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Safety and COLREG evaluation for marine collision avoidance algorithms☆

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ABSTRACT

Verification is a necessary step in system development, so too for autonomous surface vehicles (ASVs). However, formal and analytical verification methods are not well suited for such highly complex systems. Simulation-based testing has therefore been proposed as a viable approach. This would require numerous simulations to be performed and evaluated, raising the need for automatic evaluation.

An important assessment measure with regards to ASV safety, is the degree of International Regulations for Preventing Collisions at Sea (COLREG) compliance. One important aspect of these rules is vessel behavior in situations where a risk of collision with other vessels is present.

This paper presents a comprehensive method for automatic evaluation of collision avoidance maneuvers in terms of safety and compliance with a subset of the COLREG steering and sailing rules. The rules are formulated as mathematical expressions, and insight is given into the authors' interpretation of the rules through the choice of values for parameters and weights. Results from simulated encounters and from encounters between vessels in normal operation show the method's capability to correctly identify situations and detect and penalize undesired behaviors. They also evoke the need for more research into how to interpret the COLREG in terms of angles, velocities, and distance between vessels.

1. Introduction

Preventing collisions is crucial for safe navigation at sea. An important action in this regard was the 1972 adoption of the International Regulations for Preventing Collisions at Sea (COLREG) (International Maritime Organization , IMO) as a convention by the International Maritime Organization (IMO), providing a set of rules regulating traffic at sea. Since then, navigational aids such as the automatic identification system (AIS), automatic radar plotting aid (ARPA) and Electronic Chart Display and Information System (ECDIS) have become commonplace, further assisting vessels navigate increasingly congested waterways. However, a report by European Maritime Safety Agency (EMSA) (European Maritime Safety Agency, 2020) shows that human action remain a significant cause of casualties or incidents at sea, with 54% of analyzed accident events being attributed to human action. Over the period 2014–2019, a total of 13,204 incidents with a ship were reported, out of which 44% were classified as navigational, i.e., collision, contact and grounding or stranding. Reducing these numbers by increasing the autonomy level of marine vessels is one of the main objectives for research into collision avoidance methods. Removing or reducing the need for personnel onboard could also have the additional benefit of reducing the human consequences of any type of marine accident or casualty, which according to the same report (European Maritime Safety Agency, 2020), amounted to 496 fatalities and 6210 persons injured within the same period. Reducing the number of crew will furthermore alleviate the current issue of getting enough qualified personnel to work onboard marine vessels.

However, a major problem facing anyone who wish to implement a COLREG compliant collision avoidance algorithm is the rules' intentional vagueness with regards to the prescribed actions in vessel encounters. The rules were developed for manned vessels and leave room for interpretation and the use of judgment, as seen in this example from Rule 8(a): "Any action to avoid collision shall be … made in ample time and with due regard to the observance of good seamanship". Work on how to regulate the behavior of autonomous vessels is underway and the IMO recently announced the completion of a scoping exercise (International Maritime Organization (IMO), 2021) analyzing its ship safety treaties in this regard. Still, the outcome of the exercise (Maritime Safety Committee (MSC), 2021) underlines that the

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COLREG should remain the reference point and that as much as possible of its current content should be retained. This conclusion provides a rationale for further research into methods that seek to include the COLREG.

A considerable amount of literature has been published on the topic of marine collision avoidance and several review articles are available, giving an overview of the development over the years. While some articles, such as (Statheros et al., 2008; Tam et al., 2009), treat the subject as a means for supporting human operators, others, for instance (Campbell et al., 2012; Liu et al., 2016; Polvara et al., 2018) see it as a component in the development of autonomous surface vehicles (ASVs). This distinction was remarked upon in Huang et al. (2020) which seeks to find common grounds where research advances with regards to one objective could benefit the other. The methods considered are compared in terms of motion prediction, conflict detection and resolution and their strengths and weaknesses highlighted. A more recent review paper (Vagale et al., 2021b), that also views collision avoidance in the ASV perspective, focus on clarifying terminology, analyze existing regulatory framework and suggest a classification scheme for path-planning[1] algorithms. The work is supported by the accompanying paper (Vagale et al., 2021a) where 45 collision avoidance algorithms were compared based on eight properties from the literature. While such reviews give an overview over methods, trends and developments within collision avoidance research, it is remarked in Vagale et al. (2021a) that when comparing algorithms "the comparison of the considered properties only gives a partial understanding of the performance".

The trajectories produced by collision avoidance algorithms can be evaluated in terms of different properties such as time, length, smoothness, energy consumption and safety. Many algorithms also claim compliance with the COLREG, but as noted in Vagale et al. (2021a), exactly what this entails varies. Although it should be assumed that when discussing collision avoidance algorithms exclusively, rather than autonomous vessels as a whole, compliance only relates to rules concerning steering and navigation as opposed to the complete rule set. It therefore seems pertinent to identify methods for evaluating vessel behavior in terms of COLREG-compliance. Based on the conclusions of Vagale et al. (2021a) an evaluation simulator platform using multi-objective optimization (MOO) was proposed in Vagale et al. (2020), assessing performance in terms of path fitness and safety. While COLREG-compliance and seamanship is not yet included in the evaluation, it is clear that it is considered by the author as a crucial point for further research.

One example where evaluation of COLREG compliance was attempted is (Porres et al., 2020), where a neural network was employed to create scenarios likely to challenge the collision avoidance algorithms being tested. While scenario generation is the main focus, the metrics used to measure the difficulty levels of the generated scenario, namely the risk of collision and the degree of COLREG noncompliance give an impression of the performance of the algorithms. The degree of compliance is given as the percentage of simulation steps where the vessel is behaving in accordance with the COLREG. For a single simulation step, lack of compliance is defined as a give-way vessel in a crossing or head on situation not making a starboard maneuver, or as a stand-on vessel in a crossing situation making a starboard maneuver. The importance of scenario selection was also brought forward in Rye Torben et al. (2021), which proposes a methodology for automatic simulation-based testing of ASVs using a Gaussian process (GP) model to guide the scenario selection. The method is demonstrated by two case studies, where requirements for safety, mission compliance and COLREG compliance for give-way vessels are formulated in the formal specification language signal temporal logic (STL). In the paper (Nakamura and Okada, 2019), a simulation based method is proposed where the evaluation is a numerical criterion with penalties when entering the "Danger area", "Caution area", and "Safety area" defined by the relative distance and rate of change of the bearings.

While the evaluation of COLREG compliance is limited in the above mentioned works, it was the main topic of a thesis by Woerner (2016), which presents an exhaustive method for objective evaluation of COLREG compliance. The method was included in the proposition of a test framework for ASVs in Woerner et al. (2016), and further developed in Woerner and Benjamin (2018) and Woerner et al. (2019). The assessment method is based on trajectory data and can be performed either in real time or post mission. A score is assigned to each vessel based on a set of metrics that evaluate the degree of safety and COLREG compliance in an encounter, notably with regards to Rules 8 and 13–17. Its ability to detect violations of these is validated in the thesis by statistics from an extensive simulation study along with a survey of how well self-identified ship masters agree with the algorithm's rule and blame assignment in near-miss or collision scenarios. The metrics have since served as an inspiration for the evaluation methods in Minne (2017), Henriksen (2018), Kjerstad (2020), Stankiewicz and Mullins (2019) and Pedersen et al. (2020). It was also suggested in Vagale et al. (2020) that they may be used as a basis for further extension of the evaluation simulator platform.

The fact that the COLREG was written for human operators and are vague enough to allow for the use of common sense is reflected in Worner's works by the number of parameters that are left to be decided by the evaluator. The values may depend on for instance the type of encounter or present environmental conditions. This leads to the question of what value these parameters should take on for the evaluation algorithm's scores to conform with the human notion of good seamanship as prescribed by the COLREG. To get some insight into how the COLREG is practiced at sea and how that translates into the parameters of the evaluation algorithm this paper presents two case studies based on AIS-data collected from vessels in normal operation.

The purpose of this paper is to provide a comprehensive system for the evaluation of vessel behavior with regards to compliance with COLREG Rule 8a, 8b and 13–17. The algorithm presented in the following sections is founded on the method developed by Woerner, presented in Woerner et al. (2019), Woerner (2016), and includes several improvements of the evaluation algorithm itself along with a detailed description of the complete evaluation process. This also covers implementation details that were omitted in the presentation of Woerner's method.

It is desirable to test the algorithm's performance on realistic encounters to shed light on whether the scores reflect human interpretation of the COLREG. To highlight the connection between vessel behavior and the resulting scores, several scenarios have been evaluated and are presented along with their corresponding scores and the parameter values used in the evaluation.

In short the main contributions of this paper are: (a) providing an algorithm for maneuver detection, (b) mathematical expressions for all the metrics employed where some were omitted in Woerner et al. (2019) and some have been improved, (c) tentative values for parameters and weights used in the calculations, and (d) validation results showing trajectories along with the resulting scores and penalties from both simulated and real life encounters. The overreaching objective being to provide a practical tool for the development of COLREG compliant collision avoidance (COLAV) algorithms that can also act as a starting point for a more complete performance evaluation of autonomous vessels. Section 2 presents a method for maneuver detection along with the metrics used in the performance evaluation. Section 3 describes the parameters used. Results from both simulated and real data are presented and discussed in Section 4. Limitations and further work is then discussed in Section 5 and conclusions presented in Section 6.

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[1] The article labels collision avoidance as local (reactive) path-planning.

2. Method

One of the first questions that must be addressed when discussing collision avoidance is how to define an encounter. A useful concept in this context is the ship domain, which is the area around a vessel that the navigator would like to keep free of other vessels or objects. Several different shapes for this area have been proposed (Tam et al., 2009; Zhang and Qiang, 2019; Pietrzykowski and Wielgosz, 2021; Pietrzykowski and Uriasz, 2009; Hansen et al., 2013), and the size of the area will vary with factors such as the type of area, traffic density, ship length, maximum speed. However, to avoid infringement of the ship domain, action must be taken before the domain is breached. This leads to the definition of a larger domain that, when entered, requires the navigator to consider evasive maneuvers. Providing a detailed definition of such an area is outside the scope of this paper, instead four stages are defined such that different definitions of the ship domain can be accommodated. Stage 1 include vessels that have been detected but are at a distance that does not require any actions to be considered. In Stage 2 the type of encounter must be decided and evasive maneuvers must be considered. Stage 3 and 4 is only relevant for stand-on vessels and will be further explained in Section 2.5. This definition implies that an encounter occurs when two vessels enter Stage 2 range and ends when the vessels re-enter Stage 1. These instances thus define the start and end point of the trajectories that are to be evaluated. In the case of multiple encounters within the same period of time, each vessel's behavior is evaluated with regards to each of the other vessels encountered. The evaluation results in a total score which is calculated based on different scores and penalties depending on the encounter type. Scores are denoted $\mathcal{S} \in [0,1]$ where 1 is the best score and penalties are denoted $\mathcal{P} \in [0,1]$, where 1 is the highest penalty. The relation between a penalty and its corresponding score is such that $\mathcal{S} = 1 - \mathcal{P}$. When a score is calculated from multiple penalties, weights are used to balance their importance, these are denoted $\gamma \in [0,1]$.

