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The Krone Exchange Rate Puzzle: Uncovering the Power of Market Sentiment and Nonlinearity

Master's thesis in Industrial Economics and Technology Management
Supervisor: Dr. Morten Risstad
Co-supervisor: Dr. Malvina Marchese and Dr. Sjur Westgaard
June 2023

Master's thesis

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Abstract

This thesis examines if the inclusion of market sentiment and nonlinearity improves statistical modeling and forecasting of four different Norwegian exchange rates: USDNOK, EURNOK, CADNOK, and SEKNOK.

A novel approach is used to capture market sentiment with Google Trends and formulate sentiment indices. The merit of these indices is initially assessed within the frameworks of the PPP, UIRP, and monetary structural models. Then, they are integrated into linear and nonlinear autoregressive distributed lag regressions with error correction. The purpose of this methodology is twofold. First, it permits a better assessment of the sentiment indices' significance in more complex models and their interplay with other variables. Second, it inspects the nonlinear influences of commodities, global economic risks, and financial markets on improving the statistical models (in-sample) and forecasts (out-of-sample). The analysis is performed using monthly data from January 2005 until December 2022.

Our initial results indicate that our sentiment indices improve the statistical models for all examined currencies and the forecasting for some. We also provide evidence of an asymmetric relationship between Brent oil futures prices and all studied exchange rates, except for the USDNOK, which exhibits asymmetric ties with the S&P500 stock market. These results underscore the shortcomings of linear models exclusively reliant on traditional macroeconomic and monetary variables for modeling the Norwegian exchange rate, thus signifying a pivotal advancement in understanding the ongoing structural weakening of the Norwegian krone.

Sammendrag

Denne avhandlingen undersøker om inkludering av markedssentiment og ikke-lineæritet forbedrer statistisk modellering og prediksjon av fire forskjellige norske valutakurser: USDNOK, EURNOK, CADNOK og SEKNOK.

En ny tilnærming blir brukt for å fange opp markedssentiment ved hjelp av Google Trends, som benyttes til å utforme sentimentindekser. Verdien av disse indeksene blir først vurdert innenfor rammene av PPP, UIRP og monetære strukturelle modeller. Deretter blir de integrert i lineære og ikke-lineære autoregressive distribuerte lag modeller med feiljustering. Formålet med denne metodikken er todelt. For det første tillater den en bedre vurdering av sentimentindeksenes betydning i mer komplekse modeller og deres samspill med andre variabler. For det andre undersøker den de ikke-lineære påvirkningene av råvarer, globale økonomiske risikoer og finansmarkeder for å forbedre de statistiske modellene og prediksjonene. Analysen utføres med månedlige data fra januar 2005 til desember 2022.

Våre innledende resultater indikerer at sentimentindekser forbedrer statistiske modeller for alle undersøkte valutaer, og prediksjonene for noen av dem, et resultat som er i tråd med tidligere litteratur. Vi finner også et asymmetrisk forhold mellom prisene på Brent oljefutures og alle studerte valutakurser, med unntak av USDNOK, som viser et asymmetriske forhold til S&P500. Disse resultatene fremhever svakhetene ved lineære modeller som utelukkende inkluderer tradisjonelle makroøkonomiske og monetære variabler for å modellere den norske valutakursen, og bidrar dermed med en dypere forståelse av den pågående strukturelle svekkelsen av den norske kronen.

Preface

This thesis completes our Master of Science (MSc) degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). It is written independently by Jarl Christophe Skrivarhaug-Boudier and Airin Thodesen from January to June 2023.

Our motivation for writing this thesis stems from a combination of academic curiosity and professional aspiration. The opportunity to apply our knowledge of empirical finance to gain a deeper comprehension of real-world problems, has been truly rewarding. Moreover, the record low exchange rates of the Norwegian krone sparked our interest, prompting us to investigate the underlying factors driving its development. This study as equipped us with essential insights and knowledge that will be invaluable in our forthcoming professional careers.

We want to express a sincere gratitude to our primary supervisor Dr.Morten Risstad for his academic support. We have received valuable guidance in academic writing and detailed feedback throughout the semester. Additionally, we extend our thanks to Dr.Malvina Marchese for her valuable insights and expertise in empirical finance, which greatly contributed to the quality of our research. We are also grateful to Dr.Sjur Westgaard for his feedback and constructive input.

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1 Introduction

The persistent weakening of the Norwegian krone (NOK), hitting record low exchange rates against the US dollar and Euro, is sparking an increase in national and international interest. Bloomberg points out that the NOK is "the worst G-10 versus the Euro in the past decade" (Tajitsu 2023), a trend with both short- and long-term consequences[1]. In the short-term, a weak krone causes imported inflation, increased difficulty recruiting foreign workers, and higher expenses for Norwegians traveling abroad (WorldBank 2023). Long-term consequences may include a lower standard of living for Norwegian's due to costly imports, challenges for import-reliant industries, increased economic shock vulnerability, and even a potential decrease in foreign direct investment (Solheim 2023). The NOK is steadily depreciating despite a strong Norwegian economy, whose major companies are largely profitable and with the price of its major exports, oil and gas, being relatively high. This makes it difficult to explain the relative decline in the NOK, even for the chief of the Norwegian Central Bank, who, during a press conference on May 4th, 2023, stated that "The weakening we have seen [...] is larger than what we can explain with our models"[2]. The goal of this thesis is to dig deeper into the puzzling depreciation of the krone. We examine if new modelling methods and innovative variables can cast a new light on the short- and long-term movements in the NOK. Our focus lies in investigating whether factors such as market sentiment, nonlinearity in commodities, global economic risks, and financial market might significantly boost in-sample (IS) modeling and out-of-sample (OOS) forecasting capabilities, thereby giving us better tools to make sense of this challenging economic phenomenon.

Our motivation to include market sentiment in exchange rate modeling stems from the growing body of research uncovering the importance of investors' sentiment and the social mood in explaining different macroeconomic variables (see e.g., Ito et al. (2021), Bulut (2018), Castelnuovo & Tran (2017), Seabold & Coppola (2015)). This body of literature is also gaining traction thanks to the extensive access to online sources potentially revealing the public's sentiment. Existing research uses three structural models to assess market sentiment's explanatory and predictive power on the exchange rate: the purchase power parity (PPP), uncovered interest rate parity (UIRP), and the monetary model. Although theoretically sound, these models are facing criticism in empirical studies. Rossi (2013) highlights that these models never consistently outperform a random walk in exchange rate prediction and sheds doubt on the significance of the monetary predictors included in them (interest rates, prices, output, and money).

As such, even though the structural models can offer some insights into the performance of sentiment index variables, they present certain limitations causing uncertainty on the validity of the results. Notably, they use at most two regressors which causes a high risk of omitted variable bias, and their linear nature is in contrast to the strong empirical evidence of nonlinearities (see, e.g. Nusair & Olson (2022) and Akram (2004)). These limitations shape the basis for our novel modeling approach. We extend the literature on market sentiment in exchange rates, by adopting the inclusion of other, more country-specific predictor variables in our models. This includes the oil price, a variable that has been affirmed by Klovland et al. (2021) and Benedictow & Hammersland (2022) to significantly influence the Norwegian currency's fluctuations. Other pertinent predictors include variables related to financial assets, as found by (Reboredo et al. 2016), as well

[1] Norway's weak currency also offers advantages: it enhances export competitiveness, beneficial as Norway's is a net exporter, and supports key sectors i.e. fishing and aluminum production (Olsson 2023).

[2] Press conference (Bache 2023)

as variables representing financial uncertainty, as demonstrated again by (Klovland et al. 2021).

Our study is further inspired by a subset of literature focusing on nonlinearity in exchange rate modeling. Findings provide strong evidence of asymmetric effects, both in the long- and short-run (Saidu et al. 2021). Although several nonlinear models have been tested (Sarno 2003), the nonlinear autoregressive distributed lag model is generally the model of choice used to investigate the relationship between oil price, stock prices, and exchange rates across currencies (see e.g., Saidu et al. (2021), Nazeer et al. (2021), Nusair & Olson (2022)). A similar nonlinear relationship has been observed between oil price and the NOK exchange rate (Akram 2004). Despite this, a timely re-evaluation of the nonlinearities in the NOK exchange rate has yet to be conducted, indicating an unexplored area in the current body of research.

This thesis aims to bridge the gap between the bodies of exchange rate literature mentioned above, exploring a novel approach in constructing a sentiment index and modeling nonlinearity. We focus on several exchange rates, including USDNOK, CADNOK, EURNOK, and SEKNOK, chosen due to their characteristics and relation to Norway; where the US dollar (USD) and the Euro (EUR) are widely regarded as reference currencies, the Swedish krona (SEK) is the currency of Sweden, Norway's neighboring country, and the Canadian dollar (CAD) is another commodity currency. To our knowledge, the inclusion of market sentiment of any kind has never been used to model or predict the returns in the NOK exchange rate. We wish to explore if it can aid the exchange rate modeling, as it has been able to with other currency pairs. Secondly, we propose nonlinear approaches to model the relationship between the krone and a unique combination of non-monetary variables, examining both their short- and long-run relationship. The existing literature on nonlinear models for the krone is limited and mainly focuses on nonlinearity of the oil price. We provide an updated status on the nonlinear relationship between oil price and the krone, but in addition, we conduct similar analyses on other variables, such as financial assets.

Our choice of variables draws inspiration from Bulut (2018) and Ito et al. (2021) to construct a sentiment index that incorporates word categories associated with different macroeconomic phenomena. However, the way in which we define the variable diverges significantly from prior literature, thus culminating in a novel representation of market sentiment. The market sentiment is collected using the online Google Trends tool as data source. To assess the baseline performance of the indices, we commence by examining the change in IS and OOS performance of the structural models, with and without the indices. Then, we construct linear Autoregressive Distributed Lag (ARDL) and nonlinear ARDL (NARDL) regressions, keeping the sentiment indices and including several predictors that have proven to significantly impact the NOK exchange rates. Specifically, we assess whether nonlinearity for oil price, financial assets, and market risk aversion, coupled with the sentiment indices, enhance model and predictive accuracy. We focus on asymmetric ARDLs, but also provide an IS analysis using a threshold ARDL (TARDL), as other literature has obtained promising results with both asymmetric (Saidu et al. 2021) and threshold (Akram 2004) models. Finally, we evaluate the OOS predictive accuracy of our linear ARDLs and asymmetric NARDLs. We fit each model with two-thirds of the dataset and adopt a rolling window prediction approach.

A first compelling finding is that in the structural models, the sentiment index variables are statistically significant and enhance model performance for IS modeling of the EURNOK, USDNOK, and SEKNOK. Sentiment indices also improve the OOS performance for the

EURNOK, underscoring Norway's tight bond with the European economy. However, it does not consistently improve the OOS prediction for the other exchange rates.

Henceforth, the core finding in our research is to provide consistent evidence of predictors' nonlinearity for IS modeling. We find substantial evidence of nonlinear relationship between oil price and exchange rate when multiple control variables are included in the models. This finding holds true across all exchange rates studied, except for USDNOK. For the USDNOK, we note a pronounced asymmetry against the S&P500. We also provide evidence of nonlinearity between market uncertainty and the exchange rate for all but the USDNOK, for which we recommend using other proxies for market risk and volatility. OOS returns promising results for some of the exchange rates. Models including asymmetry in the oil futures and the S&P500 improve prediction for USDNOK and CADNOK. For the SEKNOK, we only observe marginal improvement with asymmetric oil price returns. Finally, it is worth noting that sentiment indices always keep their significance in our nonlinear regression models, further highlighting their potential in exchange rate modeling, both IS and OOS.

The remainder of the paper is organized as follows. Chapter 2 gives an overview of the relevant literature on exchange rate modeling. In Chapter 3, we present the variables extracted, their respective data sets, and the construction of the sentiment index variables. Chapter 4 thoroughly describes the methods used to construct and compare our various models. Our results and discussion are presented in Chapter 5. Finally, in Chapter 6, we conclude our study and suggest further work.

2 Literature Review

The prediction of exchange rates has been a subject of extensive research over the years, and for a long time, researchers agreed that the best-performing model was the driftless random walk, leading to what is known as the Meese and Rogoff puzzle. Meese & Rogoff (1983) investigated multiple candidate models for the exchange rate of major currencies during the seventies, concluding that neither of the models provided more accurate OOS predictions than a simple random walk. Since then, there has been immense research within the field, resulting in a substantial body of literature exploring diverse methodologies and predictor variables in exchange rate prediction and modeling.

Traditional exchange rate modeling has followed three fundamental models: purchase power parity (PPP), uncovered interest rate parity (UIRP), and the monetary model. These take advantage of price and inflation differentials, interest rate differentials, and money and output differentials, respectively, to predict the exchange rate between two currencies. The UIRP principle suggests that a higher interest rate in a domestic country relative to a foreign country should lead to the domestic currency depreciating. However, the recent depreciation of the NOK, despite Norway having lower interest rates than other countries, seems to contradict this. Studies by Klovland et al. (2021) and Martinsen (2017) also find that a negative interest rate difference tends to result in a krone depreciation, indicating a negative correlation. Further, the PPP model suggests that the currency of a country with a higher price level is expected to depreciate against that of a country with a lower price level, resulting in a positive correlation (Taylor 2003). Lastly, the monetary model defines the nominal exchange rate in terms of the relative demand and supply of money in two economies. It presumes that both the PPP and UIRP conditions are met.

