Master's thesis

NTNU
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Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

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Production scheduling and investment in pumped-storage hydropower under uncertainty

Master's thesis in Industrial Economics and Technology Management
Supervisor: Stein-Erik Fleten
Co-supervisor: Ane Marte Andersson
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Preface

This Master's thesis was written by three students at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology, in the spring semester of 2023. The thesis is a continuation of the authors' project report at the same department, and both works have been written in collaboration with Statkraft AS.

We would like to express our gratitude to our supervisor professor Stein-Erik Fleten at NTNU, and our co-supervisor from Statkraft Ane Marte Andersson, for their continuing guidance and counselling throughout this period. Their input has been greatly valued. For introductions to, and explanations of, their respective research we must extend our thanks to Andreas Kleiven, post-doctor from NTNU and Arild Helseth from SINTEF Energy. Professor Leif Lia at the HydroCen research centre has also been greatly appreciated for his insight into cost calculations for hydropower facilities.

Sammendrag

I en fremtid med høy andel fornybar elektrisitet vil et magasinbasert vannkraftverk med pumpefunksjon være etterspurt for å kunne levere mot-syklisk kraft. I denne oppgaven presenterer vi et beslutningsrammeverk for å beregne optimal tidspunkt for investering i et pumpekraftverk. Beslutningsrammeverket er støttet av en detaljert produksjonsplanlegger spesifikt utviklet for pumpekraftverk. Planleggeren bruker en to-trinns stokastisk lineær programmering implementert med en rullerende intrinsisk metode. Produksjonen planlegges med høy presisjon, der produksjonsbeslutninger tas for alle timer i en ettårig planleggingsperiode. Vi formulerer investeringsproblemet som et optimalt stoppeproblem, med en kostnadsfunksjon avhengig av produksjonskapasitet og en verdivurdering basert på produksjonsplanleggeren. Våre resultater indikerer at pumpekraftverk kan være svært lønnsomme gitt markedsvolatiliteten i moderne kraftmarkeder med åpen handel.

Abstract

In a future with high shares of renewable electricity sources, the ability of reservoir hydropower to provide counter-cyclical power will be in high demand. The inclusion of a pump, converting a hydropower facility to a pumped storage facility will increase this ability further. In this thesis, we present a decision framework for calculating the optimal time to invest in a pumped storage hydropower facility. The framework is supported by a detailed production scheduler, specifically modelled for pumped storage hydropower. The scheduler is implemented as a two-stage stochastic linear program in a rolling intrinsic method. Production is scheduled with high precision, with a production decision for all hours within a one-year planning period. We formulate the investment problem as an optimal stopping problem, with an upgrade-dependent cost function and a plant valuation based on the production scheduler. Our results indicate that, given the market volatility in modern openly traded power markets, pumped storage facilities could be highly profitable.

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1 Introduction

Hydropower accounts for more energy production globally than any other low-carbon energy source, and all other renewable production combined (DNV 2022). In the developed economies that rely heavily on hydropower, the ageing of facilities is becoming an increasing issue (International Energy Agency 2021). Facilities require refurbishment, replacement, or upgrading as they age and deteriorate to ensure continued functionality and efficiency. With the development of advanced production models suited for the modern openly traded power markets, upgrading production capacity could also significantly increase the profitability and value of hydropower plants (Andersson et al. 2014, Rahi and Chandel 2015). New hydropower investments could also play a key role in meeting increased electricity demand (International Energy Agency 2021). Given these challenges, it becomes relevant to develop accurate approaches to discern the optimal way to invest in both refurbishment of existing, and the development of new, hydropower plants.

Energy production from renewable sources is forecast to be a significantly larger part of the energy mix by 2050 (DNV 2022). Most renewable energy production has some common characteristics that constitute potential new challenges for power markets (Stathakis et al. 2021). Production through wind, photovoltaic and run-of-river hydro sources are highly volatile, as they all rely on uncontrollable weather conditions (Rodriguez et al. 2017). Hydropower facilities that with reservoirs can however help reduce this volatility, as production can be increased or decreased to meet market demand (Pérez-Díaz et al. 2015). By allocating water to production only during price peaks, supply can be somewhat increased to meet the consumers' varying demand, thus reducing volatility while increasing the plants' profits (Bozorg Haddad et al. 2014).

In this thesis, we introduce a method to discern the ideal timing of investment into upgrading a hydropower facility, and the related ideal production capacity to invest in. Specifically, we study the effects of installing both a pump and a turbine, a ternary unit as in Koritarov et al. (2013), which expands the production flexibility and turns the plant into a pumped storage hydropower facility[1]. Pumping water back up from a lower to a higher reservoir enables producers to exploit the price variations in the power market even further than traditional hydropower (Jain and Patel 2014). By purchasing power to pump when the market is in abundance, and producing extra when power from other sources is scarce, operators increase profits from their facility while helping reduce the market volatility. This volatility is forecast to increase with the increasing development of power generated from renewable sources (Moura and Almeida 2010). Pumping water does however accrue a net loss of total electricity produced by the pumped storage hydropower facility, usually in the range of 15-30%, which is a consideration operators must incorporate into their strategy (Deane et al. 2010).

As a case study to test our methods, we consider an upgrade of an existing facility located in the southern part of Norway, constructing a new penstock and building a new facility. This allows for ongoing production throughout the construction period, minimizing economic losses caused by production-halt. We construct a production scheduling model for the proposed facility that optimises production for different production capacities and combines it with a capacity-dependent cost function, which serves as input into a real options analysis model.

Hydropower production is mainly dependent on two external factors, electricity price and the inflow of water (Fosso et al. 1999). We model weekly electricity prices in accordance with the method of Schwartz and Smith (2000), consisting of both long- and short-term price developments. This two-factor implementation is considered more realistic than standard price models used in most other hydropower investment literature (Kleiven et al. 2022). The inflow of water to reservoirs

[1] Hydropower literature differs between "conventional" pumped storage, where the upper and lower reservoirs are in a closed loop, with no inflow. For a plant with inflow into the reservoirs, the precise definition is a "pump-back" facility, but we will in the entire thesis refer to the facility as pumped storage hydropower.

has both yearly and seasonal volatility and is dependent on rain, snow melting during spring and other climate conditions (Gjelsvik et al. 2010). Our inflow model is an adaption of Helseth et al. (2013), where we use historical inflow data to produce a set of weighted scenarios.

The product of the price and inflow models function as input to our production scheduling model, a two-stage stochastic linear programming formulation, implemented with a rolling intrinsic method. The inclusion of a stochastic forecast period increases the realism of the scheduling model, allowing for more accurate insights into optimal production patterns while also contributing to finding a more accurate valuation of the investment (Helseth et al. 2018). The plant valuation obtained from the stochastic linear programming model is used as input into our investment model, a real options analysis framework adapted from the work of Dixit and Pindyck (1994). We formulate the investment decision as an optimal stopping problem, where the value of the plant is dependent on electricity price and production capacity. In contrast to the more generally applied NPV method, the real options framework allows the investment to consider waiting for further market developments (McDonald and Siegel 1986). We formulate investment cost as a function of production capacity, using the industry-standard report from The Norwegian Energy Regulatory Authority and Norconsult AS (Norconsult AS 2016). Costs are calculated according to the location of our specific case problem. We then solve the optimal stopping problem semi-analytically to find the optimal production capacity and price conditions that indicate investment, as in Dangl (1999).

Our main findings indicate that, given the historic and forecast future volatility in the power market, pumped storage hydropower is a profitable investment for a large range of capacities. Provided our capacity-dependent cost function with decreasing marginal costs for increasing capacity, investment will be carried out at the maximum available capacity and will require a low price trigger, as demonstrated in a case in Dangl (1999). Larger pumping capabilities will also greatly increase seasonal production and reservoir level fluctuation. Further, removing intra-weekly price volatility, restricting production to only exploit seasonal volatility, results in an investment of a high capacity but with a correspondingly large price trigger. This is also consistent with earlier literature concerning investment with high demand uncertainty (Dangl 1999, Hagspiel et al. 2016).

The work done in this Master's thesis is an expansive continuation of the work done in our project report (Arnstad et al. 2022). While the novelties of our approach constitute the majority of our work, the structure of our method remains fairly similar.

The rest of the thesis is organised as follows. Section 2 begins with a review of the literature relevant to the academic topics and methods applied. In Section 3 we give a short description of the specific problem we aim to solve followed by a presentation of our modelling methods in Section 4. We then give a thorough explanation of our solution method in Section 5. In the results, Section 6, we discuss and demonstrate different aspects and effects of our model, and compare its expansions to the implementation of Kleiven et al. (2022). Finally in Section 7, we make some concluding remarks on our findings, with a discussion of possibilities for further research.

2 Literature

In this section we list and explain the literature relevant to our topics of pumped storage hydropower investment and production scheduling. In sequence, the topics discuss the hierarchical planning structure, pumped storage hydropower literature, the real options framework, methods of modelling commodities and some common optimisations methods found in hydropower production scheduling. The end of the section comprises an explanation of our contributions to the literature.

2.1 Hierarchical planning

To create models and planning methods suited for comprehending the entirety of plant management, the hydropower industry has adopted the hierarchical planning structure of Dempster et al. (1981) (Fosso et al. 1999). Long-, medium- and short-term decisions are sorted into three different planning horizons, the strategic, tactical and operational sub-problems respectively. By separating decisions, and allowing decisions at the top of the hierarchy to constrain or influence decisions below, plant managers are able to more accurately and efficiently model the different aspects of the facility (Bøckman et al. 2008).

The strategic sub-problem, with the longest horizon of up to multiple decades, includes planning for market-wide developments and investments into either new facilities or refurbishment/upgrade projects (Linnerud et al. 2014). Tactical decisions are made for a time period ranging from a few weeks to between one and three years. The objective of these models is to valuate the availability of water for each week in its scheduling period, to allocate more water to seasonal peaks and minimise the risk of spillage (Pereira and Pinto 1991, Schäffer et al. 2022). For the operational sub-problem, short-term decisions include optimal hourly production and bidding on the open spot market for the next day (Fleten and Kristoffersen 2008, Avesani et al. 2022). These operational problems are constrained by the solution to the tactical sub-problem, which gives a marginal water value as input into a bidding model (Aasgard et al. 2014). The decision framework for the operational sub-problem can be expanded when considering short-term trading on other electricity markets, such as the balancing market and the ancillary services market (Braun and Burkhardt 2015, Wen and David 2002).

2.2 Pumped Storage Hydropower

The construction of pumped storage hydropower increases the operator's production flexibility (Yang and Jackson 2011). Given adequate market conditions, flexibility is increased as a result of having more water available for production, without a significant increase in the risk of spillage or violating regulations. In general, increased production capacity allows the producer to allocate water production to periods of high electricity prices (Fertig et al. 2014). Pumped storage enables the water to be moved from a lower reservoir to a higher one when prices are low, and stored until production when prices are higher, further increasing profitability (Deane et al. 2010). This production pattern would also help decrease volatility in the power market (Rodriguez et al. 2017).

A large majority of pumped storage hydropower utilises a reversible Francis turbine for both production and pumping, known as a pump as a turbine or PAT (Deane et al. 2010, Antal 2014, Carravetta et al. 2014). This is the economically viable option in most instances, but for facilities with a significantly large head, the Francis turbine is a less viable option (Norconsult AS 2016). A solution to this issue is the construction of a ternary unit, as in Koritarov et al. (2013), where a separate turbine and pump are connected to the same electric generator. A ternary unit allows for the installation of turbines designed specifically for producing and pumping, increasing the efficiency of both. Another advantage of this design is a reduction of the time required to switch from production to pumping, and vice versa (Nag and Lee 2020). While pump and turbine technology has become more advanced, there is still a significant loss in total water utilisation, meaning there is a loss in net energy production when pumping and producing compared to singular production (Jain and Patel 2014). The energy losses due to efficiency deem the profitability of the pumped storage hydropower as highly dependent on seasonal, weekly or daily price fluctuations (Pérez-Díaz et al. 2015).

2.3 Real Options

When modelling investment decisions, real options analysis has several advantages over the more widely used net present value method (Farzin et al. 1998, McDonald 2003). However, within the hydropower industry, the standard for making refurbishment and upgrading decisions is based on the NPV method (Andersson et al. 2014). For real options to be applicable, the investment must be irreversible, future rewards must be dependent on future uncertain developments and the investor can decide to delay the investment to ascertain more information (Huisman and Kort 2002). The advantages of having the option to delay when deciding on investment under uncertainty were highlighted early by McDonald and Siegel (1986) amongst others, but a comprehensive introduction to real options theory can be found in (Dixit and Pindyck 1994). The methodology developed by Dangl (1999) extends Dixit and Pindyck (1994) by considering both optimal timing and capacity in a case where different investment opportunities of continuous capacities choices are mutually exclusive of each other once an investment is undertaken, meaning that no expansion at a later stage is possible. This is further expanded upon by Hagspiel et al. (2016), where investment in both capacity and timing is considered, but with either flexible or inflexible production.

A model for comparing the option to invest in two similar but mutually exclusive projects is introduced in Dixit (1993). It is however proven in Décamps et al. (2006) that under general conditions the findings in Dixit (1993) do not hold, and the investment region is dichotomous. In this thesis, we refrain from comparing the option value of two exclusive investments. This is a consideration that would otherwise be relevant when considering expanding from a conventional hydropower facility to a pumped storage plant, as one could invest solely in a capacity upgrade of the existing plant.

A significant portion of existing hydropower facilities in developed countries are ageing, with the average age of facilities in North America and Europe being somewhere between 45 to 50 years (International Energy Agency 2021). Demand for reinvestment is increasing along with the ageing of hydropower plants, as components deteriorate, maintenance costs increase and market conditions change (Andersson et al. 2014, Linnerud et al. 2014). The real options framework has been proven effective when applied to small hydropower projects both in the Norwegian and the Brazilian setting (Bøckman et al. 2008, Martínez-Ceseña and Mutale 2011). The real options framework has also been successfully applied to the investment and refurbishment of facilities with water reservoirs (Andersson et al. 2014, Fertig et al. 2014). Kleiven et al. (2022) considers the specific problem of investing in a production capacity upgrade, simulating production with different capacities and inputting production valuations into an optimal stopping problem. Common amongst these methods is the use of Dangl (1999) to find the price trigger at which it is optimal to invest, i.e. timing, with corresponding capacity.

2.4 Commodity modelling

Modelling the price developments of a commodity is a central aspect of evaluating projects and making informed decisions about the future (Dixit and Pindyck 1994, Schwartz and Smith 2000). A non-exhaustive list of common price process models includes Geometric Brownian Motions (GBM), Ornstein–Uhlenbeck processes (also known as mean-reverting processes) and the family of auto-regressive models as ARMA/ARIMA and ARCH/GARCH (Brooks 2019). The two-factor model of Schwartz and Smith (2000) utilises a combination of GBM in the equilibrium price of the commodity and an Ornstein-Uhlenbeck process in short-term price deviations. The two factors are estimated using spot prices and the forward curve, the latter being created using futures contracts (Fleten and Lemming 2003). The real options framework requires a stochastic process, which renders the predictive models mentioned inapplicable (Dixit and Pindyck 1994).

Industry and academia have jointly developed an alternative approach to forecast medium-term price dynamics in the form of a specialised equilibrium market model known as EFI's Multi-area Power-market Simulator (EMPS). This model generates price forecasts for power producers by simulating weather conditions and production across the entire Nordic region (SINTEF 2022b). A key characteristic of the Nordic power market is the dependence on different weather conditions, as renewable energy production such as hydro- and wind-power is a major part of the production mix (Fosso et al. 1999). EFI's Multi-area Power-market Simulator (EMPS) is a model that develops price forecasts for hydropower producers by simulating weather and production for the entire Nordics. Due to its precision and low computational demands, it has become the industry standard in Norway and is today maintained by SINTEF Energy[2] (Wolfgang et al. 2009).

In the Nordic electricity market[3], the price fluctuates according to regional and national demand and supply (NVE 2023a). Due to bottlenecks in grid capacity, there are also different price areas within the Nordics. The primary day-ahead spot market and the balancing intraday market are managed by Nord Pool, where most trading occurs (*Nord Pool* 2023). An ancillary market exists for the transmission system operator to ensure the required quality of the grid (SINTEF 2022a). A hydropower producer that can schedule production has the ability to trade on several of these markets, a well researched subject (Wen and David 2002, Fleten and Kristoffersen 2007, Aasgard et al. 2014, Braun and Burkhardt 2015). The majority of trading is however done on the day-ahead spot market, and it remains the reliable benchmark for medium- and long-term planning (Kleiven et al. 2022).

Besides electricity prices, inflow into the reservoir is the most influential exogenous uncertainty that affects hydropower production scheduling. There are multiple methods of modelling inflow scenarios used in literature and the industry. Statistical auto-regressive models are the Nordic industry standard (Helseth et al. 2018). One example is the first-order AR(1) model, described in Gjelsvik et al. (2010), which combines accuracy with computational feasibility. The AR model can be improved by including seasonality in a periodic auto-regressive model (PAR) when generating inflow scenarios, as detailed in Tilmant and Kelman (2007) and further extended with multiplicative error terms to avoid negative realisations of inflow (Shapiro et al. 2013). Historical inflow records can also be used as scenarios directly. A new fundamental market optimisation and simulation model introduced in Helseth et al. (2017) applies yearly weather measurements of inflow, wind, snow and temperature as scenarios. Utilising historical weather gives the advantage of holding correlations in time and space and keeping relationships between different types of data. This can be difficult to obtain from statistical models (Helseth et al. 2018). It is also common to include a negative correlation between price and inflow. Despite the fact that correlation exists in reality, Kleiven et al. (2023) conclude that the influence of not including the correlation is minor when regarding revenues.

[2]EFI was absorbed into SINTEF and rebranded into SINTEF Energy in 1998

[3]The Nordic power market has established power trading with other European nations, so it is not excluded from the rest of northern Europe. However, grid capacities and heavy investments in renewable sources of production create a difference between the Nordics and their neighbours. In the base case of this thesis we concern ourselves only with the Nordics.