2.1. Maneuver detection

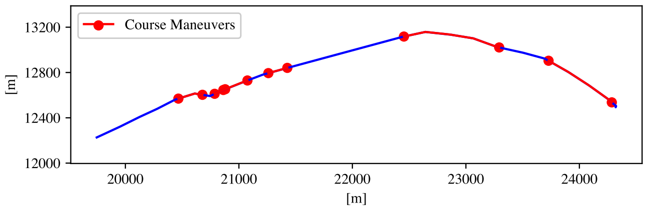
2.1.1. Pre-processing

Maneuvers can be marked by either a change in course or speed. The method for identifying maneuvers in vessel trajectories is identical for data produced in simulations or ship automation systems and data gathered from AIS. However, as the update frequency of AIS messages can vary both between vessels and depending on the current speed of a vessel some pre-processing is necessary before the evaluation is performed.

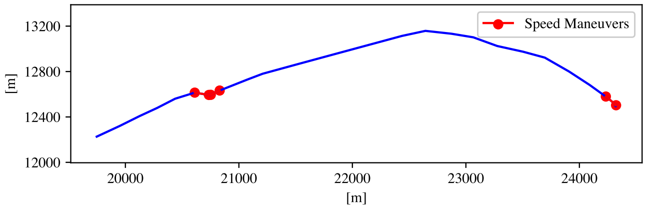
The simulation outputs are trajectories in the form of timestamped states in a local North-East-Down (NED) frame containing position (N, E), speed $(\dot{N}, \dot{E})$ and course angle (χ), from which the speed over ground (SOG), denoted U, can easily be obtained. The timestamped AIS messages contains position in the form of latitude and longitude, along with course over ground (COG) and SOG. The first step in the pre-processing is to remove any undefined (NaN) values and transform the messages into a local NED frame with positions in meters and velocities in meters per second. Constant sample frequency is then achieved by applying one-dimensional linear interpolation to each trajectory.

2.1.2. Detection

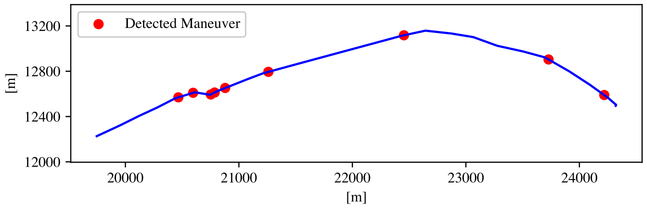
Maneuvers can be marked by either a change in course or speed. The approach employed here to detect these changes is based on the derivatives of the course and speed. Before the derivatives are calculated, a Gaussian filter is applied to both the course angle values and the speed values to assure smoothness. The derivatives are then found by central finite difference. The maneuver detection method is illustrated by Figs. 1–3 which display example data gathered from AIS messages over a thirty minute period.



(a) Detected course maneuvers.



(b) Detected speed maneuvers.



(c) Start point of detected maneuvers.

Fig. 1. Example of maneuver detection in AIS data. The vessel is moving from left to right.

A course change maneuver is detected when the following conditions are fulfilled:

$$\begin{aligned} &|U| \geq \epsilon_U \\ &|\dot{\chi}| \geq \epsilon_{\dot{\chi}}. \end{aligned} \tag{1}$$

Meaning that the vessel must be classified as moving and the first course derivatives must be above the given thresholds, see Fig. 2(b). The maneuver lasts until $\dot{\chi} = 0$ or $|U| < \epsilon_U$. The course maneuvers detected in the example data marked on the trajectory can be seen in Fig. 1(a).

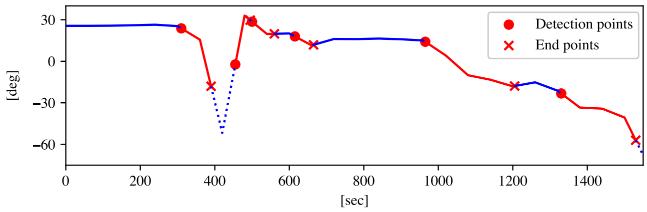
When it comes to speed changes, crossing the threshold $|U| \geq \epsilon_U$ will signify the vessel starting or stopping, see Fig. 3(a), and will thus mark either the start or end of a maneuver. If the vessel is moving, maneuver detection is triggered when acceleration exceeds the threshold $|\dot{U}| \geq \epsilon_{\dot{U}}$. The maneuver lasts until the acceleration again falls beneath the threshold or the vessel stop moving, see example in Fig. 3.

2.2. Safety score

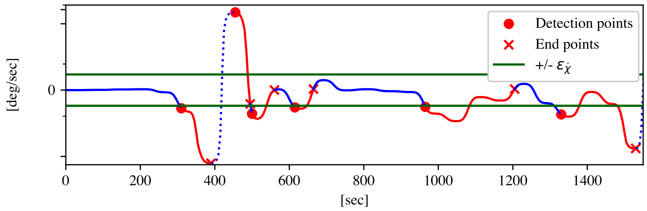
The safety score $(\mathcal{S}_{safety})$ is determined at the time of closest point of approach (CPA), i.e., when the range between vessels is at its smallest, and is calculated based on the score for pose $(\mathcal{S}_\Theta)$ and the score for range $(\mathcal{S}_r)$. Woerner et al. (2019) proposes several possible expressions for this score, the formulation presented below is an alternative to these which increases the importance of the pose as the range between the vessels decreases.

$$\mathcal{S}_{safety} = \begin{cases} 0, & \mathcal{S}_r = 0 \\ 1, & \mathcal{S}_r = 1 \\ (1 - \mathcal{S}_r)\mathcal{S}_\Theta + \mathcal{S}_r, & otherwise. \end{cases} \tag{2}$$

The first case signifies that the vessels are at a distance small enough to be considered a collision and the vessel is awarded a zero score,

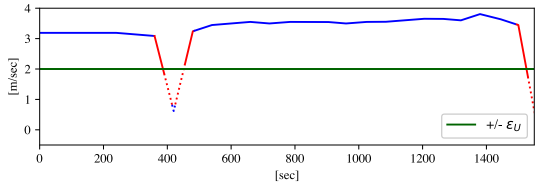


(a) Maneuver detection: Course.

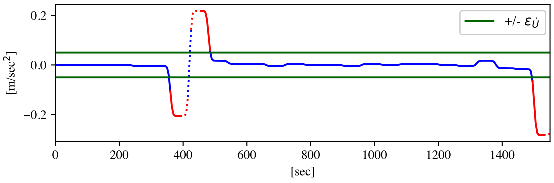


(b) Maneuver detection: First derivative of course.

Fig. 2. Detection of course changes in AIS data. Sections marked in red (——) are registered as maneuvers. Dotted sections denote that the vessel is static.



(a) Speed: A vessel moving slower than ϵ_U is considered static (dotted line).



(b) Acceleration: Speed maneuvers are registered when the acceleration is larger than $\epsilon_{\dot{U}}$.

Fig. 3. Detection of speed changes in AIS data. Sections marked in red (——) are registered as maneuvers. A dotted line signifies that the vessel is considered as static.

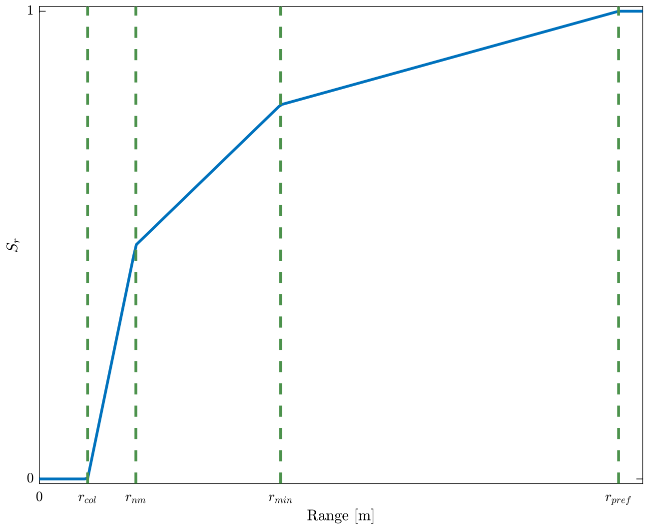


Fig. 4. Scoring function for range, S_r, along with range parameters r_{col}, r_{nm}, r_{min} and r_{pref} (- - - -).

independent of pose. In the second case, the range between the vessels at CPA is equal to or larger than the preferred range, and the pose is again disregarded. If the range at CPA is somewhere in between collision and preferred range, the score is a weighted combination of the two.

The safety score with regards to range (S_r) is, as proposed in Woerner et al. (2019), defined as a piece-wise continuous function of the range at CPA (r_{cpa}) and depends on a set of range parameters defining the preferred (r_{pref}), minimum acceptable (r_{min}), near-miss (r_{nm}) and collision (r_{col}) range. The function is given by

$$S_r = \begin{cases} 1, & r_{pref} \leq r_{cpa} \\ 1 - \gamma_{min}\left(\frac{r_{pref}-r_{cpa}}{r_{pref}-r_{min}}\right), & r_{min} \leq r_{cpa} < r_{pref} \\ 1 - \gamma_{min} - \gamma_{nm}\left(\frac{r_{pref}-r_{cpa}}{r_{pref}-r_{nm}}\right), & r_{nm} \leq r_{cpa} < r_{min} \\ 1 - \gamma_{min} - \gamma_{nm} - \gamma_{col}\left(\frac{r_{nm}-r_{cpa}}{r_{nm}-r_{col}}\right), & r_{col} \leq r_{cpa} < r_{nm} \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

with the condition $\gamma_{min} + \gamma_{nm} + \gamma_{col} = 1$ (see Fig. 4).

