Master's thesis

NTNU
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Faculty of Economics and Management
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Modelling EURNOK Returns using Genetic Programming Symbolic Regression

An examination of nonlinearities and structural shifts between 2002 and 2022

Master's thesis in Industrial Economics and Technology Management
Supervisor: Dr. Morten Risstad
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June 9, 2023

Preface

This master thesis represents the culmination of our Masters of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology, with specialisation in Financial Engineering. The thesis was written in the period January 2023 to June 2023.

This master thesis is written by Mika Myrseth, Silje Mangersnes Røstum, and Benedicte Chen Vestrum. The Norwegian krone has been steadily depreciating since 2015, particularly in 2023, making exchange rate studies a very current topic and of great academic interest to us. Applying genetic programming symbolic regression allows us to capture and study non-linear dependencies between the krone and macroeconomic fundamentals. This can provide valuable insights into the underlying factors influencing EURNOK returns, and contribute to the broader understanding of the krone. We appreciate having been able to study the krone fundamentals in detail, and have gained valuable insight into understanding the behaviour of EURNOK returns.

We would like to thank our supervisor Dr. Morten Risstad for his invaluable academic guidance, expertise, and support through the process. Our discussions have been instrumental in shaping this thesis and refining our understanding of the krone and macroeconomic fundamentals. We would also like to express our gratitude to our co-supervisor Sjur Westgaard for his feedback on our work. We are very grateful to have had such good guidance.

Mika Løset Myrseth	Silje Mangersnes Røstum	Benedicte Chen Vestrum

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Abstract

This master thesis explores nonlinearities suspected to exist between the Norwegian krone and macroeconomic fundamentals, aiming to enhance understanding of movements in the krone exchange rate. Genetic programming symbolic regression (GPSR) is employed for developing a non-linear descriptive model for daily EURNOK returns from 2002 to 2022.

The results demonstrate that GPSR successfully develops a parsimonious model for EURNOK returns that outperforms benchmark models, surpassing the limitations of traditional linear models and machine learning models. The identified non-linear relationships provide valuable insights into understanding the nature of EURNOK returns. Several structural shifts are identified between 2002 and 2022, affecting the relationships and the features' impacts on the model predictions. A large drift contributing to the krone's depreciation however remains unexplained.

Understanding the Norwegian krone dynamics is of great interest to economists, policymakers, and other stakeholders. This thesis identifies non-linear dependencies which have previously not been studied to a great extent. Further research exploring these nonlinearities could be insightful for enhancing understanding of the krone and EURNOK movements. The findings of this thesis also encourage future studies to apply GPSR to other fields in economics and finance to further examine the validity and efficiency of GPSR as an alternative to traditional modelling approaches.

Sammendrag

Denne masteroppgaven undersøker ikke-lineære sammenhenger mellom den norske kronen og makroøkonomiske faktorer for å bedre forstå bevegelser i kronekursen. *Genetic Programming Symbolic Regression* (GPSR) brukes til å utvikle en ikke-lineær beskrivende modell for daglige EURNOK-avkastninger fra 2002 til 2022.

Resultatene viser at GPSR er en effektiv og hensiktsmessig metode for utvikling av en modell for EURNOK-avkastning som overgår sammenlignbare modeller, og som overkommer begrensningene til tradisjonelle lineære modeller og maskinlæringsmodeller. De ikke-lineære sammenhengene som er identifisert gir viktige innsikter i hvordan EURNOK-valutakursen svinger. Vi identifiserer flere strukturelle endringer mellom 2002 og 2022 som påvirker forholdene mellom modellens faktorer og hvordan faktorene påvirker modellens prediksjoner. En betydelig nedadgående trend som bidrar til svekkelse av kronen forblir imidlertid uforklart.

Å forstå bevegelsene til den norske kronen er av stor interesse for økonomer, beslutningstakere og andre interessenter. Vi identifiserer ikke-lineære forhold mellom faktorer som ikke har blitt undersøkt i litteraturen. Videre forskning som undersøker disse ikke-lineæritetene kan gi verdifulle innsikter for å bedre vår forståelse av kronens svingninger og bevegelser. Våre funn underbygger GPSRs effektivitet og hensiktsmessighet som et alternativ til tradisjonelle modelleringstilnærminger, og oppfordrer til videre studier av GPSRs muligheter for anvendelse innenfor økonomi og finans.

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1 Introduction

Since the floating exchange rate was adopted in 1992, various studies present different descriptive models for explaining the relationship between the Norwegian krone (NOK) exchange rate and macroeconomic fundamentals. Understanding the mechanisms of the Norwegian krone exchange rate movements is of great interest to investors, policymakers and other interested parties, and these models have provided valuable insight on the Norwegian krone. However, as most of these models employ multivariate linear regression, this has caused a shortcoming as a limited methodology forms the basis for the academic discourse. Moreover, linear regression's assumptions of linearity and independence often do not hold for exchange rates, with several studies finding evidence of non-linear relationships. Recent studies have applied non-linear machine learning (ML) techniques, however many ML approaches suffer from non-interpretable outputs, and even with advancements in explainable AI, insight can be limited.

Symbolic regression (SR) is a ML technique that searches through possible symbolic mathematical expressions to find the equation that best fits the training data. SR requires no assumption of linearity, in contrast to multivariate linear regression models, and generates an interpretable output, in contrast to popular ML approaches. SR also requires no a priori knowledge on the data structure. Despite these properties, the application of SR within finance has so far been sparse.

Our thesis contributes to the existing literature in three ways. First, we contribute to the literature on the Norwegian krone by applying genetic programming symbolic regression (GPSR), a little widespread yet promising field of ML, to the well-studied macroeconomic field of exchange rates and foreign exchange. We use GPSR to develop a non-linear model for the krone, and examine whether this application yields new insight and proof of the method's validity. Few previous studies have applied GPSR to model exchange rates, and to our knowledge, no study has specifically applied GPSR to the krone.

Second, we contribute to the literature on nonlinearities in the krone by identifying and examining non-linear relationships that affect the krone. Several studies suspect that non-linearities exist in the relationships between different features and the krone. GPSR is interpretable and non-linear, and hence allows for identifying and studying nonlinearities, contrary to existing linear regression and ML models.

Third, we contribute to the literature by examining the krone's unexplained depreciation since the 2014-2016 oil price drop, which is causing debate amongst economists. We test for and identify structural shifts, which allow for identifying time-varying relationships and impact of features. This contributes to isolating the unexplained depreciation from the time-varying impact of observable features.

Our method implements GPSR to develop a descriptive model for daily EURNOK returns in the period 1 January 2002 to 1 August 2022. The analysis includes a representative selection of macroeconomic fundamentals, commodities, and financial assets to capture significant effects and relationships. The selected GPSR model is an optimal trade-off between accuracy and complexity. For increased robustness, the GPSR model is evaluated against linear and non-linear benchmark models. This builds credibility and examines the model's explanatory power.

After identifying structural shifts in the estimation period, the GPSR model is re-estimated to account for the shifts. This allows for capturing and identifying time-varying effects and relationships, and for studying the krone's depreciation over time.

The GPSR algorithm identifies a parsimonious interpretable non-linear model for EURNOK returns. The GPSR model's high performance when compared to benchmark models demonstrates GPSR's success as an effective tool in finance generally and exchange rate studies specifically. The GPSR model consist of three linear terms and three non-linear terms. The linear terms are commonly found in literature, and include currency volatility, Norwegian specific volatility, and 12-month interest rate difference. The non-linear relationships include Brent crude oil price, Euronext 100 Index, and currency volatility. The nonlinearities can provide an increased understanding of the nature of EURNOK returns.

Three major structural breaks are identified. These are the global financial crisis, the 2014-2016 oil price fall, and the Covid-19 pandemic. The structural shifts cause time-varying feature impact on the model's predictions, challenging established consensus that the Brent crude oil price remains the most important feature for the krone. The unexplained drift contributing to the depreciation of the krone after 2014 is however not explained by our model's macroeconomic fundamentals, encountering similar challenges as traditional models analysing the behaviour of the krone.

The paper is organised as follows. Section 2 presents the existing body of literature on modelling krone exchange rates, and GPSR. Section 3 presents the data and variables employed in the analysis and their statistical properties. Section 4 presents the methodology employed for the analysis. Section 5 presents the results of the analysis and applies economic intuition to interpret the results. Section 6 concludes the thesis.

2 Literature review

Section 2.1 introduces exchange rate fundamentals that form the basis for exchange rate studies. Section 2.2 introduces fundamental relationships affecting the Norwegian krone, as well as established multivariate linear models describing the krone. Section 2.3 introduces genetic programming symbolic regression (GPSR), the methodology which will be utilised in this study.

2.1 Exchange rate fundamentals

2.1.1 The Meese-Rogoff puzzle

Meese and Rogoff (1983a,b)'s seminal papers establish a basis for exchange rate studies. Their studies find that structural models based on macroeconomic fundamentals cannot outperform the random walk in exchange rate forecasting. This is referred to as the Meese-Rogoff puzzle (Moosa and Burns, 2015; Bacchetta et al., 2009).

The Meese-Rogoff puzzle has been examined in several papers. Cheung et al. (2005) could not find a structural model which consistently outperformed a random walk. Engel and West (2005) propose that some unidentified macroeconomic fundamental follows a random walk, causing the random walk behaviour observed. Bacchetta and van Wincoop (2004, 2013) propose a scapegoat theory to explain the Meese-Rogoff puzzle. The scapegoat theory states that exchange rate fluctuations resulting from unobserved shocks might be attributed to an observed macroeconomic fundamental feature, a "scapegoat", even if the fluctuation is unrelated to the feature. Fratzscher et al. (2015) support the scapegoat theory, and find exchange rate fluctuations are often driven by information that is not publicly available. Engel and West (2003) find that exchange rate fluctuations are caused by unobservable shocks, and differences between current and expected values of observable economic fundamentals. Andersen et al. (2003) find that exchange rates jump in reaction to news, with the jump size equalling the difference between macroeconomic expectation and realisation. This relationship is also non-linear, with bad news having a greater impact than good news.

Some economists contradict the Meese-Rogoff puzzle. Diebold and Mariano (1995) argue that disregarding models with low forecasting accuracy might lead to overlooking important insight not reflected in forecasting accuracy metrics. Similarly, Moosa and Burns (2015) criticise Meese and Rogoff (1983b) for using a narrow selection of performance metrics which only focus on forecasting error. Mark (1995); Hwang (2001) find evidence of a long-term relationship between exchange rate and macroeconomic fundamentals, and Ince and Molodtsova (2017) find that the forecasting accuracy of structural models increases with longer forecasting horizons. However, Hungnes (2020) finds that the performance of the random walk depends on how recent the data is, with more lags decreasing forecasting accuracy.

2.1.2 Macroeconomic fundamentals and parity conditions

Economic theories state that macroeconomic fundamentals determine the floating exchange rate; however in practice this has to little extent been observed (Engel and West, 2003, p.6).

2.1.2.1 Purchasing Power Parity (PPP)

Purchasing power parity (PPP) is the first principle studied when determining equilibrium exchange rates. PPP is the proposition of equilibrium between exchange rates and price levels of countries, resulting from no-arbitrage (Shapiro, 1983). This implies exchange rates are mean-reverting (MacDonald, 1999, p.676). PPP is formulated as :

$$s_t = p_t - p_t^*. \tag{1}$$

where s_t denotes nominal home currency price of a unit of foreign currency (exchange rate), and p_t and p_t^* denote home and foreign price levels respectively (MacDonald, 1999, p.676):

PPP is based on the Law of One Price (LOP) (Protopapadakis and Stoll, 1986), the economic theory that an identical asset will have the same price globally in efficient markets. Several studies examine PPP and its suspected relationship with exchange rates. Krugman (1978) find that simple regression tests lead to rejecting PPP, but tests which recognise the endogeneity of both prices and exchange rates offer more support to PPP. Cheung and Chinn (2001) find that while 40% of FX traders view PPP as having influence on exchange rates over long time horizons, few traders consider PPP useful in practice. Taylor and Taylor (2004) find that PPP's validity as a long-run concept enjoys strong academic support. Akram et al. (2009) find that LOP holds on average, but is frequently violated with increased market volatility. Grochová and Plecitá (2019) find that PPP holds for the eurozone (EZ) in the long run, but could not find evidence supporting PPP's validity in the short run.

2.1.2.2 Uncovered Interest Rate Parity (UIP)

The validity of uncovered interest rate parity (UIP) as an fundamental parity condition for exchange rates has been debated in the scientific community, and UIP is generally rejected in empirical studies. UIP states that a currency with higher interest rate will depreciate relatively to a currency with a lower interest rate (Isard, 2006; Cappiello and De Santis, 2007). UIP is formulated as:

$$\Delta s^e_{t,t+k} = (i - i^*)_{t,k}, \tag{2}$$

where $\Delta s^e_{t,t+k}$ denotes the expected change in the exchange rate from period t to period $t + k$ and $(i - i^*)_{t,k}$ denotes the current interest differential (Meredith and Chinn, 1998, p.3).

UIP is the cornerstone condition for foreign exchange market efficiency, however evidence suggests that the FX market is informationally inefficient, so that forward expectations predict future exchange rate movements in the wrong direction (Fama, 1984; Sarno, 2005). Several studies such as Bilson (1981); Meese and Rogoff (1983b); Sarno (2005); Engel et al. (2022) examine the poor empirical performance of UIP, and reject its importance in forecasting exchange rate (Isard, 2006). Rejecting UIP rejects the hypothesis of efficient FX markets. However, McCallum (1994); Bernhardsen and Røisland (2000) suggest that as interest rate differential is an endogenous variable, studies might give a misleading impression of a non-existent causal relationship, leading to the poor performance of UIP. Chaboud and Wright (2003) support UIP for very short time horizons. Chinn and Quayyum (2012) find evidence of a long-run relationship between interest rates and exchange rates.

2.1.2.3 Stock prices

Several studies find evidence supporting a relationship between stock prices and exchange rates. Hau and Rey (2005) find that global investors reduce their risk exposure to exchange rate volatility, causing outperforming markets' currencies to depreciate, if the foreign exchange exposure is incompletely hedged. However, Froot et al. (1992); Bohn and Tesar (1996); Griffin et al. (2004); Chabot et al. (2014) find that investors will increase their holdings and advance further investments in stock markets outperforming other stock markets, resulting in the outperforming markets' currencies appreciating instead (Cenedese et al., 2015). Cenedese et al. (2015) oppose both views, and find that exchange rate movements are unrelated to movements in stock price differentials.

2.2 Norwegian krone

This section will introduce background for the Norwegian krone. This includes Norwegian krone exchange rate models and variables that are common in studies explaining the Norwegian krone. The krone's general depreciation and its possible explanations are also presented.

2.2.1 Multivariate linear regression models explaining the Norwegian krone

Several models have been developed with the goal of modelling the Norwegian krone. These are generally multivariate linear regression models that investigate the relationship between the Norwegian krone and several macroeconomic variables. Our thesis will primarily focus on three models, presented in Table 2.1. These three models have all been published in recent years, show high explanatory power, build upon similar assumptions, and apply similar macroeconomic variables.

Researchers from Norges Bank that have developed models for the Norwegian krone include Akram (2020) and Martinsen (2021). Akram (2020) find that the relationship between oil price and the Norwegian krone is time-varying, and heightened geopolitical uncertainty has contributed to weakening the krone. Martinsen (2021) find that fundamental economic features are efficient in describing krone fluctuations, and that his models effectively capture and explain the krone's general depreciation. Klovland et al. (2021) use macroeconomic fundamentals to explain krone fluctuations, and find that the krone's depreciation can be attributed to an increasing price differential. Klovland et al. (2021) outperforms the other models in terms of explanatory power.

Table 2.1: List of multivariate linear models.

Multivariate linear model specifications				
Model	Model Type	Frequency	Original interval	Dependent variable
Akram (2020)	BEER[1]	Weekly	15 Jan 2010 to 31 Dec 2018	I-44
Martinsen (2021)	BEER	Weekly	01 Jan 1999 to 31 Dec 2016	I-44
Klovland et al. (2021)	VECM[2]	Monthly	1 Apr 2001 to 1 Jan 2020	TWI

2.2.2 Exchange rate fundamentals in relation to the krone

The exchange rate fundamentals PPP and UIP have both been studied in relation to the Norwegian krone. Akram (2000) examine PPP between Norway and its main trading partners, and find convergence towards the PPP in the long term. Naug (2003) identify a non-linear relationship between interest rate differentials and the Norwegian krone, and following a sharp decline in stock prices, the krone is more susceptible to changes in interest rate differentials. Benedictow and Hammersland (2022) include interest rate difference and UIP in their study, and find that in the short term interest rate differential affects the Norwegian exchange rate. However, the interest rate differential is an endogenous variable, making it difficult to prove causation and not correlation. In practice, Norges Bank raises the policy rate when aiming to appreciate the krone (Norges Bank, 2023b; Norges Bank). Increasing the policy rate makes investing in the Norwegian economy more attractive for foreign investors, driving up demand for the krone[3]. This contradicts the theoretical parity condition of UIP, which requires that increasing policy rate causes depreciation of the currency.

It has been suspected that the Norwegian krone is particularly affected by global financial unrest. Small currencies are often highly susceptible to geopolitical or financial unrest, with investors fleeing to larger and safer currencies. Bernhardsen and Røisland (2000) find that the Norwegian krone is influenced by turbulence in international financial markets in the short term. Flatner (2009) does not find empirical evidence supporting either that the krone is a safe-haven currency, nor that it is not. Aamodt (2010) find evidence of a relationship between the krone and the Norwegian stock market, and that the Norwegian krone is considered a "safe haven" in times of high unrest, contrary to other non-dominant currencies. Benedictow and Hammersland (2022) on the contrary find that the Norwegian krone depreciated during GFC due to investors fleeing to larger, safe haven currencies.

The relationship between the Norwegian krone and oil price has been extensively studied. Oil is a main Norwegian export, and the Norwegian economy is sensitive to changes in oil price, supply, and demand. Bernhardsen and Røisland (2000); Naug (2003); Akram (2020); Benedictow and Hammersland (2022) find evidence of a short-term relationship between oil price and the Norwegian krone. ter Ellen (2016) identify non-linearities in the relationship, with lower oil price causing higher krone fluctuations. Due to the extensive research and similar findings, oil price is therefore in

[1] Behavioural Equilibrium Exchange Rate
[2] Vector Error Correction Model
[3] Erikstad (2023); DNB (n.d.)

practice considered a macroeconomic fundamental for the Norwegian krone.

2.2.3 The weak krone

Following the 2014-2016 oil price drop, the krone has experienced a general depreciation, as discussed by Klovland et al. (2021), NOU 2020: 8 (2020) and Norges Bank (2023a, p.21), and as seen in Figure 3.2a. This trend of a weakening krone has been debated, and the depreciation has been attributed to a number of factors, including declining interest rate differential (Norges Bank, 2023a), increasing price differential (Klovland et al., 2021), increasing global uncertainty[4], decreasing belief in the Norwegian economic system[5], and an outdated dependence on petroleum[6]. However, many of these factors are hard to observe and lack statistical data, leading to lower explanatory power of the established models.

2.3 GPSR in Finance

This thesis chooses to model the krone using Symbolic Regression (SR), which is solved computationally using Genetic Programming (GP).

SR is a sub-field of machine learning concerned with finding a symbolic expression that matches data from an unknown function (Koza, 1994; Schmidt and Lipson, 2009; Udrescu and Tegmark, 2020). Orzechowski et al. (2018) examine the trade-off between interpretability and accuracy in ML methods (Otte, 2013), and find that SR performs well compared to popular machine learning methods in terms of predictive power. An advantage of SR is high interpretability, especially when compared to other ML models. However, extensive search through a space is also associated with high run time and computational complexity (Langdon and Poli, 2002).

Several advances have been made within the field of SR in recent years improving SR robustness, accuracy, range, and dependability (Korns, 2014, 2015a), which has contributed to strengthening the academic credibility of SR (Korns, 2015b). SR is actively applied within scientific fields, including physics and chemistry. Udrescu and Tegmark (2020) use SR to find symbolic equations matching data from unknown algebraic functions, chosen from the *Feynmann Lectures on Physics*. SR was successfully used to discover all chosen equations in the study, indicating the advantages of SR when attempting to identify unknown functions. Angelis et al. (2023) explore possible fields of SR application, and find that SR is highly suitable for application in finance, for example for econometric modelling.

SR problems can be solved using several different methods. These include neural networks (NNs), GP, and Bayesian frameworks (McRee, 2010; Udrescu and Tegmark, 2020). Traditionally, GP is the most applied method for solving SR problems (Diveev and Shmalko, 2021). GP falls under the field of Evolutionary Computation (EC), drawing inspiration from evolution to solve the central issue in

[4]Becker (2023)
[5]Nilsen and Hovland (2023)
[6]Holvik (2023)

SR of an exponentially large search space. GP is also considered a generalisation of the Genetic Algorithm (GA), but distinguishes itself from other forms of GA through its use of mutation and crossover (Langdon and Poli, 2002).

