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APPROACH TO COMPUTER MODELLING OF POWER ENERGY DEVICES' PERIODIC MODES

The increasing complexity of power energy devices poses new challenges for designers, and the solutions depend on the efficiency and adequacy of computer modeling of electromagnetic processes in real circuits of power energy devices with the required accuracy. Using known methods for constructing equivalent circuits, it is possible to obtain equivalent circuits of power energy devices. However, a high number of nodes of equivalent circuits (hence, a high order of systems of differential equations) often does not provide solving the analysis problems with the required accuracy and with time constraints which requires new, more effective approaches for modeling periodic modes of power energy devices' circuits.

Keywords: *power energy devices, periodic modes, integral equations method, energy objects dynamics modelling.*

Introduction. As a method for solving the above problem, an approach can be proposed based on dividing complex equivalent circuits into subcircuits [1-3] and using more effective methods of analysis, in particular the method of integral equations [4-9]. It is applicable to piecewise-linear systems of power energy devices containing delays. It provides an opportunity to link the properties of periodic solutions with the filtering properties of the linear part of the system and is also applicable to systems specified in operator form. The number of calculations (when using the apparatus of integral equations) slightly depends on the order of differential equations describing the dynamics of the system under consideration. From the point of view of the criterion for choosing a method, the last advantage can be decisive in computer modeling of processes in a power energy device to optimize the choice of its parameters.

Mathematical problem set up. According to [1], the mathematical model of a steady-state periodic process in the circuit of power energy devices can be written as a system of k integral and $q-1$ algebraic equations

$$A(t)Y(t)+\int_0^T G(t,\tau)Y(\tau)d\tau = E(t), \quad (1)$$

$$D(Y(\varphi_i))=0, \quad (2)$$

where: T is the period value of the process under study; q – number of conduction sites T_i, i.e. periods of time with a fixed position of the switching elements; φ_i – switching moments of the switching elements determined by the subsystem equations (2), $i=\overline{1,q-1}$; $Y(\varphi_i)$ – the solution vector to the left of the switching point, i.e. before the break, corresponding to the point φ_i; $A(t) = A_i(t)$ – piecewise-constant matrix of dimensions m, whose elements take constant values on each segment T_i; $F(t)$ – vector of dimension m, the elements of which can depend on the conduction site T_i; $G(t, \tau)$ – kernel of the Green's function type, i.e.

$$G(t,\tau)=\begin{cases} G(t-\tau) & \text{at } 0\le\tau\le t\le T, \\ G(T+t-\tau) & \text{at } 0\le t\le\tau\le T, \end{cases}$$

where $G(t)$ – matrix of dimension k with elements sufficiently smooth in t.

- D – matrix $(q-1)\times(q-1)$;
- $Y(t)$ – vector of dimension k of the desired solution, assumed to be piecewise-continuous (with discontinuities at points φ_i).

In the case of fixed switching moments φ_i system (1), (2) is a system of linear Fredholm integral equations of the 3rd kind with a discontinuous kernel.

A feature of such a system is also the presence of discontinuities in the non-integral term at moments φ_i, which causes discontinuities in the solution at the same points. The use of computer modelling makes it possible to calculate the steady-state periodic mode for a circuit with one definable switching moment (the remaining possible switching are fixed in time or absent).

Numerical calculations have shown that the error in solving such a problem is determined primarily by the error in determining the switching moment φ. The error order is determined by the grid step value H. Thus, it is advisable to use a grid with a small step, which, in turn, predetermines the choice of methods with a small enough computational error for the algorithm for solving the system (1). The latter justifies the use of the quadrature method of the 1st order (trapezoid formula).

Algorithm for determining the values of switching moments. Functionally, the algorithm consists of the following two parts:

a) a quadrature solution [2] of a linear system (1) (SINT subroutine) corresponding to the fixed switching moments;

b) search for switching moments.

