vanderbilt University, CSDI Working Paper 5-2013, available at <https://www.vanderbilt.edu/csdi/research/CSDI_WP_05-2013.pdf>.
Topics	Prospective versus retrospective voting; "blind retrospection"
Question	Do voters occasionally "punish" incumbents in elections for "acts of God" (i.e., bad events that are outside the incumbents' control)?

Week 5

Title	**Richer *Americans* are more likely to vote Republican, but richer American *states* vote more Democratic**
Duration	Two hours
Organization	Lecture and discussion
Literature	Gelman, Andrew, Boris Shor, Joseph Bafumi, and David Park. 2007. "Rich state, poor state, red state, blue state: What's the matter with Connecticut?" *Quarterly Journal of Political Science* 2, 345-367, available at <http://www.stat.columbia.edu/~gelman/research/published/rb_qjps.pdf>.
Topics	Logical/methodological fallacies (ecological v. individualistic); linear regression (slope)
Question	Can we always assume that a result obtained at one level of analysis (e.g., invididual) will be replicated at the aggregate (e.g., state) level?

Week 6

Title	**A puzzle of electoral politics in Ukraine (the 1999 v. 1994 presidential election)**
Duration	Two hours
Organization	Lecture and discussion
Literature	Fesnic, Florin. 2017. "Making Methodology Fun." Poster presented at the Annual Meeting of the American Political Science Association, San Francisco, CA. Available at <https://apsa2017-apsa.ipostersessions.com/default.aspx?s=60-59-1B-88-2E-8F-28-E9-CC-54-F2-31-3A-2D-B8-AD>.
Topics	Left versus right; nationalism; ideological dimensions
Question	Why was Kuchma's share of regional vote in the 1999 Ukraine presidential election ***negatively*** correlated with his 1994 share of the vote?

Week 7

Title	**Alabama paradox: comparing formulas for proportional representation**
Duration	Two hours
Organization	Lecture and discussion
Literature	The Institute for Mathematics and Democracy. "Apportionment." Available at <https://mathematics-democracy-institute.org/apportionment/#>. Accessed August 8, 2023.
Topics	Proportional representation: monotonic versus non-monotonic formulas; highest averages versus largest remainders
Questions	Are there non-monotonic proportional representation formulas? If so, why? What are then the empirical consequences? What are the normative consequences?

Week 8

Title	**Electoral reform in Hungary**
Duration	Two hours
Organization	Lecture and discussion
Literature	Scheppele, Kim Lane. 2014 (May 26). "Hungary and the End of Politics: How Victor Orbán launched a constitutional coup and created a one-party state." *The Nation*, <http://www.thenation.com/article/179710/hungary-and-end-politics#>. Scheppele, Kim Lane, Miklós Bánkuti, and Zoltán Réti. 2014 (April 13). "Legal But Not Fair (Hungary)." *The Conscience of a Liberal*, <http://krugman.blogs.nytimes.com/2014/04/13/legal-but-not-fair-hungary/>.
Topics	Majoritarian v. proportional electoral systems; malapportionment; gerrymandering
Questions	What is gerrymandering? What is malapportionment? Is it fair to use these in institutional design?

Week 9

Title	**Electronic voting machines and infant health (Brazil)**
Duration	Two hours
Organization	Lecture and discussion
Literature	Fujiwara, Thomas. 2017 (October 24). "Political inclusion and development outcomes: Brazil introduces electronic voting." *VoxDev*, <https://voxdev.org/topic/technology-innovation/political-inclusion-and-development-outcomes-brazil-introduces-electronic-voting>. *Recommended:* Fujiwara, Thomas. 2015. "Voting Technology, Political Responsiveness, and Infant Health: Evidence from Brazil." *Econometrica* 83, 2: 423-464. Available at <https://www.princeton.edu/~fujiwara/papers/elecvote_site.pdf>.
Topics	Natural experiment; treatment; treatment group; control group
Questions	Based on the evidence presented by Fujiwara, but also on your intuition, how persuasive do you find his argument (introduction of electronic voting machines → infant health)?

Week 10

Title	**Handwashing and puerperal fever**
Duration	Two hours
Organization	Lecture and discussion
Literature	Carey, Stephen S. 2011. *A Beginner's Guide to Scientific Method*, 4th ed. Boston, MA: Wadsworth Cengage Learning (Chapter 1, "Science," pp. 2-5).

	Last, John M. 2002. "Semmelweis, Ignaz," in Scheppele, Breslow, Lester (ed.), *Encyclopedia of Public Health.* Macmillan (pp. 1087-88). Wikipedia, "Ignaz Semmelweis", <https://en.wikipedia.org/wiki/Ignaz_Semmelweis>.
Topics	Natural experiment; treatment; treatment group; control group
Questions	Do you see any similarities between Fujiwara's "natural experiment" discussed last week and Semmelweis' work? Can we talk about an experiment here? If so, is it a "proper" experiment, or a "natural experiment"? Why?

Week 11

Title	**John Snow's "Ghost Map" of cholera and his "grand experiment" of 1854**
Duration	Two hours
Organization	Lecture and discussion
Literature	Frerichs, Ralph R. "Snow's Grand Experiment of 1854." <https://www.ph.ucla.edu/epi/snow/grand_experiment.html>. National Geographic. "Mapping A London Epidemic." <https://www.nationalgeographic.org/activity/mapping-london-epidemic>. Montelpare, William J., Emily Read, Teri McComber, Alyson Mahar, and Krista Ritchie. *Applied Statistics in Healthcare Research* (Ch. 7, "John Snow and the Natural Experiment"). <https://pressbooks.library.upei.ca/montelpare/chapter/john-snow-and-the-natural-experiment/#chapter-172-section-1>.
Topics	Natural experiment; treatment; treatment group; control group
Questions	Do you see any similarities between Fujiwara's "natural experiment" discussed two weeks ago, Semmelweis' work discussed last week, and John Snow's work? Can we talk about an experiment here? If so, is it a "proper" experiment, or a "natural experiment"? Why?

Weeks 12-14

Title	**Civic education, scientific education, and response to Covid-19**
Duration	Six hours
Organization	Lecture and discussion
Literature	Fesnic, Florin. 2022. "Three Toxic Ingredients Making the COVID-19 Pandemic Worse: United States and Romania Compared." Presented at the Annual Meeting of the Midwest Political Science Association, Chicago, IL. **Recommended:** Claessens, Michel. 2021. *The Science and Politics of Covid-19*. Springer. Lilleker, Darren (ed.) 2021. *Political Communication and COVID-19 Governance and Rhetoric*. Routledge.

	Spiegelhalter, David, and Anthony Masters. 2021. *Covid by Numbers: Making Sense of the Pandemic with Data*. Pelican.
Topics	Scientific education; civic education; public; elites; media; Covid-19
Questions	How much of an impact do you think scientific and civic education had on how various countries responded to Covid-19?

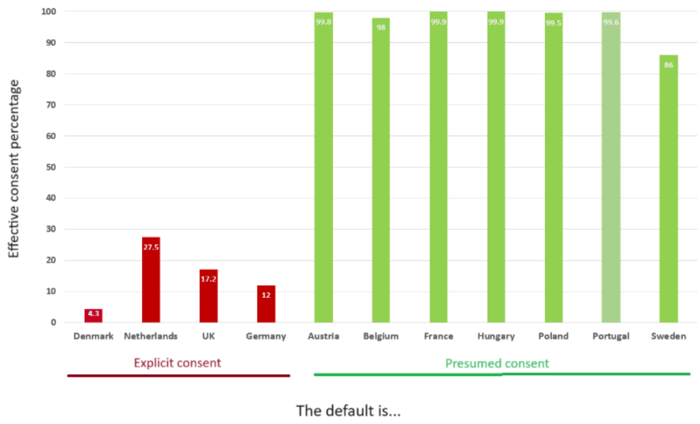
Brief summaries of weekly topics

Week 1: Introduction. What is the scientific method? Example: "Do defaults save lives?"

