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Using Stochastic Programming to Evaluate Biomass Estimation Methods in Salmon Farming

Master's thesis in Industrial Economics and Technology Management
Supervisor: Peter Schütz
June 2023

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Preface

This thesis is written in the spring of 2023 as part of the subject TIØ4905 - Managerial Economics and Operations Research, Master's thesis, at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management.

First and foremost, we wish to thank our supervisor Peter Schütz for his detailed and valuable feedback throughout the work with this thesis. Through his detailed insight into our thesis and work, we have gained a lot from Peter's input. In addition, we wish to thank our industrial partners Optoscale AS and Eidsjord Sjøfarm AS represented by Sven Jørund Kolstø and Rolf-Arne Reinholdtsen respectively. The data they provided has been vital to our results, as has the industry knowledge they have shared with us.

Fredrik Holmeide, Magnus Strand & Aksel Østmoe

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Abstract

In this thesis, we analyze the value of increased precision from recent technological advancements in biomass estimation in salmon farming. The analysis is conducted through a comparative study, where we solve a sales planning problem. By comparing solutions from instances utilizing traditional biomass estimation methods with instances that utilize real-time biomass estimation technology, the study quantifies the increased profits and determines the value of information gained from more precise biomass estimations. The instances leveraging real-time biomass estimation rely on the technological solutions provided by the company Optoscale.

We determine the optimal solution for salmon farmers' sales planning problem by defining a multi-stage stochastic programming model over a four-month planning horizon. The model optimizes sales decisions to maximize profits while accounting for production costs. It incorporates uncertainty in both price and biomass estimation development. Furthermore, the model considers the risk preference of salmon farmers, ranging from risk-neutral to risk-averse, by weighing expected profits against Conditional Value at Risk.

Results reveal an increase in profits of 4% for a risk-neutral salmon farmer utilizing Optoscale's estimations compared to traditional estimations. The difference increases to 11% for risk-averse salmon farmers, as they make a higher portion of their sales through forward contracts. The results show how increasing precision in biomass estimates helps salmon farmers make better sales decisions and generate higher profits. This thesis contributes to the existing literature by evaluating the potential for enhanced profitability in the salmon farming industry by adopting technological advancements.

Sammendrag

I denne oppgaven analyserer vi verdien av økt presisjon fra nyere teknologiske fremskritt innen biomasseestimering i lakseoppdrett. Analysen gjennomføres gjennom en komparativ studie, hvor vi løser et salgsplanleggingsproblem. Ved å sammenligne løsninger som bruker tradisjonelle biomasseestimeringsmetoder med løsninger som bruker sanntids biomasseestimeringsteknologi, kvantifiserer studien den økte fortjenesten og finner verdien av informasjon oppnådd fra mer presise biomasseestimasjoner. Instansene som utnytter sanntids biomasseestimering er basert på teknologi levert av selskapet OptoScale.

Vi finner den optimale løsningen for lakseoppdretternes salgsplanleggingsproblem ved å definere en flertrinns stokastisk programmeringsmodell over en planleggingshorisont på fire måneder. Modellen optimerer salgsbeslutninger for å maksimere fortjenesten samtidig som den tar hensyn til produksjonskostnader. Den inkorporerer usikkerhet i både pris- og biomasseutvikling. I tillegg imøtekommes risikopreferansene til lakseoppdrettere, fra risikonøytral til risikoavers, ved å veie forventet fortjeneste opp mot Conditional Value at Risk.

Resultatene viser en økning i fortjeneste på 4% for en risikonøytral lakseoppdretter som benytter Optoscales estimater sammenlignet med tradisjonelle estimater. Forskjellen øker til 11% for risikoaverse lakseoppdrettere, ettersom de gjør en høyere andel av salget gjennom terminkontrakter. Resultatene viser hvordan økt presisjon i biomasseestimering hjelper lakseoppdrettere til å ta bedre salgsbeslutninger og generere høyere fortjeneste. Denne oppgaven bidrar til den eksisterende litteraturen ved å evaluere potensialet for økt lønnsomhet i lakseoppdrettsnæringen gjennom å ta i bruk teknologiske fremskritt.

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Chapter 1

Introduction

In 2022, Norway exported almost 3 million tons of seafood, translating to more than 150 billion NOK in revenue. Of this, 105 billion came from salmon farming (Norwegian Seafood Council, 2022). The Norwegian salmon industry has proven extremely valuable, but certain environmental challenges have been a cause of concern. Lekang et al. (2016) raises two main environmental challenges the salmon industry faces. Large parasite outbreaks damage the farmed fish and the wildlife, while escaping fish lead to gene mixing between farmed and wild fish. As a result, regulations that limit the maximum allowable biomass (MAB) present at all times have been imposed by the government. Thus, the production volume has stagnated during the last decade.

Given the MAB regulations, new approaches must be developed in order to improve productivity in the industry. Multiple approaches have been studied to increase production volume while adhering to the regulations. This includes land-based and offshore salmon farming. Also, novel smolt types have been developed to shorten the time spent in the sea, and thus the production cycle. Another approach, which this thesis focuses on, is to utilize new technology to maximize the value of current production volumes.

Lafont et al. (2019) states that the aquaculture industry must make the same technological transition as agriculture did decades ago in order to become reliable and sustainable. In this setting, technological transition refers to Industry 4.0, the utilization of IoT, 5G, and machine learning. These new technologies allow salmon farmers to access accurate real-time data. Leveraging the enhanced data can improve decision-making and be a part of the solution to improve productivity in the industry.

One particular application of new technology in the salmon farming industry is biomass estimation. Information about biomass affects salmon farmers' feeding schedules, harvesting decisions, and sales decisions. Several approaches for accurate biomass estimation have recently been proposed, and some have reached the market (Li et al., 2020). However, little to no literature exists on the value that more accurate biomass estimations provide in salmon farming. Thus, in this thesis, we add to the existing literature by analyzing the value of information gained from accurate real-time biomass estimations. In particular, we study how sales decisions and profitability change with more accurate biomass data.

In order to study sales decisions and evaluate the value of information, we utilize multi-stage stochastic optimization techniques. Some attention has been given to such optimization techniques in salmon farming previously. For the most part, the existing literature on stochastic optimization in salmon farming revolves around production planning and decision-making to be used by salmon farmers. We differentiate our work by using the stochastic program to solve a sales planning problem in order to evaluate new technology for biomass estimation.

The rest of this thesis is organized as follows. We present relevant background information about salmon farming in Chapter 2. Then we provide an abstract description of the problem in Chapter 3. In Chapter 4, we review the relevant literature. Next, Chapter 5 explains the methodology used to calculate input parameters to the mathematical model. We present the mathematical model formulation in Chapter 6. Furthermore, we give an overview of the instantiation in Chapter 7. We then present the computational study in Chapter 8. After the computational study, we discuss possible future research in Chapter 9. At last, we conclude with our final remarks in Chapter 10.

Chapter 2

Background

This chapter presents background knowledge about the salmon farming industry relevant to the remainder of this thesis. Firstly, we introduce salmon farming, describing the process and the regulatory framework in which salmon farmers operate. The second section introduces sources of uncertainty in the industry, focusing on biomass uncertainty and price risk. Thirdly, we present risk management tools that farmers can utilize to handle risk.

2.1 Salmon farming

This section provides an overview of the process of salmon farming, introducing key concepts and expression in the industry. Furthermore, we present regulations present in the industry.

2.1.1 The process of salmon farming

Salmon production is a comprehensive process consisting of multiple stages, where each production cycle last approximately three years (MOWI, 2022). Firstly, salmon roe is fertilized and hatched in hatcheries, which are land-based facilities. The newly hatched salmon, known as fry, will reside within these land-based facilities for 10 to 16 months until they undergo a process called smoltification. Following the smoltification, the now-called smolt acquires the ability to survive in saltwater. The smolt typically weighs between 60 and 100 grams at this point (laks.no, 2023).

Next, the smolt is released into underwater net pens in seawater. The smolt will stay in these net pens for 14 to 22 months to become full-grown salmon ready for harvest (laks.no, 2023). Following the growth period in net pens, the salmon is transported to a processing facility through well-boats. Finally, the salmon is sorted in the facilities based on size and quality before being gutted and transported to be either sold or further processed. The process of salmon farming in its entirety is visualized in Figure 2.1.

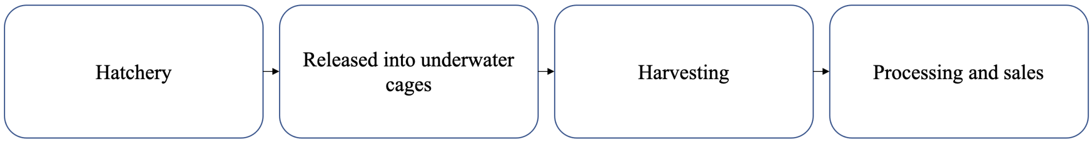


Figure 2.1: Stages of the salmon farming process (laks.no, 2023).

2.1.2 Regulatory framework

Among the most central regulations that salmon farmers need to consider, we find the MAB. This regulation states the total amount of biomass that can be present at any time. The Norwegian Directorate of Fisheries regulates the MAB through licenses. In general, each license provides a MAB of 780 tonnes, except in the most northern county, where each license provides a MAB of 945 tonnes. Then, the sum of all MAB from licenses held by a company in a particular region determines the company's MAB in that region. Additionally, each production site is associated with a MAB that restricts the total amount of fish on that site. On site level, the MAB is typically between 2,340 and 4,680 tonnes (MOWI, 2022). Salmon farming companies must report biomass monthly, and exceeding the MAB incurs fines (Fiskeridirektoratet, 2023).

Another comprehensively regulated aspect of salmon farming is diseases affecting the farmed salmon. The Norwegian government enforces many regulations to ensure the well-being of the salmon and that the salmon is safe to eat. Farmed salmon in Norway are vaccinated to limit bacterial infections and virus diseases that previously led to a high mortality rate (Fiskeridirektoratet, 2023). The use of antibiotics is also tightly regulated, and no salmon treated with antibiotics can be sold in Norway. Today's most common diseases affecting Norwegian farmed salmon are infectious salmon anemia and pancreas disease (Norges Sjømatråd, 2021). Salmon infected with a disease are prohibited from being sold.

Allowable levels of sea lice are also subject to regulations. High amounts of sea lice living on the salmon lead to deterring quality and growth. Sea lice could also have environmental impacts if the farmed salmon escapes from production facilities and pass the sea lice on to wild salmon. Therefore, the sea lice living in an underwater net pen is regulated to a maximum of 0.5 adult female lice per fish. During the breeding season for wild salmon, the regulation is even more stringent at 0.2 adult female lice per fish (Fiskeri- og kystdepartementet, 2023). If allowable sea lice levels are exceeded, the salmon must undergo treatments to reduce sea lice levels, which often causes stress and potential injuries. As a last resort, the salmon farmer might be forced to harvest the salmon prematurely (Nærings- og fiskeridepartementet, 2012). To not let sea lice contaminate across generations in a net pen, a fallowing period of normally two months is required after harvest before new smolt can be released into a net pen (Werkman et al., 2010).

As well as ensuring the well-being of farmed salmon, the aforementioned regulations are implemented to limit the environmental impact of salmon farming. Escaping salmon from salmon farming facilities that breed with wild Atlantic salmon and sea lice spreading to the wild stock of salmon are considered the most severe environmental challenges linked to salmon farming (Misund, 2023). In addition, feces and sediments left on the bottom of the ocean underneath salmon farming facilities threaten the biodiversity along the Norwegian coastline.

2.2 Uncertainty in salmon farming

This section introduces factors of uncertainty present in salmon farming. We focus on uncertainty related to biomass and future price development.

2.2.1 Biomass uncertainty

Fish are subject to changing growth conditions in the sea, which over time, causes uncertainty in the total biomass and size distribution of a fish cohort. Multiple factors affect growth conditions for salmon. Firstly, environmental factors play a significant role in growth, with water temperature having the most impact (Dwyer & Piper, 1987). Water temperature is subject to weather and seasonal variations and is difficult for salmon farmers to influence. The ideal growth temperature for salmon is between 8 and 14 degrees Celcius (MOWI, 2022). Secondly, as mentioned in Section 2.1, diseases affecting salmon are an issue for salmon farmers. An outbreak of diseases will stagnate growth in the salmon cohort. Additionally, diseases can increase mortality rates, which also affect the biomass. Thirdly, another factor that impacts the growth of salmon is gender maturation. Salmon farmers wish to delay gender maturation, as this process limits the potential for growth and often decreases the fish's quality. Several methods are used in the salmon farming industry today to delay this process, but the timing of gender maturation remains uncertain.

Uncertainty in the growth of salmon, and consequently biomass, results in challenges when trying to optimize production. For one, sales decisions become increasingly tricky as biomass uncertainty rises. Also, the feeding schedule becomes more challenging. Salmon feed is salmon farmers' most significant running expense, making up about 50% of expenses (MOWI, 2022). Underestimating biomass due to higher growth than expected could lead to underfeeding, limiting growth and profitability at harvest. On the other hand, overestimating biomass could lead to overfeeding, resulting in unnecessary feeding costs and waste, negatively affecting the environment.

Environmental factors like water temperature and diseases affect growth, introducing biomass uncertainty. This uncertainty presents challenges for salmon farmers. Next, we focus on the salmon market, examining uncertainty in salmon prices and its implication on farmers.

2.2.2 Salmon market

Variations in supply and demand for salmon affect the price. From the open database provided by Fish Pool (2022), we can analyze the historical price development of salmon. Figure 2.2 depicts the development of salmon prices in the last ten years while Figure 2.3 shows a box plot that statistically describes the weekly and four weeks price changes over the same time period. The price has varied from less than 40 NOK/kg to more than 120 NOK/kg during the last decade. Also, substantial deviations occur in relatively short periods. From Figure 2.3, we can see that almost 50% of the weeks experience a price change of more than ±5%. While over four weeks, almost 50% of the samples show a price change of more than ±10%.

Price change in itself does not imply uncertainty as long as the change is predictable. Thus, we must analyze whether or not future price development is predictable or not. Seasonal patterns exist in the salmon price. Bloznelis (2016) explains that the seasonality patterns are caused by varying growth conditions, affecting the salmon supply and thus price. However, these observed yearly seasonal patterns cannot explain the significant price changes in short time periods. Furthermore, attempts to forecast short-term price development have yet to be particularly successful. Bloznelis (2018) predicts future prices 1-5 weeks ahead using both traditional time series models and customized machine learning models trained on a series of relevant parameters. Results show some ability to predict future prices, but only minor improvements compared to simpler methods. According to Bloznelis (2018), unpredictable events largely determine the short-term price change.

MOWI, the largest Norwegian salmon farming company, present their view on factors that affect the salmon price in their industry handbook (MOWI, 2022). According to MOWI, since the production cycle is three years long, it is challenging to adjust production. As a result, supply quantity is very inelastic in the

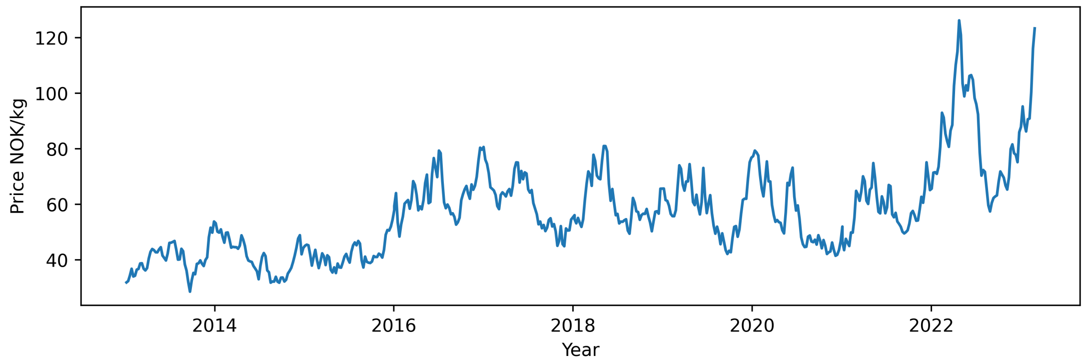


Figure 2.2: Historic development of price per kilogram for 4-5 kilogram salmon.

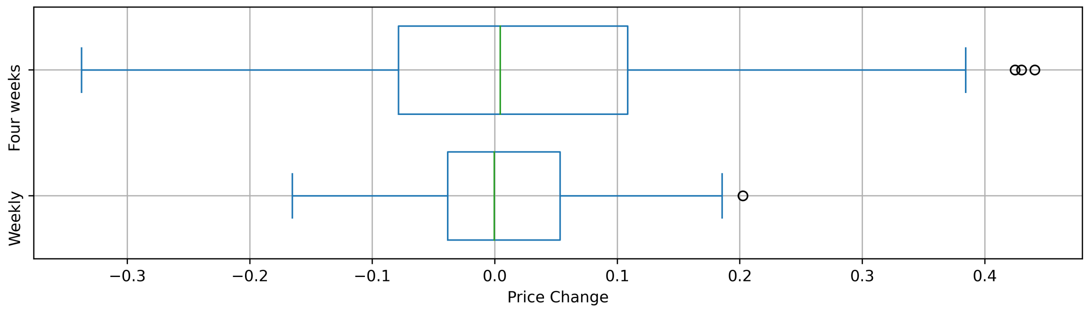


Figure 2.3: Boxplot showing the locality, spread, and skewness of the weekly and four weeks price changes in the past ten years.

short term. In their view, this is the most prominent reason for price volatility in the salmon market.

Considerable price uncertainty impacts salmon farmers' profitability. Sudden price drops can lead to substantial financial losses. With low prices, salmon farmers might struggle to cover their production costs. Furthermore, significant increases can, but not necessarily, lead to big gains. As the production quantity is hard to adjust, companies might struggle to take advantage of price spikes.

Another consequence of volatile prices is related to investments. Since future profits are uncertain, investing in new projects will require a high-risk premium. On the other hand, a lack of investments affects individual companies and limits the entire industry's potential for growth. Thus, implementing effective risk management tools can prove valuable for the industry. We address this topic in the forthcoming section.

2.3 Risk management

We begin this section by discussing tools salmon farmers can utilize to mitigate the effects of price uncertainty. Afterwards, we focus on biomass and present a new technology that can reduce biomass estimation uncertainty.

2.3.1 Forward contracts

By introducing Fish Pool in 2005, selling salmon through a regulated exchange (Fish Pool, 2022) became possible. The introduction of Fish Pool provided the option of selling salmon in future contracts as an alternative to spot sales, which is selling the salmon at the time of harvest. Future contracts are agreements to buy a predefined amount of a commodity at an agreed-upon time. The agreement is mutually binding, meaning that the seller needs to deliver what is specified in the contract while, at the same time, the buyer is obliged to buy at the previously agreed-upon price. There also exists another option for selling salmon in advance, which is the use of forward contracts. Forward and future contracts differ mainly because future contracts are traded through an exchange and are used as financial instruments. In contrast, forward contracts are agreed upon directly between the seller and the buyer (CFI, 2022). Prices on future contracts available on Fish Pool provide transparency to the salmon pricing, making the pricing of individual forward contracts less variable. The duration of forward and future contracts is called maturity. For the remainder of this thesis, we focus on the term forward contracts since we assume a direct transaction between buyer and seller. However, to determine the prices on the forward contracts, we utilize prices based on data from Fish Pool.

Through forward contracts, given that the seller can meet the criteria specified in the contract, the income is certain. Forward contracts remove the possibility of price development affecting the seller's income. However, from analyzing historical price data, the forward price, on average, tends to be lower than the spot price. Table 2.1 presents the average price and standard deviation from the last decade. The spot price indicates the highest average price, whereas the forward price declines when the contract is agreed upon further in advance. On average,

Table 2.1: Mean and standard deviation for spot prices and forward prices the last decade. Each numbered column represent the number of months in advance of delivery the forward contract is agreed upon.

	Spot	One	Two	Three	Four	Five	Six
Mean	59.82	59.07	58.58	58.02	57.62	57.33	56.93
Std	15.62	12.87	11.79	11.16	10.93	10.70	10.62

selling fish in contracts with a six month maturity will experience a 4.8% reduced price, as seen in Table 2.1. At the same time, we observe that the forward prices experience less variability than the spot price, as the standard deviation is decreases for the contracts as the maturity increases.

This brief analysis shows that forward contracts are an effective tool for mitigating price risk. However, as spot prices are on average higher than forward prices, the usage of forward contracts becomes a question of risk profile.

Through meetings and dialogue with Norwegian salmon farming companies, we have gained insight into how sales decisions are currently made within the industry (Eidsfjord, 2022). A central takeaway is that the risk profile of a salmon farming company significantly affects how they make sales decisions and that the risk profiles of different companies can vary substantially. The primary decision to be made by a salmon farmer regarding sales is how to distribute sales through forward contracts compared to spot sales. Dialogue with salmon farmers provided insight into what decisions look like for a risk-averse and risk-neutral salmon farmer. Risk-averse salmon farmers make more sales through forward contracts, while risk-neutral farmers make most sales at harvest, obtaining the spot price. Bergfjord (2009) surveyed Norwegian salmon farmers about their risk profile and management. Results show that salmon farmers view themself as moderately risk-averse compared to other industries.

