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Using Branch and Price to Optimize Land-Based Salmon Production

Master's thesis in Industrial Economics and Technology Management
Supervisor: Peter Schütz
Co-supervisor: Jørgen Skålnes
June 2021

Master's thesis

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Kunnskap for en bedre verden

Preface

This thesis is written during spring 2021 and concludes our Master of Science at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management. The thesis builds on the work conducted in the specialization project within TIØ4500 Managerial Economics and Operations Research during fall 2020.

We want to address our sincere thanks to our supervisor Peter Schütz and co-supervisor Jørgen Skålnes, for their valuable feedback and insightful discussions throughout the months working on this thesis. Your guidance has greatly contributed towards the final product. Furthermore, a special thanks to Aquaculture Innovation AS represented by Ole Aas Skålnes, who has supported the project with relevant data and knowledge into the problem presented in this thesis.

June 11th, 2021

Johan Moldeklev Føsund & Erlend Hjelle Strandkleiv

Abstract

Production of salmon on land is an exciting new trend within the salmon farming industry and may have operational synergies with traditional sea-based salmon farming. Letting the smolt grow larger on land before being released into the sea is expected to enhance production levels and mitigate the biological risk connected to farming of salmon in the sea. In this thesis we examine a tactical production planning problem at a land-based salmon facility that can combine production of post-smolt intended for further grow-out in the sea, and production of harvestable salmon intended for the consumer market.

The work of this thesis has developed a deterministic optimization model for a tactical production planning problem at a land-based facility, proposed a Dantzig-Wolfe reformulation of the model, and implemented a Branch and Price algorithm to solve the problem. The model provides decision support to a land-based salmon farmer. It includes important production dynamics with associated costs and revenues, as well as how to plan the production to fully utilize available capacity and maximize total profits over the planning horizon.

The tactical production planning problem is mathematically modeled as a mixed integer program that determines when, where, and how much smolt to deploy, whether to transfer salmon between tanks during the production cycle, when to sell post-smolt, and when to sell harvestable salmon. The model takes into account different operational and important regulatory constraints. We distribute the total weight extracted to weight classes based on a normal distribution around the expected weight of an individual salmon in the population to model differences in individual growth rates.

We find that the Branch and Price algorithm succeeds in finding close to optimal solutions with a major reduction in computational runtime compared with a commercial mixed integer programming solver. This is mainly due to the improved dual bound from the Dantzig-Wolfe reformulation of the mixed integer program, in combination with the implemented matheuristic that is able to exploit the available columns to find high quality feasible solutions. Analyzing problem instances, we find that with only a minor reduction in total biomass produced, a profit maximizing production plan gives a 15% higher expected profit than a production plan with a biomass objective. Including the flexibility of transferring salmon between tanks during the production cycle results in 10% higher expected profit, and a change in production strategy when the aim is to maximize profits. Furthermore, post-smolt production is very attractive to fully utilize the production facility.

Sammendrag

Produksjon av laks på land er en spennende ny retning innen lakseoppdrett og kan ha operasjonelle synergieffekter med tradisjonelt sjøbasert lakseoppdrett. Å la smolt vokse seg større på land før de settes ut i sjøen er forventet å kunne øke produksjonsvolumene og minske den biologiske risikoen tilknyttet oppdrett av laks i sjø. I denne avhandlingen undersøker vi et taktisk produksjonsplanleggingsproblem for et landbasert oppdrettsanlegg som kan kombinere produksjon av post-smolt for utsett i sjø med produksjon av slakteklar laks (matfisk) for forbrukermarkedet.

Vi har utviklet en deterministisk optimeringsmodell for det taktiske produksjonsplanleggingsproblemet i et landbasert oppdrettsanlegg, foreslått en Dantzig-Wolfe reformulering av modellen og implementert en Branch and Price algoritme for å løse problemet. Modellen gir beslutningsstøtte til en landbasert lakseprodusent. Den inkluderer sentrale aspekter vedrørende produksjon med tilknyttede kostnader og inntekter, samt hvordan man planlegger produksjon for å utnytte tilgjengelig kapasitet og maksimere den totale profitten over planleggingshorisonten.

Det taktiske produksjonsplanleggingsproblemet er matematisk modellert som et blandet heltallsprogram som bestemmer til hvilke tidspunkt, i hvilke tanker og mengden smolt som settes ut, om flytting av laks mellom tanker inngår i produksjonssyklusen, og når salg av post-smolt og matfisk bør skje. Modellen tar høyde for ulike operasjonelle og viktige regulatoriske begrensninger. Vi fordeler den totale ekstraherte mengden laks ut over vektklasser basert på en normalfordeling omkring forventet vekt til en individuell laks i populasjonen for å fange forskjeller i individuell vekst.

Vi finner at Branch and Price algoritmen lykkes i å finne tilnærmede optimale løsninger med en drastisk reduksjon i kjøretid sammenlignet med en kommersiell programvare for blandede heltalls-program. Dette skyldes hovedsakelig forbedringen man får i den duale grensen som en konsekvens av Dantzig-Wolfe reformuleringen, i kombinasjon med en matheuristic som utnytter tilgjengelige kolonner til å finne lovlige løsninger av høy kvalitet. Gjennom å analysere instanser av det taktiske planleggingsproblemet finner vi at en profitt-maksimerende produksjonsplan gir 15 % høyere forventet profitt enn en produksjonsplan som søker å maksimere totalt ekstrahert biomasse, med en tilsvarende minimal nedgang i totalt ekstrahert biomasse. Det å ha muligheten til å flytte laks mellom tanker i løpet av en produksjonssyklus, resulterer i 10 % høyere forventet profitt, og endring produksjonsstrategi når målet er å makismere profitt. Videre er produksjon av post-smolt svært attraktivt for å utnytte produksjonsfasiliteten.

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Chapter 1

Introduction

The global demand for Atlantic Salmon, hereafter referred to as salmon, is increasing. Salmon represents a healthy source of protein, making it a highly valued product among a growing global population with more health-conscious consumers (Moe, 2019). The Norwegian Government aims to increase salmon production volumes five-fold within 2050 (Norsk Fiskerinæring, 2021). Norwegian production of salmon amounted to more than 1.4 million tonnes in 2019 according to Statistics Norway (2020), of which around 80% was exported (Ministry of Trade, Industry and Fisheries, 2020b). The Norwegian production of salmon represents around 60% of the global salmon production in 2019 (MOWI ASA, 2020). However, production of salmon farmed in the sea has seen a stagnation in production growth due to strict governmental restrictions as a consequence of continued biological and environmental challenges (Bjørndal & Tusvik, 2019a). Thus, there is a need for alternative ways of producing farmed salmon to increase the total supply from the salmon farming industry.

Whereas sea-based salmon farming faces continued biological and environmental challenges, production of larger salmon in land-based facilities is a new and viable alternative for the salmon farming industry (Craze, 2020). Land-based production technologies have been used by traditional salmon farmers in many years to produce smolt at the early stages of the salmon life cycle (Moe, 2019). The knowledge and technology developments acquired from this juvenile salmon production are important factors for why salmon grown in tanks on land is a viable production alternative. Additionally, land-based production facilities may have synergistic effects with the established sea-based salmon farming industry through the supply of larger smolt, referred to as post-smolt. It is expected that releasing post-smolt into the open-net pens will enhance production in sea-based facilities through shorter production cycles, less mortality, and improved fish welfare (Grieg Seafood, 2021). Grieg Seafood, one of the world's leading salmon farming companies, develops post-smolt as part of their main strategy towards more sustainable farming.

As the salmon farming industry is expected to increase the supply of salmon leading to increased competition among actors, an optimal production plan becomes a key factor for profitability (Guttormsen, 2008). The production plan, mainly specifying when to release and harvest salmon, impacts the allocation of scarce production resources such as space, fish and feed. Thus, it directly impacts the cash flow from the salmon farming facilities. The salmon farming industry relies heavily on traditional experience-based approaches to salmon farming. To increase value creation, Moe (2019) concludes that the industry needs to evolve from experience-based decision making to decisions based on insights derived from data-analysis and leading practices for fish-farming technology.

Forsberg (1999) addresses the issue of production planning as one of the most important managerial activities within salmon farming. The research within production planning in salmon farming focuses on traditional sea-based facilities. However, in a land-based facility options as post-smolt production and salmon transfer between tanks during the production cycle, introduce aspects that have not been studied in previous literature on production planning within salmon farming. Moreover, land-based facilities are subject to an emission permit restricting yearly production levels in addition to the maximum allowed biomass limit also faced by sea-based salmon producers.

The purpose of this thesis is to develop a tactical production plan for a land-based facility that can produce both post-smolt intended for further grow-out in sea and harvestable salmon intended for the consumer market. For this reason, we develop a deterministic optimization model for decision support to a land-based salmon producer facing challenges of how much, when, and where to deploy smolt, whether to transfer salmon between tanks during the production cycle, when to sell post-smolt and when to sell harvestable salmon.

Modeling a production plan that includes post-smolt production and the possibility to transfer salmon throughout the production cycle at a tank-level of detail results in a complex problem, mainly in terms of computational runtime. Therefore, we apply a Dantzig-Wolfe reformulation of the problem and develop a Branch and Price algorithm to solve the tactical production planning problem. This is a novel approach to solve the production planning problem for salmon farming. The solution method enables us to evaluate different aspects of the production plan for a full-scale facility in terms of changing objective functions, the value of including the flexibility of transferring salmon, and the attractiveness of post-smolt production. Evaluating these aspects help gain managerial insight into the production planning problem for a land-based salmon producer.

The remainder of the thesis is organized as follows. In Chapter 2 we introduce the salmon farming industry with an emphasis on land-based salmon farming. We review some of the literature on production planning within aquaculture in Chapter 3. In Chapter 4 we introduce the tactical production planning problem we study in this thesis. The mathematical model that represents the problem is formulated in Chapter 5, while Chapter 6 presents the solution method applied to solve the problem. In Chapter 7 we present a case study explaining input data to the model and introducing problem instances used to derive insight into the production planning problem. Computational results are presented and analyzed in Chapter 8. In Chapter 9 we point out interesting directions for future research on the tactical production planning problem within land-based salmon farming. We conclude and present final remarks in Chapter 10.

Chapter 2

Industry Background

In this chapter, we introduce relevant aspects affecting salmon production and the production planning process. In Section 2.1 we present the value chain of salmon farming to give an understanding of the input and output of the different production stages in the salmon farming industry. We emphasize how the introduction of land-based salmon farming alters the traditional value chain of salmon farming through post-smolt production and describe how the production environment changes with land-based salmon farming. In Section 2.2 we introduce aspects affecting the biomass development at the production facility. In Section 2.3, we introduce the salmon markets and the main cost drivers in land-based salmon farming. The production process in land-based production facilities is regulated by a set of regulatory restrictions, which we introduce in Section 2.4. Finally, in Section 2.5 we give a brief introduction to the production planning process and the tactical decisions that need to be made.

2.1 Value Chain of Salmon Farming

The value chain of salmon farming is segmented into different stages that reflect the life cycle of salmon. Although it includes actors within the supply of technical solutions, biotechnology, and logistics, we only introduce the production part of the value chain, which is the focus of this thesis. Salmon is an anadromous species, meaning they are born in freshwater but spend most of their lives in saltwater (MOWI ASA, 2020). The value chain can be split into a freshwater phase and a saltwater phase, as depicted in Figure 2.1. In the following, we present the two different phases and emphasize how the introduction of land-based salmon farming alters the traditional value chain in the industry.

2.1.1 Freshwater Phase

The freshwater phase takes place in facilities located on land and has a duration of around 10-16 months (Moe, 2019). It comprises egg and spawn production and production of smolt. The egg and spawn (production) segment specializes in producing optimal genetic material for further production. Following the spawn of a broodstock salmon, the eggs are placed in incubators keeping the temperature at 8 degrees Celsius, and it takes around 60 days to hatch (SalMar ASA, 2019). At this point, the fish is referred to as fry, and in the initial period after hatching it feeds on the content of its yolk sack.

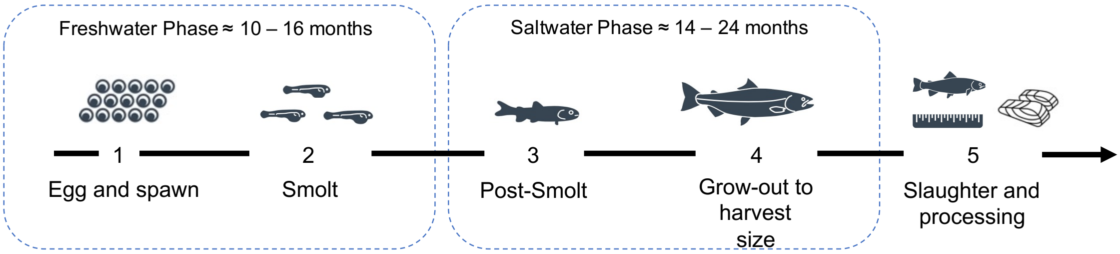


Figure 2.1: Segments in the land-based salmon farming value chain

Smolt producers manage the process from fertilized fry to delivery of smolt, covering segment two in Figure 2.1. By controlling temperature, using light manipulation, and different feeding regimes, the growth rates of the fish can be controlled (Stefansson et al., 2005). Towards the end of the smolt production cycle, the fish undergoes a smoltification process, which is a biological transition enabling the smolt to survive in a saltwater environment. In the wild, the smoltification process naturally occurs with the arrival of summer and longer days. However, in a controlled environment on land, the smoltification process can be manipulated by artificial light, meaning smolt can be delivered throughout the whole year.

Many of the companies operating in the segment that produces eggs also incorporate smolt production as part of their business (Moe, 2019). Due to the importance of the genetic material in the first stages for later successful salmon farming, large salmon farming actors have made this an integral part of their production. Traditionally, the production of eggs and smolt has easily been scaled to match demand (MOWI ASA, 2020; Asche and Bjørndal, 2011). Hence, lack of smolt is usually not a concern for a salmon producer operating in the saltwater phase.

2.1.2 Saltwater Phase

Following smoltification, the smolt are transferred into a saltwater environment for further growth. In traditional sea-based farming, this means that smolt are transferred by well-boats to open-net pens in the sea. Hence, the saltwater phase in the traditional value chain of salmon farming only consists of segment four in Figure 2.1. Due to biological constraints concerning seawater temperatures, the industry typically operates with one release window during spring and one release window during fall (MOWI ASA, 2020). In sea, salmon are fed until they reach harvestable sizes, typically ranging from 3 to 6 kg. The growing process in sea takes around 14-24 months, depending on water temperatures (Moe, 2019). Regarding the length of the growing process, a biological detail of importance is that the value of salmon deteriorates when they are sexually maturing (Asche & Bjørndal, 2011). In consequence, the growing process comprising segments three and four in Figure 2.1, usually does not exceed two years.

In land-based salmon farming, the smolt are transferred to tanks filled with saltwater on land. Up until sizes of around one kilogram, the salmon is referred to as post-smolt. There is an increased interest from sea-based salmon farmers for post-smolt (Olsen, 2020). Using post-smolt instead of smolt is expected to enhance the production volumes in sea-based salmon farming. It gives higher flexibility and reduces the time in sea during which the salmon are subject to pathogens that can cause diseases and deaths. This introduces an option for a land-based salmon producer to sell post-smolt of sizes around one kilogram to a nearby sea-based salmon farmer. Salmon typically reach sizes of around one kilogram after six to eight months of growth (Food and Agriculture Organizations of the UN, 2021).

Land-based facilities producing only harvestable salmon follow the same procedure as in sea-based facilities by feeding the salmon until it reaches harvestable sizes. Facilities that choose to control water temperatures are expected to shorten the growing process compared with sea-based facilities (Bjørndal & Tusvik, 2019a).

Upon harvesting, the salmon are typically starved for some weeks to ensure proper hygiene in the further processing (Waagbø et al., 2017). When salmon are harvested, they are transported to a processing facility. At the processing facility, the salmon are slaughtered, gutted, and processed through multiple steps before being sent out to the market (SalMar ASA, 2019). There is a harvest yield on salmon, as inedible parts of the salmon need to be removed. Typically, the harvest yield is around 85%, meaning that 85% of the salmon weight is left after removing inedible parts (Skretting, 2021a). Thus, a salmon weighing five kilograms yields 4.3 kilograms after gutting, which is the weight that is sold to the market.

2.1.3 Changes in the Value Chain Caused by Land-Based Farming

From the value chain description above, it is apparent that post-smolt production is a stage that is introduced by land-based salmon farming. Moreover, the introduction of land-based salmon farming naturally changes the environment in which the salmon grow. In the following, we emphasize how these two aspects, post-smolt production, and production environment, change the value chain compared to the traditional view where smolt are released into the sea for grow-out to harvest size.

Post-Smolt Production

The increased interest in post-smolt is clearly reflected through the large investments made in land-based post-smolt facilities (Bjørndal et al., 2018). Lerøy Seafood Group invested NOK 650 million into a post-smolt production facility in 2017 to supply the company's sea-based production sites with seven million post-smolt yearly (Berge, 2017). Moreover, Grieg Seafood attempts land-based salmon farming through a joint venture where Årdal Aqua will produce at least 3 000 tonnes post-smolt yearly and grow salmon to harvestable sizes (Salmon Business, 2021). The idea of producing post-smolt is based on a desire to improve the production efficiency of a sea-based salmon farmer (Hilmarsen et al., 2018). Deploying post-smolt will reduce the production cycle duration for a sea-based salmon farmer, thus enabling an increased number of production cycles. Increasing the number of production cycles will increase the utilization of expensive production licenses for a sea-based farmer. A shorter production cycle will also reduce the time salmon are exposed to sea-lice, and this may reduce production costs (Moe, 2019). Moreover, post-smolt will be more robust to survive the transition into sea, also at lower temperatures. This will reduce production losses connected to smolt-release in sea-based farming (Nofima, 2021). The robustness towards sea temperatures will allow sea-based salmon producers to initiate a production cycle outside the traditional release windows (Akvaplan Niva, 2021). The abovementioned benefits are expected to increase the capacity and utilization of sea-based production facilities and explain the emergence of demand for post-smolt in recent years.

Production Environment

With land-based salmon farming, the production environment in the saltwater phase changes to tanks filled with saltwater on land instead of open-net pens in the sea for traditional sea-based farming. The land-based facilities are typically designed to include separate bio-secure zones (Salmon Evolution AS, 2021). The bio-secure zones are commonly referred to as modules, where one module may comprise one or several tanks. This design choice minimizes the risk of diseases being transferred to salmon in other tanks belonging to other modules, as the salmon and the water in one module are not shared with other modules. This allows a land-based salmon producer to have salmon stemming from different generations at the same time within the same facility, whereas a sea-based producer can only have one generation of salmon in their facility at the same time due to the risk of transferring diseases across generations (MOWI ASA, 2020).

2.2 Biomass Development

Knowledge about biomass development is essential for any salmon producer to make the correct decisions in a production process as it is decisive for complying with the regulatory constraints concerning salmon production levels. In the following, we introduce some of the factors affecting salmon growth and also address the concern of production losses at a production facility operating in the saltwater phase of the value chain.

2.2.1 Factors Affecting Growth

As salmon is a cold-blooded animal, its growth is highly reliant on water temperatures (MOWI ASA, 2020). Variations in water temperatures are considered to be the single most important biophysical factor causing variations in the grow-out period of salmon (Thyholdt, 2014). Figure 2.2 illustrates how the expected salmon growth varies when the salmon is subject to different temperatures. Most efficient growth is found in experiments at water temperatures of around 13 degrees Celsius (Thyholdt, 2014). Thus, having the ability to control water temperatures in land-based facilities may enhance and stabilize growth performance. The initial production cycle from Norway's first land-based salmon farm showed some individuals growing from 100 grams to more than 7 kg in less than 12 months in a controlled temperature environment (Fletcher, 2020).

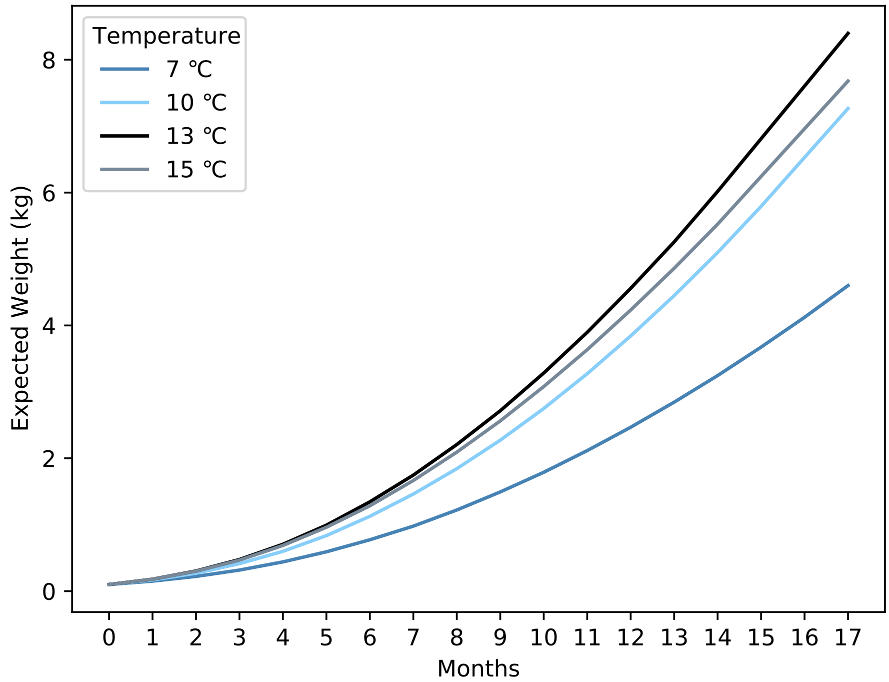


Figure 2.2: Expected salmon growth for different constant water temperatures (Skretting, 2018)

Other factors influencing salmon growth include water quality parameters such as oxygen, pH, salinity, ammonium, carbon dioxide levels, and water current rates. In land-based salmon farming, the farmer can control these factors to some degree, resulting in more control over the production process than sea-based salmon farming.

The salmon producer wants minimal handling of the salmon to improve growth rates and fish health (Salmon Evolution AS, 2021). When transferring salmon between tanks, a reduction in growth during the following weeks can be expected. This is because stress affects salmon appetite, growth rates and mortality (Basrur et al., 2010; Iversen et al., 2005). Due to the reduction in salmon growth, there are uncertainties within the industry whether salmon transfer between tanks should be included as an option in the production planning.

According to the expected growth of salmon as illustrated in Figure 2.2, we observe that it takes around 5–8 months for the salmon to reach post-smolt sizes of around 1 kg depending on the temperature profile. Further, the salmon reaches weights of around 4–8 kg after 12–18 months of growth. Remark that we here have assumed constant temperature profiles for illustrative purposes.

The general uncertainty in biomass development due to growth includes two aspects. First, there is uncertainty in the overall population's growth performance due to uncertain conditions in the environment, such as seawater temperatures. Second, there is uncertainty in the growth distribution of a salmon population due to individual differences in the growth rates of salmon of the same size (MOWI ASA, 2020). MOWI ASA (2020) expects a salmon population at harvestable sizes to exhibit a normal distribution.

2.2.2 Production Loss

Throughout the grow-out phase of a salmon population, a salmon producer needs to expect production losses. The causes of production losses in traditional salmon farming at sea typically involve sea-lice, disease outbreaks, mortality of young fish related to release into seawater, and salmon escapes. In recent years traditional sea-based producers have registered a production loss of 15–20% over a year (Mikkelsen, 2020). Some of the sources causing production losses in sea are assumed to be non-present in land-based farming, such as salmon escapes, sea-lice, and disease outbreaks. This is expected to reduce production losses in land-based salmon farming compared to sea-based salmon farming. The initial production cycle from Norway's first land-based salmon farm revealed a production loss of 5% (Fletcher, 2020).

2.3 The Economics of Salmon Farming

The general trend in salmon prices is that higher salmon weight yields higher income. In addition to generating revenues, production of salmon infers several costs that need to be accounted for in the production planning. This naturally leads to a trade-off, making it necessary to evaluate both revenues and costs related to production for a profit maximizing salmon producer. In Section 2.3.1 and Section 2.3.2 we introduce the two different markets of post-smolt and harvestable salmon. Then, in Section 2.3.3, we present the main cost drivers in land-based salmon production.

2.3.1 Post-Smolt Market

Deploying post-smolt into a sea-based facility is a new production strategy. Therefore, a land-based farmer that produces post-smolt faces a new market of which the demand and expected post-smolt prices are challenging to estimate. Post-smolt prices are not publicly available as no official marketplace exists for this product. However, one could use estimated production costs as an estimate of possible price levels. Based on estimates by Bjørndal and Tusvik (2018) and Berget (2016), a land-based salmon producer could expect the price of one kilogram post-smolt to be at least at levels around 40-50 NOK/post-smolt.

2.3.2 Harvestable Salmon Market

Salmon is sold either through negotiated contracts between a salmon producer and a wholesaler or on the spot market (MOWI ASA, 2020). The NASDAQ Salmon Index depicted in Figure 2.3, is the weighted average of weekly reported NOK/kg sales prices of fresh Atlantic Superior Salmon (NASDAQ, 2021). The prices from the salmon index are representative for the export prices out of Norway. As seen in Figure 2.3 these prices are subject to high volatility. Further, the salmon prices depend on the size of the salmon. The different weight classes are segmented from one to more than nine kilograms, and the price per kilogram increases with larger salmon weight.

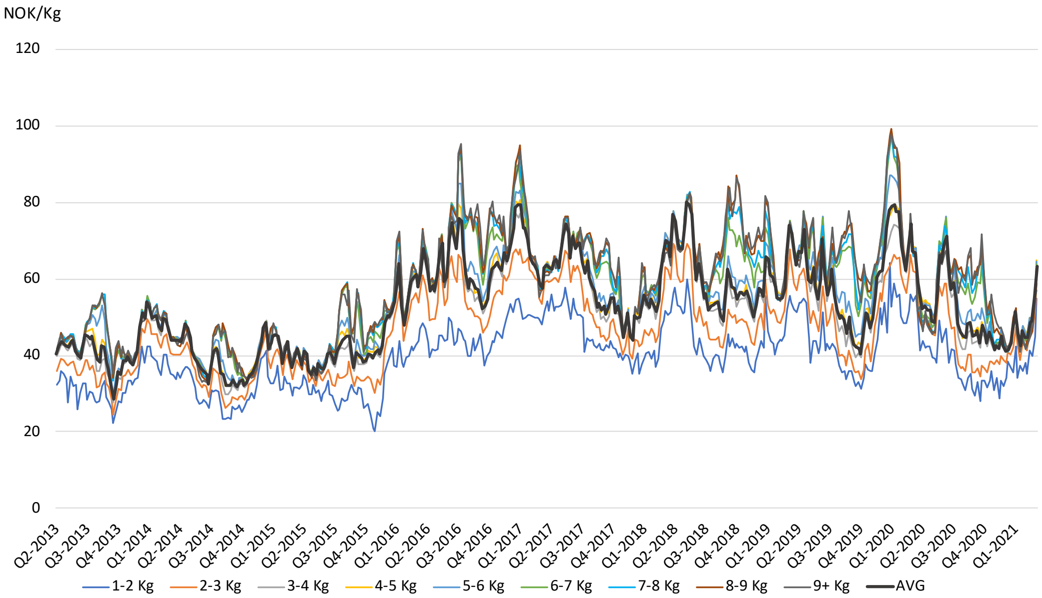


Figure 2.3: Salmon Price Index by quarter since Q2 2013 (NASDAQ, 2021)

In Norway, around 32% of salmon are harvested at target weights of four to five kilograms, pictured in Figure 2.4. However, some farmers may want to harvest at both lower and higher target weights to balance market risk and biological risk. Drivers behind smaller harvest weights can be the fear of a major incident occurring that results in mass death and huge value losses, the need for immediate cash, the fear of price decline in following months, or to release space to initiate a new production cycle. The reasons for larger harvest weights include exploitation of economies of scale, expectations of price increase in the near future, and production for niche markets.

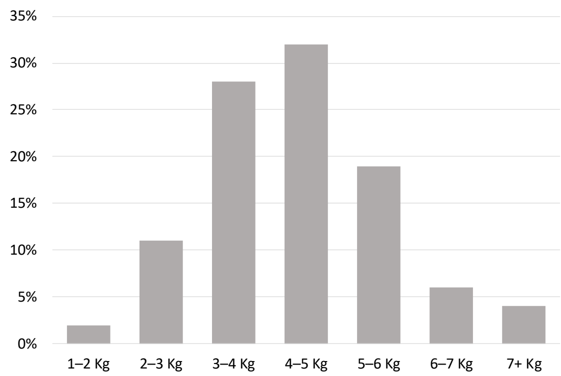


Figure 2.4: Weight distribution of salmon harvested in Norway in 2019 (MOWI ASA, 2020)

2.3.3 Cost Drivers in Land-Based Salmon Farming

Whereas market dynamics decide salmon prices, costs are more controllable for a salmon producer. Bjørndal and Tusvik (2019a) estimate that the three main production cost drivers for a land-based facility are feeding, energy, and oxygen. Smolt procurement costs will also be a major cost component for land-based facilities. Although the production costs of producing salmon in a land-based facility are highly uncertain, they are expected to be higher compared to producing salmon in sea-based facilities (Aukner & Hanstad, 2019). In the following, we introduce the main cost drivers related to the production-level in a land-based facility.