A very important and widely known cross-national study showing that an apparently innocuous bureaucratic detail (i.e., whether the national legislation makes citizens organ donors by default or not) can have a large impact on the share of the population who are actually potential organ donors. Therefore, the answer to the question above is definitely "yes".

Figure 1

Effective consent rates, by country: Explicit consent (opt-in) vs presumed consent (optout)



Note. The figure is adapted from Johnson and Goldstein (2003).

Weeks 2–3: Brief Introduction to Basic Research Methods and Methodological Concepts

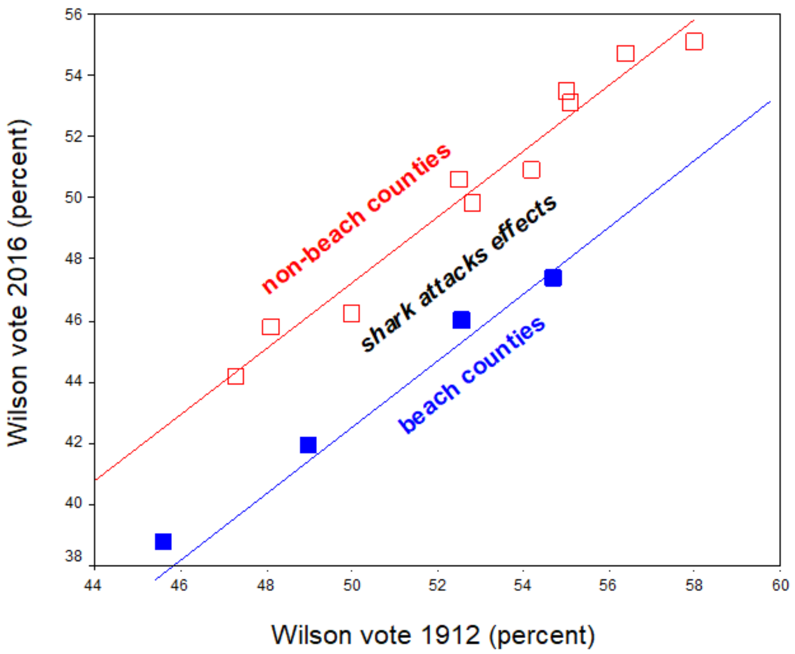
This section of the class is really important for students who did not have previous exposure to research methods, but less so for those who already took classes in research methods (or those who take this as a seminar or a completion for a research methods class). Accordingly, this section can be extended, shrunk, or skipped altogether, as the instructor sees fit.

Week 4: Blind Retrospection: Why Shark Attacks Are Bad for Democracy

The concept of retrospective voting is widely used in the voting behaviour literature. Essentially, it is the notion that voters evaluate incumbents based on their performance. If a voter is happy with the incumbent candidate's performance, they will vote for them. If not, they would not. However, it would be less intuitive if voters "punish" incumbents for events such as natural disasters that are totally out of the latter's control. Nonetheless, this is exactly what, according to Achen and Bartels, happened in the 2016 US presidential election, when voters in coastal regions in New Jersey "punished" the incumbent president Woodrow Wilson for a string of shark attacks that occurred in the area in the summer preceding the election—therefore, illustrating an instance of "blind" retrospection.

Figure 2

County-level vote for Woodrow Wilson in New Jersey, 1912 and 1916: beach vs non-beach counties



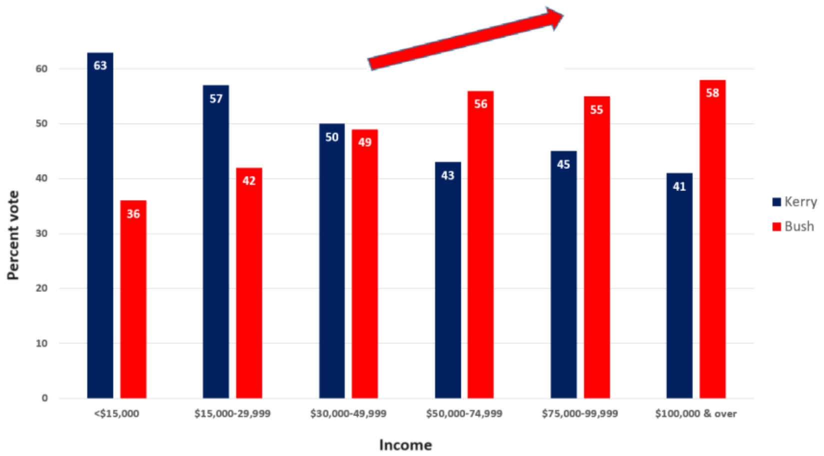
Note. The figure is adapted from Achen and Bartels (2003).

Week 5: Rich Americans Vote Republican, Rich American States Vote Democratic

The individualistic fallacy happens when a researcher assumes that a result seen at the individual level is replicated at the aggregate level (a less intuitive fallacy than its ecological counterpart). Voting in the US offers a telling example. Even though at the individual level richer Americans are more likely to vote Republican, the highest level of electoral support for the Republican party is found at the aggregate level, in poorer American states (Figure 3 vs Figure 4):

Figure 3

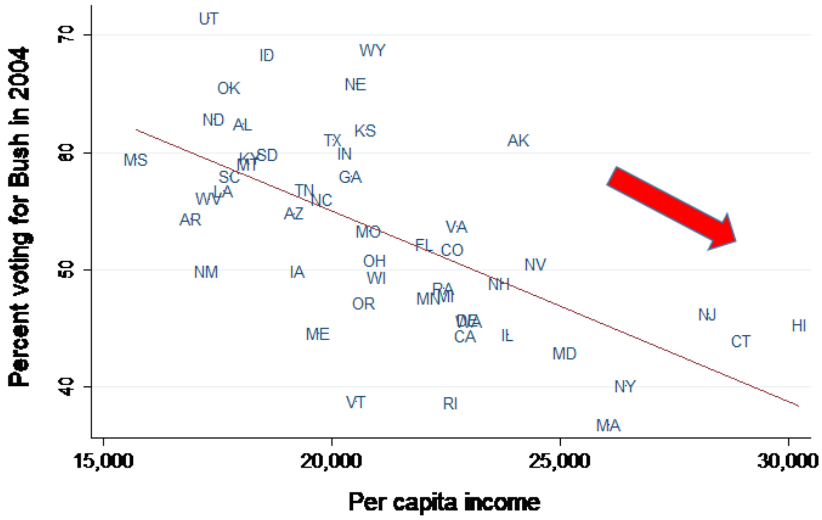
The probability of voting Republican (Bush) as a function of personal income in 2004



Note. The figure is made by the author (Florin Fesnic) using Roper Center exit poll data.