Reasons for the risk profile of a company will vary. Discussions with the industry indicated that capital structure and ownership were two essential factors. Firstly, companies with high debt-equity ratios could be forced to lower risks by securing income through forward contracts. On the other hand, a company with a more robust capital structure could be more inclined to take risks and thus sell more fish at harvest to spot price. Secondly, the number of stakeholders can affect the risk preference. Publicly traded companies have more investors to satisfy. A demand for stable positive results is often valued to obtain a stable stock price. As a result, lowering risk through selling salmon in forward contracts might be a natural strategy.

Although price risk is reduced when salmon farmers make sales through forward contracts, utilization of forward contracts leads to increased consequences of wrong biomass estimation. Entering into a forward contract implies that salmon farmers must predict future biomass to determine the contract size they believe they can deliver. As a result, farmers could fail to deliver the agreed-upon amount of salmon in case of overestimation. Consequently, fish must be bought in the market, probably from a competitor, at a premium price higher than the spot price. Next, we present a new approach to biomass estimation, aiming to lower biomass uncertainty and, thus, the possibility of making better decisions.

2.3.2 Novel biomass estimation

Multiple approaches to mitigate biomass uncertainty have been proposed and tested. Amongst the most widely utilized are mathematical formulas, a frame with sensors measuring fish passing through it, and manual sampling within net pens. Aunsmo et al. (2013) perform a statistical analysis on 240 net pens, where site managers estimate the total biomass before harvest using fish frames and manual sampling before exact results are obtained from harvesting. Results show that the mean absolute biomass estimation error was 5.1%. However, current industry agents claim that today's estimation methods result in uncertainty between 6 to 8 % (OptoScale, 2022a). Additionally, manual sampling of fish is deemed intrusive, causing stress and damage to the fish (Ashley, 2007).

Recent technological advancements in fields such as AI, IoT, and sensory technology have the potential to provide more accurate biomass estimations, which can positively affect salmon farmers' operating results and sales decisions. Representing the current technological development in biomass estimation, we now give an overview of a product providing a novel solution for biomass estimations.

The company OptoScale provides an advanced camera module called the Bio-

Figure 2.4: BioScope - A camera module to measure salmon.

Scope. The BioScope utilizes image recognition technology to identify individual salmon passing the camera and measuring each identified salmon's size and weight. The BioScope can make up to 40,000 fish measurements in a single day. All measurements are, after processing, represented to the customers, salmon farmers, in a dashboard visualising live data from the measured net pens (OptoScale, 2022b). Figure 2.4 shows a picture of the BioScope. OptoScale provides a guarantee stating that the absolute deviation between the mean weight from measurements by OptoScale and the actual mean weight at harvest is less than 3% (OptoScale, 2022b).

Chapter 3

Problem Description

This section introduces the sales planning problem faced by salmon farmers. The sales planning problem involves making critical decisions regarding how and when fish is sold and harvested within a given planning horizon. The outcome of sales and harvesting decisions directly impact the profitability of salmon farms. Specifically, the objective of the sales planning problem is to maximize the expected profit while considering the risk profile of the farmer. Farmers must also adhere to any restrictions imposed by the government and production plans.

As discussed in Section 2.2, farmers can sell fish in advance through forward contracts or after harvest, called spot sales. In this problem, the fish price depends on weight and maturity. We need to define the sales decision over both of these factors. For weight, the price per kilogram is given for a discretized set of fish weights, e.g. 4-5 kilograms. Next, maturity refers to the number of months from when the contract is entered to the delivery of the fish. Note that we define a contract with maturity zero as spot sales. Additionally, salmon is a commodity that experiences significant price volatility due to fluctuating supply and demand. Thus, the sales decisions must be defined over a set of time periods in addition to weight classes and maturities.

Harvesting decisions must also be addressed. A salmon farmer often runs operations at different sites in different locations. Each site contains a set of net pens where the fish live. In each time period, the farmer must decide whether to harvest fish from a particular net pen and, if so, how much. Thus the harvest decisions are defined over the set of net pens and the set of time periods.

In addition to sales and harvesting decisions, producers can purchase fish in the market. In this problem, if the harvested amount of fish is less than anticipated and thus fails to fulfill existing contracts, producers must buy the remaining fish in the market. Purchasing decisions are defined for time periods and weight classes. As buying fish in the market involves purchasing from competitors, we apply a price premium to the spot price in this case.

Another aspect that influences the decisions mentioned above is uncertainty. In salmon farming, sales and harvesting decisions are made repeatedly. In this problem, new information about price and biomass estimates is revealed between time periods, but decisions must be made before this revelation. Specifically, the current price is known, but the farmer must account for both positive and negative future price developments. Biomass estimates, however, will be subject to uncertainty even in the current time period.

In this problem, we handle uncertainty by discretizing and combining the possible developments of price and biomass estimates. Thus, multiple scenarios can occur with an associated probability between time periods. One scenario refers to one possible combination of price and biomass estimation. Since this problem deals with multiple time periods, we repeat the discretization process for each time period. This process is represented as a scenario tree, where each vertex in the tree represents an outcome of the uncertain parameters. The root vertex of the scenario tree represents the decision to be made here and now. It must consider all possible realizations of uncertain parameters for all time periods when making decisions.

Furthermore, salmon farmers must adhere to multiple restrictions in their operations. MAB restricts the total biomass at any time. The Norwegian government has imposed this restriction on two levels. First, based on the number of licenses a company owns, an upper limit restricts the amount of fish they can have in the water at any time at a company level. Also, each production site has an associated MAB limit.

Chapter 4

Literature Review

In this chapter, we review existing literature on optimization regarding sales planning in salmon farming. We begin the review by chronologically discussing and reviewing research on sales and production planning in salmon farming up to the most recent literature. Then, we present a section on papers discussing risk management using forward contracts in the salmon market. Next, we present papers that study and assess the value of information. More precisely, we review papers that study how more precise information can affect decisions and improve profitability. Finally, we place our contribution in the landscape of existing literature.

4.1 Sales and production planning in salmon farming

Literature on sales planning within the salmon industry is limited, with much more literature available regarding production planning. We, therefore, begin by introducing existing literature related to optimization within production planning before redirecting attention to the literature regarding sales planning.

Bjørndal (1988) analyzes optimal harvesting times for farmed salmon and turbot. The optimal harvesting time is in Bjørndal (1988) found by solving the rotation problem, initially applied in the forestry industry. The biomass modeling in Bjørndal (1988) remains deterministic, without uncertainty, highlighted as a point of further research. Arnason (1992) extends this analysis by including the optimal feeding schedule in the model. Furthermore, Arnason (1992) simplifies the model by assuming a uniform weight distribution of all fish in a cohort, together with constant prices. With these assumptions, Arnason (1992) identifies the limitations of the practical use cases of such a model.

Forsberg (1996) presents an approach for production planning in fish farms. Production planning in this setting refers to smolt deployment, biomass estimation, and harvesting schedule planning. Here, fish growth is simulated as a Markov process. In this approach, each fish belongs to a fish class before possibly moving to the next class, in the following time period, with a predetermined probability. Furthermore, Forsberg (1996) utilizes a multi-period LP formulation to maximize profits over the planning horizon. Forsberg (1999) looks further into maximizing profit by finding the optimal harvest schedule. The concept of graded harvest is introduced. This harvest strategy allows salmon producers to harvest each cohort's most profitable fish classes. Forsberg (1999) shows that graded harvest has the potential of increasing profits by 10-15%. The presented model is deterministic and will disregard all uncertain parameters like growth rates, mortality, and prices.

Guttormsen (2008) applies the optimal rotation problem in salmon farming to find optimal deployment and harvesting. Two central aspects of the salmon farming industry are emphasized. First, seasonal relative price changes are considered, meaning the difference between weight classes varies with time. Secondly, restrictions in terms of possible deployment times of smolt are considered. These aspects fundamentally make the rotational problem in salmon farming different from the application in the forestry industry. As a result, the theoretically optimal harvest weight obtained by the default rotation problem will have limited importance in salmon farming. Guttormsen (2008) extends the original rotation problem by incorporating the aforementioned aspects. Results from an empirical illustration show that optimal harvesting time changes radically with the inclusion of relative price changes and restrictions on deployment.

Hæreid et al. (2013) present a multi-staged stochastic programming model to find optimal decisions for releasing smolt and sales planning. Important uncertain variables included in the formulation are demand for salmon, price, growth, mortality, and smolt deliveries. Also, the model includes MAB restrictions, which regulate the maximum allowable biomass at all times. Including uncertainty parameters separates this paper from previous work on the topic, which mostly regarded the problem as deterministic. Hæreid et al. (2013) discuss the effects of Fish Pool on the salmon market, making it possible for salmon farmers to reduce price uncertainty by selling fish through forward contracts.

4.2 Risk management using forward contracts

Larsen & Asche (2011) investigate the use of forward contracts through data on Norwegian salmon exports to France. They find that close to 25% of all sales were through forward contracts. The result indicates that forward contracts are actively used to reduce price risk.

Misund & Asche (2016) address whether salmon futures provide a good hedge. They measure the effectiveness of futures as a hedging tool by calculating the deviation in variance from a strategy where all fish is sold spot. Results show that utilization of futures contracts reduces risk by 30-40%. They conclude that Fish Pool works better than other seafood markets, such as black tiger shrimp. On the other hand, several markets that sell agricultural commodities have proven to be even more established and have achieved a certain maturity level.

Misund & Asche (2016) also address another property futures exchanges should possess to succeed. Future prices should serve as a price discovery of future spot prices. Future contract prices have a price discovery mechanism if they predict future spot prices. Asche et al. (2016) analyze this issue and conclude that future prices in the salmon market do not provide a price discovery mechanism. However, Asche et al. (2016) point out that Fish Pool seems to move in the right direction to become a standard risk management tool in the salmon market. It is worth noting that Fischer & Lai (2016) also analyze the relationship between spot and future prices on Fish Pool and find that future prices provide a price discovery function.

Schütz & Westgaard (2018) extend the discussion regarding the future salmon market by analyzing optimal hedging strategies for salmon farmers. The problem is formulated and solved as a multi-stage stochastic programming model, where the price is modeled as the uncertain parameter. The critical decision in the problem is how much fish to sell in advance, and the objective is to maximize the weighted sum of the expected profit and the time-consistent Conditional Value at Risk (CVaR). They conclude that salmon farmers should, at reasonably low levels of risk aversion, hedge prices through futures.

4.3 Value of information

Access to more information about uncertain variables can alter decisions and improve profitability. Today, new technologies present the opportunity of gathering precise information on industry processes in real time, which was impossible only a couple of decades ago. In this section, we review the literature measuring information's value.

The value of information from new technological advancements has yet to be studied in salmon farming. However, Forsberg & Guttormsen (2006) study the value of information regarding future spot prices. More specifically, they study the production and sales plan for a fish farm given different levels of price information. Results show that profits increase proportionally with more information about prices.

4.4 Our contribution

As this chapter presents, optimization in salmon farming has already been studied for multiple decades. Early papers focus on production planning and usually model the problem deterministically. More recent studies incorporate stochasticity in the problem formulation. Also, after establishing the futures exchange Fish Pool, optimal sales planning has been studied in recent years. However, work has yet to be conducted on technological advancement in the industry and increasingly precise biomass measurements.

This thesis formulates and solves the stochastic sales planning problem for salmon farmers. We formulate the sales planning problem partly inspired by Schütz & Westgaard (2018) and extend it by incorporating essential aspects from the production side of the problem. The problem we solve is stochastic multi-stage sales planning, focusing on the value of information from better biomass estimates. We analyze this influence on the objective value and various decision variables to extend the existing literature.

Chapter 5

Methodology

In this chapter, we describe the methods used to calculate specific parameters. We elaborate on the parameters that require extra preprocessing, specifically those that are not discussed in the parameterization for various instances of the model in Chapter 7. We begin by explaining what kind of projection model we use to predict the future weight of fish, then how we distribute an expected mean weight into various weight classes. Afterward, we present analyzes of historical fish prices and price variations based on weight class. The work presented in these sections is based on Holmeide et al. (2022). Finally, we discuss the methodology for multi-stage stochastic programming.

5.1 Modeling biomass development

As described in Chapter 3, salmon farmers have the option of selling salmon in forward contracts. This means they sell salmon in advance before the actual growth of the salmon has taken place. Therefore, to make the best possible sales decisions regarding contracts, salmon farmers want the best possible insight into the current biomass and the most precise projections of biomass development.

What seems most widespread in practice are simple models that convert the amount of food fed to the fish into a biomass increase based on a feed conversion ratio. However, Aunsmo et al. (2014) show that this way of modeling biomass development is unstable for fish in different weight classes, as they will have different feed conversion ratios. In addition, we have no data describing the amount of feed or feed conversation ratios for Eidsfjord Sjøfarm.

Aunsmo et al. (2014) carried out a field validation of various projection models for biomass and concluded that using thermal growth coefficients (TGC) is among the most robust models where all coefficients are publicly published. Therefore, we have chosen to model biomass development using TGC, shown in (5.1), developed by (Iwama & Tautz, 1981). The equation calculates a TGC factor based on the relationship between the mean weight of salmon and the seawater temperature.

$$TGC = \frac{1000(w_e^{1/3} - w_0^{1/3})}{\sum_{d=1}^{e} T_d} \tag{5.1}$$

We describe the modeling of biomass development in (5.2). The next period's mean weight is calculated based on the previous mean weight, the TGC factor, the length of the time period, and the seawater temperature.

$$w_{(t+1)n} = (w_{tn}^{\frac{1}{3}} + \frac{1}{1000} \cdot TGC_{tn} \cdot L_t \cdot T_{tn})^3 \tag{5.2}$$

Equation (5.2) calculates the new expected mean weight without including a reduction in the biomass due to, for example, mortality. However, assuming a monthly mortality rate μ, we can estimate the future population more precisely. Equation (5.3) shows this, where we see that the proportion of fish that are alive $s_{(t+1)n}$ in time period *t+1* is the proportion that is alive in s_{tn}, less the mortality rate μ.

$$s_{(t+1)n} = (1 - \mu) \cdot s_{tn} \tag{5.3}$$

To determine the ratio of the mean weight in a given time period compared to the initial mean weight, we multiply w_{tn} in time period t by the proportion of fish alive s_{tn} before dividing by the initial mean weight w_{1n}, as described in Equation (5.4).

$$A_{tn} = \frac{s_{tn} \cdot w_{tn}}{w_{1n}} \tag{5.4}$$

The biomass ratio given by A_{tn}, as described in Equation (5.4), can also be used to calculate the periodic growth in percentage.

$$R_{tn} = \frac{A_{tn}}{A_{(t-1)n}} \tag{5.5}$$

Throughout the mathematical model described in Chapter 6, R_{tn}, the monthly growth rate, occurs as G^{s}_{tn}. An explanation of the various scenarios s will follow in Chapter 7.

5.2 Biomass distribution

Each contract regarding sold fish specifies the amount to be delivered within each weight class. We, therefore, have to find a distribution that distributes an expected mean weight into various weight classes.

There are different practices related to whether the distribution of fish weight follows a normal or log-normal distribution, but a normal distribution is the most widespread (MOWI, 2022). We have used time series with measurements from OptoScale to find the most suitable distribution. From these measurements, a normal distribution appears the most realistic. We visualize an example of a weight distribution in a net pen from the measurements provided by OptoScale in Figure 5.1.

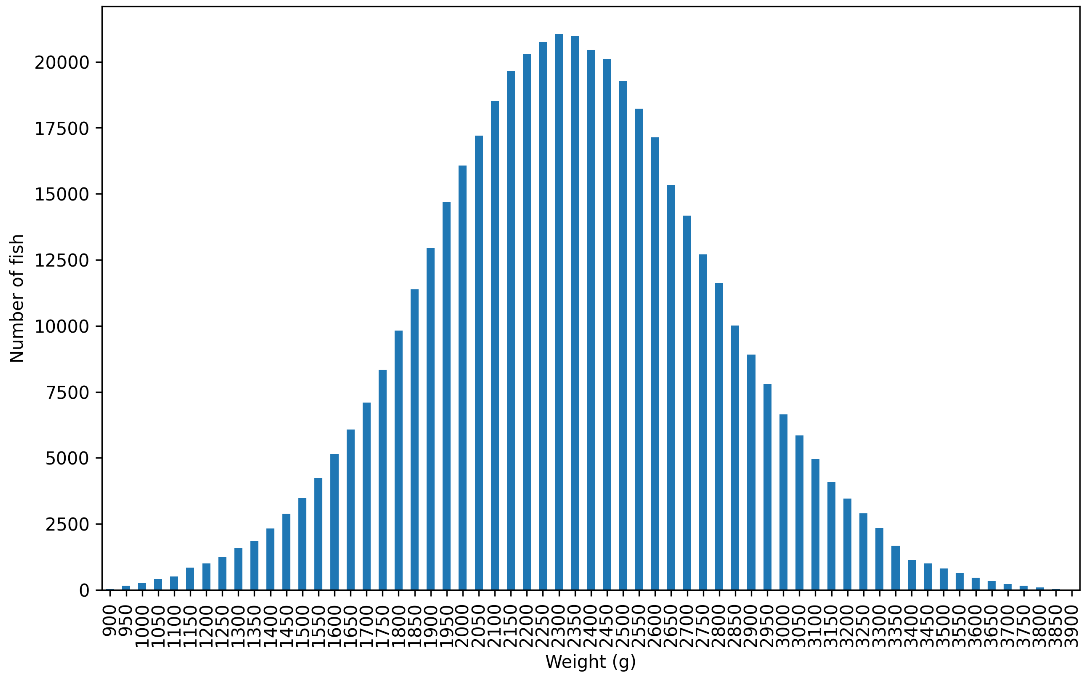


Figure 5.1: Distribution of weight for a net pen based on data from OptoScale.

When we assume what kind of distribution the fish's weight follows, we need a coefficient of variation (CV) to distribute the fish into the different weight classes. CV is the ratio between the standard deviation (SD) and the mean of a specific distribution (Birge & Louveaux, 2011), as shown in Equation (5.6).

$$CV = \frac{\sigma}{\mu} \tag{5.6}$$

We obtain the mean weight, μ, by applying the monthly growth rate described in Equation (5.5). Next, we calculate the SD using the data from OptoScale in the formula described in Equation (5.7). This equation calculates the SD by counting the number of fish in each weight class and net pen during the entire time series for which we have data. Based on this calculation, we find a value for CV for all net pens, and by taking the mean, we are left with a value of 0.217. This is somewhat lower than the CV traditionally used by salmon farmers when they distribute the weight of the fish, which is 0.225 (Forsberg, 1999). The different CVs mean we can model weight distribution differently for OptoScale and traditional methods.

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2} \tag{5.7}$$

By using the cumulative distribution function, we can find the amount of fish that falls within each weight class. To do this, we need to standardize first. We denote Φ as the standard normal cumulative distribution function, μ as the mean weight, and σ as the standard deviation. The standardization is explained in (5.8), where we find the difference between the cumulative distributions for the various weight classes. We limit them between a lower and an upper weight limit, $\mathrm{w_l}$ and $\mathrm{w_u}$ respectively, for the specific weight classes.

$$\mathrm{P(X < w_u) - P(X < w_l)} = \Phi\left(\frac{\mathrm{w_u} - \mu}{\sigma}\right) - \Phi\left(\frac{\mathrm{w_l} - \mu}{\sigma}\right) \tag{5.8}$$

Using dissimilar CVs will ensure that the fish distributions will be separate for OptoScale and traditional methods. We use this throughout the analysis to distribute the biomass into various weight classes. In the upcoming section, we discuss price modeling, as fish distributed in different weight classes will have different prices.

5.3 Price

So far, in this chapter, we have described growth development and distribution. To assess the value of information in sales planning, we depend on linking the amount of fish of a given weight with the associated price. The analysis uses public historical information from Fish Pool as a foundation. Fish Pool provides price information for the spot price development of the weight classes 3-4, 4-5, and 5-6 kilograms, as these are the most common weight classes of sold salmon (Fish Pool, 2022).