2.5 Production scheduling methods

To manage the uncertainties in inflow and price, different optimisation methods to schedule production have been developed through collaboration between the industry and academia. Originally, Bellman (1957) introduced a method to calculate a policy that recommends the optimal course of action while dealing with uncertainty, called stochastic dynamic programming (SDP). SDP has been applied to several scheduling problems and is a key part of the EMPS model (Wolfgang et al. 2009, Schäffer et al. 2022, Helseth et al. 2016). In order to avoid the issue of dimensionality associated with dynamic programming, which arises from the discretisation of state variables, Pereira and Pinto (1991) propose an alternative to SDP, labelled stochastic dual dynamic programming (SDDP), in the setting of optimising the operation of a hydrothermal system with uncertain inflow. The algorithm generates Benders' cuts from the dual solutions to each stage of the stochastic optimisation problem. Despite being a commonly used method for operational scheduling, Gjerden et al. (2015) find that cuts from the SDDP are difficult to obtain in a detailed Norwegian hydropower system using historical data. In the long-term hydrothermal scheduling model presented by Helseth et al. (2018), a two-stage stochastic model is instead employed, where the first-stage problem holds uncertainties in inflow and price and the second-stage is solved deterministically for each scenario of future prices and inflow. The method demonstrates promising results but demands a reduced number of scenarios due to high computational requirements. In Kleiven et al. (2022), a deterministic hydropower scheduling model from an investment perspective is introduced, where expected values of price and inflow replace uncertainty. The model follows the re-optimisation heuristic methodology, which reduces time complexity significantly (Lai et al. 2010, Gray and Khandelwal 2004).

2.6 Our contributions

In this section, we list and explain the contributions of this thesis to the literature on hydropower investment, production scheduling and optimisation methods within pumped storage hydropower.

A central aspect of our thesis is the consideration of investment into a pumped storage hydropower facility. Specifically, we combine detailed production scheduling for pumped hydropower with a real options investment model. Separately, these methods have been used on pumped storage hydropower before, real options in Reuter et al. (2012), finding optimal capacity in Bozorg Haddad et al. (2014) and Nasir et al. (2022), and production scheduling in Helseth et al. (2013). The totality of our work contributes to joining approaches from these different research fields and expanding on these works with the new implementations mentioned in this section.

In the hierarchical planning context of Dempster et al. (1981), we join the operational and tactical planning horizons into a single level, to more accurately calculate the value of production. The implementation draws inspiration from work by Helseth et al. (2018) and extends the model of Kleiven et al. (2022). By assuming external factors known in the immediate future, scheduling production for a dynamic amount of hour blocks, decreasing granularity as the scheduler moves further in the future and re-optimising each week, the method plans production on an operational basis with a tactical horizon.

Pumped storage hydropower facilities require price volatility to be profitable. We include in our price model intra-weekly price variations. This is an extension of the model in Kleiven et al. (2022), where intra-weekly variations consist of a calculated average and are identical for all weeks. By instead sampling historic weekly spot-price deviations the authors argue that the realism of the model is increased. This implementation is advantageous to pumped hydropower specifically, as averaged volatility would limit the model's opportunities for pumping (Rodriguez et al. 2017).

Another extension to the model of Kleiven et al. (2022) is the discretising of uncertain future inflow and price developments to scenarios. Their linear optimisation program resolves the uncertainties using expected values as forecasts for inflow and price. By including a weighted set of scenarios, the model has the opportunity to optimise for multiple outcomes at the same time, to possibly improve performance.

One crucial aspect often overlooked in hydropower investment literature, but deemed essential by the authors, is an accurate description of costs associated with the investment. Properly accounting for these costs is a fundamental component in calculating the real option value with precision. NVE has, in partnership with the Norwegian consultant firm Norconsult AS, developed a report covering estimates of most construction and component costs related to hydropower facility construction (Norconsult AS 2016). Both in the Nordic setting and more internationally, research using ROA for hydropower investment assumes fixed or simplistic cost functions (Martínez-Ceseña and Mutale 2011, Andersson et al. 2014, Reuter et al. 2012). One exception is the case-based method of cost calculations in Kvamme (2008), where investment in a specific refurbishment project is planned. These calculations are however not capacity dependent. Using the report and conversations with Professor Leif Lia (personal communication, 24.05.23) as a basis we construct a capacity-dependent cost function. This function has decreasing marginal cost as capacity increases and is similar to the concave cost function of Dangl (1999).

3 Problem description

We view the problem from the perspective of a hydropower facility owner that wishes to maximise the value of their plant. We consider a facility between two large reservoirs in southern Norway. The plant is ageing and designed for steady production in a power market of different market conditions. To achieve the highest possible profits from their facility, water must be used efficiently given uncertain external factors, minimising spillage and exploiting price variations to produce at peak prices.

The facility owner has the option to increase the capacity of the existing facility, to optimise the value of the plant. To further make use of price volatility, we consider investing in upgrading the facility to a pumped storage hydropower plant. This extends the hydropower plant with both pumping and production capabilities, enabling the producer to schedule each at low and high prices respectively. This also enables full operation of the existing plant during the construction of a new facility.

We operate as a price taker in the open market of the Nordics, specifically the NO2 region, and limit ourselves to only considering the day-ahead spot market for electricity. Although price manipulation is a technical possibility, this assumption is reasonable in accordance with the number of market participants in the market the producer operates in (Arteaga and Zareipour 2019). The producer is interested in long-term planning, and as such trading on other short-term markets is deemed out of the scope of our problem.

Available to the producer is historical data on spot prices, electricity futures and inflow into the upper reservoir. Future developments in the exogenous factors of price and inflow must be modelled. The size of the lower reservoir is so large that the availability of water for pumping can be considered uncapped. The upper reservoir level must be balanced to avoid spillage, which will induce an opportunity cost. When scheduling production, exogenous factors are considered deterministic one week ahead but are subject to uncertain developments after the first week.

The producer also holds information on the cost parameters associated with the upgrade. Construction of a pumped storage hydropower facility requires a new penstock, power station and a ternary unit, along with all electrotechnical components. A new dam is not necessary, as the upper reservoir is already dammed. The decision to invest in an upgrade can be made now or in the future, but once made it is irreversible. Costs related to the upgrade are not influenced by uncertain developments and need only be discounted by the risk-free rate. Both construction costs and plant valuation are considered fixed at the time an investment decision is made. The producer can discard maintenance costs for both the existing and the new facility.

The hydropower facility's production flexibility is limited by both physical constraints and regulations imposed by governmental agencies. There are upper and lower water levels allowed in the upper reservoir. The hourly production is bounded by a maximum production capacity. Friction loss in the penstock and efficiencies of the pump and turbine can be considered fixed.

The decision to be made comprises two related choices, the pumped storage hydropower facility's production capacity and the ideal timing of investment.

4 Modelling

This section presents the mathematical formulation and modelling decisions that form the foundation of our method. The chapter starts with the introduction of price and inflow models, which serve as input to the production valuation model. The approach uses a stochastic linear optimisation technique to assess production at various capacities. The first- and second-stage problem of the stochastic linear optimisation model is introduced, along with their respective parameters. Lastly, a cost function is defined and, together with the valuation function, incorporated into an optimal stopping problem adapted from the work of Dixit and Pindyck (1994).

4.1 Price modelling

A price modelling framework is needed as input to the optimisation program of the production scheduling problem. We apply the approach of Schwartz and Smith (2000) to build a two-factor electricity model. This framework requires a non-overlapping forward price curve of a fixed delivery-date commodity such as electricity. To form the price curve, we use historical price data from the period of 2017-2022. The data includes spot prices from the day-ahead market and futures contracts. The futures are traded as a speculative derivative of the Nordic system price, significantly more liquid than the contracts of the NO2 area price, where the hydropower plant in the case study is located. System spot prices are also utilised accordingly, obtained from Nord Pool (*Nord Pool* 2023). The futures contracts, provided by Montel (*Montel Online* 2023), have overlapping delivery periods ranging from one month to multiple years. By inserting futures and system spot prices into the method of Dietze et al. (2022), we extract estimated forward prices, with a delivery period of one day, that estimate the forward price curve.

The two-factor stochastic model of Schwartz and Smith (2000) is described by equations (1)-(3), where S_t denotes the price for time $t \in [0, \infty)$. As there are strong cyclic seasonal price fluctuations, trigonometric functions adding seasonal deviations are also included. ϕ_1 and ϕ_2 determine the magnitude of the seasonal effects, and θ define the number of periods in a cycle.

$$\ln S_t = \phi_1 \cos \frac{2\pi t}{\theta} + \phi_2 \sin \frac{2\pi t}{\theta} + \chi_t + \xi_t \tag{1}$$

$$d\chi_t = (-\kappa_\chi \chi_t - \lambda_\chi)dt + \sigma_x dz_\chi \tag{2}$$

$$d\xi_t = (\mu_\xi - \lambda_\xi - \frac{1}{2}\sigma_\xi^2)dt + \sigma_\xi dz_\xi \tag{3}$$

The stochastic factors χ_t and ξ_t in equations (1)-(3) drive the electricity price developments. The short-term factor in equation (2), χ_t, follows an Ornstein-Uhlenbeck process, also known as a mean-reverting process. The increments dz_χ and dz_ξ of the stochastic factors are correlated, given by $dz_\chi dz_\xi = \rho_{\chi\xi} dt$, and they correspond to increments of a Wiener process. The mean reversion speed of χ_t is denoted as κ_χ, while λ_χ represents the short-term risk premium, and σ_χ signifies the short-term volatility. The long-term price factor in equation (3), ξ_t, evolves as a geometric Brownian motion with drift. The long-term risk-adjusted drift of log-prices is given by $(\mu_\xi - \lambda_\xi - \frac{1}{2}\sigma_\xi^2)$, where μ_ξ represents the drift and λ_ξ is the risk adjustment term. The long-term volatility is denoted as σ_ξ.

As the estimated forward price data is discrete, the continuous-time representation of the two-factor model in (1)-(3) needs to be discretised, resulting in equations (4)-(6). In equation (5), ϵ_1 represents a standard Gaussian variable, whereas in equation (6), ϵ_2 is a Gaussian variable with a mean of ρ_{χ_ξ} and a variance of $1 - \rho_{\chi_\xi}^2$.

In the discrete model, we choose a weekly resolution of $\Delta t = \frac{1}{\theta} = \frac{1}{52}$ years, giving weekly changes in both the short- and long-term price factors, and thus also in price in week t, S_t. The seasonal factors of the model are found through sinusoidal regression of spot prices and removed from the forward prices. The remaining parameters are then estimated by Kalman filtering and maximum likelihood estimation with weekly time steps from the adjusted price data set (Goodwin 2013).

$$S_t = exp(\phi_1 \cos \frac{2\pi t}{\theta} + \phi_2 \sin \frac{2\pi t}{\theta} + \chi_t + \xi_t) \tag{4}$$

$$\xi_{t+1} = \xi_t + (\mu_\xi - \lambda_\xi - \frac{1}{2}\sigma_\xi^2)\Delta t + \sigma_\xi \epsilon_1 \sqrt{\Delta t} \tag{5}$$

$$\chi_{t+1} = \chi_t e^{-\kappa_\chi \Delta t} - \frac{\lambda_\chi}{\kappa_\chi}(1 - e^{-\kappa_\chi})\Delta t + \sigma_\chi \epsilon_2 \sqrt{\frac{1 - e^{-2\kappa_\chi \Delta t}}{2\kappa_\chi}} \tag{6}$$

With all parameters accounted for, we simulate price paths with weekly developments performing Monte Carlo simulations.

As the considered hydropower plant is located in the price area NO2, the simulated Nordic system prices need to be converted into NO2 prices. Electricity price area differentials (EPAD) are futures contracts which represent the market value of the price difference between the system price and the price of a specified area for a period in the future (Nasdaq 2023). By adding the differential, an estimation of the price level of NO2 relative to the simulated system prices is approximated.

The contract with the longest maturity listed on Montel (*Montel Online* 2023) is selected, with the assumption that an EPAD contract with a longer maturity provides a more accurate representation of future price differentials compared to a contract with a shorter maturity. This reasoning is based on the understanding that futures with shorter maturities are more susceptible to short-term market conditions, which can change rapidly. Contracts with longer maturities offer a more comprehensive view of the price difference, taking into account mostly long-term market conditions. This long-term perspective is crucial for a hydropower plant, which operates over multiple decades. This approach aligns with the presumption of Schwartz and Smith (2000) where long-maturity futures contracts provide the most reliable information regarding changes in the long-term equilibrium price.

The price process introduced in equations (4)-(6) provides electricity prices on a weekly basis. To facilitate the operational perspective of variable electricity production and consumption from the turbine and pump with hourly changes, intra-weekly variations must be accounted for. Similarly to Kleiven et al. (2022), we consider weekly profiles of hourly log price deviations from the weekly mean, focusing on the area of NO2. Instead of averaging the weekly profiles, sampling of historical profiles is instead implemented as a novel approach. The selected profiles are incorporated into each weekly price simulation. This is to preserve both low and high-volatility weeks, where especially the latter is assumed to be of high importance when both a turbine and pump are available. The sampling of historical profiles presents an alternative to Muche (2009), where a stochastic jump component is included in the price modelling to create price spikes.

4.2 Inflow modelling

The method in Helseth et al. (2017) utilises historical weather data to create scenarios. Our specific focus is on historical inflow data for a single hydropower plant. This data consists of daily records spanning several decades near the plant's geographical location. To simplify the analysis, we allocate the daily inflow uniformly across each hour of the day, assuming that hourly differences have minimal impact on hydropower plants with large reservoirs. We define an *inflow year* as one year's worth of inflow data. To ensure consistency, we scale the average sum of the inflow years to match the average yearly inflow of the reservoir for the hydropower plant. These measured inflow years can then be treated as inflow scenarios, representing a set of possible future developments of inflow. However, instead of considering every inflow year as a potential future realisation of inflow, we employ a fast-forward scenario reduction technique proposed by Heitsch and Römisch (2003) specifically for inflow years. This algorithm allows us to reduce the number of inflow scenarios while providing probabilities for each scenario based on similarities between inflow years. By doing so, we effectively simplify the stochastic problem and make it computationally feasible given the available resources, without compromising the accuracy of the optimisation results. Yearly inflow realisations are denoted λ_t.

4.3 Stochastic linear program

The goal of the optimisation model is to maximise the value of the pumped storage hydropower facility by scheduling production and pumping to optimal time periods. To facilitate this we denote a scheduling horizon of one year, termed the planning horizon, which is separated into weekly time steps $i \in \mathcal{I} = \{0, \ldots, I-1\}$. These decision periods allow for a separation of the original problem into smaller segments where exogenous information can be assumed deterministic. For each time step, we create a scheduling problem formulated as a stochastic linear program, labelled the intrinsic problem. The intrinsic problem spans two different decision periods, namely the central section and the forecast section. The central section spans one week i of the planning horizon, while the forecast section is defined as weeks $w \in \mathcal{W} = \{i+1, \ldots, W\}$, where W is the last week of the forecast section, with $W \geq I$. Production is scheduled of the entire forecast section to avoid overemphasising the amount of production in the central section. The solution to the intrinsic problem provides the optimal production schedules for both the central section and forecast section, dependent on the initial water level and the exogenous factors of price and inflow. In the central section, electricity prices and inflow are realised, meaning the model can treat both as deterministic. Conversely, scenarios of price and inflow, $s \in \{1, \ldots, N\}$, dependent on the realisations, are used in the forecast section. This is illustrated in Figure 1. N corresponds to the number of unique scenarios.

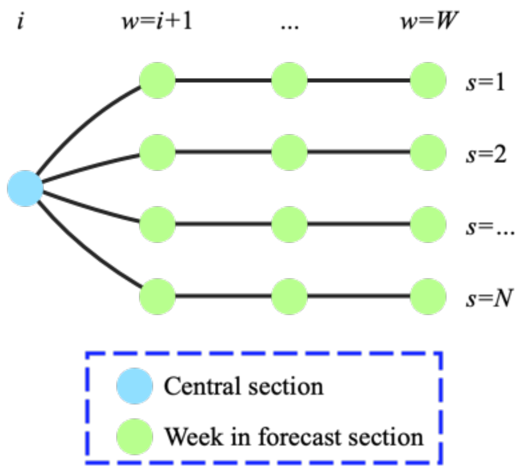


Figure 1: The intrinsic problem. The solid lines communicate time coupling between weeks.

The parameters and variables relevant to the central section of the intrinsic problem, are shown in Figure 2. Price S and inflow λ are realisations of the price process introduced in Section 4.1 and the historical inflow of Section 4.2. The subvectors discharge, $\mathbf{x}^{dis}$, pumping, $\mathbf{x}^{pump}$, reservoir level, $\mathbf{x}^{res}$ and spillage, $\mathbf{x}^{spill}$, represent the decisions variables that are made within the week. We denote the decision variable vector, that contains all decisions made for the first week

$$\mathbf{x} = (\mathbf{x}^{res}, \mathbf{x}^{dis}, \mathbf{x}^{spill}, \mathbf{x}^{pump}) \tag{7}$$

The upper reservoir has a maximum capacity denoted by R, and the ternary production capacity is represented by P. The energy coefficients E^{dis} and E^{pump} are conversion rates from discharged and pumped water volumes to corresponding amounts of electricity.

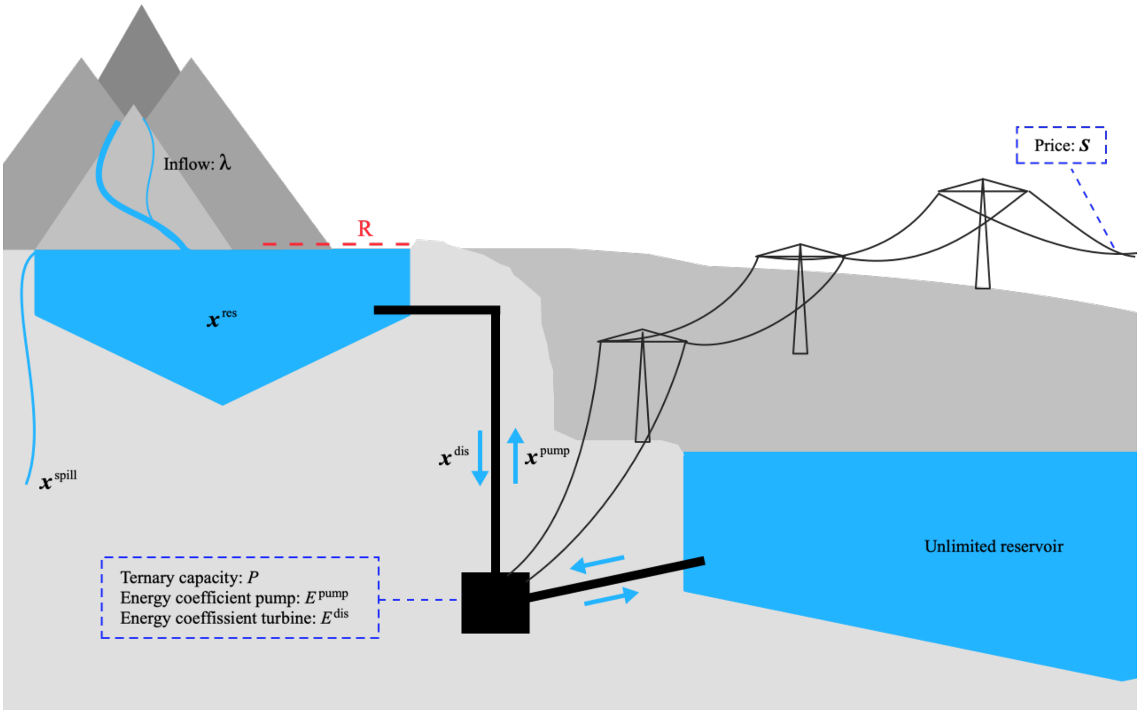


Figure 2: Schematic view of power plant with decision variables $\mathbf{x}$ and some parameters.