Figure 4

Average income in the state and state-level Republican (Bush) vote in 2004

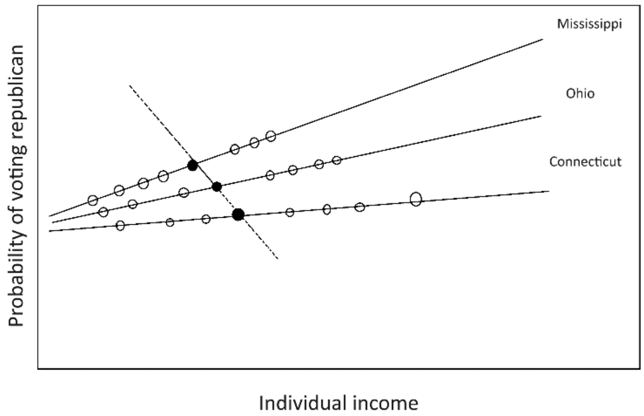


Note. The figure is made by the author (Florin Fesnic) using data from Wikipedia.

Andrew Gelman offers an explanation for this apparent paradox. If we use survey data to fit regression lines in graphs with income on the X axis and probability to vote Republican on the Y axis for various states, we see that the poorer the state, the higher (and positive) the slope is. When we aggregate the results, the pattern is reversed (see Figure 5, but also Figure 4).

Figure 5

The probability of voting Republican as a function of personal income is very different in poor vs. rich states



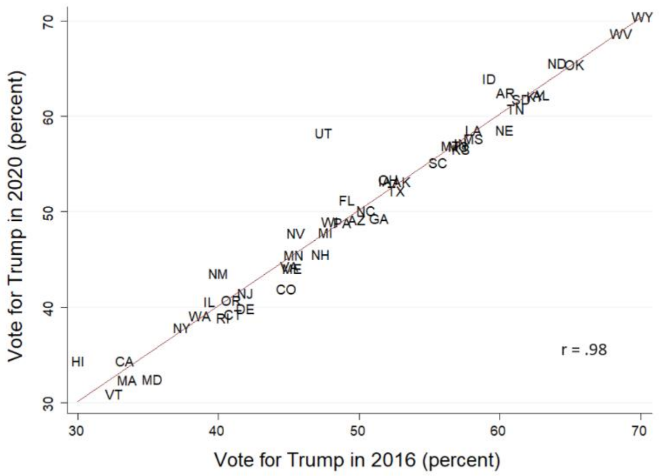
Note. The figure isa dapted from Figure 4 in Gelman et al. (2007).

Week 6: Why was Leonid Kuchma's Share of Regional Vote in the 1999 Ukraine Presidential Election *negatively* Correlated With his 1994 Share of the Vote?

Typically, regional support for a candidate in a given election is a very good predictor of his support in the following election—i.e., if V_e and V_{e+1} are candidate's X shares of the vote in elections e and e+1, then V_e and V_{e+1} will be highly and positively correlated.

Figure 6

Average income in the state and state-level Republican (Bush) vote in 2004

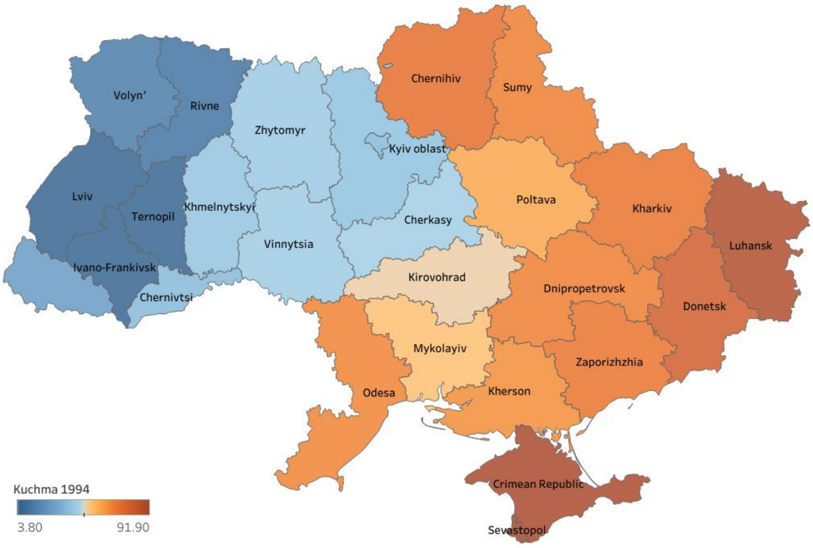


Note. The figure is made by the author (Florin Fesnic) using data from Wikipedia.

However, what happened with Leonid Kuchma's vote in the 1990's in Ukraine was the opposite: the correlation between V_{1994} and V_{1999} was -.80! The electoral map of Kuchma's results in 1999 was almost a mirror image of his results in 1994 (compare Figures 7 and 8):

Figure 7

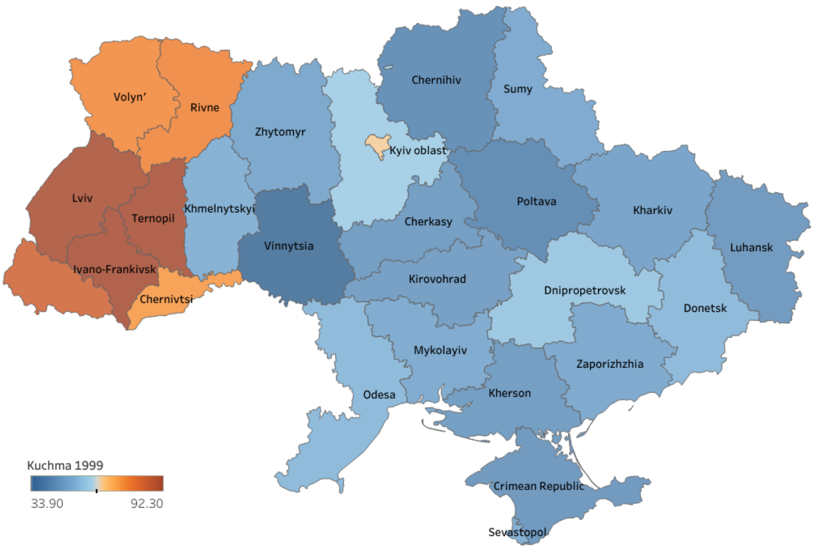
Regional vote for Leonid Kuchma in Ukraine's 1994 presidential runoff



Note. The figure is made by the author (Florin Fesnic) using data from Wikipedia.

Figure 8

Regional vote for Leonid Kuchma in Ukraine's 1999 presidential runoff

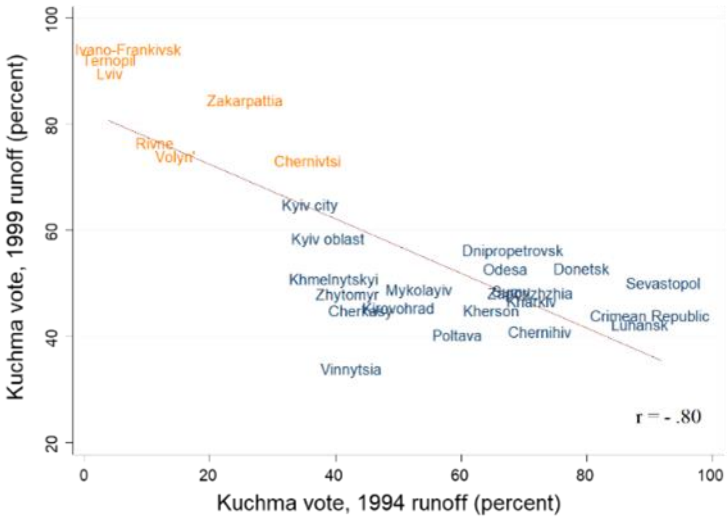


Note. The figure is made by the author (Florin Fesnic) using data from Wikipedia.

