Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

Hans Markus Jahle

Operational transportation planning of laden load carriers with empty load carrier repositioning using a network flow approach

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Supervisor: Peter Schütz
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Purpose of Master's Thesis

The purpose of this Master's Thesis is to create an operational transportation plan for laden and empty load carriers in the maritime logistics system of Norsk Hydro. The system consists of production, transsshipment, and terminal ports. The products must be transported with load carriers between the various ports, along fixed maritime routes, and must be delivered within a time window. From these transportation requirements, there arises a flow of laden load carriers and empty load carriers.

Since there are transport imbalances between the ports, some ports have more empty load carriers than they require, while other ports have fewer. The transportation plan must therefore reposition the empty load carriers from ports with excess empty load carriers to ports that lack empty load carriers. This must be accomplished while laden load carriers are transported from their origin port to their destination port. The transportation plan must minimize the relevant costs.

Preface

This Master's thesis is the concluding part of my Master of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). My field of specialization is Managerial Economics and Operations Research at the Department of Industrial Economics and Technology Management.

The Master's thesis was written during the spring semester of 2023 and is based on my work in the specialization project during the fall semester of 2022. The thesis aims to find an operational transportation plan for laden and empty load carriers in Norsk Hydros maritime logistics network.

I thank my supervisor Peter Schütz for his excellent guidance, feedback, and insight. Furthermore, I would like to thank Norsk Hydro ASA for their cooperation.

Abstract

This Master's thesis presents a network flow model for optimizing the flow of laden and empty load carriers over an operational planning horizon in the maritime logistics system of a global aluminum producer. The system consists of production, transshipment, terminal ports, and fixed maritime transport routes connecting the ports in space and time. The aim is to find a transportation plan that satisfies the transportation needs. This means delivering laden load carriers from their origin port to their destination port within their delivery time window while repositioning empty load carriers from excess ports to ports that lack empty load carriers. The transportation plan is created while minimizing transportation costs, inventory costs, costs of purchasing extra transport capacity with trucks, and the cost of purchasing new empty load carriers. The model simultaneously considers the flow of laden and empty load carriers to create the transportation plan.

I present a case study with five instances. The first four instances aim to analyze the effect of the planning horizon length when the transportation plan is created in a dynamic environment with new information arriving weekly. Planning horizons with three to ten weeks of known transportation needs are analyzed. The fifth instance aims to analyze how the model adapts to changes in known data. Both rolling horizon and finite horizon methods are utilized. I also compare the model with an algorithm that mimics the actual planning policy of the aluminum producer.

The results of my computational study show that the suggested model performs better than the company policy in terms of the required number of load carriers in the system and total costs. Also, the planning horizon length had little effect on the total costs. However, the planning horizon length somewhat affects the repositioning of empty load carriers. Further, the results show that the model requires more load carriers in the system to adapt to changes in known data.

Sammendrag

Denne masteroppgaven presenterer en nettverksflytmodell for å optimere flyten av lastede og tomme lastbærere over en operasjonell planleggingshorisont i det maritime logistikksystemet til en global aluminiumsprodusent. Systemet består av produksjon, omlasting, terminalhavner og fikserte maritime transportruter som kobler havnene sammen i rom og tid. Målet er å finne en transportplan som tilfredsstiller transportbehovene. Dette betyr å levere lastede lastbærere fra opprinnelseshavnene til destinasjonshavnen innenfor leveringstidsvinduet, samtidig som tomme lastbærere reposisjoneres fra havener med for mange tomme lastbærere til havner som mangler tomme lastbærere. Transportplanen minimerer transportkostnader, lagerkostnader, kostnader ved kjøp av ekstra transportkapasitet med lastebiler og innkjøp av nye tomme lastbærere. Modellen planlagger strømmen av lastede og tomme lastbærere samtidig for å lage transportplanen.

Jeg presenterer en casestudie med fem instanser. De fire første instansene tar sikte på å analysere effekten av planleggingshorisontens lengde når transportplanen lages i et dynamisk miljø der ny informasjon kommer ukentlig. Instansene implementerer planleggingshorisonter der tre til ti uker med transport behov er kjent. Den femte instansen har som mål å analysere hvordan modellen tilpasser seg endringer i kjente data. Både rullende horisont og endelig horisont-metoder brukes. Jeg sammenligner også modellen med en algoritme som etterligner den faktiske planleggings metoden til aluminiumsprodusenten.

Resultatene av beregningsstudien viser at den foreslåtte modellen gir bedre resultater enn selskapets policy når det gjelder nødvendig antall lastbærere i systemet og totale kostnader. Resultatene viser også at planhorisontens lengde har liten effekt på de totale kostnadene. Planleggingshorisontens lengde påvirker imidlertid omplasseringen av tomme lastbærere til en viss grad. Videre viser resultatene at modellen krever flere lastbærere i systemet for å tilpasse seg endringer i kjente data.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Aluminium is a versatile, lightweight, and strong metal used in various applications. It doesn't rust easily, which means it can last a long time and work well outside or in places where other materials might get damaged. It can conduct heat and electricity well, so it's great for making things like electronics or heat exchangers. Aluminum can also be cast into a wide range of products. The casting process allows for complex and efficient designs, bringing versatility not easily achieved by other manufacturing methods. This combination of material and process advantages has made aluminum-cast products a preferred choice in various industries. The market for aluminum cast products is expected to grow with a Compound Annual Growth Rate (CAGR) of 5.13% from 2023 to 2032(Research, 2023).

In 2022, the largest producer of primary aluminum was China, with 59.06% of the global production. However, aluminum-cast products are sold all over the world. Europe and North America are net importers of aluminum cast products, while Asia is a net exporter. Thus, maritime transportation is an important part of the aluminum value chain due to large geographical distances.

The shipping industry underpins the international economy, contributing significantly to global trade. It provides the most efficient, safe, and environmentally friendly means of transporting mass goods worldwide (Elmi et al., 2022, accounting for over 90% of world trade. Containerization, a landmark development in the mid-twentieth century, dramatically reduced transport costs, which was formerly a considerable expense (Meng et al., 2019).

Norsk Hydro ASA referred to as Hydro from here on, operates in all stages of the aluminum value chain. The company is a large global actor and is one of the most valuable companies in the aluminum industry. Hydro's primary aluminum and cast product production are located across the globe to serve the company's international customer base.

Hydro has recently invested in a new maritime logistics system to handle transportation between Norwegian production plants and European terminal ports. This system utilizes load carriers and roll-on / roll-off (ro-ro) vessels to transport goods. The load carriers transport cast products to the European transit ports, where the cast products are reloaded for further transportation. In addition, some return flows of load carriers with anodes (raw material for aluminum production) from the European terminal ports to the Norwegian plants.

The operational planning process in the maritime logistics network is challenging since the ports do not have a regular flow of goods. Thus the ports may have too many or too few load carriers when they are required for transportation. In addition, the planning must account for the repositioning of empty load carriers. The planning must be done while respecting ro-ro vessel capacities, available transport routes, and port storage capacities.

ToDo: insert paragraph about contribution here.

The report is structured as follows: chapter 2 presents relevant context and background information, chapter 4 presents a review of relevant literature, chapter 3 presents the problem description, chapter 5 provides a mathematical model, chapter 6 presents a case study of Hydro's situation, 7 provides a computational study of the model. Chapter concludes and suggests future research opportunities.

Chapter 2

Background

Hydro has recently invested in a new maritime logistics system to increase efficiency and reduce costs. While the old system had crane-based vessels, the new system uses roll-on / roll-off (ro-ro) vessels with load carriers. The shift to the new system creates a new transportation planning problem with new challenges for Hydro.

The purpose of this chapter is to provide a context for the operational transportation planning problem Hydro faces after the strategic changes. First, section 2.1 provides an overview of the global aluminum industry. Second, section 2.2 presents Hydro and its transportation requirements. Third, section 2.3 describes the maritime logistics system Hydro has designed to handle the transport requirements and the operational planning challenges this system creates.

2.1 The Global Aluminium Industry

Aluminium is a versatile, lightweight, and strong metal used in various applications. It has about one-third the weight of steel, which makes it an ideal material for use in the construction of vehicles, airplanes, and other structures where weight is a concern. Despite this lightweight property, aluminum is still a solid and durable material that can withstand substantial stress. Furthermore, the metal can be molded, shaped, and fabricated into several products, including packaging, construction materials, and automotive components. These properties make aluminum valuable and widely used in several industries (Hydro, 2022a).

First, subsection 2.1.1 presents the stages in the aluminum value chain. Then, subsection 2.1.2 describes the role of maritime transportation in that value chain.

2.1.1 The Aluminium Value chain

The aluminum value chain encompasses the production process, from mining raw materials to delivering finished aluminum products. Figure 2.1 presents the six stages in the value chain.

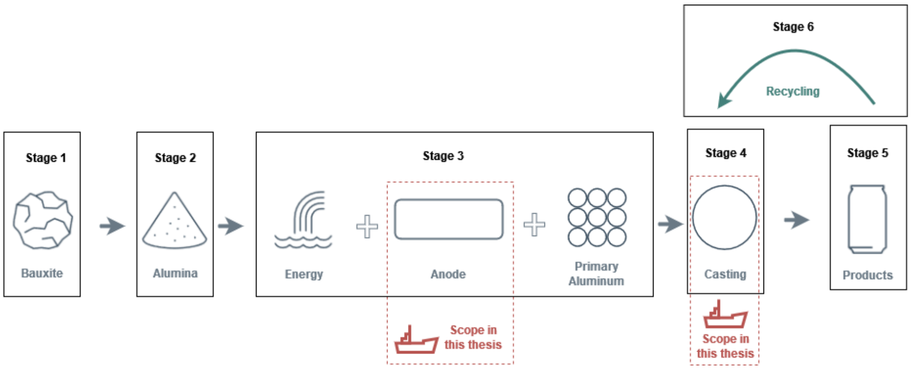


Figure 2.1: The aluminum value chain (Hydro, 2022c)

The first stage involves the extraction of bauxite (figure 2.2a). Second is the refining of bauxite into alumina (figure 2.2b). The third stage is primary aluminum (figure 2.2c) production, which takes place in production plants that transforms energy, alumina, and anodes (figure 2.2d) into primary aluminum using electrolysis. The fourth stage is to cast primary aluminum into semi-fabricated cast products. Cast products include sheet ingots, foundry alloys, extrusion ingots, and wire rods (Hydro, 2022b). These cast products are presented in figure 2.3. The fifth stage is to take semi-fabricated cast products and manufacture finished products for various industries.

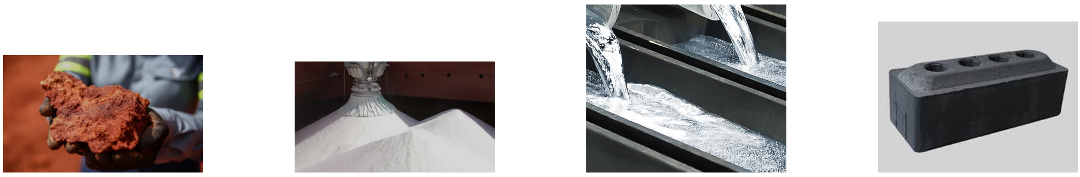

(a) Bauxite (b) Alumina (c) Primary aluminum (d) Anodes

Figure 2.2: The raw materials at each stage in the aluminum production process

The sixth and increasingly important stage in the value chain is recycling, which is an integral part of the circular economy of aluminum. This stage involves collecting and processing used aluminum products to reintroduce them into the production cycle, thus reducing the need for new bauxite mining. The recycled aluminum undergoes re-melting and re-purifying before being cast into new products, all with a significantly lower environmental impact than primary production. According to the Aluminum Association (2022), recycling aluminum saves 90% of the energy needed to produce new aluminum from raw materials. It is essential in reducing the overall carbon footprint of the aluminum industry.

(a) Sheet ingots

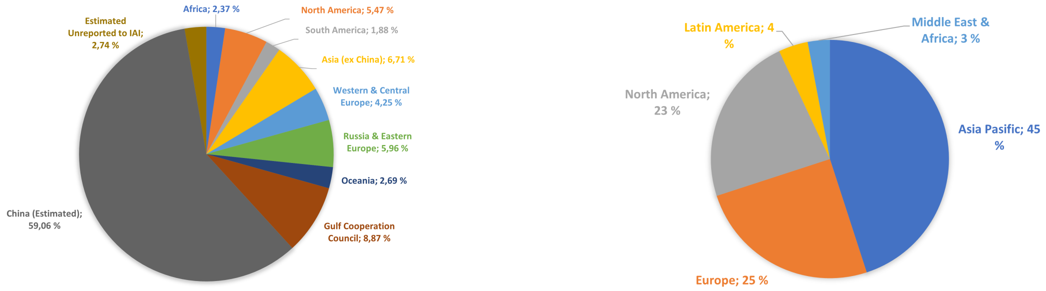
(b) Foundry alloys

(c) Extrusion ingots

(d) Wire rod

Figure 2.3: The main cast products

An overview of the geographical distribution of primary aluminum production in 2022 is provided in figure 2.4a. The most prominent market actors are China, the Gulf Cooperation Council (Middle East), and Asia, except China. The cost drivers in this production are alumina, energy, and anodes. These cost factors comprise 80-85 percent of costs in 2022 (Hydro, 2023).



(a) Geographical distribution of primary aluminum production in 2022 (International Aluminium Institute, 2022)

(b) Geographical distribution of aluminum cast product market share in 2022 (Research, 2023)

Figure 2.4: Comparison of primary aluminum production and aluminum cast product market share in 2022

The aluminum casting market was valued at approximately USD 95.34 billion in 2022 and is projected to expand to nearly USD 157.23 billion by 2032, reflecting a Compound Annual Growth Rate (CAGR) of 5.13% over the forecast period from 2023 to 2032. The growth in this sector is fueled by a diverse range of customers such as automotive, aerospace, machinery, and construction industries. This growth is geographically dispersed with significant market shares held by regions like North America, Europe, and Asia-Pacific. Specifically, countries like the United States, Germany, and China dominate the demand due to their advanced industries (Research, 2023). The geographical distribution of aluminum cast product market share is provided in figure 2.4b.

By comparing figure 2.4b and figure 2.4a, it is evident that the producers of primary aluminum and the customers of cast products are located in different parts of the world. For example, both North America and Europe have large parts of the aluminum cast market share, while they have a much smaller share of the primary aluminum production.

Since there are long distances between producers and customers in the aluminum value chain, the value chain has transportation requirements. Since maritime transportation is suitable for long-distance transportation (Rodrigue and Notteboom, 2023). Maritime transportation is relevant for large parts of the value chain. This thesis focuses on the parts in figure 2.1.

2.1.2 Maritime Transportation in the Value Chain

Maritime transportation is central to several parts of the aluminum value chain. Alumina transportation to smelters usually happens by bulk ships due to its powder-like nature and the long travel distances (International Aluminium Institute, 2022). Anodes also travel long distances to reach the smelters, but their solid shape makes general cargo vessels more suitable (GlobalForwarding, 2022). Primary aluminum production plants typically have a cast house nearby due to the inconveniences of transporting the liquid metal. Therefore, primary aluminum transportation usually happens within the infrastructure of the production plants (Hydro, 2022c). Further, due to their solid shape, cast products use general cargo vessels as a primary transport mode.

Bauxite is, however, transported from the mines with trucks or convoy belts to the alumina refineries since these are often located close to each other due to the substantial volume reduction when refining bauxite to alumina (Government of Canada, 2022).

The various products and their standard transport modes are summarized in table 2.1. The table shows that maritime transportation is common in large parts of the value chain. This thesis focuses on cast products and anode transportation (marked in table 2.1).

Table 2.1: Summary of the common transport modes for the various products. This thesis focuses on products that are colored.

Products	**Product Characteristic**	**Common Transport Mode**
Bauxite	Sedimentary rock	Truck or Convoy belt
Alumina	Dry powder	Bulk carriers
Primary aluminum	Liquid metal	Plant infrastructure
Anodes	Solid rectangular structure	General cargo vessels
Cast products	Solid products	General cargo vessel

2.2 Norsk Hydro ASA

Hydro is a Norwegian aluminum company with over 30 000 employees in over 140 locations in 40 countries. Their business operations encompass "bauxite mining, alumina refining, electrolysis of primary aluminum products and alloy technology to finished products" (Hydro, 2022b, p.29). This makes Hydro present in large parts of the aluminum value chain presented in subsection 2.1.1. The company has grown significantly since its foundation in 1905 and had a market capitalization of USD 15.054 Billion in 2022. This makes Hydro the fourth largest aluminum company in the world, ranked by market capitalization (Value.Today, 2023).

This section overviews Norsk Hydro and its operations and transport requirements.

2.2.1 Hydro's Primary Aluminum and Cast Product Business

Hydro's plants are spread across several countries. "The [primary aluminum] business area consists of wholly-owned aluminum metal plants in Norway and partly owned plants in Slovakia, Qatar, Australia, Canada, and Brazil" (Hydro, 2023, p. 22). Additionally, Hydro owns a network of casthouses, which are integrated with their primary aluminum plants (Hydro, 2023). The production plants, therefore, produce both primary aluminum (i.e., electrolysis) and cast products. The capacities of Hydro's plants are presented in table 2.2.

Hydro plants produce cast products that are utilized by a wide range of industries. Notable sectors that use Hydro's products include automotive, building and construction, packaging, and electronics manufacturing. The automotive sector, especially with the growing trend of electric vehicles, makes substantial use of the aluminum produced, benefiting from the material's lightweight and recyclable properties. This aluminum is employed in various applications, including car bodies and heat exchangers. Similarly, in the construction sector, Hydro's aluminum features prominently in various applications, ranging from window frames to structural components (Hydro, n.d.). Thus, they sell cast products to industries similar to those presented in subsection 2.1.1.

Table 2.2: Summary of the plants and their capacities (Hydro, 2022c)

Plant (ownership)	**Country**	**Electrolysis capacity (000 mt)**	**Casthouse capacity (000 mt)**	**Main products**
Karmøy (100%)	Norway	271	370	extrusion ingot, wire rod
Årdal (100%)	Norway	202	223	sheet ingot, foundry alloys
Sunndal (100%)	Norway	425	525	extrusion ingot, foundry alloys
Høyanger (100%)	Norway	66	120	sheet ingot
Husnes (100%)	Norway	195	215	extrusion ingot
Slovalco (55.3%)	Slovakia	175 (100% basis)	250 (100% basis)	extrusion ingot, foundry alloys
Tomago (12.4%)	Australia	74	75	extrusion ingot, foundry
Alouette (20%)	Canada	320	334	extrusion ingot, foundry alloys
Qatalum (50%)	Qatar	125	150	standard ingot, foundry alloys
Albras (50%)	Brazil	460 (100% basis)	460 (100% basis)	standard ingot, foundry alloys

Hydro has "more than 30,000 customers worldwide" (Hydro, 2022c, p 17). As presented in table 2.2, Hydro also has an extensive network of production plants distributed globally. Given this extensive network, the company generally aims to serve customers with the closest plants to optimize logistics and reduce transportation costs. However, the type of cast products produced varies by plant, implying that a certain product a customer needs may not be available at the nearest plant. This difference in production capabilities complicates the logistics process and necessitates long-haul transportation of certain cast products to satisfy customer needs.

Adapting production capabilities at a plant to accommodate short-term changes in customer demand is not always feasible due to the high costs and time-consuming nature of investing in new production capacities. As such, a flexible and efficient transportation strategy is critical to ensuring that customers' demands are met promptly, regardless of where the required products are produced.

2.2.2 Transportation Requirements at Hydro's Norwegian Plants

Hydro operates five plants in Norway, located in Karmøy, Årdal, Sunndal, Høyanger, and Husnes. These plants engage in the production of primary aluminum along with a wide array of cast products. As presented in subsection 2.1.1, Europe has a large share of the aluminum casting market. It is, therefore, natural to assume that the Norwegian plants produce primarily for the European market. However, as stated in subsection 2.2.1, plants may be required to serve customers globally.

Hydro has a maritime logistics system handles some of the transport requirements at the Norwegian ports. Anodes are transported to the plants from China (Hydro, 2022c) through European terminal ports. Likewise, cast products are transported to the downstream customers through the same terminal ports, where they are either loaded on trucks and driven to customers on the European continent or reloaded to other vessels and transported to other parts of the world. Nevertheless, a transportation flow in the maritime logistics system of both anodes and cast products occurs between the Norwegian and terminal ports.

Due to its geographical location, the Karmøy plant is a hub for planning in the maritime logistics system. The focus of this thesis is Hydro's transportation flow in this system. More explicitly, the flow of cast products to terminal ports in Europe and the flow of anodes to the Norwegian plants.

2.2.3 Transport Imbalance at the Norwegian Plants

Hydro experiences transportation imbalances in the transportation flow represented in figure 2.5. This imbalance is primarily shaped by the disparity between the flow of cast products from its Norwegian plants to the terminal ports in Europe and the flow of anodes from these terminal ports to the Norwegian plants. The flow of cast products, representing the movement from the production plants to the downstream customers, is larger than the reverse flow of anodes.

Figure 2.5: The flow of cast products from the Norwegian Plants and the return flow of anodes from Europe(Hydro, 2022c)

This transport imbalance creates unique logistical challenges when Hydro conducts operational planning. The overutilization of the maritime logistics system for transporting cast products to European terminal ports may lead to underutilization on the return journey when transporting anodes back to the Norwegian plants. This inefficiency can lead to wastage of resources and higher overall transportation costs. These challenges are further described in subsection 2.3.5.

2.3 Hydro's Maritime Logistics System

Hydro's maritime logistics system handles the transportation requirements described in subsection 2.2.2 (i.e., the transportation of anodes and cast products presented in figure 2.5). This section describes this system. Subsection 2.3.1 presents the old maritime logistics system Hydro had, while subsection 2.3.2 presents the new maritime logistics system Hydro has recently invested in. Further, subsection 2.3.3 describes the transportation infrastructure in the system and how Hydro handles the infrastructure planning. Last, subsection 2.3.4 presents the operational planning Hydro has to conduct, given the infrastructure.

2.3.1 Hydro's Old Maritime Logistics System

Hydro's old maritime logistics system was operated by general cargo vessels that used cranes to load and unload cargo (see figure 2.6). This type of vessel has some negative aspects. One of the significant drawbacks of using cranes is that the loading and unloading process can be time-consuming and labor-intensive, especially when dealing with large or complex cargo. This results in longer turnaround times in port, which can cause delays

Figure 2.6: Hydro's old maritime logistics system had vessels that used cranes to load cargo (Hydro, 2021).

and thus impact transport efficiency. Additionally, cranes can increase the risk of damage to cargo, personnel, and the vessel itself, as improper handling or shifting of cargo during loading or unloading can cause accidents or cargo loss. Finally, using cranes also have high labor and equipment costs associated with operating and maintaining the crane systems.

Due to the negative aspects of using crane-based vessels, Hydro invested in a new maritime logistics system.

2.3.2 Hydro's New Maritime Logistics System

Hydro's new maritime logistics system uses ro-ro (roll-on/roll-off) vessels. Ro-ro is a method of transporting goods that involves loading goods onto a vessel using specialized ramps or platforms. In contrast to Hydro's old crane-based vessels, ro-ro vessels allow for more efficient loading and unloading of goods since goods are rolled on and off. Another benefit is reduced goods handling, which minimizes the risk of damage to the goods. These properties make ro-ro vessels a convenient and efficient method of transporting large and heavy items, such as anodes or cast products (Sinay, 2022). Figure 2.7 shows the ramp on which cargo is rolled.

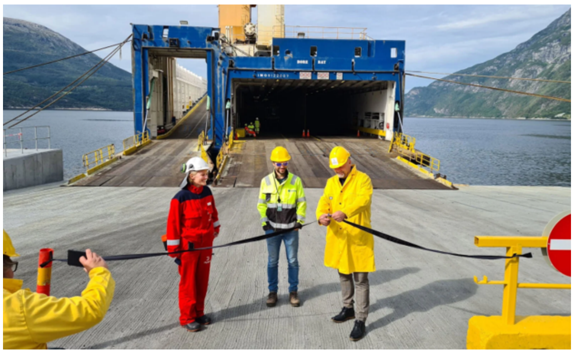


Figure 2.7: Opening of Hydro's new ro-ro based logistics system at Sunndal.

(a) Cassette **(b)** Mafi

Figure 2.8: Load carriers types used to transport goods with ro-ro vessels in Hydro's new maritime logistics network

Further, Hydro's maritime logistics network uses load carriers to transport goods. Figure 2.8 presents various load carrier types. Load carriers allow for reduced loading time compared to traditional cranes (Hydro, 2022d). They reduce handling as goods can travel with the same load carrier from point to point through transshipment. Less handling reduces the risk of damaging the goods. Load carriers can also save space compared to traditional container transportation, as the empty load carriers can be stacked on each other, taking up less space than laden load carriers. However, load carrier utilization can vary depending on the types of goods transported. For example, cast products such as wire rods and sheet ingots have different utilization due to the different shapes of the products.

2.3.3 The Maritime Infrastructure

The maritime infrastructure in Hydro's new logistics system consists of a maritime logistics network. The network includes maritime transport routes, schedules, and capacities for the ro-ro vessels. The ro-ro vessels, traveling along a transport route according to a schedule, visit several ports. The ports are terminal ports in Europe and those near the Norwegian plants (see figure 2.9). The transport routes thus connect all the ports so that the transport flow presented in figure 2.5 can be handled. Hydro outsources the operations of the maritime logistics network to third-party maritime logistics providers. For 2018 - 2024, the Norwegian company Sea Cargo AS operates the logistics network (Skipsrevyen, 2022).

Figure 2.9: The Norwegian plants and the terminal ports in Europe.

The vessels operating the transport routes have fixed capacities. This can cause problems when the available capacities provided by Sea Cargo are lower than the capacity required. In these situations, Hydro purchases extra transport capacity from truck providers. However, since trucks have lower capacities than maritime vessels, the transportation costs are higher for using this transportation mode to transport the same amount of goods.

Further, the Norwegian and terminal ports have an inventory with a certain amount of storage. When Hydro decides to use the inventory, then inventory holding costs accrue. The costs are proportional to the amount of storage used.

2.3.4 Operational Planning

The operational time horizon for planning the transportation of load carriers between ports spans three to six weeks. It involves planning the transportation of the load carriers within the framework provided by the maritime infrastructure to deliver all transport orders within a delivery time window. "Transport order" is the term used to describe a request to transport a certain number of load carriers from an origin to a destination. The delivery time window for a transport order indicates the periods in which the destination port can receive the laden load carriers. Hydro operates with delivery time windows to create flexibility regarding the delivery time. The planning at Karmøy conducts the operational planning for all the Norwegian ports.

The planning team at Karmøy creates transport orders based on transportation requirements. As mentioned in subsection 2.3.2, ro-ro vessels transport goods with load carriers. Thus, the planning team calculates the required number of load carriers. The required number depends on the amount of anode or cast product and the type of load carrier required (due to different utilization grades). Particular products (e.g., cast products) require specific load carrier types. Load carrier types are, therefore, never substituted to fulfill the number needed for a transport order.

After creating transport orders, the logistical planning team has to organize the transport to minimize transportation costs. This organization involves making sure that enough empty load carriers are present at the correct port at the time they are required. The logistical planning team can transport empty load carriers from other ports, obtain empty load carriers stored at the port inventory, or purchase new ones from load carrier providers. The load carrier providers will then deliver them at the required port. However, load carriers of all types are expensive, and they try to minimize new purchases.

Further, the load carriers become laden when the port personnel load empty carriers with anodes or cast products. The logistical planning team then decides the path for the laden load carriers from the origin port to the destination. Thus, the logistical planning team must consider two transportation flows. The flow of empty and the flow of laden load carriers.

2.3.5 Challenges in the Operational situation

The planning team at Karmøy faces two main challenges.

- The first challenge is that the ports do not have a regular flow of goods. The amount of anode or cast products can vary weekly. Thus, the number of laden load carriers can vary weekly. This variation may result in the ports having too many load carriers or too few, creating confusion and dissatisfaction from the port operators.
- The second challenge is that the planning team at Karmøy now must consider empty load carrier repositioning. This is because the new maritime logistics system introduced a flow of empty load carriers. Planning the repositioning is a challenge the planning team has little experience with and lacks decision support for because they had never had this responsibility before (since the old maritime logistics system had no concept of load carriers).

This thesis will study the flow of laden load carriers and the repositioning of empty load carriers in Hydro's maritime logistics system over an operational planning horizon.

Chapter 3

Problem Description

This chapter presents the operational planning problem that Hydro's planning team at Karmøy face. The problem is to optimize the flow of laden and empty load carriers within Hydro's maritime logistics system over an operational planning horizon. The objective is to minimize the associated costs while fulfilling all transport orders within their respective delivery time windows.

Maritime Infrastructure

Hydro's maritime logistics system infrastructure comprises a set of transport routes and a set of ports. Each transport route is operated by a single vessel. A transport route is described by a sequence of ports the vessel visits according to a fixed schedule. The schedule describes the time period when the vessel arrives at and departs from each port in the transport route. The transport routes thus create a maritime network interconnecting the ports. The storage capacity of each vessel is also fixed within the planning horizon. Hydro can acquire more transportation capacity in addition to the transport routes by utilizing trucks that can travel between any pair of ports.

Each port has certain constraints, including inventory storage capacity and the lead time required for converting empty load carriers into laden and laden into empty (i.e., loading and unloading load carriers).

Transport Orders

Hydro has a set of transport orders representing transportation needs. Each transport order states a given number of laden load carriers that should be transported from an origin port to a destination port, the time period when these laden load carriers are ready for transport at the origin port, and the delivery time window within which those laden carriers must reach their destination. Further, each transport order requires a specified load carrier type, and these types cannot be substituted.

Flow of Laden and Empty Load Carriers

The goods transported in the network must be placed on top of an empty load carrier of a given type before being transported with the transport routes. Each load carrier requires storage capacity when stored in inventory at a port or transported along a transport route with a vessel. The storage capacity required by a load carrier depends on the load carrier type and whether the load carrier is laden or empty (since empty load carriers can be stacked on top of each other and thus require less storage).

The problem is to find the most efficient flow of laden load carriers so that all transport orders are delivered within their delivery time window. Once the laden load carriers have reached their destination and are emptied, they should be repositioned to ensure that the correct type and quantity of empty load carriers are in the required port and time period.

If it is impossible to reposition the empty load carriers sufficiently so that the correct quantity is present at the required port and time, purchasing new empty load carriers and having them delivered at a port is possible.

Associated Costs

There is a cost for transporting laden and empty load carriers between ports. Two factors determine the transportation cost: the distance traveled and the amount of storage required on the vessel. Thus, laden load carriers are more expensive to transport since they require more storage. The ports also have an inventory holding cost for storing load carriers (laden or empty) in the port inventory. There is also a cost for purchasing new empty load carriers. When the vessels' capacities are lower than those required to fulfill all transport orders, Hydro has to purchase extra transport capacity from the trucks at a higher cost.

Objective

The ultimate aim is to minimize the total cost associated with the flow of laden and empty load carriers. The total costs include transporting load carriers between ports, holding load carriers at ports, purchasing new load carriers when necessary, and purchasing extra transport capacity when required. The model must optimize the flow of load carriers to meet all transport orders within the delivery time windows while adhering to all capacity and routing constraints.

Chapter 4

Literature Review

This chapter aims to provide an overview of the relevant literature on how empty load carrier repositioning can be handled. First, an introduction to empty container repositioning is presented in section 4.1. Then, this thesis' problem is classified in section 4.2 before relevant approaches to empty container repositioning for problems that have a similar classification are investigated in section 4.3.

4.1 Empty Container Repositioning (ECR)

This section focuses on the empty container repositioning problem, which shares similarities with the challenge of repositioning empty load carriers in Hydro's maritime logistics system. Both problems involve managing the movement and allocation of empty transportation units (containers or load carriers) within a logistics network to minimize costs and optimize resource utilization. By examining the empty container repositioning problem, this literature review aims to draw insights and identify potential solutions that could be adapted and applied to repositioning empty load carriers in Hydro's maritime logistics network. For comprehensive reviews on empty container repositioning, I refer to Kuzmicz and Pesch (2019), Abdelshafie et al. (2022), and Braekers et al. (2011).

The primary cause of empty-container problems is global trade imbalances. Regions with higher imports than exports grapple with an accumulation of empty containers, whereas regions with more exports than imports face a shortage. Even in countries with balanced import and export activities, empty containers accumulate due to imbalances in specific container types, notably reefer containers and special equipment (Abdelshafie et al., 2022).

Despite empty containers not generating revenue, they demand the same transportation, storage, and space resources as full containers, thus incurring significant costs. Empty containers are essential to the supply chain, facilitating port activities. They possess their supply chain comprising containers, container ports/terminal facilities, and transport means like trucks, rail, or maritime vessels (Kuźmicz and Pesch, 2017; Zain et al., 2014). Container terminals have evolved into intermodal hubs, promoting seamless interchange between various modes of transport and cargo handling (Zain et al., 2014)

4.2 Classifying the ECR

Two classification axis for empty container repositioning problems appear in the literature (Kuzmicz and Pesch, 2019; Abdelshafie et al., 2022; Braekers et al., 2011).

4.2.1 Repositioning Scale

In the context of empty container repositioning, global and regional approaches play distinct roles (Braekers et al., 2011).

Regional Repositioning

Regional repositioning focus on one geographical region to fulfill empty container demand and reduce costs. These regions have multiple shippers, inland depots, and terminals. Thus, many allocation options are available at the regional level, with many companies and consignees involved (Kuzmicz and Pesch, 2019). Often several transportation modes are involved (e.g., maritime and rail). The regional container allocation models consider factors such as determinism, the static or dynamic nature of the model, container substitution, container leasing, and street turns (Braekers et al., 2011).

Global Repositioning

Global repositioning aims to move empty containers from ports where they are abundant to ports where they are needed (Kuzmicz and Pesch, 2019). Thus, global repositioning focus on maritime transportation. The repositioning results from a global trade imbalance, which naturally leads to a surplus of containers in some areas and a deficit in others. This discrepancy necessitates carriers to reposition their empty containers to fulfill future demands.

Drewry Shipping Consultants of London has quantified this need, estimating that about 20% of all maritime container, movements are those of empty containers, a trend that has been driven by increasing trade imbalances (Boile et al., 2006; Song and Carter, 2009). Consequently, managing these empty containers presents unique challenges at the global level, given the fewer available options due to factors like limited direct connections between ports and fixed shipping schedules.

Much research has considered either single maritime service routes or service networks with specific route structures in the broader context of global repositioning. For instance, studies such as those conducted by Lai et al. (1995), Song and Zhang (2010),Lam et al. (2007) (2007), and Song and Dong (2012) have utilized various mathematical models, control policies, and optimization strategies to address the challenge of efficient empty container repositioning in specified routes and port systems.

4.2.2 Planning Horizon

Figure 4.1 summarizes the key decisions required to address the empty container repositioning issue at each planning level.

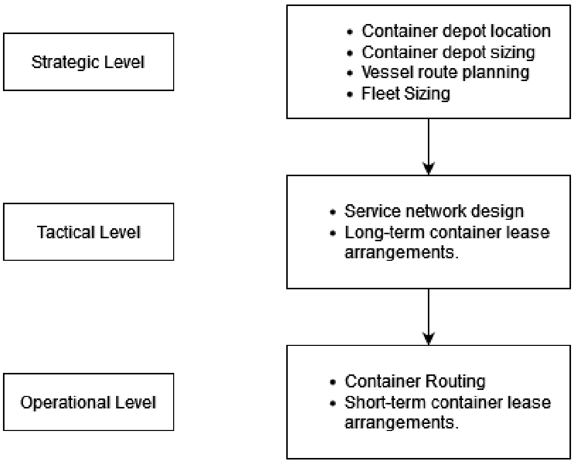


Figure 4.1: Overview of the key decisions required at each planning level (based on Braekers et al., 2011; Lam et al., 2007).

Strategic Level

Strategic planning focuses on long-term decisions like fleet sizing and location decisions. Fleet sizing determines the appropriate size and composition of the container fleet based on projected demand and cost considerations. Service network design involves selecting optimal shipping routes and port calls to meet container demand while minimizing empty container movements. These strategic decisions provide the foundation for effective empty container management (Braekers et al., 2011).

Tactical Level

Tactical planning addresses medium-term decisions to optimize the configuration and utilization of the service network. This includes optimizing the allocation of empty containers across different service routes and carriers to balance container flows and minimize repositioning costs (Braekers et al., 2011; S. Wang, 2013).

Operational Level

Operational planning focuses on short-term decisions and real-time adjustments. Container allocation involves allocating available empty containers to meet specific demand locations or customer requirements. Integrating allocation and routing decisions allows for synergies and improves overall operational efficiency. Real-time adjustments are made based on up-to-date information, such as changes in container demand or unexpected events, to adapt the repositioning plans as necessary (Braekers et al., 2011), e.g., using rolling horizon (Olivo et al., 2005).

4.2.3 Classifying this Thesis' problem

The problem considered in this thesis is a global repositioning problem since the repositioning problem arises from a trade imbalance between ports. In Hydro's maritime logistics system, more laden load carriers are transported from the Norwegian plants to the terminal ports in Europe. At the same time, laden load carriers have a substantially smaller return. Thus empty load carriers stack up at the terminal ports. Further, the planning level is operational with a short time horizon of three to six weeks. The key decision is container (i.e., load carrier) routing, which is an operational decision according to figure 4.1. Some tactical decisions are also involved since Hydro can purchase new empty load carriers. However, the main objective is to create a transportation plan over an operational planning horizon.

4.3 Approaches To Empty Container Repositioning

Several approaches to empty container repositioning are suggested in the literature. Abdelshafie et al. (2022) and Kuzmicz and Pesch (2019) classified the approaches that utilize optimization techniques into three categories: network flow models, network design models, and other models. Table 4.1 presents research papers that seek to solve the ECR. The problem considered in this thesis has, as presented in subsection 4.2.3, an operational planning level and a global repositioning scale. Thus, I have selected research articles with this classification in mind. Table 4.1 shows that variants of the network flow approach (e.g., network flow, time-space, or flow models) were the most common. Thus, this thesis will focus on the network flow approach to modeling the ECR. Subsection 4.3.1 presents the ECR from a network flow point of view.

Table 4.1: Approaches to Empty Container Modeling

Authors	**Repositioning Scale**	**Planning Level**	**Model Type**
Choong et al., 2002	Regional	Operational	Time-space network
J.-A. Li et al., 2004	Global	Operational	Inventory-based control mechanisms
Olivo et al., 2005	Regional	Operational	Minimum cost flow
Erera et al., 2009	Global	Operational	Time-expanded networks with recovery
Shintani et al., 2007	Global	Tactical	Knapsack problem then network flow problem
Ye et al., 2007	Global	Tactical	Container flow and vessel deployment
Di Francesco et al., 2009	Global	Operational	Cargo routing problem
Chou et al., 2010	Global	Operational	Container allocation problem
Brouer et al., 2011	Global	Operational	Time-expanded multi-commodity flow model
Epstein et al., 2012	Global	Operational	Multi-commodity flow model and inventory model for safety stocks
Song and Dong, 2012	Global	Operational	Cargo routing problem in a multi-service multi-voyage shipping network; cargo fleet sizing
Moon et al., 2013	Global	Tactical	Inventory model; foldable containers
Song and Dong, 2013	Global	Strategic	A single liner service route design; ship deployment
Long et al., 2013	Global	Operational	Time-space network
Chao and Chen, 2015	Global	Operational	Time-space network; minimal cost flow problem
L. Li et al., 2014	Global	Operational	Routing problem
Akyüz and Lee, 2016	Global	Operational	Simultaneous service type assignment and container routing flow problem
Sáinz Bernat et al., 2016	Global	Strategic	Inventory control problem
Zheng et al., 2016	Global	Tactical	Network design problem; mixed-integer nonlinear programming model
K. Wang et al., 2017	Global	Tactical	Network flow model; foldable containers
Neamatian Monemi and Gelareh, 2017	Global	Strategic	Network design model

4.3.1 ECR by Network Flow Models

Early research by Florez (1986) examined how a dynamic network model could be used with container leasing. Choong et al. (2002) focused on how long-term planning could impact decisions about repositioning containers in an intermodal transport network. Around the same time, Erera et al. (2009) made a model that considered decisions about booking and routing containers. Olivo et al. (2005) suggested a model that used integer programming and accounted for multiple ways containers could be moved between ports and depots (see also, Song and Dong, 2012)).