Within finance, GPSR has been used to develop econometric models and investigate relationships between economic variables. Koza (1990) uses GPSR to rediscover the equation of exchange. This equation describes the non-linear relationship between the total amount of money and macroeconomic variables. Using time series data of the macroeeconomic variables, Koza (1990) is able to derive the equation for exchange without a priori knowledge, as well as capture the non-linear relationship between the variables.

SR has also been applied to create models that describe and predict financial indices. Drachal (2022) forecasts the crude oil spot price using Bayesian Symbolic Regression (BSR), which accounts for feature selection issues in oil price forecasting. La Malfa et al. (2022) apply GPSR to obtain and investigate models for five of the largest financial indices, including the S&P500. Model results show that GP is an effective method, particularly when compared with other methods to solve SR problems.

Sheta et al. (2015) also attempt to predict the S&P500 using multi-gene GP. This study compares the proposed model to a traditional multiple linear regression model, indicating the comparability of SR results to traditional methods. The S&P500 data set consists of ten years of data. Input variables include 1-year treasury bill yield, earnings per share, dividend per share for the S&P500 and the current week's S&P500. The GP model is compared to a multivariate linear regression and a fuzzy model. Sheta et al. (2015) conclude that the GPSR model produces estimates comparable with traditional models in both training and testing samples.

3 Data and Variables

This section describes the data used in this thesis. Section 3.1 gives an overview of the variables and their statistical properties. Section 3.2 introduces the dependent variable chosen for the analysis, while Section 3.3 provides a background and context for the explanatory variables input in the model.

3.1 Data overview

The estimation period is set from 1 January 2002 to 1 August 2022 and consists of 5 370 samples. The data set is split into two sets, training and testing, with a ratio of 80:20. Respectively, the training and testing samples consist of 4 296 samples and 1 074 samples.

The data is sampled at a daily frequency, with the exception of Geopolitical Risk (GPR) Index and price differential, which are interpolated to daily frequency from weekly and monthly frequency, respectively. One disadvantage of a daily frequency is that noise may be present in the data set, however daily frequency ensures there are enough samples for adequate training and testing, which could help avoid overfitting. A daily frequency is therefore chosen.

This thesis considers 17 unique explanatory variables, included both in their original form, lagged, differentiated, and differentiated with a lag. This totals 50 explanatory variables. The data is collected from Bloomberg, Eurostat and Norges Bank, with a full list of data sets and corresponding data sources provided in Appendix A. An overview of the data set and summary statistics are also shown in Table 3.1. Stationarity is a prerequisite for many statistical tests, and the variables are tested for stationarity using Augmented Dickey-Fuller (ADF) test[7]. The time series charts for all 17 unique explanatory variables are shown in Appendix B.

In order to reduce the computational complexity, the input variables have been limited to a range of selected variables. This selection is based on fundamental exchange rate relationships in previous literature presented in Section 2.1 and Section 2.2, as well as features that have not been included in previous literature, but may have an economic relationship with the Norwegian krone. These features include key commodities, European and Norwegian stock indices, financial uncertainty and volatility indices, and macroeconomic fundamentals. The data is further pre-processed by performing feature selection. This allows for further reducing the computational complexity and model runtime. Feature selection is further elaborated in Section 4.1.

[7]The significance level in the ADF test is 0.05. A lower p-value than the significance level indicates stationarity.

Table 3.1: Data overview. Lowercase variables are in log. Differentials denote difference between Norwegian (NOK) and EU (EUR) values.

Data overview										
			Statistical properties							
		Abbr.	Mean	SD	Max	Min	Skew	Kurtosis	Stationarity	Freq.
	Δeurnok	d_eurnok	0.00004	0.00517	-0.03202	0.07540	1.06442	14.38550	Yes	Daily
	EURNOK$_{t-1}$	EURNOKlag	8.63567	0.95283	0.00000	12.53660	0.57207	0.86303	No	Daily
	Δeurnok$_{t-1}$	d_eurnoklag	0.00004	0.00517	-0.03202	0.07540	1.07289	14.62566	Yes	Daily
Commodities	Brent Crude Oil Price	OIL	68.54746	27.95504	18.41000	146.08000	0.33837	-0.79182	No	Daily
	Δbrent crude oil price	d_oil	0.00032	0.02274	-0.27976	0.19077	-0.63429	12.76355	Yes	Daily
	Δbrent crude oil price$_{t-1}$	d_oillag	0.00031	0.02274	-0.27976	0.19077	-0.63404	12.76604	Yes	Daily
	UK NBP Natural Gas Futures	GAS	49.20098	38.77580	8.74000	501.00000	4.55448	29.48258	No	Daily
	Δuk nbp natural gas futures	d_gas	0.00033	0.05971	-2.26930	2.32909	1.31630	832.37877	Yes	Daily
	Δuk nbp natural gas futures$_{t-1}$	d_gaslag	0.00029	0.05963	-2.26930	2.32909	1.31295	837.15565	Yes	Daily
	Salmon	SALMON	5.55202	1.59013	2.39900	10.34990	0.03037	-0.77678	No	Daily
	Δsalmon	d_salmon	0.00026	0.00402	-0.02230	0.02090	-0.18961	4.47506	Yes	Daily
	Δsalmon$_{t-1}$	d_salmonlag	0.00026	0.00402	-0.02230	0.02090	-0.18927	4.47630	Yes	Daily
	Gold	GOLD	1100.64227	479.38484	278.40000	2069.40000	-0.18006	-1.06620	No	Daily
	Δgold	d_gold	0.00036	0.01094	-0.09810	0.08589	-0.39921	5.68179	Yes	Daily
	Δgold$_{t-1}$	d_goldlag	0.00036	0.01094	-0.09810	0.08589	-0.39891	5.68428	Yes	Daily
Assets	MSCI World Index	MSCIW	1570.61727	562.54551	688.64000	3248.12000	0.97907	0.61809	No	Daily
	Δmsci world index	d_msciw	0.00019	0.01024	-0.10442	0.09096	-0.66686	12.20073	Yes	Daily
	Δmsci world index$_{t-1}$	d_msciwlag	0.00020	0.01024	-0.10442	0.09096	-0.66686	12.22687	Yes	Daily
	OSEAX	OSEAX	579.30300	303.53841	105.82000	1445.48000	0.57781	-0.28656	No	Daily
	Δoseax	d_oseax	0.00041	0.01318	-0.09832	0.09188	-0.74104	7.60383	Yes	Daily
	Δoseax$_{t-1}$	d_oseaxlag	0.00041	0.01318	-0.09832	0.09188	-0.74084	7.60428	Yes	Daily
	Euronext 100 Index	EURONEXT	831.20785	204.85406	419.95001	1388.08997	0.38953	-0.52150	No	Daily
	Δeuronext 100 index	d_euronext	0.00009	0.01286	-0.12752	0.10322	-0.29019	7.99605	Yes	Daily
	Δeuronext 100 index$_{t-1}$	d_euronextlag	0.00008	0.01286	-0.12752	0.10322	-0.28995	7.99696	Yes	Daily

			Statistical properties							
		Abbr.	Mean	SD	Max	Min	Skew	Kurtosis	Stationarity	Freq.
Uncertainty	Geopolitical Risk (GPR) Index	GPR	103.19388	35.51751	60.67761	354.45760	2.98897	12.76117	Yes	Weekly
	Δgeopolitical risk (gpr) index	d_gpr	-0.00003	0.01007	-0.06791	0.09321	0.66890	9.24456	Yes	Weekly
	Δgeopolitical risk (gpr) index$_{t-1}$	d_gprlag	-0.00003	0.01006	-0.06791	0.09321	0.66876	9.24938	Yes	Weekly
	Global Hazard Indicator (GHI)	GHI	0.11669	0.05052	0.03148	0.71814	3.36186	22.11378	Yes	Daily
	Δglobal hazard indicator (ghi)	d_ghi	0.00002	0.08119	-0.89485	1.12847	0.84356	16.73880	Yes	Daily
	Δglobal hazard indicator (ghi)$_{t-1}$	d_ghilag	0.00005	0.08117	-0.89485	1.12847	0.84435	16.75916	Yes	Daily
	Currency Volatility Index (CVIX)	CVIX	9.30705	2.67752	4.87000	24.23830	1.71447	5.05829	Yes	Daily
	Δcurrency volatility index (cvix)	d_cvix	-0.00000	0.02549	-0.23093	0.32850	1.09159	18.26700	Yes	Daily
	Δcurrency volatility index (cvix)$_{t-1}$	d_cvixlag	-0.00001	0.02549	-0.23093	0.32850	1.09250	18.28537	Yes	Daily
	Risk Aversion Index	RISKAVERSION	3.14135	1.41699	2.42540	32.71077	10.25579	152.92763	Yes	Daily
	risk aversion index	d_riskaversion	0.00004	0.06626	-1.17911	1.29795	0.79048	110.44867	Yes	Daily
	risk aversion index$_{t-1}$	d_riskaversionlag	0.00002	0.06625	-1.17911	1.29795	0.79114	110.50502	Yes	Daily
	Norwegian specific volatility	NOKVOL	-1.58995	2.55502	-15.80450	22.13000	1.03900	5.72193	Yes	Daily
	Δnorwegian specific volatility	d_nokvol	0.00095	0.47588	-5.39375	7.68750	0.95873	39.90731	Yes	Daily
	Δnorwegian specific volatility$_{t-1}$	d_nokvollag	0.00083	0.47580	-5.39375	7.68750	0.95953	39.93834	Yes	Daily
Macroeconomic fundamentals	Price Differential	PRICEDIFF	0.00657	0.03768	-0.05140	0.09277	0.70681	-0.82995	No	Monthly
	Δprice differential	d_pricediff	0.00001	0.00030	-0.00192	0.00192	0.16913	6.72856	Yes	Monthly
	Δprice differential$_{t-1}$	d_pricedifflag	0.00001	0.00030	-0.00192	0.00192	0.16883	6.73053	Yes	Monthly
	10-Year Interest Rate Differential	I10Y	1.45782	0.42992	0.25200	2.29800	-0.41435	-0.32920	No	Daily
	Δ10-year interest rate differential	d_10y	0.00005	0.03557	-0.25100	0.25100	-0.25609	4.50294	Yes	Daily
	Δ10-year interest rate differential$_{t-1}$	d_i10ylag	0.00005	0.03557	-0.25100	0.25100	-0.25651	4.50509	Yes	Daily
	12-Month Interest Rate Differential	I12M	1.24432	0.78223	-0.38650	4.07450	0.95980	2.19459	No	Daily
	Δ12-month interest rate differential	d_i12m	-0.00021	0.03633	-0.38400	0.32680	-1.59092	21.26885	Yes	Daily
	Δ12-month interest rate differential$_{t-1}$	d_i12mlag	-0.00021	0.03633	-0.38400	0.32680	-1.59080	21.26942	Yes	Daily
	3-Month Interest Rate Differential	I3M	1.30331	0.82960	-0.36400	4.07400	0.62760	1.54169	No	Daily
	Δ3-month interest rate differential	d_i3m	-0.00026	0.04100	-0.57000	0.58200	-0.34591	49.29620	Yes	Daily
	Δ3-month interest rate differential$_{t-1}$	d_i3mlag	-0.00026	0.04100	-0.57000	0.58200	-0.34577	49.29761	Yes	Daily

The correlation matrix in Figure 3.1 displays levels of correlation between the differentiated features presented in Table 3.1.

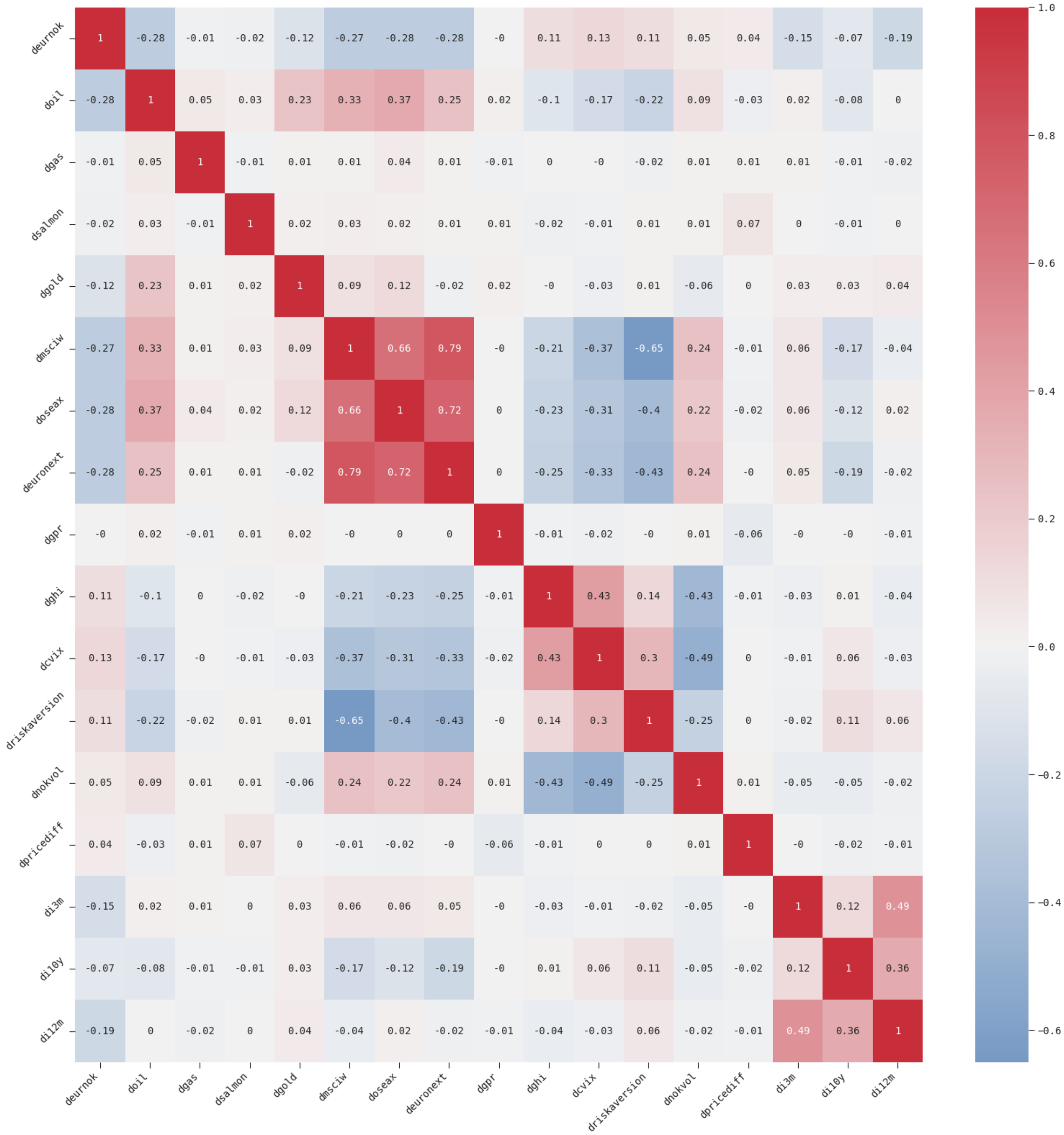
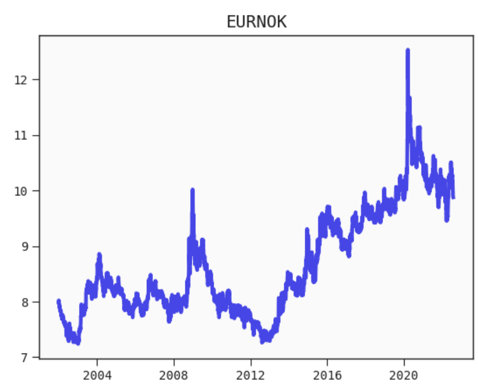


Figure 3.1: Correlation matrix.

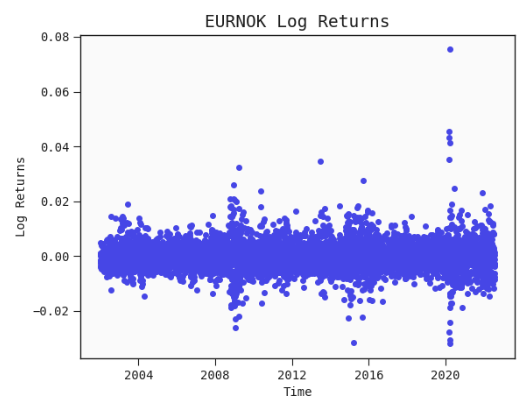
3.2 Dependent variable

EURNOK log returns, shown in Table 3.1 as Δeurnok, is chosen as the dependent variable for the model. An increase in EURNOK returns signifies a depreciation of the krone relative to the euro. Norway trades extensively with countries in the eurozone (EZ), and EZ is considered Norway's most important trading partner (Fagerli, 2023). The euro was launched in 1999 as a common currency used in EZ, and is today used by 20 European countries (European Central Bank, 2023). The EURNOK exchange rate captures movements in the krone, and makes it easy to separate NOK and EUR effects. Since this study it focused on non-linearities and possible relationships between explanatory variables, we constrain the model to a a single currency pair, instead of using weighted variables. This could make it easier to find consistent relationships in the data. Other exchange rates possible for the model include USDNOK. However, given the size of EZ as a trading partner, EURNOK is a suitable dependent variable.

EURNOK returns is also easier to implement and analyse than a trade weighted index, such as Norges Bank's trade weighted exchange rate (TWI), or an exchange rate index, such as Norges Bank's import-weighted krone exchange rate (I-44). TWI consists of Norway's 25 largest trading partners, while I-44 consists of Norway's 44 largest import partners (Norges Bank, 2020b,a). Both indices' weights change every year according to the previous year's trade and imports. Yearly adjustment of all indices and their weights in our model would therefore be required. Examples of such indices which would need yearly adjustment for the model to be correct include interest rate differentials and harmonised index consumer prices (HICP). Employing EURNOK instead of TWI or I-44 allows us to circumvent this challenge.



a EURNOK exchange rate.



b EURNOK log returns.

Figure 3.2: EURNOK exchange rate 1 January 2002 to 1 August 2022.

Source: Data from Norges Bank.

3.3 Explanatory variables

The 17 explanatory variables can be divided into four main categories: commodities, financial assets, financial uncertainty, and macroeconomic fundamentals. A diverse range of variables allows for capturing broad economic effects. This will allow this thesis to explore non-linearities identified in previous published works and identify possible relationships which have not been studied before. Explanation and proposed connections between the variables and the krone is presented in the following sections.

3.3.1 Commodities

The commodity variables include Norway's main exports. In addition, gold is also included as it is often considered an international safe haven asset.

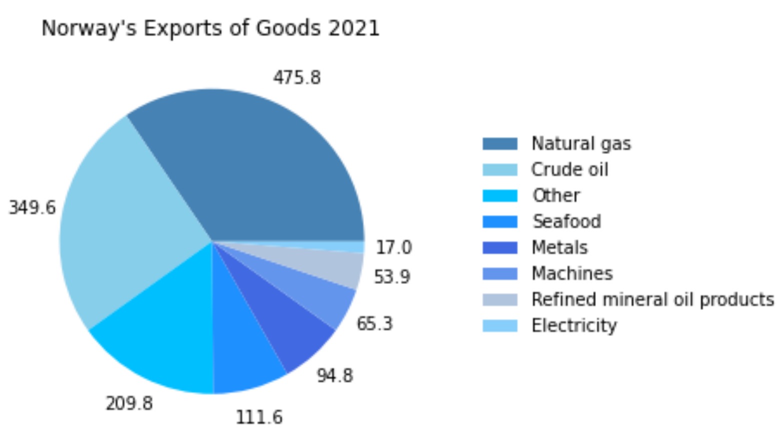


Figure 3.3: Norway's exports of goods 2021. Data are in billion NOK.
Norway's total export of goods in 2021 totalled 1 377 billion NOK.

Source: Data from Gruben et al. (2022)

Norway's main exports are crude oil and natural gas. As seen in Figure 3.3, these account for more than 60% of Norway's total export in 2021. Although Norway is a minor oil supplier internationally, oil accounts for a large part of of the Norwegian economy. Brent crude oil price is therefore included as a variable in the model. Norway is the third largest supplier of natural gas to the global markets, with almost all natural gas exported to the EU and UK (Norwegian Petroleum, 2023). The model therefore includes natural gas as a variable, with the specific natural gas price being the UK NBP Natural Gas Futures.

The oil price shows a strong inverse correlation with EURNOK returns, indicating that increasing oil price causes the krone to appreciate against the euro. This is in line with established literature discussed in Section 2.2.2, which find that there exists a relationship between the krone and oil price. Notably, ter Ellen (2016) identifies that this relationship is non-linear, with the significance of oil price varying from period to period and the existence of threshold values in the oil price. The correlation heat map shows a positive correlation between oil price and financial assets, particularly OSEAX. This is due to increased oil price increasing the value of the major petroleum companies, gas carrier shipping companies, and petroleum subcontractors on the Oslo Stock Exchange.