The intrinsic problem of week i is formulated as a stochastic linear program in (8)-(10). The first term of Equation (8) spans the central section, while the second term covers the forecast section. To distinguish between decision vectors for the different periods, the decisions of the central section are denoted by $\mathbf{x}_i$, and by $\mathbf{y}_{sw}$ for the forecast section. The cost vectors $\mathbf{c}_i$ and $\mathbf{c}_{sw}$ contain various components, including profit parameters and conversion rates from water to electricity for the intrinsic problem. Each scenario of the second-stage problem, s, has a weighted probability p_s, dependent on the scenario reduction of the historical inflow explained in Section 4.2.

$$Z_i = \max_{(\mathbf{x}_i)} \mathbf{c}_i \mathbf{x}_i + \sum_{s=1}^{N} p_s \sum_{w=i+1}^{W} \mathbf{c}_{sw} \mathbf{y}_{sw} \tag{8}$$

s.t.

$$\mathbf{x}_i \in \mathcal{X}_i \qquad \forall i \in \mathcal{I} \tag{9}$$

$$\mathbf{y}_{sw} \in \mathcal{Y}_{sw} \qquad \forall s \in \{1, \ldots, N\}, \forall w \in \mathcal{W} \tag{10}$$

The constraints (9) and (10) of the intrinsic problem define the feasible solution space for all decision vectors.

4.3.1 Decomposition

To make solving the intrinsic problem of week i computationally feasible, we decompose it into a first-stage and a second-stage problem, using Benders' decomposition (Benders 1962). The first-stage problem, formulated in (11)-(13), aims to maximise the profit of the central section, taking into account approximate future profits of the second-stage problem, denoted $\hat{\alpha}_{i+1}$. The approximate future profit is constrained from above by cuts, $cut \in \mathcal{C}_i = \{1, \dots, C_i\}$. As this is a maximisation problem, an upper bound on the optimal value of the future profit is established. The cuts are created by the solutions of the second-stage problem. C_i is the number of cuts made in week i. Cuts are generated by iteratively solving first- and second-stage problems.

$$Z_i^1 = \max_{(\mathbf{x}_i)} \mathbf{c}_i \mathbf{x}_i + \hat{\alpha}_{i+1} \tag{11}$$

s.t.

$$\mathbf{x}_i \in \mathcal{X}_i \qquad \forall i \in \mathcal{I} \tag{12}$$

$$\hat{\alpha}_{i+1} - \pi_{cut} \mathbf{x}_i \leq \beta_{cut} \qquad \forall cut \in \mathcal{C}_i \tag{13}$$

The second-stage problem consists of N independent subproblems. A subproblem solves the scheduling problem over the forecast section for a specific scenario. The end reservoir level of the first-stage problem resulting from the first-stage decisions, $\mathbf{x}_i^*$, sets the initial water level of all the subproblems in the second-stage. The objective of the subproblem (14) is similar to the second term of (8), but for a single scenario s.

$$Z_{s,i+1}^2(\mathbf{x}_i^*) = \max_{(\mathbf{y}_{sw})} \sum_{w=i+1}^{W} \mathbf{c}_{sw} \mathbf{y}_{sw} \tag{14}$$

s.t.

$$\mathbf{y}_{sw} \in \mathcal{Y}_{sw}(\mathbf{x}_i^*) \qquad (\pi_s) \quad \forall w \in \mathcal{W} \tag{15}$$

The dual value π_s, to (15), associated with the initial water level of the subproblem, is utilised for generating cuts in (16). The right-hand side parameter of (13) is calculated in (17).

$$\pi_{cut} = \sum_{s \in \mathcal{S}} p_s \pi_s \tag{16}$$

$$\beta_{cut} = \sum_{s \in \mathcal{S}} p_s (Z_{s,i+1}^2 - \pi_s \mathbf{y}_{si}) \tag{17}$$

The iterative process of Benders' decomposition is illustrated in Figure 3. For each iteration, a trial decision of the first-stage problem is found, giving an end-of-week water level, l_{i+1}^*. This is utilised as input in the second-stage problem. Next, the second-stage problem is solved providing cuts that constrain the future profit variable of the first-stage problem, which is solved again with the new information. This process is repeated until convergence.

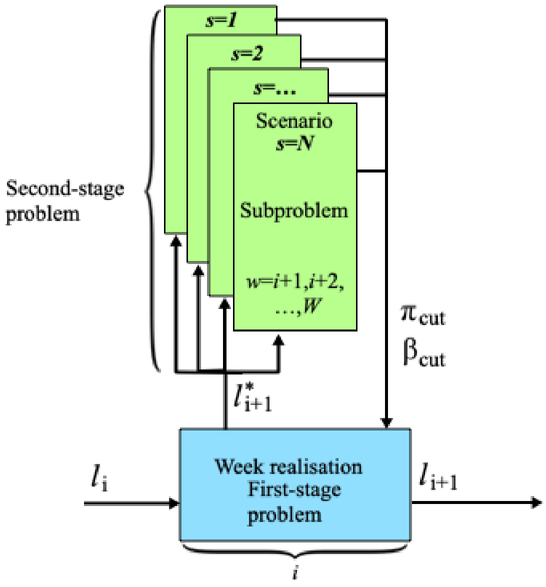


Figure 3: First-stage problem i and the N scenarios of second-stage problem. The second-stage problem exists to provide the valuation of the end water level of the first-stage problem.

$$\underline{Z}_i = c_i x_i^* + \sum_{s \in \mathcal{S}} p_s Z^2_{s,i+1}(\mathbf{x}_i^*) \tag{18}$$

$$Z_i^1 - \underline{Z}_i < \epsilon \tag{19}$$

Convergence occurs when the solution to the first-stage problem closely approximates the value of (18). The first-stage solution provides an upper bound to the two-stage problem by incorporating an optimistic approximation of future profits. (18) represents the lower bound of the problem as it combines a feasible decision set of the second-stage problem with the determined production schedules of the first-stage problem. The solution is considered optimal when the difference between the upper and lower bounds is smaller than a predefined threshold value, ϵ, as shown in (19). Upon convergence, the solution for $\mathbf{x}_i$ is stored, and the initial water level for the next week is set to the last trial value, as illustrated in (20).

$$l_{i+1} = l^*_{i+1} \tag{20}$$

4.3.2 Detailed first- and second-stage problems

The two-stage problem introduced in the last section is a condensed version of the intrinsic problem, explained in more detail here, both for the objective functions and their constraints. Each intrinsic problem of the planning horizon is solved separately, therefore we omit the index i from the decision variables in this section. The first- and second-stage problems are divided into J hour blocks to account for intra-weekly price deviations. The objective function of the first-stage problem of week i, in (21), maximises the value of operational decisions in the central section while considering approximate future profits, based on leftover water. The first-stage problem receives inflow and price realisations $\boldsymbol{S}_i$ and $\boldsymbol{\lambda}_i$, and a water level as input. Water discharged or pumped is converted into produced or consumed power by the energy coefficients, E^{dis} and E^{pump}. The objective value is discounted by a weekly discount rate, γ_2^i.

$$Z_i^1(l_i, \mathbf{S}_i, \boldsymbol{\lambda}_i) = \max_{(\mathbf{x})} \sum_{j \in \mathcal{J}} \gamma_2^i \mathbf{S}_j (E^{dis} x_j^{dis} - E^{pump} x_j^{pump}) + \hat{\alpha}_{i+1} \tag{21}$$

s.t.

$$x_0^{res} = l_i \tag{22}$$

Constraint (22) enforces that the reservoir level at the start of each week is equal to the specified initial reservoir level for that week.

$$x_{j+1}^{res} = x_j^{res} - x_j^{dis} - x_j^{spill} + x_j^{pump} + \lambda_j \qquad \forall j \in \mathcal{J} \setminus \{J-1\} \tag{23}$$

$$l_{i+1} = x_j^{res} - x_j^{dis} - x_j^{spill} + x_j^{pump} + \lambda_j \qquad j = J-1 \tag{24}$$

The reservoir level for the next hour-block is set as a result of all decisions affecting water balance in (23). l_{i+1} in (24) is an auxiliary variable, which represents the end-of-week water level.

$$0 \leq x_j^{dis} \leq |j| \frac{P}{E^{dis}} \qquad \forall j \in \mathcal{J} \tag{25}$$

$$0 \leq x_j^{pump} \leq |j| \frac{P}{E^{pump}} \qquad \forall j \in \mathcal{J} \tag{26}$$

Both (25) and (26) ensure that we do not discharge or pump more water than capacity P allows, where $|j|$ is the length of each hour block. The decision variables are non-negative.

$$x_j^{res} \leq R \qquad \forall j \in \mathcal{J} \tag{27}$$

$$x_j^{dis} \leq x_j^{res} + \lambda_j \qquad \forall j \in \mathcal{J} \tag{28}$$

$$x_j^{spill}, x_j^{res} \geq 0 \qquad \forall j \in \mathcal{J} \tag{29}$$

(27)-(28) restricts the reservoir level to its upper limit and discharge to never exceed the total available water in the reservoir. (29) adds a non-negativity constraint for the reservoir and spillage decision variables.

$$\hat{\alpha}_{i+1} \leq \beta_{cut} + \pi_{cut} l_{i+1} \qquad \forall cut \in \mathcal{C}_i \tag{30}$$

The future profit function, in (30), is constrained by the cut contributions from the second-stage solution.

We solve the second-stage problem for a set of price and inflow forecasts, $\mathbf{F}$ and $\boldsymbol{\Lambda}$ respectively, over the forecast section, $\mathcal{W}^i$. The forecast section starts at week $i+1$ and ends at week W, $\mathcal{W}^i = \{i+1, \ldots, W\}$. We implement dynamic hour-block sizing in the forecast section. As a result, the set $\mathcal{J}$ varies in length for different weeks of $\mathcal{W}^i$, $\mathcal{J}_w^i = \{0, \ldots, J_w^i\}, \forall w \in \mathcal{W}^i$. The hour block size progressively increases as we move deeper into the forecast section, starting from week i.

The objective function of each subproblem in the second-stage is quite similar to the objective of the first-stage problem. However, the approximate future profit is replaced with an end-of-horizon water value function, $\mathcal{A}(y_{end}^{res})$, valuing leftover water after the end of the finite forecast section. Most constraints in the subproblems function similarly to the ones in the first-stage problem.

$$Z_{s,i+1}^2 = \max_{(\mathbf{y})} \sum_{w \in \mathcal{W}^i} \sum_{j \in \mathcal{J}_w^i} \gamma_2^w F_{swj} * (E^{dis} y_{wj}^{dis} - E^{pump} y_{wj}^{pump}) + \gamma_2^W \mathcal{A}(y_{end}^{res}) \tag{31}$$

s.t.

$$y^{res}_{0,0} = l_{i+1} \qquad (\pi_s) \qquad (32)$$

$$y^{res}_{w,j+1} = y^{res}_{wj} - y^{dis}_{wj} - y^{spill}_{wj} + y^{pump}_{wj} + \Lambda_{swj} \qquad \forall w \in \mathcal{W}^i, j \in \mathcal{J}^i_w \setminus \{J^i_w - 1\} \qquad (33)$$

$$y^{res}_{w+1,0} = y^{res}_{wj} - y^{dis}_{wj} - y^{spill}_{wj} + y^{pump}_{wj} + \Lambda_{swj} \qquad \forall w \in \mathcal{W}^i, j = J^i_w - 1 \qquad (34)$$

$$y^{res}_{end} = y^{res}_{wj} - y^{dis}_{wj} - y^{spill}_{wj} + y^{pump}_{wj} + \Lambda_{swj} \qquad \forall w = W, j = J^i_w - 1 \qquad (35)$$

$$0 \leq y^{dis}_{wj} \leq |j| \frac{P}{E^{dis}} \qquad \forall w \in \mathcal{W}^i, j \in \mathcal{J}^i_w \qquad (36)$$

$$0 \leq y^{pump}_{wj} \leq |j| \frac{P}{E^{pump}} \qquad \forall w \in \mathcal{W}^i, j \in \mathcal{J}^i_w \qquad (37)$$

$$y^{res}_{wj} \leq R \qquad \forall w \in \mathcal{W}^i, j \in \mathcal{J}^i_w \qquad (38)$$

$$y^{dis}_{wj} \leq y^{res}_{wj} + \Lambda_{swj} \qquad \forall w \in \mathcal{W}^i, j \in \mathcal{J}^i_w \qquad (39)$$

$$y^{spill}_{wj}, y^{res}_{wj} \geq 0 \qquad \forall w \in \mathcal{W}^i, j \in \mathcal{J}^i_w \qquad (40)$$

Constraint (32) sets the initial reservoir level. The constraint (33) ensures water balance within each week, while (35) enforces water balance between weeks. (36) and (37) ensures that discharging and pumping are non-negative and within the limits of the capacity, P. (38) sets the maximum reservoir level. (39) constrains the water available for production, and (40) ensures non-negativity in the decision variables of spillage and reservoir level.

The optimal solutions to the subproblems of intrinsic problem i contribute to parameters of the cuts in (30), as described in (41) and (42).

$$\pi = \sum_{s=1}^{N} p_s \pi_s \qquad (41)$$

$$\beta = \sum_{s=1}^{N} p_s (Z^2_{s,i+1} - l_{i+1}\pi_s) \qquad (42)$$

4.4 The investment decision

The optimal timing of investment into the pumped storage hydropower facility and its capacity can be found by solving an optimal stopping problem. The timing and capacity solutions are largely dependent on the value of the existing hydropower plant, the value of the new facility and its associated investment costs. Next, we show how these factors of the investment decision process are formulated.

4.4.1 Optimal stopping

Optimal stopping is defined as the maximisation of an expected reward by choosing the timing of an action, such as the decision to invest in a pumped storage hydropower facility. In our case, a capacity decision must also be made. Alongside the value of the existing hydropower plant and the new pumped storage hydropower facility, we define a capacity-dependent cost function, $K(P)$, to formulate the optimal stopping problem.

In the existing hydropower plant, the capacity is denoted P_0. The capacity of the pumped storage hydropower project is independent of this but assumed to be no less than P_0 as of the increased flexibility from the pump. We denote the capacity of the pumped storage hydropower facility as $P \in [P_0, \overline{P}]$ measured in MW, where $\overline{P}$ is a hypothetical maximum capacity. The investment can occur at the start of each year $t \in [0, \dots, \infty)$, with the timing denoted $\tau \in [0, \dots, \infty)$. The capacity of the hydropower plant in time t is thus

$$P_t(\tau, P) = \begin{cases} P_0, & \text{for } t < \tau \\ P, & \text{for } t \geq \tau \end{cases} \tag{43}$$

The value of one year of hydropower production is denoted $V_t[\boldsymbol{\omega}_t, P_t(\tau, P)]$ in year t. The state vector $\boldsymbol{\omega}_t = (\chi_t, \xi_t, \lambda_t)$ represents the exogenous variables in year t, the short-term- and long-term price factor, and inflow. The optimal stopping problem can then be formulated as shown in (44). The solution to the problem is conditional on the exogenous variables in $\boldsymbol{\omega}_t$ and investment cost. A possibility of indefinitely postponing investment exists, meaning that the current production can continue in perpetuity.

$$D(\boldsymbol{\omega}_t) = \max_{\tau, P} \mathbb{E}\left[\int_0^\infty \gamma_1^t V_t[\boldsymbol{\omega}_t, P_t(\tau, P)] \, dt - \gamma_1^\tau K(P) \right] \tag{44}$$

The production value and investment cost are discounted using the rate of $\gamma_1 = e^{-r}$, where r is the risk-free rate. We assume the risk-free rate to be an appropriate discount factor as the price parameters in (1)-(3) are calibrated using historical futures contracts, which already are discounted for the risk of their asset (McDonald 2003).

5 Solution method

This section begins with an explanation of how the two-stage stochastic optimisation model is used to determine the annual production valuations for different capacity options. This includes a discussion of the input data required by the model, such as price and inflow, and what information is extracted. The final part of the solution is solving the optimal stopping problem of Section 4.4, in which we incorporate the yearly valuations to determine the ideal timing for investment and capacity for the pumped storage hydropower facility. The entire solution method is summarised in Figure 4.

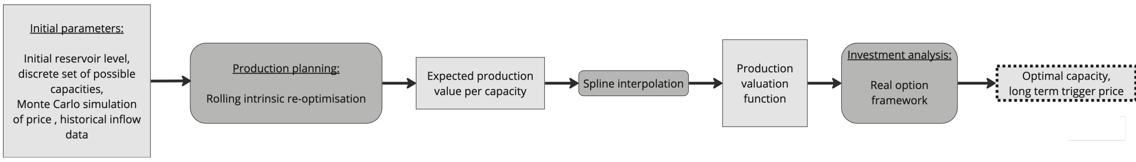


Figure 4: Solution method overview

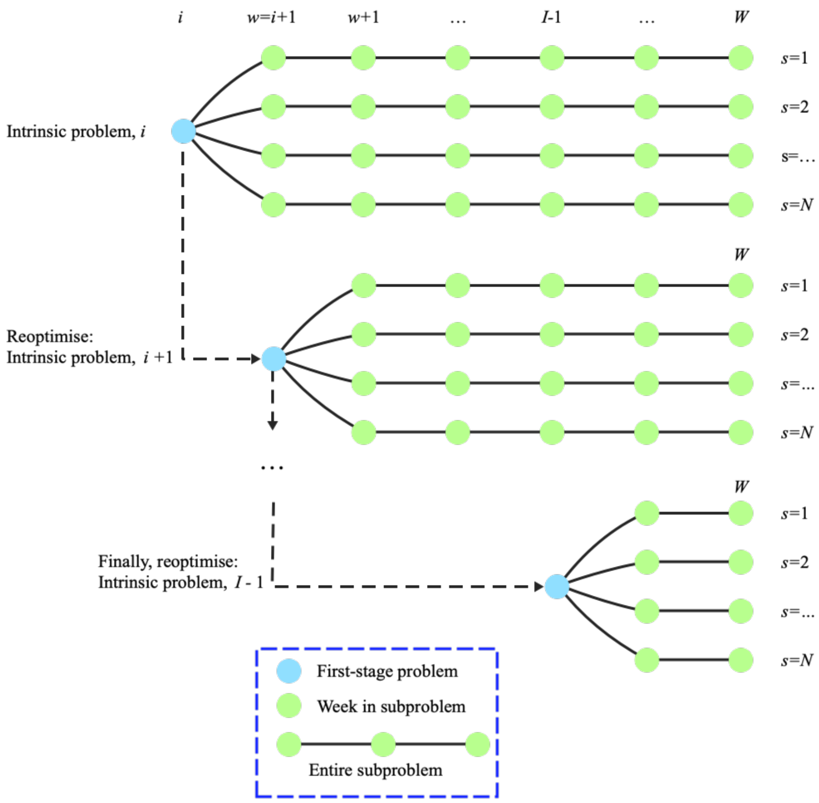


Figure 5: The rolling intrinsic methodology from an arbitrary week i. Only the first-stage solution is stored for each iteration of the rolling intrinsic.