Figure 9 offers a statistical summary of the results of the two previous elections:

Figure 9

Regional vote for Leonid Kuchma in Ukraine's presidential runoffs, 1999 vs 1994



If we try to analyse these elections in left-right terms, things make little sense. Kuchma was a centrist or centre-right candidate, while his opponents in both elections were on the left:

Figure 10

Left-right position of main presidential candidates in Ukraine, 1999 vs 1994

It is only when we take into account the second, more important line of conflict in Ukrainian politics, pitting Ukrainian nationalists against Russophiles, things start to make sense (Figure 11):

Figure 11

A two-dimensional ideological mapping of the main presidential candidates in Ukraine, 1999 & 1994

Week 7: Alabama Paradox: Some Proportional Representation Systems are Non-Monotonic!

One reasonable expectation for an electoral system, especially for any form of proportional representation, is to be monotonic. Of the two major types of PR formulas, the highest averages and the larger remainders, only the former are monotonic. This was discovered in the US in the late 19th century, at a time when it was using a form of largest remainders formula (the Hamilton Apportionment method) to allocate seats in the House of Representatives. Using this formula, in a House with 299 seats, Alabama would have got eight seats. If the size of the House were to be increased to 300 seats, Alabama would have received only seven seats.

Week 8: Malapportionment and Gerrymandering in Hungary

Malapportionment (having single-member districts with widely unequal number of voters) and gerrymandering (the design of constituencies to give an advantage to a certain political party or candidate) are, arguably, serious problems for any democratic polity where they are present. Things are much worse when they become tools in the hand of an authoritarian leader seeking to consolidate their power, as it is the case with Viktor Orbán in Hungary. Up until 2010, Hungary was using an electoral system roughly similar with the German one, but less proportional. After gaining power, Orbán made two major changes to the electoral system, (i) redesigning the borders of single-member electoral districts and (ii) shifting the share of SMD's vs PR seats in favour of the former. To these two we may add a third, equally important determinant of electoral outcomes, namely, (iii) malapportionment. Even though this was always technically present, it only became important after the electoral reform (see below, Figure 12 vs Figure 13).

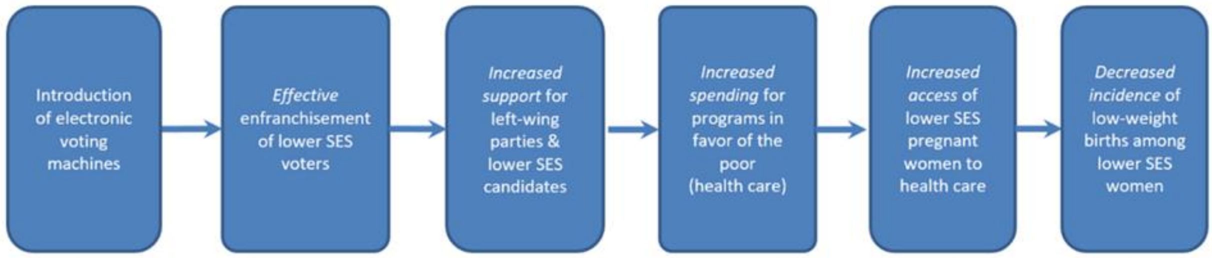
Week 9: Electronic Voting Machines and Infant Health

Any electoral reform that results in a significant increase in voter turnout (not just “formal” turnout, but also “effective” turnout) is an important step forward in a democracy. By “effective” I mean the share of the electorate who not only does vote, but has their votes counted. Brazil offers an example of a country where, due to high illiteracy rates and a complicated electoral system, a large share of the votes cast were discarded. The introduction of electronic voting machines in the 1990’s had a significant, and positive, effect: the share of invalid ballots dropped dramatically. Moreover, the introduction of e-voting machines was gradual, thus creating the setting for a wonderful natural experiment.

In addition to significantly reducing the share of invalid votes, this electoral reform had some other positive, though quite unexpected, results. It led to significant wins for left-wing, socially reformist parties and candidates. This, in turn, translated into policies that were beneficial for lower class Brazilians. One spectacular example is the significant progress made with respect to the health of infants (more specifically, an important drop in the share of underweight new-borns in lower class families):

Figure 15

From electronic voting machines to infant health



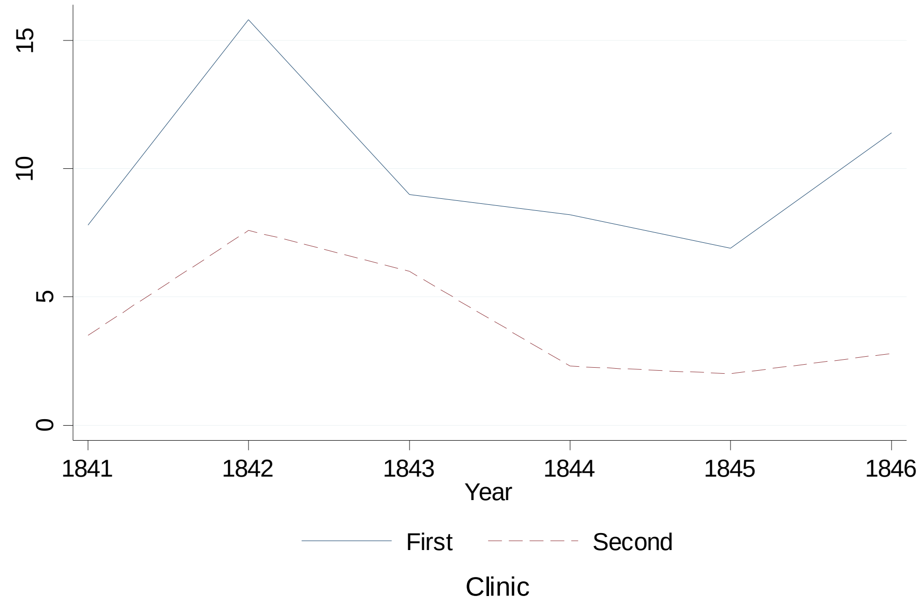
Note. The figure is made by the author (Florin Fesnic), based on a summary of Fujiwara (2015, 2017).

Week 10: Ignaz Semmelweis: Hand Disinfection and Puerperal Fever

Childbed fever is a very serious, often deadly, disease that was rampant among one of the serious diseases among women following childbirth or miscarriage up until mid-19th century. The root cause is infection caused by uncleanness. In the 1840s, the incidence of puerperal fever and deaths associated with it was significantly higher at the First Maternity Clinic in Vienna, which offered teaching service for medical students, than at the Second Maternity Clinic next door, which offered instruction for midwives.

Figure 16

Deaths from puerperal fever in Vienna clinics, 1841–1846

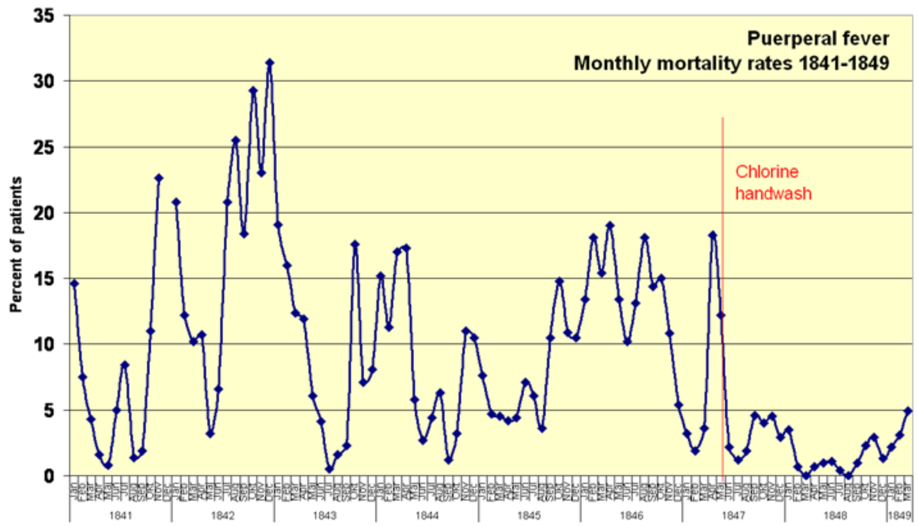


Note. The figure is made by the author (Florin Fesnic) using Wikipedia data (“Ignaz Semmelweis”).

