Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

Helle Villmones Haug
Sigrid Hallem Solum
Sanna Baug Warholm

Planning Annual Delivery Programs in the Liquefied Natural Gas Industry

Extending current models to handle multiple loading ports and speed optimization

Master's thesis in Industrial Economics and Technology Management
Supervisor: Kjetil Fagerholt
Co-supervisor: Mingyu Li and Inge Norstad
June 2023

Quorum Software

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NTNU
Kunnskap for en bedre verden

Department of Industrial Economics & Technology Management

TIØ4905 - Managerial Economics and Operations Research, Master Thesis

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Authors:
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Sanna Baug Warholm

Supervisor:
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Co-supervisors:
Mingyu Li
Inge Norstad

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Preface

This master's thesis concludes our Master of Science at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management. The work has been conducted in the course of the spring of 2023 and is a continuation of the specialization project in TIØ4500 - *Managerial Economics and Operations Research, Specialization Project* in the fall of 2022 (Haug et al., 2022). Given the similarity of the problem addressed in this master's thesis, certain chapters are revised versions of the corresponding chapters in the project thesis.

We would like to thank our supervisor, Professor Kjetil Fagerholt, and our co-supervisors, Dr. Mingyu Li and Dr. Inge Norstad, for valuable guidance and interesting discussions. We would also like to express our gratitude to our industry partner Quorum Software for providing us with real-world data and insight into the industry.

Helle Villmones Haug, Sigrid Hallem Solum & Sanna Baug Warholm

Trondheim, June 2023

Abstract

Planning long-distance transportation of liquefied natural gas (LNG) is a complex process faced by the LNG producers. This includes setting up a so-called Annual Delivery Program (ADP), a plan for the delivery of LNG from a producer over a period of one year. The ADP typically includes the scheduling of deliveries to different locations/customers, inventory control at the liquefaction plant(s), and the allocation and optimization of LNG vessel routes to minimize costs. By creating an LNG-ADP, LNG producers can more efficiently manage their supply chains and ensure that customers receive their LNG deliveries in a timely and cost-effective manner. In this thesis, we define and solve the Liquefied Natural Gas Annual Delivery Program Planning Problem with Speed Optimization and Multiple Loading Ports (LNG-ADP-SO-MLP).

The LNG-ADP-SO-MLP is a special version of the LNG-ADP, where we extend existing models for the LNG-ADP problem found in the literature by handling multiple loading ports (or liquefaction plants), optimizing vessel speeds, selling LNG in the spot market, varying production rates, and having the option of chartering out the producer's vessels. We propose a novel model for the LNG-ADP-SO-MLP, formulated as a mixed-integer linear program (MILP) based on a time-space network structure with a discrete-time representation. Due to the scale and complexity of the problem, it is challenging, if not impossible, to find optimal solutions using a commercial solver for real-world cases.

Consequently, we develop a rolling horizon heuristic (RHH) to solve the model. The RHH divides the planning horizon into sub-horizons and solves them iteratively using a commercial solver to obtain a complete solution. Our industry partner Quorum Software provided a number of realistic test instances, varying in the number of loading ports, number of vessels, customer demands, and the length of the planning horizons to test the RHH as a solution method. The computational results show that the RHH is able to find feasible and good solutions for planning horizons of 12 months within a reasonable amount of time. We also show that the application of the RHH to solve the LNG-ADP-SO-MLP yields valuable managerial insights that can inform both strategic and tactical decision-making for the producer, e.g., for evaluating the value of investing in larger storages at the liquefaction plant(s) and for having a shared fleet across the liquefaction plants instead of separate ones.

Sammendrag

Å planlegge langdistansetransport av flytende naturgass (LNG) er en kompleks prosess for LNG produsenter å håndtere. Dette innebærer å sette opp et såkalt årlig leveringsprogram (ADP), som er en plan for levering av LNG for en produsent over en periode på ett år. En ADP inkluderer vanligvis planlegging av leveranser til forskjellige kunder og lokasjoner, lagerkontroll for én eller flere produksjonsanlegg, og optimal allokering av sjøruter for LNG-skipene slik at produsentens kostnader minimeres. Ved å sette opp en LNG-ADP kan LNG-produsenten mer effektivt administrere verdikjeden sin og sikre at kundene får LNG-leveransene sine i tide og på en kostnadseffektiv måte. I denne masteroppgaven skal vi definere og løse "LNG-ADP med optimering av seilingshastighet og flere lastehaver"-problemet (LNG-ADP-SO-MLP).

LNG-ADP-SO-MLP er en spesialversjon av LNG-ADP-problemet, hvor vi utvider eksisterende modeller som finnes i litteraturen ved å håndtere flere lastehavner, optimere skipenes seilingshastighet, variere produksjonsrater, og gi produsenten muligheten til å leie ut sine egne skip. Vi foreslår videre en ny matematisk modell for LNG-ADP-SO-MLP-problemet, formulert som en blandet lineær heltallsmodell (MILP) med diskret tid. På grunn av problemets størrelse og kompleksitet er det nærmest umulig å finne optimale løsninger ved bruk av en kommersiell MILP-problemløser for å løse modellen for et helt år.

Som følge av dette implementerer vi et heuristisk rammeverk med rullende horisont (RHH) for å løse modellen. RHHen deler opp planleggingshorisonten i flere mindre subhorisonter, og løser dem iterativt ved hjelp av en kommersiell problemløser for å finne en komplett løsning. Vår industripartner Quorum Software ga oss et sett med realistiske testinstanser, som varierer i antall lastehavner, skip, etterspørsel fra kunder og lengde på planleggingshorisonten for å teste RHHen som løsningsmetode. Resultatene fra beregningsstudien viser at RHHen finner lovlige og gode løsninger for planleggingshorisonter på opp til 12 måneder, innenfor rimelige løsningstider. Vi viser også at RHHen kan brukes til å gi verdifull strategisk innsikt, som kan brukes til å informere både taktiske- og strategiske beslutninger for produsenten, for eksempel for å vurdere verdien av å investere i mer lagerkapasitet på lastehavnen(ene), og av å ha en delt flåte for lastehavnene, i stedet for å ha separate flåter.

Table of Contents

List of Figures

List of Tables

Abbreviations and Terminology

Abbreviations Used Frequently Throughout the Report

ACQ Annual Contracted Quantity. 6

ADP Annual Delivery Program. ii, 2–4, 6, 70

DES Delivered Ex-Ship. 3, 6, 7, 15, 18

FOB Free-On-Board. 3, 7, 15, 18

IRP Inventory Routing Problem. 12

ISP Inventory and Scheduling Problem. 12

LNG Liquefied Natural Gas. 1

LNG-ADP-SO-MLP Liquefied Natural Gas Annual Delivery Program Planning Problem with Speed Optimization and Multiple Loading Ports. ii, 2, 3, 9, 14–17, 23, 25, 35, 39, 42, 57, 60, 65, 67, 75–77

MILP Mixed Integer Linear Programming. 3, 17, 23

MIRP Maritime Inventory Routing Problem. 2

NG Natural Gas. 1

RHH Rolling Horizon Heuristic. 3

RSP Routing and Scheduling Problem. 12

Time partitions subsets of the planning horizon. 18

Definition of Frequently Used Terms

ballast A material used in ships to provide stability and reduce movement. 5, 51

laden A term meaning carrying a load, or being filled with a certain substance. 5, 51

operational time The number of time periods a given operation take, eg. unloading LNG at an unloading port, or the duration of maintenance.. 16, 21, 22, 51

voyage A ship sailing from one port to another port. 5

Chapter 1

Introduction

Although the earth holds vast reserves of natural gas, they are often located far from where they are in high demand. There exist comprehensive pipeline systems for transporting Natural Gas (NG) on land and shorter distances in the sea. In areas without connecting pipelines and long distances from production sites, natural gas must be transported by road or sea. In order to make transportation volume-efficient and safer, the natural gas is condensed to its liquid form before being loaded onto trucks or tank ships specifically designed for transporting Liquefied Natural Gas (LNG). After the LNG has reached its destination, it is boiled to natural gas at a regasification port. The natural gas is then ready to be used as fuel for commercial vehicles, to generate electricity for homes, or for heating. It can also be used for industrial purposes, e.g., to make products like paint and medicine (MET Group, 2020).

Today, liquefaction and regasification ports used for shipping of LNG are well-established in large parts of the world. Figure 1.1 shows the global distribution of LNG liquefaction and regasification ports, illustrating, by the size of a circle, the number of ports located in the respective area. A clear trend emerges as certain regions, notably the Middle East, USA, and Australia, serve as net exporters of LNG, while regions such as China, Japan, and Europe rely on LNG imports. The considerable geographical distances between suppliers and customers create a high demand for shipping of LNG by sea. Notably, a prevailing pattern in the LNG market is the export of liquefied natural gas to areas with higher gas prices, particularly Asia and Europe. Furthermore, there exists a significant difference between the number of regasification ports and liquefaction ports, with regasification ports outnumbering the latter. However, it is worth noting that the number of liquefaction ports is steadily increasing (GIIGNL, 2022).

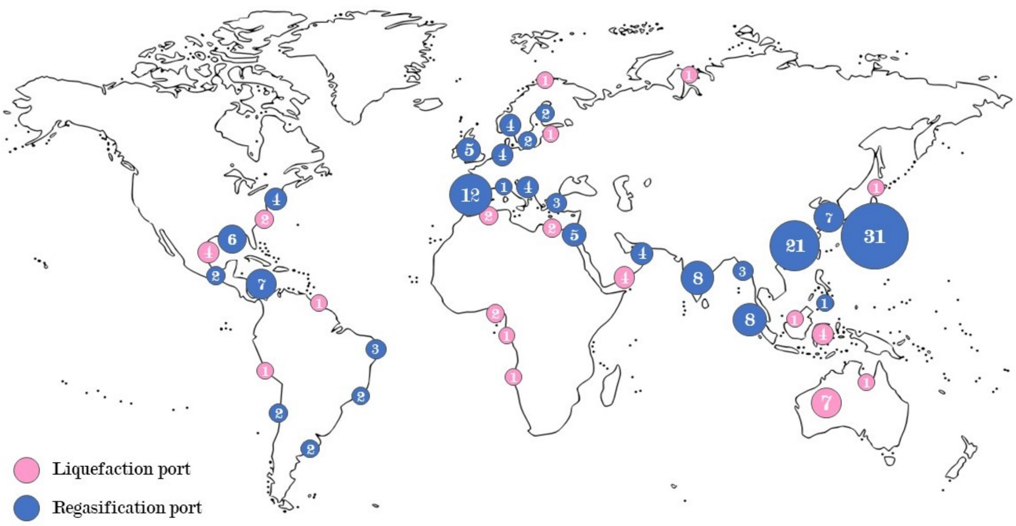


Figure 1.1: *Global distribution of LNG liquefaction and regasification ports (GIIGNL, 2022).*

The escalating global energy demand, driven by rising living standards, faces a critical challenge with the green transition. This transition necessitates a shift towards low-carbon solutions to meet energy needs. In this context, the LNG industry stands to benefit from both trends in the coming years. LNG serves as an energy source with lower emissions compared to conventional options like oil and coal. Figure 1.2 indicates a notable increasing trend in the traded volume of LNG on the global market. Moreover, with the delivery of 68 new LNG ships in 2021, the total number of operational LNG vessels reached 700 globally, representing a 9% increase in cargo capacity from the previous year (GIIGNL, 2022). This growth is primarily driven by the increasing demand for LNG, which again is driven by the overall increase in global energy consumption and a preference for low-emission energy alternatives.

Figure 1.2: *Global LNG trade for the last two decades, retrieved from GIIGNL (n.d.)*

In 2021, 36.6% of the LNG volumes were imported through spot or short-term arrangements, as depicted in Figure 1.2. As demand outpaces LNG production, spot LNG prices have experienced a prominent increase. Additionally, the average spot charter rate for a 160 000 cubic meter LNG vessel rose to approximately USD 89 000 per day in 2021, compared to around USD 59 300 per day in 2020. However, due to political tensions, such as Russia's involvement in a war leading to the cutoff of gas supplies to Europe, along with the sabotage of two pipelines in the Baltic Sea (FreightWaves, 2022), the spot charter rate skyrocketed to USD 400 000 per day in 2022. While these incidents are exceptional and may be one-time occurrences, they underscore the pressing need to optimize LNG transportation for maximizing fleet utilization among producers (GIIGNL, 2022).

In this thesis, we study a special case of the Maritime Inventory Routing Problem (MIRP) for an LNG producer. The producer must fulfill a series of long-term contracts with customers all over the world while also having the option to sell LNG in the spot market. The producer is in charge of the inventory of LNG at the liquefaction plant, the loading ports with a limited number of berths, and the routing and scheduling of a heterogeneous fleet of LNG vessels. LNG producers tend to create an Annual Delivery Program (ADP) every year. The ADP describes the delivery and receipt of LNG cargoes for the next 12 months within the boundaries that the context and the stakeholders inflict. We expand previous formulations of the problem of creating an ADP by including speed optimization of the vessels' sailing speeds. Our model also allows for multiple loading ports, which is becoming relevant for some producers as the industry develops. Hence, we refer to the problem as an Liquefied Natural Gas Annual Delivery Program Planning Problem with Speed Optimization and Multiple Loading Ports (LNG-ADP-SO-MLP). The LNG-ADP-SO-MLP aims to maximize the profits from the sale of LNG, taking revenue from several contract types into account, as well as sailing and chartering costs. Thus, the model can provide decision support for the producer when planning an ADP facing a real-world problem. Furthermore, we extend the LNG-ADP-SO-MLP by including two additional features, involving the producer's option of varying production, and chartering out its own vessels. Based on this, a Mixed Integer Linear Programming (MILP) model for the LNG-ADP-SO-MLP problem is formulated.

This thesis builds upon our specialization project (Haug et al., 2022), which includes primarily two contributions to the LNG-ADP research. Firstly, compared to related research, we have provided a rich formulation of the LNG-ADP Planning Problem. Our model is a complex mathematical model for the LNG-ADP Planning Problem, which contributes explicitly to the LNG-ADP research by including speed optimization and the possibility to have more than one loading port for a producer. The model is designed to handle each leg of a voyage separately, rather than as a round trip. This allows for multiple loading ports, as well as efficient modeling of maintenance since the vessel can sail directly to a maintenance port after unloading without returning to a loading port. Additionally, we have included both relevant spot contract types in the same model which is not done before, to the best of our knowledge. The spot contract types are called Free-On-Board (FOB) and Delivered Ex-Ship (DES), which is further elaborated upon in Background, Chapter 2. Apart from the contribution made by the specialization project Haug et al. (2022), this thesis explores two other aspects of the LNG-ADP research. Firstly, it considers the production rate as a decision variable, which makes the model more realistic as there is no guarantee of selling excess production on the spot market. Secondly, the model allows for producers to charter their own vessels, as there in reality is a possibility for the producers of chartering out vessels from their own fleet.

The MILP model is formulated and solved, based on a priori generation of all possible sailings of a vessel. Finding an optimal solution to an ADP for a producer is a complex problem, especially given the necessary level of detail. Furthermore, the LNG-ADP-SO-MLP model presented challenges for Haug et al. (2022) in terms of computational efficiency for a full planning horizon when solving with a commercial solver. This motivates the use of a heuristic solution approach, and a Rolling Horizon Heuristic (RHH) is therefore implemented and used for solving the MILP model in this thesis. The general idea with RHH is to solve shorter sub-horizons of the MILP model using a commercial solver, by splitting the full horizon and fixing variables for each iteration.

Our main findings reveal that utilizing the RHH to solve the LNG-ADP-SO-MLP effectively generates solutions for instances encompassing a full 12-month planning horizon, considering cases with one and two loading ports. Importantly, the deviation between results obtained from the commercial solver and the RHH is minimal, highlighting the RHH's ability to produce both feasible and high quality solutions with low optimality gaps. Additionally, the results indicate that incorporating speed optimization in the model enhances planning flexibility.

This report is organized as follows: Firstly, Chapter 2 presents a general overview of the LNG industry. Chapter 3 discusses the related literature to the problem, while Chapter 4 describes the problem we are solving. The mathematical model is described in Chapter 5, and Chapter 6 describes our solution method in detail. Chapter 7 presents the test instances and how the model is implemented. In Chapter 8 a computational study is presented, in addition to the managerial insights gained from running the heuristic on various test instances. Lastly, our concluding remarks and suggestions for future research are presented in Chapter 9 and Chapter 10, respectively.

Chapter 2

Background

In this chapter, a general overview of the LNG industry's development and characteristics is presented and related to the need for optimization. This chapter is retrieved from the specialization project Haug et al. (2022). Section 2.1 provides an overview of the LNG value chain, while Section 2.2 introduces the ADP for LNG shipping. Section 2.3 describes the LNG Industry's historical development, as well as some recent trends that might change the development of the industry going forward.

2.1 The Value Chain

The LNG value chain, illustrated in Figure 2.1, includes extraction and production of natural gas, liquefaction, long-distance transportation, and regasification prior to being sent to the end-users (Dobrota et al., 2013). The process is described here to introduce the reader to the industry's dynamics and characteristics.

Figure 2.1: *The LNG value chain*

Production

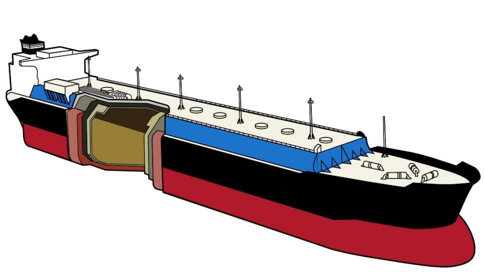
The first stage in the supply chain is the extraction of natural gas from offshore or onshore wells and the transportation of the gas via pipelines to a processing facility. Natural gas is processed in the processing facility to remove impurities and regulate the gas mixture, making the gas acceptable for usage and liquefaction. To prevent corrosion and freezing issues during the liquefaction process, the gas must be clean, dry, and impurity-free prior to liquefaction. To do this, components that would freeze during liquefaction, components that must be removed to fulfill LNG product standards, corrosive and erosive components, inert components, and oil are removed. The clean natural gas is then transported through pipes to the liquefaction site, located in large areas by the sea.

Liquefaction

The core objectives of the liquefaction process are maintaining constant composition and combustion properties of natural gas while cooling the gas to a cryogenic liquid state and loading it as LNG onto specialized means of transport to be delivered to the end consumers. By cryogenic liquid, we mean a liquid at a temperature below its boiling point at near atmospheric pressure. The natural gas is condensed by chilling it to about -162 °C, reducing its volume by 600 times. LNG is then stored as a cryogenic liquid in special storage tanks, designed to maintain safety and minimize the heat ingress into the tanks to prevent evaporation of LNG. Producers normally have multiple storage tanks, with the volume of each tank typically ranging between 80 000 and 160 000 m^3. (Dobrota et al., 2013)

Transportation

By sea, LNG is transported by specialized double-hull tank ships called LNG carriers or vessels. The double-hulled design minimizes the risk of leakage in case of collision or grounding. LNG vessels have temperature-controlled tanks inside the hull that enables transportation of LNG under the temperature of -162 °C. LNG vessels have a hybrid of conventional ship design (Dobrota et al., 2013). The most common cargo-tank designs are membrane cargo tanks, supported by the vessel's hull, and type B spherical (Moss) tanks, which are self-supporting, both illustrated in Figure 2.2 (Cameron LNG, n.d.). LNG vessels vary in cargo capacity, but the majority of modern deep-sea vessels have a capacity between 125 000 and 150 000 m^3. The size of a typical modern LNG vessel is approximately 300 meters long, 43 meters wide, and has a draft of about 12 meters. The largest vessel existing today, *Q-max*, has a capacity of 260 000 to 270 000 m^3. This means that an LNG vessel with a large capacity can empty several storage tanks at the liquefaction plant during loading one cargo. Some small LNG vessels with 1 000-25 000 m^3 in capacity, also operate in some areas, such as Norway and Japan. The standard loading and unloading rate of LNG is 10 000 to 12 000 m^3 per hour, which means that LNG vessels can load or unload 125 000-270 000 m^3 within 12-18 hours (Dobrota et al., 2013).



(a) *Membrane LNG vessel, illustration courtesy of Cameron LNG*

(b) *Moss LNG vessel, illustration courtesy of Cameron LNG*

Figure 2.2: *Illustrations of the two most typical LNG vessel cargo-tank designs*

Boil-Off

At temperatures over its boiling point, LNG evaporates like other liquids and produces boil-off gas. Although the tanks are insulated, some heat ingress will occur and evaporate some of the LNG as it reaches its boiling point. The main reason for boil-off under maritime transportation is the infiltration of heat into cargo tanks as a consequence of temperature differences with the environment outside the tanks. Other processes that increase the quantity of boil-off gas are cooling of a vessel's tank during ballast voyages, and sloshing of cargo in partially full tanks as a result of rough seas. Based on this, the amount of boil-off gas in the tank fluctuates during sailing as ambient temperatures, sea temperature, sea roughness, and the amount of LNG on board change. The amount of boil-off gas generated is typically regarded to be lost cargo in maritime transportation. This is because the boil-off gas must be removed to maintain correct pressure in the tanks. The boil-off rate, expressed as a percentage of the total volume of liquid cargo during a single day, is used to assess the quantity of cargo lost during a voyage. (Dobrota et al., 2013).

Even though the boil-off gas is presented as loss, the gas is not pure waste. LNG vessels, unlike conventional tankers, use a natural gas-powered propulsion system, which releases less greenhouse gasses. Estimates say that boil-off gas equals 80-90% of the energy needed for the LNG vessel at full power output in laden voyage (full tanks), and 40-50% in ballast voyage (almost empty tanks). Therefore, additional fuel is also required. Forcing the vaporization of LNG or using fuel is thus an economical decision as it affects the amount and quality of the LNG delivered to the customer. The boil-off gas can also be cooled down and put back in the tank as LNG. (Dobrota et al., 2013)

Cool-down Procedure

Due to the low temperatures that are required for transporting LNG, an LNG vessel with empty tanks must always go through a purge and cool-down procedure before it starts loading LNG. An empty LNG vessel is called *gas-free*, meaning tanks are full of air. When a tank is gas-free, the vessel can go through maintenance on the tanks and pumps. In this state, LNG cannot be loaded directly into the tank as a rapid temperature change caused by loading LNG at -162°C could damage the tanks. First, the oxygen in the tanks is replaced by CO_2 to remove the risk of explosion that could occur if LNG gets in contact with oxygen. Second, boiled LNG is sprayed into the tanks to replace the CO_2, leaving the tanks gassed up and warm. The last procedure before loading is cool-down, which makes the tanks reach a temperature of -140°C (Bai and Jin, 2016). LNG is then pumped from the storage tanks into the LNG tanks until at least 98% of the capacity is reached. The last 2% allows for thermal expansion/contraction of cargo. The vessels should not sail with partially filled tanks as sloshing and the free surface effect would reduce fatigue resistance and stability. The purge and cool-down procedure must be performed every time an LNG vessel empties its tanks, e.g., after maintenance and after voyages of considerable length. In these cases, the loading time is longer than normal. To avoid empty tanks, a share of the total vessel capacity of LNG is kept in the tank after unloading to keep the tanks cold by spraying the walls in the tank with the remaining LNG. The remaining LNG is also used as fuel during the ballast voyage (Dobrota et al., 2013).

Regasification

At the destination port, the LNG is transferred to a regasification plant, where it is heated and thereby converted back into its natural gaseous state. The gas is then transported to the end customer through a natural gas pipeline system.

Consumption

LNG demand comes from private consumers, using gas for e.g. heating and cooking, as well as from businesses and industrial facilities, using it for e.g. electricity production and manufacturing.

2.2 An Annual Delivery Program

The setup of an Annual Delivery Program (ADP) is an important problem on a tactical level in the liquefied natural gas supply chain. Each year, LNG suppliers and buyers agree on an ADP for the coming 12 months. The ADP plans the timely and efficient delivery and receipt of LNG cargoes within the boundaries that the context and the stakeholders inflict. Usually, the suppliers and buyers agree on an Annual Contracted Quantity (ACQ) and a specified pattern of deliveries throughout the year, with more or less flexibility. The ADPs build on long-term contracts with customers that last around 14 years on average (GIIGNL, 2022).

A producer in the liquefied natural gas supply chain is contractually committed to meeting customer demands through long-term contracts. By controlling the inventory at the liquefaction plant, the loading ports with a limited number of berths, and the routing and scheduling of a heterogeneous fleet of LNG vessels, the producer can set up an annual delivery program. As illustrated in Figure 2.3, an ADP includes the whole sailing schedule for each vessel as well as the operation to which the vessel is assigned. The upper part of Figure 2.3 illustrates the sailing schedule of each vessel. The lower part illustrates how the loading of the vessels affects the inventory level at the loading port. The producer is responsible for creating and presenting the ADP to the customers, who will either accept the plan or come up with changes. The most common adjustments are associated with the delivery time window. This is an iterative process in which the customer usually gets the last say.

The contractual customer pays for the actual quantity of LNG delivered at the customer's regasification plant. This means that the producer is accountable for the delivery and the volume supplied that may be impacted by boil-off. We say that the cargo is Delivered Ex-Ship (DES). The delivery ex-ship trade phrase requires the seller to deliver the cargo to the buyer at a predefined port of arrival. Thus, the seller bears the whole risk and cost of transporting the goods to the port. A customer contract may also specify that the customer picks up LNG from the liquefaction plant using its own vessels, called delivered Free-On-Board (FOB). In this case, the customer is responsible for transportation, risks, and costs that may be incurred. The term "free-on-board" refers to a purchase made at the liquefaction facility, in which the customer organizes shipment to the destination. Whenever the producer has a surplus of LNG, while satisfying the long-term contracts, the producer can sell LNG in the spot market. The LNG spot market is based on short-term contracts which can be delivered FOB or DES.

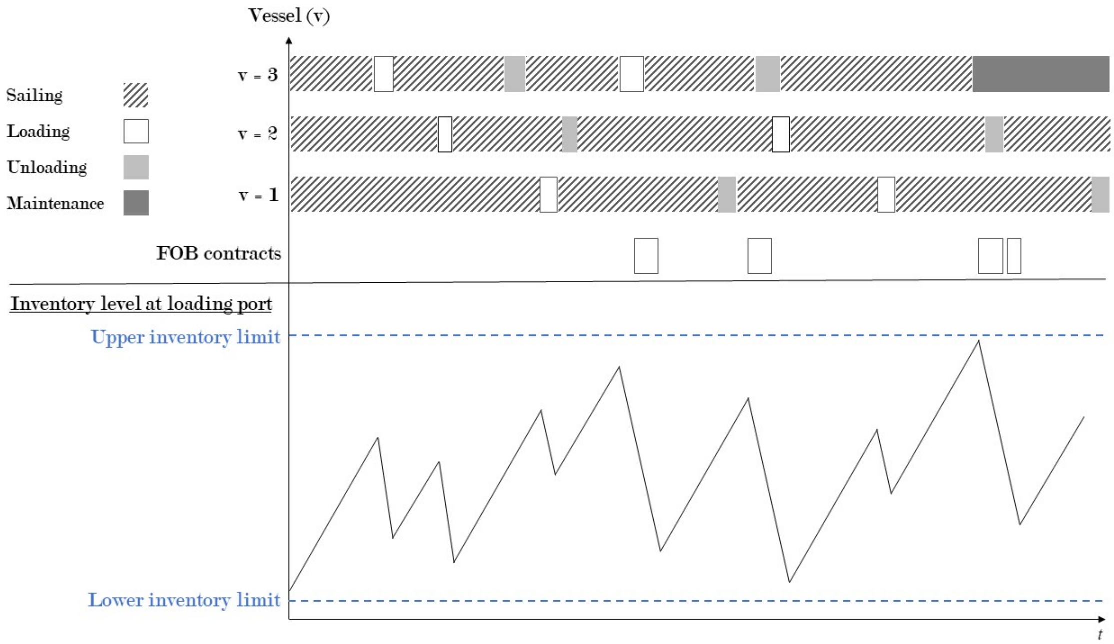


Figure 2.3: *Illustration of an ADP. The ADP contains the entire sailing schedule for each vessel. Here, the inventory level at the loading port is also included to show how the ADP affects the inventory level.*

2.3 Development of the Industry

2.3.1 Historical Development

Figure 2.4 shows that the global number of LNG vessels has grown with a compound annual growth rate close to 8% since 2016, reflecting a general growth in the global trade of natural gas transported on vessels (GIIGNL, 2016, 2017, 2018, 2019, 2020, 2021, 2022). This growth is primarily driven by an increase in LNG demand, which again is driven by an overall increase in global energy consumption, as well as a preference for low-emission energy alternatives. The potential impact of LNG shipping optimization generally grows with the industry, even if the increased market complexity presents a challenge.

An increasing diversification on the supply side has long coincided with the rising natural gas demand, laying the groundwork for healthy growth with relatively small price fluctuations (International Energy Agency, 2019). However, since the third quarter of 2021, the demand started outgrowing the supply, resulting in a hike in the LNG price (Nasdaq Inc., 2023). This was further accelerated in 2022, partly by the 2022 Russian invasion of Ukraine. Appendix A shows the development in the natural gas price index (Nasdaq Inc., 2023).

2.3.2 Projected Development

The LNG industry grows with the continuation of current large-scale trends. The global population is facing increasing energy demands as global living standards increase. The increased energy demand is challenged by the green transition, which requires switching to low-carbon solutions to fulfill energy needs. The LNG industry might benefit from both trends in the coming years as LNG

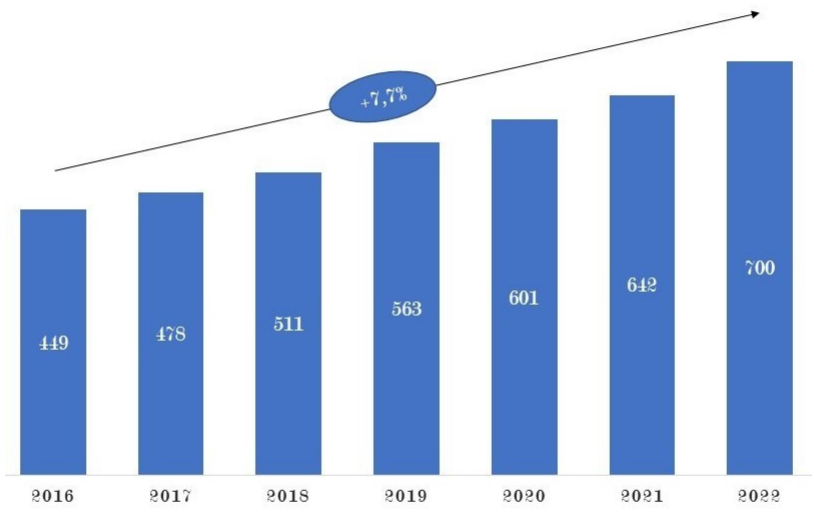


Figure 2.4: *Development in the global number of LNG vessels*

provides a source of energy with lower emissions than many other energy sources that are widely used today, like oil and coal. However, it should be mentioned that there exist other energy sources that produce less harmful emissions than LNG, so in the future, LNG too might be considered relatively polluting.

The LNG demand will remain artificially high in many years to come due to the Russia-Ukrainian conflict. The Russia-Ukrainian conflict is likely to have long-lasting effects on the global energy system (BP Plc., 2023). Therefore, Europe is expected to keep reducing its dependency on Russian gas imports in the years to come, keeping the LNG demand artificially high until at least 2026 (BloombergNEF, 2022).

The supply of natural gas is expected to lag behind demand in the current decade, which might increase the need for LNG shipping optimization. The overall market growth is likely to be constrained by lagging supply since the ability to add new gas supply infrastructure before 2026 is somewhat limited due to time consuming construction. However, some supply ramp-ups are expected, for example in the USA (BloombergNEF, 2022). The main mode of long-distance natural gas transportation, for example from the USA to Europe and Asia, is through LNG shipping (Petroleum Economist, n.d.). In other words, the gas Europe previously got from Russian pipelines might to some degree be replaced by LNG shipments going forward, resulting in a larger demand for LNG vessels. This means that in the time coming, optimization of LNG shipping will also contribute to the handling of the demanding transition away from dependency on Russian gas that Europe is currently facing.

Producers are expected to increase flexible supply. Due to the supply and demand discrepancies, prices are expected to stay above the recent historical average. A consequence of this is that LNG suppliers are expected to ramp up flexible supply (increasing the spot volume) since selling LNG in the spot market is more profitable when the prices have increased recently (BloombergNEF, 2022).

Chapter 3

Literature Review

This chapter provides a comprehensive review of the existing literature pertinent to the LNG-ADP-SO-MLP. Significant parts of this chapter are based on the specialization project Haug et al. (2022).

A systematic search was conducted using the academic search engine *Google Scholar*, primarily focusing on the key phrases "LNG Annual Delivery Program", "LNG Supply Chain Planning Problem", and "LNG Inventory Routing Problem". Both searches yielded a substantial number of papers relevant to the problem. A two-stage selection process was employed, first filtering for relevance, followed by a further selection based on titles and abstracts of the most pertinent articles for each search term. Additionally, *Connected Papers* (Connected Papers, n.d.) was utilized as a visualization tool to generate a graph illustrating relevant papers and their interconnections, as depicted in Appendix B. A total of 32 articles were deemed highly relevant and selected for an in-depth review.

Section 3.1 synthesizes the existing literature regarding the planning of the LNG Annual Delivery Program. In Section 3.2, we investigate papers addressing the relevant literature from the broader field of LNG Supply Chain Planning, concurrently situating the LNG-ADP within this more general domain. Lastly, Section 3.3 outlines our contribution to the literature.

3.1 The LNG Annual Delivery Program Planning Problem

The LNG-ADP problem combines ship routing and scheduling with inventory management and can thus be considered a subclass to the category of the Maritime Inventory Routing Problem (MIRP). MIRPs address the general problem of inventory routing in maritime transportation, which can be applied to various industries and commodities. The LNG-ADP problem distinguishes from the general MIRP by the requirement of a long planning horizon and specific challenges associated with the nature of liquefied natural gas. These challenges include handling cryogenic cargoes and addressing transportation considerations due to boil-off. These unique aspects of LNG make the problem more complex than the general MIRP. Compared to MIRPs, LNG-ADP problems are often featuring a simple network structure including only one production port. Moreover, MIRPs have been extensively studied in the literature and a comprehensive MIRP survey was recently provided by Fagerholt et al. (2023).

The LNG Annual Delivery Program Planning Problem has obtained research interest over the past decade, with Rakke et al. (2011) being a seminal early paper. In this section, we systematically examine the existing literature on the LNG-ADP Planning Problem, focusing on the various features researchers have incorporated into their models. Table 3.1 summarizes the most pertinent papers for our project, each of which is elaborated upon in this literature review. The primary features of the problems studied in each article are highlighted.