Smolt costs

Some land-based facilities aim to include smolt production as an integral part of the production (Salmon Evolution AS, 2019). Other facilities will buy the smolt from smolt suppliers. In both cases, smolt procurement infers costs on the production. The standard industry price of smolt is at around 14-16 NOK/smolt (Bjørndal & Tusvik, 2020). Smolt procurement costs will account for a larger share of the total production costs producing post-smolt compared with producing harvestable salmon.

Feeding Costs

Feeding cost is one of the main components of the costs related to salmon production, even though salmon is one of the most efficient species to convert feed into biomass (Skretting, 2021b). The industry operates with a feed conversion ratio (FCR) which measures how much food salmon need to gain one kilogram weight. This ratio increases with increasing salmon weights and is estimated to be 1.15 for a harvestable salmon of weights 3–4 kg (Skretting, 2018). This means that the salmon needs 1.15 kg feed to grow one kilogram in biomass. In comparison, the FCR of chicken is estimated to be 1.6. For land-based salmon farming, it is assumed that one of the advantages to sea-based salmon farming could be lower FCR due to the increased control over the production system (Bjørndal & Tusvik, 2019a). The typical cost of feed is at 13-14 NOK/kg (Bjørndal & Tusvik, 2020).

Energy Costs

The energy costs in a land-based facility depend on the choice of production technology. The two main production technologies used for land-based salmon farming are Recirculating Aquaculture Systems (RAS) and Flow-Through Systems (FTS), which mainly differ in the degree of water recycling (Craze, 2020). In RAS facilities up to 99% of the water is recycled, which reduces water consumption. However, RAS is a complex production technology as it requires both mechanical and biological filters to clean and recycle the water in the system (Aukner & Hanstad, 2019). FTS facilities typically keep the level of recycling below a threshold of around 70% (Salmon Evolution AS, 2020). FTS facilities will have higher water consumption and pumping costs compared with RAS facilities. On the other hand, FTS technology reduces the biological and operational risk due to less complex water-treatment units. The facility design in terms of height differences in the pipe-system will impact energy costs, as energy is required to lift the water. Deciding to control water temperatures will also naturally incur large energy costs.

Pumping water in the facility incurs energy costs that vary according to the production level. Higher densities of salmon typically require more supply of freshwater, hence causing higher pumping costs. In practice, some land-based facilities operate with a certain residence time of water in a tank. This means that all water in a tank needs to be replaced within a certain amount of time. Another way to estimate the amount of water needed is to make it a function of biomass or feed consumption. In a RAS-facility it is normal to calculate the water consumption as a function of feed consumption (Bjørndal et al., 2018). In normal production, an estimate of 300–500 liters of water per kilogram feed per 24 hours is estimated. In FTS-facilities water consumption of 0.3 liters per minute per kilogram salmon is normally estimated.

Oxygen Costs

Oxygenation of the water will be a cost driver in land-based facilities. The oxygen consumption of a salmon is dependent on the weight of the salmon and the water temperature (Fivelstad & Smith, 1991). Bjørndal et al. (2018) estimate a usage of approximately one kilogram of oxygen per kilogram harvested salmon. The typical oxygen cost is at around 2-3 NOK/kg. Up to 70 % of the total water consumption in FTS facilities can be recirculated by adding oxygen and removing carbon dioxide (Salmon Evolution AS, 2021).

2.4 The Regulatory Framework of Salmon Farming

To encourage a sustainable increase in production levels, the Norwegian Government decided in 2016 to grant land-based salmon production licenses free of charge upon application (Ministry of Trade, Industry and Fisheries, 2020a). Applications for production of salmon on land are processed at county level, and promising projects are issued permits continuously. This change in industry regulations has drastically lowered the barrier to entry compared with sea-based salmon farming, where one production license can cost around NOK 150-160 million (Tveterås et al., 2020).

However, the production at a land-based facility needs to comply with regulations that restrict production levels in the facility. Production of salmon is regulated through the Aquaculture Act and by health and welfare restrictions enforced by the Norwegian Food Safety Authority (Fjørtoft & Fondevik, 2020). In the following, we present the three most relevant regulations that decide the production levels in land-based salmon farming.

2.4.1 Maximum Allowed Biomass Limit

The maximum weight of biomass that a production facility can hold at any given time is referred to as the maximum allowed biomass (MAB) limit and applies to both sea-based and land-based salmon farming. A salmon producer can hold several production licenses distributed around on several production facilities. In sea-based salmon farming, one production license allows a

MAB limit of 780 tonnes, whereas land-based salmon producers may be granted a higher MAB limit associated with a production license (Fjørtoft & Fondevik, 2020).

2.4.2 Emission Permits

To limit the amount of wastewater released from the facility, land-based salmon farming needs emission permits (Norwegian Enviroment Agency, 2020). An emission permit restricts the total production of salmon during one year and basically acts as a yearly production limit for a land-based facility. This differs from sea-based salmon farming, which does not have emission permits restricting the yearly production of salmon (Ministry of Trade, Industry and Fisheries, 2016). Land-based salmon farming can include sludge treatment systems cleaning the wastewater before being released and therefore have less ecological footprint than farming in sea. This is expected to make it easier to obtain emission permits with acceptable conditions (Fjørtoft & Fondevik, 2020).

2.4.3 Fish Health and Welfare Restrictions

A major focus for aquaculture regulations is fish welfare (Fjørtoft & Fondevik, 2020). As stated by the Aquaculture Act, parameters as water quality, dissolved oxygen level, and water flow-through rate at the production facility must be monitored to facilitate good living conditions for the salmon (Norwegian Ministry of Fisheries and Coastal Affairs, 2005). The density of salmon is one of the factors affecting fish health. The ability to control water flow rates enables land-based salmon producers to produce at higher densities than sea-based salmon producers. In land-based production, the salmon producer can decide on the maximum level of salmon density, as long as good fish welfare is documented (Fjørtoft & Fondevik, 2020). The first results from land-based production in Norway reported successful production at densities up to 95 kg/m^3 (Fletcher, 2020). In traditional sea-based farming, salmon densities cannot exceed a maximum limit of 25 kg/m^3 within an open-net pen to ensure fish welfare.

2.5 The Production Planning Process in Land-Based Facilities

Having introduced some of the aspects within the salmon farming industry, this section will focus on the production planning process of a land-based salmon producer operating in the saltwater phase. The objective of the production plan is discussed in Section 2.5.1. We give an overview of the production planning decisions facing a land-based salmon producer in Section 2.5.2. In Section 2.5.3 we highlight complicating elements in the production planning process. Finally, the motivation for using an optimization model as decision support is presented in Section 2.5.4.

2.5.1 Production Planning Objective

Land-based facilities are expected to have a high focus on the profit margins of salmon production (O. Skålnes, personal communication, 2021). This is mainly due to the high investment costs and the increased control over the production system. In traditional sea-based salmon farming, the focus has mainly been on how to increase production levels to utilize the expensive production licenses. Due to high salmon prices, this has traditionally led to a target on maximizing total biomass production rather than maximizing total profits.

A land-based facility is faced with the option of selling post-smolt in addition to harvestable salmon. As post-smolt and harvestable salmon production represent different cost profiles and expected revenues, finding the production levels that result in the highest expected profit is a complex challenge.

2.5.2 Production Planning Decisions

For a land-based salmon producer, the main decisions in the production planning process are the timing and allocation of smolt deployments and when to extract either post-smolt or harvestable salmon.

Smolt Deployment Strategy

With the arrival of smolt, a production cycle is initiated. For all purposes in this thesis, a production cycle starts at deployment of smolt into a module and lasts until all tanks belonging to the module are emptied. The first step of the production planning is to decide on smolt deployment. This includes both the timing of the deployment and the amount of smolt to deploy in the production cycle.

Next, the salmon producer needs to decide on the allocation of smolt into specific production units. In a land-based facility, this means to decide which tanks should receive the amount of smolt to deploy. The various tanks could be of different volumes depending on the facility design. Choosing to deploy smolt into a specific tank will initiate production costs and occupy capacity until the salmon is extracted as either post-smolt or harvestable salmon. Therefore, the challenge for a salmon producer is how to allocate smolt into tanks to minimize the overall cost profile. Land-based facilities that support transfer of salmon between tanks during the production cycle will add additional complexity to this allocation decision.

Post-Smolt and Harvest Strategies

Following the deployment of smolt into a tank, the salmon producer must decide on how long the salmon should stay in the tanks to maximize overall profits. Having salmon in a tank introduces costs in terms of feed, supply of oxygen, and energy for water pumping. On the other hand, the salmon weight gain gives additional value in terms of future revenues. The process of developing a production plan is more complicated for salmon producers choosing to include production of post-smolt, as they are faced with the extra option of extracting the salmon as post-smolt. Selling post-smolt will not introduce any further production costs and will naturally also not introduce any additional revenues. If deciding not to sell post-smolt, the salmon is fed to harvestable sizes.

2.5.3 Complicating Elements in the Production Planning

The task of producing a profit maximizing plan for a land-based facility is challenging. The decisions in the production plan are associated with costs and revenues, and the production needs to adhere to a set of regulatory restrictions. Decisions on when, where, and the amount of salmon to deploy and extract at what point in time must be synchronized to not violate the tank density limit, MAB limit, or the emission permit. The option to extract both post-smolt and harvestable salmon and the option of transferring salmon between tanks during the production cycle are additional complicating factors in the production planning.

2.5.4 Optimization-Based Decision Support

Many salmon producers develop their production plans manually based on experience from sea-based production and previous successes. This may very well result in good production plans, but the process is cumbersome and repetitive. Furthermore, as land-based facilities introduce the option of post-smolt production, experience from sea-based production plans is not necessarily still valid. Using an optimization model that quickly generates production plans, can provide decision support for the production planning in a land-based salmon facility and allow the salmon producer to re-optimize the production plan as actual biomass development in the facility is observed. The common practice in the industry is to plan salmon production for the upcoming three to four years.

Chapter 3

Literature Review

In this chapter we introduce the most relevant research on production planning within aquaculture, enabling us to position this thesis into the literature. Section 3.1 limits the scope of the literature review and presents a description of the search strategy used to find existing literature. In Section 3.2, we present resulting articles and research made on production planning within the aquaculture industry. The literature on different growth modeling techniques applied to bioeconomic models is discussed in Section 3.3. In Section 3.4, we position this thesis in relation to the surveyed literature.

3.1 Literature Scope and Search Strategy

The following literature review is an updated version of the review made in Føsund and Strandkleiv (2020). The scope of the review is narrowed to focus on the use of mathematical models to find an optimal production plan within aquaculture, and how these models are integrated with the modeling of salmon growth, i.e. biomass development. The modeling of biomass development is a fundamental part of the production plan in aquaculture (Forsberg, 1996).

The search strategy for finding relevant literature has been guided by the review paper of Bjørndal et al. (2004). This paper investigates the role of operational research in the understanding and management of renewable resources within aquaculture. To find additional relevant literature, broad and general searches were conducted on Google Scholar and on NTNU's literature database, Oria. Some of the most applied broad search terms and additional filters used in combination during this search are depicted in Table 3.1.

Further literature was found by investigating the bibliography and referenced articles in the papers that were deemed most relevant in the broad search. This resulted in a list of articles enabling us to review some of the work done within the use of growth models and operational research models on production planning within aquaculture.

Table 3.1: The most applied search terms with different filters used in combination with the terms

Broad Search Terms	Additional Filters
Production Planning	Dynamic Programming
Aquaculture	Linear Programming
Salmon Farming	Mixed Integer Programming
Optimal Harvesting Strategy	Operational Research
Salmon Production	Optimization
Optimal Rotation Problem	Profit Maximization
Salmon Growth Modeling	Biomass Maximization

3.2 Production Planning in Aquaculture

When planning to maximize production over the upcoming years, salmon producers need to decide on which deployment and harvest strategy they want to follow (Forsberg, 1996). These strategies include the number of smolt to deploy, the number of salmon to harvest and the timing of these activities. Deciding on an optimal harvest strategy is closely linked to the optimal rotation problem from forestry production and was the inspiration for the first work within the literature conducted by Karp et al. (1986) to find an optimal harvest strategy for aquaculture (Guttormsen, 2008). The optimal rotation problem was originally introduced by Faustmann (1849) and consists of finding the optimal harvest time and harvest level to maximize the net present value of a standing stock of timber.

Karp et al. (1986) established the link between the rotation problem in forestry and finding the optimal deployment and harvest sequence, i.e. the optimal rotation, in aquaculture. They also expanded the problem to let the optimality conditions not only include optimal harvest levels but also include optimal restocking levels. Bjørndal (1988) presents an optimal harvesting strategy for fish farming using a bioeconomic model. The paper links the optimal rotation problem with a biological model on fish growth. The rotation problem aims to find the start periods of production cycles and the production cycle length that maximizes the present value of an investment. In light of this, one could say that Bjørndal (1988) views the optimal harvesting strategy of salmon as nothing more than finding the period of harvesting that maximizes the net present value of the cash flows given a specific smolt deployment. Later, several authors extended the model introduced by Bjørndal (1988). Arnason (1992) includes the modeling of an optimal feeding schedule together with an optimal harvest strategy to maximize profits, and Hean (1994) extends the model to include the optimal number of smolt to deploy, a feature which was not included in the model presented by Bjørndal (1988).

Guttormsen (2008) argues that the shortcoming of most research within optimal harvesting, including the literature mentioned above, is that the analyses omit the aspect that harvesting salmon releases capacity for a new deployment of smolt. He claims that focusing on a one-shot decision rather than focusing on decisions for an optimal rotation where harvesting makes room for new releases, at best gives a rough estimate of the optimal harvest time. Guttormsen (2008) solves the optimal rotation problem by presenting a dynamic programming model that incorporates smolt release in specific periods and relative price relationship between fish of different sizes. Guttormsen (2008) also includes seasonal variations by having a growth function dependent on salmon weight and seawater temperatures. Based on empirical data, his extended model shows that including relative price relationships affects the timing of harvesting.

Forsberg (1996) introduces a multi-period linear programming model that aims at optimizing the number of smolt released and the number of salmon harvested to maximize profits. Salmon is structured into size classes dependent on weight, and salmon growth is modeled using the Markovian assumption. This assumption implies that all salmon in a size-class, even large and small salmon within a particular size-class, have an equal probability of reaching the next size-class, irrespective of the length of time the various fish took to reach their present size. A size-structured approach is advantageous when size-grading equipment allows the fish farmer to harvest various size-classes separately, and in cases where transfer or removal of individual salmon into or out of a size-class, is important for the management of the stock. Forsberg (1999) further extends the model to include batch harvesting and demonstrate that it is more profitable to size-grade fish prior to harvest compared to harvesting a batch of fish with similar size distribution as the standing stock.

The bioeconomic models introduced by Bjørndal (1988), Guttormsen (2008) and Forsberg (1996) focuses on the optimal deployment and harvesting strategy for a single production unit. However, the introduction of a maximum allowed biomass limit on Norwegian salmon production makes it essential to capture all production units involved in the production system. Replicating the optimal production plan found for one production unit to all production units may violate the overall regulatory restrictions.

Hæreid (2011) models the production planning problem of sea-based salmon farming while maintaining the MAB limit and taking uncertain growth and price development into account using a multi-stage stochastic mixed integer model. The model includes decisions on when to deploy and harvest salmon and how to allocate sales between available contracts and sales in the spot market to maximize profits. Using discretized fish classes, Hæreid (2011) models biomass development by distributing salmon into the two fish classes closest to the estimated salmon growth. Langan and Toftøy (2011) introduce a two-stage stochastic linear programming model for sea-based salmon farming with a cost related to breaking the MAB-limit. However, this cost is set to infinity to ensure the limit is never surpassed. They pre-decide several operational decisions, as the length of the production cycles, which allow them to avoid the computational complexity of a mixed integer program. Based on the work of Hæreid (2011), Rynning-Tønnesen and Øveraas (2012) present a multi-stage stochastic model with a more realistic decision process for ordering and deployment of smolt and time-dependent aggregation of sea-based production sites. Their key finding is that to increase profit the average harvest weight should be adjusted according to seasonal temperature variations.

A limitation of the mixed integer programs above is that decisions on fallowing, which is necessary in sea-based salmon production to let the sea-bed recover, are predetermined. Having decisions allowing for different lengths of the fallowing period provides more flexibility to the salmon producer with respect to maximize production. Næss and Patricksson (2019) introduce a mixed integer model with the possibility of deciding the fallowing duration. Such increased flexibility increases model complexity, and site aggregation is necessary to solve stochastic instances.

3.3 Biomass Modeling

The models developed by Forsberg (1996), Hæreid (2011) and Rynning-Tønnesen and Øveraas (2012) structure salmon into discretized size classes. A disadvantage of having a growth model with distinct size classes is the large number of variables needed in the mixed integer programming model, which restricts the size of the problem that can be solved within a reasonable amount of time (Langan & Toftøy, 2011). Growth models which only keep track of the mean size of the salmon population will reduce the number of variables in the linear programming model and increase computational tractability. Langan and Toftøy (2011) model the production planning problem for a sea-based producer using the mean weight of the salmon population in a production unit with the objective of maximizing total profits. A limitation of their model is that they assume all salmon in a population to have a weight equal to the mean weight of the population. However, the size within a salmon population when reaching harvest weights is normally distributed. This can result in different revenues since the revenue per kilogram salmon is dependent on the weight of the salmon.

Yu and Leung (2005) include the size distribution when harvesting shrimps. They introduce a linear network formulation of the optimal scheduling model for a multi-cycle and multi-pond shrimp operation based on the original optimal harvesting theory proposed by Bjørndal (1988) for a single production unit. The optimal grow-out schedule comprises restocking and harvesting time maximizing the total profits. Yu and Leung (2005) include harvest size distribution in a shrimp population by introducing a size distribution function which transforms the total weight of the shrimp population into a distribution of weights dependent on the mean shrimp weight in the population. This technique is also applicable for modeling salmon production. Aasen (2020) introduces a growth model which uses mean population size and transforms the weight of harvested salmon to distinct weight classes according to a size distribution that is dependent on the mean weight of the harvested salmon. This model was used for a sea-based salmon production problem with novel smolt types, i.e. having the possibility of deploying different types of smolt into the production units.

3.4 Our Contribution to Existing Literature

This chapter reviews the most relevant literature within the field of production planning for aquaculture. Most of the early research focused on finding the optimal deployment and harvest strategy for a single production cycle using economical models with extensions of the optimal rotation model for aquaculture introduced by Bjørndal (1988). Guttormsen (2008) further extends the problem by including seasonality and planning for an infinite amount of production cycles using dynamic programming. The literature mentioned above focuses on a single production unit and excludes regulatory restrictions which affect the production on a facility and on a company level. Hæreid (2011) introduces a mixed integer programming model inspired by Forsberg (1996), which includes multiple production units and the regulatory MAB restrictions both on a facility and on a company level (Ministry of Trade, Industry and Fisheries, 2008). However, there are model features that are specific for land-based salmon production, which are not covered in the existing literature that mainly focuses on traditional sea-based salmon production.

In land-based salmon farming, the salmon producer can decide to produce post-smolt. We contribute to the existing literature by including this option in the optimization model. Producing post-smolt can be seen as having an early harvest possibility. However, such a view is too simplified because if the salmon producer decides not to extract post-smolt, the salmon need to remain in the tank for a minimum number of months until reaching harvest weights.

Moreover, land-based producers also have a yearly production limit which restricts the total biomass produced per year, and therefore differently restricts the production compared to only having a MAB limit. If the yearly production limit is more restrictive than the MAB limit, the resulting production plan might differ from a production plan that is only restricted by the MAB limit.

Another aspect that differentiates land-based salmon production from sea-based salmon production is the possibility of transferring salmon between tanks. The flexibility of transferring salmon increases the computational complexity of a mixed integer programming model (Føsund & Strandkleiv, 2020). Thus, more advanced solution methods than standard mixed integer programming solvers are needed in order to solve the problem within a reasonable amount of time. This thesis applies a decomposition approach to solve the production planning problem for land-based salmon production, which is a novel solution method compared to the literature reviewed above.

Chapter 4

Problem Description

In this chapter, we describe the tactical production planning problem in land-based salmon farming. The problem involves decisions on when, where, and how much smolt to deploy, whether to transfer salmon between tanks during the production cycle, and when to sell post-smolt and harvestable salmon. All these decisions need to be determined in order to maximize profits over the planning horizon without breaking operational and regulatory constraints. The planning horizon is chosen long enough to capture at least two production cycles of harvestable salmon. The revenues obtained depend on the weight of the post-smolt, and the harvestable salmon after extraction. Post-smolt is sold to a salmon producer operating a sea-based facility, whereas harvestable salmon is sold to the consumer market.

The land-based facility consists of a set of modules creating separate bio-secure zones. Each module consists of one or several tanks with a given volume. The tanks are the actual production units in which salmon grow. As modules are bio-secure zones, a module is only considered available for deployment whenever none of the tanks in the module contain salmon and all tanks have been cleaned. Any smolt deployed into the tanks are ordered in advance and infer a purchase cost. Deployments of smolt are limited to release windows.

Having salmon in a tank infer costs related to operating the tank, feeding the salmon, and adding oxygen into the water. The costs related to operating the tank are mainly driven by pumping costs, which are dependent on the water flow rates necessary to maintain salmon welfare. The water flow rate is dependent on the density of salmon within the tank, where higher densities require higher water flow rates. Feeding costs are linked to the growth and the feed conversion rate of the salmon. The feed conversion rate given a specific salmon weight is known to the salmon producer. Further, the salmon population needs supply of oxygen to thrive as some of the water in the facility is recirculated. The oxygen consumption rate of the salmon depends on both weight and water temperatures.

The growth of the salmon in terms of weight increase is dependent on seawater temperatures. Given the temperature profile, the salmon producer can estimate the growth of the salmon population. The salmon producer makes decisions on when to extract salmon based on the expected weight of an individual salmon in the population. When the expected weight of an individual salmon in a tank is at post-smolt sizes, the salmon producer needs to decide whether to sell the post-smolt to a sea-based producer or let the salmon continue to grow to harvestable sizes in the production facility. We assume the salmon population is provided with sufficient feed and oxygen. Hence, decisions regarding feeding regimes and the amount of oxygen to supply to optimize growth are not part of the tactical production planning problem.

Whenever the salmon producer extracts salmon from a tank, whether it be post-smolt or harvestable salmon, the whole tank needs to be emptied and cleaned. Therefore, the salmon producer needs to ensure that the production plan includes at least one full month between the time a tank is emptied until salmon are transferred or deployed into the tank. Extraction of harvestable salmon introduces a packaging and processing cost and the actual salmon weight sold to the market is affected by a harvest yield as inedible parts of the salmon are removed. We assume that there is unlimited demand for all extracted post-smolt and harvested salmon. Throughout the production cycle some salmon will die, leading to production losses throughout the planning horizon.

The production level in the facility is restricted by three different production regulations. The emission permit restricts the total weight of salmon extracted from the facility during a one-year period, acting as a yearly production limit. The two other production regulations apply at any given time. The MAB limit restricts the total aggregated weight of salmon the facility can contain at any given time. Additionally, the density of salmon within one tank at any given time cannot exceed a specified limit to ensure a healthy environment for the salmon. Note, the emission permit and the MAB limit apply to the production facility, while the density limit restricts the density within each tank. We assume the salmon producer manages to comply with salmon welfare restrictions whenever salmon densities within each tank are kept below the limit.

To produce efficiently and to comply with the restrictions mentioned above, the salmon producer may choose to transfer salmon during a production cycle. Transfer of salmon is only allowed between tanks belonging to the same module and requires the receiving tank to be empty. Transfer induces stress among the salmon population and results in a growth setback for the salmon being transferred. Salmon can only be transferred when the expected weight of an individual salmon in the population is at specific weights. Further, any population being transferred needs to stay at least one month in the tank after the transfer, since the salmon producer needs to monitor and get control of the actual level of salmon within the tanks following a transfer.

All logistics connected to transportation of smolt to the facility and transportation of salmon out of the facility are kept out of the problem scope. Further, the problem does not capture daily tasks such as controlling water quality and water flow rates. These activities are regarded to belong to the operational planning level concerning day-to-day operations.

Chapter 5

Mathematical Model

In this chapter, we present the mathematical model of the tactical production planning problem faced by a land-based producer described in Chapter 4. We start by presenting our modeling approach in Section 5.1, where we explain some key aspects of the modeling choices and assumptions made. In Section 5.2, we introduce the mathematical notation, while Section 5.3 presents the mathematical model.

5.1 Modeling Approach

In this section, we present the modeling choices and assumptions we apply to capture relevant aspects of the production planning problem. We start this section by describing how we model biomass development in Section 5.1.1. Then, in Section 5.1.2, we describe our approach to model revenues and costs before we explain properties of the planning horizon in Section 5.1.3. Further, we discuss different ways to approach end-of-horizon modeling in Section 5.1.4, as we assume the salmon producer wants to continue production at the end of the planning horizon. Finally, in Section 5.1.5 we describe how we model the possibility of transferring salmon between tanks within the facility.

5.1.1 Biomass Modeling

An essential component of the production plan is the modeling of growth development for a salmon population. This is incorporated as part of data preprocessing, but explicitly treated here to ease the understanding of how we model salmon growth. The biomass development is modeled using specific growth rates given by a growth table. The growth table specifies the growth for salmon with a certain weight subject to a specific water temperature. As we introduced in Section 3.3, there are different ways to model growth. Inspired by Yu and Leung (2005) and Aasen (2020), we base the biomass modeling on expected weight developments.

A growth factor, $G_{\hat{p}p}$, specifies the expected relative increase in weight during period p for salmon deployed in period $\hat{p}$. In the numerical example in Figure 5.1, the weight of the salmon population is expected to grow 1.3 times during period p when originally deployed in period $\hat{p}$. Thus, $G_{\hat{p}p}$ is 1.3 in this specific example. Given the expected weight of the salmon population in period p, we calculate the biomass development within a tank from period p to period $p+1$ by multiplying the expected weight of the population in a tank by the corresponding growth factor $G_{\hat{p}p}$. Thus, in Figure 5.1 the expected weight of the salmon population increases from 4 635 kg in period p to 6 026 kg in period $p+1$.

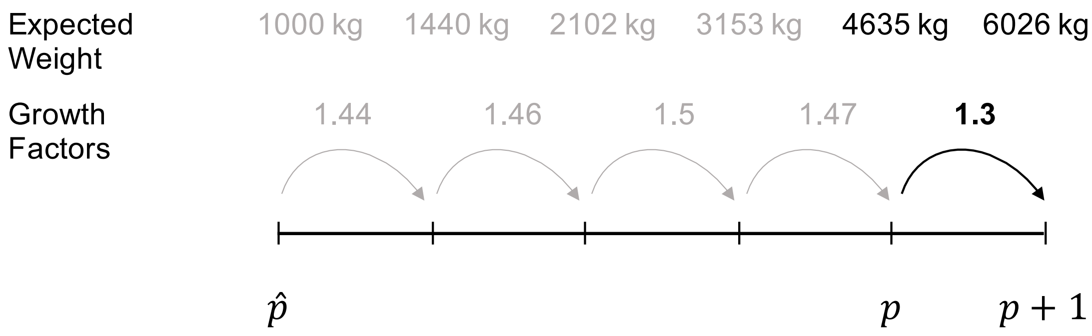


Figure 5.1: A numerical example on how we calculate the biomass development from p to $p+1$ for a salmon population deployed in $\hat{p}$

The expected weight of an individual salmon in the population decides whether certain actions can be performed during a period. Given the period an individual salmon was deployed, we can calculate its expected weight $w^{\mu}_{\hat{p}p}$ in any given period as:

$$w^{\mu}_{\hat{p}p} = \prod_{i=\hat{p}}^{p} G_{\hat{p}i} * w^{smolt},$$

where w^{smolt} is the weight of an individual smolt. Thus, we know the expected weight of an individual salmon in every period following deployment. This allows for defining sets that specify when salmon can be transferred, when salmon can be extracted as post-smolt, and when salmon can be extracted as harvestable salmon.

As individual salmon grow at different speeds, there will be weight variations within the salmon population (MOWI ASA, 2020). After extracting salmon from a tank, we distribute the total extracted weight into weight classes using the expected weight of an individual salmon in the population. We define a set of possible weight classes, $\mathcal{W}$. The weight classes, $w \in \mathcal{W}$, represent the weights salmon can take after extraction. A distributional factor, $Z_{w\hat{p}p}$, specifies the portion of the total extracted weight that falls into a specific weight class w.