5.1 Production planning

The pumped storage hydropower facility's evaluation of different capacities involves solving I intrinsic problems, one for each time step in the planning horizon. In Section 4.3, we explain in detail the solution method for a single intrinsic problem, which provides optimal operation schedules for both the central section and forecast section. However, only the schedule of the central

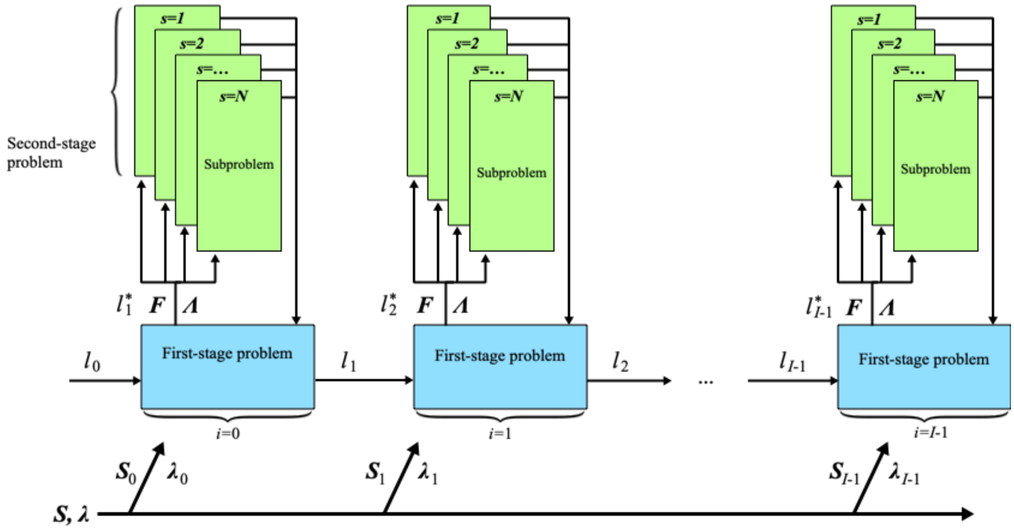


Figure 6: A different perspective of the rolling horizon. Exogenous factors are revealed each week to the first-stage problem, and based on those, forecast scenarios are generated for the second-stage problem.

section is utilised for the facility valuation. This approach follows the rolling intrinsic methodology, where each intrinsic problem's solution is reoptimised based on the previous problem's results and new price and inflow information.

To implement the rolling intrinsic method, we start with the first intrinsic problem and find a solution that provides operational schedules for the central section ($i = 0$) and the forecast section from week 1 to week W. The schedule for the central section is then fixed and stored as part of the final solution. Then, the rolling intrinsic method moves to the next intrinsic problem. In the subsequent iteration, there is a reoptmisation of the schedules from the solution to the first iteration. Now, the central section's schedule spans week $i = 1$, and the forecast section covers week 2 to W. Figure 5 illustrates the general procedure of the rolling intrinsic method from week i. After I iterations, each with a new week in the central section, we obtain a planning schedule for every hour of the planning horizon.

A critical aspect of the rolling intrinsic method is how we determine the initial reservoir level for each intrinsic problem. We achieve time-coupling between intrinsic problems by using the end water level to the first-stage problem solution in the previous iteration as the initial reservoir level. For the first week of the planning horizon (i.e., the first intrinsic problem with $i = 0$), the water level is initialised to what we refer to as the steady-state water level, denoted l_0.

Another central aspect of the rolling intrinsic methodology is the realisation of the exogenous variables price and inflow for each intrinsic problem. These factors are described in Sections 4.1 and 4.2 and are used deterministically in the first-stage problem. The forecast scenarios provided for each subproblem in the second-stage problem are dependent on these realisations. This is illustrated in Figure 6.

The realisation vector of electricity prices of week i, $\mathbf{S}_i$, is given from

$$\boldsymbol{S}_i = (S_i + EPAD) * \exp(\alpha_j) \qquad \forall j \in \mathcal{J} \tag{45}$$

In the equation above, S_i represents the price of week i simulated from Equation (4), as a continuation of the realisation of week $i - 1$. EPAD is the futures contract of the price differential between the system price and NO2-prices, and α_j denotes the hourly log-price deviation from the weekly mean of a sampled historical week for hour block j. The same historical week is used for all hourly price deviations within the week, which constitute a weekly profile. The realisation of inflow, $\boldsymbol{\lambda}_i$, follows a predefined historical inflow year, the same year as the realisation $\boldsymbol{\lambda}_{i-1}$. The realisation consists of seven days of inflow records, where the records correspond to the same week as i. Inflow is uniformly distributed within each day, for all hour blocks j.

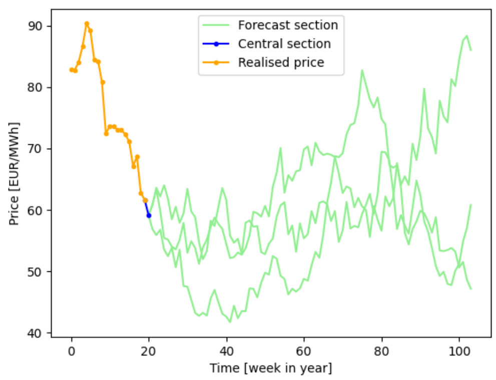
New forecasts of the exogenous variables are generated for each price and inflow realisation. The price forecast for every week $w \in \mathcal{W}^i = \{i+1, \ldots, W\}$ of the forecast period is dependent on the newly realised values of the short-term- and long-term price factors of week i. Utilising (4)-(6), we generate N distinct price scenario paths, one for each scenario of the forecast section, to obtain $F_{sw}(\xi_i, \chi_i), \forall s \in \{1, \ldots, N\}, \forall w \in \mathcal{W}^i$. Figure 7 illustrates the scenario generation at two time steps, $i = 20$ and $i = 40$, for one simulation. Further, we add the EPAD contract and the expectation of all available weekly profiles, $\mathbb{E}(\alpha_j), \forall j \in \mathcal{J}$. The price forecast vector of week w can thus be written as

$$\boldsymbol{F}_{sw} = (F_{sw}(\xi_i, \chi_i) + EPAD) * \exp(\overline{\alpha}_j) \qquad \forall j \in \mathcal{J}, \forall s \in \{1, \ldots, N\}, \forall w \in \mathcal{W}^i \tag{46}$$

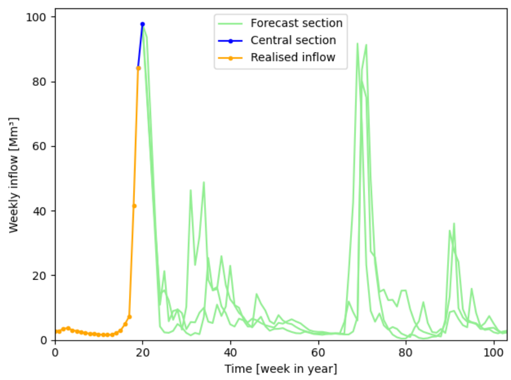
where $\overline{a}_j$ denotes the average log-price deviation for hour block j. The average weekly profile is plotted in Figure 11. The inflow forecasts are based on the new inflow realisation, $\boldsymbol{\lambda}_i$, and the other scenario-reduced inflow years. Section 4.2 explains our approach to managing the uncertainty of future inflow by utilising the historical inflow years as scenarios. These inflow years exhibit somewhat similar patterns across different seasons, but they also demonstrate significant variations for the same week in different years. While rainfall changes rapidly, we have observed a notable serial correlation in inflow within our available data set. Consequently, we employ a method called smoothing, outlined in Helseth et al. (2017), to condition the forecast inflow on the past realisation. This approach helps maintain the temporal correlation between realisations and future scenarios. After each new weekly inflow realisation, we apply smoothing to the future inflow scenarios to obtain inflow forecasts $\boldsymbol{\Lambda}_{sw}$.

Along with the scenario reduction performed on the historical inflow records to limit the number of future scenarios, the use of dynamic sizing of hour blocks reduces the computational complexity of our problem. Having fewer time steps reduces the number of decisions the model has to make. In general, operational decisions in the near future has more influence on cuts than decisions taken in the distant future (Helseth et al. 2018). This enables a less detailed time granularity as we move further away from the present. In the central section, where price, $\boldsymbol{S}_i$, and inflow, $\boldsymbol{\lambda}_i$, are realised, every hour is represented by a one-hour block. Through the weeks of the forecast section, the size of each block gradually changes from a three-hour block to an eight-hour block. The block size is increased after week i, meaning that dynamic block-sizing only affects the weeks in the forecast section. Aggregating the hourly inflows of a scenario to larger hour blocks is not expected to have a substantial impact on the operation, as hourly variations in inflow should have minimal effect on the operation of plants with large reservoirs. When considering forecast prices, $\boldsymbol{F}_{sw}$, it is important to note that the prices are averaged over the entire hour block. This approach reduces the ability to leverage intra-weekly price volatility, which is assumed to be particularly important in the utilisation of a pump. The ability diminishes progressively deeper into the forecast horizon, with fewer hour blocks available. Hence, the incorporation of dynamic block-sizing decreases the complexity of the problem but sacrifices the opportunity to take full advantage of the price volatility.

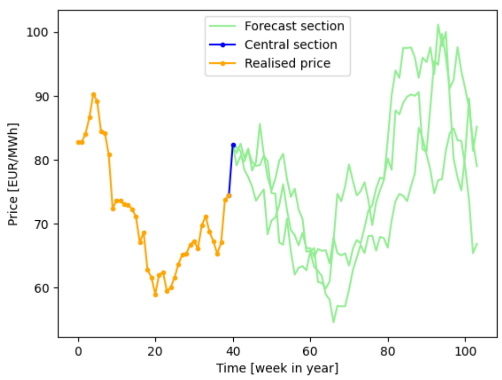
With realised prices and inflow and their forecast values used as input to the optimisation model of Section 4.3, we obtain Y different yearly production valuations for each capacity, P. Y represents the total of unique combinations of price and inflow realisations. The valuations are conditional on two crucial factors that need to be considered. First, we need to determine the *end-of-horizon*



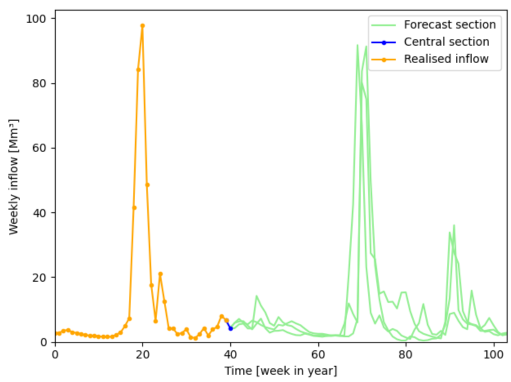
(a) Price realisation for week $i = 20$



(b) Inflow realisation for week $i = 20$



(c) Price realisation for week $i = 40$



(d) Inflow realisation for week $i = 40$

Figure 7: Realisations and forecasts of exogenous factors. The number of forecast scenarios graphed is set to three for the illustration.

value, which accounts for the valuation of leftover water after the forecast section of each intrinsic problem. Second, we must establish a *steady-state* water level to define the initial water level at the beginning of the first year. These additions are vital in obtaining representative yearly valuations that enable comparison between the different capacities.

To ensure a fair assessment of the value of water over time, it is important to avoid overemphasising the value of the first year of production compared to subsequent years. If we allocate the discharge of the entire reservoir and inflow to production within the central- and forecast section of the intrinsic problem, it can lead to an inflated production valuation for those years, leaving no value for the years following the forecast section, except for inflow. To address this issue, we need to assign a positive value to the water remaining in the reservoir after the forecast section, an end-of-horizon water value (Helseth et al. 2017). To compute this value, we create a water value function using the objective value and corresponding dual value obtained from running a subproblem with an extended forecast section starting from the end of our original forecast section. This process is carried out for various initial water levels and all capacities, P. By extending the period, we mitigate the bias towards maximising the value of the nearest years without considering the subsequent years within the forecast horizon. The end-of-horizon value is then used to evaluate the remaining water after the forecast period in our original problem, ensuring that the water consumption in the first year is not disproportionately high compared to the subsequent years. Extending the forecast section of our original problem presents an alternative solution to the requirement for an end-of-horizon value. However, it is crucial to acknowledge that this extension will significantly increase the computational workload associated with the problem.

The comparison of yearly production valuations for different capacities, P, also relies on finding steady-state water levels. After a year of production with capacity, P, we end up with an end water level. If this level is below the initial reservoir level, it indicates a positive net water discharge. However, repeating this process over multiple years becomes impractical as we run out of water. Thus, in order to obtain a representative valuation for a given capacity based on the first year of production, it is important to ensure that the initial water level and the average final reservoir level of different simulations are approximately equal. With representative valuations for each capacity, their magnitude can be compared.

After making these considerations, the expected yearly profit for capacity P is given as the average of the Y valuations in

$$V_P(\boldsymbol{\omega}_0) = \frac{1}{Y} \sum_{n=1}^{Y} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \gamma_2^i S_{ij}^n y_{ij}^{nP} \tag{47}$$

with γ_2 as the weekly discount rate. We employ the method of *random common numbers* for the evaluation of the various capacities. This approach involves using the same price and inflow realisations and forecasts for each capacity, leading to positive correlations between their valuations. Consequently, the use of random common numbers reduces the number of simulations needed to make valid comparisons (Figueira and Almada-Lobo 2014).

5.2 Optimal stopping problem

The objective of the optimal stopping problem introduced in Section 4.4 is to determine the optimal timing for exercising the investment option, and the corresponding capacity. For this problem, a continuous function is required to estimate the valuation, $V_t[\boldsymbol{\omega}_t, P_t(\tau, P)]$ in year t, for all practical capacities, P. The evaluation of the stochastic linear program is limited to a discrete number of capacity alternatives. To address this limitation, we adopt spline interpolation on the values obtained from Equation (47), following the proposition by Kleiven et al. (2022) regarding a concave yearly valuation function for capacities ranging from P_0 to $\overline{P}$. This interpolation method adheres to the constraints of concavity and monotonic increase, as explained in the approach outlined by Pya and Wood (2014). The resulting valuation function $V_0(\boldsymbol{\omega}_0, P)$ serves as the input to the optimal stopping problem in Equation (44).

Under a risk-neutral expectation of futures price, Kleiven et al. (2022) show that the optimal solution of the weekly intrinsic problems is independent of the initial long-term price factor. The valuation function of the first year can thus be written as

$$V_0(\boldsymbol{\omega}_0, P) = \exp(\xi_0) G_0(\chi_0, \lambda_0, P) \tag{48}$$

The valuation function is thus only dependent on the short-term deviations in electricity price, χ_0, and the uncertainty of inflow, λ_0, besides capacity. These parameters are considered short-term factors. Schwartz and Smith (2000) find that the valuations of a generalisable long-term investment in oil properties are insensitive to short-term price deviations. This is due to a long productive lifetime and a construction lag. Kleiven et al. (2022) also prove that the potential approximation error of the valuation function depending on short-term price deviation is extremely small. Further, it is assumed that the same argument is valid for inflow deviation, thus concluding that short-term factors do not affect the strategic decision of upgrading a hydropower plant. Similarly, considering the project's long production lifetime, we assume that the production valuation function of the pumped storage hydropower project is independent of short-term factors, which can then be approximated as

$$V_t[\boldsymbol{\omega}_t, P_t(\tau, P)] \approx \exp(\xi_t) G_t[P_t(\tau, P)] \tag{49}$$

where $G_t(P_t(\tau, P))$ is continuous, concave, monotonically increasing, similarly to $V_t(\cdot)$, and also independent of the long-term price level. The optimal stopping problem of (44) can then be reformulated to

$$D(\xi_t) = \max_{\tau,\ P} \mathbb{E}\left[\int_0^\infty \gamma_1^t \exp(\xi_t) G_t[P_t(\tau, P)]\, dt - \gamma_1^\tau K(P) \right], \tag{50}$$

The optimal stopping problem is now feasible to solve to obtain the price trigger and optimal capacity to maximise the real options value of the new project by following the approach of Dangl (1999). Consider a case where the long-term price is above the trigger price, $\xi \geq \xi^*$. In such a scenario, the optimal course of action would be to "stop" and promptly invest in the new project. The value of the real option approximates thus to the perpetual net present value (NPV) of the hydropower plant, as of an extensive production lifespan, with the new capacity, P, minus the investment cost, $K(P)$, given in

$$NPV(\xi, P) = \frac{\exp(\xi) G(P)}{\rho} - K(P) \tag{51}$$

The discount factor, $\rho = r - (\mu_\xi - \lambda_\xi)$, denotes the risk-free rate subtracted by the expected growth rate of the project, i.e. the risk-adjusted drift of the long-term price.