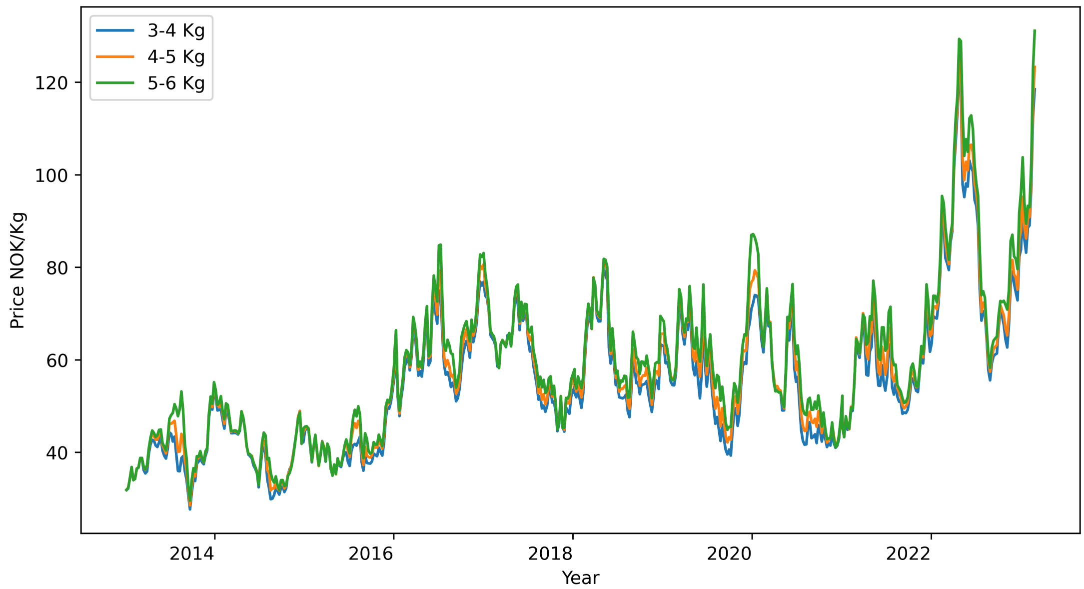


Figure 5.2: Salmon Price development since 2015 for different weight classes.

The historical price development for the mentioned weight classes is visualized in Figure 5.2. We observe a volatile price development. The price has, on average, increased during the last decade. However, an almost equally significant decrease often follows the large increases, and we do not see any immediate seasonal adjustments that explain the volatile price. We also note that the various weight class prices closely follow each other. Still, some variations exist, especially at the price development's most prominent peaks and depths.

Usually, the larger weight classes have a higher price per kilogram than the lower weight classes. We analyze this further by visualizing the percentage deviation of weight classes compared to the 4-5 kilogram class since 2015 in Figure 5.3. The differences vary from 0 to over 10% in brief periods. Note that the largest differences occur in the same time periods as the large price fluctuations. When the deviation for the 5-6 kilogram class increases towards 10%, we see a tendency of mirroring by the 3-4 kilogram class, as it often increases towards -10%. On average, the 5-6 kilogram class is 2.8% higher per kilogram in price than the 4-5 kilogram, while the 3-4 kilogram class is 3.2% lower.

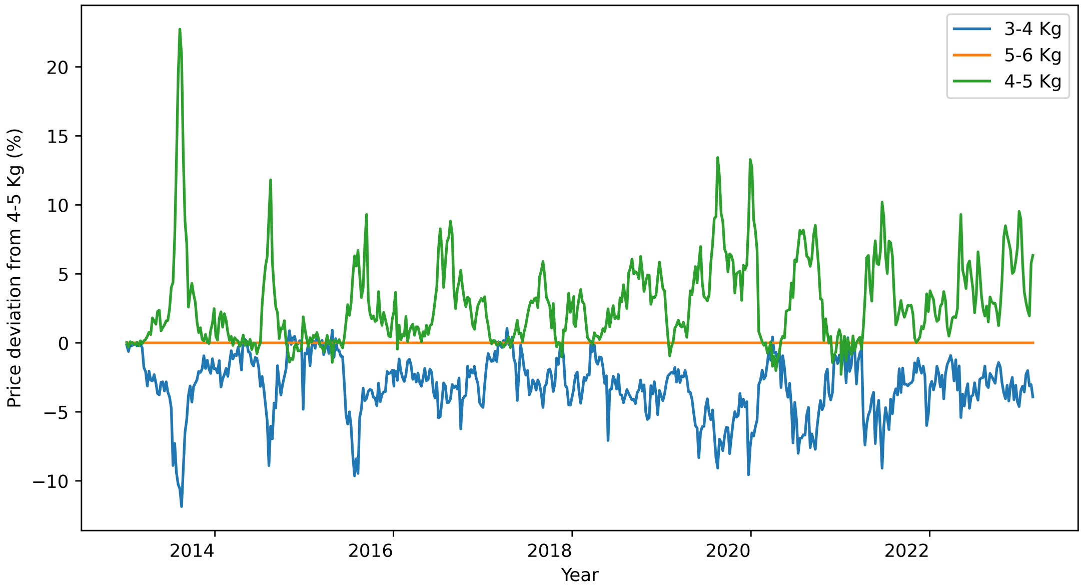


Figure 5.3: Deviation in percent from 4-5 kilogram price.

So far in this subsection, all price history obtained from Fish Pool has been linked to spot prices. Fish Pool also provides information regarding forward prices but only has price information for the 4-5 kilogram class. Therefore, we have to consider the 3-4 and 5-6 kilogram classes and fish outside these weight classes as well. To model this, we give all fish larger than 6 kilos the same price as 5-6 kilos, while we not give any price for fish under 3 kilos, as the volumes traded in these weights are, in practice, negligible. For the fish in the 3-4 and 5-6 kilogram classes, we apply an average deviation of -3.2% and 2.8%, as demonstrated in the previous paragraph and Figure 5.3.

5.4 Stochastic programming

As described in Chapter 3, we want to model different degrees of risk preferences. We do this by introducing CVaR. To explain CVaR, we depend on Value at Risk (VaR) being a familiar term. Therefore, we start by explaining VaR. VaR is a statistical measure used to quantify the potential losses on investments for a given confidence level over a time period. Financial institutions and portfolio managers often use this measure to monitor their risk exposure. A shortcoming with VaR is that it fails to incorporate the losses that exceed the confidence level. CVaR, however, calculates the expected loss that exceeds the confidence level. CVaR provides a more comprehensive measure of downside risk, thus well-suited for optimization problems (Artzner et al., 1999). Another benefit of using CVaR is the possibility of formulating it linearly, as shown in Birge & Louveaux (2011). As the stochastic problem spans several time periods, we must ensure that the CVaR implementation is time-consistent. Time consistency refers to the principle that decision-making processes should not incorporate outcomes that are impossible to occur in the future (Shapiro, 2009). To ensure time consistency, we introduce a nested CVaR implementation.

Furthermore, we want to model the problem as a multi-stage problem. This entails several different requirements for the mathematical modeling, including non-anticipativity constraints, which will ensure feasibility between the various vertices in the scenario tree (Birge & Louveaux, 2011). The non-anticipativity constraints must ensure that each action is a function of previous decisions and realized uncertainty. In other words, there is no possibility of reversing previous decisions. Where non-anticipativity ensures feasibility, time consistency in each time period ensures optimality for the decisions. We follow the principles of multi-stage programming, non-anticipativity, and time-consistent CVaR in the model formulation and implementation described in the upcoming chapter.

Chapter 6

Mathematical Model

This chapter outlines the methodology for solving the sales planning problem utilizing a multi-stage stochastic programming model. The objective is to maximize the weighted average of expected profits and CVaR over a designated planning horizon. In order to facilitate clear and concise communication, we introduce the notation used in the model. Finally, we present the complete model formulation, which includes the objective function and the constraints.

6.1 Notation

This section introduces the necessary notation needed to understand the model formulation. This includes sets, parameters, and decision variables.

6.1.1 Sets

Symbol	Description
$\mathcal{L}$	Set of all sites.
$\mathcal{I}$	Set of all stages.
$\mathcal{V}$	Set of all vertices in the scenario tree.

Symbol	Description
$\mathcal{V}_i$	Set of all vertices in stage i. $\mathcal{V}_i \subset \mathcal{V}$.
$\mathcal{C}_v$	Set of children vertices of v. $\mathcal{C}_v \subset \mathcal{V}$.
$\mathcal{N}$	Set of all net pens.
$\mathcal{N}_l$	Set of all net pens at site l. $\mathcal{N}_l \subseteq \mathcal{N}$.
$\mathcal{T}$	Set of all time periods during the planning horizon.
$\mathcal{T}_i$	Set of time periods in stage i. $\mathcal{T}_i \subset \mathcal{T}$.
$\mathcal{M}$	Set of maturities for contracts.
$\mathcal{M}_t$	Set of available maturities in time period t. $t \in \mathcal{T}$.
$\mathcal{W}$	Set of weight classes.
$\mathcal{S}$	Set of all scenarios.
$\mathcal{S}(v)$	Set of all scenarios present in vertex v.

6.1.2 Parameters

Symbol	Description
P^s_{tmw}	Price per kilogram of salmon in time period t in a contract with maturity m for weight class w in scenario s. $t \in \mathcal{T}$, $m \in \mathcal{M}$, $w \in \mathcal{W}$, $s \in \mathcal{S}$.
MAB_{lt}	Maximum allowable biomass at site l. $l \in \mathcal{L}$, $t \in \mathcal{T}$.
MAB^{COMP}	Maximum allowable biomass for a company.
$\hat{B}_n$	Estimated initial total amount of biomass at net pen n. $n \in \mathcal{N}$.
C	Cost per kilogram for keeping biomass at sea for one time period.
G^s_{tn}	Percentage increase in biomass taking place in time period t, in net pen n and scenario s. $t \in \mathcal{T}$, $n \in \mathcal{N}$, $s \in \mathcal{S}$.
Z^s_{tnw}	The fraction of biomass in net pen n and time period t that belongs to weight class w for scenario s. $t \in \mathcal{T}$, $n \in \mathcal{N}$, $w \in \mathcal{W}$, $s \in \mathcal{S}$.
γ	Scaling factor to the spot price resulting in the premium price producers must pay to fulfill contracts. $\gamma > 1$.
π_v	Probability of reaching vertex v from it's predecessor.
α	Confidence level of CVaR.

Symbol	Description
λ	Level of risk aversion for the producer. Risk neutrality appears at $\lambda = 0$. $\lambda \in [0, 1]$.

6.1.3 Decision variables

Symbol	Description
b_{tn}^s	Amount of biomass in kilograms in net pen n in the beginning of time period t in scenario s. $t \in \mathcal{T}$, $n \in \mathcal{N}$, $s \in \mathcal{S}$.
f_{iv}	Variable used to model CVaR. Approaches VaR for the optimal solution. $i \in \mathcal{I}$, $v \in \mathcal{V}_i$.
k_{iv}	Objective value at stage i and vertex v. $i \in \mathcal{I}$, $v \in \mathcal{V}_i$.
p_{iv}	Loss of profit with respect to CVaR at stage i and vertex v. $i \in \mathcal{I}, v \in \mathcal{V}_i$.
v_{tw}^s	Amount of salmon in kilograms bought in the market to fulfill contracts in time period t per weight class w in scenario s. $t \in \mathcal{T}$, $w \in \mathcal{W}$, $s \in \mathcal{S}$.
w_{tn}^s	Amount of biomass in kilograms harvested from net pen n in the beginning of time period t in scenario s. $t \in \mathcal{T}$, $n \in \mathcal{N}$ $s \in \mathcal{S}$.
y_{tmw}^s	Amount of salmon in kilograms of weight class w sold in time period t, in a contract with maturity m, in scenario s. $t \in \mathcal{T}$, $m \in \mathcal{M}$, $w \in \mathcal{W}$ $s \in \mathcal{S}$.

6.2 Objective function

$$\begin{aligned}
&\frac{1}{|S|}\sum_{s\in S}\sum_{t\in\mathcal{T}_1}\bigg(\sum_{w\in\mathcal{W}}(\sum_{m\in\mathcal{M}_t} P^s_{tmw}y^s_{tmw} - P^s_{t0w}\gamma v^s_{tw}) - C\sum_{n\in\mathcal{N}}(b^s_{tn} - w^s_{tn})\bigg) \\
&+ (1-\lambda)\sum_{v\in\mathcal{V}_2}\pi^v k_{2v} + \lambda\bigg(f_{11} - \frac{1}{1-\alpha}\sum_{v\in\mathcal{V}_2}\pi^v p_{2v}\bigg)
\end{aligned} \tag{6.1}$$

The first term in the objective function is the profit in the first time period. It consists of two terms; the first is revenue from fish sales $P^s_{tmw}y^s_{tmw}$, less money spent on fish bought in the market $P^s_{t0w}\gamma v^s_{tw}$. The second term represents the cost of holding fish for the first time period. Thus the cost C is multiplied by the remaining fish in the net pens after harvest $b^s_{tn} - w^s_{tn}$. Next, we recursively call the objective function in the next stage k_{2v}, scaled by the risk preference, and summed over all vertices in the second stage. Finally, the last term represents the CVaR. For a detailed description of optimization using CVaR, see Rockafellar et al. (2000).

6.3 Constraints

Several regulatory, operational, and biological factors must be considered when producing and selling salmon. In this section, we show how the mathematical model addresses these factors. We begin by presenting constraints related to the objective function and CVaR in the remaining stages. Next, we focus on regulatory constraints such as the maximum amount of fish allowed. Finally, we present the biological restrictions and how sold fish relates to the fish a salmon farmer harvests. It is worth pointing out that we assume all harvesting occurs at the beginning of a time period, while growth occurs at the end of a time period. This implies that fish does not grow in the time period they are harvested.

6.3.1 Objective value for each stage

$$\begin{aligned}
&\frac{1}{|S(v)|}\sum_{s\in S(v)}\sum_{t\in\mathcal{T}_i}\Bigg(\sum_{w\in\mathcal{W}}\Big(\sum_{m\in\mathcal{M}_t}P^s_{tmw}y^s_{tmw}-P^s_{t0w}\gamma v^s_{tw}\Big)-C\sum_{n\in\mathcal{N}}(b^s_{tn}-w^s_{tn})\Bigg)\\
&+(1-\lambda)\sum_{v'\in\mathcal{C}_v}\pi^{v'}k_{(i+1)v'}\\
&+\lambda\Bigg(f_{iv}-\frac{1}{1-\alpha}\sum_{v'\in\mathcal{C}_v}\pi^{v'}p_{(i+1)v'}\Bigg)=k_{iv}\quad i=2\ldots|\mathcal{I}|-1, v\in\mathcal{V}_i
\end{aligned}\tag{6.2}$$

Constraint (6.2) is a recurrence of the objective function for each stage i and vertex $v\in\mathcal{V}_i$. Note that k_{iv} depends on all children vertices in the next stage $\sum_{v'\in\mathcal{C}_v}\pi^{v'}k_{(i+1)v'}$.

$$\sum_{s\in S(v)}\sum_{t\in\mathcal{T}_{|\mathcal{I}|}}\Bigg(\sum_{w\in\mathcal{W}}\Big(\sum_{m\in\mathcal{M}_t}P^s_{tmw}y^s_{tmw}-P^s_{t0w}\gamma v^s_{tw}\Big)-C\sum_{n\in\mathcal{N}}(b^s_{tn}-w^s_{tn})\Bigg)=k_{|\mathcal{I}|,v}\quad v\in\mathcal{V}_{|\mathcal{I}|}\tag{6.3}$$

No recursive call is made for the last stage, and no CVaR is calculated. Thus, only the expected profit in the last stage is calculated.

6.3.2 CVaR constraint

$$f_{iv}-k_{(i+1)v'}\leq p_{(i+1)v'}\quad i=1\ldots|\mathcal{I}|-1, v\in V_i, v'\in\mathcal{C}_v\tag{6.4}$$

Constraint (6.4) is the required CVaR constraint. It ensures that auxiliary variable f_{iv} less the objective value $k_{(i+1)v'}$ at stage $i+1$ and vertex *v'* is less than or equal to the shortfall of revenues $p_{(i+1)v'}$ at stage $i+1$ and vertex *v'*. We define the constraint for all stages except the last one.

6.3.3 MAB constraints

In order to adhere to biomass limitations imposed by the government, we restrict the total biomass volume present in the net pens at any time.

$$\sum_{n \in \mathcal{N}_l} b^s_{tn} \leq MAB_{lt} \qquad t \in \mathcal{T}, l \in \mathcal{L}, s \in \mathcal{S} \tag{6.5}$$

Constraint (6.5) restricts the biomass at each location l, in all time periods t for all scenarios s. Since multiple net pens can be present at each location, we sum the biomass from all net pens at one location b^s_{tn} and restrict it to be smaller than the MAB at that location.

$$\sum_{n \in \mathcal{N}} b^s_{tn} \leq MAB^{COMP} \qquad t \in \mathcal{T}, s \in \mathcal{S} \tag{6.6}$$

In addition to the location-dependent Constraint (6.5), the government imposes a biomass restriction on a company level. We restrict the total biomass to be smaller than the company MAB.

6.3.4 Biomass constraints

The fish grows in each time period with a growth rate G^s_{tn}. Constraint (6.7) ensures that the biomass that grows is the existing fish in the net pen at the start of the period, less what is harvested in that period. Note that harvesting happens before any growth happens.

$$b^s_{(t+1)n} = G^s_{(t+1)n}(b^s_{tn} - w^s_{tn}) \qquad t = 1 \ldots |\mathcal{T}|{-}1, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.7}$$

The biomass balance in Constraint (6.8) initiates the decision variable b^s_{1n} in the first time period based on the known initial biomass $\hat{B}_s$.

$$b_{1n}^{s} = G_{1n}^{s} \hat{B}_{s} \qquad n \in \mathcal{N}, s \in \mathcal{S} \tag{6.8}$$

6.3.5 Harvesting and Sales constraints

Since fish can be sold in advance, we must ensure that the contracted fish is harvested in the specified time period. Also, harvesting capacities for each location need to be ensured.

$$\sum_{t'=1}^{t} y_{t'(t-t')w}^{s} = v_{tw}^{s} + \sum_{n \in \mathcal{N}} Z_{tnw}^{s} w_{tn}^{s} \quad t \in \mathcal{T}, w \in \mathcal{W}, s \in \mathcal{S} \tag{6.9}$$

Constraint (6.9) is defined for each time period t, weight class w, and scenario s. The left-hand side of the equation is the sum of all fish of weight class w sold with delivery in time period t, for scenario s. This sum needs to correspond to the amount of harvested fish w_{tn}^{s}, plus fish bought in the market v_{tw}^{s}. To find the amount of harvested fish in weight class w, we apply the scaling parameter Z_{tnw}^{s}.

$$w_{tn}^{s} \leq b_{tn}^{s} \quad t \in \mathcal{T}, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.10}$$

Harvesting should never exceed the biomass in a net pen. Constraint (6.10) makes sure this always holds.

6.3.6 Non-Anticipativity

$$\frac{1}{|S(v)|} \sum_{s' \in S(v)} y_{tmw}^{s'} = y_{tmw}^{s} \quad t \in \mathcal{T}, m \in \mathcal{M}, v \in \mathcal{V}, s \in \mathcal{S}(v), w \in \mathcal{W} \tag{6.11}$$

$$\frac{1}{|S(v)|} \sum_{s' \in S(v)} b_{tn}^{s'} = b_{tn}^{s} \quad t \in \mathcal{T}, v \in \mathcal{V}, s \in \mathcal{S}(v), n \in \mathcal{N} \tag{6.12}$$

$$\frac{1}{|S(v)|}\sum_{s'\in S(v)} v_{tw}^{s'} = v_{tw}^{s} \quad t \in \mathcal{T}, v \in \mathcal{V}, s \in \mathcal{S}(v), w \in \mathcal{W} \tag{6.13}$$

$$\frac{1}{|S(v)|}\sum_{s'\in S(v)} w_{tn}^{s'} = w_{tn}^{s} \quad t \in \mathcal{T}, v \in \mathcal{V}, s \in \mathcal{S}(v), n \in \mathcal{N} \tag{6.14}$$

Non-Anticipativity constraints ensure that decisions made in each time period only depend on currently available information. We formulate these constraints by forcing all decisions of different scenarios in the same vertex to be equal.

6.3.7 Requirements of decision variables

$$b_{tn}^{s} \geq 0 \quad t \in \mathcal{T}, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.15}$$

$$v_{tw}^{s} \geq 0 \quad t \in \mathcal{T}, w \in \mathcal{W}, s \in \mathcal{S} \tag{6.16}$$

$$w_{tn}^{s} \geq 0 \quad t \in \mathcal{T}, n \in \mathcal{N}, s \in \mathcal{S} \tag{6.17}$$

$$y_{tmw}^{s} \geq 0 \quad t \in \mathcal{T}, m \in \mathcal{M}, w \in \mathcal{W}, s \in \mathcal{S} \tag{6.18}$$

$$p_{iv} \geq 0 \quad i \in \mathcal{I}, v \in \mathcal{V}_i \tag{6.19}$$

$$k_{iv} \quad \textit{free}, \quad i \in \mathcal{I}, v \in \mathcal{V}_i \tag{6.20}$$

$$f_{iv} \quad \textit{free}, \quad i = 1 \ldots |\mathcal{I}|-1, v \in V_i \tag{6.21}$$

Chapter 7

Case Study

This chapter explains and visualizes the parameters we use in the model instances, which we present and discuss in Chapter 8. We begin with an overview of the production system of Eidsfjord Sjøfarm, as well as the planning horizon of the problem. We then describe how we have modeled uncertainty in price and biomass estimation and the discretization of the two parameters. Afterward, we explain the scenario generation procedure, which gives the combined scenario tree. Finally, we give an overview of the remaining parameters.