We illustrate how we calculate this distributional factor through an example. In this example we extract a salmon population from a tank in period p, originally deployed in $\hat{p}$. The total extracted weight is denoted as E. The first step to calculate the distributional factor is to estimate the number of salmon distributed to the different weight classes. We apply the normal probability density function, $h(W_w)$:

$$h(W_w) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{1}{2}(\frac{W_w - w^{\mu}_{\hat{p}p}}{\sigma})^2, \qquad w \in \mathcal{W},$$

where W_w is the weight of the weight class w in consideration, $\mathcal{W}$ is the set of weight classes, and $w^{\mu}_{\hat{p}p}$ is the expected weight of an individual salmon in the population with corresponding growth variance σ. The density values $h(W_w)$ for all weight classes $w \in \mathcal{W}$ are normalized to sum to one. This gives the normalized density values $h^*(W_w)$ for all $w \in \mathcal{W}$. The estimate on the number of salmon in each weight class, N_w, is found through

$$N_w = \frac{E}{w^{\mu}_{\hat{p}p}} h^*(W_w), \qquad w \in \mathcal{W}.$$

Having established the number of salmon in the different weight classes, the next step is to multiply the number of salmon in a weight class, N_w, by the weight of the weight class W_w. This gives the distributed weight, D_w, in each weight class after extraction:

$$D_w = N_w W_w, \qquad w \in \mathcal{W}.$$

The last step is to divide the resulting distributed weight, D_w, by the total extracted weight, E. This gives the distributional factor

$$Z_{w\hat{p}p} = \frac{D_w}{E}, \qquad w \in \mathcal{W}.$$

The problem with this example is that we assume a value on the total extracted weight, E. As we preprocess the distributional factor, $Z_{w\hat{p}p}$, the information on the total extracted weight is not known. However, inserting the expression for D_w, and then the expression for N_w into the above expression, we arrive at a formula for the distributional factor that does not assume a value on E:

$$Z_{w\hat{p}p} = h^*(W_w)\frac{W_w}{w^{\mu}_{\hat{p}p}}, \qquad w \in \mathcal{W}.$$

Hence, we can preprocess the distributional factor $Z_{w\hat{p}p}$ for salmon extracted in period p, originally deployed in period $\hat{p}$.

5.1.2 Revenue and Cost Modeling

The land-based facility includes production of post-smolt. Combining the production of post-smolt with production of harvestable salmon gives two possible sources of income. Post-smolt is sold to a sea-based salmon farmer. Aquaculture Innovation expects the sea-based salmon farmer to carry any transportation costs connected to a post-smolt delivery (O. Skålnes, personal communication, 2021). Therefore, to model the revenues from selling post-smolt, we simply multiply the total extracted weight by the distributional factor and the post-smolt price associated with a given weight class.

The revenues obtained from selling harvestable salmon to the consumer market are modeled differently. We need to account for a processing cost that includes slaughtering and packaging of the salmon before it reaches the market. This processing cost applies per kilogram salmon produced. We therefore subtract the cost directly from the selling price per kilogram for the different weight classes. In the slaughtering, inedible parts of the salmon are removed. Hence, we multiply the extracted weight of harvestable salmon by a harvest yield.

For salmon to grow, they need feed. Recall that the ability for salmon to convert feed into biomass is known as the feed conversion ratio (FCR). The FCR-value specifies how many kilogram feed is needed for the salmon to gain one kilogram in biomass. The FCR-value is dependent on the salmon weight. The feed cost per kilogram salmon is dependent on the period, p, and the release period, $\hat{p}$.

*Feed cost = FCR * (Growth Factor - 1) * Expected weight of population * Feed Price per kg*

To calculate the feed costs we apply the weight-dependent FCR-value on the expected weight increase of the salmon population during the period. Then, we multiply by the feed price per kilogram to get an estimate of the feed costs needed for salmon to grow as expected.

The oxygen consumption of salmon depends on both weight and temperature. Therefore, we model oxygen costs through a parameter that specifies the cost of oxygen for a given period connected to a release period. The oxygen cost is found by multiplying the weight and temperature-dependent oxygen consumption by the price of oxygen.

*Oxygen cost = Oxygen Consumption in kg * Oxygen Price per kg*

To estimate the water pumping costs we need to apply assumptions. According to Hilmarsen et al. (2018), the water flow rate in land-based salmon production can be estimated as a function of biomass. Therefore, we assume a constant marginal increase in pumping cost per kilogram increase in weight of salmon in the tank, as seen by C^{MC} in Figure 5.2. We set a minimum cost level of initiating the use of a tank, C^{min}, which is dependent on the water flow rate needed when choosing to deploy salmon into a tank. This gives a cost function as depicted in Figure 5.2.

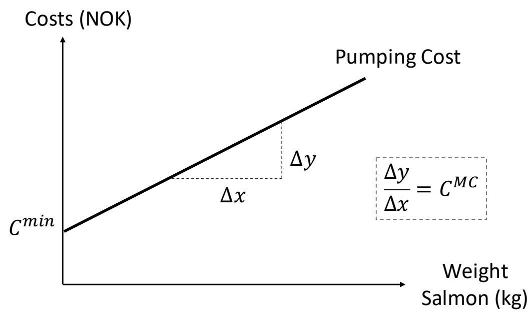


Figure 5.2: Pumping costs of water into a tank as a function of total salmon weight

5.1.3 Properties of the Planning Horizon

The planning horizon is discretized into a set of periods where each period represents one month. In collaboration with Aquaculture Innovation, we agreed that a monthly resolution of the planning horizon is sufficient to capture the impact of decisions made on a tactical level. Decisions regarding smolt deployment, transfer of salmon and extraction of salmon are made at the beginning of each period.

The planning horizon comprises several production cycles generating revenues and costs. Figure 5.3 illustrates the occurrence of costs and revenues within a planning horizon stretching from p to $p + L$, where p is the starting period and L is the length of the planning horizon.

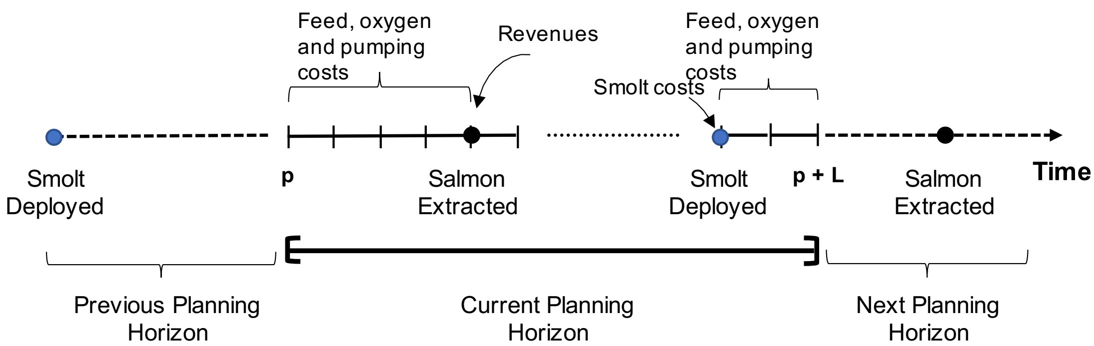


Figure 5.3: Occurrence of revenues and costs within a planning horizon stretching from p to $p + L$

Revenues and costs are accounted for when they arise in the current planning horizon. As illustrated in Figure 5.3, smolt costs occur for the smolt deployment in the current planning horizon, but not for the deployment in the previous planning horizon. Water pumping costs and oxygen costs arise in each period in which the tank contains salmon. Feed costs occur in each period where salmon grow. Feed costs are not included for the period when salmon are extracted, due to the industry practice of starving salmon for some weeks ahead of extraction.

5.1.4 End-of-Horizon Modeling

We assume the salmon producer wants to continue the production of salmon beyond the original planning horizon and into the next planning horizon. A mathematical model with the objective of maximizing profits over a finite planning horizon is not compatible with this assumption. Therefore, we need to apply some sort of end-of-horizon condition to maintain production at the end of the planning horizon.

If the optimal production plan has a cyclic behavior, then a reasonable approach can be to force a certain biomass level in the facility at the final period of the planning horizon. Without a cyclic behavior, the major drawback of forcing a production level at the end of the planning horizon is that decisions throughout and towards the end of the planning horizon are affected by the condition. Føsund and Strandkleiv (2020) applied an end-of-horizon condition that forced the production level in the final period to be at least as high as the average production level during years in the middle of the planning horizon. The results indicated that production during the middle of the planning horizon was adjusted to allow for large extractions towards the end of the planning horizon.

Rather than forcing a certain biomass level, an approach that incentivizes to maintain production may mitigate the influence on otherwise optimal behavior. This can be obtained through the salvage procedure suggested by Grinold (1983), which is one of the techniques he suggests to deal with the end effects arising from not having an infinite planning horizon. To incorporate the salvage procedure in our model, we would in the objective function include the future value of having salmon in the facility at the final period of the planning horizon. However, an acceptable valuation of the salmon is hard to perform and is further complicated by the option to produce post-smolt as part of the production cycle.

One way around the challenge of valuating the stock of salmon is to extend the planning horizon by a set of periods to allow for further production beyond the original planning horizon. The extended set of periods should be of such length that salmon deployed in the last release period of the original planning horizon can reach harvestable sizes in the final added period. When evaluating the results from the model, the periods belonging to the extended planning horizon are excluded.

Modeling end-of-horizon in this manner allows for the unfolding of any decision towards the end of the original planning horizon. Another advantage with this end-of-horizon modeling is that we are ensured a valid state for further production at the end of the original planning horizon. Through this approach for end-of-horizon modeling, the original problem remains, but the end-of-horizon effects are hopefully delayed into the extended planning horizon. However, some influence on the decisions in the original planning horizon will persist from the fact that the mathematical model seeks to empty the facility towards the end of the extended planning horizon.

The major drawback of this approach is that we increase the computational complexity of the model as we extend the planning horizon. Despite the increase in computational complexity, we choose to implement this approach for end-of-horizon modeling. This is because we expect less influence on otherwise optimal production behavior compared to forcing a given biomass-level as the end-of-horizon condition.

5.1.5 Salmon Transfer

Transferring salmon introduces flexibility into the production planning but also increases stress levels for the salmon being transferred. Hence, we introduce a growth factor that ensures a reduction in growth during the period salmon is transferred. Salmon can be transferred between tanks within the same module. The flexibility of salmon transfer allows the salmon producer to deploy salmon into a subset of the tanks, and then distribute salmon to several tanks later in the production cycle. Supporting salmon transfer in all directions between every tank within a module is not necessary due to the expected practical use of salmon transfer (O. Skålnes, personal communication, 2021).

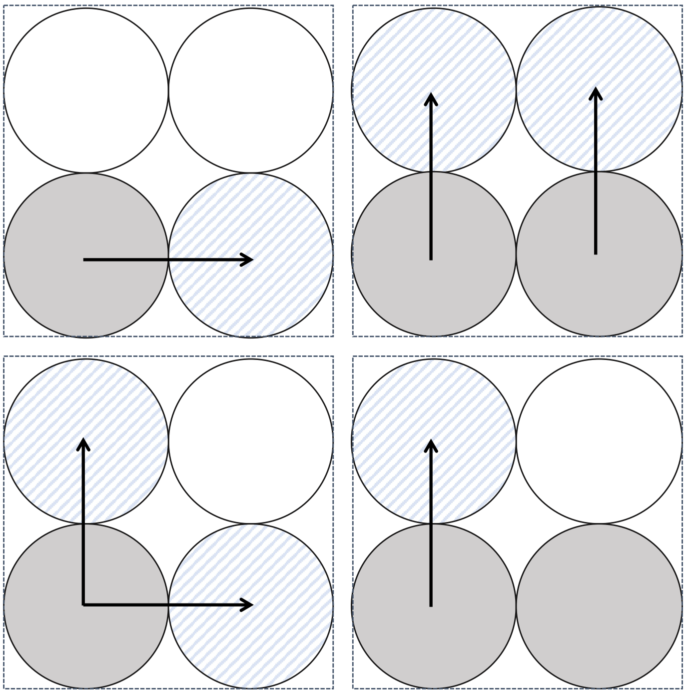


Figure 5.4: Four transfer examples for a module consisting of four tanks

We introduce a non-symmetric transfer pattern that distributes salmon in a breadth-first manner. This transfer pattern implies that salmon can only be transferred between certain tanks within a module. Choosing to deploy salmon into one tank, salmon can be transferred to its neighboring tanks. To illustrate the pattern we give an example with four tanks. The transfer pattern implies that the salmon producer can distribute salmon from one tank to two tanks, as in the upper left example of Figure 5.4. Thereafter, at a later stage in the production cycle salmon can be distributed from the two tanks into all four tanks, as in the upper right example in Figure 5.4. Hence, the transfer pattern allows the salmon producer to distribute salmon from one tank to all tanks throughout the production cycle. This also implies that the salmon producer can extract post-smolt from one or more tanks, and thereafter distribute salmon intended for harvesting into the tanks that were emptied due to post-smolt extraction. However, the transfer pattern is a simplification, and one of the consequences are that two transfers are needed to employ all four tanks, if salmon are initially only deployed into one tank. Technical benefits of applying this transfer pattern are reduced presence of symmetry within a module, and a reduced number of transfer variables. Both these aspects decrease computational complexity.

5.2 Mathematical Notation

In this section we introduce our mathematical model notation. Table 5.1 defines the sets, parameters and decision variables used to formulate the mathematical model. To ease the mathematical notation and formulation, we include the extended set of periods that arise as a consequence of the end-of-horizon modeling as part of the defined planning horizon.

Table 5.1: Notation applied in the mathematical model

Sets		
$\mathcal{W}$	Set of all weight classes	
$\mathcal{M}$	Set of all modules	
$\hat{\mathcal{T}}_t$	Set of all tanks that can receive salmon from tank t	
$\mathcal{T}_t$	Set of all tanks that can transfer salmon to tank t	
$\mathcal{T}_m$	Set of all tanks in module m	
$\mathcal{P}$	Set of all periods in the planning horizon	
$\mathcal{P}^R$	Set of all release periods in the planning horizon	
$\mathcal{P}^{R-}$	Set of periods salmon were released in the previous planning horizon	
$\mathcal{Y}$	Set of all years in the planning horizon	
$\mathcal{P}_y$	Set of all periods in year y	
$\mathcal{P}_p^D$	Set of all periods salmon could have been deployed when the current period is p	
$\mathcal{P}_p^{DE}$	Set of all periods salmon could have been deployed when they can be extracted in period p	
$\mathcal{P}_p^{DT}$	Set of all periods salmon could have been deployed when they can be transferred in period p	
$\mathcal{P}_{\hat{p}}^{PS}$	Set of all periods salmon can be extracted as post-smolt when originally deployed in period $\hat{p}$	
$\mathcal{P}_{\hat{p}}^H$	Set of all periods salmon can be harvested when originally deployed in period $\hat{p}$	
$\mathcal{P}_{\hat{p}}^E$	Set of all periods salmon can be extracted when originally deployed in period $\hat{p}$, $\mathcal{P}_{\hat{p}}^E = \mathcal{P}_{\hat{p}}^{PS} \cup \mathcal{P}_{\hat{p}}^H$	
$\mathcal{P}_{\hat{p}}^G$	Set of all periods salmon cannot be extracted when originally deployed in period $\hat{p}$, $\mathcal{P}_{\hat{p}}^G = \mathcal{P} \setminus \{\mathcal{P}_{\hat{p}}^E\}$	
$\mathcal{P}_{\hat{p}}^T$	Set of all periods salmon can be transferred when originally deployed in period $\hat{p}$	
Parameters		**Unit**
C^D	Purchase price of smolt.	NOK/kg
$C_{\hat{p}p}^F$	Feed cost in period p for salmon deployed in period $\hat{p}$	NOK/kg
$C_{\hat{p}p}^O$	Oxygen cost in period p for salmon deployed in period $\hat{p}$	NOK/kg

Table 5.1 continued from previous page

C^{min}	Minimum monthly cost of operating a tank	NOK
C^{MC}	Marginal increase in monthly pumping costs connected to a tank	NOK/kg
D^{max}	Maximum weight of smolt deployed in a module	kg
D^{min}	Minimum weight of smolt deployed in a module	kg
E^{max}	Maximum weight of salmon extracted from a tank	kg
Y^{max}	Maximum weight of salmon transferred from a tank	kg
Y^{min}	Minimum weight of salmon transferred from a tank	kg
$Z_{w\hat{p}p}$	Fraction of extracted weight that falls into weight class w in period p when originally deployed in period $\hat{p}$	
$G_{\hat{p}p}$	Growth factor for salmon in period p, originally deployed in period $\hat{p}$	
$G^{T}_{\hat{p}p}$	Growth factor for salmon transferred in period p, originally deployed in period $\hat{p}$	
L^{den}	Maximum allowed density of salmon in a tank at any given time	kg/m^3
L^{mab}	Maximum allowed biomass at any given time in the facility	kg
L^{prod}	Maximum total production during a year	kg
R^{PS}_{w}	Revenue for post-smolt in weight class w	NOK/kg
R^{H}_{w}	Revenue for harvested salmon in weight class w	NOK/kg
V_t	Inverse volume of tank t	m^{-3}
P^{loss}	Monthly rate of production loss	
Y^{H}	Harvest yield of slaughtered salmon	
Initial Conditions		
$x_{\hat{p}t1}$	Initial weight of salmon in tank t, defined for $\hat{p} \in \mathcal{P}^{R-}$, $m \in \mathcal{M}$, $t \in \mathcal{T}_m$	
α_{t0}	Initial use of tank t in the period prior to the planning horizon, defined for $m \in \mathcal{M}$, $t \in \mathcal{T}_m$	
Continuous Variables		
$e_{\hat{p}tp}$	Weight of salmon extracted from tank t in period p deployed in period $\hat{p}$	
$x_{\hat{p}tp}$	Weight of salmon population in tank t in period p deployed in period $\hat{p}$	
$y_{\hat{p}\hat{t}tp}$	Weight of salmon transferred from tank $\hat{t}$ to tank t in period p deployed in period $\hat{p}$	
Binary Variables		
α_{tp}	1 if tank t contains salmon in period p, 0 otherwise	
δ_{mp}	1 if smolt are deployed in module m in period p, 0 otherwise	
ϵ_{tp}	1 if salmon are extracted from tank t in period p, 0 otherwise	
σ_{tp}	1 if salmon are transferred to tank t in period p, 0 otherwise	

5.3 Mathematical Model

In this section, we present the mathematical model representing the tactical production planning problem described in Chapter 4. First, we present two possible objective functions for the model. Then, we introduce the different constraints that any feasible solution needs to adhere to. A compact version of the mathematical model is attached in Appendix A.

Profit Objective Function

In the profit objective function, we maximize profits from operating the facility over the planning horizon. In the following, we explain the five components of the profit objective function which comprises revenues, smolt costs, feed costs, oxygen costs, and tank costs. Due to the span of the planning horizon, incorporating discount rates on future revenues and costs is deemed irrelevant.

The first component finds the total revenues obtained from selling post-smolt and harvestable salmon. The post-smolt price of a weight class, R_w^{PS}, or harvest price R_w^H is multiplied by the distributional factor and the total extracted weight of post-smolt or total weight of salmon harvested. When harvestable salmon are extracted, we adjust the weight by including the harvest yield. We sum over all possible release periods, all tanks, all periods where salmon can be extracted as post-smolt and harvested given a release period, and all weight classes to find the total revenues.

$$\sum_{\hat{p}\in\{\mathcal{P}^R\cup\mathcal{P}^{R-}\}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\left(\sum_{p\in\mathcal{P}_{\hat{p}}^{PS}}\sum_{w\in\mathcal{W}}R_w^{PS}Z_{w\hat{p}p}e_{\hat{p}tp}+\sum_{p\in\mathcal{P}_{\hat{p}}^{H}}\sum_{w\in\mathcal{W}}R_w^{H}Z_{w\hat{p}p}Y^He_{\hat{p}tp}\right) \tag{5.1}$$

In the second component we account for smolt costs by multiplying the weight of smolt deployed by the purchase price, C^D. We sum over all tanks and all release periods in the current planning horizon to find the total smolt costs.

$$-\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\sum_{\hat{p}\in\mathcal{P}^R}C^Dx_{\hat{p}t\hat{p}}$$

The third component includes the costs connected to feeding the salmon. We multiply the monthly cost of feeding salmon by the weight of salmon in the tank subject to growth. This gives an estimate on the feeding costs during the period. In case salmon are extracted in a period, we adjust the weight of salmon in the tank correspondingly. We sum over all release periods, all tanks and all periods connected to a release period to find the total feeding costs.

$$-\sum_{\hat{p}\in\{\mathcal{P}^R\cup\mathcal{P}^{R-}\}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\left(\sum_{p\in\mathcal{P}_{\hat{p}}^{G}}C_{\hat{p}p}^Fx_{\hat{p}tp}+\sum_{p\in\mathcal{P}_{\hat{p}}^{E}}C_{\hat{p}p}^F(x_{\hat{p}tp}-e_{\hat{p}tp})\right)$$

In the fourth component, we account for costs related to inducing oxygen into the tanks. We multiply the monthly cost of oxygen per kilogram salmon during a period by the weight of salmon in the tank in the period. By summing over all release periods, all tanks, and all periods connected to a release period, we get the total oxygen costs.

$$-\sum_{\hat{p}\in\{\mathcal{P}^R\cup\mathcal{P}^{R-}\}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\sum_{p\in\{\mathcal{P}^G_{\hat{p}}\cup\mathcal{P}^E_{\hat{p}}\}} C^O_{\hat{p}p}x_{\hat{p}tp}$$

The fifth component finds the costs related to operating all tanks throughout the planning horizon. Whenever a tank contains salmon, a minimum monthly cost is inferred. As higher densities of salmon require more water flow through, we multiply the marginal increase in monthly pumping costs by the weight of salmon in the tank in a period. We sum over all periods, all tanks, and all possible release periods connected to a period to find the total tank-operating costs.

$$-\sum_{p\in\mathcal{P}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\left(C^{min}\alpha_{tp}+\sum_{\hat{p}\in\mathcal{P}^D_p}C^{MC}x_{\hat{p}tp}\right),$$

Biomass Objective Function

Setting the price of all weight classes to one and all cost parameters to zero, we arrive at a biomass objective function. In the biomass objective function, we maximize the total extracted weight of salmon from the facility over the planning horizon. This objective function is introduced here and used for analytical purposes later in the thesis.

$$max \sum_{\hat{p}\in\{\mathcal{P}^R\cup\mathcal{P}^{R-}\}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\sum_{p\in\mathcal{P}^E_{\hat{p}}}\sum_{w\in\mathcal{W}} e_{\hat{p}tp}z_{w\hat{p}p}, \tag{5.2}$$

Smolt Deployment Constraints

Whenever deploying smolt into a tank, we need to deploy at least a weight of D^{min} and at most a weight of D^{max}. The maximum weight of smolt to deploy in a module is to have every tank at the density limit in the first possible extraction period. The minimum weight of smolt to deploy is set at a reasonable lower limit. Further, we ensure that δ_{mp} is forced to 1 when smolt are deployed into one of the tanks belonging to the module. This is maintained through constraints (5.3).

$$D^{min}\delta_{m\hat{p}} \leq \sum_{t\in\mathcal{T}_m} x_{\hat{p}t\hat{p}} \leq D^{max}\delta_{m\hat{p}}, \qquad m\in\mathcal{M}, \hat{p}\in\mathcal{P}^R, \tag{5.3}$$

Next, constraints (5.4) ensure that smolt cannot be deployed in a module unless all tanks connected to the module are empty in the preceding period. This is to allow for cleaning and maintenance of the tanks following an extraction.

$$\delta_{m\hat{p}} + \alpha_{t(\hat{p}-1)} \leq 1, \qquad m\in\mathcal{M}, t\in\mathcal{T}_m, \hat{p}\in\mathcal{P}^R, \tag{5.4}$$

Extraction Constraints

Constraints (5.5) ensure that ϵ_{tp} is set to 1 whenever an extraction is performed from a tank. E^{max} is the maximum weight that can be extracted from a tank at once, which depends on the volume of the tank and the allowed density of salmon in the tank.

$$\sum_{\hat{p} \in \mathcal{P}_p^{DE}} e_{\hat{p}tp} \leq E^{max} \epsilon_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{5.5}$$

Whenever an extraction is performed, the whole tank is emptied. Therefore, constraints (5.6) force $\alpha_{t(p+1)}$ to be 0 whenever ϵ_{tp} is 1. In combination with constraints (5.11), this ensures that the tank is empty in the following period of an extraction.

$$\alpha_{t(p+1)} + \epsilon_{tp} \leq 1, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{5.6}$$

Salmon Transfer Constraints

When transferring salmon from one tank to another tank within the same module, the receiving tank needs to be empty. Hence, we include constraints (5.7), forcing σ_{tp} to 0 unless $\alpha_{t(p-1)}$ is 0.

$$\sigma_{tp} + \alpha_{t(p-1)} \leq 1, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{5.7}$$

A tank cannot both receive and extract salmon during the same period, due to the necessary monitoring of actual transferring amounts. Thus, we ensure through constraints (5.8) that both σ_{tp} and ϵ_{tp} cannot take the value 1 in the same period.

$$\sigma_{tp} + \epsilon_{tp} \leq 1, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{5.8}$$

Transferring all salmon from a tank is a decision that is not practical in real life. Thus, we ensure through constraints (5.9) that when σ_{tp} is one, the tank $\hat{t}$ where salmon were transferred from remains in use the next period.

$$\sigma_{tp} - \alpha_{\hat{t}(p+1)} \leq 0, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{t} \in \hat{\mathcal{T}}_t, p \in \mathcal{P}, \tag{5.9}$$

The weight of salmon transferred from tank $\hat{t}$ to tank t needs to be within an interval specified by Y^{min} and Y^{max}. The maximum weight of salmon to transfer is set to the maximum weight of salmon that can be contained within a tank, while the minimum weight is set to a reasonable lower limit. Constraints (5.10) force σ_{tp} to 1 whenever salmon are transferred to tank t in period p.

$$Y^{min} \sigma_{tp} \leq \sum_{\hat{p} \in \mathcal{P}_p^{DT}} y_{\hat{p}\hat{t}tp} \leq Y^{max} \sigma_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{t} \in \mathcal{T}_t, p \in \mathcal{P}, \tag{5.10}$$

Salmon Density and Tank Activation Constraints

To comply with salmon welfare restrictions, we require the density of salmon within a tank to be below L^{den} at any given period. We find the weight of salmon within a tank at a given period and multiply by the reciprocal of the tank-volume, V_t. Then, we require this density to be less than or equal to L^{den}. Constraints (5.11) force α_{tp} to be 1 whenever the tank contains salmon in a given period. Equivalently, they force the tank to be empty whenever α_{tp} is 0. We include the weight of salmon transferred into a tank in a given period to update the value of α_{tp} in the corresponding period.

$$\left(\sum_{\hat{p} \in \mathcal{P}_p^D} x_{\hat{p}tp} + \sum_{\hat{p} \in \mathcal{P}_p^{DT}} \sum_{\hat{t} \in \mathcal{T}_t} y_{\hat{p}\hat{t}tp} \right) V_t \leq L^{den} \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{5.11}$$

Regulatory Constraints

Next, we need to comply with the regulatory restrictions on the production level. Through constraints (5.12) we ensure that the weight of salmon within the facility at any given period is below the MAB limit, L^{mab}.

$$\sum_{\hat{p} \in \mathcal{P}_p^D} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}_m} x_{\hat{p}tp} \leq L^{mab}, \qquad p \in \mathcal{P}, \tag{5.12}$$

The production plan also needs to comply with the emission permit that basically acts as a restriction on the yearly production level. We sum the weight of salmon extracted throughout the periods belonging to a year and require this total weight to be below the production level limit, L^{prod}.

$$\sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}_m} \sum_{p \in \mathcal{P}_y} \sum_{\hat{p} \in \mathcal{P}_p^{DE}} e_{\hat{p}tp} \leq L^{prod}, \quad y \in \mathcal{Y}, \tag{5.13}$$

Biomass Development Constraints

We need to incorporate growth and production loss among the salmon population throughout the periods following deployment. To model this, we use the growth factor $G_{\hat{p}p}$, specifying how much larger an average salmon become during period p, when originally deployed in period $\hat{p}$. Constraints (5.14)–(5.18) all serve the same purpose of modeling biomass development, but their components are different depending on the periods they apply to. As it is hard to follow the mathematical notation in constraints (5.14)–(5.18), we illustrate how these constraints apply for a given configuration of weight intervals in Figure 5.5.