We assume that the initial values of the switching moments (breaks or fractures φ_i) are given. These values can be determined approximately based on a qualitative analysis of the nature of electromagnetic processes of the considered class of power energy devices' switching circuits and written as follows $\varphi_H \in [\varphi_i, \varphi_{i+1}]$, where φ_i, φ_{i+1}, – are the known switching points. Thus, we obtain the following algorithm:

1. A system of integral equations is solved with respect to $Y(t)$ according to the specified or obtained initial values of switching moments.
2. The residual vector Φ of the system of equations is determined describing the switching conditions based on previously defined decisions $Y(t)$ in p.1.
3. The residual functional is formed according to some of the given criteria

$$\Phi_1 = \sum_{i=1}^{n} |\Phi_i|, \ \Phi_2 = \sum_{i=1}^{m} (\Phi_i)^2, \ \Phi_3 = \sum_{i=1}^{m} \Phi_i$$

and its numerical value is determined for the initial switching moments φ_H

4. The ratio $\Phi \le \varepsilon$ is verified, where ε – a preselected sufficiently small value. If the conditions are met, then the values of φ_H correspond to the systems of integral and algebraic equations and the solution to the problem is determined. Otherwise, i.e. if the condition $\Phi \le \varepsilon$ is not met, a search for refined values of φ_H is carried out.
5. The values of the derivatives $\left.\frac{\partial \Phi}{\partial \varphi}\right|_{\varphi_H} \cong \frac{\Delta \Phi}{\Delta \varphi}$ are determined numerically. The values of the derivatives $\Delta\varphi$ are determined numerically and sequentially solving a system of integral equations for $Y(t)$ and a system of algebraic equations describing the conditions for switching valves, a new value of the residual functional is determined.

Based on the calculated values according to the expression

$$\frac{\Delta \Phi}{\Delta \varphi} = \frac{\Phi(\varphi_H) - \Phi(\varphi_H + \Delta\varphi)}{\Delta \varphi}$$

the numerical values of the derivatives are determined, as well as the correction value $\delta\varphi$.

6. The components of the switching moment vector $\varphi = \varphi - \delta\varphi$ are corrected.
7. The vector of adjusted switching moments obtained in this way is used to organize the next refinement step by referring to step п.1. This cycle is carried out until the condition $\Phi \le \varepsilon$ is satisfied. At the same time, it is also checked that each component of the required switching moment

vector belongs to the corresponding range of values of the known switching moments $\varphi_m \in [\varphi_{m-1}, \varphi_{m+1}]$.

Failure to satisfy the membership condition may indicate the following:

a) insufficient accuracy of solving a system of integral equations with a correctly specified sequence of switching moments, which can be used to change the parameters of the method for solving the system of integral equations in order to increase the accuracy of its solution (reducing the grid step, changing the type of quadrature form, etc.);
b) incorrect setting of the sequence of switching moments, subject to high accuracy of solving the system of integral equations, which indicates a discrepancy with the selected mode.

Algorithm implementation. Thus, we obtain the following expression as a result of partitioning the segment T and replacing the integral with the quadrature obtained from (1) for the lth equation of the system and $t = (i-1)H$:

$$\sum_{s=1}^{q}\sum_{k=1}^{m}\alpha_{lk}^{s}Y_k^i + 0.5\sum_{k=1}^{m}g_{lk}^{i}Y_k^1 H + \\ +\sum_{j=1}^{i-1}\sum_{k=1}^{m}g_{lk}^{i-j+1}Y_k^j H + 0.5\sum_{k=1}^{m}g_{lk}^{i}Y_k^1 H + 0.5\sum_{k=1}^{m}g_{lk}^{n}Y_k^1 H + \\ +\sum_{j=1}^{n-1}\sum_{k=1}^{m}g_{lk}^{n+i-j}Y_k^j H + 0.5\sum_{k=1}^{m}g_{lk}^{i}Y_k^n H = f_l^{si}, \quad (3)$$

where l is the number of the equation in the system (1), i – node number for variable t, j – node number for variable τ, s – conductivity section number, m – number of equations in the system (1), n – number of points in the partition grid T, q – number of conduction sections, $H = T / (n-1)$ – grid pitch.

The matrices included in (1) are equal, respectively

$$A(t) = \begin{pmatrix} \alpha_{11}^{s} & \cdots & \alpha_{1m}^{s} \\ \vdots & \ddots & \vdots \\ \alpha_{m1}^{s} & \cdots & \alpha_{mm}^{s} \end{pmatrix}, \; G(t) = \begin{pmatrix} g_{11} & \cdots & g_{1m} \\ \vdots & \ddots & \vdots \\ g_{m1} & \cdots & g_{mm} \end{pmatrix}.$$

The upper indices (3) correspond to time variables and conductivity sections, the lower ones – to the number of variables in (1).