On the other hand, there is a negative correlation between oil price and volatility, particularly the risk aversion index. Oil price is vulnerable to changes in supply and demand caused by global and geopolitical tensions (Kolaczkowski and White, 2022). While the global demand for oil is more volatile, the European demand for natural gas is more stable due to natural gas being used as a substitute for electricity in large parts of Europe (IEA, 2020). Therefore, natural gas shows no discernible correlation with uncertainty assets.

Seafood is another main export, as seen in Figure 3.3, with salmon alone accounting for 61 billion NOK. In order to minimise the number of variables, only salmon price is included in the model. Salmon price shows no discernible correlation with the other variables.

In total, crude oil, natural gas and salmon account for almost 65% of Norway's total exports, allowing for the model to capture sufficient effects.

Studies have found that investors consider gold a hedge or safe haven asset during financial unrest due to a negative correlation to stock markets (Beckmann et al., 2014). The gold price is therefore included as a variable. The correlation matrix shows a weak correlation between gold price and financial assets for our data set, and strongest correlation with OSEAX. The correlation matrix also shows a positive correlation between oil price and gold price. Shahbaz et al. (2017) found evidence suggesting a frequent positive correlation between oil price and gold price. The gold price is weakly inverse correlated with EURNOK returns, indicating that increasing gold price causes the krone to appreciate. Surprisingly, the gold price shows no discernible correlation with uncertainty assets.

3.3.2 Financial assets

With the krone being a non-dominant currency, studies such as Klovland et al. (2021) find that the krone will depreciate at times of financial unrest, due to investors fleeing to larger and safer markets, as discussed in Section 2.2.2. Stock market indices are included in the model in order to capture financial unrest and forecast the direction of the economy (The Conference Board, 2023). The stock market index Euronext 100 captures trends in the European market, a major trading partner for Norway and a financial superpower, and the global equity index MSCI captures trends in the global economy. The Euronext 100 Index comprises the 100 largest and most liquid stocks traded on European New Exchange Technology (Euronext N.V.), including seven of Norway's major companies. The MSCI World Index is a broad global equity index and comprises 1 509 constituents across 23 developed markets (MSCI, 2022).

The OSEAX index native to Norway is included in order to account for financial effects and trends in Norway. Many of Norway's largest companies are registered on Oslo Stock Exchange, and the OSEAX index comprises all shares listed on Oslo Stock Exchange. As major Norwegian petroleum companies heavily influence the stock exchange's movements due to their major size and impact on the Norwegian economy, there is a strong positive correlation between oil price and OSEAX.

The correlation matrix in Figure 3.1 shows a strong positive correlation between the financial asset variables, as expected. There is also a positive correlation between the MSCI World Index, the Euronext 100 Index, and Brent crude oil price, reflecting that oil prices are heavily affected by the global economic outlook (Kolaczkowski and White, 2022). The correlation matrix shows that financial assets are strongly negatively correlated to CVIX, GHI, and the risk aversion index, which is also as expected due to increasing volatility and unrest decreasing stock market values. There is also a negative correlation between interest rate differentials and financial assets, reflecting that decreasing stock market values lead investors to put more emphasis on longer-term macroeconomic fundamentals such as interest rate differentials.

3.3.3 Financial uncertainty

The GPR Index uses newspaper articles from ten different newspapers to measure geopolitical risk (Caldara and Iacoviello, 2022). The index captures the geopolitical tension as felt by investors, and as such captures subjective effects that reflect investor's sentiments.

GHI is calculated from the implied volatilities of currency options (Brousseau and Scacciavillani, 1999). GHI is an indicator of hazard (risk) in the foreign exchange markets, and reflects the market's expectations of volatility for the underlying exchange rate. GHI comprises of the currency pairs USD/EUR, USD/JPY and EUR/JPY. GHI is included in several exchange rate models, including Naug (2003).

Deutsche Bank's CVIX comprises the volatility of the major G7 currencies (Saravelos, 2007). CVIX is an indicator of volatility in currency markets. Low volatility indicates that moving between currencies is easy, while high volatility indicates that moving between currencies is riskier.

The Bekaert-Engstrom-Xu U.S. Risk Aversion Index (Bekaert et al., 2022) is a measure of risk-aversion over time, which is calculated from observable financial information at high frequencies. High index values indicate high economic uncertainty and high risk aversion. The index is updated at daily and monthly frequencies.

The Norwegian-specific volatility, from Martinsen (2021, p.6), represents the Norwegian krone risk premium relative to major dominant currencies, e.g. the euro or the US dollar, by taking the difference between the Norwegian implied volatility from global implied volatility.

The correlation matrix shows that GHI, CVIX and the risk aversion index are positively correlated and co-moving, as expected, however GPR does not show a correlation with any of the other variables for financial uncertainty, which is surprising. The weak negative correlation between GHI, CVIX, the risk aversion index and oil price, and GHI, CVIX, the risk aversion index and financial assets, indicate that increasing oil prices decrease uncertainty in markets, increasing stock markets. Norwegian specific volatility is strongly negatively correlated with both GHI, CVIX, and the risk aversion index, while GHI, CVIX and the risk aversion index show a weak positive correlation with EURNOK returns. This suggests that an increase in global implied volatility increases the risk premium on the Norwegian krone, depreciating the krone. Norwegian specific volatility also shows a positive correlation with financial assets, indicating that increasing stock prices increase the risk premium on NOK demanded by investors.

3.3.4 Macroeconomic fundamentals

Most exchange rate models include price differential as a variable, as PPP is considered a macroeconomic fundamental and has been studied to a great extent. Price differential between Norway and EZ is therefore included as an explanatory variable. The price differential is the difference in the harmonised index of consumer prices (HICP) between Norway and EZ. The price differential shows weak correlation with other variables in the correlation matrix. As most studies find that PPP does not hold in the short term, a weak correlation between price differential and EURNOK is as expected.

Since UIP has not been entirely discarded by the scientific community, 3-month, 12-month and 10-year interest rate differentials are included in the model. This allows for capturing both long and short-term effects. As discussed in Section 2.1, UIP has shown more promising results in the long term. However, the correlation matrix does not reflect this, with 12-month and 3-month interest rates showing higher inverse correlation with EURNOK returns than 10-year interest rates. Inverse correlation suggests that increasing interest rates cause the krone to appreciate. This is contrary to the theory of UIP, but in line with current practice, as discussed in Section 2.2.2. There is also a negative correlation between financial assets and 10-year interest rate difference, but little discernible correlation between short-term interest rates and other variables.

4 Methodology

Our methodology follows five steps. Section 4.1 explains how feature selected will be performed to reduce the search space of the SR problem. Section 4.2 presents how the SR problem is solved using *PySR*, and how the optimal GPSR model is selected using model selection criterion. Section 4.3 presents the benchmark models and performance metrics which the selected GPSR model is evaluated against.

4.1 Feature selection

Feature selection is the process of removing features from a dataset that may be considered irrelevant or redundant (Guyon and Elisseef, 2003). Feature selection is a necessary step in the process of solving a SR problem, as SR problems can be computationally intensive. Although GPSR is considered to have a built-in feature selection ability, it is not always sufficient. Implementing feature selection in the data pre-processing stage can therefore improve the efficiency when solving SR problems (Chen et al., 2017), as the search space is reduced. The explanatory features presented in Section 3.3 represent a wide range of suspected effects and relationships, and some of the features might be redundant, while others are of significance.

Several feature selection methods can be applied to GPSR problems. Chen et al. (2017) also propose GP with feature selection (GPWFS). This is a two-stage feature seletion method for high-dimensional SR. Other methods include information gain. A study by Yang et al. (2021) use Permutation Feature Importance (PFI) as a feature selection method. However, there is no standard method for feature selection in GPSR, allowing users to choose from a range of feature selection methods. Stijven et al. (2011) compare random forest regression and SR as feature selection methods. The study concludes that random forest is an efficient method, but caution should be extended in the case of spurious variables. SR is also an effective method, but model quality must be verified.

In order to increase the robustness of feature selection, we employ two different methods for feature selection: random forest and LASSO. Both random forest and LASSO are embedded feature selection methods. Appendix E specifies implementation details for both the random forest and LASSO feature selection methods in this thesis.

Random forest is a commonly accepted feature selection method, and therefore chosen for this thesis. An advantage of random forest is that increasing the number of estimators does not lead to overfitting (Breiman, 2001, p.7). On the other hand, allowing the trees to grow to infinite size may cause overfitting. In order to better fit the number of input features and to reduce the impact of randomness, we increase the number of estimators and set a maximum tree depth.

Least Absolute Shrinkage and Selection Operator (LASSO) regression (Tibshirani, 1996) is another commonly accepted feature selection method, and subsequently also chosen for this thesis. LASSO minimises its cost function by means of an $l1$ penalty term causing the method to discard insignificant and redundant features, reducing the number of features to a significant subsample.

It is suspected that the methods will yield similar, but not identical results. The results from both methods must therefore be adhered to. To ensure that feature selection correctly reduces the search space while not simplifying the SR problem, the results from both methods are combined, and the ten most significant features from each method are selected.

4.2 Model generation and selection

The genetic algorithm generates a Pareto frontier of equations with Pareto-optimal complexity and accuracy for the GPSR problem. A model selection criteria is used to select the optimal GPSR model. This section gives an introduction to SR and GP, as well as describing the chosen setup for solving the GPSR problem.

4.2.1 Model generation

Symbolic regression

SR generates expressions that match data from an unknown function (Koza, 1994). For a given data set (X, y) where each point $X_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}$, SR searches over a space of mathematical functions to identify a function $f : \mathbb{R}^n \to \mathbb{R}$ that best fits the data set, where f is a closed-form mathematical expression (Petersen et al., 2019, p.1). Figure 4.1 shows a computational tree encoding SR models using GP (Virgolin and Pissis, 2022).

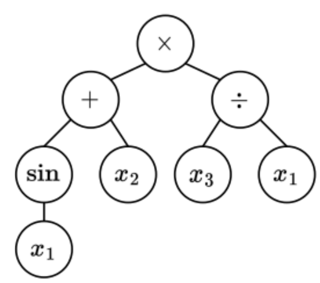


Figure 4.1: Example of a computational tree that encodes $f(x) = (\sin x_1 + x_2) \cdot x_3/x_1$.

Source: Virgolin and Pissis (2022, p.2)

SR is widely used for inferring complex models, due to the ability to recognise data correlations and define interpretable models, as well as high efficiency at identifying and evolving nonlinear relationships (La Malfa et al., 2022). In SR, no equation bias or assumptions on the function form is required, which is useful for data sets where there is little forehand knowledge on the data distribution (Angelis et al., 2023). No assumption of stationarity is therefore either required (Kefalas et al., 2022; Wang et al., 2019; Murari et al., 2019), and non-stationary time series data does not need to be transformed in order to apply SR.

SR models have high interpretability due to the model's algorithmic simplicity (Otte, 2013). This makes SR models highly suitable for analysis and gaining useful insight of the underlying data (Virgolin and Pissis, 2022), and builds trustworthiness and credibility.

SR's most severe drawback stems from computational costs. High data dimensionality poses a challenge for SR algorithms, as search space grows exponentially with dimensionality and operators (Icke and Bongard, 2013). Performing SR is a computationally intensive task, and SR is an NP-hard problem (Virgolin and Pissis, 2022). As SR also suffers from a lack of learning and improvement, with models employing SR not becoming better at solving SR problems over time (Biggio et al., 2021), the computational cost cannot be reduced by training the model on similar data. Furthermore, as users cannot pre-define suitable equations in relation to the nature of the data, due to lack of inductive bias, the computational costs increase further (Cranmer et al., 2020; Biggio et al., 2021). A lack of common benchmarking has also been attributed the failure of widespread adoption of SR methods (La Cava et al., 2021).

However, several studies have aimed to overcome these drawbacks, such as Udrescu and Tegmark (2020), Biggio et al. (2021), Cranmer et al. (2020), and La Cava et al. (2021). Increased user involvement can also be used for overcoming these drawbacks. Reducing the number of input features and possible operators and functions in cases where the user has prior knowledge on the output data or practical applications further allows limiting the search space.

Genetic programming

GP is the most common method for solving SR problems. The use of evolution to create computer programs was first proposed by Alan Turing in 1950 (Turing, 1950). Researchers, including John Koza, have been responsible for developments within the field. Although GP is a relatively new field, researchers have found it to be an efficient way to search a state of possible equations, allowing it to be applied to problems such as SR.

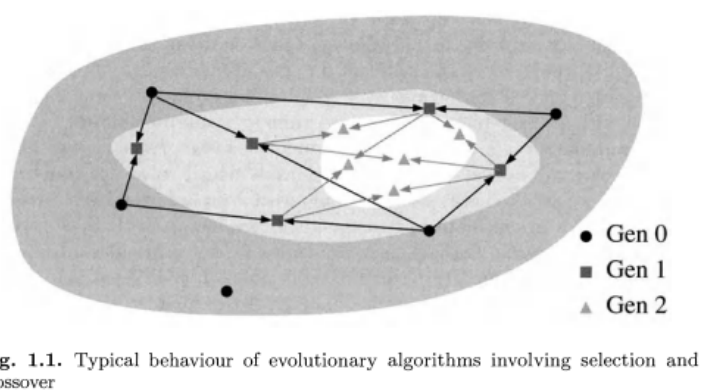


Figure 4.2: GP search behaviour.

Source: Koza (1994, p.3)

GP search behaviour is shown in Figure 4.2. GP starts by defining two sets, the terminal set and the function set. Together, these two sets form the search space that GP explores. The terminal set consists of the program's external inputs, functions with no arguments, and constants. These are used by GP to create programs. The function set consists of functions and terminals that are used to solve the identified problem. Arithmetic functions are commonly used as a function set. In order to evaluate the search space and to identify a good solution, a fitness measure is applied. The fitness measure can be quantified in several ways, such as by measuring the error between the actual output and the desired output. A problem can also be constrained using GP parameters. Parameters include population size, probability of performing the genetic operators, and maximum size for programs. This allows a user to define an optimal solution. The steps of the GP algorithm are shown in Figure 4.3.

1: Randomly create an *initial population* of programs from the available primitives (more on this in Section 2.2).
2: **repeat**
3:    *Execute* each program and ascertain its fitness.
4:    *Select* one or two program(s) from the population with a probability based on fitness to participate in genetic operations (Section 2.3).
5:    Create new individual program(s) by applying *genetic operations* with specified probabilities (Section 2.4).
6: **until** an acceptable solution is found or some other stopping condition is met (e.g., a maximum number of generations is reached).
7: **return** the best-so-far individual.

Algorithm 1.1: Genetic Programming

Figure 4.3: Pseudocode of GP algorithm.

Source: Poli et al. (2008, p.3)

GPSR setup

The GPSR problem is implemented using the Python library *PySR*. Alternative packages solving GPSR problems have been developed, but *PySR* is selected as it is efficent, open-source, and has a configurable Python interface. Usage of *PySR* is based on Cranmer (2020). Computing specifications can be found in Appendix F.

The time series data is split into two parts: training and testing. As shown in Figure 4.4, the first 80% of the data is assigned to model training and the remaining 20% to model testing.

Figure 4.4: Train-test split.

PySR uses various genetic mechanisms to define the evolutionary process. The mechanisms follow a typical tournament selection genetic algorithm customised to work with equations in tree format. A detailed explanation of the genetic mechanisms relevant to *PySR* are explained in Appendix D. The chosen configuration for the GPSR problem is shown in Table 4.1. The chosen loss function is mean squared error (MSE).

Parameter	**Value**
Populations	50
Population size	75
Tournament selection size	23
Mutation cycles per iteration	100
Loss function	MSE

Table 4.1: Hyperparameter specifications of the genetic algorithm.

GPSR is initialised with a set of operators that can be chosen by the program in order to discover a best solution. The initial set of operators available for the program to choose from, with associated maximum complexity and nesting constraints, are listed in Appendix C. *PySR* allows users to define a set of operators, which includes both standard and custom operators.

An advantage of choosing a range of operators is the possibility to explore non-linear properties that are of interest to this thesis. However, the use of operators must be constrained for two main reasons. The first is to ensure that the generated equations remain directly interpretable. With complicated nestings of many variables, interpretability can be lost. The second reason is to restrict over-fitting through complex factorisation. Additionally, restricting operator use reduces the search space of the SR problem. Similarly to the challenge with feature selection in Section 4.1, each additional operator exponentially increases the search space of the SR problem.

4.2.2 Model selection

Model selection is performed after generating the Pareto frontier consisting of optimal equations using MSE as the loss function. The Bayesian Information Criteria (BIC) is chosen as the model selection criterion. The generated GPSR model on the Pareto frontier with the lowest in-sample BIC is selected.

Several model selection techniques can be applied in GP to promote generalisation and control complexity. Structural Risk Minimisation (SRM) methods are shown to perform efficiently, but may be difficult to apply to standard GP-based tree problems, such as in this thesis. Traditional model selection techniques such as Akaike Information Criterion (AIC) and BIC perform sufficiently well for GP-based tree problems (Le et al., 2016). AIC is more suitable for large data sets, and typically chooses a more complex model, but with higher prediction accuracy. BIC more strictly penalises increasing features, and therefore typically chooses a more parsimonious model but with lower prediction accuracy (Kuha, 2004). As our data set is relatively small, and economic interpretability is essential for the selected GPSR model, BIC is chosen for this thesis.

4.3 Model evaluation

The selected GPSR model's performance is compared to different benchmarking models using performance metrics. The deviance between actual returns and predicted returns is examined. The cumulative predicted returns are also studied. This allows for identifying forecast biasness present in the GPSR model.

Benchmarking is performed with respect to models that are of economic interest, ML interest, and previous models developed for the Norwegian krone. Table 4.2 shows a list of all benchmark models with both complexity and symbolic form.

Machine-learning (ML) benchmark model

Showing that GPSR produces comparable results whilst being more interpretable than other ML methods, indicates the advantages of GPSR as a method, and shows robustness of methodology. The chosen ML method is a random forest regression.

Random forest is an ML method, and is commonly applied in finance for financial time series forecasting[8]. Other alternative ML methods include LSTM, which is often used in finance, however LSTM might underperform when the data is highly non-linear. Furthermore, training LSTMs is difficult and time consuming (Pascanu et al., 2013). Random forest is less time consuming to implement correctly, and is therefore chosen for the ML benchmark.

Linear benchmark models

The GPSR model is evaluated against fundamental models from economic and exchange rate theory. These include the random walk prediction of no change, and three multivariate linear models consisting of specific features.

The three multivariate linear models are created based on (1) all features listed as input in Table 3.1, (2) the features selected from feature selection, and (3) the features selected by the GPSR algorithm. The models therefore have decreasing complexity due to a decreasing number of input features. These models allow for evaluating the features selected in the different steps of the methodology.

Published multivariate linear models

The GPSR model is also benchmarked against three published multivariate linear models, as described in Section 2.2.1.

The multivariate linear regression models from literature in Table 2.1 have different sampling frequencies, making benchmarking difficult. Akram (2020), Martinsen (2021), and Klovland et al. (2021) have a monthly frequency. A re-estimation method involving re-estimating the coefficients of the GPSR model is therefore used. The re-estimated model has monthly frequency.

[8] IBM (n.d.)

Table 4.2: Benchmark models that are compared to the GPSR model. N is the number of features presented in Table 3.1. n is the number of selected features found using feature selection. m is the number of features in the selected GPSR model.

	Benchmark models		
	Benchmark model	Complexity	Symbolic form (Δeurnok_t)
ML	Random forest	N/A	N/A
Linear models	Random walk	0	0
	Linear multivariate regression model of all explanatory variables[9]	$4N-1$	$\boldsymbol{\beta_N} \cdot \boldsymbol{x_{Nt}}$
	Linear multivariate regression model of selected features[10]	$4n-1$	$\boldsymbol{\beta_n} \cdot \boldsymbol{x_{nt}}$
	Linear multivariate regression model of features in selected GPSR model[11]	$4m-1$	$\boldsymbol{\beta_m} \cdot \boldsymbol{x_{mt}}$
Published monthly models	Akram (2020)[12]	29	$\alpha + c_1\Delta\text{i12m} + c_2\Delta\text{i10y} + c_3\Delta\text{gpr} + c_4\Delta\text{fxv-em} + c_5\Delta\text{oil\_demand} + c_6\Delta\text{oil\_supply} + c_7\Delta\text{oil\_residual}$
	Martinsen (2021)	17	$\alpha + c_1\Delta\text{i12m} + c_2\Delta\text{i10y} + c_3\Delta\text{oil} + c_4\Delta\text{nokvol}$
	Klovland et al. (2021)	39	$\alpha_1 + c_1\Delta\text{eurnok}_{t-1} + c_2\Delta\text{i12m} + c_3\Delta\text{oil\_price} + c_4\Big(\text{eurnok}_{t-1} - (\alpha_2 + c_5\text{i12m} + c_6\text{pricediff} - c_7\text{oil} + c_8\text{cvix} + c_9\text{sp500})_{t-1}\Big)$

[9]In Table 3.1.
[10]From feature selection.
[11]From model interpretation.
[12]Akram (2020) uses decomposed oil price, from New York Fed (n.d.), and Barclays' emerging economies currency volatility index (FXV-EM).