The results from the structural models vary and depend on the currency choice, sample frequency, and time period. Clark & West (2006) and Molodtsova & Papell (2009) report positive findings for the UIRP model at short horizons for some countries. Molodtsova & Papell (2009) find limited empirical evidence in favor of the monetary model, while Mark (1995) finds strong and significant evidence in favor of this model at very long horizons. However, the robustness of these positive findings has been questioned, and according to a survey conducted by Rossi (2013) on the literature on predicting exchange rates, the overall conclusion is that these structural models are not sufficient to predict significantly better than a random walk. That survey aims to offer a critical view of the literature on exchange rate prediction for the last ten years.

Rossi (2013) argues that the choice of predictors is crucial for the degree of success in forecasting exchange rates OOS and that the overall empirical evidence is not favorable to traditional predictors, such as interest rates, prices, output, and money. However, a large body of literature also includes other predictor variables, which in many cases are more country-specific. An example is commodity prices, which, e.g., Chen & Rogoff (2003) investigate, focusing on exchange rates for countries where primary commodities constitute a significant share of exports. Norway is a prime example of such a country, given that oil is a major part of its export. More recent studies (see, e.g., Ferraro et al. (2015), Salisu et al. (2019) and Liu et al. (2020)), find strong evidence that commodity prices, amongst them oil price, can predict exchange rates both IS and OOS. For the case of Norway, both Klovland et al. (2021) and Benedictow & Hammersland (2022) include oil price in their models and conclude that oil price has a determining effect on the movements of the Norwegian currency measured against other currencies. Klovland et al. (2021) make a model for the trade-weighted index, which is the nominal effective krone exchange rate calculated on the basis of NOK exchange rates against the currencies of Norway's 25 main trading partners, while Benedictow & Hammersland (2022) try to predict EURNOK. Both conclude that an increase in the oil price generally leads to an appreciation of the krone measured against the respective currencies.

Researchers have also examined financial assets as a predictor variable in regards to their relationship with exchange rates. Given that stock price movements have an impact on capital movements and that changes in currency values have an impact on trade flows, stock, and exchange rate markets are naturally linked. Reboredo et al. (2016) investigate this relationship for several emerging economies, concluding that there indeed exists a positive relationship between stock prices and currency values with respect to the USD and EUR. Similar findings have been conducted for the NOK. Naug (2003), Klovland et al. (2021) and Bernhardsen & Røisland (2000) all investigate if the S&P500 has an impact on the krone exchange rate, concluding that an increase of the stock index will result in a depreciation of the krone. In addition to the findings regarding financial assets, it has also been found that financial uncertainty and volatility affect the krone, see, e.g., Klovland et al. (2021), Benedictow & Hammersland (2022) and Naug (2003).

In addition to the various choice of predictor variables included for exchange rate prediction, model choice also matters significantly. Rossi (2013) compares multiple econometric models, both linear and nonlinear, with the conclusion that linear econometric models, in general, perform better than their nonlinear counterparts, with some exceptions. The most successful models are the single-equation error correction model (ECM) and the panel ECM, although there is some disagreement among researchers regarding its degree of robustness (Rossi 2013). Most of the models that attempt to predict the Norwegian exchange rate also use linear models, e.g., Klovland et al. (2021) and Benedictow & Hammersland (2022) both employ versions of a vector error correction model (VECM). An advantage of this model is that it can capture both short- and long-run effects.

Since the findings in Rossi (2013), the field of exchange rate forecasting has received much new empirical evidence. There has been immense research on nonlinear methods such as neural networks and machine learning, but also the combination of classical economic fundamental-based models with neural network approaches and autoregressive models (Vasconcelos & Hadad Júnior 2023). Furthermore, research shows that using nonlinearities in the relationship between oil prices and exchange rates increases predictive performance. Salisu et al. (2019) build upon the work by Ferraro et al. (2015), who conclude that oil price and other commodities are promising predictor variables for commodity currencies. Salisu et al. (2019) further argue that including nonlinearities in these models, such as asymmetries and structural breaks, offers enhanced forecast results.

Other literature (see, e.g., Saidu et al. (2021), Nazeer et al. (2021), Nusair & Olson (2022)) employs both a linear and nonlinear autoregressive distributed lag (ARDL) model to investigate the relationship between oil price, stock prices, and exchange rates. The linear ARDL is the bound testing approach established by Pesaran et al. (2001), and the nonlinear extension is developed by Shin et al. (2014). These models have shown promising results in several studies, given that they offer several major advantages, e.g., they are applicable regardless of the stationarity properties and can measure both short- and long-run effects. Saidu et al. (2021) employ this model to examine whether oil price has a significant asymmetric long- and short-run effect on the exchange rate of six African net oil importers. The findings suggest an asymmetric effect present, both in the long- and short-run, with some variations across exchange rates.

The findings regarding a potential nonlinear relationship between oil price and exchange rates are important for the NOK exchange rate, and we find some similar results for the Norwegian currency. Akram (2004) finds strong evidence of a nonlinear relationship between the value of the NOK and crude oil price. His findings suggest that the strength of the negative relationship varies with the level and trend in oil price and that the effect on the exchange rate is strongest when prices are below certain thresholds. Ellen (2016) also concludes that the relation between oil price changes and the NOK is nonlinear, and her findings are similar to those of Akram (2004), especially regarding the threshold levels. Contrary to the findings of Saidu et al. (2021), however, they conclude that including asymmetric oil prices does not improve the models significantly for the Norwegian exchange rate.

In addition to the traditional predictor variables, there is an increasing interest in sentiment analysis in the literature on macroeconomic studies. We find a series of papers emphasizing the importance of investor's sentiment and the social mood in explaining different variables, such as unemployment rate, price level, business cycle, and exchange rates (see, e.g., Ito et al. (2021), Bulut (2018), Castelnuovo & Tran (2017), Seabold & Coppola (2015)). They challenge traditional efficient market theory, which relies solely on macro and financial metrics, but fails to consistently create models with performance better than a random walk.

Bulut (2018) and Ito et al. (2021) both use the structural exchange models, PPP, UIRP, and monetary model, to assess the added value of Google Trends-based market sentiment proxies. They reach similar conclusions: the inclusion of market sentiment can outperform structural models and a random walk in many cases. Bulut (2018) carefully selects a set of queries and turns their Google search frequencies into separate variables. These queries are proxies for three major macroeconomic fundamentals believed to impact the exchange rate: 1) the relative pricing in a country, supported by the Purchase Power Parity theory); 2) real income in a country, which in turn affect the money demand and supply; 3) liquidity and credit needs in a country. Ito et al. (2021) builds a sentiment index by pooling together the search frequencies from a set of search queries. The idea of representing market sentiment as an index is inspired by Da et al. (2014), which creates a FEARS index with Google Trends to capture investor pessimism in the United States. Bulut (2018) emphasizes that Google Trends proxies increase correct prediction of the direction of change for the exchange rate. Neither of these papers concludes that Google Trends will solve the problem of exchange rate forecasting, but that it is a useful tool that can contribute to more accurate forecasts.

This study presents a novel approach to analyzing the Norwegian krone exchange rate by integrating previous research elements and incorporating a new sentiment index while accounting for nonlinearity. Our unique index construction derives from Bulut (2018) and Ito et al. (2021) but with notable deviations, yielding an innovative sentiment representation. We also utilize the ARDL and NARDL bounds testing techniques by Pesaran et al. (2001) and Shin et al. (2014), examining the nonlinear relationship between exchange rates, oil prices, and financial assets, as well as their short- and long-term effects.

3 Data and Variables

This chapter presents the data and variables included in the study. We first present all variables included in our models in Section 3.1. In Section 3.2, we offer an overview of all data sets, together with some preliminary data analysis. Section 3.3 justifies using Google Trends to capture market sentiment, while Section 3.4 explains how we construct the sentiment index variables.

3.1 Variables

We study four exchange rates: EURNOK, USDNOK, CADNOK, and SEKNOK. The examination of multiple exchange rates allows us to draw broader conclusions and gain a deeper understanding of how the Norwegian krone fluctuates against various currencies. The USD and EUR are both major currencies and are often regarded as reference currencies, which is why we include them in this study. CAD is also a large currency and a commodity currency, similar to the Norwegian krone. Lastly, we include the SEK, a minor currency compared to the others. However, being Norway's neighboring country and one of its biggest trade partners, we consider their currency dynamic interesting. For the exchange rates, we use Norway as the home country and define the exchange rate as the Norwegian krone value of one unit of foreign currency.

One of the primary objectives of the analysis is to investigate if the inclusion of market sentiment improves IS and OOS performance of various models. As such, we include market sentiment proxies in all models, whose construction is explained in Section 3.4.

To explore the explanatory power of market sentiment, we first develop three structural models commonly utilized in existing literature ((Bulut 2018), (Ito et al. 2021)). These models input the disparities in interest rates, prices, total output, and money between Norway and the other target countries as regressors. These disparities are again created with data sets similar to the ones employed in the above literature: for the interest rate, we employ the three-month interbank offered rate[3]; for the money supply we employ M1, which also is the most liquid money supply class (CFI 2023); for the prices we use the monthly harmonized consumer price index (HCPI) indexed 2015=100; for total output we use the industrial production, with base year 2015=100.

While the variables to include in the structural models are predetermined, we carefully select the most appropriate variables for our extended ARDL and NARDL models. Since the oil industry makes up a large part of Norway's total export, we expect the oil price to be one of the key drivers for the Norwegian exchange rate, as supported by previous literature, such as Klovland et al. (2021) and Benedictow & Hammersland (2022). We incorporate the ICE Brent Crude Future (LCOc1) in USD as our oil price variable. The reason for using this and not another oil price is that ICE's Brent benchmark reflects global oil market fundamentals and around 78% of globally traded physical crude oil is priced off the Brent benchmark, either directly or indirectly (Wittner 2020). We also aim to include financial assets in our models, considering that other literature finds it to be an important driver for many exchange rates, including the NOK. Therefore, we include both the S&P500 and OSEBX. The S&P500 is one of the financial assets most commonly included in models for exchange rates, both for the Norwegian currency and other exchange rates. Additionally,

[3]NIBOR (Norway), EURIBOR (Eurozone), AMERIBOR (US), CDOR (Canada) and STIBOR (Sweden)

due to the substantial impact of the American market on the global economy, we consider it a reliable proxy for measuring global market movements. Furthermore, we incorporate the OSEBX to enhance our ability to capture fluctuations in the Norwegian market, given that it is the 55 most traded stocks on the Oslo Stock Exchange. Although we expect the S&P500 and OSEBX to be highly correlated, we include both to test whether OSEBX is more influential, e.g., on SEKNOK. Notably, while some other literature has found a positive correlation ((Klovland et al. 2021), (Naug 2003)), Thodesen & Thune (2022) find the opposite, especially when including more recent data. Lastly, we include a proxy for market risk, motivated by the findings of Thodesen & Thune (2022), that the correlation between exchange rate returns and oil price, and exchange rate returns and financial assets are greatest when the market uncertainty and risk aversion are high. We use a US risk aversion index recently developed by Bekaert et al. (2021) as a proxy for market risk aversion. It is constructed from a set of six observable financial variables: the term spread, credit spread, detrended dividend yield, realized and risk-neutral equity return variances, and realized corporate bond return variance. This approach makes it possible to distinguish the time variation in economic uncertainty ("the amount of risk") from time variation in risk aversion ("the price of risk"), and offers an unbiased measurement of time-varying risk aversion in financial markets.

In addition to the abovementioned variables, several other variables could be good candidates as additional control variables. However, since our estimation window limits the data points, we struggle with overfitting if we include too many variables in the models.

3.2 Data Overview

Table 1 offers an overview of all the data in this study, alongside some descriptive statistics and the expected sign of their coefficients, which is based on findings in other literature on the NOK exchange rate. All data sets are at a monthly frequency, with data from January 2005 until December 2022. The source of each data set can be found in Appendix A.