In their work, Song and Dong (2015) presented two interconnected components within the container transport process. Firstly, laden containers, whose movements are primarily dictated by external customer demand. Secondly, empty containers, whose positioning is determined mainly by internal shipping company decisions. These elements function within a shared transportation network, utilizing the same resources, yet different operational drivers direct them. To appropriately address and model the repositioning of empty containers, it is vital to concurrently model the laden container routing within the same transport network. This dual modeling approach is crucial because the movements of laden containers, influenced by trade imbalances, directly dictate the repositioning of empty containers. Therefore they suggested building models that consider the trade imbalance and the changing nature of operations. They proposed a time-space network flow model based on Brouer et al. (2011) where customer demand is certain but changes over time. This model aims to reduce the costs associated with moving both loaded and empty containers and the costs associated with lost sales.

A significant amount of research has been done to find ways to improve how shipping companies manage the movement of empty containers on international trade routes. Ships usually follow a regular route, returning to the starting port every four weeks. Long et al. (2013) defined "a service" as all the operations connected to a standard route through a set of ports. They developed a linear network flow model that used variables to represent the number of empty containers loaded or unloaded at a specific stop at a particular time. Over a planning horizon of three weeks, they aimed to minimize the total operations costs. These include handling, storage, and transportation costs for empty containers and penalties for not meeting demand. The model's limitations reflect a ship's weight and capacity limits for empty containers. They balance the flow of empty containers and ensure that the number of unloaded containers does not exceed the available amount on a vessel.

Thus, a network flow approach to the global and operational ECR should model both the flow of laden and empty containers while minimizing the relevant operational costs.

4.3.2 Network Flow Based Models

Ahuja et al. (1993, p. 4) states that "the minimum cost flow problem is the most fundamental of all network flow problems." Thus, each network flow-based approach to solving the ECR should use the minimum cost flow problem as a starting point.

Ahuja et al. (1993) describes the general minimum cost flow problem. Consider a directed network, denoted as $G = (\mathcal{N}, \mathcal{A})$, where N represents a set of n nodes and $\mathcal{A}$ represents a set of m directed arcs. Each arc (i, j) in $\mathcal{A}$ is assigned a cost C_{ij}, indicating the cost per unit flow on that arc. The cost is assumed to vary linearly with the amount of flow. Additionally, every arc (i, j) is associated with a capacity u_{ij}, representing the maximum flow it can carry, and a lower bound l_{ij}, indicating the minimum amount that must flow on the arc.

Within this network, each node i in $\mathcal{N}$ is assigned an integer value b_i, which characterizes its supply or demand. If b_i is positive, node i is considered a supply node. Conversely, if b_i is negative, node i is a demand node with a demand of $-b_i$. If b_i equals zero, node i is classified as a transshipment node.

In the minimum cost flow problem, the decision variables correspond to the flow on the arcs. We represent the flow on an arc (i, j) in $\mathcal{A}$ as x_{ij}. The minimum cost flow problem is an optimization model that seeks to minimize the total cost while satisfying the flow requirements and constraints of the network.

Objective:

$$\min \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ij}$$

Constraints:

$$\begin{aligned} \sum_{j:(i,j)\in\mathcal{A}} x_{ij} - \sum_{j:(j,i)\in\mathcal{A}} x_{ji} = b_i \quad & i \in \mathcal{N} \\ l_{ij} \le x_{ij} \le u_{ij} \quad & (i,\ j) \in \mathcal{A} \\ \sum_{i\in\mathcal{N}} b_i = 0 & \end{aligned}$$

Further, the minimum cost flow problems can be extended based on various characteristics and specific problem variations (Ahuja et al., 1993).

1. Single-commodity vs. Multi-commodity: In single-commodity minimum cost flow problems, there is only one type of flow being considered, such as the transportation of a single product or the movement of a single resource. In contrast, multi-commodity minimum cost flow problems involve the simultaneous flow of multiple commodities, where each commodity may have different origins, destinations, and cost structures (Ahuja et al., 1993). Since multiple load carrier types correspond to multiple commodities that must flow through the network, this thesis' problem has multiple commodities.

2. Static vs. Dynamic: Static minimum cost flow problems consider a fixed network and do not account for changes over time. Dynamic minimum cost flow problems, on the other hand, incorporate time-varying elements, such as varying demands, capacities, or costs over different time periods (Ahuja et al., 1993). This allows for more realistic modeling of flow dynamics in dynamic systems, such as transportation networks with varying traffic patterns. Since the number of laden load carriers that must be transported can vary weekly, the problem in this thesis is dynamic.

3. Time-space minimum cost flow problems: Time-space minimum cost flow problems extend the traditional minimum cost flow framework by incorporating spatial and temporal dimensions (Ahuja et al., 1993). These problems consider the flow movement over time and space, considering time-dependent costs, time windows, and other temporal constraints. This is the case for the problem considered in this thesis since laden load carriers must be transported from an origin port to a destination port (space) and reach the destination port within a delivery time window (time).

4. Source and Sink Constraints: Some minimum cost flow problems include constraints on the sources and sinks of the flow. For example, the problem may require that certain nodes in the network act as sources with fixed supply or sinks with fixed demand. These constraints can model scenarios where flow originates from specific locations (e.g., manufacturing plants) or is destined for specific locations (e.g., distribution centers or customers) (Ahuja et al., 1993). This thesis' problem involves ensuring that laden load carriers are transported from an origin port to a destination port. Thus, a flow of laden load carrier originates at the origin port (source) and is destined for the destination port (sink).

5. Capacity Constraints: Capacity constraints limit the flow traversing an edge in the network. Minimum cost flow problems can involve different capacity constraints, such as edge capacity limits, node capacity limits, or dynamic capacity constraints that vary over time. These constraints ensure that the flow in the network does not exceed the available resources or infrastructure capacity (Ahuja et al., 1993). The problem in this thesis has capacity constraints on transport routes (due to vessel capacities) and port inventory capacities.

6. Multi-modal or Multi-layer Networks: Minimum cost flow problems can also consider networks with multiple modes of transportation or different infrastructure layers. For example, a problem may involve flows transported via road, rail, or air, each with costs and capacities. Alternatively, a problem may consider flows that can traverse different communication network layers, such as wired and wireless connections (Ahuja et al., 1993). Since ECR problems should model both the flow of laden and empty containers (i.e., load carriers in the case of the problem considered in this

thesis) (Song and Dong, 2015), this can be regarded as two layers of the network. The empty load carrier layer and the laden load carrier layer.

The various characteristics of extensions to the minimum cost flow problem are summarized in table 4.2.

Table 4.2: Categorization of Minimum Cost Flow Problems based on Ahuja et al. (1993)

Category	**This Thesis' Problem**
Single-commodity vs. Multi-commodity	Multiple load carrier types
Static vs. Dynamic	Dynamic laden load carrier transportation requirements
Time-space minimum cost flow problems	Laden load
Source and Sink Constraints	Laden load carrier origins and destinations are sources and sinks repectively
Capacity Constraints	Vessel capacities on transport routes and inventory capacities at ports
Multi-modal or Multi-layer Networks	Two layers: one for empty and one for laden load carriers.

4.3.3 Rolling Horizon Planning

When using a network flow approach to solve the ECR, choosing the planning horizon is an important decision (Olivo et al., 2005; Long et al., 2013). Finite horizon planning makes an independent plan for certain periods (from 1 to T), after which a new plan is made for the next set of periods (from $1 + T$ to $2T$), and so on. However, because these plans often need to be updated with new information, such as changes in demand forecasts or actual demand, they are typically only used for the upcoming periods.

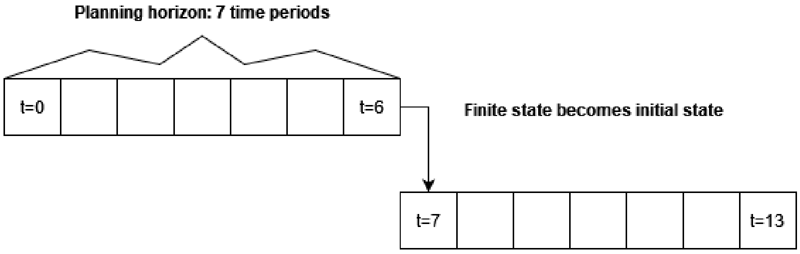


Figure 4.2: Finite horizon

On the other hand, rolling horizon planning works slightly differently. It creates a plan for a set of periods (from 1 to T), but only the decisions for the first ΔT periods are enacted. Then, the planning horizon 'rolls forward' by ΔT periods, leading to a revised plan for the period from $1 + \Delta T$ to $T + \Delta T$. This strategy allows for delaying decisions about future periods until new information becomes available. Additionally, rolling horizon planning can lead to more accurate production plans using actual demand data rather than demand forecasts (Long et al., 2013). Figure 3.3 illustrates the differences between finite and rolling horizon planning, emphasizing the enhanced adaptability provided by the latter method.

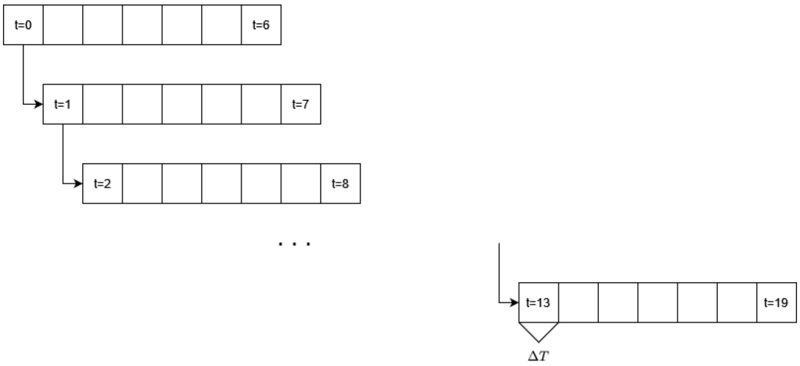


Figure 4.3: Rolling horizon

Chapter 5

Mathematical Model

In this section, the mathematical model that addresses the problem laid out in chapter 3 is presented. The model minimizes the cost of managing load carriers so that all orders are delivered within their time window.

5.1 Modelling Approach

The following sample illustrates the modeling approach by serving as a representative subset of the larger system that the model aims to describe.

5.1.1 Sample Network

The sample network has four ports and two transport routes. The ports are represented as nodes, and the transport is represented as arcs that connect the nodes.

In the sample network, transport route R1 visits P1, P2, P4, and P3 before it travels back to P1. Likewise, the transport route R2 visits P1 and P2 before it travels back to P1. However, the transport route schedule is also essential. The schedule states the time period when the transport routes visit each port in their sequence. Let the transport routes have the following schedules:

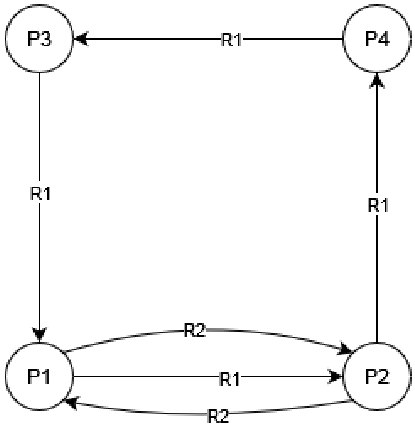


Figure 5.1: Sample network

Schedule for transport route R1:

- Arrive at P1 at time period 0
- Depart from P1 at time period 0
- Arrive at P2 at time period 1
- Depart from P2 at time period 2
- Arrive at P4 time period 3
- Depart from P4 at time period 3
- Arrive at P3 at time period 4
- Depart from P3 at time period 4
- Arrive at P1 at time period 5

Schedule for transport route R2:

- Arrive at P1 at time period 0
- Depart from P1 at time period 2
- Arrive at P2 at time period 3
- Depart from P2 at time period 5
- Arrive at P1 at time period 6

Further, let the transport routes have a capacity of six storage units, and let each laden load carrier takes up two storage units while one empty load carrier takes up one storage unit. Thus, each transport route has a capacity of at most three laden load carriers, six empty load carriers, or a combination of the two.

Let there be one transport order that states that five laden load carriers should be transported from P1 to P2. The laden load carriers are ready for transportation at $t = 0$, and the delivery time window at P2 is $[t = 2,\ t = 4]$.

Further, let there be five empty load carriers in the inventory of port P1 at $t = 0$ and let both P1 and P2 have a lead time of zero time periods for converting empty load carriers to laden and laden to empty (i.e., loading and unloading load carriers).

5.1.2 Modelling the Sample Network as a Time-Space Network

A time-space network is a graphical representation that helps visualize and solve routing and scheduling problems where time plays a significant role. It extends a traditional network diagram by adding the time dimension (Bai et al., 2017).

A possible solution to the sample presented in subsection 5.1.1 can be seen in figure 5.2.

Sample Time-Space Network Structure

The transport route schedules are modeled as paths in the time-space network. In this time network, nodes represent ports and specific time periods at those ports.

The colored arcs represent the transport route paths (i.e., orange for R1 and blue for R2). Load carrier flow along these arcs indicates that the load carriers are transported with the vessel from one port at a time period to another port at a later time period. Thus, the colored arcs connect the port visiting sequence of the transport route according to the schedule.

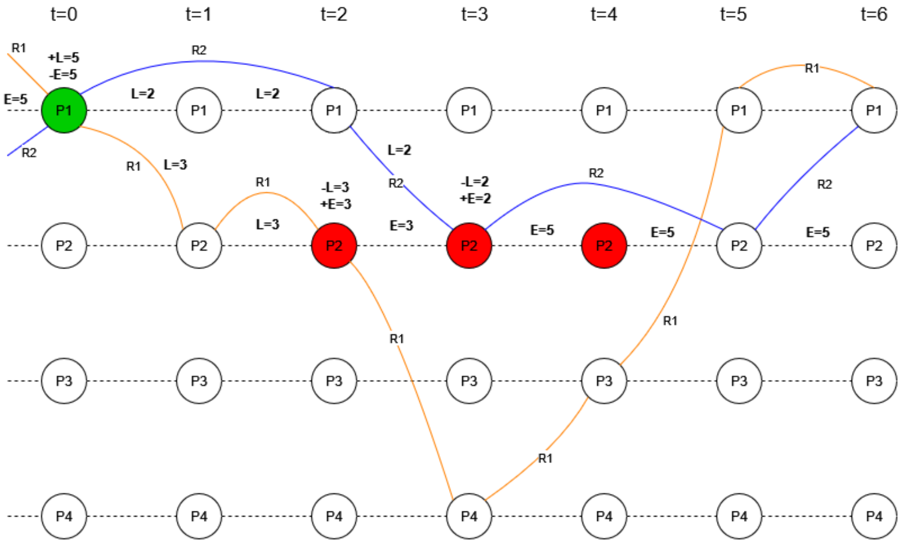


Figure 5.2: The time-space representation of the sample, with a possible solution.

Inventory is modeled similarly to transport routes and is represented by dashed arcs. Load carrier flow along these arcs indicates that load carriers are stored in inventory at a port from one time to the following. Modeling inventory this way is common in the literature (e.g., see: Olivo et al., 2005). The arcs can be thought of as inventory routes.

The orange-colored arc between P2 at $t = 1$ and P2 at $t = 2$ are used by load carriers that stay on board the vessel when anchored at P2 from $t = 1$ to $t = 2$. Only load carriers that arrive at P2 from the orange-colored arc at $t = 1$ can flow through that arc. The load carriers rolled off the vessel are stored in inventory and thus flow through the dashed arc from P2 at $t = 1$ to P2 at $t = 2$. Only load carriers that arrive with a transport route can stay on board the vessel when it docks at a port because the cost of storing load carriers on the vessel and in inventory at the port may differ.

Sample Time-Space Network Decisions

The decision is to find a flow through the time-space network so that the laden load carriers of the transport order reach their destination in the delivery time window. Flow conservation constraints handle the flow at each node.

The laden load carriers belonging to the transport order are ready for transportation at P1 at the time period $t = 0$ (i.e., the green node). Thus, five empty load carriers must be converted to laden load carriers at this node (indicated by the $+L = 5$ and the $-E = 5$). The five empty load carriers arrive from the inventory of $P1$, as indicated by the $E = 5$ flow in the dashed arc arriving at P1 at $t = 0$. Thus, five laden load carriers must depart from P1 at $t = 0$ to ensure that the flow is conserved.

The laden load carriers of the transport order flow from P1 at $t = 0$ and must reach P2 within the delivery time window (i.e., red nodes). In the sample solution, three (resp. two) laden load carriers flow to P2 along transport route R1 (resp. R2). The load carriers arriving with R1 (resp. R2) start unloading at time period $t = 2$ (resp. $t = 3$). Thus, the number of laden load carriers decreases by three (resp. two) at P2 at $t = 2$ (resp. P2 at $t = 3$), while the number of empty load carriers increases by three (resp. two). Since the flow is conserved at each node, the empty load carriers are stored in inventory at P2.

5.2 Notation

Sets

$\mathcal{D}_{rt}$ - Set of arcs $(i,\ j)$ in route r that departs from a port i at time period t

$\mathcal{L}$ - Set of load carrier types

$\mathcal{O}$ - Set of transport orders

$\mathcal{P}$ - Set of ports

$\mathcal{R}$ - Set of routes

$\mathcal{R}^{\mathcal{T}}$ - Set of transport routes. $\mathcal{R}^{\mathcal{T}} \subset \mathcal{R}$

$\mathcal{R}^{\mathcal{A}}_{jit}$ - Set of routes, arriving at port i from port j at time period t.

$\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit}$ - Set of transport routes, arriving at port i from port j at time period t, and that docks at port i for at least one time period (i.e., does not depart from i in the same time period as it arrives at i). $\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit} \subset \mathcal{R}^{\mathcal{A}}_{jit}$

$\mathcal{R}^{\mathcal{D}}_{ijt}$ - Set of routes, departing from port i to port j at time period t.

$\mathcal{R}^{\mathcal{T}\mathcal{D}}_{jit}$ - Set of transport routes, departing from port i to port j at time period t, and that have docked at port i for at least one time period (i.e., does not depart in the same time period as it arrived at i). $\mathcal{R}^{\mathcal{T}\mathcal{D}}_{jit} \subset \mathcal{R}^{\mathcal{D}}_{jit}$

$\mathcal{T}$ - Set of time periods

Parameters

C_{ijr} - Cost of transporting one storage unit between port $i \in \mathcal{P}$ and port $j \in \mathcal{P}$ with route $r \in \mathcal{R}$

C^E_l - Cost of purchasing a new empty load carrier of type $l \in \mathcal{L}$.

D_o - The destination port for order $o \in \mathcal{O}$

D^{TL}_o - Earliest possible delivery time for order $o \in \mathcal{O}$ (i.e., lower part of the time window for order $o \in \mathcal{O}$)

D^{TU}_o - Latest possible delivery time for order $o \in \mathcal{O}$ (i.e., upper part of the time window for order $o \in \mathcal{O}$)

K_r - Maximum storage capacity for route $r \in \mathcal{R}$

L_{it} - Lead time at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

O_o - Origin port of order $o \in \mathcal{O}$

S^L_{ilot} - The number of empty load carriers of type $l \in \mathcal{L}$ that have completed loading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and have become laden load carriers belonging to transport order $o \in \mathcal{O}$

T_{ijr} - The number of time periods required to travel between port $i \in \mathcal{P}$ and $j \in \mathcal{P}$ with route $r \in \mathcal{R}$

T^D_{irt} - The number of time periods the vessel serving transport route $r \in \mathcal{R}^{\mathcal{T}}$ has been in the dock at port $i \in \mathcal{P}$ when it departs from port i at time period $t \in \mathcal{T}$.

T^S_o - The time period where the laden load carriers belonging order $o \in \mathcal{O}$ are ready for transportation.

W^E_l - The number of storage units required by an empty load carrier of type $l \in \mathcal{L}$

W^L_l - The number of storage units required by a laden load carrier of type $l \in \mathcal{L}$

X_{jilrot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to transport order $o \in \mathcal{O}$ transported with route $r \in \mathcal{R}$ that arrives at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and were transported from port $j \in \mathcal{O}$ before the beginning of the planning horizon.

Y_{jilrt} - The number of empty load carriers of type $l \in \mathcal{L}$ transported with route $r \in \mathcal{R}$ that arrive at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and were transported from port $j \in \mathcal{P}$ before the beginning of the planning horizon.

Decision Variables

u^L_{ilot} - The number of laden load carriers of type $l \in \mathcal{L}$ of order $o \in \mathcal{O}$ that starts unloading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

x_{ijlrot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to order $o \in \mathcal{O}$ that depart from port $i \in \mathcal{P}$ to port $j \in \mathcal{P}$ with route $r \in \mathcal{R}$, at time period $t \in \mathcal{T}$

y_{ijlrt} - The number of empty load carriers of type $l \in \mathcal{L}$ that depart from port $i \in \mathcal{P}$ to port $j \in \mathcal{P}$ with route $r \in \mathcal{R}$, at time period $t \in \mathcal{T}$

y^P_{ilt} - The number of new empty load carriers of type $l \in \mathcal{L}$ purchased and delivered port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

Auxiliary Variables

a^E_{ilt} - The number of empty load carriers of type $l \in \mathcal{L}$ that arrive at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

a^L_{ilot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to transport order $o \in \mathcal{O}$ that arrive at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

d^E_{ilt} - The number of empty load carriers of type $l \in \mathcal{L}$ that depart from port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

d^L_{ilot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to transport order $o \in \mathcal{O}$ that depart from port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

s^E_{ilt} - The number of laden load carriers of type $l \in \mathcal{L}$ that have completed unloading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and thus have become empty.

u^E_{ilt} - The number of empty load carriers of type $l \in \mathcal{L}$ that starts loading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

5.3 Model Description

Flow Conservation Constraints

Flow of Laden Load Carriers

$$\sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}^{\mathcal{A}}_{jit} \mid (t-T_{jir}) \geq 0} x_{jilro(t-T_{jir})} + \sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}^{\mathcal{A}}_{jit}} X_{jilrot} = a^{L}_{ilot} \qquad t \in \mathcal{T},\ i \in \mathcal{P},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.1}$$

$$\sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}^{\mathcal{D}}_{ijt}} x_{ijlrot} = d^{L}_{ilot} \qquad t \in \mathcal{T},\ i \in \mathcal{P},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.2}$$

$$S^{L}_{ilot} - u^{L}_{ilot} = d^{L}_{ilot} - a^{L}_{ilot} \qquad t \in \mathcal{T},\ i \in \mathcal{P},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.3}$$

The constraint (5.1) has two terms. The first term encompasses the laden load carriers that were transported from $j \in \mathcal{P}$ to $i \in \mathcal{P}$ in the planning horizon. It states that laden load carriers arriving at a port $i \in \mathcal{P}$ can arrive from any other port $j \in \mathcal{P}$ if there is a route arriving at i from j at a time period t and that the departure time period from j is in the planning horizon. The second term encompasses laden load carriers that were transported from j to i and departed from j before the beginning of the planning horizon. It states that laden load carriers arriving at a port $i \in \mathcal{P}$ can arrive from any other port $j \in \mathcal{P}$ if there is a route arriving at i from j at a time period t. If $i = j$ then the load carriers arrive from inventory. These load carriers arrive at i at the exact time when time period t starts.

The constraint (5.2) states that laden load carriers departing from port $i \in \mathcal{P}$ can be transported to any other port $j \in \mathcal{P}$ if there is a route departing from i to j at time period t. If $i = j$ then the laden load carriers are stored in inventory. These load carriers depart from i at the end of time period t.

Constraints (5.3) represent the flow balance for laden load carriers at each port. It states that the number of laden load carriers that depart less those that arrive should be equal to the number of empty load carriers that have completed loading and thus become laden, less the laden load carriers that have started unloading.

$$x_{iilrot} - x_{jilro(t-\mathcal{T}_{jir})} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}_{jit}^{\mathcal{T}\mathcal{A}} \mid t - T_{jir} \geq 0\},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.4}$$

$$x_{iilrot} - X_{jilrot} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}_{jit}^{\mathcal{T}\mathcal{A}} \mid t - T_{jir} < 0\},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.5}$$

$$x_{iilro(t-T_{irt}^{D})} - x_{ijlrot} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}_{ijt}^{\mathcal{T}\mathcal{D}} \mid t - T_{jir}^{D} \geq 0\},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.6}$$

$$X_{iilrot} - x_{ijlrot} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}_{ijt}^{\mathcal{T}\mathcal{D}} \mid t - T_{jir}^{D} < 0\},\ l \in \mathcal{L},\ o \in \mathcal{O} \tag{5.7}$$

Constraints (5.4)–(5.7) represent the flow conservation for laden load carriers that stay on the vessel when it docks at a port. These constraints are to enforce two things. First, only laden load carriers that arrive with a vessel at a port can stay on board the vessel. Second, laden load carriers that stay on board a vessel that docks at a port must also depart with the same vessel.

Flow of Empty Load Carriers

The flow constraints for empty load carriers are essentially the same as those for laden load carriers.

$$\sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}_{jit}^{\mathcal{A}} \mid (t - T_{jir}) \geq 0} y_{jilr(t-T_{jir})} + \sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}_{jit}^{\mathcal{A}}} Y_{jilrt} = a_{ilt}^{E} \qquad t \in \mathcal{T},\ i \in \mathcal{P},\ l \in \mathcal{L} \tag{5.8}$$

$$\sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}_{ijt}^{\mathcal{D}}} y_{ijlrt} = d_{ilt}^{E} \qquad t \in \mathcal{T},\ i \in \mathcal{P},\ l \in \mathcal{L} \tag{5.9}$$

$$s_{ilt}^{E} + y_{ilt}^{P} - u_{ilt}^{E} = d_{ilt}^{E} - a_{ilt}^{E} \qquad t \in \mathcal{T},\ i \in \mathcal{P},\ l \in \mathcal{L}, \tag{5.10}$$

The constraint (5.8) has two terms. The first term encompasses the empty load carriers transported from $j \in \mathcal{P}$ to $i \in \mathcal{P}$ in the planning horizon. It states that empty load carriers arriving at a port $i \in \mathcal{P}$ can arrive from any other port $j \in \mathcal{P}$ if there is a route arriving at i from j at a time period t and that the departure time period from j is in the planning horizon. The second term encompasses empty load carriers that were transported from j to i and departed from j before the beginning of the planning horizon. It states that empty load carriers arriving at a port $i \in \mathcal{P}$ can arrive from any other port $j \in \mathcal{P}$ if there is a route arriving at i from j at a time period t. If $i = j$, then the load carriers arrive from inventory. These load carriers arrive at i at the exact time when time period t starts.

The constraint (5.9) states that empty load carriers departing from port $i \in \mathcal{P}$ can be transported to any other port $j \in \mathcal{P}$ if there is a route departing from i to j at time period t. If $i = j$, the laden load carriers are stored in inventory. These load carriers depart from i at the end of the time period t.

Constraints (5.10) represent the flow balance for empty load carriers at each port. It states that the number of empty load carriers that depart less those that arrive should be equal to the number of laden load carriers that have completed unloading and thus become empty plus the new empty load carriers purchased, less the empty load carriers that have started loading (and will become laden after the port lead time).

$$y_{iilrt} - y_{jilr(t-\mathcal{T}_{jir})} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}^{\mathcal{T}^{\mathcal{A}}}_{jit} \mid t - T_{jir} \geq 0\},\ l \in \mathcal{L} \tag{5.11}$$

$$y_{iilrt} - Y_{jilrt} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}^{\mathcal{T}^{\mathcal{A}}}_{jit} \mid t - T_{jir} < 0\},\ l \in \mathcal{L} \tag{5.12}$$

$$y_{iilr(t-T^{D}_{irt})} - y_{ijlrt} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}^{\mathcal{T}^{\mathcal{D}}}_{ijt} \mid t - T^{D}_{jir} \geq 0\},\ l \in \mathcal{L} \tag{5.13}$$

$$Y_{iilrt} - y_{ijlrt} \leq 0 \quad t \in \mathcal{T},\ i \in \mathcal{P},\ j \in \mathcal{P} \setminus \{i\},\ r \in \{\mathcal{R}^{\mathcal{T}^{\mathcal{D}}}_{ijt} \mid t - T^{D}_{jir} < 0\},\ l \in \mathcal{L} \tag{5.14}$$

Constraints 5.11–5.14 represent the flow conservation for empty load carriers that stay on the vessel when it docks at a port. These constraints are to enforce two things. First, only laden load carriers that arrive with a vessel at a port can stay on board the vessel. Second, laden load carriers that stay on board a vessel that docks at a port must also depart with the same vessel.

Connecting the Flow of Laden and Empty Load Carriers

$$s^E_{ilt} = \sum_{o \in \mathcal{O}} u^L_{ilo(tL_{it})} \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t - L_{it} \geq 0\,\},\ l \in \mathcal{L} \tag{5.15}$$

$$s^E_{ilt} = 0 \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t - L_{it} < 0\,\},\ l \in \mathcal{L} \tag{5.16}$$

$$u^E_{ilt} = \sum_{o \in \mathcal{O}} S^L_{ilo(t+L_{it})} \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t + L_{it} \leq |\mathcal{T}|\,\},\ l \in \mathcal{L} \tag{5.17}$$

$$u^E_{ilt} = 0 \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t + L_{it} > |\mathcal{T}|\,\},\ l \in \mathcal{L} \tag{5.18}$$

The equation (5.15)-(5.16) states that the number of laden load carriers that have completed unloading at a port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ is equal to the number of laden load carriers that started unloading at port $i \in \mathcal{P}$ at time period $i - L_{it} \in \mathcal{T}$. The equation thus states that empty load carriers appear exactly L_{it} time periods after the laden load carriers start unloading. The variable s^E_{ilot} thus states the number of empty load carriers that have completed converting from laden.

The equation (5.17)-(5.18) states that the number of empty load carriers that starts unloading at a port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ is equal to the number of empty load carriers that have completed unloading at $i \in \mathcal{P}$ at $t + L_{it}$. The equation thus states that empty load carriers must start loading L_{it} time periods before the empty load carriers have completed loading and have become laden load carriers ready for transportation. The variable u^E_{ilot} thus states the number of empty load carriers that start the process of being converted to laden (i.e., starts loading).

Capacity constraint

$$\sum_{(i,j) \in \mathcal{D}_{rt}} \sum_{l \in \mathcal{L}} \sum_{o \in \mathcal{O}} W^L_l x_{ijlrot} + \sum_{(i,j) \in \mathcal{D}_{rt}} \sum_{l \in \mathcal{L}} W^E_l y_{ijlrt} \leq\ K^r \qquad r \in \mathcal{R}, t \in T \tag{5.19}$$

Constraint (5.19) ensures that the storage capacity on a route is never exceeded. This constraint enforces vessel capacity in the case of transport routes. If $r \in \mathcal{R}$ is an inventory route belonging to port $i \in \mathcal{P}$, then constraint (5.19) ensures that the inventory capacity at i is never exceeded.

Time window constraint

$$S_{O_o l o T_o^S} = \sum_{t'=T_o^S+D_o^{TL}}^{T_o^S+D_o^{TU}} u^L_{D_o l o t'} \qquad o \in \{\mathcal{O} \mid T_o^S \in \mathcal{T}\}, l \in \mathcal{L} \tag{5.20}$$

$$\sum_{t'=0}^{D_o^{TU}} \sum_{j\in\mathcal{P}} \sum_{i\in\mathcal{P}} \sum_{r\in\mathcal{R}^A_{jit}} X_{jilrot'} = \sum_{t'=T_o^S+D_o^{TL}}^{T_o^S+D_o^{TU}} u^L_{D_o l o t'} \qquad o \in \{\mathcal{O} \mid T_o^S \notin \mathcal{T}\}, l \in \mathcal{L} \tag{5.21}$$

Constraint (5.20) and (5.21) ensures that the laden load carriers belonging to a transport order start unloading in the time window of that order. There are two situations to handle. Constraint (5.20) handles the situation where the laden load carriers belonging to the order start their transportation journey in the planning horizon. Constraint (5.21) handles the situation where the laden load carriers belonging to the transport order start their transportation journey before the beginning of the planning horizon but where the delivery time window lies within the planning horizon.

Equilibrium constraints

$$\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{P}} u^L_{ilot} = \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{P}} S_{ilot} \qquad o \in \{\mathcal{O} \mid T_o^S \in \mathcal{T}\}, l \in \mathcal{L} \tag{5.22}$$

$$\sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{P}} u^L_{ilot} = \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{P}} \sum_{i\in\mathcal{P}} \sum_{r\in\mathcal{R}^A_{jit}} X_{jilrot'} \qquad o \in \{\mathcal{O} \mid T_o^S \notin \mathcal{T}\}, l \in \mathcal{L} \tag{5.23}$$

Constraint (5.22)-(5.23) ensures that no more load carriers can be unloaded than what has been loaded.

Non-negativity constraints

$$u^L_{ilot} \in \mathbb{Z}^+ \quad o \in \mathcal{O}, l \in \mathcal{L}, i \in \mathcal{P},\ t \in \mathcal{T} \tag{5.24}$$

$$x_{ijlrot} \in \mathbb{Z}^+ \quad o \in \mathcal{O},\ l \in \mathcal{L}, j \in \mathcal{P}, i \in \mathcal{P}, r \in \mathcal{R},\ t \in \mathcal{T} \tag{5.25}$$

$$y_{ijlrt} \in \mathbb{Z}^+ \quad l \in \mathcal{L}, j \in \mathcal{P}, i \in \mathcal{P}, r \in \mathcal{R},\ t \in \mathcal{T} \tag{5.26}$$

$$y^P_{ilt} \in \mathbb{Z}^+ \quad l \in \mathcal{L}, i \in \mathcal{P},\ t \in \mathcal{T} \tag{5.27}$$

Constraint (5.24)-(5.27) ensures that all decision variables must be non-negative integers.

Objective Function

$$
\begin{aligned}
min\, z\ =& \sum_{t\in \mathcal{T}} \sum_{r\in \mathcal{R}} \sum_{(i,j)\in \mathcal{D}_{rt}} \sum_{l\in \mathcal{L}} \sum_{o\in \mathcal{O}} C_{ijr} W_l^L x_{ijlrot} \\
&+ \sum_{t\in \mathcal{T}} \sum_{r\in \mathcal{R}} \sum_{(i,j)\in \mathcal{D}_{rt}} \sum_{l\in \mathcal{L}} C_{ijr} W_l^E y_{ijlrt} \\
&+ \sum_{t\in \mathcal{T}} \sum_{i\in \mathcal{P}} \sum_{l\in \mathcal{L}} C_l^E y_{ilt}^P
\end{aligned}
\tag{5.28}
$$

The objective function presented in equation (5.28) represents the total costs of managing load carriers. The equation has three terms. The first two calculate costs for handling laden and empty load carriers. In the case of transport routes, $C_{ijr}W_l^L$ (resp. $C_{ijr}W_l^E$) is the cost of transporting one laden (resp. empty) load carrier of type $l \in \mathcal{L}$ between ports $i \in \mathcal{P}$ and $j \in \mathcal{P}$. In the case of inventory routes, $C_{iir}W_l^L$ (resp. $C_{iir}W_l^E$) is the cost of storing one laden (resp. empty) load carrier of type $l \in \mathcal{L}$ in inventory at port $i \in \mathcal{P}$. The third term calculates the cost of purchasing new empty load carriers.

Chapter 6

Case Study

The mathematical model presented in chapter 5 is tested on real data. Hydro provides a realistic data set called the *source data set.* The source data set contains the number of laden load carriers planned to be transported between the Norwegian plants and the terminal ports each week in 2022. It also contains information about transport routes that were planned to be used and the port sequence and schedules of these transport routes. However, the source data set does not contain the actual transportation data of laden load carriers (i.e., the path each load carrier took to get from the origin to the destination), the delivery time windows, or any information about transportation or inventory levels of empty load carriers.

6.1 Input Data

6.1.1 General Data

Planning Horizon

For the analysis in this thesis, time is divided into distinct periods, with four time periods per day. Each time period thus corresponds to a duration of 6 hours. This division is based on the schedules of transport routes presented below (see subsection 6.1.1). The length of the planning horizon studied in this thesis stretches from 173 time periods to 341 time periods (see section 6.4).

Plants and Ports

This section describes the port input data. Five ports are associated with Hydro's Norwegian production plants: Karmøy, Husnes, Årdal, Høyanger, and Sunndal. These ports reflect Hydro's domestic operations within Norway. There are also five terminal ports in

Europe: Rotterdam, Swinoujscie, Gothenburg, Immingham, and Esbjerg. Finally, there are three transshipment ports: Haugesund, Bergen, and Tananger. These are all ports $p \in \mathcal{P}$

As presented in section 5.1.1, the inventory at each port is modeled as an inventory route that has arcs that connect a port at a time period to the same port in the following time period. Thus, each port $p \in \mathcal{P}$ has an inventory route, and the storage capacity K_r is the inventory capacity when r is an inventory route.

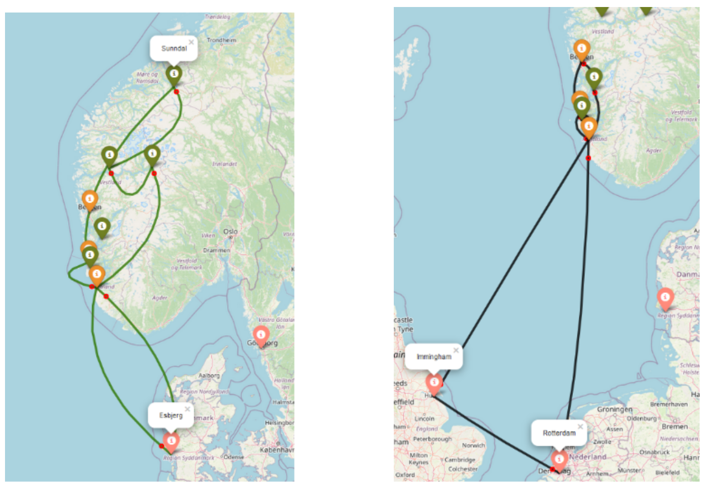
The inventory capacity at each port is not provided in the source data set, nor publicly available information. Discussions with Hydro revealed that inventory capacity is not generally a limiting factor. Thus, K_r is set to a large number of 5 000 storage units for each inventory route (i.e., the inventory of each port $p \in \mathcal{P}$). Storage units are nominal values. When laden and empty load carriers are stored in inventory, they use up storage units according to the factor provided in Table 6.2.