The safety score with regards to pose at CPA (S_Θ) is a weighted combination of contact angle score ($S_{\alpha_{cpa}}$) and relative bearing score ($S_{\beta_{cpa}}$), and is given by

$$S_\Theta = \gamma_\alpha S_{\alpha_{cpa}} + \gamma_\beta S_{\beta_{cpa}}. \tag{4}$$

This is the same formulation as used in Woerner et al. (2019) with the addition of weights that can be adjusted to place more importance on the own ship's pose by setting $\gamma_\beta > \gamma_\alpha$. The weights must fulfill the condition $\gamma_\beta + \gamma_\alpha = 1$.

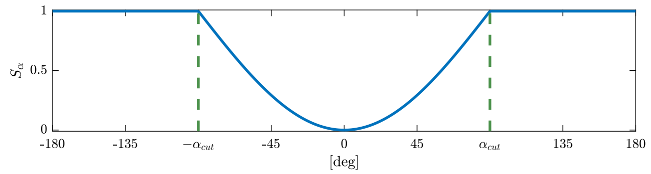
The contact angle $\alpha \in [-180°, 180°)$, and the relative bearing $\beta \in [0°, 360°)$ are defined as the angle between course and line-of-sight (LOS) to the other vessel as seen from the obstacle and the own ship's point of view respectively. The importance of α_{cpa} and β_{cpa} with regards to the safety of a passing is illustrated by the situations shown in Fig. 6. The calculation of $S_{\alpha_{cpa}}$ and $S_{\beta_{cpa}}$ is shown in Eqs. (5) and (6), where the values for the cut-off angles α_{cut}, β_{cut}^{min} and β_{cut}^{max} can be chosen to reward beam and stern contact, see Table 4. Plots of these functions can be seen in Fig. 5.

$$S_{\alpha_{cpa}} = \begin{cases} \frac{1-\cos(\alpha_{cpa})}{1-\cos(\alpha_{cut})}, & |\alpha_{cpa}| < \alpha_{cut} \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

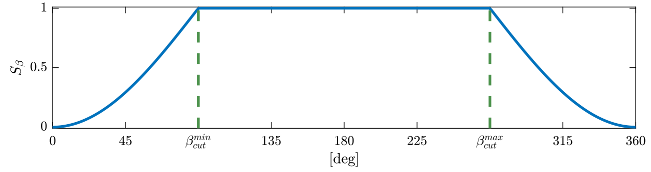
$$S_{\beta_{cpa}} = \begin{cases} \frac{1-\cos(\beta_{cpa})}{1-\cos(\beta_{cut}^{min})}, & \beta_{cpa} < \beta_{cut}^{min} \\ \frac{1-\cos(\beta_{cpa})}{1-\cos(\beta_{cut}^{max})}, & \beta_{cpa} > \beta_{cut}^{max} \\ 1 - \cos(\beta_{cut}^{min}), & \text{otherwise} \end{cases} \tag{6}$$

2.3. Encounter classification

The COLREG specify the required actions for vessels in head-on situations and for the give-way and stand-on vessels in crossing and overtaking situations, see Fig. 7. The method used for determining which rule to apply in a given situation follows Woerner's entry criteria (Woerner et al., 2019), but is outlined here for completeness. The applicable rule is predominantly determined by the relative poses of the vessels involved at entry time into Stage 2, illustrated in Fig. 8. The COLREG does in general not use numerical values in its definitions, the exception is overtaking situations where a vessel is said to be overtaking another when it approaches from a direction more than 22.5° abaft from her beam. For the vessel being overtaken this corresponds to a relative bearing of $\beta \in (\theta_{min}^{15}, \theta_{max}^{15})$, where $\theta_{min}^{15} = 112.5°$ and $\theta_{max}^{15} = 247.5°$. To assure that the other vessel is approaching, a limit has also been set on the contact angle $\alpha \in (-\theta_{crit}^{13}, \theta_{crit}^{13})$ and it has been

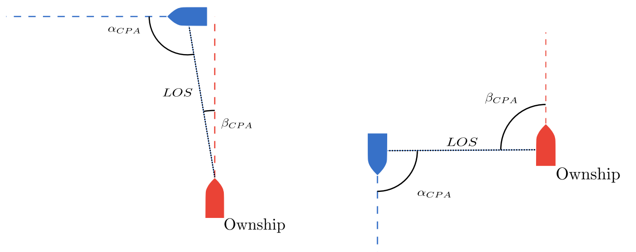


(a) Scoring function for contact angle at CPA, $\mathcal{S}_{\alpha_{cpa}}$.



(b) Scoring function for relative bearing at CPA, $\mathcal{S}_{\beta_{cpa}}$.

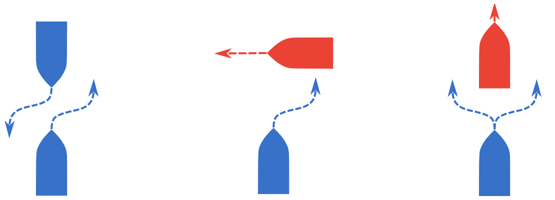
Fig. 5. Scoring functions for vessel pose at time of CPA, along with cut-off angles (-----).



(a) Near perpendicular pose at CPA, $\mathcal{S}_{\alpha_{cpa}} \approx 1$ and $\mathcal{S}_{\beta_{cpa}} \approx 0.1$.

(b) Parallel pose at CPA, $\mathcal{S}_{\alpha_{cpa}} \approx 1$ and $\mathcal{S}_{\beta_{cpa}} \approx 1$.

Fig. 6. The relative bearing angle β and contact angle α at CPA convey the difference between two encounters with equal range at CPA.



(a) Head-on: Both vessels should alter their course to starboard.

(b) Crossing: The vessel which has the other on its starboard side should keep out of the way and preferably avoid passing ahead of the other vessel.

(c) Overtaking: The vessel doing the overtaking should keep out of the way of the vessel being overtaken.

Fig. 7. Qualitative behavior as prescribed by the COLREG.

added that the overtaking vessel must keep a higher speed than the own ship.

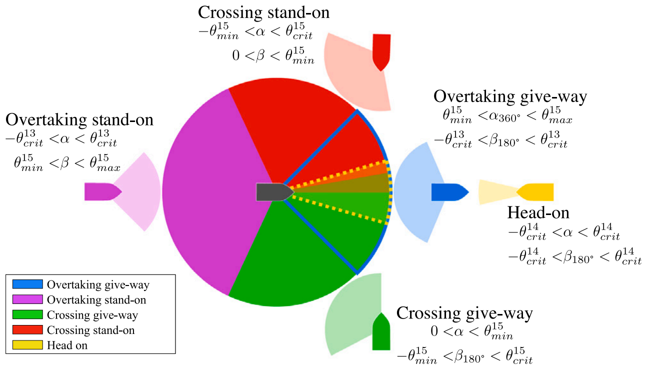


Fig. 8. Situation classification from the own ship's viewpoint. The mappings α_{360} : $[-180°, 180°) \to [0°, 360°)$ and β_{180} : $[0°, 360°) \to [-180°, 180°)$ are used to better display the inherent symmetry in the situation classification.

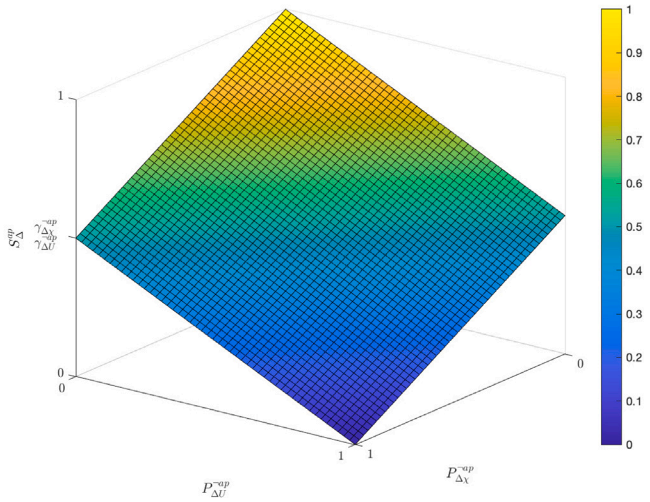


Fig. 9. Apparent maneuver score, $\mathcal{S}_{\Delta}^{ap}$, as a function of the penalties for non-apparent course change, $\mathcal{P}_{\Delta\chi}^{\neg ap}$, and non-apparent speed change, $\mathcal{P}_{\Delta U}^{\neg ap}$. Weights are $\gamma_{\Delta\chi}^{\neg ap} = \gamma_{\Delta U}^{\neg ap} = 0.5$.

Rule 13 also fixes one side of the limits for crossing situations. When a vessel is approaching from port the lower limit is set to zero such that $\beta \in (0°, \theta_{min}^{15})$, for a vessel approaching from starboard the limits are $\beta_{180} \in (-\theta_{min}^{15}, \theta_{crit}^{15})$, where β_{180} : $[0°, 360°) \to [-180°, 180°)$. The corresponding limits for the contact angles are reciprocate of the relative bearing for these two situations. Note that the sector defining a crossing from starboard, i.e., a give-way situation, is larger than the crossing from port section. For head-on situations the limits for both the relative bearing and contact angle have been set to $\alpha, \beta_{180} \in (-\theta_{crit}^{14}, \theta_{crit}^{14})$.

2.4. Rule 16 - give way

Rule 16 concerns the behavior of vessels that have give-way responsibilities towards another vessel. The give-way vessel must then keep well clear by taking early and substantial action. As suggested by Woerner in Woerner et al. (2019), the behavior is evaluated by penalizing late and not-readily apparent maneuvers. The formulation of the give-way score is given by

$$\mathcal{S}_{16} = \mathcal{S}_{safety} \mathcal{S}_{\Delta}^{ap} (1 - \mathcal{P}_{delay}), \tag{7}$$

where the penalty $\mathcal{P}_{delay}$ is based on the timeliness of the action and the score $\mathcal{S}_{\Delta}^{ap}$ (see Fig. 9) on how readily apparent the maneuver is, both are explained in the following sections. The chosen formulation for $\mathcal{S}_{16}$ places equal importance on the different factors and requires that all factors are high for a good overall score. In his thesis Woerner (2016) also applies a penalty for hindrance of stand-on vessel, i.e., the failure to stay well clear, but does not present a definition for this penalty. As Rule 16 applies in both crossing and overtaking situations that have quite different vessel configurations this penalty is in our work included in the scores for the specific situations.