When the long-term price is lower than the trigger price, $\xi < \xi^*$, it is not optimal to invest. Instead, it is advantageous to continue with the existing production and wait to exercise the option until the long-term price reaches the trigger level. To determine the continuation value of the optimal stopping problem, we utilise the framework introduced by Dixit and Pindyck (1994) for evaluating a project without operating costs. By applying this framework to the optimal stopping problem, we derive the Bellman equation

$$\frac{1}{2}\sigma_\xi^2[\exp(\xi)G(P_0)]^2 D''(\xi) + (\mu_\xi - \lambda_\xi)[\exp(\xi)G(P_0)]D'(\xi) - rD(\xi) + \exp(\xi)G(P_0) = 0 \tag{52}$$

The solution to the differential equation (52) with its general and particular solutions is on the form

$$D(\xi) = C_1[\exp(\xi)G(P_0)]^{\beta_1} + C_2[\exp(\xi)G(P_0)]^{\beta_2} + \frac{\exp(\xi)G(P_0)}{\rho} \tag{53}$$

where the last term may be seen as the perpetual value of production with the current hydropower plant, while the two other terms indicate the additional value of waiting to obtain more information. β_1 and β_2 represent the positive and negative roots, respectively, of the fundamental quadratic equation

$$\frac{1}{2}\sigma_\xi^2 \beta(\beta - 1) + (\mu_\xi - \lambda_\xi)\beta - r = 0. \tag{54}$$

In order to prevent the option value from becoming infinitely large for values of $\exp(\xi)$ close to zero when $\xi < \xi^*$, it is necessary to set $C_2 = 0$. This condition ensures that the option value remains negligible at low prices, consistent with the boundary condition $D(0) = 0$. C_1 is given by the smooth-pasting condition $D'(\xi^*) = NPV'(\xi^*, P)$ in (55).

$$C_1 = \frac{G(P) - G(P_0)}{\beta_1 \rho [\exp(\xi^*)]^{\beta_1 - 1}} \tag{55}$$

For the two different cases, $\xi \geq \xi^*$ and $\xi < \xi^*$, the option value of the project is thus

$$D(\xi) = \begin{cases} \frac{\exp(\xi)G(P)}{\rho} - K(P), & \text{for } \xi \geq \xi^* \\ \frac{\exp(\xi)G(P_0)}{\rho} + C_1[\exp(\xi)]^{\beta_1}, & \text{for } \xi < \xi^* \ . \end{cases} \tag{56}$$

Each case in (56) are defined to have the same value in $\xi = \xi^*$ by value-matching, $D(\xi^*) = NPV(\xi^*)$. This property allows us to solve for $\exp(\xi^*)$ to obtain (57), i.e. the corresponding trigger price to a specific production capacity.

$$\exp(\xi^*) = \frac{\beta_1}{\beta_1 - 1} \frac{K(P)\rho}{G(P) - G(P_0)} \tag{57}$$

Further, to maximise the NPV of the investment, the marginal value of production with capacity P must equal the marginal instalment cost of such capacity. We then obtain the equation:

$$\frac{\exp(\xi)}{\rho} \frac{G(P)}{\partial P}|_{P^*} = \frac{K(P)}{\partial P}|_{P^*} \tag{58}$$

With the two expressions in (57) and (58), we can implicitly find the optimal production capacity and price trigger. First, we make an initial capacity guess of the new project, $\hat{P}$. By inserting the guess into (57), we find the corresponding trigger price. Then (58) is evaluated, indicating whether the optimal capacity, P^*, is bigger or smaller than our guess. If the marginal instalment cost of capacity is bigger/smaller than the marginal value of production, the guess, $\hat{P}$, is too large/low. Through iterative updates of the capacity using progressively smaller steps until reaching satisfactory convergence, we can identify a near-optimal price trigger, ξ^*, and its corresponding capacity P^*.

6 Base Case and Results

In this section, we present the results of our study that revolves around a two-stage stochastic optimization model, leading to the real options valuation through the optimal stopping problem. The primary focus of our analysis is to determine the optimal timing and capacity for investing in a pumped storage hydropower facility. We begin by providing an overview of the central parameters utilised as inputs to both the optimisation model and the optimal stopping problem. These parameters form the foundation of our decision-making process. Subsequently, we delve into the examination of various aspects, including production patterns and valuations, which are central to a comprehensive analysis. Finally, we explore the sensitivity of our solutions, discussing how changes in key variables and parameters impact the outcomes.

6.1 The case

To provide context for our results it is relevant to explain our base case setting and parameters. Figure 8 depicts the location of our facility's penstock, while Table 1 lists the values of the parameters relevant to the case. Reservoir capacity, average annual inflow and the initial production capacity of the existing facility are all publicly available information. The remaining parameters needed to provide the production scheduler and optimal stopping problem with proper input is explained in this section. To adjust for design considerations and simplifications we set penstock length L to 23500 meters.

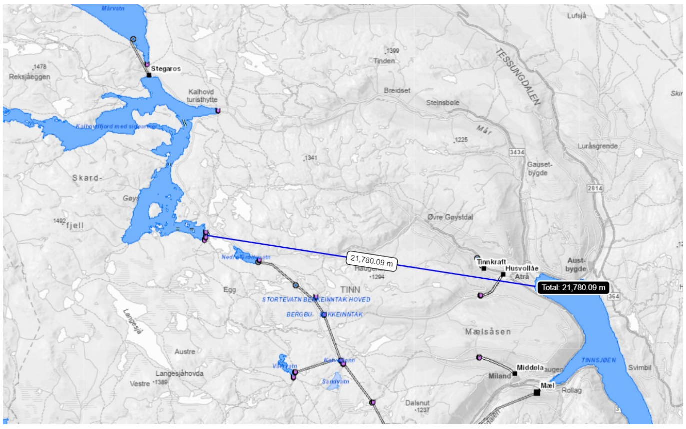


Figure 8: Simple visualisation of the penstock location between the two reservoirs

6.1.1 Energy coefficients

The generation of electricity in a hydropower facility depends on its design. The facility has a specific conversion rate, a certain amount of water flowing through the turbine will produce a corresponding amount of electricity. This conversion rate is commonly denoted e or E in literature, we use the latter in this thesis. E is dependent on the net head and the efficiency of the turbine, electric generator, and transformer when producing electricity. When pumping, the turbine and the generator are switched to a pump and a motor. As such we get E_{dis} and E_{pump} for water discharged and water pumped respectively. To calculate the energy coefficients we must first calculate the net head of the penstock, ΔH, and then use the formulas found in Antal (2014):

$$P_{prod} = E_{dis} \times Q = \rho g \gamma_{dis} \Delta H \eta_t \times Q \tag{59}$$

Table 1: Facility parameter values

Parameter description	Symbol	Value	Unit
Reservoir capacity	R	595	Mm^3
Average annual inflow		560	Mm^3
Initial production capacity	P_0	180	MW
Weekly inflow of inflow years	λ_t	Figure 9	Mm^3
Initial reservoir level	l_0	Figure 10	Mm^3
Average intra-weekly hourly log price deviation	$\overline{\alpha}_j$	Figure 11	€/MWh (log)
Water velocity	v	1.7	m/s
Net head	ΔH	874	m
Energy coefficient production	E_{dis}	1.911	kWh/m^3
Energy coefficient pumping	E_{pump}	2.120	kWh/m^3
Total penstock length	L	23500	m
Transformer efficiency	η_t	0.95	
Electric generator efficiency	η_g	0.95	
Motor efficiency	η_m	0.90	
Pelton turbine efficiency	η_{pt}	0.91	
Francis turbine efficiency	η_{ft}	0.95	

$$P_{pump} = E_{pump} \times Q = \frac{\rho g \gamma_{pump} \Delta H}{\eta_p} \times Q \tag{60}$$

The above equations describe the relationship between the power produced and water discharged, where P is power and Q is water flow in the penstock. The variables ρ, g, η and γ are water density, gravity, turbine-/pump-efficiency and the the product of generator-/motor- and transformer-efficiency, respectively. The efficiencies in Table 1 are chosen using conventional values and from literature (Norconsult AS 2016). From these energy coefficients the roundtrip efficiency of the proposed facility is about 75%, in the standard range for pumped storage hydropower facilities (Deane et al. 2010).

To calculate net head we apply Manning's equation to determine the friction loss in the penstock. Manning's equation is on the form

$$h_f = \frac{L \times v^2}{M^2 \times r_h^{4/3}} \tag{61}$$

where L is the length of the penstock, v is the water speed, M is the Manning-coefficient and r_h is the hydraulic radius. By subtracting h_f from the total vertical drop H we get net head, ΔH. E_{dis} and E_{pump} are both applied in the stochastic linear program of Section 4.3.

6.1.2 Investment cost

To estimate the cost of investing in a pumped hydropower production facility, we use NVE's "Cost basis for hydropower"[4], authored by Norconsult AS (Norconsult AS 2016). The objective of this thesis does not involve evaluating a particular investment; rather, it aims to enhance our understanding of the profitability associated with a pumped storage hydropower facility. To achieve this, we conduct basic calculations to estimate the potential expenses associated with such an investment. We study the scenario of constructing a new facility, with a new penstock that connects

[4]We have translated the title from Norwegian to English. In Norwegian the report is called "Kostnadsgrunnlag for vannkraft"

two larger bodies of water, allowing for high utilization of the pumping technology, given adequate price conditions.

A capacity upgrade will inflict an investment cost. The costs are formulated after our specific case. The proposed facility requires a new penstock, a power station with all necessary electrotechnical components, and a ternary unit for pumping and producing. Turbine parameters, penstock diameter, power station size and electrotechnical components are all dependent on either the chosen production capacity or maximum water flow in the penstock (Norconsult AS 2016). The costs need to be expressed as a function of production capacity P, therefore we define $B_{ter}(P)$, $B_{el}(P)$, $B_{pen}(P)$ and $B_{st}(P)$ as the costs related to purchasing and constructing the ternary unit, electrotechnical components, the penstock and the power station respectively. According to Professor Leif Lia (personal communication, 24.05.23), the total of these four functions should be multiplied by a factor of 1.4 to adjust for simplifications and omissions. We then get the cost formulated as presented in equation (62).

$$K(P) = [B_{ter}(P) + B_{el}(P) + B_{pen}(P) + B_{st}(P)] \times 1.4 \tag{62}$$

The cost functions are affected by several varying conditions. As "Cost basis for hydropower" was last updated in 2015, the numbers are index-adjusted to today's prices. We define ψ as a variable that index-adjusts prices to an updated value and adds relevant taxes. The costs of the ternary unit, the power station and the penstock are dependent on the amount of water the penstock is able to transport per second, maximum water flow, Q. Q is dependent on the velocity of water in the penstock and the size of its cross-section. Water velocity is dependent on penstock slope, water head, friction and pressure on each side of the penstock, but as we study economic effects we assume fixed water velocity. We can express the relationship between Q and P using the formula

$$P = E_{dis} \times 3600Q \tag{63}$$

where P is power $[MW]$, E_{dis} is the energy coefficient of production $[kWh/m^3]$ and Q is maximum discharge per second $[m^3/s]$ (Antal 2014). From this, we calculate the price of production per MW. Equation (64) denotes the formula for the cost of the ternary unit.

$$B_{ter}(P) = k_{pel} \times (\frac{P/2}{E_{dis} \times 3600})^{u_{pel}} \times P \times \psi_{ter} + k_{fra} \times (\frac{P/3}{E_{pump} \times 3600})^{u_{fra}} \times P \times \psi_{ter} \tag{64}$$

In equation (64), k_{pel}, u_{pel}, k_{fra} and u_{fra} are variables dependent on the vertical drop from the upper reservoir to the turbine, or water head. The first two are for a Pelton turbine and the latter two for a Francis turbine. A ternary unit is comprised of both. Due to the significant head of our proposed facility, a ternary unit is considered more appropriate than using the more common solution of a reversible Francis turbine. The head is both larger than what a conventional Francis turbine is able to pump and above the ideal head for using Francis in production (Koritarov et al. 2013, Nag and Lee 2020). Therefore, we model the cost of multiple turbines in a sequence, both in production and pumping, which is why P is divided by 2 in the first term and 3 in the second term, for two Pelton and three Francis turbines respectively.

Norconsult AS (2016) also provides an approximation of total electrotechnical costs. The cost is expressed as a function of the rotational speed of the electric generator, n, and production capacity. k_{el}^n and u_{el}^n are variables dependent on the choice of n and the chosen number of electric generators.

$$B_{el}(n, P) = 10^6 \times (k_{el}^n \times P^{u_{el}^n}) \times \psi_{el} \tag{65}$$

The cost of the penstock is dependent on the length of the penstock and the area of its cross-section. Length is fixed, while the size of the cross-section is dependent on maximum water flow

F and velocity v. Due to friction losses in the penstock, the optimal cross section size is found by the intersection of marginal cost and marginal revenue of increasing penstock area. As we are interested in large-scale economic effects, we choose to define a fixed v. The cross-section can then be expressed with the equation $A = F/v$ and penstock costs as

$$B_{pen}(P) = k_{pen} + u_{pen} \times (\frac{P}{v \times E_{dis} \times 3600}) \times L \times \psi_{pen} \tag{66}$$

Station cost is a function of V, the total demolition volume necessary to build the station, where $V = 78 \times H^{0.5} \times Q^{0.7} \times n^{0.1}$. We also define a variable for concrete volume, $B = 0.2V$. Equation (67) is a function of V and B and includes the demolition costs, the cost of concrete, reinforcement, formwork, securing the stone, brick & plaster and decoration costs. To account for unforeseen costs we multiply all of the above with 1.1 and add a fixed cost of 13MNOK for ventilation, heat, lighting, water and other electric necessities, before index-adjustment (Norconsult AS 2016).

$$B_{st}(V) = ((405V + 6990B) \times 1.1 + 1.3 \times 10^6) \times \psi_{sta} \tag{67}$$

By substituting Q in the formula for V with $P/(e \times 3600)$, and inserting the equation for V into (67), we get the cost of building the power station as a function of P.

As calculating accurate costs for an entire hydropower facility is far more complicated than what can be expressed through a single equation, (62) is considered an approximation of the total costs. From the many components included in facility design, we choose the cost-bearing elements above. Specific to our case of upgrading an existing facility is that the reservoirs are already dammed. As such we do not model costs related to dam construction.

By solving Equations (59)-(61) and inserting the parameters found in Appendix A into Equations (62)-(67) we find that our capacity-dependent cost function is an increasing and concave function, i.e., the marginal cost of installed capacity is decreasing. This is consistent with the model of Dangl (1999). The design of the facility is dependent on a distinct number of turbines, generators and transformers, and the producer has the option to build multiple penstocks as well. Considering this, a more realistic function would be step-wise and dependent on these distinct elements. Comparing costs to value in the optimal stopping problem does however require a continuous cost function, and based on conversations with Leif Lia (personal communication, 04.06.23) we deem it adequate for our purpose.

6.1.3 Parameter calculations

Even though the average inflow is known, as mentioned in Section 6.1, the distribution of inflow through the year is not. The historical measurements of inflow recorded near the hydropower plant are instead used as an estimate for this. We adjust these historic inflow years to match the average annual inflow to the existing facility. All inflow years are plotted in Figure 9.

Moreover, the initial reservoir level, l_0, is dependent on facility capacity, P. Instead of choosing an initial reservoir level by random, we follow the reasoning in Section 5 to find differing steady-state water levels. These are key to ensuring comparable yearly production valuations for different capacities. For each evaluated capacity, the steady-state water level is presented in Figure 10.

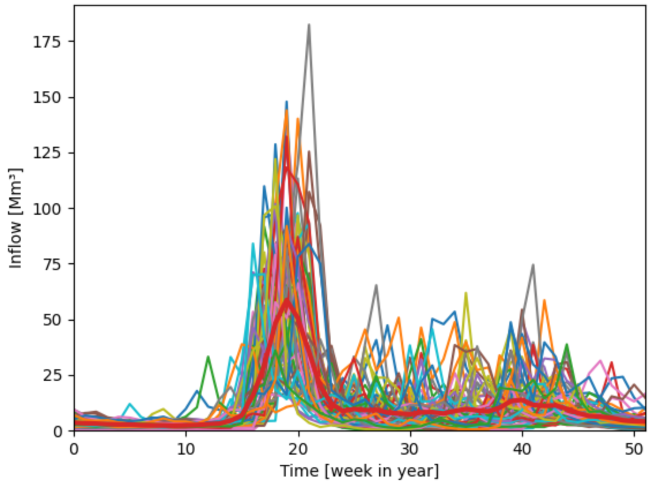


Figure 9: The inflow each week of the year from 73 years of inflow records. The bold red line represents the average weekly inflow.

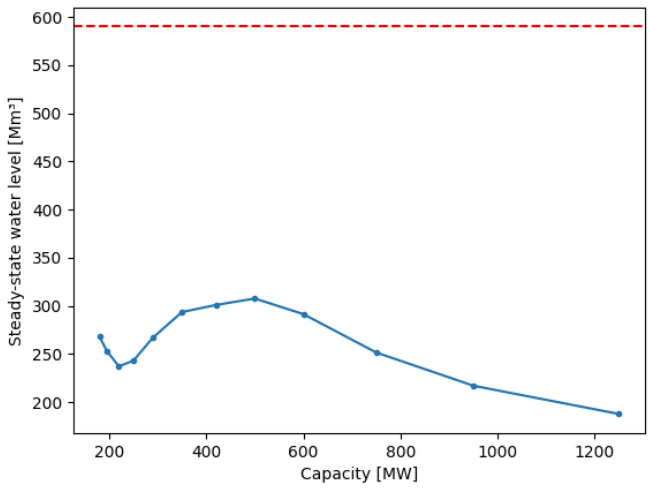


Figure 10: The steady-state water level, l_0, for each evaluated capacity. The red line represents the maximum reservoir capacity.

In Section 4, the parameters of the two-factor stochastic price model of Schwartz and Smith (2000) are introduced. The parameters are calibrated using Kalman filtering and maximum likelihood estimation, yielding an initial long-term price of 45€/MWh. The seasonality parameters, ϕ_1 and ϕ_2 are found through sinusoidal regression. The price parameters provide a basis for the Monte Carlo price path simulations. The EPAD contract with a maturity of three years, referencing the expected differential between the system and NO2 spot prices with a value of 30€/MWh, is then added to the long-term price, giving $\exp(\xi_0) = 75$€/MWh. Further, sampled historical weekly log price deviations from NO2 are incorporated in the price realisations. As we consider the period from 2017-2022, 312 different weekly profiles are sampled. The average intra-weekly log price deviation profile is shown in Figure 11.