Transport Routes

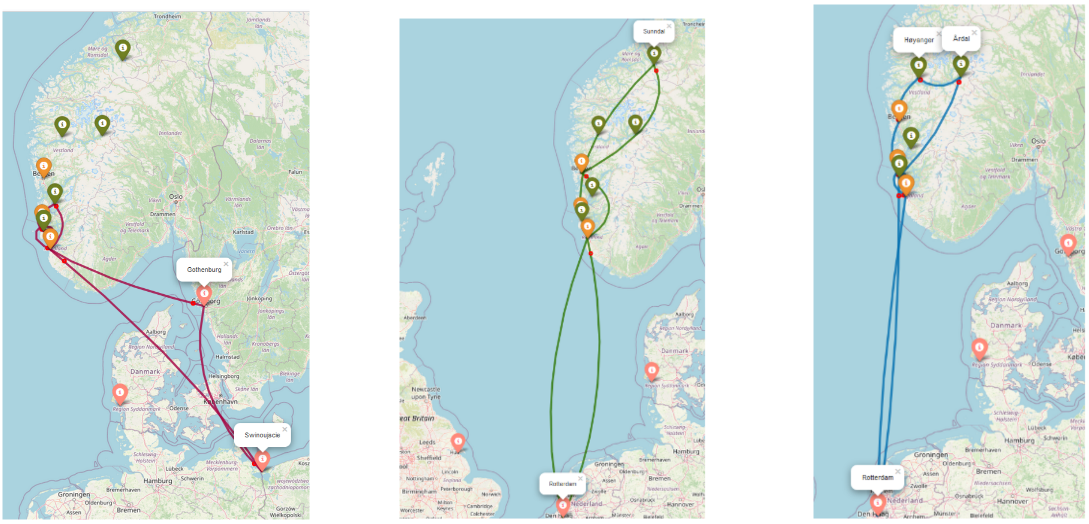
The transport routes that operate Hydro's maritime transportation system are presented in Table 6.1. The source data set does not provide the available capacity on board each vessel. However, since data about planned laden transportation along each transport route is given, the available storage has therefore been set not to limit transportation planned for in the past. Figure 6.1 shows a map of each transport route.

Table 6.1: Input data for the transport routes.

Transport Route	**Vessel Name**	**Storage K_r**	**Plants visited**	**Terminal ports visited**
Coastal Interplant	Trans Carrier	200	Årdal Høyanger Sunndal	Esbjerg
Northern Trade	Misida	300	Sunndal	Rotterdam
Sognefjord Trade	Misana	300	Karmøy Årdal Høyanger	Rotterdam
Poland Trade	SC Ahtela	325	Karmøy Husnes	Gothenburg Swinoujscie
Southern Trade 1	Bore Bay	200	Husnes Karmøy	Rotterdam Immingham
Southern Trade 2	SC Connector	250	Husnes Karmøy	Rotterdam Immingham



(a) Costal Interplant **(b)** Southern Trade 2 **(c)** Southern Trade 1

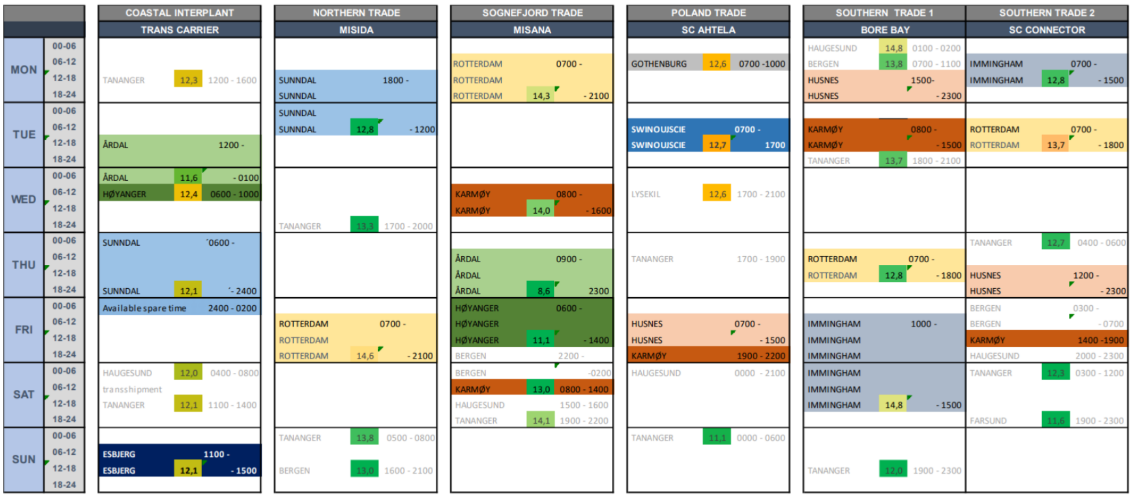


(d) Poland Trade **(e)** Northern Trade **(f)** Sognefjord Trade

Figure 6.1: Map of the transport routes

Schedules for Transport Routes

The schedule for each transport route is publicly available information and can be found online through services like Vessel Finder. Schedules were also provided in the source data set. The schedule for each transport route repeats every week (i.e., every 28th time period). Figure 6.2 shows the weekly schedules for the transport routes. The T^{D}_{irt} parameters can also be derived from the weekly schedule and repeated every week. Further, the sets $\mathcal{R}^{\mathcal{A}}_{jit}$, $\mathcal{R}^{\mathcal{T}^{\mathcal{A}}}_{jit}$, $\mathcal{R}^{\mathcal{D}}_{ijt}$, and $\mathcal{R}^{\mathcal{T}^{\mathcal{D}}}_{jit}$ are also derived from the schedule.



		COASTAL INTERPLANT	NORTHERN TRADE	SOGNEFJORD TRADE	POLAND TRADE	SOUTHERN TRADE 1	SOUTHERN TRADE 2
		TRANS CARRIER	MISIDA	MISANA	SC AHTELA	BORE BAY	SC CONNECTOR
MON	00-06					HAUGESUND 14,8 0100 - 0200	
	06-12			ROTTERDAM 0700 -	GOTHENBURG 12,6 0700 -1000	BERGEN 13,8 0700 - 1100	IMMINGHAM 0700 -
	12-18	TANANGER 12,3 1200 - 1600	SUNNDAL 1800 -	ROTTERDAM		HUSNES 1500-	IMMINGHAM 12,8 - 1500
	18-24		SUNNDAL	ROTTERDAM 14,3 - 2100		HUSNES - 2300	
TUE	00-06		SUNNDAL				
	06-12		SUNNDAL 12,8 - 1200		SWINOUJSCIE 0700 -	KARMØY 0800 -	ROTTERDAM 0700 -
	12-18	ÅRDAL 1200 -			SWINOUJSCIE 12,7 1700	KARMØY - 1500	ROTTERDAM 13,7 - 1800
	18-24					TANANGER 13,7 1800 - 2100	
WED	00-06	ÅRDAL 11,6 - 0100					
	06-12	HØYANGER 12,4 0600 - 1000		KARMØY 0800 -	LYSEKIL 12,6 1700 - 2100		
	12-18			KARMØY 14,0 - 1600			
	18-24		TANANGER 13,3 1700 - 2000				
THU	00-06	SUNNDAL ´0600 -					TANANGER 12,7 0400 - 0600
	06-12			ÅRDAL 0900 -	TANANGER 1700 - 1900	ROTTERDAM 0700 -	
	12-18			ÅRDAL		ROTTERDAM 12,8 - 1800	HUSNES 1200 -
	18-24	SUNNDAL 12,1 ´- 2400		ÅRDAL 8,6 2300			HUSNES - 2300
FRI	00-06	Available spare time 2400 - 0200		HØYANGER 0600 -			BERGEN 0300 -
	06-12		ROTTERDAM 0700 -	HØYANGER	HUSNES 0700 -	IMMINGHAM 1000 -	BERGEN - 0700
	12-18		ROTTERDAM	HØYANGER 11,1 - 1400	HUSNES - 1500	IMMINGHAM	KARMØY 1400 -1900
	18-24		ROTTERDAM 14,6 - 2100	BERGEN 2200 -	KARMØY 1900 - 2200	IMMINGHAM	HAUGESUND 2000 - 2300
SAT	00-06	HAUGESUND 12,0 0400 - 0800		BERGEN -0200	HAUGESUND 0000 - 2100	IMMINGHAM	TANANGER 12,3 0300 - 1200
	06-12	transshipment		KARMØY 13,0 0800 - 1400		IMMINGHAM	
	12-18	TANANGER 12,1 1100 - 1400		HAUGESUND 1500 - 1600		IMMINGHAM 14,8 - 1500	
	18-24			TANANGER 14,1 1900 - 2200			FARSUND 11,6 1900 - 2300
SUN	00-06		TANANGER 13,8 0500 - 0800		TANANGER 11,1 0000 - 0600		
	06-12	ESBJERG 1100 -					
	12-18	ESBJERG 12,1 - 1500	BERGEN 13,0 1600 - 2100			TANANGER 12,0 1900 - 2300	
	18-24						

Figure 6.2: Schedule for the routes

Load Carrier Types

Hydro has two load carrier types, mafis, and cassettes. These are shown in Figure 2.7. As presented in chapter 3, empty load carriers have a smaller storage requirement than laden load carriers because empty load carriers can be stacked on top of each other during transportation. For example, five empty cassettes can be stacked on each other to take up the same amount of storage as one laden cassette. Both mafis and cassettes take up one storage unit when they are empty and five storage units when they are laden (i.e., $W^{E}_{l} = 1$ and $W^{L}_{l} = 5$ for both).

As presented in chapter 3, the operators at the ports require some lead time to convert laden load carriers into empty, and empty into laden takes time. Discussions with Hydro revealed a lead time of one time period at each port would be suitable for both load carrier types (cassettes and mafis), given a granularity of four time periods per day. This lead time gives the port operators enough time to load (resp. unload) empty (resp. laden) load carriers.

Since the properties of cassettes and mafis are indistinguishable, this thesis treats them as identical. This is done by replacing mafis with cassettes in the source data set and initializing the model with one load carrier type (i.e., cassettes). Table 6.2 presents the parameter values used for cassettes.

Table 6.2: Values for the load carrier parameters.

Parameter	**Value**
W_l^E	1
W_l^L	5
L_{it}	1

6.1.2 Costs Structure

Operational Costs

The operational costs consist of the transportation and inventory costs. The exact inventory cost for each port is not public information nor provided by the source data set. Likewise, the exact transportation costs are sensitive information. However, discussions with Hydro revealed that transportation costs dominate inventory costs. Thus, storing laden and empty load carriers in port inventory is cheaper than on vessels.

The cost parameter C_{iir} describes the inventory costs when r is an inventory route. Thus, if r is the inventory route of port i, then C_{iir} describes the cost of storing one storage unit in the inventory of port i for one time period.

I pick higher inventory costs for the terminal and transshipment ports than for the ports belonging to the Norwegian plants. The reason for this is that this cost structure favors storing load carriers close to the production plants. This will facilitate the repositioning of empty load carriers from the terminal ports back to the Norwegian plants. Therefore, the cost parameter C_{iir} is set to 1 NOK in the Norwegian plants and 100 NOK in the case of the terminal and transshipment ports. Thus, it costs 1 NOK to store one storage unit in one time period in the inventory of a Norwegian plant, while it costs 100 NOK to do the same at a transshipment or terminal port.

The cost parameter C_{ijr} describes the transportation costs when r is a transport route. Transportation costs depend on the storage required and the distance (as described in chapter 3). I assume that the vessels serving the transport routes travel steadily and that the travel time between ports is proportional to the distance. Therefore, the cost parameters C_{ijr} are equal to the time periods it takes to travel between port i and port j with transport route r scaled with a factor of 1000 NOK. The scaling of the transport costs is necessary to ensure that transportation load carriers are more expensive than storing them in inventory. Transport times between ports are found in the source data set (see

Figure 6.2).

The transport costs C_{ijr}, and the inventory costs C_{iir} are given in storage units. To find the cost of storing load carriers, the storage units each load carrier requires should be considered (see Table 6.2).

Penalty Costs

The penalty costs consist of the cost of purchasing new empty load carriers and the costs of purchasing extra capacity with trucks. The model presented in chapter 5 handles the option to purchase extra capacity with trucks through an "extra capacity route." I initialize the extra capacity route with an arc between every pair of ports daily (i.e., every fourth period) and a large capacity of $K_r = 10000$ storage units. The specific cost details for acquiring this additional capacity are confidential and not openly accessible. However, discussions with Hydro revealed that purchasing extra capacity is expensive and should be avoided. Further, the specific cost details for purchasing new empty load carriers are confidential and not openly accessible.

However, the ratio between the penalty costs is more important than the absolute values. Purchasing new empty load carriers should be more expensive than transporting empty load carriers between ports. This facilitates empty load carrier repositioning with the transport routes. Further, the number of load carriers in the maritime logistics system should not be a limiting factor. Therefore, transporting load carriers with trucks should be more expensive than purchasing empty load carriers. This prohibits repositioning empty load carriers with trucks, which would indicate too few load carriers in the system.

To enforce the penalty cost structure, the cost of purchasing one extra capacity is set to a large number of 1 000 000 NOK. Further, the cost parameter C_{ijr} is set equal to the time periods it takes to travel between port i and port j with trucks (i.e., r is the extra capacity route) and scaled with a factor of 10 000 000 NOK. This ensures that purchasing a new empty load carrier at j is cheaper than transporting one from i to j.

6.1.3 Overview of Input Data

Table 6.3 provides an overview of the value ranges for the sets and parameters discussed in this section.

Table 6.3: Overview of the input data

Parameter or Set		Value Range
Ports	$\mathcal{P}$	13 ports
Load Carrier Types	$\mathcal{L}$	1 load carrier type
Transport Routes	$\mathcal{R}^{\mathcal{T}}$	6 transport routes
Routes	$\mathcal{R}$	20 routes (6 transport routes, 13 inventory routes, and 1 extra capacity route)
Time periods	$\mathcal{T}$	173 - 341 time periods
Route capacity	K_r	200 - 10 000 storage units
Number of storage units required by an empty load carrier	W_l^E	1 storage unit
Number of storage units required by a laden load carrier	W_l^L	5 storage units
Port lead time	L_{it}	1 time period
Cost of transporting one storage unit from port i to j with route r	C_{ijr}	1 - 10 000 000 NOK

6.2 Transport Orders

Transport orders represent transportation needs and are described by the parameters in Table 7.2. I pick only one $S^L_{ilot} > 0$ for each transport order since transport orders represent the transport of laden load carriers from one place and time to an other place and time. The source data set has no concept of transport orders. However, the origin port, destination port, the weekly number of laden load carriers to transport between the origin and the destination, and the planned transport route, are provided in the source data set. The time period when the laden load carriers are ready for transportation and the delivery time windows is not provided.

Since the source data set only contained parts of the information necessary to construct transport orders, I had to make assumptions about the missing information. Discussions with Hydro revealed that the laden load carriers that departed from an origin port became ready for transportation the week before. I assume that the load carriers from the weekend and Monday complete loading on Monday evening, that load carriers from Tuesday and Wednesday complete loading on Wednesday evening, and that load carriers from Thursday and Friday complete loading on Friday evening. Thus, I create one transport order each week on Monday, Wednesday, and Friday. Further, discussions with Hydro revealed that a delivery time window of 3 days - 15 days is reasonable for each transport order. Therefore,

Table 6.4: Parameters and their descriptions

Parameter	**Description**	**Provided in the Source Data Set**
O_o	Origin port	Yes
D_o	Destination port	Yes
S^L_{ilot}	The number of laden load carriers to transport	Yes
T_o	The time period when the laden load carriers are ready for transportation	No
$[D^{TL}_o,\ D^{TU}_o]$	The delivery time window	No

I pick a delivery time window of 12 - 60 time periods for each transport order.

The procedure for creating transport orders is provided in Algorithm 1. The algorithm is written in pseudocode, and the time periods for T_o, D^{TL}_o, and D^{TU}_o are determined based on the start of the planning horizon.

The source data set provided data for 2022. I created transport orders from January 1st, 2022. Thus, the first week of transport orders is created for week 52 in 2021.

Algorithm 1: Creating transport orders

```
Input  : The origin port, destination port, the weekly number of laden load carriers to transport between
         the origin and the destinations
Output : Set of transport orders
for each week $w$ that has transport of laden load carriers of type $l$ between origin $i$ and destination $j$ do
    amount $\leftarrow$ the number of laden load carriers of type $l$ to transport between $i$ and $j$ for $w$  ;
    $n \leftarrow$ amount // 3 ;
    $r \leftarrow$ amount % 3 ;
    for day in {Monday, Wednesday, Friday} of the week before $w$ do
        Create a transport ordero;
        $t \leftarrow$ time period of day
        transportOrderAmount $\leftarrow n$ ;
        if day = Monday and $r \leq 2$ then
            transportOrderAmount $\leftarrow$ transportOrderAmount + 1;
        end
        if day = Wednesday and $r = 2$ then
            o.transportOrderAmount $\leftarrow$ o.transportOrderAmountt + 1;
        end
        $T_o \leftarrow t$;
        $D^{DL}_o \leftarrow$ day + 3 days;
        $D^{DU}_o \leftarrow$ day + 15 days;
        $S^L_{ilot} \leftarrow$ transportOrderAmount
    end
end
```

6.3 Initialization

To transport laden load carriers from the origin ports to the destination ports, empty load carriers are required at the origin ports. However, the source data did not contain information about transportation or inventory levels of empty load carriers. To compensate for this missing data, I developed a strategy to stimulate the circulation of empty load carriers. This was accomplished by running the model for a period corresponding to 12 weeks. This duration was chosen to ensure that the delivery time window for nine weeks of transport orders would fall within the planning horizon (since transport orders have a delivery time window of 3 - 15 days, three additional weeks are required). These were the first nine weeks of transport orders that were created from the source data set. Thus, the initialization contained transport orders that had load carriers ready for transportation from Monday, December 27th, 2021 to Sunday, February 20th, 2022. During these 12 weeks, the model purchased new empty load carriers where they were required. This method allowed for the integration of empty load carriers into the model despite the lack of direct data.

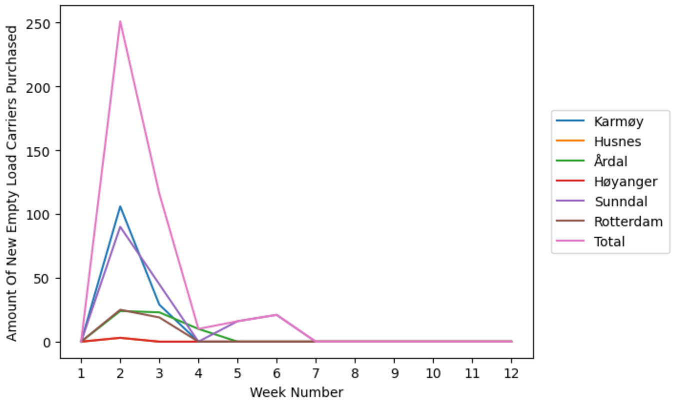


Figure 6.3: The number of new empty load carriers purchased at the various ports under the initialization phase.

The results from the initialization of empty load carriers demonstrate an initial high frequency of new empty load carrier purchases in the starting weeks. Figure 6.3 shows that this trend found stability around weeks six and seven. The total number of new empty load carriers obtained over the 12 weeks is 414.

Hydro's transportation needs may be higher in some periods and lower in others (e.g., because of seasonal changes). Therefore, more empty load carriers were included in the port inventories to adapt the initialization to this variation in transportation needs. This measure ensured that the number of empty load carriers would not become a limiting factor under different demand conditions. Therefore, 86 additional load carriers were added, totaling 500 load carriers.

Moreover, all subsequent instances proceeded from week ten. Thus, the starting point for all future model runs was the status at the start of week ten (i.e. Monday, February 28th, 2022), including the distribution of empty and laden load carriers. This approach allows for a smooth transition between model runs, each starting with an accurate representation of the current empty load carrier distribution. The inventories of empty load carriers at the various ports at the beginning of week ten can be seen in Table 6.5, while those on vessels can be seen in Table 6.6.

Table 6.5: Initial status of empty load carriers in inventory at the ports

Port	**Empty Load Carriers in Inventory**
Haugesund	28
Gothenburg	15
Karmøy	30
Høyanger	7
Husnes	61
Sunndal	30
Årdal	21
Rotterdam	15

Table 6.6: Empty load carriers on vessels at the beginning of the planning horizon

Transport Route	**From**	**To**	**Empty Load Carriers On The Vessels**
Coastal Interplant	Esbjerg	Tananger	23

6.4 Instances

This section presents an overview of the five instances used in this thesis. The purpose of each of the five instances is to create a ten-week transportation plan. All five instances are initialized with the data from the starting point selected in the initialization phase (see section 6.3). The starting point is Monday, February 28th, 2022. The instances thus use transport order data from Monday, February 28th, 2022, and ten weeks onward. The number of laden load carriers ready for transportation at each port (i.e., the values of the S^L_{ilot} parameters) for these ten weeks is presented in Figure 6.4.

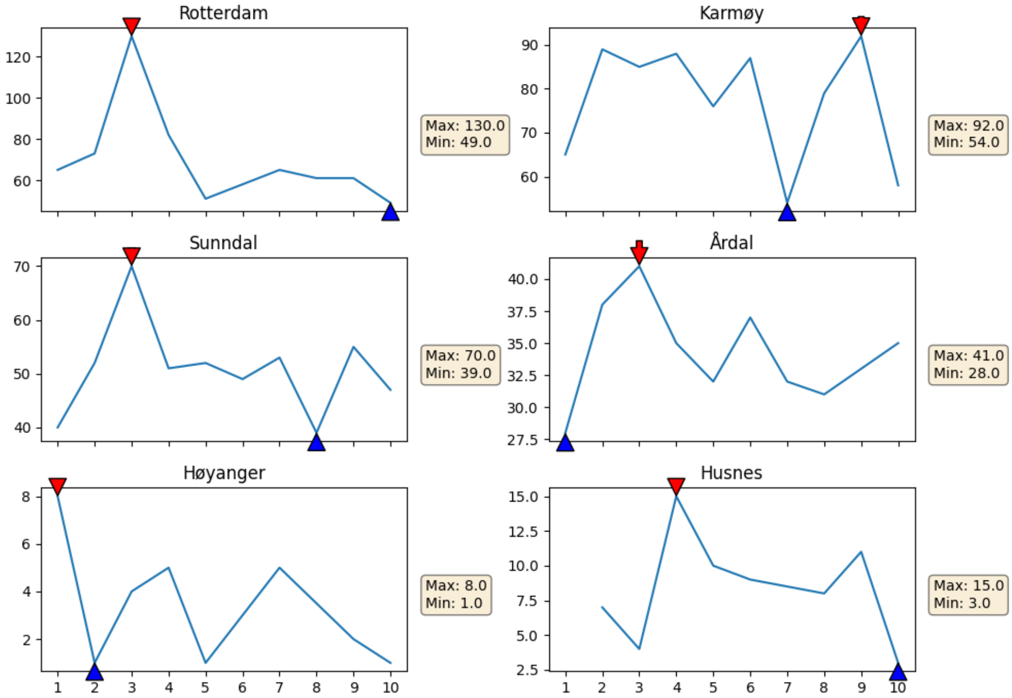


Figure 6.4: The number of laden load carriers ready for transportation by week.

6.4.1 Instances without changes to known data

This subsection presents four instances. These instances create a ten-week transportation plan without changes to the known data. These instances aim to analyze the effects of the planning horizon length.

Instance 1: Finite Horizon Ten Weeks

In instance 1, all ten weeks of transport orders are known. This instance uses a finite horizon. This means the model creates a ten-week transportation plan, and then the decisions for all ten weeks are applied. To make sure that the transport orders with T_o in the tenth week are included, the planning horizon must be ten weeks (280 time periods) plus the delivery time window (60 time periods) plus the largest port lead time (1 time period). It is necessary to add the port lead time to the planning horizon so laden load carriers that start unloading in the 340th time period have enough time (i.e., the lead time) to become empty. The planning horizon thus becomes 341 time periods. With this planning horizon length, 558 transport orders are included.

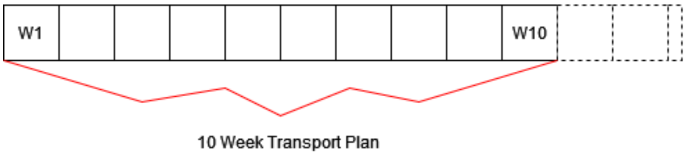


Figure 6.5: Instance 1 use a finite horizon where transport orders for all ten weeks are known

Instance 2: Rolling Horizon Six Weeks

In instance 2, six weeks of transport orders are known, but new information arrives weekly. Thus, transportation orders for weeks one to six are known at week one, but in week two, information about transportation orders for week seven arrives. Similarly to instance 1, instance 2 must ensure that the transport orders with T_o in the sixth week are included. Thus the planning horizon has 229 time periods (six weeks plus delivery time window and lead time). The rolling horizon is run with $\Delta t = 1$ *week*. Thus, the horizon is rolled forward one week when the model is run and the first week's decisions are applied. The port inventories and load carriers on board vessels are updated after each roll. The model runs until ten weeks of transport orders are known. Figure 6.6 shows that this requires five model runs and four rolls.

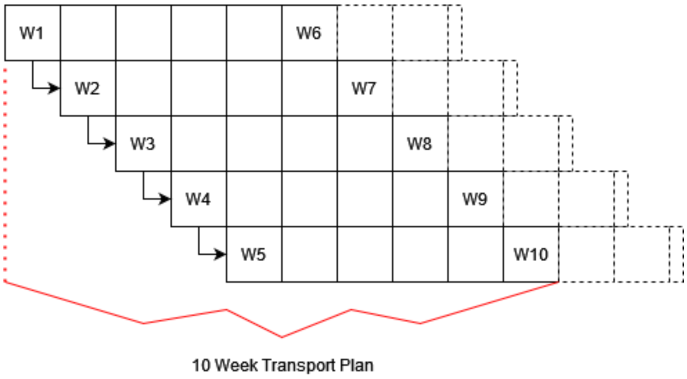


Figure 6.6: Instance 2 uses a rolling horizon where six weeks of transport orders are known.

Instance 3: Rolling Horizon Four Weeks

In instance 3, four weeks of transport orders are known, but new information arrives weekly. Thus, at week one, transportation orders for weeks one to four are known, but in week two, information about transportation orders for week five arrives. Similarly to instances 1 and 2, instance 3 must ensure that the transport orders with T_o in the fourth week are included. Thus the planning horizon has 173 time periods (4 weeks plus delivery time window and lead time). The model is run in a rolling horizon manner to deal with the dynamic environment. The rolling horizon is run with $\Delta t = 1\ week$. Thus, the horizon is rolled forward one week when the model is run and the first week's decisions are applied. The port inventories and load carriers on board vessels are updated after each roll. The model runs until ten weeks of transport orders are known. Figure 6.7 shows that this requires seven model runs and six rolls.

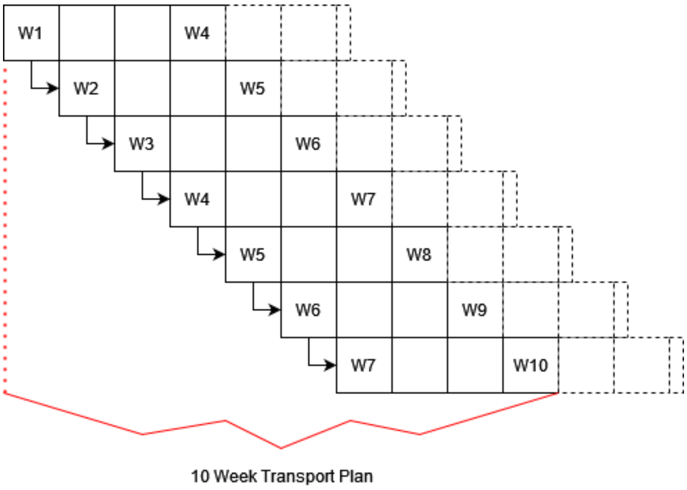


Figure 6.7: Instance 3 use a rolling horizon where four weeks of transport orders are known.

Instance 4: Rolling Horizon Three Weeks

In instance 4, three weeks of transport orders are known, but new information arrives weekly. Thus, at week one, transportation orders for weeks one to three are known, but in week two, information about transportation orders for week four arrives. Instance 4 must ensure that the transport orders with T_o in the third week are included. Thus the planning horizon has 145 time periods (3 weeks plus delivery time window and lead time). The model is run in a rolling horizon manner to deal with the dynamic environment. The rolling horizon is run with $\Delta t = 1\ week$. Thus, the horizon is rolled forward one week when the model is run and the first week's decisions are applied. The port inventories and load carriers on board vessels are updated after each roll. The model runs until ten weeks of transport orders are known. Figure 6.8 shows that this requires eight model runs and seven rolls.

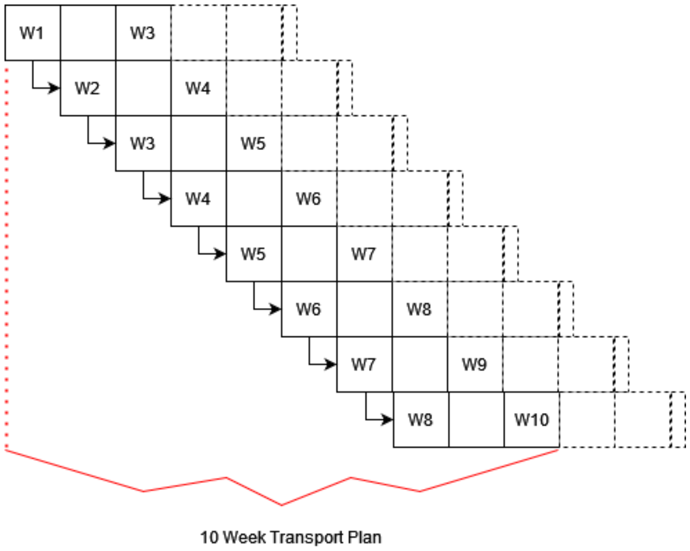


Figure 6.8: Instance 4 use a rolling horizon where three weeks of transport orders are known.

6.4.2 Instance with Changes to Known Data

This subsection presents an instance where changes happen to the known data during the planning. This instance aims to analyze how the model adapts to such changes.

Instance 5: Rolling Horizon Four Weeks with Data change

Instance 5 is similar to instance 3, but at each roll, there are changes to the known data. In week one, transport orders for weeks one to four are known, but in week two, information about transportation orders for week five arrives. When information about week five arrives, roughly 10% of the number of laden load carriers originally belonging to transport orders in week five is subtracted from week five. Then, roughly 10% of them are added back to week four. This changes the already known data since the model created a plan where the content of the transport orders for week four was considered known. The changing process repeats itself for each roll. There are some rounding errors in the change process. However, these errors are not corrected since the purpose of instance 5 is to analyze how the model adapts to changes in data.

Instance 5 must also ensure that the transport orders with T_o in the fourth week are included. Thus the planning horizon has 173 time periods (4 weeks plus delivery time window and lead time). The model runs until ten weeks of transport orders are known. Similar to instance 3, this requires seven model runs and six rolls. Figure 6.9 shows how the number of laden load carriers is changed between each roll.

Table 6.7 shows the changes in each roll, and Subfigures 6.10a-6.10f show the actual changes for each port. These changes can, for example, represent changes in customer

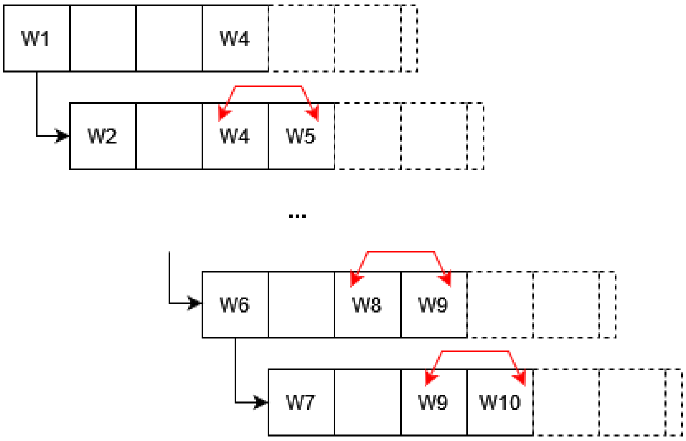


Figure 6.9: Instance 5 use a rolling horizon where four weeks of transport orders are known but changes to the known data happens at each roll.

Table 6.7: The number of laden load carriers changed at each roll

	Roll 1	**Roll 2**	**Roll 3**	**Roll 4**	**Roll 5**	**Roll 6**
Laden load carriers removed from transport orders in week n	32	24	17	20	19	20
Laden load carriers added to transport orders in week n - 1	25	27	20	16	20	17

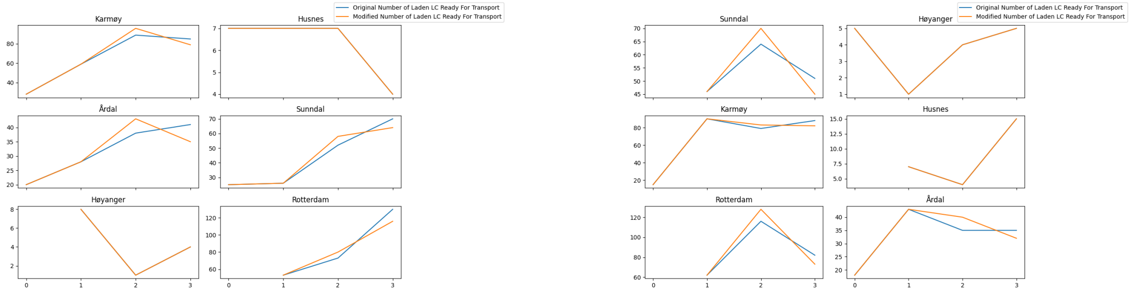
requests.

6.4.3 Overview of Instances

An overview of the instances presented in this section is provided in Table 6.8.

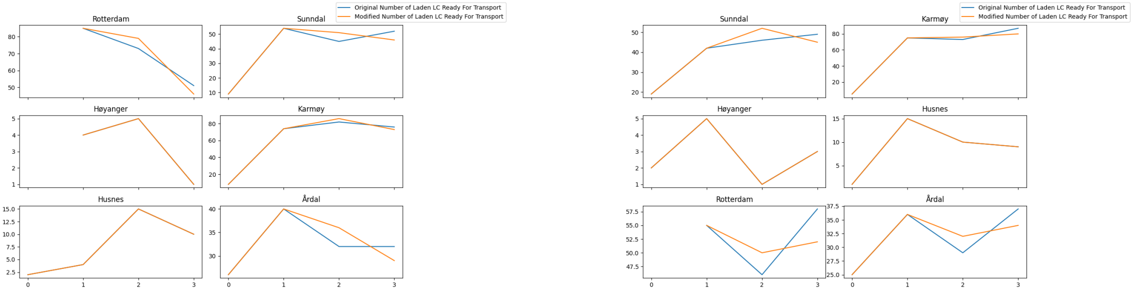
Table 6.8: Overview of the five instances considered in this thesis.

Instance	**Weeks of Transport Orders Known**	**Number of Time Periods** $\|\mathcal{T}\|$	**Number of Transport Orders** $\|\mathcal{O}\|$
1	Ten weeks	341	558
2	Six weeks (at each run)	229 (each run)	362 (average)
3	Four weeks (at each run)	173 (at each run)	261 (average)
4	Three weeks (at each run)	145 (at each run)	208.5
5	Four weeks (with 10% change at last week each roll)	173 (at each run)	261 (average)



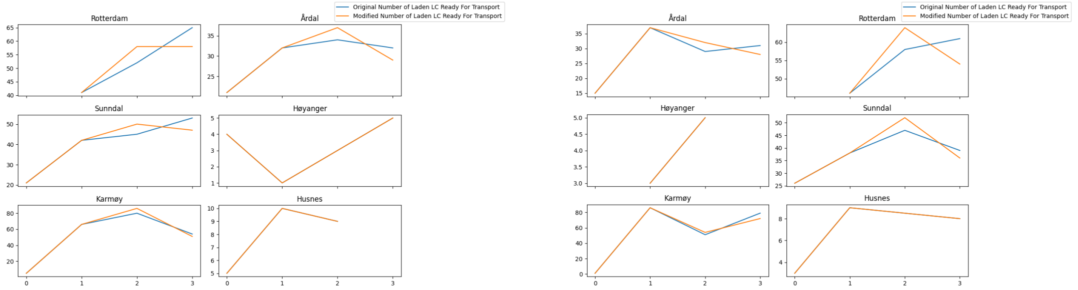
(a) Change in laden load carriers (LC) ready for transport at roll 1

(b) Change in laden load carriers (LC) ready for transport at roll 2



(c) Change in laden load carriers (LC) ready for transport at roll 3

(d) Change in laden load carriers (LC) ready for transport at roll 4



(e) Change in laden load carriers (LC) ready for transport at roll 5

(f) Change in laden load carriers (LC) ready for transport at roll 6

Figure 6.10: The difference between the original (blue) and modified (orange) laden load carriers ready for transport at each of the six rolls in instance 5.

Chapter 7

Computational Study

This chapter presents the results of the computational study of is presented. The model has been written in Python and implemented with Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64). All instances have been run on a computer with Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz, using four physical cores, eight logical processors, and 16Gb RAM.

7.1 Instance Results

This section presents the results of the five instances presented in chapter 6.

7.1.1 Instance 1

Instance 1 was solved to optimality in 31.55 minutes. The model had 2 937 018 constraints and 12 906 673 variables. Table 7.9 presents the results. Transport costs and inventory costs account for 85.11% and 14.89% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.1: Costs for instance 1

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 874 006	14 360 700	2 513 306	0	0
100,00 %	85,11 %	14,89 %	0 %	0 %

The transportation plan for the ten-week horizon is a description of how all laden load carriers are transported from their origin port (i.e., O_o) at the time period when they are ready for transportation (i.e., T_o) to the destination port (i.e., D_o) within the delivery time window (i.e., $[D_o^{TL}, D_o^{TU}]$). Since instance 1 created a total of 558 transport orders, the solution for one transport order, transport order 538, is presented here for illustration.

Table 7.2: Parameter values for transport order 538

Parameter	**Value**
O_o	Karmøy
D_o	Rotterdam
S_{ilot}^{L}	16 laden load carriers
T_o	Time period 59
$[D_o^{TL}, D_o^{TU}]$	[time period 71, time period 119]

Figure 7.1 shows the paths taken for transport order 538. The 16 laden load carriers are stored in inventory at the origin port Karmøy ('Kar') until time period 77. Then, the laden load carriers take two different paths. Eleven laden load carriers are transported along the transport route Sognefjord Trade through the transshipment ports Haugesund ('Hau') and Tananger ('Tan') before they reach the destination Rotterdam ('Rot') at time period 85. Five laden load carriers remain in the inventory at Karmøy until time period 90. Then the five laden load carriers are transported along the transport route Southern Trade 1 through the transshipment port Tananger before they reach the destination Rotterdam at time period 97. The laden load carriers take two different paths because of capacity limits on board the vessel that serves the transport route Sognefjord Trade (i.e., Misana).

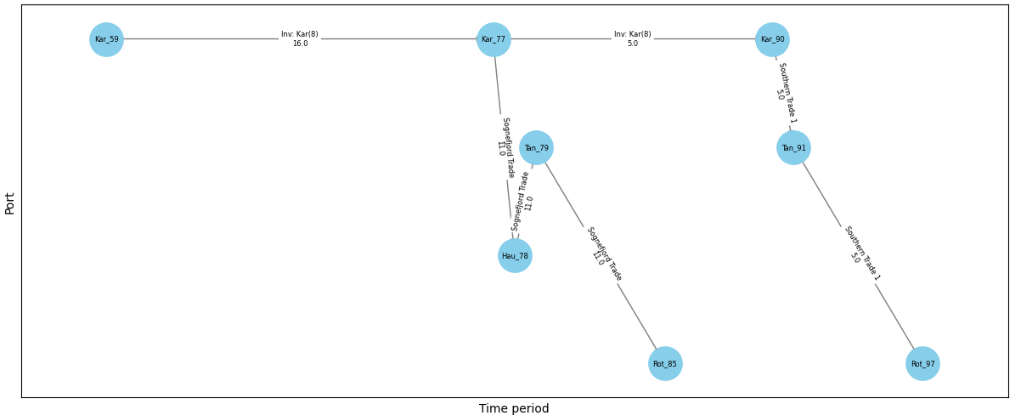


Figure 7.1: The transport paths taken by transport order 538.