4.4 Adjusting the GPSR model for Structural Shifts

As discussed in Section 2.2.3, the krone has been persistently weak since the 2014-2016 oil price drop. This indicates that structural shifts exist, which can change the nature of EURNOK over time. To understand these changes, we identify the most significant structural shifts and adjust the selected GPSR model accordingly.

The Chow test (Chow, 1960) is used to identify statistically significant breakpoints. The breakpoints are used to split the data into subsamples. On the largest subsample, the same procedure is repeated until we identify the three most significant breakpoints, resulting in four subsamples. Restricting the model to a maximum of four subsamples avoids overfitting.

The coefficients for the GPSR model are re-estimated for the subsamples. The resulting piecewise regression model can be studied for determining time-varying characteristics of the exchange rate.

5 Results and Discussion

5.1 Feature selection

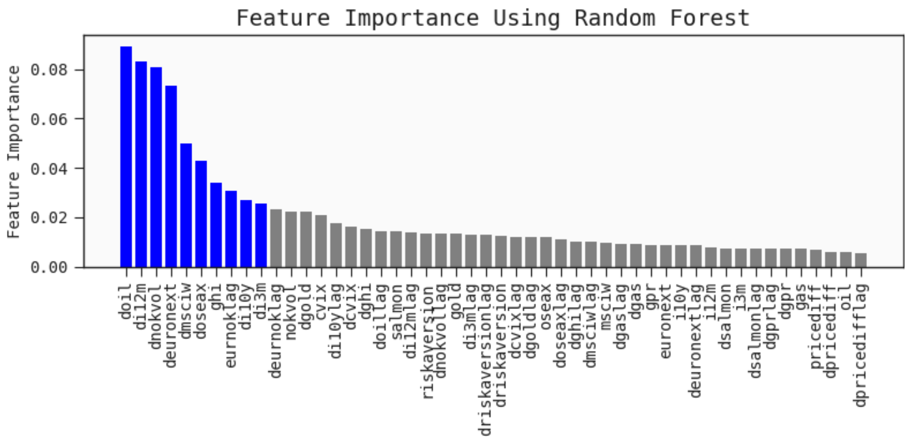
As described in Section 4.1, random forest regression and LASSO regression is used to select the most significant features from the initial set of explanatory variables. It is important to note that due to randomness, the selected features are not guaranteed to be the same in each random forest regression. Figure 5.1 illustrates the importance of each feature for both feature selection methods. Table 5.1 lists the 10 most important features for each method. Six of the features are selected by both methods, resulting in a total of 14 selected features.

Table 5.1: 10 most significant features found by random forest and LASSO regression in prioritised order. The six features marked with an asterisk (*) are selected by both methods, yielding 14 features selected for the GPSR algorithm.

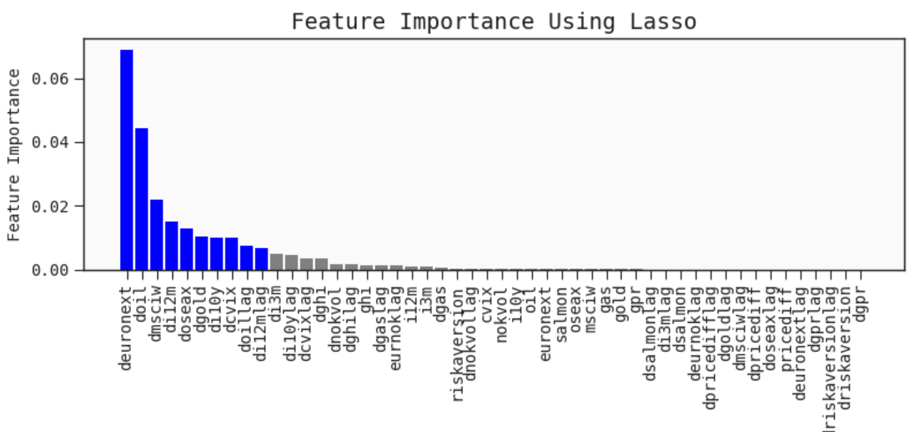
Selected Features	
Random Forest Regression	LASSO Regression
Brent crude oil price returns*	Euronext 100 Index returns*
Change in 12-month interest rates*	Brent crude oil price returns*
Norwegian specific volatility returns	MSCI World Index returns*
Euronext 100 Index returns*	Change in 12-month interest rates*
MSCI World Index returns*	OSEAX returns*
OSEAX returns*	Gold price returns
GHI	Change in 10-year interest rates*
Lagged EURNOK	Change in CVIX
Change in 10-year interest rates*	Lagged Brent crude oil price returns
Change in 3-month interest rates	Lagged change in 12-month interest rates

The features selected by both models are all in line with expectations. Brent crude oil price, interest rate differentials, and equity market returns are well established in literature and included in most existing models.

Random forest also selects lagged EURNOK and GHI, which are non-differentiated variables. This could be because ML methods better utilise nominal data. LASSO regression selects some lagged iterations of interest rates and Brent crude oil price, suggesting that there could be autoregressive characteristics in the exchange rate. The different characteristics of features selected by the two methods illustrate the importance of evaluating different feature selection methods.



a Random forest.



b LASSO regression.

Figure 5.1: Feature selection using random forest and LASSO regression.

Correlation between features is not taken into account by the random forest, but tends to be accounted for by LASSO regression. Both feature selection methods choose all of the highly correlated financial assets: MSCI World Index returns, OSEAX returns, and Euronext 100 Index returns. The correlation between these features were shown in Figure 3.1 and explained in Section 3.3.2. Significantly correlated input features may not contribute to improving a model, as they are more likely to capture the same effects, thus not contributing additional information. The inclusion of all three indices divide the financial assets effects into global, European, and Norwegian segments, which may lead to more detailed analysis pertaining to economic areas when analysing the final GPSR model. This might yield new insight in to what extent global markets affect the krone.

The selection methods select several uncertainty features. Norwegian specific volatility returns is found to be significant using random forest. Random forest also selects GHI as a significant feature. LASSO regression finds CVIX returns to be significant. The Norwegian specific volatility is strongly negatively correlated with CVIX. This correlation suggests that the krone might be considered a safe haven, with the krone risk premium decreasing with increasing currency volatility.

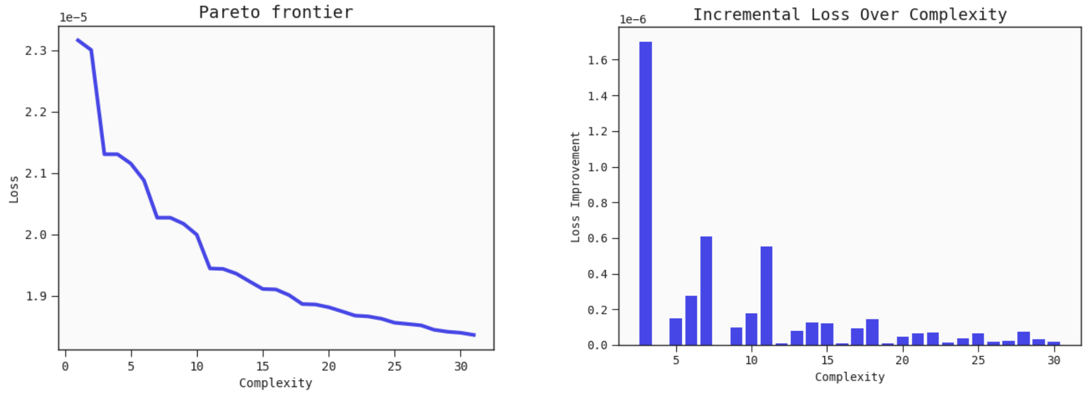
Both feature selection methods find that 10-year interest rate differential returns, 12-month interest rate differential returns, and 3-month interest rate differential returns are significant. LASSO regression also finds lagged 12-month interest rate differential returns to be significant. This is consistent with recent literature, as described in Section 2.2.2. Considering the ranking of the interest rate differential's significance, this indicates that 12-month interest rate most closely relates to the future expectations of the economic conditions in Norway. 3-month interest rates might be susceptible to short-term fluctuations that are not necessarily related to daily fluctuations in the Norwegian Krone, while 10-year interest rates might be inflexible in the short term, becoming therefore less relevant for daily predictions. Additionally, this could suggest that 12-month interest rates are the most representative of the central banks' monetary policy stances. LASSO regression also selects lagged 12-month interest rates as a significant feature.

Neither feature selection method find that price differential is significant. While there is much evidence supporting PPP in the long run, short term PPP has less support, as discussed in Section 2.1. Furthermore, price differential is supplied on a monthly basis, with the model calculating daily returns. This might make price differential superfluous and insignificant for modelling daily returns.

5.2 Model generation and selection

5.2.1 Model generation

The output generated by running the GPSR model is 30 equations with increasing complexity. The equations are shown in Appendix G with corresponding complexity and loss. An analysis of recurring features across generated models is also supplied in Appendix H. Figure 5.2a shows the Pareto frontier with decreasing loss as complexity increases. Figure 5.2b shows that loss improvement diminishes exponentially as complexity increases.



a Best loss (MSE) per complexity. b Incremental MSE improvements per complexity.

Figure 5.2: Pareto frontier of symbolic regression.

Figure 5.2b also shows critical points at complexities 3, 7 and 11, with significantly large loss improvements. These critical points correspond to new variables being added the equations, namely Δeuronext, Δoil, and Δi12m. All three equations consist only of linear components. The pattern of large improvements every four complexity stems from the fact that adding a new linear variable requires increasing the equation complexity by four. A more detailed explanation of complexity is found in Appendix D.

Some non-linear combinations are also consistently seen across the frontier. Multiplying the Euronext 100 Index and the lagged EURNOK is first seen at complexity 4. This is also performed on Brent crude oil price, starting at complexity 20. At greater complexities the Euronext 100 Index is also cubed. In the range from 22-30, there are also various non-linear terms such as $\Delta\text{gold} \cdot \Delta\text{cvix}$, $(\Delta\text{i12m})^3 \cdot \text{GHI}$, and $(\Delta\text{cvix})^3$.

5.2.2 Model selection

The Pareto frontier and generated models discussed in Section 5.2.1 illustrate the trade-off between complexity and accuracy. BIC is used to select an equation on the frontier with an optimal balance between complexity and accuracy. The model with the lowest in-sample BIC is selected.

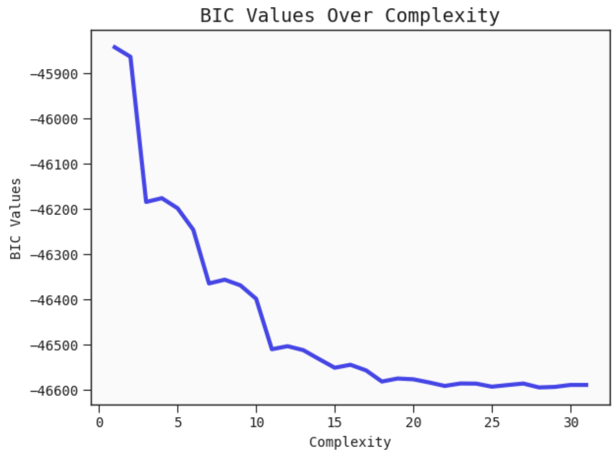


Figure 5.3: Loss in validation sample for different complexity values.

Figure 5.3 shows the BIC values for each equation complexity. The BIC values decrease with increasing equation complexity. A threshold value is found at a global minimum of equation complexity 28. After this complexity, BIC values begin to increase with equation complexity. This indicates that the equation with complexity 28 is the optimal solution for the GPSR problem.

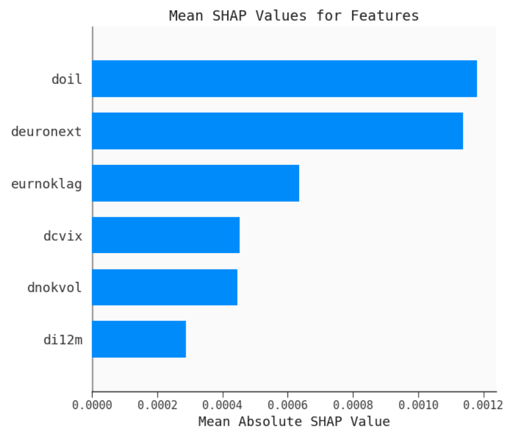
5.3 Model interpretation

This section presents a qualitative interpretation of the selected GPSR model, consisting of reasoning associated with the features and the relationships between the model components. GPSR contrary to ML approaches allows for interpreting and explaining model features, providing increased understanding of features affecting the krone behaviour.

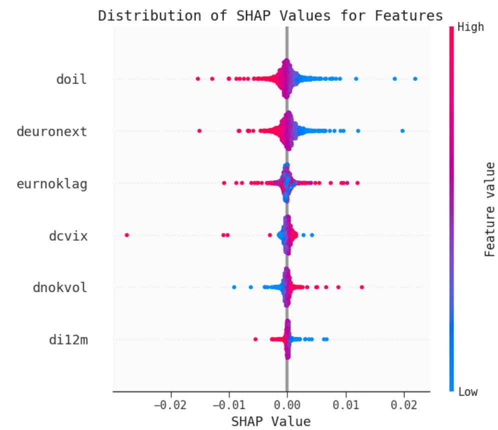
The selected equation, shown in Equation 3, has a complexity of 28, and is composed of three linear and three non-linear terms. A negative prediction indicates the krone appreciating against the euro, whilst a positive prediction indicates a depreciation.

$$\begin{aligned}\Delta\text{eurnok} = {} & (0.0016730422520128164\ \Delta\text{nokvol}) \\ & + (-0.022186430754550842\ \Delta\text{i12m}) \\ & + (0.02415359996911089\ \Delta\text{cvix}) - (\Delta\text{cvix})^3 \\ & + (-7.807286138088565 \cdot 10^{-5}\ \Delta\text{oil} \cdot\ (\text{EURNOK}_{t-1})^3) \\ & + (-0.00016478763632213396\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)\end{aligned} \tag{3}$$

The interpretation is supported by a SHapley Additive exPlanations (SHAP) analysis (Lundberg and Lee, 2017) providing insight into the quantitative impact of the features. SHAP analysis is widely used in ML to determine how each player in a coalition contribute to the gained payoff. This is useful in our GPSR model for inferring how the different terms are contributing to the model predictions. The results of the SHAP analysis are shown in Figure 5.4.



a Average impact on model output for each feature.



b Distribution of features impact on model output.

Figure 5.4: SHAP values of the selected GPSR model.

Figure 5.4a shows that the most important feature is Brent crude oil price returns, closely followed by Euronext 100 Index returns.

An important part of a SHAP analysis that provides further insight into individual feature contribution is a SHAP beeswarm graph, as depicted in Figure 5.4b. For each feature, a point on the graph is an observation indicating the impact of the feature on the prediction. The graph allows for comparing magnitude and directionality of the Shapley values for each feature. SHAPLEY values for each feature, including spread of values, can be found in Appendix L and Appendix M.

5.3.1 Change in Norwegian specific volatility

The change in Norwegian specific volatility is included as a linear term with a positive weight in Equation 3. When Norwegian-specific volatility rises, it is expected that the krone will depreciate or appreciate proportionally to the change. This is also shown in Figure 5.4b, which shows that the change in the Norwegian specific volatility has a larger positive impact on model predictions. Feature value is generally higher for each positive SHAP value observation. Figure 5.4b also shows that the feature has a large spread, indicating that the feature has high variability.

This relationship is supported by literature discussed in Section 2.1, which states that an increase in the volatility differential leads to investors demanding a higher krone risk premium compared to other global currencies. The risk premium will typically increase as expected future exchange rates decrease, for example due to national political uncertainty, increased expectations of inflation, or decreasing interest rate expectations (Engel, 2016). An increasing risk premium indicates that the krone is considered a more risky currency, and risk-averse foreign investors would rather invest in safer and larger currencies. This leads to the krone depreciating against the euro, a safer currency.

5.3.2 Change in 12-month interest rate differential

The change in the 12-month interest rate differential is included as a linear term with a negative weight. This indicates that the krone is expected to appreciate proportionally to positive changes and depreciate with negative changes.

An increasing interest rate differential signifies that the Norwegian interest rate is higher and increases more rapidly in comparison to the European interest rate. In accordance with UIP theory, the krone would be expected to depreciate following an increase in exchange rate and interest rate, in order for there to not be arbitrage. However, as discussed in Section 2.2.2, in practice it is found that increasing interest rates cause a currency to appreciate due to increased foreign investment and demand for the currency[13]. Existing exchange rate models[14] include the interest rate differential with negative weights, supporting this view. This can also be seen in the krone, as Norwegian economists have used a decreasing interest rate differential and a historically weak krone to argue for increasing the policy rate (Noem, 2023). Equation 3 supports this relationship, as increasing change in the 12-month interest rate differential leads to an appreciation of the krone.

5.3.3 Change in cvix

Equation 3 includes change in CVIX as both a linear and a non-linear term. The linear term has a positive weight suggesting that the krone depreciates with positive changes. However, the non-linear term has a negative weight, indicating that the krone appreciates with positive returns.

The inclusion of CVIX returns as both a linear and non-linear term and their relationship could indicate a threshold relationship. When changes to the CVIX are small, the linear term outweighs the non-linear term. However, when CVIX returns are significantly large, the non-linear term outweighs the linear term. This results in the krone depreciating with small increases in currency volatility but appreciating with significantly large increases in currency volatility. The appreciation of the krone in times of high CVIX returns is captured in Figure 5.4b. As can be seen, points with high CVIX returns have a strong negative SHAP value. These points most likely correspond to identified volatile events, including GFC and Covid-19 pandemic.

Although this could support the "safe haven" theory discussed by Benedictow and Hammersland (2022), the model still predicts overall depreciation in times of significant volatility, such as the GFC and Covid-19. Another possible explanation for the cubed CVIX term is that it corrects for volatility effects such as volatility spillover. Volatility spillover is a contagion effect, and occurs when volatility shocks from one market transmit to other markets. During extreme global events, volatility might already be accounted for in the model's other features, for example the Brent crude oil price and Euronext 100 terms. If not for the corrective term, this could cause the model to

[13]HSBC Bank (2020)
[14]Klovland et al. (2021); Akram (2020); Martinsen (2021)

overestimate the importance of volatility in times of significant uncertainty.

5.3.4 Brent crude oil price returns and lagged EURNOK

Equation 3 includes a non-linear term that multiplies Brent crude oil price returns and cubed lagged EURNOK. The effect of the features individually and the effect of the term as a whole provide interesting economic interpretation.

As seen in Figure 5.4a, Brent crude oil price returns makes the largest contribution to model prediction. This is unsurprising, as many economists find evidence supporting the relationship between the krone and Brent crude oil price, such as ter Ellen (2016); Akram (2020). Equation 3 provides support for a positive relationship between Bent crude oil price returns and the krone. Increasing Brent crude oil price returns will lead to an appreciation of the krone. The plotted directionality impact in Figure 5.4b also shows that higher value of Brent crude oil price returns causes a lower prediction, and subsequently a predicted appreciation of the krone. The extreme outliers seen for Brent crude oil price returns likely coincide with the 2014-2016 oil price shock, and the rapid decline in oil prices following Covid-19 fears and lockdown, as well as the Russia-Saudi Arabia oil price war.

The non-linear relationship between Brent crude oil price and lagged EURNOK indicate that changes to the oil price have a greater impact on model predictions when the exchange rate is high and the krone is weak. This relationship between Brent crude oil price returns and the nominal exchange rate can in part be explained by effects arising from Brent crude oil price being denoted USD, but Norwegian economy income being denoted NOK. A price increase has a greater impact on the NOK value of oil revenues when the krone is weak, as each dollar of revenue is converted to a greater amount of NOK. The nature of the non-linear relationship indicates that the importance of this effect increases rapidly as the exchange rate weakens, creating a threshold effect. In Figure 5.4b this can be seen by the fact that Brent crude oil price returns' contribution to the model's predictions is low, except for when the feature value is high.

5.3.5 Euronext 100 Index and lagged eurnok

Euronext 100 Index returns is identified as the second most important feature in the SHAP analysis. Few economist have studied the relationship between the krone and European stocks, and more often point to oil price or interest rate differential as the main features of the krone development, as discussed in Section 2.2.1.