Doctor Ignaz Semmelweis, who was working in the First clinic at the time, had the intuition that this was the result of poor hygiene. Doctors and students at the First Clinic were performing autopsies, then examining women without washing their hands. When Semmelweis introduced compulsory disinfection of hands with chlorine, the results were spectacular.

Figure 17

Deaths from puerperal fever in the First Vienna clinic, 1841–1849



Note. The figure is taken from Wikipedia, “Ignaz Semmelweis”.

Week 11: John Snow's "Ghost Map" of Cholera and his "Grand Experiment" of 1854

In mid-19th century, the prevalent theory about the cause of cholera, even among doctors and scientists, was "miasma theory." Doctor John Snow, one of the founding fathers of public health and germ theory, had the intuition that the cause was something in the water, rather than in the air. The 1853–1854 epidemic in London gave him the opportunity to support this theory. By carefully tracing the evolution of the epidemic, up until the very first case, he theorized that the epidemic was started from a very specific water pump, the Broad Street water pump.

Chance provided Snow a very nice natural experiment. At the time, there were several companies providing water to Londoners. Snow was able to compare the death rates due to cholera for the customers of two of these companies, Lambeth and Southwark, in two consecutive cholera epidemics. During the first epidemic, both companies drew their water from Thames, and they did so inside London. Thus, their water was very polluted, and a very fertile ground for spreading cholera. During this first epidemic, the death rates for customers of the two companies were very similar. In between the 1849 and the 1853 epidemics, Lambeth moved their water intake upstream. Thus, their water was now much cleaner. In the second epidemic, the death rate for Lambeth customers was lower than that for Southwark by almost an order of magnitude.

Figure 18

Deaths from cholera in the "treatment" vs "control" group

Source of water	Houses supplied	Cholera deaths	Proportion/100,000
Southwark	40,046	1,263	**315**
Lambeth	26,107	98	**38**
Other	256,423	1,422	56

Note. The figure is adapted from Montelpare et al. (n.d., Chapter 7, Table 7.2).

Weeks 12–14: Scientific Education, Civic Education, and Response to Covid-19

One way of thinking about the Covid-19 pandemic is as a mirror highlighting political, economic, social and cultural/attitudinal shortcomings in all polities affected. Some countries responded much better than others, and merely looking at economic development (as a rough proxy for the quality of the healthcare system) does not go very far in explaining why countries responded so differently. Clearly, the answer of a country such as the United States was much worse than that of comparable countries (in terms of social and economic development or democratic traditions), such as Finland or New Zealand.

In the last few weeks of the class we will discuss a few important determinants of the success (or failure) to how countries addressed the pandemic. The focus will be on Romania and the United States (a most different systems design), the core argument being that, in spite of the major differences between the two countries, the actions and rhetoric of political elites, the media, and the general public was unsatisfactory, leading to unfortunate outcomes.

Figure 20

Response to Covid-19: The impact of elites, media, and the public

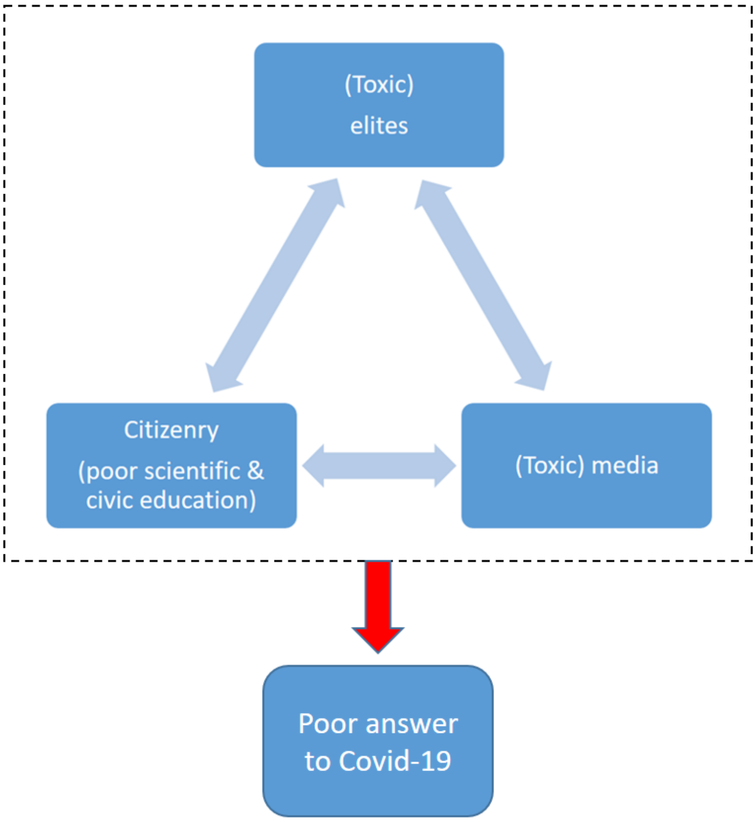
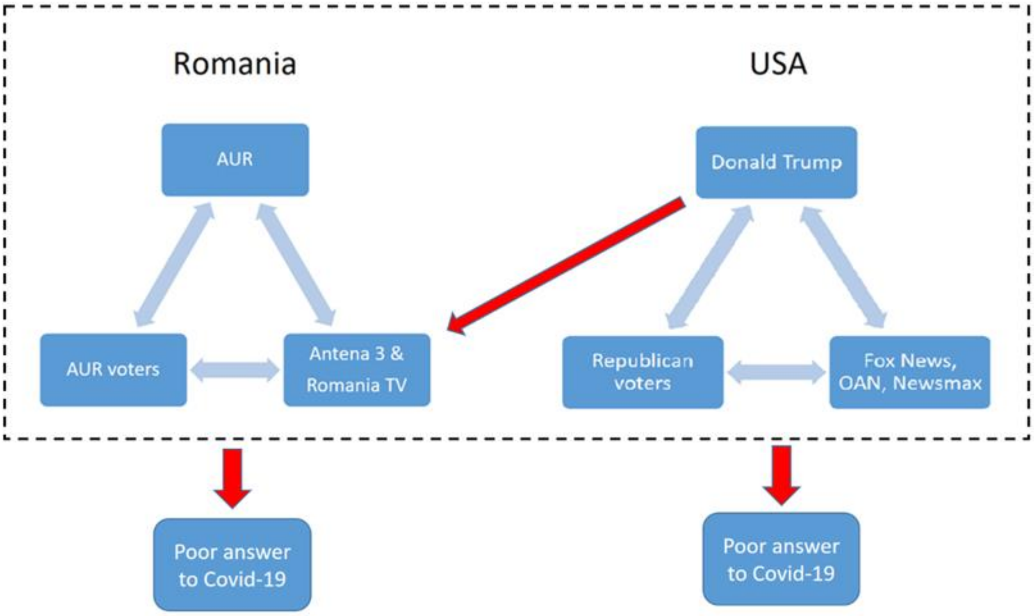

(Toxic)
elites
Citizenry
(poor scientific &
civic education)
(Toxic) media
Poor answer
to Covid-19

Figure 21

How countries answered to Covid-19: Romania vs the US, and Romania and the US



Moreover, not only that we have the cumulative and interactive effects of the three variables within the two countries. Occasionally, actors from one country learn from the actions and/or rhetoric of actors from the other country. One such example, discussed in more detail later in the paper, was when in the Summer of 2020, then-President Trump presented an argument (a very specific one) during two interviews. A few days later, the exact same argument was repeated during a show on one of the main Romanian "news" channels (Antena 3, roughly the functional equivalent of Fox News).