7.1 Production system

Eidsfjord Sjøfarm is a Norwegian salmon farming company of moderate size. We use their production data as the foundation of the analysis in this report. The company's farming facilities are in the regions of Senja, Vesterålen, and Nord-Troms. These facilities have several production sites, each subject to a MAB restriction that limits the permissible biomass quantity. Additionally, Eidsfjord, as a whole, operates under a total MAB restriction set at 10,902 tonnes of biomass. A comprehensive overview of Eidsfjord's facility data can be found in Table 7.1.

The data acquired from Eidsfjord consists of MAB values at the production site level, the total biomass, and the mean weight of fish in each site. Additionally,

Table 7.1: An overview of Eidsfjord's farming facilities (Aasen, 2021).

ID	Site	Region	MAB	Biomass (tonnes)	Mean weight (grams)	Months at sea	Net pens
1	Bremnesøya	Vesterålen	3900	1864	2207	10	5
2	Daljorda	Vesterålen	3120	2316	3273	13	5
3	Innerbrokløysa	Vesterålen	3120	0	0	NA	NA
4	Kuneset	Vesterålen	3120	0	0	NA	NA
5	Langholmen	Vesterålen	3120	0	0	NA	NA
6	Pollneset	Vesterålen	3120	0	0	NA	NA
7	Reinsnesøya	Vesterålen	3120	1007	1464	5	4
8	Sandan Sø	Vesterålen	2340	0	0	NA	NA
9	Stretarneset	Vesterålen	3120	1033	5261	17	2
10	Trolløya Sv	Vesterålen	3120	999	2641	5	3
11	Flesen	Senja	2700	0	0	NA	NA
12	Kvenbukta V	Senja	2700	225	317	2	5
13	Lavika	Senja	2700	571	689	2	5
14	Hagebergan	Nord-Troms	3600	1497	6000	17	2
15	Haukøya Ø	Nord-Troms	3600	0	0	NA	NA
16	Russelva	Nord-Troms	3500	1298	1591	5	5

the data includes the number of months the fish has been in the seawater phase. To determine the number of net pens per site, we make an estimation based on the available total biomass. By dividing the total biomass by the average weight, we approximate the fish population at each site. Assuming an upper capacity of 175,000 fish per net pen (Taranger et al., 2014), we compute the number of net pens for each site and add the data to Table 7.1. In the analysis, we have excluded sites lacking current biomass deployment.

7.2 Planning horizon

To decide on a suitable planning horizon for the problem, we must consider the computational complexity and provide a sufficiently long planning horizon to describe reality. From dialogue with a Norwegian salmon farmer, a four-month planning horizon with monthly increments of time periods seemed sufficient to account for a substantial part of sales decisions done by salmon farmers (Lerøy, 2022). An increase in the planning horizon would result in an exponential increase in complexity, making it significantly more computationally demanding to add more time periods to the planning horizon. A more in-depth explanation of the computational demands of problem instances will follow in Section 8.1.

Since the model has a limited planning horizon of four months, forward contracts must be restricted such that the delivery month is within the planning horizon. In order to uphold this restriction, each time period is associated with a set of available maturities, i.e., months until the delivery of salmon. For example, in a planning horizon of four months, the first month will have available maturities from zero to three, while the second month will have zero to two, as illustrated in Figure 7.1. It is possible to sell fish in the spot market in each time period, which we have modeled as a forward contract with maturity zero.

Time period	1				2			3		4
Maturities available	0	1	2	3	0	1	2	0	1	0

Figure 7.1: The planning horizon for the problem with maturities for each time period.

7.3 Biomass development

This section starts with an introduction to the properties of deterministic growth. As elaborated in Chapter 5, we need seawater temperature, TGC and mortality rate to calculate the growth rate. Therefore, we introduce the parametrization of these factors. Then, we explain how we have modeled the uncertain biomass estimations and the development of biomass estimations. Furthermore, we describe the discretization of biomass estimation uncertainty.

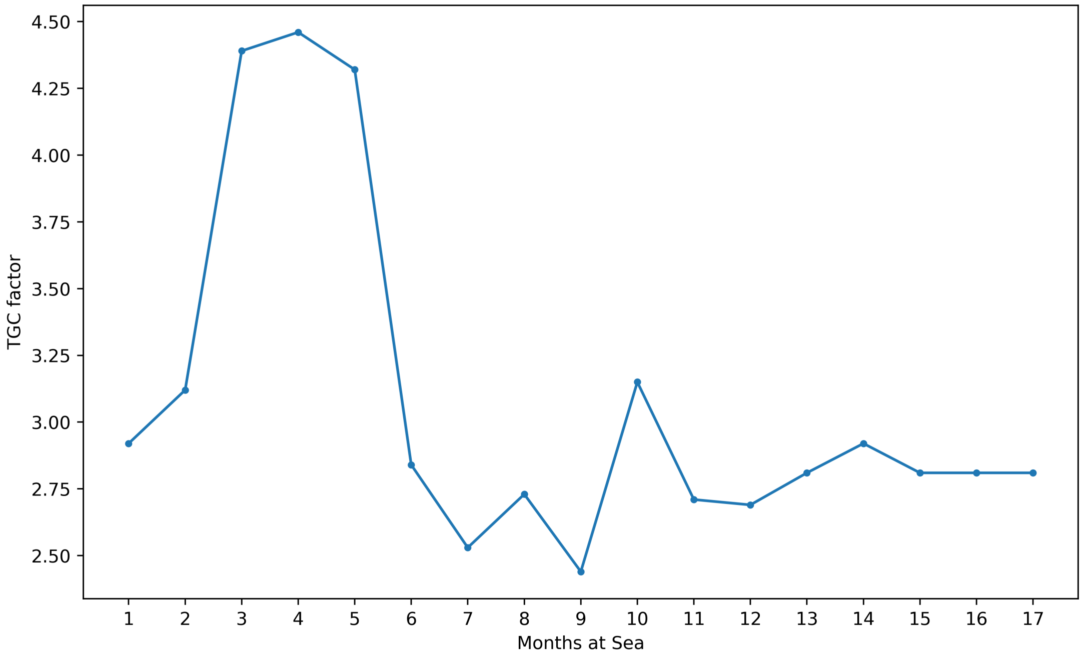


Figure 7.2: TGC factor as a function of months at sea (Aasen, 2021).

7.3.1 Growth

The growth modeling in this thesis relies on a TGC model, which requires two key parameters; seawater temperature and a TGC factor. We utilize TGC factors collected from Eidsfjord spanning 14 months. However, we need TGC factors that cover the longest time at sea present in the production system presented in Table 7.1. This makes it possible to calculate the TGC for all sites in the production system. From Table 7.1, we find 17 months to be the longest time at sea across sites. Therefore, we extrapolate the average TGC factor from the last three months to cover the remaining three months, as seen in Figure 7.2.

Accurate temperature information is vital for TGC in growth modeling. Equation (5.1) outlines the significance of temperature in this context. We utilize temperature data computed in Aasen (2021), which determines the monthly average temperature for each region present in Eidsfjord's production system. The temperature data is presented in Figure 7.3. The growth modeling treats these monthly average temperatures as deterministic parameters. We assume that the seawater temperatures follow an annual cycle.

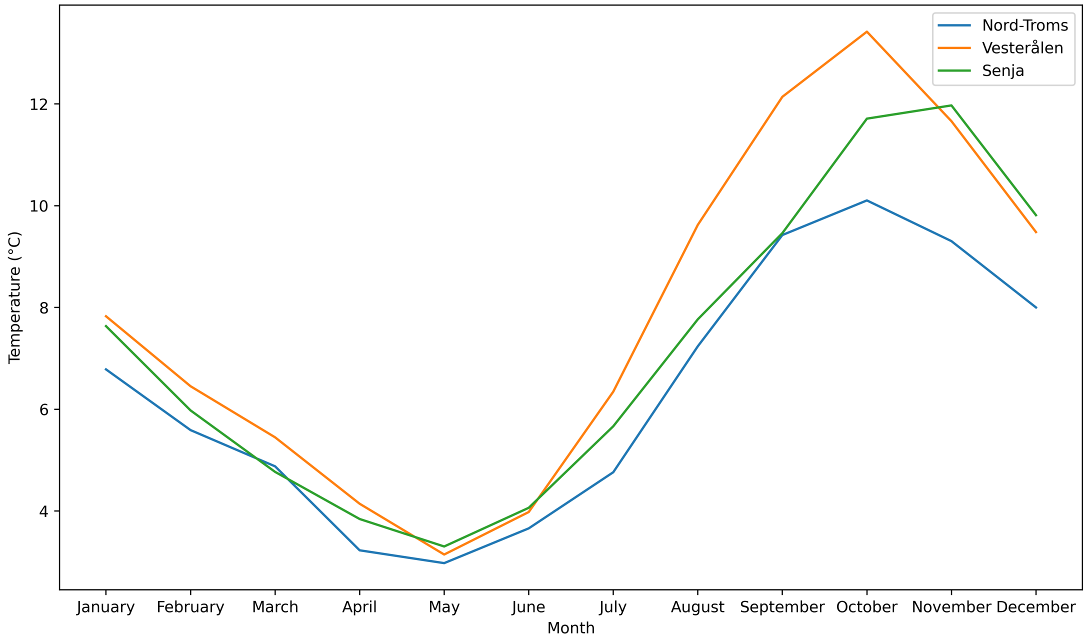


Figure 7.3: Average monthly sea temperatures in Senja, Vesterålen, and Nord-Troms. (Aasen, 2021).

In addition to temperature and the TGC factor, we require a mortality rate to calculate the periodic increase in biomass. Norwegian Veterinary Institute (2021) estimates the average loss of fish during the seawater phase in Norwegian aquaculture to be 15%. To consider this in the modeling, we calculate a monthly mortality rate of 0.99%, assuming a constant mortality rate while the fish are at sea, which lasts 16 months according to Bang Jensen et al. (2020). The constant mortality rate of 0.99% will ensure that 85% of the fish will be present at the end of the seawater phase.

All factors used in the growth computation are modeled deterministically. For the model, we distinguish between deterministic growth and uncertain biomass estimates. In this way, the difference between estimation methods is the only source of uncertainty related to available biomass that the model considers. The following section explains further how we parametrize biomass estimates.

7.3.2 Biomass Estimation

Although we model growth deterministic, we introduce uncertainty in biomass estimation. No estimation method exists today that can measure biomass with 100% accuracy. In this thesis, we consider a traditional method based on mathematical formulas in order to estimate the biomass. We also study an estimation method utilizing OptoScale's camera module. This section describes how we model uncertainty with the two estimation methods. Furthermore, we describe how we discretize biomass estimation uncertainty into scenarios.

We model the uncertainty for the traditional biomass estimation in a way that gradually increases from certain biomass when the smolt is initially released into net pens. This means the error in biomass estimations is 0% when the time at sea is zero months. As time at sea increases, so does the uncertainty in biomass estimations. We have modeled the traditional measurements to imitate manual biomass estimations traditionally used, such as mathematical formulas for calculating the biomass in net pens. Such approaches will have propagated uncertainty as time at sea increases, as there is never any real-time reveal of the actual biomass in net pens before harvest. Using this estimation on net pens in the data from Eidsfjord, where the months at sea are larger than zero, means we need to calculate a start uncertainty based on the time at sea. This way, we consider the propagation of uncertainty that has already occurred. Figure 7.4 describes the linear uncertainty increase. For simplicity, we also illustrate biomass development linearly to highlight the propagation of estimation uncertainty.

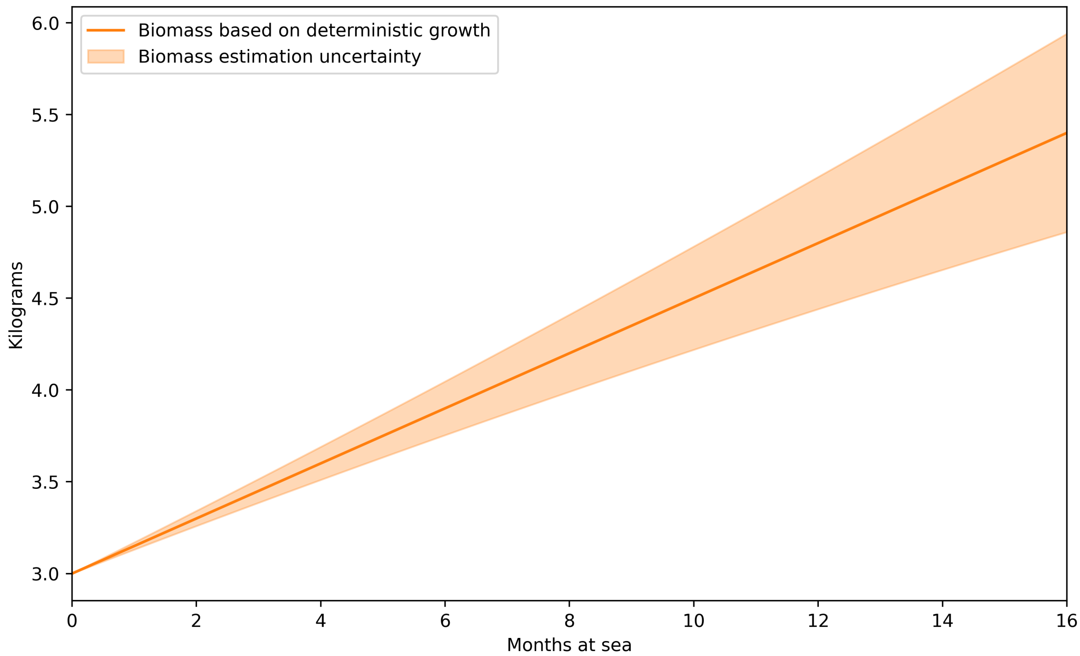


Figure 7.4: Uncertainty with traditional biomass estimation as months at sea increases.

Generating scenarios for the traditional biomass estimations depends on the value of end uncertainty, which is where the linear increase of estimation uncertainty ends after the 16-month total duration of the seawater phase. We use 16 months as the total duration since this is a common time span for the seawater phase (Bang Jensen et al., 2020). Estimate errors from traditional biomass estimates vary, with sources ranging from $\pm 6\%$ (OptoScale, 2022a) to $\pm 15 - 20\%$ (Klontz & Kaiser, 1993). While this means several values could be used for modeling traditional estimation, for the base instance with traditional methods, we use $\pm 12\%$. As previously described, the end uncertainty and the months at sea for a given net pen decide the initial estimation uncertainty. Determining the propagated error in estimations is done by dividing the months at sea by the total duration before multiplying this fraction with the end uncertainty. Figure 7.5 shows two examples of net pens with different initial months at sea, where the planning horizon of the problem in both instances is four months. In the right part of the figure, we see a net pen with initial months at sea of 12 months. Therefore, the initial estimation uncertainty will become $\pm 12\% \cdot \frac{12}{16} = \pm 9\%$. The uncertainty in estimations at the end of the planning horizon becomes equal to the end uncertainty of ±12% as the planning horizon ends at 16 months at sea, which is the same as the total duration. In the left part of the figure, we see an example of a net pen with initial months at sea of four months. This means the initial estimation uncertainty becomes $\pm 12\% \cdot \frac{4}{16} = \pm 3\%$, and the uncertainty at the end of the four-month planning horizon becomes 6%.

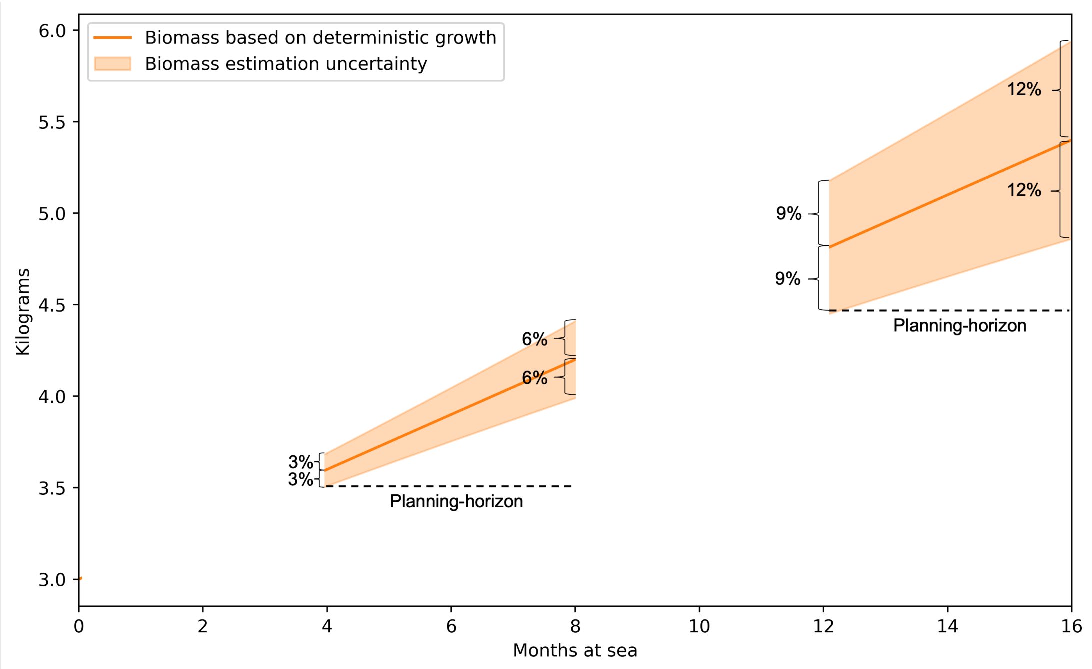


Figure 7.5: Two net pens with different initial months at sea, modeled with traditional biomass estimation and a four-month planning horizon.

Estimations when utilizing OptoScale's estimation method are calculated differently from traditional estimations. As OptoScale's estimations are based on real-time information about the fish within net pens, there is no uncertainty propagating as in the traditional instance. Instead, the uncertainty from the estimation is the same in the first time period, regardless of months at sea. We then calculate the monthly increase in the interval of biomass estimation uncertainty based on the start and end uncertainty set as parameters in the model. The monthly increase is based on the increase necessary to get from the start to the end uncertainty, given the total duration. This calculation fixates the monthly increase of the uncertain estimation interval for all instances utilizing OptoScale's estimations, regardless of the time at sea.

For OptoScale's estimations, we parametrize both the start and end uncertainty directly without calculations. We set the start uncertainty to $\pm 3\%$ due to the previously mentioned guarantee of maximum deviation of OptoScale's estimations at harvest time. Considering the initial estimation uncertainty and the propagated error of future estimation developments, we decide on $\pm 6\%$ for the end uncertainty. The combination of start and end uncertainties for OptoScale's estimations creates a monthly increase in the uncertain biomass estimation interval of $\pm \frac{6\%-3\%}{16} = \pm 0.1875\%$, which is smaller than the monthly increase in the uncertain interval when using traditional methods as explained earlier in this section. We considered increasing uncertain intervals equally across estimation methods but decided against it. We make the increase in uncertain intervals for OptoScale's estimation smaller to account for the real-time aspect of OptoScale's estimations. The future development of OptoScale's estimations will be more certain as the uncertainty interval would, in reality, be reset to the initial estimation uncertainty each month. Figure 7.6 shows two different net pens with different initial months at sea modeled with OptoScale's estimations. The initial estimation uncertainty is the same for both, as is the monthly increase of the uncertainty interval. Consequently, the end uncertainty is equal in both cases.

We divide the uncertainty interval into a finite set of realizations to discretize the uncertain biomass estimations into scenarios. We assume that the biomass estimation uncertainty is uniformly distributed, meaning large deviations within the uncertainty interval are as likely as minor deviations. We make this assumption since we lack data for analyzing the distribution of biomass estimation error. Thus biomass estimation scenarios are distributed uniformly within the uncertainty interval. This approach is applied to both traditional and OptoScale's estimation methods.