Rakke et al. (2011) were among the pioneering papers addressing the problem of devising an ADP. The problem encompasses limited berth and inventory capacities, a heterogeneous fleet of vessels, vessel maintenance, the option to charter vessels, and selling LNG in the spot market. The LNG

Study	Objective function	Time	Speed Opt.	Loading ports	FOB&DES spot cargoes	Time-space network	Charter vessels	Split deliveries	Boil -off	Var. prod.
Rakke et al. (2011)	Min costs	Discrete	No	1	No	No	In	No	Yes	No
Halvorsen-Weare and Fagerholt (2010)	Min costs	Discrete	No	1	No	No	-	No	No	No
Stålhane et al. (2012)	Min costs	Discrete	No	1	No	No	In	No	Yes	No
Halvorsen-Weare et al. (2013)	Min costs	Discrete	No	1	No	No	In	No	No	No
Mutlu et al. (2016)	Min costs	Discrete	No	1	No	No	-	Yes	No	Yes
Al-Haidous et al. (2016)	Min fleet size	Discrete	No	1	No	Yes	In	No	Yes	No
Andersson et al. (2017)	Min costs	Discrete	No	1	No	No	In	No	Yes	No
Li and Schütz (2020)	Min costs	Continuous	No	1	No	No	In	No	Yes	No
Li et al. (2022)	Min costs	Discrete	No	1	No	No	In	No	Yes	No
This work	**Max profit**	**Discrete**	**Yes**	**Multiple**	**Yes**	**Yes**	**In, Out***	**No**	**Yes**	**Yes***

Table 3.1: *Summary of the most relevant papers for the LNG-ADP Planning Problem. The last row summarizes the main features of our model. Charter vessels: "In" means chartering in vessels is permitted, "Out" means chartering out own vessels is permitted. *Included as extensions of the basic version of the model.*

spot market has gained prominence in recent years, as outlined in Chapter 2, necessitating its inclusion in the modeling of LNG-ADP problems. Both Rakke et al. (2011) and Stålhane et al. (2012) tackle the challenge of constructing a cost-effective ADP while considering the opportunity to sell LNG in the spot market, an aspect further investigated in subsequent studies, as indicated in Table 3.1. Additionally, Rakke et al. (2011) introduce penalty costs for under and over-delivery of long-term contracts into the objective function, penalizing the objective value when exceeding the upper and lower bounds of customer demand. The problem formulations of Rakke et al. (2011) and Stålhane et al. (2012) are time-discrete and feature predefined sailing times, allowing for periodic changes in sailing conditions. This formulation does not qualify as speed optimization since sailing speeds are implicitly predefined through the given sailing times and are not modeled as decisions. Moreover, Rakke et al. (2011) incorporate time partitions, enabling variations in the size and placement of the time windows for LNG customer demand specifications. We have also adopted the formulation with time partitions in this work. Rakke et al. (2011) propose a rolling horizon heuristic (RHH) that iterates through sub-problems with shorter planning horizons. Stålhane et al. (2012) employ a construction and improvement heuristic (CIH) that generates a set of solutions using a greedy insertion procedure, refining them with a first-descent neighborhood search and/or branch-and-bound on a mathematical formulation.

In contrast to penalizing under- and over-delivery, Halvorsen-Weare and Fagerholt (2010) impose penalties for violating customers' delivery time windows. As outlined in Table 3.1, this problem diverges from Rakke et al. (2011), Stålhane et al. (2012), and our model by neither including the possibility of chartering vessels nor accepting contracts in the spot market. Halvorsen-Weare and Fagerholt (2010) present an arc-flow model wherein binary flow variables directly depict the flow of vessels. Conversely, Halvorsen-Weare et al. (2013) address the same problem as Halvorsen-Weare and Fagerholt (2010) while reducing the number of variables in the model by formulating a cargo-based assignment model. This formulation portrays the transportation of a specific cargo rather than the flow of individual vessels. The problem structure permits Halvorsen-Weare et al. (2013) to solve the model with a decomposition scheme, wherein routing and scheduling decisions are considered separately.

Mutlu et al. (2016) distinguishes itself by incorporating three different extensions. The first treats the production rate at the production port as a decision variable in the model in a similar way as we do in this study. Their problem definition also includes the possibility of splitting cargoes, meaning that a vessel can unload partial cargoes at more than one unloading port. Most LNG-related papers only permit full shiploads due to the effects of sloshing and the free surface effect. Consequently, Mutlu et al. (2016) deviate from other papers, including ours, by allowing split-deliveries in LNG-ADP planning. The final extension entails flexibility by permitting a vessel to wait outside a port by adding a *waiting port* adjacent to each port. Our model addresses waiting differently. It is worth noting that Mutlu et al. (2016) only allow maintenance to be performed at a specific port and models it as a round trip from the loading port, meaning it cannot be executed as a detour from an unloading port as in our model. Unlike Rakke et al. (2011) and Stålhane et al. (2012), Mutlu et al. (2016) assume only one sailing time between two ports, without periodically different ones, and does not provide for charter vessel usage. Mutlu et al. (2016) model the possibility of selling LNG in the spot market whenever excess production capacity arises but does not consider the revenues spot contracts can generate. Mutlu et al. (2016) contend that estimating the amount and price of spot sales is challenging, hence excluding the parameter, which sets their study apart from most LNG-ADP problems, including our work. The problem is solved using a heuristic that swiftly constructs multiple solutions that can serve as starting solutions for commercial solvers.

One of the major concerns when preparing ADPs is managing excess production after fulfilling demand from long-term contracts. In our basic version of the model, as in most previous studies, the production rate is a given parameter, and excess production is assumed to be sold on the spot market. This assumption, however, may not be valid if there is insufficient spot demand or available shipping (vessel) capacity (Mutlu et al., 2016). Furthermore, a lack of available storage space combined with a lack of available vessel capacity can result in negative LNG prices, as seen in the LNG market fall 2022 (SMM, 2022). As a result, there is no guarantee that excess production can be sold on the spot market. Being able to reduce the production rate can thus be advantageous. To the best of our knowledge, Mutlu et al. (2016) and our work presented in this report are the only contributions to the LNG-ADP research that treat the production rate as a decision variable. However, in the short term LNG-IRP literature there exist several papers that incorporate variable production including Cho et al. (2018), Andersson et al. (2015) and Sheikhtajian et al. (2020).

Al-Haidous et al. (2016) base their model on the same assumptions as Rakke et al. (2011) but specifically examine the case of using a homogeneous fleet of LNG-delivery vessels with the goal of minimizing fleet size. Additionally, their study incorporates bunkering restrictions. Compared to previous LNG-ADP problem studies, Al-Haidous et al. (2016) introduce a MIP model that can be regarded as a restricted alternative to the general model delineated by Rakke et al. (2011) and Stålhane et al. (2012). Al-Haidous et al. (2016) construct a time-space graph that enables a compact mixed-integer programming formulation with a polynomial number of variables and constraints, accounting for all problem features and restrictions. This compact model allows for optimal solutions to large-scale, realistic cases, which has typically been a limitation of similar studies.

Andersson et al. (2017) model the LNG-ADP problem similarly to Rakke et al. (2011) but incorporate four families of valid inequalities to enhance the lower bounds on the problem's optimal value. One family is concerned with the penalty pricing of over- and under-delivery, another with the quantity of LNG delivered, the third with delivery timing, and the last with symmetry-breaking constraints. The problem is solved using a branch-and-cut algorithm.

Li and Schütz (2020) are the first to include transshipment in the LNG-ADP problem, motivated by the Yamal LNG case with ice-breaking ships. In the case of Li and Schütz (2020), a transshipment port helps the producer avoid longer-than-necessary voyages with ice-breaking LNG vessels, as regular LNG vessels can take over the LNG and deliver it to customers, lowering sailing costs by reducing the use of ice-breaking vessels. Li and Schütz (2020) consider one production port, one transshipment port, and multiple customer ports divided into groups depending on how a customer port can be reached from the production port. Their work includes two types of vessels, differing in size, cost, and operation area. One ice-breaking vessel type is dedicated to loading LNG at the loading port and transporting it to the transshipment port and customers near the loading port. The other type operates between the transshipment port and customers that can only be reached through the transshipment port due to long distances. While continuous-time formulations have been modeled for quite some time in maritime inventory and routing problems (MIRP), Li and Schütz (2020) introduce the first continuous-time formulation for an LNG-ADP problem. In a discrete-time formulation, the link between decisions and time is strong, while continuous-time formulations provide weaker links between decisions and time. To connect routing and inventory management with time, big-M restrictions are necessary, which makes the formulations weaker and the linear relaxations worse. The problem is solved with a rolling horizon heuristic.

Li et al. (2022) present a novel discrete-time formulation for the LNG-ADP problem with transshipment and waiting at customer ports. The waiting time at the customer port is limited. Li et al. (2022) model the waiting by adding a decision variable with two time indices. The two time indices denote loading time and unloading time. The result is multiple decision variables for all delivery voyages that start on the same day but deliver on different days. In our model, waiting is modeled differently, without the separate decision variable. The problem of Li et al. (2022) is solved with different rolling horizon configurations.

Uncertainty plays a significant role in the LNG-ADP Planning Problem, as is the case for most maritime transportation issues. However, for the LNG-ADP problem, uncertainties are frequently disregarded in the literature or modeled robustly. Halvorsen-Weare et al. (2013) take into account the uncertainty of sailing times and daily LNG production. To render the cargo-based assignment model more robust concerning uncertainties, Halvorsen-Weare et al. (2013) introduce four robustness strategies.

3.2 Additional Relevant LNG Supply Chain Planning problems

The LNG-ADP typically serves as a decision support model at a tactical level. In this section, we examine relevant papers from the LNG industry at the operational and strategic planning levels. Also, Fagerholt et al. (2023) provide a comprehensive review that covers several segments of the literature on routing problems that are considered tightly related to the LNG routing and scheduling problem.

3.2.1 The Operational Planning Level

Various types of LNG supply chain planning problems are studied at the operational level. Peña-Zarzuelo et al. (2020) classify the LNG operational supply chain planning field into two primary categories: the LNG Routing and Scheduling Problem (RSP) and the LNG Inventory and Scheduling Problem (ISP). LNG-ADP may consider all three factors (routing, inventory, and scheduling) and can thus be seen as a specific instance of an LNG inventory routing and scheduling problem, commonly referred to as an Inventory Routing Problem (IRP) (scheduling is usually included). Even though the LNG-ADP is a special case of the LNG-IRP, there are some notable differences. The most distinguishing factor of the LNG-ADP from the LNG-IRP is the requirement of a long planning horizon. The LNG-ADP is typically modeled in a less complex manner to account for the long planning horizon and potentially a large fleet of vessels while maintaining a reasonable solution time.

The LNG-IRP was to our knowledge first studied by Grønhaug and Christiansen (2009). Their problem includes several features in addition to the primary inventory management, routing, and scheduling of LNG vessels. It incorporates decision variables determining both sale quantities and production volumes of LNG, allowing for partial unloading of LNG and accounting for boil-off. This problem instance has been further developed by several authors, for example, by Fodstad et al. (2010), who differentiate their problem from Grønhaug and Christiansen (2009) by including the sale of spot cargoes and having multiple contract requirements. Andersson et al. (2015) build on Grønhaug and Christiansen (2009) by exploiting the characteristics of LNG transportation, reformulating the routes and schedules into *duties*, each duty having a set of visited nodes and a start time. To solve the problem, duties are generated a priori, and the formulation is strengthened by valid inequalities.

At a tactical and operational level, Ghiami et al. (2019) investigate an LNG deteriorating inventory routing problem (LNG-DIRP) for an inland LNG distribution network. While most LNG papers consider boil-off LNG during transportation, Ghiami et al. (2019) consider boil-off at all facilities in the distribution network. Thus, inventory levels are also affected by the boil-off, as they regard the boil-off as a loss. Ghiami et al. (2019) introduce a matheuristic that combines the mixed integer programming formulation with an adaptive large neighborhood search.

Msakni and Haouari (2018) were the first to address speed optimization in the context of LNG delivery planning. The paper investigates speed control decisions in the design of short-term delivery plans of LNG for a typical horizon of three months. This study considers both mandatory long-term contracts and spot contracts for a heterogeneous fleet of vessels, with the possibility of varying speeds for deliveries of both contract types. This paper extends the time-space network formulation of Al-Haidous et al. (2016) to integrate a heterogeneous fleet of vessels and the ability to control sailing speeds while retaining a polynomial-sized MIP model. The problem of Msakni and Haouari (2018) is represented as a graph where two nodes are linked with multiple arcs, each corresponding to a vessel sailing at a specific speed. The problem is solved using an optimization-based variable neighborhood search procedure.

Optimizing vessel speed significantly impacts operational efficiency in planning maritime transportation (Msakni and Haouari, 2018). It is easy to justify that for short-term planning, varying speeds can improve the flexibility of the planning problem and enable better utilization of spot contract opportunities. For long-term planning, the goal is to present a feasible, low-detailed plan with the most relevant problem requirements, which may explain why, to the best of our knowledge, no paper has yet been published that studies speed optimization in the context of long-term LNG-ADP Planning Problems.

In examples such as Grønhaug and Christiansen (2009) and Fodstad et al. (2010), the LNG-IRP allows for multiple loading and unloading ports. Typically, LNG-ADPs have been modeled with one loading port, but as the industry evolves, modeling the LNG-ADP with multiple loading ports is realistic (confirmed by industry partner). In this project, we explore the effects of the multiple-loading port formulation on the LNG-ADP.

Nikhalat-Jahromi et al. (2016) contribute to the research on LNG-IRP at an operational level by proposing a novel MIP model from a corporate finance perspective. The model aims to suggest a short-term trade policy for Middle Eastern LNG producers regarding the option to sell LNG in the spot market of either Japan or the UK. Specifically, they propose that producers dispatch their product to whichever market has the higher current spot price, regardless of the variability of transport expenses. Nikhalat-Jahromi et al. (2016) introduce an optimization model that helps decide when and where to deliver LNG by coordinating various factors such as tanker type, assignment and routing, inventory management, contract obligations, arbitrage, and uncommitted LNG. In addition, their work also includes the possibility of chartering out the producer's vessels, which we also have included as an extension to our model.

Maritime planning problems are inherently uncertain. According to Christiansen et al. (2007), the most significant uncertainties in the maritime sector on the tactical planning level are most likely related to weather and port conditions. Some LNG-ADP and LNG-IRP papers address uncertainty using robust optimization, and a few LNG supply chain papers explicitly handle uncertainties stochastically. Examples include Cho et al. (2018), who consider the uncertain event of bad weather that may disrupt LNG production, storage, and shipping. They use a two-stage MIP model to maximize the expected revenue and minimize the disruption cost. Khalilpour and Karimi (2012) handle the LNG buyer's contract selection problem under demand and price uncertainty. They use a two-stage MIP model to minimize total procurement costs. Sheikhtajian et al. (2020) compare shipping cost of split and non-split delivery in deterministic and uncertain settings by building on the LNG-ADP model by Mutlu et al. (2016). Vessel speed is considered a fuzzy parameter, meaning that the parameter refers to a connected set of possible, weighted speed values instead of just one value. They aim to minimize total costs and solve the problem with the algorithm proposed by Mutlu et al. (2016) combined with a genetic algorithm (GA). C. Zhang et al. (2018) study a Maritime Inventory Routing Problem with Time Windows (MIRPTW) for deliveries with uncertain disruptions that increase travel times between ports. C. Zhang et al. (2018) want to identify flexible solutions that can accommodate unforeseen disturbances and solve the problem by proposing a Lagrangian heuristic algorithm and soft constraints incorporated in the objective functions with Lagrange multipliers.

3.2.2 The Strategic Planning Level

Several strategic decisions on smaller-scale LNG supply chains have been considered with MIP models. Jokinen et al. (2015) consider a supply chain consisting of a distribution terminal, smaller satellite terminals, and a customer network, with the distribution involving vessels and trucks. A slightly different problem was considered by Koza et al. (2017), where LNG vessels supply LNG-fueled container vessels, and strategic decisions regarding investments are to be made, as both investment cost and operational costs are to be considered.

Goel et al. (2012) present an LNG-IRP for developing LNG vessel schedules for supply chain design analysis. Goel et al. (2015) address the scalability issues of the Goel et al. (2012) MIP model by introducing a constraint programming (CP) approach based on disjunctive scheduling. Since the

LNG-IRP contains precedence relations between tasks and time windows, CP can be efficient for larger instances, as it depends on the number of tasks and resources rather than the number of time points. In this case, the approach finds better solutions faster. Shao et al. (2015) also build on the LNG-IRP by Goel et al. (2012), but suggest a hybrid heuristic strategy combining mathematical programming with Greedy Randomized Adaptive Search Procedure (GRASP). The studies by Papageorgiou et al. (2018) and Munguia et al. (2019) investigate solution techniques for long-horizon strategic LNG-IRP problems with relatively few constraints compared to most MIRP models. The characteristics of the instances tested by these authors are similar to those examined by Goel et al. (2015) and Shao et al. (2015), but with some notable distinctions.

Eriksen et al. (2022) consider the design of a mid-scale LNG supply chain that includes an overseas sourcing location, coastal storage facilities, and land transportation to industrial customers. The model helps strategic decision-making regarding the import of LNG, investments in floating storage units, and customer distribution systems. The demand uncertainty is captured by a multi-stage stochastic programming model, and the solution turns out to have a significant Value of Stochastic Solution. Scenario trees are approximated with Monte Carlo sampling techniques, and the problem is solved with a commercial MIP-solver.

Other papers handling uncertainty in the LNG supply chain at a strategic level are H. Zhang et al. (2017) and Cardin et al. (2015). H. Zhang et al. (2017) study a supply chain along the Yangtze River in China and create a multi-scenario MIP formulation that they perform sensitivity analysis on to see the effect of different LNG prices. They exemplify the use of their analysis by considering the long-term deployment of liquefied natural gas (LNG) technology to supply the transportation market. Cardin et al. (2015) consider the value of the ability to adapt to changes in the market due to realizations of uncertainties. The paper considers large-scale capital-intensive projects.

3.3 Our Contribution

In this thesis, we present a comprehensive model that stands apart from existing literature due to its incorporation of several new features. The last line in Table 3.1 sums up the main features included in the problem definition and modeling of the LNG-ADP-SO-MLP.

Our primary contributions to LNG-ADP research can be summarized as follows. Firstly, compared to related research, we have provided a rich formulation of the LNG-ADP Planning Problem, which contributes explicitly to the LNG-ADP research by including speed optimization and the possibility to have more than one loading port for a producer. By formulating a time-space network, we can handle each voyage leg separately, allowing for efficient management of sailing between various loading ports and maintenance. This enables the inclusion of multiple loading ports and facilitates direct sailing to maintenance ports after unloading, without the need to return to a loading port first. Additionally, we have included both relevant spot contract types (FOB and DES) in the same model which is, to the best of our knowledge, not done before.

Furthermore, we introduce two extensions that enhance the model's realism. The first extension involves treating the production rate at the production port as a decision variable, inspired by the work of Mutlu et al. (2016). To the best of our knowledge, Mutlu et al. (2016) is the only study in the LNG-ADP field that incorporates this feature. The second extension explores the option of chartering out the producer's vessels, a concept not explored in previous studies. This extension offers several potential benefits, such as generating additional revenue, optimizing vessel utilization, accessing new markets, and mitigating market risk by diversifying revenue streams to include shipping services.

To summarize, the LNG-ADP-SO-MLP is, to the best of our knowledge, the first model in the LNG-ADP planning scope including all the following five features:

1. Sailing speed optimization
2. The handling of multiple loading ports
3. Two types of spot contracts (FOB and DES)
4. The option of chartering out the producer's own vessels
5. The option of varying production rates

Chapter 4

Problem Description

The Liquefied Natural Gas Annual Delivery Program Planning Problem with Speed Optimization and Multiple Loading Ports (LNG-ADP-SO-MLP) refers to the challenge faced by LNG producers in planning the supply of LNG to customers over a 12-month planning period, using a given fleet of LNG vessels. While this chapter builds upon the specialization project Haug et al. (2022), it also explores additional aspects of the LNG-ADP-SO-MLP that this thesis aims to study in greater detail. These include the opportunity to adjust production rates and charter out the producer's vessels.

The LNG producer has one or multiple loading ports. Each loading port has a set of berths from which LNG vessels can load. The number of available berths may change during the planning horizon due to maintenance. Each loading port also has storage tanks with given maximum and minimum capacities for holding LNG.

LNG producers offer customers two types of contracts - Delivered Ex-Ship (DES) and Free-On-Board (FOB). Under DES, the producer delivers the cargo to a designated unloading port with a vessel from its own fleet, while under FOB, the customer picks up the cargo at a designated loading port using their own vessel. Each contract has specific partitions defined by the customer for the time of delivery or pick-up. A customer can for example have annual, quarterly, and/or monthly partitions with a given delivery requirement, which can be fulfilled by one or several deliveries or pick-ups of cargoes. For DES contracts, a minimum demand over the 12-month period must be met, and a pre-defined revenue is associated with deliveries above this minimum up to a specified maximum demand. Also, each DES contract can only receive cargoes from a pre-defined loading port. For FOB contracts, each delivery must be of the size of the picking-up vessel. Optional one-time deliveries of *spot cargoes* with associated customer destinations and revenues can also be made, which must be delivered as either DES or FOB cargo. Each customer specifies how often a delivery can be made at its port. e.g., a minimum number of days between each delivery. It is assumed that the unloading ports always have available inventory and berth capacities.

The LNG producer's heterogeneous fleet of vessels remains constant throughout the planning horizon, with each vessel having a specific fuel cost that is determined by factors including fuel consumption, distance traveled, bunker cost per ton, and speed. The speed of each vessel must fall within specified upper and lower limits. All other costs, such as crew costs and time-charter costs of the vessels, are assumed sunk. If the producer needs additional vessels, they can be chartered at a predefined daily rate. Each vessel has a unique loading capacity and is always loaded up to capacity due to physical limitations. Additionally, each vessel must unload its entire cargo at the unloading port, except for a minimum tank volume share that is needed to maintain cool temperatures in the tanks. Therefore, the actual delivery volume is determined by the vessel's capacity, as well as the boil-off effect and sailing distance. Each vessel has an individual starting position in a port at the beginning of the planning horizon and becomes available for a new voyage only when it returns to a loading port. As a result, vessels may become available at different locations and times in the beginning of the planning period.

In addition, some vessels may require maintenance during the planning horizon. When a vessel is scheduled for maintenance, a specific time window and maintenance port are designated for the maintenance to begin and take place. Due to the varied needs of each vessel in the heterogeneous fleet, a given maintenance duration is assigned for each vessel scheduled for maintenance. After maintenance is completed, the vessel must undergo a purge and cool-down process before embarking on the next voyage. This process is assumed to take a specific amount of time and must be carried out at a loading port. All vessels in the fleet are subject to the same operational time for loading, unloading, and possibly purging and cool-down at each port.

The LNG-ADP-SO-MLP is responsible for determining the optimal allocation of vessels to ports i.e., where to send each vessel at what times and at what speeds. These decisions are made while taking into account several constraints. Some vessels are unable to visit certain ports or serve specific customers due to port incompatibility. Additionally, some vessels are restricted to serving a subset of contracts and unloading ports. Furthermore, since each loading port has a limited number of berths, the number of vessels that can load on any given day cannot exceed the number of available berths. Due to the upper and lower limits of LNG storage tanks at the loading ports, there may be waiting time before a vessel can pick up a cargo.

The LNG-ADP-SO-MLP also determines the number of spot cargoes to sell FOB and DES, with specific demand and revenue. Spot cargoes can only be sold if LNG production exceeds the total contractual minimum demand. The producer is aware beforehand of the potential spot contracts available, including details such as associated customers, time periods, revenues, and amounts. The FOB spot cargoes impact berth capacity and inventory constraints, while DES cargoes also affect the number of available vessels.

Two additional features are added to the LNG-ADP-SO-MLP, referred to as *extensions*. The first extension, *Extension 1*, includes that each loading port has a specific daily production rate that can be adjusted on a daily basis. The second extension, *Extension 2*, allows the producer to charter out their own vessels at a predetermined daily rate when there is surplus capacity. Within the planning horizon, the producer can only charter out each vessel for a single uninterrupted period, and this period must exceed a given number of days.

The objective of the model is to maximize the gross margin, which is the total revenue minus the transportation costs in the planning period, hereby referred to as "profit". Revenue comes from spot cargoes transported FOB and DES, long-term contracted LNG deliveries, chartering out vessels, and the remaining LNG's value at the end of the planning period. The costs are the transportation costs from the producer's fleet during the planning period, as well as chartering costs. The resulting plan is an Annual Delivery Program that outlines the fleet's schedules, routes, and deliveries to customers for the coming year.

Chapter 5

Mathematical Model

The LNG-ADP-SO-MLP is mathematically formulated as a time-discrete Mixed Integer Linear Programming (MILP) model based on a time-space network structure. Before running the MILP model, an arc-generation procedure generates feasible arcs with respect to some of the problem's constraints and our modeling assumptions. These arcs and their associated data points are input data to the MILP model. Furthermore, the MILP model finds the most profitable combination of arcs while satisfying the constraints that the arc generation procedure does not account for. In Section 5.1, we present the modeling assumptions. Section 5.2 outlines the network structure that forms the foundation of the mathematical model. Section 5.3 explains how arcs, nodes, and corresponding discrete data are generated. The notation used in the mathematical model is described in Section 5.4, and a basic version of the mathematical model is presented and explained in Section 5.5.

The content in sections 5.1 through 5.5 is based on the specialization project Haug et al. (2022). The basic version of the model presented in Section 5.5 is based on the same model as in Haug et al. (2022), with the addition of new constraints that spread the deliveries to each customer to make the model more realistic. Section 5.6 and Section 5.7 present new extensions to the basic version of the model. These extensions involve the producer's options of varying the production and chartering out its own vessels.

5.1 Modeling Assumptions

This section introduces the fundamental assumptions for modeling the LNG-ADP-SO-MLP, which is based on an arc structure where each arc represents a one-way sailing between two nodes. On the contrary, a node represents a port at a specific moment in time.

5.1.1 Discrete Time

In the model, time is discretized with uniform time steps, meaning that the planning horizon of the ADP is partitioned into a sequence of successive time intervals. A time-discrete model facilitates the easy handling of time-dependent decisions and parameters, such as varying production rates and sailing speeds. The precision and effectiveness of the model are influenced by the duration of the time periods. Long time intervals would lead to less precise modeling of vessel activities, reducing the overall model accuracy. Conversely, very short time intervals would result in a large number of constraints and variables, making the model considerably more challenging to solve. For the LNG-ADP, a time period of one day is deemed sufficiently accurate.

Continuous-time models often assume constant parameters due to the added complexity involved in accounting for variations, which may require the use of additional variables and Big-M constraints. Consequently, this approach may result in weaker linear relaxations and a larger problem size. Nevertheless, continuous-time models are generally smaller compared to their discrete-time counterparts. However, the advantage of a smaller formulation is somewhat offset by the growing intricacy of time-dependent parameters (Li et al., 2022).

5.1.2 Waiting

It is possible to model short-term waiting because of the time-space structure described in Section 5.2. The vessels are permitted to wait outside all ports for a predetermined number of days before proceeding with port operations. This significantly expands the model's opportunity space but could increase the solution time.

5.1.3 Length of Planning Horizon

Demanding that all vessels stop sailing at the end of the planning horizon is assumed to be bad fleet utilization. To avoid this, vessels are allowed to sail beyond the planning horizon, which is extended over multiple time periods. The planning horizon is based on the producer's perspective, requiring all customer demand to be picked up within this horizon, called *loading days*. LNG can still be delivered during the extended *unloading days*. If planning for 365 days, vessels can pick up LNG before day 365 and deliver during the *unloading days*. Loading and unloading days comprise the total number of days the model plan for, denoted *all days*.

5.1.4 Handling FOB and DES Contract Types

As explained in Chapter 4, LNG can be sold through different types of contracts. These contracts can be short-term (spot) or long-term, delivered by the producer (DES) or picked up by the customer (FOB). Since no papers within the scope of LNG-ADP appear to have included all contract types yet, as presented in Chapter 3, an overview of the different contracts is presented in Table 5.1

	Short-term	**Long-term**	**Vessel that can serve**
DES	DES spot contract	long-term DES contract	Producer's and chartered vessels
FOB	FOB spot contract	long-term FOB contract	Customer's vessels

Table 5.1: *Overview of the contract types and which vessels can be used to serve the contracts*

A long-term DES contract specifies an Annual Contracted Quantity (ACQ), which is the amount of LNG that the producer must deliver to the customer's port. The contract specifies the port's demand within upper and lower limits, allowing for flexibility. The ACQ is split into smaller demand quantities with delivery time requirements specified as subsets of the planning horizon (Time partitions). However, the time partitions can vary, and anything from quarterly or semi-annual delivery windows to completely custom delivery windows is handled by the model. One customer can also have several types of time partitions within one contract, for example by specifying an ACQ between 7 and 10 cargoes, and also specifying that at least 3 cargoes must be delivered in Q1 (the first quarter of a year). The producer also has the opportunity to accept or decline DES contracts in the spot market, which are similar to long-term contracts but only require one delivery. Additional vessels can be chartered for a predefined daily rate if needed.

Transport of FOB contracts is the customer's responsibility and are not linked to a customer's port, so they only impact inventory and berth availability at a loading port. It is important to highlight that, in contrast to DES contracts, each FOB contract comprises a single cargo. Therefore, if a customer intends to collect multiple FOB cargoes during the planning horizon, each cargo is handled as an entirely independent contract. A customer can have both DES and FOB contracts, but they are treated as separate contracts.

5.1.5 Chartering Vessels

In cases where the producer has insufficient vessels to fulfill the demand of DES contracts, additional vessels may be chartered at a fixed daily charter rate. It is assumed that there is always a charter vessel available at each loading port, capable of carrying the cargo size within an upper and lower capacity limit. Charter vessels have a predetermined speed, and fuel costs are already included in the daily charter rate. In the model, we only consider the voyage from the loading port to the customer port for a charter vessel, as we are not concerned with its activities beyond that.

Given that the vessels must travel to the producer's loading port, the cost of chartering a vessel for a single voyage is estimated by doubling the one-way cost.

5.1.6 Artificial Spot FOB Contracts

Given the LNG producer's ability to either halt production or accept additional spot contracts throughout the year, an assumption about defining an *artificial spot FOB contract* is made. The artificial spot FOB contract is characterized by zero revenue, a default demand equivalent to the average cargo size, and is applicable across all time periods, thus enabling the potential sale of surplus LNG throughout the planning period. Every cargo loaded through the artificial spot FOB contract in the final ADP is viewed as a potential opportunity for the LNG producer to identify a corresponding real-life spot contract for the specific cargo's pick-up time interval. The aim of this strategy is to prevent the upper inventory level from being treated as a hard constraint. By incorporating the option of selling surplus inventory beyond the predefined demands, the model may require the LNG producer to embrace an artificial spot FOB contract in the event of the inventory level exceeding its upper inventory limit, thereby expanding the feasible solution space.

5.1.7 Maximizing Gross Margin Less Production Costs

The objective of the model is to optimize the relevant contribution margin obtained from the sale of LNG, which is referred to as "profit" in this thesis. The contribution margin only takes into account variable transportation costs, while other variable costs, such as LNG production costs and labor, are disregarded. Production costs are considered sunk costs, as the production levels are assumed predefined and not handled as decision variables. Similarly, some vessel operating costs, such as onboard labor and consumables, are also assumed to be sunk costs since the vessels must remain operational throughout the planning horizon, regardless of their transportation missions. Additionally, all fixed costs, including overhead costs, cost of capital, insurance, maintenance, and others, are also considered sunk. Based on the assumption that berth costs are negligible, the model aims to maximize revenue minus transportation costs. This is a reasonable approach assuming fixed production because the producer prefers to undertake any mission that yields a positive contribution margin, unless it affects a more profitable mission (assuming profit-maximizing behavior).

5.2 Definition of Node and Arc Structure

The model is based on a time-space network where nodes represent locations and times, while arcs connect them.

5.2.1 Adapting Physical Ports and Associated Customer Contracts to Model Ports

In our MILP model, a physical port can serve multiple customers who receive LNG cargoes at the same location. To represent this scenario, we associate each customer with a distinct *model port* as illustrated in Figure 5.1. A *model port* in our model comprises a physical location and a flag that designates it as a customer port, producer port, or maintenance port. If a physical location has multiple customers, it is represented by several model ports, one for each customer, even though it corresponds to a single physical location. As illustrated in the figure, if the Port of Brazil has three contracts requiring deliveries, our set of customer model ports includes three different ports, each with the same distance data associated with the physical Port of Brazil. The differences between these customer model ports are the customers' delivery requirements and associated revenues. It is important to note that each physical *loading* port only has one corresponding *model* port in our model.

In the remainder of this thesis, we use the term *port* to refer to a *model port* and the term *location* to refer to a *physical port*.

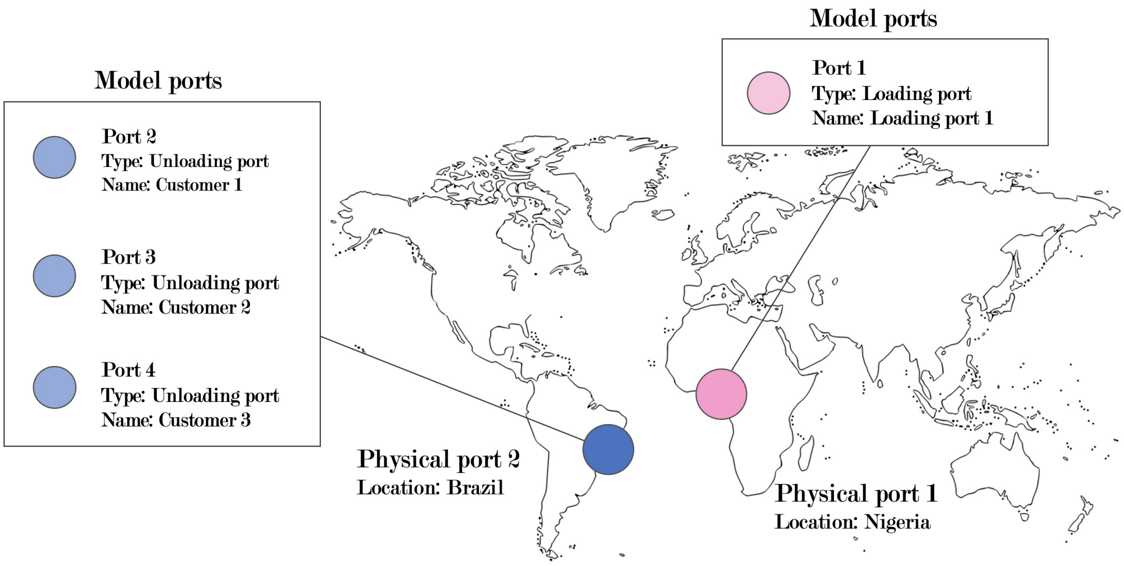


Figure 5.1: *Illustration of the difference between physical ports and model ports.*

5.2.2 Nodes

A node in the time-space network is a port that a vessel can visit at a specific time period, which is one of the time periods in the planning horizon (e.g., one day out of 365). We denote a node as (i, t) or (j, t'), where i and j denote ports, and t and t' denote time periods. Each port represents one of the following five port types:

- Loading port: producer operated port where LNG is liquefied and loaded onto vessels before transportation, represented by l_1 and l_2 in Figure 5.2
- Unloading port: LNG is unloaded from the vessel and regasified here before being transported to the customer of the unloading port, exemplified by u_3 to u_6 in Figure 5.2
- Maintenance port: vessels that require maintenance can dry-dock and perform maintenance here, represented by m_7 in Figure 5.2
- Spot unloading port: similar to an unloading port, except only optional cargoes can be unloaded here
- Artificial initialization and destination port: only defined in $t = 0$ and $t = |\mathcal{T}|+1$, represented by i_0 in Figure 5.2 and given more detail in Section 5.2.4

If a single port serves multiple functions (e.g., unloading and maintenance), the port is represented in the time-space network by one individual port for each port type. Figure 5.2 presents the structure for a scenario with an artificial port (i_0, only defined in (i_0,t_0) and (i_0,t_{9+1})) two loading ports (l_1 and l_2), four unloading ports (u_3 to u_6) and one maintenance port (m_7) in a planning horizon of nine time periods.