Country	Variable	Min	Max	Mean	Std.dev	Expected sign
Euro Area	Exchange rate	7.322	11.323	8.787	0.971	-
	Interest rate diff	-0.218	2.251	1.254	0.553	-
	Price diff	-0.01	0.093	0.010	0.041	+
	Industrial production diff	-0.111	0.307	0.031	0.077	-
	M1 diff	-0.450	0.146	-0.169	0.207	+
	SI	-7.406	7.089	0.024	2.385	+
	SI^*	-5.565	6.816	-0.019	1.664	+
US	Exchange rate	5.085	10.882	7.168	1.405	-
	Interest rate diff	-2.370	3.820	0.651	1.471	-
	Price diff	-0.058	0.064	0.006	0.032	+
	Industrial production diff	-0.084	0.272	0.063	0.084	-
	M1 diff	-1.328	0.157	-0.261	0.449	+
	SI	-11.070	12.368	0.036	2.691	+
	SI^*	-2.725	4.405	-0.041	0.937	+
Canada	Exchange rate	4.991	7.723	6.048	0.597	-
	Interest rate diff	-1.388	3.601	0.490	1.036	-
	Price diff	-0.036	0.066	0.010	0.030	+
	Industrial production diff	-0.146	0.268	0.047	0.099	-
	M1 diff	-0.472	0.154	-0.136	0.188	+
	SI	-10.806	14.866	0.028	2.717	+
	SI^*	-4.305	4.341	-0.022	1.147	+
Sweden	Exchange rate	0.793	1.050	0.911	0.063	-
	Interest rate diff	-0.050	3.290	1.386	0.594	-
	Price diff	-0.057	0.066	-0.006	0.039	+
	Industrial production diff	-0.184	0.168	-0.025	0.073	-
	M1 diff	-0.448	0.145	-0.173	0.162	+
	SI	-7.961	11.820	0.003	2.478	+
	SI^*	-7.035	5.247	-0.038	1.937	+
Other	Brent oil futures	26.350	139.830	76.173	24.506	-
	S&P500	735.090	4766.180	2057.783	979.332	-
	OSEBX	191.570	1178.510	542.632	241.088	-
	Risk aversion index	2.479	8.030	3.086	0.841	-

Table 1: Summary Statistics
The table shows the summary statistics for all variables included in the analysis. Expected sign is based on findings in other literature.

When developing the structural models with and without sentiment indices, we closely adhere to existing literature. Consequently, the selection of variables to be included in the models and the decision not to differentiate them are predetermined. This is based on the understanding that the long-run differences in interest rates, prices, industrial production, money supply, and, not least, the exchange rate between Norway and the other countries are considered stationary. This assumption serves as a fundamental basis for these models. However, it is important to acknowledge that the assumption of stationarity in the long-term has been a subject of criticism in the literature. As can be found in Appendix B, several of the variables are non-stationary for the time horizon considered. This disparity between the assumption of stationarity and the actual lack of stationarity in the data is a limitation of the structural models.

Variable	ADF		PP		KPSS	
	level	1st diff	level	1st diff	level	1st diff
EURNOK	-2.06	-5.73***	-10.1	-149.06***	3.28	0.11***
USDNOK	-2.79	-5.46***	12.4	-212.88***	3.43	0.10***
CADNOK	-3.29*	-7.02***	-28.69***	-	3.79	0.02***
SEKNOK	-2.89	-6.25***	-21.35**	-	3.15	0.06***
Brent	-2.63	-6.13***	-14.86	-156.58***	0.52*	0.06***
$Brent^{+}$	-0.93	-5.63***	-2.82	-190.54***	4.28	0.22***
$Brent^{-}$	-2.53	-6.04***	-12.86	-162.14***	4.35	0.08***
S&P500	-2.45	-6.18***	-8.36	-211.13***	3.93	0.12***
$S\&P500^{+}$	-1.61	-5.29***	-5.11	-231.44***	4.37	0.38**
$S\&P500^{-}$	-1.92	-5.22***	-4.18	-175.82***	4.17	0.14***
OSEBX	-3.17*	-6.51***	-18.96*	-187.43***	3.97	0.04***
Risk	-4.01***	-	-212.35***	-	0.09***	-
SI_{EUR}	-8.97***	-	-218.38***	-	0.04***	-
SI^{*}_{EUR}	-8.55***	-	-252.72***	-	0.03***	-
SI_{USD}	-7.67***	-	-212.35***	-	0.03***	-
SI^{*}_{USD}	-7.66***	-	-300.31***	-	0.09***	-
SI_{CAD}	-8.33***	-	-280.76***	-	0.12***	-
SI^{*}_{CAD}	-10.59***	-	-256.29***	-	0.09***	-
SI_{SEK}	-8.22***	-	-209.61***	-	0.05***	-
SI^{*}_{SEK}	-7.11***	-	-270.58***	-	0.12***	-

Table 2: Unit Root Tests

The table summarizes the result of the three unit root tests ADF, PP and KPSS for all variables included in the models. *, ** and *** indicate a significance level of 10, 5 and 1 percent, respectively.

For the comparison of linear and nonlinear models, however, we need to perform a more thorough preliminary analysis to ensure stationarity and decide which variables are appropriate to test for nonlinear behavior. A prerequisite for the ARDL and NARDL bound testing approach is that all variables are stationary either at level *I(0)* or at first difference *I(1)*. To assess stationarity, we perform three different unit roots tests, namely the Augmented Dickey-Fuller (ADF), the Phillips-Perron (PP), and the Kwiatkowski-Philips-Schmidt-Shi (KPSS) test. ADF and PP test the null hypothesis of a unit root, whereas the KPSS tests the null hypothesis of stationarity. Table 2 presents the results of these tests. We perform the tests for all variables, and for those that are non-stationary in levels, we differentiate them and repeat the test. The three tests mainly agree regarding which variables are stationary in both level and first difference. However, there are slight discrepancies for some variables and the significance level for determining stationarity. As an example, the PP test rejects the null hypothesis that both CADNOK and SEKNOK are non-stationary in levels, at a 1 and 5 percent level, respectively, whereas both the ADF and KPSS fail to reject nonstationarity in levels. Nonetheless, the three tests agree that all variables are stationary, at least in their first differences, which rules out the possibility of *I(2)* variables. Hence, all variables can be included in our models.

Next, we investigate which variables might inhibit asymmetric behavior, and we do so by creating plots that illustrate their relationship with the exchange rates. Outputs for the USDNOK and the EURNOK are presented in Figure 1, and similar visualizations can be found in Appendix C for CADNOK and SEKNOK. We use these plots to assess the asymmetric behavior of the Brent oil futures and the S&P500 prices. The largest movements in the time series are highlighted with pairwise blue and black arrows. The blue arrows represent the level of change in the differentiated regressors, while the black arrows measure the simultaneous movement in the exchange rate return.

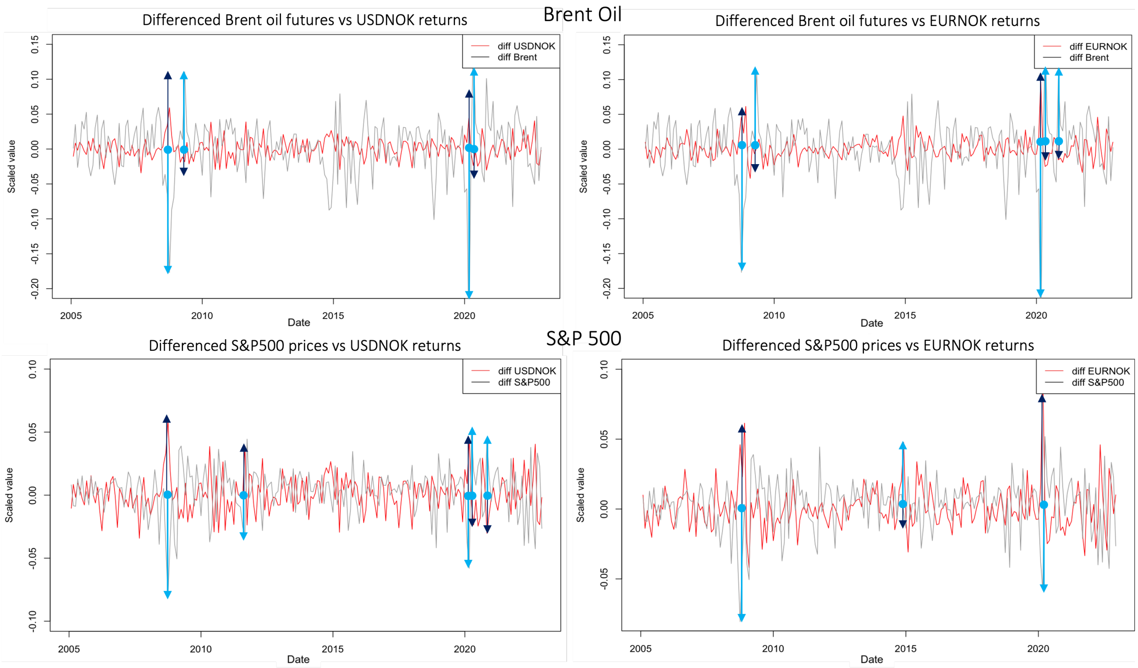

Figure 1: Asymmetries in Brent Oil Futures and S&P500 Prices.

Plots showing the timeseries of the exchange rate returns (in red) and the differenced Brent futures or S&P500 (in grey) to assess whether up and down movements in the latters are correlated with asymmetric behaviors in the movement of the former. Blue arrows show the amplitude of a movement in the regressors and black arrows highlight the simultaneous movement in oil returns.

We observe that the exchange rates and Brent oil futures often move in opposite directions, suggesting a negative correlation. This is visible when comparing the position of the red (exchange rate return) time series with the grey (differentiated oil futures) time series at different points in time and can also be observed with the opposite facing black and blue arrows. In addition, we observe that a negative swing in the oil futures tend to be associated with a more significant movement in the exchange rates than a positive swing does. This suggests a degree of asymmetry in the Brent oil futures, which is visible for all four exchange rates. Similar tendencies are visible for the S&P500 regressor. It also looks negatively correlated to the exchange rate returns, and its negative swings correlate with bigger reactions in the exchange rate returns than positive swings. These observations, along with the body of literature (e.g., Saidu et al. (2021), Nazeer et al. (2021), Nusair & Olson (2022)) supporting the existence of asymmetry in commodities' and stock exchanges' effect on foreign exchange rates makes it sensible to inspect whether asymmetry will improve IS modeling and OOS predictions.

Similarly, we wish to investigate if there are any visual indications that low and high risk levels influence the different regressors' impact on the exchange rates. This hypothesis is motivated by the finding of Thodesen & Thune (2022), that the correlation between exchange rate returns and oil price, and exchange rate returns and financial assets are greatest when the uncertainty and risk aversion in the market is high. Therefore, we graph the risk aversion index against each regressor and response variable. See plots for USDNOK and EURNOK in Figure 2. It is possible, but not obvious, to trace potential risk thresholds in these graphs, for which the variables seem to behave differently above and under the threshold. This observation and the findings of Thodesen & Thune (2022) suggest that risk might impact other regressors worth exploring.

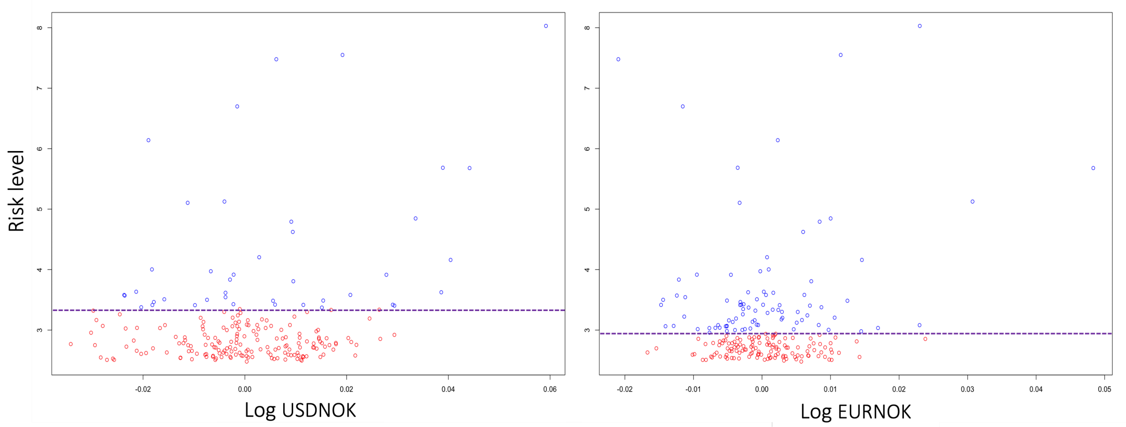

Figure 2: Threshold with Risk Aversion Index on Exchange Rates
Plots with potential thresholds (purple lines) that separate the data into 2 data sets with different regressions, with the risk aversion index of Bekaert et al. (2021) as threshold

3.3 Google Trends

We wish to evaluate the prediction power of market sentiment on the NOK exchange rate against a set of foreign currencies. With access to ever larger datasets on the internet, market sentiment's role in financial decision-making and behavioral finance has become a rising area of research. Three prominent types of online data sources are used to create market sentiment proxies in financial analysis: social media feeds, search engine data, and news sites. Due to its several advantages, we decide to construct a market sentiment index using Google's search engine.

Google is the dominant search engine in all countries of interest for this study. More than 90% of all internet search goes through Google for Norway, Sweden, Europe, USA, and Canada (Statista 2023). Social media platforms are not as suitable for international comparison as they tend to have a regional focus. For instance, Twitter, commonly employed as a proxy for market sentiment, has until recently mostly had an American audience, making it inadequate for measuring market sentiment of the total population outside of the US. Further, although news sites are perceived to be an important source for the investor's sentiment, the large number of new sites necessary to monitor and the subsequent natural language processing necessary to interpret their information makes this data source inconvenient. In contrast, Google provides the convenient Google Trends tool that lets you extract the monthly search frequency for a query and specify a desired period and region. These search frequencies are normalized to take values from zero to 100 (with 100 representing the month with the word's highest relative search frequency) and are known as the Search Volume Index (SVI). The SVI also accounts for any apparent increase in search volume due to an overall increase in internet users. Research in behavioral economics indicates that people tend to query things that concern them, and, as such, SVIs can be considered as a measure of revealed expectations (Woloszko 2020). Our study explores the possibility of using the Google Trends tool on judiciously selected queries to capture the market sentiment.