Figure 7.2 shows the transport imbalance in Hydro's maritime logistics system. The blue graphs illustrate the laden load carriers that are ready for transport. Thus the blue graph illustrates the number of empty load carriers required at each port. The orange graphs illustrate the number of laden load carriers that starts unloading (because they have reached their destination). Thus, this graph illustrates the number of empty load carriers available at each port since the laden load carriers that start unloading become empty after the port lead time.

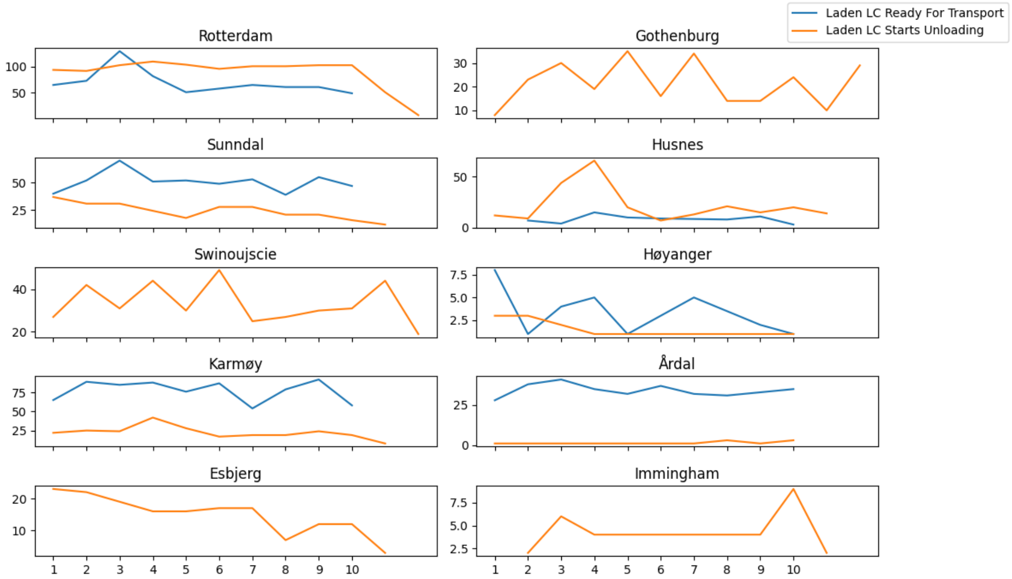


Figure 7.2: The difference between laden load carriers (LC) that starts unloading and the laden load carriers ready for transport at each port by week.

Figure 7.2 shows that the terminal ports Rotterdam, Esbjerg, Immingham, Swinoujscie, and Gothenburg have more available empty load carriers than the number required. This is illustrated with the blue graph being below the orange graph. The imbalance between available and required empty load carriers is most significant at Esbjerg, Immingham, Swinoujscie, and Gothenburg, where no empty load carriers are required. The imbalance is more minor in Rotterdam, which has some requirements for empty load carriers. Further, the Norwegian plants Sunndal, Høyanger, Karmøy, and Årdal have fewer available empty load carriers than the number required. This is illustrated by the blue graph being above the orange graph. The Norwegian plant Husnes have roughly the same number of available and required empty load carriers, although the difference is significant in the earlier weeks.

The transport imbalance requires empty load carriers to be repositioned between the terminal ports Rotterdam, Esbjerg, Immingham, Swinoujscie, and Gothenburg and the Norwegian plants Sunndal, Høyanger, Karmøy, and Årdal. Figure 7.3 illustrates the empty load carrier repositioning between ports. The figure plots the number of empty load carriers on board each vessel for the first 200 time periods. Further, the top of the figure shows a color code matching each port's color code in Figure 6.2. The colored bars in the six transport route plots show when the vessel visits each port in the port sequence of the transport route. Thus, if the number of empty load carriers on board a vessel increases at a port (i.e., there is a spike in the graph), then empty load carriers are loaded on the vessel from the port.

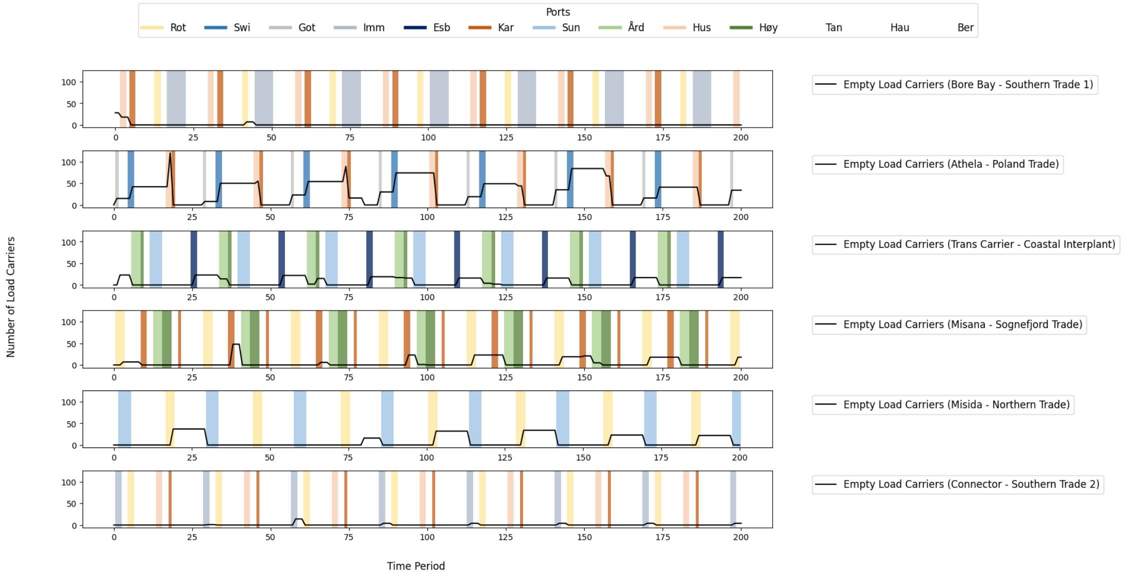


Figure 7.3: The number of empty load carriers on board each vessel at the first 200 time periods.

Figure 7.3 shows that empty load carriers from Rotterdam are primarily repositioned to Sunndal with the Northern Trade transport route. This is illustrated by the increase in empty load carriers at the yellow bar and the decrease in empty load carriers at the blue bar in the Northern Trade plot. There is also some repositioning from Immingham to Rotterdam along Southern Trade 2. These empty load carriers may therefore end up in Sunndal after visiting Rotterdam.

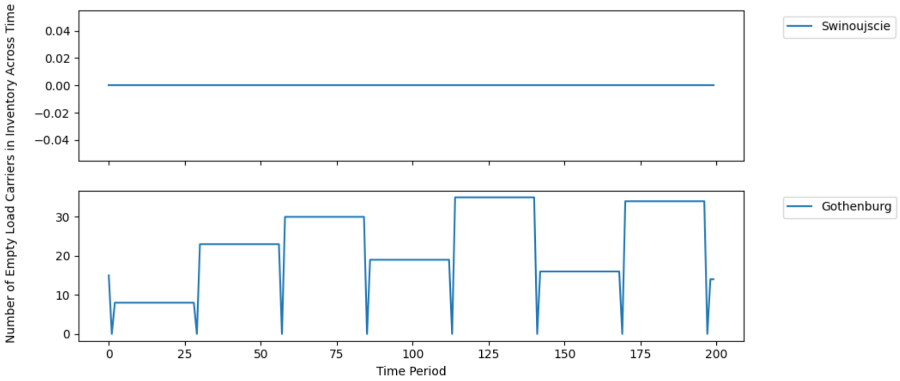


Figure 7.4: The number of empty load carriers in inventory at Gothenburg and Swinoujscie for the first 200 time periods of instance 1.

Likewise, empty load carriers are repositioned from Gothenburg and Swinoujscie to Karmøy with the Poland Trade transport route. Since the lead time of all ports is one time period (see subsection 6.1.1) and the schedule of the Poland Trade transport route states that the vessel Athela should only stop at Gothenburg for one time period, the empty load carriers that are rolled on Athela at Gothenburg must be arriving from the ports inventory. In the case of Swinoujscie, Athela stops for two time periods. This reasoning is backed by Figure 7.4, which shows that Gothenburg stores empty load carriers in inventory while Swinoujscie does not.

Further, empty load carriers to Årdal arrive from Karmøy via the Sognefjord Trade and Esbjerg via the Costal Interplant. The same is true for Høyanger, the stop after Årdal for both Sognefjord Trade and Costal Interplant. Since Karmøy primarily receives empty load carriers from Gothenburg and Swinoujscie, the empty load carriers that reach Årdal and Høyanger are also likely repositioned from these ports.

Table 7.3 summarizes which ports the empty load carriers are primarily repositioned from and the ports they are repositioned to.

Table 7.3: Summary of where the empty load carriers are primarily repositioned from and to.

Plant With Empty Load Carrier Surplus	**Empty Load Carrier Repositioned To**	**Transport Routes Involved**
Rotterdam	Sunndal, Karmøy, Årdal	Northern Trade to Sunndal Sognefjord Trade to Karmøy and Årdal
Immingham	Sunndal (via Rotterdam)	Southern Trade 2 to Rotterdam then Northern Trade to Sunndal
Esbjerg	Karmøy, Årdal or Høyanger	Costal Interplant to Karmøy, then Sognefjord Trade to Årdal or Høyanger
Gothenburg	Karmøy, Årdal or Høyanger	Poland Trade to Karmøy, then Sognefjord Trade to Årdal or Høyanger
Swinoujscie	Karmøy, Årdal or Høyanger	Poland Trade to Karmøy, then Sognefjord Trade to Årdal or Høyanger

Figure 7.5 shows the last 141 time periods and indicates an interesting end-of-horizon effect: Empty load carriers are repositioned from Rotterdam to Husnes ('Hus') with transport route Southern Trade 1 to a greater degree than at the beginning of the planning horizon. In the last 61 time periods, there have been no transport orders with load carriers ready for transport. These time periods exist only to allow the load carriers to reach their destination port within the delivery time window. Thus, if no more transportation orders require empty load carriers, then empty load carriers are transported to Husnes, where the inventory cost is lower than Rotterdam.

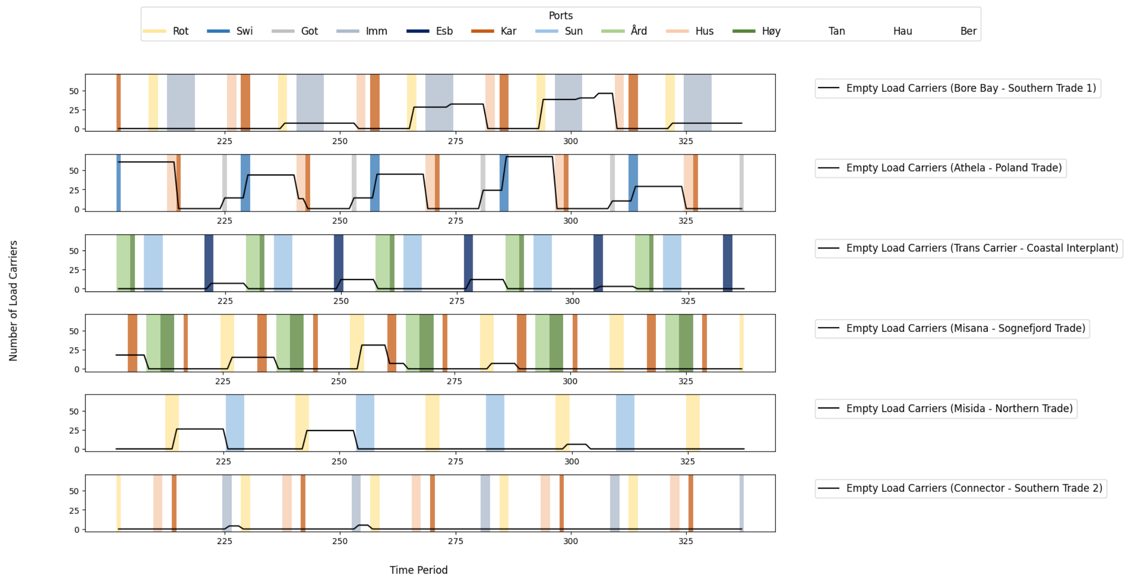


Figure 7.5: The number of empty load carriers on board each vessel at the last 141 time periods of instance 1.

7.1.2 Instance 2

Instance 2 was solved with five runs in a rolling horizon manner. Each of the five runs was solved to optimality in an average of 15 minutes. The model runs averaged 1 284 662 constraints and 5 644 433 variables. Table 7.4 presents the results. Transport and inventory costs account for 85.03% and 14.97% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.4: Costs for instance 2

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 874 006	14 347 200	2 526 806	0	0
100.00 %	85.03 %	14.97 %	0 %	0 %

Instance 2 also created a transportation plan for the ten-week horizon. Thus it included the same transport orders as instance 1 (although each model run has a different number). The solution for transport order 538 is therefore also included here for illustration. Recall Table 7.2 for details about transport order 538.

Figure 7.6 shows the path taken for transport order 538. The 16 laden load carriers are stored in inventory at the origin port Karmøy ('Kar') until time period 77. Then the laden load carriers are transported along the transport route Sognefjord Trade through the transshipment ports Haugesund ('Hau') and Tananger ('Tan') before they reach the destination Rotterdam ('Rot') at time period 85.

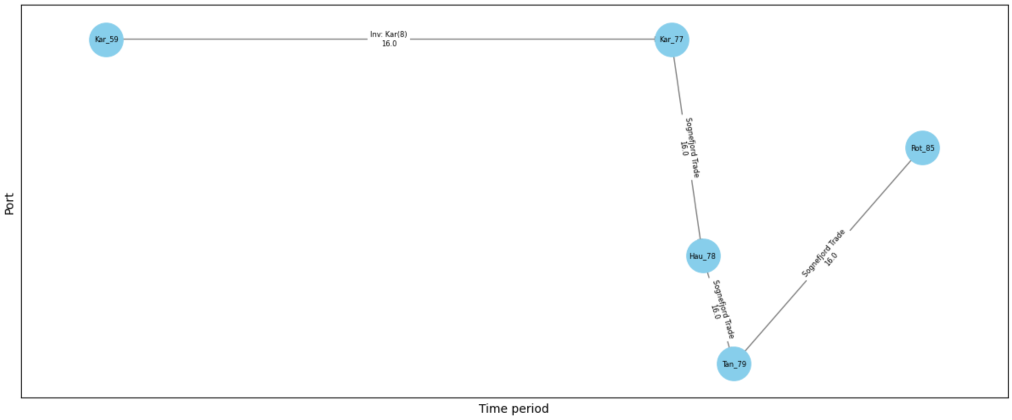


Figure 7.6: The transport path taken by transport order 538 in instance 2.

The transportation imbalance in instance 2 is similar to that of instance 1 (see Figure 7.2). The empty load carrier repositioning is also handled similarly to that presented in Table 7.3. Therefore, these details are not repeated here. For instance 2, the end-of-horizon effect is similar to the effect shown in Figure 7.5. Therefore, it is not repeated here.

7.1.3 Instance 3

Instance 3 was solved with seven runs in a rolling horizon manner. Each of the seven runs was solved to optimality in an average of 8 minutes. The model runs had an average of 702 780 constraints and 3 086 404 variables. Table 7.5 presents the results. Transport and inventory costs account for 85.05% and 14.95% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.5: Costs for instance 3

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 876 811	14 354 400	2 522 411	0	0
100,00 %	85,05 %	14,95 %	0 %	0 %

Instance 3 also created a transportation plan for the ten-week horizon. Thus it included the same transport orders as instance 1 and instance 2 (although each model run has a different number). The solution for transport order 538 is the same as the solution found in instance 1 and is therefore not repeated here (instead, recall 7.1)

The transportation imbalance in instance 3 is similar to that of instance 1 and instance 2 (see Figure 7.2). The empty load carrier repositioning is also handled similarly to that presented in Table 7.3. Therefore, these details are not repeated here. For instance 3, the end-of-horizon effect is similar to the effect shown in Figure 7.5. Therefore, it is not repeated here.

7.1.4 Instance 4

Instance 4 was solved with eight runs in a rolling horizon manner. Each of the eight runs was solved to optimality in an average of 5 minutes. The model runs had an average of 472 386.5 constraints and 2 073 211 variables. Table 7.6 presents the results. Transport and inventory costs account for 85.05% and 14.95% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.6: Costs for instance 4

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 876 811	14 354 400	2 522 411	0	0
100,00 %	85,05 %	14,95 %	0 %	0 %

Instance 4 also created a transportation plan for the ten-week horizon. Thus, it included the same transport orders as instance 1, instance 2, and instance 3 (although each model run has a different number). The solution for transport order 538 differs in instance 4 and is therefore included here for illustration. Recall Table 7.2 for details about transport order 538.

Figure 7.1 shows the paths taken for transport order 538. The 16 laden load carriers are stored in inventory at the origin port Karmøy ('Kar') until time period 62. Then, the laden load carriers take two different paths. One laden load carrier is transported along the transport route Southern Trade 1 through the transshipment ports Tananger ('Tan') before it reaches the destination Rotterdam ('Rot') at time period 69. Since the lower bound of the delivery time window for transport order 538 is time period 71 (see Table 7.2), the laden load carrier will have to stay in inventory at Rotterdam until this time period.

Further, fifteen laden load carriers remain in the inventory at Karmøy until time period 77. Then the fifteen laden load carriers are transported along the transport route Sognefjord

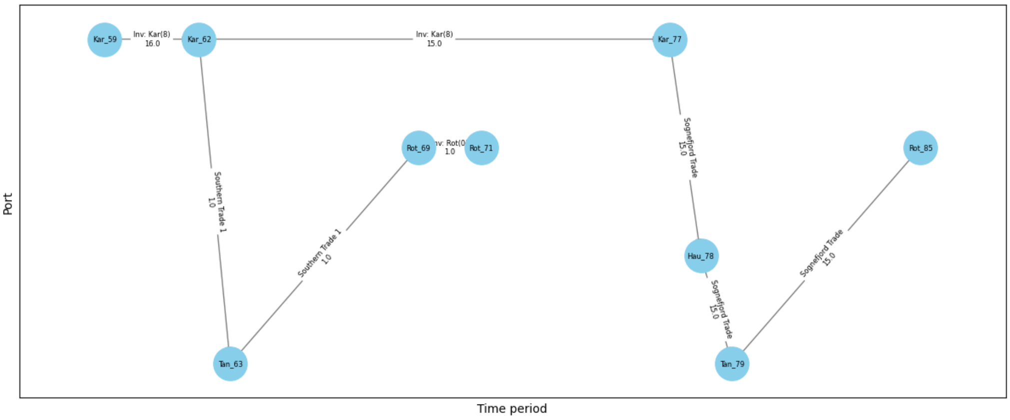


Figure 7.7: The transport paths taken by transport order 538 in instance 4.

Trade through the transshipment port Haugesund ('Hau') and Tananger ('Tan') before they reach the destination Rotterdam at time period 85. The laden load carriers take two different paths because of capacity limits on the routes. The most cost-efficient path is for all the laden load carriers to take the Southern Trade 1 transport route since this results in less inventory cost at Karmøy. However, this path is not feasible due to the capacity constraints on board the vessel Bore Bay which serves the transport route Southern Trade 1.

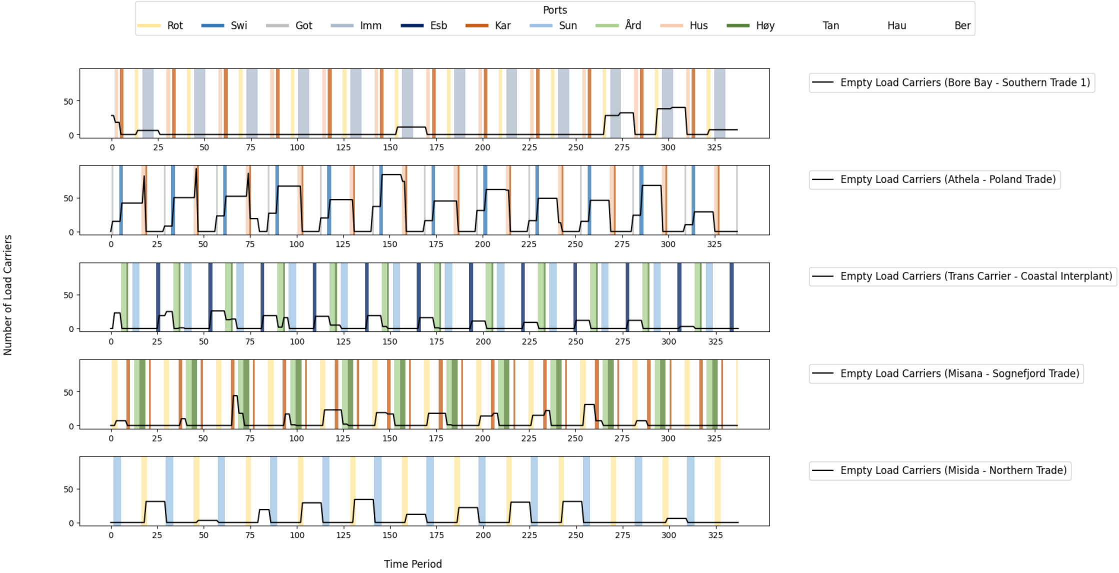


Figure 7.8: The number of empty load carriers on board each vessel for all time periods in instance 4.

The transportation imbalance in instance 4 is similar to that of instance 1, instance 2, and instance 3 (see Figure 7.2). The empty load carrier repositioning is also handled similarly. Therefore, these details are not repeated here. The end-of-horizon effect is, however, slightly different. In Figure 7.8, the empty load carriers are repositioned from

Rotterdam to Husnes ('Hus') with transport route Southern Trade 1 at the end of the planning horizon. However, Figure 7.8 indicates that this repositioning also starts during the planning horizon. Two additional spikes of empty load carriers are being repositioned from Rotterdam to Husnes with Southern Trade 1: the spike around time period 20 and the spike around time period 150.

7.1.5 Instance 5

Table 7.7: Costs for instance 5

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
176 849 973	14 352 100	2 497 873	0	130 000 000
100,00 %	8,12 %	1,41 %	0 %	73,51 %

Instance 5 was solved with seven runs in a rolling horizon manner. Each of the seven runs was solved to optimality in an average of 8 minutes. The model runs had an average of 705 432 constraints and 3 098 167 variables. Table 7.7 presents the results. Transport, inventory costs, and costs of purchasing new empty load carriers account for 8.12%, 1.41%, and 73.51% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is high. This cost comes from purchasing 26 new empty load carriers. Thus, the model did not manage to reposition the empty load carriers to meet the required number at each port.

7.2 Analysis

This section analyses the instance results.

7.2.1 Analysis of Effect of Planning Horizon Length

Figure 7.9 shows that the total costs of instance 1 to instance 4 are increasing as the planning horizon length drops from 341 time periods and ten weeks of transport orders known (instance 1) to 173 time periods and three weeks of transport orders known (instance 4). However, Table 7.9 shows that the relative difference is small.

However, the planning horizon length may affect the empty load carrier repositioning. Figure 7.5 shows that empty load carriers are repositioned from Rotterdam toward Husnes with the Southern Trade 1 transport route and from Rotterdam to Karmøy at the end of the planning horizon. The repositioning starts around time period 240, with about 100 time periods left of the planning horizon. This repositioning happens because the empty load carriers are repositioned to a cheaper inventory (Husnes has lower inventory costs

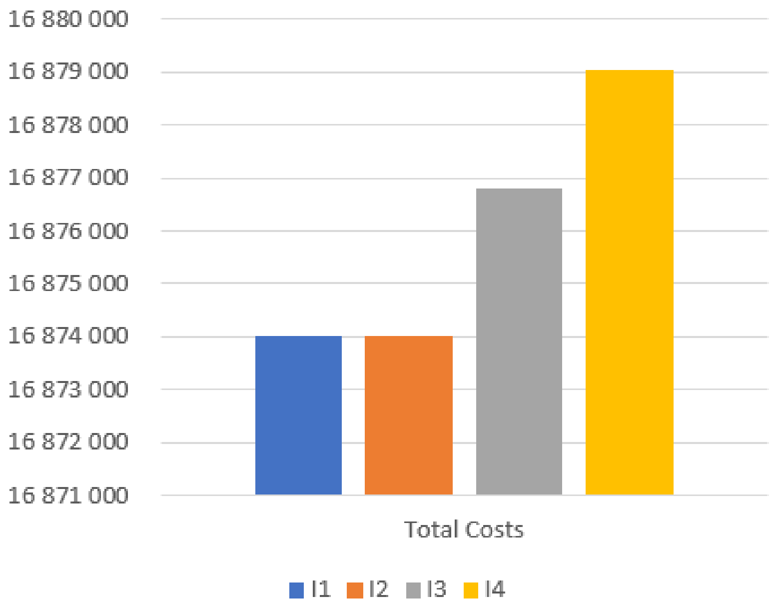


Figure 7.9: The total costs for instance 1 to 4.

than Rotterdam) when they are not needed for transport orders.

Table 7.8: Costs for instance 1 to 4.

Cost	**Instance 1**	**Instance 2**	**Instance 3**	**Instance 4**
Total Costs	16 874 006	16 874 006	16 876 811	16 879 055
Transport Costs	14 360 700	14 347 200	14 354 400	14 351 800
Inventory Costs	2 513 306	2 526 806	2 522 411	2 527 255

When the planning horizon length decrease and the model are solved in a rolling horizon manner, slightly more empty load carriers are repositioned throughout the planning horizon. In instance 4, at the first run, there are no more transport orders starting transportation after week 3. Thus, the empty load carrier repositioning toward Husnes start earlier. Since only the decisions of the first week are applied to each roll, the model has time to correct the early repositioning. However, the two spikes shown in the graph of empty load carriers being repositioned along Southern Trade 1 around time periods 25 and 150 in Figure 7.8 may indicate that a planning horizon of three weeks is too small to correct all the early repositioning.

7.2.2 Benchmark - Comparison with Hydro's Planning Policy

To compare the model to Hydro's existing planning policy, I have developed an algorithm that captures the traits of this policy. This is done because the source data did not contain the actual flow of the load carriers for all transport orders. Hydro's planning process follows a straightforward approach to finding the shortest route from the origin port to the destination port for the laden load carriers. After planning the laden load carriers, the process assigns the empty carriers at the destination ports to return to the origin ports. Suppose there is a shortage of empty carriers at the origin ports. In that case, additional carriers are purchased to meet the demand, which indicates potential shortages of load carriers in the system. The transportation of laden load carriers from the ports is planned so that the transport order with the earliest T_o is planned first. The ports are planned in the following order: Karmøy, Husnes, Årdal, Høyanger, Sunndal, and Rotterdam.

The algorithm is initialized with a time-space network with nodes for each (port, time period) pair and arcs corresponding to the transport routes. The arcs have weights that correspond to the transportation costs along the arc (i.e., C_{ijr}) and capacities (i.e., K_r). The algorithm solves the planning in three phases.

1. The first phase plans the flow of laden load carriers. The origin node (O_o, T_o) is found for each transport order. Then all possible destination nodes $(D_o, [D_o^{TL}, D_o^{TU}])$ are looped through, starting with (D_o, D_o^{TL}), and the weighted shortest path between the origin node and destination node that does not involve trucks is found. Then, the laden load carriers belonging to the transport order (i.e., S_{ilot}) are transported along the path. Suppose the path does not have enough capacity. In that case, the maximum allowed number is transported along the path (or the number of laden load carriers belonging to the transport order if this is smaller than the capacity). If there are laden load carriers left, then the weighted shortest path to the destination node $(D_o, D_o^{TL} + 1)$ is evaluated, and so on. If all possible destination nodes have been evaluated and there are still laden load carriers left, then the weighted shortest path that involves trucks is found.

2. After the laden load carriers have been accounted for, the algorithm tries to return the empty load carriers to the origin port after the laden load carriers for each transport order have been unloaded at the destination port. The path from the destination port back to the origin is found with the same shortest path procedure as the laden load carriers. There are two differences: the first is that the empty load carriers do not have a time window for all possible destination nodes up until the end of the planning horizon is evaluated, while the second is that if the capacity

constraints ensure that not all empty load carriers can return to the origin, then the rest are left in the inventory at the destination port.

3. Then, if capacity constraints ensure that some transportation orders lack empty load carriers at the origin port O_o at time period T_o, then new empty load carriers are purchased.

Benchmark Instance

The benchmark instance is implemented similarly to instance 1. There is a finite horizon with the transport orders for ten weeks. The planning horizon is 341 time periods to include the transport orders with T_o in week ten. The benchmark instance includes the same 558 transport orders included in instance 1. The benchmark instance is initialized from the same point as instance 1 - instance 5.

Benchmark Results

The benchmark instance was solved in 4 hours and 30 minutes. Table 7.9 presents the results. The cost of purchasing new empty load carriers and the cost of transporting load carriers with trucks dominate the total costs. The cost of purchasing new empty load carriers arises from purchasing 501 new empty load carriers. The cost of transporting load carriers with trucks arises from 282 laden load carriers being transported with trucks.

Table 7.9: Costs for the benchmark instance

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
28 077 296 369	15 397 700	6 898 669	25 550 000 000	2 505 000 000
100,00 %	0.05 %	0.02 %	91 %	8.92 %

Instances 1 - 4 are initialized with 500 load carriers (see section 6.3) and solved without purchasing new empty load carriers. However, the benchmark instance solved with an algorithm based on Hydro's planning policy requires 501 additional load carriers, totaling 1001. This result is consistent with the actual situation for Hydro, which reports having 1200 load carriers. If we assume that roughly 200 load carriers are at any time nonoperative (e.g., due to maintenance), then around 1000 load carriers would be close to the actual number of operative laden load carriers Hydro has.

Comparing the total costs of the benchmark instance with instance 1 - instance 4 is irrelevant because the cost of purchasing new empty load carriers and transporting trucks dominates the total costs in instance 1. However, the transport and inventory costs are more similar. Figure 7.10 shows that the transportation and inventory costs are higher for the benchmark instance than for instance 1. Transportation and inventory costs are

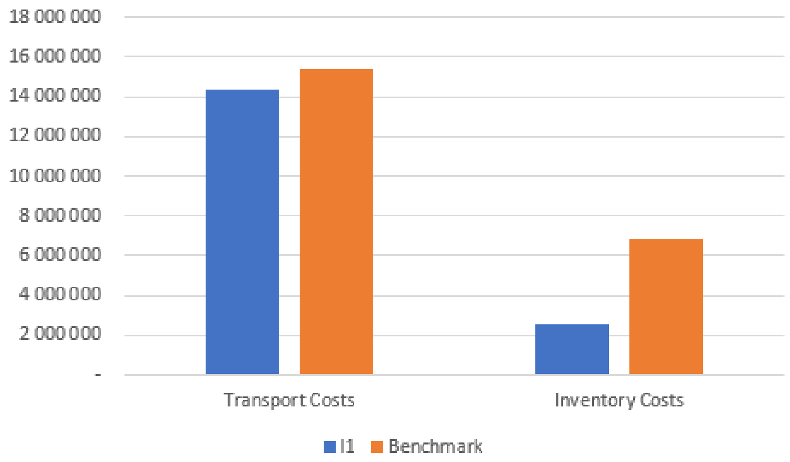


Figure 7.10: The difference between transport costs and inventory costs for instance 1 (I1) and the benchmark instance.

7.22% and 174.5% (respectively) higher for the benchmark instance.

The drastic increase in inventory costs for the benchmark instance is natural because the system has more load carriers. The 501 additional load carriers that were purchased have to be stored. Further, the increase in transportation costs may be because Hydro's planning policy selects less efficient routes for transportation. Figure 7.11 illustrates the differences between the benchmark instance and instance 1 in the number of storage units used on each transport route (i.e., the amount of capacity used). The figure shows that Southern Trade 1 reaches the storage limit in the benchmark instance than with instance 1. This is probably because Hydro's planning policy plans the transport orders for Karmøy first, and Southern Trade 1 is the earliest transport route that departs from Karmøy (see Figure 6.2). Thus, this transport route is filled up. This may again affect the possibilities of transporting transport orders from other ports along Southern Trade 1. While Figure 7.11 shows that other transport routes have more storage available, the delivery time windows of the transport orders may still not be met if these are used.

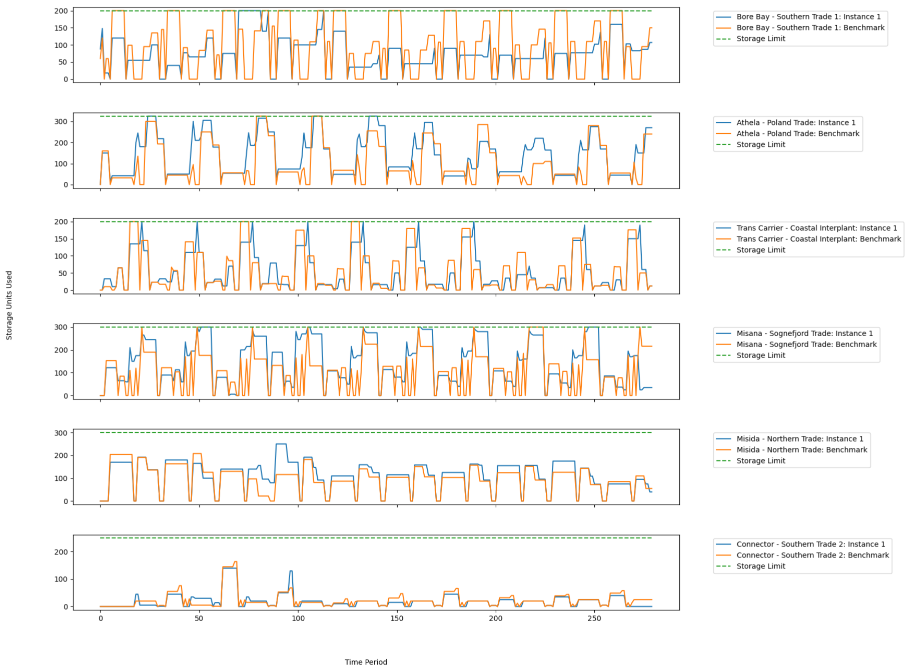


Figure 7.11: The difference between available storage in instance 1 and the benchmark instance.

7.2.3 Analysis of Model Adaptation to Changes in Known Data

The purpose of instance 5 is to analyze how the model adapts to changes in known data. The result of the changes is that 26 new empty load carriers were purchased.

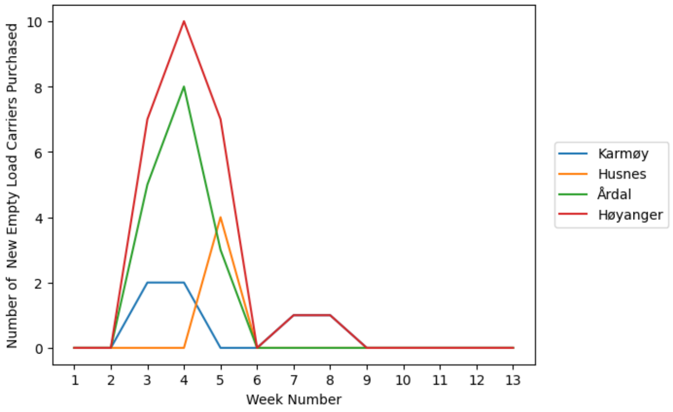


Figure 7.12: The number of new empty load carriers purchased in instance 5.

Figure 7.12 shows when the 26 new empty load carriers are purchased in instance 5. The figure shows that most new empty load carriers are purchased around week 4. This week, 20 new empty load carriers are purchased (ten at Høyanger, eight at Årdal, and two at Karmøy). This coincides with the first change in the known data after the first roll. Recall from Table 6.7 that 25 laden load carriers are added to week 4 in roll 1. The fact that 20 laden load carriers had to be purchased to adapt to this change indicates a lack of flexibility concerning the number of empty load carriers at each port. However, Table 6.7 shows that between 16 and 27 laden load carriers are added at each roll in instance 5. Still, there is only one big round of new empty load carrier purchases at around week 4 (although there are some purchases in weeks 7 and 8). This indicates that the model is able to handle changes in known data with a higher level of empty load carriers.

Chapter 8

Concluding Remarks And Future Research

This thesis develops a network flow model for optimizing the flow of laden and empty load carriers over an operational planning horizon. The system consists of production, transshipment, terminal ports, and fixed maritime transport routes connecting the ports in space and time. The aim is to find a transportation plan that ensures that all transportation orders are transported from the origin port and reach the destination port within their delivery time window. The transportation plan also repositions empty load carriers from excess ports to ports that lack empty load carriers. The transportation plan is created while minimizing transportation costs, inventory costs, costs of purchasing extra transport capacity with trucks, and the cost of purchasing new empty load carriers.

Five instances are presented to test the model. The first four instances aim to analyze the effect of the planning horizon length when the transportation plan is created in a dynamic environment with new information arriving weekly. Instance 1 has ten weeks of known transport order data, while instances 1, 2, and 4 have six, four, and three weeks of known transport orders with additional information arriving each week. Instance 1 is run in a finite horizon, while instances 2 - 4 use a rolling horizon approach. The fifth instance aims to analyze how the model adapts to changes in known data. Both rolling horizon and finite horizon methods are utilized. I also compare the model with an algorithm that mimics Hydro's planning policy.

The results of the computational study show that the suggested model performs better than Hydro's planning policy in terms of the required number of load carriers in the system and total costs. Hydro's planning policy had 7.22% higher transport costs than instance 1 and required 1001 load carriers in the system to meet the demand for empty load carriers while instances 1- 4 manages to satisfy empty load carrier demand with 500 load carriers in the system.

The results from instances 1 - 4 show that the planning horizon length affects the total costs to a minor degree. However, the planning horizon length somewhat affects the repositioning of empty load carriers since more empty load carriers are being repositioned earlier when the planning horizon is low.

Further, the results from instance 5 show that the model requires more load carriers in the system to adapt to changes in known data. Therefore, a subject for further research is to develop a stochastic model that takes multiple scenarios into account when determining the empty load carrier repositioning. An other direction for future research is to incorporate inventory models to analyse potential minimum levels of empty load carriers in the port inventories.

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Purpose of Master's Thesis

The purpose of this Master's Thesis is to create an operational transportation plan for laden and empty load carriers in the maritime logistics system of Norsk Hydro. The system consists of production, transsshipment, and terminal ports. The products must be transported with load carriers between the various ports, along fixed maritime routes, and must be delivered within a time window. From these transportation requirements, there arises a flow of laden load carriers and empty load carriers.

Since there are transport imbalances between the ports, some ports have more empty load carriers than they require, while other ports have fewer. The transportation plan must therefore reposition the empty load carriers from ports with excess empty load carriers to ports that lack empty load carriers. This must be accomplished while laden load carriers are transported from their origin port to their destination port. The transportation plan must minimize the relevant costs.

Preface

This Master's thesis is the concluding part of my Master of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). My field of specialization is Managerial Economics and Operations Research at the Department of Industrial Economics and Technology Management.