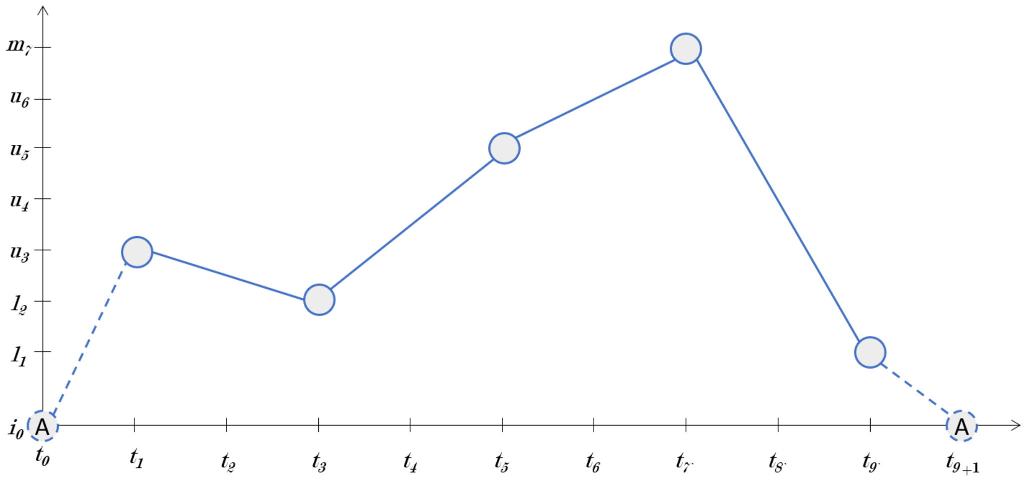


Figure 5.2: *Example of a time-space network and a possible vessel itinerary.*

5.2.3 Arcs

Every arc connects two nodes and is defined by the start and end nodes, which are denoted as $((i,t),(j,t'))$. The distance traveled is calculated using a given distance matrix between each port i and j. The point in time when the vessel begins sailing from the start port defines the start time of an arc. The end time of an arc is the arc's start time plus the time required to complete four vessel activities:

1. Sailing
2. Potential waiting
3. Potential purge- and cool down procedure
4. Operational time (docking and the relevant port process, as well as preparation for the next journey)

The blue arc in Figure 5.3 represents the arc $((i,t),(j,t'))$, and the pink lines represent the actual processes that the arc implies. The first dotted line corresponds to potential waiting time, and the second to the purge and cool down procedure.

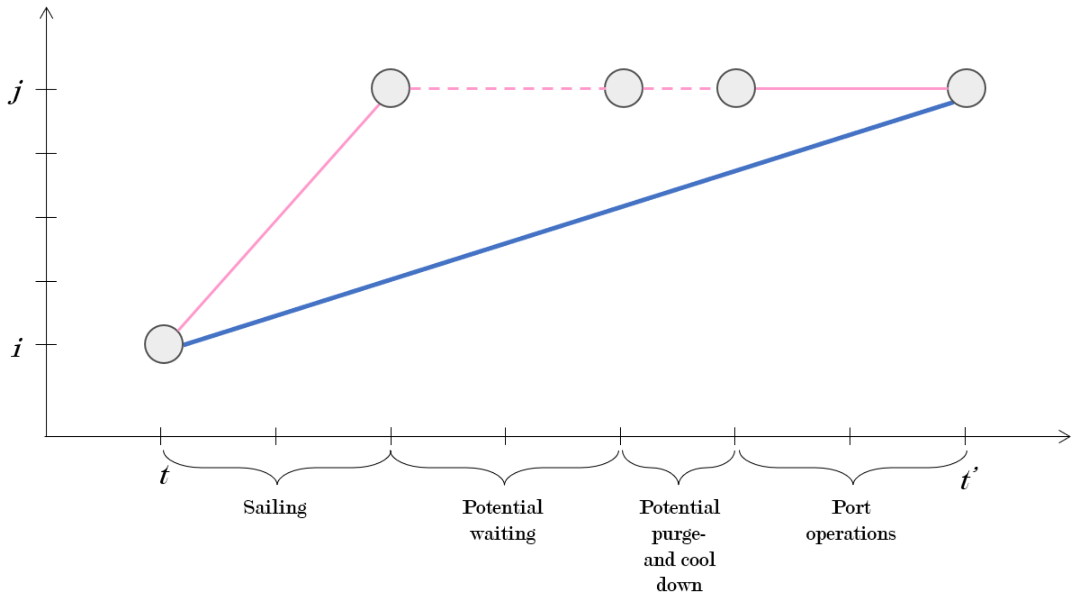


Figure 5.3: *Illustration of an arc $((i,t),(j,t'))$ and the vessel activities it represents.*

Waiting becomes necessary if the distance and time required to sail a voyage imply a speed that is less than the vessel's minimum speed. In this case, the vessel sails at its slowest speed and waits outside the port until the required time is reached. Waiting for berth availability is only relevant when the destination node is classified as a *loading port*, because we always assume berth availability at unloading and maintenance ports.

The process of purging and cooling down is only applicable if a vessel has recently visited a maintenance port.

Figure 5.3 shows how the arcs are structured such that sailing occurs prior to waiting and operations. This modeling decision was made to simplify berth constraints at the loading port and reduce symmetry. Following a maintenance visit, vessels require additional time in the loading port due to the purge and cool-down process. The arcs provide information on the type of node from which a vessel has departed (the i in $((i,t),(j,t')))$, allowing us to determine whether a purge-and-cool-down process is required and the duration for which the berth will be occupied. In contrast, if the arcs were defined such that operational time preceded sailing, knowledge about the previous arc would be necessary to determine the appropriate operation for the vessel at the port and the duration required before it could set sail again.

5.2.4 Artificial Nodes and Arcs

Artificial nodes and their associated arcs are included for two purposes:

1. A vessel can only start one journey
2. A vessel is not necessarily used at all

The first is to ensure that each vessel departs only on one voyage. The single electable arc between the artificial origin node and the time and place the vessel first becomes available is used for this purpose. The location of the artificial origin node is $(i,t) = (0,0)$. There are no associated expenses for the arcs from the artificial origin node. Using this modeling approach, we are able to remove a number of restrictions that would have otherwise prevented any vessel from being in more than one place at any given time.

The second reason is that the model allows for not using a vessel at all. For each vessel, there is an arc going from the artificial origin node in $(i,t) = (0,0)$ to the artificial destination node in $(i,t) = (0,|\mathcal{T}|+1)$. These arcs have 0 associated costs.

Figure 5.4 illustrates the artificial nodes and their associated arcs for initialization and for not using a vessel in a time-space network. The example includes two vessels represented by two different colors. The solid lines illustrate the possible initialization arcs the MILP model can choose for each vessel. The dotted lines illustrate possible arc choices for the MILP model after a vessel is initialized and chosen to be used.

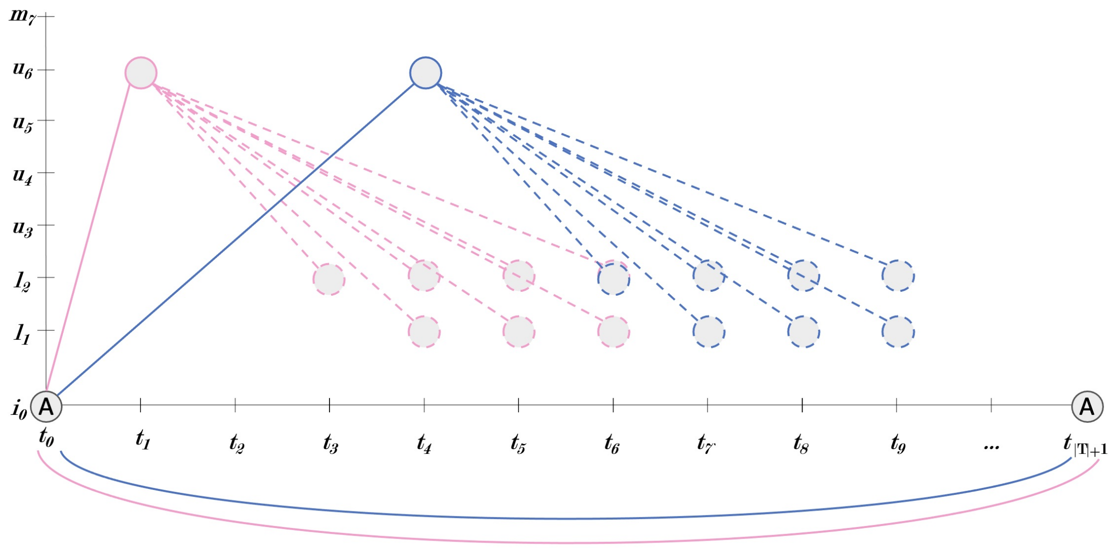


Figure 5.4: *Illustration of the artificial nodes, denoted A, and the initialization arcs, as well as the arcs that represent not using a vessel, illustrated for two example vessels in the time-space network described in Section 5.2. Step 1 for the MILP model is to choose if the vessel is to be used or not (solid lines). If it is decided to use the vessel, the MILP model's second step is to select additional arcs (dotted lines). In this case, the first example vessel first becomes available at location* u_6 *at time* t_1 *(pink) and the second example vessel first becomes available at location* u_6 *at time* t_4 *(blue).*

5.3 Data Preparation Procedure

The input to the MILP model is generated through an arc-generation procedure, thus reducing the number of problem constraints the model has to take into account. The following subsections describe how the data is prepared and what constraints are considered while doing so.

5.3.1 Arc-Generation Procedure

All feasible arcs for each vessel are generated in the arc generation procedure. The vessels that receive arcs include all the vessels that the LNG producer operates and the charter vessels. The feasible arcs are the input to the mathematical model of the LNG-ADP-SO-MLP, which is responsible for selecting the profit-maximizing combination of arcs, implicitly optimizing the speed. The algorithm that generates the arcs is described in Algorithm 1.

Some waiting- and speed terms used in the pseudo-code:

- Arc sailing+waiting **time**: The total duration of the arc minus the associated arc operational time, i.e., t'-t-operational time
- Arc sailing **time**: Arc sailing+waiting time minus waiting time, i.e., t'-t-operational time-waiting time
- Arc **speed**: The arc speed, including sailing time and waiting time, i.e., $\frac{\text{distance(i,j)}}{\text{arc sailing+waiting time}}$
- Arc sailing **speed**: The arc sailing speed, only including sailing time, i.e. either vessel minimum speed, or arc speed.

Algorithm 1 Arc Generation Procedure

Input: A vessel and its associated sets and parameters, fuel prices, length of planning horizon, and allowed waiting time

Output: Set of feasible arcs for the vessel

```
initialize the vessel's set of arcs as an empty set
create and add arcs from the artificial start node to the vessel's start position from day 0 to the first available day for the vessel
create and add arcs from the artificial start node to itself from day 0 to the last day
if the vessel needs maintenance during the planning period then
    create and add arcs associated with the vessel's planned maintenance port
end if
initialize vessel's port alternatives as an empty dictionary, and set port alternatives for first time period to the vessel's start position
for t in vessel's set of available time periods do
    for i in vessel's set of port alternatives do
        for j in vessel's set of compatible ports do
            if moving from i to j is not allowed then
                continue to next j
            end if
            for t' in vessel's set of available days, starting from t+1, do
                if j is a loading port and t' is after the last loading day + 1 then
                    continue to next t'
                end if
                if j is a customer port and t' is not in the customers' unloading days then
                    continue to next t'
                end if
                create arc (i,t,j,t')
                calculate distance between port i and j
                calculate arc sailing+waiting time and arc speed
                if arc sailing+waiting time and arc speed is feasible then
                    initialize exit arc for port j
                    if arc speed is larger than the vessel's minimum speed then
                        set arc sailing speed as arc speed
                        set arc waiting time to zero
                        calculate the arc cost given i, arc sailing speed and speed profile
                        add arc to vessel's set of arcs
                        if j is not a loading port then
                            add exit arc to vessel's feasible arcs
                            set exit arc's sailing costs and sailing time to zero
                        end if
                    else
                        calculate arc waiting time for arc
                        if arc waiting time is less than maximum allowed waiting time then
                            set arc sailing speed as vessel's minimum speed
                            calculate the arc cost given i, arc sailing speed and speed profile
                            add arc to vessel's set of arcs
                            if j is not a loading port then
                                add exit arc to vessel's feasible arcs
                                set exit arc's sailing costs and sailing time to zero
                            end if
                        end if
                    end if
                    if t' is in loading days then
                        add j as a port alternative for t' in port alternatives
                    end if
                end if
            end for
        end for
    end for
end for
```

The arc-generating procedure handles several constraints, summarized in the list that follows.

- Line 2 generates the initialization arcs, which ensure only one journey is started for each vessel, as described in Section 5.2.4.
- A vessel can only be assigned to a mission after it has finished the missions it was assigned to in the previous planning horizon. This is handled with only generating arcs from the artificial start node in t=0 to the first available day for the vessel in line 2, and no time periods before that.
- Line 3 generates the arcs that represent not using a vessel, as described in Section 5.2.4.
- Each vessel can only visit compatible ports. This is handled in lines 8 and 9.
- We only consider full shiploads, and the vessel must be empty when going into maintenance. A vessel must therefore travel in the following order: Loading - Unloading - (Maintenance) - Loading, meaning arcs between two loading ports or two unloading ports, as well as arcs from a loading port to a maintenance port, are not permitted. This is handled in line 11.
- The sailing speeds must be less than or equal to the actual vessel's maximum sailing speed limit. This is handled in line 21.
- The time difference of t and t' in the arcs must be long enough to account for the operational times of the different ports in addition to the sailing. Only the arcs that fulfill this requirement are generated. The operational time is longer if the arc starts in a maintenance node or if the distance is longer than a defined limit because the vessel then sails warm and needs to perform a purge and cool-down procedure in port before loading. This is handled in line 21.
- Vessels are not allowed to have a loading port as the final location. This is handled in line 25.
- Vessels are not allowed to wait more than a specified number of days. This is handled in line 35.
- The model can choose not to use a vessel for the rest of the planning period at any time, by sending it to the artificial destination node. This is handled by adding an exit arc to the artificial destination node from port j if an arc to port j is added to the vessel's set of feasible arcs. This is handled in lines 18, 30, and 41.

5.3.2 Cost Calculations

Prior to executing the LNG-ADP-SO-MLP, the cost calculations are performed. The cost assigned to each arc is based solely on fuel consumption, while disregarding other expenses that are deemed insignificant. There are numerous arcs that connect every possible port combination i-j, and each arc corresponds to a distinct time period, as depicted in Figure 5.5. As outlined in Section 5.3.1, an arc is not generated if the speed at which a vessel sails exceeds the maximum speed of the corresponding vessel. If the implied speed is lower than the minimum speed of the vessel, the minimum speed of the vessel is employed in the cost function, along with the sailing time implied at that speed, and the remaining time units are regarded as waiting days. Since only sunk costs are linked with these waiting days, they do not result in any additional expenses being added to the total cost of the arc.

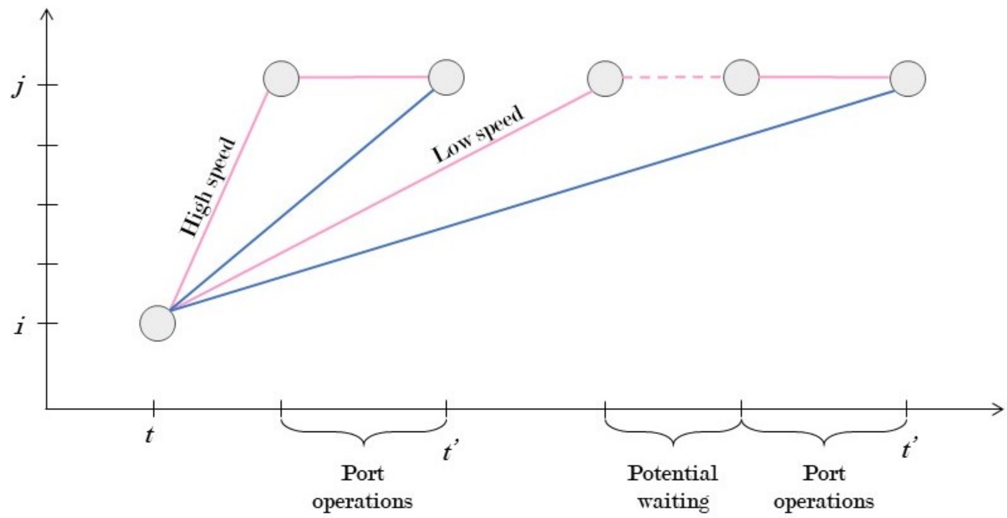


Figure 5.5: *A vessel can sail at different speeds from port i to j.*

The fuel consumption of each arc depends on the sailing speed that the time and distance of each arc imply, as illustrated in Figure 5.6. Fuel consumption is found from a non-linear fuel consumption approximation as the curve in Figure 5.6, but in the model this has been made discrete. The cost generation function assumes that the arc is sailed at the lowest possible speed, with sailing time calculated as the difference between the start and finish times of the arc minus the time required for port operations. The resulting speed in knots (kn) is then calculated by dividing the distance between ports i and j in nautical miles (NM) by the number of hours in the sailing time. To calculate fuel consumption for a given arc, Algorithm 2 is used, which takes into account the arc's start location and sailing speed, as well as the vessel's speed profile. If the vessel is loaded and starts sailing from port i, then the laden speed profile is used, whereas if the vessel is empty, the ballast speed profile is used.

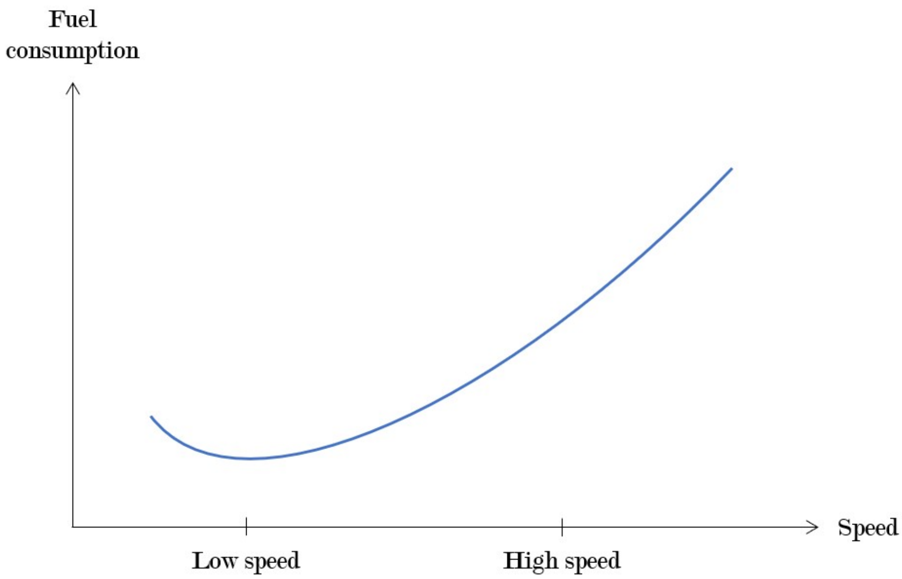


Figure 5.6: *A general illustration of fuel consumption as a function of speed. Costs increase with higher speed, but the graph also shows that costs begin to rise as speed falls below a certain threshold. The example in Figure 5.5 with two possible speeds, low and high speed, shows that the choice of speed results in significant differences in sailing costs.*

Algorithm 2 Fuel Consumption Calculation

Input: the arc's start location and sailing speed, and the vessel's speed profile
Output: the arc's daily fuel consumption

```
if start location of the arc is a loading port then
    set lower speed and lower fuel consumption based on vessel's laden speed profile
    for s in the vessel's set of laden speeds do
        if arc sailing speed is larger than s then
            set lower speed to s
        else if arc sailing speed is equal to s then
            return fuel consumption associated with s
        else
            calculating fuel consumption associated with arc sailing speed based on interpolation
            return interpolated fuel consumption
        end if
    end for
else
    set lower speed and lower fuel consumption based on vessel's ballast speed profile
    for s in the vessel's set of ballast speeds do
        if arc sailing speed is larger than s then
            set lower speed to s
        else if arc sailing speed is equal to s then
            return fuel consumption associated with s
        else
            calculating fuel consumption associated with arc sailing speed based on interpolation
            return interpolated fuel consumption
        end if
    end for
end if
```

5.4 Notation

The notation used in this project is mostly based on the existing model from the specialization project Haug et al. (2022). However, a new parameter denoted P_j^{MIN} has been added to the model and allows for setting a minimum number of days between each delivery at a customer's port. The complete MILP-model is presented in Appendix C.

Sets and Indices

$\mathcal{V}$	Set of producer-operated vessels that are available during the planning horizon, each vessel indexed by v
$\mathcal{V}^M$	Set of vessels that require maintenance during the planning horizon, $\mathcal{V}^M \subset \mathcal{V}$
$\mathcal{V}_i$	Set of vessels that can serve node i, $\mathcal{V}_i \subset \mathcal{V}$

All vessels operated by the producer that is used to transport LNG are included in the set $\mathcal{V}$. Furthermore, all producer vessels that need maintenance are included in the set $\mathcal{V}^M$ which is a subset of $\mathcal{V}$. Not all vessels are compatible with all ports, therefore the sets $\mathcal{V}_i$ contain all vessels that can serve node i. The set $\mathcal{V}_i$ is a subset of $\mathcal{V}$.

$\mathcal{N}$ Set of ports, each port indexed by i, j

$\mathcal{N}^L$ Set of loading ports, $\mathcal{N}^L \subset \mathcal{N}$

$\mathcal{N}^U$ Set of unloading ports, $\mathcal{N}^U \subset \mathcal{N}$

$\mathcal{N}^S$ Set of spot unloading ports, $\mathcal{N}^S \subset \mathcal{N}$

$\mathcal{N}^M$ Set of maintenance ports, $\mathcal{N}^M \subset \mathcal{N}$

The set $\mathcal{N}$ contains all of the model's ports. All loading, unloading, spot unloading, and maintenance ports are included here. Furthermore, for each port type, there exists a set that is a subset of $\mathcal{N}$. The subset $\mathcal{N}^L$ is a set that contains all of the producer's loading ports. $\mathcal{N}^U$ contains all of the contractual customers' unloading ports, and $\mathcal{N}^M$ contains all ports where maintenance can be performed. The set $\mathcal{N}^S$ contains all ports with a spot DES contract, which means that the producer is responsible for transporting the LNG to the port if the producer accepts the contract. A FOB cargo, on the other hand, must be picked up by the customer at the loading port. The sets $\mathcal{N}^S$ and $\mathcal{N}^M$ can, as mentioned in Section 5.2.1, include the same physical ports as the customer ports in the set $\mathcal{N}^U$, but they are defined as independent ports for modeling purposes.

$\mathcal{A}_v$ Set of feasible arcs vessel v can sail, each arc indexed by $((i,t),(j,t'))$

$\mathcal{A}_v^M$ Set of feasible maintenance arcs vessel v can sail to start maintenance, $\mathcal{A}_v^M \subset \mathcal{A}_v$

$\mathcal{A}_v^U$ Set of feasible arcs vessel v can sail to deliver a DES long-term contracted cargo, $\mathcal{A}_v^U \subset \mathcal{A}_v$

$\mathcal{A}_v^S$ Set of feasible arcs vessel v can sail to deliver a DES spot cargo, $\mathcal{A}_v^S \subset \mathcal{A}_v$

All predefined feasible arcs that vessel v can sail are included in the set $\mathcal{A}_v$. The set $\mathcal{A}_v^M$ is a subset of $\mathcal{A}_v$ and includes all arcs vessel v, in the set $\mathcal{V}^M$, can start sailing to perform maintenance. $\mathcal{A}_v^U$ contains all the arcs that vessel v can sail to deliver a long-term contracted DES cargo. There is also a set $\mathcal{A}_v^S$ with all the arcs that a vessel v can sail to carry out a spot DES contract.

$\mathcal{F}_i$ Set of FOB cargoes of LNG that want to be picked up at loading port i, indexed by f

$\mathcal{F}_i^U$ Set of long-term contracted FOB cargoes of LNG that must be picked up at loading port i, indexed by f, $\mathcal{F}_i^U \subset \mathcal{F}_i$

$\mathcal{F}_i^S$ Set of Spot FOB cargoes whose load can be picked up by a FOB vessel at loading port i, indexed by f, also including the artificial spot FOB cargo, $\mathcal{F}_i^S \subset \mathcal{F}_i$

The set $\mathcal{F}_i$ contains all LNG cargoes that can or shall be picked up at the loading ports. $\mathcal{F}_i^U$ contains the long-term FOB contract cargoes that *must* be picked up. $\mathcal{F}_i^S$ contains all FOB spot cargoes that *can* be picked up by a customer's vessel at loading port i, including the artificial spot FOB cargo.

$\mathcal{P}_j$	Set of time partitions where customer j has DES contracts, each partition indexed by p.
$\mathcal{T}$	Set of time periods in all days, each time period, indexed by t, $\mathcal{T} = \{1, 2, ..., \lvert\mathcal{T}\rvert\}$ where $\lvert\mathcal{T}\rvert$ is the last time period in the planning horizon.
$\mathcal{T}^L$	Set of time periods in loading days, where the vessels can lift LNG from a loading port, each time period indexed by t, $\mathcal{T}^L \subset \mathcal{T}$
$\mathcal{T}^U$	Set of time periods in unloading days, where vessels can deliver LNG to a customer, each time period indexed by t, $\mathcal{T}^U \subset \mathcal{T}$
$\mathcal{T}_{jp}$	Set of time periods within partition p for customer j, $\mathcal{T}_{jp} \subset \mathcal{T}$
$\mathcal{T}_f^{FOB}$	Set of time periods where FOB cargo f can be picked up, $\mathcal{T}_f^{FOB} \subset \mathcal{T}$
$\mathcal{T}_v$	Set of time periods where vessel v is available to be scheduled, $\mathcal{T}_v \subset \mathcal{T}$
$\mathcal{T}_v^M$	The time period where maintenance of vessel v is scheduled to start, $\mathcal{T}_v^M \subset \mathcal{T}_v$

The set $\mathcal{T}$ contains all time periods in the planning horizon (loading days) $\mathcal{T}^L$, and the time periods in the unloading days $\mathcal{T}^U$: $\mathcal{T} = \mathcal{T}^L \cup \mathrm{T}^U$. The planning horizon is divided into partitions of time periods. Each customer has a demand of DES cargoes specified for a partition (e.g., quarterly, semi-annually, yearly). The set $\mathcal{P}_j$ contains the partitions, and the set $\mathcal{T}_{pj}$ contains the time periods in a partition for a DES customer j. The FOB contracts, like the DES contracts, have time windows for when they can be picked up. Therefore, the set $\mathcal{T}_f^{FOB}$ contains time periods during which a FOB cargo can be picked up. A vessel can be unavailable at the start of the planning horizon, therefore the time periods for when a vessel is available have a set $\mathcal{T}_v$. Lastly, the set $\mathcal{T}_v^M$ contains the time periods where a vessel must start maintenance.

Parameters

$C^{S}_{vitjt'}$	Sailing cost of each feasible arc $((i,t),(j,t'))$ for vessel v, $((i,t),(j,t')) \in \mathcal{A}_v, v \in \mathcal{V}$
C^{C}_{itj}	Costs of using a charter vessel to deliver a cargo at port j, loading the cargo at loading port i at time t, $i \in \mathcal{N}^L, t \in \mathcal{T}, i \in \mathcal{N}^U$
$\mathcal{N}^{START}_v$	Start port of vessel v, $v \in \mathcal{V}$
T^{START}_v	First time period where vessel v is available to be scheduled, $v \in \mathcal{V}$, $\mathrm{T}^{START}_v \subset \mathcal{T}_v$
T^{O}_{vij}	Operational time associated with sailing from port location i to port location j for vessel v, $i,j \in \mathcal{N}, v \in \mathcal{V}$
T^{C}_{ij}	Sailing time for a charter vessel sailing from loading port i to unloading port j, $i \in \mathcal{N}^L, j \in \mathcal{N}^U$
T^{OFOB}_{fi}	Operational time associated to port location i for FOB cargo f, $f \in \mathcal{F}_i, i \in \mathcal{N}^L$
$T^{charter}_{ij}$	Sailing time for a charter vessel between port i and j, $i \in \mathcal{N}^L, j \in \mathcal{N}^U$
L_v	Capacity of vessel v, $v \in \mathcal{V}$
$\overline{L}^C$	Upper limit for capacity of a charter vessel
$\underline{L}^C$	Lower limit for capacity of a charter vessel
L^{FOB}_f	Loading quantity of FOB cargo f, $f \in \mathcal{F}_i, i \in \mathcal{N}^L$
$\overline{D}_{jp}$	Maximum demand of unloading port j in partition p, $j \in \mathcal{N}^U, p \in \mathcal{P}_j$
$\underline{D}_{jp}$	Minimum demand of unloading port j in partition p, $j \in \mathcal{N}^U, p \in \mathcal{P}_j$
R^{SFOB}_f	Revenue per volume unit of LNG loaded for FOB spot contract f, $f \in \mathcal{F}^S_i, i \in \mathcal{N}^L$
R^{UFOB}_f	Revenue per volume unit of LNG loaded for long-term FOB contract f, $f \in \mathcal{F}^S_i, i \in \mathcal{N}^L$
$R^{DES}_{jt'}$	Revenue per volume unit of LNG for delivering DES contract to customer j at time t', $j \in \mathcal{N}^U, t' \in \mathcal{T}$
R^{END}_i	Unit value of LNG left in storage tanks at loading port i at the end of the planning horizon, $i \in \mathcal{N}^L$
$B_{jt'}$	Berth capacity at port j at time t', $j \in \mathcal{N}^L, t' \in \mathcal{T}$
Q^{P}_{it}	Produced quantity of LNG in loading port i in time period t, $i \in \mathcal{N}^L, t \in \mathcal{T}$
$\overline{S}_i$	Maximum storage level of LNG at loading port i, $i \in \mathcal{N}^L$
$\underline{S}_i$	Minimum storage level of LNG at loading port i, $i \in \mathcal{N}^L$
S_i	Initial storage level of LNG at the start of the planning horizon at loading port i, $i \in \mathcal{N}^L$
E	Boil-off rate in percent of total vessel capacity
P^{MIN}_j	Minimum number of time periods between deliveries for customer j, $j \in \mathcal{N}^U$

Variables

$x_{vitjt'}$ 1 if vessel v sail arc $((i, t), (j, t'))$, and 0 otherwise

$z_{ft'}$ 1 if FOB cargo f is done loading in time period t', and 0 otherwise

w_{itj} 1 if a charter vessel starts sailing from loading port i at time t to deliver a cargo at j, and 0 otherwise

g_{itj} Amount loaded by a charter vessel in loading port i at time t to deliver in unloading port j

s_{it} Remaining storage at loading port i at the end of time period t

5.5 Basic Version of the Mathematical Model

We formulate our model using the notation described in Section 5.4. The objective function is introduced in Section 5.5.1, where each term is explained. Section 5.5.2 presents the constraints in the model. All constraints are presented mathematically and explained. The following mathematical model is based on our specialization project Haug et al. (2022), except for constraints (5.9) which are new.

5.5.1 Objective Function

$$
\begin{aligned}
max z = &\sum_{v\in\mathcal{V}} \sum_{((i,t),(j,t'))\in\mathcal{A}_v^U} R_{jt'}^{DES} L_v (1-(t'-t)E) x_{vitjt'} + \sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}^L}\sum_{j\in\mathcal{N}^U} R_{j,t+T_{ij}^C}^{DES}(1-T_{ij}^C E) g_{itj} \\
&+ \sum_{v\in\mathcal{V}} \sum_{((i,t),(j,t'))\in\mathcal{A}_v^S} R_{jt'}^{DES} L_v (1-(t'-t)E) x_{vitjt'} + \sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}^L}\sum_{j\in\mathcal{N}^S} R_{j,t+T_{ij}^C}^{DES}(1-T_{ij}^C E) g_{itj} \\
&+ \sum_{i\in\mathcal{N}^L}\sum_{f\in\mathcal{F}_i^U}\sum_{t'\in\mathcal{T}} R_f^{UFOB} L_f^{FOB} z_{ft'} + \sum_{i\in\mathcal{N}^L}\sum_{f\in\mathcal{F}_i^S}\sum_{t'\in\mathcal{T}} R_f^{SFOB} L_f^{FOB} z_{ft'} + \sum_{i\in\mathcal{N}^L} R_i^{END} s_{i,|\mathcal{T}|} \\
&- \sum_{v\in\mathcal{V}} \sum_{((i,t),(j,t'))\in\mathcal{A}_v} C_{vitjt'}^S x_{vitjt'} - \sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{N}^U} C_{itj}^C w_{itj}
\end{aligned}
\tag{5.1}
$$

The objective function (5.1) aims to maximize profit, which is represented by revenue less vessel sailing costs. The first seven terms are all revenue terms. The first two terms represent the revenue from the long-term DES contracts. The first term is the revenue from long-term DES contract shipments shipped by own vessels, and the second term is the revenue from long-term DES contract shipments shipped by charter vessels. The next two terms are revenue from DES *spot* contracts, formulated in the same way as for the *long-term* DES contracts. Since there must be a certain amount of LNG in the vessel's tanks after unloading at a customer port, the boil-off effect affects the amount of LNG in the tanks at both laden and ballast voyages. In terms one to four this is accounted for by including the total boil-off for both voyages in the boil-off rate E. The next two revenue terms, number five and number six, are the revenue from the FOB sale of LNG. Term five is the revenue from the long-term FOB contracts while term six is the revenue from the spot FOB contracts. The last revenue term is term seven, which represents the value of remaining LNG in the storage tanks at the end of the planning horizon. The last two terms are the cost terms. The first cost term is the cost of sailing the producer-operated vessels. Fuel and bunker costs are included in the total sailing costs while other operating costs are ignored as they are considered sunk. The cost of using charter vessels is the ninth and final term. The daily charter rate and sailing costs from the entire round trip of the charter vessel are included.