3.4 Construction of Sentiment Indices

We combine the method of Bulut (2018) and Ito et al. (2021) to create a sentiment index to capture the market sentiment in each country. For every exchange rate under study, we construct two indices. This results in a total of eight unique indices. The first index is specifically tailored to represent market sentiment in Norway, while the second index encapsulates the sentiment within the respective other country or region corresponding to the currency studied. To clarify, these indices are designed to uniquely correspond to a specific exchange rate. For instance, when examining the USDNOK, we establish one tailored index for Norway and another distinct index for the US. This approach ensures that our indices are acutely sensitive to the distinct variables and sentiments influencing each exchange rate.

Bulut (2018) proposes categories of words that fit different macroeconomic phenomena worth including in a sentiment index. We adopt his categorization scheme and expands it with search words used in other research as proxies for these categories. We also append words classified as finance or economy-related in the Oxford dictionary and assess whether they fit one of the fundamentals and are common enough to be used by the wider population. Finally, we include two classes of words 1) Oil and natural gas, and 2) Stock markets, as the Norwegian economy is heavily reliant on its exports of these commodities and a body of research ((Reboredo et al. 2016), (Naug 2003), (Klovland et al. 2021), (Bernhardsen & Røisland 2000)) finds a significant correlation between stock prices and exchange rate movements. Our motivation to go beyond the current market sentiment extraction is that no existing methodology outperforms the others, and a standard has yet to be established.

We first construct the American vocabulary and translate it to create a unique vocabulary for each language. This process yields vocabularies consisting of 85 words in English, which are then matched in size for Norwegian, French, German, and Swedish. We extract the English vocabulary for each region, as English is the most or second most spoken language in all countries. If a word is identical in different languages in a region, we only query it once. We also extract the French and German vocabularies as languages in the Euro region, as France and Germany are the two biggest economies in the Euro region and comprise over half of the Euro area GDP (Capitalist 2023).

Nonetheless, this methodology requires a significant number of Google Trends queries to obtain the desired data. See Table 3 for the English vocabulary. The vocabularies for the remaining languages can be found in Appendix D.

Categories	Respective Vocabulary
Relative pricing	Inflation, Prices, CPI, Cheap, Interest rate, Electricity, Expensive, Deflation, Quantitative easing, Conversion rates, Exchange rate, Euro nok, Euro dollar, Dollar nok
Real income	Buy, Spend, Stocks, Save, Donate, Job, Foreclosure, Vacation, Layoff, Invest, Economy, Shopping, Loan, Debt, Budget, Economic recovery, Upturn, Bear market, Bull market, Easy money, Import, Export, Patent, Intellectual property, Luxury, Gift, Shortage, Bet, Gamble, Deferred payment, Insurance, Social support, Social benefits, GDP, Privileged, Pollution, Co2 emissions
Liquidity and risk	Bankruptcy, Cash, Credit, ATM, Withdrawal, Liquidation, Risk, Instability, Economic bubble, Economic Depression, Protectionism, Financial crisis
Oil and natural gas	Oil, Oil price, Crude oil, Brent oil, Natural gas, LNG, Oil futures
Stock markets	S&P, NYSE, Nasdaq, Euronext, London Stock Exchange, Deutsche Börse, Stock exchange, ECB, Federal Reserve, Norwegian oil fund, Norwegian pension fund, Oslo Stock Exchange, FTSE, MSCI, Oljefondet

Table 3: Vocabulary of English Search Words

Vocabulary of English search words whose SVIs are extracted from Google Trend. The search words are proxies to different macroeconomic fundamentals

We retrieve all search words at once. As Google Trends randomly samples 30% of Google's total searches each day, there is a slight difference in data downloaded at different times. However, Da et al. (2014) show that the SVI downloaded at different times has a correlation of 97% or higher, suggesting that the effect of sampling error will be limited. Still, we follow the best practice from earlier literature ((Ito et al. 2021), (Bulut 2018)) and retrieve all data on the same day to try to keep the error as small as possible.

Then, we follow the index construction approach outlined in Ito et al. (2021) to turn each region's SVIs into a market sentiment index. First, we remove SVIs with frequent null values. We differ from Ito et al. (2021) in that we do not remove all data with null entrances, as we quickly observe that for smaller nations (Norway, Sweden), there is a tendency for more null values, especially in the first years. A null value is also not the same as a missing value. Instead, null values reveal a relatively low search frequency, which can be interesting information. With that reasoning, one might be tempted to keep all search words independent of their distribution. However, high levels of insignificance with a sudden peak imply that the search word is strongly related to a particular event in time, making it a poor predictor for forecasting future events. We observe that setting a 40% maximum threshold of null values is preferable, as it balances the need to include SVIs that may have become more prominent over the last decade while avoiding including search words with high insignificance punctuated by sudden relevance.

When visualizing the time series for each search word, we usually see abnormally strong volatility in their search frequency between 2004 and 2005. This is the first year Google started to measure SVI, and with still relatively few people using the search engine in that year, values become slightly biased. Therefore, we gather time series for January 2005 to December 2022, removing the first year of faulty data. We observe the presence of yearly seasonality for each of the words. Hence, as our second step, we remove the seasonality component from the keywords' time series. Third, we check the stationarity of all time series and observe that the majority are nonstationary. This finding is similar to Ito et al. (2021) findings for the English and Japanese vocabulary, and they decide to differentiate all variables to stationarize. Although this leads to some information being lost for already stationary keywords, this is necessary for the subsequent index creation, as all values should be scaled similarly to have the same level of importance. Thus, we differentiate all variables as well. With data cleaned and all SVIs preprocessed, we start building the sentiment indices for all regions.

We begin with the US and calculate the "predictive" correlation between the country's SVIs and the USDNOK exchange rate, meaning the correlation between SVI_t and $USDNOK_{t+1}$. We differ from Ito et al. (2021), which calculate the correlation of both parameters at time t. However, since we are interested in the predictive power of the sentiment indices, we would rather construct them using SVIs that demonstrate increasing popularity prior to specific movements in the exchange rate rather than those that react simultaneously with the exchange rate.

We select the 15 SVIs with the highest positive correlation and the 15 SVIs with the highest negative correlation for each country, discarding the remaining SVIs. A total of 30 SVIs is kept, since Ito et al. (2021), who experimented with various sizes, conclude that it constitutes the optimal number for constructing an index. Finally, we construct the US sentiment index by combining the high correlation SVIs as shown in formula 1:

$$SI_t^{US} = \sum_{i=1}^{15} \Delta SVI_{i,t} - \sum_{j=1}^{15} \Delta SVI_{j,t} \tag{1}$$

where $\Delta SVI_{i,t}$ denotes the log difference SVI for the term ranked i in the top 15 most positively correlated words, while $\Delta SVI_{j,t}$ is the same for the most negatively correlated words. The SI_t^{US} is defined as this until December 2014. After that, we update the 30 words by recalculating the correlations, expanding the period to the most recent month. We continue to expand the window and update the word list every 12 months until we have sentiment indices for the whole sample period. This process is then repeated for the Norwegian sentiment index against the USDNOK and the three remaining exchange rate pairs.

4 Methodology

Having introduced all variables, we now describe our methodology. First, we test if the inclusion of the sentiment indices enhances predictive performance of three structural models. This assessment mirrors the methodology typically employed in determining the effectiveness of market sentiment proxies, as detailed in previous studies (Bulut (2018) and Ito et al. (2021)). However, the models in these studies are linear, focus solely on monetary fundamentals, and do not include any control variables. As such, we extend our analysis by constructing regression models that include additional predictors with established significance, such as oil prices and financial assets. We also incorporate nonlinearity in our models, given the empirical evidence of asymmetries (e.g., Nusair & Olson (2022), Saidu et al. (2021)) and threshold values (e.g., Akram (2004), Ellen (2016)) in exchange rate modeling. The emphasis in our nonlinear models is on IS modeling, with a secondary focus on forecasting. As we continue to include the sentiment indices, we can investigate if these variables are suitable in models with variables other than monetary fundamentals. Section 4.1 describes constructing the benchmark structural models with sentiment indices. In Section 4.2, we describe our extended approach to nonlinear modeling.

4.1 Comparison of Structural and Market Sentiment Models

Similar to Bulut (2018), we use the Purchasing Power Parity (PPP), the Uncovered Interest Rate Parity (UIRP), and the monetary model to act as base models for comparison. An important assumption made by the studies employing these models is that exchange rates have a constant fundamental value, that it often diverts from in the short- or even medium-run, but that it always returns to in the long-run. We define s_t as the natural logarithm of the nominal spot exchange rate, with Norway as the home country. Then, we model the exchange rate via a single equation model of the following form:

$$s_t = c + f_t, \tag{2}$$

where c is a constant term, and f_t is the model-specific fundamental of the log exchange rate, which will be the price differential, interest rate differential, and the difference between money input and output in the three different structural models. To forecast the exchange rates, we then plug s_t into the following equation:

$$y_{t+1} = s_{t+1} - s_t = \alpha + \beta x_t + \epsilon_t, \tag{3}$$

where x_t is the deviation of the log exchange rate from its fundamental value, $f_t - s_t$, and ϵ captures the stationary disturbance term. In each structural model, we compare Equation 3 to its counterpart, Equation 4, which includes sentiment indices of the two countries.

$$y_{t+1} = s_{t+1} - s_t = \alpha + \beta x_t + \gamma_1 SI_t + \gamma_2 SI_t^* + \epsilon_t, \tag{4}$$

In the PPP model, the exchange rate fundamental takes the value $f_t = p_t - p_t^* - s_t$, where p_t is the price level in Norway, and p_t^* is the price level in the foreign country. This gives us the following models with and without the sentiment indices:

$$y_{t+1} = \alpha + \beta(p_t - p_t^* - s_t) + \epsilon_t \tag{5}$$

$$y_{t+1} = \alpha + \beta(p_t - p_t^* - s_t) + \gamma_1 SI_t + \gamma_2 SI_t^* + \epsilon_t \tag{6}$$

where SI_t and SI_t^* are the sentiment indices for Norway and the foreign country, respectively.

For the UIRP model, $f_t = i_t - i_t^* + s_t$, where i_t and i_t^* are the interest rates. The model formulation in Equation 7 suggests that when the interest rate is higher in the home country compared to the foreign country, the home country's currency depreciates. The regression models for UIRP are expressed as follows:

$$y_{t+1} = \alpha + \beta(i_t - i_t^*) + \epsilon_t \tag{7}$$

$$y_{t+1} = \alpha + \beta(i_t - i_t^*) + \gamma_1 SI_t + \gamma_2 SI_t^* + \epsilon_t \tag{8}$$

Lastly, in the monetary model, the exchange rate fundamental takes the following form, $f_t = m_t - m_t^* - k(g_t - g_t^*)$, where m_t and m_t^* are the natural log of the money supply in Norway and abroad, respectively, and g_t and g_t^* indicate the natural log of real income. k is the income elasticity of money demand. Following both Mark (1995) and Bulut (2018), we assume that k is constant at one. This gives us the following models:

$$y_{t+1} = \alpha + \beta(m_t - m_t^* - (g_t - g_t^*) - s_t) + \epsilon_t \tag{9}$$

$$y_{t+1} = \alpha + \beta(m_t - m_t^* - (g_t - g_t^*) - s_t) + \gamma_1 SI_t + \gamma_2 SI_t^* + \epsilon_t \tag{10}$$

We adopt four different tests to compare the performance of the structural models with and without sentiment indices: the Mean Squared Prediction Error (MSPE), the Clark and West test (CW test), Theil's U-test, and the sign test as a DOC measurement. The CW test is a statistical significance test for the equal predictability of a structural model and a martingale difference model. Statistically significant positive CW test statistics indicate that model i performs better than the benchmark model b. The null is rejected at a 10% level if the statistic is greater than 1.282 or at a 5% level if it is greater than 1.645 (Clark & West 2007). The Theil's U (TU) statistic compares the MSPE of model i to that of a driftless random walk. If the value of TU is less than one, the model under consideration gives more accurate forecasts than the benchmark. Lastly, the DOC test indicates whether model i can predict the correct direction of change. We provide technical details of these tests in Appendix E.

In addition to comparing the models with and without sentiment indices with each other, we compare them to a benchmark RW without drift. All the tests are based on OOS forecasts. We calculate the OOS forecasts via the rolling regression method, following Bulut (2018). We divide the complete set into a training and testing window, with the initial training window consisting of approximately two-thirds of the dataset, corresponding to a training sample of 180 observations, to produce 80 OOS forecasts.