The Master's thesis was written during the spring semester of 2023 and is based on my work in the specialization project during the fall semester of 2022. The thesis aims to find an operational transportation plan for laden and empty load carriers in Norsk Hydros maritime logistics network.

I thank my supervisor Peter Schütz for his excellent guidance, feedback, and insight. Furthermore, I would like to thank Norsk Hydro ASA for their cooperation.

Abstract

This Master's thesis presents a network flow model for optimizing the flow of laden and empty load carriers over an operational planning horizon in the maritime logistics system of a global aluminum producer. The system consists of production, transshipment, terminal ports, and fixed maritime transport routes connecting the ports in space and time. The aim is to find a transportation plan that satisfies the transportation needs. This means delivering laden load carriers from their origin port to their destination port within their delivery time window while repositioning empty load carriers from excess ports to ports that lack empty load carriers. The transportation plan is created while minimizing transportation costs, inventory costs, costs of purchasing extra transport capacity with trucks, and the cost of purchasing new empty load carriers. The model simultaneously considers the flow of laden and empty load carriers to create the transportation plan.

I present a case study with five instances. The first four instances aim to analyze the effect of the planning horizon length when the transportation plan is created in a dynamic environment with new information arriving weekly. Planning horizons with three to ten weeks of known transportation needs are analyzed. The fifth instance aims to analyze how the model adapts to changes in known data. Both rolling horizon and finite horizon methods are utilized. I also compare the model with an algorithm that mimics the actual planning policy of the aluminum producer.

The results of my computational study show that the suggested model performs better than the company policy in terms of the required number of load carriers in the system and total costs. Also, the planning horizon length had little effect on the total costs. However, the planning horizon length somewhat affects the repositioning of empty load carriers. Further, the results show that the model requires more load carriers in the system to adapt to changes in known data.

Sammendrag

Denne masteroppgaven presenterer en nettverksflytmodell for å optimere flyten av lastede og tomme lastbærere over en operasjonell planleggingshorisont i det maritime logistikksystemet til en global aluminiumsprodusent. Systemet består av produksjon, omlasting, terminalhavner og fikserte maritime transportruter som kobler havnene sammen i rom og tid. Målet er å finne en transportplan som tilfredsstiller transportbehovene. Dette betyr å levere lastede lastbærere fra opprinnelseshavnene til destinasjonshavnen innenfor leveringstidsvinduet, samtidig som tomme lastbærere reposisjoneres fra havener med for mange tomme lastbærere til havner som mangler tomme lastbærere. Transportplanen minimerer transportkostnader, lagerkostnader, kostnader ved kjøp av ekstra transportkapasitet med lastebiler og innkjøp av nye tomme lastbærere. Modellen planlagger strømmen av lastede og tomme lastbærere samtidig for å lage transportplanen.

Jeg presenterer en casestudie med fem instanser. De fire første instansene tar sikte på å analysere effekten av planleggingshorisontens lengde når transportplanen lages i et dynamisk miljø der ny informasjon kommer ukentlig. Instansene implementerer planleggingshorisonter der tre til ti uker med transport behov er kjent. Den femte instansen har som mål å analysere hvordan modellen tilpasser seg endringer i kjente data. Både rullende horisont og endelig horisont-metoder brukes. Jeg sammenligner også modellen med en algoritme som etterligner den faktiske planleggings metoden til aluminiumsprodusenten.

Resultatene av beregningsstudien viser at den foreslåtte modellen gir bedre resultater enn selskapets policy når det gjelder nødvendig antall lastbærere i systemet og totale kostnader. Resultatene viser også at planhorisontens lengde har liten effekt på de totale kostnadene. Planleggingshorisontens lengde påvirker imidlertid omplasseringen av tomme lastbærere til en viss grad. Videre viser resultatene at modellen krever flere lastbærere i systemet for å tilpasse seg endringer i kjente data.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Aluminium is a versatile, lightweight, and strong metal used in various applications. It doesn't rust easily, which means it can last a long time and work well outside or in places where other materials might get damaged. It can conduct heat and electricity well, so it's great for making things like electronics or heat exchangers. Aluminum can also be cast into a wide range of products. The casting process allows for complex and efficient designs, bringing versatility not easily achieved by other manufacturing methods. This combination of material and process advantages has made aluminum-cast products a preferred choice in various industries. The market for aluminum cast products is expected to grow with a Compound Annual Growth Rate (CAGR) of 5.13% from 2023 to 2032(Research, 2023).

In 2022, the largest producer of primary aluminum was China, with 59.06% of the global production. However, aluminum-cast products are sold all over the world. Europe and North America are net importers of aluminum cast products, while Asia is a net exporter. Thus, maritime transportation is an important part of the aluminum value chain due to large geographical distances.

The shipping industry underpins the international economy, contributing significantly to global trade. It provides the most efficient, safe, and environmentally friendly means of transporting mass goods worldwide (Elmi et al., 2022, accounting for over 90% of world trade. Containerization, a landmark development in the mid-twentieth century, dramatically reduced transport costs, which was formerly a considerable expense (Meng et al., 2019).

Norsk Hydro ASA referred to as Hydro from here on, operates in all stages of the aluminum value chain. The company is a large global actor and is one of the most valuable companies in the aluminum industry. Hydro's primary aluminum and cast product production are located across the globe to serve the company's international customer base.

Hydro has recently invested in a new maritime logistics system to handle transportation between Norwegian production plants and European terminal ports. This system utilizes load carriers and roll-on / roll-off (ro-ro) vessels to transport goods. The load carriers transport cast products to the European transit ports, where the cast products are reloaded for further transportation. In addition, some return flows of load carriers with anodes (raw material for aluminum production) from the European terminal ports to the Norwegian plants.

The operational planning process in the maritime logistics network is challenging since the ports do not have a regular flow of goods. Thus the ports may have too many or too few load carriers when they are required for transportation. In addition, the planning must account for the repositioning of empty load carriers. The planning must be done while respecting ro-ro vessel capacities, available transport routes, and port storage capacities.

ToDo: insert paragraph about contribution here.

The report is structured as follows: chapter 2 presents relevant context and background information, chapter 4 presents a review of relevant literature, chapter 3 presents the problem description, chapter 5 provides a mathematical model, chapter 6 presents a case study of Hydro's situation, 7 provides a computational study of the model. Chapter concludes and suggests future research opportunities.

Chapter 2

Background

Hydro has recently invested in a new maritime logistics system to increase efficiency and reduce costs. While the old system had crane-based vessels, the new system uses roll-on / roll-off (ro-ro) vessels with load carriers. The shift to the new system creates a new transportation planning problem with new challenges for Hydro.

The purpose of this chapter is to provide a context for the operational transportation planning problem Hydro faces after the strategic changes. First, section 2.1 provides an overview of the global aluminum industry. Second, section 2.2 presents Hydro and its transportation requirements. Third, section 2.3 describes the maritime logistics system Hydro has designed to handle the transport requirements and the operational planning challenges this system creates.

2.1 The Global Aluminium Industry

Aluminium is a versatile, lightweight, and strong metal used in various applications. It has about one-third the weight of steel, which makes it an ideal material for use in the construction of vehicles, airplanes, and other structures where weight is a concern. Despite this lightweight property, aluminum is still a solid and durable material that can withstand substantial stress. Furthermore, the metal can be molded, shaped, and fabricated into several products, including packaging, construction materials, and automotive components. These properties make aluminum valuable and widely used in several industries (Hydro, 2022a).

First, subsection 2.1.1 presents the stages in the aluminum value chain. Then, subsection 2.1.2 describes the role of maritime transportation in that value chain.

2.1.1 The Aluminium Value chain

The aluminum value chain encompasses the production process, from mining raw materials to delivering finished aluminum products. Figure 2.1 presents the six stages in the value chain.

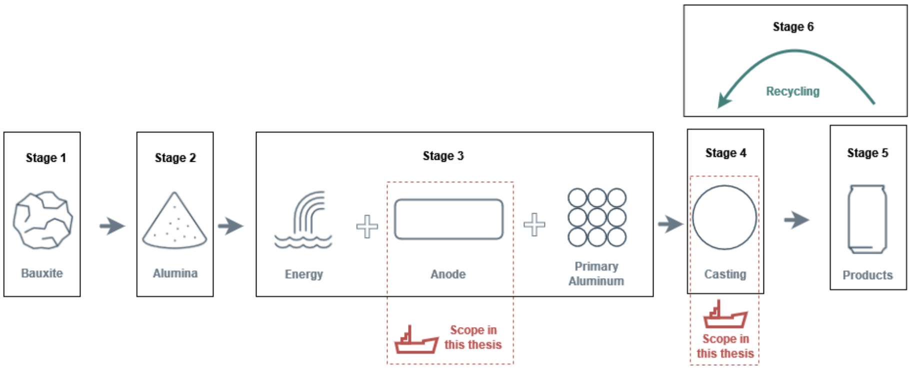
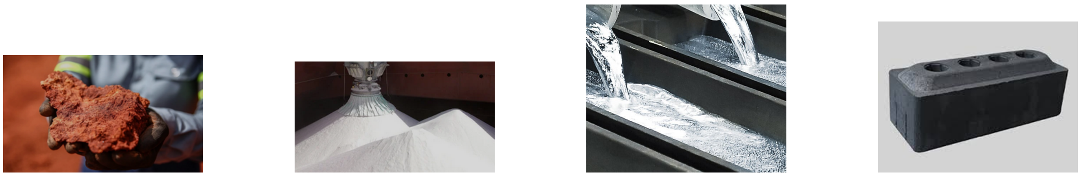


Figure 2.1: The aluminum value chain (Hydro, 2022c)

The first stage involves the extraction of bauxite (figure 2.2a). Second is the refining of bauxite into alumina (figure 2.2b). The third stage is primary aluminum (figure 2.2c) production, which takes place in production plants that transforms energy, alumina, and anodes (figure 2.2d) into primary aluminum using electrolysis. The fourth stage is to cast primary aluminum into semi-fabricated cast products. Cast products include sheet ingots, foundry alloys, extrusion ingots, and wire rods (Hydro, 2022b). These cast products are presented in figure 2.3. The fifth stage is to take semi-fabricated cast products and manufacture finished products for various industries.



(a) Bauxite **(b)** Alumina **(c)** Primary aluminum **(d)** Anodes

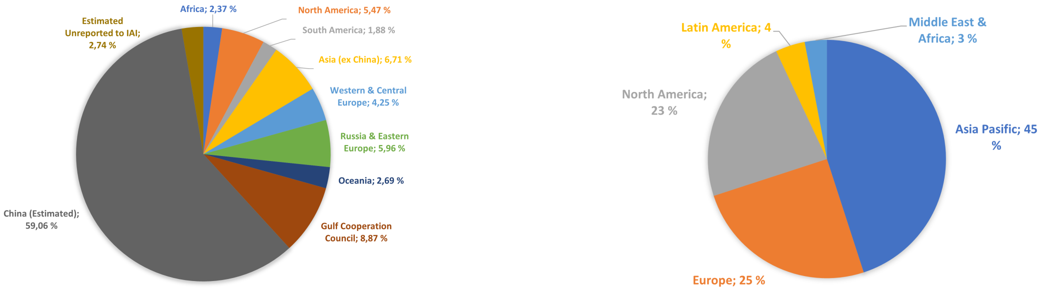
Figure 2.2: The raw materials at each stage in the aluminum production process

The sixth and increasingly important stage in the value chain is recycling, which is an integral part of the circular economy of aluminum. This stage involves collecting and processing used aluminum products to reintroduce them into the production cycle, thus reducing the need for new bauxite mining. The recycled aluminum undergoes re-melting and re-purifying before being cast into new products, all with a significantly lower environmental impact than primary production. According to the Aluminum Association (2022), recycling aluminum saves 90% of the energy needed to produce new aluminum from raw materials. It is essential in reducing the overall carbon footprint of the aluminum industry.

(a) Sheet ingots **(b)** Foundry alloys **(c)** Extrusion ingots **(d)** Wire rod

Figure 2.3: The main cast products

An overview of the geographical distribution of primary aluminum production in 2022 is provided in figure 2.4a. The most prominent market actors are China, the Gulf Cooperation Council (Middle East), and Asia, except China. The cost drivers in this production are alumina, energy, and anodes. These cost factors comprise 80-85 percent of costs in 2022 (Hydro, 2023).



(a) Geographical distribution of primary aluminum production in 2022 (International Aluminium Institute, 2022)

(b) Geographical distribution of aluminum cast product market share in 2022 (Research, 2023)

Figure 2.4: Comparison of primary aluminum production and aluminum cast product market share in 2022

The aluminum casting market was valued at approximately USD 95.34 billion in 2022 and is projected to expand to nearly USD 157.23 billion by 2032, reflecting a Compound Annual Growth Rate (CAGR) of 5.13% over the forecast period from 2023 to 2032. The growth in this sector is fueled by a diverse range of customers such as automotive, aerospace, machinery, and construction industries. This growth is geographically dispersed with significant market shares held by regions like North America, Europe, and Asia-Pacific. Specifically, countries like the United States, Germany, and China dominate the demand due to their advanced industries (Research, 2023). The geographical distribution of aluminum cast product market share is provided in figure 2.4b.

By comparing figure 2.4b and figure 2.4a, it is evident that the producers of primary aluminum and the customers of cast products are located in different parts of the world. For example, both North America and Europe have large parts of the aluminum cast market share, while they have a much smaller share of the primary aluminum production.

Since there are long distances between producers and customers in the aluminum value chain, the value chain has transportation requirements. Since maritime transportation is suitable for long-distance transportation (Rodrigue and Notteboom, 2023). Maritime transportation is relevant for large parts of the value chain. This thesis focuses on the parts in figure 2.1.

2.1.2 Maritime Transportation in the Value Chain

Maritime transportation is central to several parts of the aluminum value chain. Alumina transportation to smelters usually happens by bulk ships due to its powder-like nature and the long travel distances (International Aluminium Institute, 2022). Anodes also travel long distances to reach the smelters, but their solid shape makes general cargo vessels more suitable (GlobalForwarding, 2022). Primary aluminum production plants typically have a cast house nearby due to the inconveniences of transporting the liquid metal. Therefore, primary aluminum transportation usually happens within the infrastructure of the production plants (Hydro, 2022c). Further, due to their solid shape, cast products use general cargo vessels as a primary transport mode.

Bauxite is, however, transported from the mines with trucks or convoy belts to the alumina refineries since these are often located close to each other due to the substantial volume reduction when refining bauxite to alumina (Government of Canada, 2022).

The various products and their standard transport modes are summarized in table 2.1. The table shows that maritime transportation is common in large parts of the value chain. This thesis focuses on cast products and anode transportation (marked in table 2.1).

Table 2.1: Summary of the common transport modes for the various products. This thesis focuses on products that are colored.

Products	**Product Characteristic**	**Common Transport Mode**
Bauxite	Sedimentary rock	Truck or Convoy belt
Alumina	Dry powder	Bulk carriers
Primary aluminum	Liquid metal	Plant infrastructure
Anodes	Solid rectangular structure	General cargo vessels
Cast products	Solid products	General cargo vessel

2.2 Norsk Hydro ASA

Hydro is a Norwegian aluminum company with over 30 000 employees in over 140 locations in 40 countries. Their business operations encompass ”bauxite mining, alumina refining, electrolysis of primary aluminum products and alloy technology to finished products” (Hydro, 2022b, p.29). This makes Hydro present in large parts of the aluminum value chain presented in subsection 2.1.1. The company has grown significantly since its foundation in 1905 and had a market capitalization of USD 15.054 Billion in 2022. This makes Hydro the fourth largest aluminum company in the world, ranked by market capitalization (Value.Today, 2023).

This section overviews Norsk Hydro and its operations and transport requirements.

2.2.1 Hydro’s Primary Aluminum and Cast Product Business

Hydro’s plants are spread across several countries. ”The [primary aluminum] business area consists of wholly-owned aluminum metal plants in Norway and partly owned plants in Slovakia, Qatar, Australia, Canada, and Brazil” (Hydro, 2023, p. 22). Additionally, Hydro owns a network of casthouses, which are integrated with their primary aluminum plants (Hydro, 2023). The production plants, therefore, produce both primary aluminum (i.e., electrolysis) and cast products. The capacities of Hydro’s plants are presented in table 2.2.

Hydro plants produce cast products that are utilized by a wide range of industries. Notable sectors that use Hydro’s products include automotive, building and construction, packaging, and electronics manufacturing. The automotive sector, especially with the growing trend of electric vehicles, makes substantial use of the aluminum produced, benefiting from the material’s lightweight and recyclable properties. This aluminum is employed in various applications, including car bodies and heat exchangers. Similarly, in the construction sector, Hydro’s aluminum features prominently in various applications, ranging from window frames to structural components (Hydro, n.d.). Thus, they sell cast products to industries similar to those presented in subsection 2.1.1.

Table 2.2: Summary of the plants and their capacities (Hydro, 2022c)

Plant (ownership)	**Country**	**Electrolysis capacity (000 mt)**	**Casthouse capacity (000 mt)**	**Main products**
Karmøy (100%)	Norway	271	370	extrusion ingot, wire rod
Årdal (100%)	Norway	202	223	sheet ingot, foundry alloys
Sunndal (100%)	Norway	425	525	extrusion ingot, foundry alloys
Høyanger (100%)	Norway	66	120	sheet ingot
Husnes (100%)	Norway	195	215	extrusion ingot
Slovalco (55.3%)	Slovakia	175 (100% basis)	250 (100% basis)	extrusion ingot, foundry alloys
Tomago (12.4%)	Australia	74	75	extrusion ingot, foundry
Alouette (20%)	Canada	320	334	extrusion ingot, foundry alloys
Qatalum (50%)	Qatar	125	150	standard ingot, foundry alloys
Albras (50%)	Brazil	460 (100% basis)	460 (100% basis)	standard ingot, foundry alloys

Hydro has "more than 30,000 customers worldwide" (Hydro, 2022c, p 17). As presented in table 2.2, Hydro also has an extensive network of production plants distributed globally. Given this extensive network, the company generally aims to serve customers with the closest plants to optimize logistics and reduce transportation costs. However, the type of cast products produced varies by plant, implying that a certain product a customer needs may not be available at the nearest plant. This difference in production capabilities complicates the logistics process and necessitates long-haul transportation of certain cast products to satisfy customer needs.

Adapting production capabilities at a plant to accommodate short-term changes in customer demand is not always feasible due to the high costs and time-consuming nature of investing in new production capacities. As such, a flexible and efficient transportation strategy is critical to ensuring that customers' demands are met promptly, regardless of where the required products are produced.

2.2.2 Transportation Requirements at Hydro's Norwegian Plants

Hydro operates five plants in Norway, located in Karmøy, Årdal, Sunndal, Høyanger, and Husnes. These plants engage in the production of primary aluminum along with a wide array of cast products. As presented in subsection 2.1.1, Europe has a large share of the aluminum casting market. It is, therefore, natural to assume that the Norwegian plants produce primarily for the European market. However, as stated in subsection 2.2.1, plants may be required to serve customers globally.

Hydro has a maritime logistics system handles some of the transport requirements at the Norwegian ports. Anodes are transported to the plants from China (Hydro, 2022c) through European terminal ports. Likewise, cast products are transported to the downstream customers through the same terminal ports, where they are either loaded on trucks and driven to customers on the European continent or reloaded to other vessels and transported to other parts of the world. Nevertheless, a transportation flow in the maritime logistics system of both anodes and cast products occurs between the Norwegian and terminal ports.

Due to its geographical location, the Karmøy plant is a hub for planning in the maritime logistics system. The focus of this thesis is Hydro's transportation flow in this system. More explicitly, the flow of cast products to terminal ports in Europe and the flow of anodes to the Norwegian plants.

2.2.3 Transport Imbalance at the Norwegian Plants

Hydro experiences transportation imbalances in the transportation flow represented in figure 2.5. This imbalance is primarily shaped by the disparity between the flow of cast products from its Norwegian plants to the terminal ports in Europe and the flow of anodes from these terminal ports to the Norwegian plants. The flow of cast products, representing the movement from the production plants to the downstream customers, is larger than the reverse flow of anodes.

Figure 2.5: The flow of cast products from the Norwegian Plants and the return flow of anodes from Europe(Hydro, 2022c)

This transport imbalance creates unique logistical challenges when Hydro conducts operational planning. The overutilization of the maritime logistics system for transporting cast products to European terminal ports may lead to underutilization on the return journey when transporting anodes back to the Norwegian plants. This inefficiency can lead to wastage of resources and higher overall transportation costs. These challenges are further described in subsection 2.3.5.

2.3 Hydro's Maritime Logistics System

Hydro's maritime logistics system handles the transportation requirements described in subsection 2.2.2 (i.e., the transportation of anodes and cast products presented in figure 2.5). This section describes this system. Subsection 2.3.1 presents the old maritime logistics system Hydro had, while subsection 2.3.2 presents the new maritime logistics system Hydro has recently invested in. Further, subsection 2.3.3 describes the transportation infrastructure in the system and how Hydro handles the infrastructure planning. Last, subsection 2.3.4 presents the operational planning Hydro has to conduct, given the infrastructure.

2.3.1 Hydro's Old Maritime Logistics System

Hydro's old maritime logistics system was operated by general cargo vessels that used cranes to load and unload cargo (see figure 2.6). This type of vessel has some negative aspects. One of the significant drawbacks of using cranes is that the loading and unloading process can be time-consuming and labor-intensive, especially when dealing with large or complex cargo. This results in longer turnaround times in port, which can cause delays

Figure 2.6: Hydro's old maritime logistics system had vessels that used cranes to load cargo (Hydro, 2021).

and thus impact transport efficiency. Additionally, cranes can increase the risk of damage to cargo, personnel, and the vessel itself, as improper handling or shifting of cargo during loading or unloading can cause accidents or cargo loss. Finally, using cranes also have high labor and equipment costs associated with operating and maintaining the crane systems.

Due to the negative aspects of using crane-based vessels, Hydro invested in a new maritime logistics system.

2.3.2 Hydro's New Maritime Logistics System

Hydro's new maritime logistics system uses ro-ro (roll-on/roll-off) vessels. Ro-ro is a method of transporting goods that involves loading goods onto a vessel using specialized ramps or platforms. In contrast to Hydro's old crane-based vessels, ro-ro vessels allow for more efficient loading and unloading of goods since goods are rolled on and off. Another benefit is reduced goods handling, which minimizes the risk of damage to the goods. These properties make ro-ro vessels a convenient and efficient method of transporting large and heavy items, such as anodes or cast products (Sinay, 2022). Figure 2.7 shows the ramp on which cargo is rolled.

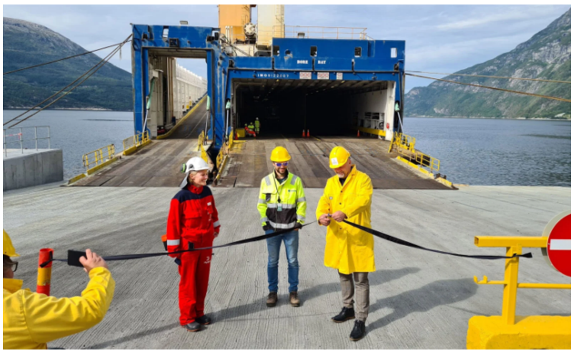


Figure 2.7: Opening of Hydro's new ro-ro based logistics system at Sunndal.

(a) Cassette (b) Mafi

Figure 2.8: Load carriers types used to transport goods with ro-ro vessels in Hydro's new maritime logistics network

Further, Hydro's maritime logistics network uses load carriers to transport goods. Figure 2.8 presents various load carrier types. Load carriers allow for reduced loading time compared to traditional cranes (Hydro, 2022d). They reduce handling as goods can travel with the same load carrier from point to point through transshipment. Less handling reduces the risk of damaging the goods. Load carriers can also save space compared to traditional container transportation, as the empty load carriers can be stacked on each other, taking up less space than laden load carriers. However, load carrier utilization can vary depending on the types of goods transported. For example, cast products such as wire rods and sheet ingots have different utilization due to the different shapes of the products.

2.3.3 The Maritime Infrastructure

The maritime infrastructure in Hydro's new logistics system consists of a maritime logistics network. The network includes maritime transport routes, schedules, and capacities for the ro-ro vessels. The ro-ro vessels, traveling along a transport route according to a schedule, visit several ports. The ports are terminal ports in Europe and those near the Norwegian plants (see figure 2.9). The transport routes thus connect all the ports so that the transport flow presented in figure 2.5 can be handled. Hydro outsources the operations of the maritime logistics network to third-party maritime logistics providers. For 2018 - 2024, the Norwegian company Sea Cargo AS operates the logistics network (Skipsrevyen, 2022).

Figure 2.9: The Norwegian plants and the terminal ports in Europe.

The vessels operating the transport routes have fixed capacities. This can cause problems when the available capacities provided by Sea Cargo are lower than the capacity required. In these situations, Hydro purchases extra transport capacity from truck providers. However, since trucks have lower capacities than maritime vessels, the transportation costs are higher for using this transportation mode to transport the same amount of goods.

Further, the Norwegian and terminal ports have an inventory with a certain amount of storage. When Hydro decides to use the inventory, then inventory holding costs accrue. The costs are proportional to the amount of storage used.

2.3.4 Operational Planning

The operational time horizon for planning the transportation of load carriers between ports spans three to six weeks. It involves planning the transportation of the load carriers within the framework provided by the maritime infrastructure to deliver all transport orders within a delivery time window. "Transport order" is the term used to describe a request to transport a certain number of load carriers from an origin to a destination. The delivery time window for a transport order indicates the periods in which the destination port can receive the laden load carriers. Hydro operates with delivery time windows to create flexibility regarding the delivery time. The planning at Karmøy conducts the operational planning for all the Norwegian ports.

The planning team at Karmøy creates transport orders based on transportation requirements. As mentioned in subsection 2.3.2, ro-ro vessels transport goods with load carriers. Thus, the planning team calculates the required number of load carriers. The required number depends on the amount of anode or cast product and the type of load carrier required (due to different utilization grades). Particular products (e.g., cast products) require specific load carrier types. Load carrier types are, therefore, never substituted to fulfill the number needed for a transport order.

After creating transport orders, the logistical planning team has to organize the transport to minimize transportation costs. This organization involves making sure that enough empty load carriers are present at the correct port at the time they are required. The logistical planning team can transport empty load carriers from other ports, obtain empty load carriers stored at the port inventory, or purchase new ones from load carrier providers. The load carrier providers will then deliver them at the required port. However, load carriers of all types are expensive, and they try to minimize new purchases.

Further, the load carriers become laden when the port personnel load empty carriers with anodes or cast products. The logistical planning team then decides the path for the laden load carriers from the origin port to the destination. Thus, the logistical planning team must consider two transportation flows. The flow of empty and the flow of laden load carriers.

2.3.5 Challenges in the Operational situation

The planning team at Karmøy faces two main challenges.

- The first challenge is that the ports do not have a regular flow of goods. The amount of anode or cast products can vary weekly. Thus, the number of laden load carriers can vary weekly. This variation may result in the ports having too many load carriers or too few, creating confusion and dissatisfaction from the port operators.
- The second challenge is that the planning team at Karmøy now must consider empty load carrier repositioning. This is because the new maritime logistics system introduced a flow of empty load carriers. Planning the repositioning is a challenge the planning team has little experience with and lacks decision support for because they had never had this responsibility before (since the old maritime logistics system had no concept of load carriers).

This thesis will study the flow of laden load carriers and the repositioning of empty load carriers in Hydro's maritime logistics system over an operational planning horizon.

Chapter 3

Problem Description

This chapter presents the operational planning problem that Hydro's planning team at Karmøy face. The problem is to optimize the flow of laden and empty load carriers within Hydro's maritime logistics system over an operational planning horizon. The objective is to minimize the associated costs while fulfilling all transport orders within their respective delivery time windows.

Maritime Infrastructure

Hydro's maritime logistics system infrastructure comprises a set of transport routes and a set of ports. Each transport route is operated by a single vessel. A transport route is described by a sequence of ports the vessel visits according to a fixed schedule. The schedule describes the time period when the vessel arrives at and departs from each port in the transport route. The transport routes thus create a maritime network interconnecting the ports. The storage capacity of each vessel is also fixed within the planning horizon. Hydro can acquire more transportation capacity in addition to the transport routes by utilizing trucks that can travel between any pair of ports.

Each port has certain constraints, including inventory storage capacity and the lead time required for converting empty load carriers into laden and laden into empty (i.e., loading and unloading load carriers).

Transport Orders

Hydro has a set of transport orders representing transportation needs. Each transport order states a given number of laden load carriers that should be transported from an origin port to a destination port, the time period when these laden load carriers are ready for transport at the origin port, and the delivery time window within which those laden carriers must reach their destination. Further, each transport order requires a specified load carrier type, and these types cannot be substituted.

Flow of Laden and Empty Load Carriers

The goods transported in the network must be placed on top of an empty load carrier of a given type before being transported with the transport routes. Each load carrier requires storage capacity when stored in inventory at a port or transported along a transport route with a vessel. The storage capacity required by a load carrier depends on the load carrier type and whether the load carrier is laden or empty (since empty load carriers can be stacked on top of each other and thus require less storage).

The problem is to find the most efficient flow of laden load carriers so that all transport orders are delivered within their delivery time window. Once the laden load carriers have reached their destination and are emptied, they should be repositioned to ensure that the correct type and quantity of empty load carriers are in the required port and time period.

If it is impossible to reposition the empty load carriers sufficiently so that the correct quantity is present at the required port and time, purchasing new empty load carriers and having them delivered at a port is possible.

Associated Costs

There is a cost for transporting laden and empty load carriers between ports. Two factors determine the transportation cost: the distance traveled and the amount of storage required on the vessel. Thus, laden load carriers are more expensive to transport since they require more storage. The ports also have an inventory holding cost for storing load carriers (laden or empty) in the port inventory. There is also a cost for purchasing new empty load carriers. When the vessels' capacities are lower than those required to fulfill all transport orders, Hydro has to purchase extra transport capacity from the trucks at a higher cost.

Objective

The ultimate aim is to minimize the total cost associated with the flow of laden and empty load carriers. The total costs include transporting load carriers between ports, holding load carriers at ports, purchasing new load carriers when necessary, and purchasing extra transport capacity when required. The model must optimize the flow of load carriers to meet all transport orders within the delivery time windows while adhering to all capacity and routing constraints.

Chapter 4

Literature Review

This chapter aims to provide an overview of the relevant literature on how empty load carrier repositioning can be handled. First, an introduction to empty container repositioning is presented in section 4.1. Then, this thesis' problem is classified in section 4.2 before relevant approaches to empty container repositioning for problems that have a similar classification are investigated in section 4.3.

4.1 Empty Container Repositioning (ECR)

This section focuses on the empty container repositioning problem, which shares similarities with the challenge of repositioning empty load carriers in Hydro's maritime logistics system. Both problems involve managing the movement and allocation of empty transportation units (containers or load carriers) within a logistics network to minimize costs and optimize resource utilization. By examining the empty container repositioning problem, this literature review aims to draw insights and identify potential solutions that could be adapted and applied to repositioning empty load carriers in Hydro's maritime logistics network. For comprehensive reviews on empty container repositioning, I refer to Kuzmicz and Pesch (2019), Abdelshafie et al. (2022), and Braekers et al. (2011).

The primary cause of empty-container problems is global trade imbalances. Regions with higher imports than exports grapple with an accumulation of empty containers, whereas regions with more exports than imports face a shortage. Even in countries with balanced import and export activities, empty containers accumulate due to imbalances in specific container types, notably reefer containers and special equipment (Abdelshafie et al., 2022).

Despite empty containers not generating revenue, they demand the same transportation, storage, and space resources as full containers, thus incurring significant costs. Empty containers are essential to the supply chain, facilitating port activities. They possess their supply chain comprising containers, container ports/terminal facilities, and transport means like trucks, rail, or maritime vessels (Kuźmicz and Pesch, 2017; Zain et al., 2014). Container terminals have evolved into intermodal hubs, promoting seamless interchange between various modes of transport and cargo handling (Zain et al., 2014)

4.2 Classifying the ECR

Two classification axis for empty container repositioning problems appear in the literature (Kuzmicz and Pesch, 2019; Abdelshafie et al., 2022; Braekers et al., 2011).

4.2.1 Repositioning Scale

In the context of empty container repositioning, global and regional approaches play distinct roles (Braekers et al., 2011).

Regional Repositioning

Regional repositioning focus on one geographical region to fulfill empty container demand and reduce costs. These regions have multiple shippers, inland depots, and terminals. Thus, many allocation options are available at the regional level, with many companies and consignees involved (Kuzmicz and Pesch, 2019). Often several transportation modes are involved (e.g., maritime and rail). The regional container allocation models consider factors such as determinism, the static or dynamic nature of the model, container substitution, container leasing, and street turns (Braekers et al., 2011).

Global Repositioning

Global repositioning aims to move empty containers from ports where they are abundant to ports where they are needed (Kuzmicz and Pesch, 2019). Thus, global repositioning focus on maritime transportation. The repositioning results from a global trade imbalance, which naturally leads to a surplus of containers in some areas and a deficit in others. This discrepancy necessitates carriers to reposition their empty containers to fulfill future demands.

Drewry Shipping Consultants of London has quantified this need, estimating that about 20% of all maritime container, movements are those of empty containers, a trend that has been driven by increasing trade imbalances (Boile et al., 2006; Song and Carter, 2009). Consequently, managing these empty containers presents unique challenges at the global level, given the fewer available options due to factors like limited direct connections between ports and fixed shipping schedules.

Much research has considered either single maritime service routes or service networks with specific route structures in the broader context of global repositioning. For instance, studies such as those conducted by Lai et al. (1995), Song and Zhang (2010),Lam et al. (2007) (2007), and Song and Dong (2012) have utilized various mathematical models, control policies, and optimization strategies to address the challenge of efficient empty container repositioning in specified routes and port systems.

4.2.2 Planning Horizon

Figure 4.1 summarizes the key decisions required to address the empty container repositioning issue at each planning level.

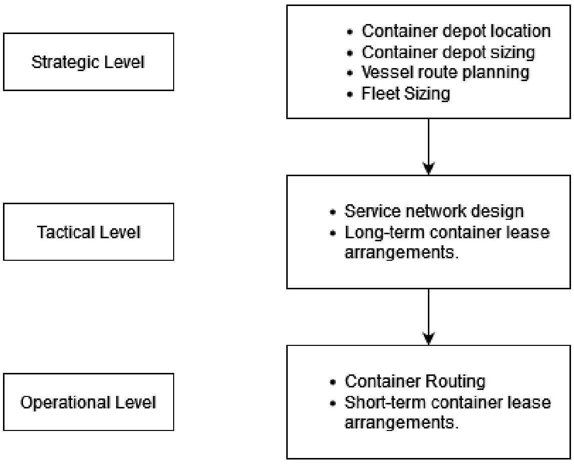


Figure 4.1: Overview of the key decisions required at each planning level (based on Braekers et al., 2011; Lam et al., 2007).

Strategic Level

Strategic planning focuses on long-term decisions like fleet sizing and location decisions. Fleet sizing determines the appropriate size and composition of the container fleet based on projected demand and cost considerations. Service network design involves selecting optimal shipping routes and port calls to meet container demand while minimizing empty container movements. These strategic decisions provide the foundation for effective empty container management (Braekers et al., 2011).

Tactical Level

Tactical planning addresses medium-term decisions to optimize the configuration and utilization of the service network. This includes optimizing the allocation of empty containers across different service routes and carriers to balance container flows and minimize repositioning costs (Braekers et al., 2011; S. Wang, 2013).

Operational Level

Operational planning focuses on short-term decisions and real-time adjustments. Container allocation involves allocating available empty containers to meet specific demand locations or customer requirements. Integrating allocation and routing decisions allows for synergies and improves overall operational efficiency. Real-time adjustments are made based on up-to-date information, such as changes in container demand or unexpected events, to adapt the repositioning plans as necessary (Braekers et al., 2011), e.g., using rolling horizon (Olivo et al., 2005).

4.2.3 Classifying this Thesis' problem

The problem considered in this thesis is a global repositioning problem since the repositioning problem arises from a trade imbalance between ports. In Hydro's maritime logistics system, more laden load carriers are transported from the Norwegian plants to the terminal ports in Europe. At the same time, laden load carriers have a substantially smaller return. Thus empty load carriers stack up at the terminal ports. Further, the planning level is operational with a short time horizon of three to six weeks. The key decision is container (i.e., load carrier) routing, which is an operational decision according to figure 4.1. Some tactical decisions are also involved since Hydro can purchase new empty load carriers. However, the main objective is to create a transportation plan over an operational planning horizon.

4.3 Approaches To Empty Container Repositioning

Several approaches to empty container repositioning are suggested in the literature. Abdelshafie et al. (2022) and Kuzmicz and Pesch (2019) classified the approaches that utilize optimization techniques into three categories: network flow models, network design models, and other models. Table 4.1 presents research papers that seek to solve the ECR. The problem considered in this thesis has, as presented in subsection 4.2.3, an operational planning level and a global repositioning scale. Thus, I have selected research articles with this classification in mind. Table 4.1 shows that variants of the network flow approach (e.g., network flow, time-space, or flow models) were the most common. Thus, this thesis will focus on the network flow approach to modeling the ECR. Subsection 4.3.1 presents the ECR from a network flow point of view.

Table 4.1: Approaches to Empty Container Modeling

Authors	**Repositioning Scale**	**Planning Level**	**Model Type**
Choong et al., 2002	Regional	Operational	Time-space network
J.-A. Li et al., 2004	Global	Operational	Inventory-based control mechanisms
Olivo et al., 2005	Regional	Operational	Minimum cost flow
Erera et al., 2009	Global	Operational	Time-expanded networks with recovery
Shintani et al., 2007	Global	Tactical	Knapsack problem then network flow problem
Ye et al., 2007	Global	Tactical	Container flow and vessel deployment
Di Francesco et al., 2009	Global	Operational	Cargo routing problem
Chou et al., 2010	Global	Operational	Container allocation problem
Brouer et al., 2011	Global	Operational	Time-expanded multi-commodity flow model
Epstein et al., 2012	Global	Operational	Multi-commodity flow model and inventory model for safety stocks
Song and Dong, 2012	Global	Operational	Cargo routing problem in a multi-service multi-voyage shipping network; cargo fleet sizing
Moon et al., 2013	Global	Tactical	Inventory model; foldable containers
Song and Dong, 2013	Global	Strategic	A single liner service route design; ship deployment
Long et al., 2013	Global	Operational	Time-space network
Chao and Chen, 2015	Global	Operational	Time-space network; minimal cost flow problem
L. Li et al., 2014	Global	Operational	Routing problem
Akyüz and Lee, 2016	Global	Operational	Simultaneous service type assignment and container routing flow problem
Sáinz Bernat et al., 2016	Global	Strategic	Inventory control problem
Zheng et al., 2016	Global	Tactical	Network design problem; mixed-integer nonlinear programming model
K. Wang et al., 2017	Global	Tactical	Network flow model; foldable containers
Neamatian Monemi and Gelareh, 2017	Global	Strategic	Network design model

4.3.1 ECR by Network Flow Models

Early research by Florez (1986) examined how a dynamic network model could be used with container leasing. Choong et al. (2002) focused on how long-term planning could impact decisions about repositioning containers in an intermodal transport network. Around the same time, Erera et al. (2009) made a model that considered decisions about booking and routing containers. Olivo et al. (2005) suggested a model that used integer programming and accounted for multiple ways containers could be moved between ports and depots (see also, Song and Dong, 2012)).