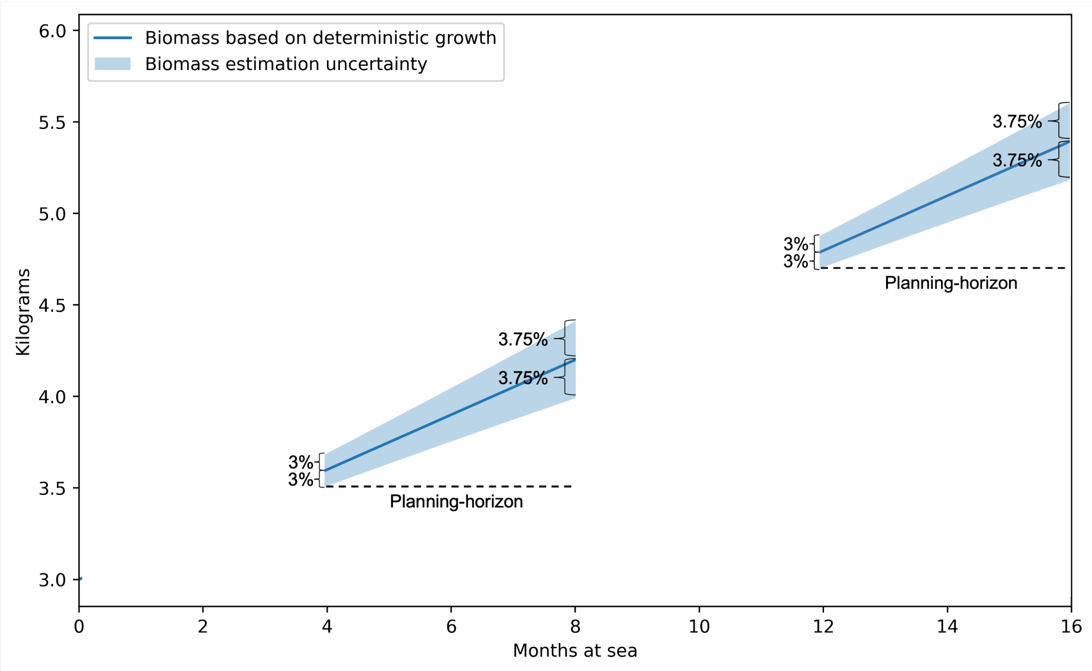


Figure 7.6: Two net pens with different initial months at sea, modeled with OptoScale's biomass estimation and a four-month planning horizon.

7.4 Price

This section presents the uncertainty in spot price development and how we discretize it. Furthermore, it discusses how forward prices are modeled based on the spot price.

We generate price scenarios based on historical salmon prices (Fish Pool, 2022). We use historical data as there are substantial amounts readily available for analysis provided by Fish Pool, and we assume historical price developments provide a good indicator of future developments. The starting price in all instances is set to 60 NOK, close to the average price in the last decade. Since each stage in the model formulation lasts one month, we analyze the historical one-month changes in the spot price. Figure 7.7 shows the distribution of all one-month price changes observed in the last decade. In order to create reasonable scenarios based on this distribution, we use the theory of quantiles. Quantiles are cut points that divide the observations into equal-sized segments. To create scenarios, we calculate the

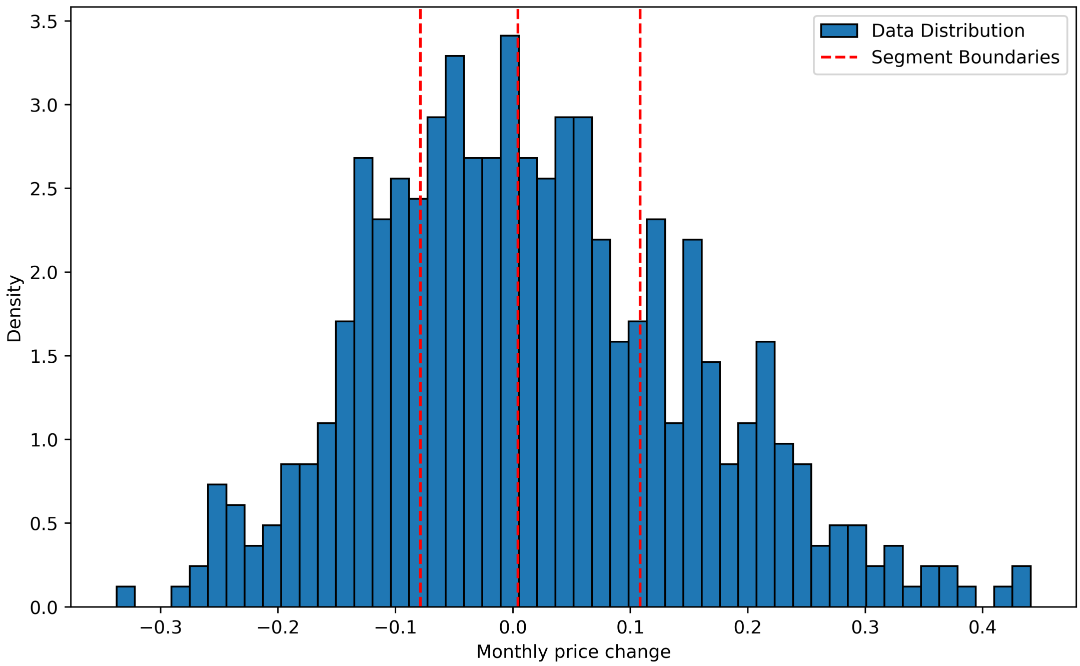


Figure 7.7: Histogram showing the density distribution of one month spot price change the last 10 years. The red dashed lines show each quartile.

expected value for each segment. For instance, four future price scenarios would require three quartiles at the 25th, 50th, and 75th percent highest observation, as shown with red dashed lines in Figure 7.7. Then four expected values are calculated based on the observations in each segment. The four possible price scenarios are $[-0.145, -0.036, 0.052, 0.200]$. This approach captures the significant trends in the underlying data. It is also very interpretive since each scenario is simply the average of each segment. However, few scenarios lead to a poor approximation of the continuous nature of future price development. This is a central aspect of this thesis and most work on stochastic programming. Optimally we would prefer finer granularity of price development, but the computational complexity increases too fast. Consequently, we must settle for a few price scenarios in each stage.

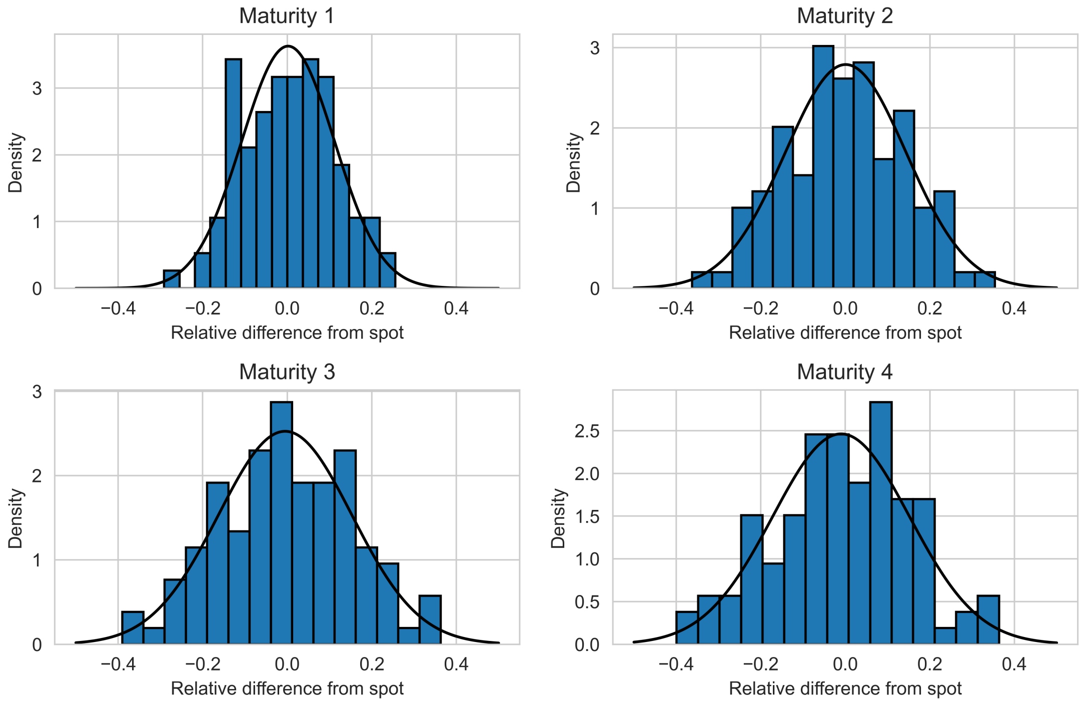


Figure 7.8: Histogram of the relative difference between forward and spot prices. Each subplot shows the samples for a specific maturity. The black lines show the associated normal distribution.

Table 7.2: Mean and standard deviation of the relative difference between spot prices and earlier forward prices. Maturity 1 means the forward price one month ahead of the spot price.

	Maturity 1	Maturity 2	Maturity 3	Maturity 4
Mean	0.002	0.001	-0.005	-0.010
Std	0.110	0.143	0.158	0.162

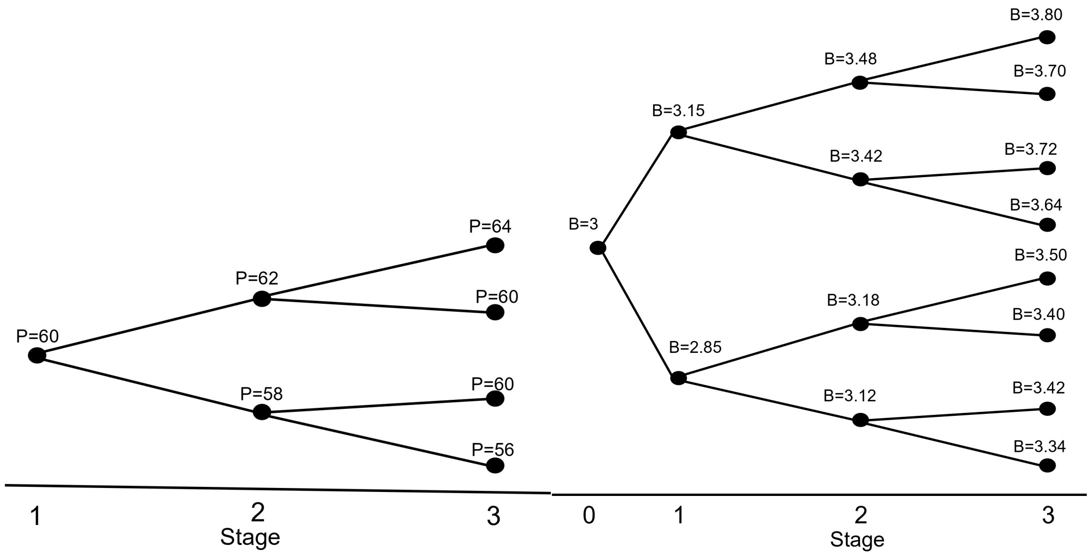
In addition to the spot price development, the model needs associated forward prices in each vertex. A forward price aims to predict the future spot price. To analyze how well forward prices are for prediction, we compare forward prices with the spot prices of the same month as the expiration of the forward contract. If forward contracts predict future spot prices accurately, then the forward price and the future spot price at expiration should be similar. However, analyzing the historical data shows that forward prices are inaccurate at predicting future realizations of spot prices. Figure 7.8 shows the relative difference between forward prices and future spot prices at the time of expiration. Each subplot shows the histogram for a specific maturity. Although all maturities show inaccuracies, there is a trend in the data implying that longer maturities are more inaccurate. We support this claim with Table 7.2, which presents the mean and standard deviation between forward prices and coherent spot prices. While the inaccuracy increases with longer maturities, the increase becomes less notable as the maturities become longer. The mean relative difference is close to zero and looks relatively unbiased for all maturities. However, the standard deviation increases with higher maturities, suggesting more spread in longer maturities. We use this information to generate forward prices. Assuming that the relative difference between forward and spot price follows a normal distribution, which is reasonable given the shape of histograms in Figure 7.8, we draw random samples from each normal distribution. These random samples are applied to future spot prices to generate forward prices. This approach captures the primary relationship between forward and spot prices. Namely, forward prices are related but deviate from the actual future spot price, and longer maturities imply more deviation.

One final aspect of price modeling is necessary to address. In contrast to the spot price, historical forward prices do not contain information on weight classes other than 4-5 kilograms. We have therefore used the average relative spot price differences between the various weight classes, as discussed in Section 5.3. Forward prices for 5-6 kilograms are increased by 2.8%, while 3-4 kilograms are reduced by 3.2% compared to 4-5 kilograms. As also addressed in Section 5.3, there are considerable fluctuations in the relative price difference. Thus, the scaling will simplify the model but still creates a difference in prices for the weight classes.

7.5 Scenario generation

So far, we have discussed how the stochastic parameters price and biomass estimation are affected by uncertainty and how we discretize them separately. Now, we explain how the scenarios from the two parameters are combined, resulting in the whole scenario tree.

We start by exemplifying with a basic instance. The basic instance has three



(a) Price development scenario tree. (b) Biomass estimation scenario tree.

Figure 7.9: A basic instance showing the scenario trees for the uncertain parameters. The biomass estimation scenario tree has an extra stage to account for the initial biomass uncertainty in stage one.

stages, and each uncertain parameter has two realizations of uncertainty between stages. The scenario tree for the price parameter is shown in Figure 7.9a. Prices in the figure represent spot prices. As explained in Section 7.4, we calculate forward prices in each vertex based on the spot price and the historical deviation for each maturity. Since the price in the first stage is known, the tree follows the expected structure of a multi-stage scenario tree, i.e., two realizations of uncertain parameters between each stage, resulting in four scenarios in the last stage.

Contrary to the price scenario tree, the biomass estimation is uncertain in the first stage due to uncertainties in current biomass estimations, as discussed in Section 7.3. Thus, to account for this uncertainty, we add stage 0, where the biomass estimation is certain. Therefore, the scenario tree for biomass estimation contains one more stage than the price scenario tree. From stages 0 to 1, we add the initial biomass estimation uncertainty for the start of the planning horizon. However, first-stage decisions must be made prior to the revelation. Figure 7.9b shows the biomass estimation scenario tree.

Combining the two separate scenario trees results in Figure 7.10. Each vertex is split into four realizations except the vertex in stage zero. Here, only biomass estimation uncertainty is resolved, leading to only two splits. This basic instance creates a scenario tree with 32 unique price and biomass estimation scenarios.

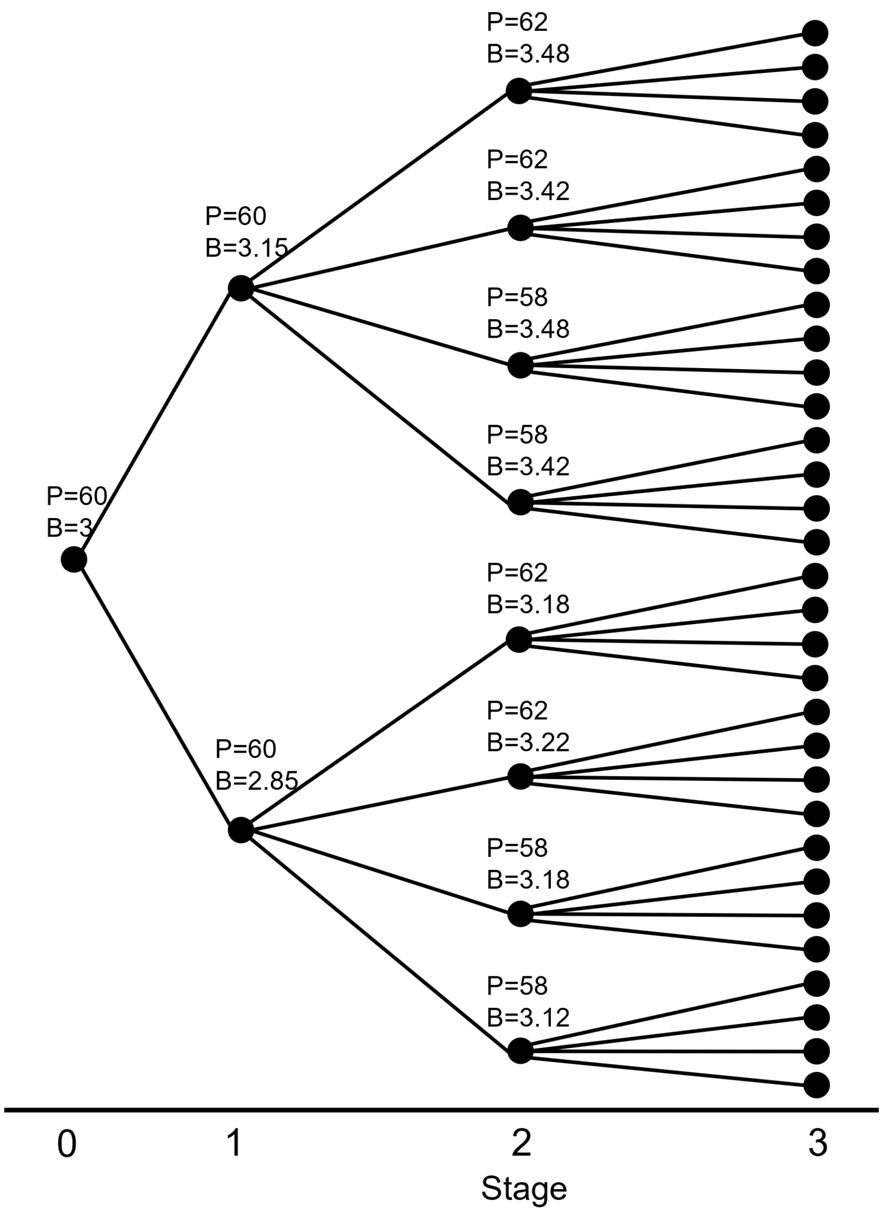


Figure 7.10: Combined scenario tree for price and biomass estimation in a basic instance.

7.6 Other parameters

In this section, we present the remaining parameters. These include a cost for keeping the fish in net pens, a premium if the farmer fails to fulfill their forward contracts, and a confidence level for modeling CVaR. We present an overview of these parameters and their values in Table 7.3.

If a salmon farmer has fish in net pens, a cost is incurred, mainly related to feeding the fish. Using MOWI (2022) cost analysis related to producing salmon, we see that the cost of keeping fish in net pens and growing them is, on average, NOK 22 per extra kilogram of salmon. To convert this into a monthly cost C per kilogram for keeping the fish in a net pen, we need the duration it takes to grow fish by one kilogram. The average of all monthly growths based on TGC during the planning period results in an increase in mean weight of 283 grams per month. We have found this increase using Equation (5.2) on the data provided by Eidsfjord. The calculation results in 3.54 months to grow a kilogram. By distributing the cost of NOK 22 evenly over the 3.535 months, we see that the cost of keeping fish in net pens is an average of NOK 6.22 monthly per kilogram of salmon. This becomes the value of parameter C.

When a salmon farmer sells fish through forward contracts, he risks a shortfall in biomass, which implies failure to fulfill the contracts. In this situation, fish can be bought in the spot market from a competitor. From talks with the industry, we learned that fish bought in the market from competitors is costly (Lerøy, 2022). Thus, we multiply the spot price with a premium γ. The exact premium varies, depending on the supply and demand for salmon. However, according to Lerøy (2022), a premium of 1.3 is a reasonable approximation. In other words, the price to buy fish from competitors will be 30% higher than the actual spot price. Furthermore, we assume that there is an unlimited supply of fish in the market. Consequently, the risk of failing to fulfill forward contracts is purely economical since fish can always be bought in the market. In reality, a shortage of fish could damage customer relations and have long-term effects. As a result, the usage of forward contracts could be more restrictive in reality since the consequences of not fulfilling them are larger. Nevertheless, we make the assumption to simplify the modeling.

Table 7.3: Overview of parameters and their values.

Parameter	Value
C	6.22
γ	1.3
α	0.9375

To model different risk profiles, we use CVaR. For the modeling, we need a confidence level α, which says that the intent is to maximize the $1-\alpha$ worst scenarios. The most used values for α range from 0.99 to 0.90 (Uryasev & Rockafellar, 2001). The analysis of risk levels uses λ to vary preferences related to the expected value versus CVaR, while the confidence level is constant at 0.9375. That is, we want to maximize the objective value in the worst scenario given 16 splits in the combined scenario tree.

Chapter 8

Computational Study

In this chapter, we present and discuss the results from solving the sales planning problem formulated in Chapter 6 by applying the parametrizations presented in Chapter 7. We first perform a technical analysis that examines the computational complexity of problem instances. To be able to increase the size of instances, we propose an alternative model formulation. We then compare the two formulations and move forward in the computational study with the most suitable one. As the primary motivation is to assess the value of accurate real-time biomass estimation in salmon farming, the subsequent sections will mainly focus on comparing and discussing the differences between results from the two types of biomass estimation methods. We study the base instance of the problem, where the stochastic solution is compared to the deterministic one. We then examine how the risk profile of the salmon producer affects decisions. To enhance the robustness of the results, we perform a sensitivity analysis of uncertain parameters. Furthermore, we examine what influence the structure of the scenario tree has on results. Finally, we discuss the potential impact of the recently introduced resource rent tax on the Norwegian salmon farming industry.