4.2 Comparison of Linear and Nonlinear Models

While the structural models offer some insights into the performance of sentiment index variables, they also present certain limitations that introduce uncertainty to the validity of the results. Notably, these models assume exchange rate stationarity, a condition that does not seem to hold true for many exchange rates over extended time periods. The UIRP and PPP use a single regressor, while the monetary model uses only two regressors in each regression, causing a high risk of omitted variable bias. Moreover, their linear nature contrasts the strong empirical evidence of nonlinearities ranging from asymmetries (Nusair & Olson 2022), to regime switches (Akram 2004).

This study addresses these limitations as we develop models that relax the stationarity assumption, incorporate additional regressors and test for nonlinearity. Consistent with the methodological approach of preceding research, we conduct a thorough IS analysis. Finally, we further extend these studies with OOS predictions.

Our methodology aligns with several papers which examine the asymmetric effect of oil prices on exchange rates through the ARDL and NARDL bound testing approach (see, e.g., Saidu et al. (2021), Nazeer et al. (2021), Nusair & Olson (2022)). A study of particular interest is that of Saidu et al. (2021), investigating this effect for six African net oil-importing countries. However, their study is limited to IS modeling. In contrast, we expand their approach to include prediction using the same methodology to determine whether asymmetry can enhance predictive performance. Additionally, we build upon the linear specification and add threshold values. Lastly, while Saidu et al. (2021) only consider the asymmetric relationship between oil price and exchange rates, we investigate if there is a nonlinear relationship between exchange rates and other variables, such as financial assets.

Section 4.2.1 describes the linear and nonlinear ARDL bound testing approach, while section 4.2.2 explains our approach to the TARDL.

4.2.1 Asymmetric Model

We build an alternative model for the effect of oil price and financial assets on exchange rates based on the theory of one price. The augmented model can be described as follows:

$$s_t = \alpha + \beta_1 op_t + \beta_2 s\&p_t + \beta_3 obx_t + \beta_4 SI_t + \beta_5 SI_t^* + \epsilon_t \tag{11}$$

where s_t is the exchange rate, op_t is the oil price, $s\&p_t$ is the S&P500, obx_t is the OSEBX, and SI_t and SI_t^* are the sentiment indices. We define all variables in logarithmic form. The βs are the long-run estimates of the variables. To account for asymmetry, Equation 11 can be expressed as:

$$s_t = \alpha + \beta_1^+ op_{t-1}^+ + \beta_2^- op_{t-1}^- + \beta_3 s\&p_t + \beta_4 obx + \beta_5 SI_t + \beta_6 SI_t^* + \epsilon_t \tag{12}$$

where op_{t-1}^+ and op_{t-1}^- are positive and negative oil price decomposition using partial sum process. We construct the same equation where we decompose the S&P500 into one negative and one positive sum.

Our primary interest is to test whether asymmetry for oil price and financial assets, and the inclusion of sentiment indices results in more accurate models and predictions. However, to avoid the issue of omitted variable bias, we include every variable in all the models, contrary to some other literature that only includes, e.g., the oil price when testing for asymmetry.

Next, we specify the linear ARDL model. We employ the ARDL bound testing approach established by Pesaran et al. (2001), which is a cointegration method developed to test the presence of a long-run cointegration relationship between variables through a bounds test. Compared to a normal ARDL, an advantage of this technique is that it is applicable regardless of the stationarity properties. Furthermore, it is a convenient approach because it can estimate both short- and long-run effects through an error correction model (ECM) and tends to perform better when dealing with small sample sizes (Pesaran et al. 2001). This gives us the following equation for the ARDL ECM:

$$\begin{aligned}\Delta s_t = \alpha + \lambda_1 s_{t-1} + \lambda_2 op_{t-1} + \lambda_3 s\&p_{t-1} + \lambda_4 obx_{t-1} + \lambda_5 SI_{t-1} + \lambda_6 SI^*_{t-1} \\ + \sum_{k=1}^{n} \beta_k \Delta s_{t-k} + \sum_{k=0}^{n} \delta_k \Delta op_{t-k} + \sum_{k=0}^{n} \gamma_k \Delta s\&p_{t-k} + \sum_{k=0}^{n} \eta_k \Delta obx_{t-k} \\ + \sum_{k=0}^{n} \zeta_k \Delta SI_{t-k} + \sum_{k=0}^{n} \zeta^*_k \Delta SI^*_{t-k} + \epsilon_t\end{aligned} \tag{13}$$

where Δ is the first difference operator. The λs are the long-run estimates of the variables, while the short-run effects are presented using the coefficient for each first difference. We estimate the model using maximum likelihood estimation (MLE). We can now calculate the F-statistic of Pesaran et al. (2001) to establish cointegration between our variables. The null hypothesis of no cointegration is $H_0 = \lambda_1 = ... = \lambda_6 = 0$. The F-statistic is compared to the critical value tabulated in Pesaran et al. (2001), Table (C1.iii), Case III: Unrestricted intercept and no trend. The values we use for comparing the specific models in this study can also be found in Appendix F. If the F-statistic is smaller than the lower bound value, the null hypothesis cannot be rejected, while if it is above, we reject the null to establish long-run cointegration. The result is believed to be inconclusive if the value falls between the lower and upper bound.

Shin et al. (2014) extend this method to allow for asymmetries, which give us the NARDL ECM. The way we allow for this is to decompose the variable, in our case, Brent oil futures and S&P500, into positive and negative partial sum processes. We carry out the decomposition (here for oil futures) as follows:

$$op_t^+ = \sum_{j=1}^{t} \Delta op_j^+ = \sum_{j=1}^{t} max(\Delta op_j, 0) \tag{14}$$

$$op_t^- = \sum_{j=1}^{t} \Delta op_j^- = \sum_{j=1}^{t} min(\Delta op_j, 0) \tag{15}$$

We substitute the new asymmetric variables into Equation 13, which results in the complete asymmetric expansion of the linear ARDL ECM model:

$$\begin{aligned}\Delta s_t = \alpha + \lambda_1 s_{t-1} + \lambda_2 op_t^+ + \lambda_3 op_t^- + \lambda_4 s\&p_{t-1} + \lambda_5 obx_{t-1} + \lambda_6 SI_{t-1} + \lambda_7 SI^*_{t-1} \\ + \sum_{k=1}^{n} \beta_k \Delta s_{t-k} + \sum_{k=0}^{n} \delta_k^+ \Delta op_{t-k}^+ + \sum_{k=0}^{n} \delta_k^- \Delta op_{t-k}^- + \sum_{k=0}^{n} \gamma_k \Delta s\&p_{t-k} \\ + \sum_{k=0}^{n} \eta_k \Delta obx_{t-k} + \sum_{k=0}^{n} \zeta_k \Delta SI_{t-k} + \sum_{k=0}^{n} \zeta^*_k \Delta SI^*_{t-k} + \epsilon_t\end{aligned} \tag{16}$$

We now perform the Wald test for asymmetry. The null hypothesis of no asymmetry is $H_0 : \sum_{k=0}^{n} \delta_k^{+} = \sum_{k=0}^{n} \delta_k^{-}$ and $H_0 : \lambda_2 = \lambda_3$, for short-run and long-run asymmetry, respectively. Furthermore, similar to the linear specification, we calculate the F-statistic to check for long-run cointegration. According to Shin et al. (2014), we can use the same critical values for the null hypothesis as in the linear ARDL ECM.

4.2.2 Threshold ARDL

The next step is to perform a threshold regression. The primary purpose with this is to determine whether incorporating a threshold can enhance model performance, rather than solely aiming to create the most effective model. Therefore, we employ a linear ARDL without error correction as a benchmark model for comparison purposes and include a threshold to this model. We test several variables, including Brent oil futures and the risk aversion index by Bekaert et al. (2021) as a threshold value. This gives us the following regression model:

$$\Delta s_t = \begin{cases} \alpha + \sum_{k=1}^{n} \beta_k \Delta s_{t-k} + \sum_{k=0}^{n} \delta_k \Delta op_{t-k} + \sum_{k=0}^{n} \gamma_k \Delta s\&p_{t-k} \\ + \sum_{k=0}^{n} \eta_k \Delta obx_{t-k} + \sum_{k=0}^{n} \zeta_k SI_{t-k} + \sum_{k=0}^{n} \zeta_k^{*} SI_{t-k}^{*} + \epsilon_t & \text{if } x_t > c \\ \alpha + \sum_{k=1}^{n} \beta_k \Delta s_{t-k} + \sum_{k=0}^{n} \delta_k \Delta op_{t-k} + \sum_{k=0}^{n} \gamma_k \Delta s\&p_{t-k} \\ + \sum_{k=0}^{n} \eta_k \Delta obx_{t-k} + \sum_{k=0}^{n} \zeta_k SI_{t-k} + \sum_{k=0}^{n} \zeta_k^{*} SI_{t-k}^{*} + \epsilon_t & \text{if } x_t < c \end{cases} \tag{17}$$

where x_t represents the threshold variable in the different models, i.e., both oil futures and risk aversion, and c is a constant representing the optimal value for the threshold. Contrary to the ARDL bound testing approach, where we could include both $I(0)$ and $I(1)$ variables, this model requires stationary variables to obtain valid results. Therefore, we differentiate all variables that are not stationary in levels.

5 Results and Discussion

This chapter presents our numerical results alongside a discussion regarding our main findings. We begin with the results obtained from comparing the structural models with and without sentiment indices in Section 5.1. Section 5.2 presents the results from comparing linear and nonlinear models.

5.1 Comparison of Structural and Market Sentiment Models

In this section, we present the results of the structural models with and without sentiment indices. Following earlier literature, we focus on OOS predictions. However, since the subsequent comparisons between linear and nonlinear models are mainly IS analyses, we conduct an IS analysis of the structural models as well, in order to get a deeper understanding of the significance of the sentiment indices.

Exchange Rate	Variable	PPP Model		UIRP Model		Monetary Model	
		PPP	With SI	UIRP	With SI	Monetary	With SI
EURNOK	intercept	0.0456	0.0353	-0.0167***	-0.0160***	0.0404**	0.0361**
	f_{t-1}	0.0206	0.0158	0.0127***	0.0122***	0.0168**	0.0150**
	SI_{t-1}	-	0.0021***	-	0.0020***	-	0.0020***
	SI^*_{t-1}	-	0.0026	-	0.0017	-	0.0016
	AIC	-954.5	**-964.7**	-965.2	**-975.6**	-958.3	**-968.2**
USDNOK	intercept	-0.0009	-0.0017	0.0005	0.0009	0.0054	0.0046
	f_{t-1}	-0.0018	-0.0022	0.0022	0.0018	0.0013	0.0009
	SI_{t-1}	-	0.0026***	-	0.0026***	-	0.0026***
	SI^*_{t-1}	-	0.0081***	-	0.0081***	-	0.0081***
	AIC	-814.8	**-835.2**	-816.7	**-836.6**	-814.9	**-835.2**
CADNOK	intercept	0.0357	0.0342	0.0015	0.0016	0.0054	0.0059
	f_{t-1}	0.0189	0.0181	0.0005	0.0005	0.0018	0.0020
	SI_{t-1}	-	0.0012	-	0.0012	-	0.0012
	SI^*_{t-1}	-	0.0023	-	0.0024	-	0.0024
	AIC	**-941.8**	-942.6	**-941.1**	941.8	**-941.0**	-941.9
SEKNOK	intercept	-0.0053*	-0.0049*	-0.0040	-0.0033	0.0004	0.0003
	f_{t-1}	0.0627**	0.0572**	0.0028	0.0023	0.0057	0.0058
	SI_{t-1}	-	0.0011*	-	0.0011*	-	0.0011*
	SI^*_{t-1}	-	0.0021***	-	0.0022***	-	0.0022***
	AIC	-1010	**-1021**	-1007	**-1018**	-1006	**-1010**

Table 4: IS Result of Complete Sample Period

The table presents the coefficients for all structural models with and without the sentiment indices. *, ** and *** indicate a significance level of 10, 5 and 1 percent, respectively. The AIC is used as a measure for comparison, and the value is in the bottom row for each exchange rate. We compare each structural model with its equivalent sentiment index based model, and the best performing model for each pair is marked in bold.

Table 4 presents the results from the IS regression of the whole sample period. The most noteworthy discovery from these regressions is that the sentiment indices are statistically significant for EURNOK, USDNOK, and SEKNOK. Additionally, the AIC shows an improvement in model performance for these respective currencies when including the indices, which suggests that they inhibit explanatory power that the monetary fundamentals lack.

We proceed with the OOS forecast with a rolling window, similar to the approach of Bulut (2018) and Ito et al. (2021). Table 5 presents the MSPE for all the structural models with and without the sentiment indices. As indicated by the lowest MSPE, the most accurate prediction is highlighted in bold for each exchange rate, and the best-performing model varies across exchange rates. This aligns with other literature that also finds that the best-performing model tends to vary between exchange rates and sample periods.

	Null	PPP Model		UIRP Model		Monetary Model	
	RW	Without SI	With SI	Without SI	With SI	Without SI	With SI
Euro area	4.18	4.18	**4.15**	4.32	4.25	4.42	4.33
US	6.69	6.95	7.02	**6.61**	6.62	7.44	7.35
Canada	**3.34**	3.44	3.48	3.34	3.39	3.35	3.39
Sweden	2.72	**2.69**	2.81	2.79	2.90	2.96	3.08

Table 5: MSPE OOS Results
The table presents the MSPE for each structural model and the sentiment index models.