In their work, Song and Dong (2015) presented two interconnected components within the container transport process. Firstly, laden containers, whose movements are primarily dictated by external customer demand. Secondly, empty containers, whose positioning is determined mainly by internal shipping company decisions. These elements function within a shared transportation network, utilizing the same resources, yet different operational drivers direct them. To appropriately address and model the repositioning of empty containers, it is vital to concurrently model the laden container routing within the same transport network. This dual modeling approach is crucial because the movements of laden containers, influenced by trade imbalances, directly dictate the repositioning of empty containers. Therefore they suggested building models that consider the trade imbalance and the changing nature of operations. They proposed a time-space network flow model based on Brouer et al. (2011) where customer demand is certain but changes over time. This model aims to reduce the costs associated with moving both loaded and empty containers and the costs associated with lost sales.

A significant amount of research has been done to find ways to improve how shipping companies manage the movement of empty containers on international trade routes. Ships usually follow a regular route, returning to the starting port every four weeks. Long et al. (2013) defined "a service" as all the operations connected to a standard route through a set of ports. They developed a linear network flow model that used variables to represent the number of empty containers loaded or unloaded at a specific stop at a particular time. Over a planning horizon of three weeks, they aimed to minimize the total operations costs. These include handling, storage, and transportation costs for empty containers and penalties for not meeting demand. The model's limitations reflect a ship's weight and capacity limits for empty containers. They balance the flow of empty containers and ensure that the number of unloaded containers does not exceed the available amount on a vessel.

Thus, a network flow approach to the global and operational ECR should model both the flow of laden and empty containers while minimizing the relevant operational costs.

4.3.2 Network Flow Based Models

Ahuja et al. (1993, p. 4) states that "the minimum cost flow problem is the most fundamental of all network flow problems." Thus, each network flow-based approach to solving the ECR should use the minimum cost flow problem as a starting point.

Ahuja et al. (1993) describes the general minimum cost flow problem. Consider a directed network, denoted as $G = (\mathcal{N}, \mathcal{A})$, where N represents a set of n nodes and $\mathcal{A}$ represents a set of m directed arcs. Each arc (i, j) in $\mathcal{A}$ is assigned a cost C_{ij}, indicating the cost per unit flow on that arc. The cost is assumed to vary linearly with the amount of flow. Additionally, every arc (i, j) is associated with a capacity u_{ij}, representing the maximum flow it can carry, and a lower bound l_{ij}, indicating the minimum amount that must flow on the arc.

Within this network, each node i in $\mathcal{N}$ is assigned an integer value b_i, which characterizes its supply or demand. If b_i is positive, node i is considered a supply node. Conversely, if b_i is negative, node i is a demand node with a demand of $-b_i$. If b_i equals zero, node i is classified as a transshipment node.

In the minimum cost flow problem, the decision variables correspond to the flow on the arcs. We represent the flow on an arc (i, j) in $\mathcal{A}$ as x_{ij}. The minimum cost flow problem is an optimization model that seeks to minimize the total cost while satisfying the flow requirements and constraints of the network.

Objective:

$$\min \sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ij}$$

Constraints:

$$\begin{aligned} \sum_{j:(i,j)\in\mathcal{A}} x_{ij} - \sum_{j:(j,i)\in\mathcal{A}} x_{ji} = b_i \quad & i \in \mathcal{N} \\ l_{ij} \leq x_{ij} \leq u_{ij} \quad & (i,\ j) \in \mathcal{A} \\ \sum_{i\in\mathcal{N}} b_i = 0 & \end{aligned}$$

Further, the minimum cost flow problems can be extended based on various characteristics and specific problem variations (Ahuja et al., 1993).

1. Single-commodity vs. Multi-commodity: In single-commodity minimum cost flow problems, there is only one type of flow being considered, such as the transportation of a single product or the movement of a single resource. In contrast, multi-commodity minimum cost flow problems involve the simultaneous flow of multiple commodities, where each commodity may have different origins, destinations, and cost structures (Ahuja et al., 1993). Since multiple load carrier types correspond to multiple commodities that must flow through the network, this thesis' problem has multiple commodities.

2. Static vs. Dynamic: Static minimum cost flow problems consider a fixed network and do not account for changes over time. Dynamic minimum cost flow problems, on the other hand, incorporate time-varying elements, such as varying demands, capacities, or costs over different time periods (Ahuja et al., 1993). This allows for more realistic modeling of flow dynamics in dynamic systems, such as transportation networks with varying traffic patterns. Since the number of laden load carriers that must be transported can vary weekly, the problem in this thesis is dynamic.

3. Time-space minimum cost flow problems: Time-space minimum cost flow problems extend the traditional minimum cost flow framework by incorporating spatial and temporal dimensions (Ahuja et al., 1993). These problems consider the flow movement over time and space, considering time-dependent costs, time windows, and other temporal constraints. This is the case for the problem considered in this thesis since laden load carriers must be transported from an origin port to a destination port (space) and reach the destination port within a delivery time window (time).

4. Source and Sink Constraints: Some minimum cost flow problems include constraints on the sources and sinks of the flow. For example, the problem may require that certain nodes in the network act as sources with fixed supply or sinks with fixed demand. These constraints can model scenarios where flow originates from specific locations (e.g., manufacturing plants) or is destined for specific locations (e.g., distribution centers or customers) (Ahuja et al., 1993). This thesis' problem involves ensuring that laden load carriers are transported from an origin port to a destination port. Thus, a flow of laden load carrier originates at the origin port (source) and is destined for the destination port (sink).

5. Capacity Constraints: Capacity constraints limit the flow traversing an edge in the network. Minimum cost flow problems can involve different capacity constraints, such as edge capacity limits, node capacity limits, or dynamic capacity constraints that vary over time. These constraints ensure that the flow in the network does not exceed the available resources or infrastructure capacity (Ahuja et al., 1993). The problem in this thesis has capacity constraints on transport routes (due to vessel capacities) and port inventory capacities.

6. Multi-modal or Multi-layer Networks: Minimum cost flow problems can also consider networks with multiple modes of transportation or different infrastructure layers. For example, a problem may involve flows transported via road, rail, or air, each with costs and capacities. Alternatively, a problem may consider flows that can traverse different communication network layers, such as wired and wireless connections (Ahuja et al., 1993). Since ECR problems should model both the flow of laden and empty containers (i.e., load carriers in the case of the problem considered in this

thesis) (Song and Dong, 2015), this can be regarded as two layers of the network. The empty load carrier layer and the laden load carrier layer.

The various characteristics of extensions to the minimum cost flow problem are summarized in table 4.2.

Table 4.2: Categorization of Minimum Cost Flow Problems based on Ahuja et al. (1993)

Category	**This Thesis' Problem**
Single-commodity vs. Multi-commodity	Multiple load carrier types
Static vs. Dynamic	Dynamic laden load carrier transportation requirements
Time-space minimum cost flow problems	Laden load
Source and Sink Constraints	Laden load carrier origins and destinations are sources and sinks repectively
Capacity Constraints	Vessel capacities on transport routes and inventory capacities at ports
Multi-modal or Multi-layer Networks	Two layers: one for empty and one for laden load carriers.

4.3.3 Rolling Horizon Planning

When using a network flow approach to solve the ECR, choosing the planning horizon is an important decision (Olivo et al., 2005; Long et al., 2013). Finite horizon planning makes an independent plan for certain periods (from 1 to T), after which a new plan is made for the next set of periods (from $1 + T$ to $2T$), and so on. However, because these plans often need to be updated with new information, such as changes in demand forecasts or actual demand, they are typically only used for the upcoming periods.

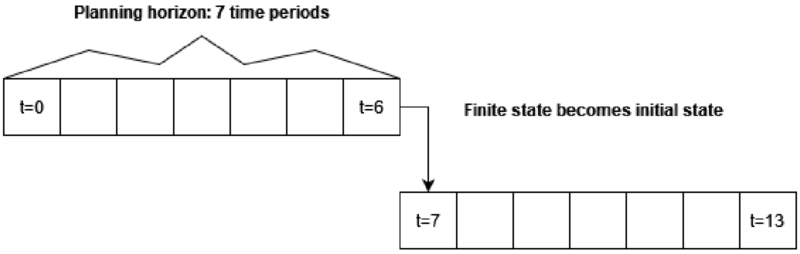


Figure 4.2: Finite horizon

On the other hand, rolling horizon planning works slightly differently. It creates a plan for a set of periods (from 1 to T), but only the decisions for the first ΔT periods are enacted. Then, the planning horizon 'rolls forward' by ΔT periods, leading to a revised plan for the period from $1 + \Delta T$ to $T + \Delta T$. This strategy allows for delaying decisions about future periods until new information becomes available. Additionally, rolling horizon planning can lead to more accurate production plans using actual demand data rather than demand forecasts (Long et al., 2013). Figure 3.3 illustrates the differences between finite and rolling horizon planning, emphasizing the enhanced adaptability provided by the latter method.

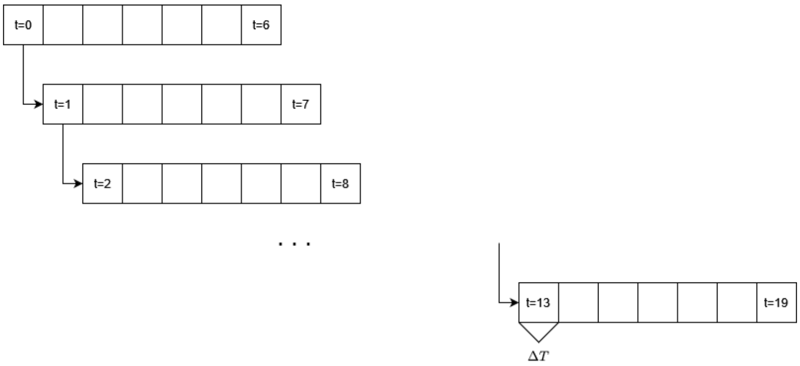


Figure 4.3: Rolling horizon

Chapter 5

Mathematical Model

In this section, the mathematical model that addresses the problem laid out in chapter 3 is presented. The model minimizes the cost of managing load carriers so that all orders are delivered within their time window.

5.1 Modelling Approach

The following sample illustrates the modeling approach by serving as a representative subset of the larger system that the model aims to describe.

5.1.1 Sample Network

The sample network has four ports and two transport routes. The ports are represented as nodes, and the transport is represented as arcs that connect the nodes.

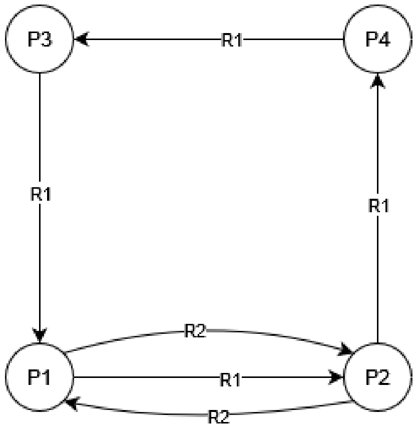


Figure 5.1: Sample network

In the sample network, transport route R1 visits P1, P2, P4, and P3 before it travels back to P1. Likewise, the transport route R2 visits P1 and P2 before it travels back to P1. However, the transport route schedule is also essential. The schedule states the time period when the transport routes visit each port in their sequence. Let the transport routes have the following schedules:

Schedule for transport route R1:

- Arrive at P1 at time period 0
- Depart from P1 at time period 0
- Arrive at P2 at time period 1
- Depart from P2 at time period 2
- Arrive at P4 time period 3
- Depart from P4 at time period 3
- Arrive at P3 at time period 4
- Depart from P3 at time period 4
- Arrive at P1 at time period 5

Schedule for transport route R2:

- Arrive at P1 at time period 0
- Depart from P1 at time period 2
- Arrive at P2 at time period 3
- Depart from P2 at time period 5
- Arrive at P1 at time period 6

Further, let the transport routes have a capacity of six storage units, and let each laden load carrier takes up two storage units while one empty load carrier takes up one storage unit. Thus, each transport route has a capacity of at most three laden load carriers, six empty load carriers, or a combination of the two.

Let there be one transport order that states that five laden load carriers should be transported from P1 to P2. The laden load carriers are ready for transportation at $t = 0$, and the delivery time window at P2 is $[t = 2,\ t = 4]$.

Further, let there be five empty load carriers in the inventory of port P1 at $t = 0$ and let both P1 and P2 have a lead time of zero time periods for converting empty load carriers to laden and laden to empty (i.e., loading and unloading load carriers).

5.1.2 Modelling the Sample Network as a Time-Space Network

A time-space network is a graphical representation that helps visualize and solve routing and scheduling problems where time plays a significant role. It extends a traditional network diagram by adding the time dimension (Bai et al., 2017).

A possible solution to the sample presented in subsection 5.1.1 can be seen in figure 5.2.

Sample Time-Space Network Structure

The transport route schedules are modeled as paths in the time-space network. In this time network, nodes represent ports and specific time periods at those ports.

The colored arcs represent the transport route paths (i.e., orange for R1 and blue for R2). Load carrier flow along these arcs indicates that the load carriers are transported with the vessel from one port at a time period to another port at a later time period. Thus, the colored arcs connect the port visiting sequence of the transport route according to the schedule.

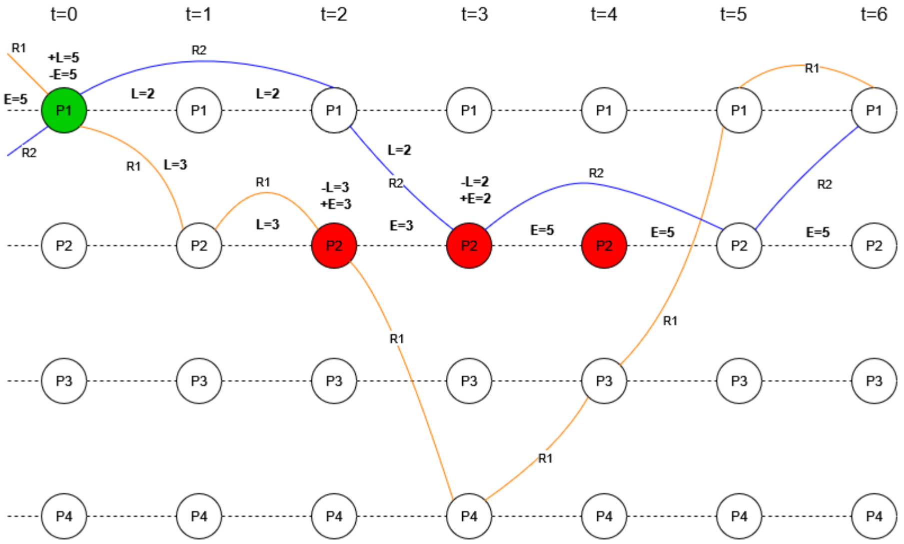


Figure 5.2: The time-space representation of the sample, with a possible solution.

Inventory is modeled similarly to transport routes and is represented by dashed arcs. Load carrier flow along these arcs indicates that load carriers are stored in inventory at a port from one time to the following. Modeling inventory this way is common in the literature (e.g., see: Olivo et al., 2005). The arcs can be thought of as inventory routes.

The orange-colored arc between P2 at $t = 1$ and P2 at $t = 2$ are used by load carriers that stay on board the vessel when anchored at P2 from $t = 1$ to $t = 2$. Only load carriers that arrive at P2 from the orange-colored arc at $t = 1$ can flow through that arc. The load carriers rolled off the vessel are stored in inventory and thus flow through the dashed arc from P2 at $t = 1$ to P2 at $t = 2$. Only load carriers that arrive with a transport route can stay on board the vessel when it docks at a port because the cost of storing load carriers on the vessel and in inventory at the port may differ.

Sample Time-Space Network Decisions

The decision is to find a flow through the time-space network so that the laden load carriers of the transport order reach their destination in the delivery time window. Flow conservation constraints handle the flow at each node.

The laden load carriers belonging to the transport order are ready for transportation at P1 at the time period $t = 0$ (i.e., the green node). Thus, five empty load carriers must be converted to laden load carriers at this node (indicated by the $+L = 5$ and the $-E = 5$). The five empty load carriers arrive from the inventory of $P1$, as indicated by the $E = 5$ flow in the dashed arc arriving at P1 at $t = 0$. Thus, five laden load carriers must depart from P1 at $t = 0$ to ensure that the flow is conserved.

The laden load carriers of the transport order flow from P1 at $t = 0$ and must reach P2 within the delivery time window (i.e., red nodes). In the sample solution, three (resp. two) laden load carriers flow to P2 along transport route R1 (resp. R2). The load carriers arriving with R1 (resp. R2) start unloading at time period $t = 2$ (resp. $t = 3$). Thus, the number of laden load carriers decreases by three (resp. two) at P2 at $t = 2$ (resp. P2 at $t = 3$), while the number of empty load carriers increases by three (resp. two). Since the flow is conserved at each node, the empty load carriers are stored in inventory at P2.

5.2 Notation

Sets

$\mathcal{D}_{rt}$ - Set of arcs $(i,\ j)$ in route r that departs from a port i at time period t

$\mathcal{L}$ - Set of load carrier types

$\mathcal{O}$ - Set of transport orders

$\mathcal{P}$ - Set of ports

$\mathcal{R}$ - Set of routes

$\mathcal{R}^{\mathcal{T}}$ - Set of transport routes. $\mathcal{R}^{\mathcal{T}} \subset \mathcal{R}$

$\mathcal{R}^{\mathcal{A}}_{jit}$ - Set of routes, arriving at port i from port j at time period t.

$\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit}$ - Set of transport routes, arriving at port i from port j at time period t, and that docks at port i for at least one time period (i.e., does not depart from i in the same time period as it arrives at i). $\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit} \subset \mathcal{R}^{\mathcal{A}}_{jit}$

$\mathcal{R}^{\mathcal{D}}_{ijt}$ - Set of routes, departing from port i to port j at time period t.

$\mathcal{R}^{\mathcal{T}\mathcal{D}}_{jit}$ - Set of transport routes, departing from port i to port j at time period t, and that have docked at port i for at least one time period (i.e., does not depart in the same time period as it arrived at i). $\mathcal{R}^{\mathcal{T}\mathcal{D}}_{jit} \subset \mathcal{R}^{\mathcal{D}}_{jit}$

$\mathcal{T}$ - Set of time periods

Parameters

C_{ijr} - Cost of transporting one storage unit between port $i \in \mathcal{P}$ and port $j \in \mathcal{P}$ with route $r \in \mathcal{R}$

C_l^E - Cost of purchasing a new empty load carrier of type $l \in \mathcal{L}$.

D_o - The destination port for order $o \in \mathcal{O}$

D_o^{TL} - Earliest possible delivery time for order $o \in \mathcal{O}$ (i.e., lower part of the time window for order $o \in \mathcal{O}$)

D_o^{TU} - Latest possible delivery time for order $o \in \mathcal{O}$ (i.e., upper part of the time window for order $o \in \mathcal{O}$)

K_r - Maximum storage capacity for route $r \in \mathcal{R}$

L_{it} - Lead time at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

O_o - Origin port of order $o \in \mathcal{O}$

S_{ilot}^L - The number of empty load carriers of type $l \in \mathcal{L}$ that have completed loading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and have become laden load carriers belonging to transport order $o \in \mathcal{O}$

T_{ijr} - The number of time periods required to travel between port $i \in \mathcal{P}$ and $j \in \mathcal{P}$ with route $r \in \mathcal{R}$

T_{irt}^D - The number of time periods the vessel serving transport route $r \in \mathcal{R}^{\mathcal{T}}$ has been in the dock at port $i \in \mathcal{P}$ when it departs from port i at time period $t \in \mathcal{T}$.

T_o^S - The time period where the laden load carriers belonging order $o \in \mathcal{O}$ are ready for transportation.

W_l^E - The number of storage units required by an empty load carrier of type $l \in \mathcal{L}$

W_l^L - The number of storage units required by a laden load carrier of type $l \in \mathcal{L}$

X_{jilrot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to transport order $o \in \mathcal{O}$ transported with route $r \in \mathcal{R}$ that arrives at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and were transported from port $j \in \mathcal{O}$ before the beginning of the planning horizon.

Y_{jilrt} - The number of empty load carriers of type $l \in \mathcal{L}$ transported with route $r \in \mathcal{R}$ that arrive at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and were transported from port $j \in \mathcal{P}$ before the beginning of the planning horizon.

Decision Variables

u^{L}_{ilot} - The number of laden load carriers of type $l \in \mathcal{L}$ of order $o \in \mathcal{O}$ that starts unloading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

x_{ijlrot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to order $o \in \mathcal{O}$ that depart from port $i \in \mathcal{P}$ to port $j \in \mathcal{P}$ with route $r \in \mathcal{R}$, at time period $t \in \mathcal{T}$

y_{ijlrt} - The number of empty load carriers of type $l \in \mathcal{L}$ that depart from port $i \in \mathcal{P}$ to port $j \in \mathcal{P}$ with route $r \in \mathcal{R}$, at time period $t \in \mathcal{T}$

y^{P}_{ilt} - The number of new empty load carriers of type $l \in \mathcal{L}$ purchased and delivered port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

Auxiliary Variables

a^{E}_{ilt} - The number of empty load carriers of type $l \in \mathcal{L}$ that arrive at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

a^{L}_{ilot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to transport order $o \in \mathcal{O}$ that arrive at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

d^{E}_{ilt} - The number of empty load carriers of type $l \in \mathcal{L}$ that depart from port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

d^{L}_{ilot} - The number of laden load carriers of type $l \in \mathcal{L}$ belonging to transport order $o \in \mathcal{O}$ that depart from port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$.

s^{E}_{ilt} - The number of laden load carriers of type $l \in \mathcal{L}$ that have completed unloading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ and thus have become empty.

u^{E}_{ilt} - The number of empty load carriers of type $l \in \mathcal{L}$ that starts loading at port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$

5.3 Model Description

Flow Conservation Constraints

Flow of Laden Load Carriers

$$\sum_{j\in\mathcal{P}}\sum_{r\in\mathcal{R}^{\mathcal{A}}_{jit}\,|\,(t-T_{jir})\geq 0} x_{jilro(t-T_{jir})} + \sum_{j\in\mathcal{P}}\sum_{r\in\mathcal{R}^{\mathcal{A}}_{jit}} X_{jilrot} = a^{L}_{ilot} \qquad t\in\mathcal{T},\ i\in\mathcal{P},\ l\in\mathcal{L},\ o\in\mathcal{O} \tag{5.1}$$

$$\sum_{j\in\mathcal{P}}\sum_{r\in\mathcal{R}^{\mathcal{D}}_{ijt}} x_{ijlrot} = d^{L}_{ilot} \qquad t\in\mathcal{T},\ i\in\mathcal{P},\ l\in\mathcal{L},\ o\in\mathcal{O} \tag{5.2}$$

$$S^{L}_{ilot} - u^{L}_{ilot} = d^{L}_{ilot} - a^{L}_{ilot} \qquad t\in\mathcal{T},\ i\in\mathcal{P},\ l\in\mathcal{L},\ o\in\mathcal{O} \tag{5.3}$$

The constraint (5.1) has two terms. The first term encompasses the laden load carriers that were transported from $j\in\mathcal{P}$ to $i\in\mathcal{P}$ in the planning horizon. It states that laden load carriers arriving at a port $i\in\mathcal{P}$ can arrive from any other port $j\in\mathcal{P}$ if there is a route arriving at i from j at a time period t and that the departure time period from j is in the planning horizon. The second term encompasses laden load carriers that were transported from j to i and departed from j before the beginning of the planning horizon. It states that laden load carriers arriving at a port $i\in\mathcal{P}$ can arrive from any other port $j\in\mathcal{P}$ if there is a route arriving at i from j at a time period t. If $i=j$ then the load carriers arrive from inventory. These load carriers arrive at i at the exact time when time period t starts.

The constraint (5.2) states that laden load carriers departing from port $i\in\mathcal{P}$ can be transported to any other port $j\in\mathcal{P}$ if there is a route departing from i to j at time period t. If $i=j$ then the laden load carriers are stored in inventory. These load carriers depart from i at the end of time period t.

Constraints (5.3) represent the flow balance for laden load carriers at each port. It states that the number of laden load carriers that depart less those that arrive should be equal to the number of empty load carriers that have completed loading and thus become laden, less the laden load carriers that have started unloading.

$$x_{iilrot} - x_{jilro(t-\mathcal{T}_{jir})} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit} \,|\, t - T_{jir} \geq 0\},\, l \in \mathcal{L},\, o \in \mathcal{O} \tag{5.4}$$

$$x_{iilrot} - X_{jilrot} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit} \,|\, t - T_{jir} < 0\},\, l \in \mathcal{L},\, o \in \mathcal{O} \tag{5.5}$$

$$x_{iilro(t-T^{D}_{irt})} - x_{ijlrot} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{D}}_{ijt} \,|\, t - T^{D}_{jir} \geq 0\},\, l \in \mathcal{L},\, o \in \mathcal{O} \tag{5.6}$$

$$X_{iilrot} - x_{ijlrot} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{D}}_{ijt} \,|\, t - T^{D}_{jir} < 0\},\, l \in \mathcal{L},\, o \in \mathcal{O} \tag{5.7}$$

Constraints (5.4)–(5.7) represent the flow conservation for laden load carriers that stay on the vessel when it docks at a port. These constraints are to enforce two things. First, only laden load carriers that arrive with a vessel at a port can stay on board the vessel. Second, laden load carriers that stay on board a vessel that docks at a port must also depart with the same vessel.

Flow of Empty Load Carriers

The flow constraints for empty load carriers are essentially the same as those for laden load carriers.

$$\sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}^{\mathcal{A}}_{jit} \,|\, (t - T_{jir}) \geq 0} y_{jilr(t-T_{jir})} + \sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}^{\mathcal{A}}_{jit}} Y_{jilrt} = a^{E}_{ilt} \qquad t \in \mathcal{T},\, i \in \mathcal{P},\, l \in \mathcal{L} \tag{5.8}$$

$$\sum_{j \in \mathcal{P}} \sum_{r \in \mathcal{R}^{\mathcal{D}}_{ijt}} y_{ijlrt} = d^{E}_{ilt} \qquad t \in \mathcal{T},\, i \in \mathcal{P},\, l \in \mathcal{L} \tag{5.9}$$

$$s^{E}_{ilt} + y^{P}_{ilt} - u^{E}_{ilt} = d^{E}_{ilt} - a^{E}_{ilt} \qquad t \in \mathcal{T},\, i \in \mathcal{P},\, l \in \mathcal{L}, \tag{5.10}$$

The constraint (5.8) has two terms. The first term encompasses the empty load carriers transported from $j \in \mathcal{P}$ to $i \in \mathcal{P}$ in the planning horizon. It states that empty load carriers arriving at a port $i \in \mathcal{P}$ can arrive from any other port $j \in \mathcal{P}$ if there is a route arriving at i from j at a time period t and that the departure time period from j is in the planning horizon. The second term encompasses empty load carriers that were transported from j to i and departed from j before the beginning of the planning horizon. It states that empty load carriers arriving at a port $i \in \mathcal{P}$ can arrive from any other port $j \in \mathcal{P}$ if there is a route arriving at i from j at a time period t. If $i = j$, then the load carriers arrive from inventory. These load carriers arrive at i at the exact time when time period t starts.

The constraint (5.9) states that empty load carriers departing from port $i \in \mathcal{P}$ can be transported to any other port $j \in \mathcal{P}$ if there is a route departing from i to j at time period t. If $i = j$, the laden load carriers are stored in inventory. These load carriers depart from i at the end of the time period t.

Constraints (5.10) represent the flow balance for empty load carriers at each port. It states that the number of empty load carriers that depart less those that arrive should be equal to the number of laden load carriers that have completed unloading and thus become empty plus the new empty load carriers purchased, less the empty load carriers that have started loading (and will become laden after the port lead time).

$$y_{iilrt} - y_{jilr(t-\mathcal{T}_{jir})} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit} \mid t - T_{jir} \geq 0\}, \quad l \in \mathcal{L} \tag{5.11}$$

$$y_{iilrt} - Y_{jilrt} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{A}}_{jit} \mid t - T_{jir} < 0\}, \quad l \in \mathcal{L} \tag{5.12}$$

$$y_{iilr(t-T^{D}_{irt})} - y_{ijlrt} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{D}}_{ijt} \mid t - T^{D}_{jir} \geq 0\}, \quad l \in \mathcal{L} \tag{5.13}$$

$$Y_{iilrt} - y_{ijlrt} \leq 0 \quad t \in \mathcal{T},\, i \in \mathcal{P},\, j \in \mathcal{P} \setminus \{i\},\, r \in \{\mathcal{R}^{\mathcal{T}\mathcal{D}}_{ijt} \mid t - T^{D}_{jir} < 0\}, \quad l \in \mathcal{L} \tag{5.14}$$

Constraints 5.11–5.14 represent the flow conservation for empty load carriers that stay on the vessel when it docks at a port. These constraints are to enforce two things. First, only laden load carriers that arrive with a vessel at a port can stay on board the vessel. Second, laden load carriers that stay on board a vessel that docks at a port must also depart with the same vessel.

Connecting the Flow of Laden and Empty Load Carriers

$$s^E_{ilt} = \sum_{o \in \mathcal{O}} u^L_{ilo(tL_{it})} \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t - L_{it} \geq 0\,\},\ l \in \mathcal{L} \tag{5.15}$$

$$s^E_{ilt} = 0 \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t - L_{it} < 0\,\},\ l \in \mathcal{L} \tag{5.16}$$

$$u^E_{ilt} = \sum_{o \in \mathcal{O}} S^L_{ilo(t+L_{it})} \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t + L_{it} \leq |\mathcal{T}|\,\},\ l \in \mathcal{L} \tag{5.17}$$

$$u^E_{ilt} = 0 \qquad t \in \mathcal{T},\ i \in \{\mathcal{P} \mid\ t + L_{it} > |\mathcal{T}|\,\},\ l \in \mathcal{L} \tag{5.18}$$

The equation (5.15)-(5.16) states that the number of laden load carriers that have completed unloading at a port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ is equal to the number of laden load carriers that started unloading at port $i \in \mathcal{P}$ at time period $i - L_{it} \in \mathcal{T}$. The equation thus states that empty load carriers appear exactly L_{it} time periods after the laden load carriers start unloading. The variable s^E_{ilot} thus states the number of empty load carriers that have completed converting from laden.

The equation (5.17)-(5.18) states that the number of empty load carriers that starts unloading at a port $i \in \mathcal{P}$ at time period $t \in \mathcal{T}$ is equal to the number of empty load carriers that have completed unloading at $i \in \mathcal{P}$ at $t + L_{it}$. The equation thus states that empty load carriers must start loading L_{it} time periods before the empty load carriers have completed loading and have become laden load carriers ready for transportation. The variable u^E_{ilot} thus states the number of empty load carriers that start the process of being converted to laden (i.e., starts loading).

Capacity constraint

$$\sum_{(i,j) \in \mathcal{D}_{rt}} \sum_{l \in \mathcal{L}} \sum_{o \in \mathcal{O}} W^L_l x_{ijlrot} + \sum_{(i,j) \in \mathcal{D}_{rt}} \sum_{l \in \mathcal{L}} W^E_l y_{ijlrt} \leq\ K^r \qquad r \in \mathcal{R}, t \in T \tag{5.19}$$

Constraint (5.19) ensures that the storage capacity on a route is never exceeded. This constraint enforces vessel capacity in the case of transport routes. If $r \in \mathcal{R}$ is an inventory route belonging to port $i \in \mathcal{P}$, then constraint (5.19) ensures that the inventory capacity at i is never exceeded.

Time window constraint

$$S_{O_o l o T^S_o} = \sum_{t'=T^S_o+D^{TL}_o}^{T^S_o+D^{TU}_o} u^L_{D_o l o t'} \quad o \in \{\mathcal{O} \mid T^S_o \in \mathcal{T}\}, l \in \mathcal{L} \tag{5.20}$$

$$\sum_{t'=0}^{D^{TU}_o} \sum_{j \in \mathcal{P}} \sum_{i \in \mathcal{P}} \sum_{r \in \mathcal{R}^A_{jit}} X_{jilrot'} = \sum_{t'=T^S_o+D^{TL}_o}^{T^S_o+D^{TU}_o} u^L_{D_o l o t'} \quad o \in \{\mathcal{O} \mid T^S_o \notin \mathcal{T}\}, l \in \mathcal{L} \tag{5.21}$$

Constraint (5.20) and (5.21) ensures that the laden load carriers belonging to a transport order start unloading in the time window of that order. There are two situations to handle. Constraint (5.20) handles the situation where the laden load carriers belonging to the order start their transportation journey in the planning horizon. Constraint (5.21) handles the situation where the laden load carriers belonging to the transport order start their transportation journey before the beginning of the planning horizon but where the delivery time window lies within the planning horizon.

Equilibrium constraints

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} u^L_{ilot} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} S_{ilot} \quad o \in \{\mathcal{O} \mid T^S_o \in \mathcal{T}\}, l \in \mathcal{L} \tag{5.22}$$

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} u^L_{ilot} = \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{P}} \sum_{i \in \mathcal{P}} \sum_{r \in \mathcal{R}^A_{jit}} X_{jilrot'} \quad o \in \{\mathcal{O} \mid T^S_o \notin \mathcal{T}\}, l \in \mathcal{L} \tag{5.23}$$

Constraint (5.22)-(5.23) ensures that no more load carriers can be unloaded than what has been loaded.

Non-negativity constraints

$$u^L_{ilot} \in \mathbb{Z}^+ \quad o \in \mathcal{O}, l \in \mathcal{L}, i \in \mathcal{P},\ t \in \mathcal{T} \tag{5.24}$$

$$x_{ijlrot} \in \mathbb{Z}^+ \quad o \in \mathcal{O},\ l \in \mathcal{L}, j \in \mathcal{P}, i \in \mathcal{P}, r \in \mathcal{R},\ t \in \mathcal{T} \tag{5.25}$$

$$y_{ijlrt} \in \mathbb{Z}^+ \quad l \in \mathcal{L}, j \in \mathcal{P}, i \in \mathcal{P}, r \in \mathcal{R},\ t \in \mathcal{T} \tag{5.26}$$

$$y^P_{ilt} \in \mathbb{Z}^+ \quad l \in \mathcal{L}, i \in \mathcal{P},\ t \in \mathcal{T} \tag{5.27}$$

Constraint (5.24)-(5.27) ensures that all decision variables must be non-negative integers.

Objective Function

$$
\begin{aligned}
min\, z \; = & \sum_{t\in \mathcal{T}} \sum_{r\in \mathcal{R}} \sum_{(i,j)\in \mathcal{D}_{rt}} \sum_{l\in\mathcal{L}} \sum_{o\in\mathcal{O}} C_{ijr} W_l^L x_{ijlrot} \\
& + \sum_{t\in \mathcal{T}} \sum_{r\in \mathcal{R}} \sum_{(i,j)\in \mathcal{D}_{rt}} \sum_{l\in\mathcal{L}} C_{ijr} W_l^E y_{ijlrt} \\
& + \sum_{t\in \mathcal{T}} \sum_{i\in \mathcal{P}} \sum_{l\in \mathcal{L}} C_l^E y_{ilt}^P
\end{aligned}
\tag{5.28}
$$

The objective function presented in equation (5.28) represents the total costs of managing load carriers. The equation has three terms. The first two calculate costs for handling laden and empty load carriers. In the case of transport routes, $C_{ijr}W_l^L$ (resp. $C_{ijr}W_l^E$) is the cost of transporting one laden (resp. empty) load carrier of type $l \in \mathcal{L}$ between ports $i \in \mathcal{P}$ and $j \in \mathcal{P}$. In the case of inventory routes, $C_{iir}W_l^L$ (resp. $C_{iir}W_l^E$) is the cost of storing one laden (resp. empty) load carrier of type $l \in \mathcal{L}$ in inventory at port $i \in \mathcal{P}$. The third term calculates the cost of purchasing new empty load carriers.

Chapter 6

Case Study

The mathematical model presented in chapter 5 is tested on real data. Hydro provides a realistic data set called the *source data set*. The source data set contains the number of laden load carriers planned to be transported between the Norwegian plants and the terminal ports each week in 2022. It also contains information about transport routes that were planned to be used and the port sequence and schedules of these transport routes. However, the source data set does not contain the actual transportation data of laden load carriers (i.e., the path each load carrier took to get from the origin to the destination), the delivery time windows, or any information about transportation or inventory levels of empty load carriers.

6.1 Input Data

6.1.1 General Data

Planning Horizon

For the analysis in this thesis, time is divided into distinct periods, with four time periods per day. Each time period thus corresponds to a duration of 6 hours. This division is based on the schedules of transport routes presented below (see subsection 6.1.1). The length of the planning horizon studied in this thesis stretches from 173 time periods to 341 time periods (see section 6.4).

Plants and Ports

This section describes the port input data. Five ports are associated with Hydro's Norwegian production plants: Karmøy, Husnes, Årdal, Høyanger, and Sunndal. These ports reflect Hydro's domestic operations within Norway. There are also five terminal ports in

Europe: Rotterdam, Swinoujscie, Gothenburg, Immingham, and Esbjerg. Finally, there are three transshipment ports: Haugesund, Bergen, and Tananger. These are all ports $p \in \mathcal{P}$

As presented in section 5.1.1, the inventory at each port is modeled as an inventory route that has arcs that connect a port at a time period to the same port in the following time period. Thus, each port $p \in \mathcal{P}$ has an inventory route, and the storage capacity K_r is the inventory capacity when r is an inventory route.

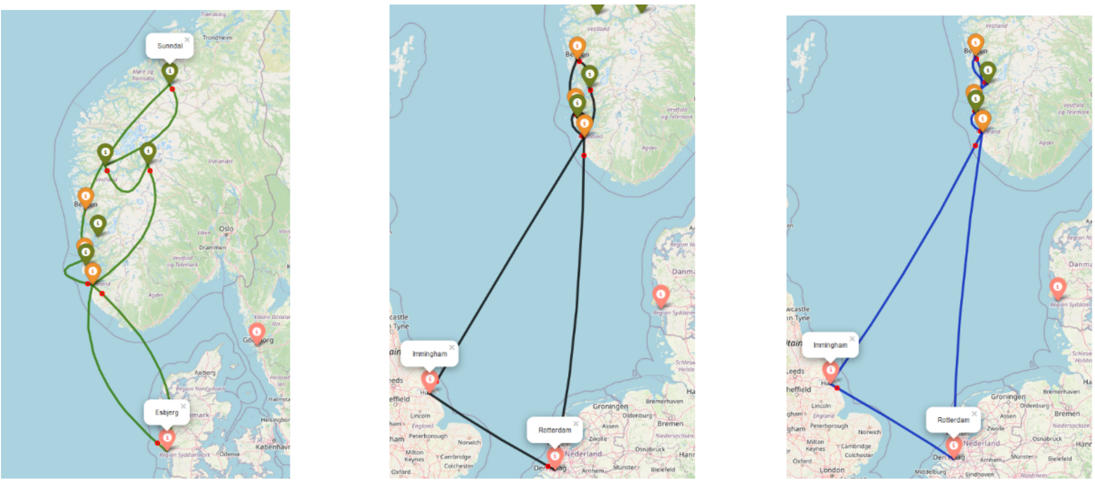
The inventory capacity at each port is not provided in the source data set, nor publicly available information. Discussions with Hydro revealed that inventory capacity is not generally a limiting factor. Thus, K_r is set to a large number of 5 000 storage units for each inventory route (i.e., the inventory of each port $p \in \mathcal{P}$). Storage units are nominal values. When laden and empty load carriers are stored in inventory, they use up storage units according to the factor provided in Table 6.2.