Constraints (5.14) cover growth of the salmon population in periods where the salmon cannot be extracted or transferred due to the expected weight. As shown in Figure 5.5 these are periods where the expected weight of an individual salmon is between deploy weight and smallest transfer weight, and when the expected weight is between largest transfer weight and smallest

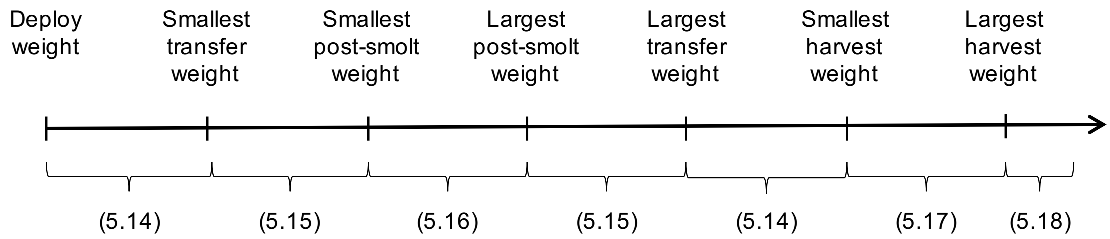


Figure 5.5: An example illustrating when the different biomass development constraints apply throughout the production cycle

harvest weight. We multiply the growth rate by the weight of salmon in the tank going into the period p, and adjust by production loss. This gives the weight of salmon in the tank at the beginning of period $p+1$ stemming from deployment in period $\hat{p}$.

$$
\begin{aligned}
&x_{\hat{p}t(p+1)} = (1-P^{loss})G_{\hat{p}p}x_{\hat{p}tp}, \\
&\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \{\mathcal{P}^{G}_{\hat{p}} \setminus \{\mathcal{P}^{T}_{\hat{p}}\}\},
\end{aligned}
\tag{5.14}
$$

Constraints (5.15) cover growth in periods salmon can be transferred, but not extracted. As shown in Figure 5.5 these periods are those where the expected weight is between smallest transfer weight and smallest post-smolt weight, and when the expected weight is between largest post-smolt weight and largest transfer weight. The weight remaining in a tank after transfer is multiplied with the normal growth factor, while any weight of salmon transferred is subject to a reduced growth factor. Also, we adjust for production loss.

$$
\begin{aligned}
&x_{\hat{p}t(p+1)} = (1-P^{loss})(G_{\hat{p}p}(x_{\hat{p}tp} - \sum_{\hat{t}\in\hat{\mathcal{T}}_t} y_{\hat{p}t\hat{t}p}) + G^{T}_{\hat{p}p}\sum_{\hat{t}\in\mathcal{T}_t} y_{\hat{p}\hat{t}tp}), \\
&\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \{\mathcal{P}^{G}_{\hat{p}} \cap \mathcal{P}^{T}_{\hat{p}}\},
\end{aligned}
\tag{5.15}
$$

Constraints (5.16) cover periods where the salmon can be extracted and transferred. As shown in Figure 5.5, this applies to the periods where the expected weight is between the smallest post-smolt weight and the largest post-smolt weight. We apply the same logic as above in constraints (5.15), but also account for any weight of salmon extracted from the tank. We exclude the last element of the set $\mathcal{P}^{E}_{\hat{p}}$, because this is the last period salmon can occupy a tank when originally deployed in $\hat{p}$. In this case, constraint (5.18) applies.

$$
\begin{aligned}
&x_{\hat{p}t(p+1)} = (1-P^{loss})(G_{\hat{p}p}(x_{\hat{p}tp} - \sum_{\hat{t}\in\hat{\mathcal{T}}_t} y_{\hat{p}t\hat{t}p} - e_{\hat{p}tp}) + G^{T}_{\hat{p}p}\sum_{\hat{t}\in\mathcal{T}_t} y_{\hat{p}\hat{t}tp}), \\
&\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \{\mathcal{P}^{E}_{\hat{p}} \cap \mathcal{P}^{T}_{\hat{p}}\} \setminus max\{\mathcal{P}^{E}_{\hat{p}}\},
\end{aligned}
\tag{5.16}
$$

Constraints (5.17) cover periods where the salmon can be extracted but not transferred when originally deployed in release period $\hat{p}$. These are periods where the expected weight of an individual salmon is above smallest harvest weight and below largest harvest weight, as seen in Figure 5.5. We exclude the last element of $\mathcal{P}^{E}_{\hat{p}}$ for the same reason as above.

$$x_{\hat{p}t(p+1)} = (1 - P^{loss})G_{\hat{p}p}(x_{\hat{p}tp} - e_{\hat{p}tp}),$$
$$\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}^E_{\hat{p}} \setminus \{\mathcal{P}^T_{\hat{p}}, max\{\mathcal{P}^E_{\hat{p}}\}\}, \tag{5.17}$$

We introduce a final harvest period connected to a deployment since salmon need to be harvested before reaching sexual maturation. Thus, when reaching the last period p connected to a deployment in period $\hat{p}$, the salmon cannot grow any larger. We ensure that any potential salmon in the tank are extracted through constraints (5.18).

$$x_{\hat{p}tp} - e_{\hat{p}tp} = 0, \qquad \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in max\{\mathcal{P}^E_{\hat{p}}\}, \tag{5.18}$$

Non-Negativity and Binary Requirements

Finally, we set the non-negativity and binary requirements on all decision variables involved in the mathematical model.

$$\begin{aligned}
e_{\hat{p}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}^E_{\hat{p}},\\
x_{\hat{p}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P},\\
y_{\hat{p}\hat{t}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{t} \in \mathcal{T}_t, p \in \mathcal{P}^T_{\hat{p}},\\
\alpha_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P},\\
\delta_{mp} &\in \{0,1\}, && m \in \mathcal{M}, p \in \mathcal{P}^R,\\
\epsilon_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P},\\
\sigma_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P},
\end{aligned} \tag{5.19}$$

Valid Inequalities

We introduce valid inequalities to improve the linear relaxation of the mathematical model. Improved linear relaxation reduces the size of the branch and bound tree employed by commercial mixed integer programming solvers, which in turn reduces the computational runtime. Preliminary results from running the model indicate that including these valid inequalities reduce the computational runtime.

The valid inequalities in (5.20) ensure that α_{tp} is set to zero if tank t was empty in the preceding period and neither a deployment into the module or a transfer into the tank occur in the current period. Further, α_{tp} is forced to zero if the tank was in use and an extraction was performed in the preceding period. The valid inequalities in (5.21) are equivalent but only apply to those periods that are not in the set of release periods.

$$\delta_{mp} + \sigma_{tp} + \alpha_{t(p-1)} - \epsilon_{t(p-1)} \geq \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}^R \setminus \{1\}, \tag{5.20}$$

$$\sigma_{tp} + \alpha_{t(p-1)} - \epsilon_{t(p-1)} \geq \alpha_{tp}, \qquad m \in \mathcal{M}, p \in \mathcal{P} \setminus \mathcal{P}^R, \tag{5.21}$$

The valid inequalities in (5.22) ensure that whenever salmon is deployed into module m at period $\hat{p}$, at least one tank is in use until the first possible extraction opportunity, which is the earliest extraction period in the set $\mathcal{P}^E_{\hat{p}}$.

$$\delta_{m\hat{p}} \leq \sum_{t \in \mathcal{T}_m} \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{p} \in \mathcal{P}^R, p \in \{\hat{p}, ..., \min(\mathcal{P}^E_{\hat{p}})\}. \tag{5.22}$$

The last valid inequalities (5.23) set the value of α_{tp} based on multiple factors for each pair of release period $\hat{p}$ and period connected to the release period. If smolt was deployed into tank t in period $\hat{p}$, then α_{tp} is forced to one in all periods until extraction is performed. If salmon is transferred to tank t at a later period in the production cycle, then α_{tp} is forced to one from the period transfer occurred until extraction is performed. If there are no deployment into the module in the release period, then the constant -2 ensures that the valid inequality has no effect.

$$2\delta_{m\hat{p}} - 2 + \alpha_{t\hat{p}} + \sum_{\bar{p}=\hat{p}+1}^{p} \sigma_{t\bar{p}} - \sum_{\bar{p}=\hat{p}+1}^{p-1} \epsilon_{t\bar{p}} \leq \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{p} \in \mathcal{P}^R, p \in \{\mathcal{P}^G_{\hat{p}} \cup \mathcal{P}^E_{\hat{p}}\}, \tag{5.23}$$

Chapter 6

Solution Method

In this chapter, we introduce the solution method applied to enhance the computational tractability of the mathematical model presented in Chapter 5. We start with discussing how we can exploit the problem structure in Section 6.1. In Section 6.2, we introduce two different decomposition methods that can be applied to our problem structure. We then present the Dantzig-Wolfe decomposed model in Section 6.3. To overcome the problem of originally having a mixed integer program (MIP) in the mathematical model, we apply Branch and Price as a solution method. This is presented in Section 6.4.

6.1 Exploiting the Problem Structure

In the mathematical model presented in Section 5.3, we identify a primal block angular structure where the regulatory constraints (5.12) and (5.13) constitute the set of linking constraints. These regulatory constraints connect all modules in the facility, while the remaining constraints can be isolated per module, as illustrated in Figure 6.1. By relaxing constraints (5.12) and (5.13) the problem is separable per module $m \in \mathcal{M}$, creating $|\mathcal{M}|$ independent subproblems.

The main motivation for decomposing a problem is that we can solve a sequence of smaller and less complex subproblems faster than solving one large problem (Lundgren et al., 2010). In our problem structure, the subproblems have an easy interpretation as a solution to one subproblem is a production plan for a specific module. Thus, a subproblem reflects a tactical production planning problem for one module over the entire planning horizon. This includes decisions on when and how much smolt to deploy into the tanks, whether to transfer salmon during the production cycle, and when to sell post-smolt and harvestable salmon.

Another advantage of applying decomposition techniques is that we get a better description of the convex hull of the original problem (Vanderbeck, 2000). When solving the linear program (LP) relaxation of the original problem, we simply remove any integer or binary requirements on the decision variables. Thus, the optimal solution to the LP-relaxation may be far away from the optimal solution to the original MIP. A tighter LP-relaxation is obtained in a decomposed formulation, as many of the integer and binary requirements from the original MIP are maintained in the subproblems of the reformulation.

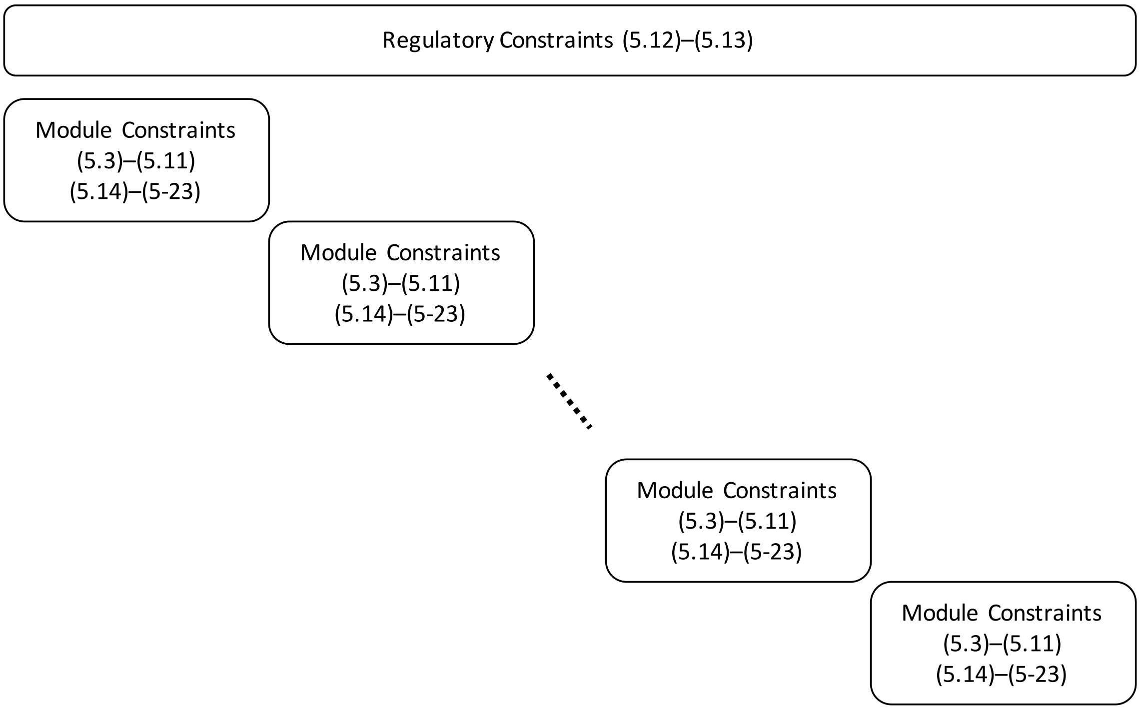


Figure 6.1: Primal block angular structure of the problem formulation with independent subsystems and linking constraints

The problem of relaxing the linking constraints arises when the combination of the solutions from the subproblems is infeasible in the original problem as they violate the linking constraints. Therefore, we need to ensure a mechanism to comply with the regulatory constraints (5.12) and (5.13) in which all the modules are tied together. Lagrangian relaxation and Dantzig-Wolfe decomposition are two methods that can exploit this kind of problem structure and additionally address the issue of subproblem solutions being infeasible.

6.2 Decomposition Methods

In this section, we introduce two different decomposition methods that can be applied to our problem structure. We briefly introduce Lagrangian relaxation in Section 6.2.1. Then, in the beginning of Section 6.2.2, we motivate why we choose to apply Dantzig-Wolfe decomposition in this thesis. The section continues with a more detailed introduction to Dantzig-Wolfe decomposition.

To help describe the two different decomposition methods we use the following integer program (IP) as an example:

$$
\begin{aligned}
(P) \qquad & \max c^T x \\
s.t. \qquad & Ax \leq b \\
& Dx \leq e \\
& x \in \{0,1\},
\end{aligned}
$$

where $Ax \leq b$ constitute the linking constraints that tie the independent subsystems together, creating the primal block angular structure depicted in Figure 6.1. For the sake of simplicity we assume constraints $Dx \leq e$ consitute a single subsystem. Further, (P) could also be represented as a linear program or a mixed integer program without loss of generality.

6.2.1 Lagrangian Relaxation

The main idea behind Lagrangian relaxation is to consider the linking constraints implicitly by moving them to the objective function of the problem (Lundgren et al., 2010). Moving the linking constraints of (P) to the objective function results in the Lagrangian dual function. The Lagrangian dual function, denoted $h(v)$ can be defined as the following optimization problem

$$
\begin{aligned}
h(v) = \max L(x,v) = c^T x - v^T(Ax - b) & \\
s.t. \qquad Dx \leq e & \\
x \in \{0,1\} &
\end{aligned}
$$

where $L(x,v)$ denotes the Lagrangian function in which any violation of the linking constraints, $Ax \leq b$, is penalized by a set of Lagrangian multipliers, $v \geq 0$. Any objective functi on value to the Lagrangian dual function, $h(v)$, is an upper bound to the objective value of the original problem (P) since we reward slack in the relaxed constraints. This optimization problem involves two sets of variables, x and v. Evaluating the Langrangian dual function given a configuration of Lagrangian multipliers $\bar{v}$, results in the Lagrangian subproblem. The Lagrangian subproblem is assumed to be faster to solve than the original problem (P), and is defined as

$$
\begin{aligned}
(LS) \qquad & \max L(x,\bar{v}) \\
& Dx \leq e \\
& x \in \{0,1\}.
\end{aligned}
$$

The solution from (LS) may be feasible in the original problem (P), in which we can use the solution to calculate a lower bound for (P). To find the multipliers, v, that yield the best possible upper bound to the original problem (P) we solve the Lagrangian dualproblem. The Lagrangian dualproblem can be expressed as

$$
\begin{aligned}
(LD) \qquad & \min h(v) \\
& v \geq 0
\end{aligned}
$$

Instead of solving the Langrangian dual problem, we evaluate the Lagrangian dual function for fixed values of the multipliers, v. Then, we use the information we get from the solution of (LS) to calculate a bound and a new value of the multiplier to evaluate. Subgradient optimization is a traditional method to update multiplier values, although it can be slow and have issues with convergence for larger optimization problems (Barnhart et al., 1998).

The Lagrangian relaxation method revolves around solving Lagrangian subproblems and updating the values of the Lagrangian multipliers until the stopping criterion is met, in which the lower bound is sufficiently close to the upper bound. The interpretation of applying a Lagrangian relaxation to our problem is to use the solutions from the subproblems to calculate a new upper bound and new multiplier values to evaluate.

6.2.2 Dantzig-Wolfe Decomposition

Rather than focusing on finding the optimal multipliers in Lagrangian relaxation where convergence might be an issue, using Dantzig-Wolfe decomposition is a matter of how to combine solutions from the subproblems in a master problem containing the linking constraints (Dantzig & Wolfe, 1960). As the solution of a subproblem is a production plan for a specific module, the idea of combining production plans in such a way that the regulatory constraints are satisfied yields a very intuitive interpretation. Further, Dantzig-Wolfe decomposition is closely related to Column Generation, which is a very useful tool for solving large and structured linear programming problems (Lundgren et al., 2010). Therefore, we choose to apply Dantzig-Wolfe decomposition as the method to exploit the structure of our original problem formulation.

The core idea in Dantzig-Wolfe decomposition is to reformulate the original problem into an equivalent interior representation that uses extreme points. Starting from the original IP problem (P) we identify $Ax \leq b$ as linking constraints. Further, we assume $X_D = \{x \mid Dx \leq e \wedge x \in \{0,1\}\}$ is a bounded convex set and denote the extreme points as $x^{(k)}$, where $k = 1, ..., K$, and K is the number of extreme points.

Barnhart et al. (1998) apply Minkowski´s representation theorem to represent any point $x \in X_D$ as a convex combination of the extreme points,

$$x = \sum_{k=1}^{K} \lambda_k x^{(k)}, \qquad \sum_{k=1}^{K} \lambda_k = 1 \wedge \lambda_k \geq 0, k = 1, ..., K.$$

Now, we can reformulate the original problem (P) into an equivalent interior representation as:

$$
\begin{aligned}
(MP) \qquad & max \sum_{k=1}^{K} (c^T x^{(k)})\lambda_k \\
s.t. \qquad & \sum_{k=1}^{K} (Ax^{(k)})\lambda_k \leq b \qquad |v \qquad (1) \\
& \sum_{k=1}^{K} \lambda_k = 1 \qquad |u \qquad (2) \\
& \sum_{k=1}^{K} \lambda_k x^{(k)} = z \\
& z \in \{0, 1\} \\
& \lambda_k \geq 0,\ k = 1, \dots, K.
\end{aligned}
$$

This problem is called the Master Problem (MP), where v and u are dual variables associated with constraints (1) and (2). Notice that the decision variables in (MP) are the weighting variables λ_k, and that the convexity constraint (2) ensures a convex combination of these variables. Further, notice that only the linking constraints, $Ax \leq b$, remain in the master problem. However, constraints $Dx \leq e$ are implicitly considered as x is expressed as a convex combination of the extreme points from the set X_D. (MP) is a problem with fewer constraints, but considerably more variables compared to the original problem formulation.

This is where the idea of Column Generation arises. Column Generation exploits that most variables of a large problem will be non-basic and take a value of zero in the optimal solution (Lübbecke & Desrosiers, 2005). This is leveraged by only producing extreme points that potentially increase the objective function of the master problem. Starting from a Restricted Master Problem (RMP) with only a subset of the extreme points in the set X_D, we generate new extreme points with a positive reduced cost, as the problem is a maximization problem. Adding an extreme point from the set X_D to the RMP is equivalent to adding a new column in the constraint matrix. Entering columns are found in a separate subproblem. The subproblem exploits information on the value of the dual variables v and u to find a new extreme point from the set X_D with a positive reduced cost. This exchange of information between the master problem and the subproblem is depicted in Figure 6.2.

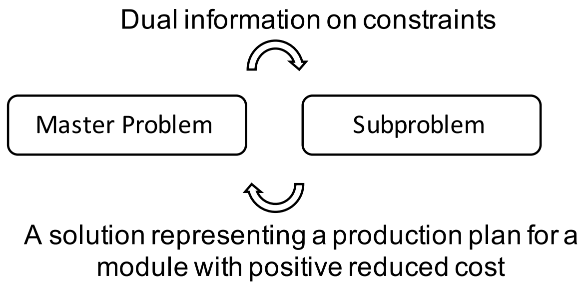


Figure 6.2: Exchange of information in Dantzig-Wolfe decomposition between the master problem and the subproblem

The expression for reduced cost is given as:

$$\bar{c}_k = (c^T - \bar{v}^T A)x^{(k)} - \bar{u}.$$

Then, to find the solution, i.e. the values of the decision variables, associated with the largest reduced cost, we solve the subproblem that generates a new extreme point of the set X_D through

$$\begin{aligned} (SP) \qquad & max\ (c^T - \bar{v}^T A)x - \bar{u} \\ & s.t. \qquad Dx \leq e \\ & \qquad\qquad x \in \{0, 1\}. \end{aligned}$$

When the optimal objective value of the subproblem is positive, we extend the RMP with the corresponding extreme point and re-optimize the RMP. Whenever the subproblem cannot find any solution with a positive reduced cost, we have that the solution to the RMP is the optimal solution to the original problem (P) (Lundgren et al., 2010).

6.3 Dantzig-Wolfe Reformulation of the Mathematical Model

In this section, we present how we decompose the original problem formulation in Section 5.3, using Dantzig-Wolfe decomposition. In Section 6.3.1, we start by presenting the subproblems that generate production plans based on the dual information from the master problem. In Section 6.3.2, we introduce the master problem that combines these solutions to comply with the linking constraints and maximizes the overall objective function.

6.3.1 Subproblem

When relaxing the regulatory constraints (5.12) and (5.13), we get one subproblem for each module $m \in \mathcal{M}$. As the original problem is a maximization problem, we search for the solution with the largest value for reduced cost in each subproblem. We let u_p^* be the optimal dual variables connected to the MAB limit constraints (6.22) in the master problem, v_y^* be the optimal dual variables connected to the yearly production limit constraints (6.23) and $v_m^{\lambda_*}$ be the optimal dual variable connected to the convexity constraint for module m in the master problem. This allows for transforming the general expression for reduced cost into the reduced cost objective in (6.1). The constraints in the subproblem are identical to constraints (5.3)–(5.11) and (5.14)–(5.23) in the original problem formulation, except for the scope of a specific module rather than the scope of all modules. For each of the subproblems, we only adapt the objective function based on the dual information from the master problem. Constraints (6.2)–(6.20) are included here for the sake of completeness. To find the production plan for module m that produces the largest reduced cost, we solve the following subproblem:

Reduced Cost Objective

$$\begin{aligned} max \quad & \sum_{t\in\mathcal{T}_m}\Bigg[\sum_{\hat{p}\in\{\mathcal{P}^R\cup\mathcal{P}^{R-}\}}\Bigg(\sum_{p\in\mathcal{P}^{PS}_{\hat{p}}}\sum_{w\in\mathcal{W}} R^{PS}_w Z_{w\hat{p}p} e_{\hat{p}tp} + \sum_{p\in\mathcal{P}^H_{\hat{p}}}\sum_{w\in\mathcal{W}} R^H_w Z_{w\hat{p}p} Y^H e_{\hat{p}tp} \\ & - \sum_{p\in\mathcal{P}^G_{\hat{p}}} C^F_{\hat{p}p} x_{\hat{p}tp} + \sum_{p\in\mathcal{P}^E_{\hat{p}}} C^F_{\hat{p}p}(x_{\hat{p}tp} - e_{\hat{p}tp}) - \sum_{p\in\{\mathcal{P}^G_{\hat{p}}\cup\mathcal{P}^E_{\hat{p}}\}} C^O_{\hat{p}p} x_{\hat{p}tp}\Bigg) \\ & - \sum_{\hat{p}\in\mathcal{P}^R} C^D x_{\hat{p}t\hat{p}} - \sum_{p\in\mathcal{P}}\Bigg(C^{min}\alpha_{tp} + \sum_{\hat{p}\in\mathcal{P}^D_p} C^{MC} x_{\hat{p}tp}\Bigg)\Bigg] \\ & - \sum_{p\in\mathcal{P}}\Bigg(\sum_{\hat{p}\in\mathcal{P}^D_p}\sum_{t\in\mathcal{T}_m} x_{\hat{p}tp}\Bigg)u^*_p - \sum_{y\in\mathcal{Y}}\Bigg(\sum_{t\in\mathcal{T}_m}\sum_{p\in\mathcal{P}_y}\sum_{\hat{p}\in\mathcal{P}^{DE}_p} e_{\hat{p}tp}\Bigg)v^*_y - v^{\lambda*}_m \end{aligned} \tag{6.1}$$

Smolt Deployment Constraints

$$D^{min}\delta_{m\hat{p}} \le \sum_{t\in\mathcal{T}_m} x_{\hat{p}t\hat{p}} \le D^{max}\delta_{m\hat{p}}, \qquad \hat{p}\in\mathcal{P}^R, \tag{6.2}$$

$$\delta_{m\hat{p}} + \alpha_{t(\hat{p}-1)} \le 1, \qquad t\in\mathcal{T}_m, \hat{p}\in\mathcal{P}^R, \tag{6.3}$$

Extraction Constraints

$$\sum_{\hat{p}\in\mathcal{P}^{DE}_p} e_{\hat{p}tp} \le E^{max}\epsilon_{tp}, \qquad t\in\mathcal{T}_m, p\in\mathcal{P}, \tag{6.4}$$

$$\alpha_{t(p+1)} + \epsilon_{tp} \le 1, \qquad t\in\mathcal{T}_m, p\in\mathcal{P}, \tag{6.5}$$

Salmon Transfer Constraints

$$\sigma_{tp} + \alpha_{t(p-1)} \le 1, \qquad t\in\mathcal{T}_m, p\in\mathcal{P}, \tag{6.6}$$

$$\sigma_{tp} + \epsilon_{tp} \le 1, \qquad t\in\mathcal{T}_m, p\in\mathcal{P}, \tag{6.7}$$

$$\sigma_{tp} - \alpha_{\hat{t}(p+1)} \leq 0, \qquad t \in \mathcal{T}_m, \hat{t} \in \hat{\mathcal{T}}_t, p \in \mathcal{P}, \tag{6.8}$$

$$Y^{min}\sigma_{tp} \leq \sum_{\hat{p} \in \mathcal{P}_p^{DT}} y_{\hat{p}\hat{t}tp} \leq Y^{max}\sigma_{tp}, \qquad t \in \mathcal{T}_m, \hat{t} \in \mathcal{T}_t, p \in \mathcal{P}, \tag{6.9}$$

Salmon Density and Tank Activation Constraints

$$\left(\sum_{\hat{p} \in \mathcal{P}_p^D} x_{\hat{p}tp} + \sum_{\hat{p} \in \mathcal{P}_p^{DT}} \sum_{\hat{t} \in \mathcal{T}_t} y_{\hat{p}\hat{t}tp} \right) V_t \leq L^{den}\alpha_{tp}, \qquad t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{6.10}$$

Biomass Development Constraints

$$\begin{aligned} & x_{\hat{p}t(p+1)} = (1 - P^{loss})G_{\hat{p}p}x_{\hat{p}tp}, \\ & \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^R\}, t \in \mathcal{T}_m, p \in \{\mathcal{P}_{\hat{p}}^G \setminus \{\mathcal{P}_{\hat{p}}^T\}\}, \end{aligned} \tag{6.11}$$

$$\begin{aligned} & x_{\hat{p}t(p+1)} = (1 - P^{loss})(G_{\hat{p}p}(x_{\hat{p}tp} - \sum_{\hat{t} \in \hat{\mathcal{T}}_t} y_{\hat{p}t\hat{t}p}) + G_{\hat{p}p}^T \sum_{\hat{t} \in \mathcal{T}_t} y_{\hat{p}\hat{t}tp}), \\ & \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^R\}, t \in \mathcal{T}_m, p \in \{\mathcal{P}_{\hat{p}}^G \cap \mathcal{P}_{\hat{p}}^T\}, \end{aligned} \tag{6.12}$$

$$\begin{aligned} & x_{\hat{p}t(p+1)} = (1 - P^{loss})(G_{\hat{p}p}(x_{\hat{p}tp} - \sum_{\hat{t} \in \hat{\mathcal{T}}_t} y_{\hat{p}t\hat{t}p} - e_{\hat{p}tp}) + G_{\hat{p}p}^T \sum_{\hat{t} \in \mathcal{T}_t} y_{\hat{p}\hat{t}tp}), \\ & \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^R\}, t \in \mathcal{T}_m, p \in \{\mathcal{P}_{\hat{p}}^E \cap \mathcal{P}_{\hat{p}}^T\} \setminus max\{\mathcal{P}_{\hat{p}}^E\}, \end{aligned} \tag{6.13}$$

$$\begin{aligned} & x_{\hat{p}t(p+1)} = (1 - P^{loss})G_{\hat{p}p}(x_{\hat{p}tp} - e_{\hat{p}tp}), \\ & \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^R\}, t \in \mathcal{T}_m, p \in \mathcal{P}_{\hat{p}}^E \setminus \{\mathcal{P}_{\hat{p}}^T, max\{\mathcal{P}_{\hat{p}}^E\}\}, \end{aligned} \tag{6.14}$$

$$x_{\hat{p}tp} - e_{\hat{p}tp} = 0, \qquad \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^R\}, t \in \mathcal{T}_m, p \in max\{\mathcal{P}_{\hat{p}}^E\}, \tag{6.15}$$

Non-Negativity and Binary Requirements

$$
\begin{aligned}
e_{\hat{p}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, t \in \mathcal{T}_m, p \in \mathcal{P}^{E}_{\hat{p}}, \\
x_{\hat{p}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, t \in \mathcal{T}_m, p \in \mathcal{P}, \\
y_{\hat{p}\hat{t}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, t \in \mathcal{T}_m, \hat{t} \in \mathcal{T}_t, p \in \mathcal{P}^{T}_{\hat{p}}, \\
\alpha_{tp} &\in \{0,1\}, && t \in \mathcal{T}_m, p \in \mathcal{P}, \\
\delta_{mp} &\in \{0,1\}, && p \in \mathcal{P}^{R}, \\
\epsilon_{tp} &\in \{0,1\}, && t \in \mathcal{T}_m, p \in \mathcal{P}, \\
\sigma_{tp} &\in \{0,1\}, && t \in \mathcal{T}_m, p \in \mathcal{P},
\end{aligned}
\tag{6.16}
$$

Valid Inequalities

$$\delta_{mp} + \sigma_{tp} + \alpha_{t(p-1)} - \epsilon_{t(p-1)} \geq \alpha_{tp}, \qquad t \in \mathcal{T}_m, p \in \mathcal{P}^{R} \setminus \{1\}, \tag{6.17}$$

$$\sigma_{tp} + \alpha_{t(p-1)} - \epsilon_{t(p-1)} \geq \alpha_{tp}, \qquad t \in \mathcal{T}_m, p \in \mathcal{P} \setminus \mathcal{P}^{R}, \tag{6.18}$$

$$\delta_{m\hat{p}} \leq \sum_{t \in \mathcal{T}_m} \alpha_{tp}, \qquad \hat{p} \in \mathcal{P}^{R}, p \in \{\hat{p}, ..., \min(\mathcal{P}^{E}_{\hat{p}})\}, \tag{6.19}$$

$$2\delta_{m\hat{p}} - 2 + \alpha_{t\hat{p}} + \sum_{\bar{p}=\hat{p}+1}^{p} \sigma_{t\bar{p}} - \sum_{\bar{p}=\hat{p}+1}^{p-1} \epsilon_{t\bar{p}} \leq \alpha_{tp}, \qquad t \in \mathcal{T}_m, \hat{p} \in \mathcal{P}^{R}, p \in \{\mathcal{P}^{G}_{\hat{p}} \cup \mathcal{P}^{E}_{\hat{p}}\}. \tag{6.20}$$

6.3.2 Master Problem

In the master problem we let $\mathcal{K}_m$ represent the set of solutions found in subproblem m. When solving subproblem m to obtain a solution with positive reduced cost, we extend the master problem by a column consisting of the following parameters for module m:

$$\text{Column for module m} = [e^{k}_{\hat{p}tp}, x^{k}_{\hat{p}tp}, \alpha^{k}_{tp}, \delta^{k}_{mp}, \epsilon^{k}_{tp}, \sigma^{k}_{tp}].$$

The parameters in column $k \in \mathcal{K}_m$, are the values of the optimal decision variables for solution k found in subproblem m. In the master problem, we combine the columns from the set of all solutions,

$$\bigcup_{m \in \mathcal{M}} \mathcal{K}_m,$$

to find a profit maximizing production plan for the facility that complies with the maximum allowed biomass limit and the yearly production limit. The decision variables in the master problem are the weighting variables λ_{mk}, defined for all $m \in \mathcal{M}$ and $k \in \mathcal{K}_m$.