There is an improvement in MSPE for EURNOK for all the models when including the sentiment indices. This indicates that the indices can capture market information which improves predictive performance, which the monetary fundamentals do not capture. For all the other currencies, however, there is no improvement besides for the monetary model for USDNOK.

We double-check our findings with the Theil's U statistic and Clark West test. The results align with the findings in Table 5 and can be found in Appendix G. The DOC statistic is presented in Table 6. Regarding the EURNOK exchange rate, incorporating sentiment indices leads to an improvement in terms of MSPE and enhances the direction of change for two out of three models. Without the inclusion of sentiment indices, the accuracy of predicting the direction of change using PPP and UIRP is below 50%. However, when these indices are integrated into the models, the accuracy improves to over 50%. For the remaining exchange rates, the inclusion of sentiment indices improves the DOC statistic in some cases, while it does not have a significant impact on other models. Since there is no notable improvement in MSPE, it is understandable that we do not observe significant enhancements in the DOC statistic either.

	PPP Model		UIRP Model		Monetary Model	
	Without SI	With SI	Without SI	With SI	Without SI	With SI
Euro area	0.432	0.580	0.469	0.506	0.432	0.432
US	0.481	0.481	0.543	0.543	0.469	0.494
Canada	0.469	0.494	0.543	0.543	0.506	0.519
Sweden	0.556	0.494	0.420	0.444	0.407	0.449

Table 6: DOC OOS Results
The table presents the DOC statistic for the structural models. A value greater than 0.5 indicates that the respective model predicts the correct direction of change more than 50% of the times.

The next step is to check whether the positive results for EURNOK are robust and to do so, we test with several different time periods and window sizes for the rolling window regression. Some of these are in Appendix H. These results show that the sentiment index-based models consistently outperform the structural models in terms of MSPE and DOC. The same is true for USDNOK for the monetary model. This suggests that the results for EURNOK are robust and indicate that sentiment indices indeed can be a good candidate variable for making more accurate forecasts.

A question that remains is why the sentiment indices have a positive effect for EURNOK, but does not seem to improve predictive performance for any of the other currencies. There could be many reasons for this. The research conducted by Bulut (2018) yields mixed results, suggesting that sentiment index-based models do not consistently outperform structural models across all exchange rates. This implies that sentiment indices may only be suitable variables to include in prediction models under some circumstances, depending on the exchange rate being analyzed. As other literature finds, the most significant variables for predicting exchange rates vary between countries. Norway is a small nation, and the social mood in our market, which is what we are trying to extract from the sentiment indices, might not be strong enough to have a determining effect on the exchange rate. The reason why we still see an improvement for EURNOK when including the indices might be because the Norwegian economy is heavily linked to the European economy, and that the market sentiment in Europe has a significant effect on both the NOK and the Euro. Another explanation could be that important variables must be added to the structural models. Omitting important control variables might lead to an underestimation (downward bias) of the coefficient of our sentiment indices for the USDNOK, CADNOK, and SEKNOK and even possibly an overestimation for the EURNOK. A lack of control variables can therefore be a limitation of the body of literature studying the explanatory and predictive power of market sentiment on exchange rates.

Based on these findings, especially those from IS modeling, we believe that the sentiment indices also serve as promising variables to include in other models. Previous literature (Bulut 2018) has emphasized the need for further research in this area, and we aim to do so by including the indices in more advanced models with variables other than monetary fundamentals. The subsequent section explores this further.

5.2 Comparison of Linear and Nonlinear Models

This section presents the various results from comparing linear and nonlinear models. We begin with the ARDL ECM and NARDL ECM in Sections 5.2.1 and 5.2.2, respectively. Both these sections consider IS modeling only, and the idea is to establish a potential asymmetric relationship between the variables. We employ the same models in a forecasting exercise for the next part, Section 5.2.3. Then we move on to the TARDL in Section 5.2.4, where we only consider IS modeling. Section 5.2.5 summarizes the comparison of all linear and nonlinear models we include in the abovementioned sections.

5.2.1 Linear ARDL ECM

We begin with the two different bounds test cointegration techniques, ARDL and NARDL, to investigate the existence of long-run cointegration relationship between the exchange rate and its determinants. We start with the linear model, and its results are presented in Table 7, including the coefficients, significance level, R^2_{adj}, AIC, BIC, and F-statistic.

We estimate each model using optimal variable lag, leading to some empty table entries as the optimal number of lags vary across exchange rates.

Variable	EURNOK	USDNOK	CADNOK	SEKNOK
Intercept	0.0425**	-0.0096	3.4e-02	-0.0240
s_{t-1}	-0.0659***	-0.0679***	-1.1e-01***	-0.1231***
SI_t	-	0.0009***	-	-
SI_{t-1}	0.0010***	-	1.2e-03***	0.0009***
SI_t^*	-0.0001	-0.0007	-	-
SI_{t-1}^*	-	-	1.4e-03	0.0010**
obx_t	0.0096	0.0022	1.2e-02	-
obx_{t-1}	-	-	-	0.0099
$s\&p_t$	0.0042	-	8.9e-03	-
$s\&p_{t-1}$	-	0.0259	-	0.0034
op_{t-1}	-0.0105**	-0.012	-6.0e-03	-0.0102***
Δs_{t-1}	-0.1450**	-	2.3e-01***	0.1878***
ΔSI_t	0.0001	-	4.2e-04*	0.0004**
ΔSI_t^*	-	-	7.2e-05	0.0001
Δobx_t	-	-	-	0.0132
Δobx_{t-1}	-	-	-	-0.0570**
$\Delta s\&p_t$	-	-0.278***	-	0.0368
$\Delta s\&p_{t-1}$	-	-	-	0.0825***
Δop_t	-0.0965***	-0.156***	-5.8e-02***	-0.0590***
R^2_{adj}	0.380	0.537	0.239	0.338
AIC	-1549.5	-1351.3	-1465.8	-1581.4
BIC	-1512.4	-1317.6	-1425.4	-1527.5
F-test	3.559	4.025	5.601	8.570

Table 7: ARDL ECM IS Results

The table present the coefficients for the ARDL in error correction form. R^2_{adj}, AIC and BIC is used as measures for comparison. The F-test is used to test long-run cointegration. *, ** and *** indicate a significance level of 10, 5 and 1 percent, respectively.

To check for the presence of cointegration in the models, we compute the F-statistic as recommended by Pesaran et al. (2001), tabulated in the bottom row of Table 7. The results suggest that the null hypothesis of no cointegration can be rejected for USDNOK, CADNOK, and SEKNOK since the critical value is above the upper bound. For EURNOK, however, the value falls between the lower and upper bound, suggesting inconclusive results, and we cannot establish a long-run cointegration relationship. Thus, it may not be necessary to utilize a model such as the ARDL ECM, which considers cointegration relationships. Instead, an ordinary ARDL model that only includes stationary variables might be equally effective. We will address if the lack of a long-run cointegration relationship is due to neglected nonlinearities when performing a NARDL ECM.

The significance of variables varies across different exchange rates. For instance, the first difference of the S&P500 in the same period shows high significance for the USDNOK exchange rate, whereas this relationship does not hold true for the other exchange rates. This may be attributed to the fact that the S&P500 only includes American companies, thereby effectively capturing changes in the American market and the value of the USD.

Its negative coefficient suggests that an increase in the S&P500 results in an appreciation of the krone against the USD. These results differ from some existing literature on the NOK exchange rate, such as the studies by Naug (2003) and Benedictow & Hammersland (2022), which suggest a positive correlation between the S&P500 and the NOK exchange rate. A reason for the different results could be the use of older data in their studies. The Norwegian economy has changed over the last decade, and the factors exerting the greatest influence on the krone are not necessarily the same now as it were before. Our results are, however, consistent with more recent literature and news publications regarding the krone. Ingvild Borgen[4], an analyst in DNB Markets, argues that the correlation between global assets and the NOK changed after the financial crisis in 2008. According to her analysis, the Norwegian krone is more heavily linked to financial assets now than prior to 2008, especially when compared to the USD. Currency and interest rate strategist in Nordea, Lars Mouland[5], has also conducted studies that are in line with these statements. One of his explanations for the weak NOK is the uncertainty in the world economy and financial markets. The Norwegian krone, often being pro-cyclical, tends to weaken when the stock market falls.

The other stock exchange we include in the model is OSEBX. This variable has limited significance for any of the currencies, except for its lagged first difference, which is significant at a 5 percent level for SEKNOK. The reason could be that this stock exchange is not large enough to capture the overall market, as it is only the 55 most traded stocks on the Oslo Stock Exchange. Additionally, the index consists of several oil and gas companies. Consequently, it might be correlated with both the S&P500 and the Brent oil futures, making this variable less significant. Hence, it is unsurprising that the stock exchange has limited impact on the Norwegian currency when measured against major currencies like the EUR, USD, and CAD. However, considering the Swedish krona's status as a smaller currency and its proximity as our neighboring country, our findings suggesting that the stock exchange is of more importance in regards to the movements of SEKNOK are reasonable.

The statistical significance of the first difference in Brent oil futures is observed at a 1 percent level across all exchange rates. Furthermore, the significance of the level value holds true for both EURNOK and SEKNOK. The sign of all oil price variables is negative, suggesting a negative correlation between the oil price and the krone exchange rate. As a result, a fall (rise) in the oil price leads to a depreciation (appreciation) of the krone against the respective currencies. These findings align with other literature ((Klovland et al. 2021), (Benedictow & Hammersland 2022)) on the NOK exchange rate, which also finds oil price to be an important driver for the development of the krone. Given Norway's status as a commodity nation heavily reliant on oil exports, a strong connection between oil futures fluctuations and the NOK exchange rate is expected.

Lastly, the sentiment indices are statistically significant at a 1 percent level for all exchange rates, and for three out of four, it is the lagged index that is of significance. This is not surprising, as we specifically design the indices to correlate with the exchange rate return in the subsequent period. These results are consistent with the findings from comparing market index-based models with the structural models, given that many of the sentiment indices are statistically significant at a conventional level in the IS analysis. The findings presented in Table 7 indicate that incorporating sentiment indices into models that consider factors beyond monetary fundamentals may yield positive outcomes as well.

[4]DNB article (Furuseth 2022)
[5]Nordnet article (Nordnet 2023)

For comparison reasons, we also run a linear ARDL model without error correction. These results are presented in Appendix I. According to the AIC and BIC values, the ARDL ECM consistently outperforms the ARDL for all exchange rates. This outcome is reasonable given that the ARDL ECM technique presents numerous benefits. It allows us to capture both the short- and long-run effects and is applicable regardless of stationarity properties. Furthermore, when two variables are cointegrated, it implies some adjustment process prevents the errors in the long-run relationship from becoming larger and larger.

5.2.2 Nonlinear ARDL ECM

We now focus on the asymmetric model of Equation 16, continuing with IS modeling. We model the asymmetry for both the Brent oil futures and the S&P500. Table 8 presents the results for asymmetric S&P500. We estimate the models with optimal lag here as well, and for the variables where none of the exchange rates include the respective variable in their optimal model, we do not include the variable in the table either. This is the case for, e.g., the op_t^- in Table 9. Similar to the ARDL, we begin by establishing the cointegration relationship. This is done by the F-statistic bound test approach, established by Shin et al. (2014). The F-statistic is above the upper bound for USDNOK, CADNOK, and SEKNOK, suggesting that there indeed exists an asymmetrical long-run cointegration relationship among the selected variables. However, similar to the ARDL ECM, we cannot establish a cointegration relationship for EURNOK, as the F-statistic remains between the lower and upper bound, suggesting inconclusive results.

Next, we perform the Wald test for asymmetry. The results suggest no short- or long-run asymmetry for EURNOK. Therefore, it is reasonable that including asymmetric S&P500 does not result in a cointegration relationship in the nonlinear specification either. For the remaining exchange rates, the Wald test reveals short-run asymmetry for USDNOK and SEKNOK. Concerning the long-run, the Wald test demonstrates long-run asymmetry for USDNOK and CADNOK, but not for SEKNOK. Furthermore, the AIC and BIC suggest no improvement compared to the ARDL model for SEKNOK, while the NARDL model outperforms the ARDL for both USDNOK and CADNOK.