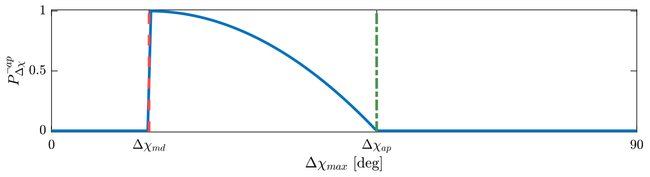


Fig. 10. Penalty function for non-apparent course changes, $\mathcal{P}^{\neg ap}_{\Delta\chi}$ (——), along with the limits for minimum detectable course change, $\Delta\chi_{md}$ (- - - -), and readily apparent course change, $\Delta\chi_{ap}$ (-·-·-).

2.4.1. Delayed action

Maneuvers to avoid collision should be made in ample time, failing to do so is a breach of both Rule 16 and 8a. In this implementation the parameter r_{detect}, used in Woerner et al. (2019) to signify the range at time of detection, has been exchanged with r_{entry} which signifies the range when the vessels enter Stage 2 and is expected to take action if required by the situation. The reason is in part that the time of detection is not known in cases where the data originates from AIS, but also that common marine radars can detect vessels at ranges far beyond what would be a reasonable distance for collision avoidance maneuvers. In addition, late maneuvers are only penalized if the range at the time of the maneuver, denoted r_{man}, is less than the defining range of Stage 3, r_{Stage3}. The concept of Stages is further explained in Section 2.5 but in short this requires that the maneuver is made before the lack of action requires the stand-on vessel to consider evasive measures.

$$\mathcal{P}_{delay} = \begin{cases} 0, & r_{man} > r_{Stage3} \\ \frac{r_{entry} - r_{man}}{r_{entry}}, & r_{Stage3} > r_{man} > r_{cpa}, \end{cases} \tag{8}$$

2.4.2. Non-apparent maneuver

Rule 16 also prescribes that maneuvers should, if possible, be substantial. This is clarified by Rule 8b which states that a change in course or speed should be large enough to be readily apparent to another vessel observing, either visually or by radar. The score consists of a course component $\mathcal{P}^{\neg ap}_{\Delta\chi}$ and a speed component $\mathcal{P}^{\neg ap}_{\Delta U}$ and is calculated by the following equation:

$$S^{ap}_{\Delta} = 1 - (\gamma^{\neg ap}_{\Delta\chi}\mathcal{P}^{\neg ap}_{\Delta\chi} + \gamma^{\neg ap}_{\Delta U}\mathcal{P}^{\neg ap}_{\Delta U}), \tag{9}$$

where the weights $\gamma^{\neg ap}_{\Delta\chi}$ and $\gamma^{\neg ap}_{\Delta U}$ must be chosen so that $\gamma^{\neg ap}_{\Delta\chi} + \gamma^{\neg ap}_{\Delta U} = 1$. This deviates from the penalty function presented in Woerner et al. (2019) by also penalizing non-apparent speed changes when there are no penalty due to course changes. Using a weighted addition instead of multiplication also makes it possible to achieve the minimum score if the vessel is given the maximum penalty for both course and speed changes.

The penalty for non-apparent course change ($\mathcal{P}^{\neg ap}_{\Delta\chi}$) is based on the maximum course change ($\Delta\chi_{max}$) between the time of entry into Stage 2, denoted t_0, and the time of CPA, denoted t_{cpa}.

$$\begin{aligned} \Delta\chi_{max} = \max\big(&|\chi(t_0) - \chi(t_1)|,\ |\chi(t_0) - \chi(t_2)|, \\ &\dots, |\chi(t_0) - \chi(t_{cpa-1})|,\ |\chi(t_0) - \chi(t_{cpa})|\big) \end{aligned} \tag{10}$$

The penalty is applied if $\Delta\chi_{max}$ is above the threshold for minimum detectable course change ($\Delta\chi_{md}$), and below the threshold of what is considered readily apparent ($\Delta\chi_{ap}$), where $\Delta\chi_{md} < \Delta\chi_{ap}$. The penalty is given by

$$\mathcal{P}^{\neg ap}_{\Delta\chi} = \begin{cases} 1 - \left(\frac{\Delta\chi_{max} - \Delta\chi_{md}}{\Delta\chi_{ap} - \Delta\chi_{md}}\right)^2, & \Delta\chi_{md} < \Delta\chi_{max} < \Delta\chi_{ap} \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

which is a slightly stricter penalty function than Woerner's (Woerner et al., 2019) linear version. The quadratic term causes higher penalties on course changes that are close to the detection limit. A plot of the penalty as a function of course change can be seen in Fig. 10.

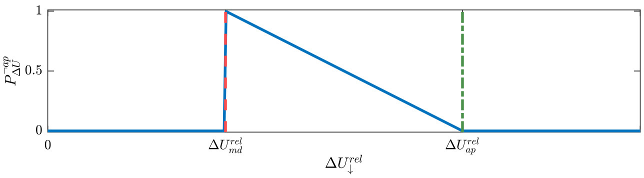


Fig. 11. Penalty function for non-apparent speed changes, $\mathcal{P}^{\neg ap}_{\Delta U}$ (——), as a function of the relative speed reduction, $\Delta U^{rel}_{\downarrow}$, along with the limits for minimum detectable speed change, ΔU^{rel}_{md} (- - - -), and readily apparent speed change, ΔU^{rel}_{ap} (-·-·-).

The penalty for non-apparent speed change ($\mathcal{P}^{\neg ap}_{\Delta U}$) is based on the relative speed reduction ($\Delta U^{rel}_{\downarrow}$) which is given by

$$\Delta U^{rel}_{\downarrow} = \Delta U_{\downarrow}/U(t_0) \tag{12}$$

where

$$\begin{aligned} \Delta U_{\downarrow} = \max\big(&U(t_0) - U(t_1),\ U(t_0) - U(t_2), \\ &\dots, U(t_0) - U(t_{cpa-1}),\ U(t_0) - U(t_{cpa}),\ 0\big). \end{aligned} \tag{13}$$

The penalty is applied if $\Delta U^{rel}_{\downarrow}$ lies between the thresholds for minimum detectable (ΔU^{rel}_{md}) and readily apparent (ΔU^{rel}_{ap}) speed changes, where $0 < \Delta U^{rel}_{md} < \Delta U^{rel}_{ap}$. The penalty is given by

$$\mathcal{P}^{\neg ap}_{\Delta U} = \begin{cases} \frac{\Delta U^{rel}_{ap} - \Delta U^{rel}_{\downarrow}}{\Delta U^{rel}_{ap} - \Delta U^{rel}_{md}}, & \Delta U^{rel}_{md} < \Delta U^{rel}_{\downarrow} < \Delta U^{rel}_{ap} \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

This formulation is equal to Woerner's (Woerner et al., 2019) apart from the addition of the lower threshold ΔU^{rel}_{md}. This was added to avoid putting heavy penalties on very small speed changes that will neither be noticed, nor affect the situation in any significant manner. A plot of this penalty as a function of the relative speed reduction can be seen in Fig. 11. As in the original formulation, only non-apparent speed reductions are penalized. The reasoning behind this was not explained, but for give-way vessels in overtaking or crossing situations, where this penalty is applicable, it appears that an additional penalty for non-apparent speed increase is superfluous. With regards to overtaking situations, a small speed increase by the give-way vessel is unlikely to have negative effects, as it will only reduce the duration of the encounter. In crossing situations, the give-way vessel is expected to pass behind the stand-on vessel. Any negative effect in the form of a less than preferred range at CPA will then be penalized by the range safety score, see Eq. (3). On the other hand, passing ahead of the stand-on vessel will incur its own penalty, see Section 2.8, possibly in conjunction with a reduced safety range score.

2.5. Rule 17 - stand on

Rule 17 is concerned with vessels that have stand-on responsibilities to another vessel and implicitly defines four zones or stages for stand-on vessel responsibilities based on range between vessels. The circular representation of these stages, used in this work, is shown in Fig. 12. However more complex shapes where the stage-defining ranges vary according to relative bearing and vessel speed can also be applied. While vessels in Stage 1 are considered too distant for the COLREG to apply, a stand-on vessel in Stage 2 is required to maintain its course and speed. However, if the give-way vessel fails to take appropriate action the vessels enter Stage 3 where the stand-on vessel may take action to avoid collision. Further, if it becomes apparent that collision cannot be avoided by the actions of the give-way vessel alone, the vessels enter Stage 4 where the stand-on vessel must take action to avoid collision. Even so, she should, as far as the situation allows avoid turning port for a vessel on her port side. The total score for rule 17 is thus determined

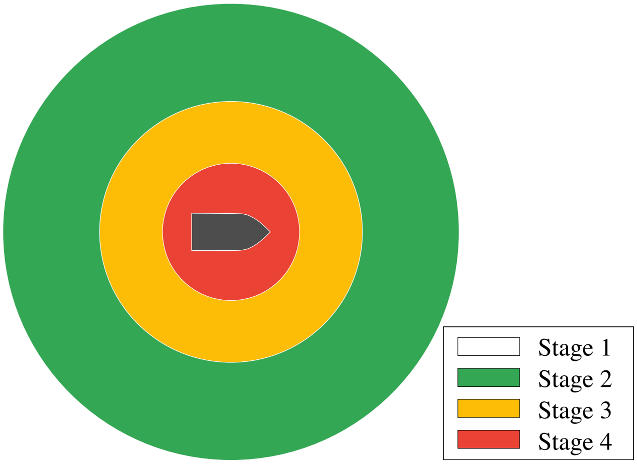


Fig. 12. Circular representation of the different stages in an encounter. Vessels in Stage 1 are detected, but at a distance that does not require further action to be taken. If a vessel enters Stage 2 the navigators must consider evasive maneuvers. Stage 3 and 4 only concerns stand-on vessels.

by any penalties accumulated in Stage 2 ($\mathcal{P}_2$) and 3 ($\mathcal{P}_3$), along with the penalty on port turns from Stage 4 ($\mathcal{P}_{pt}$) and is given by:

$$\mathcal{S}_{17} = \mathcal{S}_{safety}\mathcal{S}_2\mathcal{S}_3\mathcal{S}_{pt}. \tag{15}$$

This formulation places equal importance in the different scores and require high scores in all stages for a good total score. Note that if a stage is not entered during an encounter, the vessel will receive a score of 1 in this stage. While Woerner et al. (2019) does not include an equivalent of Stage, 3 his definition of *in extremis* corresponds to our Stage 4.