Transport Routes

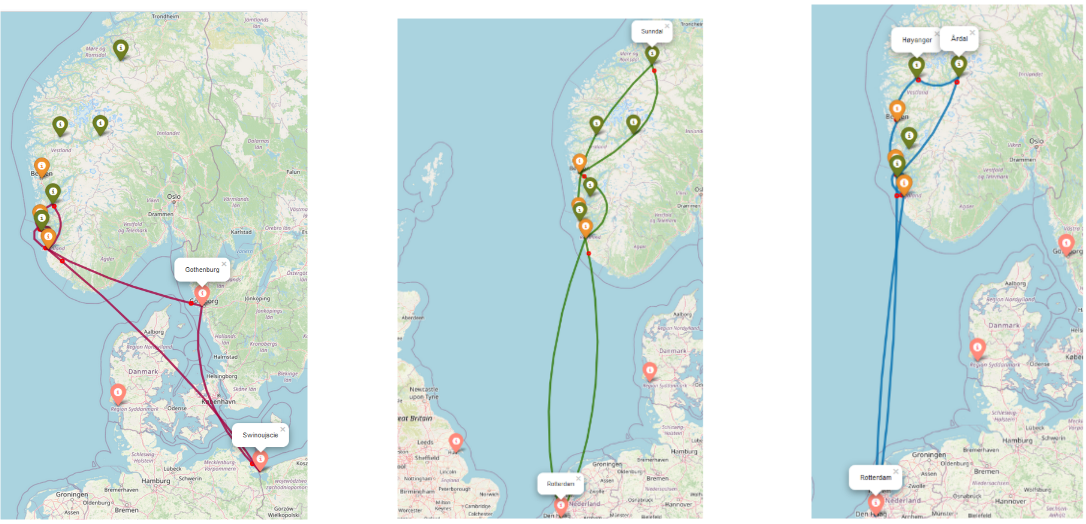
The transport routes that operate Hydro's maritime transportation system are presented in Table 6.1. The source data set does not provide the available capacity on board each vessel. However, since data about planned laden transportation along each transport route is given, the available storage has therefore been set not to limit transportation planned for in the past. Figure 6.1 shows a map of each transport route.

Table 6.1: Input data for the transport routes.

Transport Route	**Vessel Name**	**Storage K_r**	**Plants visited**	**Terminal ports visited**
Coastal Interplant	Trans Carrier	200	Årdal Høyanger Sunndal	Esbjerg
Northern Trade	Misida	300	Sunndal	Rotterdam
Sognefjord Trade	Misana	300	Karmøy Årdal Høyanger	Rotterdam
Poland Trade	SC Ahtela	325	Karmøy Husnes	Gothenburg Swinoujscie
Southern Trade 1	Bore Bay	200	Husnes Karmøy	Rotterdam Immingham
Southern Trade 2	SC Connector	250	Husnes Karmøy	Rotterdam Immingham



(a) Costal Interplant **(b)** Southern Trade 2 **(c)** Southern Trade 1

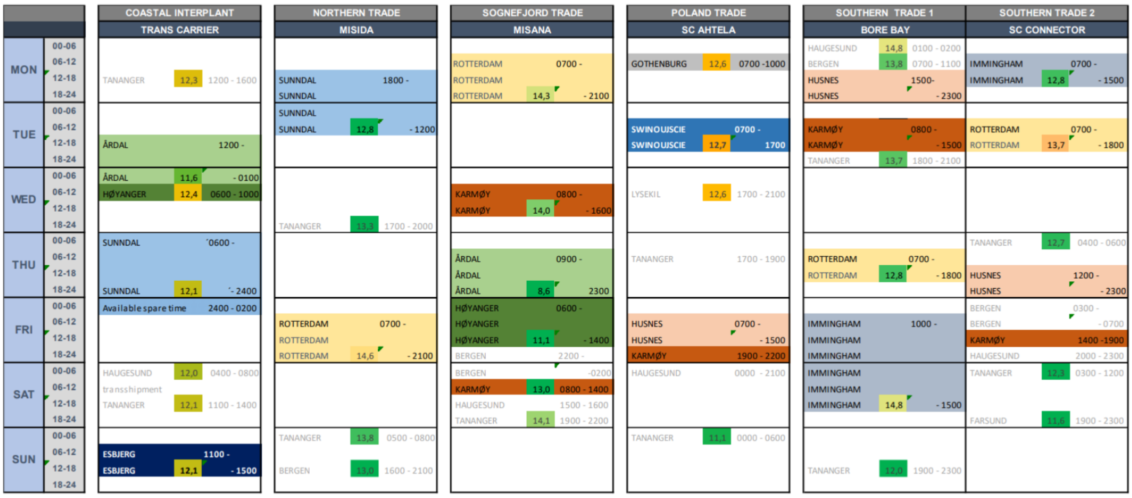


(d) Poland Trade **(e)** Northern Trade **(f)** Sognefjord Trade

Figure 6.1: Map of the transport routes

Schedules for Transport Routes

The schedule for each transport route is publicly available information and can be found online through services like Vessel Finder. Schedules were also provided in the source data set. The schedule for each transport route repeats every week (i.e., every 28th time period). Figure 6.2 shows the weekly schedules for the transport routes. The T^D_{irt} parameters can also be derived from the weekly schedule and repeated every week. Further, the sets $\mathcal{R}^{\mathcal{A}}_{jit}$, $\mathcal{R}^{\mathcal{T}^{\mathcal{A}}}_{jit}$, $\mathcal{R}^{\mathcal{D}}_{ijt}$, and $\mathcal{R}^{\mathcal{T}^{\mathcal{D}}}_{jit}$ are also derived from the schedule.



		COASTAL INTERPLANT	NORTHERN TRADE	SOGNEFJORD TRADE	POLAND TRADE	SOUTHERN TRADE 1	SOUTHERN TRADE 2
		TRANS CARRIER	MISIDA	MISANA	SC AHTELA	BORE BAY	SC CONNECTOR
MON	00-06					HAUGESUND 14,8 0100 - 0200	
	06-12			ROTTERDAM 0700 -	GOTHENBURG 12,6 0700 -1000	BERGEN 13,8 0700 - 1100	IMMINGHAM 0700 -
	12-18	TANANGER 12,3 1200 - 1600	SUNNDAL 1800 -	ROTTERDAM		HUSNES 1500-	IMMINGHAM 12,8 - 1500
	18-24		SUNNDAL	ROTTERDAM 14,3 - 2100		HUSNES - 2300	
TUE	00-06		SUNNDAL				
	06-12		SUNNDAL 12,8 - 1200		SWINOUJSCIE 0700 -	KARMØY 0800 -	ROTTERDAM 0700 -
	12-18	ÅRDAL 1200 -			SWINOUJSCIE 12,7 1700	KARMØY - 1500	ROTTERDAM 13,7 - 1800
	18-24					TANANGER 13,7 1800 - 2100	
WED	00-06	ÅRDAL 11,6 - 0100					
	06-12	HØYANGER 12,4 0600 - 1000		KARMØY 0800 -	LYSEKIL 12,6 1700 - 2100		
	12-18			KARMØY 14,0 - 1600			
	18-24		TANANGER 13,3 1700 - 2000				
THU	00-06	SUNNDAL ´0600 -					TANANGER 12,7 0400 - 0600
	06-12			ÅRDAL 0900 -	TANANGER 1700 - 1900	ROTTERDAM 0700 -	
	12-18			ÅRDAL		ROTTERDAM 12,8 - 1800	HUSNES 1200 -
	18-24	SUNNDAL 12,1 ´- 2400		ÅRDAL 8,6 2300			HUSNES - 2300
FRI	00-06	Available spare time 2400 - 0200		HØYANGER 0600 -			BERGEN 0300 -
	06-12		ROTTERDAM 0700 -	HØYANGER	HUSNES 0700 -	IMMINGHAM 1000 -	BERGEN - 0700
	12-18		ROTTERDAM	HØYANGER 11,1 - 1400	HUSNES - 1500	IMMINGHAM	KARMØY 1400 -1900
	18-24		ROTTERDAM 14,6 - 2100	BERGEN 2200 -	KARMØY 1900 - 2200	IMMINGHAM	HAUGESUND 2000 - 2300
SAT	00-06	HAUGESUND 12,0 0400 - 0800		BERGEN -0200	HAUGESUND 0000 - 2100	IMMINGHAM	TANANGER 12,3 0300 - 1200
	06-12	transshipment		KARMØY 13,0 0800 - 1400		IMMINGHAM	
	12-18	TANANGER 12,1 1100 - 1400		HAUGESUND 1500 - 1600		IMMINGHAM 14,8 - 1500	
	18-24			TANANGER 14,1 1900 - 2200			FARSUND 11,6 1900 - 2300
SUN	00-06		TANANGER 13,8 0500 - 0800		TANANGER 11,1 0000 - 0600		
	06-12	ESBJERG 1100 -					
	12-18	ESBJERG 12,1 - 1500	BERGEN 13,0 1600 - 2100			TANANGER 12,0 1900 - 2300	
	18-24						

Figure 6.2: Schedule for the routes

Load Carrier Types

Hydro has two load carrier types, mafis, and cassettes. These are shown in Figure 2.7. As presented in chapter 3, empty load carriers have a smaller storage requirement than laden load carriers because empty load carriers can be stacked on top of each other during transportation. For example, five empty cassettes can be stacked on each other to take up the same amount of storage as one laden cassette. Both mafis and cassettes take up one storage unit when they are empty and five storage units when they are laden (i.e., $W^E_l = 1$ and $W^L_l = 5$ for both).

As presented in chapter 3, the operators at the ports require some lead time to convert laden load carriers into empty, and empty into laden takes time. Discussions with Hydro revealed a lead time of one time period at each port would be suitable for both load carrier types (cassettes and mafis), given a granularity of four time periods per day. This lead time gives the port operators enough time to load (resp. unload) empty (resp. laden) load carriers.

Since the properties of cassettes and mafis are indistinguishable, this thesis treats them as identical. This is done by replacing mafis with cassettes in the source data set and initializing the model with one load carrier type (i.e., cassettes). Table 6.2 presents the parameter values used for cassettes.

Table 6.2: Values for the load carrier parameters.

Parameter	**Value**
W_l^E	1
W_l^L	5
L_{it}	1

6.1.2 Costs Structure

Operational Costs

The operational costs consist of the transportation and inventory costs. The exact inventory cost for each port is not public information nor provided by the source data set. Likewise, the exact transportation costs are sensitive information. However, discussions with Hydro revealed that transportation costs dominate inventory costs. Thus, storing laden and empty load carriers in port inventory is cheaper than on vessels.

The cost parameter C_{iir} describes the inventory costs when r is an inventory route. Thus, if r is the inventory route of port i, then C_{iir} describes the cost of storing one storage unit in the inventory of port i for one time period.

I pick higher inventory costs for the terminal and transshipment ports than for the ports belonging to the Norwegian plants. The reason for this is that this cost structure favors storing load carriers close to the production plants. This will facilitate the repositioning of empty load carriers from the terminal ports back to the Norwegian plants. Therefore, the cost parameter C_{iir} is set to 1 NOK in the Norwegian plants and 100 NOK in the case of the terminal and transshipment ports. Thus, it costs 1 NOK to store one storage unit in one time period in the inventory of a Norwegian plant, while it costs 100 NOK to do the same at a transshipment or terminal port.

The cost parameter C_{ijr} describes the transportation costs when r is a transport route. Transportation costs depend on the storage required and the distance (as described in chapter 3). I assume that the vessels serving the transport routes travel steadily and that the travel time between ports is proportional to the distance. Therefore, the cost parameters C_{ijr} are equal to the time periods it takes to travel between port i and port j with transport route r scaled with a factor of 1000 NOK. The scaling of the transport costs is necessary to ensure that transportation load carriers are more expensive than storing them in inventory. Transport times between ports are found in the source data set (see

Figure 6.2).

The transport costs C_{ijr}, and the inventory costs C_{iir} are given in storage units. To find the cost of storing load carriers, the storage units each load carrier requires should be considered (see Table 6.2).

Penalty Costs

The penalty costs consist of the cost of purchasing new empty load carriers and the costs of purchasing extra capacity with trucks. The model presented in chapter 5 handles the option to purchase extra capacity with trucks through an "extra capacity route." I initialize the extra capacity route with an arc between every pair of ports daily (i.e., every fourth period) and a large capacity of $K_r = 10000$ storage units. The specific cost details for acquiring this additional capacity are confidential and not openly accessible. However, discussions with Hydro revealed that purchasing extra capacity is expensive and should be avoided. Further, the specific cost details for purchasing new empty load carriers are confidential and not openly accessible.

However, the ratio between the penalty costs is more important than the absolute values. Purchasing new empty load carriers should be more expensive than transporting empty load carriers between ports. This facilitates empty load carrier repositioning with the transport routes. Further, the number of load carriers in the maritime logistics system should not be a limiting factor. Therefore, transporting load carriers with trucks should be more expensive than purchasing empty load carriers. This prohibits repositioning empty load carriers with trucks, which would indicate too few load carriers in the system.

To enforce the penalty cost structure, the cost of purchasing one extra capacity is set to a large number of 1 000 000 NOK. Further, the cost parameter C_{ijr} is set equal to the time periods it takes to travel between port i and port j with trucks (i.e., r is the extra capacity route) and scaled with a factor of 10 000 000 NOK. This ensures that purchasing a new empty load carrier at j is cheaper than transporting one from i to j.

6.1.3 Overview of Input Data

Table 6.3 provides an overview of the value ranges for the sets and parameters discussed in this section.

Table 6.3: Overview of the input data

Parameter or Set		**Value Range**
Ports	$\mathcal{P}$	13 ports
Load Carrier Types	$\mathcal{L}$	1 load carrier type
Transport Routes	$\mathcal{R}^{\mathcal{T}}$	6 transport routes
Routes	$\mathcal{R}$	20 routes (6 transport routes, 13 inventory routes, and 1 extra capacity route)
Time periods	$\mathcal{T}$	173 - 341 time periods
Route capacity	K_r	200 - 10 000 storage units
Number of storage units required by an empty load carrier	W_l^E	1 storage unit
Number of storage units required by a laden load carrier	W_l^L	5 storage units
Port lead time	L_{it}	1 time period
Cost of transporting one storage unit from port i to j with route r	C_{ijr}	1 - 10 000 000 NOK

6.2 Transport Orders

Transport orders represent transportation needs and are described by the parameters in Table 7.2. I pick only one $S_{ilot}^L > 0$ for each transport order since transport orders represent the transport of laden load carriers from one place and time to an other place and time. The source data set has no concept of transport orders. However, the origin port, destination port, the weekly number of laden load carriers to transport between the origin and the destination, and the planned transport route, are provided in the source data set. The time period when the laden load carriers are ready for transportation and the delivery time windows is not provided.

Since the source data set only contained parts of the information necessary to construct transport orders, I had to make assumptions about the missing information. Discussions with Hydro revealed that the laden load carriers that departed from an origin port became ready for transportation the week before. I assume that the load carriers from the weekend and Monday complete loading on Monday evening, that load carriers from Tuesday and Wednesday complete loading on Wednesday evening, and that load carriers from Thursday and Friday complete loading on Friday evening. Thus, I create one transport order each week on Monday, Wednesday, and Friday. Further, discussions with Hydro revealed that a delivery time window of 3 days - 15 days is reasonable for each transport order. Therefore,

Table 6.4: Parameters and their descriptions

Parameter	**Description**	**Provided in the Source Data Set**
O_o	Origin port	Yes
D_o	Destination port	Yes
S^L_{ilot}	The number of laden load carriers to transport	Yes
T_o	The time period when the laden load carriers are ready for transportation	No
$[D_o^{TL}, D_o^{TU}]$	The delivery time window	No

I pick a delivery time window of 12 - 60 time periods for each transport order.

The procedure for creating transport orders is provided in Algorithm 1. The algorithm is written in pseudocode, and the time periods for T_o, D_o^{TL}, and D_o^{TU} are determined based on the start of the planning horizon.

The source data set provided data for 2022. I created transport orders from January 1st, 2022. Thus, the first week of transport orders is created for week 52 in 2021.

Algorithm 1: Creating transport orders

Input : The origin port, destination port, the weekly number of laden load carriers to transport between the origin and the destinations
Output : Set of transport orders

```
for each week $w$ that has transport of laden load carriers of type $l$ between origin $i$ and destination $j$ do
    amount ← the number of laden load carriers of type $l$ to transport between $i$ and $j$ for $w$  ;
    $n$ ← amount // 3 ;
    $r$ ← amount % 3 ;
    for day in {Monday, Wednesday, Friday} of the week before $w$ do
        Create a transport ordero;
        $t$ ← time period of day
        transportOrderAmount ← $n$ ;
        if day = Monday and $r \leq 2$ then
            transportOrderAmount ← transportOrderAmount + 1;
        end
        if day = Wednesday and $r = 2$ then
            o.transportOrderAmount ← o.transportOrderAmountt + 1;
        end
        $T_o \leftarrow t$;
        $D_o^{DL}$ ← day + 3 days;
        $D_o^{DU}$ ← day + 15 days;
        $S^L_{ilot}$ ← transportOrderAmount
    end
end
```

6.3 Initialization

To transport laden load carriers from the origin ports to the destination ports, empty load carriers are required at the origin ports. However, the source data did not contain information about transportation or inventory levels of empty load carriers. To compensate for this missing data, I developed a strategy to stimulate the circulation of empty load carriers. This was accomplished by running the model for a period corresponding to 12 weeks. This duration was chosen to ensure that the delivery time window for nine weeks of transport orders would fall within the planning horizon (since transport orders have a delivery time window of 3 - 15 days, three additional weeks are required). These were the first nine weeks of transport orders that were created from the source data set. Thus, the initialization contained transport orders that had load carriers ready for transportation from Monday, December 27th, 2021 to Sunday, February 20th, 2022. During these 12 weeks, the model purchased new empty load carriers where they were required. This method allowed for the integration of empty load carriers into the model despite the lack of direct data.

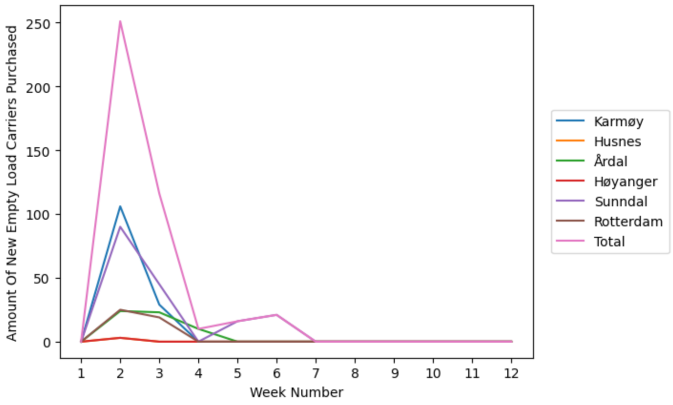


Figure 6.3: The number of new empty load carriers purchased at the various ports under the initialization phase.

The results from the initialization of empty load carriers demonstrate an initial high frequency of new empty load carrier purchases in the starting weeks. Figure 6.3 shows that this trend found stability around weeks six and seven. The total number of new empty load carriers obtained over the 12 weeks is 414.

Hydro's transportation needs may be higher in some periods and lower in others (e.g., because of seasonal changes). Therefore, more empty load carriers were included in the port inventories to adapt the initialization to this variation in transportation needs. This measure ensured that the number of empty load carriers would not become a limiting factor under different demand conditions. Therefore, 86 additional load carriers were added, totaling 500 load carriers.

Moreover, all subsequent instances proceeded from week ten. Thus, the starting point for all future model runs was the status at the start of week ten (i.e. Monday, February 28th, 2022), including the distribution of empty and laden load carriers. This approach allows for a smooth transition between model runs, each starting with an accurate representation of the current empty load carrier distribution. The inventories of empty load carriers at the various ports at the beginning of week ten can be seen in Table 6.5, while those on vessels can be seen in Table 6.6.

Table 6.5: Initial status of empty load carriers in inventory at the ports

Port	**Empty Load Carriers in Inventory**
Haugesund	28
Gothenburg	15
Karmøy	30
Høyanger	7
Husnes	61
Sunndal	30
Årdal	21
Rotterdam	15

Table 6.6: Empty load carriers on vessels at the beginning of the planning horizon

Transport Route	**From**	**To**	**Empty Load Carriers On The Vessels**
Coastal Interplant	Esbjerg	Tananger	23

6.4 Instances

This section presents an overview of the five instances used in this thesis. The purpose of each of the five instances is to create a ten-week transportation plan. All five instances are initialized with the data from the starting point selected in the initialization phase (see section 6.3). The starting point is Monday, February 28th, 2022. The instances thus use transport order data from Monday, February 28th, 2022, and ten weeks onward. The number of laden load carriers ready for transportation at each port (i.e., the values of the S^L_{ilot} parameters) for these ten weeks is presented in Figure 6.4.

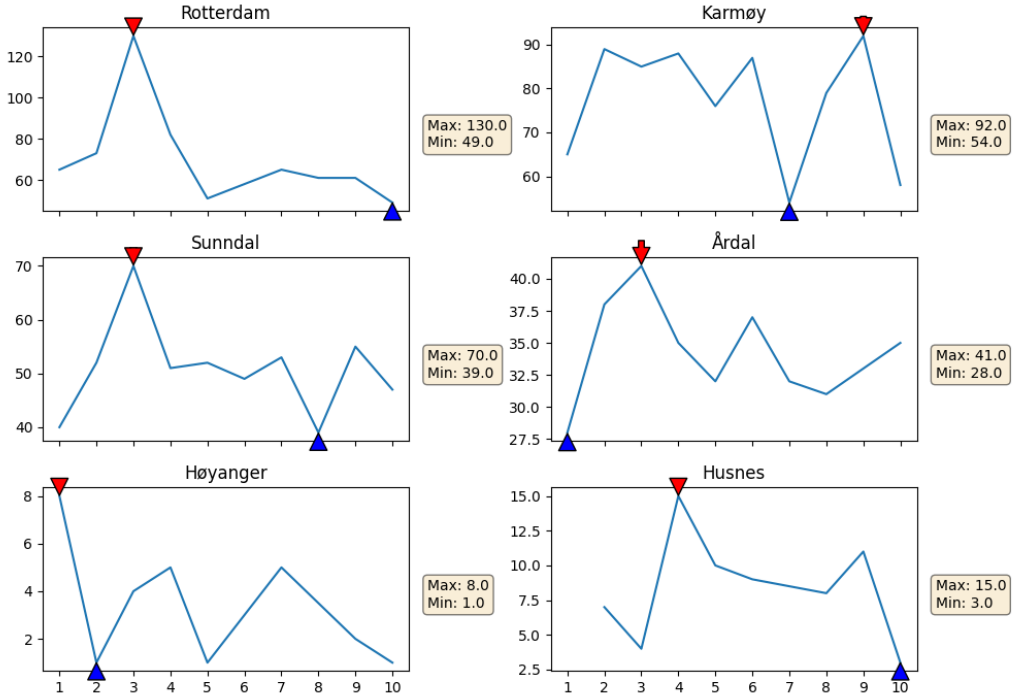


Figure 6.4: The number of laden load carriers ready for transportation by week.

6.4.1 Instances without changes to known data

This subsection presents four instances. These instances create a ten-week transportation plan without changes to the known data. These instances aim to analyze the effects of the planning horizon length.

Instance 1: Finite Horizon Ten Weeks

In instance 1, all ten weeks of transport orders are known. This instance uses a finite horizon. This means the model creates a ten-week transportation plan, and then the decisions for all ten weeks are applied. To make sure that the transport orders with T_o in the tenth week are included, the planning horizon must be ten weeks (280 time periods) plus the delivery time window (60 time periods) plus the largest port lead time (1 time period). It is necessary to add the port lead time to the planning horizon so laden load carriers that start unloading in the 340th time period have enough time (i.e., the lead time) to become empty. The planning horizon thus becomes 341 time periods. With this planning horizon length, 558 transport orders are included.

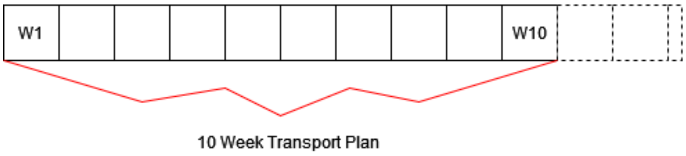


Figure 6.5: Instance 1 use a finite horizon where transport orders for all ten weeks are known

Instance 2: Rolling Horizon Six Weeks

In instance 2, six weeks of transport orders are known, but new information arrives weekly. Thus, transportation orders for weeks one to six are known at week one, but in week two, information about transportation orders for week seven arrives. Similarly to instance 1, instance 2 must ensure that the transport orders with T_o in the sixth week are included. Thus the planning horizon has 229 time periods (six weeks plus delivery time window and lead time). The rolling horizon is run with $\Delta t = 1$ *week*. Thus, the horizon is rolled forward one week when the model is run and the first week's decisions are applied. The port inventories and load carriers on board vessels are updated after each roll. The model runs until ten weeks of transport orders are known. Figure 6.6 shows that this requires five model runs and four rolls.

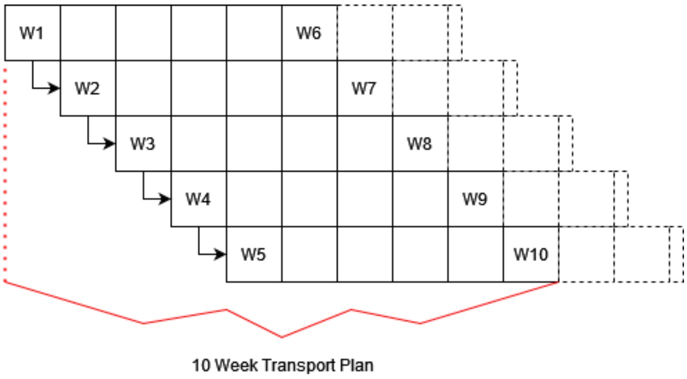


Figure 6.6: Instance 2 uses a rolling horizon where six weeks of transport orders are known.

Instance 3: Rolling Horizon Four Weeks

In instance 3, four weeks of transport orders are known, but new information arrives weekly. Thus, at week one, transportation orders for weeks one to four are known, but in week two, information about transportation orders for week five arrives. Similarly to instances 1 and 2, instance 3 must ensure that the transport orders with T_o in the fourth week are included. Thus the planning horizon has 173 time periods (4 weeks plus delivery time window and lead time). The model is run in a rolling horizon manner to deal with the dynamic environment. The rolling horizon is run with $\Delta t = 1\ week$. Thus, the horizon is rolled forward one week when the model is run and the first week's decisions are applied. The port inventories and load carriers on board vessels are updated after each roll. The model runs until ten weeks of transport orders are known. Figure 6.7 shows that this requires seven model runs and six rolls.

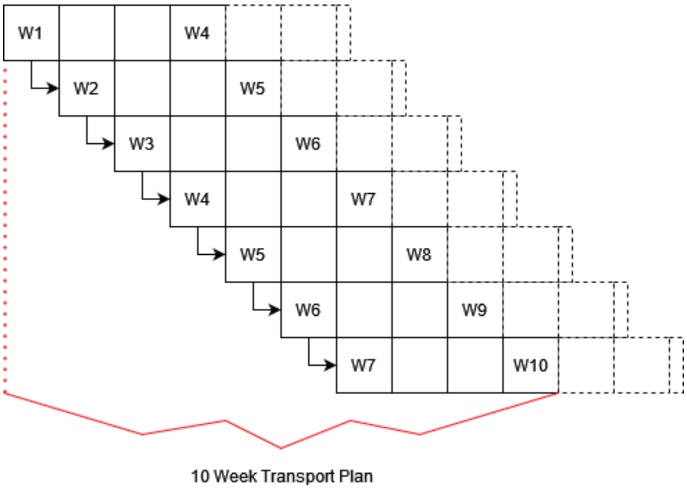


Figure 6.7: Instance 3 use a rolling horizon where four weeks of transport orders are known.

Instance 4: Rolling Horizon Three Weeks

In instance 4, three weeks of transport orders are known, but new information arrives weekly. Thus, at week one, transportation orders for weeks one to three are known, but in week two, information about transportation orders for week four arrives. Instance 4 must ensure that the transport orders with T_o in the third week are included. Thus the planning horizon has 145 time periods (3 weeks plus delivery time window and lead time). The model is run in a rolling horizon manner to deal with the dynamic environment. The rolling horizon is run with $\Delta t = 1\ week$. Thus, the horizon is rolled forward one week when the model is run and the first week's decisions are applied. The port inventories and load carriers on board vessels are updated after each roll. The model runs until ten weeks of transport orders are known. Figure 6.8 shows that this requires eight model runs and seven rolls.

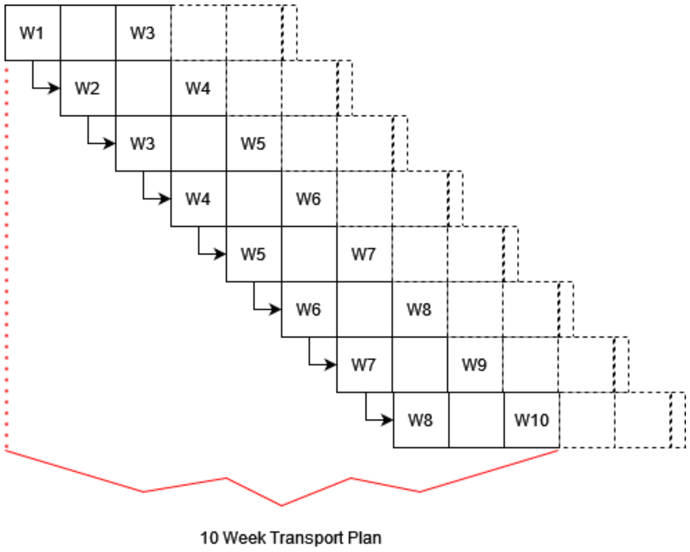


Figure 6.8: Instance 4 use a rolling horizon where three weeks of transport orders are known.

6.4.2 Instance with Changes to Known Data

This subsection presents an instance where changes happen to the known data during the planning. This instance aims to analyze how the model adapts to such changes.

Instance 5: Rolling Horizon Four Weeks with Data change

Instance 5 is similar to instance 3, but at each roll, there are changes to the known data. In week one, transport orders for weeks one to four are known, but in week two, information about transportation orders for week five arrives. When information about week five arrives, roughly 10% of the number of laden load carriers originally belonging to transport orders in week five is subtracted from week five. Then, roughly 10% of them are added back to week four. This changes the already known data since the model created a plan where the content of the transport orders for week four was considered known. The changing process repeats itself for each roll. There are some rounding errors in the change process. However, these errors are not corrected since the purpose of instance 5 is to analyze how the model adapts to changes in data.

Instance 5 must also ensure that the transport orders with T_o in the fourth week are included. Thus the planning horizon has 173 time periods (4 weeks plus delivery time window and lead time). The model runs until ten weeks of transport orders are known. Similar to instance 3, this requires seven model runs and six rolls. Figure 6.9 shows how the number of laden load carriers is changed between each roll.

Table 6.7 shows the changes in each roll, and Subfigures 6.10a-6.10f show the actual changes for each port. These changes can, for example, represent changes in customer

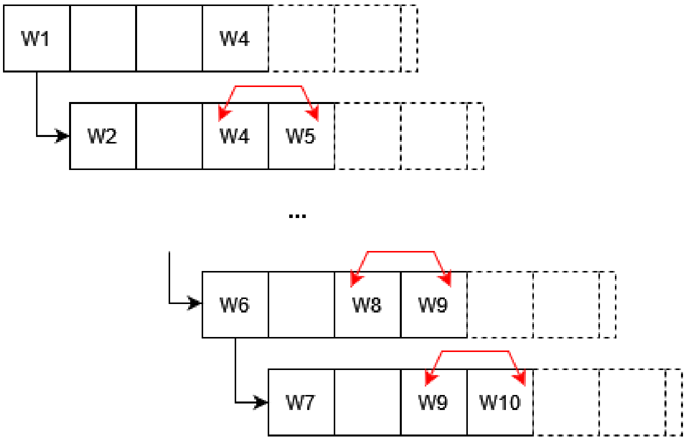


Figure 6.9: Instance 5 use a rolling horizon where four weeks of transport orders are known but changes to the known data happens at each roll.

Table 6.7: The number of laden load carriers changed at each roll

	Roll 1	**Roll 2**	**Roll 3**	**Roll 4**	**Roll 5**	**Roll 6**
Laden load carriers removed from transport orders in week n	32	24	17	20	19	20
Laden load carriers added to transport orders in week n - 1	25	27	20	16	20	17

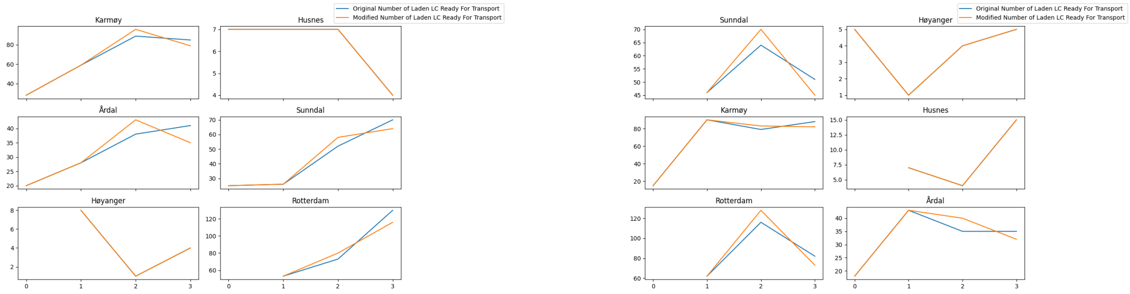
requests.

6.4.3 Overview of Instances

An overview of the instances presented in this section is provided in Table 6.8.

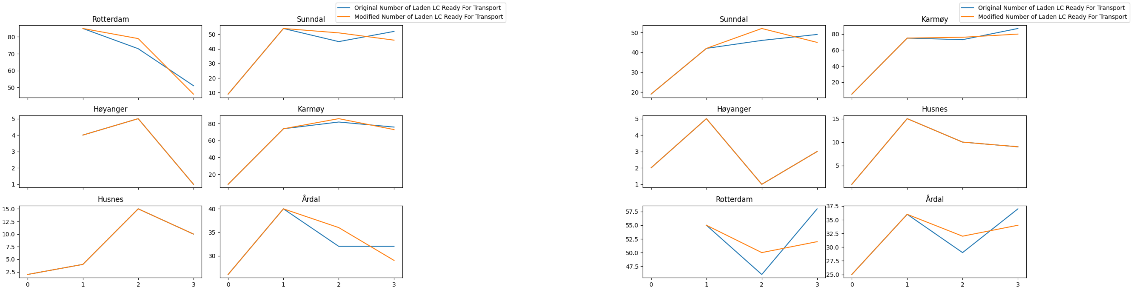
Table 6.8: Overview of the five instances considered in this thesis.

Instance	**Weeks of Transport Orders Known**	**Number of Time Periods $\|\mathcal{T}\|$**	**Number of Transport Orders $\|\mathcal{O}\|$**
1	Ten weeks	341	558
2	Six weeks (at each run)	229 (each run)	362 (average)
3	Four weeks (at each run)	173 (at each run)	261 (average)
4	Three weeks (at each run)	145 (at each run)	208.5
5	Four weeks (with 10% change at last week each roll)	173 (at each run)	261 (average)



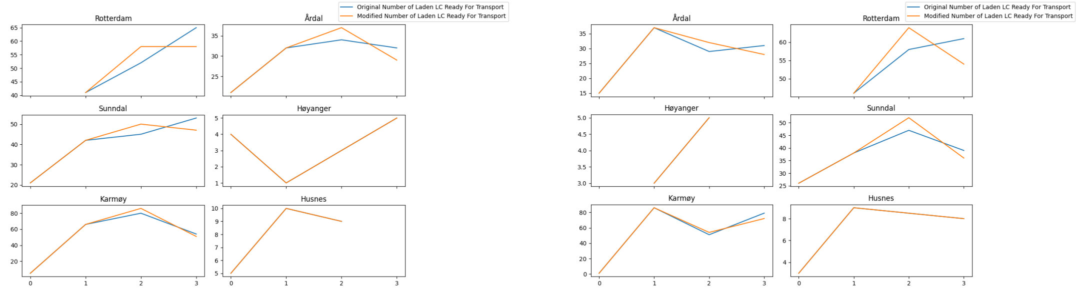
(a) Change in laden load carriers (LC) ready for transport at roll 1

(b) Change in laden load carriers (LC) ready for transport at roll 2



(c) Change in laden load carriers (LC) ready for transport at roll 3

(d) Change in laden load carriers (LC) ready for transport at roll 4



(e) Change in laden load carriers (LC) ready for transport at roll 5

(f) Change in laden load carriers (LC) ready for transport at roll 6

Figure 6.10: The difference between the original (blue) and modified (orange) laden load carriers ready for transport at each of the six rolls in instance 5.

Chapter 7

Computational Study

This chapter presents the results of the computational study of is presented. The model has been written in Python and implemented with Gurobi Optimizer version 10.0.0 build v10.0.0rc2 (win64). All instances have been run on a computer with Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz, using four physical cores, eight logical processors, and 16Gb RAM.

7.1 Instance Results

This section presents the results of the five instances presented in chapter 6.

7.1.1 Instance 1

Instance 1 was solved to optimality in 31.55 minutes. The model had 2 937 018 constraints and 12 906 673 variables. Table 7.9 presents the results. Transport costs and inventory costs account for 85.11% and 14.89% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.1: Costs for instance 1

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 874 006	14 360 700	2 513 306	0	0
100,00 %	85,11 %	14,89 %	0 %	0 %

The transportation plan for the ten-week horizon is a description of how all laden load carriers are transported from their origin port (i.e., O_o) at the time period when they are ready for transportation (i.e., T_o) to the destination port (i.e., D_o) within the delivery time window (i.e., $[D_o^{TL}, D_o^{TU}]$). Since instance 1 created a total of 558 transport orders, the solution for one transport order, transport order 538, is presented here for illustration.

Table 7.2: Parameter values for transport order 538

Parameter	**Value**
O_o	Karmøy
D_o	Rotterdam
S_{ilot}^L	16 laden load carriers
T_o	Time period 59
$[D_o^{TL}, D_o^{TU}]$	[time period 71, time period 119]

Figure 7.1 shows the paths taken for transport order 538. The 16 laden load carriers are stored in inventory at the origin port Karmøy ('Kar') until time period 77. Then, the laden load carriers take two different paths. Eleven laden load carriers are transported along the transport route Sognefjord Trade through the transshipment ports Haugesund ('Hau') and Tananger ('Tan') before they reach the destination Rotterdam ('Rot') at time period 85. Five laden load carriers remain in the inventory at Karmøy until time period 90. Then the five laden load carriers are transported along the transport route Southern Trade 1 through the transshipment port Tananger before they reach the destination Rotterdam at time period 97. The laden load carriers take two different paths because of capacity limits on board the vessel that serves the transport route Sognefjord Trade (i.e., Misana).