Variable	EURNOK	USDNOK	CADNOK	SEKNOK
Intercept	6.6e-02*	0.1280**	1.2e-0***	-0.0010
s_{t-1}	-7.5e-02***	-0.0848***	-1.6e-01***	-0.1358***
SI_t	-	0.0008***	-	-
SI_{t-1}	9.2e-04***	-	1.2e-03***	0.0009***
SI^*_t	-6.8e-05	-0.0007	-	-
SI^*_{t-1}	-	-	1.1e-03	0.0011**
$s\&p^+_t$	-3.9e-03	-	1.4e-02	-
$s\&p^+_{t-1}$	-	-0.0323*	-	-0.0059
$s\&p^-_t$	2.5e-03	-	5.2e-03	-
$s\&p^-_{t-1}$	-	0.0228	-	0.0034
obx_t	-	-0.0135	1.7e-03	-
obx_{t-1}	9.6e-03	-	-	0.0050
op_t	-	-	-8.1e-03	-
op_{t-1}	-1.2e-02**	-0.0157	-	-0.0109***
Δs_{t-1}	1.1e-01**	-	2.5e-01***	0.1965***
ΔSI_t	7.1e-05	-	4.4e-04*	0.0004**
ΔSI^*_t	-	-	-4.7e-05	0.0001
$\Delta s\&p^+_t$	-	-0.3850***	-	0.0061
$\Delta s\&p^+_{t-1}$	-	-	-	0.0652
$\Delta s\&p^-_t$	-	-0.2016***	-	0.0623
$\Delta s\&p^-_{t-1}$	-	-	-	0.0903*
Δobx_t	-1.1e-05	-	-	0.0075
Δobx_{t-1}	-3.4e-02*	-	-	-0.0592**
Δop_t	-8.8e-02***	-0.1561***	-5.5e-02***	-0.0574***
R^2_{adj}	0.382	0.545	0.259	0.333
AIC	-1547.3	-1353.4	-1470.5	-1576.8
BIC	-1500.1	-1313.0	-1426.7	-1512,8
F-test	3.00	3.75**	5.85***	7.41***
W_{SR}	-	< 0.01	-	0.05
W_{LR}	0.85	0.04	0.03	0.58

Table 8: NARDL ECM IS Results with Asymmetric S&P500

The table presents the coefficients for the NARDL in error correction form. R^2_{adj}, AIC and BIC are measures used for comparison. The F-test is used to test long-run cointegration. *, ** and *** indicate a significance level of 10, 5 and 1 %, respectively. W_{SR} and W_{LR} are the short- and long-run Wald tests for asymmetry, respectively, and it is the p-value of these tests that are included in the bottom two rows.

The improved model for USDNOK with asymmetric S&P500 can be attributed to the high significance of this variable in the linear specification as well. Dividing it into negative and positive partial sums allows for extracting more information, thereby improving model performance. Considering the short-run estimates, both are significant at a 1 percent level and have a negative sign, suggesting a negative correlation. Thus, a rise in the S&P500 leads to an appreciation, while a price fall leads to a depreciation of the krone measured against the USD. Furthermore, the absolute value is greater for a positive change than a negative one. This suggests that USDNOK is affected more by a rise in the S&P500 than by a fall. A possible explanation for this asymmetric effect could be tied to market risk. The S&P500 is often strongly correlated to the wider global financial markets. Thus, a rise in the S&P500 could indicate that financial assets are generally doing well, contributing to lower risk aversion in the market. This could, in turn, make investors allocate their investments towards riskier assets, e.g., the Norwegian krone, a smaller currency than the USD, which could lead to an appreciation. On the other hand, a fall in the S&P500 could make market participants more hesitant, holding back on their investments, resulting in less movement in the exchange rate. Because of this, and other findings discussed later, we explore market volatility and risk aversion further in the last section.

While the S&P500 does not appear to significantly impact the CADNOK in the linear model, its significance emerges when the model is allowed to be nonlinear. Given the substantial interconnection between the Canadian and American economies, segmenting the variable into positive and negative components could enable more accurate extraction of the most impactful information for the exchange rate.

Several potential explanations exist for the lack of improvement in model performance when incorporating asymmetry in the S&P500 for the EURNOK and SEKNOK exchange rates. Firstly, the S&P500 did not exhibit statistical significance in the linear model. Consequently, it is not surprising that the asymmetric S&P500 variables also remained insignificant. The lack of significance is reflected in the AIC and BIC scores, which penalize the inclusion of weak variables in a model. Moreover, the EURNOK exchange rate is significantly influenced by monetary policies and economic conditions within the Eurozone to a greater extent than the USDNOK. The decisions made by the European Central Bank, economic indicators from Eurozone countries, and region-specific events can therefore play a more substantial role in determining the movements of the EURNOK or SEKNOK. As a consequence, the relationship between the S&P500 and EURNOK, and SEKNOK may be less pronounced or have different dynamics compared to USDNOK. The results might differ if we include other financial assets in the model, e.g., some European stock indices. Such a variable might be significant in the ARDL ECM and, therefore, also improve model performance by including asymmetry.

Next, we move on to the NARDL with asymmetric Brent oil futures, presented in Table 9. Again, we begin by testing the long-run cointegration relationship. The F-statistic is above the upper bound for all exchange rates, confirming an asymmetrical long-run cointegration relationship among the selected variables. For EURNOK, we cannot establish a long-run cointegration relationship when all variables are linear, but with asymmetric oil futures, we can reject the null and confirm the existence of cointegration. Therefore, the lack of cointegration for EURNOK could be due to neglected nonlinearities in the oil price.

Variable	EURNOK	USDNOK	CADNOK	SEKNOK
Intercept	0.0569**	0.0083	9.9e-02***	-3.9e-02**
s_{t-1}	-0.0948***	-0.0926***	-1.7e-01***	-1.2e-01***
SI_t	-	0.0008***	-	-
SI_{t-1}	0.0010***	-	1.3e-03***	9.1e-04***
SI_t^*	-0.0002	-0.0007	-	-
SI_{t-1}^*	-	-	1.0e-03	9.0e-04**
op_t^+	-0.0101**	-	-3.4e-03	-8.3e-03**
op_{t-1}^+	-	-0.0153	-	-
op_{t-1}^-	-0.0120**	-0.0190*	-8.8e-03*	-8.3e-03**
obx_t	0.0059	-0.0088	1.5e-03	-
obx_{t-1}	-	-	-	1.1e-02
$s\&p_t$	0.0042	-	7.0e-03	-
$s\&p_{t-1}$	-	0.0283	-	1.4e-03
Δs_{t-1}	0.1612***	-	2.3e-01***	1.8e-01***
ΔSI_t	0.0001	-	4.4e-04*	4.2e-04**
ΔSI_t^*	-	-	-6.9e-05	4.7e-05***
Δop_t^+	-	-0.1582***	-	-
Δop_t^-	-0.1405***	-0.1557***	-8.5e-02***	-9.0e-02***
Δobx_t	-	-	-	9.7e-03
Δobx_{t-1}	-	-	-	-4.9e-02**
$\Delta s\&p_t$	-	-0.2753***	-	4.8e-02
$\Delta s\&p_{t-1}$	-	-	-	8.2e-02***
R^2_{adj}	0.415	0.538	0.278	0.352
AIC	-1560.8	-1350.0	-1476.0	-1584.8
BIC	-1520.4	-1309.6	-1432.2	-1527.6
F-test	4.13 (0.019)	3.82 (0.035)	6.42 (0.0005)	6.96 (0.0001)
W_{SR}	< 0.01	< 0.01	< 0.01	< 0.01
W_{LR}	0.07	0.13	0.01	0.12

Table 9: NARDL ECM IS Results with Asymmetric Brent Oil Futures

The table presents the coefficients for the NARDL in error correction form. R^2_{adj}, AIC and BIC are measures used for comparison. The F-test is used to test long-run cointegration. *, ** and *** indicate a significance level of 10, 5 and 1 %, respectively. W_{SR} and W_{LR} are the short- and long-run Wald tests for asymmetry, respectively, and it is the p-value of these tests that are included in the bottom two rows.

We continue with the Wald test as recommended by Shin et al. (2014) to check for long- and short-run asymmetry. The result shows a short-run asymmetric effect at a 1 percent significance level for all exchange rates. However, we can only establish a long-run asymmetry for CADNOK at a 1 percent level and at a 10 percent level for EURNOK. This suggests that the inclusion of asymmetry in the models might improve performance. To confirm this, we compare R^2_{adj}, AIC, and BIC with the ARDL ECM. The R^2_{adj} is higher for all exchange rates when including asymmetry, while the AIC and BIC improve for all besides USDNOK. Based on these metrics, it seems like significant short-run asymmetry does not improve model performance as much as long-run asymmetry. USDNOK has the highest p-value for the long-run Wald test, and there is no model improvement according to neither AIC or BIC. Furthermore, the Wald test did not suggest long-run asymmetric significance for SEKNOK, and here there is only a minor improvement in performance by including asymmetry. Additionally, those exchange rates where the model improves by including asymmetric S&P500 are only those with long-run asymmetric significance. An explanation could be that for the long-term persistence of the short-term asymmetric effect, it is necessary for the variables to exhibit cointegration and for the long-run coefficient estimates to display statistical significance.

Since there is an overall improvement for three of the currencies, we investigate their properties further. First, their short-run coefficient for an oil price fall is statistically significant at a 1 percent level, and the sign is negative. A positive change is insignificant and is not even included in the model. This suggests that a drop in the oil price significantly affects the respective exchange rates, while a rise in the price does not. Furthermore, since the sign is negative, it indicates a negative correlation. Thus, a drop in the oil price results in a krone depreciation, measured against the other currencies. Next, we consider the long-run coefficient, as we have determined that the long-term asymmetry likely plays a crucial role in enhancing model performance. For EURNOK, CADNOK, and SEKNOK, the falling (op^-) oil prices have a greater effect than rising (op^+) prices. Additionally, the asymmetric variables take different lag orders, further strengthening the argument that there is a significant long-run asymmetric significance.

According to both AIC and BIC, there is no model improvement for USDNOK when considering asymmetric oil price. Comparing the values, however, the difference in AIC is only 1.3, while it is slightly larger for BIC, which is reasonable since BIC penalizes complex models more than AIC. The lack of model improvement is probably because we do not have significant long-run asymmetry, as both the long-run asymmetric oil prices are of the same lag, and are not significantly different. This suggests that falling and rising oil prices affect USDNOK quite similar, which can be explained by several factors. Firstly, the fact that Brent oil futures are denominated in USD could play a role. There will naturally be a relationship between commodity prices and currency exchange rates. USDNOK may exhibit a different level of sensitivity to the asymmetry in oil price than other exchange rates because USD is already a reference currency for the oil price. Furthermore, the USD is widely regarded as the global reserve currency and plays a central role in international trade and finance. Consequently, factors influencing USDNOK can be more complex and influenced by a broader range of global factors, such as geopolitical events and market sentiment towards the USD. The Norwegian sentiment index and the first difference of the S&P500 are highly significant, suggesting that both market sentiment and financial assets are determining for USDNOK.

Although the asymmetric model does not improve significantly for USDNOK, the other currencies' results suggest a nonlinear relationship between oil price and exchange rate. By incorporating asymmetry into the oil price, the models seem to better capture the complex relationship between Brent oil and the Norwegian exchange rate. What is evident is that negative changes in the oil price affect the exchange rates more significantly than positive changes. There could be several explanations for this. An oil price fall will directly impact the revenue generated from Norwegian oil exports. This will result in less earnings from export, which will negatively affect the balance of trade for Norway. Therefore, there will be less demand for the Norwegian krone, leading to a depreciation consistent with our findings. Market expectations and investor sentiment could also play a crucial role. Since a fall in oil price could have an immediate and negative impact on Norway's trade balance, this can trigger market concerns about the country's economic outlook, leading to an even higher degree of currency depreciation. This is something the central bank chief in Norway, Ida Wolden Bache[6], recently emphasized in regards to the weakening

[6] NRK article (Skårdalsmo & Hjellen 2023)

of the Norwegian krone. She states that if the players in the market lose confidence in Norwegian monetary policy, the krone will depreciate even more. Lastly, a significant fall in the oil price will not only affect Norway but could also affect the world economy. A price fall could lead to increased market uncertainty and financial turbulence, and the Norwegian krone, a small and peripheral currency, will not necessarily be considered a safe haven, which will be the case, e.g., for the USD. Therefore, high financial turbulence could contribute to a depreciation of the krone.

On the other hand, a rise in the oil price will benefit the Norwegian economy from increased oil revenues, which will strengthen the trade balance and the Norwegian currency. This can also impact the market participants' expectations of Norway's economic outlook in a positive manner. However, people tend to react more strongly to negative than positive news. Furthermore, building on the argument that a negative oil price change can be correlated to market uncertainty and financial turbulence, the opposite could be true for high oil prices. Analyst in DNB Markets, Ingvild Borgen[7], argues that in periods with low market uncertainty and risk aversion, the Norwegian krone tends to follow the oil price less than in a turbulent market. This could also be an argument that supports why positive oil price changes do not have an equally strong effect on the krone as negative changes.

5.2.3 Forecasting with Asymmetry

Until this point, we have been concerned with IS modeling to establish a long-run cointegration relationship and test whether asymmetry can increase model performance. So far, the results suggest that nonlinear relationships exist between the explanatory variables and the krone exchange rate. Therefore, we also want to examine whether this relationship holds OOS and investigate whether asymmetry can contribute to enhanced predictive performance.

We proceed to evaluate the predictive accuracy of our ARDLs and NARDLs. We fit each model on two-thirds of the dataset, specifically from January 2005 to December 2014. Then, we adopt a rolling window prediction approach with a one-year step. In this approach, we use the initial fitted model to forecast foreign exchange rates for 2015. We then refit the model using the available data for 2015 before proceeding to predict rates for 2016. This process will be repeated until the end of 2022. Ultimately, we merge our predictions and compare them against the real exchange rate values. To compare the models, we calculate the MSE and DOC values.