2.5.1. Stage penalties

The penalty for port turns in Stage 4 is only applied if the give-way vessel is on the port side of own ship when entering Stage 4, i.e., the contact angle $\alpha < 0°$ and the relative bearing $\beta < 180°$. A port turn is defined as present if the stand on vessel moves more than two ship widths to port between the time of entry into Stage 4 and time of CPA. Such a penalty is also included in Woerner et al. (2019), but its definition is not presented. The definition used in this paper is given by

$$\mathcal{P}_{pt} = \begin{cases} 1, & \alpha < 0°,\ \beta < 180°,\ \text{and port turn} \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

The calculation of $\mathcal{P}_x$, $x \in \{2, 3\}$ is based on the penalties for course change ($\mathcal{P}_{x,\Delta\chi}$), speed increase ($\mathcal{P}_{x,\Delta U\uparrow}$) and speed decrease ($\mathcal{P}_{x,\Delta U\downarrow}$) from the respective stages. A give-way compensation ($C_{x,gw}$) will be given in situations where the stand-on vessel has give-way responsibilities to another vessel. For both stages, the penalty is calculated as follows:

$$\begin{aligned}\mathcal{P}_x = \min(1, (&\gamma_{\Delta\chi}\mathcal{P}_{x,\Delta\chi} + \gamma_{\Delta U\uparrow}\mathcal{P}_{x,\Delta U\uparrow} \\ &+ \gamma_{\Delta U\downarrow}\mathcal{P}_{x,\Delta U\downarrow})(1 + C_{x,gw})).\end{aligned} \tag{17}$$

2.5.2. Give-way compensation

In Woerner's algorithm (Woerner et al., 2019), vessels are compensated for all maneuvers required of normal navigation, but the method for calculating the compensation is not presented. While such maneuvers may be ascribed to many things, e.g., grounding hazards, sea marks, shipping lanes, this paper is limited to encounters with no such restrictions. Compensation is therefore only given if the stand-on vessel finds itself in a multi-vessel encounter where it has give-way responsibilities to other vessels. The compensation given is

$$C_{x,gw} = \begin{cases} \gamma_c, & \text{if give-way responsibilities} \\ 0, & \text{otherwise,} \end{cases} \tag{18}$$

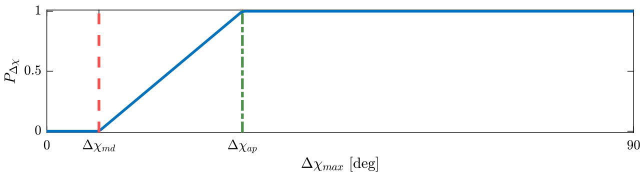


Fig. 13. Penalty function for course change, $P_{\Delta\chi}$ (——), along with limits for minimum detectable course change, $\Delta\chi_{md}$ (- - - -), and readily apparent course change, $\Delta\chi_{ap}$ (-·-·-).

where the γ_c parameter determines the size of the compensation. For the present, this value is a fixed parameter but can in future work be set to vary according to the situation,

2.5.3. Course change penalty for stage 2 and 3

For each course measurement within the stage, the angular difference with regards to the course at the time of entry is calculated. The largest of these values, denoted $\Delta\chi_{max}$, is then used for the calculation of the penalty ($P_{\Delta\chi}$), in the following expression:

$$\mathcal{P}_{\Delta\chi} = \begin{cases} \min\left(1,\ \frac{\Delta\chi_{max} - \Delta\chi_{md}}{\Delta\chi_{ap} - \Delta\chi_{md}}\right), & \Delta\chi_{max} > \Delta\chi_{md} \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

As in the calculation of penalties for non-apparent course changes (see Section 2.4.2), course changes above $\Delta\chi_{ap}$ are considered readily apparent and will in this case receive the full penalty, while changes smaller than $\Delta\chi_{md}$ are considered insignificant. A plot of the function can be seen is Fig. 13.

2.5.4. Speed change penalties

The calculations for speed change penalties for Stage 2 and 3 are based on the relative speed increase and decrease within each stage, given by

$$\begin{aligned}\Delta U_{\uparrow}^{rel} &= \Delta U_{\uparrow}/U(t_0) \\ \Delta U_{\downarrow}^{rel} &= \Delta U_{\downarrow}/U(t_0),\end{aligned} \tag{20}$$

where t_0 denotes time of entry into the respective stage. For each stage the speed increase ($\Delta U_{\uparrow}$) and the speed decrease ($\Delta U_{\downarrow}$) are given by

$$\begin{aligned}\Delta U_{\uparrow} = \max(&U(t_1) - U(t_0),\ U(t_2) - (t_0), \dots, \\ &U(t_{end-1}) - U(t_0)),\ U(t_{end}) - U(t_0))\end{aligned} \tag{21}$$

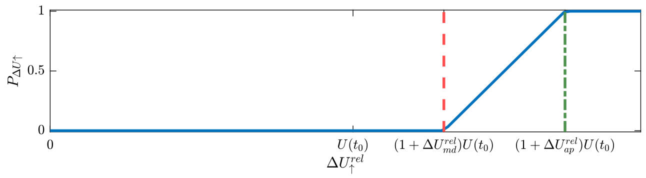
$$\begin{aligned}\Delta U_{\downarrow} = \max(&U(t_0) - U(t_1),\ U(t_0) - U(t_2), \dots, \\ &U(t_0) - U(t_{end-1}),\ U(t_0) - U(t_{end})),\end{aligned} \tag{22}$$

where t_{end} denotes either time of exit from the stage, or time of CPA, depending on which event occurs first. Both increasing and decreasing the speed can be penalized if the relative change is above the threshold for minimum detectable speed change (ΔU_{md}^{rel}). The penalties are given by Eqs. (23) and (24), the plots of which can be seen in Fig. 14.

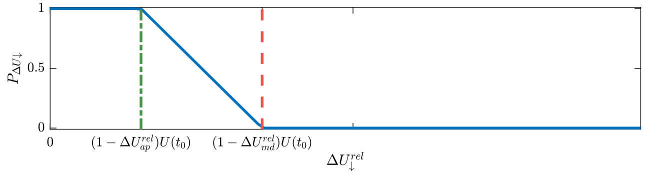
$$\mathcal{P}_{\Delta U\uparrow} = \begin{cases} 0, & \Delta U_{\uparrow}^{rel} < \Delta U_{md}^{rel} \\ 1 - \frac{\Delta U_{ap}^{rel} - \Delta U_{\uparrow}^{rel}}{\Delta U_{ap}^{rel} - \Delta U_{md}^{rel}}, & \Delta U_{md}^{rel} < \Delta U_{\uparrow}^{rel} < \Delta U_{ap}^{rel} \\ 1, & \text{otherwise,} \end{cases} \tag{23}$$

and

$$\mathcal{P}_{\Delta U\downarrow} = \begin{cases} 0, & \Delta U_{\downarrow}^{rel} < \Delta U_{md}^{rel} \\ 1 - \frac{\Delta U_{ap}^{rel} - \Delta U_{\downarrow}^{rel}}{\Delta U_{ap}^{rel} - \Delta U_{md}^{rel}}, & \Delta U_{md}^{rel} < \Delta U_{\downarrow}^{rel} < \Delta U_{ap}^{rel} \\ 1, & \text{otherwise,} \end{cases} \tag{24}$$



(a) Penalty function for speed increase, $P_{\Delta U\uparrow}$ (——).



(b) Penalty function for speed decrease, $P_{\Delta U\downarrow}$ (——).

Fig. 14. Penalties for speed change as functions of relative speed change along with limits for minimum detectable speed change, ΔU^{rel}_{md} (- - - -), and readily apparent speed change, ΔU^{rel}_{ap} (-·-·-).

2.6. Rule 13 - Overtaking

In overtaking situations, the COLREG deem the vessel being overtaken as the stand-on vessel, she is required to keep her course and speed, while the overtaking vessel must keep out of the way until she is past and clear. The score calculation for this rule thus contains two cases and is given by the following expression

$$\mathcal{S}_{13} = \begin{cases} \mathcal{S}_{16} - \gamma_{ah13}\mathcal{P}_{ah13}, & \text{if give-way} \\ \mathcal{S}_{17}, & \text{if stand-on,} \end{cases} \tag{25}$$

where $\mathcal{P}_{ah13}$ is a penalty for passing ahead of, and thus being a hindrance to the stand-on vessel, and γ_{ah13} the weight deciding the influence of this penalty on the total score. This penalty, though left undefined, is also used in Woerner's algorithm (Woerner et al., 2019), where it is included in the give-way score $\mathcal{S}_{16}$. Our definition of the penalty is based on the pose of the give-way vessel and is given by:

$$\mathcal{P}_{ah13} = \begin{cases} 1, & |\alpha_{cpa}| < \alpha_{ah13} \\ 0, & \text{otherwise,} \end{cases} \tag{26}$$

where α_{ah13} marks the limits for the undesired contact angles.

2.7. Rule 14 - Head on

In head on situations, both vessels have equal responsibility for avoiding collision by changing their course towards starboard, and pass on the port side of each other. The maneuver must be made in ample time and be readily apparent to the other vessel. Applicable penalties for head on situations are therefore a penalty for non-starboard turns ($\mathcal{P}_{nsb}$), a penalty for starboard-to-starboard passing ($\mathcal{P}_{sts}$) and a non-apparent course change penalty ($\mathcal{P}^{\neg ap}_{\Delta\chi}$) in combination with the penalty for delayed action ($\mathcal{P}_{delay}$), see Sections 2.4.2 and 2.4.1. The score is calculated using the following expression

$$\begin{aligned}\mathcal{S}_{14} = &(1-\gamma_{nsb}\mathcal{P}_{nsb}-\gamma_{sts}\mathcal{P}_{sts})(1-\mathcal{P}^{\neg ap}_{\Delta\chi}) \\ &(1-\mathcal{P}_{delay}),\end{aligned} \tag{27}$$

where γ_{nsb} and γ_{sts} are weights and should be chosen so that $\gamma_{nsb}+\gamma_{sts} = 1$.