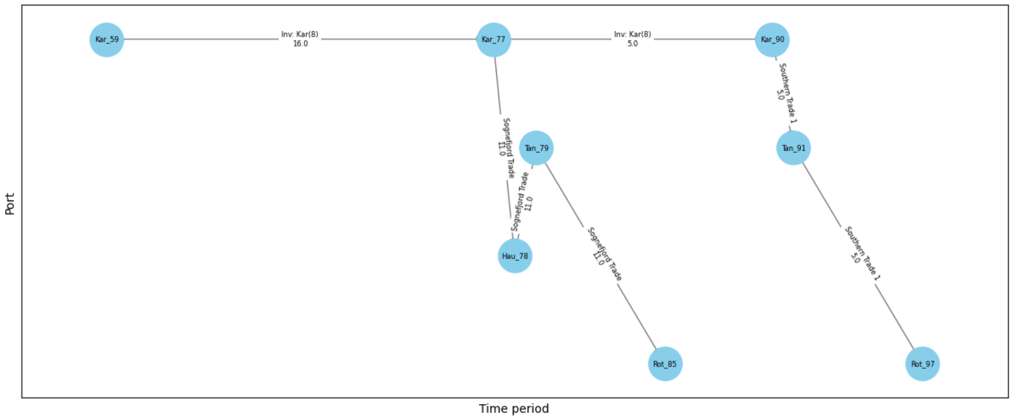


Figure 7.1: The transport paths taken by transport order 538.

Figure 7.2 shows the transport imbalance in Hydro's maritime logistics system. The blue graphs illustrate the laden load carriers that are ready for transport. Thus the blue graph illustrates the number of empty load carriers required at each port. The orange graphs illustrate the number of laden load carriers that starts unloading (because they have reached their destination). Thus, this graph illustrates the number of empty load carriers available at each port since the laden load carriers that start unloading become empty after the port lead time.

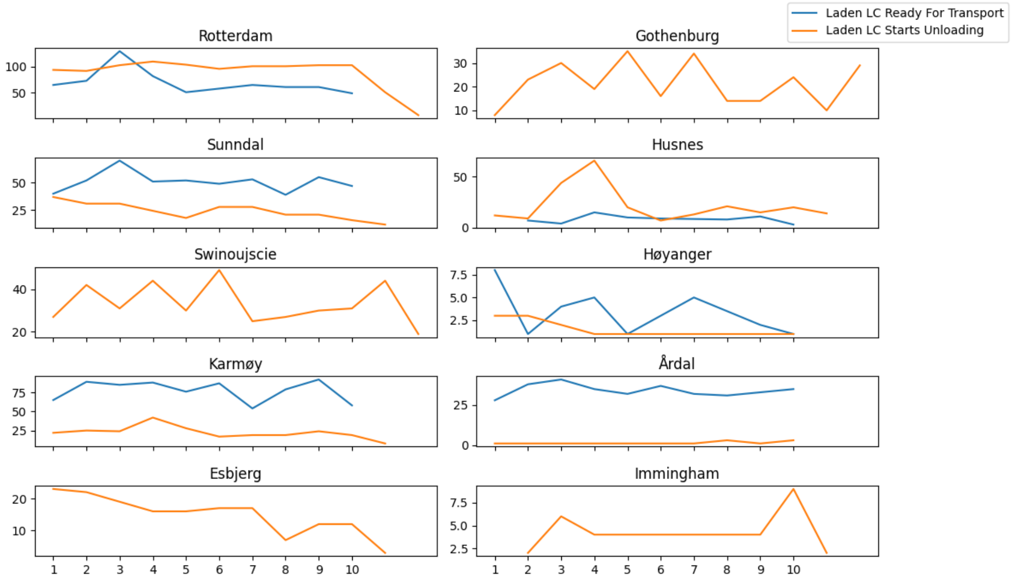


Figure 7.2: The difference between laden load carriers (LC) that starts unloading and the laden load carriers ready for transport at each port by week.

Figure 7.2 shows that the terminal ports Rotterdam, Esbjerg, Immingham, Swinoujscie, and Gothenburg have more available empty load carriers than the number required. This is illustrated with the blue graph being below the orange graph. The imbalance between available and required empty load carriers is most significant at Esbjerg, Immingham, Swinoujscie, and Gothenburg, where no empty load carriers are required. The imbalance is more minor in Rotterdam, which has some requirements for empty load carriers. Further, the Norwegian plants Sunndal, Høyanger, Karmøy, and Årdal have fewer available empty load carriers than the number required. This is illustrated by the blue graph being above the orange graph. The Norwegian plant Husnes have roughly the same number of available and required empty load carriers, although the difference is significant in the earlier weeks.

The transport imbalance requires empty load carriers to be repositioned between the terminal ports Rotterdam, Esbjerg, Immingham, Swinoujscie, and Gothenburg and the Norwegian plants Sunndal, Høyanger, Karmøy, and Årdal. Figure 7.3 illustrates the empty load carrier repositioning between ports. The figure plots the number of empty load carriers on board each vessel for the first 200 time periods. Further, the top of the figure shows a color code matching each port's color code in Figure 6.2. The colored bars in the six transport route plots show when the vessel visits each port in the port sequence of the transport route. Thus, if the number of empty load carriers on board a vessel increases at a port (i.e., there is a spike in the graph), then empty load carriers are loaded on the vessel from the port.

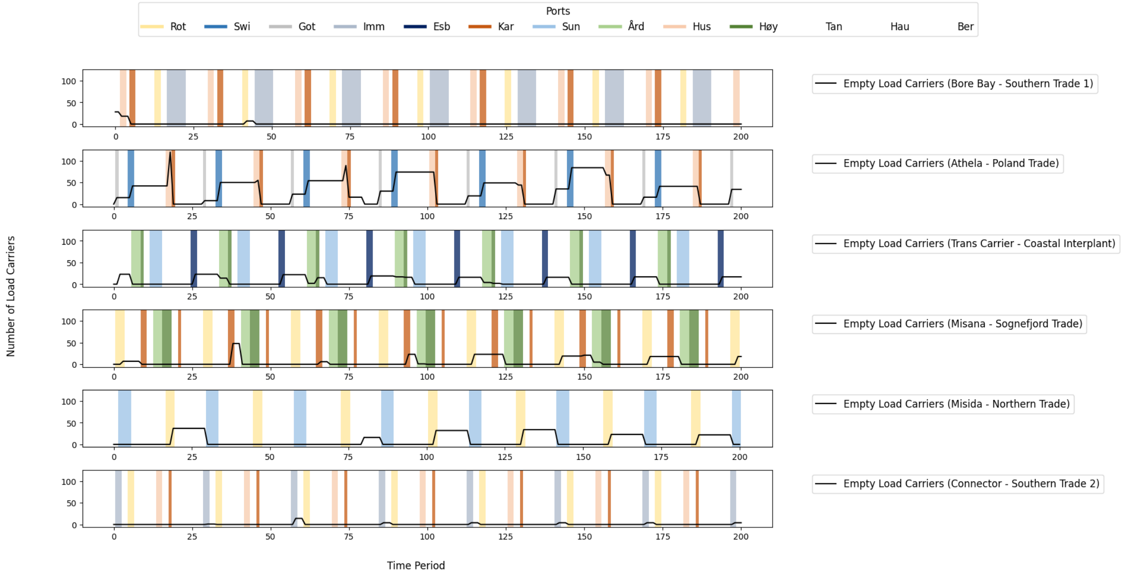


Figure 7.3: The number of empty load carriers on board each vessel at the first 200 time periods.

Figure 7.3 shows that empty load carriers from Rotterdam are primarily repositioned to Sunndal with the Northern Trade transport route. This is illustrated by the increase in empty load carriers at the yellow bar and the decrease in empty load carriers at the blue bar in the Northern Trade plot. There is also some repositioning from Immingham to Rotterdam along Southern Trade 2. These empty load carriers may therefore end up in Sunndal after visiting Rotterdam.

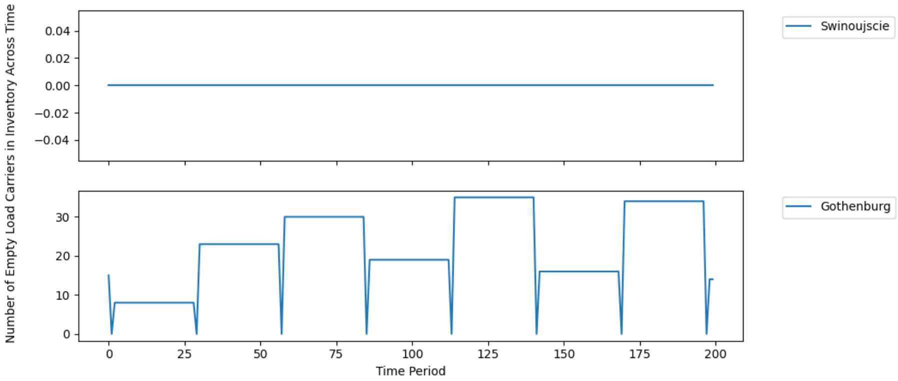


Figure 7.4: The number of empty load carriers in inventory at Gothenburg and Swinoujscie for the first 200 time periods of instance 1.

Likewise, empty load carriers are repositioned from Gothenburg and Swinoujscie to Karmøy with the Poland Trade transport route. Since the lead time of all ports is one time period (see subsection 6.1.1) and the schedule of the Poland Trade transport route states that the vessel Athela should only stop at Gothenburg for one time period, the empty load carriers that are rolled on Athela at Gothenburg must be arriving from the ports inventory. In the case of Swinoujscie, Athela stops for two time periods. This reasoning is backed by Figure 7.4, which shows that Gothenburg stores empty load carriers in inventory while Swinoujscie does not.

Further, empty load carriers to Årdal arrive from Karmøy via the Sognefjord Trade and Esbjerg via the Costal Interplant. The same is true for Høyanger, the stop after Årdal for both Sognefjord Trade and Costal Interplant. Since Karmøy primarily receives empty load carriers from Gothenburg and Swinoujscie, the empty load carriers that reach Årdal and Høyanger are also likely repositioned from these ports.

Table 7.3 summarizes which ports the empty load carriers are primarily repositioned from and the ports they are repositioned to.

Table 7.3: Summary of where the empty load carriers are primarily repositioned from and to.

Plant With Empty Load Carrier Surplus	**Empty Load Carrier Repositioned To**	**Transport Routes Involved**
Rotterdam	Sunndal, Karmøy, Årdal	Northern Trade to Sunndal Sognefjord Trade to Karmøy and Årdal
Immingham	Sunndal (via Rotterdam)	Southern Trade 2 to Rotterdam then Northern Trade to Sunndal
Esbjerg	Karmøy, Årdal or Høyanger	Costal Interplant to Karmøy, then Sognefjord Trade to Årdal or Høyanger
Gothenburg	Karmøy, Årdal or Høyanger	Poland Trade to Karmøy, then Sognefjord Trade to Årdal or Høyanger
Swinoujscie	Karmøy, Årdal or Høyanger	Poland Trade to Karmøy, then Sognefjord Trade to Årdal or Høyanger

Figure 7.5 shows the last 141 time periods and indicates an interesting end-of-horizon effect: Empty load carriers are repositioned from Rotterdam to Husnes ('Hus') with transport route Southern Trade 1 to a greater degree than at the beginning of the planning horizon. In the last 61 time periods, there have been no transport orders with load carriers ready for transport. These time periods exist only to allow the load carriers to reach their destination port within the delivery time window. Thus, if no more transportation orders require empty load carriers, then empty load carriers are transported to Husnes, where the inventory cost is lower than Rotterdam.

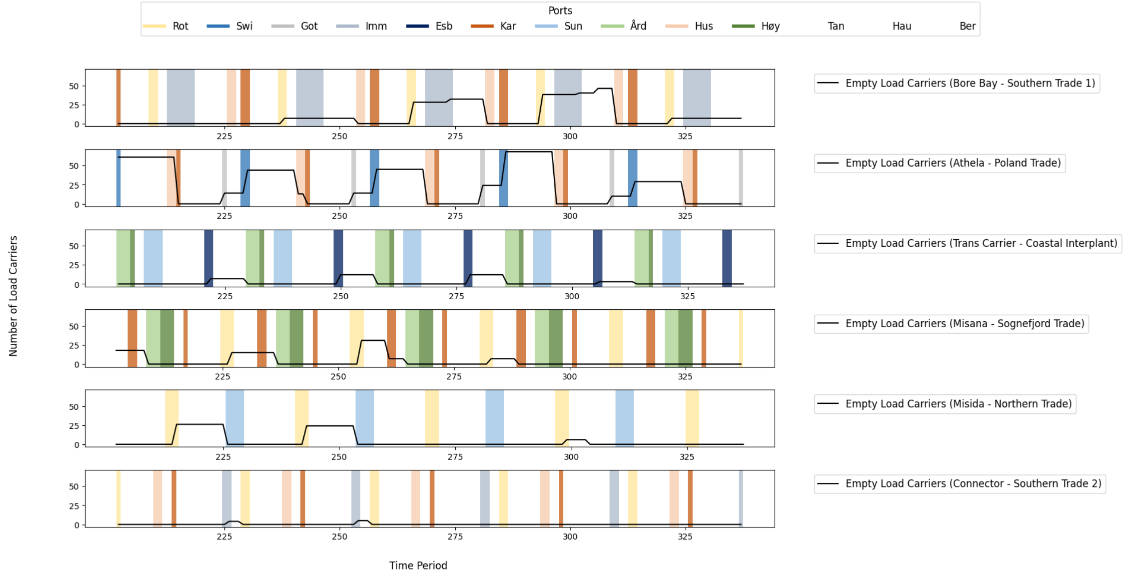


Figure 7.5: The number of empty load carriers on board each vessel at the last 141 time periods of instance 1.

7.1.2 Instance 2

Instance 2 was solved with five runs in a rolling horizon manner. Each of the five runs was solved to optimality in an average of 15 minutes. The model runs averaged 1 284 662 constraints and 5 644 433 variables. Table 7.4 presents the results. Transport and inventory costs account for 85.03% and 14.97% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.4: Costs for instance 2

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 874 006	14 347 200	2 526 806	0	0
100.00 %	85.03 %	14.97 %	0 %	0 %

Instance 2 also created a transportation plan for the ten-week horizon. Thus it included the same transport orders as instance 1 (although each model run has a different number). The solution for transport order 538 is therefore also included here for illustration. Recall Table 7.2 for details about transport order 538.

Figure 7.6 shows the path taken for transport order 538. The 16 laden load carriers are stored in inventory at the origin port Karmøy ('Kar') until time period 77. Then the laden load carriers are transported along the transport route Sognefjord Trade through the transshipment ports Haugesund ('Hau') and Tananger ('Tan') before they reach the destination Rotterdam ('Rot') at time period 85.

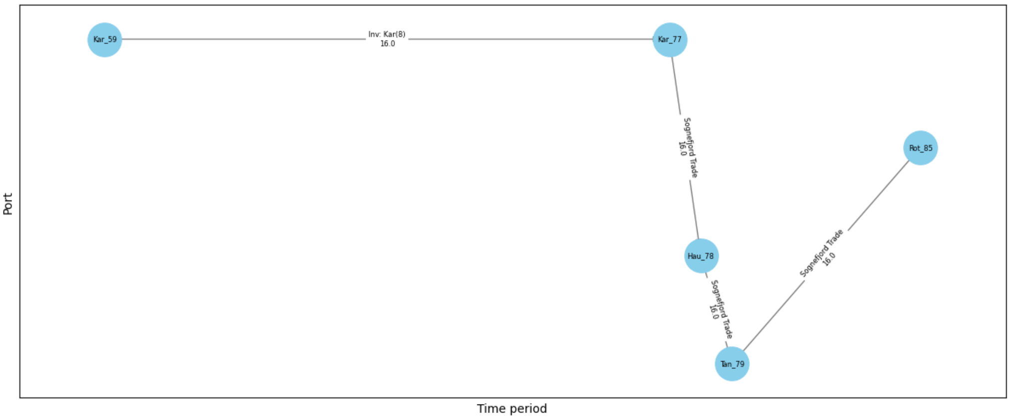


Figure 7.6: The transport path taken by transport order 538 in instance 2.

The transportation imbalance in instance 2 is similar to that of instance 1 (see Figure 7.2). The empty load carrier repositioning is also handled similarly to that presented in Table 7.3. Therefore, these details are not repeated here. For instance 2, the end-of-horizon effect is similar to the effect shown in Figure 7.5. Therefore, it is not repeated here.

7.1.3 Instance 3

Instance 3 was solved with seven runs in a rolling horizon manner. Each of the seven runs was solved to optimality in an average of 8 minutes. The model runs had an average of 702 780 constraints and 3 086 404 variables. Table 7.5 presents the results. Transport and inventory costs account for 85.05% and 14.95% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.5: Costs for instance 3

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 876 811	14 354 400	2 522 411	0	0
100,00 %	85,05 %	14,95 %	0 %	0 %

Instance 3 also created a transportation plan for the ten-week horizon. Thus it included the same transport orders as instance 1 and instance 2 (although each model run has a different number). The solution for transport order 538 is the same as the solution found in instance 1 and is therefore not repeated here (instead, recall 7.1)

The transportation imbalance in instance 3 is similar to that of instance 1 and instance 2 (see Figure 7.2). The empty load carrier repositioning is also handled similarly to that presented in Table 7.3. Therefore, these details are not repeated here. For instance 3, the end-of-horizon effect is similar to the effect shown in Figure 7.5. Therefore, it is not repeated here.

7.1.4 Instance 4

Instance 4 was solved with eight runs in a rolling horizon manner. Each of the eight runs was solved to optimality in an average of 5 minutes. The model runs had an average of 472 386.5 constraints and 2 073 211 variables. Table 7.6 presents the results. Transport and inventory costs account for 85.05% and 14.95% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is zero. This means the model successfully repositions the empty load carriers to the required ports.

Table 7.6: Costs for instance 4

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
16 876 811	14 354 400	2 522 411	0	0
100,00 %	85,05 %	14,95 %	0 %	0 %

Instance 4 also created a transportation plan for the ten-week horizon. Thus, it included the same transport orders as instance 1, instance 2, and instance 3 (although each model run has a different number). The solution for transport order 538 differs in instance 4 and is therefore included here for illustration. Recall Table 7.2 for details about transport order 538.

Figure 7.1 shows the paths taken for transport order 538. The 16 laden load carriers are stored in inventory at the origin port Karmøy ('Kar') until time period 62. Then, the laden load carriers take two different paths. One laden load carrier is transported along the transport route Southern Trade 1 through the transshipment ports Tananger ('Tan') before it reaches the destination Rotterdam ('Rot') at time period 69. Since the lower bound of the delivery time window for transport order 538 is time period 71 (see Table 7.2), the laden load carrier will have to stay in inventory at Rotterdam until this time period.

Further, fifteen laden load carriers remain in the inventory at Karmøy until time period 77. Then the fifteen laden load carriers are transported along the transport route Sognefjord

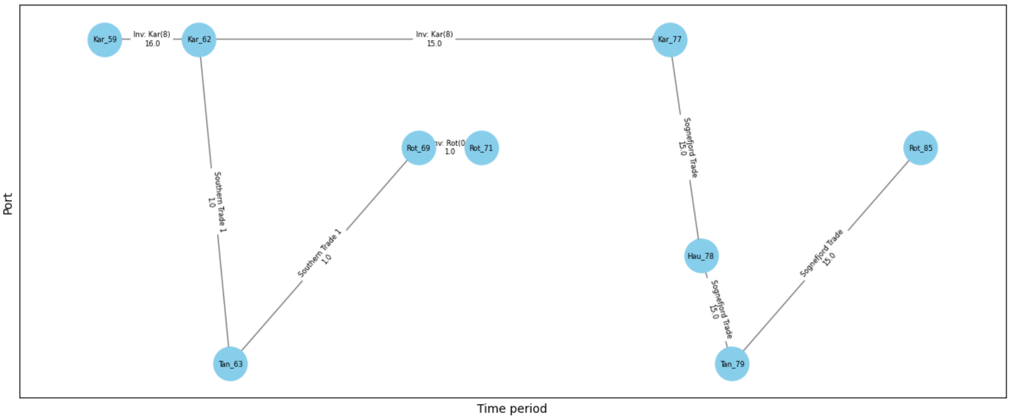


Figure 7.7: The transport paths taken by transport order 538 in instance 4.

Trade through the transshipment port Haugesund ('Hau') and Tananger ('Tan') before they reach the destination Rotterdam at time period 85. The laden load carriers take two different paths because of capacity limits on the routes. The most cost-efficient path is for all the laden load carriers to take the Southern Trade 1 transport route since this results in less inventory cost at Karmøy. However, this path is not feasible due to the capacity constraints on board the vessel Bore Bay which serves the transport route Southern Trade 1.

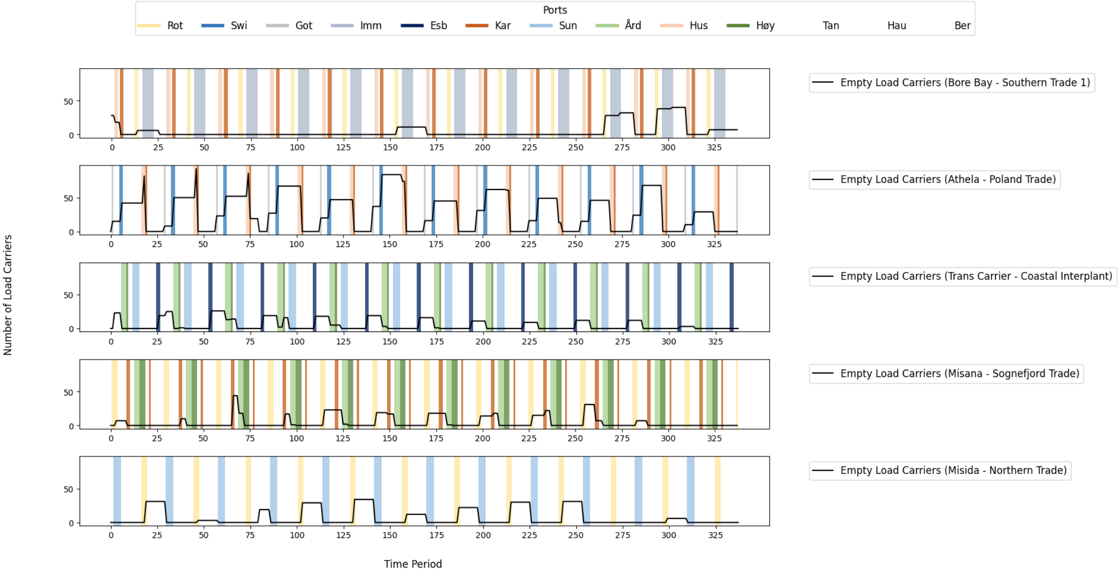


Figure 7.8: The number of empty load carriers on board each vessel for all time periods in instance 4.

The transportation imbalance in instance 4 is similar to that of instance 1, instance 2, and instance 3 (see Figure 7.2). The empty load carrier repositioning is also handled similarly. Therefore, these details are not repeated here. The end-of-horizon effect is, however, slightly different. In Figure 7.8, the empty load carriers are repositioned from

Rotterdam to Husnes ('Hus') with transport route Southern Trade 1 at the end of the planning horizon. However, Figure 7.8 indicates that this repositioning also starts during the planning horizon. Two additional spikes of empty load carriers are being repositioned from Rotterdam to Husnes with Southern Trade 1: the spike around time period 20 and the spike around time period 150.

7.1.5 Instance 5

Table 7.7: Costs for instance 5

Total Costs	Transport Costs	Inventory Costs	Cost of Transporting with Trucks	Cost of Purchasing New Empty Load Carriers
176 849 973	14 352 100	2 497 873	0	130 000 000
100,00 %	8,12 %	1,41 %	0 %	73,51 %

Instance 5 was solved with seven runs in a rolling horizon manner. Each of the seven runs was solved to optimality in an average of 8 minutes. The model runs had an average of 705 432 constraints and 3 098 167 variables. Table 7.7 presents the results. Transport, inventory costs, and costs of purchasing new empty load carriers account for 8.12%, 1.41%, and 73.51% of the total costs, respectively. The cost of purchasing extra capacity with trucks is zero. Thus, the model manages to transport all laden load carriers from their origin port to their destination port without using expensive extra capacity. The cost of purchasing new empty load carriers at each port is high. This cost comes from purchasing 26 new empty load carriers. Thus, the model did not manage to reposition the empty load carriers to meet the required number at each port.

7.2 Analysis

This section analyses the instance results.

7.2.1 Analysis of Effect of Planning Horizon Length

Figure 7.9 shows that the total costs of instance 1 to instance 4 are increasing as the planning horizon length drops from 341 time periods and ten weeks of transport orders known (instance 1) to 173 time periods and three weeks of transport orders known (instance 4). However, Table 7.9 shows that the relative difference is small.

However, the planning horizon length may affect the empty load carrier repositioning. Figure 7.5 shows that empty load carriers are repositioned from Rotterdam toward Husnes with the Southern Trade 1 transport route and from Rotterdam to Karmøy at the end of the planning horizon. The repositioning starts around time period 240, with about 100 time periods left of the planning horizon. This repositioning happens because the empty load carriers are repositioned to a cheaper inventory (Husnes has lower inventory costs

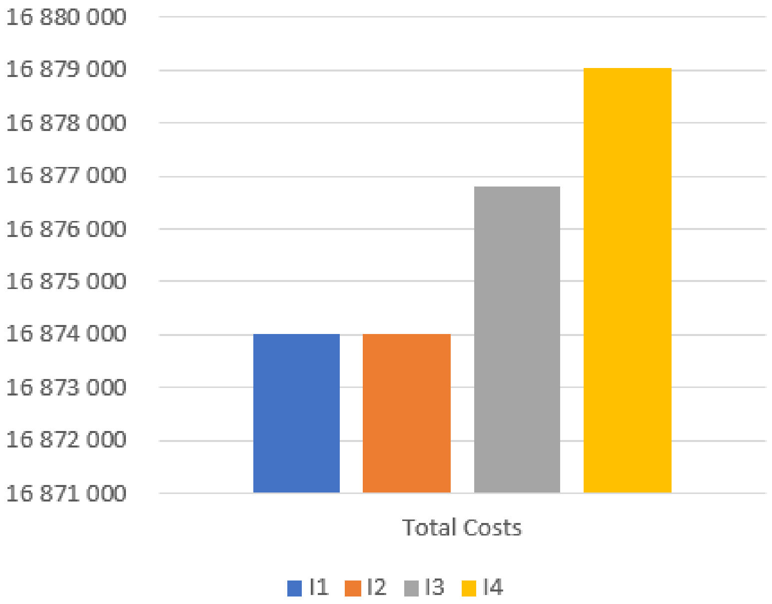


Figure 7.9: The total costs for instance 1 to 4.

than Rotterdam) when they are not needed for transport orders.

Table 7.8: Costs for instance 1 to 4.

Cost	**Instance 1**	**Instance 2**	**Instance 3**	**Instance 4**
Total Costs	16 874 006	16 874 006	16 876 811	16 879 055
Transport Costs	14 360 700	14 347 200	14 354 400	14 351 800
Inventory Costs	2 513 306	2 526 806	2 522 411	2 527 255

When the planning horizon length decrease and the model are solved in a rolling horizon manner, slightly more empty load carriers are repositioned throughout the planning horizon. In instance 4, at the first run, there are no more transport orders starting transportation after week 3. Thus, the empty load carrier repositioning toward Husnes start earlier. Since only the decisions of the first week are applied to each roll, the model has time to correct the early repositioning. However, the two spikes shown in the graph of empty load carriers being repositioned along Southern Trade 1 around time periods 25 and 150 in Figure 7.8 may indicate that a planning horizon of three weeks is too small to correct all the early repositioning.

7.2.2 Benchmark - Comparison with Hydro's Planning Policy

To compare the model to Hydro's existing planning policy, I have developed an algorithm that captures the traits of this policy. This is done because the source data did not contain the actual flow of the load carriers for all transport orders. Hydro's planning process follows a straightforward approach to finding the shortest route from the origin port to the destination port for the laden load carriers. After planning the laden load carriers, the process assigns the empty carriers at the destination ports to return to the origin ports. Suppose there is a shortage of empty carriers at the origin ports. In that case, additional carriers are purchased to meet the demand, which indicates potential shortages of load carriers in the system. The transportation of laden load carriers from the ports is planned so that the transport order with the earliest T_o is planned first. The ports are planned in the following order: Karmøy, Husnes, Årdal, Høyanger, Sunndal, and Rotterdam.

The algorithm is initialized with a time-space network with nodes for each (port, time period) pair and arcs corresponding to the transport routes. The arcs have weights that correspond to the transportation costs along the arc (i.e., C_{ijr}) and capacities (i.e., K_r). The algorithm solves the planning in three phases.

1. The first phase plans the flow of laden load carriers. The origin node (O_o, T_o) is found for each transport order. Then all possible destination nodes $(D_o, [D_o^{TL}, D_o^{TU}])$ are looped through, starting with (D_o, D_o^{TL}), and the weighted shortest path between the origin node and destination node that does not involve trucks is found. Then, the laden load carriers belonging to the transport order (i.e., S_{ilot}) are transported along the path. Suppose the path does not have enough capacity. In that case, the maximum allowed number is transported along the path (or the number of laden load carriers belonging to the transport order if this is smaller than the capacity). If there are laden load carriers left, then the weighted shortest path to the destination node $(D_o, D_o^{TL} + 1)$ is evaluated, and so on. If all possible destination nodes have been evaluated and there are still laden load carriers left, then the weighted shortest path that involves trucks is found.

2. After the laden load carriers have been accounted for, the algorithm tries to return the empty load carriers to the origin port after the laden load carriers for each transport order have been unloaded at the destination port. The path from the destination port back to the origin is found with the same shortest path procedure as the laden load carriers. There are two differences: the first is that the empty load carriers do not have a time window for all possible destination nodes up until the end of the planning horizon is evaluated, while the second is that if the capacity

constraints ensure that not all empty load carriers can return to the origin, then the rest are left in the inventory at the destination port.

3. Then, if capacity constraints ensure that some transportation orders lack empty load carriers at the origin port O_o at time period T_o, then new empty load carriers are purchased.

Benchmark Instance

The benchmark instance is implemented similarly to instance 1. There is a finite horizon with the transport orders for ten weeks. The planning horizon is 341 time periods to include the transport orders with T_o in week ten. The benchmark instance includes the same 558 transport orders included in instance 1. The benchmark instance is initialized from the same point as instance 1 - instance 5.

Benchmark Results

The benchmark instance was solved in 4 hours and 30 minutes. Table 7.9 presents the results. The cost of purchasing new empty load carriers and the cost of transporting load carriers with trucks dominate the total costs. The cost of purchasing new empty load carriers arises from purchasing 501 new empty load carriers. The cost of transporting load carriers with trucks arises from 282 laden load carriers being transported with trucks.

Table 7.9: Costs for the benchmark instance

Total Costs	**Transport Costs**	**Inventory Costs**	**Cost of Transporting with Trucks**	**Cost of Purchasing New Empty Load Carriers**
28 077 296 369	15 397 700	6 898 669	25 550 000 000	2 505 000 000
100,00 %	0.05 %	0.02 %	91 %	8.92 %

Instances 1 - 4 are initialized with 500 load carriers (see section 6.3) and solved without purchasing new empty load carriers. However, the benchmark instance solved with an algorithm based on Hydro's planning policy requires 501 additional load carriers, totaling 1001. This result is consistent with the actual situation for Hydro, which reports having 1200 load carriers. If we assume that roughly 200 load carriers are at any time nonoperative (e.g., due to maintenance), then around 1000 load carriers would be close to the actual number of operative laden load carriers Hydro has.

Comparing the total costs of the benchmark instance with instance 1 - instance 4 is irrelevant because the cost of purchasing new empty load carriers and transporting trucks dominates the total costs in instance 1. However, the transport and inventory costs are more similar. Figure 7.10 shows that the transportation and inventory costs are higher for the benchmark instance than for instance 1. Transportation and inventory costs are

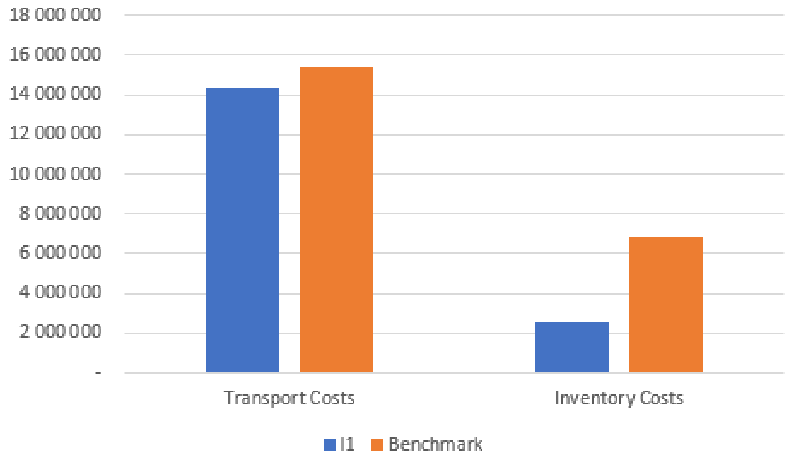


Figure 7.10: The difference between transport costs and inventory costs for instance 1 (I1) and the benchmark instance.

7.22% and 174.5% (respectively) higher for the benchmark instance.

The drastic increase in inventory costs for the benchmark instance is natural because the system has more load carriers. The 501 additional load carriers that were purchased have to be stored. Further, the increase in transportation costs may be because Hydro's planning policy selects less efficient routes for transportation. Figure 7.11 illustrates the differences between the benchmark instance and instance 1 in the number of storage units used on each transport route (i.e., the amount of capacity used). The figure shows that Southern Trade 1 reaches the storage limit in the benchmark instance than with instance 1. This is probably because Hydro's planning policy plans the transport orders for Karmøy first, and Southern Trade 1 is the earliest transport route that departs from Karmøy (see Figure 6.2). Thus, this transport route is filled up. This may again affect the possibilities of transporting transport orders from other ports along Southern Trade 1. While Figure 7.11 shows that other transport routes have more storage available, the delivery time windows of the transport orders may still not be met if these are used.

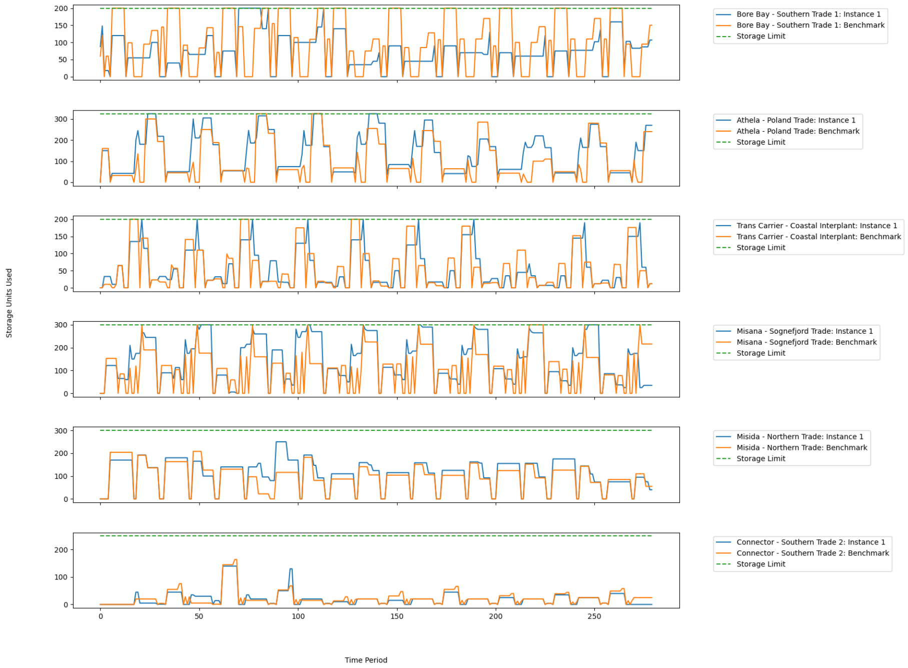


Figure 7.11: The difference between available storage in instance 1 and the benchmark instance.

7.2.3 Analysis of Model Adaptation to Changes in Known Data

The purpose of instance 5 is to analyze how the model adapts to changes in known data. The result of the changes is that 26 new empty load carriers were purchased.

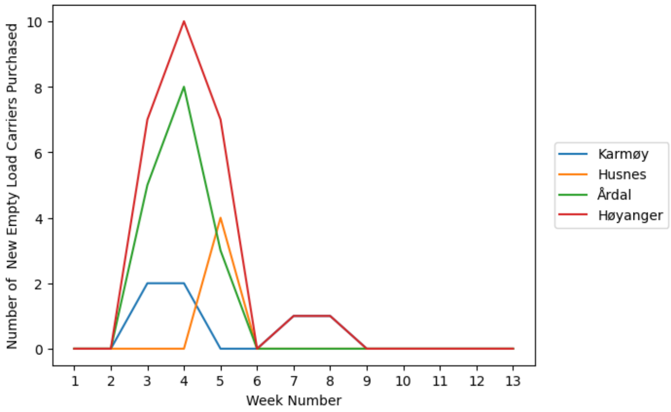


Figure 7.12: The number of new empty load carriers purchased in instance 5.

Figure 7.12 shows when the 26 new empty load carriers are purchased in instance 5. The figure shows that most new empty load carriers are purchased around week 4. This week, 20 new empty load carriers are purchased (ten at Høyanger, eight at Årdal, and two at Karmøy). This coincides with the first change in the known data after the first roll. Recall from Table 6.7 that 25 laden load carriers are added to week 4 in roll 1. The fact that 20 laden load carriers had to be purchased to adapt to this change indicates a lack of flexibility concerning the number of empty load carriers at each port. However, Table 6.7 shows that between 16 and 27 laden load carriers are added at each roll in instance 5. Still, there is only one big round of new empty load carrier purchases at around week 4 (although there are some purchases in weeks 7 and 8). This indicates that the model is able to handle changes in known data with a higher level of empty load carriers.

Chapter 8

Concluding Remarks And Future Research

This thesis develops a network flow model for optimizing the flow of laden and empty load carriers over an operational planning horizon. The system consists of production, transshipment, terminal ports, and fixed maritime transport routes connecting the ports in space and time. The aim is to find a transportation plan that ensures that all transportation orders are transported from the origin port and reach the destination port within their delivery time window. The transportation plan also repositions empty load carriers from excess ports to ports that lack empty load carriers. The transportation plan is created while minimizing transportation costs, inventory costs, costs of purchasing extra transport capacity with trucks, and the cost of purchasing new empty load carriers.

Five instances are presented to test the model. The first four instances aim to analyze the effect of the planning horizon length when the transportation plan is created in a dynamic environment with new information arriving weekly. Instance 1 has ten weeks of known transport order data, while instances 1, 2, and 4 have six, four, and three weeks of known transport orders with additional information arriving each week. Instance 1 is run in a finite horizon, while instances 2 - 4 use a rolling horizon approach. The fifth instance aims to analyze how the model adapts to changes in known data. Both rolling horizon and finite horizon methods are utilized. I also compare the model with an algorithm that mimics Hydro's planning policy.

The results of the computational study show that the suggested model performs better than Hydro's planning policy in terms of the required number of load carriers in the system and total costs. Hydro's planning policy had 7.22% higher transport costs than instance 1 and required 1001 load carriers in the system to meet the demand for empty load carriers while instances 1- 4 manages to satisfy empty load carrier demand with 500 load carriers in the system.

The results from instances 1 - 4 show that the planning horizon length affects the total costs to a minor degree. However, the planning horizon length somewhat affects the repositioning of empty load carriers since more empty load carriers are being repositioned earlier when the planning horizon is low.

Further, the results from instance 5 show that the model requires more load carriers in the system to adapt to changes in known data. Therefore, a subject for further research is to develop a stochastic model that takes multiple scenarios into account when determining the empty load carrier repositioning. An other direction for future research is to incorporate inventory models to analyse potential minimum levels of empty load carriers in the port inventories.

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