5.5.2 Constraints

Inventory Constraints

Constraints (5.2) to (5.4) handle the inventory at the loading ports. The inventory control constraints (5.2) and (5.3) ensure that the inventory level left at a given day is the inventory level from the day before plus quantity produced that day, minus LNG picked up that day both with the producer's vessels, charter vessels, and FOB, including the artificial spot FOB contract. Constraints (5.2) initialize the constraint for each loading node, while constraints (5.3) make sure the inventory requirement is fulfilled for each day afterward. Constraints (5.4) ensure the inventory level is within its upper and lower storage capacity bounds at all times.

$$
s_{i1} = S_i + Q_{i1}^P - \sum_{j\in\mathcal{N}^U}\sum_{t'\in\mathcal{T}}\sum_{v\in\mathcal{V}_i} L_v x_{vi1jt'} - \sum_{j\in\mathcal{N}^U\cup\mathcal{N}^S} g_{i1j} - \sum_{f\in\mathcal{F}_i} L_f^{FOB} z_{f1}, \qquad i\in\mathcal{N}^L \tag{5.2}
$$

$$s_{it} = s_{i,t-1} + Q_{it}^P - \sum_{j \in \mathcal{N}^U} \sum_{t' \in \mathcal{T}} \sum_{v \in \mathcal{V}_i} L_v x_{vitjt'} - \sum_{j \in \mathcal{N}^U \cup \mathcal{N}^S} g_{itj} - \sum_{f \in \mathcal{F}_i} \sum_{t' \in \mathcal{T}_f^{FOB}} L_f^{FOB} z_{ft'}, \quad i \in \mathcal{N}^L, t \in \mathcal{T}^L \backslash \{1\} \tag{5.3}$$

$$\underline{S_i} \leq s_{it} \leq \overline{S_i}, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L \tag{5.4}$$

Maintenance Constraints

Constraints (5.5) state that all the maintenance vessels need to perform maintenance exactly one time.

$$\sum_{((i,t),(j,t')) \in \mathcal{A}_v^M} x_{vitjt'} = 1, \qquad v \in \mathcal{V}^M \tag{5.5}$$

Flow Constraints

Constraints (5.6) ensure that each vessel either starts at its start position at its first available day, or is not used at all during the planning period.

$$x_{v,0,0,N_v^{START},T_v^{START}} + x_{v,0,0,0,|\mathcal{T}|+1} = 1, \qquad v \in \mathcal{V} \tag{5.6}$$

Constraints (5.7) are flow constraints, ensuring that if a vessel sails to a port j, it must subsequently sail from the same port j.

$$\sum_{i \in \mathcal{N}} \sum_{t=0}^{t'-1} x_{vitjt'} = \sum_{i \in \mathcal{N}} \sum_{t=t'+1}^{|\mathcal{T}|} x_{vjt'it}, \qquad v \in \mathcal{V}, j \in \mathcal{N}, t' \in \mathcal{T} \tag{5.7}$$

Demand Constraints

Constraints (5.8) are demand constraints ensuring that demand is satisfied for each DES contract, both long-term and spot, in each time partition. The constraints account for the difference in loading and unloading volumes by subtracting the boil-off.

$$\underline{D}_{jp} \leq \sum_{v \in \mathcal{V}_i} \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}_v} \sum_{t' \in \mathcal{T}_{jp}} L_v (1 - (t' - t)E) x_{vitjt'} + \sum_{i \in \mathcal{N}^L} \sum_{t \in (\mathcal{T}_{jp} - T_{ij}^C)} g_{itj} (1 - T_{ij}^C E) \leq \overline{D}_{jp}, \quad j \in \mathcal{N}^U \cup \mathcal{N}^S, \quad p \in \mathcal{P}_j \tag{5.8}$$

Constraints (5.9) ensure that the deliveries of cargoes to an unloading port are spread throughout the planning horizon.

$$\sum_{v \in \mathcal{V}_i} \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}_v} \sum_{\tau = t'}^{t' + P_j^{MIN}} x_{vitj\tau} + \sum_{i \in \mathcal{N}^L} \sum_{\tau = t' - T_{ij}^{charter}}^{t' - T_{ij}^{charter} + P_j^{MIN}} w_{i\tau j} \leq 1, \quad j \in \mathcal{N}^U \cup \mathcal{N}^S, t' \in \mathcal{T}^U \backslash \{|\mathcal{T}^U| - \mathcal{P}_j^{MIN}\} \tag{5.9}$$

Constraints (5.10) and (5.11) are the FOB contract constraints. Constraints (5.10) make sure each long-term FOB cargo contract is fulfilled once in the relevant time window, while constraints (5.11) make sure each spot FOB cargo contract is fulfilled at most once in the relevant time window, except for the artificial spot FOB contracts (contract 1), which can be fulfilled as many times as wanted.

$$\sum_{t' \in \mathcal{T}_f^{FOB}} z_{ft'} = 1, \qquad j \in \mathcal{N}^L, f \in \mathcal{F}_j^U \tag{5.10}$$

$$\sum_{t' \in \mathcal{T}_f^{FOB}} z_{ft'} \leq 1, \qquad j \in \mathcal{N}^L, f \in \mathcal{F}_j^S \backslash \{1\} \tag{5.11}$$

Berth Constraints

Constraints (5.12) are berth constraints, ensuring that each loading port does not have more docked vessels than available berths. The constraints sum over all the future time points where an arrival would imply that the berth is occupied in t'.

$$\sum_{v \in \mathcal{V}^P} \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} \sum_{\tau = t'+1}^{t' + T_{vij}^O} x_{vitj\tau} + \sum_{j' \in \mathcal{N}^U \cup \mathcal{N}^S} w_{jt'j'} + \sum_{f \in \mathcal{F}_j} \sum_{\tau = t'+1}^{t' + T_{fj}^{OFOB}} z_{f\tau} \leq B_{jt'}, \qquad j \in \mathcal{N}^L, \quad t' \in \mathcal{T}^U \tag{5.12}$$

Charter Constraints

Constraints (5.13) ensure that if a charter vessel loads a cargo, the amount loaded is bounded with an upper and a lower limit. This allows flexibility in the capacities of the charter vessels.

$$\underline{L}^C w_{itj} \leq g_{itj} \leq \overline{L}^C w_{itj}, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L, j \in \mathcal{N}^U \cup \mathcal{N}^S \tag{5.13}$$

Variable Constraints

Constraints (5.14) to (5.18) specify the domains of our three binary variables $x_{vitjt'}$, $z_{ft'}$ and w_{itj} as well as our two continuous variables g_{itj} and s_{it}. $z_{ft'}$ is only defined for loading ports and the relevant time periods so that the variable cannot take values outside this domain. Note that w_{itj}, the decision variable for using a charter vessel, is only defined for arcs that go from a loading node to an unloading node.

$$x_{vitjt'} \quad \in \quad \{0,1\}, \qquad v \in \mathcal{V}, ((i,t),(j,t')) \in \mathcal{A}_v \tag{5.14}$$

$$z_{ft'} \in \{0,1\}, \qquad j \in \mathcal{N}^L, f \in \mathcal{F}_j, t' \in \mathcal{T}_f^{FOB} \tag{5.15}$$

$$w_{itj} \in \{0,1\}, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L, j \in \mathcal{N}^U \cup \mathcal{N}^S \tag{5.16}$$

$$g_{itj} \geq 0, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L, j \in \mathcal{N}^U \cup \mathcal{N}^S \tag{5.17}$$

$$s_{it} \geq 0, \qquad i \in \mathcal{N}, t \in \mathcal{T}^L \tag{5.18}$$

5.6 Extension 1: Variable Production

As touched upon in Chapter 3, one major concern in preparing ADPs is managing excess production after fulfilling long-term contracts. In our basic model, excess production is assumed to be sold on the spot market through *artificial spot FOB contracts* generated by the model. However, this assumption may not be valid if spot demand or shipping (vessel) capacity is insufficient (Mutlu et al., 2016). Therefore, reducing the production rate could be beneficial as there is no guarantee of selling excess production on the spot market.

We extend the model for the basic version of the LNG-ADP-SO-MLP, inspired by Mutlu et al. (2016), to include variable production, minimizing the number of artificial spot FOB contracts and thus making the model more realistic. The production rate is constrained by an upper and lower limit, and production decisions can be made for each time period.

5.6.1 Changes to Notation

Parameters

Q_{it}^{MIN} Minimum production rate at production port i in time period t, $i \in \mathcal{N}^L, t \in \mathcal{T}^L$

Q_{it}^{MAX} Maximum production rate at production port i in time period t, $i \in \mathcal{N}^L, t \in \mathcal{T}^L$

Variables

q_{it} Production rate at production port i at time period t

5.6.2 Changes to Mathematical Model

As production costs are considered sunk, the variable production extension has no effect on the objective function, only the model's constraints.

Constraints

In the inventory constraints (5.2) and (5.3) from the basic version of the model, the production rate parameter, Q_{it}^P, is replaced by the production rate variable q_{it}, resulting in the constraints (5.19) and (5.20).

$$s_{i1} = S_i + q_{i1} - \sum_{j \in \mathcal{N}^U} \sum_{t' \in \mathcal{T}} \sum_{v \in \mathcal{V}_i} L_v x_{vi1jt'} - \sum_{j \in \mathcal{N}^U} g_{i1j} - \sum_{f \in \mathcal{F}_i} \sum_{t' \in \mathcal{T}_f^{FOB}} L_f^{FOB} z_{ft'}, \qquad i \in \mathcal{N}^L \tag{5.19}$$

$$s_{it} = s_{i,t-1} + q_{it} - \sum_{j \in \mathcal{N}^U} \sum_{t' \in \mathcal{T}} \sum_{v \in \mathcal{V}_i} L_v x_{vitjt'} - \sum_{j \in \mathcal{N}^U \cup \mathcal{N}^S} g_{itj} - \sum_{f \in \mathcal{F}_i} L_f^{FOB} z_{ft}, \quad i \in \mathcal{N}^L, t \in \mathcal{T}^L \backslash \{1\} \tag{5.20}$$

Constraints (5.21) ensures that the production rate at each loading port i must lie within the limits at all times t, while constraints (5.22) specify the domain of the new continuous decision variables, q_{it}.

$$Q_i^{MIN} \leq q_{it} \leq Q_i^{MAX}, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L \tag{5.21}$$

$$q_{it} \geq 0, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L \tag{5.22}$$

5.7 Extension 2: Chartering Out Own Vessels

In reality, as mentioned in Chapter 2, producers can always charter out their own vessels. As a result, the model must account for this option in order to provide more profitable annual delivery programs in practice. To keep the model efficient, the extended model only allows for one chartering out term, of at least a given number of time periods, per vessel in each planning horizon. According to our industry partner, this is also a reasonable assumption.

5.7.1 Changes to Arc and Node Structure

In each time period, every unloading and maintenance port has an arc that goes to the artificial node in the same time period. This implies that a vessel can choose to be chartered out after unloading or after maintenance. If the model chooses this arc, it implies that the vessel is chartered out for at least a minimum number of days. In Figure 5.7 a vessel sails from an unloading port to an artificial node, and is chartered out from time period t_2 to t_6, before returning to a loading port and is ready for a new voyage on behalf of the LNG producer.

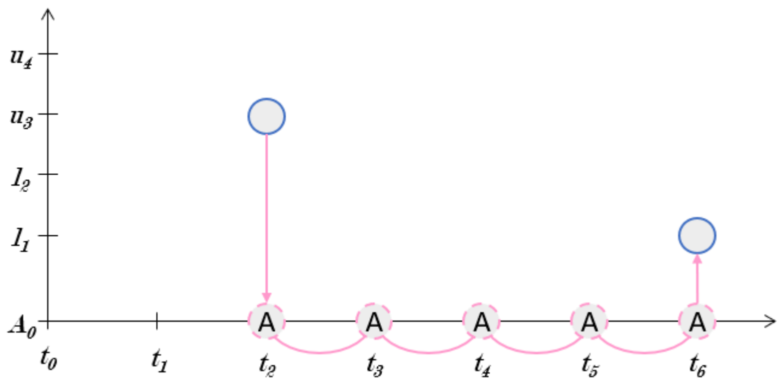


Figure 5.7: *A producer vessel can be chartered out for a minimum number of time periods, here from t_2 to t_6. After being chartered out, the vessel must return to a loading port.*

5.7.2 Changes to Notation

Parameters

$R_{vt}^{Charter}$ Daily revenue of chartering out vessel v on day t, $v \in \mathcal{V}, t \in \mathcal{T}^L$

$\underline{M}$ Minimum number of time periods the vessel can be chartered out for

Variables

y_v 1 if vessel v is chartered out during the planning horizon, 0 otherwise

5.7.3 Changes to Mathematical Model

Objective Function

As chartering out a vessel is a potential source of income, a term (marked in red) considering the revenue from chartering out is added to the objective function (5.23).

$$
\begin{aligned}
max z = & \sum_{v \in \mathcal{V}} \sum_{((i,t),(j,t')) \in \mathcal{A}_v^U} R_{jt'}^{DES} L_v (1 - (t' - t)E) x_{vitjt'} + \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}^L} \sum_{j \in \mathcal{N}^U} R_{j,t+T_{ij}^C}^{DES} (1 - T_{ij}^C E) g_{itj} \\
& + \sum_{v \in \mathcal{V}} \sum_{((i,t),(j,t')) \in \mathcal{A}_v^S} R_{jt'}^{DES} L_v (1 - (t' - t)E) x_{vitjt'} + \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}^L} \sum_{j \in \mathcal{N}^S} R_{j,t+T_{ij}^C}^{DES} (1 - T_{ij}^C E) g_{itj} \\
& + \sum_{i \in \mathcal{N}^L} \sum_{f \in \mathcal{F}_i^U} \sum_{t' \in \mathcal{T}} R_f^{UFOB} L_f^{FOB} z_{ft'} + \sum_{i \in \mathcal{N}^L} \sum_{f \in \mathcal{F}_i^S} \sum_{t' \in \mathcal{T}} R_f^{SFOB} L_f^{FOB} z_{ft'} + \sum_{i \in \mathcal{N}^L} R_i^{END} s_{i,|\mathcal{T}|} \\
& + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}^L} R_{vt}^{Charter} x_{v0t0,t+1} \\
& - \sum_{v \in \mathcal{V}} \sum_{((i,t),(j,t')) \in \mathcal{A}_v} C_{vitjt'}^S x_{vitjt'} - \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{N}^U} C_{itj}^C w_{itj}
\end{aligned}
\tag{5.23}
$$

Constraints

Charter Out Constraints

Constraints (5.24) ensure that each vessel can only be chartered out one term during the planning horizon.

$$\sum_{i \in \mathcal{N}} \sum_{t=0}^{|\mathcal{T}^L|} x_{vit0t} \leq y_v, \qquad v \in \mathcal{V} \tag{5.24}$$

Constraints (5.25) ensures that if a vessel is chartered out, it must be chartered out for at least a certain number of time periods, here denoted $\underline{M}$.

$$\sum_{t=0}^{|\mathcal{T}^L|} x_{v0t0,t+1} \geq \underline{M} y_v, \qquad v \in \mathcal{V} \tag{5.25}$$

Flow Constraints

Constraints (5.26) show the changes (marked in red) made to flow constraints (5.7) from the basic version of the model presented in Section 5.5, which ensure that if a vessel sails to a port j, then the vessel must sail from the very same port j afterward. The changes make sure a vessel can go directly from an artificial node in time t to the next artificial node in the next time period, skipping port operations which is not relevant in an artificial port.

$$\sum_{i \in \mathcal{N}} \sum_{t=0}^{t' \cancel{-1}} x_{vitjt'} = \sum_{i \in \mathcal{N}} \sum_{t=t' \cancel{+1}}^{|\mathcal{T}|} x_{vjt'it}, \qquad v \in \mathcal{V}, j \in \mathcal{N}, t' \in \mathcal{T} \tag{5.26}$$

Constraints (5.27) are based on the initialization constraints (5.6) from the basic version of the LNG-ADP model. Constraints (5.27) ensure that each vessel either starts at its start position on its first available day or starts being chartered out (marked in red).

$$x_{v,0,0,N_v^{START},T_v^{START}} + x_{v,0,0,0,T_v^{START}} = 1, \qquad v \in \mathcal{V} \tag{5.27}$$

Constraints (5.28) are necessary to ensure that vessels that go to the artificial node start their journey at a loading port after the chartering out period. This is to ensure that the model does not move the vessels freely after chartering out.

$$x_{vit0t} = \sum_{\tau=t}^{|\mathcal{T}^L|} \sum_{j \in \mathcal{N}^L} x_{v0\tau j\tau}, \qquad v \in \mathcal{V}, i \in \mathcal{N}, t \in \mathcal{T}^L \tag{5.28}$$

Variable Constraints

Constraints (5.29) specify the domains of the new binary variables y_v.

$$y_v \quad \in \quad \{0,1\}, \qquad v \in \mathcal{V} \tag{5.29}$$

Chapter 6

Rolling Horizon Heuristic (RHH)

The objective of our industry partner is to find the best possible solution to the LNG-ADP-SO-MLP within a practical amount of time. In practice, this means that the solution time can not surpass 12 hours and should ideally be less than 3 hours. In our project thesis Haug et al. (2022) we showed that the commercial solver Gurobi alone is an insufficient solution method for full-sized instances with a planning horizon of 12 months within the time limits required. Therefore we propose a new solution method involving a version of a rolling horizon heuristic (RHH). In addition, we present a customized construction heuristic that warm-starts the RHH with a feasible solution.

The RHH has previously proven useful for solving problem instances where the complexity increases with a longer planning horizon. LNG ADP-specific examples include Rakke et al., 2011 and Li et al., 2022. In both papers, the RHH is applied to an ADP model, but their mathematical models differ notably from ours by not including hard demand constraints and speed optimization. These differences have implications for the application of a rolling horizon heuristic. They are discussed in this chapter.

Section 6.1 presents a general overview of the main features of the RHH heuristic. Section 6.2 dives deeper into the features of the different horizons, while Section 6.3 explains the necessary constraint relaxations in each iteration of the RHH and their implications. Finally, Section 6.4 describes a construction heuristic that finds an initial feasible solution that allows for a warm start.

6.1 Overview of the Rolling Horizon Heuristic

The general idea of the RHH is to solve shorter sub-horizons (subproblems) of the MILP using a commercial solver. This is done by splitting the entire planning horizon into several parts, solving the subproblems iteratively until a complete solution is generated, as illustrated in Figure 6.1. A forecast horizon (FH) is included in each subproblem to make the current central horizon (CH) influenced by what is to come, as illustrated with a dotted line in each iteration k in Figure 6.1. When the algorithm moves from one sub-horizon to the next (iteration k to $k+1$), some variables from the central horizon CH_k are frozen, while the sub-horizon shift so that the whole or the first part of FH_k becomes CH_{k+1}.

Please note that in previous literature, e.g., in Rakke et al. (2011), the following terminology has been used to describe the sub-horizons; the central period, the forecasting period, and the frozen period. However, as *periods* is used to describe the time increments in the mathematical formulation of the LNG-ADP-SO-MLP, we avoid the dual use of the word "period" by using the following terminology instead; the central horizon, the forecasting horizon, and the frozen horizon, respectively.

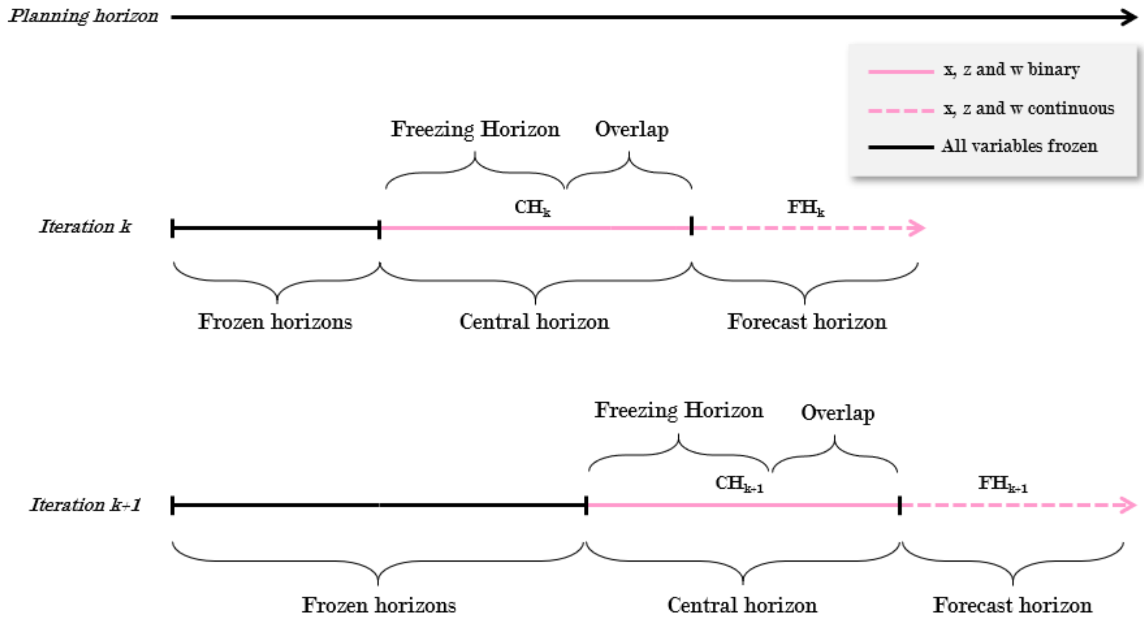


Figure 6.1: *The rolling horizon heuristic.*

Due to the problem characteristics, three additional features are included in the RHH to avoid infeasibility and far-from-optimal solutions: a relaxation of the demand- and berth constraints an extension of the very last forecast horizon, and an extension of the very last central horizon. These features are elaborated upon in the coming section. The RHH algorithm is described in pseudo-code in Algorithm 3. Please note that in the pseudo-code, the "overlap" denotes the time in the central horizon that is not a part of the freezing horizon, and the "central horizon" in the pseudo-code only encapsulates the freezing horizon.

The Rolling Horizon Heuristic (RHH) detailed in Algorithm 3 solves the problem iteratively over a sequence of horizons. The algorithm starts by initializing the model for the first central horizon and forecast horizon. The variables, horizon extension (β), stop status (δ), iteration count (ι), and constraints stop time (υ) are all initialized at the outset.

During each iteration of the algorithm, the model is solved using a commercial solver. If the stop status (δ) has been set to True, the while loop is terminated. The stop time for the constraints (υ) is updated to reflect the current iteration's time limit. The optimized decision variables for the central horizon are frozen, and the forecast horizon variables are removed. When the next iteration's central horizon would reach near the end of the loading days, β is updated to reflect the remaining length of the loading days after the next central horizon, ensuring that the central horizon is extended if there is less than half a forecast horizon left. Subsequently, the model variables for the next central horizon are added. These variables represent the days in the next central horizon, the overlap, and the possible extension. If there is still room for a forecast horizon (implying that β is zero), the β is updated to reflect the remaining length of the loading days after the next central horizon, overlap, and forecast horizon. Subsequently, the forecast horizon variables are added to the model. The extended horizon (β) is then reset to zero in preparation for the next iteration.

Algorithm 3 Rolling Horizon Heuristic

Input: Length of loading days, length of all days, central horizon length $|CH|$, forecast horizon length $|FH|$, overlap length, $|O|$

Output: Best solution

Initialise model for the first central horizon and forecast horizon
Initialise variable horizon extension, $\beta \leftarrow 0$
Initialise stop status, $\delta \leftarrow$ False
Initialise variable iteration count, $\iota \leftarrow 0$
Initialise variable for constraints stop time, υ
while entire horizon not covered **do**
    optimize model with commercial solver
    if δ = True **then**
        end while
    end if
    $\upsilon \leftarrow |CH| \cdot (\iota{+}2) + |FH|$ **if** $|CH| \cdot (\iota{+}2){+}|FH| <$ length of loading days **else** $\upsilon \leftarrow$ length of all days
    Freeze central horizon variables
    Remove forecast horizon variables
    if $|CH| \cdot (\iota{+}2) >=$ length of loading days - $(|FH|\ /\ 2)$ **and** $|CH| \cdot (\iota{+}2) <$ length of loading days **then**
        β = length of loading days - $(|CH| \cdot (\iota{+}2))$
    end if
    Add variables for days from $|CH| \cdot (\iota{+}1)$ to $|CH| \cdot (\iota{+}2) + |O| + \beta$
    if $|CH| \cdot (\iota{+}2) + \beta <$ length of loading days **then**
        if $|CH| \cdot (\iota{+}2) + |FH| >=$ length of loading days - $|FH|$ **and** $|CH| \cdot (\iota{+}2) + |FH| <$ length of loading days **then**
            β = length of loading days - $(|CH| \cdot (\iota{+}2) + |FH|)$
        end if
        Add variables for days from $|CH| \cdot (\iota{+}2) + |O|$ to $|CH| \cdot (\iota{+}2) + |O| + |FH| + \beta$
        $\beta = 0$
    end if
    Remove constraints
    Re-initialise constraints based on new variables and υ
    ι += 1
    if $|CH| \cdot (\iota{+}1){+}\beta >=$ length of loading days **then**
        δ = True
    end if
end while

Please note that the variable β, representing the horizon extension, does not have a value for the forecast horizon if it has a value for the central horizon. This is because β is only modified if there is room for a forecast horizon, and if the central horizon β is activated, it automatically covers the rest of the loading days. Therefore, these two horizons (central and forecast) never share an extension value within the same iteration.

All the constraints from the previous iteration are subsequently removed. These constraints are then reinitialized based on the newly added variables and the updated stop time for constraints (υ). The iteration count (ι) is incremented at the end of each cycle. If the next central horizon (including any possible extension) would surpass the total length of the loading days, the stop status (δ) is set to True. This indicates that the entire horizon has been covered, signaling the end of the iterations in the next cycle.

6.2 The RHH iteration horizons

As explained in Section 6.1, each iteration of the algorithm has three horizons; the central horizon, the forecasting horizon, and the frozen horizon. The characteristics of each horizon type and how they affect the decision variables are elaborated upon in the following sections.

6.2.1 The Central Horizon

The x, z, and w variables are treated as binary in the entire central horizon. Variables defined in the mathematical model as continuous, like the g- and s-variables, remain continuous. All variables in the freezing horizon will be frozen in the next iteration and thereby be a part of the final solution. For all horizons except the first, the commercial solver starts every central horizon with variable hints from the past forecast horizon. This means that the model is initialized with a near-feasible solution, which speeds up the simplex algorithm prior to the branch and bound.

6.2.2 The Forecasting Horizon

To increase the likelihood of solutions that are both feasible and have high objective values in the central horizon, a forecasting horizon is applied. This way solutions that are obviously sub-optimal beyond the central horizon are to a larger extent avoided. The forecasting horizon is relaxed by making the binary variables continuous since it could make the problem easier to solve as it could limit the computational time for the branch & bound in the Gurobi solver. Below is an overview of the relaxed variables in this period, and the expected effects of relaxing them.

- **The x-variables:** As the number of x-variables outnumbers the other variables by far, the relaxation of the x-variables is likely the most effective relaxation. In effect, this allows that one vessel can follow several arcs out of one node and deliver parts of the loads in the forecasting horizon.
- **The z-variables:** The z-variables are also relaxed. In effect, this relaxation allows a FOB cargo to pick up part of the loads.
- **The w-variables:** The w-variables are relaxed, enabling partial charter cargo loads. The relaxation allows the g-variable to freely accommodate loads ranging in size from 0 to the upper capacity limit when it is constrained in constraints (5.13).

6.2.3 The Frozen Horizon

When the algorithm reaches the computational time limit, finds the optimal solution within a given horizon, or finds a feasible solution within the pre-defined optimality gap limit, the variables within the central horizon are subject to a freezing strategy before proceeding to the next iteration. Various freezing strategies have been examined in the literature, with Rakke et al. (2011) highlighting two of them: freezing all decision variables from the preceding central horizons, or solely freezing some of them. In the context of the multi-item capacitated lot-sizing problem introduced by Mercé and Fontan (2003), the latter strategy has demonstrated superior efficiency. However, Rakke et al. (2011) argue that implementing this freezing strategy in the LNG-ADP Planning Problem becomes impractical due to the problem's size, thereby opting for the first strategy.

Unlike Rakke et al. (2011), who are also addressing the LNG-ADP problem, we have adopted a freezing strategy that incorporates different lengths for the freezing horizon and the central horizon. By doing so, we maintain binary variables for the initial days of the forecast horizon, thereby reducing the chances of encountering infeasible solutions. This approach also allows us to avoid suboptimal solutions by not relaxing all of the time periods past the freezing horizons.

6.3 Constraint Relaxations

The direct application of the RHH heuristic used by Rakke et al. (2011) on the mathematical model for the LNG-ADP-SO-MLP is not always feasible. In Rakke et al. (2011), the demand that is not fulfilled in the partitions overarching the horizons is transferred to the next horizon. This is possible because Rakke et al. (2011)'s mathematical model contains a relaxation of the demand constraint, where over-and under-delivery is permitted, but punished in the objective function. With hard upper and lower demand constraints, the model risks having a very small delivery window at the end of each horizon to fulfill the upper and lower demand limits, especially if the minimum spread constraint stops tight delivery schedules. The problem is illustrated in Figure 6.2, where a demand partition is only barely present in the current horizons. To avoid this issue, we apply a demand relaxation of the partitions that have an ending period far outside the current forecast horizon.

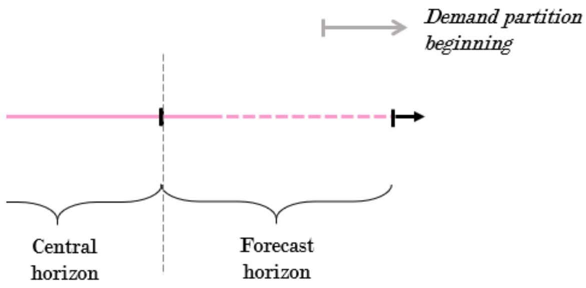


Figure 6.2: *The figure illustrates a partition that is only partially present in the horizon and gets too tight as a consequence*

Constraints that sum over time partitions into the future are relaxed if the end of the partition is too far outside of the current forecast horizon. Figure 6.3 shows one iteration of the RHH with three examples of demand constraints that all sum over time periods within a partition. "Demand partition example 1" in Figure 6.3 exemplifies a constraint that is relaxed because its ending point is outside of the current forecast horizon. "Demand partition example 2" exemplifies an active constraint with its ending point inside the current central and forecast horizons.

An advantagee of this relaxation strategy compared to Rakke et al. (2011) is that the objective function is kept free of non-monetary "punishing" terms. A disadvantage is that the relaxation strategy sets certain boundaries on the RHH parameters. Two consequences of the constraint relaxation if we want to ensure feasibility are:

- The central horizon must be longer than the longest partition
- The forecast horizon must be longer than the longest partition plus the longest sailing arc

"Demand partition example 3" in Figure 6.3 is longer than the current central horizon and illustrates the issue. In the previous iteration, the constraint summing over the partition would have been relaxed since it ends outside of the forecast horizon. In the current iteration, the constraint is activated, but some variables from the previous central horizon have been frozen without this constraint being active. Since the constraint was not active before these variables were frozen, the likelihood of infeasibility and poor solutions is large. This is particularly a problem if the horizon is significantly smaller than the partition. This is handled in the RHH by making sure the number of time periods in the central horizon and forecasting horizon is similar in size or greater than the number of time periods in the largest partition in the model. The problem also to applies to the sum of future periods that happens in the berth constraints.

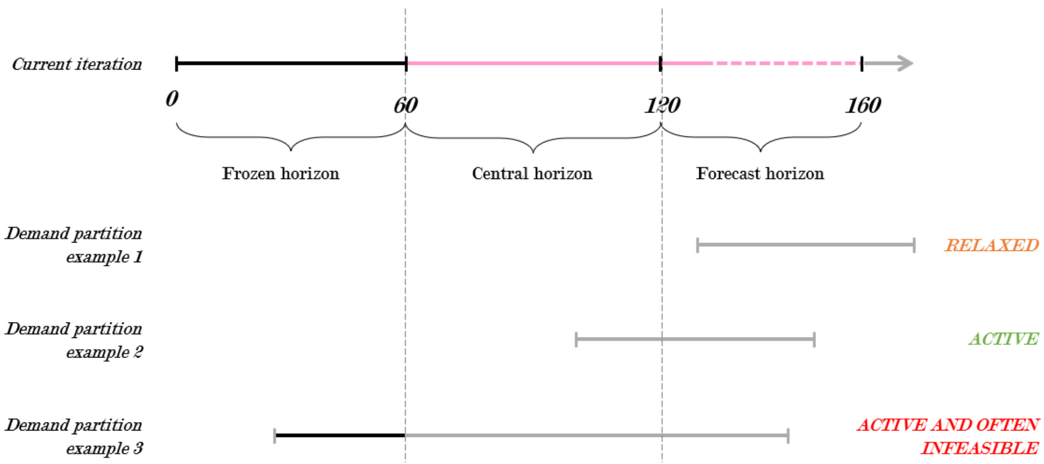


Figure 6.3: *The figure illustrates why the central horizon must be larger than the largest partition to minimize the likelihood of infeasibilities*

Both the demand and berth constraints are affected by the issue exemplified in "Demand partition example 3" in Figure 6.3. The demand constraints (5.8) and (5.10) sum over the time periods in the demand partitions of the DES long-term contracts and the FOB long-term contracts. This means that horizons shorter than the demand partitions and FOB time windows might lead to infeasibility and/or worse solutions. The berth constraints (5.12) sum over all future time periods where an arrival implies that the berth is occupied in the current time period. This implies that the horizon size should be larger than the longest sailing arc or at least similar in size.

Due to the constraint relaxations described above, a dynamic extension of the last forecasting horizon is performed for the second to last iteration. This is illustrated in Figure 6.4. If this was not performed, some constraints would only be active for a short period at the end which often leads to infeasibility. In addition, a dynamic extension of the last central horizon is performed for the last iteration to increase the model efficiency.