The matrix of system (3) splits into $m \times m$ blocks with numbers (l, k), corresponding to n (for all i) by n (for all i). Each block is a matrix consisting of 5 types of elements:

1) angular:

$$a_{11} = \alpha^1 + 0{,}5g^1H; \; a_{nn} = \alpha^q + 0{,}5g^nH;$$

2) diagonal:

$$a_{ii} = \alpha^s + 0{,}5g^1H + 0{,}5g^nH,\ i = \overline{2,n-1},\ s = \overline{1,q};$$

3) elements of the outer columns:

$$a_{1i} = 0{,}5g^iH,\ i = \overline{2,n};$$

$$a_{ni} = 0{,}5g^iH,\ i = \overline{1,n-1}.$$

4) elements above the diagonal:

$$a_{ij} = g^{1+i-j}H,\ i = \overline{3,n},\ j = \overline{2,i-1}.$$

5) elements under the diagonal

$$a_{ij}=g^{n+i-j}H,\ i = \overline{1,n-2},\ j = \overline{i-1,n-1}.$$

In all expressions, the subscripts *lk* corresponding to the block number are omitted.

By solving the resulting system of equations, we obtain an approximation to the solution of the integral problem with fixed switching moments.

Next, consider the following case: the algebraic part of system (2) consists of one equation, i.e. all switching moments, except one moment ψ, are fixed in time or absent: $D(Y(\psi)) = 0$.

The search for the moment in time ψ is carried out iteratively. To find the initial approximation ψ_0 we build a grid with a large step H^*. Setting sequentially $\psi_j = \frac{x_j + x_j + 1}{2}$, $j = \overline{1,n-1}$, x_i – grid nodes, and solving system (3) for each ψ_j, we find the minimum of the function $\Phi(j) = \left|D(Y(\overline{x_j}))\right|$, and set $\psi_0 = \psi_j$.

Next, we set the small grid spacing H and search for the best approximations of the form $\frac{x_j + x_j + 1}{2}$, x_j – grid nodes, iteratively based on the initial approximation ψ_0.

At each i-th step of the iteration, we solve system (3) twice with the switching moments ψ_i and $\psi_1 + H$, then we determine the numerical value of the derivative

$$\frac{dD}{d\psi} = \frac{\Delta D}{\Delta \psi} = \frac{D(Y_2(x_{i+1})) - D(Y_1(x_i))}{H},$$

where Y_1 – vector of the system (3) solution for switching ψ_i and Y_2 – vector of (3) solution for switching $\psi_1 + H$.

Then we calculate the value $\psi_i^* = \psi_i - D(Y(x_j)) \cdot H / \frac{\Delta D}{\Delta \psi}(\psi_i)$. Then, we take the value $\frac{x_j + x_j + 1}{2}$ closest to ψ^*_{i+1} as the ψ_{i+1} – approximation

of the next iteration step. The iteration process ends when $\psi_{i+1} = \psi_i$. Finally, the switching point should be considered as ψ^*_{i+1}, which is a more accurate approximation than ψ_{i+1}. The block diagram of the above algorithm is presented in Fig. 1.