Profit Maximization Objective

The expression for profit maximization is equivalent to the original problem formulation, except that we now sum over all modules $m \in \mathcal{M}$, all columns $k \in \mathcal{K}_m$, and multiply with the respective weighting variable λ_{mk}.

$$
\begin{aligned}
max \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}_m} \sum_{k\in\mathcal{K}_m} \lambda_{mk} \Bigg[\sum_{\hat{p}\in\{\mathcal{P}^R \cup \mathcal{P}^{R-}\}} \Bigg(& \sum_{p\in\mathcal{P}^{PS}_{\hat{p}}} \sum_{w\in\mathcal{W}} R^{PS}_w Z_{w\hat{p}p} e^k_{\hat{p}tp} + \sum_{p\in\mathcal{P}^H_{\hat{p}}} \sum_{w\in\mathcal{W}} R^H_w Z_{w\hat{p}p} Y^H e^k_{\hat{p}tp} \\
& - \sum_{p\in\mathcal{P}^G_{\hat{p}}} C^F_{\hat{p}p} x^k_{\hat{p}tp} + \sum_{p\in\mathcal{P}^E_{\hat{p}}} C^F_{\hat{p}p} (x^k_{\hat{p}tp} - e^k_{\hat{p}tp}) - \sum_{p\in\{\mathcal{P}^G_{\hat{p}} \cup \mathcal{P}^E_{\hat{p}}\}} C^O_{\hat{p}p} x^k_{\hat{p}tp} \Bigg) \\
& - \sum_{\hat{p}\in\mathcal{P}^R} C^D x^k_{\hat{p}t\hat{p}} - \sum_{p\in\mathcal{P}} \Bigg(C^{min} \alpha^k_{tp} + \sum_{\hat{p}\in\mathcal{P}^D_p} C^{MC} x^k_{\hat{p}tp} \Bigg) \Bigg],
\end{aligned}
\tag{6.21}
$$

Regulatory Constraints

The maximum allowed biomass and emission permit restrictions are now extended to sum over all solutions k in $\mathcal{K}_m$ multiplied with the corresponding weighting variable λ_{mk}.

$$
\sum_{m\in\mathcal{M}} \sum_{\hat{p}\in\mathcal{P}^D_p} \sum_{t\in\mathcal{T}_m} \sum_{k\in\mathcal{K}_m} \lambda_{mk} x^k_{\hat{p}tp} \leq L^{mab}, \qquad p \in \mathcal{P}, \qquad | \quad u^*_p \tag{6.22}
$$

$$
\sum_{m\in\mathcal{M}} \sum_{p\in\mathcal{P}_y} \sum_{\hat{p}\in\mathcal{P}^{DE}_p} \sum_{t\in\mathcal{T}_m} \sum_{k\in\mathcal{K}_m} \lambda_{mk} e^k_{\hat{p}tp} \leq L^{prod}_y, \quad y \in \mathcal{Y}, \qquad | \quad v^*_y \tag{6.23}
$$

Convexity Constraints

To ensure a convex combination of the decision variables, we include constraints (6.24) that enforce the sum of all λ_{mk} to be equal to 1 for each module.

$$
\sum_{k\in\mathcal{K}_m} \lambda_{mk} = 1, \qquad m \in \mathcal{M}, \qquad | \quad v^{\lambda_*}_m \tag{6.24}
$$

As a column represents a production plan for a module, we require the convex combination of the λ_{mk} variables for a module to enforce binary values for tank activation α_{tp}, smolt deployment δ_{mp}, extraction of salmon ϵ_{tp} and transfer of salmon σ_{tp}. This is ensured through constraints (6.25)–(6.30). Performing an LP-relaxation of constraints (6.30), makes the convexity contraints (6.25)–(6.28) redundant. Therefore, the constraints have no connected dual variables.

$$
\sum_{k\in\mathcal{K}_m} \lambda_{mk} \delta^k_{mp} = \delta_{mp}, \qquad m \in \mathcal{M}, p \in \mathcal{P}, \tag{6.25}
$$

$$\sum_{k \in \mathcal{K}_m} \lambda_{mk} \alpha_{tp}^k = \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{6.26}$$

$$\sum_{k \in \mathcal{K}_m} \lambda_{mk} \epsilon_{tp}^k = \epsilon_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{6.27}$$

$$\sum_{k \in \mathcal{K}_m} \lambda_{mk} \sigma_{tp}^k = \sigma_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{6.28}$$

Non-Negativity and Binary Requirements

We set non-negativity requirements on the decision variables λ_{mk} in constraints (6.29). Moreover, we set binary requirements on the original binary decision variables for tank activation, smolt deployment, extraction of salmon and transfer of salmon in constraints (6.30).

$$\lambda_{mk} \geq 0, \qquad m \in \mathcal{M}, k \in \mathcal{K}_m, \tag{6.29}$$

$$\begin{aligned} \alpha_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \\ \delta_{mp} &\in \{0,1\}, && m \in \mathcal{M}, p \in \mathcal{P}^R, \\ \epsilon_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \\ \sigma_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \end{aligned} \tag{6.30}$$

6.4 Branch and Price Algorithm

The Branch and Price (B&P) algorithm is a solution method that can be applied to solve a mixed integer Dantzig-Wolfe decomposed model. The B&P algorithm is an extension of the more common Branch and Bound (B&B) algorithm, where each node in the B&B tree is solved using the Column Generation algorithm. In the Column Generation algorithm, the dual values of the linking constraints are passed on from the RMP to the subproblems. Hence, the method requires the RMP to be LP-relaxed. Therefore, we relax the binary requirements in (6.30). In Section 6.4.1 we give a brief overview of the B&P algorithm. Section 6.4.2 explains the implemented algorithmic configurations, while Section 6.4.3 introduces the extensions we have implemented into the B&P algorithm.

6.4.1 Overview of Branch and Price Algorithm

The solution process of the B&P algorithm is depicted in Figure 6.3. The algorithm is initialized by creating the root node and adding it to the queue. The queue contains all unexplored nodes, meaning nodes that have been created but not solved yet. In each node, we solve the restricted master problems and corresponding subproblems according to the Column Generation algorithm depicted in Algorithm 1. The first step is to set up initial columns. In the root node, initial columns are generated, while in other nodes the Column Generation algorithm starts from a set of columns already generated.

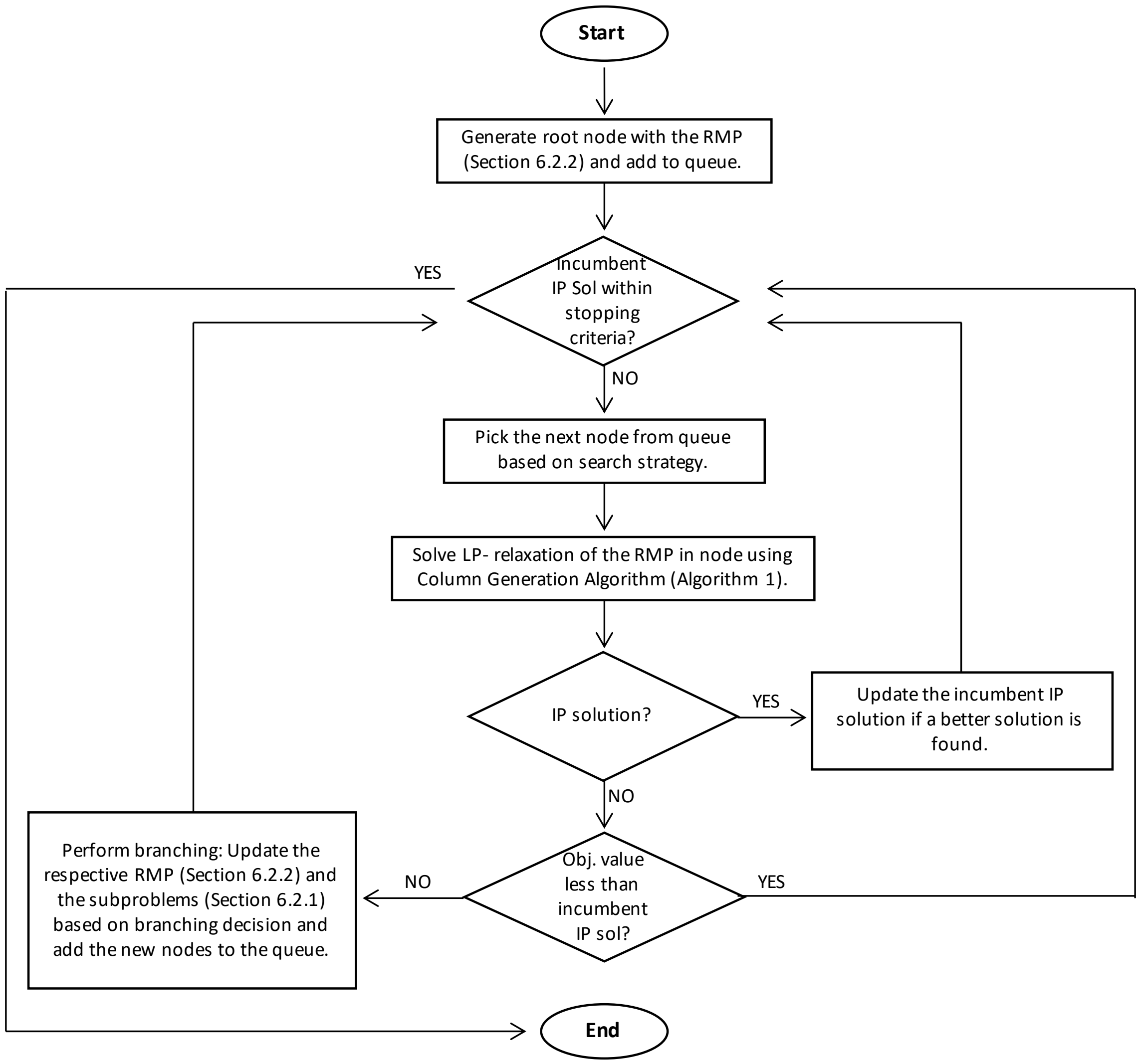


Figure 6.3: Solution process Branch and Price algorithm

After each node is solved using the Column Generation algorithm, the solution of the RMP is checked for integer feasibility, by checking the binary requirements of constraints (6.30). The solution is not integer feasible when it includes a convex combination violating any binary requirements on the original variables. If the node is infeasible or the objective value of the node is less than the current best integer feasible solution (incumbent IP solution), it is pruned. If the solution is not integer feasible and has a higher objective value than the current best integer feasible solution, new nodes are created through branching and then added to the queue. The RMP and subproblems in the new nodes are updated based on the branching decision.

Algorithm 1: Pseudocode for the implementation of Column Generation in every node of the B&P tree

```
Set up initial columns;
Optimize restricted master problem;
Extract dual information from restricted master problem;
Optimize subproblems based on dual information from restricted master problem;
while Positive reduced cost for any subproblem do
    Add solutions with positive reduced cost as columns to restricted master problem;
    Re-optimize restricted master problem;
    Extract new dual information from restricted master problem;
    Re-optimize subproblems based on new dual information;
end
Result: λ_mk variables are the optimal combination of columns.
```

The incumbent IP solution is updated if the solution of a node is integer feasible and is better than the current best integer feasible solution. The incumbent IP solution is a feasible solution that is stored globally. New nodes are picked from the queue following a search strategy. The algorithm continues until the incumbent IP solution satisfy any stopping criteria.

6.4.2 Algorithmic Configurations

Different strategies can be applied to the B&P algorithm, that can have a significant impact on the performance of the algorithm (Morrison et al., 2016). The first configuration we decide on in the B&P algorithm is the search strategy, i.e. the order in which nodes in the tree are explored. The next is the branching strategy, namely which fractional decision variables to branch on. The branching strategy specifies how the solution space is partitioned to produce new nodes in the search tree. We also describe the stopping criterion employed for the B&P algorithm and how we generate initial columns in the root node of the search tree.

Search Strategy

The choice of search strategy has potentially significant consequences for memory requirements and amount of computation time needed for the B&P algorithm. As we perform a Dantzig-Wolfe reformulation of the original problem, we expect a tight gap between the solution in the root node of the B&P tree and the optimal integer feasible solution. Since we extend the B&P algorithm with a primal heuristic for finding good integer feasible solutions, which is introduced in Section 6.4.3, the focus in the B&P tree is on improving the dual bound. Therefore, our implementation employs the best-first strategy. Savelsbergh (1997) finds the best-first strategy to be effective in combination with a primal heuristic. This, because it tends to a scattered exploration of the B&P tree, that produces a variety of different columns for a primal heuristic to work on. Hence, the next node that is picked from the queue is the node with the highest objective value. Note that unexplored nodes have the objective value of their parent node as an objective value estimate. If there are multiple nodes with equal objective values, we pick the node with the least number of fractional values on the original binary variables in the subproblems.

Branching Strategy

In the master problem, constraints (6.25)—(6.30) force the convex combination of the binary parameters from the subproblem to be binary. We branch on the original binary variables in the subproblems if the solution to the LP-relaxation of the RMP is not integer feasible, instead of branching on the weighting variables. These binary variables in the subproblem are deployment in a module δ_{mp}, extraction from a tank ϵ_{tp} and transfer to tank σ_{tp}. The value of the tank active variable α_{tp} is unambiguously defined by the values of δ_{mp}, ϵ_{tp} and σ_{tp} as a consequence of the constraints in the subproblems. Therefore, we do not branch on α_{tp}.

The branching strategy is to branch on the most fractional original binary variable according to their assigned priority. The most fractional variable is the variable which in the solution of the LP-relaxed RMP has a value closest to 0.5. The binary variable δ_{mp} is assigned the highest priority and is branched on first. Thereafter ϵ_{tp}, and if all δ_{mp} and ϵ_{tp} in the relaxed RMP solution are binary, we branch on σ_{tp}. When we branch on a binary variable two new nodes are created. In each node, the variable that is branched on is set to either zero or one. All columns in the RMP previously generated in the parent nodes that violate the imposed value of the branching variable are excluded from the set $\bigcup_{m\in\mathcal{M}}\mathcal{K}_m$. Thus, each branch in the search tree is either a zero or one branch following a standard binary branching strategy (Morrison et al., 2016).

Stopping Criterion

The algorithm also includes a stopping criterion when the optimality gap is less than 1%. An optimality gap of less than 1% is sufficient for all practical purposes of the tactical production planning problem. The optimality gap applied in this thesis is defined in (6.31). The best upper bound is the lowest objective function value of a node in the B&P tree that acts as an upper bound to all nodes in the queue. The incumbent IP solution is the current best integer feasible solution, representing a lower bound for the objective value of the optimal solution.

$$\textit{Optimality Gap} = \frac{\textit{Best Upper Bound} - \textit{Incumbent IP solution}}{\textit{Incumbent IP solution}} \tag{6.31}$$

Generating Initial Columns

In the root node of the B&P tree, there exists no previously generated columns. Therefore, to initialize the Column Generation algorithm, we initiate the mathematical model presented in Section 5.3 using a MIP solver. The first integer feasible solution found is added as initial columns, one column per module, to the RMP.

6.4.3 Algorithmic Extensions

We have implemented extensions to the B&P algorithm. We expand the B&P algorithm by a matheuristic to quickly find good integer feasible solutions. Furthermore, we add several columns from each subproblem iteration. Another extension is to parallelize the solution process when the subproblems are solved to optimality using a MIP solver.

Inclusion of a Matheuristic

We implement a matheuristic into the B&P algorithm. The reason for including a matheuristic is to quickly find integer feasible solutions. How the matheuristic is included into the B&P algorithm is depicted in Figure 6.4. The RMP is solved using a MIP solver with the binary requirements in constraints (6.30), after the Column Generation algorithm produces an LP solution in a node. If the RMP is not solved to optimality within 5 seconds, we terminate the process. Whenever the RMP is infeasible, or no integer feasible solution is found after 5 seconds, the B&P algorithm continues to the next step. An integer feasible solution can be interpreted as the best possible combination of the available columns in the RMP. The incumbent IP solution is updated if the integer feasible solution from the matheuristic is better than the current best integer feasible solution. Following the matheuristic, the next step of the algorithm is to return to the solution of the LP-relaxation of the RMP to assess whether to branch or not.

Generating Additional Columns

The next extension is the addition of up to ten columns from each subproblem in each iteration of the Column Generation algorithm, depicted in Algorithm 1. When a subproblem is solved to optimality, the algorithm collects the ten best feasible solutions. All solutions with positive reduced costs are then added as columns to the master problem. This extension is included since there is no guarantee that the optimal solution for a subproblem is part of the optimal solution in the master problem. Adding several columns increase the probability that one of the columns added is part of the optimal solution to the master problem.

For each original column added to the RMP, we find two additional columns by using the values in the column stemming from the binary variables in the corresponding subproblem. We solve two optimization problems with all constraints from the subproblem, and all binary variables locked to the value specified by the original column. One optimization problem is solved with the objective to maximize total biomass produced, and one optimization problem is solved with the objective to minimize total biomass produced. As the values of the binary variables are locked in these problems, we apply a standard LP-solver. We get one column representing the maximum weight of salmon possible to produce given the values of the locked binary variables, and another column representing the minimum weight of salmon possible to produce given the values of the locked binary variables. The main motivation for adding these additional columns is that the matheuristic can now adjust the production of salmon in a module given the values on the original binary variables with more flexibility, through the weighting variables λ_{km}. Preliminary results indicate that these additional columns are highly utilized by the matheuristic to produce good integer feasible solutions.

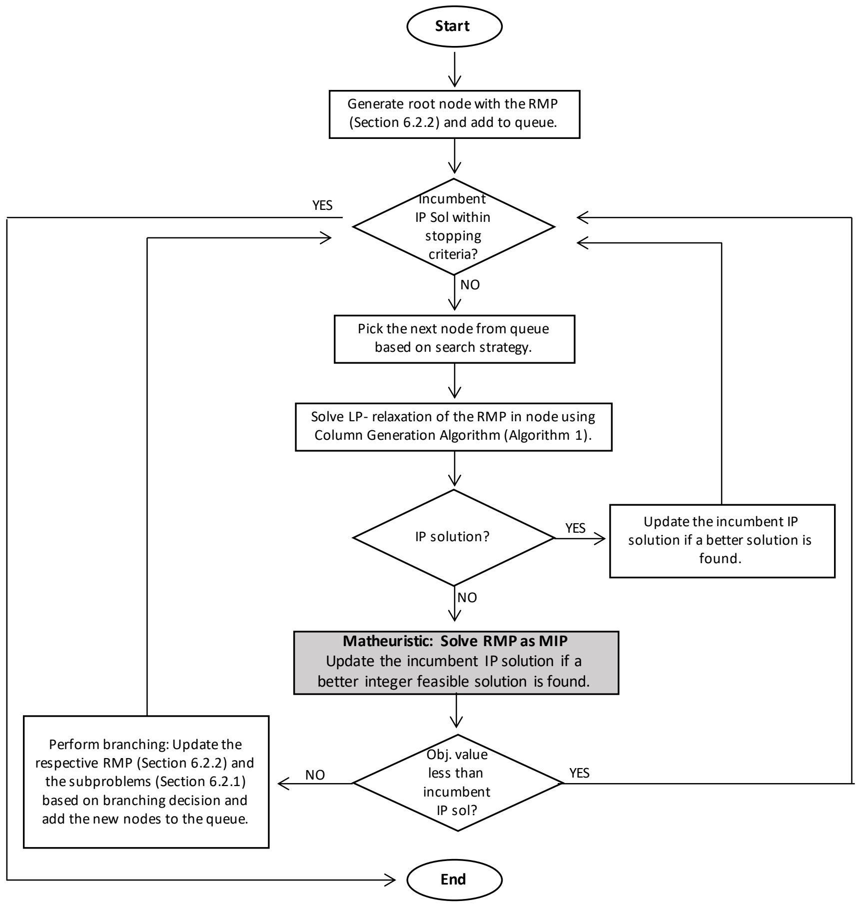


Figure 6.4: The Branch and Price algorithm with the extension of a matheuristic

Parallelization of Subproblems

To exploit the problem structure, in which we have one subproblem per module, we choose to solve the subproblems in parallel to speed up each iteration of the Column Generation algorithm, described in Algorithm 1. In each iteration, we update the subproblems with the dual information extracted from the RMP. Thereafter, we initiate the solution process of the subproblems in parallel. The Column Generation algorithm waits until all subproblems have been solved to optimality, before continuing to the next step, which is to re-optimize the RMP if any positive reduced cost is found. A potential improvement to the suggested implementation would be to dynamically solve and update the RMP and the subproblems. Whenever a subproblem would have been solved to optimality, columns could have been added to the RMP immediately and the RMP could have been re-optimized before all subproblems would have been solved to optimality. However, this potential improvement introduces additional aspects to consider in order to make it work, and it has not been tested as the current implementation is sufficient.

Chapter 7

Model Application - a Case Study

In this chapter, we present the case study that forms the basis for analyzing the tactical production planning problem, and the computational results in Chapter 8. In Section 7.1 we introduce the land-based production facility. Then, in Section 7.2 we present input data not explicitly given by the facility design and regulatory restrictions. These include the planning horizon, biomass data, revenue data, cost data, and other input parameters. Finally, in Section 7.3 we present instances used in the presentation of the computational results in Chapter 8.

7.1 Production Facility

In this section, we present the land-based production facility used to generate problem instances in this thesis. We start by introducing the facility design in Section 7.1.1. Then, we introduce the regulatory restrictions that the production facility needs to operate within in Section 7.1.2.

7.1.1 Facility Design

The land-based production facility is presented in Figure 7.1. The facility consists of seven modules, where each module contains four tanks. Thus, the facility comprises in total 28 tanks, each with a volume of 3 500 m^3. Seawater from 20–25 meters depth is lifted a certain height through large pumps to reach an elevation pool. In the elevation pool, seawater is mixed with recycled water, and oxygen is added. Then, the water is further distributed into the tanks. The recycled water only needs lifting from tank level up to the elevation pool. For reasons of confidentiality, we cannot specify the different lifting heights, but they are an essential part in the calculation of water pumping costs. Salmon transfer only occurs between specific tanks within a module. Numbering the tanks within a module from one to four, transfer is only allowed from tank one to tanks two and three, and from tank two to tank four. This information is captured through the sets $\hat{\mathcal{T}}_t$ and $\mathcal{T}_t$. Naturally, deployments into all tanks directly, are also allowed.

Figure 7.1: An illustration of the production facility with seven modules, where each module contains four tanks

7.1.2 Regulatory Restrictions

The acquired production license allows the land-based salmon producer to have a maximum of 3 900 tonnes of salmon in the facility at any given time. Hence, the maximum allowed biomass limit, L^{mab}, is at 3 900 tonnes. Further, the County Governor has issued an emission permit corresponding to a yearly production of 5 500 tonnes of salmon. Consequently, the yearly production limit, L^{prod}, is at 5 500 tonnes. These are production parameters for the full-scale production facility with seven modules containing four tanks each. To model instances with different total number of tanks, we scale L^{mab} and L^{prod} linearly according to the total number of tanks in the problem instance. For example, an instance with ten modules and four tanks in each module will have a MAB limit of 40/28 times L^{mab}, yielding 5 000 tonnes. The yearly production limit is scaled similarly to 7 857 tonnes. Further, we assume the facility to comply with regulatory fish welfare requirements for densities of salmon up to 45 kg/m^3 in each tank. Hence, L^{den}, is set to 45 kg/m^3.

7.2 Preprocessing of Input Data

This section presents input data that is not explicitly given from the production facility. In Section 7.2.1, we introduce the planning horizon that is implemented along with the initial conditions. All data belonging to biomass modeling is presented in Section 7.2.2. Then, we present the data on revenues in Section 7.2.3 and the data on costs in Section 7.2.4. In Section 7.2.5 we present the value on other parameter values that need to be stated for the implementation of the mathematical model.

7.2.1 Planning Horizon

The planning horizon for the production plan comprises 48 months constituting what is defined as the original planning horizon in Figure 7.2. The first month of the planning horizon is March, since March is the first possible release period. Therefore a production year, for which L^{prod} is applicable, is defined as starting from March and last to the end of February. The production facility operates with two release windows, where the first release window spans from March to May, and the second release window spans from September to November. Hence, the set of release periods, $\mathcal{P}^R$, contains periods corresponding to months within a release window. This is represented by green and blue boxes with vertical and horizontal lines in Figure 7.2. The blue boxes with horizontal lines are release periods belonging to the extended planning horizon. To model end-of-horizon, we expand the planning horizon by 17 months to capture the full effect of releasing smolt in the last release period belonging to the original planning horizon. When releasing smolt in release period 45, the salmon needs 17 months to reach an expected weight of 5,5 kg according to our biomass modeling. The set $\mathcal{P}$ comprises a total of 62 months, i.e. all months of the original planning horizon and all months of the extended planning horizon.

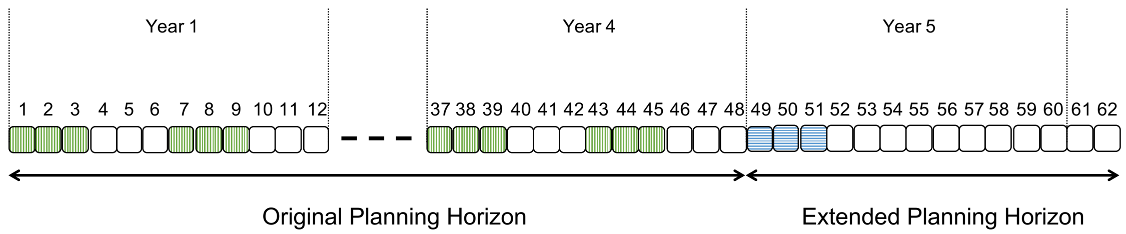


Figure 7.2: Illustration of the planning horizon

Generating Initial Conditions

We want an initial condition that reflects a stable state of production. As the production facility is not yet built, and we do not possess a fully developed production plan for the facility, we generate an initial condition for the production plan. Therefore, we run our model with the objective to maximize profits on an initially empty facility and save the production state at month 25. Then, we use this production state as an initial condition in the next run of the model. This process is repeated three times. The resulting production state at month 25 is used as the initial condition in all of the problem instances to make them comparable.

7.2.2 Biomass Data

This section introduces the data used to model biomass development in the mathematical model.

Defining Weight Ranges

Certain actions are available only during periods where the expected weight of an individual salmon in the population is at specific weights. In Table 7.1 we define the weight ranges where salmon is smolt, post-smolt, transferable and harvestable. These weight ranges are provided by Aquaculture Innovation, and define together with the expected weight development, the periods in the sets $\mathcal{P}_p^D, \mathcal{P}_p^{DE}, \mathcal{P}_p^{DT}, \mathcal{P}_{\hat{p}}^{PS}, \mathcal{P}_{\hat{p}}^{H}, \mathcal{P}_{\hat{p}}^{G}, \mathcal{P}_{\hat{p}}^{T}$.

Table 7.1: Defining the weight range for different salmon classes

Salmon Class	Expected Weight Range
Smolt	0.1 kg
Post-Smolt	0.8 kg – 1.5 kg
Transferable	0.5 kg – 2.5 kg
Harvestable	4.0 kg – 5.5 kg

Temperature Data

The temperature data we use is provided by the Norwegian Institute of Marine Research (Institute of Marine Research, 2020). The temperatures reflect the historical monthly seawater temperatures from 2014 to 2019 at a research station close to the production facility. These temperatures are measured at 20 and 30 meters depth. To approximate the water temperatures at 25 meters depth we calculate the average of the temperatures provided by the data set. Figure 7.3 illustrates the approximated temperatures from the research station at 25 meters depth. The actual temperatures applied in the implementation of the model are depicted by the black line in Figure 7.3, reflecting the historical mean temperature in a given month. The first month in the planning horizon is March. Hence, the temperature in the first month of the planning horizon is six degrees Celsius. We replicate these monthly mean temperatures in every year of the planning horizon.