8.1 Technical Analysis

We begin the computational study by examining the performance of the model. By increasing the number of stages, and splits in each stage, we analyze how the model scales regarding the number of constraints, variables, scenarios, and runtime. The results determine the most suitable model instance for the following sections of the computational study, referred to in forthcoming sections as the base instance.

The mathematical program is implemented in Python 3.10, and solved in Gurobi 9.5. All program instances are solved using a Lenovo NextScale nx360 M5 system, with 2x 2.3GHz Intel E5-2670v3 – 12 core processors, with 64-512 GB RAM.

Table 8.1: Technical results

Stages	Splits	Num Scenario	Num Constrs (10^3)	Num Vars (10^3)	Runtime (s)
3	4	32	7	4	0.1
3	9	243	150	69	3.51
3	16	1024	633	289	289
3	25	3125	1,932	882	2619
3	36	7776	4,806	2,193	NA
4	4	128	104	47	1.9
4	9	2187	1,778	807	1175
4	16	16,384	NA	NA	NA

Table 8.1 presents eight model instances that differ in the number of stages and splits in each stage. In this technical analysis, each stage represents one time period of one month. Thus, more stages lead to a longer planning horizon. Regarding splits, we would like to point out, as discussed in Chapter 7, that the number of splits is the product of the number of price and biomass estimation splits. This means that 4, 9, and 16 total splits correspond to 2, 3, and 4 splits for both price and biomass estimation. In all instances presented in the technical analysis, we emphasize that the decision maker has a risk-neutral risk preference.

Increasing splits and stages of the scenario trees results in an exponential growth in problem size. Going from four to nine splits in the three-stage instance increases the number of scenarios from 32 to 243. A scenario increase of 659%, when the number of splits increased by 125%. This trend is also present for the number of constraints, variables, and runtime. Furthermore, including an additional stage only enhances this trend. Going from four to nine splits in the four-stage instance increases the number of scenarios by 1609% when the number of splits increases by 125%. As shown in Table 8.1, the problem size quickly grows beyond what is computationally possible to solve. The current problem formulation fails to solve the three-stage instance with 36 splits. We also note that the four-stage instance with 16 splits fails when defining constraints, suffering from a memory error, even with 512 GB RAM memory.

To solve problem instances with longer planning horizons or finer granularity on uncertain parameters, we introduce an alternative problem formulation.

8.1.1 Vertex formulation

As shown in Chapter 6, we formulate non-anticipativity constraints in order to restrict decisions from depending on future realizations of uncertain variables. In the original problem formulation, all decision variables defined over the same sets must be equal for every scenario. A consequence of this formulation is that the non-anticipativity constraints account for a large portion of the total constraints in the problem. Specifically, in the case of four stages and nine splits, the non-anticipativity constraints account for 41% of all constraints.

We now present an alternative to the original formulation. In the new formulation, decision variables previously defined for a set of scenarios will instead be defined for a single vertex. Figure 8.1 illustrates the difference between the original formulation and the alternative. In the original formulation (left tree), each vertex is defined for a set of scenarios, depicted by the braces above each vertex. The root vertex in the left tree is defined for all scenarios. Since the available biomass and price information is identical for all scenarios in the root vertex, non-anticipativity constraints are necessary to enforce equality for all decision variables in the root vertex. However, in the alternative formulation (right tree), we do not define decision variables over scenarios. Instead, the decision variables are defined for a single vertex in the tree. In this manner, non-anticipatively constraints become redundant. Consequently, the size of the problem instances

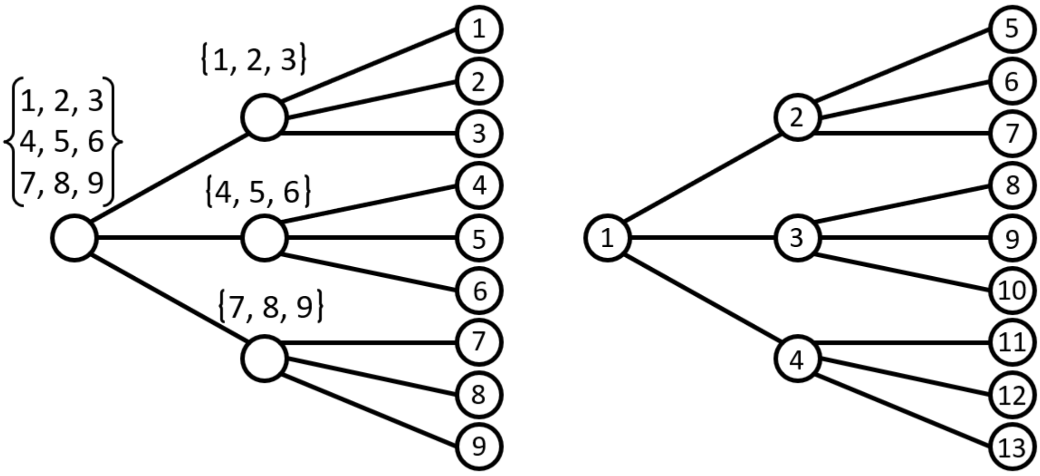


Figure 8.1: Illustration of the difference between scenario formulation (left tree) and vertex formulation (right tree).

is greatly reduced. We apply some minor changes to the mathematical model formulation. The models are practically equivalent, but we have to make some changes to the sets of the mathematical model to keep the structure of the vertices in order when transitioning from the scenario formulation.

Constraint (8.1) and (8.2) show how the harvesting constraint changes in the new formulation. We only present the changes made on this constraint for brevity, as changes made to most constraints in the original formulation follow the same procedure. The full alternative mathematical formulation can be found in Appendix A. The harvesting constraint, shown in Constraint (8.1) and (8.2), restricts the amount of fish harvested in each time period for each weight class to be equal to the amount of fish sold with delivery for that time period and weight class. Note that the formulation changes slightly from Constraint (8.1) to (8.2). Instead of defining the constraints for each scenario, the alternative formulation utilizes a set of vertices per time period. To correctly sum the y variables, we iterate through the set of parent vertices $v' \in \mathcal{P}_v$ in order to capture sales in the previous time periods. In addition, we have added the variable y^v_{t0w} to account for spot sales in time period t.

$$\sum_{t'=1}^{t} y^s_{t'(t-t')w} = v^s_{tw} + \sum_{n\in\mathcal{N}} Z^s_{tnw} w^s_{tn} \quad t \in \mathcal{T}, w \in \mathcal{W}, s \in \mathcal{S} \tag{8.1}$$

$$\sum_{v'\in\mathcal{P}_v} \sum_{t'\in\mathcal{T}_{v'}} y^{v'}_{t'(t-t')w} + y^v_{t0w} = v^v_{tw} + \sum_{n\in\mathcal{N}} Z^v_{tnw} w^v_{tn} \quad t \in \mathcal{T}, w \in \mathcal{W}, v \in \mathcal{V}_t \tag{8.2}$$

The alternative formulation remains practically identical regarding solutions, but the structure changes dramatically. For the most complex problem instance that we were able to solve with both formulations, nearly identical solutions were produced. Only a slight difference of 0.05% could be observed in the objective value. Since the two formulations produce nearly identical solutions, we can compare the problem structure between formulations and how they scale in terms of constraints and variables.

We solve the model in Gurobi. Two algorithms are mainly used to solve optimization problems in Gurobi: Simplex and Barrier (Gurobi, 2023). Concurrent optimizers can also be used, which run different solvers simultaneously. Barrier is recommended for large, complicated models that require more memory. A possible downside of Barrier is that it can be slow and struggle with numerical issues. Concurrent optimizers use different threads for each solver. It chooses the first to finish, using Barrier, primal Simplex, and dual Simplex. Concurrent optimizers are recommended if the model can handle the challenges concerning memory and numerical issues, as it runs faster. We have tried the various solution algorithms for the scenario and vertex formulations and concluded that Barrier performs best for the scenario formulation. At the same time, concurrent optimizers work best for the vertex formulation. It is worth noting that concurrent optimizers can produce different optimal bases in different runs (Gurobi, 2023). We assume that this is the reason why the objective values slightly differ between the two problem formulations.

Table 8.2 presents several problem instances solved by both versions of the problem formulation. Solving the two problem formulations results in remarkably different results regarding problem size and runtime. First, the scenario formulation creates approximately four times as many constraints and more than two times as many variables as the vertex formulation. The difference is even more striking comparing the runtime. The vertex formulation shows clear advantages over the scenario formulation. Most importantly, the vertex formulation is able to handle much larger problem instances than the scenario formulation. Given the large computational advantages the alternative problem formulation shows, we choose to move forward in the analysis with this formulation.

Going forward with our analysis, we use the instance with four stages and 16 splits, divided into four splits for both price and biomass estimation, of the vertex formulation as our base instance. We use the base instance in different analyzes in the upcoming sections. We chose this instance as it accounts for a reasonable amount of uncertainty while being computationally manageable.

Table 8.2: Technical results, scenario and vertex model.

Model formulation	Stages	Splits	Num Constrs (10^3)	Num Vars (10^3)	Runtime (s)
Scenario	3	9	150	69	3.51
Vertex	3	9	39	31	0.21
Scenario	3	16	633	289	289
Vertex	3	16	158	126	1.1
Scenario	3	25	1,932	882	2619
Vertex	3	25	474	377	5
Scenario	3	36	4,806	2,193	NA
Vertex	3	36	1,168	928	16
Scenario	4	9	1,778	807	1175
Vertex	4	9	352	279	3.6
Scenario	4	16	NA	NA	NA
Vertex	4	16	2,529	2,001	62

8.2 Value of the stochastic solution

In this section, we introduce a deterministic solution called the expected value solution (EV) and study how this solution performs when introducing uncertainty. We compare this solution to the performance of the stochastic model's solution. We analyze the objective values, which provide information about the value of solving the problem stochastically. The stochastic model will be referred to as the recourse problem (RP) for the remainder of this comparison between deterministic and stochastic results.

8.2.1 Evaluation of the expected value solution

Escudero et al. (2007) provide calculations to determine the value of introducing stochasticity in optimization for multi-stage problems. To perform the calculations, Escudero et al. (2007) introduce a series of terms and calculations. First, we must find a deterministic solution to the problem by calculating the EV. We do this by solving an instance where all uncertain parameters, namely price and biomass estimation, take their expected value. To assess how the decisions from the EV perform when we introduce stochasticity, we calculate the evaluation of the expected value solution (EEV). In a two-stage stochastic optimization problem, the EEV is found by fixating the first-stage decision variables from the EV before realizing each possible scenario in the second stage. This calculation provides an objective value for each scenario. Averaging the objective values results in the EEV.

For multi-stage stochastic optimization problems like the one presented in this thesis, determining the EEV is more complex. Escudero et al. (2007) proposes an approach to calculating the EEV for multi-stage problems. The approach begins by calculating the EV. Then we find the EEV by solving the RP with all decisions forced equal to those in the EV until a specific stage. This calculation provides different values of EEV for each stage, as shown in equation (8.3). In equation (8.3), $\bar{x}_1$, $\bar{x}_2$, ..., $\bar{x}_{i-1}$ refer to the optimal values of decision variables found from the EV.

$$EEV_i = \begin{cases} RP & \text{model} \\ \text{s.t.} & x_1^v = \bar{x}_1 \quad \forall v \in \mathcal{V}_1, \\ & \ldots \\ & x_{i-1}^v = \bar{x}_{-1} \quad \forall v \in \mathcal{V}_{i-1}. \end{cases} \tag{8.3}$$

EEV$_i$ for a given stage i denotes the solution where all decision variables until stage $i-1$ are forced equal to the decisions in the EV. Consequently, EEV$_1$ becomes a problem where no decision variables are locked, which is identical to the RP. The more stages' decision variables are locked, the less uncertainty is considered. Therefore, EEV$_{|\mathcal{I}|}$, which locks all decision variables expect the last stage, takes the least amount of uncertainty into consideration.

8.2.2 Objective values

Table 8.3: Overview of EEV$_i$ values.

Instance	Objective value
EEV$_2$	103,784,810
EEV$_3$	100,215,037
EEV$_{\lvert\mathcal{I}\rvert}$	96,347,131

Table 8.3 shows objective values for different EEV$_i$ instances. We remove EEV$_1$ from the analysis as this problem is, as mentioned, equal to the RP problem. The more stages have locked decision variables, the less uncertainty is accounted for, and consequently, the solution should perform worse. We can see this effect clearly in Table 8.3.

Having computed EEV$_i$, we can calculate the value of the stochastic solution, VSS$_i$. The VSS$_i$ is calculated as the difference between the objective values found when solving the RP and the EEV$_i$, as shown in equation (8.4). Consequently, the VSS$_i$ must differ based on which EEV$_i$ we use for comparison, with VSS$_{|\mathcal{I}|}$ being the difference when the least uncertainty is accounted for. Birge & Louveaux (2011) define VSS as the cost of ignoring uncertainty when making decisions. Following this interpretation, Escudero et al. (2007) define VSS$_{|\mathcal{I}|}$ as the maximum cost a salmon producer would be willing to take to ignore uncertainty.

$$VSS_i = RP - EEV_i \quad i \in \mathcal{I} \tag{8.4}$$

Table 8.4 shows objective values for RP instances using traditional and OptoScale's biomass estimations and associated VSS_i values. Because the RP instances have different objective values, the VSS_i will also vary between the two RP instances. The objective values for the two RP instances are in a risk-neutral instance. The OptoScale instance of the RP outperforms the traditional, resulting in higher VSS_i values for every stage i. We observe an approximately linear increase in VSS_i for both RP instances as i increases. Escudero et al. (2007) state that VSS_i values that become approximately similar for increasing i indicate that there is non-significant gain from adding additional stages to the model, and therefore additional stages are unnecessary. We do not see this effect within the four stages of the RP instances, meaning the model becomes an increasingly realistic approximation for each stage.

Table 8.4: VSS_i values for both RP instances.

Instance	Objective value	VSS_2	VSS_3	$\text{VSS}_{\lvert\mathcal{I}\rvert}$
RP Traditional	110,347,865	6,563,055	10,132,828	14,000,734
RP OptoScale	114,577,085	10,792,275	14,362,048	18,229,954

Looking at the objective value for $\text{EEV}_{\lvert\mathcal{I}\rvert}$ in Table 8.3, we can see the performance of sales decisions disregarding uncertainty. Salmon farmers ignoring uncertainty obtain a profit of 96.4 million NOK. However, compared to the RP with the traditional estimation method, the cost of ignoring uncertainty is 14 million NOK. Such a large cost underpins the significance of solving the sales planning problem stochastically. Having confirmed significant benefits to solving the problem stochastically, we look at the additional value from OptoScale's estimation method. The objective value with OptoScale's estimations outperforms $\text{EEV}_{\lvert\mathcal{I}\rvert}$ by 18.9%, compared to the traditional estimates who outperform $\text{EEV}_{\lvert\mathcal{I}\rvert}$ by 14.5%. Using OptoScale's estimations outperform the solution using traditional estimates by 3.8%. This value shows that there is an additional gain to be made from using OptoScale's estimations compared to traditional estimations.

8.3 Risk profiles

This section introduces different levels of risk aversion to the instances. As briefly described in Chapter 6, risk aversion is modeled through the parameter $\lambda \in [0, 1]$. By increasing λ, the weighting of expected profit compared to CVaR decreases. Risk levels vary from completely risk-neutral when $\lambda = 0$ to completely risk-averse when $\lambda = 1$. As the risk preference in the model moves from risk-neutral to risk-averse, we analyze how the value of information between biomass estimation methods changes. We provide this analysis first for the objective values before looking at decision variables. The motivation for studying risk preference and its implication on sales decisions stems from talks with the salmon farming industry. Apparently, risk profile has large consequences for a salmon farmer's operation and decisions, making this analysis important (Eidsfjord, 2022).

8.3.1 Objective values and risk profiles

Table 8.5 presents the objective values for problem instances with different λ-values. We study how the objective value changes, both for the traditional and OptoScale's estimation methods. The objective value decreases with higher levels of risk aversion. Higher λ-values increase the weighting on maximizing CVaR, which represents the expected profits in the worst-case scenarios. Thus, potential profit is sacrificed for secure income, which reduces the objective value. This trend holds for both biomass estimation methods.

When comparing biomass estimation methods across risk levels, OptoScale achieves better objective values for all risk levels compared to the traditional instances. Not only is the objective value larger, but the difference increases with higher λ-values. Figure 8.2 illustrates the objective value as a function of λ for both biomass estimation methods. The two solid lines show the decline in objective value as risk aversion increases. Furthermore, we illustrate the percentage difference for different risk levels between the two estimation methods, shown by the red dashed line. As shown, the difference increases linearly with higher risk aversion, starting at 3.8% for the risk-neutral case and reaching 10.8% in the completely risk-averse case. This result highlights the advantages salmon

Table 8.5: Objective values for different risk profiles for the two biomass estimation methods.

λ	Biomass estimation	Objective value (NOK 10^6)
0	Traditional	110.3
0.25	Traditional	98.8
0.5	Traditional	92.2
0.75	Traditional	88.4
1.0	Traditional	85.1
0	OptoScale	114.6
0.25	OptoScale	104.2
0.5	OptoScale	98.8
0.75	OptoScale	96.4
1.0	OptoScale	94.3

farmers can obtain by utilizing OptoScale's technology for biomass estimation, especially when risk-averse. In order to understand why the technology proves more valuable in the risk-averse case, we analyze the decisions for different levels of risk aversion.

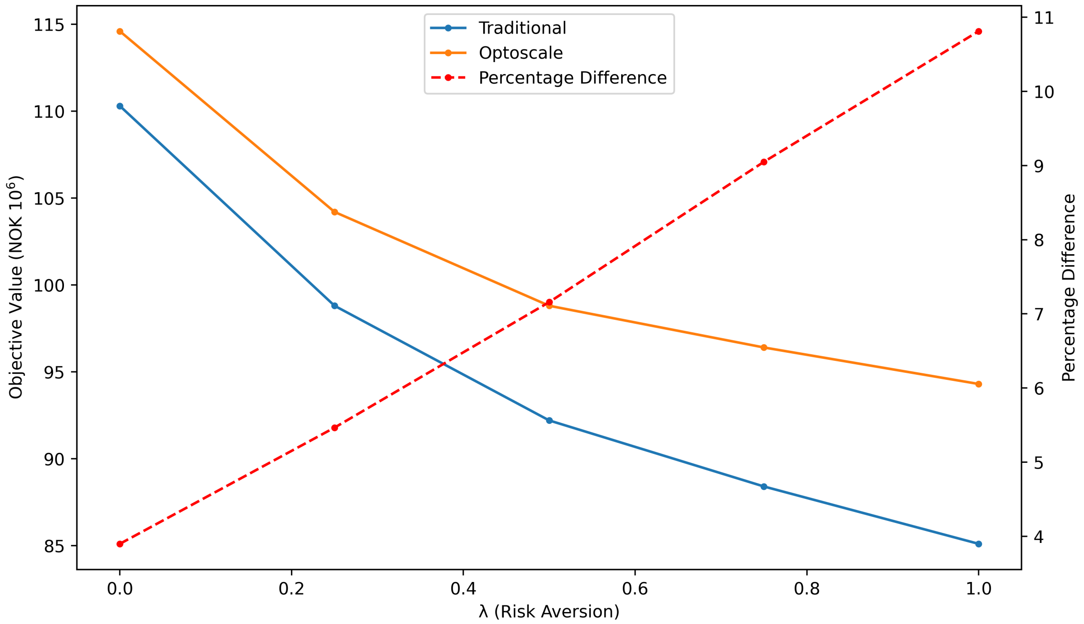


Figure 8.2: Illustration of the objective values from the two estimation methods for different risk profiles. The red line shows the difference between the estimation methods in percent.

8.3.2 Decision variables and risk profiles

As risk aversion increases, we expect more fish to be sold early in the planning horizon and through an increased amount of forward contracts. Selling fish in advance through contracts guarantees income and enables farmers to mitigate the risk of price drops while sacrificing the possibility of price increases. This follows the theory of risk aversion, where a risk-averse agent is expected to sacrifice potential revenue in order to secure income.

Figure 8.3 shows that the percentage of sales made in the first time period increases with higher levels of risk aversion. The trend holds for both OptoScale and traditional estimation methods. For the risk-neutral salmon farmer, just 30% of sales are made during the first time period, while the share increases to around 80% for moderate and high levels of risk aversion. Furthermore, Figure 8.4 illustrates the forward contract utilization as a percentage of total sales for different levels of risk aversion in the first time period. With increasing levels of risk aversion, a larger share of the sales made in the first time period is done through forward contracts.