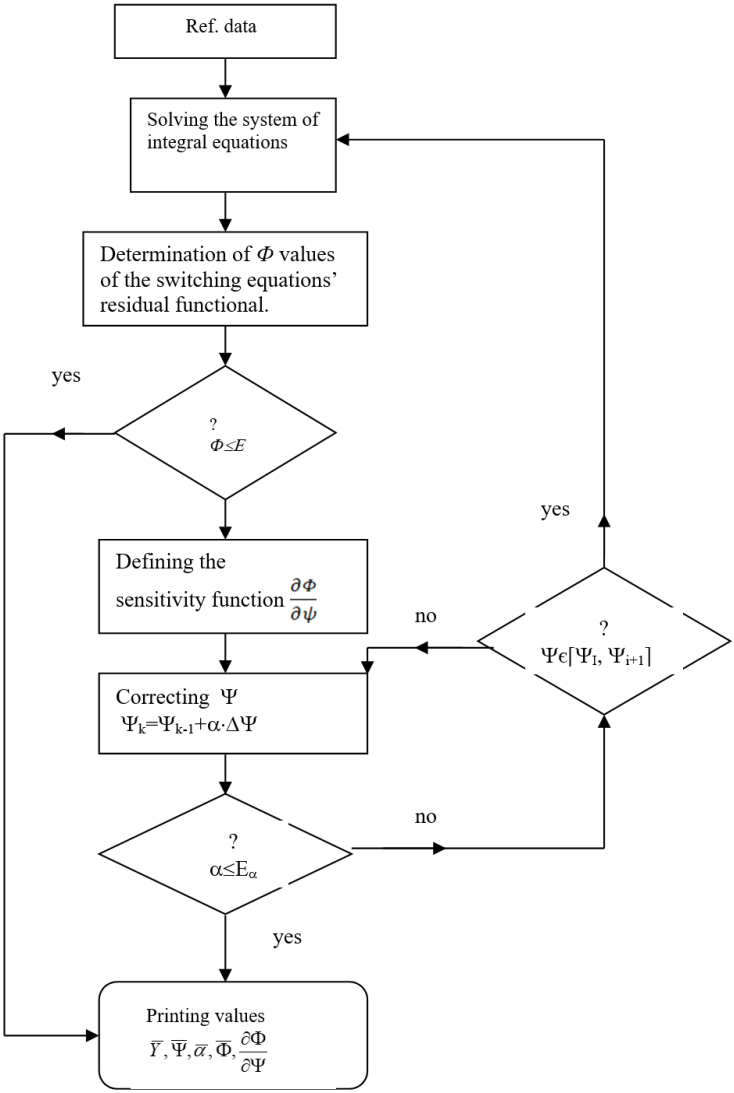

Fig. 1. *Block diagram of the program algorithm for clarifying the values of switching moments*

As an illustration, consider the following problem with two independent variables Y_1, Y_2:

$$\begin{cases} A(t)Y(t)+\int\limits_0^t G(t-\tau)Y(\tau)d\tau+\int\limits_t^T G(T+t-\tau)Y(\tau)d\tau+=F(t), \\ Y_2(\psi^-)-Y_1(\varphi^-)+6e^{-1/2}+1=0; \qquad T=[0,1]; \end{cases}$$

$$A(t)=\begin{cases} \begin{pmatrix} \alpha_{11}^{(1)} & \alpha_{12}^{(1)} \\ \alpha_{21}^{(1)} & \alpha_{22}^{(1)} \end{pmatrix}, \quad t\in T_1=[0,\psi], \\ \begin{pmatrix} \alpha_{11}^{(2)} & \alpha_{12}^{(2)} \\ \alpha_{21}^{(2)} & \alpha_{22}^{(2)} \end{pmatrix}, \; t\in T_2=[\varphi,T]; \end{cases}$$

$$\alpha_{11}^{(1)}=-2+e^{-1}/6;\; \alpha_{12}^{(1)}=-2+e^{-1/2};$$

$$\alpha_{21}^{(1)}=-2;\; \alpha_{22}^{(1)}=-2+e^{-1/2}+7e^{-1}/6;$$

$$\alpha_{21}^{(2)}=+5/2+6e^{1/2}-5e^{-1};\; \alpha_{12}^{(2)}=-7/2+6e^{1/2}-2e^{-1};$$

$$\alpha_{11}^{(2)}=-5/2+6e^{1/2}-3e^{-1};\; \alpha_{22}^{(2)}=1/2+6e^{1/2}-2e^{-1};$$

$$G(t)=\begin{pmatrix} e^{-t} & 2e^{-t} \\ -e^{-t} & 2e^{-t} \end{pmatrix};$$

$$F(t)=\begin{cases} \begin{pmatrix} -3te^{-t}(1-e^{-1})-3+4e^{-1/2}+29e^{-1}/6 \\ -9te^{-t}(1-e^{-1})-1+4e^{-1/2}+16e^{-1}/3 \end{pmatrix}, & \text{at } t\in T_1, \\ \begin{pmatrix} 2te^{-1}(1-e^{-1}) \\ 2te^{-1}(1-e^{-1}) \end{pmatrix}, & \text{at } t\in T_2. \end{cases}$$

The precise solution that satisfies the reduced system has the form:

$$\psi=0{,}5,\; Y(t)=\begin{cases} \begin{pmatrix} 3e^{-t}-1 \\ -3e^{-t}-2 \end{pmatrix} & \text{at } t\in T_1=[0,\,0.5], \\ \begin{pmatrix} -1 \\ e^{-t}+1 \end{pmatrix} & \text{at } t\in T_2=[0.5,1]. \end{cases}$$

The dependence of the functional D on the switching moment ψ, which obtained numerically for this problem, is shown in Fig. 2. The graph shows that there is a second solution $\psi = 0{,}125$. Obtaining a particular solution depends on the initial conditions of the problem, and in the case of a numerical solution, on the choice of the initial approximation ψ_0.