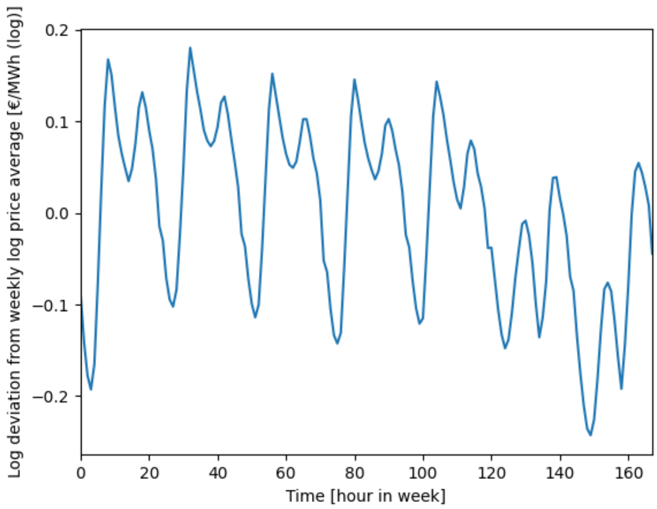


Figure 11: Average log deviation from average weekly log price for all 168 hours during the week. Based on NO2 spot price data from 2017-2022.

Using PWC (2022) we find a risk-free rate, r, of 3.4%, which is used to discount the yearly valuation function and cost of investment. This rate is determined based on the yield of the Norwegian 10-year government bond. The decision aligns with the findings from the PWC yearly report on the risk premium in the Norwegian market, where 50% of the respondents believe that the yield of the Norwegian 10-year government bond should be utilised as the risk-free rate for Norwegian companies (PWC 2022).

6.2 Results and discussion

In this section, we present our final results. The section begins with an examination of the variations in production and pumping patterns, average water levels, and spillage across different capacities. These factors play a crucial role in the valuation of the pumped storage hydropower facility. This valuation is compared to the valuation of the method in Arnstad et al. (2022), where expectancy is used instead of scenarios. We also demonstrate how our valuation method differentiates from the upper bounds associated with our model. As for the real options analysis we assess the valuation and cost functions simultaneously, to determine the optimal timing of investment and capacity of the facility. Finally, we present a discussion on the sensitivity of the final results, with particular emphasis on the impact of intra-weekly price volatility.

We analyse how the facility operates in a single week to gain a fundamental understanding of how water is discharged and pumped throughout the week. Figure 12 graphs a price realisation based on Equation (45), showcasing noticeable price peaks during the day and lower prices during the night and weekends. For the same week, the hourly discharge of the different capacities $P = 250$ MW and $P = 1250$ MW are plotted in Figures 13a and 13b, respectively. Additionally, Figures 14a and 14b illustrate the corresponding pumping hours for the same week and capacities. Evidently, discharge is scheduled during periods of high prices, while pumping takes place during times of low prices. In the case of low capacity, production is scheduled for the first four days of the week, whereas high capacity only produces for the first three days, before both transition to pumping. This highlights the significant influence of capacity on the optimal schedule, emphasising the different valuations of water for production and pumping for each capacity. The switch from production to pumping occurs when the price peak-to-off-peak ratio surpasses the ratio between the energy coefficient of pumping and production.

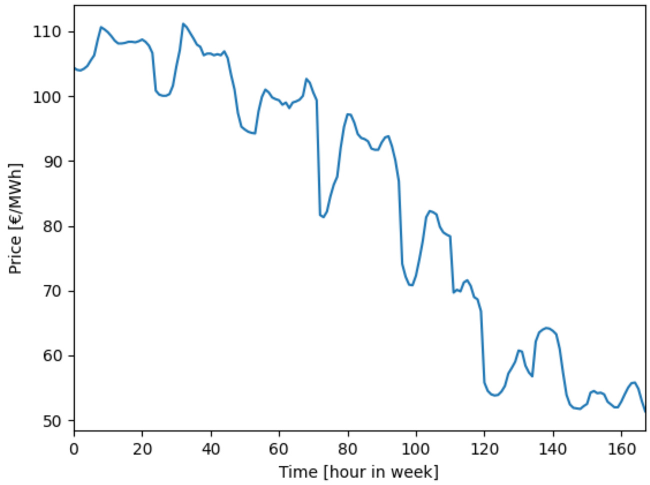
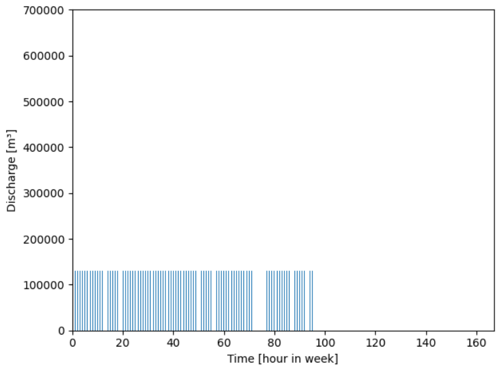
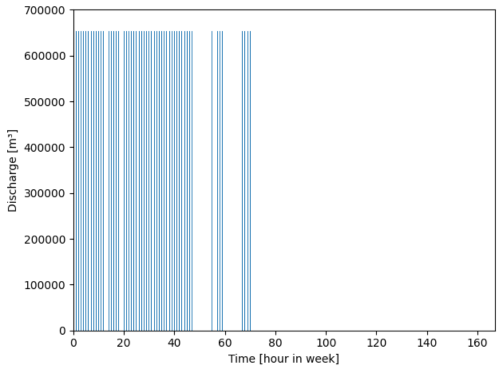


Figure 12: Price realisation of a single week with sampled intra-weekly price volatility.

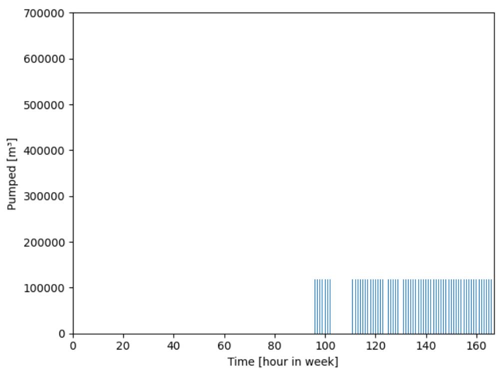


(a) Production capacity $P = 250$ MW.

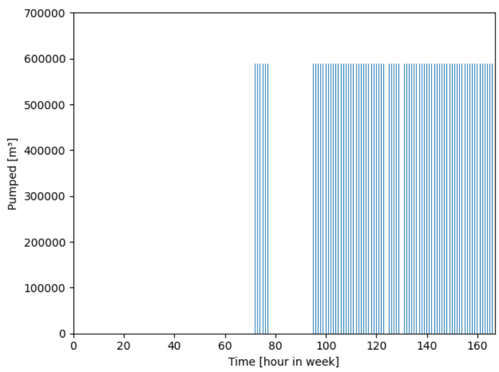


(b) Production capacity $P = 1250$ MW.

Figure 13: Illustration of which hours of the week the model has scheduled discharge for production.



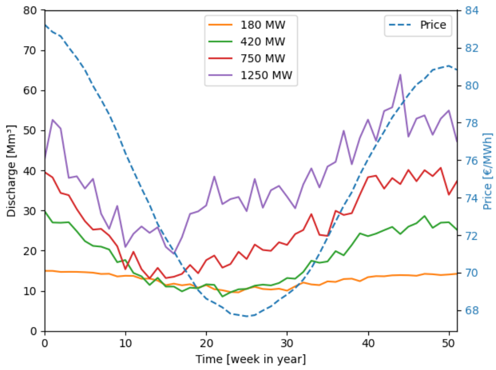
(a) Pumping capacity $P = 250MW$



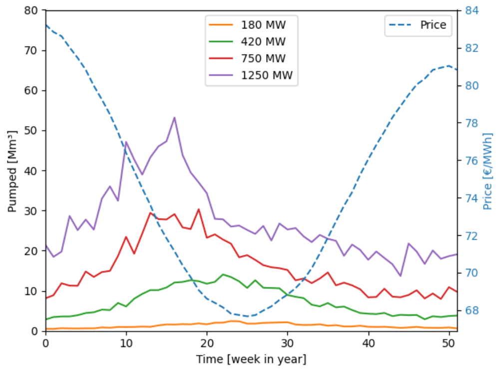
(b) Pumping capacity $P = 1250MW$

Figure 14: Illustration of which hours of the week the model has scheduled for pumping.

Aggregating operations data provide further insights into the patterns of discharging and pumping. Figures 15a and 15b display the average weekly amounts of water discharged and pumped, along with their corresponding average prices throughout the year. During the winter, when prices are high, production reaches its peak. In the spring, the discharge falls and is replaced by pumping and inflow, as depicted in Figure 9. From spring until fall the production rises again, while pumping decreases to its off-peak at the end of the year. Given a steady-state production pattern, where the end water level approximates the initial water level, this pattern is expected to repeat annually.



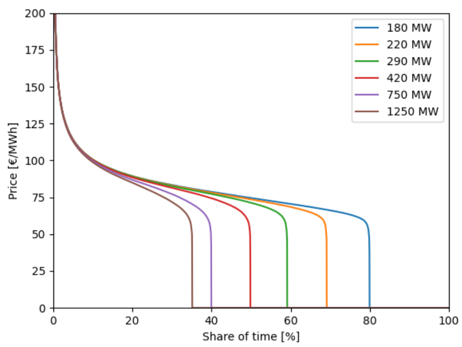
(a) Weekly discharge over one year for different capacities with average price.



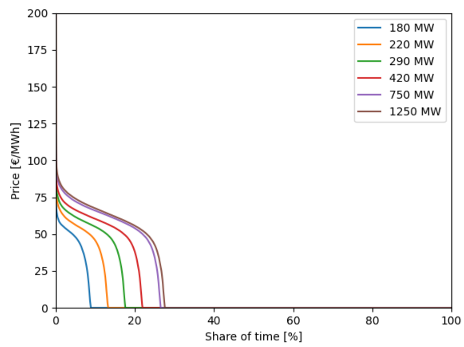
(b) Weekly pumping over one year for different capacities with average price.

Figure 15: Discharging and pumping patterns for some capacities for each week of the year in the planning horizon, after the rolling intrinsic method has scheduled production for one year.

The duration curves presented in Figures 16a and 16b offer a clear overview of the price levels at which various capacities engage in either production or pumping throughout the year. For low capacities, a substantial portion of the year requires maintained production at a high level due to spillage considerations. This means that low capacities are forced to produce at low prices. In contrast, higher capacities have the flexibility to choose advantageous production times with most production above 75€/MWh. As the capacity increases, the impact of yearly inflow on the overall production decreases in importance. Instead, the emphasis shifts towards the quantity of water being pumped, which becomes increasingly significant. Although all capacities engage in pumping during periods of low electricity prices, higher capacities also pump water when prices are higher. This indicates a higher water valuation for high capacities than low, in at least parts of the year.



(a) Duration curve for discharging



(b) Duration curve for pumping

Figure 16: The duration curves illustrate the proportion of time the electricity price during production and pumping exceeds a certain price level.

Figure 17 illustrates the average water level of the reservoir throughout the year for different capacities. The water level is impacted by inflow and production and pumping patterns. The plot aligns with the expectation that facilities with higher capacities will exhibit more pronounced fluctuations in water levels. This is due to the fact that larger production capabilities can leverage larger volumes of water, and exploit seasonal volatility. The low capacities are required to produce for most of the year simply to discharge the yearly inflow. Large capacity allows for significant pumping when water is in abundance and prices are low and is also capable of discharging large amounts to produce at peak prices and avoiding spillage. Facilities with lower capacities must maintain a more stable water level to prevent potential spillages caused by excessively high levels, while also avoiding significant depletion of the reservoir due to insufficient inflow for replenishment before the next season of high prices.

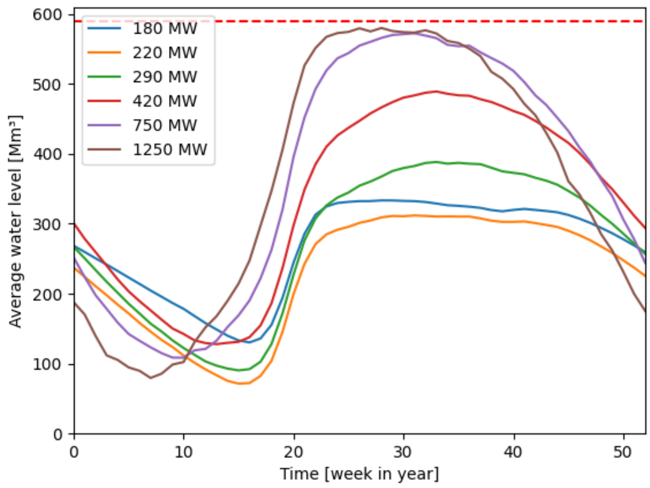


Figure 17: Average water level through the year for different capacities within the highest (red line) and lowest regulated water levels.

To maximise profits from water utilisation throughout the year, each capacity adjusts its water level to find a balance between having ample water during periods of high prices and avoiding wasteful spillages. However, some spillages may still occur due to uncertainty in future inflow realisations. Figure 18 presents the average yearly spillage for each capacity. When considering low capacities, the limited water level leads to relatively low spillage. In the case of medium capacities, a higher water level in the fall increases the risk of spillage. For the highest capacities, the risk of spillage decreases, despite the average water level during the fall being close to the maximum, due to an increased ability to discharge water to production. It is worth noting that the capacity with the highest spillage, $P = 420$ MW, has a spillage amounting to only 0.163% of the average yearly inflow. This indicates that spillage does not appear to be a significant concern in the valuation of each capacity.

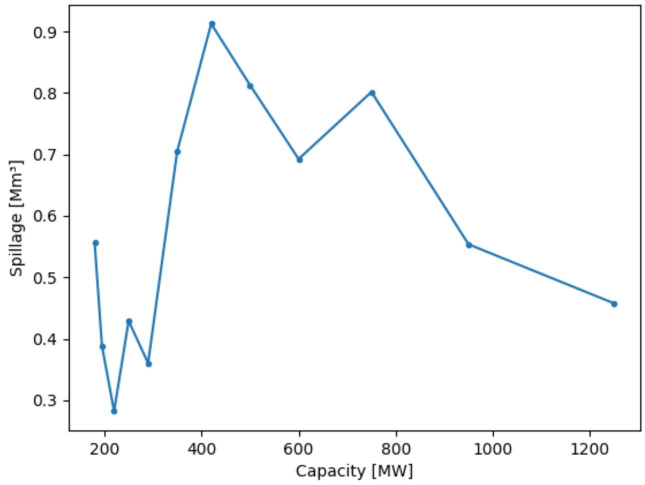


Figure 18: Average spillage for different capacities.

In Figure 19, we present the yearly valuation, V_P, of the pumped storage hydropower facility calculated using two different methodologies, namely the stochastic linear program of this thesis, and the method of Arnstad et al. (2022). The latter method has been altered to include a pump and samples of historical intra-weekly price profiles, instead of average weekly profiles, but uses the expectancy of inflow and prices in the forecast section of the intrinsic problem, instead of scenarios. The comparison aims to evaluate the additional value of a stochastic optimisation method, with profit maximisation over different future price and inflow scenarios. The plot demonstrates that the methods' performances are nearly identical for this case, within their standard deviations shown in Table 2.

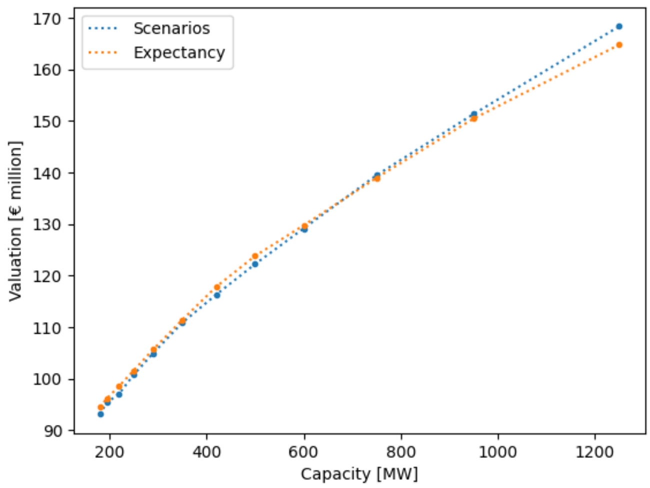


Figure 19: Valuation of the facility using scenarios in the forecast section compared to the methodology of using expected values of the exogenous factors.

Figure 20 shows the valuation, V_P for each evaluated capacity, P of the pumped storage hydropower, in comparison to the perfect information upper bound and a dual penalty upper bound.

Table 2: Standard deviation of capacity valuations.

Capacity [MW]	**Scenarios [%]**	**Expectancy [%]**
180	1.2	3.6
195	1.4	3.7
220	1.3	3.8
250	1.4	3.8
290	1.5	3.8
350	1.5	3.8
420	1.5	3.7
500	1.6	3.6
600	1.6	3.4
750	1.6	3.3
950	1.6	3.2
1250	1.5	3.4

The perfect information upper bound is found by solving the deterministic case where all future realisations of inflow and prices are known in advance, i.e. all non-anticipativity constraints are relaxed. This is a relatively weak upper bound, and we find a better one by employing dual-bound penalties (Brown et al. 2010). The dual upper bound assumes perfect information but penalises the relaxation. The dual penalty is calculated from the equations

$$d_1 = \delta_1 \sum_{i \in I} \sum_{j \in J} (x_{ij}^{dis} + x_{ij}^{pump})(S_{ij} - \mathbb{E}(S_{ij}|\chi_0, \xi_0)) \tag{68}$$

$$d_2 = \delta_2 \sum_{i \in I} \sum_{j \in J} (x_{ij}^{dis} + x_{ij}^{pump})(\lambda_{ij} - \mathbb{E}(\lambda_{ij}|\lambda_0)) \tag{69}$$

with penalty coefficients $\delta_1 = 4 * 10^{-4}$ and $\delta_2 = 2 * 10^{-8}$ found to provide good penalties. The dual penalties are subtracted from the objective when solving for the dual bound. The last factor in (68) and (69) values the difference between realisation and expected realisation given the initial parameters of the stochastic price process and the inflow year. The optimality gap between our valuations and the upper bounds is presented in Table 3 for all capacities.

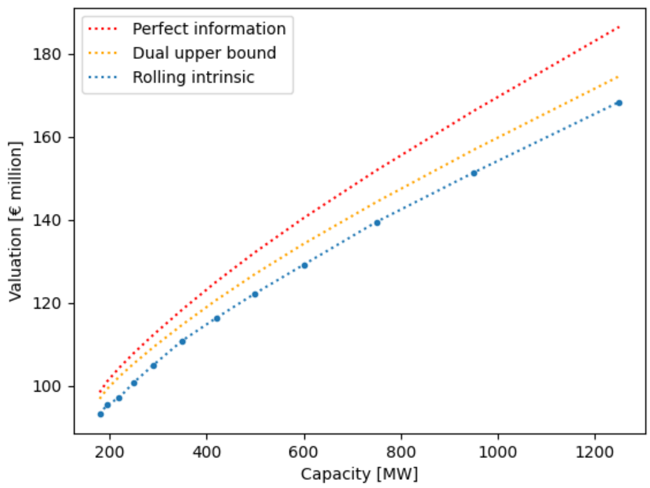


Figure 20: Rolling intrinsic valuation compared to its upper bounds.