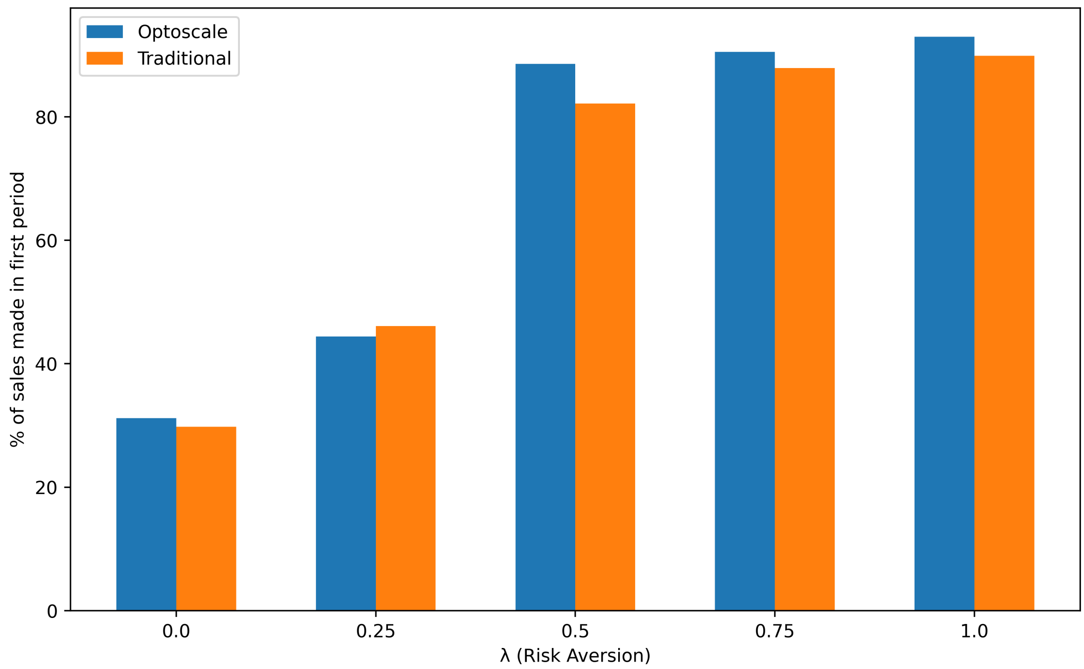


Figure 8.3: Share of total sales done in the first time period, for different levels of risk aversion.

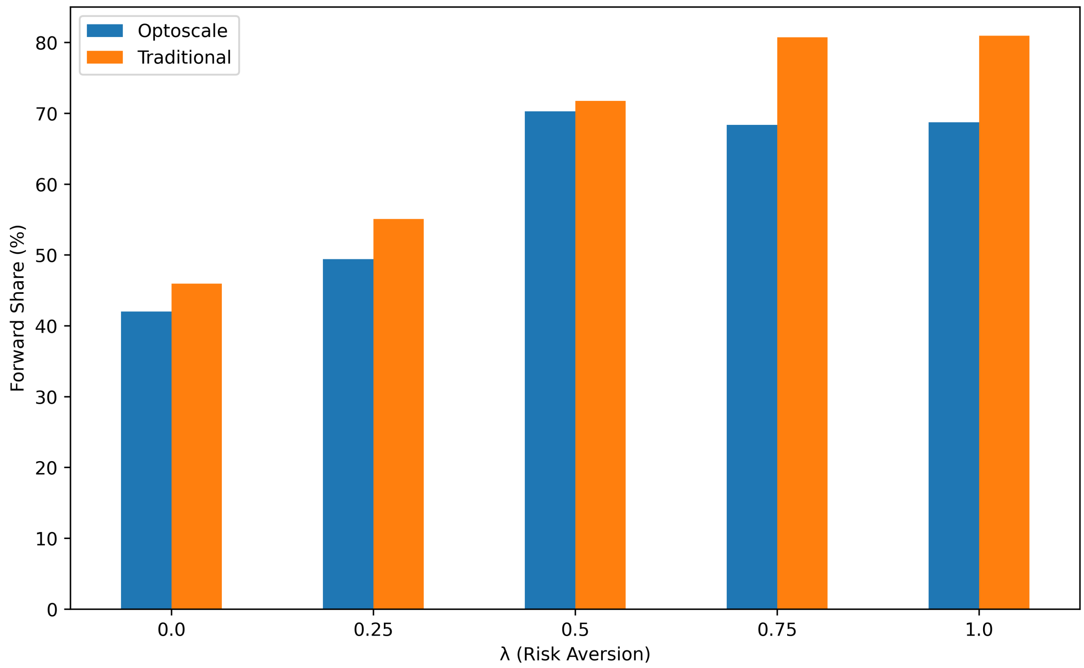


Figure 8.4: Share of fish sold through forward contracts as a percentage of total sales in the first time period for different levels of risk aversion.

Figure 8.5 shows the amount of fish bought in the market for different levels of risk aversion. The result shows that there is little to no buying for low and moderate levels of risk aversion. However, for high levels of risk aversion, the amount of fish bought in the market increases rapidly. The increase in fish bought stems from the increase in forward contracts since a large amount of biomass sold in forward contracts increases the exposure to the risk of overestimating future biomass, which demands purchases in the market to fulfill contracts. With the traditional estimation method, the amount of fish bought exceeds 60,000 kilograms in the completely risk-averse instance. This is approximately 50% more than when using OptoScale's estimation method. It is evident that by using OptoScale's real-time biomass estimation, the salmon producer is able to enter more precise forward contracts, which ultimately leads to fewer unfulfilled contracts. The difference in the amount of fish bought in the market contributes to the difference we observed in the objective value in Figure 8.2. This result highlights the value of information producers gain with more precise biomass estimations.

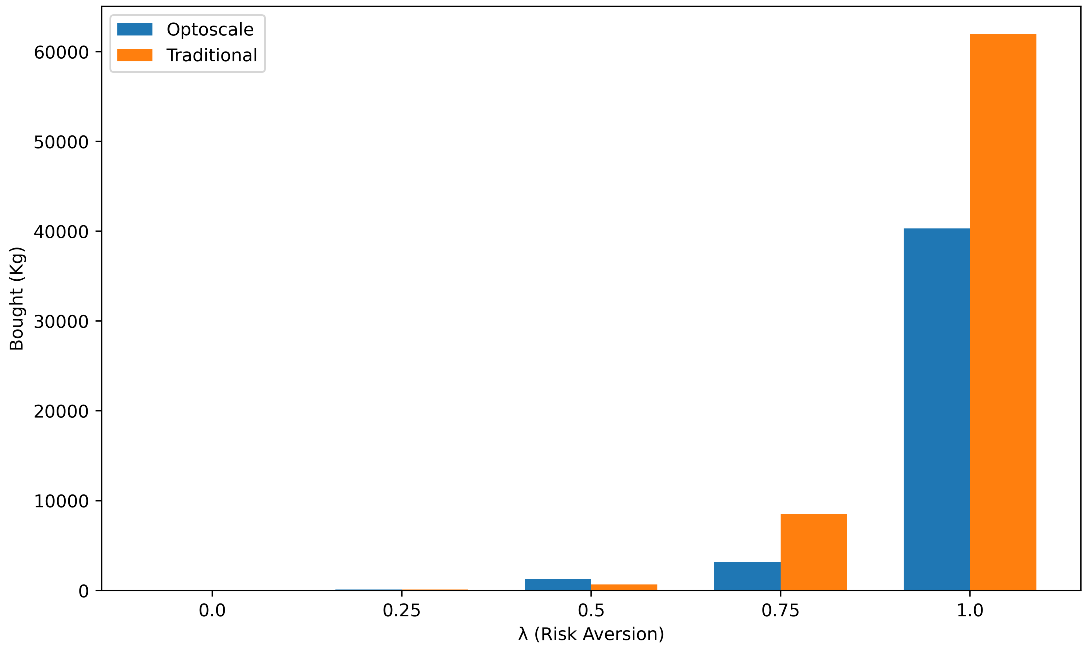


Figure 8.5: Amount of salmon in kilograms bought in the market for different levels of risk aversion.

From the analysis of risk profiles, we can conclude that the value of information gained from OptoScale's estimations comes into full effect when salmon farmers are risk averse. Looking into decision variables helps to explain the reason behind this. As the sales in the first period increase along with the share of forward contracts as a salmon farmer becomes risk-averse, the increased precision of Optoscale's biomass estimations becomes more important. This is because the increased precision helps avoid entering into contracts that the salmon farmer cannot fulfill. Therefore, the analysis of risk preferences unveiled behaviors where precise biomass estimations are important.

8.4 Sensitivity analysis of uncertain parameters

In this section, we provide an overview of how the parameterization of the uncertain parameters, namely biomass estimation and price, affect the model's results. Through this sensitivity analysis, we intend to shed light on the relationship between parameters and the solution found by the model. By analyzing various instances for price and biomass estimation, we gain insight into the robustness and reliability of the model. We also learn how the value of information gained from OptoScale's estimations changes due to differences in parameterization.

8.4.1 Biomass estimation

We present the biomass estimation parameters first introduced in Section 7.3 in Table 8.6. Using OptoScale's estimations, biomass estimation development depends on the start and end uncertainty. The start uncertainty is the initial accuracy of biomass estimation, while the end uncertainty is the propagated uncertainty of initial estimation and future development in biomass estimates. The start uncertainty of 3% originates from a guarantee OptoScale provides for the maximum error of their measurements compared to the mean weight at harvest (OptoScale, 2022b). By varying the value of this number, we consequently must vary the end uncertainty as well to take into account that biomass estimation development in future time periods will have a propagating effect on the start uncertainty. By varying start and end uncertainties, we can analyze how the OptoScale instance would perform compared to the traditional instance if OptoScale's product were more or less precise in its measurements. Furthermore, we can analyze the importance of the quality of OptoScale's product in providing value of information compared to traditional estimations.

Table 8.6: Biomass estimation parameters used in the base instances.

Estimation	Initial uncertainty	End uncertainty
Traditional	-	12%
OptoScale	3%	6%

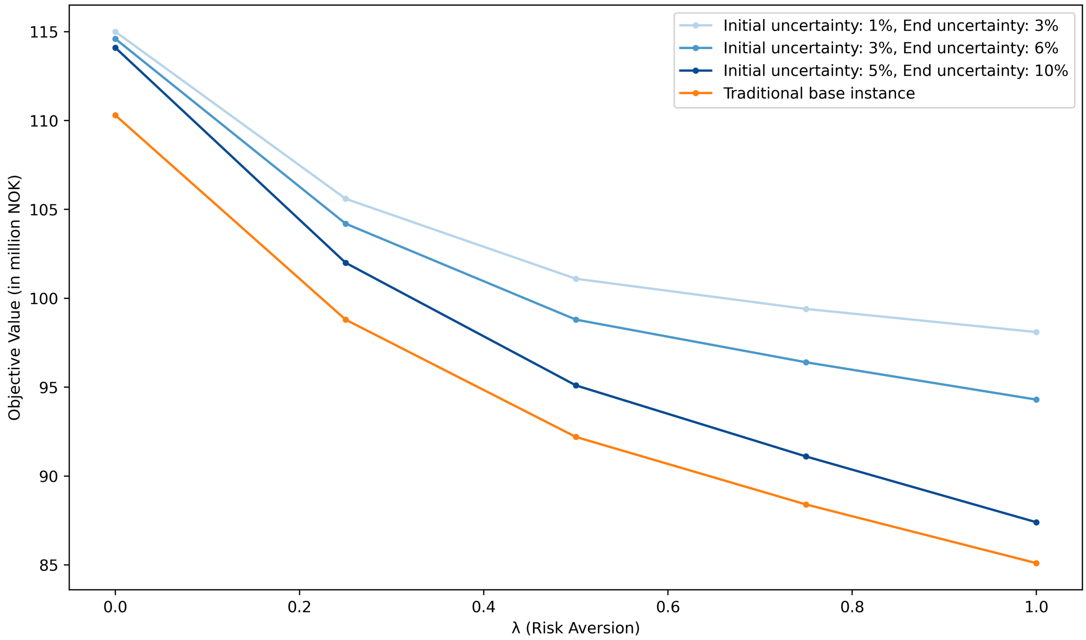


Figure 8.6: Objective values for varying initial and end uncertainties of biomass estimation per time period for estimation methods and risk levels.

Figure 8.6 shows how different start and end uncertainties for OptoScale perform with different levels of risk aversion. As expected, the smaller the uncertainty in biomass estimation and its development, the better the objective value becomes for each value of λ. Knowing with increased precision what the future amount of biomass will become provides the model with greater certainty when entering forward contracts. The importance of certainty in biomass estimates becomes increasingly prominent with higher risk aversion since higher risk aversion leads to more sales made through forward contracts, as shown in Figure 8.4. This can be seen in Figure 8.6 by the increasing distance between the lines of the OptoScale instances as λ increases. From the results of the OptoScale biomass estimation instances, we can conclude that higher precision in OptoScale's estimations becomes increasingly important the more risk-averse a salmon farmer is.

The orange line in Figure 8.6 shows the objective value as λ increases for the traditional instance, as presented in Table 8.5. We observe that neither of the OptoScale biomass estimation instances takes on an objective value lower than the traditional instance. This holds true even though the worst-performing OptoScale instance has a start uncertainty of 5%, which is substantially higher than the guaranteed 3% from OptoScale's product. Thus, OptoScale outperforms the traditional instance even if OptoScale's product performs significantly worse than claimed.

8.4.2 Price

The development of prices, regardless of biomass estimation method, used in instances presented up until this point is explained in detail in Section 7.4. In this section, we assume a symmetric maximum relative change of prices between time periods. By varying this value, we see both narrow and wide scenario trees for price development throughout the time periods. This means that the price, and consequently forward prices and premium prices for buying salmon in the market, can experience more or less significant changes between time periods. At the same time, the expected value over all price realizations in a time period will be the same across instances. This analysis shows how sensitive the results of the model are to price changes and how these changes affect the value of information gained by utilizing OptoScale's estimations.

Figure 8.7 shows how the objective value varies with different biomass estimation methods and maximum price changes per time period for different risk levels. In the risk-neutral instances, all values for the same biomass estimation method are similar. This is to be expected, as the risk-neutral salmon farmer wants to maximize its expected profit and not account for CVaR, and the expectation of price in each time period is unchanged even though the interval in which price scenarios are spread varies. It is also evident for all price uncertainties that utilizing OptoScale's estimations provides better results than their traditional equivalent with equal price uncertainty. Furthermore, the interval between biomass estimations with equal price uncertainty appears similar across all levels of price uncertainty. This implies that the results between different biomass estimation methods for different price developments are similar to the results presented in Section 8.3, where OptoScale's estimations provide an increase in objective value from 3.8% when risk-neutral to 10.8% when risk-averse. From this, we can conclude that the value of information gained by utilizing OptoScale's estimations is present regardless of uncertainty in price development.

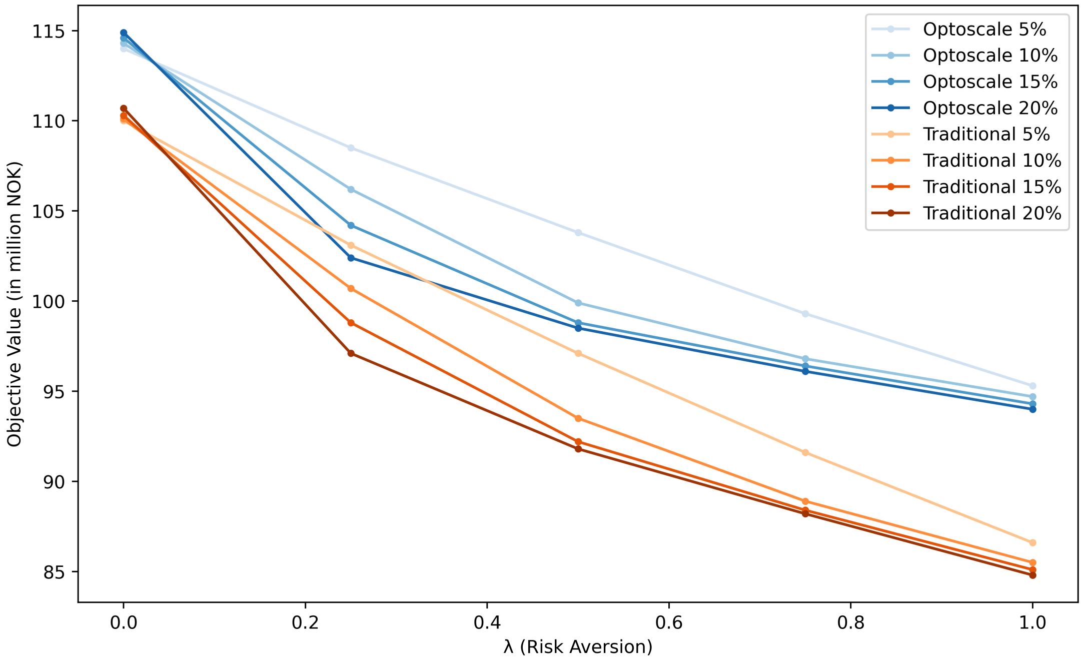
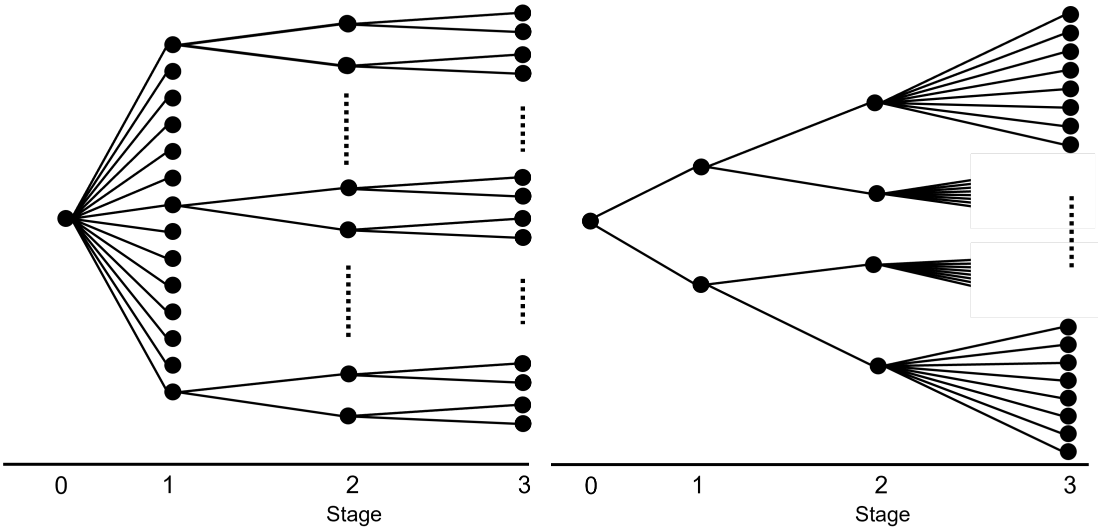


Figure 8.7: Objective values for varying maximum price difference per time period for estimation methods and risk levels.

Another observation in Figure 8.7 is an almost linear decline in objective values for increasing risk aversion when the price uncertainty is at its lowest level, namely 5%. As the price uncertainty increases, the graphs defer from the linear decrease. Instead, they have a sharper decrease for λ values in the interval [0, 0.25] before it diminishes and ends at almost the same value as the linear decrease when λ=1. The initial sharp decrease of objective values is the most prominent for both biomass estimation methods when the price uncertainty is at its highest level. In the instances with the most price uncertainty, objective values decrease by over 10% for both estimation methods when moving from λ=0 to λ=0.25. This means that even if salmon farmers are just somewhat risk-averse, they will be willing to sacrifice significant amounts of expected profits in order to secure their income if the price uncertainty becomes too substantial. The similarities in objective values when λ approaches 1 can be explained by the salmon farmer's decision-making when there is a high level of risk aversion. As presented in Section 8.3, the amount of sales done in the first period increases with increasing risk aversion. Regardless of the uncertainty of price development, the price in the first time period remains the same. Therefore, for each of the biomass estimations, a substantial part of earnings will become the same across price uncertainties resulting in the objective values approaching each other when λ approaches 1. The effect discussed in this paragraph is present for both biomass estimation methods and does not change the conclusion from the previous paragraph, that the value of information gained from OptoScale compared to traditional methods withstands regardless of the level of price uncertainty.

8.5 Scenario tree structure

Thus far, the computational study has focused on scenario trees with equal splits in each stage. The number of splits should ideally be large enough to represent a realistic development of the uncertain parameters. However, increasing the number of splits in a multi-stage scenario tree will quickly lead to computational issues, as discussed in Section 8.1. Consequently, we simplify by reducing the number of splits and, thus, the number of vertices in the scenario tree such that the problem becomes simpler to solve. For a salmon farmer, solving the sales planning problem with few vertices entails some notable drawbacks. Failing to represent all possible future developments may result in unforeseen scenarios that were not considered in the problem instance, potentially causing suboptimal sales decisions and loss of profits. Although the total number of vertices in the scenario tree is limited due to computational complexity, changing the tree's structure is possible. This section studies how the objective value and decision variables are affected by different tree structures. We perform the study to examine whether alternative tree structures are more applicable to this sales planning problem. By performing this study, we answer whether or not salmon farmers can increase their expected profit by concentrating on a precise representation of uncertainty in the near or far future.