Equation 3 finds that increasing European stock prices appreciate the krone. This further supports the theory that the krone is affected by global financial markets and unrest, following the krone being a small and non-dominant currency (Aamodt, 2010). However, similarly to the Norwegian specific volatility returns term, this also opposes Benedictow and Hammersland (2022)'s theory of the krone being a "safe haven" during financal unrest. The model finds that a decrease in the Euronext 100 Index returns causes the krone to depreciate, as investors flee to larger currencies.

Similar to Brent crude oil price returns, Equation 3 identifies a non-linear relationship between

European financial markets and the krone. This relationship indicates that changes in European stock prices are more important when EURNOK is high. When the krone is weak, the krone appears to be more susceptible to changes in stock price. This might indicate that investors are more risk-seeking and willing to invest in the Norwegian market and the weak krone when global stock prices increase, but will flee sooner to more dominant currencies and markets following volatility. This is further substantiated by Figure 5.4b, which shows that lagged EURNOK only has an impact on the model when its value is high.

The Euronext 100 Index's large impact likely arises from its relevance to the euro, as well as its global reach. The Euronext 100 Index represents some of the largest companies in the world, allowing it to capture general stock market movements, global effects and unrest. These characteristics may explain why the model does not choose the MSCI World Index or the OSEAX index. Furthermore, when evaluating the model as a whole, Figure 3.1 shows there is strong correlation between Brent crude oil price and Norwegian stocks. Although OSEAX was found to be significant after feature selection, Brent crude oil price might sufficiently capture the same effects observed in OSEAX.

The non-linear relationship between Euronext 100 and lagged EURONOK can also be viewed in the context of the Government Pension Fund Global (the Oil Fund) and with a similar argument as for the oil revenues. The values in the fund are denoted in foreign currencies such as USD and EUR. When the krone is weak, a change to the underlying value has a greater impact on the NOK value of the fund than when the krone is strong. The value of the fund can impact the NOK exchange rate through the fiscal spending rule.

5.4 Model evaluation

5.4.1 Evaluation metrics

Table 5.2 and Table 5.3 display the performance of the selected GPSR model and the relevant benchmark models. The re-estimated monthly GPSR model can be found in Appendix J.

For daily models, the random forest regression outperforms the other models for all performance metrics in-sample. The selected GPSR model is the second-best model, however interestingly only slightly better than the linear regression model of selected features.

Out-of-sample, the selected GPSR model outperforms all benchmark models in terms of performance metrics, except the direction of change. This indicates that the GPSR model strikes a good balance between complexity and training fit, in line with our findings using BIC in Section 5.2.2. The results demonstrate GPSR's competitive power, and supports GPSR's validity and efficiency as an alternative to linear regression or ML models.

While the random forest regression severely reduces its performance out-of-sample, indicating overfitting, the random forest regression demonstrates superior performance in predicting the correct direction out-of-sample. This indicates that while the model might err on prediction size, the predicted direction is mostly correct. The non-linear models outperform the linear models out-of-sample, indicating that capturing nonlinearities is crucial for models analysing the krone behaviour. However, the linear models demonstrate accuracy in predicting direction of change, similarly to the random forest regression.

For monthly models, the re-estimated GPSR model outperforms published multivariate linear models for all performance metrics except Akram (2020). Out-of-sample, the re-estimated GPSR model demonstrates superior performance for all performance metrics. This further supports that the GPSR methodology decreases the risk of overfitting, and outperforms multivariate linear regression models which do not capture non-linear relationships. The re-estimated GPSR model's particularly high performance predicting correct direction of change is notable.

In order to evaluate the robustness of the model across different estimation periods, a four-fold time series cross validation in the training data is also estimated in Appendix N. The re-estimated GPSR model outperforms the alternative benchmarks, indicating that the superior out-of-sample performance is not specific to the out-of-sample time period.

Table 5.2: Evaluation of model and benchmark performances in-sample (train).

Model evaluation in-sample						
		Evaluation metrics				
Daily models	Complexity	R^2	MSE	RMSE	MAE	Direction of change
Selected GPSR model	28	0.20366	0.00002	0.00430	0.00317	0.61453
Random forest	N/A	**0.56080**	**0.00001**	**0.00319**	**0.00249**	**0.71136**
Random walk	0	-0.00007	0.00002	0.00481	0.00346	0.00047
Linear multivariate regression model of all explanatory variables[15]	199	0.21564	0.00002	0.00426	0.00315	0.62966
Linear multivariate regression model of selected features[16]	39	0.19729	0.00002	0.00431	0.00317	0.62547
Linear multivariate regression model of features in selected GPSR model[17]	23	0.17999	0.00002	0.00436	0.00322	0.60894
		Evaluation metrics				
Monthly models	Complexity	R^2	MSE	RMSE	MAE	Direction of change
Re-estimated monthly GPSR model	32	**0.576396**	**0.000194**	**0.013911**	**0.010880**	0.687943
Akram (2020)	29	0.504142	0.000228	0.015104	0.011546	**0.714286**
Martinsen (2021)	17	0.548067	0.000208	0.014419	0.011094	0.678571
Klovland et al. (2021)	39	0.550442	0.000207	0.014398	0.011002	0.712230

[15] In Table 3.1.

[16] From feature selection, Section 5.1. The selected features are: lagged EURNOK, Brent crude oil price returns, lagged Brent crude oil price returns, gold price returns, MSCI World Index returns, OSEAX returns, Euronext 100 Index returns, GHI, CVIX returns, Norwegian specific volatility returns, 10-year interest rate differential returns, 12-month interest rate differential returns, lagged 12-month interest rate differential returns, and 3-month interest rate differential returns.

[17] From model interpretation, (3). The selected features are: 12-month interest rate differential returns, Norwegian specific volatility returns, CVIX returns, Brent crude oil price returns, Euronext 100 Index returns, and lagged EURNOK.

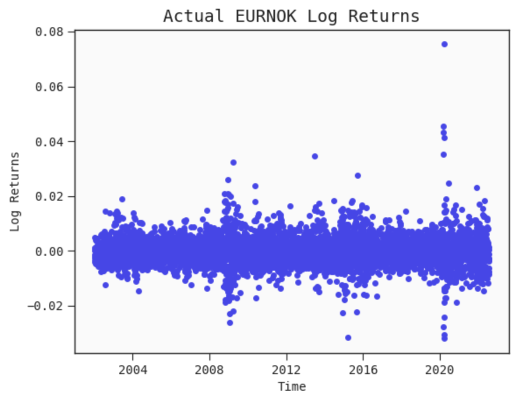
Table 5.3: Evaluation of model and benchmark performances out-of-sample (test).

Model evaluation out-of-sample						
		Evaluation metrics				
Daily models	Complexity	R^2_{OOS} [18]	MSE	RMSE	MAE	Direction of change
Selected GPSR model	28	**0.40315**	**0.00002**	**0.00496**	**0.00332**	0.67877
Random forest	N/A	0.32145	0.00003	0.00529	0.00344	**0.68343**
Random walk	0	0.00000	0.00004	0.00642	0.00412	0.00000
Linear multivariate regression model of all explanatory variables [19]	199	0.36181	0.00003	0.00513	0.00341	0.66201
Linear multivariate regression model of selected features[20]	39	0.35860	0.00003	0.00514	0.00342	0.67784
Linear multivariate regression model of features in selected GPSR model[21]	23	0.35190	0.00003	0.00517	0.00344	0.67877
		Evaluation metrics				
Monthly models	Complexity	R^2_{OOS}	MSE	RMSE	MAE	Direction of change
Re-estimated monthly GPSR model	32	**0.763774**	**0.000218**	**0.014755**	**0.011186**	**0.805556**
Akram (2020)	29	0.511089	0.000464	0.021541	0.016431	0.628571
Martinsen (2021)	17	0.746400	0.000241	0.015514	0.011657	0.742857
Klovland et al. (2021)	39	0.694965	0.000297	0.017220	0.013397	0.705882

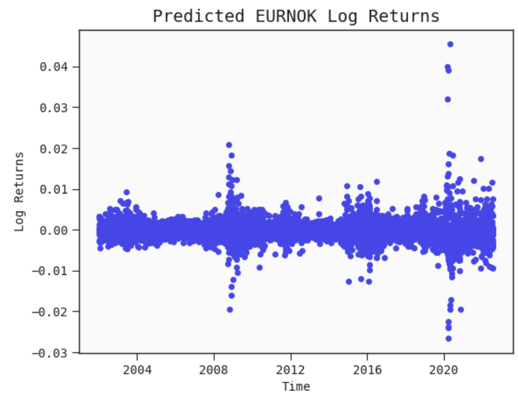
[18] R^2, as proposed by Campbell and Thompson (2008) for evaluating models out-of-sample, with random walk as benchmark.
[19] In Table 3.1.
[20] From feature selection, Section 5.1.
[21] From model interpretation, (3).

5.4.2 Comparison of actual returns and predicted returns

The actual EURNOK returns and the GPSR model's predicted EURNOK returns are shown in Figure 5.5. The predictions are heavily concentrated around the mean prediction of 0.00, while the actual returns have a larger spread. The GPSR model identifies structural shifts and accounts for periods of extreme volatility, such as GFC and Covid-19. However, the model underestimates the magnitude of highly volatile returns, predicting less spread returns. This decreased spread might arise from the model being trained on MSE as the loss function. The model is more severely penalised for larger errors than smaller errors.



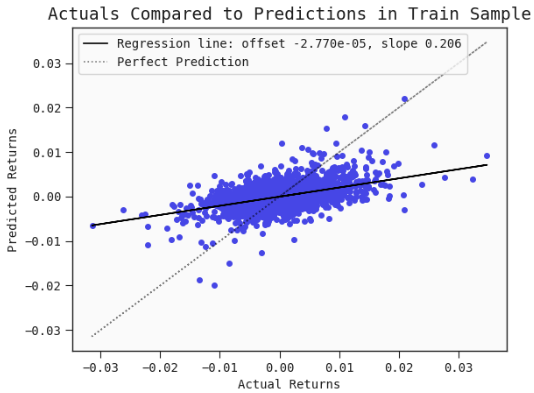
a Actual EURNOK returns 2002 to 2021.



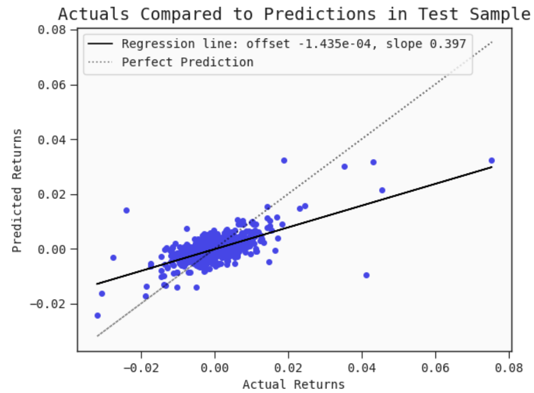
b Predicted EURNOK returns 2002 to 2021.

Figure 5.5: Comparison of actual and predicted EURNOK returns over time.

Figure 5.6 illustrates that the GPSR model's predictions deviate from the actual returns. Figure 5.6b is closer in resemblance to a perfect plot than Figure 5.6a, indicating overfitting in-sample. However, both figures show that the model makes less accurate predictions for larger returns, and tends to underestimate the daily returns, particularly outliers. This negative bias is also supported by the analysis of the residuals in Appendix I. The residuals show a slight positive mean, indicating a negative bias is present in the selected GPSR model.



a Training (in-sample).



b Testing (out-of-sample).

Figure 5.6: Comparison of actual and predicted EURNOK returns in training (in-sample) and testing (out-of-sample) samples.

Figure 5.7 further supports that the selected GPSR model is negatively biased. An increase in the cumulative sums corresponds to a depreciation of the krone against the euro. Although the GPSR model performs well at explaining the general trend of the EURNOK exchange rate until 2013, after that it accumulates a large prediction error, failing to explain the significant depreciation of the krone in the period from 2013 to 2022. This suggests that the model has difficulty capturing economic or political factors that might have contributed to the unexpected changes in the exchange rate during this period. Klovland et al. (2021) argue that an increase in the price difference between Norway and other countries is the main reason for the significant weakening between 2012 to 2015. Other economists offer other possible explanations, as discussed in Section 2.2.3.

Despite this limitation, the selected GPSR model appears to accurately explain a weakening krone during times of uncertainty. This might suggest that the model is better suited for capturing short-term fluctuations in the exchange rate that are driven by specific shocks, rather than long-term trends that are influenced by broader economic or political factors. This limitation relates to the model's daily frequency, for which broader trends can be difficult to capture.

The GPSR model's difficulty in explaining long-term trends might arise from not adequately accounting for structural shifts in market sentiment that have occurred from 2013 to 2022. Another reason for the negative bias might be attributed to effects not reflected in the model's features. The model might not capture all relevant economic or political factors that have influenced the exchange rate during this period, causing the model's predictions to deviate from the actual values. Structural changes in the Norwegian economy have been described by researchers including Akram (2020) and Klovland et al. (2021). Figure 5.7 supports these possible explanations, as following 2013 the GPSR model captures the movement of the krone, but is unable to capture the general depreciation. This results in the GPSR model becoming increasingly erroneous, and with time the model will become superfluous. This trend of general depreciation identified in the selected GPSR model allows for a study of possible structural shifts in the krone, as will be discussed in Section 5.5. In the next section, the model is adjusted for structural shifts.

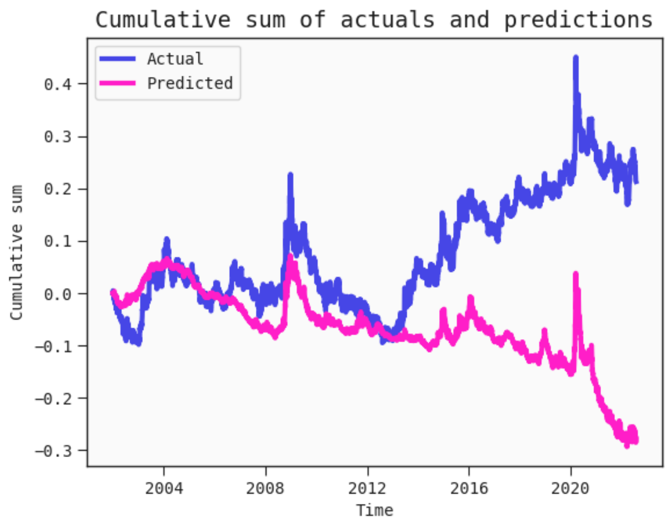


Figure 5.7: Cumulative sum of predicted and actual EURNOK returns.

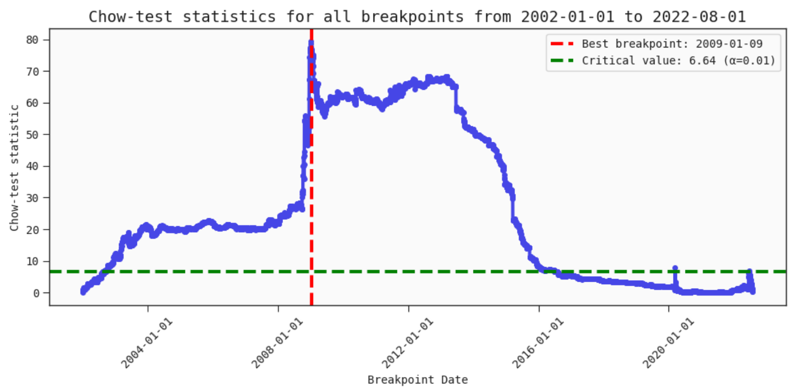
5.5 Adjusting the GPSR model for Structural Shifts

This section studies structural shifts, as identified in Section 5.4.2, and suggested to exist by economists due to the unexplained depreciation of the krone (Klovland et al., 2021). Figure 5.7 also alludes to the existence of a structural shift, as seen by a large cumulative error building up in the model.

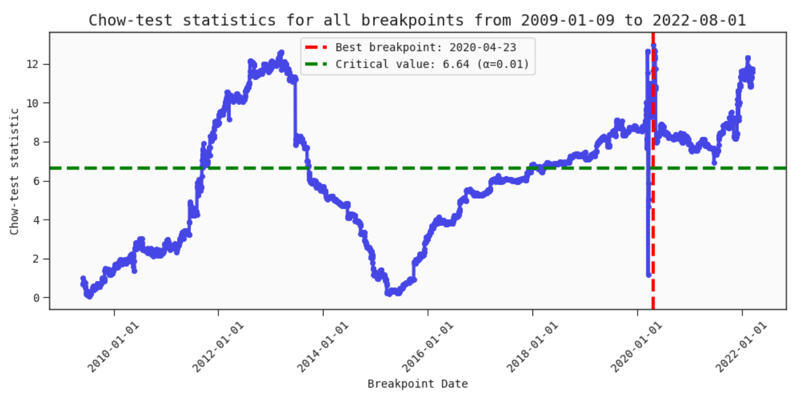
Figure 5.8 shows the results from the Chow tests. The most significant breakpoint is GFC (9 January 2009). The subsequent Chow test is performed on the largest subsample (2009 to 2022), where COVID-19 (23 April 2020) is selected as the breakpoint. Both these structural shifts are rooted in global extreme events. Chow test is lastly performed on the largest remaining sample (2009 to 2020), selecting 13 March 2013 as the final breakpoint. This breakpoint can be motivated both by the 2014-2016 oil price fall, and the exchange rate being close to its strongest since 2003.

Following the identified structural shifts, the selected GPSR model is re-estimated for the four subsamples; 1 January 2002 to 9 January 2009, 9 January 2009 to 13 March 2013, 13 March 2013 to 23 April 2020, and 23 April 2020 to 1 August 2022. This allows the re-estimation to account for all identified structural shifts. The piecewise model can better adjust the intercept term to each period, likely improving cumulative error. However, this increases the risk of overfitting, particularly for the fourth and shortest subsample. Figure 5.9 shows the cumulative predictions of the piecewise model. It shows a clear improvement in cumulative error, compared to cumulative predictions in Figure 5.7.

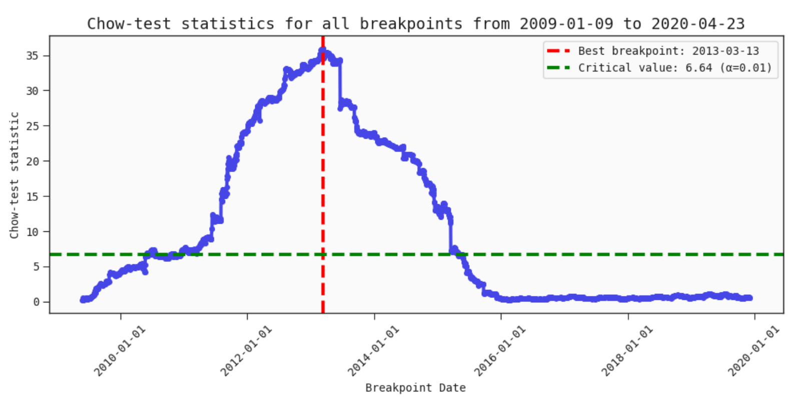
The re-estimated subsample models' coefficients and performances are shown in Appendix J. The re-estimation increases performance measured by R^2 from 0.275 to 0.304, a modest increase. This



a Full sample: 1 Jan 2002 to 1 Aug 2022. Best breakpoint: 9 Jan 2009.



b Subsample: 9 Jan 2009 to 1 Aug 2022. Best breakpoint: 23 Apr 2020.



c Subsample: 9 Jan 2009 to 23 Apr 2020. Best breakpoint: 13 Mar 2013.

Figure 5.8: The Chow test statistic for all possible breakpoints for subsamples of the available data. A higher Chow test statistic indicates a more significant rejection of the null hypothesis, which is used to select the best breakpoint. The critical value is marked in green, and shows the test statistic required for rejecting the null hypothesis with a significance level of 0.01.

suggests that the model was already relatively adapt at generalising across multiple structural shifts. A possible explanation can be the non-linear scaling of Δoil and Δeuronext variables.

The SHAP analyses of the re-estimated subsample models are supplied in Appendix K. The analysis shows how the importance of different features varies in the subsamples. Oil prices and the Euronext 100 index are generally important in all periods, although the Euronext 100 Index emerges as the most important feature in the period 2013 to 2020, and even more so in 2020 to 2022. Interestingly, the interest rate differential has the largest impact on the model in the period 2009 to 2013.

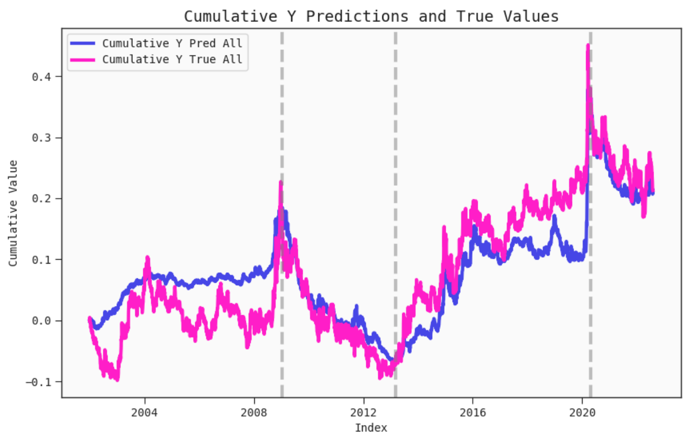


Figure 5.9: Cumulative sum of predictions and actual returns of piecewise regression model. Dotted gray lines mark the structural shifts.