Table 3: Rolling intrinsic valuation and optimality gaps.

Capacity [MW]	V_P [**€ million**]	**Perfect information** [%]	**Dual bound** [%]
180	93.1	5.67	4.00
195	95.4	5.88	3.99
220	97.1	7.42	5.21
250	100.8	7.01	4.52
290	104.9	6.94	4.09
350	110.9	6.75	3.46
420	116.3	7.49	3.70
500	122.2	8.79	4.47
600	129.2	10.31	5.42
750	139.5	12.53	6.89
950	151.3	12.60	6.25
1250	168.4	10.76	3.69

The valuation, V_P, exhibits an upward trend for larger capacities, but the increase rate does not diminish as much as observed in Arnstad et al. (2022), which considers a conventional hydropower facility with no pump. The combination of low-cost pumping and high-price production appears to establish a nearly linear relation between capacity and valuation, especially for larger capacities. To examine this relationship further, we explore how the intra-weekly price profiles affect the value of the pumped storage hydropower for three different cases. The first is the valuation resulting from the sampling of actual historical weekly profiles, $\alpha_j, \forall j \in \mathcal{J}$, which are added to the weekly simulated price, referred to as 1.0α. For the next valuation, each historical intra-weekly profile is scaled down by 50%, giving 0.5α. Last, the 0.0α refers to the valuation in a situation where the simulated week price is used for the entire week, with no hourly deviations. From our rolling intrinsic methodology, we obtain three different yearly valuations, V_P. Using the approach in Section 5.2, we find the smoothed, continuous valuation functions, $V_0(\boldsymbol{\omega}_0, P)$, for each case. Next, we evaluate each function to find the optimal timing and corresponding capacity for the pumped storage hydropower investment.

In Table 4, the trigger price and capacity for each case are presented along with perpetual facility valuations. We observe that the semi-analytical process of Section 5.2 did not reach convergence for any capacities where $P \in$ (180MW, 1250MW), neither for 1.0α nor for 0.5α. The reason for this is that the marginal value of production with capacity, $P \in$ (180MW, 1250 MW), decreases at a slower rate than the marginal capacity investment cost for the same region. As discussed in Dangl (1999), this yields immediate investment in maximum capacity, and we set the installed capacity to the max, $P = 1250$ MW.

While the cost function of Equation (62) differs from the ones found in other hydropower investment literature, such as Kleiven et al. (2022) or Bøckman et al. (2008), the authors argue for its increased realism compared to their functions (Norconsult AS 2016). Not achieving convergence may be viewed as an argument against the usefulness of real options analysis when applied to our problem. However, as mentioned in Section 6.1.2, decreasing marginal costs of installed capacity is similar to cost functions found in earlier literature on real options theory (Dangl 1999, Hagspiel et al. 2016). Hagspiel et al. (2016)(p.103) argue that "There are important economic reasons pleading for such a concave investment costs function, such as indivisibilities, use of information, fixed costs of ordering and quantity discounts". In Dangl (1999), investment timing and optimal capacity are studied under varying demand conditions, especially the amount of uncertainty involved with demand. For larger demand uncertainty (or price volatility), the investment will occur only for a large production capacity but will be postponed until price requirements are sufficiently high. This is consistent with our findings in Table 4 and Figure 22.

To further ascertain the useful applications of real options analysis, we study the case of no intra-weekly deviations, 0.0α, where convergence is reached with the trigger price, $\exp(\xi^*) = 167$ €/MWh and optimal capacity, $P^* = 930$ MW. As the trigger is significantly higher than the current price level $\exp(\xi_0) = 75$ €/MWh and the long-term risk-adjusted drift is negative, investment is unlikely in the near future. The real options valuation of the investment project is thus dominated by the NPV of the existing facility (with 180 MW turbine capacity), which is also higher than the NPV of the optimal investment in a pumped storage hydropower plant at today's price level. In Figure 21 the valuations of the three cases are shown at the price level, $\exp(\xi)$, set to the trigger price of 0.0α. The cost function and 0.0α has the same slope in P^*.

Table 4: Results

	1.0α	0.5α	0.0α	**Unit**
Trigger price, $\exp(\xi^*)$	40.29	77.00	166.8	€/MWh
Facility capacity (at price trigger)	> 1250	> 1250	930.2	MW
Facility capacity (invest immediately)	> 1250	> 1250	542.4	MW
NPV (existing facility)	2197	2157	2074	€ (million)
NPV (invest immediately)	3450	2513	2020	€ (million)
Real options value, $D(\xi)$	3450	2527	2131	€ (million)
Investment probability (10 years)	100	87.13	2.18	%
Investment probability (40 years)	100	90.95	12.71	%

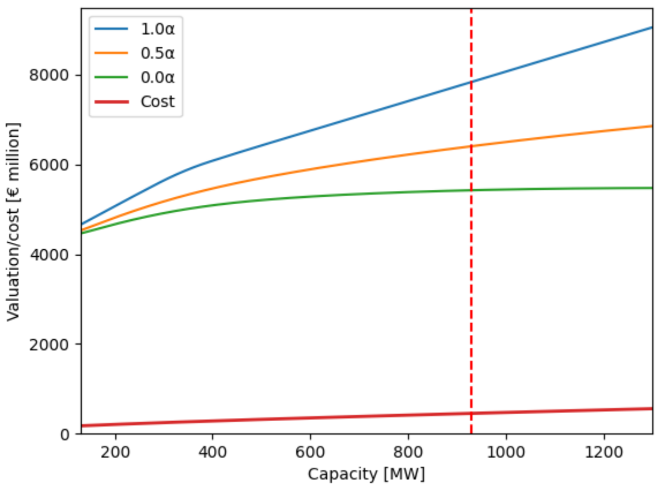


Figure 21: Valuations with different intra-weekly volatility at the trigger price level, $\exp(\xi^*)$, of 0.0α. The dashed line is the corresponding optimal capacity.

While convergence may not be achieved when including intra-weekly price deviations, it is evident that the market conditions characterised by high volatility within the week present a highly profitable opportunity for investment in pumped storage hydropower facilities. Due to the profitability, the real options value equals the NPV of immediate investment for 1.0α, as the option should be exercised immediately. Considering 0.5α, immediate investment is nearly as profitable as delaying investment until the long-term price surpasses the trigger point slightly above the current price level.

Figure 22 depicts the sensitivity of the price trigger in long-term price volatility, σ_ξ, for the three different cases. The shaded areas indicate when it is optimal to invest, given the magnitude of intra-weekly price deviations. In the case of 0.0α, a substantial price level is necessary for immediate investment. Conversely, for 1.0α and 0.5α, the trigger prices are significantly lower. Irrespective of the case, heightened long-term price volatility corresponds to higher trigger prices. Increased price volatility introduces greater uncertainty regarding future price levels, consequently amplifying the value of waiting to obtain more reliable information, thus giving higher trigger prices. Notably, for all values of long-term price volatility, the optimal capacity corresponding to the trigger prices of 1.0α and 0.5α is the maximum capacity, i.e. in our case $P = 1250$MW.

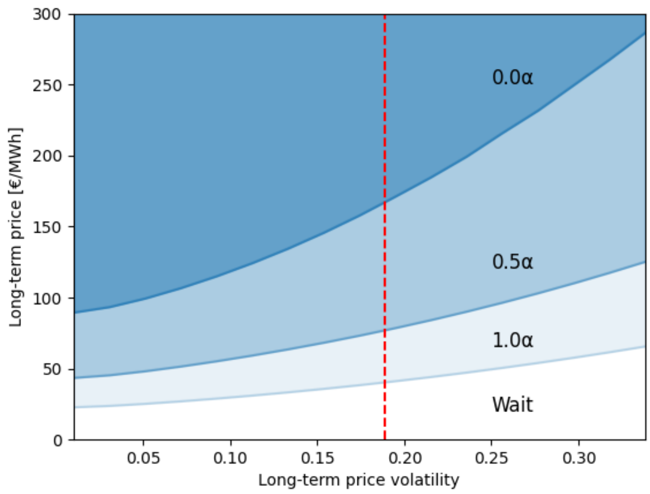


Figure 22: Sensitivity of the long-term price volatility. The red line represents the estimated parameter value of the two-factor price model.

7 Concluding remarks

Our results and findings necessitate some remarks and a discussion of possibilities for further research. In this section, we summarise our work and comment on the aspects of our model that are relevant for further work.

7.1 Conclusion

In conclusion, our method has provided some interesting and noteworthy results. First and foremost is the demonstration of the profitability of pumped storage hydropower for a single producer with a price-taker perspective. The combination of our cost function and facility valuation suggests investing in a production capacity as high as possible, given the present price level. While lower intra-week volatility may result in a decrease in profitability, investments can still remain profitable due to seasonal price differences. However, it is important to note that an increased number of pump investments could limit this profitability, as this could reduce intra-weekly market volatility.

Our model demonstrates how increased production and pumping capacity impacts production patterns and reservoir management, giving varying valuations. The valuation function from our stochastic model is also closely aligned with its upper bounds. One result that was somewhat surprising to the authors is the negligible effect of uncertainty included in the linear program. Our model, and the model in Kleiven et al. (2022) performed similarly, indicating that using expectations in the second-stage problem provides adequate results.

We have presented a general production planning model, a general investment model, and applied them to a case with a case-dependent cost function. Detailed cost calculations of any hydropower investment are heavily case specific, but our method for discerning said function has general applications. The use of real options analysis as an investment model is widely suggested by academia as an improvement upon the NPV method and other methods. It is however slow to be adopted by the industry. When considering pumped storage hydropower facilities, which currently appear highly profitable within the existing market conditions, the application of the real options methodology does not provide additional value to the investment problem. The analysis yields the same investment recommendation as the traditional NPV analysis. However, under different market conditions, and for other investment problems, the real options framework does present an advantage.

Another important question to address regarding the indicated profitability of pumped storage hydropower under the current market conditions is the lack of construction of such facilities, specifically in Norway, the location of our case problem. In their review of pumped storage hydropower in Norway, Pitorac et al. (2020) note a decrease in hydropower investment after the market deregulation of 1991, both pumped storage and conventional hydropower. This is because the market was saturated with supply exceeding demand significantly. Prior market conditions, how power prices were defined and requirements for security in supply resulted in investments in overcapacity (Bye and Hope 2005). These considerations extend from conventional hydropower to pumped storage (Pitorac et al. 2020). It is also worth noting that from the perspective of the regulatory bodies, excess pumping reduces the total net energy produced. The Nordic market is however forecast to experience grid imbalance by 2030 (NVE 2022). Pumping capabilities would give hydropower facilities the ability to counteract this imbalance, and so may warrant investment and further research in pumped storage hydropower.

7.2 Further work

The main finding of our thesis, that pumped storage hydropower warrants high valuation and considerable profit, hinges on the current market conditions. As long as price volatility enables profits from pumping, investment in pumped storage hydropower will be profitable. An interesting direction for research would however be to study how the market would be affected by the inclusion of a large amount of pumping capabilities. As pumped storage hydropower contributes to reducing market price volatility, this would be interesting from a socioeconomic perspective.

An important consideration for further investigation is whether the intra-weekly volatility can be modelled as a process or discretised into scenarios with different probabilities. This prompts the exploration of implementing a two-stage stochastic model, where the initial stage focuses on optimising capacity upgrades, followed by optimisation over scenario realisations. By examining these possibilities, we can enhance decision-making by accounting for the dynamic nature of intra-weekly volatility and capturing a range of potential outcomes, enabling more informed and effective decision-making regarding the investment problem.

Our price model considers solely trading on the day-ahead spot market. The increased production flexibility of pumped storage hydropower makes it ideal for trading on the ancillary and balancing markets (Braun and Burkhardt 2015). Including these trading opportunities in our model could yield interesting insight. A topic that is somewhat related to the research proposed in the paragraph above would be researching the inclusion of large pumping capabilities in the other power trading markets, and the effects it would have on these markets.

We consider in this thesis the investment into a brand-new facility. The construction of our proposal entails that the old facility would be decommissioned. We do however avoid comparing the option value of refurbishing the existing facility with our proposal, as Décamps et al. (2006) ascertains that such a comparison is inadequate. A method of comparing two discrete projects with discrete costs would assist facility owners even further when considering refurbishing old plants. Additionally, maintenance costs for old facilities are generally larger than for newer plants, and including this would further increase the realism of our investment model.

By including stochastic developments of our cost function, for instance using the approach in Pindyck (1993), the accuracy of the options model could be further improved. This would require solving the real option numerically, which is possible using the Least Squares Monte Carlo method in Longstaff and Schwartz (2001).

In our production scheduling model we disregard several operational constraints that are not relevant to our case problem. We operate with a fixed net head, disregarding how the head is affected by the reservoir levels due to the large head of our case. This is a relevant constraint for pumped storage hydropower with a smaller head. Our problem description dismisses ramping constraints and constraints imposed by regulatory services due to environmental concerns, as our case problem consists of an underground penstock connecting two large reservoirs. In addition to these decisions, we also assume head-loss and turbine/pump efficiency to be constant. The inclusion of all these considerations would further increase the general applicability of our model.

In a more detailed production scheduler, some of the variables discussed above would provide incentives for production at less than maximum capacity. Restricting the model to only producing at maximum capacity is however a necessary modelling constraint. The alternative would make the problem non-linear and significantly more difficult to solve. However, including volume flexibility in the real options model is done in Hagspiel et al. (2016). Their case differs from ours somewhat, but implementing aspects of their real options method could provide further insight into optimal investment strategies.

Bibliography

Aasgard, Ellen K., Gorild S. Andersen, Stein-Erik Fleten and Daniel Haugstvedt. 'Evaluating a Stochastic-Programming-Based Bidding Model for a Multireservoir System'. In: *IEEE Transactions on Power Systems* 29.4 (July 2014), pp. 1748–1757. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2014.2298311.

Andersson, Ane Marte, Morten Elverhøi, Stein-Erik Fleten, Sabine Fuss, Jana Szolgayová and Ole Christian Troland. 'Upgrading hydropower plants with storage: timing and capacity choice'. In: *Energy Systems* 5.2 (June 2014), pp. 233–252. ISSN: 1868-3967, 1868-3975. DOI: 10.1007/s12667-013-0112-2.

Antal, Brandi A. 'Pumped Storage Hydropower: A Technical Review'. In: *B.S., University of Colorado – Boulder, 2004* (2014).

Arnstad, Fredrik Malt, Thomas Bruun Karud and Nicolai Bugge Lysaa. 'Optimal production scheduling and refurbishment timing for hydropower plants'. In: *Project report - NTNU* (15th Dec. 2022).

Arteaga, Juan and Hamidreza Zareipour. 'A Price-Maker/Price-Taker Model for the Operation of Battery Storage Systems in Electricity Markets'. In: *IEEE Transactions on Smart Grid* 10.6 (2019), pp. 6912–6920. DOI: 10.1109/TSG.2019.2913818.

Avesani, Diego, Ariele Zanfei, Nicola Di Marco, Andrea Galletti, Francesco Ravazzolo, Maurizio Righetti and Bruno Majone. 'Short-term hydropower optimization driven by innovative time-adapting econometric model'. In: *Applied Energy* 310 (Mar. 2022), p. 118510. ISSN: 03062619. DOI: 10.1016/j.apenergy.2021.118510.

Bellman, Richard. *Dynamic programming*. Princeton, NJ: Dover Publications, 1957. 339 pp. ISBN: 978-0-691-07951-6.

Benders, J F. 'Partitioning procedures for solving mixed-variables programming problems'. In: *Numerische Mathematik* 4 (1962), pp. 238–252.

Bøckman, Thor, Stein-Erik Fleten, Erik Juliussen, Håvard J. Langhammer and Ingemar Revdal. 'Investment timing and optimal capacity choice for small hydropower projects'. In: *European Journal of Operational Research* 190.1 (Oct. 2008), pp. 255–267. ISSN: 03772217. DOI: 10.1016/j.ejor.2007.05.044.

Bozorg Haddad, Omid, Parisa-Sadat Ashofteh, Solmaz Rasoulzadeh-Gharibdousti and Miguel A. Mariño. 'Optimization Model for Design-Operation of Pumped-Storage and Hydropower Systems'. In: *Journal of Energy Engineering* 140.2 (June 2014), p. 04013016. ISSN: 0733-9402, 1943-7897. DOI: 10.1061/(ASCE)EY.1943-7897.0000169.

Braun, Sebastian M. and Michael Burkhardt. 'Trading of pumped storage hydropower plants on energy only and ancillary services markets'. In: *2015 International Conference on Renewable Energy Research and Applications (ICRERA)*. 2015 4th International Conference on Renewable Energy Research and Applications (ICRERA). Palermo: IEEE, Nov. 2015, pp. 649–653. ISBN: 978-1-4799-9982-8. DOI: 10.1109/ICRERA.2015.7418492.

Brooks, Chris. *Introductory Econometrics for Finance*. 4th ed. Cambridge University Press, 2019. DOI: 10.1017/9781108524872.

Brown, David B., James E. Smith and Peng Sun. 'Information Relaxations and Duality in Stochastic Dynamic Programs'. In: *Operations Research* 58.4 (Aug. 2010), pp. 785–801. ISSN: 0030-364X, 1526-5463. DOI: 10.1287/opre.1090.0796.

Bye, Thorstein and Einar Hope. 'Deregulation of Electricity Markets: The Norwegian Experience'. In: *Economic and Political Weekly* (2005).

Carravetta, A., O. Fecarotta, R. Martino and L. Antipodi. 'PAT Efficiency Variation with Design Parameters'. In: *Procedia Engineering* 70 (2014), pp. 285–291. ISSN: 18777058. DOI: 10.1016/j.proeng.2014.02.032.

Dangl, Thomas. 'Investment and capacity choice under uncertain demand'. In: *European Journal of Operational Research* 117.3 (Sept. 1999), pp. 415–428. ISSN: 03772217. DOI: 10.1016/S0377-2217(98)00274-4.

Deane, J.P., B.P. Ó Gallachóir and E.J. McKeogh. 'Techno-economic review of existing and new pumped hydro energy storage plant'. In: *Renewable and Sustainable Energy Reviews* 14.4 (May 2010), pp. 1293–1302. ISSN: 13640321. DOI: 10.1016/j.rser.2009.11.015. (Visited on 2nd May 2023).