The penalty for a starboard-to-starboard passing is based on the involved vessels' pose at CPA, see Fig. 15, and is given by

$$\mathcal{P}_{sts} = 1 - \left(\frac{\sin(\alpha_{cpa})-1}{2}\right)^2 \left(\frac{\sin(\beta_{cpa})-1}{2}\right)^2, \tag{28}$$

which is the same expression as used in Woerner's evaluation (Woerner et al., 2019), except formulated as a penalty.

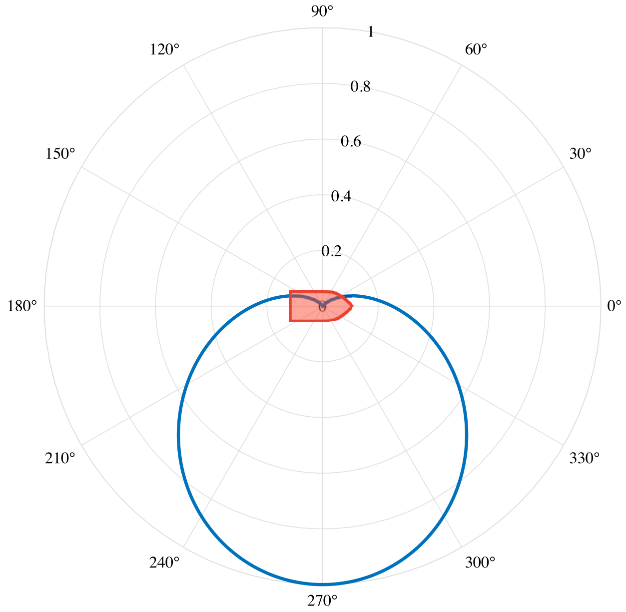


Fig. 15. Polar plot of $\left(\frac{\sin(\theta)-1}{2}\right)^2$ (——), used in Eq. (28) to penalize starboard-to-starboard poses at CPA.

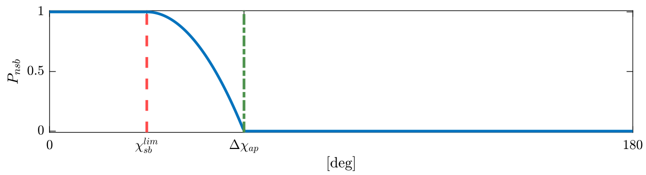


Fig. 16. Non-starboard turn penalty, $\mathcal{P}_{nsb}$ (——), as a function of course change along with limits for what is considered a starboard turn, χ^{lim}_{sb} (- - - -), and readily apparent speed change, ΔU_{ap} (-·-·-).

The penalty for non-starboard turns ($\mathcal{P}_{nsb}$) is based on the starboard course change ($\Delta\chi_{sb}$), which is given by

$$\begin{aligned}\Delta\chi_{sb} = \max\big(&\chi(t_1)-\chi(t_0),\ \chi(t_2)-\chi(t_0), \\ &\dots, \chi(t_{cpa-1})-\chi(t_0),\ \chi(t_{cpa})-\chi(t_0)\big),\end{aligned} \tag{29}$$

assuming that starboard course changes are defined as positive. A full penalty is given if the course change is less than the limit χ^{lim}_{sb} and no penalty is given if the starboard course change is readily apparent. The penalty is given by

$$\mathcal{P}_{nsb} = \begin{cases} 1, & \Delta\chi_{sb} < \chi^{lim}_{sb} \\ 0, & \Delta\chi_{sb} > \Delta\chi \\ 1-\left(\frac{\Delta\chi_{sb}-\chi^{lim}_{sb}}{\Delta\chi-\chi^{lim}_{sb}}\right)^2, & \text{otherwise.} \end{cases} \tag{30}$$

A plot of this function can be seen in Fig. 16.

2.8. Rule 15 - Crossing

In crossing situations the vessel that has the other on her port side is required to stand on, while the other must give way and keep well clear. The score is thus dependent on the vessel's give-way or stand-on behavior, the evaluation of which is described in Sections 2.4 and 2.5 respectively. As in the case of overtaking situations a penalty is applied if the give-way vessel crosses in front of the stand-on vessel. The score is given by the following

$$\mathcal{S}_{15} = \begin{cases} \mathcal{S}_{16} - \gamma_{ah15}\mathcal{P}_{ah15}, & \text{if give way} \\ \mathcal{S}_{17}, & \text{if stand on,} \end{cases} \tag{31}$$

Table 1
Adjustable range parameters.

Parameter	Description
r_{Stage2}	Range at which two vessels are considered in an encounter.
r_{Stage3}	Range at which a stand-on vessel may take action if the give-way vessel fails to do so.
r_{Stage4}	Range at which the stand-on vessel must take action to avoid collision.
r_{pref}	Preferred range between two vessels at CPA.
r_{min}	Minimum acceptable range between two vessels at CPA.
r_{nm}	Range between two vessels at CPA considered as a near-miss.
r_{col}	Range between two vessels at CPA considered as a collision.

Table 2
Situation classification parameters.

Parameter	Value	Description
θ^{13}_{crit}	45.0°	Angle defining an overtaking situation.
θ^{14}_{crit}	13.0°	Angle defining a head-on situation.
θ^{15}_{min}	112.5°	Angle used in definition of crossing and overtaking situations.
θ^{15}_{max}	247.5°	Angle used in definition of crossing and overtaking situations.
θ^{15}_{crit}	10.0°	Angle used in definition of crossing situation.

Table 3
Maneuver detection parameters.

Parameter	Value	Description
ϵ_U	2.0 m/s	Minimum speed for vessel to be considered as moving.
$\epsilon_{\dot{U}}$	0.05 m/s^2	Acceleration threshold.
$\epsilon_{\dot{\chi}}$	0.6 deg/s	Course change threshold.

where γ_{ah15} is a weight defining the importance of the crossing ahead penalty ($\mathcal{P}_{ah15}$), which is given by

$$\mathcal{P}_{ah15} = \begin{cases} 1, & \alpha^{min}_{ah15} < \alpha_{cpa} < \alpha^{max}_{ah15} \\ 0, & \text{otherwise.} \end{cases} \tag{32}$$

3. Parameter values

The COLREG rely on the experience and judgment of sailors, and contains few guidelines when it comes to quantifying the required actions. A thorough investigation into quantification of the COLREG is outside the scope of this article but values used for parameters and weights are included here for completeness. Note that the presented values have been chosen based on the study of a limited amount of AIS data and is accompanied by a high degree of uncertainty.

The set of parameters can be divided into four subsets. The first, containing range parameters that are likely to vary according to situation specific factors, has been classified as adjustable parameters. These parameters along with a description of each is listed in Table 1.

The second set contains angular values used for classifying an encounter as either head-on, crossing (stand-on or give-way) or overtaking (stand-on or give-way) and can be found in Table 2. The values chosen for θ^{15}_{min} and θ^{15}_{max} have their basis in COLREG Rule 13.

Third, the set of parameters used for maneuver detection are displayed in Table 3.

Table 4
Rule specific parameters.

Parameter	Value	Description
α_{cut}	90°	Cut-off angle, used in contact angle score $\mathcal{S}_{\alpha_{cpa}}$, Eq. (5).
β^{min}_{cut}	90°	Minimum cut-off angle, used in relative bearing score $\mathcal{S}_{\beta_{cpa}}$, Eq. (6).
β^{max}_{cut}	270°	Maximum cut-off angle, used in relative bearing score $\mathcal{S}_{\beta_{cpa}}$, Eq. (6).
$\Delta\chi_{md}$	2°	Minimum detectable course change, used in course change penalties $\mathcal{P}^{\neg ap}_{\Delta\chi}$, Eq. (11), and $\mathcal{P}_{\Delta\chi}$, Eq. (19).
$\Delta\chi_{ap}$	30°	Minimum course change considered readily apparent, used in penalty for non-apparent course change penalty $\mathcal{P}^{\neg ap}_{\Delta\chi}$, Eq. (11).
ΔU^{rel}_{md}	0.07	Minimum detectable relative speed change, used in speed change penalties $\mathcal{P}_{\Delta U\uparrow}$, Eq. (23), and $\mathcal{P}_{\Delta U\downarrow}$, Eq. (24).
ΔU^{rel}_{ap}	0.25	Minimum relative speed change considered readily apparent, used in speed change penalties $\mathcal{P}_{\Delta U\uparrow}$, Eq. (23), and $\mathcal{P}_{\Delta U\downarrow}$, Eq. (24)
α_{ah13}	45°	Contact angle defining an ahead passing in overtaking situations, Eq. (26).
$\Delta\chi^{lim}_{sb}$	10°	Minimum course change considered a starboard maneuver.
α^{min}_{ah15}	−25°	Minimum contact angle defining an ahead passing in a crossing situation.
α^{max}_{ah15}	165°	Maximum contact angle defining an ahead passing in a crossing situation.
α_{ah13}	45°	Contact angle defining an ahead passing in an overtaking situation.

Last comes the rule specific parameters, see Table 4. All depend on the evaluator's interpretation of the COLREG, and while the presented values provide a basic tuning for the algorithm, they should not be viewed as the final answer. For instance, the combination of visual observations and heavy fog may require a much larger course change for the maneuver to be readily apparent than the $\Delta\chi = 30°$ used here.

The weights, balancing the importance of different scores or penalties, can be found in Table 5.

4. Results

The following sections present the evaluation scores for three test cases. This was chosen over the alternative option of presenting statistics from multiple evaluations as it better demonstrates how the evaluation works and how the scores are influenced by the choice of parameter values.

The first case presented shows the results from a simulated encounter, where one of the vessels is running the scenario based model predictive control (SBMPC) collision avoidance algorithm from Hagen et al. (2018). For this scenario the range parameters were adjusted to fit those of the algorithm. This approach is useful for testing if an algorithm produces the expected behavior, or for comparing two algorithms with the same tuning.

The second and third test case, presented in Section 4.2, display the behavior of vessels in normal operation. The behavior displayed by these vessels will therefore reflect how the COLREG are interpreted by professional mariners. For these cases, the range parameters (see Table 1) influencing the range score at CPA have been adjusted according

Table 5
Weights used in calculations of scores and penalties.