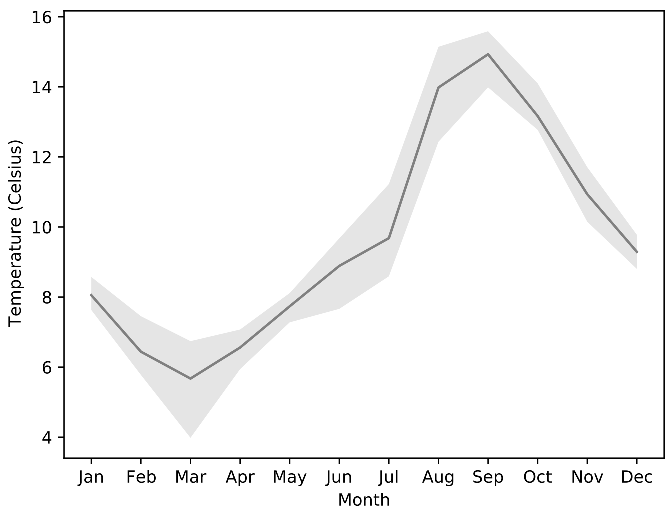


Figure 7.3: Approximated temperatures at 25 meters of depth. The shaded area illustrate the variance in measured temperatures between 2014 to 2019

Growth Data

The data used to calculate growth originates from a growth table developed by Skretting (2018), which is attached in Appendix B. The table entries describe the daily percentage weight increase for salmon with a given weight in different seawater temperatures. To comply with a monthly time resolution, we transform each of the table entries into monthly growth in kilograms. Using the initial weight of the table entry as a starting point, we calculate the salmon's daily weight development over 30 days given the temperature from the temperature profile above in Figure 7.3. Whenever the salmon weight enters a new entry in the growth table during the 30 days, the daily growth factor is updated to the corresponding value in the new entry. By dividing the salmon's resulting weight after 30 days by the initial weight, we get the monthly expected growth factor $G_{\hat{p}p}$ for salmon in period p originally deployed in period $\hat{p}$. An excerpt of the modified Skretting growth table is shown in Table 7.2, where each entry describes the monthly growth factor $G_{\hat{p}p}$. Smolt deployed in the first month exceeds the largest harvest weight at 5.5 kg in month 18. Therefore, growth factors are only defined for 17 months in this case.

According to Aquaculture Innovation, salmon that are transferred are expected to lose two weeks of growth. Therefore, the growth factor for salmon that are transferred in period p, when originally deployed in period $\hat{p}$, is given by,

$$G_{\hat{p}p}^{T} = 1 + (\frac{G_{\hat{p}p} - 1}{2}).$$

Table 7.2: Salmon growth factor in period p based on deployment period $\hat{p}$

p	**1**	**2**	**3**	...	**15**	**16**	**17**
$\hat{p} = 1$	1.45	1.46	1.50	...	1.13	1.13	1.13
p	**2**	**3**	**4**	...	**16**	**17**	**18**
$\hat{p} = 2$	1.51	1.53	1.50	...	1.14	1.14	1.14
p	**3**	**4**	**5**	...	**16**	**17**	**18**
$\hat{p} = 3$	1.57	1.55	1.56	...	1.15	1.15	1.14
⋮				⋮			
p	**43**	**44**	**45**	...	**56**	**57**	**58**
$\hat{p} = 43$	1.78	1.69	1.54	...	1.17	1.15	1.12
p	**44**	**45**	**46**	...	**57**	**58**	**59**
$\hat{p} = 44$	1.80	1.64	1.48	...	1.16	1.13	1.12
p	**45**	**46**	**47**	...	**59**	**60**	**61**
$\hat{p} = 45$	1.73	1.54	1.46	...	1.12	1.09	1.09
⋮				⋮			

Distribution of Salmon into Weight Classes after Extraction

The distributional factor, $Z_{w\hat{p}p}$, specifies the fraction of the total weight extracted that is distributed into weight class w. The weight classes span from 0.5 kg to 9 kg to cover all possible weights of salmon when extracted either as post-smolt or harvestable salmon. An interval of 0.1 kg between each weight class yields a sufficient approximation of a normal distribution. This gives a total of 86 weight classes. By analyzing harvest reports provided by Aquaculture Innovation, we estimate that the variations in growth rates among salmon with a mean weight of four to five kilograms can be approximated by a normal distribution with a 15% coefficient of variation. We assume an equivalent coefficient of variation when extracting post-smolt. The coefficient of variation and the expected weight of an individual salmon in the population are used to calculate the variance in growth in the normal distribution. Figure 7.4 illustrates three examples on how a total extracted weight of 100 000 kg is distributed to the different weight classes given three expected individual salmon weights.

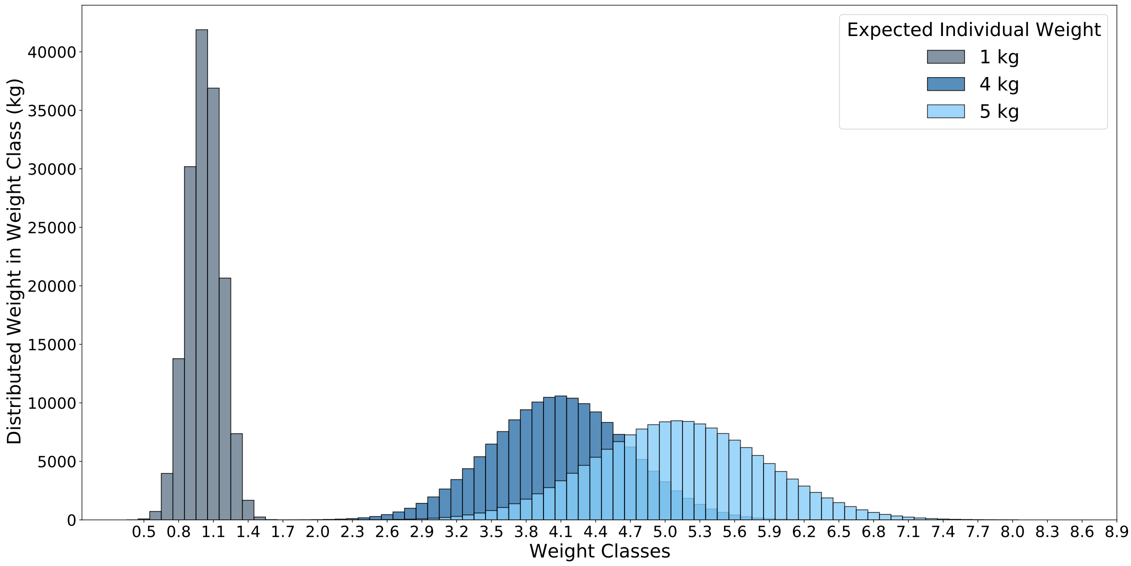


Figure 7.4: Distributing total extracted weight of 100 000 kg salmon into weight classes around three expected weights of an individual salmon

Production Loss

There are few empirical results on production loss in land-based salmon farming. Fredrikstad Seafood is the first and only land-based producer in Norway that has completed a full production cycle (Fletcher, 2020). They have reported a production loss of five percent during the first year of production. In lack of other data for production loss in land-based salmon farming, we apply the same value for production loss. As every period represents one month, we transform this yearly production loss to a monthly production loss P^{loss}. This is done through (7.1), where P^{year} is the yearly production loss and M is the number of months in a year.

$$P^{loss} = 1-(1-P^{year})^{1/M} = 1-(1-0.05)^{1/12} = 0.0043 \tag{7.1}$$

7.2.3 Revenue Data

We apply the average NASDAQ salmon prices from January 2016 to March 2021 to estimate the salmon price for different weight classes (NASDAQ, 2021). The salmon price parameters R_w^H are adjusted by a processing and packaging cost of 4.5 NOK/kg, according to Aquaculture Innovation, resulting in prices for harvestable salmon as presented in Table 7.3a. Additionally, the total weight of harvested salmon is adjusted by a harvest yield of 85 %, which is the current industry standard (Skretting, 2021a). Historical data on post-smolt prices is lacking, but Aquaculture Innovation estimates prices as given in Table 7.3b. Notice that these are prices per post-smolt, not per kg as for the salmon that are harvested. Hence, to get the parameter values of R_w^{PS}, the post-smolt prices are adjusted by dividing the prices by the weight of each weight class w.

Table 7.3: Salmon prices

(a) Harvested salmon prices

Salmon Weight	Price
2 kg - 3 kg	47 NOK/kg
3 kg - 4 kg	53 NOK/kg
4 kg - 5 kg	55 NOK/kg
5 kg - 6 kg	57 NOK/kg
6 kg - 7 kg	60 NOK/kg
7 kg - 8 kg	60 NOK/kg
8 kg - 9 kg	61 NOK/kg

(b) Post-smolt prices

Post-Smolt Weight	Price
0.5 kg - 1.0 kg	40 NOK/post-smolt
1.0 kg - 2.0 kg	50 NOK/post-smolt

7.2.4 Cost Data

The smolt cost is at 16 NOK/smolt, which implies that the purchase price of smolt, C^D, is 160 NOK/kg, as we operate with 0.1 kg smolt-deliveries. We use a feed price of 14 NOK per kilogram feed, and FCR-values between 0.8 to 1.35 as given by the Skretting growth table, attached in Appendix B. The FCR-values increase with increasing salmon weight.

The price of oxygen is at 2.6 NOK/kg which is an industry standard (Bjørndal & Tusvik, 2019b). The oxygen cost parameter $C_{\hat{p}p}^O$ is calculated through the price of oxygen and the monthly oxygen consumption per kilogram salmon for salmon with a given expected weight. To estimate the oxygen consumption for salmon, we apply Liao´s (1971) formula estimating the oxygen consumption per minute for salmon with a given weight in a given water temperature:

$$0_2 = \frac{K}{0.144} 32 + 1.8T^n (0.454W)^m,$$

where O_2 is the oxygen consumption per minute per kilogram salmon with weight W, T is the water temperature in degrees Celsius, and K, m, n are constants provided by Aquaculture Innovation that are undisclosed for reasons of confidentiality. To find the monthly oxygen consumption for salmon with an expected weight, we multiply this value by the number of minutes in a month.

Estimates on the water pumping costs in a land-based facility are hard to find due to lack of available data. We choose an estimate that requires a certain minimum level of water pumping, in addition to extra water flow per kilogram salmon in the tank. We apply an estimate on the pumping costs per cubic meters of water per meter lifting height given by Aquaculture Innovation. This estimate is multiplied by the tank-volume and the lifting height to give a minimum level of water pumping cost, C^{min}, at 30 030 NOK per tank per month. According to Hilmarsen et al. (2018) a land-based facility based on Flow-Through (FTS) technology, has an estimated water usage of 0.3 L/min/kg salmon. This information is applied to calculate the additional marginal increase in monthly pumping costs connected to a tank C^{MC} at 0.231 NOK per kilogram per month. A tank containing 100 000 kg salmon will thus have a total of 53 130 NOK in water pumping costs for that month.

7.2.5 Other Input Parameters

Finally, we present the values of the big-M coefficients in the mathematical model. The value of E^{max}, which is the maximum weight of salmon extracted from a tank, is simply calculated through the inverse tank-volume V_t and the maximum allowed density L^{den}, resulting in a value of 157 500 kg. When deploying smolt into a module, the smolt will have to grow at least 8 times larger before it can be extracted as post-smolt. A full module contains 630 000 kg salmon. Dividing 630 000 kg by 8 gives 78 750 kg. As of temperature variations and loss of growth when transferring, we set D^{max} to 85 000 kg to ensure it does not affect any optimal solution. To estimate a value for D^{min} we assume that operating less than two of the four tanks within a module from deployment until salmon are harvested at the maximum harvest weight of 5.5 kg is considered inefficient. Two full tanks contain 315 000 kg, and when the expected weight is at 5.5 kg, the smolt are 55 times larger than when deployed. Hence, D^{min} is set to 5 725 kg. The maximum weight to transfer from a tank, Y^{max}, is set to the same value as E^{max}. For the minimum weight to transfer from a tank, Y^{min}, we set this value equal to 14 300 kg. Transferring salmon at 0.5 kg, which is the lowest possible transfer weight, the salmon are 11 times larger when they reach the maximum harvest weight at 5.5 kg. Hence, we arrive at a value of 14 300 kg for Y^{min}. A summary of the parameter values is given in Table 7.4.

Table 7.4: Overview of parameter values used in the computational study

Parameter	Description	Value/Range
C^D	Purchase price of smolt [NOK/kg]	160
R_w^{PS}	Post-smolt revenues [NOK/post-smolt]	40–50
R_w^H	Harvest revenues [NOK/kg]	47–61
-	Food Conversion Ratio	0.8–1.35
-	Feed Price [NOK/kg]	14
-	Oxygen Price [NOK/kg]	2.6
C^{min}	Minimum monthly cost of operating a tank [NOK]	30 030
C^{MC}	Marginal increase in monthly pumping costs for a tank [NOK/kg]	0.231
L^{den}	Maximum allowed density at any given time [kg/m^3]	45
L^{mab}	Maximum allowed biomass at any given time [tonnes]	3900
L^{prod}	Yearly production limit due to emission permit [tonnes]	5500
P^{loss}	Monthly rate of production loss	0.43%
Y^H	Harvest yield of slaughtered salmon	85%
V_t	Inverse volume of a tank [m^{-3}]	3500
E^{max}	Maximum weight of salmon extracted from a tank [kg]	157 500
D^{max}	Maximum weight of smolt deployed in a module [kg]	85 000
D^{min}	Minimum weight of smolt deployed in a module [kg]	5 725
Y^{max}	Maximum weight of salmon transferred from a tank [kg]	157 500
Y^{min}	Minimum weight of salmon transferred from a tank [kg]	14 300

7.3 Problem Instances

In this section we present problem instances of the tactical production planning problem for land-based salmon farming. The purpose of these instances is to analyze how different instance characteristics affect the suggested production plans for the land-based facility. An overview of the main problem instances is given in Table 7.5.

Table 7.5: Overview of the main problem instances analyzed in the computational study

Instance	Objective	Transfer allowed
Profit Transfer	Profit	Yes
Biomass Transfer	Biomass	Yes
Profit No Transfer	Profit	No
Biomass No Transfer	Biomass	No

All instances reflect a full-scale production facility with seven modules and four tanks in each module. The Profit Transfer instance includes the option of transferring salmon as part of the production cycle with an overall objective to maximize total profits over the planning horizon.

In practice, most of the production plans within salmon farming are developed to maximize the total biomass production over the planning horizon (O. Skålnes, personal communication, 2021). To analyze how the production plan changes with a biomass objective rather than a profit objective, we generate the Biomass Transfer instance where Equation (5.2) is applied as the objective for the mathematical model.

There are uncertainties regarding the benefits of salmon transfer and whether it should be included in the production planning due to the trade-off between increased flexibility and loss in growth. To analyze the benefits of transferring salmon, we include instances Profit No Transfer and Biomass No Transfer where transferring salmon is not allowed.

Chapter 8

Computational Results

In this chapter, we discuss and analyze the results obtained from solving different problem instances of the tactical production planning problem within land-based salmon farming. In Section 8.1, we study the effect of applying Branch and Price (B&P) as a solution method and analyze technical aspects regarding the B&P algorithm applied in this thesis. Then, in Section 8.2 we focus on creating managerial insight into the problem by analyzing how the suggested production changes with different instance characteristics.

8.1 Technical Analysis

In this section, we analyze technical aspects regarding the B&P algorithm. In Section 8.1.1 we introduce the problem size of the instances of the mathematical model presented in Chapter 5 and evaluate the computational performance from applying a straightforward MIP solver on these instances. Then, in Section 8.1.2 we analyze the effect of applying the proposed B&P algorithm as a solution method. In Section 8.1.3 we discuss the performance of the B&P algorithm. Finally, in Section 8.1.4 we study the scalability of the B&P algorithm in terms of how large problem instances the current implementation can solve.

The B&P algorithm is implemented using Java version 11.0.2 as programming language, while all mixed integer programs and linear programs are solved with Gurobi 9.0.2. The input data is preprocessed in a script written in Python version 3.7. The problem instances are solved on a computer with a linux64 operating system and 2 x 2.3 GHz Intel E5-2670v3 12 Core CPU and 64 GB of installed RAM.

8.1.1 Solving Problem Instances Using Branch and Bound

We start by evaluating the computational runtime and solution quality when solving the problem instances presented in Section 7.3 with Gurobi's Branch and Bound (B&B) algorithm. The maximum computational runtime is set to two days (172 800 seconds). We terminate when reaching an optimality gap of less than 1%, as this is the stopping criterion proposed in the B&P algorithm.

To give an overview of the problem sizes, the number of constraints (Rows) and the number of decision variables (Columns) in the different problem instances before presolve in Gurobi are shown in Table 8.1. The number of binary variables is indicated in parenthesis. Recall that the instances are solved over a planning horizon of 62 months as a consequence of the end-of-horizon modeling explained in Section 5.1.4 and Section 7.2.1.

Table 8.1: Overview on the problem size of the instances before presolve in Gurobi

Instance	Rows	Columns	Non-zero Elements
Profit Transfer	41 298	27 478 (5 337)	215 121
Profit No Transfer	34 998	22 057 (3 717)	148 386
Biomass Transfer	41 298	27 478 (5 337)	215 121
Biomass No Transfer	34 998	22 057 (3 717)	148 386

Note that the problem sizes of Profit Transfer and Biomass Transfer are equal, as we only change the objective function in the two instances. Excluding the option to transfer salmon reduces the number of constraints (Rows) by 15 % and the number of decision variables (Columns) by 20 %.

The problem complexity becomes evident when analyzing the computational results obtained by the B&B algorithm on the profit objective instances. These results are presented in Table 8.2. Table 8.2 includes the lower bound (LB) and the upper bound (UB) of the objective value obtained over 62 months.

Table 8.2: Computational results for the profit objective instances solved with the B&B algorithm

Instance	Solution Method	Runtime [s]	Gap [%]	LB [10^6 NOK]	UB [10^6 NOK]
Profit Transfer	B&B	172 800	19.1	621.09	739.65
Profit No Transfer	B&B	172 800	15.0	534.34	614.67

We observe a considerable optimality gap when the computational runtime limit of 172 800 seconds is reached for both Profit Transfer and Profit No Transfer. Either the B&B algorithm fails in finding good integer feasible solutions, or the resulting LP-relaxation of the MIP is weak. Including revenues and costs into the production planning could lead to a weak dual bound on the objective value, as a given biomass production level can be associated with different cost and revenue profiles. Anyhow, the problem instances are too complex for the B&B algorithm to be able to close the optimality gap within two days. This indicates the need for a more sophisticated solution method for these problem instances.

Table 8.3 shows that the B&B algorithm succeeds in solving the Biomass Transfer and Biomass No Transfer instances to a tight optimality gap, even though it struggles to close the optimality gap below 1.02 % in the Biomass No Transfer instance within the computational runtime limit. The solution with an optimality gap of 1.02% in the Biomass No Transfer instance was found after 5 067 seconds.

Table 8.3: Computational results for the biomass objective instances solved with the B&B algorithm

Instance	Solution Method	Runtime [s]	Gap [%]	LB [10^3 kg]	UB [10^3 kg]
Biomass Transfer	B&B	2 178	0.98	28 142	28 417
Biomass No Transfer	B&B	172 800	1.02	28 130	28 417

Analyzing the upper bound in Table 8.3, we find that this value corresponds to the objective value obtained from producing at the yearly production limit in every year. The instances are solved over 62 months, resulting in a maximum possible production of 28 417 tonnes over these months assuming a yearly production limit at 5 500 tonnes. Hence, the yearly production limit acts as a strong dual bound for the Biomass Transfer and Biomass No Transfer instances, strengthening the suggestion that including revenues and costs into the production planning leads to a weaker dual bound. As the B&B algorithm finds good integer feasible solutions fast, the yearly production limit results in a tight optimality gap for the biomass objective instances.

To test the hypothesis of the yearly production limit acting as a strong dual bound, we re-run the Biomass Transfer instance from an empty facility. As it is impossible to reach the yearly production limit during the first year starting from an empty facility, we get a weaker dual bound in this case. As Table 8.4 shows, the B&B algorithm does not succeed in finding a lower optimality gap than 2.79 % after 172 800 seconds in this case. This observation indicates that without a strong dual bound, the problem is too complex for the B&B algorithm to be able to close the optimality gap.

Table 8.4: Computational results for the Biomass Transfer instance starting from an empty facility

Instance	Solution Method	Runtime [s]	Gap [%]	LB [10^3 kg]	UB [10^3 kg]
Biomass Transfer Empty Facility	B&B	172 800	2.79	27 326	28 088

In Føsund and Strandkleiv (2020) a biomass objective instance with only two modules and a total of eight tanks solved over 48 months ended at an optimality gap of 39 % after 85 368 seconds. Comparing the overall results presented thus far against those obtained in Føsund and Strandkleiv (2020) it is evident that the mathematical model suggested in this thesis enables the B&B algorithm to handle considerably larger dimensions of the land-based facility at a tank-level of detail. The major reason behind the improved computational performance is the new approach for modeling biomass development in this thesis, reducing the number of decision variables.

8.1.2 Effect of Branch and Price as Solution Method

In this section, we solve the problem instances using the proposed B&P algorithm to analyze the effect of applying this solution method against the results obtained by the B&B algorithm. The computational results of applying the B&P algorithm on the profit objective instances are presented in Table 8.5. The change in lower bound (△ LB [%]) denotes the increase in lower bound for the instance solved by B&P relative to the lower bound obtained by the B&B algorithm. The change in the upper bound (△ UB [%]) denotes the reduction in the upper bound for the instance solved by the B&P algorithm relative to the upper bound obtained by the B&B algorithm.

Table 8.5: Computational results for profit objective instances solved with B&P

Instance	Solution Method	Runtime [s]	Gap [%]	LB [10^6 NOK]	UB [10^6 NOK]	△ LB [%]	△ UB [%]
Profit Transfer	B&P	1 555	0.38	623.36	625.75	0.4	15.4
Profit No Transfer	B&P	684	0.33	534.30	536.08	0.0	12.8

Solving the profit objective instances with the proposed B&P algorithm yields a reduction in computational runtime and an improvement in optimality gap. The Profit Transfer instance is solved within 26 minutes to an optimality gap of 0.38%. This is a considerable improvement compared to solving the instance with a regular B&B algorithm which ended at an optimality gap of 19.1%. The Profit No Transfer instance is solved within 12 minutes to an optimality gap of 0.33%, where the B&B algorithm ends at 15.0% optimality gap. Note the considerable reduction in the upper bound values for the Profit Transfer and Profit No Transfer instances. This indicates that the Dantzig-Wolfe reformulation gives a strong description of the convex hull of the original mixed integer problem. This reformulation yields a strong LP-relaxation in the root node of the B&P tree. Thus, the B&P algorithm succeeds in improving the dual bound, causing considerably tighter optimality gaps for the profit objective instances.

For the Biomass Transfer instance, we observe in Table 8.6 that the B&P algorithm finds a good solution within the stopping criterion of 1% with a major reduction in computational runtime. There are no improvements in the upper bounds due to the yearly production limit constraint, which already acts as a strong upper bound. Note that in the Biomass Transfer Empty Facility instance, in which the dual bound is weaker, the B&P algorithm finds a solution with an optimality gap of 0.33% within 1 272 seconds. The B&B algorithm ended at an optimality gap of 2.79% after the computational runtime limit of 172 800 seconds. However, the lower bound at 27 326 tonnes found by the B&B algorithm is by all practical means the optimal solution considering the upper bound at 27 327 tonnes found by the B&P algorithm.

Table 8.6: Computational results for biomass objective instances solved with B&P

Instance	Solution Method	Runtime [s]	Gap [%]	LB [10^3 kg]	UB [10^3 kg]	△ LB [%]	△ UB [%]
Biomass Transfer	B&P	1 663	0.78	28 197	28 417	0.2	0.0
Biomass Transfer Empty Facility	B&P	1 272	0.33	27 234	27 327	-0.3	2.7
Biomass No Transfer	B&P	7 253	0.98	28 130	28 405	0.0	0.0

The increased computational runtime when solving the Biomass No Transfer instance is due to branching. The B&P algorithm finds a solution with an optimality gap of 1.32% already within 304 seconds, but does not succeed in closing this optimality gap below the stopping criterion of 1 % before reaching 7 253 seconds. The performance of the B&P algorithm will be discussed more closely in Section 8.1.3.

While the improvement in the upper bound is considerable in the B&P algorithm, the B&B algorithm succeeds in finding equally good lower bounds. This indicates that one of the strengths of the B&P algorithm resides in the dual bound improvement. This dual bound improvement drastically reduces the computational runtime needed to find a solution within the stopping criterion.

8.1.3 Technical Aspects of Branch and Price

In this subsection, we analyze technical aspects regarding the B&P algorithm in more detail. In Table 8.7 we present how the B&P algorithm performs in terms of the number of nodes it visits in the B&P tree (# Nodes), total number of RMP solved (# Iterations), how many subproblems it solves (# SP Solved), and some statistics regarding the solution time in the subproblems. We solve the subproblems in parallel at every iteration between the master and the subproblems as described in Section 6.4.3. Thus, when optimizing the subproblems we wait until all subproblems are solved before we re-optimize the RMP. The time spent in the subproblems (Time in SP [s]) therefore indicates the total time, as a sum over all iterations, for the last subproblem in every iteration to finish. We also report the time spent in the subproblems as a percentage of the total runtime (Time in SP [%]), and the minimal (Min SP [s]) and maximal (Max SP [s]) time spent solving a subproblem.

Table 8.7: Information regarding algorithmic aspects of the B&P Algorithm

Instance	# Nodes	# Iterations	# SP solved	Time in SP [s]	Time in SP [%]	Min SP [s]	Max SP [s]
Profit Transfer	1	4	28	1 383	89	142	384
Profit No Transfer	1	7	41	310	45	10	85
Biomass Transfer	1	4	28	1 241	75	10	488
Biomass No Transfer	421	475	2 836	1 375	19	0.006	26

Notice that the instances Profit Transfer, Profit No Transfer, and Biomass Transfer are solved in the root node of the B&P tree. Having a strong dual bound, we solve the problem in the root node since we have columns, i.e. production plans for modules, of sufficient quality for the matheuristic to find an integer feasible solution with an optimality gap within the stopping criterion of 1 %. Recall that columns are added through the Column Generation procedure, Algorithm 1, and the generation of additional columns described in Section 6.4.3. The matheuristic solves the resulting RMP as a MIP after terminating Algorithm 1.

Only four iterations are needed to solve the Profit Transfer and Biomass Transfer instances. During these four iterations a total of 28 subproblems are solved. Every masterproblem is solved within a second, whereas most of the computational runtime is spent solving the subproblems. The time spent solving the subproblems varies depending on the reduced cost objective. In the Profit Transfer instance, it takes between 142 seconds and 384 seconds to solve the different subproblems, and 89 % of the total computational runtime is spent solving the subproblems. In the Biomass Transfer instance, 75 % of the total runtime is spent solving the subproblems. This indicates that solving the subproblems represents the computational bottleneck in the Profit Transfer and the Biomass Transfer instances. Thus, an interesting area for future research could be to explore possible solution methods for solving the subproblems faster.

In the Profit No Transfer instance seven iterations are performed solving a total of 41 subproblems in the root node of the B&P tree. The time spent in the subproblems relative to the total computational runtime is significantly less than in the Profit Transfer and Biomass Transfer instances. The Profit No Transfer instance subproblems are solved within 85 seconds, indicating that excluding transfer variables yields considerably easier subproblems. This is also observed for the Biomass No Transfer instance, where a total of 2 836 subproblems are solved but only 19% of the total runtime is spent in the subproblems.

In the Biomass No Transfer instance, branching is necessary to find a solution satisfying the stopping criterion. Figure 8.1 illustrates the development of the upper and lower bound as a function of solution time in seconds. The matheuristic extension closes the optimality gap to 1.32% after 363 seconds in the solution process. The different jumps in the lower bound occur as the matheuristic finds better integer feasible solutions as more columns are generated. As described in Section 6.4.2, the B&P algorithm is implemented with standard binary branching on the original binary variables in the subproblems. Through branching, better columns are added to the RMP, allowing the matheuristic to find improving integer feasible solutions throughout the solution process. From Figure 8.1 we observe that the matheuristic finds a total of six improving integer solutions. Eventually, after 7 253 seconds the dual bound is improved, reaching an optimality gap of 0.98%.