Décamps, Jean-Paul, Thomas Mariotti and Stéphane Villeneuve. 'Irreversible investment in alternative projects'. In: *Economic Theory* 28.2 (June 2006), pp. 425–448. ISSN: 0938-2259, 1432-0479. DOI: 10.1007/s00199-005-0629-2.

Dempster, M. A. H., M. L. Fisher, L. Jansen, B. J. Lageweg, J. K. Lenstra and A. H. G. Rinnooy Kan. 'Analytical Evaluation of Hierarchical Planning Systems'. In: *Operations Research, 29(4), 707–716.* (1981).

Dietze, Marina, Iago Chavarry, Ana Carolina Freire, Davi Valladao, Alexandre Street and Stein-Erik Fleten. 'A Novel Semiparametric Structural Model for Electricity Forward Curves'. In: *IEEE Transactions on Power Systems* (2022), pp. 1–11. ISSN: 1558-0679. DOI: 10.1109/TPWRS.2022.3197982.

Dixit, A. K. and S Pindyck. *Investment under uncertainty.* Princeton University Press, 1994.

Dixit, Avinash. 'Choosing among alternative discrete investment projects under uncertainty'. In: *Economics Letters* 41.3 (Jan. 1993), pp. 265–268. ISSN: 01651765. DOI: 10.1016/0165-1765(93)90151-2.

DNV. *ENERGY TRANSITION OUTLOOK 2022.* 2022. URL: https://www.dnv.com/energy-transition-outlook/download.html.

Farzin, Y.H., K.J.M. Huisman and P.M. Kort. 'Optimal timing of technology adoption'. In: *Journal of Economic Dynamics and Control* 22.5 (May 1998), pp. 779–799. ISSN: 01651889. DOI: 10.1016/S0165-1889(97)00097-3.

Fertig, Emily, Ane Marte Heggedal, Gerard Doorman and Jay Apt. 'Optimal investment timing and capacity choice for pumped hydropower storage'. In: *Energy Systems* 5.2 (June 2014), pp. 285–306. ISSN: 1868-3967, 1868-3975. DOI: 10.1007/s12667-013-0109-x.

Figueira, Gonçalo and Bernardo Almada-Lobo. 'Hybrid simulation–optimization methods: A taxonomy and discussion'. In: *Simulation Modelling Practice and Theory* 46 (Aug. 2014), pp. 118–134. ISSN: 1569190X. DOI: 10.1016/j.simpat.2014.03.007.

Fleten, S-E. and T. K. Kristoffersen. 'Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer'. In: *European Journal of Operational Research* (2007).

Fleten, Stein-Erik and Trine Krogh Kristoffersen. 'Short-term hydropower production planning by stochastic programming'. In: *Computers & Operations Research* 35.8 (Aug. 2008), pp. 2656–2671. ISSN: 03050548. DOI: 10.1016/j.cor.2006.12.022.

Fleten, Stein-Erik and Jacob Lemming. 'Constructing forward price curves in electricity markets'. In: *Energy Economics* (2003).

Fosso, O.B., A. Gjelsvik, A. Haugstad, B. Mo and I. Wangensteen. 'Generation scheduling in a deregulated system. The Norwegian case'. In: *IEEE Transactions on Power Systems* 14.1 (Feb. 1999), pp. 75–81. ISSN: 08858950. DOI: 10.1109/59.744487.

Gjelsvik, A., B. Mo and A. Haugstad. 'Long- and Medium-term Operations Planning and Stochastic Modelling in Hydro-dominated Power Systems Based on Stochastic Dual Dynamic Programming.' In: *Handbook of Power Systems I. Energy Systems.* (2010).

Gjerden, Knut Skogstrand, Arild Helseth, Birger Mo and Geir Warland. 'Hydrothermal scheduling in Norway using stochastic dual dynamic programming; a large-scale case study'. In: *2015 IEEE Eindhoven PowerTech.* 2015, pp. 1–6. DOI: 10.1109/PTC.2015.7232278.

Goodwin, Dominice. *Schwartz-Smith 2-factor model - Parameter estimation.* 2013. URL: https://www.mathworks.com/matlabcentral/fileexchange/43352-schwartz-smith-2-factor-model-parameter-estimation (visited on 15th June 2023).

Gray, J and P Khandelwal. 'Realistic gas storage models II: Trading strategies'. In: *Commodities Now* September (2004), pp. 1–5.

Hagspiel, Verena, Kuno J.M. Huisman and Peter M. Kort. 'Volume flexibility and capacity investment under demand uncertainty'. In: *International Journal of Production Economics* 178 (2016), pp. 95–108. ISSN: 0925-5273. DOI: https://doi.org/10.1016/j.ijpe.2016.05.007.

Heitsch, Holger and Werner Römisch. 'Scenario Reduction Algorithms in Stochastic Programming'. In: *Computational Optimization and Applications* 24.2 (1st Feb. 2003), pp. 187–206. ISSN: 1573-2894. DOI: 10.1023/A:1021805924152.

Helseth, A., M. Fodstad and B. Mo. 'Optimal Medium-Term Hydropower Scheduling Considering Energy and Reserve Capacity Markets'. In: *IEEE TRANSACTIONS ON SUSTAINABLE ENERGY, VOL. 7, NO. 3* (2016).

Helseth, Arild, Anders Gjelsvik, Birger Mo and Úlfar Linnet. 'A model for optimal scheduling of hydro thermal systems including pumped-storage and wind power'. In: *IET Generation, Transmission & Distribution* 7.12 (Dec. 2013), pp. 1426–1434. ISSN: 1751-8695, 1751-8695. DOI: 10.1049/iet-gtd.2012.0639.

Helseth, Arild, Birger Mo, Arild L Henden and Geir Warland. *SOVN Model Implementation.* SINTEF Energi, 2017, p. 47.

Helseth, Arild, Birger Mo, Arild Lote Henden and Geir Warland. 'Detailed long-term hydro-thermal scheduling for expansion planning in the Nordic power system'. In: *IET Generation, Transmission & Distribution* 12.2 (Jan. 2018), pp. 441–447. ISSN: 1751-8695, 1751-8695. DOI: 10.1049/iet-gtd.2017.0903.

Huisman, Kuno J.M. and Peter M. Kort. 'Strategic technology investment under uncertainty'. In: *OR Spectrum* 24.1 (1st Feb. 2002), pp. 79–98. ISSN: 0171-6468, 1436-6304. DOI: 10.1007/s291-002-8201-0.

International Energy Agency. *Hydropower Special Market Report: Analysis and forecast to 2030.* OECD, 9th July 2021. ISBN: 978-92-64-48263-0. DOI: 10.1787/07a7bac8-en.

Jain, Sanjay V. and Rajesh N. Patel. 'Investigations on pump running in turbine mode: A review of the state-of-the-art'. In: *Renewable and Sustainable Energy Reviews* 30 (Feb. 2014), pp. 841–868. ISSN: 13640321. DOI: 10.1016/j.rser.2013.11.030.

Kleiven, A., S. Risanger and S-E. Fleten. 'Co-movements between forward prices and resource availability in hydro-dominated electricity markets'. In: *NTNU Doctoral theses* (2023).

Kleiven, Andreas, Selvaprabu Nadarajah and Stein-Erik Fleten. 'Revisiting hierarchical planning for hydropower plant upgrades using semi-analytical policies and reinforcement learning'. In: *Decision analytics in hydropower: Investment and operational planning under uncertainty, PHD-Thesis* (2022).

Koritarov, Vladimir, Leah Guzowski, James Feltes, Yuriy Kazachov, Bo Gong, Bruno Trouille, Peter Donalek and Vahan Gevorgian. *Modeling Ternary Pumped Storage Units.* Aug. 2013.

Kvamme, Christina. 'Investering i vannkraftverk'. In: *NTNU Master's thesis* (2008).

Lai, Guoming, François Margot and Nicola Secomandi. 'An Approximate Dynamic Programming Approach to Benchmark Practice-Based Heuristics for Natural Gas Storage Valuation'. In: *Operations Research* 58.3 (June 2010), pp. 564–582. ISSN: 0030-364X, 1526-5463. DOI: 10.1287/opre.1090.0768.

Linnerud, K., A. M. Andersson and S-E. Fleten. 'Investment timing under uncertain renewable energy policy: An empirical study of small hydropower projects'. In: *Energy* (2014).

Longstaff, Francis A. and Eduardo S. Schwartz. 'Valuing American Options by Simulation: A Simple Least-Squares Approach'. In: *Review of Financial Studies* 14.1 (Jan. 2001), pp. 113–147. ISSN: 0893-9454, 1465-7368. DOI: 10.1093/rfs/14.1.113.

Martínez-Ceseña, E. A. and J. Mutale. 'Application of an advanced real options approach for renewable energy generation projects planning'. In: *Renewable and Sustainable Energy Reviews* (2011).

McDonald, R.L. *Derivatives Markets*. Addison-Wesley series in finance. Addison Wesley, 2003. ISBN: 978-0-321-21072-2.

McDonald, Robert and Daniel Siegel. 'The Value of Waiting to Invest'. In: *The Quarterly Journal of Economics* 101.4 (Nov. 1986), p. 707. ISSN: 00335533. DOI: 10.2307/1884175.

Montel Online. URL: https://app.montelnews.com/en/nordicoverview?31382 (visited on 29th May 2023).

Moura, Pedro and Aníbal de Almeida. 'Large Scale Integration of Wind Power Generation'. In: June 2010, pp. 95–119. DOI: 10.1007/978-3-642-02493-1_5.

Muche, Thomas. 'A real option-based simulation model to evaluate investments in pump storage plants'. In: *Energy Policy* 37.11 (Nov. 2009), pp. 4851–4862. ISSN: 03014215. DOI: 10.1016/j.enpol.2009.06.041.

Nag, Soumyadeep and Kwang Y. Lee. 'Power System Resiliency Enhancement with Ternary Pumped – Storage Hydropower'. In: *IFAC-PapersOnLine* 53.2 (2020), pp. 12714–12718. ISSN: 24058963. DOI: 10.1016/j.ifacol.2020.12.1875.

Nasdaq. *Power futures and options for convenient trading and hedging*. URL: https://www.nasdaq.com/solutions/nordic-european-power (visited on 23rd May 2023).

Nasir, Jehanzeb, Adeel Javed, Majid Ali, Kafait Ullah and Syed Ali Abbas Kazmi. 'Capacity optimization of pumped storage hydropower and its impact on an integrated conventional hydropower plant operation'. In: *Applied Energy* 323 (Oct. 2022), p. 119561. ISSN: 03062619. DOI: 10.1016/j.apenergy.2022.119561.

Norconsult AS. *Cost basis for hydropower*. 46-2016. Norges vassdrags- og energidirektorat, Apr. 2016, p. 197.

Nord Pool. URL: https://www.nordpoolgroup.com/en/ (visited on 29th May 2023).

NVE. *Hvordan fungerer kraftmarkedet?* URL: https://www.nve.no/reguleringsmyndigheten/slik-fungerer-kraftsystemet/hvordan-fungerer-kraftmarkedet/ (visited on 12th May 2023).

— *Kostnadsgrunnlag for vasskraft*. URL: https://www.nve.no/energi/energisystem/vannkraft/kostnadsgrunnlag-for-vasskraftverk/ (visited on 29th May 2023).

— *Norsk og nordisk effektbalanse fram mot 2030*. 2022. URL: https://publikasjoner.nve.no/rapport/2022/rapport2022_20.pdf (visited on 15th June 2023).

Pereira, M. V. F. and L. M. V. G. Pinto. 'Multi-stage stochastic optimization applied to energy planning'. In: *Mathematical Programming* 52.1 (May 1991), pp. 359–375. ISSN: 0025-5610, 1436-4646. DOI: 10.1007/BF01582895.

Pérez-Díaz, Juan I., M. Chazarra, J. García-González, G. Cavazzini and A. Stoppato. 'Trends and challenges in the operation of pumped-storage hydropower plants'. In: *Renewable and Sustainable Energy Reviews* 44 (Apr. 2015), pp. 767–784. ISSN: 13640321. DOI: 10.1016/j.rser.2015.01.029.

Pindyck, S. 'Investments of uncertain cost'. In: *Massachusetts Institute of' Technologv. Cambridge, MA 02139, USA* (1993).

Pitorac, Livia, Kaspar Vereide and Leif Lia. 'Technical Review of Existing Norwegian Pumped Storage Plants'. In: *Energies* 13.18 (19th Sept. 2020), p. 4918. ISSN: 1996-1073. DOI: 10.3390/en13184918.

PWC. *Risikopremien i det norske markedet*. Survey. Dec. 2022. URL: https://www.pwc.no/no/publikasjoner/pwc-risikopremie-2022.pdf.

Pya, N. and S. Wood. 'Shape constrained additive models'. In: *Statistics and Computing* 2014 (Feb. 2014). DOI: 10.1007/s11222-013-9448-7.

Rahi, O.P. and A.K. Chandel. 'Refurbishment and uprating of hydro power plants—A literature review'. In: *Renewable and Sustainable Energy Reviews* 48 (Aug. 2015), pp. 726–737. ISSN: 13640321. DOI: 10.1016/j.rser.2015.04.033.

Reuter, Wolf Heinrich, Sabine Fuss, Jana Szolgayová and Michael Obersteiner. 'Investment in wind power and pumped storage in a real options model'. In: *Renewable and Sustainable Energy Reviews* 16.4 (May 2012), pp. 2242–2248. ISSN: 13640321. DOI: 10.1016/j.rser.2012.01.025.

Rodriguez, Paulo, Vasco Pesquita and J. P. Pereira. 'The effect of hydro and wind generation on the mean and volatility of electricity prices in Spain'. In: (2017), pp. 1–8. DOI: 10.1109/EEM.2017.7981915.

Schäffer, Linn Emelie, Arild Helseth and Magnus Korpås. 'A stochastic dynamic programming model for hydropower scheduling with state-dependent maximum discharge constraints'. In: *Renewable Energy* 194 (2022), pp. 571–581. ISSN: 0960-1481. DOI: https://doi.org/10.1016/j.renene.2022.05.106.

Schwartz, Eduardo and James E. Smith. 'Short-Term Variations and Long-Term Dynamics in Commodity Prices'. In: *Management Science* 46.7 (July 2000), pp. 893–911. ISSN: 0025-1909, 1526-5501. DOI: 10.1287/mnsc.46.7.893.12034.

Shapiro, Alexander, Wajdi Tekaya, Joari Paulo da Costa and Murilo Pereira Soares. 'Risk neutral and risk averse Stochastic Dual Dynamic Programming method'. In: *European Journal of Operational Research* 224.2 (2013), pp. 375–391. ISSN: 0377-2217. DOI: https://doi.org/10.1016/j.ejor.2012.08.022.

SINTEF. *Ancillary system services*. URL: https://www.sintef.no/en/expertise/sintef-energy-research/ancillary-system-services/ (visited on 9th Dec. 2022).

— *Kraftmarkeder og markedsmodeller for vannkraft*. URL: https://www.sintef.no/fagomrader/vannkraft/kraftmarkeder-og-markedsmodeller-for-vannkraft/ (visited on 22nd Nov. 2022).

Stathakis, Efthymios, Theophilos Papadimitriou and Periklis Gogas. 'Forecasting Price Spikes in Electricity Markets'. In: *Review of Economic Analysis* 13.1 (24th Mar. 2021), pp. 65–87. ISSN: 1973-3909. DOI: 10.15353/rea.v13i1.1822.

Tilmant, A. and R. Kelman. 'A stochastic approach to analyze trade-offs and risks associated with large-scale water resources systems'. In: *Water Resources Research* 43.6 (2007). _eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2006WR005094. DOI: https://doi.org/10.1029/2006WR005094.

Wen, Fushuan and A.K. David. 'Coordination of bidding strategies in day-ahead energy and spinning reserve markets'. In: *International Journal of Electrical Power & Energy Systems* 24.4 (May 2002), pp. 251–261. ISSN: 01420615. DOI: 10.1016/S0142-0615(01)00038-2.

Wolfgang, Ove, Arne Haugstad, Birger Mo, Anders Gjelsvik, Ivar Wangensteen and Gerard Doorman. 'Hydro reservoir handling in Norway before and after deregulation'. In: *Energy* 34.10 (Oct. 2009), pp. 1642–1651. ISSN: 03605442. DOI: 10.1016/j.energy.2009.07.025.

Yang, Chi-Jen and Robert B. Jackson. 'Opportunities and barriers to pumped-hydro energy storage in the United States'. In: *Renewable and Sustainable Energy Reviews* 15.1 (Jan. 2011), pp. 839–844. ISSN: 13640321. DOI: 10.1016/j.rser.2010.09.020.

Appendix

A Case parameters

The parameters of our case not considered relevant for apprehending our results are placed below in Table 5. Cost parameters are all derived from the report of Norconsult AS (2016). Index adjustments are calculated using NVE's adjustment report (NVE 2023b). The parameters of the two-factor price model are calibrated as described in Section 4.1.

Table 5: Parameter values

Parameter description	Symbol	Value	Unit
Penstock cost parameter	k_{pen}	12190	
Penstock cost parameter	u_{pen}	196.3	
Pelton turbine cost parameter	k_{pel}	2304	
Pelton turbine cost parameter	u_{pel}	-0.522	
Francis turbine cost parameter	k_{fra}	1036	
Francis turbine cost parameter	u_{fra}	-0.304	
Electrotechnical cost parameter	k_{el}^{n}	5.643	
Electrotechnical cost parameter	u_{el}^{n}	0.655	
Mechanical index adjustment	ψ_{ter}	1.79	
Electrotechnical index adjustment	ψ_{el}	1.81	
Construction index adjustment	ψ_{sta}	1.75	
Penstock index adjustment	ψ_{pen}	1.95	
Initial long-term price factor	$\exp(\xi_0)$	45.0	€/MWh
Initial short-term price factor	χ_0	0.00	€/MWh (log)
Short-term price mean reversion	κ_χ	1.768	
Short-term price risk premium	λ_χ	0.113	
Short-term price volatility	σ_χ	0.505	
Long-term risk-adjusted drift	$\mu_\xi - \lambda_\xi$	-0.008	
Long-term price volatility	σ_ξ	0.189	
Price factor correlation	$\rho_{\chi\xi}$	-0.207	
Risk-free rate	r	0.034	
Seasonality price parameter	ϕ_1	-0.028	
Shifted seasonality price parameter	ϕ_2	0.160	
Electricity price area differential	EPAD	30.0	€/MWh

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