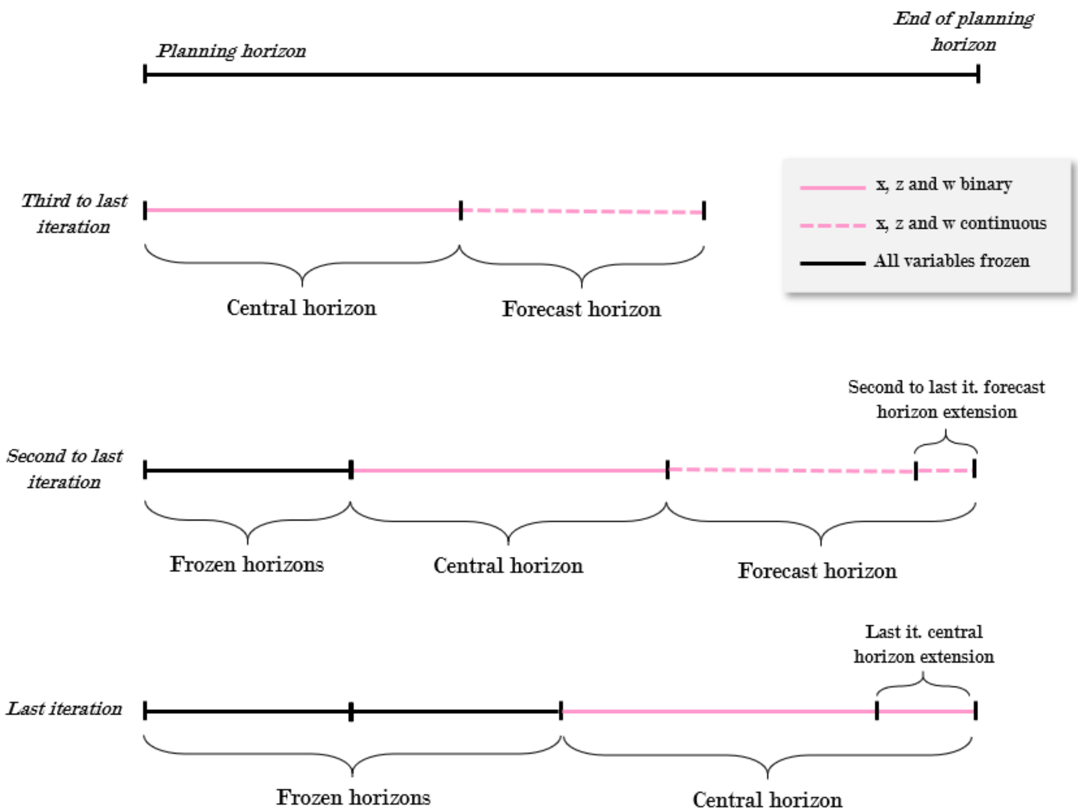


Figure 6.4: *A dynamic extension of the last forecasting horizon is performed for the second to last iteration to avoid infeasibility, and the last central horizon is extended for model efficiency.*

6.4 Incorporating Warm Start into the RHH

As mentioned in Section 6.2, the first iteration of the RHH algorithm is not started warm since there is no previous forecast horizon to base the warm start on. This will typically lead to a higher solution time than compared to the succeeding iterations. To test how a warm start for the first iteration affects the performance of the RHH, we, therefore, devised a greedy construction heuristic, outlined in Algorithm 4, to generate an initial solution for the problem.

The greedy construction heuristic takes advantage of that the mathematical model always assumes there is an available charter vessel for a given cargo, described in Section 5.1.5, and that loading ports can ship a pre-defined amount of LNG every day to an artificial FOB spot customer, mentioned in Section 5.1.6. Consequently, the initial solution obtained through this heuristic involves the non-utilization of any vessel, except those requiring maintenance, while the producer exclusively relies on charter vessels to meet the demand specified in the DES contract.

Algorithm 4 Greedy Randomized Construction Heuristic

Input: Decision variables, and sets and parameters
Output: Decisions variables with updated values

```
for vessel in set of vessels do
    if vessel requires maintenance then
        find shortest feasible route for vessel, set all other arcs to 0
    else
        set exit arc to 1, and all other arcs to 0
    end if
end for
for g,w,z in sets of g-, w- and z-variables do
    set g, w and z to 0
end for
for i,t in set of s-variables' keys do
    s[i,t] = s[i,t-1] + lng produced at day t at loading port i
end for
all demand satisfied = False
allocate a random pick-up day for each FOB contract f in FOB contract IDs
for loading day t in all loading days do                    ▷ satisfy DES- and FOB-demand
    for loading port l in all loading ports do
        for b in range(0, number of berths for loading port i) do
            check if inventory and berth availability is feasible at day t for loading port l
            for FOB contract f in FOB contract IDs do
                if pick-up for FOB contract f is scheduled at day t at l then
                    if allocating f is feasible then
                        z[f, t] = 1 and update inventory for l
                    else
                        schedule pick-up for f to t+1 and continue
                    end if
                end if
            end for
            best partition, contract = Find Best Partition (Algorithm 5)
            missing demand = lower demand for best partition - amount chartered[best partition]
            if ⌈missing demand / lower charter amount⌉ == ⌈missing demand / upper charter amount⌉ then
                amount = lower charter amount
            else
                amount = ⌈missing demand / ⌈missing demand / upper charter amount⌉⌉
            end if
            if chartering amount to best partiton from loading port l on day t is feasible then
                g[l, t, contract] = amount and w[l, t, contract] = 1
                Update inventory for l and amount chartered for best partition
                if demand is satisfied for contract c then
                    remove c from DES-contract ids
                end if
                if all demand is satisfied then
                    all demand satisfied = True
                    break
                end if
            end if
        end for
        if inventory for l in day t > upper inventory limit then
            z[artificial FOB for l, t] = 1 and update inventory for l
        end if
    end for
end for
if not all demand satisfied then                            ▷ try to satisfy leftover DES-demand
    Satisfy Leftover Demand (Algorithm 6)
end if
```

As implied in line 15, FOB contracts get allocated random pick-up dates from their pre-defined set of feasible pick-up dates. More specifically, each FOB contract f is assigned a pick-up day randomly selected from f's pick-up days$[\lceil \frac{\mu}{2} \rceil : \mu - \alpha]$, where μ is the length of f's pick-up days. In practice, this means that the algorithm tries to create solutions where FOB contracts get picked up relatively late. This intuitively makes sense since LNG sent to a FOB contract f on day t also satisfies demand on day t, while LNG sent to DES contracts on day t satisfies demand on day $t +$ the corresponding charter sailing time. Note that α is subtracted from the end-point of the pick-up range to ensure that the pick-up date can be postponed with α days if the original allocation is infeasible.

Lines 30-34 define the charter amount for a chosen best partition from line 29. This is done by trying to satisfy the partition's lower demand by sending as little LNG and as few cargoes as possible. In line 53-54 the algorithm tries to satisfy leftover demand if not all the contracts are satisfied, by calling Algorithm 6, presented in the Appendix D.

Algorithm 5 presents the greedy element of the Algorithm 4 (line 29), where the partition with the highest amount missing from satisfying minimum required demand to the number of days left of delivery-ratio, referred to as *score* in the pseudo-code, gets prioritized when allocating charter cargoes.

Algorithm 5 Find Best Partition

Input: loading day t, loading port l
Output: the best partition and the corresponding contract c

```
best partition = None
best score = 0
charter amount = upper charter amount
for c in set of DES-contracts do
    if minimum spread is infeasible for contract c then
        Continue
    end if
    for p in set of partitions for contract c do
        last day = last element in p's set of partition days
        amount missing = p's lower partition demand - amount chartered to p
        days left = last day - charter sailing time from l to c - t
        score = amount missing / days left
        if chartering charter amount to partition p from l is feasible then
            if score > best score then
                best partition = p
                best score = score
            end if
        end if
    end for
end for
```

Note that this greedy assumption not always finds a feasible solution. This is because the problem does not hold the "Greedy Property", which means being globally solvable optimally by making optimal decisions locally. Due to close bounds between upper- and lower partition demand and inventory constraints, allocating a charter cargo to a partition on a given day can in practice prevent a later partition from satisfying its demand within its partition days.

In some special cases, there can a surplus of either FOB or DES demands for the algorithm to satisfy, where Algorithm 4 is insufficient in finding a feasible initial solution to use for warm start. Because of this, an alternative greedy construction heuristic, Algorithm 8, was implemented, where both FOB- and DES contracts get assigned scores and are scored against each other for each loading day. This algorithm is presented and elaborated upon in Appendix E.

Chapter 7

Data and Test Instances

This chapter presents the characteristics and modifications of the data sets used to test the model. Section 7.1 and Section 7.2 present the two real-life producer cases the test instances are based on and their relevant data, including loading and customer ports and vessels. Section 7.3 describes how the data was modified based on the mathematical extensions described in Chapter 5. Lastly, Section 7.4 presents the test instances and how they are generated.

7.1 The Nigeria Case

Nigeria LNG Limited (NLNG) is one of the world's top suppliers of LNG. NLNG was incorporated as a Limited Liability Company on May 17, 1989, to harness Nigeria's vast natural gas resources and produce Liquefied Natural Gas and Natural Gas Liquids for export. Today, NLNG has a total production capacity of 22 Million Tons Per Annum (mtpa) of LNG and 5 mtpa of Natural Gas Liquids from its six-train plant complex (Nigeria LNG Ltd., 2023).

7.1.1 NLNG's Loading Ports and Customer Ports

Data from NLNG is used to create instances with one loading port. This loading port, referred to as 'NGBON', is located at Bonny Island in Nigeria, and is marked with a pink dot in Figure 7.1.

Figure 7.1: *Map of Nigeria LNG's loading port.*

In all the test instances based on the NLNG case, NGBON produces 127 250 m^3 of LNG daily and has minimum- and maximum inventory limits of 40 000 and 336 000, respectively. The loading port also operates two berths, meaning a maximum of two vessels can be docked on the same day. This information is summarized in Table 7.1.

Loading port	**Production rate** [m^3 of LNG]	**Min inventory limit** [m^3 of LNG]	**Max inventory limit** [m^3 of LNG]	**Number of berths** [berths]
NGBON	127 250	40 000	336 000	2

Table 7.1: *NLNG loading port properties: Production rate, minimum- and maximum inventory limit, and number of berths.*

NLNG has a total of 22 potential customers, serving the European, South American, Middle East, and Far East markets. These customers have the flexibility to engage in both ordinary- and spot DES- and FOB contracts. Furthermore, NLNG's fleet of vessels undergoes maintenance at a port located in Singapore. A summary of this information is presented in Table 7.2, and a visualization of all the ports on a world map is presented in Figure 7.2.

Port ID	**Location**	**Port type**
NGBON	Nigeria	Loading port
AE	United Arab Emirates	Unloading port
AR	Argentina	Unloading port
BD	Bangladesh	Unloading port
BE	Belgium	Unloading port
BR	Brazil	Unloading port
CN	China	Unloading port
ES	Spain	Unloading port
FR	France	Unloading port
GB	Great Britain	Unloading port
IN	India	Unloading port
IT	Italia	Unloading port
JP	Japan	Unloading port
KR	South Korea	Unloading port
KW	Kuwait	Unloading port
MX	Mexico	Unloading port
NL	Netherlands	Unloading port
PT	Portugal	Unloading port
TH	Thailand	Unloading port
TR	Turkey	Unloading port
TW	Taiwan	Unloading port
US	USA	Unloading port
SG	Singapore	Maintenance port

Table 7.2: *Overview of all ports in the NLNG-case.*

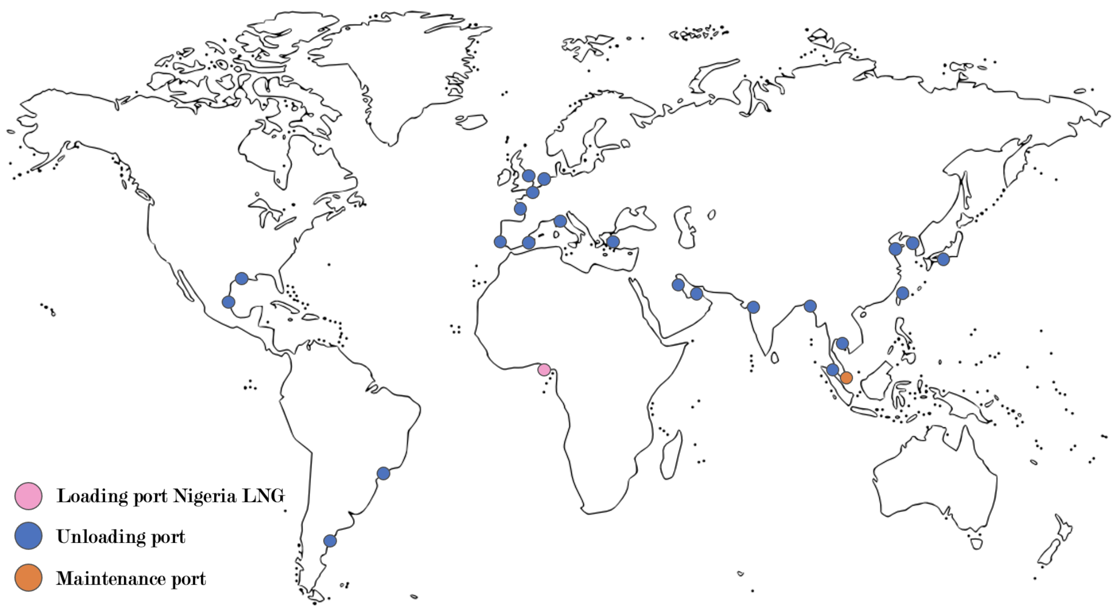


Figure 7.2: *Map of Nigeria LNG's loading-, customer- and maintenance ports.*

All data describing customers' locations, partitions, and demands are provided for each test instance. Due to confidentiality requirements regarding contracts between the producer and customers, modified (but still realistic) LNG prices were used in the instances, ranging from 60 - 155 USD/m^3 for ordinary DES- and FOB contracts, dependent on the customer and time period. For DES- and FOB spot contracts, prices range from 90 - 200 USD/m^3.

Sailing distances among all loading and unloading ports are provided, as well as distances to and from the maintenance port. The longest distance between two ports in the NLNG data is 13 088 nautical miles (or 24 239 kilometers), which is the distance between the unloading port in the United States and the maintenance port in Singapore. The lowest possible sailing speed is nine knots, which makes the maximum travel time in the instances 60 sailing days each way. All distances between ports are presented in Appendix F.

Data describing the artificial spot contract mentioned in Section 5.1.6 is created manually for each instance, and typical values are presented in Table 7.3. Each loading port has a default loading quantity, corresponding to the size of a typical cargo of LNG, and this is used as the demand input for the artificial spot contracts. The artificial spot contract revenue is set conservatively to 0 USD/m^3 since the producers are uncertain whether or not they can actually sell the LNG in the spot market or must handle the excess inventory by cutting production.

Customer type	**Demand** [m^3 of LNG]	**Price** [1000 USD/day]
Artificial spot FOB	150 000	0

Table 7.3: *Artificial spot properties: demand and price.*

7.1.2 NLNG's Vessels

Producer vessels

The complete list of NLNG's 23 vessels is presented in Table 7.4. These correspond to actual vessels operated by the producer in real life. A majority of the vessels have different capacities, which makes the fleet heterogeneous. Furthermore, most of the vessels have different speed profiles, with varying minimum and maximum speeds, in addition to a service speed, which is the vessel's default sailing speed.

Vessel Name	**Capacity** [m^3 of LNG]	**Min speed** [knots]	**Max speed** [knots]	**Service speed** [knots]
LNG Bayelsa	137 000	10	20	17
LNG Sokoto	137 000	10	20	17
LNG Rivers	137 000	10	20	17
LNG Cross River	141 000	12	20	16
LNG River Niger	141 000	12	20	16
LNG Adamawa	141 000	12	20	16
LNG Akwa Ibom	141 000	12	20	16
LNG Abalamabie	175 000	9	20	16
LNG Abuja II	175 000	9	20	16
LNG Finima II	175 000	9	20	16
LNG Port Harcourt II	175 000	9	20	16
LNG Bonny II	177 000	9	20	16
LNG Lagos II	177 000	9	20	16
LNG Borno	150 000	14	20	16
LNG Ogun	150 000	14	20	16
LNG Benue	146 000	14	20	16
LNG Enugu	146 000	14	20	16
LNG Oyo	146 000	14	20	16
LNG River Orashi	146 000	14	20	16
LNG Imo	148 000	14	20	16
LNG Kano	148 000	14	20	16
LNG Lokoja	148 000	14	20	16
LNG Ondo	148 000	14	20	16

Table 7.4: *The complete lists of NLNG's LNG vessels, with different capacities, and minimum-, maximum-, and service speeds.*

Data describing the vessels' start location, availability throughout the planning period, boil-off rate, and compatible ports were all given for each NLNG test instance by Quorum and corresponded to real-life information about the vessels. Information about which vessels require maintenance and the corresponding start times, locations, and durations is also given in each NLNG test instance. There is normally one ship per instance that require maintenance.

Charter vessels

As mentioned in Section 5.1, we assume that there is an available charter vessel with the same capacity as the cargo the producer wants to be transported, as long as it is within the bounds. This is implemented by defining upper and lower capacity limits for charter vessels, ranging from the smallest to the largest possible cargo of LNG in the data sets. All other charter vessel attributes are specified directly in the instances. The daily cost of chartering a vessel can vary significantly throughout the year. The producer's charter price assumptions are included in the test instances and are based on real-life data from the industry. Fuel costs for charter vessels are included in the daily charter rate, and therefore the charter vessels sail at a given default sailing speed. Typical values for charter vessels for a given instance are presented in Table 7.5.

Vessel type	**Capacity** [m^3 of LNG]	**Price range** [1000 USD/day]	**Default speed** [knots]
Charter vessel	100 000 - 180 000	60 - 80	17.5

Table 7.5: *Charter vessel properties: Capacity, price range, and default speed.*

The vessels have different ballast- and laden fuel consumption rates, based on their given ballast- and laden speed profile. A summary of the range of different ballast- and laden fuel consumptions is given by Table 7.6 and Table 7.7. The fuel consumption for ballast speed is lower than for laden speed because the vessel is lighter without cargo. The fuel consumption rates are used to calculate the costs of the arcs in the network based on the given sailing speed, by multiplying the corresponding fuel consumption per day by the number of days and the fuel price. The fuel price is also given and is 350 USD for all instances, as suggested in the data provided by our industry partner.

Ballast speed [knots]	**Fuel consumption range** [tonnes/day]
9	24.0 - 28.0
10	33.0 - 57.8
11	34.0 - 64.2
12	36.0 - 72.8
13	51.0 - 81.9
14	63.0 - 103.0
15	72.0 - 107.1
16	85.0 - 119.0
17	98.0 - 133.0
18	104.0 - 144.9
19	107.9 - 159.6
20	120.0 - 183.0

Table 7.6: *Ballast speed consumption NLNG*

Laden speed [knots]	**Fuel consumption range** [tonnes/day]
9	28.0 - 33.0
10	34.0 - 66.3
11	43.0 - 73.8
12	39.0 - 82.4
13	51.0 - 91.0
14	60.0 - 105.0
15	71.0 - 112.4
16	90.0 - 125.2
17	97.0 - 139.1
18	106.0 - 155.2
19	115.0 - 173.3
20	122.0 - 192.6

Table 7.7: *Laden speed consumption NLNG*

Vessel operational time

For each loading-, unloading- and maintenance operation there is an associated operational time, describing the time needed for loading-, unloading- and maintenance operations in the ports, respectively. This is normally computed by using data describing vessels' and ports' loading- and unloading rates, which is provided in the test instances. Since the operational times vary only slightly from vessel to vessel in the test instances, and the length of our time periods is relatively long (1 day), a simplification was made by fixing the operational time in each port. As presented in Table 7.8, all unloading and loading operations are set to take 24 hours, i.e., one time period. As for maintenance, when a vessel returns to a loading port after performing maintenance, it has to cool down for an extra time period making the operational time 48 hours instead, i.e., two time periods. Lastly, operational time corresponding to maintenance duration varies a lot from vessel to vessel, so this is an input parameter that comes directly from the data in the instances.

From port	**To** port	**Operational time** [days]
Loading port	Unloading port	1
Unloading port	Loading port	1
Maintenance port	Loading port	2
Unloading port	Maintenance port	pre-defined

Table 7.8: *Different port combinations and corresponding operational times for the operation performed in the port in the **To**-column, if you sail from the port in the **From**-column.*

As described in Section 5.1.2, the vessels are allowed to wait for a number of days outside all ports, increasing the flexibility of the model. The maximum allowed number of waiting time periods is set to seven for all test instances, i.e., a week.

7.2 The Abu Dhabi Case

Abu Dhabi National Oil Company LNG (ADNOC LNG) is a subsidiary of Abu Dhabi National Oil Company, which is wholly owned by the Abu Dhabi Government. The company operates as an oil and gas company that produces and markets LNG and LPG gas products (Offshore Technology, 2023).

All assumptions regarding available data for ports and vessels for NLNG made in Section 7.1 applies for the ADNOC LNG case as well. This section only presents data that differ from the NLNG case.

7.2.1 ADNOC LNG's Loading Ports and Customer Ports

Data from ADNOC LNG is used to create test instances with two loading ports. These loading ports, referred to as 'DI' and 'FU', are located at Das Island and Fujairah respectively, both locations in the United Arab Emirates. Das Island is an island and is marked with the leftmost green dot in Figure 7.3, while Fujairah is marked with the rightmost dot.

Figure 7.3: *Map of ADNOC LNG's loading ports.*

FU and DI have individual production rates, minimum- and maximum inventory limits, and number of berths. This information is summarized in Table 7.9.

Loading port	**Production rate** [m^3 of LNG]	**Min inventor limit** [m^3 of LNG]	**Max inventory limit limit** [m^3 of LNG]	**Number of berths** [berths]
FU	56 000	50 000	300 000	2
DI	34 000	30 000	250 000	1

Table 7.9: *ADNOC loading ports' properties: Production rate, minimum inventory limit, and maximum inventory limit.*

ADNOC LNG has six customer ports, located in European and Far East markets. The maintenance port is the same as for the NLNG case, located in Singapore. A summary of this information is presented in Table 7.10, and a visualization of all the ports on a world map is presented in Figure 7.4.

Port ID	Location	Port type
DI	Das Island	Loading port
FU	Fujairah	Loading port
BE	Belgium	Unloading port
CN	China	Unloading port
ES	Spain	Unloading port
IN	India	Unloading port
JP	Japan	Unloading port
KR	South Korea	Unloading port
SG	Singapore	Maintenance port

Table 7.10: *Overview of all ports in the ADNOC LNG case.*

Figure 7.4: *Map of ADNOC LNG's loading-, customer- and maintenance ports.*

In the ADNOC LNG case, prices range from 70 - 130 USD/m^3 for ordinary DES- and FOB contracts, and 70 - 200 USD/m^3 for DES- and FOB spot contracts.

7.2.2 ADNOC LNG's Vessels

ADNOC LNG operates a fleet of 15 LNG vessels with different capacities, and minimum- and maximum- and service speeds. A summary of this data is presented in Table 7.11.

The vessels' ballast- and laden fuel consumptions vary less than for the NLNG case and are presented in Table 7.12 and Table 7.13.

Charter capacities, -price range, and -default speeds, as well as vessel operational times, for the ADNOC LNG case are identical to the NLNG case's values presented in Table 7.5 and Table 7.8.

Vessel Name	Capacity [m^3 of LNG]	Min speed [knots]	Max speed [knots]	Service speed [knots]
AD-1	137 000	10	20	17
AD-2	137 000	10	20	17
AD-3	137 500	10	20	17
AD-4	137 000	12	20	16
AD-5	136 000	12	20	16
AD-6	137 500	12	20	16
AD-7	137 000	12	20	16
AD-8	165 000	12	20	16
AD-9	168 000	12	20	16
AD-10	168 000	12	20	16
AD-11	168 000	12	20	16
AD-12	156 000	12	20	16
AD-13	168 000	12	20	16
AD-14	168 000	12	20	16
AD-15	170 000	12	20	16

Table 7.11: *The complete lists of the ADNOC LNG's vessels, with different capacities, and minimum, maximum, and service speeds.*

Ballast speed [knots]	Fuel consumption range [tonnes/day]
10	57.8
11	64.2
12	71.4 - 72.8
13	80.3 - 81.9
14	89.9 - 95.55
15	99.5 - 107.1
16	11.3 - 199.7
17	125.2 - 132.3
18	139.1 - 144.9
19	156.0 - 159.6
20	175.5 - 174.3

Table 7.12: *Ballast speed consumption ADNOC*

Laden speed [knots]	Fuel consumption range [tonnes/day]
10	66.3
11	73.8
12	75.3 - 82.4
13	86.9 - 91.0
14	99.6 - 101.7
15	112.4 - 112.5
16	125.1 - 125.2
17	137.8 - 139.1
18	151.6 - 155.2
19	166.4 - 173.3
20	182.3 - 192.6

Table 7.13: *Laden speed consumption ADNOC*

7.3 Problem Extensions

As described in Section 5.6 and Section 5.7, the mathematical model was extended with the possibility to both vary production throughout the planning period and charter out the producer's vessels. This section presents data and assumptions made for the implementation of the new extensions.

7.3.1 Variable Production

We decided that the lower limit for production should be 30% of normal production in collaboration with our industry partner, based on what they thought would be most realistic. Values for minimum and maximum production for each loading port for the two cases are presented in Table 7.14

Loading port	**Production rate** [m^3 of LNG]	**Min production** [m^3 of LNG]	**Max production** [m^3 of LNG]
NGBON	127 250	38 175	127 250
FU	56 000	16 800	56 000
DI	34 000	10 200	34 000

Table 7.14: *NGBON's, FU's, and DI's values for minimum- and maximum production rate.*

7.3.2 Chartering Out Vessels

We chose the minimum period for a vessel to be chartered to be 60 days. This duration corresponds to an ordinary roundtrip and is considered to be the most realistic option since customers want to charter a vessel for some time horizon. Therefore, it is not feasible to charter a vessel for a shorter period.

Also, we set the daily charter revenue to be 5 % of the charter costs presented in Table 7.5, in order to prevent the model from chartering out all the producer vessels. As long as the revenue is positive, the model would incentivize to charter out vessels with a positive contribution to the objective function, but only if there is surplus fleet capacity.

7.4 Test Instances

Based on the presented data, Quorum Software provided us with six different datasets, three from each producer case. Table 7.15 presents the eight different datasets, with the corresponding number of loading- and unloading ports $|\mathcal{N}^L|$ and $|\mathcal{N}^U|$, long-term FOB contracts $|\mathcal{F}^U|$, and vessels $|\mathcal{V}|$. Note that all datasets have a number of time periods equal to 365 days, in other words, a full year. Dataset **A-2L-D** and **A-2L-E**, each marked with a star *****, was created based on datasets **A-2L-C** and **A-2L-B**, respectively, by both removing and altering ordinary DES- and FOB contracts.

Dataset ID	$|\mathcal{N}^L|$	$|\mathcal{N}^U|$	$|\mathcal{F}^U|$	$|\mathcal{V}|$	$|\mathcal{T}|$
N-1L-A	1	16	33	23	365
N-1L-B	1	13	32	23	365
N-1L-C	1	14	40	23	365
A-2L-A	2	6	41	15	365
A-2L-B	2	6	55	15	365
A-2L-C	2	6	23	15	365
A-2L-D*	2	6	22	15	365
A-2L-E*	2	6	51	15	355

Table 7.15: *Overview of the different datasets' problem sizes, differing in the number of loading ports $|\mathcal{N}^L|$, the average number of unloading ports $|\mathcal{N}^U|$, average number of long-term FOB contracts $|\mathcal{F}^U|$, average number of vessels $|\mathcal{V}|$ and number of time periods $|\mathcal{T}|$.*

The black horizontal line splits the datasets into two different sets, where the upper set of datasets is based on data from NLNG, and the set at the bottom is based on data from ADNOC LNG. This is also indicated by the dataset's name, where datasets based on NLNG start with 'N' and have one loading port (1L), while datasets based on ADNOC LNG start with 'A' and have two loading ports (2L).

For each dataset, four test instances were created, with 4-, 6-, 8-, and 12- months in the planning horizon. This was done by shortening down the planning period in the original dataset and removing contracts extending the new planning horizon. An example of this is provided in Table 7.16, where dataset **A-2L-B** has been broken down into four test instances: **A-2L-B-4M**, **A-2L-B-6M**, **A-2L-B-8M**, and **A-2L-B-12M**. The observant reader may notice that test instance **A-2L-B-12M** corresponds to the dataset **A-2L-B** 's original data.

Instance ID	$\lvert\mathcal{N}^L\rvert$	$\lvert\mathcal{N}^U\rvert$	$\lvert\mathcal{F}^U\rvert$	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{T}\rvert$
A-2L-B-4M	2	6	20	15	120
A-2L-B-6M	2	6	30	15	280
A-2L-B-8M	2	6	38	15	240
A-2L-B-12M	2	6	55	15	365

Table 7.16: *Overview of one dataset's corresponding instances' problem sizes, differing in the number of loading ports $|\mathcal{N}^L|$, average number of unloading ports $|\mathcal{N}^U|$, average number of long-term FOB contracts $|\mathcal{F}^U|$, average number of vessels $|\mathcal{V}|$ and number of time periods $|\mathcal{T}|$.*

A full overview of all the test instances is presented in Appendix G. Results from the testing of different datasets is presented in Chapter 8.

Chapter 8

Computational Study

In this chapter, we present the computational study of the RHH heuristic's performance when solving instances of the LNG-ADP-SO-MLP. Section 8.1 describes the test environment and technical implementation details, as well as the test approach. Section 8.2 explains the parameter tuning procedure which involves studying different sets of RHH parameters as well as the effect of the warm start algorithm. An in-depth computational description of the best configurations is presented. Section Section 8.3 presents the performance of the heuristic compared to that of a commercial solver. Lastly, Section 8.4 presents managerial insights.

8.1 Test Environment and Approach

Table 8.1 presents the hardware and software utilized in the computational study of the LNG-ADP-SO-MLP. The experiments were performed on uniform hardware with identical processing power and software configurations. When employing the commercial solver Gurobi, the tests were executed until either an optimal solution was found or the maximum time limit of 10 800 seconds was reached, depending on the size of the instance. Conversely, in the case of the RHH, the runs were conducted until either the time limit of 10 800 seconds per iteration was reached or a solution within a 1.5% gap relative to the optimal solution was found. For some of the test instances, the total time for all iterations subsequently ends up above 10 800 seconds. Orientation of the code in the GitHub repository is presented in Appendix H.

Processor	2 x Intel Xeon Gold 5115 @ 2.40GHz
Memory	96 GB
Commercial Solver	Gurobi v9.5.0
Gurobi Licence Type	Academic
Python version	3.9.6
Github repository	https://github.com/hellevhaug/master-lng-adp

Table 8.1: *Description of hardware and software used for the computational study*

As mentioned in Chapter 7, a total of eight data sets were supplied by our industry partner. Three of these were used for parameter tuning of the RHH parameters, while the remaining five were used to test the RHH and compare it to the Gurobi results.

The complexity of the LNG-ADP-SO-MLP is assumed to be affected by the number of loading and unloading ports, the number of vessels, and the number of time periods. The test instances presented in Section 7.4 have different values for these features. The complexity is particularly sensitive to the number of time periods, so even though the primary objective is to solve for a full year (365 time periods), test instances with shorter horizons are also included. The ADNOC LNG test instances have fewer vessels and customers than the Nigeria LNG test instances, shown in Chapter 7. Therefore, it is expected that the full-horizon Nigeria LNG cases will stress-test the solution methods to a larger extent than the shorter horizons and the ADNOC LNG cases.

8.2 Tuning Parameters

Chapter 6 provides an overview of the parameters utilized in the RHH. To determine optimal values for these parameters, a thorough tuning process is undertaken. The specific parameters that undergo tuning are presented in Table 8.3. For a comprehensive understanding of each parameter, including their detailed descriptions, please refer to Section 6.2.

Parameter	Description
Central Horizon (CH) Length	The number of periods where no relaxation strategy is applied
Forecast Horizon (FH) Length	The forecast horizon where the relaxation strategy is applied
Freezing Horizon Length	The time periods inside the central horizon which are frozen in the next horizon
Warm Start Initiation	The application of a warm start construction heuristic

Table 8.2: *The parameters that underwent tuning.*

Some parameters in the model are not selected for tuning but have been set by trial and error while developing the algorithm as these parameters are less decisive for the end result. These parameters include the last-horizon central horizon extension decision rule, the second-to-last-horizon forecast horizon extension rule, and the requirement for a constraint to be relaxed.

8.2.1 Theoretical Lower Limits to the RHH Parameters for Feasibility Assurance

The parameter values are bounded by certain limits due to data-specific constraints. As explained in Section 6.3, a partition or a sum of a set of future periods plus the longest arc can not be longer than the forecast horizon if we want to ensure feasibility. The central horizon must, in turn, be longer than the longest partition or sum of future periods. This implies the following feasibility ensuring lower limits for our specific data sets;

Parameter	ADNOC LNG	NLNG
Central Horizon (CH)	32 days	49 days
Forecast Horizon (FH)	63 days	80 days

Table 8.3: *Theoretical lower limits to the RHH parameters for ensuring feasibility for both the ADNOC LNG case and the Nigeria LNG case .*

However, as the chances of infeasibility are quite low when the horizon lengths are only somewhat shorter, the lower limits required for the feasibility guarantee are not treated as absolute limits in the parameter tuning. Our preliminary parameter tests showed that significantly longer horizons than the specific values above are rarely solved within reasonable time limits.

8.2.2 Warm Start Initiation

As mentioned in Section 6.4, the first iteration of the RHH is not started warm, and can therefore potentially take longer time than the other iterations. Because of this, we included a warm start in the preliminary testing, and the results showed that it in some cases improved the performance of the RHH. We, therefore, chose to include warm start as a parameter in the parameter tuning as well, by using the greedy construction algorithm described in Algorithm 4.

8.2.3 RHH Parameter Configurations

Four RHH parameter configurations were identified through a preliminary rough analysis of diverse parameters: configurations **1**, **1-WS**, **2**, and **3**. These configurations are presented in Table 8.4, and shows each configuration's parameter values for the parameters presented in Table 8.3. Note that configuration 1-WS is essentially the same as configuration 1, with the addition of using warm start.

RHH configuration ID	1	1-WS	2	3
Central Horizon Length	85	85	65	90
Forecast Horizon Length	50	50	80	110
Frozen Horizon Length	70	70	50	80
Warm start	No	Yes	No	No

Table 8.4: *Parameter configurations for parameter tuning of the RHH algorithm. WS is short for "Warm Start" and denotes that the configuration is used with the warm start algorithm.*

8.2.4 Parameter Tuning Results

The parameter tuning process involved evaluating the performance on three specific test instances: **A-2L-D-12M**, **A-2L-E-12M**, and **N-1L-A-12M**, presented in Section 7.4. The results of the parameter tuning process, including the objective value, total time, and solver time, are presented in Table 8.5. The *objective value* represents the value of the objective when a solution below the 1.5% gap is found in the final iteration of the RHH. *Total time* indicates the total time required by the RHH procedure to find a solution, encompassing the initialization time for each iteration as well as the solution time of the commercial solver. In addition, the table supplies the *solver time*, which refers to the time it takes the commercial solver to solve each iteration, summarized for all iterations. Here, the initialization time for each iteration is excluded. The two different time measures are included to highlight the difference in solution time that is caused by relatively slow initializations by the commercial solver.

	Configuration 1			Configuration 1-WS			Configuration 2			Configuration 3		
Instance ID	Obj. [USD]	Solver time [s]	Total time [s]	Obj. [USD]	Solver time [s]	Total time [s]	Obj. [USD]	Solver time [s]	Time time [s]	Obj. [USD]	Solver time [s]	Total time [s]
A-2L-D-12M	3 429 904'	2 585	13 741	3 475 600'	2 312	11 650	3 442 164'	5 750	19 182	na	na	na
A-2L-E-12M	3 432 439'	2 720	11 829	3 436 200'	3 539	12 737	3 444 015'	1 678	15 891	na	na	na
N-1L-A-12M	4 679 471'	1 434	41 150	4 806 700'	22 005	63 860	na	na	na	na	na	na

Table 8.5: *Total time for each iteration includes solver time and the time needed for reconfiguring the variables and constraints in-between each iteration. na stands for "time limit reached", indicating that the RHH did not find a feasible solution within the given time limit.*

The results obtained from tuning parameters demonstrate that configuration 1 outperforms configuration 2 in terms of time. However, Configuration 1 has a slightly lower objective function value for the instances that Configuration 2 manages to solve. Configuration 1 the RHH achieves a feasible solution within a 1.5% gap faster than configuration 2, and configuration 2 exceeds the time limit for **N-1L-A-12M**. However, configuration 2 adheres to the limits that the feasibility guarantee requires. Configuration 1 does not, as a fifty-day forecast horizon is too short for both ADNOC LNG and NLNG. In other words, Configuration 1 is inherently prone to infeasibility while Configuration 2 is not.