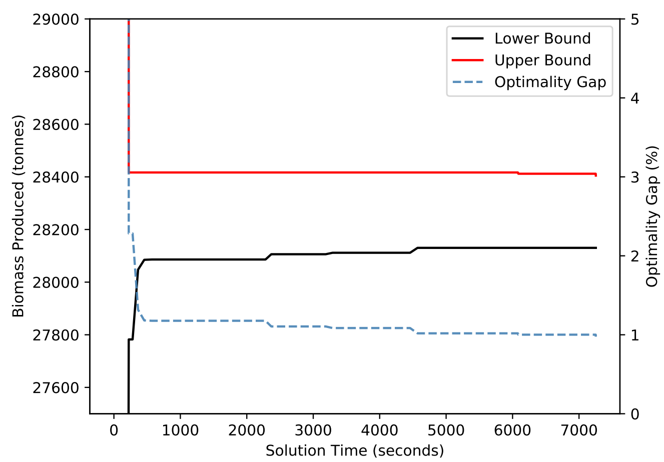


Figure 8.1: Development of upper bound, lower bound and optimality gap for the Biomass No Transfer instance throughout the B&P solution process

8.1.4 Scalability of Current Branch and Price Algorithm

To analyze the scalability of the current B&P algorithm with respect to larger problem instances we introduce additional instances of the Profit Transfer instance as shown in Table 8.8. We focus on the scalability of the Profit Transfer instance since this instance includes profits and transfer of salmon, both of which increase the time spent in the subproblems. The M7T4 instance is equivalent to the original Profit Transfer instance, but we start from an empty facility to make it comparable to the other instances. The name of the instance indicates the number of modules and the number of tanks per module. The MAB limit and the yearly production limit are scaled as described in Section 7.1.2.

Table 8.8: Overview of the instances run to analyze the scalability of the B&P algorithm.

Instance	# Modules	# Tanks per module	# Total tanks	L^{mab}	L^{prod}
M7T4	7	4	28	3 900	5 500
M14T4	14	4	56	7 800	11 000
M21T4	21	4	84	11 700	16 500
M28T4	28	4	112	15 600	22 000
M7T5	7	5	35	4 875	6 875
M7T6	7	6	42	5 850	8 250
M7T7	7	7	49	6 825	9 625
M7T8	7	8	56	7 800	11 000

Increasing the number of modules but not the number of tanks per module, increases the size of the master problem and the total number of subproblems. However, the size of each subproblem remains unchanged. As we are able to solve the masterproblem fast and exploit the structure of the problem by solving the subproblems in parallel, we expect moderate increases in computational runtime with increasing number of modules.

More interesting is an increase in number of tanks per module, as this increases the complexity of the subproblems. As the subproblems already represent the bottleneck for computational performance in the Profit Transfer instance, we expect a large increase in computational runtime with an increasing number of tanks per module.

The computational results from solving the additional instances are shown in Table 8.9. The instances were solved in the root node of the B&P tree. As expected, an increase in number of modules leads to moderate increases in computational runtime. The time spent solving the different subproblems does not change across the M7T4, M14T4, M21T4 and M28T4 instances. The increased runtime is due to the increased size of the master problem, which requires more computational time on storing columns and updating the respective master and subproblems. Note that we experience a constant increase in computational runtime of around 700 seconds when adding 7 modules. This indicates that the B&P algorithm is able to scale well in the number of modules.

Table 8.9: Computational results on larger problem instances

Instance	Runtime [s]	Gap [%]	Time in SP [s]	Min SP [s]	Max SP [s]
M7T4	1 840	0.47	1 019	20	234
M14T4	2 528	0.08	1 124	20	205
M21T4	3 286	0.27	1 061	21	208
M28T4	3 884	0.08	1 133	21	197
M7T5	6 698	0.17	4 718	59	1 195
M7T6	8 859	0.45	7 644	102	2 046
M7T7	43 980	0.08	42 754	243	15 444
M7T8	-	-	-	-	-

We observe a significant increase in computational runtime for the instances with an increased number of tanks per module. The maximum time spent on solving the subproblems increases more than five-fold from the M7T4 instance to the M7T5 instance. At the M7T7 instance we experience a major increase in time spent solving the different subproblems, and at the M7T8 instance we run into memory-issues causing the program to halt. This observation strengthens the suggestion of considering alternative solution methods for solving the subproblems in order to decrease computational runtime and solve larger problem instances.

The current implementation of the B&P algorithm scales well in the number of modules, but less well in the number of tanks per module. We solve the M28T4 instance to an optimality gap of 0.08 % within 3 884 seconds, whereas a computational runtime of 6 698 seconds is needed to solve the M7T5 instance.

8.2 Analyzing the Production Planning Problem

In this section, we analyze the results from solving the instances presented in Section 7.3. We present an overview of the results in Section 8.2.1. Note that while being solved over 62 months, these results are for a planning horizon over 48 months as we develop production plans spanning a total of four years.

Maximizing biomass has, with success, been the focus for production planning in traditional sea-based salmon farming. However, land-based salmon farming introduces additional aspects as post-smolt production, salmon transfer, and yearly production limits, whose effect on production planning has not been evaluated. Hence, to evaluate how the objective of the production planning affects the proposed production strategy at the land-based facility, we compare a profit maximizing production plan with a biomass maximizing production plan in Section 8.2.2.

Transferring salmon between tanks during the production cycle introduces flexibility into the production planning. On the other hand, it also introduces stress that, in principle, is undesirable due to the impact on growth performance and salmon welfare. Therefore, we assess the value of including salmon transfer by comparing production plans with and without the flexibility of transfer in Section 8.2.3.

The option to produce post-smolt is arguably the most distinct factor in the production planning of a land-based facility to the production planning of a sea-based facility. Therefore, in Section 8.2.4, we evaluate the attractiveness of post-smolt production.

8.2.1 Overview of Results

Table 8.10 presents an overview of the results in terms of the profits obtained, total biomass produced, level of post-smolt production, and capacity utilization rates over the four year planning horizon. The level of post-smolt production (Post-Smolt [%]) indicates the percentage of total weight extracted over the four years that are post-smolt. The MAB utilization rate (MAB [%]), is the average weight of biomass in the facility over the four year planning horizon relative to the MAB limit of 3 900 tonnes. Similarly, the yearly production limit utilization rate (Prod-Limit [%]) indicates the average yearly production over the four years relative to the yearly production limit of 5 500 tonnes. The boldface indicates the actual objective of the instance, while the other objective is postprocessed for the reason of comparison.

Table 8.10: Overview of the results from the problem instances presented in Section 7.3

Instance	Profit Objective [MNOK]	Biomass Objective [Tonnes]	Post-smolt [%]	MAB [%]	Prod-Limit [%]
Profit Transfer	**411**	21 528	56.1	49.0	97.9
Profit No Transfer	**373**	21 356	88.2	38.4	97.1
Biomass Transfer	329	**21 904**	88.5	40.0	99.6
Biomass No Transfer	356	**22 000**	97.1	37.8	100.0

Note that the general tendency across all instances is that the yearly production limit is close to fully utilized. The MAB utilization rate is considerably lower, indicating that the yearly production limit is more restricting to the production than the MAB limit.

Post-smolt production seems to be the preferred production strategy. However, in the Profit Transfer instance we have an almost even production level of post-smolt and harvestable salmon. The Profit Transfer instance yields the highest expected profit of NOK 411 million, whereas the Biomass No Transfer instance yields the highest expected total biomass produced of 22 000 tonnes.

In the Biomass Transfer instance transferring salmon is included as an option. Therefore, we may conclude that a production plan over four years with the option of transferring salmon is at least as good as a production plan without this option. The reason why Biomass No Transfer produces more than Biomass Transfer in the first 48 months is that the instances are solved over 62 months to deal with end-of-horizon effects. In the following analysis when comparing the effects that occur by changing objectives, we use the results from the Profit Transfer instance and the Biomass No Transfer instance.

8.2.2 Effects of Changing Production Objective

In this section, we analyze the resulting production plans from the Profit Transfer and Biomass No Transfer instances. We start by introducing the two suggested production plans to expose typical production patterns, and to highlight differences across the production plans. Then, in light of the suggested production, we analyze the components behind the expected profit.

Production Strategies

The suggested production plan for the Profit Transfer instance is depicted in Figure 8.2. The typical production pattern for each module seems to be to combine production cycles solely producing post-smolt with production cycles producing both post-smolt and harvestable salmon. This production pattern seems to be replicated in all of the modules, with a shift in the timing of deployments and extractions. Although we identify a production pattern, there are different variants of how to combine post-smolt production cycles with production cycles producing both post-smolt and harvestable salmon across the modules. Bjørndal (1988) and Guttormsen (2008) focused on finding the optimal rotation to solve the production planning problem within aquaculture. The production plan in Figure 8.2 includes different sequences of production cycles across all modules. This suggests that focusing on finding the optimal rotation for one module and apply this rotation to all modules is a rough simplification of the land-based tactical production planning problem and would imply a violation of production regulations.

An example of the typical production pattern can be seen in module three, containing tanks 9–12. The deployment in May-21, indicated by blue dots, initiates a production cycle where we combine production of post-smolt in tanks 11 and 12, indicated by green dots in October-21, with the production of harvestable salmon extracted from all tanks in July-22, as indicated by the large black dots. In January-22 the flexibility of transferring salmon during the production cycle is exploited to accommodate the combination of post-smolt and harvestable salmon production. Having extracted post-smolt from tanks 11 and 12 in October-21, these tanks receive salmon from tanks 9 and 10 that has grown larger since deployment in May-21. Then, salmon are harvested in July-22 when the density within all tanks are reached. Following a month without any salmon in the module containing tanks 9–12 due to cleaning and maintenance, a production cycle solely intended for post-smolt production is initiated in September-22.

We observe that shifts in deployments and extractions are necessary to distribute the total weight of salmon extracted throughout the production years and to avoid violating the MAB limit in certain months. Figure 8.3a illustrates the total weight of salmon extracted during each production year and Figure 8.3b illustrates the total weight of salmon in the facility.

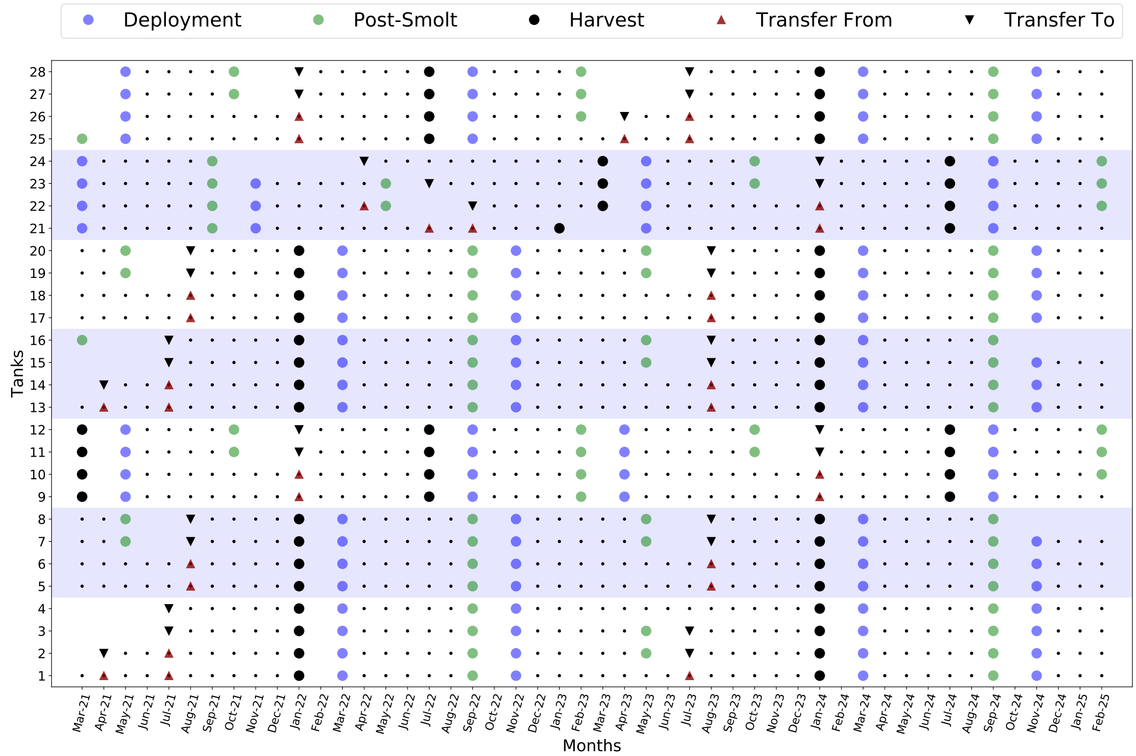
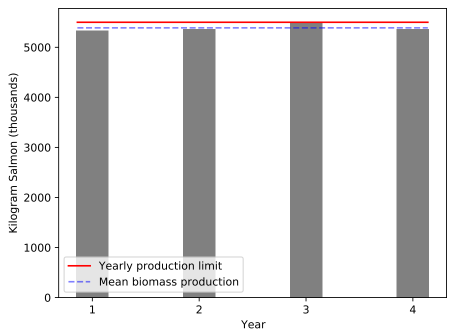
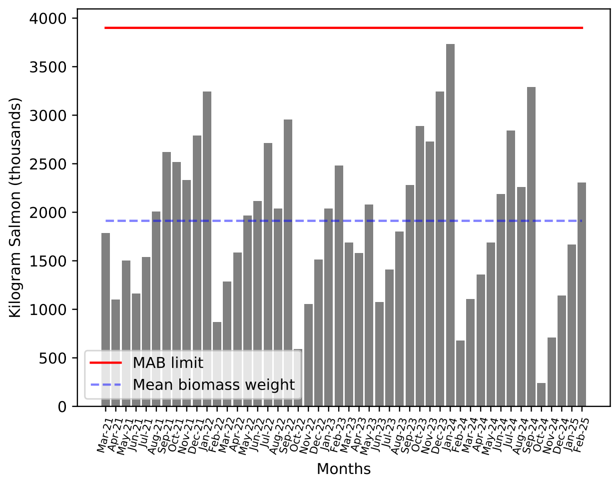


Figure 8.2: Production plan for the Profit Transfer instance



(a) Yearly production Profit Transfer instance



(b) Yearly biomass development Profit Transfer instance

Figure 8.3: Utilization of yearly production limit and MAB limit for the Profit Transfer instance

Note in Figure 8.2, that the production cycle initiated with a deployment in November-21 into tanks 21–23 exhibits a different transfer pattern than the other production cycles. Given the suggested production in the other modules during the second production year, stretching from March-22 to February-23, the yearly production limit allows for a total of three extractions from tanks 21–24. Thus, this production cycle exploits the flexibility of transfer to accommodate two extractions of post-smolt and one extraction of harvestable salmon. Increasing the yearly production limit to 15 000 tonnes, in which it may be regarded as redundant, removes this unusual transfer pattern. This production plan is attached in Appendix C. Therefore, we can trace the transfer pattern observed in the second production cycle in module six containing tanks 21–24 back to the yearly production limit.

Figure 8.4 illustrates the large structural change that occurs when changing the objective of the production plan to maximize total biomass produced. In this production plan we observe a production pattern of solely combining production cycles producing post-smolt. The salmon that are harvested from tanks 9–12 in module three can be attributed to the initial conditions. Considering Figure 8.5, we observe that shifts in deployments and extractions are necessary to avoid breaking the yearly production limit and the MAB limit in certain periods.

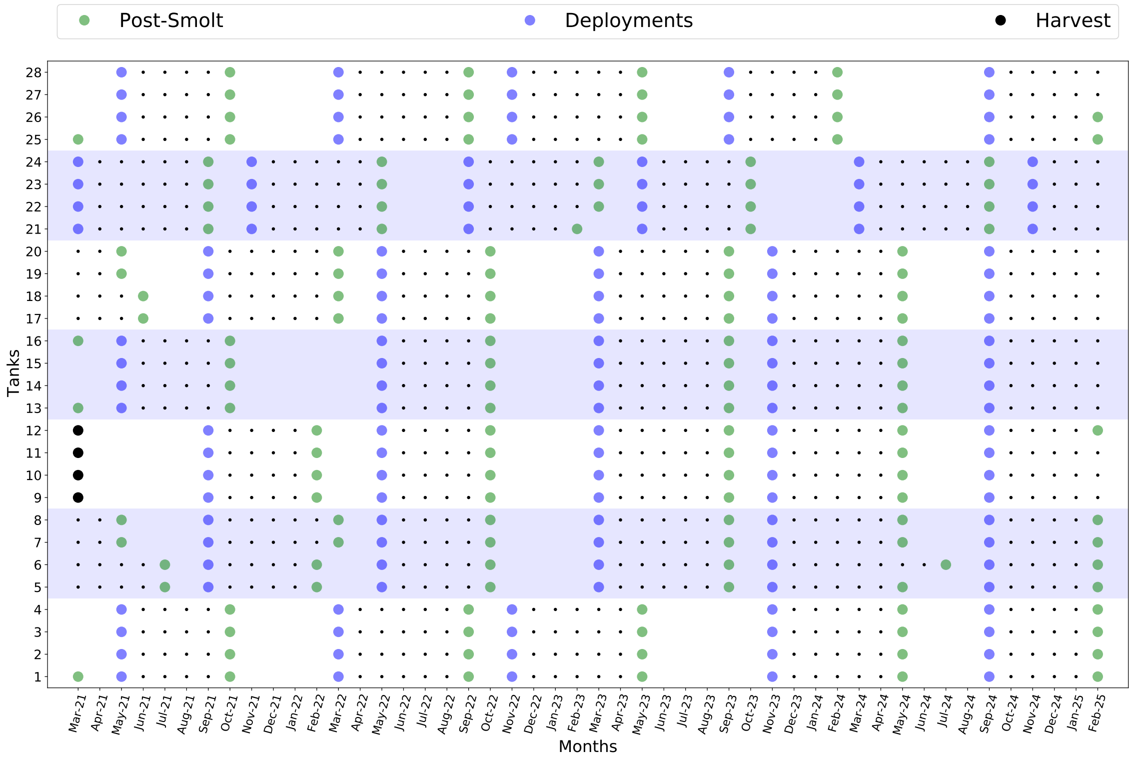
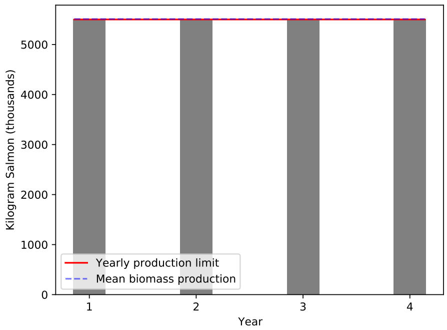
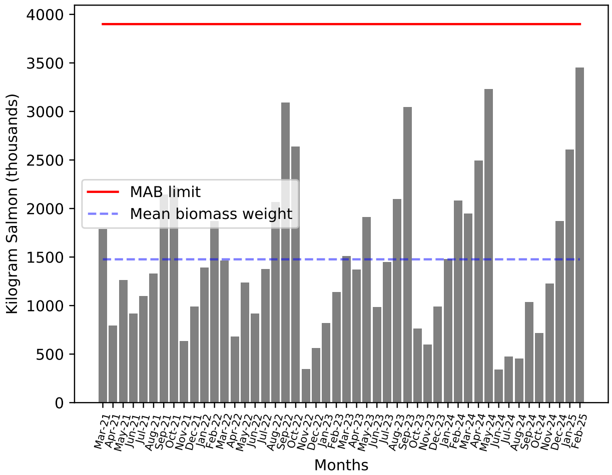


Figure 8.4: Production plan for Biomass No Transfer instance

The most obvious reason for why post-smolt is attractive when maximizing biomass is the difference in average production cycle length between post-smolt and harvestable salmon. The



(a) Yearly production Biomass No Transfer instance



(b) Yearly biomass development Biomass No Transfer instance

Figure 8.5: Utilization of yearly production limit and MAB limit for the Biomass No Transfer instance

average production cycle length for post-smolt in Figure 8.4 is 6.7 months, while the average production cycle length for harvestable salmon in the Profit Transfer instance is 15.7 months. Irrespective of whether extracting post-smolt or harvestable salmon, a full tank yields 157 500 kg in biomass, resulting from the density limit at 45 kg/m^3 and the tank volume of 3 500 m^3. As the yearly production limit covers a 12 month period, and the average production cycle length for harvestable salmon is 15.7 months, producing harvestable salmon results in low utilization of this production limit compared to producing post-smolt.

Another evident change in the production plan for the Biomass No Transfer instance compared to the production plan for the Profit Transfer instance, is the increased number of months where modules are empty. We observe in Figure 8.4, that the module containing tanks 13–16 remains empty in six months from November-21 to April-22. In the Profit Transfer instance, at most one month elapses between two production cycles, and this is required due to cleaning and maintenance. Considering the yearly production limit for the Biomass No Transfer, depicted in Figure 8.5a, there is no capacity available to increase the number of post-smolt production cycles. This explains why we observe the large gaps between each production cycle in Figure 8.4.

Revenue and Cost Breakdown

The consequences of changing the objective of the production plan are further emphasized by investigating the numbers behind the expected profits. The expected profit for the Profit Transfer instance is at NOK 411 million, while the expected profit for the Biomass No Transfer is at NOK 356 million. The expected profit includes revenues from post-smolt and harvestable salmon sales and costs concerning smolt procurement, feed, oxygen, and water pumping. Table 8.11 presents the numbers behind the expected profit in the two different instances.

Table 8.11: Revenue and cost breakdown from the results of Profit Transfer and Biomass No Transfer instances

Instance	Post-Smolt Revenues	Harvest Revenues	Smolt Costs	Feed Costs	Tank Costs	Oxygen Costs
Profit Transfer	569 MNOK	456 MNOK	279 MNOK	262 MNOK	56 MNOK	31 MNOK
Biomass No Transfer	1 037 MNOK	31 MNOK	403 MNOK	237 MNOK	46 MNOK	27 MNOK

The Profit Transfer instance shows considerably smaller smolt costs than the Biomass No Transfer instance. In light of the proposed production strategies and the projected biomass development, this is expected as a production cycle producing post-smolt results in a larger number of smolt deployed.

On the other hand, feed costs, tank costs, and oxygen costs are higher in the Profit Transfer instance compared to the Biomass No Transfer instance. This is expected due to the less efficient FCR value for larger salmon, the increased utilization of available tanks in the Profit Transfer instance, and the weight-dependent oxygen costs.

The expected profits are 15% higher in the production plan that maximizes profits compared to the production plan that maximizes total biomass produced. On the other hand, the difference in total biomass produced between the two production plans amount to 2%. The average profit margin of producing harvestable salmon is calculated to be 26 NOK/kg, while the average profit margin of producing post-smolt amounts to 16 NOK/kg. While still maintaining high production levels, production cycles that combine the post-smolt production with the production of harvestable salmon yield higher profit margins per kilogram salmon produced. This aspect is ignored when the objective of the production plan is to maximize total biomass produced, which explains why we observe these large structural differences between the two production plans. Thus, accounting for costs and revenues in a land-based facility with the option to produce post-smolt, heavily influences production planning and the resulting production strategy.

8.2.3 Value of Transferring Salmon During a Production Cycle

The production plan for the Biomass Transfer instance, attached in Appendix D, includes salmon transfer during production cycles. However, the Biomass No Transfer instance results prove that the possible production over four years is maximized without this flexibility. Consequently, when the production's objective is to maximize total biomass produced, the facility is by all practical means a post-smolt facility without any need to support salmon transfer.

More interesting is to evaluate how including salmon transfer affect the production plan when the objective is to maximize profits. To assess the value of including salmon transfer in the production planning when maximizing profits, we investigate a production plan where this flexibility is excluded. The effect of excluding salmon transfer from the production planning is illustrated in Figure 8.6. We observe that every production cycle only produce either post-smolt or harvestable salmon, and that the production strategy changes towards post-smolt production. The production plan includes a total of three full production cycles producing harvestable salmon over the four years.

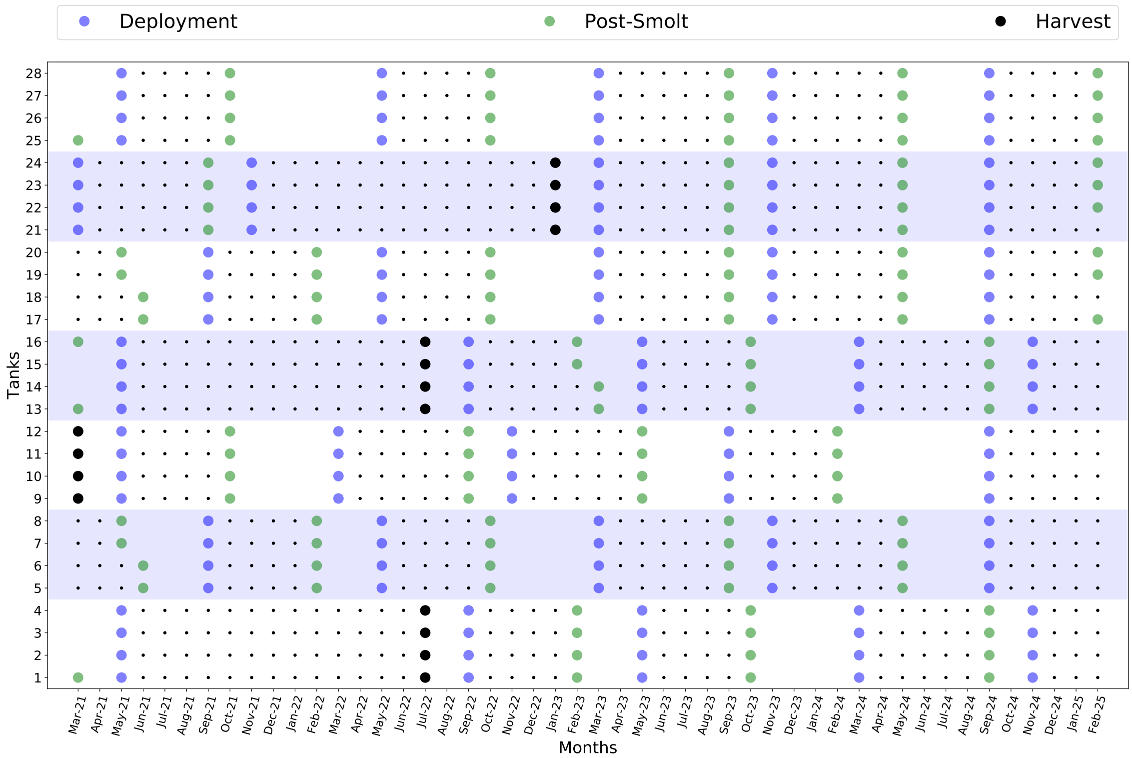


Figure 8.6: Production plan for Profit No Transfer instance

When the yearly production limit allows for two production cycles of post-smolt, this is preferred as they yield higher combined profits compared to one production cycle of harvestable salmon. However, when the yearly production limit forces the choice between a production cycle of post-smolt and a production cycle of harvestable salmon, then harvestable salmon is preferred due to the higher profit margin per kilogram salmon produced. To confirm this hypothesis, we run a sub-instance of Profit No Transfer, where we increase the yearly production limit to 15 000 tonnes. The resulting production plan is attached in Appendix E, and shows that in this case two production cycles of post-smolt are always preferred.

Transferring salmon introduces flexibility into the production planning. As previously discussed, the main reason for exploiting salmon transfer is to efficiently produce both post-smolt and harvestable salmon during the same production cycle. Production cycles combining the production of post-smolt and harvestable salmon are not present when excluding salmon transfer. The production plan for the Profit No Transfer instance gives an expected profit of NOK 373 million. Including salmon transfer gives an expected profit of NOK 411 million, which is an increase of 10% in total profits over a four year planning horizon. This indicates that the benefits from the increased flexibility of transferring salmon are greater than the cost due to loss of salmon growth.

8.2.4 The Attractiveness of Post-Smolt Production

The overall results indicate that post-smolt is a very attractive production possibility in a land-based facility given the assumed price configurations. There are two major reasons why post-smolt represents such an attractive production alternative. The first reason is the relatively short production cycle of post-smolt, enabling an effective utilization of the yearly production limit. The second reason is the relatively high profit margin per kilogram from producing post-smolt, considering the difference in duration between the two production cycles. However, as post-smolt production is a new business strategy, the actual demand for post-smolt is highly uncertain. The assumption of being able to sell all post-smolt produced is important to keep in mind for the results obtained from these problem instances.

To illustrate the effect of not including post-smolt production as an option, we generate instances where the option to produce post-smolt is excluded for Profit Transfer and Biomass Transfer. Table 8.12 illustrates the immediate consequences of not producing post-smolt with a drastic reduction in both total biomass produced and profits obtained. When the salmon producer can sell all post-smolt produced, we observe an increase in profits of 39% compared to only producing harvestable salmon and the objective of the production plan is to maximize profits. When the objective of the production plan is to maximize total biomass, post-smolt production yields an increase in total biomass produced of 98% compared to only producing harvestable salmon. Moreover, the yearly production limits are poorly utilized, illustrating how post-smolt production enables this limit to be efficiently utilized. If the business strategy is to produce only harvestable salmon, the dimensions of the facility should be increased, or the facility should investigate how to produce at densities higher than 45 kg/m^3, to better utilize allocated production permissions.

Table 8.12: Overview of results with no post-smolt production

Instance	Runtime [s]	Gap [%]	Profit	Biomass	MAB [%]	Prod-Limit [%]
Profit Transfer No Post-Smolt	1 463	0.29	**295 MNOK**	10 959 tonnes	34.5	49.9
Biomass Transfer No Post-Smolt	969	0.0	252 MNOK	**11 131 tonnes**	32.9	50.7

Anyhow, our results indicate that a land-based facility should investigate the market of post-smolt. Including post-smolt production as part of their business strategy will better utilize the production facility and result in higher profits.