5.5.1 Structure of EURNOK prediction across time

The cumulative of contributions from each feature in Figure 5.10 show the impact on the model prediction across time. This allows for examining the structure of predictions across time. To better compare the relative importance of features, cumulative feature contributions are also shown together in Figure 5.11. This allows for identifying periods where specific variables are particularly contributing to krone appreciation or depreciation, and can give a better understanding of underlying factors of EURNOK fluctuations. This also allows for identifying and isolating the unexplained depreciation (Klovland et al., 2021) from effects arising from changing feature impacts.

The constant α represents an unexplained drift. The drift is positive between 2002 to 2009, indicating the krone is depreciating due to an unobserved factor. From 2009 to 2013, the drift is negative, indicating that the krone is appreciating, and this appreciation can to a greater extent be explained by fundamentals. After 2013, the drift contributes significantly to weakening the krone. This can be seen in Figure 5.11, where the drift contributes significantly more than other features. This indicates weakness in the model, as a large part of the movements remain unexplained. Klovland et al. (2021) argues that this depreciation might arise from increasing price differential. Other possible explanations include increasing global uncertainty, decreasing faith in the Norwegian economic

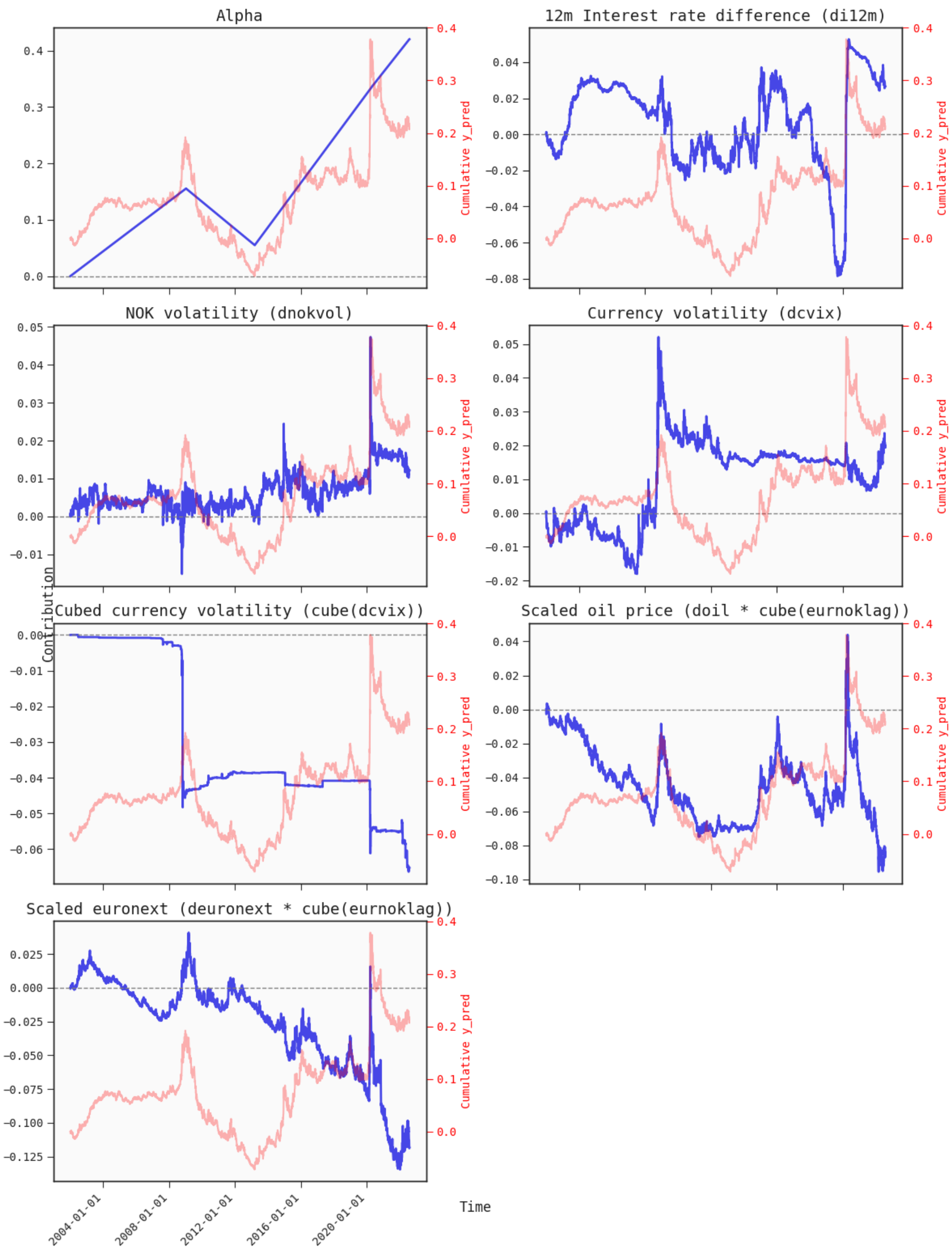


Figure 5.10: The cumulative sum of the contribution of each term to the total prediction for each term in the model. The total model prediction is shown in red in each subplot. The sum of all blue feature contribution plots equals the red prediction plot.

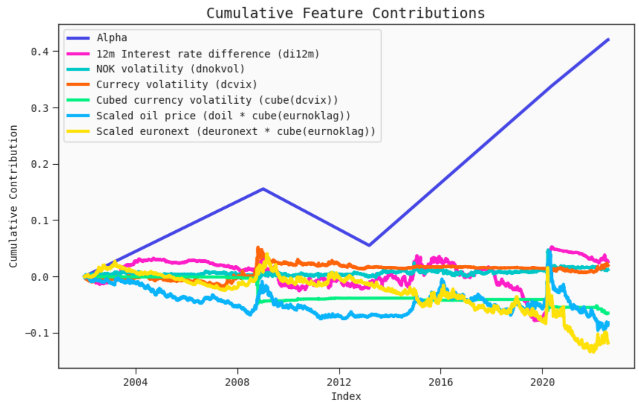


Figure 5.11: Comparison of cumulative sums of the contribution of each term to the total prediction, for each term in the re-estimated GPSR model.

system, and an outdated dependence on petroleum, as previously discussed in Section 2.2.3. These explanations are specific to the krone and the Norwegian economy. Possible explanations not focused on krone fundamentals include Bacchetta and van Wincoop (2004, 2013)'s scapegoat theory and Engel and West (2003)'s theory of unobservable shocks, as discussed in Section 2.1.1.

Following 2013 and the 2014-2016 oil price fall, the Euronext 100 Index emerges as a major net contributor to the exchange rate, and surpasses oil price as the biggest net contributor to EURNOK appreciation. This might correlate with the increasing value of the Oil Fund to the Norwegian economy (NBIM, n.d.), and might suggest that the existing consensus of oil price being the most important predictor for the exchange rate should be further studied. Following Covid-19, oil and stock prices cause a rapid depreciation of the krone, with a similarly rapid appreciation during Covid-19 recovery. This same pattern is evident for GFC, though not as significantly.

The interest rate differential is characterised by large sudden changes, while the Norwegian specific volatility has a large daily variance. Between 2013 and 2020, interest rates contribute a net appreciation of the krone, which is offset by a rapid depreciation due to Covid-19. The volatility differential has a momentary impact during GFC, the 2014-2016 oil price fall, and Covid-19.

Currency volatility appreciates the krone until 2009, at which point it causes a large depreciation. Δcvix has minimal impact on the model after 2013, indicating that the direct impact of volatility has diminished over time. $(\Delta\text{cvix})^3$ has little impact on the model except for when the change in volatility is very large. Figure 5.10 shows that this contributes to a significant strengthening of the krone during GFC and Covid-19. During these periods of high volatility, most of the other factors contribute to depreciation. $(\Delta\text{cvix})^3$ can therefore be interpreted as a corrective term adjusting for the correlation of other explanatory variables during times of extreme volatility.

5.6 Our limitations

GPSR is not a widely-used method in finance, with relatively few researchers and research papers, and few resources exploring similar issues as our thesis. However, there exist several excellent publicly available GPSR tools, allowing for GPSR to be successfully employed in our thesis. Benchmarking against an established ML method and linear regression models also lends GPSR increased credibility.

There exists several alternative ML methods to random forest, such as LSTM or other NNs. LSTM (Long Short-Term Memory) has been used extensively in finance, and is often the preferred ML method. However, LSTMs are less suitable with highly nonlinear data containing several structural breaks. Random forest is therefore chosen for ML benchmarking. Examining whether other NNs are suitable for explaining the EURNOK or ML benchmarking would be interesting for further works.

The issue of run time is a challenge for GPSR. In theory, GPSR can run infinitely if not ended manually or by a stop criteria. However, GPSR quickly converges, with improvements to the models decaying exponentially with time. While we test running our GPSR algorithm for over ten days, the results yielded were marginal improvements compared to the results obtained after running the algorithm for 24 hours. Therefore, while lack of a common run time standard poses a disadvantage, by testing different lengths of run time we conclude that our run time of 24 hours is sufficient.

Data on a daily frequency is less available than data with lower frequencies, possibly leading to omitted variable bias. For instance, the selected GPSR model does not include price differential, which is supplied at a monthly frequency, though its inclusion is motivated by theory. There is also a risk we may omit important variables that could capture relationships not highlighted in this study.

5.7 Further works

Our findings demonstrate GPSR's success in application to nonlinear problems, and encourages further application of GPSR in finance, especially in fields where the nature of the nonlinear relationships are unknown. Nonlinearities are often observed in financial economics, risk management, option pricing and derivatives, and behavioural finance, and GPSR can yield valuable insight.

The non-linear relationships identified in our GPSR model have not been previously studied in literature. Further examining the nature of the relationships using different methods and approaches can increase understanding of krone fluctuations, and would benefit policymakers, industrial players, and stakeholders. Understanding reasons behind krone fluctuations can also give Norges Bank new tools for managing krone behaviour, and allow for implementing more precise and direct measures. This is also useful for managing the krone's depreciation, which has caused major discourse amongst economist, and remains a challenge. While we identify time-varying relationships between the krone and macroeconomic features resulting from structural shifts, the nature of the krone' depreciation in recent years remains mostly unknown, and no definite cause has been identified. Further studies examining features omitted from this study might yield new light on the krone's behaviour.

6 Conclusion

Linear descriptive models for explaining the relationship between the Norwegian krone exchange rate and macroeconomic fundamentals inefficiently describe the nonlinearities suspected to exist between the krone and the fundamentals. However, non-linear models developed using ML methods may be uninterpretable and yield little insight for explaining the nature of the non-linear relationships. Symbolic regression addresses the shortcomings of linear regression and ML methods.

This thesis contributes to existing literature on the Norwegian krone, particularly nonlinearities in the krone, by applying GPSR, a method not previously used for modelling the krone, for developing a parsimonious and interpretable exchange rate model for EURNOK returns. Our GPSR model outperforms alternative linear and non-linear models, demonstrating GPSR's competitive power. When re-estimated with monthly data, the GPSR model outperforms published monthly models from the literature.

We further contribute to the literature by identifying several nonlinear properties that have previously not been studied. We find that the GPSR model scales the stock market and oil price variables by the nominal value of the exchange rate, indicating that changes to these variables are more important when the exchange rate is high. The model also includes a cubed term for the change in currency volatility, a corrective term for adjusting the model in times of high volatility. Lastly, the GPSR model includes linear terms that support the existing consensus of interest rates and the krone's perceived riskiness affecting the krone fluctuations.

Finally, we contribute to the literature by examining the krone's unexplained depreciation, which is plaguing economists and also affects our model's performance. The structural shifts affect the nonlinearities and features' impacts on the model's predictions over time. Stock markets have an increasing impact on the krone, challenging the existing consensus of Brent crude oil price being the most important feature for the krone. However, a large and increasing negative drift remains unexplained. The drift is a major contributor to the depreciation of the krone, but is not explained by our GPSR model or macroeconomic fundamentals. Literature suggests several possible reasons for this unidentified depreciation, but has yet to identify a definitive cause. This indicates that although our GPSR model is powerful and yields new insight into the Norwegian krone, it is unable to overcome all challenges plaguing traditional exchange rate models.

This study has limitations that should be acknowledged. As GPSR is not a widely-used method in finance, there is little similar research available for comparison. This might have affected the implementation of the GPSR algorithm, and subsequently the choice of run time, for which we had little research to support our views. Further extensive benchmarking with other non-linear methods such as NNs would therefore be beneficial. The model's daily frequency reduces the data available, and might cause omitting important variables capturing relationships not highlighted in this study.

We encourage future studies to apply GPSR to other fields in economics and finance to further examine the validity and efficiency of GPSR as an alternative to linear regression or ML methods.

This thesis highlights GPSR as an efficient way to explore economic relationships where the nature of the relationships are unclear or unknown. This thesis also identifies non-linear relationships which have previously not been studied. Further research exploring these nonlinearities could motivate possible explanations for the relationships. Validating possible explanations can be meaningful for explaining how international investors view the Norwegian economy, and provide new insight into the krone and EURNOK movements.

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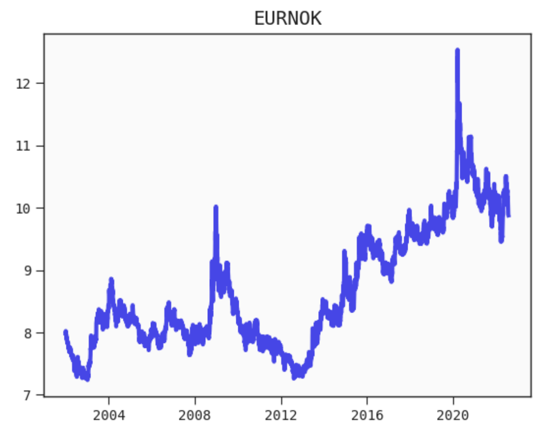
Appendix

A Data sources

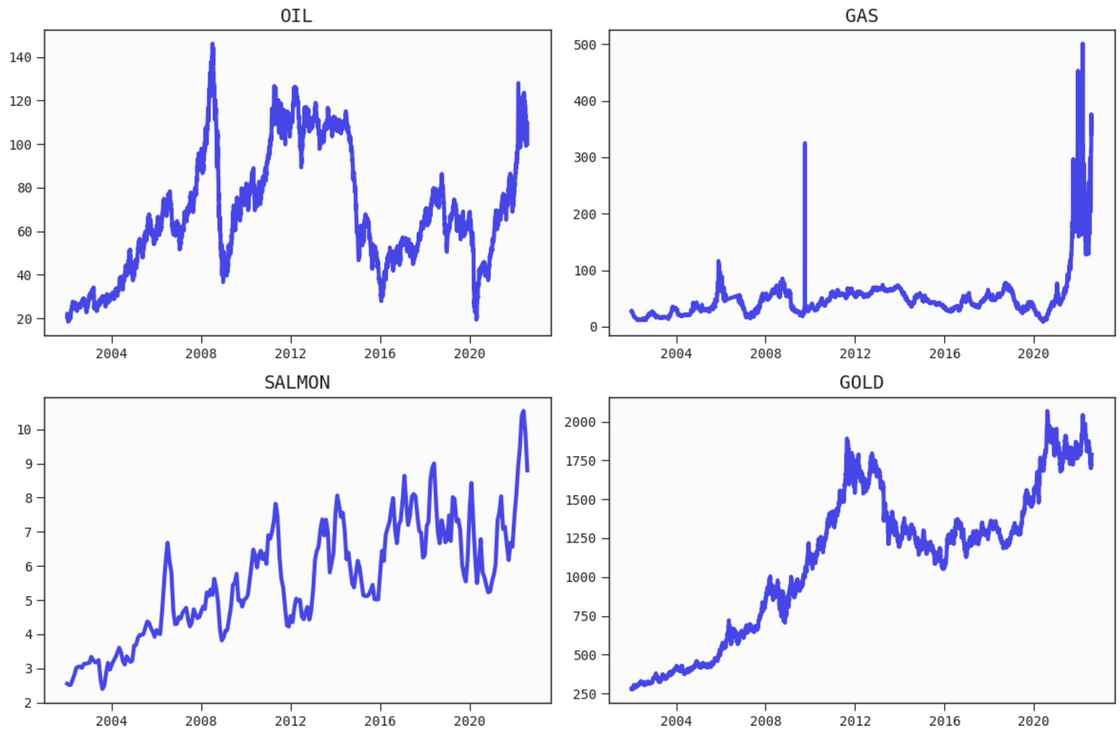
List of datasets

Datasets			
Abbr.	Description	Source	Frequency
EURNOK	Euro/krone exchange rate	Bloomberg	Daily
GAS	UK NBP Natural Gas Futures	Bloomberg	Daily
GOLD	Salmon prices	Bloomberg	Daily
OIL	Brent crude oil price	Bloomberg	Daily
SALMON	Salmon prices	Bloomberg	Daily
EURONEXT	EURONEXT 100 Index	Bloomberg	Daily
MSCI	MSCI World Index	Bloomberg	Daily
OSEAX	OSEAX Index	Bloomberg	Daily
CVIX	Currency Volatility Index	Bloomberg	Daily
GHI	Global Hazard Indicator	Bloomberg	Daily
GPR	Geopolitical risk index	Caldara and Iacoviello (2022)	Weekly
NOKVOL	Norwegian specific volatility	Martinsen (2021)	Daily
RISKAVERSION	U.S. Risk Aversion Index	Bekaert et al. (2022)	Daily
I10Y	10-year interest rate differential between Norway and EU	Bloomberg	Daily
I3M	3-month interest rate differential between Norway and EU	Bloomberg	Daily
I12M	12-month interest rate differential between Norway and EU	Bloomberg	Daily
PRICEDIFF	Price differential between Norway and EU	Eurostat	Monthly

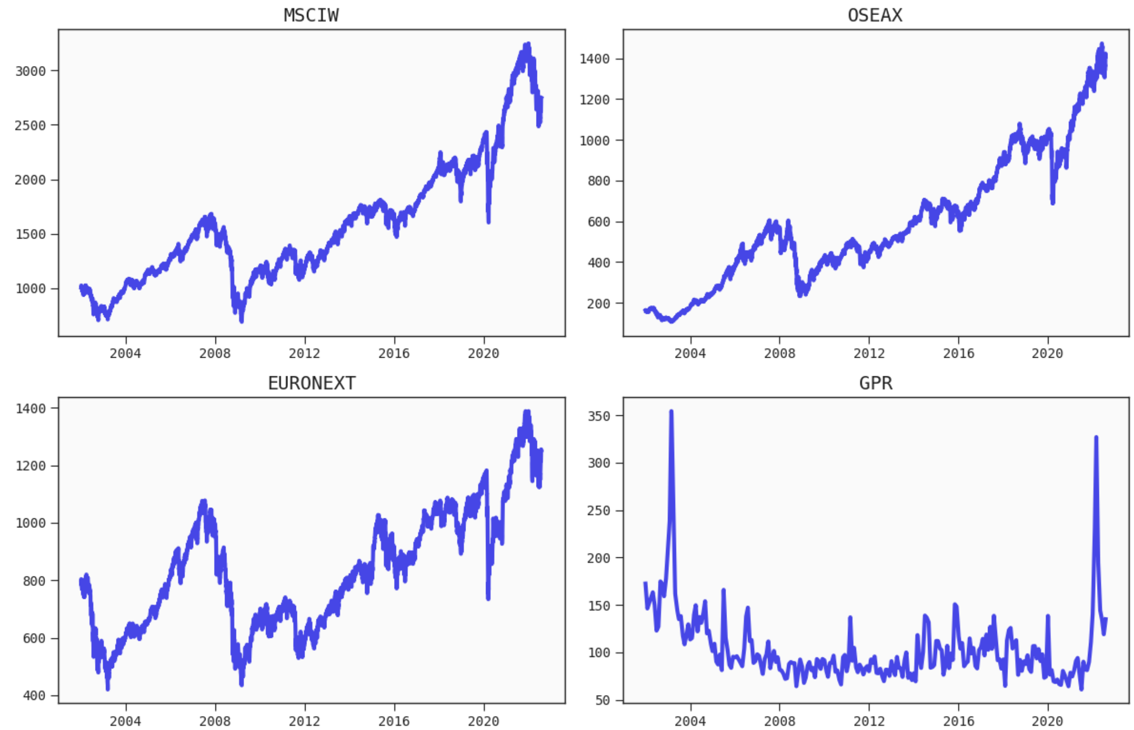
B Time series charts for explanatory variables



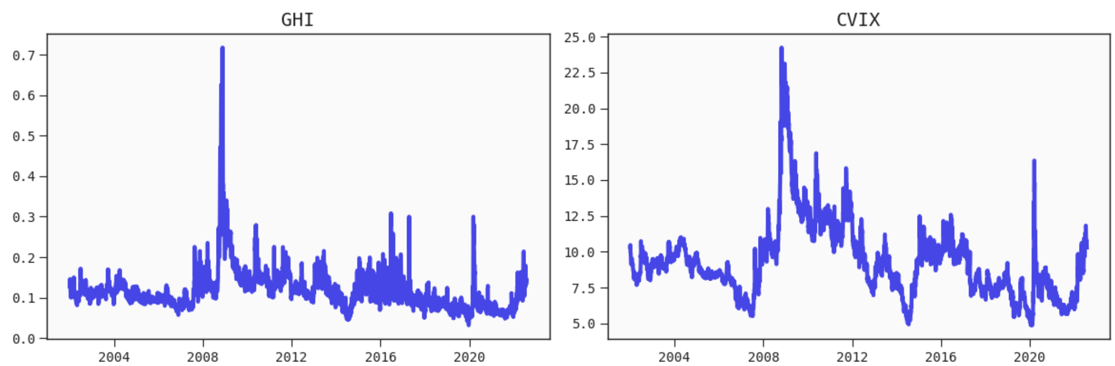
EURNOK time series from 1 Jan 2002 to 1 Aug 2022.