The warm start enhancement is aimed at assisting the RHH in finding a feasible solution by leveraging initial starting points. In addition to making it easier to find a feasible solution, configuration 1-WS (with warm start) also finds a slightly higher objective value than configuration 1 (without warm start) for all test instances. However, the total time and solver time to sub 1.5% gap is higher with configuration 1-WS compared to configuration 1 in two out of three instances.

Configuration 3 reached the time limit for all tested test instances. This signifies that the horizon lengths are too long to solve within the preferred time limits of the industry partner and that shorter horizons must be applied.

We choose configuration 1 and 1-WS to our computational study of the model due to the drawbacks of configuration 2 where it can not find a feasible solution within the time limits in all cases provided for this tuning. As the objective functions of Configuration 1-WS are higher than in Configuration 1, we are using Configuration 1-WS first. However, in the case of **N-1L-A-12M**, Configuration 1 severely outperformed Configuration 1-WS in terms of time. We, therefore, kept both configurations to use when suitable in the computational study.

Table 8.6 shows an example of a breakdown of the solver times for the iterations of the RHH. The example is Configuration 1 and 1-WS applied to **A-2L-D-12M**, which results in 5 iterations. The table shows that the solver time is highest for the middle iterations, which makes sense considering that the middle sections have the most variables due to the arc formulation of the problem. We can see that Configuration 1-WS uses less total time than Configuration 1.

	A-2L-D-12M	
	Configuration 1	Configuration 1-WS
Iteration 1	63	70
Iteration 2	730	174
Iteration 3	662	963
Iteration 4	1082	936
Iteration 5	48	169
Solver time	2 585	2 312
Total time	13 741	11 650
Objective value	3 429 904'	3 475 600'

Table 8.6: *Solver time per iteration for solving the RHH, comparing the configurations with and without warm start, configuration 1 and 1-WS respectively.*

8.3 Comparing the RHH to Gurobi

In this computational study, a comparison is drawn between the efficiency of the commercial solver Gurobi and the RHH when tackling various LNG-ADP-SO-MLP problems, represented by different test instances described in Section 7.4. Both methods were applied to all instances, and the results offer insights into their performance. Solution times and objective function values are used to compare the performance of the different instances and solution methods.

8.3.1 Results of Gurobi and RHH

This section presents the results from the commercial solver Gurobi and the RHH solving the test instances described in Section 7.4, focusing on solution times, objective values and problem sizes. The results from running the instances are presented in Table 8.7 and the problem sizes are summarized in Table 8.8.

When subjected to a time limit of 3600 seconds, Gurobi was able to solve specific instances, particularly **N-1L-B-4M**, **N-1L-D-4M**, and **A-2L-C-4M**, as well as **A-2L-B-6M**. These results alone suggest the solver's strengths in solving these types of instances within a relatively short duration. However, it reached its time limit without providing solutions for **A-2L-A-4M** and **A-2L-B-4M**, among others, denoted as "na" (time limit reached).

Upon extending the time limit to 10 800 seconds, Gurobi's performance improved, solving instances such as **A-2L-A-4M**, **A-2L-B-4M** and **N-1L-D-6M**, which previously resulted in a time limit reached. Nevertheless, many problem instances remained unsolved, particularly those with higher complexity, denoted by longer planning horizons (8M and 12M).

RHH, on the other hand, while subjected to different time constraints, managed to find solutions for all time horizons for at least four out of five instances. For all solvable instances with the RHH, all solutions were found with a lower solver time than 10 800 seconds. With the highest solver time (the time it takes the commercial solver to solve each iteration, summarized for all iterations) of 7 319 seconds, this is significantly lower than the preferred time limit of 3 hours. The results are shown in Table 8.7.

Test instance	Gurobi				RHH			
	3600 s		10 800 s			Solver time		Deviation from
	Gap	Objective [USD]	Gap	Objective [USD]	Config.	[s]	Objective [USD]	com. solv.
N-1L-B-4M	0.33%	1 695 100’	0.31%	1 695 220’	1-WS	237	1 700 070’	+ 0.29%
N-1L-D-4M	0.28%	1 711 800’	0.27%	1 712 038’	1-WS	97	1 664 000’	- 2.80%
A-2L-A-4M	na	na	0.57%	812 751’	1	1 152	793 135’	- 2.49%
A-2L-B-4M	na	na	0.94%	1 022 702’	1-WS	1 207	1 019 508’	- 0.32%
A-2L-C-4M	0.62%	1 148 166’	0.57%	1 148 512’	1-WS	951	1 145 203’	- 0.29%
N-1L-B-6M	na	na	na	na	1	inf.	inf.	-
N-1L-D-6M	na	na	0.57%	2 551 691’	1-WS	2 553	2 529 653’	- 0.88%
A-2L-A-6M	na	na	na	na	1-WS	1 236	1 259 544’	-
A-2L-B-6M	3.25%	1 498 835’	0.91%	1 553 663’	1	2 410	1 522 697’	- 2.05%
A-2L-C-6M	na	na	na	na	1-WS	235	1 689 506’	-
N-1L-B-8M	na	na	na	na	1	inf.	inf.	-
N-1L-D-8M	na	na	na	na	1	7 319	3 385 896’	-
A-2L-A-8M	na	na	na	na	1	1 018	1 896 686’	-
A-2L-B-8M	na	na	na	na	1-WS	927	2 188 142’	-
A-2L-C-8M	na	na	na	na	1	398	2 349 206’	-
N-1L-B-12M	na	na	na	na	1	inf.	inf.	-
N-1L-D-12M	na	na	na	na	1-WS	1 881	5 063 835’	-
A-2L-A-12M	na	na	na	na	1-WS	5 048	2 402 100’	-
A-2L-B-12M	na	na	na	na	1	3 027	3 365 856’	-
A-2L-C-12M	na	na	na	na	1-WS	2 563	3 287 314’	-

Table 8.7: *Results from solving the model with a commercial solver and the RHH. ”na” denotes that the time limit of the respective solution method was reached for the different instances, and ’inf.’ is short for infeasible.*

The total time (the time required by the RHH procedure to find a solution, encompassing the initialization time for each iteration as well as the solution time of the commercial solver) varied considerably among the instances, reflecting the method’s adaptability to problem complexity, though at the cost of higher computation time. It is important to note that the RHH required significantly more time than 10 800 seconds in total time in some cases, but never more than 35 269 seconds, which is within the upper acceptable time limit for the industry partner of 12 hours. It is worth noticing that solver time and total time differ when solving the problem with Gurobi as well. For the Nigeria LNG instances **N-1L-B-12M** and **N-1L-D-12M**, additional initialization times of up to 9 000 seconds must be added to total time, summing up to 20 000 seconds, given the instances were solvable. For the largest Abu Dhabi instances, total time is typically between 13 - 14 000 seconds. An overview of solver time vs total times for both Gurobi and RHH is presented in Appendix I.

Additionally, the RHH deviates from the objective values found by Gurobi in the range of + 0.29% to - 2.80%, which proves that the heuristic solution method can find both feasible and also good solutions with low optimality gaps. This indicates that the RHH constrains the problem enough to make it solvable, while not heavily at the expense of solution quality. Although, the most valuable feature of the RHH is that it can solve instances for up to one year, which is the main purpose of an Annual Delivery Program and has shown to be a challenge in the literature.

Furthermore, instances from the dataset **N-1L-B** resulted in infeasible solutions for both methods, marked as ”inf.” in the table. The instances **N-1L-B-6M**, **N-1L-B-8M**, and **N-1L-B-12M** all reached their time limits with Gurobi, and were proven infeasible with the RHH. This might signify that the instances **N-1L-B-6M**, **N-1L-B-8M**, and **N-1L-B-12M** are inherently infeasible. However, RHH algorithms naturally constrain the opportunity space compared to solving the entire horizon at once due to the segmentation of the problem horizon and the sequential decision-making process. In effect, the application of the algorithm makes global optimization impossible, so if there are very tight volume intervals in the demand partitions, infeasibilities might arise even if the data set itself is feasible.

Table 8.8 provides a summary of the average number of variables and constraints associated with each instance and model type when the problem is solved with a commercial solver. The "Integer variables" column specifies the count of integer variables, which are binary variables exclusively used in our model. The "Continuous variables" column denotes the number of continuous variables, while the "Constraints" column represents the total number of constraints for each test instance. For detailed values of all instances, please refer to the comprehensive table available in Appendix J.

Instance ID	Model Type	Integer variables	Continuous variables	Constraints
N-1L-B-4M	Basic	922 369	1 650	66 614
N-1L-B-4M	Variable production	922 369	1 774	66 862
N-1L-B-4M	Charter out	968 047	1 650	109 647
N-1L-B-4M	Combined	968 047	1 774	109 895
N-1L-B-6M	Basic	1 597 449	2 535	127 458
N-1L-B-8M	Basic	2 273 909	3 413	191 379
N-1L-B-12M	Basic	3 648 296	5 183	335 362
A-2L-C-4M	Basic	344 022	1 044	31 218
A-2L-C-4M	Variable production	344 022	1 290	31 710
A-2L-C-4M	Charter out	360 482	1 044	44 729
A-2L-C-4M	Combined	360 482	1 290	45 221
A-2L-C-6M	Basic	644 818	1 616	54 545
A-2L-C-8M	Basic	938 509	2 188	83 361
A-2L-C-12M	Basic	1 531 109	3 323	158 996

Table 8.8: *Overview of two of the test instances' number of variables and constraints for each model type, run with Gurobi.*

8.3.2 Details of an Example Solution

Figure 8.1 presents a Gantt chart for instance **A-2L-C-12M**, showing delivery dates at customer ports for DES contracts and pick-up dates at the loading port for FOB contracts. Note that contracts starting with ”98..” corresponds to DES spot contracts.

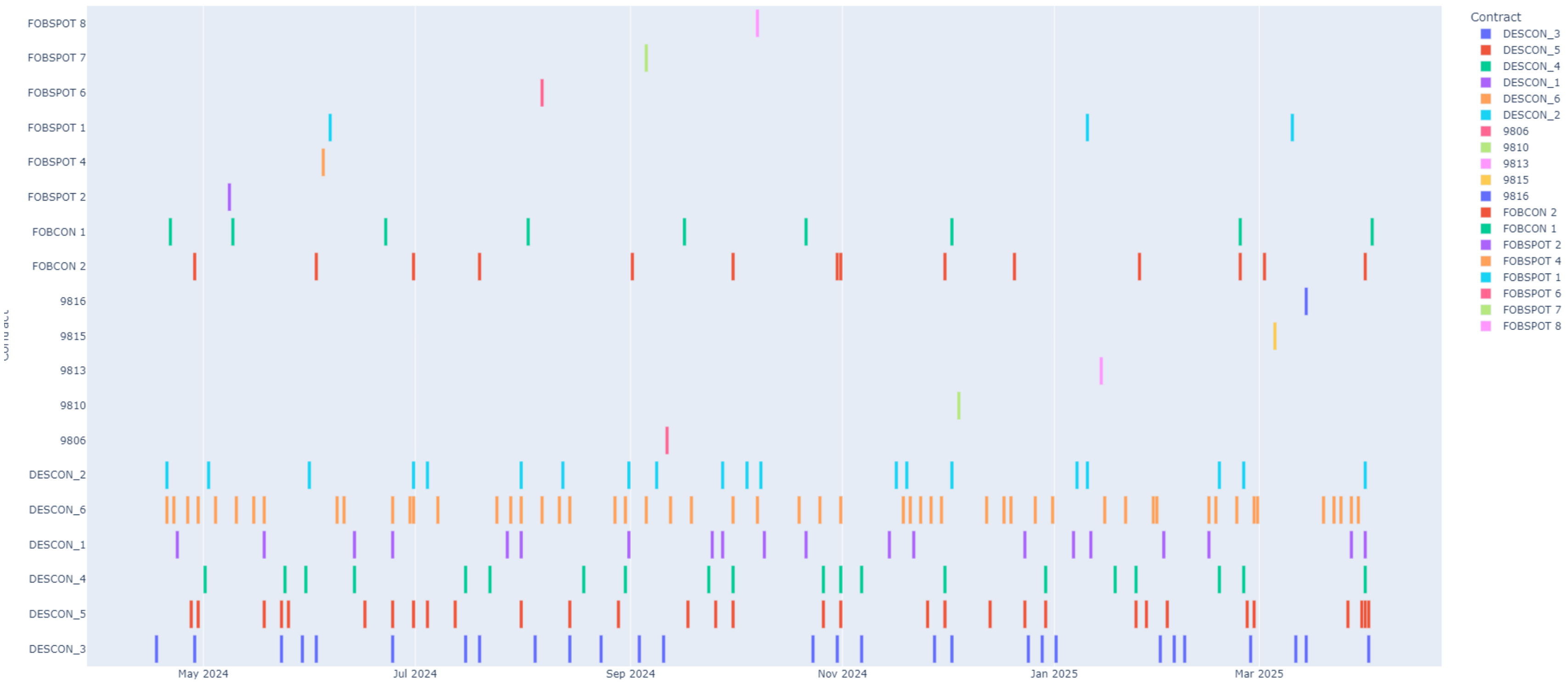


Figure 8.1: *Example of an ADP presented as Gantt chart, showing delivery dates for DES contracts and pick-up dates for FOB contracts*

The Gantt chart visually shows LNG delivery schedules for each contract, distinguishing delivery (DES) and pickup (FOB) contracts. A one-day minimum gap between deliveries prevents multiple deliveries in a single day for a contract.

8.3.3 Single- vs Multiple Loading Ports

In this section, we investigate the consequences of incorporating multiple loading ports. As indicated in Chapter 3, one of the major contributions of this thesis lies in the possibility of handling multiple loading ports. It should be noted that none of the LNG producers among Quorom's customers utilize more than two loading ports. Consequently, this section is focused on instances with one and two loading ports.

A comparable analysis was conducted in the specialization project Haug et al. (2022) utilizing a commercial solver and focusing on instances characterized by shorter planning horizons (2 months). While the present analysis centers around an RHH applied to larger instances, there are notable parallels in terms of key findings and conclusions.

Our evaluation of varying numbers of loading ports fulfills two objectives. Firstly, it evaluates the performance of the RHH in solving the mathematical model, described in Chapter 5, with both one and two loading ports. Secondly, it examines the increase in complexity tied to an increasing number of loading ports. To facilitate a precise comparison, we contrast instances **A-2L-C-6M** and **A-1L-C-6M**. The sole differences between these two setups are that in **A-1L-C-6M**, one loading port has been removed, while the production rate of the remaining loading port is adjusted to match the combined production rate of the two loading ports in **A-2L-C-6M**. The inventory limits remain the same. The results from running the two instances with one and two loading ports are presented in Table 8.9.

Instance ID	Total time	Solver time	Objective value
A-1L-C-6M	613	69	1 689 424'
A-2L-C-6M	845	235	1 689 506'

Table 8.9: *Results for the RHH on two test instances with one and two loading ports.*

By the results from Table 8.9, we observe that the objective values the RHH achieves, 1 689 424' and 1 689 506' USD, are by all practical purposes identical, especially when operating with heuristic solutions. The tiny increase in objective value for **A-2L-C-6M** can be a result of the two loading ports increasing the producer's flexibility through a reduced waiting time at loading ports due to increased berth capacity, as well as the fact that vessels can sail shorter distances when they have the option to pick the nearest port. However, it is worth pointing out that the loading ports are located in close proximity to each other in our test instances, so the last effect might be limited. Because of the added flexibility, the producer may be able to fulfill more contracts and reduce sailing costs. In both instances, the contracts remain unchanged, offering an equal chance to generate the same revenues. This explains why the difference observed in the objective value is almost insignificant.

The solver time for the one-loading-port instance, denoted as **A-1L-C-6M**, was recorded at 69 seconds, while the two-loading-ports instance (**A-2L-C-6M**) required 235 seconds for completion. Moreover, the total time for solving the instances was 613 seconds and 845 seconds, respectively. These findings show that the RHH can solve both instances within a 15-minute timeframe. Hence, the inclusion of an extra loading port does not significantly enhance the problem's complexity when considering the given modeling approach and test instances. It is worth mentioning that if we had chosen to model the problem with only one loading port, we could have modeled the vessel trips as round trips, reducing the number of variables, as Rakke et al. (2011) have done. But again, this would exclude the opportunity for optimizing speeds for both legs of the voyage.

Instance ID	Loading ports	Integer variables	Continuous variables	Constraints
A-1L-C-6M	1	446 419	1 368	50 603
A-2L-C-6M	2	658 926	1 642	54 267

Table 8.10: *Description of the problem sizes for the mathematical model for two test instances with one and two loading ports. The numbers are from the last iteration of the RHH.*

Table 8.10 includes the same test instances as Table 8.9. The leftmost column indicates the test instance, and in each column, the number of integer variables, continuous variables, and constraints are listed for one and two loading ports. The numbers are from the last iteration of the RHH, which are elaborated upon in Chapter 6. Table 8.10 shows that the instance with one loading port has 446 419 integer variables and 1 368 continuous variables, whereas the instance with two loading ports has 658 926 integer variables and 1 642 continuous variables in the last iteration. As observed, the inclusion of additional loading ports leads to a substantial increase in the number of variables. This relationship is logical since the addition of a port generates a greater number of arcs and, consequently, increases the number of integer variables. Particularly, the number of arcs generated is significantly higher due to the introduction of a loading port, which tends to have higher traffic compared to customer ports from the producer's perspective. Furthermore, when comparing instances with two loading ports to those with just one, the count of continuous variables increases remarkably. This correlation is expected as the continuous variables are influenced by the presence of loading ports. Consequently, the increase in loading ports has a significant impact on the overall complexity of the problem, thus leading to higher solution times for the RHH.

8.3.4 The Effect of Modeling Extensions

In Chapter 5, two new extensions were introduced to the basic version of the LNG-ADP-SO-MLP. These extensions include enabling variable production rate, denoted *Extension 1* and the option of chartering out own vessels, denoted *Extension 2*, presented in Section 5.6 and Section 5.7 respectively. The extensions were run with the commercial solver for instance **N-1L-B-4M** and **A-2L-C-4M**. Table 8.11 presents the findings.

Instance ID	Model type	300 s	3600 s	10800 s	Objective value USD
N-1L-B-4M	Basic	1.8%	0.33%	0.31%	1 695 220'
	Variable prod.	0.44%	0.32%	0.31%	1 695 537'
	Charter out	1.59%	0.41%	0.36%	1 697 338'
	Combined	1.59%	0.41%	0.36%	1 697 338'
A-2L-C-4M	Basic	0.93%	0.62%	0.57%	1 148 512'
	Variable prod.	1.32%	0.71%	0.57%	1 148 557'
	Charter out	1.24%	0.56%	0.5%	1 150 797'
	Combined	1.24%	0.56%	0.5%	1 150 777'

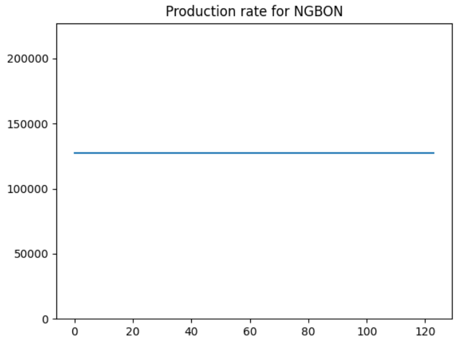
Table 8.11: *Overview of the results from running model extensions with Gurobi.*

Effects of Extension 1: Variable production

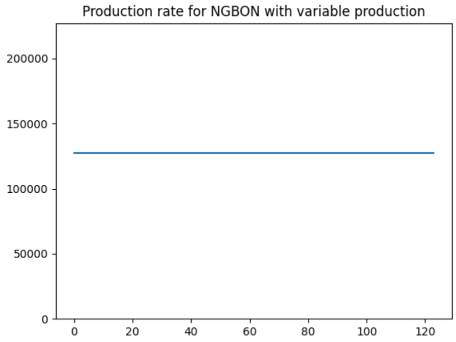
From Table 8.8 we see that implementing Extension 1, variable production rate, increases the number of continuous variables and constraints compared to the basic version of the model, which is natural as a new continuous variable comes with the extension. For both the basic version and Extension 1 the gaps are the same when reaching the running time limit of 10 800 seconds, with values of 0.31% and 0.57% for **N-1L-B-4M** and **A-2L-C-4M**, respectively. As production costs are neglected, varying the production rate does not directly affect the profit-maximizing objective value negatively. Regardless, implementing variable production gives a slightly higher objective value in these two cases. Figure 8.2 show that for a problem of four months, the model does not vary the production rate, but maximizes the production capacity for the whole planning horizon to be able to satisfy demand.

Effects of Extension 2: Chartering out

By incorporating Extension 2, enabling chartering out producer vessels, to the basic version of the LNG-ADP-SO-MLP model, the complexity of the model increases significantly. This is evident through the increase in the number of constraints presented in Table 8.8, rising from 66 614 to 109 647 for test instance **N-1L-B-4M** and 31 218 to 44 729 for test instance **A-2L-C-4M**. With Extension 2, the objective value experiences a slight increase compared to the basic version of the model in these cases, reflecting the higher potential for profit that comes with the option to charter the producer's own vessels.



(a) *Production rate for loading port NGBON with constant production rate.*



(b) *Production rate for loading port NGBON with variable production rate.*

Figure 8.2: *Production rate in solutions, run with Gurobi.*

Effects of the combination of extensions

The results in Table 8.11 show that there is no big difference between using extension 2 alone and using a combination of extensions 1 and 2. This suggests that the extensions do not have any significant combined benefits. Therefore, exploring the relationship between these extensions further may not provide very interesting findings.

8.3.5 The Effects of Warm Start

As shown in Table 8.7, configuration 1-WS was used for several of the test instances. To show how using warm start affects solving the mathematical model, test instances **A-2L-B-4M**, **A-2L-B-6M**, **A-2L-B-8M** and **A-2L-B-12M** were run with Gurobi with and without warm start, to explore how the gap progresses with solution time. This is in practice the same as running the first iteration of the RHH with and without warm start, only without an optimality gap as a termination criteria. The results are presented in Table 8.12.

	Without warm start				**With warm start**			
	A-2L-B-4M	**A-2L-B-6M**	**A-2L-B-8M**	**A-2L-B-12M**	**A-2L-B-4M**	**A-2L-B-6M**	**A-2L-B-8M**	**A-2L-B-12M**
	[Gap]	[Gap]	[Gap]	[Gap]	[Gap]	[Gap]	[Gap]	[Gap]
60 s	-	-	-	-	40.8%	39.1%	34.4%	29.6%
300 s	-	-	-	-	21.1%	11.0%	12.1%	14.6%
1 800 s	-	-	-	-	1.38%	11.0%	12.1%	14.6%
3 600 s	-	3.25%	-	-	1.22%	3.22%	4.98%	14.6%
7 200 s	1.85%	1.04%	-	-	1.03%	3.22%	4.98%	14.6%
10 800 s	0.94%	0.91%	-	-	0.98%	3.22%	2.71%	14.6%

Table 8.12: *Gurobi gaps with and without warm start*

As presented in the table, it seems like including warm start helps the commercial solver find solutions with low optimality gaps faster than when not using warm start. Meanwhile, when the solution time is approaching its maximum limit of 10 800 seconds, not using warm start seems to yield solutions with lower gaps. Since the iterations in the RHH generally aim to find solutions with gaps lower than the gap limit in the shortest amount of time possible, this motivates the use of warm start on instances where Gurobi struggles to find a feasible initial solution. Also, note that the chosen parameter configurations, including warm start, only find a solution within the gap limit of 1.5% for instance **A-2L-B-4M** with a planning horizon of four months. Therefore, the warm start is typically used to find initial feasible solutions for problems with no longer than a four-month planning horizon in the first iteration.

8.3.6 Partitions and Infeasibility: Studying N-1L-B

As presented in Table 8.7, test instances **N-1L-B-6M**, **N-1L-B-8M** and **N-1L-B-12M** were infeasible when solving with the RHH. Meanwhile, when running the instances with a commercial solver, the model did not find a feasible solution within the time limits.

We suspect that the infeasibility of these test instances is likely due to DES contract partitions with too tight intervals between lower and upper demand. If, for example, the interval is smaller than one typical cargo size, the freezing of variables in the RHH could create infeasibilities. For the commercial solver, tight partitions only make the model hard to find feasible solutions to. In order to test this theory, we created relaxed versions of the test instances, where all partition intervals between lower and upper partition demand were at least one typical cargo size, in other words, 150 000 m^3 of LNG. The results from running the RHH on the relaxed test instances are presented in Table 8.13.

Instance ID	Solver time [s]	Objective value [USD]
N-1L-B-6M	269	2 589 500’
N-1L-B-8M	4 356	3 509 500’
N-1L-B-12M	4 538	5 231 200’

Table 8.13: *Results for running the RHH on a relaxed version of dataset **N-1L-B**.*

As it appears in Table 8.13, all instances were solvable after relaxing the partitions. This confirms our hypothesis regarding the increased difficulty faced by the RHH in solving test instances with tight partitions.

8.4 Managerial Insights

This section presents some proposals on how the LNG-ADP-SO-MLP can be used to gain managerial insights. In particular, how an LNG producer can use the mathematical model as a tool when looking into the effect of model extensions, speed optimization, and the value of larger storage among others.

8.4.1 Breakdown of Profit for the LNG Producer

Table 8.14 present a breakdown of the optimal objective value instance **A-2L-C-12M**.

	Value [USD]	%
Revenue, FOB contracts	285 800’	7.99%
Revenue, FOB spot	160 200’	4.48%
Revenue, DES spot producer vessels	143 469’	4.01%
Revenue, DES spot charter	114 064’	3.19%
Revenue, DES contract charter	1 753 056’	48.99%
Revenue, DES contract producer vessels	1 102 138’	30.8%
Revenue, tank left-over value	19 360’	0.54%
Total revenue	**3 578 087’**	100 %
Cost, producer vessel sailing costs	(86 013’)	29.58%
Cost, charter-in costs	(204 760’)	70.42%
Total costs	**(290 773’)**	100%
Total profit	**3 287 314’**	

Table 8.14: *Breakdown of profit for instance **A-2L-C-12M**, showing revenues, costs and profit.*

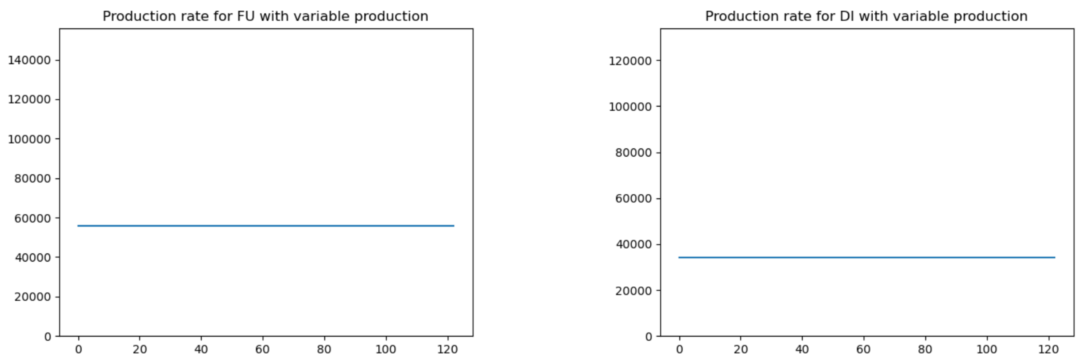
As shown in the table, revenue dominates costs. This signifies that revenue maximization is more important than cost minimization for the LNG producer. For this test instance, DES contracts generate the most revenue, while chartering in vessels incurs the highest costs.

8.4.2 Effect of Extensions

Variable production

As presented in Section 8.3, running **A-2L-C-4M** and **N-1L-B-4M** with the modeling extension of varying production did not affect the production rate, which was shown in Figure 8.4b. This is not surprising, due to the artificial spot cargoes discussed Section 5.1.6. The model can create an artificial spot cargo whenever there is excess capacity, resulting in less need of varying the production rate. To test this theory, instances **A-2L-C-4M** and **N-1L-B-4M** were re-run without the option to generate artificial spot cargoes. Also, as long as there are ordinary spot contracts not yet satisfied, the model will always gain on satisfying these contracts before lowering the production rate. Test instance **A-2L-A-4M** was therefore also re-run with extension 1 and without artificial spot cargoes since the test instance does not contain spot contracts.

For instances **A-2L-C-4M** and **N-1L-B-4M**, excluding the possibility of generating artificial spot cargoes did not affect the production rates. This is shown in Figure 8.3, where the production rates for the two loading ports in **A-2L-C-4M** are presented and appear constant.

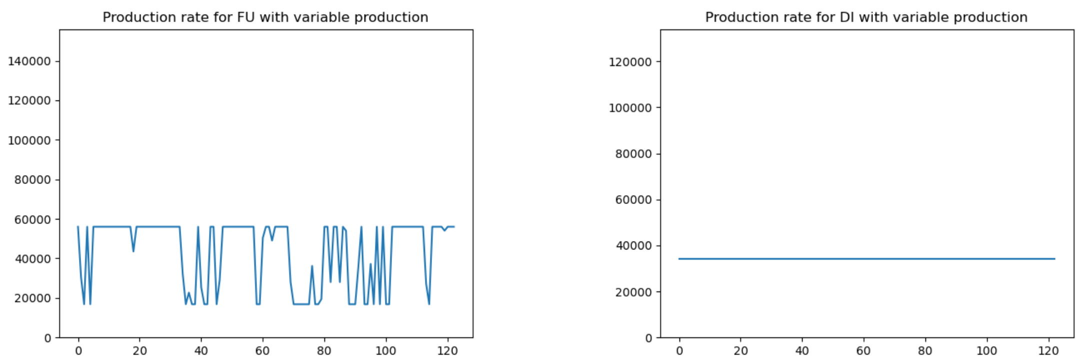


(a) *Production rate for loading port FU with variable production rate.*

(b) *Production rate for loading port DI with variable production rate.*

Figure 8.3: *Effects of varying production rates when running* ***A-2L-C-4M*** *with Gurobi.*

For instance **A-2L-A-4M**, excluding artificial spot cargoes had an effect, which is shown in Figure 8.4 where the loading port FU's production rate varies.



(a) *Production rate for loading port FU with variable production rate.*

(b) *Production rate for loading port DI with variable production rate.*

Figure 8.4: *Effects of varying production rates when running* ***A-2L-A-4M*** *with Gurobi.*

Based on these results, it seems suitable to model with variable production in problems without spot markets. Otherwise, the possibility of varying the production rate is redundant.

Chartering out

Table 8.15 shows a comparison of the number of vessels the producer charters both out and in, with and without the model extension of chartering out the producer's vessel. The comparison is done with one instance for each of the producer cases, **N-1L-B-4M** and **A-2L-C-4M**.

Instance ID	Model type	Number of vessels chartered out	Number of vessels chartered in
N-1L-B-4M	Basic	-	53
	Charter out	10	54
A-2L-C-4M	Basic	-	31
	Charter out	5	34

Table 8.15: *Number of vessels chartered in and chartered out when running instances **N-1L-B-4M** and **A-2L-C-4M** with Gurobi.*

Note that each vessel chartered out corresponds to a reduction in the fleet for the chartering-out period, whereas each vessel chartered in corresponds to a cargo delivered by a charter vessel. As we can see, in both test instances the model chooses to charter out producer vessels when given the opportunity. In the Abu Dhabi case, five vessels are chartered out, accounting for approximately 33% of the total fleet, and 10 vessels in the Nigeria case, accounting for roughly 43%. In both cases, all the vessels are chartered out for the whole planning horizon, and the number of vessels chartered in increases. This intuitively makes sense since the customer has fewer producer vessels to use to satisfy the required customer demand.

In the discussion in Section 5.1.5, the daily charter out revenue was set to 5% of the charter in rate, in order to prevent the model from chartering out all the producer's vessels. One might argue that based on this assumption, it can appear odd that the model chooses to charter out producer vessels when it in return must charter in additional charter vessels. To show that this can be profitable to do for this model, we show an objective function breakdown in terms of charter costs and revenues for **A-2L-C-4M** with and without Extension 2 in Table 8.16.

Revenue/Cost type	**A-2L-C-4M** Basic	**A-2L-C-4M** Charter out	**Difference** [%]
Revenue, charter out own vessels	0	2 025'	
Costs, own vessels	32 309'	28 924'	**-** 10.5%
Costs, chartering vessels	46 720'	52 360'	+ 12.7%
Total costs	79 029'	81 284'	+ 0.3%
→ adjusted for charter out revenue	79 029'	79 259'	
Other revenue terms	1 227 542'	1 230 057'	+ 0.2%
Total profit	1 148 512'	1 150 797'	+ 0.2%

Table 8.16: *Objective function breakdown comparing the basic version of the model with the charter out model extension. The instances were run with Gurobi.*

As it appears in the table, chartering out vessels leads to higher total profit. However, when we examine the different profit factors closely, we find that even after accounting for revenue from chartering, the total costs are slightly higher when vessels are chartered out. This indicates that the increased profit is rather a result of increased revenue, in other words, the producer is able to deliver more LNG to customers. Considering the charter modeling assumption presented in Section 5.1.5, saying that there always is an available charter vessel to charter in, this increase is logical, because delivering to DES contracts becomes more flexible.

It is clear that the different charter vessel assumptions heavily affect the optimal solution. If, for example, we remove the assumption that there always is an available vessel to charter in, the effects of increased flexibility, and therefore also revenue, would become less significant. Furthermore, if we do not assume that we can charter out all the producer's own vessels for the whole planning horizon, the possibility of only chartering out vessels in cases of excess fleet capacity also somewhat diminishes in value. Setting the charter-out revenue price low enough is also essential for not creating an unrealistic high preference for chartering out own vessels and at the same time getting the increased flexibility with using charter vessels to satisfy DES contracts.

8.4.3 Fleet Analysis: Shared or Separated Fleets?

Introducing the arc-flow model described in Section 5.2 also introduces increased model complexity. In order to test if modeling with arc flow instead of round trips provides enough flexibility and value to account for the increased complexity, a version of test instance **A-2L-C-4M** where the fleet is split between the two loading ports was created. The splitting of the fleet was done by allocating more vessels to the loading port with the highest total demand. In combination with that each DES contract has a pre-defined loading port, this splitting results in the test instance having two separate loading ports, with both separate fleets and sets of contracts. In this scenario, using round trips as a modeling alternative could simplify the model and reduce its complexity. The results from running the modified version of **A-2L-C-4M** are presented in Table 8.17, and compared to the original results for the instance.