Name	Value	Description
γ_{min}	0.20	Weight on minimum acceptable range in score for range safety $\mathcal{S}_r$, Eq. (3).
γ_{nm}	0.30	Weight on near miss range in score for range safety $\mathcal{S}_r$, Eq. (3).
γ_{col}	0.50	Weight on collision range in score for range safety $\mathcal{S}_r$, Eq. (3).
γ_{α}	0.25	Weight on contact angle in pose score $\mathcal{S}_{\Theta_{cpa}}$, Eq. (4).
γ_{β}	0.75	Weight on relative bearing in pose score $\mathcal{S}_{\Theta_{cpa}}$, Eq. (4).
$\gamma_{\Delta\chi}^{\neg ap}$	0.50	Weight on non-apparent course changes in score for apparent maneuvers $\mathcal{S}_{\Delta}^{\neg ap}$, Eq. (9)
$\gamma_{\Delta U}^{\neg ap}$	0.50	Weight on non-apparent speed changes in score for apparent maneuvers $\mathcal{S}_{\Delta}^{\neg ap}$, Eq. (9)
$\gamma_{\Delta\chi}$	0.50	Weight on penalties for course changes in stage penalties $\mathcal{P}_x$, Eq. (17).
$\gamma_{\Delta U\uparrow}$	0.25	Weight on penalties for speed increase in stage penalties $\mathcal{P}_x$, Eq. (17).
$\gamma_{\Delta U\downarrow}$	0.25	Weight on penalties for speed decrease in stage penalties $\mathcal{P}_x$, Eq. (17).
γ_{c}	0.20	Weight on give-way compensation in stage penalties $\mathcal{P}_x$, Eq. (17).
γ_{ah13}	0.30	Weight on penalty for passing ahead in overtaking score $\mathcal{S}_{13}$, Eq. (25).
γ_{nsb}	0.30	Weight on penalty for non-starboard turns in head-on score $\mathcal{S}_{14}$, Eq. (27).
γ_{sts}	0.40	Weight on penalty for starboard-to-starboard passing in head-on score $\mathcal{S}_{14}$, Eq. (27).
γ_{ah15}	0.50	Weight on penalty for give-way vessel passing ahead in crossing score $\mathcal{S}_{15}$,Eq. (31).

Table 6
Parameters used for simulated scenario 1.

Parameter	Value
r_{Stage2}	1900 m
r_{Stage3}	700 m
r_{Stage4}	200 m

Parameter	Value
r_{pref}	200 m
r_{min}	100 m
r_{nm}	50 m
r_{col}	35 m

Table 7
Scores and Penalties for simulated scenario 1.

(a) Ship 0: Give-way crossing with regards to Ship 1.

Score/penalty	Value
$\mathcal{S}_{15}$	0.96
$\mathcal{P}_{ahead15}$	0.00
$\mathcal{S}_{16}$	0.96
$\mathcal{S}_{safety}$	1.00
$\mathcal{S}_{\Theta}$	0.52
$\mathcal{S}_{r}$	1.00
$\mathcal{P}_{delay}$	0.00
$\mathcal{P}_{\Delta}^{\neg ap}$	0.04
$\mathcal{P}_{\Delta U}^{\neg ap}$	0.07
$\mathcal{P}_{\Delta\chi}^{\neg ap}$	0.00

(b) Ship 0: Stand-on crossing with regards to Ship 2.

Score/penalty	Value	
$\mathcal{S}_{15}$	0.35	
$\mathcal{S}_{safety}$	1.00	
$\mathcal{S}_{\Theta}$	0.83	
$\mathcal{S}_{r}$	0.99	
$\mathcal{S}_{17}$	0.35	
$\mathcal{P}_{pt}$	0.00	
Stage	**2**	**3**
$\mathcal{P}_{x}$	0.42	0.40
$\mathcal{P}_{x,\Delta\chi}$	1.00	1.00
$\mathcal{P}_{x,\Delta U\uparrow}$	0.05	0.00
$\mathcal{P}_{x,\Delta U\downarrow}$	0.02	0.00
$\mathcal{C}_{x,gw}$	0.20	0.20

(c) Ship 1: Stand-on crossing with regards to Ship 0.

Score/penalty	Value	
$\mathcal{S}_{15}$	1.00	
$\mathcal{S}_{safety}$	1.00	
$\mathcal{S}_{\Theta}$	0.84	
$\mathcal{S}_{r}$	1.00	
$\mathcal{S}_{17}$	1.00	
$\mathcal{P}_{pt}$	0.00	
Stage	**2**	**3**
$\mathcal{P}_{x}$	0.00	0.00
$\mathcal{P}_{x,\Delta\chi}$	0.00	0.00
$\mathcal{P}_{x,\Delta U\uparrow}$	0.00	0.00
$\mathcal{P}_{x,\Delta U\downarrow}$	0.00	0.00
$\mathcal{C}_{x,gw}$	0.20	0.20

(d) Ship 1: Head-on with regards to Ship 2.

Score/penalty	Value
$\mathcal{S}_{14}$	1.00
$\mathcal{P}_{sts}$	0.00
$\mathcal{P}_{nsb}$	0.00
$\mathcal{P}_{\Delta\chi}^{\neg ap}$	0.00
$\mathcal{P}_{delay}$	0.00

(e) Ship 2: Give-way crossing with regards to Ship 0.

Score/penalty	Value
$\mathcal{S}_{15}$	0.89
$\mathcal{P}_{ahead15}$	0.00
$\mathcal{S}_{16}$	0.89
$\mathcal{S}_{safety}$	0.99
$\mathcal{S}_{\Theta}$	0.49
$\mathcal{S}_{r}$	0.99
$\mathcal{P}_{delay}$	0.10
$\mathcal{P}_{\Delta}^{\neg ap}$	0.00
$\mathcal{P}_{\Delta U}^{\neg ap}$	0.00
$\mathcal{P}_{\Delta\chi}^{\neg ap}$	0.00

(f) Ship 2: Head-on with regards to Ship 1.

Score/penalty	Value
$\mathcal{S}_{14}$	1.00
$\mathcal{P}_{sts}$	0.00
$\mathcal{P}_{nsb}$	0.00
$\mathcal{P}_{\Delta\chi}^{\neg ap}$	0.00
$\mathcal{P}_{delay}$	0.00

to the displayed behavior. Notably, r_{pref} is set equal or slightly lower than the distance between the vessels at CPA and r_{Stage2} and r_{Stage3} is chosen that the distance between the vessels when a maneuver is made falls within the interval between them. This is justified by neither vessel making an effort to increase the distance.

4.1. Simulation results

4.1.1. Simulated scenario 1

While encounters between multiple vessels are relatively rare in open waters, it is nevertheless important to be able to evaluate such situations in a sensible manner. The situation shown in Fig. 17 consists of a head on encounter between Ship 1 and Ship 2 while Ship 0 is in a crossing encounter with both vessels. The COLREG does not specify how to handle contradicting responsibilities, therefore the evaluation algorithm gives each vessel an independent score with regards to each other vessel encountered. The only consideration made for multi-vessel encounters is the give-way compensations ($\mathcal{C}_{x,gw}$) that can be given if a vessel has contradicting responsibilities. Note that for this case the range parameters (see descriptions in Table 1) have been set to the corresponding values (see Table 6) used in a collision avoidance algorithm running on Ship 0 to test if the algorithm performs as expected. With these parameters, Ship 0 enters Stage 2 with regards to both the other vessels shortly after the simulation is started, while Ship 1 and 2 do not enter each other's Stage 2 until after Ship 2 has completed her maneuver and returned to her original course.

From the trajectory plot (Fig. 17) it appears that Ship 0 has prioritized her give-way responsibilities to Ship 1 over her stand-on responsibilities to Ship 2. This is reflected in the resulting scores, see Table 7a and 7b.

Ship 2, which is the give-way vessel in the crossing situation with Ship 0, makes a large starboard turn to pass behind, but receives a small penalty for delayed action ($\mathcal{P}_{delay}$). The scores for this situation is shown in Table 7e.

In the head-on situation between Ship 1 and Ship 2, both vessels receive full scores (see Table 7d and 7f) despite not making any avoidance maneuvers. This is because the range at CPA is larger than the preferred range (r_{pref}).

Ship 1 keeps constant course and speed throughout the simulation and therefore receives a full score for her stand-on behavior in the crossing situation with Ship 0, see Table 7c.

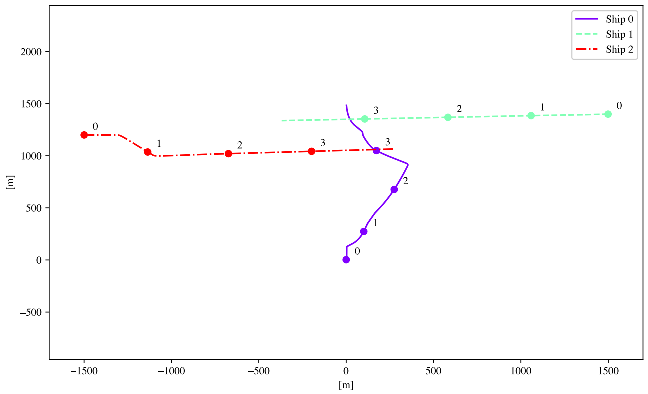


Fig. 17. Simulated scenario 1, multi-vessel encounter. The positions of the vessels are marked at four instances during the encounter.

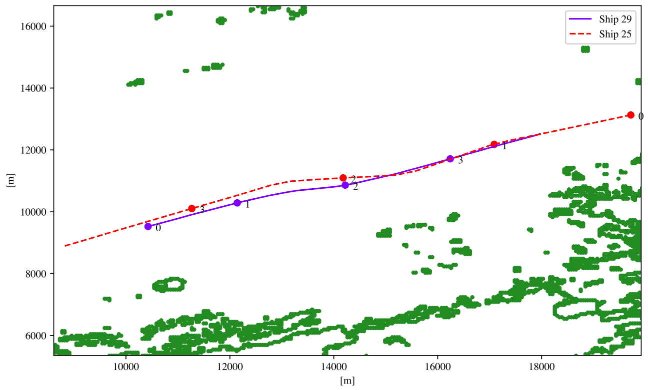


Fig. 18. AIS scenario 1, head-on. The positions of the vessels are marked at four instances during the encounter.

4.2. On-water results

The trajectories used for testing the evaluation method with regards to human behavior were extracted from AIS data gathered by AIS Norway, a network consisting of about 90 base stations, established by the Norwegian Coastal Administration (The Norwegian Coastal Administration) and covers an area stretching from the Norwegian baseline to 40–60 nautical miles from the coast.