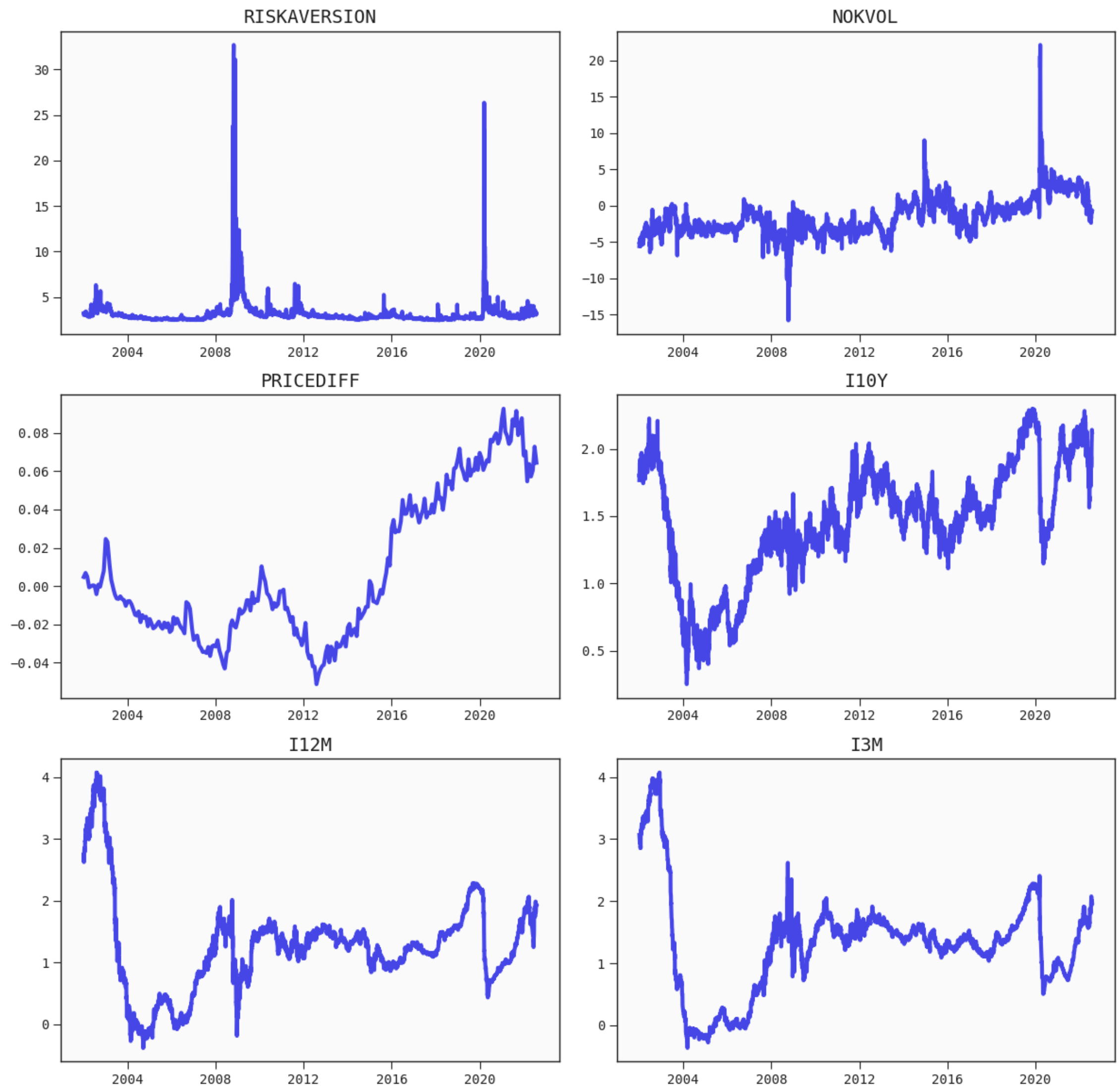
Commodity variables from 1 Jan 2002 to 1 Aug 2022.



Asset variables from 1 Jan 2002 to 1 Aug 2022.



Uncertainty variables from 1 Jan 2002 to 1 Aug 2022.



Macroeconomic fundamental variables from 1 Jan 2002 to 1 Aug 2022.

C GPSR Operators

Operators included in the GPSR model

Operator	Symbolic form	Max complexity	Nesting constraints
Binary operators			
addition	$x + y$	(4, 4)	
subtraction	$x - y$	(4, 4)	
multiplication	$x \cdot y$	(4, 4)	addition, subtraction
division	$\frac{x}{y}$	(4, 4)	addition, subtraction
Unary operators			
square	x^2	4	addition, subtraction
cube	x^3	4	addition, subtraction
exp	e^x	4	addition, subtraction
inv	$1/x$	4	addition, subtraction
tanh	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	4	addition, subtraction
gaussian	e^{-x^2}	4	addition, subtraction

Each operator, constant, or instance of variables is assigned a complexity of one. Total complexity is the sum of the complexities of all terms in an expression. Although the default complexity is one, it is possible to assign operators, constants, and variables different complexities.

D Genetic Mechanisms

Equations are modeled in a tree structure. parent nodes are always operators, while leaf nodes can be explaining variables, constants, or sub-trees with new parent and leaf nodes. The trees represent equations solving for the dependent variable. Trees can be rewritten to symbolic form, and is what we generally refer to as "models" elsewhere in this paper. The equations are restricted to a maximum value of *complexity*.

Complexity is a proxy measure for the complexity of an equation. In this setup, a score of one is assigned per operator, variable, and constant. For example, the equation ax+b, has a complexity of 4.

Populations are distinct sets of equations, in which*tournaments* are held.

Tournaments are when a set number of equations are chosen from the population. The winner of the tournament is chosen for *crossover*. The chance of winning the tournament increases the better the equation is, as measured by the lowest possible *loss*. The frequency of the complexity of the equation in the populations also has an impact on which equations win the tournament, working to keep populations diverse in terms of complexities.

Loss is used to evaluate equations in *tournaments*. The loss evaluates how well the equation fits the data.

Crossover randomly combines two equations by attaching a random sub-tree of one equation and attaching it to a random node of the other.

When **mutations** happen, equations are changed. This can be, swapping operators, adding or deleting nodes, mutating constants, or deleting and randomly rewriting the equation.

Migrations are when a set number of the best equations of each population have a chance to move into other populations once every *iteration*.

Age defines which equations in each population will be replaced by *crossover* and *migration*. Age is defined as *iterations* since the equation was created.

In a single **iteration** a set number of cycles with mutations occur in the population, followed by a tournament and migrations. The process does not have to run for a set number of iterations, and iterations can be repeated until results stagnate or results are satisfying.

In the **hall of fame**, the best equations for each given complexity are added if they score better than the existing equations. Migrations also happen between the hall of fame into populations in order to keep the best equations alive in the populations. The hall of fame is at a given iteration the current Pareto-frontier of the symbolic regression problem.

E Feature selection configuration

The random forest configuration chosen for feature selection in this thesis.

Parameter	**Value**
Number of estimators	5 000
Max depth	3
Criterion	Gini importance

The LASSO configuration chosen for feature selection in this thesis.

Parameter	**Value**
Number of estimators	5 000
Max depth	3
Penalty term	L1 regularisation

F Computing specifications

Computing solutions

In equations, constants can be used in non-linear ways. This means linear optimisation methods such as OLS will not work to optimise constants. Instead, the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm is used. OLS finds a global optimal solution, whilst BFGS finds a local optimal solution. As a result, equations may re-optimise constants using BFGS with random initial conditions, which could result in better solutions.

Computing specifications

The GPSR algorithm is set to run for 24 hours.

PySR allows for multi-threading computations, significantly speeding up computation time.

The algorithm is run on NTNU compute nodes with two 2x Intel Xeon Gold 5115 processors.

The genetic algorithm is set to a maximum capacity of 30.

Operator selection

The program is allowed to choose from a set of binary and unary operators. An advantage of including unary operators is that non-linear economic relationships in the solution may be found. *Square*, *cube*, and *exp* capture polynomial and exponentially increasing impact on the model output as the value of a variable increase. Inverse and Gaussian capture relationships where the impact on model output decreases as the value of a variable increases. Gaussian can be described as a bell curve with a diminishing impact as the value moves away from zero. *Tanh* is a hyperbolic function and captures increasing importance from a first threshold to a second threshold, after which the importance will not increase further. An advantage of this combination of both binary and unary operators will allow the program to cover natural occurring dependencies (abzu, 2020).

The constraints are imposed in two ways. The first is constraining the maximum complexity of the terms that the operators are allowed to be used on. For example, the square operator cannot be applied to terms with a larger complexity than four. The most important restrictions are the ones used for addition and subtraction. This effectively constrains the solution to be linear combinations of expressions with complexities of four or less. Additionally, nesting constraints restrict the use of certain operators within an operator. For instance, no multiplication can be performed on terms containing either addition or subtraction. These restrictions are also listed in Appendix C.

G Model generation

Model generation		
Complexity	Loss	Equation
1	$2.3165728569073557e^{-5}$	$4.010577130897364e^{-5}$
2	$2.3007183659543342e^{-5}$	$(\Delta\text{msciw})^2$
3	$2.130887219330615e^{-5}$	$(-0.10546023206099785\ \Delta\text{euronext})$
4	$2.130872654953825e^{-5}$	$(-0.10551897225824527\ \tanh \Delta\text{euronext})$
5	$2.1157452646064837e^{-5}$	$(-0.013313286791027723\ \Delta\text{euronext} \cdot \text{EURNOK}_{t-1})$
6	$2.0883324998171553e^{-5}$	$(-2.012333664661083e^{-5}\ \Delta\text{euronext} \cdot \exp \text{EURNOK}_{t-1})$
7	$2.027488982763106e^{-5}$	$(-0.08505392861034694\ \Delta\text{euronext}) + (-0.05035740620283159\ \Delta\text{oil})$
8	$2.027465504563859e^{-5}$	$(-0.05035889171617561\ \Delta\text{oil}) + (-0.08510562408181077\ \tanh\ \Delta\text{euronext})$
9	$2.0176068677776036e^{-5}$	$(-0.049196148883940506\ \Delta\text{oil}) + (-0.010795736609653347\ \Delta\text{euronext} \cdot \text{EURNOK}_{t-1})$
10	$1.9998018080396257e^{-5}$	$(-0.046984698276337236\ \Delta\text{oil}) + (-1.6467431673503336e^{-5}\ \Delta\text{euronext} \cdot \exp \text{EURNOK}_{t-1})$
11	$1.9447556909236194e^{-5}$	$(-0.023401471702697334\ \Delta\text{i12m}) + (-0.0862117015433864\ \Delta\text{euronext}) + (-0.050135944584624864\ \Delta\text{oil})$
12	$1.9440145546357203e^{-5}$	$(-0.023747804097475178\ \tanh\ \Delta\text{i12m}) + (-0.08623721683971577\ \Delta\text{euronext}) - (0.05013553941751746\ \Delta\text{oil})$
13	$1.9362921508715538e^{-5}$	$(-0.023197833358032983\ \Delta\text{i12m}) + (-0.04905820422783943\ \Delta\text{oil}) + (-0.010868473858907962\ \Delta\text{euronext} \cdot\ \text{EURNOK}_{t-1}))$
14	$1.923689056052512e^{-5}$	$(-0.02275086465356777\ \Delta\text{i12m}) + (-0.04736654912286654\ \Delta\text{oil}) + (-0.00016235403956912993\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
15	$1.9113762372844187e^{-5}$	$(-0.02314892261115526\ \Delta\text{i12m}) + (-0.05091885221172723\ \Delta\text{oil}) + (0.0012876054505496003\ \Delta\text{nokvol}) - (0.0971386739246355\ \Delta\text{euronext})$
16	$1.9105727194345143e^{-5}$	$(-0.02350254048964614\ \tanh\ \Delta\text{i12m}) + (0.0012887937275112514\ \Delta\text{nokvol}) + (-0.050919093743341755\ \Delta\text{oil}) + (-0.09717456912536576\ \Delta\text{euronext})$
17	$1.901306579688382e^{-5}$	$(-0.02291199553107491\ \Delta\text{i12m}) + (-0.0497572574371233316\ \Delta\text{oil}) + (0.001319374878124692\ \Delta\text{nokvol}) + (-0.012252420245830188\ \Delta\text{euronext} \cdot \text{EURNOK}_{t-1})$
18	$1.8867916000996375e^{-5}$	$(-0.022399760197550345\ \Delta\text{i12m}) + (-0.047945695715077355\ \Delta\text{oil}) + (0.0013556179330956992\ \Delta\text{nokvol}) + (-0.00018277061126267916\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
19	$1.8860324351969066e^{-5}$	$(-0.022742750206710197 \tanh\ \Delta\text{i12m}) + (-0.04794798713292391\ \Delta\text{oil}) + (0.0013566645621407754\ \Delta\text{nokvol}) + (-0.00018280196068836093\ \Delta\text{euronext} \cdot \text{EURNOK}_{t-1})^3$
20	$1.881599612288696e^{-5}$	$(-0.02235152023854781\ \Delta\text{i12m}) + (-0.0057867712281107105\ \Delta\text{oil} \cdot \text{EURNOK}_{t-1}) + (0.0013497723622760685\ \Delta\text{nokvol}) + (-0.00018074732609902683\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
21	$1.8749258356799997e^{-5}$	$(-0.022268328799045643\ \Delta\text{i12m}) + (-7.94994369337317e^{-5}\ \Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3) + (0.001334431808076171\ \Delta\text{nokvol}) + (-0.0001781260212713044\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$

Model generation

Complexity	Loss	Equation
22	$1.8679109635157403e^{-5}$	$(-0.02286973534524461\ \Delta\text{i12m}) + (-0.0471832122003601\ \Delta\text{oil}) + (0.001345102338979817\ \Delta\text{nokvol}) - (\ \Delta\text{gold} \cdot\ \Delta\text{cvix}) + (-0.00018423648143165775\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
23	$1.8666256469890837e^{-5}$	$(-0.029497060350919962\ \Delta\text{i12m}) + ((\Delta\text{i12m})^3 \cdot \text{GHI}) + (-0.0484099943250809\ \Delta\text{oil}) + (0.0013786048204090366\ \Delta\text{nokvol}) + (-0.00018642002118506962\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
24	$1.8628611678920495e^{-5}$	$(-0.02282226742974025\ \Delta\text{i12m}) - (\ \Delta\text{cvix} \cdot\ \Delta\text{gold}) + (-0.005695396311754282\ \Delta\text{oil} \cdot \text{EURNOK}_{t-1}) + (0.0013393538608786927\ \Delta\text{nokvol}) - (0.00018224107755194946\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
25	$1.8563015028500635e^{-5}$	$(-0.022740335014623095\ \Delta\text{i12m}) + (-7.82809148275497e^{-5}\ \Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3) + (0.0013242645797958315\ \Delta\text{nokvol}) - (\ \Delta\text{gold} \cdot\ \Delta\text{cvix}) + (-0.00017964224015472850\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
26	$1.8542474149600333e^{-5}$	$(-0.029364206900470754\ \Delta\text{i12m}) + ((\Delta\text{i12m})^3 \cdot \text{GHI}) + (-8.037558927625353e^{-5}\ \Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3) + (0.0013572400873777559\ \Delta\text{nokvol}) + (-0.00018167581551172817\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
27	$1.852097901044322e^{-5}$	$(-0.029967042260720264\ \Delta\text{i12m}) + ((\Delta\text{i12m})^3 \cdot \text{GHI}) + (-0.04764751525118519\ \Delta\text{oil}) + (0.0013680894040649704\ \Delta\text{nokvol}) - (\ \Delta\text{gold} \cdot\ \Delta\text{cvix}) + (-0.000187885895637249\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$
28	$1.8447816380095166e^{-5}$	$(-0.022186430754550842\ \Delta\text{i12m}) + (-7.807286138088565e^{-5}\ \Delta\text{oil} \cdot\ (\text{EURNOK}_{t-1})^3) + (0.0016730422520128164\ \Delta\text{nokvol}) + (0.02415359996911089\ \Delta\text{cvix}) + (-0.00016478763632213396\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3) - (\Delta\text{cvix})^3$
29	$1.8416947356827762e^{-5}$	$(-0.022779398482893082\ \Delta\text{i12m}) - (-0.024415640840414157\ \Delta\text{cvix}) + (-0.045934657704201236\ \Delta\text{oil}) + (0.001688805703535278\ \Delta\text{nokvol}) - (\ \Delta\text{gold} \cdot\ \Delta\text{cvix}) - (0.0001709356150734925\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3) - (\Delta\text{cvix})^3)$
30	$1.8399827367717917e^{-5}$	$(-0.02983621223584316\ \Delta\text{i12m}) + ((\Delta\text{i12m})^3 \cdot \text{GHI}) + (-7.915708213099362e^{-5}\ \Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3) + (0.0013470728117145703\ \Delta\text{nokvol}) - (\ \Delta\text{gold} \cdot\ \Delta\text{cvix}) + (-0.00018319202514309395\ \Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3)$

H Analysis of recurring features in generated models

The generated GPSR models explore all operators and almost all features.

At complexity 1, the optimal equation is the mean of EURNOK returns, as expected.

In Section 5.1, both feature selection methods find Brent crude oil price returns significant. As discussed in Section 2.2, Brent crude oil price may be considered one of the most important macroeconomic features for the krone. However, Brent crude oil price returns is not included in any equations before complexity 7. This might suggest that while Brent crude oil price returns might not be the most significant feature, Brent crude oil price returns has a key fundamental relationship with the krone. However, lagged Brent crude oil price returns, which was found to be significant using LASSO regression, is not included in any of the generated models.

Gold price returns is only included in some of the generated models of highest complexity, and then in a non-linear relationship with CVIX returns.

The MSCI World Index is the first included feature, in equation 2. This is unsurprising, as the MSCI World Index is representative of large and mid-cap companies across 23 developed countries, including Norway. It therefore may be the best choice for explaining the general trend of the returns of the Norwegian krone, and the MSCI World Index is likely a good indicator of global uncertainty. However, as the equations continue to evolve, the MSCI World Index is replaced by the Euronext 100 Index. The Euronext 100 Index is included in all following generated equations, and is likely a better representation of EURNOK effects. However, the native OSEAX index is not included in any generated GPSR models. It was expected that OSEAX would be better at explaining movements in the krone due to its high correlation with Brent crude oil price, however this significant correlation might have caused the GPSR algorithm to dismiss the feature in favour of more significant features.

GHI is included in few equations, and only of high complexity, showing that GHI is not selected when the model is restricted. The models instead include other uncertainty features, or avoid uncertainty features altogether. CVIX returns is included in all equations from complexity 22 and after, except complexities 23 and 26. Only LASSO regression found CVIX returns to be significant. The models therefore only include CVIX when not limited by complexity. 1-month volatility differential returns is included in all equations of complexity 15 and higher. Prior to this, no uncertainty asset is included in any of the generated models, and the equations prioritise other features of higher importance. This indicates that the risk premium is a good indicator of krone risk for low complexity models.

10-year interest rate differential returns, 3-month interest rate differential returns, 12-month interest rate differential returns, and lagged 12-month interest rate differential returns were found to be significant in Section 5.1. However, the generated GPSR models only include 12-month interest rate differential returns. 12-month interest rate differential returns is included in all equations after complexity 11. Moreover, 12-month interest rate differential returns is included twice in equations complexity 23, 26, 27, and 30. The inclusion of only 12-month interest rate differential returns is unsurprising, as 12-month interest rate differential returns was found to have the highest correlation with EURNOK returns of the interest rate differential returns in Figure 3.1.