	Shared fleet	**Separate fleet**
Integer variables	344 002	189 379
Continious variables	1 044	1 044
Constraints	31 218	31 218
Upper bound	1 155 082'	1 154 256'
Objective function value	1 148 512'	1 148 343'
Gap	0.57%	0.51%

Table 8.17: *Results from sharing fleet vs splitting fleet, presenting integer variables, continuous variables, constraints, upper bound, objective function value, and optimality gap. The instances were run with Gurobi.*

According to the table, the key distinction between sharing and separating the fleet is the number of integer variables, or in other words - arcs. The value for separate is nearly half of the shared fleet value, which intuitively aligns with the fact that creating separate port sets roughly reduces the number of potential ports for each vessel by half. Furthermore, there is a tiny decrease in both upper bound and objective function values when separating the fleet.

As described in Section 7.2, the loading ports in the Abu Dhabi case are located close to each other in distance. This is not an optimal producer case when illustrating the value of an arc-flow model, because the difference in distance for vessels choosing which port to sail to and from is relatively small. That is to say, in cases where the loading ports are close to each other in distance, like the Abu Dhabi case, a modeling approach using round trips could be sufficient. However, this approach would overlook the benefits of speed optimization.

8.4.4 The Value of Larger Storage

In this section, we conduct an analysis that explores the impact of expanding storage capacity at a loading port on the Annual Delivery Program (ADP). According to the given mathematical model outlined in Chapter 5, compliance with the storage tank capacities (maximum and minimum) is a hard constraint. This potential investment aims to increase the available storage capacity, thereby relaxing the existing constraints (5.4) outlined in Chapter 5 for the cases in this analysis.

In the case of Nigeria LNG (NLNG), one storage tank can hold 68 000 m^3 of LNG. For the purpose of this analysis, we consider this standard storage tank capacity as applicable to both NLNG and ADNOC LNG cases. When a new storage tank is added to a port, the minimum storage safe limit also increases by 10 000 m^3. It is important to note that the investment cost is regarded as sunk, as it is not affecting the operating profits considered in this study.

Table 8.18 showcases the influence of adding an extra storage tank at the NLNG loading port for instance **N-1L-D-6M** run with RHH. By investing in this additional storage tank, providing a capacity of 68 000 m^3, NLNG could experience a 0.5% increase in profits. This percentage increase could indicate a financial gain for the producer, given the significant amounts of money involved in the LNG industry. However, the increased profit must be evaluated against the increased cost of adding another storage tank.

Increase of number of storage tanks	Total volume at loading port [m^3]	Objective value [USD]	Change in obj.
0	336 000	2 529 653'	-
+1	404 000	2 542 300'	+ 0.5%

Table 8.18: *Overview of how larger storage capacity affects the objective value for* ***N-1L-D-6M*** *solved with the RHH*

In our analysis of the ADNOC LNG case, we have examined four scenarios to assess the value of larger storage capacity for instance **A-2L-C-6M**, as summarized in Table 8.19.

In the first scenario, referred to as *Scenario 1*, we considered the original storage capacity at the loading ports, reflecting the current state. In *Scenario 2* and *Scenario 3*, we introduced an additional storage tank with a capacity of 68 000 m^3 at one of the two loading ports. Finally, in *Scenario 4*, we combined the changes from *Scenario 2* and *Scenario 3*, resulting in both loading ports having an extra storage tank. The objective values corresponding to these different storage capacity scenarios are summarized in Table 8.19, demonstrating an increase in the objective value as the storage capacity expands.

	Lading port "DI"		Loading port "FU"		
Scenario	Increase of number of storage tanks	Total storage volume at loading port [m^3]	Increase of number of storage tanks	Total storage volume at loading port [m^3]	Objective value [USD]
Scenario 1	0	250 000	0	300 000	1 689 506'
Scenario 2	+1	318 000	0	300 000	1 699 000'
Scenario 3	0	250 000	+1	368 000	1 700 200'
Scenario 4	+1	318 000	+1	368 000	1 704 609'

Table 8.19: *Overview of how larger storage capacity affects the objective value for* ***A-2L-C-6M*** *and ADNOC LNG.* The scenarios were run with RHH.

To further examine the impact, we present an objective function breakdown in Table 8.20, focusing on *Scenario 1* (original storage capacity) and *Scenario 4* (additional storage tanks at both loading ports) of ADNOC LNG. The breakdown involves nine terms, as described in the objective function of the mathematical model presented in Chapter 5.

Referring to Table 8.20, it becomes evident that a larger storage capacity leads to a significant reduction in the reliance on charter vessels. Specifically, the revenue generated from long-term DES contracts with charter vessels shows a decline of 21.98%, while the revenue from spot contracts using chartered vessels experiences a decrease of 40.04%. Simultaneously, the costs associated with chartering vessels are lowered by 28.98%.

However, despite these revenue reductions, the overall profit of the producers actually increases. This finding demonstrates that the expansion of storage capacity enables more efficient utilization of the fleet, resulting in improved profitability. This improvement is further supported by the rise in revenue derived from contracts utilizing the producer's own vessels. It is worth noting that there is a notable increase of 49.47% in the cost of using its own vessels, suggesting a higher level of utilization for the producer's fleet. Despite the rise in sailing costs associated with own vessels, the daily charter rate for external vessels is expected to be so high that utilizing the producer's own

Revenue/Cost type [USD]	A-2L-C-6M	A-2L-C-6M Scenario 4	Difference
Revenue, DES long-term contracts, own vessels	579 857’	754 74’	+ 30.17%
Revenue, DES long-term contracts, chartered	884 810’	690 491’	- 21.98%
Revenue, DES spot contracts, own vessels	29 421’	68 034’	+ 131.16%
Revenue, DES spot contracts, chartered	97 570’	58 500’	- 40.08%
Revenue, FOB long-term contracts	139 700’	139 700’	0.00%
Revenue, FOB spot contracts	94 550’	117 440’	+ 24.23%
Revenue, tank left-over LNG	3 200’	4 400’	+ 37.50%
Costs, own vessels	37 682’	56 337’	+ 49.47%
Costs, chartering vessels	101 920’	72 360’	- 28.98%
Total profit	1 689 506’	1 704 609’	+ 0.89%

Table 8.20: *Objective function breakdown for the current storage capacity scenario of ADNOC LNG and a scenario with one extra storage tank at each loading port.*

vessels remains advantageous. If charter rates were to increase drastically, larger storage could turn out even more valuable than for these instances. As seen, larger storage provides greater flexibility in managing LNG shipments, allowing for improved coordination between production and transportation schedules. This, in turn, optimizes shipping operations by accumulating LNG during periods of low demand and loading larger vessels for more efficient transportation.

Expanding the storage capacity not only allows the producer to store more LNG but also opens up opportunities to engage in a greater number of spot contracts. This becomes evident when examining the FOB spot revenue term in the objective function breakdown, which experiences a significant increase from 94 550’ to 117 440’ USD, representing a growth of 24.23%. This trend highlights the advantage of having expanded storage capacity as it empowers the producer to strategically time their LNG sales according to favorable market conditions. By effectively managing their inventory and leveraging the flexibility provided by larger storage, the producer can optimize profitability and generate higher revenue. This capability is particularly advantageous for timing the sales of spot contracts, ensuring maximum benefit from market fluctuations, and capturing increased revenue opportunities.

8.4.5 Exploring DES Contract Loading Port Requirements

As mentioned in Chapter 4, each DES contract has a pre-defined loading port, usually because different loading ports can produce LNG with different qualities. This requirement is redundant in the Nigeria case with one loading port, but for the Abu Dhabi case, it constrains the problem in which contracts each loading port can deliver cargoes to. To explore the effect of relaxing this requirement, instance **A-2L-C-4M** was run without *DES contract loading port requirements*, abbreviated as DCLPR. The results are presented in Table 8.21.

	With DCLPR	Without DCLPR
Integer variables	344 002	460 336
Continuous variables	1 044	1 757
Constraints	31 218	32 644
Upper bound [USD]	1 155 082’	1 158 456’
Objective function value [USD]	1 148 512’	-
Gap	0.57%	-

Table 8.21: *Results from running **A-2L-C-4M** with and without DES contract loading port requirements, presenting integer variables, continuous variables, constraints, upper bound, objective function value, and optimality gap. The instances were run with Gurobi.*

The result from running **A-2L-C-4M** without DCLPR was that the RHH did not find a feasible solution within the time limits. As it appears in the table, the upper bound increases without DCLPR, but also the number of both variables and constraints. This is reasonable since the producer now has more options in how to satisfy customer demands. The results may indicate that the problem becomes too large for the model to solve without DCLPR.

In the Abu Dhabi case, DES contracts loading port requirements are implemented because the two loading ports produce LNG with different qualities. As a result, the customer contracts need to specify a loading port, to ensure that they get the contracted LNG quality delivered. The results in Table 8.21 indicate that there is no need for ADNOC LNG to explore the opportunity to produce both qualities for LNG at both loading ports, since it increases the model complexity too much. Also, since the customers still order LNG with a specific quantity, this would require additional mathematical modeling of variables indicating what quality type each vessel is shipping, which would cause additional complexity.

8.4.6 Speed Optimization

In order to evaluate the impact of speed optimization, a test was conducted where each instance is tested with both speed optimization and the service speed only. The test is performed on instances **A-2L-B-6M** and **A-2L-C-6M** and solved with the RHH. The results are summarized in Table 8.22.

In the specialization project Haug et al. (2022), a similar analysis was performed using a commercial solver and focusing on instances with shorter planning horizons of up to two months. Although our current analysis focuses on larger instances and utilizes an RHH approach, there are similarities in terms of the main findings and conclusions drawn from the study.

	A-2L-B-6M		**A-2L-C-6M**	
	Service speed	Speed optimization	Service speed	Speed optimization
Objective value [USD]	1 501 444’	1 522 697’	1 679 000’	1 689 506’
Solver time [s]	280	2 410	43	235
Total time [s]	1 061	3 011	820	844
Integer variables	311 168	669 069	319 441	658 926
Continuous variables	1 635	1 635	1 642	1 642
Constraints	54 278	54 278	54 267	54 267

Table 8.22: *Test results of the performance of speed optimization by running the RHH on test instance **A-2L-B-6M** and **A-2L-C-6M**. The complexity of the problems under consideration is quantified by the number of variables and constraints.*

In Table 8.22 the results from each test instance **A-2L-B-6M** and **A-2L-C-6M** are listed in separate columns. Each column is divided into two with titles ”Service speed” and ”Speed optimization”. ”Speed optimization” represents the test results when including speed optimization, and ”Service speed” represents the test results when each vessel has one sailing speed for laden and one for ballast, corresponding to their pre-stated default service speed. The results are listed in rows with the following information: ”Objective value” after termination, ”Solver time”, which refers to the time it takes the commercial solver to solve each iteration to the first sub 1.5% gap, summarized for all iterations, and ”Total time” that indicates the total time required by the RHH procedure to find a solution where each iteration is below the 1.5% gap, encompassing initialization times. To illustrate problem complexity the number of ”Integer variables”, ”Continuous variables” and ”Constraints” are also included.

The findings indicate that solving the problem with a fixed speed is quicker and takes less time compared to solving the problem with speed optimization for the given instances. When speed optimization is implemented, the number of integer variables in the problem doubles or more, while the number of continuous variables and constraints remains the same because they are not dependent on vessel speeds. The substantial increase in the number of integer variables in the speed optimization problem suggests that the problem becomes more complex. This complexity is also related to the time it takes for the RHH to find a solution.

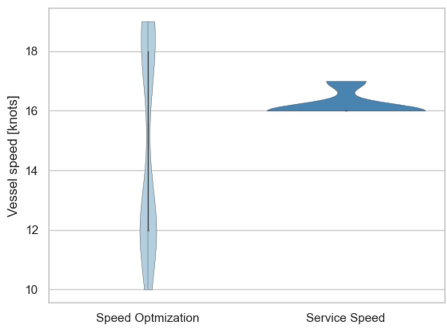
Furthermore, incorporating speed optimization seems to lead to a higher objective value, as presented in Table 8.22. Specifically, the profits increase by 1.42% and 0.62% for **A-2L-B-6M** and **A-2L-C-6M**, respectively. Speed optimization generates more arcs, as shown in Table 8.22, which in turn increases the number of integer variables, thus expanding the solution space and providing opportunities for higher profits. To see what drives the objective value, Table 8.23 presents an objective value breakdown for instance **A-2L-B-6M** with and without speed optimization.

Revenue/Cost type [USD]	**With default speed**	**With speed optimization**	**Difference** [%]
Revenue, DES long-term contracts, own vessels	440 165'	553 123'	+ 25.68%
Revenue, DES long-term contracts, chartered	516 254'	399 529'	- 22.61%
Revenue, DES spot contracts, chartered	140 250'	122 485'	- 12.66%
Revenue, DES spot, own vessels	0	0	0.00%
Revenue, FOB long-term contracts	379 300'	379 300'	0.00%
Revenue, FOB spot contracts	0	176 370'	
Revenue, tank left-over LNG	7 130'	3 200'	- 55.13%
Costs, own vessels	38 067'	49 270'	+ 29.40%
Costs, chartering vessels	86 360'	62 040'	- 28.10%
Total profit	1 501 444'	1 522 697'	+ 1.42%

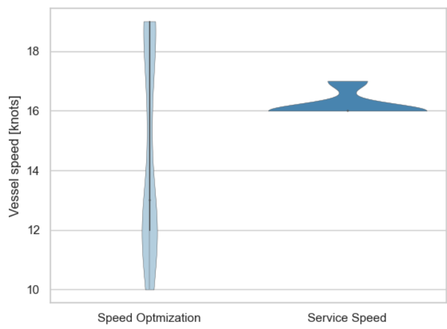
Table 8.23: *Objective function breakdown for instance **A-2L-B-6M** with and without speed optimization.*

Referring to Table 8.23, by using speed optimization the producer enables to increase the revenues from long-term contracts delivered with own vessels by 25.68%, including reducing the charter costs by 28.10%. This indicates that by using speed optimization the producer achieves better fleet utilization and is less need of chartering in vessels. Reducing the number of charter vessels is beneficial as argued upon in Section 8.4.4. The producer is also able to take on FOB spot contracts in the case of speed optimization, which it was not able to do with only service speed. This indicates the flexibility choosing sailing speeds brings improves the planning and scheduling of the ADP, thus resulting in higher amounts of LNG sold and thus higher profits.

Figure 8.5 illustrates the speed distribution of vessels in the optimal solution for instances **A-2L-B-6M** and **A-2L-C-6M**. Each plot includes two violins, representing the two cases: speed optimization and default speeds. The graphs reveal that the speed optimization model utilizes the entire range of permitted vessel speeds, displaying a slight concentration around 12 and 18 knots for both instances. On average, the optimized speeds are slightly lower than the default service speeds, suggesting a potential reduction in fuel costs per unit of distance traveled.



(a) *Instance **A-2L-B-6M***



(b) *Instance **A-2L-C-6M***

Figure 8.5: *Distribution of vessel speeds with- and without speed optimization for two instances.*

Chapter 9

Concluding Remarks

This master's thesis explores an operational approach for solving the Liquefied Natural Gas Annual Delivery Program Planning Problem with Speed Optimization and Multiple Loading Ports (LNG-ADP-SO-MLP), a tactical-level planning problem faced by LNG producers. Our study focuses on presenting a comprehensive LNG-ADP Planning Problem that incorporates multiple loading ports, optimized vessel speeds, and the ability to sell LNG in the spot market. The primary objective of the LNG-ADP-SO-MLP is to maximize profits, considering both revenue from sales and the costs associated with vessel sailing.

The LNG market has witnessed consistent growth in sales volumes in recent years, largely attributed to the global surge in energy consumption and the growing demand for environmentally friendly energy sources. Notably, the share of LNG traded on the spot market has reached one-third of the total global trade volume, reflecting the increasing significance of this segment. Furthermore, the price of LNG has experienced an upward trend over the past years. This combination underscores the critical need for flexibility among LNG producers, enabling them to allocate capacity for additional spot sales. Moreover, the rising charter vessel prices further emphasize the importance of optimizing the utilization of available producer fleets.

To the best of our knowledge, the existing literature on LNG-ADP lacks any inclusion of speed optimization or the consideration of multiple loading ports. Our research contributes significantly to this field by presenting a comprehensive formulation of the LNG-ADP Planning Problem as a mixed-integer linear program (MILP). Notably, our model takes a unique approach by treating each leg of the voyage separately, accommodating multiple loading ports, rather than assuming a round trip. This approach allows for different sailing speeds to and from the production ports, which can be useful to reduce fuel costs and enhance planning flexibility. Additionally, treating the voyage legs separately allows for efficient modeling of maintenance activities, as vessels can proceed directly to maintenance ports after unloading, without returning to loading ports. By incorporating multiple loading ports and speed optimization, our model allows for a more comprehensive representation than those found in the existing literature on the subject. Also, our model incorporates two additional features, namely the inclusion of variable production and the option to charter out the producer's vessels.

To test the model in a realistic setting, data sets provided by our industry partner Quorum Software were modified to test different aspects of the problem. In the test instances, we vary the length of the planning horizon to test how well the model can be solved for different instances up to a realistic planning horizon of 12 months. We used the commercial solver Gurobi to test and solve our MILP model. The solver found solutions with small optimality gaps for instances with planning horizons of up to 120 days. For the larger instances, the solver was not able to find any feasible solutions within the time limits. Due to the comprehensive modeling of the LNG-ADP-SO-MLP, the exact solution method developed in this thesis is not sufficient as an operational tool.

In response to this, a rolling horizon heuristic (RHH) was created to be able to solve the LNG-ADP-SO-MLP for longer planning horizons. The RHH divides the planning horizon into sub horizons and solves them iteratively using a commercial solver to obtain a complete solution. The RHH successfully generates solutions for instances spanning a full planning horizon of 12 months within the industry partner's maximum acceptable time limit of 12 hours. As the RHH iterates through the sub horizons, new constraints must be initialized for each sub horizon. When excluding initialization time, the solution times remained under three hours for all instances. Notably, the deviation between the results obtained from the instances the commercial solver solved and the RHH was minimal, demonstrating that the heuristic method produces both feasible and high-quality solutions with low optimality gaps. Furthermore, the application of the RHH to solve the LNG-ADP-SO-MLP yielded valuable managerial insights that can inform strategic and tactical decision-making for the producer. Consequently, the model and solution method presented in this thesis serves as a comprehensive framework for decision-making beyond the tactical level.

Chapter 10

Future Research

Upon evaluating the research conducted on the LNG-ADP-SO-MLP, we have identified three primary areas for future research: extending the current mathematical model, improving the RHH, and exploring other heuristic solution methods.

Extending the current mathematical model with additional features could make the problem more realistic. Although our thesis presents a comprehensive formulation and model of the LNG-ADP Planning Problem, there are certain aspects that we have not considered. One potential extension of the model is incorporating split deliveries of LNG. In the LNG-ADP-SO-MLP, vessels are currently required to deliver a full shipload to a customer. Allowing for split deliveries, where vessels can distribute their capacity across multiple unloading ports, has been shown to improve solutions, as demonstrated by Mutlu et al. (2016). However, this is not as per now a common practice in the LNG industry. Additionally, imposing demand partitions as strict constraints proves to restrict the problem making it hard to find feasible solutions. Alternatively, penalizing deliveries outside the partitions can provide a more relaxed approach, which is commonly used in the literature. This would however result in a new challenge with weighting the penalties in the objective funtion. Furthermore, to improve model realism, we should limit the assumption of always having a charter vessel available at a loading port to a specific number per day or time period. By introducing such constraints, we can ensure a more accurate representation of the practical availability of charter vessels in the model.

Improving the RHH. First, using the test instances to dynamically determine appropriate RHH parameters could resolve some of the infeasibilities the algorithm encounters for certain instances and RHH parameter combinations. This dynamic parameter determinant would have to be set so that the horizon and forecast horizon is longer than the longest sum over time periods in the problem (longest partition and longest arc length), but otherwise as small as possible. Secondly, the Gurobi solver requires a re-initialization of the constraints in each iteration, which is shown to be a time-consuming process. Researching ways of making a commercial solver speed up this process, or finding a way of pre-initializing and storing all possible constraints before running the algorithm could have a serious impact on the total solution time.

Exploring other heuristic solution methods. As discussed in Chapter 6, the arc-flow-implementation and DES contract partitions in LNG-ADP-SO-MLP made the matheurstic sensitive to infeasibilities. There are several other heuristic solution methods that hold the potential for not facing the same difficulties as the RHH, some of them mentioned in Chapter 3. One such method is the Adaptive Large Neighborhood Search (ALNS) algorithm, which is highly suitable for this problem due to its capacity to handle complexity, adapt to dynamic environments, and efficiently explore large solution spaces. This solution method was applied in Ghiami et al. (2019). In general, other methods that require less domain knowledge for implementation can be suitable for the proposed mathematical model.

Bibliography

Al-Haidous, S., Msakni, M. K., & Haouari, M. (2016). Optimal planning of liquefied natural gas deliveries. *Transportation Research Part C: Emerging Technologies*, *69*, 79–90. https://doi.org/10.1016/j.trc.2016.05.017

Andersson, H., Christiansen, M., & Desaulniers, G. (2015). A new decomposition algorithm for a liquefied natural gas inventory routing problem. *International Journal of Production Research*, *54*(2), 564–578. https://doi.org/10.1080/00207543.2015.1037024

Andersson, H., Christiansen, M., Desaulniers, G., & Rakke, J. G. (2017). Creating annual delivery programs of liquefied natural gas. *Optimization and Engineering*, *18*(1), 299–316. https://link.springer.com/article/10.1007/s11081-015-9305-y

Bai, Y., & Jin, W.-L. (2016). Chapter 4 - LNG carrier. In Y. Bai & W.-L. Jin (Eds.), *Marine structural design* (Second Edition, pp. 49–71). Butterworth-Heinemann. https://doi.org/https://doi.org/10.1016/B978-0-08-099997-5.00004-6

BloombergNEF. (2022). *Global LNG outlook overview: Tight supply expected until 2026*. Retrieved 10th September 2022, from https://www.bloomberg.com/professional/blog/global-lng-outlook-overview-tight-supply-expected-until-2026/

BP Plc. (2023). Bp energy outlook 2023 edition.

Cameron LNG. (n.d.). *Natural gas and the liquefaction process*. https://cameronlng.com/lng-facility/lng-and-liquefaction/

Cardin, M.-A., Ranjbar-Bourani, M., & De Neufville, R. (2015). Improving the lifecycle performance of engineering projects with flexible strategies: Example of on-shore LNG production design. *Systems Engineering*, *18*(3), 253–268. https://doi.org/10.1002/sys.21301

Cho, J., Lim, G. J., Kim, S. J., & Biobaku, T. (2018). Liquefied natural gas inventory routing problem under uncertain weather conditions. *International Journal of Production Economics*, *204*, 18–29. https://doi.org/10.1016/j.ijpe.2018.07.014

Christiansen, M., Fagerholt, K., Nygreen, B., & Ronen, D. (2007). Maritime transportation. *Handbooks in operations research and management science*, *14*, 189–284. https://doi.org/10.1016/S0927-0507(06)14004-9

Connected Papers. (n.d.). *About*. https://www.connectedpapers.com/about

Dobrota, D., Lalić, B., & Komar, I. (2013). Problem of boil-off in LNG supply chain. *Transactions on maritime science*, *2*, 91–100. https://doi.org/10.7225/toms.v02.n02.001

Eriksen, U., Kristiansen, J., Fagerholt, K., & Pantuso, G. (2022). Planning a maritime supply chain for liquefied natural gas under uncertainty. *Maritime Transport Research*, *3*, 100061. https://doi.org/10.1016/j.martra.2022.100061

Fagerholt, K., Hvattum, L. M., Papageorgiou, D. J., & Urrutia, S. (2023). Maritime inventory routing: Recent trends and future directions. *International Transactions in Operational Research, forthcoming*. https://doi.org/10.1111/itor.13313

Fodstad, M., Uggen, K. T., Rømo, F., Lium, A.-G., Stremersch, G., & Hecq, S. (2010). LNG scheduler: A rich model for coordinating vessel routing, inventories and trade in the liquefied natural gas supply chain. *The journal of energy markets*, *3*(4), 31–64.

FreightWaves. (2022). *LNG shipping rates 'shooting for the stars' at $500,000 per day*. Retrieved 4th December 2022, from https://www.freightwaves.com/news/lng-shipping-rates-shooting-for-the-stars-at-500000-per-day

Ghiami, Y., Demir, E., Van Woensel, T., Christiansen, M., & Laporte, G. (2019). A deteriorating inventory routing problem for an inland liquefied natural gas distribution network. *Transportation Research Part B: Methodological*, *126*, 45–67. https://doi.org/10.1016/j.trb.2019.05.014

GIIGNL. (n.d.). *Market trends*. Retrieved 1st December 2022, from https://giignl.org/market-trends/

GIIGNL. (2016). GIIGNL Annual Report 2016. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

GIIGNL. (2017). GIIGNL Annual Report 2017. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

GIIGNL. (2018). GIIGNL Annual Report 2018. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

GIIGNL. (2019). GIIGNL Annual Report 2019. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

GIIGNL. (2020). GIIGNL Annual Report 2020. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

GIIGNL. (2021). GIIGNL Annual Report 2021. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

GIIGNL. (2022). GIIGNL Annual Report 2022. The International Group of Liquefed Natural Gas Importers, Neuilly-sur-Seine, France.

Goel, V., Furman, K., Song, J.-H., & El-Bakry, A. S. (2012). Large neighborhood search for LNG inventory routing. *Journal of Heuristics*, *18*(6), 821–848. https://link.springer.com/article/10.1007/s10732-012-9206-6

Goel, V., Slusky, M., van Hoeve, W.-J., Furman, K. C., & Shao, Y. (2015). Constraint programming for LNG ship scheduling and inventory management. *European Journal of Operational Research*, *241*(3), 662–673. https://doi.org/10.1016/j.ejor.2014.09.048

Grønhaug, R., & Christiansen, M. (2009). Supply chain optimization for the liquefied natural gas business. In *Innovations in distribution logistics* (pp. 195–218). Springer. https://link.springer.com/chapter/10.1007/978-3-540-92944-4_10

Halvorsen-Weare, E. E., & Fagerholt, K. (2010). Routing and scheduling in a liquefied natural gas shipping problem with inventory and berth constraints. *Annals of Operations Research*, *203*(1), 167–186. https://link.springer.com/article/10.1007/s10479-010-0794-y

Halvorsen-Weare, E. E., Fagerholt, K., & Rönnqvist, M. (2013). Vessel routing and scheduling under uncertainty in the liquefied natural gas business. *Computers and Industrial Engineering*, *64*(1), 290–301. https://doi.org/10.1016/j.cie.2012.10.011

Haug, H. V., Solum, S. H., & Warholm, S. B. (2022). Planning annual delivery programs with speed optimization and multiple loading ports in the liquefied natural gas business.

International Energy Agency. (2019). LNG market trends and their implications. https://www.iea.org/reports/lng-market-trends-and-their-implications

Jokinen, R., Pettersson, F., & Saxén, H. (2015). An MILP model for optimization of a small-scale LNG supply chain along a coastline. *Applied Energy*, *138*, 423–431. https://doi.org/10.1016/j.apenergy.2014.10.039

Khalilpour, R., & Karimi, I. (2012). Contract selection under uncertainty: LNG buyers' perspective. *31*, 1487–1491. https://doi.org/10.1016/B978-0-444-59506-5.50128-0

Koza, D. F., Ropke, S., & Molas, A. B. (2017). The liquefied natural gas infrastructure and tanker fleet sizing problem. *Transportation Research Part E: Logistics and Transportation Review*, *99*, 96–114. https://doi.org/10.1016/j.tre.2017.01.003

Li, M., Fagerholt, K., & Schütz, P. (2022). Maritime inventory routing with transshipment: The case of yamal LNG. *Flexible Services and Manufacturing Journal*. https://doi.org/10.1007/s10696-022-09476-5

Li, M., & Schütz, P. (2020). Planning annual LNG deliveries with transshipment. *Energies*, *13*(6), 1490. https://doi.org/10.3390/en13061490

MET Group. (2020). *Natural gas transportation: How is natual gas transported?* https://group.met.com/en/media/energy-insight/natural-gas-transportation

Msakni, M. K., & Haouari, M. (2018). Short-term planning of liquefied natural gas deliveries. *Transportation Research Part C: Emerging Technologies*, *90*, 393–410. https://doi.org/10.1016/j.trc.2018.03.013

Munguia, L.-M., Ahmed, S., Bader, D. A., Nemhauser, G. L., Shao, Y., & Papageorgiou, D. J. (2019). Tailoring parallel alternating criteria search for domain specific mips: Application to maritime inventory routing. *Computers & Operations Research*, *111*, 21–34.

Mutlu, F., Msakni, M. K., Yildiz, H., Sönmez, E., & Pokharel, S. (2016). A comprehensive annual delivery program for upstream liquefied natural gas supply chain. *European Journal of Operational Research*, *250*(1), 120–130. https://doi.org/10.1016/j.ejor.2015.10.031

Nasdaq Inc. (2023). *Natural gas (ng:nmx)*. Retrieved 19th May 2023, from https://www.nasdaq.com/market-activity/commodities/ng:nmx

Nigeria LNG Ltd. (2023). *Who we are*. https://www.nigerialng.com/the-company/Pages/Home.aspx

Nikhalat-Jahromi, H., Bell, M. G., Fontes, D. B., Cochrane, R. A., & Angeloudis, P. (2016). Spot sale of uncommitted LNG from middle east: Japan or the uk? *Energy Policy*, *96*, 717–725. https://doi.org/10.1016/j.enpol.2016.06.027

Offshore Technology. (2023). *Oil gas field profile: Adnoc lng conventional gas field, uae*. https://www.offshore-technology.com/marketdata/oil-gas-field-profile-adnoc-lng-conventional-gas-field-uae/

Papageorgiou, D. J., Cheon, M.-S., Harwood, S., Trespalacios, F., & Nemhauser, G. L. (2018). Recent progress using matheuristics for strategic maritime inventory routing. *Modeling, computing and data handling methodologies for maritime transportation*, 59–94.

Peña-Zarzuelo, I. D. L., Freire-Seoane, M. J., & Pais-Montes, C. (2020). Optimisation of the LNG supply chain: A literature overview. *International Journal of Oil, Gas and Coal Technology*, *24*(4), 523–542.

Petroleum Economist. (n.d.). *Interactive world gas map*. https://energywebatlas.com/interactiveworldgasmap.html

Rakke, J., Stålhane, M., Moe, C. R., Christiansen, M., Andersson, H., Fagerholt, K., & Norstad, I. (2011). A rolling horizon heuristic for creating a liquefied natural gas annual delivery program. *Transportation Research Part C: Emerging Technologies*, *19*(5), 896–911. https://doi.org/10.1016/j.trc.2010.09.006

Shao, Y., Furman, K. C., Goel, V., & Hoda, S. (2015). A hybrid heuristic strategy for liquefied natural gas inventory routing. *Transportation Research Part C: Emerging Technologies*, *53*, 151–171. https://doi.org/10.1016/j.trc.2015.02.001

Sheikhtajian, S., Nazemi, A., & Feshari, M. (2020). Marine inventory-routing problem for liquefied natural gas under travel time uncertainty. *International Journal of Supply and Operations Management*, *7*(1), 93–111. https://www.proquest.com/scholarly-journals/marine-inventory-routing-problem-liquefied/docview/2413996855/se-2

SMM. (2022). *European natural gas prices dropped astonishingly to negative zone, but are likely to rebound shortly*. https://news.metal.com/newscontent/101982927/european-natural-gas-prices-dropped-astonishingly-to-negative-zone-but-are-likely-to-rebound-shortly/

Stålhane, M., Rakke, J. G., Moe, C. R., Andersson, H., Christiansen, M., & Fagerholt, K. (2012). A construction and improvement heuristic for a liquefied natural gas inventory routing problem. *Computers and Industrial Engineering*, *62*(1), 245–255. https://doi.org/10.1016/j.cie.2011.09.011

Zhang, C., Nemhauser, G., Sokol, J., Cheon, M.-S., & Keha, A. (2018). Flexible solutions to maritime inventory routing problems with delivery time windows. *Computers and Operations Research*, *89*, 153–162. https://doi.org/10.1016/j.cor.2017.08.011

Zhang, H., Liang, Y., Liao, Q., Yan, X., Shen, Y., & Zhao, Y. (2017). A three-stage stochastic programming method for LNG supply system infrastructure development and inventory routing in demanding countries. *Energy*, *133*, 424–442. https://doi.org/10.1016/j.energy.2017.05.090

Appendix

A Development in the LNG Price

Figure 1 shows the development in the LNG price from 2018-2022.

Figure 1: *Development in the LNG price (NG:NMX index), retrieved from Nasdaq Inc. (2023)*

B Connected Papers

As mentioned in Chapter 3, Connected Papers was used to visualize how the different papers are connected. Figure 2 is an example of how we created a graph based on Msakni and Haouari (2018).