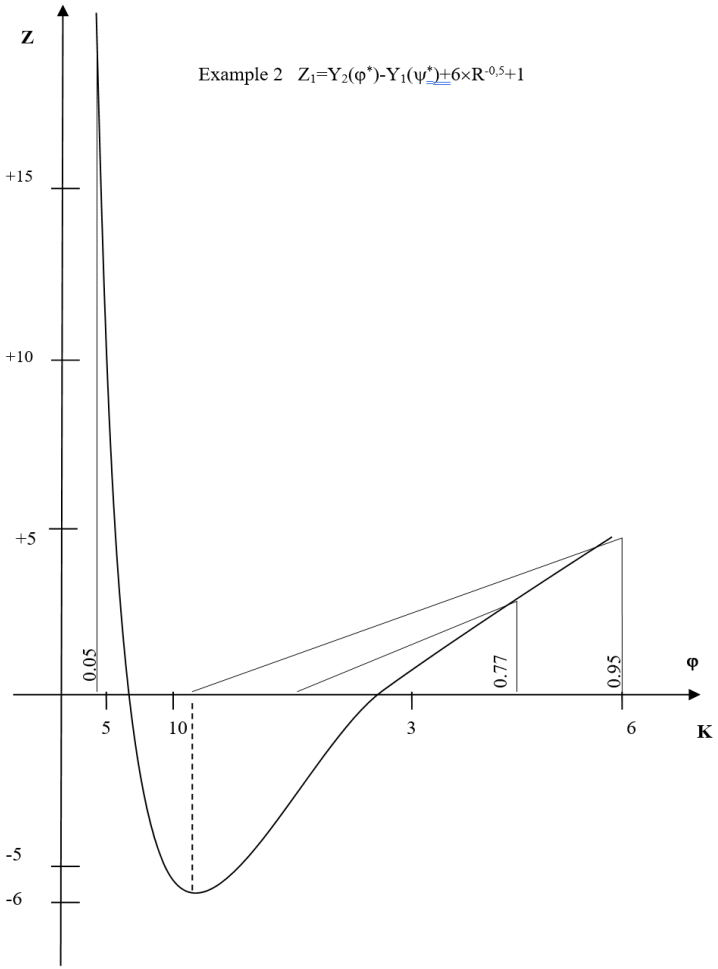

Fig. 2. *Dependence of the functional D values on the switching moments* ψ

Numerical calculations were carried out on a grid of 64 points (H = 0.015873) with initial approximation ψ_0 = 0.77. At the first step of the iteration, according to the algorithm, system (3) is solved for ψ^-= 48H (Table 1).

Functional $D(\psi) = Y_2(\psi) - Y_1(\psi) + 6e^{-1/2} + 1$ value, for given switching moment it is equal to D(48H) = 4.1699.

For ψ = 49H we have the following values Y_1 и Y_2 (Table 1, iteration step №1) corresponding to D(49H) = 4.3544.

Table 1

Iteration step	**0**	**8H**	**16H**	**24H**
1	1,02400 –3,5587 0,9853 –3,5068	0,6320 –3,0020 0,5915 –2,9414	0,2951 –2,5030 0,2533 –2,4343	0,0091 –2,0597 –0,0336 –1,9835
2	2,5196 –5,8082 2,4268 –5,6635	2,1763 –5,5503 2,0809 –5,3878	1,8689 –5,3263 1,7722 –5,1476	1,5920 –5,1295 1,495 –4,9365
3	2,0801 –5,1240 2,00013 –5,00024	1,7246 –4,7817 1,64236 –4,64249	1,4107 –4,4807 1,32726 –4,32740	1,3334 –4,2157 1,04973 –4,04988
Analytical solution	2,00000 –5,00000	1,64224 –4,64224	1,32715 –4,32715	1,04963 –4,04963
Iteration step	**32H**	**40H**	**48H**	**56H**
1	–0,2300 –1,6707 –0,2730 –1,5877	–0,4257 –1,3346 –0,4683 1,2458	–0,5811 –1,0504 –0,6229 –0,9656	–1,6923 2,2211 –1,7211 2,2559
2	–0,6305 1,1409 –0,6906 1,2229	–0,6305 1,0891 –0,6965 1,1676	–0,6366 1,0428 –0,7014 1,1184	–0,6417 1,0015 –0,7056 1,0745
3	–0,9418 1,5302 –0,99991 1,60164	–0,9429 1,4616 –0,99991 1,52988	–0,9438 1,4011 –0,99991 1,46669	–0,9446 1,3477 –0,99991 1,41103
Analytical solution	–1,00000 1,60174	–1,00000 1,52998	–1,00000 1,46678	–1,00000 1,41111

Based on the above values, we calculate

$$\psi_1^* = \psi_0 - D(\psi_0) : \frac{\Delta D}{\Delta \psi}(\psi_0) = 0.4033$$

and rounding to the nearest value $(x_i + x_{i+1}) / 2$, we obtain an approximate value of the switching moment $\psi = 25.5\text{H} = 0.4048$ for the second iteration step.