Citizens with poor civic and scientific education are skeptical about the dangers of Covid, about the usefulness of social distancing and wearing masks, and significantly less likely to get vaccinated. Figures 22 and 23 place Romania in comparative perspective, illustrating these claims and showing the consequences.

Figure 22

Opinions about vaccines and vaccination rates

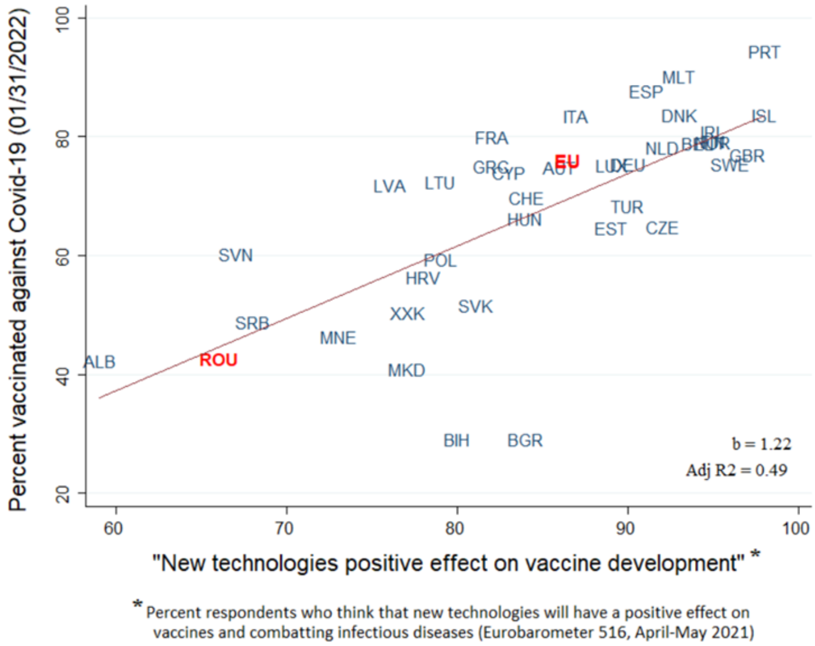
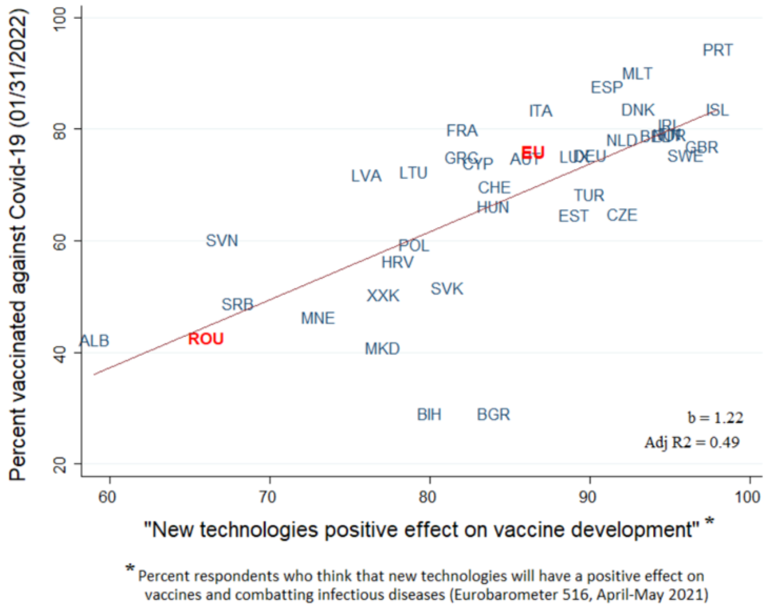


Figure 23

Vaccination rates and Covid-19 death rates



In both countries, voters with poor civic and scientific education were more likely to vote for anti-vaxx candidates and parties (see Figures 24 and 25 for the US, then Table 2 for Romania).

Figure 24

State-level vote for Biden in 2020 and percent of adults vaccinated against Covid-19

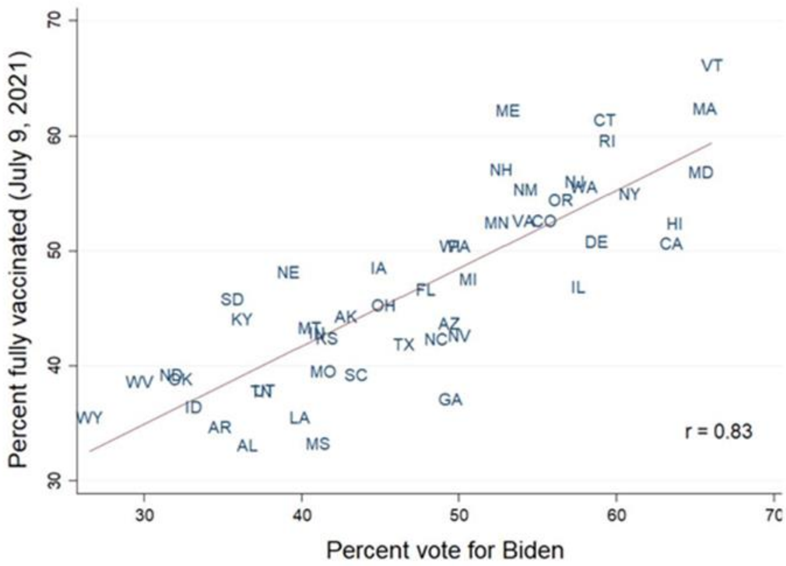


Figure 25

County-level vote for Biden in 2020 and percent of adults vaccinated against Covid-19

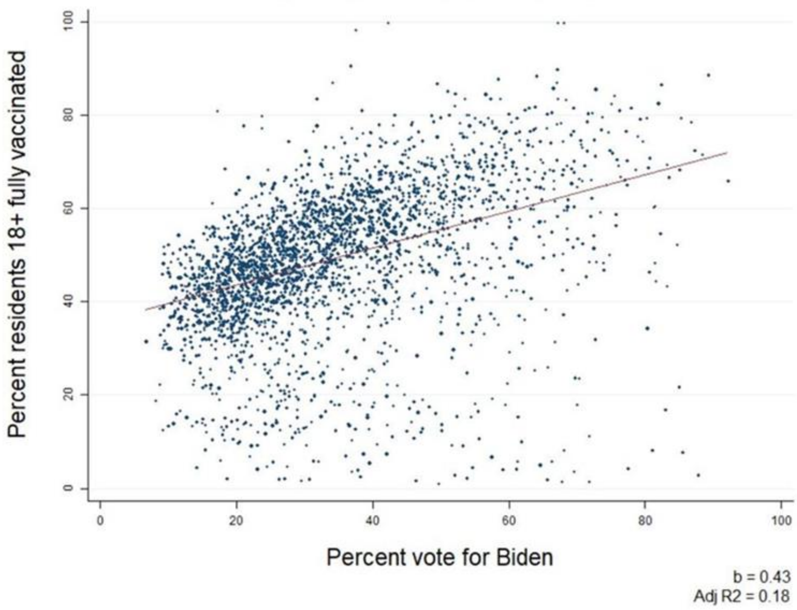


Table 2

Partisanship and vaccination rates in Romania (locality-level data, 2020 parliamentary elections)

	Vaccination			
	USR+	PSD	PNL	**AUR**
Development Party*	.39 **.18**	.47 .06	.44 .03	.43 **-.33**
Adjusted R^2	.42	.42	.41	.45

Note. *Vote for party X (2020 parliamentary elections, Chamber of Deputies) as predictor of vaccination rate. Unstandardized coefficients of linear regression. N = 3,079.