(a) Scenario tree with many splits in the first stage, and fewer in the latter.

(b) Scenario tree with few splits in the first stages, and many in the latter.

Figure 8.8: Examples of different tree structures with varying numbers of splits in each stage.

We introduce two new tree structures and compare solutions using these structures to the solution using the tree with an equal number of splits in each stage. Figure 8.8 displays the two new structures. In Figure 8.8a, we present a scenario tree with more splits in the first stage than in the subsequent stages. This structure represents a situation where the importance of an accurate representation of uncertainty in the near future is emphasized, while the representation in the far future is less prioritized. The second structure, presented in Figure 8.8b, consists of a few splits in the first stages while representing the uncertainty realization in the last stage more precisely with an increased number of splits. This structure emphasizes the importance of accurately representing uncertainty in the far future.

Table 8.7: Results from solving instances with different tree structures.

Tree structure name	Number of splits			Number of vertices	Objective value (NOK 10^6)
	Stage 1	Stage 2	Stage 3		
A	4	9	121	17,588	115.0
B	16	16	16	17,476	114.6
C	100	9	4	18,404	114.6

In order to evaluate and compare the different scenario tree structures, we use trees of similar sizes. Size in this setting refers to the total number of vertices in the tree. Table 8.7 presents the objective values and size for the two instances described above, along with the base instance with an equal number of splits in each stage. Note that we name the different tree structures to make it easy to reference them later in the analysis. Results show that the objective values in all three instances are similar, with a maximum difference of 0.4% between the different instances. The similar objective values indicate no significant gain from detailed uncertainty realizations in a particular stage in the tree.

Table 8.8: Spot and forward sales in the first stage for different tree structures.

Stage	Tree structure	Sales Type	Sales (Tons)
1	A	Spot	488
		Forward	354
	B	Spot	488
		Forward	354
	C	Spot	488
		Forward	354

We look into the decision variables from solutions using each tree to further analyze the different tree structures. Table 8.8 shows each tree structure's first-stage decision variables for sales divided into spot sales and forward contracts. For all tree structures, the first-stage sales decisions are identical. This holds for amounts bought in the market as well. For decision variables in the later stages, there are minor differences between tree structures. However, we can still conclude that the model finds similar solutions regardless of the tree structure.

From the analysis of objective values and decision variables for different tree structures, it is evident that they do not produce notably different results. This implies that the model is not sensitive to where uncertainty is modeled precisely. The most important aspect is considering uncertainty, not how it is modeled. The value of introducing uncertainty is elaborated through the calculations of VSS in Section 8.2.

8.6 Implications of the resource rent tax

The results presented in this chapter point to an increased value of information from OptoScale's estimations when the salmon farmer is risk-averse compared to risk-neutral. The increasing difference is due to the increased utilization of forward contracts compared to spot sales in risk-averse instances. Therefore, the value of information discovered in this thesis relies on the possibility of forward contract utilization to provide the maximum benefit.

In September 2022, the Norwegian Ministry of Finance published the first draft of a proposed resource rent tax on aquaculture in Norway. For salmon farming, the first draft proposed a tax level of 40% using norm prices as the value for taxation on sold salmon. Using norm prices means that the price agreed upon in a forward contract would not be the taxed value, but salmon farmers would, instead, be taxed based on a price calculated from the spot price of salmon at the time of maturity of the contract (Finansdepartementet, 2023b). This means the price earned from a forward contract would be independent of the tax on the same contract. From talks with the salmon farming industry, it became clear that there was a concern for the usability of forward contracts as a hedging alternative. If the spot market experiences a price increase in the period between entering into a forward contract and its expiration date, the salmon farmer would be taxed based on a higher price than the price earned from the forward contract. Therefore, forward contracts no longer remove the risk of price changes in the market, which is the opposite of the intended purpose of forward contracts as a hedging alternative. In such a scenario, the value of information from OptoScale's estimations would not be used to its full effect as no salmon farmers would utilize forward contracts in sales decisions.

In March 2023, a revised proposal was published. After negotiations between political parties throughout the spring of 2023 resulted in additional changes to the proposal, it was accepted in parliament on May 31st, 2023. Among the key differences to the initial proposal was a change in taxation level from 40% to 25% and deciding against using norm prices. Instead, the tax's price basis will be the salmon's market value at the time of harvest. In 2023, the market value is determined by the salmon farmers themselves, but for 2024 and onward, an independent council will determine the market value (Finansdepartementet, 2023a). The revised proposal accepted in parliament is, as the initial proposal, subject to debate. There is still a lot of uncertainty regarding how forward contracts will work after implementing the resource rent tax. The market value at the time of harvest can still be different from the price in a forward contract agreed in advance, posing the same challenges as norm prices did to the usage of forward contracts.

Evidently, there is still a lot of uncertainty regarding how the resource rent tax will affect behaviors in the Norwegian salmon farming industry. If the tax prevents salmon farmers from utilizing forward contracts, the value of the information presented in this thesis will not be possible to utilize fully, and the potential of the technology OptoScale provides will decrease.

Chapter 9

Future Research

The model we have developed takes the stochastic development of biomass estimation and price into consideration to determine the value of information from utilizing OptoScale's estimations compared to traditional estimations. We find the model suitable to determine that there exists an additional value for salmon farmers in using OptoScale's biomass estimations. Still, naturally, there are limitations to the model that influence the preciseness of results and conclusions.

One of the most central aspects affecting the model is how we have parametrized the uncertain parameters. When working with biomass estimation parametrization, we investigated how we could model the real-time aspect of OptoScale's measurements. In reality, for each realization, the uncertainty of OptoScale's estimations should be $\pm 3\%$. The multi-stage scenario tree for OptoScale's biomass estimates does not have this property. Creating a stochastic modeling of biomass estimation that does include this would be a more precise approach to how OptoScale's product works in reality.

A natural extension of the model would be considering more stochastic parameters in addition to price and biomass estimation. One potential stochastic addition to the model is an uncertain amount of salmon available for purchase in the market if the salmon farmer cannot fulfill an expiring forward contract. The current model assumes an infinite amount of available salmon for purchase as long as it pays the premium price. To approximate the real world more precisely, the available salmon for purchase could be stochastically varied. Consequently, the premium price could vary based on the availability of salmon in the market. The risk of being unable to buy fish at a premium price to fulfill forward contracts could make the model less willing to use forward contracts, as there is an additional risk introduced with such decisions in this case.

Several factors that influence biomass development are not considered in this thesis. Introducing breakouts of salmon lice or diseases would affect biomass development substantially. Furthermore, the introduction of uncertain water temperatures could impact the TGC parameter. Other factors, such as gender maturation, also affect biomass development. All events mentioned could be stochastically introduced to the model to make it more realistic. Many of the events also impact the quality of the salmon. Therefore it is also suitable to introduce quality classes into the model, affecting the prices obtained from different shares of biomass. To introduce quality classes, information regarding shares of biomass belonging to different quality classes must be gathered from the industry. From this, the model can be extended to include quality classes existing for each weight class in net pens.

The implications of the resource rent tax, as discussed in Section 8.6, are unclear as of completing this thesis. The impact of the tax can potentially change behaviors in sales planning in the Norwegian salmon farming industry. Introducing the tax as a cost to the model would make the model more accurate and could change its behavior and results.

Chapter 10

Conclusion

In this thesis, we determine the value of information that more precise biomass estimation methods provide to salmon farmers. To do this, we construct a multi-stage stochastic optimization model that solves the sales planning problem for salmon farmers. The model determines the optimal combination of selling salmon in the spot market and forward contracts over a planning horizon of four months. The model maximizes profit while accounting for production costs and shortcomings on forward contracts. By solving the model with different biomass estimation methods, the value of information salmon farmers can gain by utilizing OptoScale's measurements is assessed. First, we formulate a model based on previous literature regarding multi-stage stochastic optimization. Then, to run larger instances, we modify the formulation and make a more computationally effective model leveraging the vertex structure of the scenario tree. Including CVaR in the objective function and the ability to weight CVaR against expected value enables the modeling of risk preferences. We also perform a sensitivity analysis of uncertain parameters, scenario tree size, and structure.

The results of the model show the value of considering uncertainty when making sales decisions for salmon farmers. Compared to a deterministic solution disregarding uncertain parameters, the solutions utilizing traditional biomass estimations achieve an increase in the objective value of 15%. This result emphasizes the value of utilizing stochastic programming to solve the sales planning problem.

Our results highlight the value of information gained by utilizing OptoScale biomass estimations compared to traditional methods. While the traditional estimations increase objective value by 15% compared to a deterministic solution, OptoScale's estimations provide an increase of 19%. Furthermore, when comparing the results of the two estimation methods, the gain obtained from OptoScale becomes increasingly evident as the salmon farmer becomes more risk-averse. The difference in objective values when comparing OptoScale to traditional estimations ranges from an increase of 4% when risk-neutral to 11% when completely risk-averse. The increase is mainly due to the increased use of forward contracts as a salmon farmer becomes risk-averse, and consequently, the increased value of more precise biomass estimations. From the sensitivity analysis of the uncertain parameters price and biomass estimation, OptoScale performs better and provides value of information, regardless of price uncertainty levels. OptoScale also outperforms the traditional estimation methods even if their estimation precision is substantially worse than they guarantee. Furthermore, we find no evidence that the structure of the scenario tree affects the solution notably.

Based on all analyses presented in this thesis, it is evident that biomass estimations from OptoScale outperform traditional biomass estimations and provide a substantial and measurable improvement in profits for salmon farmers. Although the model is subject to assumptions and simplifications compared to the real-world process of salmon farming, we can safely state that the Norwegian salmon farming industry could benefit from utilizing precise real-time biomass estimations when making sales decisions. Thus, integrating new biomass estimation technology will play an important role in future value creation in the industry.

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Appendix

A Node Mathematical Model

Symbol	Description
$\mathcal{L}$	Set of all sites.
$\mathcal{V}$	Set of all vertices in the scenario tree.
$\mathcal{V}^i$	Set of all vertices in stage i. $\mathcal{V}^i \subset \mathcal{V}$.
$\mathcal{V}_t$	Set of all vertices in time period t. $\mathcal{V}_t \subset \mathcal{V}$.
$\mathcal{C}_v$	Set of children vertices of v. $\mathcal{C}_v \subset \mathcal{V}$.
$\mathcal{P}_v$	Set of all ancestors of vertex v. $\mathcal{P}_v \subset \mathcal{V}$.
$\mathcal{N}$	Set of all net pens.
$\mathcal{N}_l$	Set of all net pens at site l. $\mathcal{N}_l \subseteq \mathcal{N}$.
$\mathcal{T}$	Set of all time periods during the planning horizon.
$\mathcal{T}_i$	Set of time periods in stage i. $\mathcal{T}_i \subset \mathcal{T}$.
$\mathcal{T}^v$	Time period in vertex v. $\mathcal{T}^v \subset \mathcal{T}$.
$\mathcal{M}$	Set of maturities for contracts.
$\mathcal{M}_t$	Set of available maturities in time period t. $t \in \mathcal{T}$.
$\mathcal{W}$	Set of weight classes.
$\mathcal{I}$	Set of stages.

Symbol	Description
P^v_{tmw}	Price per kilogram of salmon at time t in a contract with maturity m for weight class w in vertex v. $t \in \mathcal{T}$, $m \in \mathcal{M}$, $w \in \mathcal{W}$, $v \in \mathcal{V}_t$. obs; t = i når vi implementerer i koden
MAB_{lt}	Maximum allowed biomass at site l. 0 for time periods t when site l is left fallowed. $l \in \mathcal{L}$, $t \in \mathcal{T}$.

Symbol	Description
MAB^{COMP}	Maximum allowed biomass for a company.
$\hat{B}_n$	Estimated initial total amount of biomass at net pen n. $n \in \mathcal{N}$.
C	Cost per kilogram for keeping biomass at sea one time period.
G^v_{tn}	Percentage increase in biomass taking place going in to time period t, in net pen n and vertex v. $t \in \mathcal{T}$, $n \in \mathcal{N}$, $v \in \mathcal{V}_t$. obs; samme som P
Z^v_{tnw}	The fraction of biomass in net pen n and time period t that belongs to weight class w for vertex v. $t \in \mathcal{T}$, $n \in \mathcal{N}$, $w \in \mathcal{W}$, $v \in \mathcal{V}_t$. obs; samme som G
D_{tn}	Amount of biomass deployed in time period t in net pen n. $t \in \mathcal{T}$, $n \in \mathcal{N}$.
H_l	Harvesting capacity at site l. $l \in \mathcal{L}$.
γ	Scaling factor to the spot price representing the premium price producers has to pay to fulfill contracts. $\gamma > 1$.
π_v	Probability of reaching vertex v from it's predecessor.
α	Confidence level of VaR and CVaR.
λ	Level of risk aversion for the producer. Risk neutrality appears at $\lambda = 0$. $\lambda \in [0, 1]$.

Symbol	Description
b^v_{tn}	Amount of biomass in kilograms in net pen n in the beginning of time period t in vertex v. $t \in \mathcal{T}$, $n \in \mathcal{N}$, $v \in \mathcal{V}_t$.
f_{iv}	Variable used to model CVaR. Approaches VaR for the optimal solution. $i \in \mathcal{I}$, $v \in \mathcal{V}^i$.
k_{iv}	Objective value at stage i and vertex v. $i \in \mathcal{I}$, $v \in \mathcal{V}^i$.
p_{iv}	Loss of profit with respect to CVaR at stage i and vertex v. $i \in \mathcal{I}, v \in \mathcal{V}^i$.
v^v_{tw}	Amount of salmon in kilograms bought in the market to fulfill contracts in time period t per weight class w in vertex v. $t \in \mathcal{T}$, $w \in \mathcal{W}$, $v \in \mathcal{V}_t$.
w^v_{tn}	Amount of biomass in kilograms harvested from net pen n in the beginning of time period t in vertex v. $t \in \mathcal{T}$, $n \in \mathcal{N}$ $v \in \mathcal{V}_t$.

Symbol	Description
y^{v}_{tmw}	Amount of salmon in kilograms of weight class w sold in time period t, in a contract with maturity m, in vertex v. $t \in \mathcal{T}$, $m \in \mathcal{M}$, $w \in \mathcal{W}$ $v \in \mathcal{V}_t$.

A.1 Objective function

$$
\begin{aligned}
&\frac{1}{|V_1|}\sum_{v\in V_1}\sum_{t\in\mathcal{T}_1}\Bigg(\sum_{w\in\mathcal{W}}(\sum_{m\in\mathcal{M}_t} P^{v}_{tmw}y^{v}_{tmw} - P^{v}_{t0w}\gamma v^{v}_{tw}) - C\sum_{n\in\mathcal{N}}(b^{v}_{tn} - w^{v}_{tn})\Bigg) \\
&+ (1-\lambda)\sum_{v\in\mathcal{V}_2}\pi^{v}k_{2v} + \lambda\Bigg(f_{11} - \frac{1}{1-\alpha}\sum_{v\in\mathcal{V}_2}\pi^{v}p_{2v}\Bigg)
\end{aligned}
\tag{1}
$$

A.2 Constraints

$$
\begin{aligned}
&\sum_{t\in\mathcal{T}_i}\Bigg(\sum_{w\in\mathcal{W}}(\sum_{m\in\mathcal{M}_t} P^{v'}_{tmw}y^{v'}_{tmw} - P^{v'}_{t0w}\gamma v^{v'}_{tw}) - C\sum_{n\in\mathcal{N}}(b^{v'}_{tn} - w^{v'}_{tn})\Bigg) \\
&+ (1-\lambda)\sum_{v''\in\mathcal{C}_v}\pi^{v''}k_{(i+1)v''} \\
&+ \lambda\Bigg(f_{iv} - \frac{1}{1-\alpha}\sum_{v''\in\mathcal{C}_v}\pi^{v''}p_{(i+1)v''}\Bigg) = k_{iv} \quad i = 2\ldots|\mathcal{I}|-1, v\in\mathcal{V}_i
\end{aligned}
\tag{2}
$$

$$
\sum_{t\in\mathcal{T}_{|\mathcal{I}|}}\Bigg(\sum_{w\in\mathcal{W}}(\sum_{m\in\mathcal{M}_t} P^{v}_{tmw}y^{v}_{tmw} - P^{v}_{t0w}\gamma v^{v}_{tw}) - C\sum_{n\in\mathcal{N}}(b^{v}_{tn} - w^{v}_{tn})\Bigg) = k_{|\mathcal{I}|,v} \quad v\in\mathcal{V}_{|\mathcal{I}|} \tag{3}
$$

$$
f_{iv} - k_{(i+1)v'} \leq p_{(i+1)v'} \quad i = 1\ldots|\mathcal{I}|-1, v\in V_i, v'\in\mathcal{C}_v \tag{4}
$$

$$
\sum_{n\in\mathcal{N}_l} b^{v}_{tn} \leq MAB_{lt} \qquad t\in\mathcal{T}, l\in\mathcal{L}, v\in\mathcal{V}_t \tag{5}
$$

$$\sum_{n \in \mathcal{N}} b_{tn}^{v} \leq MAB^{COMP} \qquad t \in \mathcal{T}, v \in \mathcal{V}_t \tag{6}$$

$$b_{(t+1)n}^{v'} = G_{(t+1)n}^{v'}(b_{tn}^{v} - w_{tn}^{v}) + D_{tn} \qquad t = 1 \ldots \mathcal{T} - 1, n \in \mathcal{N}, v \in \mathcal{V}_t, v' \in \mathcal{C}_v \tag{7}$$

$$b_{1n}^{v} = G_{1n}^{v} \hat{B}_n + D_{0n} \qquad n \in \mathcal{N}, v \in \mathcal{V} \tag{8}$$

$$\sum_{v' \in \mathcal{P}_v} \sum_{t' \in \mathcal{T}_{v'}} y_{t'(t-t')w}^{v'} + y_{t0w}^{v} = v_{tw}^{v} + \sum_{n \in \mathcal{N}} Z_{tnw}^{v} w_{tn}^{v} \quad t \in \mathcal{T}, w \in \mathcal{W}, v \in \mathcal{V}_t \tag{9}$$

$$\sum_{n \in \mathcal{N}_l} w_{tn}^{v} \leq H_l \quad t \in \mathcal{T}, l \in \mathcal{L}, v \in \mathcal{V}_t \tag{10}$$

$$w_{tn}^{v} \leq b_{tn}^{v} \quad t \in \mathcal{T}, n \in \mathcal{N}, v \in \mathcal{V}_t \tag{11}$$

$$\frac{1}{|V_1|} \sum_{v' \in \mathcal{V}_1} y_{tmw}^{v'} = y_{tmw}^{v} \quad t \in \mathcal{T}_1, m \in \mathcal{M}, v \in \mathcal{V}_1, w \in \mathcal{W} \tag{12}$$

$$\frac{1}{|V_1|} \sum_{v' \in \mathcal{V}_1} v_{tw}^{v'} = v_{tw}^{v} \quad t \in \mathcal{T}_1, v \in \mathcal{V}_1, w \in \mathcal{W} \tag{13}$$

$$\frac{1}{|V_1|} \sum_{v' \in \mathcal{V}_1} w_{tn}^{v'} = w_{tn}^{v} \quad t \in \mathcal{T}_1, n \in \mathcal{N}, v \in \mathcal{V}_1 \tag{14}$$

$$b_{tn}^{v} \geq 0 \quad t \in \mathcal{T}, n \in \mathcal{N}, v \in \mathcal{V}_t \tag{15}$$

$$v_{tw}^{v} \geq 0 \quad t \in \mathcal{T}, w \in \mathcal{W}, v \in \mathcal{V}_t \tag{16}$$

$$w_{tn}^{v} \geq 0 \quad t \in \mathcal{T}, n \in \mathcal{N}, v \in \mathcal{V}_t \tag{17}$$

$$y_{tmn}^{v} \geq 0 \quad t \in \mathcal{T}, m \in \mathcal{M}, w \in \mathcal{W}, v \in \mathcal{V}_t \tag{18}$$

$$p_{iv} \geq 0 \quad i \in \mathcal{I}, v \in \mathcal{V}^i \tag{19}$$

$$k_{iv} \quad \textit{free} \quad i \in \mathcal{I}, v \in \mathcal{V}^i \tag{20}$$

$$f_{iv} \quad \textit{free} \quad i = 1 \ldots |\mathcal{I}|-1, v \in \mathcal{V}^i \tag{21}$$

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