Chapter 9

Future Research

In this thesis, we apply a Branch and Price (B&P) algorithm to solve the tactical production planning problem faced by a land-based salmon producer. The model proposed in this thesis does not capture every aspect of the real-world production planning problem. Therefore, a set of model extensions are suggested in Section 9.1 to increase the applicability of the model for salmon producers. In Section 9.2 we identify potential directions for improving the current implementation of the B&P algorithm.

9.1 Model Extensions

This thesis considers a deterministic planning problem in which we have perfect information about seawater temperatures, growth, and salmon prices. Føsund and Strandkleiv (2020) did not succeed in solving the problem for a full-scale facility, whereas the B&P algorithm applied in this thesis solves the deterministic variant of the tactical production planning problem for all practical means. As seawater temperatures, growth, and salmon prices are uncertain in real life, a natural extension of the problem is to include uncertainty into the modeling to enhance the model for real-life application. Uncertainty in biomass development in sea-based salmon farming has an effect on the optimal amount of smolt deployed and the time of harvesting Næss and Patricksson (2019). The salmon market exhibits volatile salmon prices, with seasonality in the spread between the different harvest sizes Asche et al. (2016). Schütz and Westgaard (2018) finds that the salmon producer chooses to reduce exposure in the spot market and enters into futures contracts at quite low levels of risk-aversion. The salmon producer continues to do so as the degree of risk-aversion increases. Hence, the volatile prices affect the decisions made by the salmon producer. Therefore, including uncertain prices with a price-modeling technique and the possibility to enter into futures contracts, could impact the optimal production strategy for a land-based salmon producer.

The production planning problem introduced in this thesis assumes an infinite demand for post-smolt. There is a demand for post-smolt in the industry, but the amount of post-smolt demanded is still highly uncertain (Olsen, 2020). We expect the demand for post-smolt to be dependent on the geographical location of the land-based facility since post-smolt are delivered to sea-based producers in the area. An important aspect of post-smolt production is the coordination between the land-based producer and the sea-based producer that receives the post-smolt. The sea-based producer needs to have available production units and capacity for the post-smolt delivered. Hence, we suggest enriching the model with a post-smolt demand function defined for all periods.

9.2 Improved Solution Methods

The results in Section 8.1 indicate that the implemented B&P algorithm can reach sufficient solution quality for the problem instances we wanted to solve in this thesis. However, the model extensions introduced above can result in increased computational complexity, introducing a potential need for improved solution methods. As presented in Section 8.1.4, increasing the size of the subproblem drastically increases the required runtime for solving the model.

The subproblems are complex and their runtimes vary according to the reduced cost objective. A potential improvement is to use a heuristic to generate better initial columns. With better initial columns the number of iterations needed in the Column Generation algorithm can be reduced. Thus, as we solve fewer subproblems, the total subproblem solution time is potentially reduced.

As mentioned in Section 3.2, dynamic programming has been used to solve the optimal rotation problem in aquaculture which exhibits strong similarities with the production planning problem in land-based salmon farming. Guttormsen (2008) implements a dynamic programming model for a single production unit, where only the regulatory density limit is enforced. In each subproblem of our Dantzig-Wolfe decomposed model, the density limit is the only density limit that is enforced. Thus, the subproblem shares characteristics with the problem studied by Guttormsen (2008). However, the subproblem is more complex as a module contains four tanks and salmon can be transferred between tanks. Anyhow, we believe a dynamic programming approach could solve the subproblem faster than a standard MIP solver, since a production plan for a module can be separated into smaller segments that can be re-used when generating production plans for the module. This could allow for re-using the production plan found for one tank to other tanks. Another possibility is to re-use specific production cycles. A production cycle last from smolt are deployed in the module until the module is empty.

Chapter 10

Concluding Remarks

In this thesis, we develop production plans for a land-based salmon farming facility. The tactical production planning problem involves decisions on when, where, and how much smolt to deploy, whether to transfer salmon between tanks during the production cycle, when to sell post-smolt, and when to sell harvestable salmon. We model the problem through a deterministic mixed integer programming model and apply a Dantzig-Wolfe reformulation that exploits the structure of the problem. Then, we use a Branch and Price algorithm with the extension of a matheuristic to solve the tactical production planning problem. The matheuristic checks whether the available columns at a given time in the solution process can give an improving integer feasible solution.

We introduce instances of the tactical production planning problem representing a full-scale production facility. The instances are solved over 62 months to deal with end-of-horizon effects, and the production plan spans a total of four years. We first try to solve the instances by a regular Branch and Bound algorithm employed by a commercial mixed integer programming solver. Branch and Bound algorithm succeeds in finding good feasible solutions. However, the problem is too complex for the Branch and Bound algorithm to reduce the upper bound. Solving the Profit Maximization instance the Branch and Bound algorithm reaches an optimality gap of 19.1% after a runtime of two days. Applying Branch and Price as the solution method solves the Profit Maximization instance to an optimality gap of 0.38% within 26 minutes. The solution method solves the tactical production planning problem for the facility studied in this thesis and is highly scalable in terms of increasing the number of modules. Hence, future research can extend the model with uncertainty into the biomass development and price estimates to enhance the model for real-life application. Exploring alternative solution methods for solving the resulting subproblems is an interesting direction for future research in terms of improving computational runtime.

The objective of the production plan is decisive for the production strategy at the land-based facility. If the salmon producer wants to maximize total profits, our results suggest a production strategy with an even production of both post-smolt and harvestable salmon. A production plan that maximizes total profits results in a minor decrease in total biomass produced, but an increase of 15% in expected profit compared to a production plan that maximizes total biomass produced. If the aim of the salmon producer is to maximize total biomass produced, the proposed production strategy changes towards production of post-smolt. According to our results, a facility that only produces post-smolt does not need to support salmon transfer.

When the objective of the production plan is to maximize profits, our results indicate that the flexibility of transferring salmon between tanks is exploited to efficiently produce both post-smolt and harvestable salmon during a production cycle. This flexibility allows the salmon producer to distribute salmon intended for harvest into tanks from which post-smolt were extracted previously in the production cycle. This opportunity increases the attractiveness of producing harvestable salmon and leads to an increase of 10% in expected profits compared to a production plan not having the flexibility of transferring salmon.

As post-smolt is an attractive production alternative to utilize the land-based facility, we suggest that land-based salmon producers investigate the post-smolt market to get better estimates on prices and demand. Including post-smolt as part of the production strategy will according to our results increase facility utilization, total biomass produced, and overall expected profits compared to only producing harvestable salmon.

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Appendix A

Compact Model Formulation

Profit Objective Function

$$\begin{aligned} max \quad & \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}_m}\Bigg[\sum_{\hat{p}\in\{\mathcal{P}^R\cup\mathcal{P}^{R-}\}}\Bigg(\sum_{p\in\mathcal{P}^{PS}_{\hat{p}}}\sum_{w\in\mathcal{W}}R^{PS}_w Z_{w\hat{p}p}e_{\hat{p}tp}+\sum_{p\in\mathcal{P}^H_{\hat{p}}}\sum_{w\in\mathcal{W}}R^H_w Z_{w\hat{p}p}Y^H e_{\hat{p}tp} \\ & -\sum_{p\in\mathcal{P}^G_{\hat{p}}}C^F_{\hat{p}p}x_{\hat{p}tp}+\sum_{p\in\mathcal{P}^E_{\hat{p}}}C^F_{\hat{p}p}(x_{\hat{p}tp}-e_{\hat{p}tp})-\sum_{p\in\{\mathcal{P}^G_{\hat{p}}\cup\mathcal{P}^E_{\hat{p}}\}}C^O_{\hat{p}p}x_{\hat{p}tp}\Bigg) \\ & -\sum_{\hat{p}\in\mathcal{P}^R}C^D x_{\hat{p}t\hat{p}}-\sum_{p\in\mathcal{P}}\Bigg(C^{min}\alpha_{tp}+\sum_{\hat{p}\in\mathcal{P}^D_p}C^{MC}x_{\hat{p}tp}\Bigg)\Bigg] \end{aligned} \tag{A.1}$$

Smolt Deployment Constraints

$$D^{min}\delta_{m\hat{p}} \le \sum_{t\in\mathcal{T}_m}x_{\hat{p}t\hat{p}} \le D^{max}\delta_{m\hat{p}}, \qquad m\in\mathcal{M},\hat{p}\in\mathcal{P}^R, \tag{A.2}$$

$$\delta_{m\hat{p}}+\alpha_{t(\hat{p}-1)} \le 1, \qquad m\in\mathcal{M},t\in\mathcal{T}_m,\hat{p}\in\mathcal{P}^R, \tag{A.3}$$

Extraction Constraints

$$\sum_{\hat{p}\in\mathcal{P}^{DE}_p}e_{\hat{p}tp} \le E^{max}\epsilon_{tp}, \qquad m\in\mathcal{M},t\in\mathcal{T}_m,p\in\mathcal{P}, \tag{A.4}$$

$$\alpha_{t(p+1)}+\epsilon_{tp} \le 1, \qquad m\in\mathcal{M},t\in\mathcal{T}_m,p\in\mathcal{P}, \tag{A.5}$$

Salmon Transfer Constraints

$$\sigma_{tp}+\alpha_{t(p-1)} \le 1, \qquad m\in\mathcal{M},t\in\mathcal{T}_m,p\in\mathcal{P}, \tag{A.6}$$

$$\sigma_{tp}+\epsilon_{tp} \le 1, \qquad m\in\mathcal{M},t\in\mathcal{T}_m,p\in\mathcal{P}, \tag{A.7}$$

$$\sigma_{tp}-\alpha_{\hat{t}(p+1)} \le 0, \qquad m\in\mathcal{M},t\in\mathcal{T}_m,\hat{t}\in\hat{\mathcal{T}}_t,p\in\mathcal{P}, \tag{A.8}$$

$$Y^{min}\sigma_{tp} \leq \sum_{\hat{p}\in\mathcal{P}_p^{DT}} y_{\hat{p}\hat{t}tp} \leq Y^{max}\sigma_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{t} \in \mathcal{T}_t, p \in \mathcal{P}, \tag{A.9}$$

Salmon Density and Tank Activation Constraints

$$\left(\sum_{\hat{p}\in\mathcal{P}_p^{D}} x_{\hat{p}tp} + \sum_{\hat{p}\in\mathcal{P}_p^{DT}} \sum_{\hat{t}\in\mathcal{T}_t} y_{\hat{p}\hat{t}tp}\right) V_t \leq L^{den}\alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \tag{A.10}$$

Regulatory Constraints

$$\sum_{\hat{p}\in\mathcal{P}_p^{D}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}_m} x_{\hat{p}tp} \leq L^{mab}, \qquad p \in \mathcal{P}, \tag{A.11}$$

$$\sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}_m} \sum_{p\in\mathcal{P}_y} \sum_{\hat{p}\in\mathcal{P}_p^{DE}} e_{\hat{p}tp} \leq L^{prod}, \quad y \in \mathcal{Y}, \tag{A.12}$$

Biomass Development Constraints

$$\begin{aligned} &x_{\hat{p}t(p+1)} = (1-P^{loss})G_{\hat{p}p}x_{\hat{p}tp}, \\ &\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \{\mathcal{P}_{\hat{p}}^{G} \setminus \{\mathcal{P}_{\hat{p}}^{T}\}\}, \end{aligned} \tag{A.13}$$

$$\begin{aligned} &x_{\hat{p}t(p+1)} = (1-P^{loss})(G_{\hat{p}p}(x_{\hat{p}tp} - \sum_{\hat{t}\in\hat{\mathcal{T}}_t} y_{\hat{p}t\hat{t}p}) + G_{\hat{p}p}^{T} \sum_{\hat{t}\in\mathcal{T}_t} y_{\hat{p}\hat{t}tp}), \\ &\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \{\mathcal{P}_{\hat{p}}^{G} \cap \mathcal{P}_{\hat{p}}^{T}\}, \end{aligned} \tag{A.14}$$

$$\begin{aligned} &x_{\hat{p}t(p+1)} = (1-P^{loss})(G_{\hat{p}p}(x_{\hat{p}tp} - \sum_{\hat{t}\in\hat{\mathcal{T}}_t} y_{\hat{p}t\hat{t}p} - e_{\hat{p}tp}) + G_{\hat{p}p}^{T} \sum_{\hat{t}\in\mathcal{T}_t} y_{\hat{p}\hat{t}tp}), \\ &\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \{\mathcal{P}_{\hat{p}}^{E} \cap \mathcal{P}_{\hat{p}}^{T}\} \setminus max\{\mathcal{P}_{\hat{p}}^{E}\}, \end{aligned} \tag{A.15}$$

$$\begin{aligned} &x_{\hat{p}t(p+1)} = (1-P^{loss})G_{\hat{p}p}(x_{\hat{p}tp} - e_{\hat{p}tp}), \\ &\hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}_{\hat{p}}^{E} \setminus \{\mathcal{P}_{\hat{p}}^{T}, max\{\mathcal{P}_{\hat{p}}^{E}\}\}, \end{aligned} \tag{A.16}$$

$$x_{\hat{p}tp} - e_{\hat{p}tp} = 0, \qquad \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in max\{\mathcal{P}_{\hat{p}}^{E}\}, \tag{A.17}$$

Non-Negativity and Binary Requirements

$$
\begin{aligned}
e_{\hat{p}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}^{E}_{\hat{p}}, \\
x_{\hat{p}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \\
y_{\hat{p}\hat{t}tp} &\geq 0, && \hat{p} \in \{\mathcal{P}^{R-} \cup \mathcal{P}^{R}\}, m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{t} \in \mathcal{T}_t, p \in \mathcal{P}^{T}_{\hat{p}}, \\
\alpha_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \\
\delta_{mp} &\in \{0,1\}, && m \in \mathcal{M}, p \in \mathcal{P}^{R}, \\
\epsilon_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}, \\
\sigma_{tp} &\in \{0,1\}, && m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P},
\end{aligned}
\tag{A.18}
$$

Valid Inequalities

$$\delta_{mp} + \sigma_{tp} + \alpha_{t(p-1)} - \epsilon_{t(p-1)} \geq \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P}^{R} \setminus \{1\}, \tag{A.19}$$

$$\sigma_{tp} + \alpha_{t(p-1)} - \epsilon_{t(p-1)} \geq \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, p \in \mathcal{P} \setminus \mathcal{P}^{R}, \tag{A.20}$$

$$\delta_{m\hat{p}} \leq \sum_{t \in \mathcal{T}_m} \alpha_{tp}, \qquad m \in \mathcal{M}, \hat{p} \in \mathcal{P}^{R}, p \in \{\hat{p}, ..., \min(\mathcal{P}^{E}_{\hat{p}})\}, \tag{A.21}$$

$$2\delta_{m\hat{p}} - 2 + \alpha_{t\hat{p}} + \sum_{\bar{p}=\hat{p}+1}^{p} \sigma_{t\bar{p}} - \sum_{\bar{p}=\hat{p}+1}^{p-1} \epsilon_{t\bar{p}} \leq \alpha_{tp}, \qquad m \in \mathcal{M}, t \in \mathcal{T}_m, \hat{p} \in \mathcal{P}^{R}, p \in \{\mathcal{P}^{G}_{\hat{p}} \cup \mathcal{P}^{E}_{\hat{p}}\}. \tag{A.22}$$

Appendix B

Growth Table

Table B.1 defines feed conversion ratios and specific growth rates in percent for an Atlantic salmon with a specific weight and in a specific water temperature.

Table B.1: Skretting SGR table

Temperature												
Weight	5	6	7	8	9	10	11	12	13	14	15	FCR
100	1.09	1.26	1.42	1.56	1.7	1.82	1.92	2.01	2.06	2.08	2.04	0.81
110	1.08	1.25	1.41	1.55	1.68	1.79	1.9	1.98	2.03	2.05	2.01	0.81
120	1.07	1.24	1.39	1.53	1.66	1.77	1.87	1.95	2	2.02	1.98	0.81
130	1.05	1.22	1.37	1.51	1.64	1.75	1.85	1.93	1.98	1.99	1.95	0.81
140	1.04	1.21	1.36	1.5	1.62	1.73	1.83	1.9	1.95	1.96	1.92	0.81
150	1.03	1.2	1.34	1.48	1.6	1.71	1.8	1.88	1.93	1.94	1.9	0.81
200	0.97	1.13	1.27	1.4	1.51	1.62	1.7	1.77	1.81	1.82	1.78	0.82
250	0.92	1.07	1.21	1.33	1.44	1.54	1.62	1.68	1.71	1.72	1.68	0.82
300	0.88	1.02	1.15	1.27	1.37	1.47	1.54	1.6	1.63	1.63	1.59	0.83
350	0.84	0.98	1.1	1.22	1.32	1.4	1.48	1.53	1.56	1.56	1.52	0.83
400	0.81	0.94	1.06	1.17	1.27	1.35	1.42	1.47	1.5	1.49	1.45	0.84
450	0.77	0.9	1.02	1.13	1.22	1.3	1.37	1.42	1.44	1.44	1.4	0.84
500	0.75	0.87	0.99	1.09	1.18	1.26	1.32	1.37	1.39	1.39	1.35	0.85
550	0.72	0.84	0.95	1.05	1.14	1.22	1.28	1.32	1.35	1.34	1.3	0.85
600	0.7	0.82	0.93	1.02	1.11	1.18	1.24	1.28	1.3	1.3	1.26	0.86
650	0.68	0.79	0.9	0.99	1.08	1.15	1.21	1.25	1.27	1.26	1.22	0.86
700	0.66	0.77	0.87	0.97	1.05	1.12	1.17	1.21	1.23	1.22	1.19	0.87
750	0.64	0.75	0.85	0.94	1.02	1.09	1.14	1.18	1.2	1.19	1.15	0.87
800	0.62	0.73	0.83	0.92	1	1.06	1.12	1.15	1.17	1.16	1.12	0.88
850	0.6	0.71	0.81	0.9	0.97	1.04	1.09	1.12	1.14	1.13	1.09	0.88
900	0.59	0.7	0.79	0.88	0.95	1.01	1.06	1.1	1.11	1.11	1.07	0.89
950	0.58	0.68	0.77	0.86	0.93	0.99	1.04	1.08	1.09	1.08	1.04	0.89
1000	0.56	0.66	0.76	0.84	0.91	0.97	1.02	1.05	1.07	1.06	1.02	0.9

Table B.1 continued from previous page

1250	0.51	0.6	0.69	0.76	0.83	0.88	0.93	0.96	0.97	0.96	0.92	0.92
1500	0.46	0.55	0.63	0.7	0.76	0.81	0.85	0.88	0.89	0.88	0.85	0.95
1750	0.43	0.51	0.58	0.65	0.71	0.76	0.8	0.82	0.83	0.82	0.78	0.97
2000	0.4	0.48	0.55	0.61	0.67	0.71	0.75	0.77	0.77	0.76	0.73	1.00
2250	0.38	0.45	0.52	0.58	0.63	0.67	0.7	0.72	0.73	0.72	0.69	1.02
2500	0.35	0.42	0.49	0.55	0.6	0.64	0.67	0.69	0.69	0.68	0.65	1.05
2750	0.34	0.4	0.46	0.52	0.57	0.61	0.64	0.65	0.66	0.65	0.62	1.07
3000	0.32	0.38	0.44	0.5	0.54	0.58	0.61	0.62	0.63	0.62	0.59	1.10
3250	0.31	0.37	0.43	0.48	0.52	0.56	0.58	0.6	0.6	0.59	0.56	1.12
3500	0.3	0.35	0.41	0.46	0.5	0.54	0.56	0.58	0.58	0.57	0.54	1.15
3750	0.28	0.34	0.39	0.44	0.48	0.52	0.54	0.55	0.56	0.54	0.52	1.17
4000	0.27	0.33	0.38	0.43	0.47	0.5	0.52	0.53	0.54	0.52	0.5	1.20
4250	0.27	0.32	0.37	0.41	0.45	0.48	0.51	0.52	0.52	0.51	0.48	1.22
4500	0.26	0.31	0.36	0.4	0.44	0.47	0.49	0.5	0.5	0.49	0.46	1.25
4750	0.25	0.3	0.35	0.39	0.43	0.45	0.48	0.49	0.49	0.47	0.45	1.27
5000	0.24	0.29	0.34	0.38	0.41	0.44	0.46	0.47	0.47	0.46	0.44	1.30
5250	0.24	0.28	0.33	0.37	0.4	0.43	0.45	0.46	0.46	0.45	0.42	1.33
5500	0.23	0.28	0.32	0.36	0.39	0.42	0.44	0.45	0.45	0.44	0.41	1.35

Appendix C

Results - Profit Transfer Sub-Instance

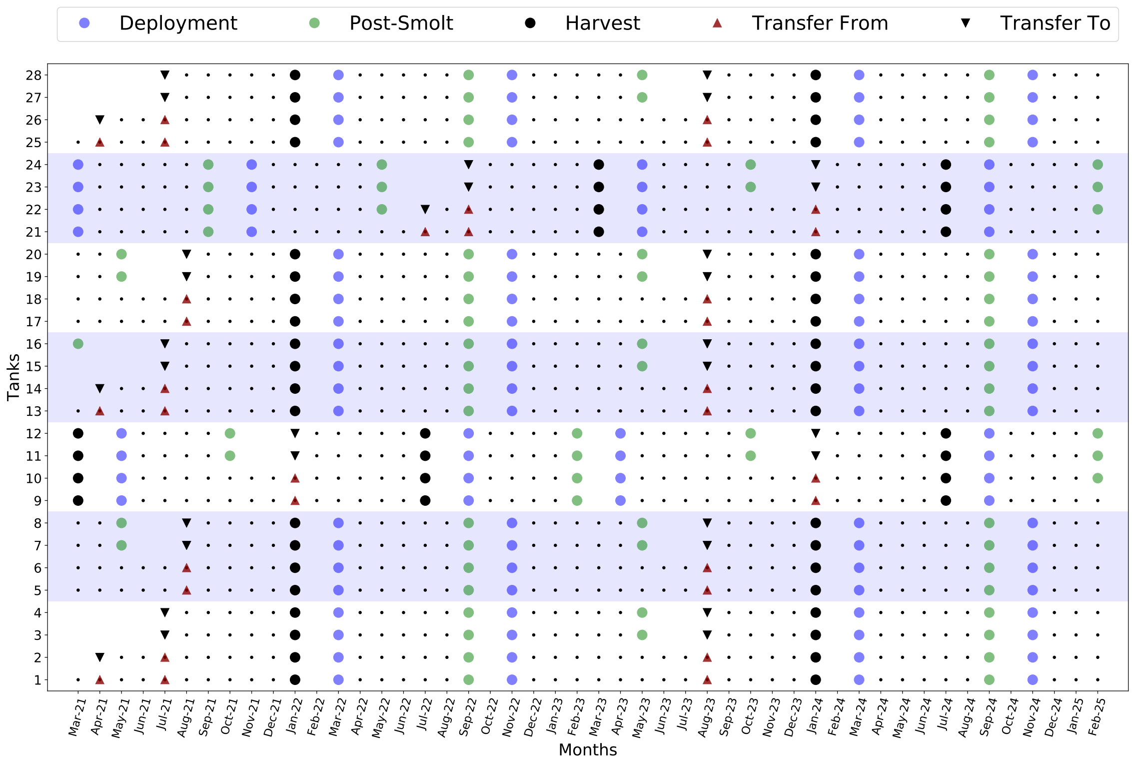

Figure C.1: Production plan for Profit Transfer with increased yearly production limit to 15 000 tonnes. Optimality gap of 0.5 % after 1 065 seconds

Appendix D

Results - Biomass Transfer Instance

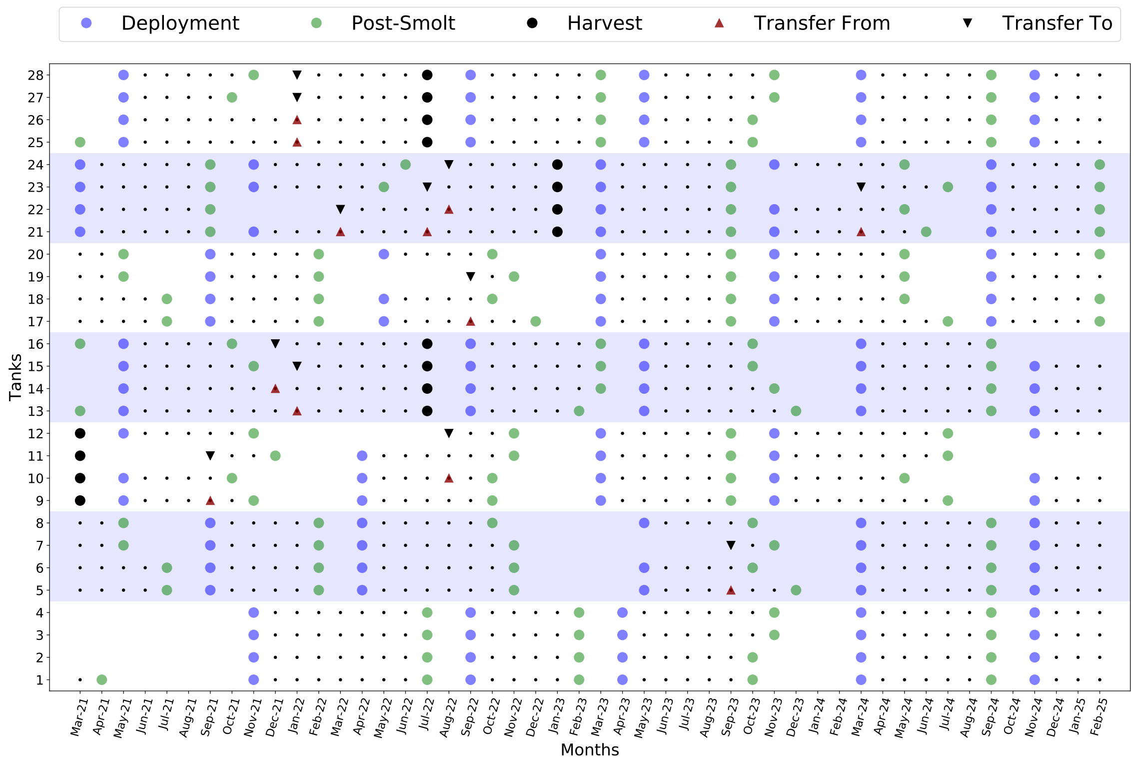

Figure D.1: Production plan for Biomass Transfer instance

Appendix E

Results - Profit No Transfer Sub-Instance

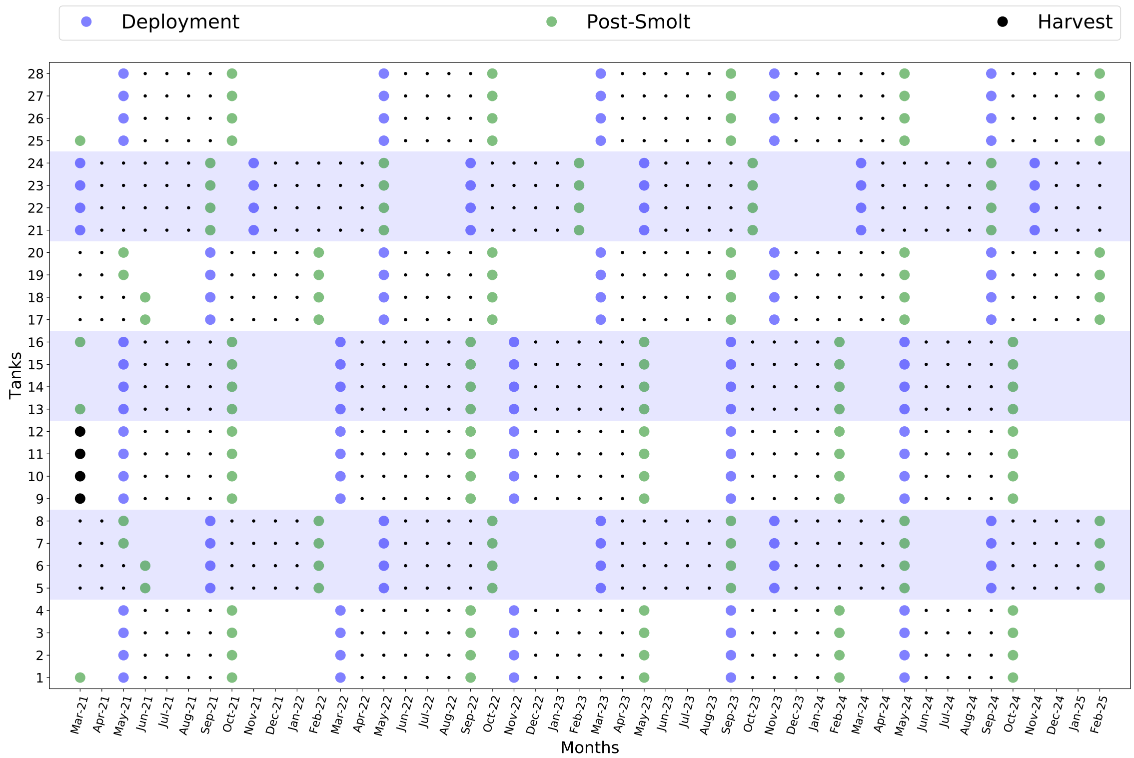

Figure E.1: Production plan for Profit No Transfer with increased yearly production limit to 15 000 tonnes. Optimality gap of 0.07% after 183 seconds

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Optimizing Land-Based Salmon Production

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