Some combinations of variables are consistently seen in each generation. This indicates that there may exist specific relationships between variables that have economic interpretability.

A non-linear relationship between the Euronext 100 Index and lagged EURNOK is identified in the generated models, and all equations after complexity 18 include the non-linear term. The non-linear term is also included in many equations of lower complexity. The non-linear term remaining constant though the models' complexities increase, indicates that the cubed relationship is a more accurate representation of the non-linear relationship than other possible representations.

A non-linear relationship between Brent crude oil price and lagged EURNOK is also identified, of similar nature to the relationship between the Euronext 100 Index and lagged EURNOK. This relationship is included in all equations from complexity 25 and after, indicating that this relationship is less prevalent than the relationship with the Euronext 100 Index, which is included in earlier models of lower complexity.

The equations become harder to interpret and connect to existing literature and practice as complexity increases. To ensure economic interpretability of the models, maximising complexity is not preferred.

I Residuals

The residuals are tested for violation of statistical assumptions. The residuals for the selected GPSR model are also plotted over time, and for in- and out-of-sample.

Assumption of normality

All models are normally distributed both in- and out-of-sample, with the exception of random forest, which is approximately normally distributed in-sample. The deviation from normality is not too extreme. The plot of the selected GPSR model's residuals over time shows that the residuals are approximately normally distributed around the horizontal line of a mean residual of zero. This also supports that the selected GPSR model's assumption of normality holds.

The residuals from the models with monthly frequency do not follow a normal distribution in-sample, but more closely resemble a normal distribution out-of-sample. This violation of the normality assumption might arise from the significantly smaller sample size, making it difficult to identify the normal distribution. The violation also indicates that alternative modelling approaches should be considered for the monthly frequency. Performing GPSR on this SR problem instead of re-estimating the existing GPSR equation might improve the models.

Assumption of mean zero

The residuals have an in-sample mean of zero, in line with the statistical assumption, with the exception of the GPSR model, which has a mean slightly deviating from zero. However, the plot of the selected GPSR model's residuals over time shows that the residuals are heavily concentrated around zero, indicating that the mean is approximately very close to zero.

Out-of-sample, the models have a mean deviating slightly more from zero, with all models having a positive mean. This indicates a slight negative bias in the models' predictions, and they tend to underestimate predictions, and predicting a weaker depreciation than what is the case. The models lack a positive component. This positive mean indicates that there may be omitted variables which our models do not capture. We previously discussed that this possibly relates to the drift identified in the EURNOK exchange rate which has so far not been explained by economists and traditional macroeconomic fundamentals and features, that has caused a trend of depreciation of the krone.

Assumption of homoscedasticity

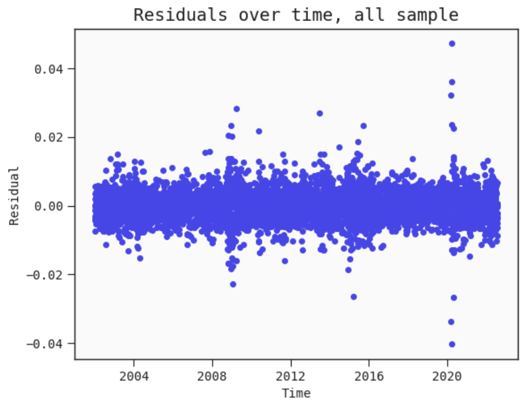
The assumption of same but unknown variance is verified by considering the selected GPSR model's plotted residuals in-sample and out-of-sample. The plots indicate that in-sample, the assumption of homoscedasticity holds. However out-of-sample there is a weak trend of heteroscedasticity, with lower predictions having more variance in residuals that equally higher predictions. This supports that the GPSR model might be biased, with the model being more accurate with predictions of depreciation, and less accurate with predictions of appreciation, resulting from the drift previously discussed. However, the trend of heteroscedasticity is very weak, and closely approximates homoscedasticity.

The models' low standard deviations in- and out-of-sample indicate that the variability in the residuals is small, and that the models perform well. The monthly models have a slightly larger standard deviance.

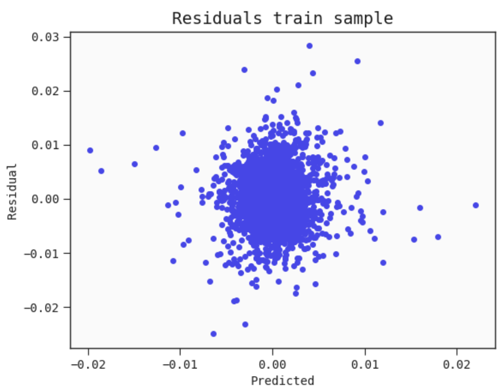
Assumption of independence of errors

The Ljung-Box test is performed to test for correlation between the residuals and the lagged residuals. The p-values indicate that the monthly models, the random walk, and the linear multivariate model of all explanatory variables have independence of errors in-sample. Out-of-sample, the p-values indicate that all the monthly models have independence of errors, while the daily models do not have independence of errors. If errors are dependent and correlated, this might affect the model's performance, and the model's performance might be overestimated.

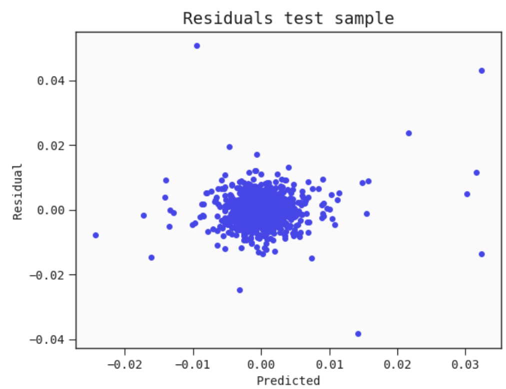
However, the residual plots for the selected GPSR model in- and out-of-sample do not indicate a lack of independence, as there is no clustering of residuals consistently positive or negative. This indicates that the assumption of independence holds for the selected GPSR-model, despite the p-value indicating otherwise.



a Residuals over time from 1 Jan 2002 to 1 Aug 2022.



b Residuals for in-sample (train) for 1 Jan 2002 to 1 Aug 2022.



c Residuals for out-of-sample (test) for 1 Jan 2002 to 1 Aug 2022.

Residuals evaluation for in-sample (train) for 1 Jan 2002 to 1 Aug 2022						
	Statistical properties					
Daily models	Mean	SD	Skew	Kurtosis	Normality	Autocorrelation 1 lag
Selected GPSR model	0.00007	0.00429	0.21814	5.89181	0.00000	0.00007
Random forest	0.00000	0.00319	0.10158	3.31652	0.00002	0.00033
Random walk	0.00000	0.00481	0.33674	6.90816	0.00000	0.49322
Linear multivariate regression model of all explanatory variables [22]	-0.00000	0.00426	0.22237	5.91027	0.00000	0.49582
Linear multivariate regression model of selected features[23]	-0.00000	0.00431	0.17678	6.17043	0.00000	0.00034
Linear multivariate regression model of features in selected GPSR model[24]	-0.00000	0.00436	0.17143	5.98135	0.00000	0.00046
	Statistical properties					
Monthly models	Mean	SD	Skew	Kurtosis	Normality	Autocorrelation 1 lag
Re-estimated monthly GPSR model	0.00000	0.013911	0.157455	3.122526	0.616653	0.820126
Akram (2020)	0.00000	0.015104	0.300414	4.222862	0.019442	0.547330
Martinsen (2021)	0.00000	0.014419	0.040310	3.941834	0.126723	0.237544
Klovland et al. (2021)	0.00000	0.014398	0.377364	3.580329	0.060898	0.489756

[22] In Table 3.1.
[23] From feature selection, Section 5.1.
[24] From model interpretation, (3).

Residuals evaluation out-of-sample (train) for 1 Jan 2002 to 1 Aug 2022

	Statistical properties					
Daily models	Mean	SD	Skew	Kurtosis	Normality	Autocorrelation 1 lag
Selected GPSR model	0.00015	0.00496	0.53161	21.21042	0.00000	0.00000
Random forest	0.00024	0.00528	2.19442	31.58850	0.00000	0.00000
Random walk	0.00000	0.00642	2.22015	27.94665	0.00000	0.00004
Linear multivariate regression model of all explanatory variables [25]	0.00040	0.00511	1.76553	27.21822	0.00000	0.00000
Linear multivariate regression model of selected features[26]	0.00026	0.00513	0.82639	23.71541	0.00000	0.00000
Linear multivariate regression model of features in selected GPSR model[27]	0.00029	0.00516	0.98616	24.71395	0.00000	0.00000
	Statistical properties					
Monthly models	Mean	SD	Skew	Kurtosis	Normality	Autocorrelation 1 lag
Re-estimated monthly GPSR model	0.001498	0.014679	-0.869460	4.105141	0.020550	0.682616
Akram (2020)	0.000179	0.021541	-0.311452	3.158641	0.542826	0.776453
Martinsen (2021)	0.000999	0.015482	-0.974719	4.741016	0.005581	0.925423
Klovland et al. (2021)	0.006005	0.016139	-0.177098	2.965665	0.807122	0.829393

[25] In Table 3.1.
[26] From feature selection, Section 5.1.
[27] From model interpretation, (3).

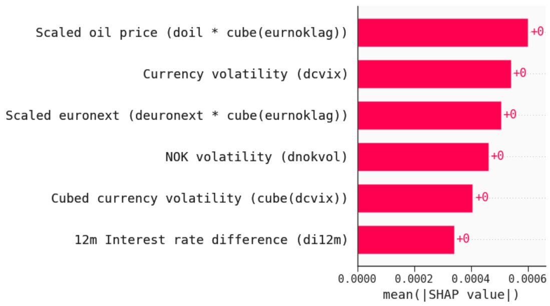
J Re-estimated subsample models

Re-estimated models							
Subsample	α	Δi12m	Δnokvol	Δcvix	$(\Delta\text{cvix})^3$	Δoil $\cdot\,(\text{EURNOK}_{t-1})^3)$	Δeuronext $\cdot(\text{EURNOK}_{t-1})^3)$
1 Jan 2002 - 9 Jan 2009	0.000085	-0.010339	0.001482	0.003744	-0.000367	-0.000067	-0.000103
9 Jan 2009 - 13 Mar 2013	-0.000092	-0.039021	0.001650	0.001789	0.000377	-0.000084	-0.000164
13 Mar 2013 - 23 Apr 2020	0.000151	-0.080072	0.001742	0.000602	-0.000197	-0.000066	-0.000174
23 Apr 2020 - 1 Aug 2022	0.000142	-0.017474	0.001208	0.002744	-0.002914	-0.000060	-0.000171

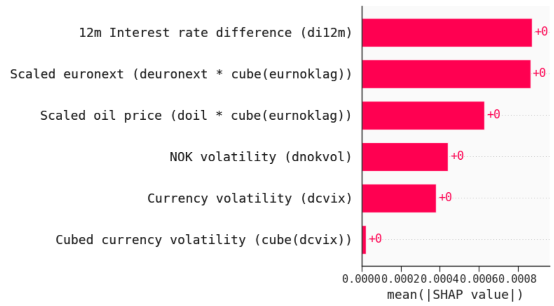
Evaluation of re-estimated subsample models					
	Evaluation metrics				
Subsample	R^2	MSE	RMSE	MAE	Direction of change
1 Jan 2002 - 9 Jan 2009	0.154884	0.000016	0.003969	0.002954	0.579607
9 Jan 2009 - 13 Mar 2013	0.245178	0.000019	0.004348	0.003250	0.639118
13 Mar 2013 - 23 Apr 2020	0.385245	0.000021	0.004572	0.003143	0.666667
23 Apr 2020 - 1 Aug 2022	0.376898	0.000020	0.004466	0.003454	0.684654
2002 - 2022 All subsamples [28]	0.303634	0.000019	0.004317	0.003135	0.633352
2002 - 2022 No subsamples	0.274643	0.000019	0.004407	0.003191	0.625885

[28] Combined model of all four subsample models.

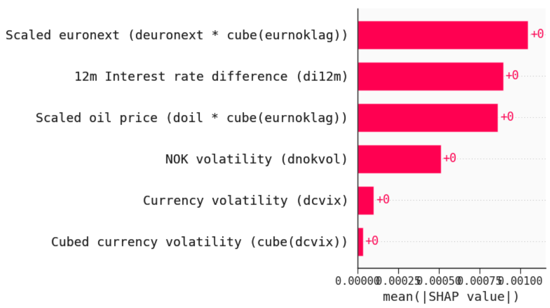
K SHAP-analysis of re-estimated subsample models



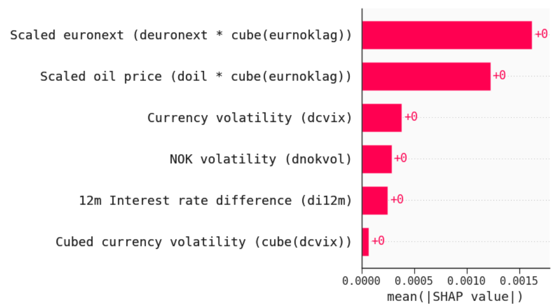
a 1 Jan 2002 to 1 Sept 2009



b 1 Sept 2009 to 13 Mar 2013



c 13 Mar 2013 to 23 Apr 2020



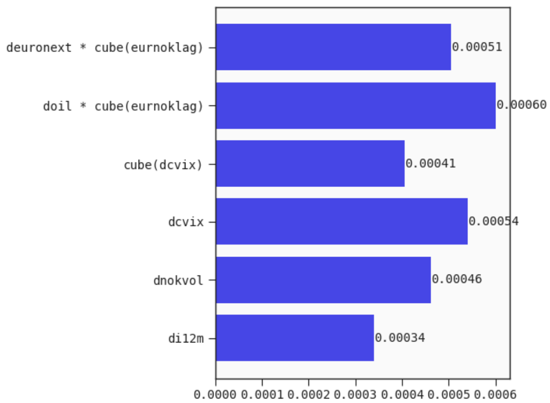
d 23 Apr 2020 to 1 Aug 2022

L SHAP values of features in GPSR model

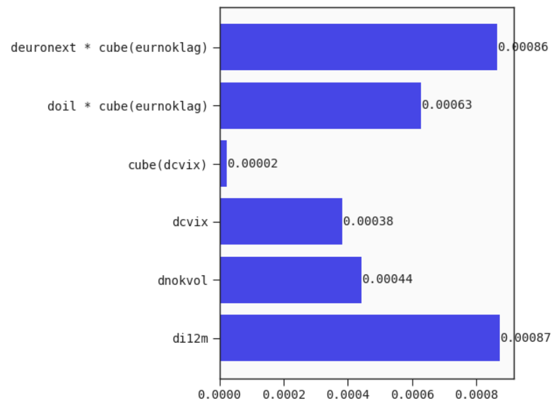
SHAP values of GPSR model features

SHAP values			
Feature	Max SHAP value	Min SHAP value	Spread
Δoil	-0.01534026060544727	0.02192448954673848	0.03726475015221112
EURNOK$_{t-1}$	-0.01153964668706599	0.011623300273313344	0.023162946960379335
Δi12m	-0.005387713656352864	0.007043778636063758	0.012431492292416621
Δoseax	-0.0001989681810672197	0.0005560785445019797	0.0007550467255691994
Δgold	-0.0008635924653626302	0.00023791908376039062	0.0011015115491230208
Δmsciw	-0.00019759653962709974	0.00024679809083144946	0.0004443946214585492
GHI	-0.000509867965031969	0.0007421514152858319	0.001252019380317801
Δi12m$_{t-1}$	-0.0009162289634848534	0.0003360304985747936	0.001252259462059647

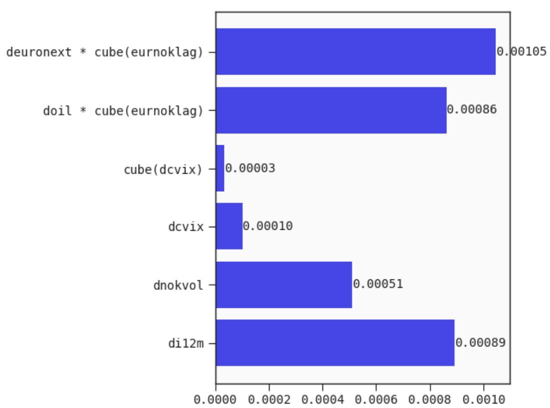
M SHAP-analysis values of of re-estimated subsample model features



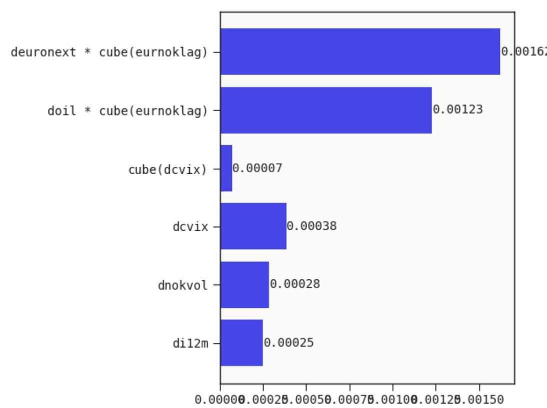
a 01.01.2002 to 09.01.2009



b 09.01.2009 to 13.04.2013



c 13.04.2013 to 23.04.2020



d 23.04.2020 to 01.08.2022

SHAP values in time period 09.01.2009 to 13.04.2013			
Feature	Max SHAP value	Min SHAP value	Spread
Δi12m	-0.01713081045651053	0.032339503898926904	0.04947031435543743
Δnokvol	-0.021799682849686468	0.03856660345642969	0.06036628630611616
Δcvix	-0.009042845118470871	0.03513493135133881	0.044177776469809686
$(\Delta\text{cvix})^3$	-0.01631099061041851	0.0005491512985370489	0.01686014190895556
$\Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3$	-0.007928488598878623	0.025138945663117657	0.03306743426199628
$\Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3$	-0.00259097096553927	0.007683694884255096	0.010274665849794366

SHAP values 2009 to 2013			
Feature	Max SHAP value	Min SHAP value	Spread
Δi12m	-0.018608430113491205	0.02332632893810394	0.04193475905159515
Δnokvol	-0.006816486881874233	0.005094577462635522	0.011911064344509757
Δcvix	-0.005630194894789084	0.005094577462635522	0.014151421788425832
$(\Delta\text{cvix})^3$	-0.005623691915234316	0.0026728625915553228	0.00829655450678964
$\Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3$	-0.0016815143384690792	0.00060833389095 29299	0.002289848229422009
$\Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3$	-0.0008278563466898591	0.001053010989032368	0.001880867335722227

SHAP values 2013 to 2020			
Feature	Max SHAP value	Min SHAP value	Spread
Δi12m	-0.008381369462808364	0.022788351449799665	0.03116972091260803
Δnokvol	-0.03606453427937986	0.056593397914985445	0.0926579321943653
Δcvix	-0.01170432402810761	0.0346972122779319	0.046401536306039506
$(\Delta\text{cvix})^3$	-0.05125452315231	0.0028258736329892903	0.054080396785299294
$\Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3$	-0.007256217507350758	0.02677944020851276	0.03403565771586352
$\Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3$	-0.015411346497823688	0.03641481744422543	0.051826163942049114

SHAP values 2020 to 2022			
Feature	Max SHAP value	Min SHAP value	Spread
Δi12m	-0.033215024600896134	0.036956705756848666	0.0701717303577448
Δnokvol	-0.005076963539386131	0.006562495308011726	0.011639458847397857
Δcvix	-0.008006410040883318	0.007535386725177972	0.015541796766061291
$(\Delta\text{cvix})^3$	-0.021942618959297038	0.02132574723315282	0.04326836619244986
$\Delta\text{oil} \cdot (\text{EURNOK}_{t-1})^3$	-0.05099408411318765	0.01609104480920057	0.06708512892238822
$\Delta\text{euronext} \cdot (\text{EURNOK}_{t-1})^3$	-0.015315600531781126	0.0101617312084903	0.025477331740271426

N Time series cross validation

Mean scores across four folds					
	Evaluation metrics				
Daily models	R^2_{OOS}	MSE	RMSE	MAE	Direction of change
Re-estimated GPSR model	**0.173062**	**0.000020**	**0.004458**	**0.003268**	**0.614959**
Linear multivariate regression model of all explanatory variables [29]	-0.266830	0.000027	0.005152	0.003898	0.588149
Linear multivariate regression model of selected features[30]	0.147594	0.000021	0.004520	0.003319	0.603774
Linear multivariate regression model of features in selected GPSR model[31]	0.153726	0.000021	0.004516	0.003324	0.613797
Random walk	0.000000	0.000025	0.004950	0.003621	0.000295
Random forest	0.134984	0.000021	0.004561	0.003350	0.603774

[29] In Table 3.1.
[30] From feature selection, Section 5.1.
[31] From model interpretation, (3).

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