Even though this is a very brief and sketchy presentation of what will be a much more extensive discussion, it offers suggestive, but strong, evidence for the assertion that the position of political elites, the media, and the education of the public can make an important difference—either in the right or, as it was largely the case in both the US and Romania, in the wrong direction.

References

Achen, C. H., & Bartels, L. B. (2003). *Blind retrospection: Why shark attacks are bad for democracy*. Vanderbilt University, CSDI Working Paper 5-2013. https://www.vanderbilt.edu/csdi/research/CSDI_WP_05-2013.pdf

Carey, S. S. (2011). *A beginner's guide to scientific method* (4th ed.). Boston, MA: Wadsworth Cengage Learning (Chapter 1, "Science," pp. 2–5).

Fesnic, F. (2017). *Making methodology fun*. Poster presented at the Annual Meeting of the American Political Science Association, San Francisco, CA. https://apsa2017-apsa.ipostersessions.com/default.aspx?s=60-59-1B-88-2E-8F-28-E9-CC-54-F2-31-3A-2D-B8-AD

Fesnic, F. (2022). *Three toxic ingredients making the COVID-19 pandemic worse: United States and Romania compared*. Presented at the Annual Meeting of the Midwest Political Science Association, Chicago, IL.

Frerichs, R. R. (n.d.). *Snow's grand experiment of 1854*. https://www.ph.ucla.edu/epi/snow/grand_experiment.html

Fujiwara, T. (2015). *Voting technology, political responsiveness, and infant health: Evidence from Brazil*. *Econometrica*, *83*(2), 423–464. https://www.princeton.edu/~fujiwara/papers/elecvote_site.pdf

Fujiwara, T. (2017, 24 October). *Political inclusion and development outcomes: Brazil introduces electronic voting*. *VoxDev*. https://voxdev.org/topic/technology-innovation/political-inclusion-and-development-outcomes-brazil-introduces-electronic-voting

Gelman, A., Shor, B., Bafumi, J., & Park, D. (2007). Rich state, poor state, red state, blue state: What's the matter with Connecticut? *Quarterly Journal of Political Science* 2, 345–367. http://www.stat.columbia.edu/~gelman/research/published/rb_qjps.pdf

Hague, R., Harrop, M, & McCormick, J. (2019). *Comparative government and politics: An introduction* (11th ed.). Red Globe Press (Chapter 3, "Comparative Methods: An Overview").

Ignaz Semmelweis. (2021, 24 June). In *Wikipedia*. https://no.wikipedia.org/w/index.php?title=Ignaz_Semmelweis&oldid=21625554

Johnson, E. J., & Goldstein, D. (2003, 21 November). Do defaults save lives? *Science*, *302*(5649), 1338–1339. https://www.science.org/doi/10.1126/science.1091721

Last, J. M. (2002). Semmelweis, Ignaz. In B. Lester (Ed.), *Encyclopedia of public health* (pp. 1087–1088). New York: Macmillan.

Montelpare, W. J., Read, E., McComber, T., Mahar, A., & Ritchie, K. (n.d.). *Applied statistics in healthcare research* (Chapter 7, "John Snow and the Natural Experiment"). https://pressbooks.library.upei.ca/montelpare/chapter/john-snow-and-the-natural-experiment/#chapter-172-section-1

National Geographic. *Mapping a London epidemic*. https://media.nationalgeographic.org/assets/file/Mapping_a_London_Epidemic_1.pdf

Scheppele, K. L. (2014, 26 May). Hungary and the end of politics: How Victor Orbán launched a constitutional coup and created a one-party state. *The Nation*. http://www.thenation.com/article/179710/hungary-and-end-politics#

Scheppele, K. L., Miklós Bánkuti, M., & Zoltán Réti, Z. (2014, 13 April). Legal but not fair (Hungary). *The Conscience of a Liberal Blog*. http://krugman.blogs.nytimes.com/2014/04/13/legal-but-not-fair-hungary/

The Institute for Mathematics and Democracy. *Apportionment*. https://mathematics-democracy-institute.org/apportionment/#

Wynn, C. M., & Wiggins, A. W. (2017). *Quantum leaps in the wrong direction: Where real science ends... and pseudoscience begins* (2nd ed.). New York, NY: Oxford University Press (Chapter 1, "The Road to Reality: Scientific Method").

7.1. Didactic Commentary

Didactic commentary UBB

Florin Fesnic

Center for the Study of Democracy, Department of Political Science, Babeș-Bolyai University Cluj, Romania

The potential public for this class (or, rather, for the forthcoming book based on it) is wide-ranging. The initial public for the course, intended to be offered as a semester-long elective, are undergraduate students in social sciences (e.g., political science or sociology). It can be either a standalone class or an addition to more traditional introductory courses in research methods. Nonetheless, the public can be much broader than this, such as students from various fields (e.g., medicine), high school students wanting to continue their education in universities, or the general public. One of the most important messages of this class is that, while the examples discussed are very diverse, the logic of scientific research is very similar across scientific fields.

Even though the class was not offered as a formal class so far, from informal presentations for, and interaction with, students from my home university/department (the Department of Political Science at Babes-Bolyai University in Cluj, Romania), as well as formal presentations ("Stories about scientific research" at Antalya International University, September 26, via an Erasmus+ teaching mobility, <https://www.youtube.com/watch?v=IoFrVST6mDw>, and "Stories about Elections and War in Ukraine" at Kokugakuin University, Tokyo, July 14, 2023, <https://www.youtube.com/watch?v=RhW408z3T-4>, a presentation made during a research and teaching mobility funded by CiviMatics), the feedback was very positive. Both students from my home university, as well as students from Turkey and Japan thought such a class would be very interesting and useful.

CHAPTER 8

Summary and Outlook

Bastian Vajen, Heidi Strømskag & Florin Fesnic

The goal of this handbook is to offer educational tools for enhancing the competences of future teachers to address complex societal challenges in their classrooms and to combine socio-scientific and mathematical perspectives to help their students understand various aspects connected to these issues. The modules outlined in this book address these issues from different disciplinary viewpoints and use diverse approaches to combine mathematics education and civic education.