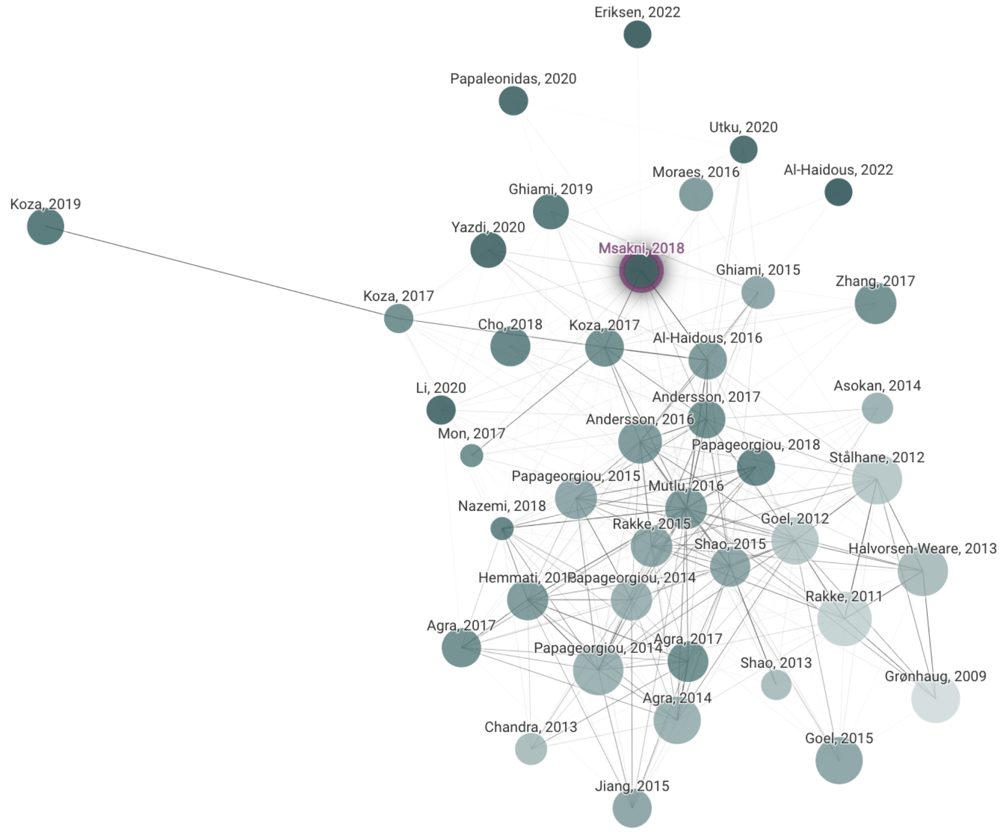


Figure 2: *Graph based on Msakni and Haouari (2018), where node size is number of citations, node color correlates with publishing year, and connected papers have strong connected lines and are clustered together.*

C Mathematical Model

C.1 Basic version

Sets and Indices

$\mathcal{V}$	Set of producer-operated vessels that are available during the planning horizon, each vessel indexed by v
$\mathcal{V}^M$	Set of vessels that require maintenance during the planning horizon, $\mathcal{V}^M \subset \mathcal{V}$
$\mathcal{V}_i$	Set of vessels that can serve node i, $\mathcal{V}_i \subset \mathcal{V}$
$\mathcal{N}$	Set of ports, each port indexed by i, j
$\mathcal{N}^L$	Set of loading ports, $\mathcal{N}^L \subset \mathcal{N}$
$\mathcal{N}^U$	Set of unloading ports, $\mathcal{N}^U \subset \mathcal{N}$
$\mathcal{N}^S$	Set of spot unloading ports, $\mathcal{N}^S \subset \mathcal{N}$
$\mathcal{N}^M$	Set of maintenance ports, $\mathcal{N}^M \subset \mathcal{N}$
$\mathcal{A}_v$	Set of feasible arcs vessel v can sail, each arc indexed by $((i,t),(j,t'))$
$\mathcal{A}_v^M$	Set of feasible maintenance arcs vessel v can sail to start maintenance, $\mathcal{A}_v^M \subset \mathcal{A}_v$
$\mathcal{A}_v^U$	Set of feasible arcs ship v can sail to deliver a DES long-term contracted cargo, $\mathcal{A}_v^U \subset \mathcal{A}_v$
$\mathcal{A}_v^S$	Set of feasible arcs ship v can sail to deliver a DES spot cargo, $\mathcal{A}_v^S \subset \mathcal{A}_v$
$\mathcal{F}_i$	Set of FOB cargoes of LNG that want to be picked up at loading port i, indexed by f
$\mathcal{F}_i^U$	Set of long-term contracted FOB cargoes of LNG that must be picked up at loading port i, indexed by f, $\mathcal{F}_i^U \subset \mathcal{F}_i$
$\mathcal{F}_i^S$	Set of Spot FOB cargoes who's load can be picked up by a FOB vessel at loading port i, indexed by f, also including the artificial spot FOB-pickup, $\mathcal{F}_i^S \subset \mathcal{F}_i$

$\mathcal{P}_j$	Set of time partitions where customer j has DES contracts, each partition indexed by p.
$\mathcal{T}$	Set of time periods in all days, each time period, indexed by t, $\mathcal{T} = \{1, 2, ..., \lvert\mathcal{T}\rvert\}$ where $\lvert\mathcal{T}\rvert$ is the last time period in the planning horizon.
$\mathcal{T}^L$	Set of time periods in loading days, where the vessels can lift LNG from a loading port, each time period indexed by t, $\mathcal{T}^L \subset \mathcal{T}$
$\mathcal{T}^U$	Set of time periods in unloading days, where vessels can deliver LNG to a customer, each time period indexed by t, $\mathcal{T}^U \subset \mathcal{T}$
$\mathcal{T}_{jp}$	Set of time periods within partition p for customer j, $\mathcal{T}_{jp} \subset \mathcal{T}$
$\mathcal{T}_f^{FOB}$	Set of time periods where FOB cargo f can be picked up, $\mathcal{T}_f^{FOB} \subset \mathcal{T}$
$\mathcal{T}_v$	Set of time periods where vessel v is available to be scheduled, $\mathcal{T}_v \subset \mathcal{T}$
$\mathcal{T}_v^M$	The time period where maintenance of vessel v is scheduled to start, $\mathcal{T}_v^M \subset \mathcal{T}_v$

Parameters

$C^S_{vitjt'}$	Sailing cost of each feasible arc $((i,t),(j,t'))$ for vessel v, $((i,t),(j,t')) \in \mathcal{A}_v, v \in \mathcal{V}$
C^C_{itj}	Costs of using a charter vessel to deliver a cargo at port j, loading the cargo at loading port i at time t, $i \in \mathcal{N}^L, t \in \mathcal{T}, i \in \mathcal{N}^U$
$\mathcal{N}^{START}_v$	Start port of vessel v, $v \in \mathcal{V}$
T^{START}_v	First time period where vessel v is available to be scheduled, $v \in \mathcal{V}$, $\mathrm{T}^{START}_v \subset \mathcal{T}_v$
T^O_{vij}	Operational time associated with sailing from port location i to port location j for vessel v, $i,j \in \mathcal{N}, v \in \mathcal{V}$
T^C_{ij}	Sailing time for a charter vessel sailing from loading port i to unloading port j, $i \in \mathcal{N}^L, j \in \mathcal{N}^U$
T^{OFOB}_{fi}	Operational time associated to port location j for FOB cargo f, $f \in \mathcal{F}_j, i \in \mathcal{N}^L$
$T^{charter}_{ij}$	Sailing time for a charter vessel between port i and j, $i \in \mathcal{N}^L, j \in \mathcal{N}^U$
L_v	Capacity of vessel v, $v \in \mathcal{V}$
$\overline{L}^C$	Upper limit for capacity of a charter vessel
$\underline{L}^C$	Lower limit for capacity of a charter vessel
L^{FOB}_f	Loading quantity of FOB cargo f, $f \in \mathcal{F}_i, i \in \mathcal{N}^L$
$\overline{D}_{jp}$	Maximum demand of unloading port j in partition p, $j \in \mathcal{N}^U, p \in \mathcal{P}_j$
$\underline{D}_{jp}$	Minimum demand of unloading port j in partition p, $j \in \mathcal{N}^U, p \in \mathcal{P}_j$
R^{SFOB}_f	Revenue per volume unit of LNG loaded for FOB spot contract f, $f \in \mathcal{F}^S_i, i \in \mathcal{N}^L$
R^{UFOB}_f	Revenue per volume unit of LNG loaded for long-term FOB contract f, $f \in \mathcal{F}^S_i, i \in \mathcal{N}^L$
$R^{DES}_{jt'}$	Revenue per volume unit of LNG for delivering DES contract to customer j at time t', $j \in \mathcal{N}^U, t' \in \mathcal{T}$
R^{END}_i	Unit value of LNG left in storage tanks at loading port i at the end of the planning horizon, $i \in \mathcal{N}^L$
$B_{jt'}$	Berth capacity at port j at time t', $j \in \mathcal{N}^L, t' \in \mathcal{T}$
Q^P_{it}	Produced quantity of LNG in loading port i in time period t, $i \in \mathcal{N}^L, t \in \mathcal{T}$
$\overline{S}_i$	Maximum storage level of LNG at loading port i, $i \in \mathcal{N}^L$
$\underline{S}_i$	Minimum storage level of LNG at loading port i, $i \in \mathcal{N}^L$
S_i	Initial storage level of LNG at the start of the planning horizon at loading port i, $i \in \mathcal{N}^L$
E	Boil-off rate in percent of total vessel capacity
P^{MIN}_j	Minimum number of time periods between deliveries for customer j, $j \in \mathcal{N}^U$

Variables

$x_{vitjt'}$	1 if vessel v sail arc $((i,t),(j,t'))$, and 0 otherwise
$z_{ft'}$	1 if FOB cargo f is done loading in time period t', and 0 otherwise
w_{itj}	1 if a charter vessel starts sailing from loading port i at time t to deliver a cargo at j, and 0 otherwise
g_{itj}	Amount loaded by a charter vessel in loading port i at time t to deliver in unloading port j
s_{it}	Remaining storage at loading port i at the end of time period t

Objective Function

$$
\begin{aligned}
max z = &\sum_{v\in\mathcal{V}} \sum_{((i,t),(j,t'))\in\mathcal{A}_v^U} R_{jt'}^{DES} L_v (1-(t'-t)E) x_{vitjt'} + \sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}^L}\sum_{j\in\mathcal{N}^U} R_{j,t+T_{ij}^C}^{DES}(1-T_{ij}^C E) g_{itj} \\
&+ \sum_{v\in\mathcal{V}} \sum_{((i,t),(j,t'))\in\mathcal{A}_v^S} R_{jt'}^{DES} L_v (1-(t'-t)E) x_{vitjt'} + \sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}^L}\sum_{j\in\mathcal{N}^S} R_{j,t+T_{ij}^C}^{DES}(1-T_{ij}^C E) g_{itj} \\
&+ \sum_{i\in\mathcal{N}^L}\sum_{f\in\mathcal{F}_i^U}\sum_{t'\in\mathcal{T}} R_f^{UFOB} L_f^{FOB} z_{ft'} + \sum_{i\in\mathcal{N}^L}\sum_{f\in\mathcal{F}_i^S}\sum_{t'\in\mathcal{T}} R_f^{SFOB} L_f^{FOB} z_{ft'} + \sum_{i\in\mathcal{N}^L} R_i^{END} s_{i,|\mathcal{T}|} \\
&- \sum_{v\in\mathcal{V}} \sum_{((i,t),(j,t'))\in\mathcal{A}_v} C_{vitjt'}^S x_{vitjt'} - \sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{N}^U} C_{itj}^C w_{itj}
\end{aligned}
\tag{1}
$$

Constraints

$$
s_{i1} = S_i + Q_{i1}^P - \sum_{j\in\mathcal{N}^U}\sum_{t'\in\mathcal{T}}\sum_{v\in\mathcal{V}_i} L_v x_{vi1jt'} - \sum_{j\in\mathcal{N}^U\cup\mathcal{N}^S} g_{i1j} - \sum_{f\in\mathcal{F}_i} L_f^{FOB} z_{f1}, \qquad i\in\mathcal{N}^L \tag{2}
$$

$$
\begin{aligned}
s_{it} = s_{i,t-1} + Q_{it}^P - \sum_{j\in\mathcal{N}^U}\sum_{t'\in\mathcal{T}}\sum_{v\in\mathcal{V}_i} L_v x_{vitjt'} - \sum_{j\in\mathcal{N}^U\cup\mathcal{N}^S} g_{itj} - \sum_{f\in\mathcal{F}_i}\sum_{t'\in\mathcal{T}_f^{FOB}} L_f^{FOB} z_{ft'}, \\
i\in\mathcal{N}^L, t\in\mathcal{T}^L\backslash\{1\}
\end{aligned}
\tag{3}
$$

$$
\underline{S_i} \le s_{it} \le \overline{S_i}, \qquad i\in\mathcal{N}^L, t\in\mathcal{T}^L \tag{4}
$$

$$
\sum_{((i,t),(j,t'))\in\mathcal{A}_v^M} x_{vitjt'} = 1, \qquad v\in\mathcal{V}^M \tag{5}
$$

$$
x_{v,0,0,N_v^{START},T_v^{START}} + x_{v,0,0,0,|\mathcal{T}|+1} = 1, \qquad v\in\mathcal{V} \tag{6}
$$

$$\sum_{i\in\mathcal{N}}\sum_{t=0}^{t'-1} x_{vitjt'} = \sum_{i\in\mathcal{N}}\sum_{t=t'+1}^{|\mathcal{T}|} x_{vjt'it}, \qquad v\in\mathcal{V}, j\in\mathcal{N}, t'\in\mathcal{T} \tag{7}$$

$$\underline{D}_{jp} \leq \sum_{v\in\mathcal{V}_i}\sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}_v}\sum_{t'\in\mathcal{T}_{jp}} L_v(1-(t'-t)E)x_{vitjt'} + \sum_{i\in\mathcal{N}^L}\sum_{t\in(\mathcal{T}_{jp}-T_{ij}^C)} g_{itj}(1-T_{ij}^C E) \leq \overline{D}_{jp}, \quad j\in\mathcal{N}^U\cup\mathcal{N}^S, \quad p\in\mathcal{P}_j \tag{8}$$

$$\sum_{v\in\mathcal{V}_i}\sum_{i\in\mathcal{N}^L}\sum_{t\in\mathcal{T}_v}\sum_{\tau=t'}^{t'+P_j^{MIN}} x_{vitj\tau} + \sum_{i\in\mathcal{N}^L}\sum_{\tau=t'-T_{ij}^{charter}}^{t'-T_{ij}^{charter}+P_j^{MIN}} w_{i\tau j} \leq 1, \quad j\in\mathcal{N}^U\cup\mathcal{N}^S, t'\in\mathcal{T}^U\backslash\{|\mathcal{T}^U|-\mathcal{P}_j^{MIN}\} \tag{9}$$

$$\sum_{t'\in\mathcal{T}_f^{FOB}} z_{ft'} = 1, \qquad j\in\mathcal{N}^L, f\in\mathcal{F}_j^U \tag{10}$$

$$\sum_{t'\in\mathcal{T}_f^{FOB}} z_{ft'} \leq 1, \qquad j\in\mathcal{N}^L, f\in\mathcal{F}_j^S\backslash\{1\} \tag{11}$$

$$\sum_{v\in\mathcal{V}^P}\sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{\tau=t'+1}^{t'+T_{vij}^O} x_{vitj\tau} + \sum_{j'\in\mathcal{N}^U\cup\mathcal{N}^S} w_{jt'j'} + \sum_{f\in\mathcal{F}_j}\sum_{\tau=t'+1}^{t'+T_{fj}^{OFOB}} z_{f\tau} \leq B_{jt'}, \qquad j\in\mathcal{N}^L, \quad t'\in\mathcal{T}^U \tag{12}$$

$$\underline{L}^C w_{itj} \leq g_{itj} \leq \overline{L}^C w_{itj}, \qquad i\in\mathcal{N}^L, t\in\mathcal{T}^L, j\in\mathcal{N}^U\cup\mathcal{N}^S \tag{13}$$

$$x_{vitjt'} \in \{0,1\}, \qquad v\in\mathcal{V}, ((i,t),(j,t'))\in\mathcal{A}_v \tag{14}$$

$$z_{ft'} \in \{0,1\}, \qquad j\in\mathcal{N}^L, f\in\mathcal{F}_j, t'\in\mathcal{T}_f^{FOB} \tag{15}$$

$$w_{itj} \in \{0,1\}, \qquad i\in\mathcal{N}^L, t\in\mathcal{T}^L, j\in\mathcal{N}^{\mathcal{U}}\cup\mathcal{N}^{\mathcal{S}} \tag{16}$$

$$g_{itj} \geq 0, \qquad i\in\mathcal{N}^L, t\in\mathcal{T}^L, j\in\mathcal{N}^{\mathcal{U}}\cup\mathcal{N}^{\mathcal{S}} \tag{17}$$

$$s_{it} \geq 0, \qquad i \in \mathcal{N}, t \in \mathcal{T}^L \tag{18}$$

C.2 Extension 1: Variable Production

Parameters

Q_{it}^{MIN} Minimum production rate at production port i in time period t, $i \in \mathcal{N}^L, t \in \mathcal{T}^L$

Q_{it}^{MAX} Maximum production rate at production port i in time period t, $i \in \mathcal{N}^L, t \in \mathcal{T}^L$

Variables

q_{it} Production rate at production port i at time period t

Constraints

$$s_{i1} = S_i + q_{i1} - \sum_{j \in \mathcal{N}^U} \sum_{t' \in \mathcal{T}} \sum_{v \in \mathcal{V}_i} L_v x_{vi1jt'} - \sum_{j \in \mathcal{N}^U} g_{i1j} - \sum_{f \in \mathcal{F}_i} \sum_{t' \in \mathcal{T}_f^{FOB}} L_f^{FOB} z_{ft'}, \qquad i \in \mathcal{N}^L \tag{19}$$

$$s_{it} = s_{i,t-1} + q_{it} - \sum_{j \in \mathcal{N}^U} \sum_{t' \in \mathcal{T}} \sum_{v \in \mathcal{V}_i} L_v x_{vitjt'} - \sum_{j \in \mathcal{N}^U \cup \mathcal{N}^S} g_{itj} - \sum_{f \in \mathcal{F}_i} L_f^{FOB} z_{ft}, \quad i \in \mathcal{N}^L, t \in \mathcal{T}^L \backslash \{1\} \tag{20}$$

$$Q_i^{MIN} \leq q_{it} \leq Q_i^{MAX}, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L \tag{21}$$

$$q_{it} \geq 0, \qquad i \in \mathcal{N}^L, t \in \mathcal{T}^L \tag{22}$$

C.3 Extension 2: Chartering out own vessels

Parameters

$R_{vt}^{Charter}$ Daily revenue of chartering out ship v on day t, $v \in \mathcal{V}, t \in \mathcal{T}^L$

$\underline{M}$ Minimum number of time periods the vessel can be chartered out for

Variables

y_v 1 if vessel v is chartered out during the planning horizon, 0 otherwise

Objective function

$$
\begin{aligned}
max z = & \sum_{v \in \mathcal{V}} \sum_{((i,t),(j,t')) \in \mathcal{A}_v^U} R_{jt'}^{DES} L_v (1-(t'-t)E) x_{vitjt'} + \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}^L} \sum_{j \in \mathcal{N}^U} R_{j,t+T_{ij}^C}^{DES} (1-T_{ij}^C E) g_{itj} \\
& + \sum_{v \in \mathcal{V}} \sum_{((i,t),(j,t')) \in \mathcal{A}_v^S} R_{jt'}^{DES} L_v (1-(t'-t)E) x_{vitjt'} + \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}^L} \sum_{j \in \mathcal{N}^S} R_{j,t+T_{ij}^C}^{DES} (1-T_{ij}^C E) g_{itj} \\
& + \sum_{i \in \mathcal{N}^L} \sum_{f \in \mathcal{F}_i^U} \sum_{t' \in \mathcal{T}} R_f^{UFOB} L_f^{FOB} z_{ft'} + \sum_{i \in \mathcal{N}^L} \sum_{f \in \mathcal{F}_i^S} \sum_{t' \in \mathcal{T}} R_f^{SFOB} L_f^{FOB} z_{ft'} + \sum_{i \in \mathcal{N}^L} R_i^{END} s_{i,|\mathcal{T}|} \\
& + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}^L} R_{vt}^{Charter} x_{v0t0,t+1} \\
& - \sum_{v \in \mathcal{V}} \sum_{((i,t),(j,t')) \in \mathcal{A}_v} C_{vitjt'}^S x_{vitjt'} - \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{N}^U} C_{itj}^C w_{itj}
\end{aligned}
\tag{23}
$$

Constraints

$$\sum_{i \in \mathcal{N}} \sum_{t=0}^{|\mathcal{T}^L|} x_{vit0t} \leq y_v, \qquad v \in \mathcal{V} \tag{24}$$

$$\sum_{t=0}^{|\mathcal{T}^L|} x_{v0t0,t+1} \geq \underline{M} y_v, \qquad v \in \mathcal{V} \tag{25}$$

$$\sum_{i \in \mathcal{N}} \sum_{t=0}^{t' \cancel{-1}} x_{vitjt'} = \sum_{i \in \mathcal{N}} \sum_{t=t' \cancel{+1}}^{|\mathcal{T}|} x_{vjt'it}, \qquad v \in \mathcal{V}, j \in \mathcal{N}, t' \in \mathcal{T} \tag{26}$$

$$x_{v,0,0,N_v^{START},T_v^{START}} + x_{v,0,0,0,T_v^{START}} = 1, \qquad v \in \mathcal{V} \tag{27}$$

$$x_{vit0t} = \sum_{\tau=t}^{|\mathcal{T}^L|} \sum_{j \in \mathcal{N}^L} x_{v0\tau j\tau}, \qquad v \in \mathcal{V}, i \in \mathcal{N}, t \in \mathcal{T}^L \tag{28}$$

$$y_v \quad \in \quad \{0,1\}, \qquad v \in \mathcal{V} \tag{29}$$

D Greedy Randomized Construction Support Functions

This Appendix presents the additional support functions and corresponding logic presented in Algorithm 4.

D.1 Satisfy Leftover Demand Pseudo Code

Algorithm 6 is implemented to try to satisfy leftover demand after all cargoes for all loading days is allocated.

Algorithm 6 Satisfy Leftover Demand

Input: not satisfied partitions
Output:

```
for partition p in all partitions still not satisfied do
    Calculate missing from satisfying lower demand, missing demand
    for (i, t, j,) in g-variables' keys where j is p's corresponding contract do
        if missing demand + inventory level for i on day t < upper charter capacity then
            Try to satisfy p by increasing g[i,t,j] with missing demand
        else
            feasible amount = upper charter capacity − inventory level for i on day t
            Try to satisfy p by increasing g[i,t,j] with feasible amount
        end if
    end for
end for
```

E Greedy Score-based Construction Heuristic

As mentioned in Section 6.4, sometimes Algorithm 4 is insufficient in finding a feasible initial solution to use for warm start. This is usually due to one of three characteristics with the test instance:

1. Surplus of FOB demand compared to DES demand
2. Surplus of DES demand compared to FOB demand
3. Either DES or FOB contracts having few days to deliver and being high in demand at the same time

Algorithm 4 is lacking in this context mainly due to the random allocation and prioritization of FOB cargoes. This both hinders the algorithm from prioritizing DES cargoes, but also from strategically allocating FOB cargoes to either early pick-up days if FOB contracts are high in demand, or late days if DES contracts are high in demand. To address this issue, Algorithm 8 was implemented, which shares much of the same logic as the randomized construction algorithm. The main difference is that instead of randomly allocating FOB cargoes, the algorithm chooses a best FOB contract in a similar way as it chooses the best DES contract. This logic is shown in Algorithm 7.

Algorithm 7 Find Best FOB contract

Input: loading day t, loading port l
Output: the best partition

```
best FOB contract = None
best score = 0
for f in set of FOB-contracts do
    last day = last element in f's set of FOB days
    amount missing = f's contracted FOB amount
    days left = last day - loading day t
    score = amount missing / days left
    if pick-up of amount missing at l is feasible then
        if score > best score then
            best FOB contract = f
            best score = score
        end if
    end if
end for
```

As shown in lines 19-21, Algorithm 8 finds the DES- and FOB contracts with the highest scores, and then scores them against each other to decide which contract to allocate a cargo to. In order to account for all three demand cases described above, the algorithm is implemented with three configurations. The main difference between the configurations is the charter amount used to find the best DES contract, described in line 3 in Algorithm 5.

Prioritizing DES: In order to prioritize DES contracts, the charter amount set in Algorithm 5 is set in the same manner as the amount set in lines 23-26 in Algorithm 8. In simple terms, the charter amount is set in order to minimize the number of cargoes needed to satisfy lower required demand, with as little LNG as possible. This gives the allocation of DES contracts the benefit of being feasible for more loading days, due to the flexibility in regard to the loading ports' inventory. In contrast, FOB contracts have a set amount that must be picked up.

Prioritizing FOB: To prioritize FOB, the algorithm runs the Algorithm 5 algorithm in the same way as Algorithm 4, with charter amount set to upper charter amount. In this way, the algorithm avoids the effects of DES contracts having a greater number of feasible pick-up dates than FOB contracts.

Prioritizing least number of feasible delivery days left: In this case, the charter amount is set in the same manner as when the algorithm prioritizes DES contracts. The main difference is that the *days left* term used for scoring DES and FOB scores is squared, to prioritize contracts with few days left to a greater extent.

Algorithm 8 Greedy Score-based Construction Heuristic

Input: Decision variables, and sets and parameters
Output: Decisions variables with updated values

```
for vessel in set of vessels do
    if vessel requires maintenance then
        find shortest feasible route for vessel, set all other arcs to 0
    else
        set exit arc to 1, and all other arcs to 0
    end if
end for
for g,w,z in sets of g-, w- and z-variables do
    set g, w and z to 0
end for
for i,t in set of s-variables' keys do
    s[i,t] = s[i,t-1] + lng produced at day t at loading port i
end for
all demand satisfied = False
for loading day t in all loading days do                    ▷ satisfy DES- and FOB-demand
    for loading port l in all loading ports do
        for b in range(0, number of berths for loading port i) do
            check if inventory and berth availability is feasible at day t for loading port l
            best FOB contract = Find best FOB contract (Algorithm 7)
            best partition, contract = Find Best Partition (Algorithm 5)
            if best partition's score >= best FOB contract's score then
                missing = lower demand for best partition - amount chartered[best partition]
                if ⌈missing / lower charter amount⌉ == ⌈missing / upper charter amount⌉ then
                    amount = lower charter amount
                else
                    amount = ⌈missing / ⌈missing / upper charter amount⌉⌉
                end if
                if chartering amount to best partiton on day t is feasible then
                    g[l, t, contract] = amount and w[l, t, contract] = 1
                    Update inventory for l and amount chartered for best partition
                    if demand is satisfied for contract c then
                        remove c from DES-contract ids
                    end if
                    if all demand is satisfied then
                        all demand satisfied = True
                        break
                    end if
                end if
            else
                if allocating best FOB contract is feasible then
                    z[best FOB contract, t] = 1 and update inventory for l
                end if
            end if
        end for
        if inventory for l in day t > upper inventory limit then
            z[artificial FOB for l, t] = 1 and update inventory for l
        end if
    end for
end for
if not all demand satisfied then                    ▷ try to satisfy leftover DES-demand
    Satisfy Leftover Demand (Algorithm 6)
end if
```

F Distance Between Ports

The table is this Appendix presents all the relevant distances between ports used in the test instances described in Chapter 7. The distance unit is nautical miles.

	AE	**AR**	**BD**	**BE**	**BR**	**CN**	**ES**	**FR**	**GB**	**IN**	**IT**	**JP**	**KR**	**KW**	**MX**	**MY**	**NL**	**PT**	**SG**	**TH**	**TR**	**TW**	**US**
NGBON	7308	4650	7928	4414	3584	10600	3700	4000	4400	7000	4200	10500	10300	7600	7000	7935	4420	3304	8054	8794	5051	9440	6200
FU	-	-	-	6560	-	6655	5260	-	-	2850	-	6880	6450	-	-	-	-	-	3890	-	-	-	-
DI	-	-	-	6210	-	6305	4910	-	-	2500	-	6530	6100	-	-	-	-	-	3540	-	-	-	-
SG	3430	9417	1517	11689	8896	2230	11000	11330	11693	1586	11547	2896	2552	3807	13014	120	11755	10730	-	810	12484	1621	13088
AE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3430	-	-	-	-
AR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9417	-	-	-	-
BD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1517	-	-	-	-
BE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11689	-	-	-	-
BR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8896	-	-	-	-
CN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2230	-	-	-	-
ES	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11000	-	-	-	-
FR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11330	-	-	-	-
GB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11693	-	-	-	-
IN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1586	-	-	-	-
IT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11547	-	-	-	-
JP	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2896	-	-	-	-
KR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2552	-	-	-	-
KW	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3807	-	-	-	-
MX	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13014	-	-	-	-
MY	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	120	-	-	-	-
NL	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11755	-	-	-	-
PT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10730	-	-	-	-
TH	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	810	-	-	-	-
TR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	12484	-	-	-	-
TW	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1612	-	-	-	-
US	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13088	-	-	-	-

Table 1: *Full overview of distances between ports. The distance unit is nautical miles*

Note that all the empty cells corresponds to distances that are never used in the model, mainly distances between two customers.

G Test Instances

Table 2 shows a full overview of the test instances that are used for testing the performances of the solution methods presented in this thesis. The double horizontal line separates the instances generated from the Nigeria case from instances created from the Abu Dhabi case. The single horizontal lines separate test instances with a different number of time periods from each other.

Instance ID	$\lvert\mathcal{N}^L\rvert$	$\lvert\mathcal{N}^U\rvert$	$\lvert\mathcal{F}^U\rvert$	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{T}\rvert$
N-1L-B-4M	1	13	10	23	120
N-1L-C-4M	1	14	14	23	120
N-1L-B-6M	1	13	15	23	180
N-1L-C-6M	1	14	21	23	180
N-1L-A-8M	1	16	22	23	240
N-1L-B-8M	1	13	20	23	240
N-1L-C-8M	1	14	27	23	240
N-1L-A-12M	1	16	33	23	365
N-1L-B-12M	1	13	32	23	365
N-1L-C-12M	1	14	40	23	365
A-2L-A-4M	2	6	17	15	120
A-2L-B-4M	2	6	20	15	120
A-2L-C-4M	2	6	8	15	120
A-2L-A-6M	2	6	23	15	180
A-2L-B-6M	2	6	30	15	180
A-2L-C-6M	2	6	11	15	180
A-2L-A-8M	2	6	29	15	240
A-2L-B-8M	2	6	38	15	240
A-2L-C-8M	2	6	15	15	240
A-2L-D-8M	2	6	15	15	240
A-2L-E-8M	2	6	36	15	240
A-2L-A-12M	2	6	41	15	365
A-2L-B-12M	2	6	55	15	365
A-2L-C-12M	2	6	23	15	365
A-2L-D-12M	2	6	22	15	365
A-2L-E-12M	2	6	51	15	365

Table 2: *Overview of one configuration's corresponding instances' problem sizes, differing in the number of loading ports* $\lvert\mathcal{N}^L\rvert$*, average number of unloading ports* $\lvert\mathcal{N}^U\rvert$*, average number of long-term FOB contracts* $\lvert\mathcal{F}^U\rvert$*, average number of vessels* $\lvert\mathcal{V}\rvert$ *and number of time periods* $\lvert\mathcal{T}\rvert$*.*

H Code and Test Instances

The code and test instances can be found at https://github.com/hellevhaug/master-lng-adp. The GitHub repository contains two folders:

1. **commercial solver**: The implementation of the exact model described in Chapter 5.

2. **RHH**: The implementation of the Rolling Horizion Heuristic.

All test instances described in Chapter 7 are to be found in both folders under the *testData* folder. An explanation of the code is also provided in the README.md file in the repository.

H.1 Commercial solver

The code for the arc-flow model presented in Chapter 5 is found in the folder ”commercial solver”. Code for reading data from is located at ”commercial solver/readData”, and includes reading data for contracts, ports, vessels, spot and other relevant sets. Code for initializing arcs, constraints and the Gurobi model is located at ”commercial solver/runModel/”, in addition to files for running the model. Files for initialzing the greedy construction heuristic described in Section 6.4 is also located in this folder. Gurobi logfiles is located at ”commercial solver/logFiles”, and model output files is saved in the ”commercial solver/jsonFiles” folder. Different constants set before running the model are specified in the file ”commercial solver/supportFiles/constants.py”.

H.2 RHH

The code structure for the Rolling Horizon Heuristic is identical to the commercial solver folder structure described above. The main difference in located in the ”RHH/runModel” folder, where the files for both initialization and running is implemented for the RHH instead. Particulary, the implementation of the different compontents of the Rolling Horizon Heuristic is located in the files ”RHH/runModel/runModel.py” and ”RHH/runModel/initModelRHH.py”

I Solver Times and Total Times

This appendix presents an overview of solver times and total times for running all test instances for datasets **N-1L-D** and **A-2L-B** for both Gurobi and the RHH.

Solver times and total times for Gurobi

The table below presents initialization time, solver time, total time, and increase in time when adding initialization time when running the instances with Gurobi.

	Test instance	Initialization time [s]	Solver time [s]	Total time [s]	Increase in time
NLNG	N-1L-D-4M	282	10 800	11 082	2.61%
	N-1L-D-6M	1009	10 800	11 809	9.34%
	N-1L-D-8M	2562	10 800	13 362	23.72%
	N-1L-D-12M	8826	10 800	19 626	81.72%
ADNOC LNG	A-2L-B-4M	84	10 800	10 884	0.78%
	A-2L-B-6M	258	10 800	11 058	2.39%
	A-2L-B-8M	608	10 800	11 408	5.36%
	A-2L-B-12M	2318	10 800	13 118	21.46%

Table 3: *An overview of initialization time, solver time, total time and increase in time for all test instances from datasets* ***N-1L-D*** *and* ***A-2L-B*** *when running the instances with the Gurobi.*

As shown in the table, the initialization time, and therefore also total time, steadily increases with problem size when solving the model with Gurobi.

Solver times and total times for the RHH

The table below presents initalization time, solver time, total time and increase in time when adding initialization time when running the instances with the RHH.

	Test instance	Initialization time [s]	Solver time [s]	Total time [s]	Increase in time
NLNG	N-1L-D-4M	697	97	794	718.57%
	N-1L-D-6M	2643	2 553	5 196	103.53%
	N-1L-D-8M	7 782	7 319	15 101	106.33%
	N-1L-D-12M	33 388	1 881	35 269	1 775.01%
ADNOC LNG	A-2L-B-4M	612	1 207	1 819	50.70%
	A-2L-B-6M	901	2 410	3 311	2.39%
	A-2L-B-8M	4045	927	4 972	37.39%
	A-2L-B-12M	12 870	3 027	15 906	425.47%

Table 4: *An overview of initialization time, solver time, total time and increase in time for all test instances from datasets* ***N-1L-D*** *and* ***A-2L-B*** *when running the instances with the RHH.*

The table shows that the increase in time caused by initialization is in most cases significantly higher when solving the model with the RHH.

J Results

Table 5 presents an overview of integer variables, continuous variables and constraints for all test instances run with Gurobi, both with and without the model extensions describes in Chapter 5.

Instance ID	Model Type	Integer variables	Continuous variables	Constraints
N-1L-B-4M	Basic	922 369	1 650	66 614
N-1L-B-4M	Variable production	922 369	1 774	66 862
N-1L-B-4M	Charter out	968 047	1 650	109 647
N-1L-B-4M	Combined	968 047	1 774	109 895
N-1L-B-6M	Basic	1 597 449	2 535	127 458
N-1L-B-8M	Basic	2 273 909	3 413	191 379
N-1L-B-12M	Basic	3 648 296	5 183	335 362
N-1L-D-4M	Basic	998 721	1 773	71 041
N-1L-D-6M	Basic	1 713 527	2 696	133 681
N-1L-D-8M	Basic	234 675	3 505	193 111
N-1L-D-12M	Basic	3 531 163	5 109	290 880
A-2L-A-4M	Basic	34 196	953	23 071
A-2L-A-6M	Basic	625 708	1 470	35 462
A-2L-A-8M	Basic	890 579	1 956	46 969
A-2L-A-12M	Basic	1 387 751	2 844	64 445
A-2L-B-4M	Basic	352 357	1 037	31 220
A-2L-B-6M	Basic	654 791	1 609	54 556
A-2L-B-8M	Basic	962 091	2 181	83 377
A-2L-B-12M	Basic	1 572 631	3 294	158 969
A-2L-C-4M	Basic	344 022	1 044	31 218
A-2L-C-4M	Variable production	344 022	1 290	31 710
A-2L-C-4M	Charter out	360 482	1 044	44 729
A-2L-C-4M	Combined	360 482	1 290	45 221
A-2L-C-6M	Basic	644 818	1 616	54 545
A-2L-C-8M	Basic	938 509	2 188	83 361
A-2L-C-12M	Basic	1 531 109	3 323	158 996

Table 5: *Overview of test instances' total number of variables and constraints*

As shown in the table, both integer variables, continuous variables and constraints increases with the length of the planning horizon.

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