AIS is a transceiver/receiver system for tracking and monitoring of vessels at sea. The fitting of an AIS transceiver is required by the International Convention for the Safety of Life at Sea (SOLAS) (International Maritime Organization (IMO), 1974) for all vessels of 300 gross tonnage and above traveling internationally and above 500 gross tonnage not traveling internationally, and on all passenger vessels. In addition, many non-SOLAS vessels are fitted with the simpler and less expensive Class B transceivers that provide much of the same functionality including the publication of the vessel's position, speed, unique id (maritime mobile service identity (MMSI)), type and dimensions. Recorded AIS can be useful for extracting information about the behavior of human operated vessels.

4.2.1. AIS scenario 1

The head-on encounter shown in Fig. 18 is between two relatively large vessels, a passenger ship (Ship 25) and a cargo ship (Ship 29) with lengths of 136 and 67 meters respectively. When the distance between the vessels is 3079 meters the cargo ship makes a starboard maneuver of 10°, and at 2410 meters the passenger ship one of 25.5°. At the time of CPA each vessel has the other on their port side at a distance of 235 m.

Table 8
Parameters used for AIS scenario 1.

Parameter	Value
r_{Stage2}	3500 m
r_{Stage3}	2000 m
r_{Stage4}	700 m
r_{pref}	200 m
r_{min}	100 m
r_{nm}	50 m
r_{col}	35 m

Table 9
Scores and Penalties for AIS scenario 1.

(a) Ship 25

Score/penalty	Value
$\mathcal{S}_{14}$	0.77
$\mathcal{P}_{sts}$	0.01
$\mathcal{P}_{nsb}$	0.00
$\mathcal{P}^{\neg ap}_{\Delta\chi}$	0.23
$\mathcal{P}_{delay}$	0.00

(b) Ship 29

Score/penalty	Value
$\mathcal{S}_{14}$	0.13
$\mathcal{P}_{sts}$	0.01
$\mathcal{P}_{nsb}$	0.00
$\mathcal{P}^{\neg ap}_{\Delta\chi}$	0.87
$\mathcal{P}_{delay}$	0.00

Table 10
Parameters used for AIS scenario 2.

Parameter	Value
r_{Stage2}	3500 m
r_{Stage3}	2000 m
r_{Stage4}	700 m
r_{pref}	1200 m
r_{min}	1000 m
r_{nm}	800 m
r_{col}	200 m

The parameters used for the evaluation of this encounter can be found in Table 8 and the resulting scores in Table 9. As the distance between the vessels at the CPA is larger than the preferred distance, no penalty has been given for delayed action ($\mathcal{P}_{delay}$). It is safe port-to-port passing leading to a negligible penalty for the vessels' pose at the CPA ($\mathcal{P}_{sts}$). As neither vessel makes a port maneuver no penalty is given for non starboard maneuvers ($\mathcal{P}_{nsb}$). However, both vessels receive a penalty for non-apparent maneuver ($\mathcal{P}^{\neg ap}_{\Delta\chi}$) as the maneuvers made are below the threshold for what is considered readily apparent ($\Delta\chi_{app}$). The difference in the magnitude of the maneuvers is reflected in both the penalty and the total score for this encounter ($\mathcal{S}_{14}$). While this head-on situation may be an exception, the fact that both vessels are penalized for making non-apparent maneuvers may indicate that the threshold is set too high. Lowering this limit from $\Delta\chi_{app} = 30°$ to $\Delta\chi_{app} = 25°$ results in a total score of $\mathcal{S}_{14} = 0.99$ for Ship 25, and a total score of $\mathcal{S}_{14} = 0.20$ for Ship 29 and highlights the importance of selecting appropriate parameter values.

4.2.2. AIS scenario 2

The crossing encounter shown in Fig. 19 is between a bulk carrier (Ship 0) and an offshore supply ship (Ship 1). These are again relatively large vessels, their respective lengths being 118 and 90 m. Additionally, the encounter takes place in open waters which is reflected in the high values chosen for the range parameters in this evaluation, see Table 10. When the distance between the vessels is 2157 meters the carrier makes a starboard maneuver of 35° allowing the supply ship to pass ahead with the range at CPA being 1218 m, the carrier then returns to its original course. The supply ship keeps constant course and speed throughout the duration of the encounter.

The behavior of both vessels are in line with the COLREG which is reflected in the scores in Table 11. The give-way vessel (Ship 0) makes a substantial maneuver in good time which is rewarded by a zero penalty for non-apparent maneuver ($\mathcal{P}^{\neg ap}_{\Delta}$) and delayed action ($\mathcal{P}_{delay}$). It then passes behind the other vessel, thus the penalty for passing ahead ($\mathcal{P}_{ah15}$) is also zero. This behavior allows the stand-on vessel (Ship 1) to keep constant course and speed leading to no maneuvering penalties in Stage 2 or 3 ($\mathcal{P}_x$). The penalty for port-turns in Stage 4 remains zero as the vessels never enter this stage. Both vessels do however receive a penalty for their pose at time of CPA, but as the distance between

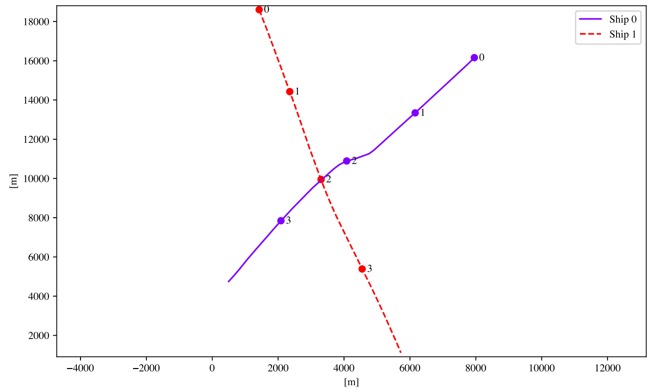


Fig. 19. AIS scenario 2, crossing. The positions of the vessels are marked at four instances during the encounter.

Table 11
Scores and penalties for AIS scenario 2.

(a) Ship 0

Score/penalty	Value
$\mathcal{S}_{15}$	1.00
$\mathcal{P}_{ahead15}$	0.00
$\mathcal{S}_{16}$	1.00
$\mathcal{S}_{safety}$	1.00
$\mathcal{S}_{\Theta}$	0.31
$\mathcal{S}_{r}$	1.00
$\mathcal{P}_{delay}$	0.00
$\mathcal{P}_{\Delta}^{\neg ap}$	0.00
$\mathcal{P}_{\Delta U}^{\neg ap}$	0.00
$\mathcal{P}_{\Delta \chi}^{\neg ap}$	0.00

(b) Ship 1

Score/penalty	Value	
$\mathcal{S}_{15}$	1.00	
$\mathcal{S}_{safety}$	1.00	
$\mathcal{S}_{\Theta}$	0.77	
$\mathcal{S}_{r}$	1.00	
$\mathcal{S}_{17}$	1.00	
$\mathcal{P}_{pt}$	0.00	
Stage	**2**	**3**
$\mathcal{P}_{x}$	0.00	0.00
$\mathcal{P}_{x,\Delta\chi}$	0.00	0.00
$\mathcal{P}_{x,\Delta U\uparrow}$	0.00	0.00
$\mathcal{P}_{x,\Delta U\downarrow}$	0.00	0.00
$\mathcal{C}_{x,gw}$	0.00	0.00

them is larger than the preferred range this is not included in the total safety score ($\mathcal{S}_{safety}$). They thus receive the maximum total score ($\mathcal{S}_{15}$) for this encounter.

5. Discussion

The presented method is only concerned with a subset of the COLREG and is limited to test a finite number of scenarios. As such, it will never definitively prove that a collision avoidance algorithm is safe or abiding by the COLREG. If used for verification purposes, it must be implemented as a part of a framework capable of assuring sufficient coverage of the test space. This includes the implementation of a systematic method for generating test cases. The evaluation method can however, be used on its own to identify issues in the behavior produced by COLAV algorithms and compare the performance of different COLAV algorithms or the behavior produced by algorithms with that of human operators.

Note that one should show caution when comparing scores produced by the evaluation algorithm, as these depend completely on the parameters used for the evaluation, which can be tuned to favor the evaluator's preferences. More research is needed concerning the behavior of human-controlled vessels and how the COLREG are interpreted in different situations before parameter values can be finally agreed upon. As an example; The range parameters are likely dependent on factors such as the type, size and speed of the vessels, but also on the type of encounter, and geographical and meteorological factors, thus one set of parameters may not be suitable for all situations.

The algorithm is, as mentioned above, not an exhaustive test for COLREG compliance. It could easily be extended to include COLREG rule 18 by incorporating navigational status (fishing, sailing, etc.) in the situation classification, but a thorough discussion around how to include a larger part of the COLREG in evaluation algorithms can be found in Woerner's paper (Woerner et al., 2019). Other objectives, such as temporal efficiency and energy efficiency, could also be included to provide a more comprehensive evaluation scheme.

Further work also includes the quantification of COLREG in terms such as angles, velocities and distances, along with the identification of their dependencies on encounter specific factors. The implementation of these within the evaluation method will enable useful comparisons between autonomous and human collision avoidance behavior.

6. Conclusion

A method for evaluation of COLAV behavior with regards to safety and compliance with the COLREG Rules 8(a, b), 13, 14, 15, 16 and 17 has been presented. The method is based on metrics developed by Woerner (2016) and Woerner et al. (2019) but include mathematical expressions for the complete evaluation process along with the method employed for maneuver detection. Specifically, some score/penalty terms were not formulated in Woerner et al. (2019). They are now provided in Eqs. (7), (18) and (26). Moreover, other scores/penalties from Woerner et al. (2019). have been improved, in particular Eqs. (2), (11), and (14). Additionally the values of all tunable parameters and weights are presented. Results from three case studies have been included to provide more insight into the evaluation process, and highlight the effect of tuning choices. The presented method allows for unbiased evaluation and comparison of COLAV algorithms and can help identify issues with the behaviors produced by said algorithms.

CRediT authorship contribution statement

Inger B. Hagen: Developed and implemented the method, Concepts of stage-wise evaluation, Simulation study and tuning of the method, Wrote the paper. **Olav Vassbotn:** Method for prepossessing the AIS-data was developed and implemented, Provided the data for the real-life encounters. **Morten Skogvold:** Provided useful inputs to the development, Reviewed the paper. **Tor A. Johansen:** Supervision, Reviewed the paper. **Edmund F. Brekke:** Supervision, Reviewed the paper.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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