At the second iteration step for $\psi^- = 25\text{H}$ we obtain new values Y_1 и Y_2 (Table 1, iteration step № 2) corresponding to $D(25\text{H}) = -2.0267$.

Similar to the first iteration step for $\psi^*_2 = 0.4878$ and corresponding values Y_1 and Y_2 (Table 1, iteration step № 3) we obtain the value of the functional $D(30\text{H}) = -0.3451$.

For $\psi^1 = 31\text{H}$ and corresponding values Y_1 and Y_2 (Table 1, iteration step № 3) the value of the functional is $D(31\text{H}) = -0.0293$, i.e. Φ close to minimum.

We calculate then $\psi^*_3 = 0.5013$, $\psi_3 = 31.5$, H = 0.5.

Value $\psi_3 = 0.5$ coincides with the precise value of the switching moment.

Calculation of the fourth iteration step gives the value ψ_4, corresponding to ψ_3, and thus the fourth iteration step is the last one. First line of values Y for this step coincides with the values of solutions for Y_1 of the 3rd iteration step, and the second has an auxiliary meaning, and therefore they are not given. Thus, as the final search result we have $\psi = 0.5013$, as well as values Y_1, Y_2 for $\psi = 31$H (Table 1, iteration step № 3), which, together with the analytical solution given in the next row of the Table 1 ($\psi = 0.5$), justifies to conclude that with the precise value of the switching moment it is possible to obtain four correct significant digits of the solution.

Conclusion. An algorithm for calculating periodic modes of power energy device circuits based on the method of integral equations is considered and studied. The algorithm is reduced to solving systems of linear integral equations that have discontinuous solutions. An algorithm has been developed for solving the nonlinear problem of searching and clarifying the values of switching moments. Computational experiments confirm the performance and efficiency of the algorithm when calculating periodic modes of power energy devices.

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ПІДХІД ДО КОМП'ЮТЕРНОГО МОДЕЛЮВАННЯ ПЕРІОДИЧНИХ РЕЖИМІВ ЕНЕРГЕТИЧНИХ ПРИСТРОЇВ

Зростаюча складність силових енергетичних установок ставить перед конструкторами нові завдання, вирішення яких залежить від

ефективності та адекватності комп'ютерного моделювання електромагнітних процесів у реальних схемах енергетичних пристроїв з необхідною точністю. Використовуючи відомі методи побудови схем заміщення, можна отримати схеми заміщення силових енергетичних установок. Однак велика кількість вузлів еквівалентних схем (отже, високий порядок систем диференціальних рівнянь) часто не забезпечує вирішення задач аналізу з необхідною точністю та часовими обмеженнями, що вимагає нових, більш ефективних підходів до моделювання періодичних режимів силових енергетичних установок.

Ключові слова: *силові енергетичні установки, періодичні режими, метод інтегральних рівнянь, моделювання динаміки енергетичних об'єктів.*

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МОДЕЛЮВАННЯ КОЛИВАЛЬНИХ ПРОЦЕСІВ ВІБРОУДАРНИХ СИСТЕМ

З метою підвищення ефективності методів і засобів математичного моделювання віброударних систем розроблено узагальнену функцію періодичного режиму руху виконавчого органу у формі залежності ударного імпульсу від співвідношення кутових швидкостей лінійної консервативної системи та власної. При отриманні даної функції було використано інтегральну функцію одиничного стрибка Хевісайда та періодичну функцію Гріна. Функція залежності частоти коливань від ударного імпульсу визначена із умов співударяння для функції реакції системи на періодичну послідовність імпульсів. Розглянута розрахункова модель віброударної системи, як з одним ударним елементом та обмежувачем руху, так із двосторонньою ударною парою при почергових ударних взаємодіях з обмежувачами із лінійною силовою взаємодією в проміжках між ударами. При розробці математичної моделі було використано стереомеханічну модель удару, яка характеризується коефіцієнтом відновлення швидкості після удару. Аналіз функції залежності частоти коливань від ударного імпульсу дозволив отримати скелетні діаграми резонансних та квазірезонансних