The first module, created by Heidi Strømskag, uses two inquiries structured as study and research paths (SRPs), both centred on the topic of climate change. Additionally, the course includes several other inquiries that delve into various subjects, like thermal insulation capabilities of two brands of thermoses, the likelihood of having a disease in the event of a positive test result, and the braking distance of a vehicle suddenly braking on a horizontal surface, among others. The course is part of the education or preservice teachers and as such, the use of an SRP serves a dual purpose. Firstly, it helps future teachers understand the underlying reasons for the knowledge they need to become familiar with in order to address specific research questions. Secondly, it aims to provide them with essential skills to effectively utilize SRPs as a teaching tool, especially when guiding students in exploring open modelling questions. Both aspects were highlighted during the implementation of this course. As part of the evaluation of the course, students highlighted how the course had enhanced their understanding of mathematical modelling. For example, algebra was perceived not merely as an abstract domain but as an important modelling tool, echoing the anthropological dimension of the ATD. The feedback on modelling assignments underlines the course's efficacy in fostering praxeological practices, with a notable emphasis on parameters. For the successful execution of such a module, or student research projects more broadly, it is important to note that neither the instructor nor the students are expected to possess prior expertise in the specific disciplines relevant to the generating (research) question. In the context of this module, for instance, pre-existing knowledge in geology, physics, or chemistry is not a prerequisite. Both the teacher and the students embark on a shared learning journey, embodying what Chevallard (2015) describes as a Herbartian and precognitive mindset. This collaborative exploration can be demanding, as it reshapes traditional roles and expectations, necessitating both parties to step into unfamiliar territories, thereby fostering collaboration and continuous discovery. However, as the results show, such a course has the potential to engage students in interdisciplinary learning processes and help develop crucial competences for teaching mathematics in the future.

The second module, developed by Frode Rønning, uses a different approach to teach about (normative) mathematical modelling and is aimed at teachers with insufficient formal background in mathematics to teach at the secondary school level, but who do so despite of lacking the formal qualifications. As such, it is part of a professional development programme to qualify in-service teachers to teach mathematics. The approach to modelling taken in this module is based on the modelling cycle, which entails taking a real situation and problem as a starting point and through a process of simplifying, structuring and mathematising, bringing the problem on a mathematical form so that it becomes a mathematical model and problem. From this, the process of working mathematically starts, and mathematical results are obtained. These are interpreted and validated, and through this process, the cycle has brought the problem and its solution(s) back to the real world. Although the modelling done in this course is descriptive, it has a clear normative aspect. For example, the course is connected to learning goals about sustainability in the Norwegian National Curriculum and starts by quoting the secretary-general of the UN and goals set by the Norwegian government about reducing CO2 emissions by a certain percentage. The practical implementation of this module showed that the participants were actively engaged in finding situations that could be modelled with functions, but these situations always involved descriptive modelling. This can be in part be explained by the National Curriculum, which focusses on descriptive modelling. However, the examples discussed in the module highlight the possibility to include prescriptive modelling processes and highlight normative aspects.

The third module, developed by Jakob Steinbachner and Nicola Nagy, takes a different perspective on the topic of climate change and the interdisciplinary perspectives connected to it. Based on the social science issues approach, which expands the scope of citizenship education by integrating scientific issues and their interconnections with ethical, moral, and political dimensions, the module aims to cultivate critical awareness about climate change and the relationship between science and politics among learners. The overarching question about the fairness of climate change asks learners to engage with natural science concepts—especially ones routed in mathematical modelling—related to climate change and its underlying causes, but also touches on philosophical notions of equality, equity, and justice as they relate to concepts of fairness. Furthermore, the module includes citizenship education approaches as that address the problem of climate change and its perceived fairness in the global political system. These topics and the corresponding disciplinary perspectives serve as the primary focal points of this module, thus combining perspectives of political science, philosophy and also mathematics. The practical implementation showed appreciation by the students to be able to address pressing issues like climate change from multiple disciplinary perspectives and highlighted the possibility to insert certain parts and sessions from the module into other courses.

The fourth module, developed by Bastian Vajen, uses perspectives from citizenship education but highlights the potential of its cooperation with other subjects. Especially when teaching about complex issues like climate change, interdisciplinary approaches are necessary to enable the students to understand various different aspects connect to them. As such, the course discusses citizenship education as a subject, as a whole school approach and as an element of other subjects and uses examples connected to climate change to facilitate a discourse among students about the possibility to connect mathematics education and citizenship education on a theoretical and practical level. The implementation of the module shows that the students appreciated to learn about citizenship education as a component of other subjects, both from the perspective of citizenship education itself as well as from the perspective of other disciplines. Also, they were able to better understand the principles of mathematics education, its relevance for a democratic society and democratic citizenship and identified possible connections between citizenship education and mathematics education.

The fifth module, created by Lara Gildehaus and Michael Liebendörfer, addresses the connection between citizenship education and mathematics education from the perspective of mathematics and mathematics education. It contains two submodules which integrate the aspect of normative modelling in three different university courses. The first submodule contains two sessions on normative modelling within a general course introducing mathematical thinking, the second one contains a homework-based implementation of normative modelling within a general course on formal mathematics and the third module contains two sessions on modelling and normative modelling as part of a course on teaching stochastics. From their perspective, the integration of normative modelling into existing courses offers the advantage that teaching can be directly enriched without having to change the formal rules of the courses and, although normative modelling occupies only a small part of each course, it highlights the potential of introducing this aspect in regular modules of mathematics and mathematics education without altering their central learning objectives. Although their experiences implementing this course was promising, the necessity to address students' attitudes can be highlighted as an important issue for further implementation. Learning in the context of normative modelling requires not only to gain knowledge, but also to reflect one's own attitudes, and if students do neither have enough knowledge nor an open mind to normative aspects of modelling and its relation to societal issues, discussions about normative modelling can be very challenging.

Lastly, the sixth module, developed by Florin Fesnic, focusses on science education as an important element of informed decisions in democratic societies. It offers examples of scientific research to illustrate the logic of scientific inquiry and the normative and civic aspects connected to it. Hence, a central aspect of the module is the selection of the examples that relate to the social, economic, political and/or medical wellbeing of citizens and societies. The potential target group for this class is wide ranging. Although it is intended to be taught as a semester-long elective course to undergraduate students in social sciences (e.g., political science or sociology) or as an addition to more traditional introductory courses in research methods, it is also suitable for students from various fields (e.g., medicine), high school students wanting to continue their education in universities, or the general public. One of the most important messages of this module is that, while the examples discussed are very diverse, the logic of scientific research is very similar across scientific fields. Its understanding is of importance for democratic societies, especially when faced with epochal problems like climate change or societal crisis situations, like the COVID-19 pandemic.

All in all, these modules highlight different areas of the connection between mathematical and civic issues and are designed for a wide variety of students with various disciplinary backgrounds. As most of them are designed for teacher education, they can be seen as a first step in transforming educational processes connected to complex societal issues, such as climate change, by actively connecting two or more disciplines. The general idea underlying this approach posits that sound judgment is unattainable without the ability to adequately analyse and comprehend key societal and political issues. Since these problems often require a deep and multifaceted understanding across various disciplines, it is not practical to confine their educational treatment to a single school subject. The ambition to address these topics in an interdisciplinary manner hinges on teachers capable of fostering such collaboration, and consequently, on teacher education programmes that impart the necessary competences. In a complex world where many interrelations are best understood through mathematical

Co-funded by the European Union

reasoning, both mathematics education and citizenship education must play pivotal roles in these processes. The presented modules can be seen as proposals for combining aspects of citizenship education and mathematics education at the university level and to broaden the perspectives of preservice teachers regarding the possibilities of interdisciplinary learning processes. Although the results of their implementation at the respective universities have shown promise, they have to be seen as a first step.

To successfully orchestrate interdisciplinary education, it is imperative to roll out modules that synergise pedagogical methodologies across a range of academic fields. This must be executed alongside the crucial task of creating well-suited didactic resources. Moreover, it is advisable to undertake empirical studies designed to evaluate the effectiveness of such interdisciplinary initiatives on learner outcomes, spanning both secondary schools and higher education institutions. Given the escalating complexity and urgency of challenges faced by democratic societies in Europe, there exists an acute need to pioneer new pedagogical approaches. These should aim to furnish students with the requisite skill set to navigate and address contemporary societal issues effectively.