Table 10 presents the forecasting results from the ARDL ECM and NARDL ECM with the asymmetric oil price and S&P500. On the left-hand side, the MSEs are tabulated, while the right-hand side presents the proportion of times the model predicts the correct direction of change. The bold MSE values represent the best-performing models in terms of MSE. An exception is SEKNOK, where the ARDL and NARDL errors are similar. However, since the directional change for the NARDL is better, we conclude that the asymmetric model performs marginally better.

In terms of MSE, the model improves for both USDNOK and CADNOK by including asymmetric oil price and S&P500. Additionally, predictive performance improves marginally for SEKNOK, but only in the NARDL with asymmetric oil price. Furthermore, there is a directional improvement for both CADNOK and SEKNOK in the asymmetric models. These results suggest that in addition to an asymmetric IS relationship, asym-

[7] DNB article (Furuseth 2022)

	MSE			Direction		
	$ARDL$	$NARDL_{op}$	$NARDL_{s\&p}$	$ARDL$	$NARDL_{op}$	$NARDL_{s\&p}$
EURNOK	**0.72**	0.89	0.80	0.55	0.55	0.46
USDNOK	3.03	**2.88**	2.99	0.46	0.46	0.45
CADNOK	0.98	0.96	**0.93**	0.52	0.52	0.53
SEKNOK	0.62	**0.62**	0.64	0.53	0.55	0.56

Table 10: Forecasting Results
The table presents the MSE and direction of change for all linear and asymmetric models. The bold numbers represent the best performing model for each exchange rate.

metry can also improve predictions. The above section discussed why these asymmetric relationships exist and are of significance. Therefore, we will not detail that here since the reason for an OOS causality can be explained by many of the same factors as the IS relationships.

These results demonstrate that a good IS fit does not necessarily guarantee an accurate OOS forecast. When performing IS modeling, the asymmetric oil price model did not outperform the linear specification for USDNOK. However, the predictive performance improves by including asymmetric Brent oil futures, with a reduction in MSE by almost 5%. Additionally, although the model improves IS by including asymmetric oil prices for EURNOK, the OOS predictive performance does not. While a well-performing IS model can be a solid foundation for exchange rate forecasting and establish relationships between variables, accurately predicting OOS data requires evaluating model performance on unseen data.

5.2.4 Threshold ARDL

Finding that asymmetry can improve both IS models and OOS predictions motivates us to model nonlinearity with a regime-shifting threshold model to investigate if that, too, could improve model performance. The results when using first difference of the oil futures as threshold variable are presented in Table 11. The optimal threshold value for each exchange rate is used and is presented in the bottom second row. The regression results when the log difference of the oil price is below (above) this threshold are in the table's top (bottom) rows. This model is compared to a benchmark ARDL, and we employ the AIC of the models for comparison. When the difference between them, which is in the bottom row, is negative, the threshold model outperforms the ARDL. The TARDL performs better for all exchange rates besides USDNOK. This is similar to the NARDL model with the asymmetric oil price, where the NARDL outperforms compared to the linear specification for every exchange rate except for USDNOK as well. This further strengthens the theory that nonlinearity in the oil price does not enhance model performance significantly when the USDNOK is under consideration.

Variable	EURNOK	USDNOK	CADNOK	SEKNOK
Intercept	-0.00580**	0.0013*	0.0008	-0.0046**
Δs_{t-1}	-0.4104*	0.0586	-0.2081	-0.0255
SI_t	0.0002	0.0011***	0.0008	-0.0017***
SI_{t-1}	-0.0011	0.0003	0.0012	0.0007
SI^*_t	-0.0007	-0.0002	-0.0014	0.0028***
SI^*_{t-1}	0.0038**	0.0014	0.0016	0.0007
$\Delta s\&p_t$	-0.0089	-0.2425***	-0.0011	-0.0057
$\Delta s\&p_{t-1}$	-0.0625	0.1498***	0.0400	0.0265
Δobx_t	0.0843	-0.0306	-0.0868	0.0312
Δop_t	-0.1600***	-0.1491***	-0.0636*	-0.1423***
Intercept	-0.0008	-0.0001	-0.0006	-0.0005
Δs_{t-1}	0.2021***	-2.2445	0.2160***	0.1577***
SI_t	0.0004	0.0000	0.0004*	0.0007***
SI_{t-1}	0.0007***	-0.0006	0.0008***	0.0005***
SI^*_t	-0.0002	-0.0024	0.0003	-0.0000
SI^*_{t-1}	0.0003	0.0084***	0.0012*	0.0012***
$\Delta s\&p_t$	0.0634*	-0.4238*	-0.0079	-0.0563
$\Delta s\&p_{t-1}$	0.0538**	-0.0327	0.0037	0.0413*
Δobx_t	0.0012	0.0225	0.0738*	0.0005
Δop_t	-0.0273	-0.0588	-0.0335	-0.0338**
Threshold value	-0.05	-0.04	-0.03	-0.04
AIC vs. benchmark	-14.4	2.6	-3.0	-19.5

Table 11: TARDL IS Results with Brent Oil Futures as Threshold
The table presents the coefficients for the threshold ARDL using the Brent oil price as a threshold variable. *, ** and *** indicate a significance level of 10, 5 and 1 percent, respectively.

A key finding by performing the TARDL is that the first difference in the oil price is consistently significant when the change is below the threshold, while this is not the case for values above the threshold. These results are also consistent with the findings in the NARDL. It suggests that when there is a large fall in the oil price, the price will be more determining for the exchange rates than when the price is more stable, or there are positive changes. When above the threshold, the variables that seem most significant for EURNOK, CADNOK and SEKNOK, which are those with a performance improvement, are the lagged exchange rate and the lagged Norwegian sentiment index. These indices are built based on monetary fundamentals, but also stock, oil, and gas terms. It could be the case that when oil price changes are positive or just slightly negative, which might indicate periods with less market uncertainty, the words that are most correlated with the exchange rates are the monetary fundamentals. On the other hand, when the market is more volatile, words related to oil price and financial markets could be more correlated. According to both Ingvild Borgen[8] and Lars Mouland[9], the krone is more affected by financial markets, risks, and oil price in times of macroeconomic unrest. Therefore, we can assume that the index will be more correlated to the monetary fundamentals in a less turbulent market, which could explain why the sentiment indices matter more.

[8] DNB article (Furuseth 2022)
[9] Nordnet article (Nordnet 2023)

Since we suspect that market volatility and risk could impact the exchange rate, we next perform a new TARDL with the risk aversion index by Bekaert et al. (2021) being the threshold variable. Table 12 presents the result from this model. As a benchmark for comparison purposes, we use a linear ARDL, including the risk aversion index as an explanatory variable. Based on the AIC, the TARDL outperforms the benchmark for EURNOK, CADNOK and SEKNOK.

Variable	EURNOK	USDNOK	CADNOK	SEKNOK
Intercept	0.0064	0.0082	0.0110*	-0.0039
Δs_{t-1}	0.1786***	0.0068	0.1248*	0.1543**
SI_t	0.0002	0.0010***	0.0004*	0.0005***
SI_{t-1}	0.0007***	0.0002	0.0007***	0.0004**
SI_t^*	0.0000	-	0.0002	-
SI_{t-1}^*	0.0004	0.0019**	0.0013**	0.0009***
r_t	-0.0021	-0.0025	-0.0039*	0.0013
$\Delta s\&p_t$	-0.4414	-0.2306***	-0.0063	0.0461
$\Delta s\&p_{t-1}$	-	0.1377***	-	0.0165
Δobx_t	0.0065	0.0052	0.0284	0.0103
Δop_t	-0.0609***	-0.1570***	-0.053***	-0.0483***
Intercept	0.0124**	0.0289**	0.0149**	0.0184**
Δs_{t-1}	-0.0091	-0.2098	0.1835	-0.0149
SI_t	-0.0005	0.0027	0.0021	-0.0001
SI_{t-1}	0.0014*	0.0028	0.0035**	0.0021**
SI_t^*	0.0017	-	0.0030	-
SI_{t-1}^*	0.0046*	0.0011	0.0038	0.0005
r_t	-0.0026**	-0.0062**	-0.0023*	-0.0005**
$\Delta s\&p_t$	-0.1961	-0.4670***	0.0133	-0.0874
$\Delta s\&p_{t-1}$	-	-0.1829	-	0.0103
Δobx_t	0.2515***	-0.0430	-0.0235	0.1183*
Δop_t	-0.2025***	-0.0572	-0.0766	-0.1325***
Threshold value	3.581	3.616	3.580	3.616
AIC vs. benchmark	-19.01	6.50	-3.57	-4.48

Table 12: TARDL IS Results with Risk Aversion Index as Threshold
The table presents the coefficients for the threshold ARDL using the risk aversion index, r_t, as a threshold variable. *, ** and *** indicate a significance level of 10, 5 and 1 percent, respectively.

One thing that separates USDNOK from the other exchange rates is the significance of the financial assets when below the threshold value. The S&P500, both lagged and contemporary period, is highly significant for USDNOK, while none of the financial assets are significant for the other exchange rates. However, when above the threshold, OSEBX is significant for EURNOK and SEKNOK. This might be why the risk threshold works well for these currencies. It is also consistent with the analysis of Ingvild Borgen[10], who stated that the Norwegian krone follows the stock market more closely in times of uncertainty. For the USDNOK, however, it might be the case that the set threshold value is not optimal for capturing this difference since the S&P500 is highly significant both above and under the threshold. Another explanation could be that the risk aversion index is not a suitable

[10]DNB article (Furuseth 2022)

threshold value for this exchange rate. It might be the case that, e.g., the VIX, which measures the volatility of the S&P500, is a better candidate for threshold value for the USD, as it might better capture the market volatility in the US. What strengthens this hypothesis is that when we perform the NARDL ECM, there is an improvement in model performance for the USDNOK when including asymmetric S&P500. Thus, both the VIX and the S&P500 might work better as threshold variables modeling the USDNOK.

Another aspect that all models besides the USDNOK have in common is that the lagged exchange rate is significant when there is low market risk aversion, while it is not in times of more uncertainty. This suggests that there is a stronger positive, significant correlation between the exchange rate and its own lag in times of low market risk aversion. Therefore, the historical value of the exchange rate is a better indicator of its development, while when the market is more turbulent, other factors, such as financial assets, will have a more determining effect. The risk aversion index itself also seems to be of greater significance in times of high market uncertainty, and these findings are in line with Ida Wolden Bache's[11] statements regarding the recent developments of the krone. She states that the krone tends to depreciate when there is high market turbulence.

5.2.5 Summary of the Comparisons

We primarily focus on IS modeling to discern nonlinear relationships between variables, and we evaluate the performance of our IS linear and nonlinear models by computing their AIC and BIC values. However, not all models are directly comparable due to their different characteristics. We begin with the ARDL and NARDL bound testing approach, both in error correction form, making them applicable for direct comparison. By doing so, we conclude that asymmetry, in many cases, has a positive effect on model performance, which is why we move on to testing a TARDL. However, this model is not in error correction form, so we cannot employ the AIC and BIC values to compare it directly with the ARDL ECM and NARDL ECM. Therefore, we only compare it with an ordinary ARDL, intending to investigate whether thresholds can improve model performance.

The comparisons make it evident that nonlinearities can improve model performance. The NARDL with asymmetric oil futures is the best performing model for EURNOK, CADNOK, and SEKNOK, which is largely attributed to the fact that a fall in the oil price affects the exchange rates more significantly than a positive change. This could have several explanations, including market expectations, investor sentiment, and market risk. As these exchange rates react more to negative fluctuations, and a fall in oil price leads to a krone depreciation, we anticipate that the krone will continue to depreciate below current levels - levels that already are triggering substantial media attention and nationwide concern. A weaker currency is also predicted by some industry leaders[12], who state that NOK could reach levels as low as 15 NOK per Euro (against the current 12 NOK per Euro). However, financial experts and the government disagree with this claim and project that the NOK exchange rate will stabilize at current levels against the Euro[13].

The best performing model for USDNOK is the NARDL with asymmetric S&P500. Contrary to the asymmetric oil price effect, a rise in the S&P500 has a greater effect on USDNOK than a fall in the index. The reasons that this model performs best for USDNOK can be vast. One possibility we have emphasized is that S&P500 is also highly

[11]NRK article (Skårdalsmo & Hjellen 2023)
[12]DN article (Helle 2023)
[13]NRK article (Olsson 2023)

significant for USDNOK in the linear specification. Additionally, similar to asymmetric oil price, we believe that market risk and volatility also play a crucial role. If this positive asymmetry in S&P500 prices is to continue, our models suggest that the NOK will appreciate against the USD. In other words, the NOK might be undervalued against the dollar, a claim shared by the Chief Economist at DNB markets Kjersti Haugland[14].

For the regime shifting TARDLs, we find that the model with risk aversion as a threshold variable performs best for EURNOK and CADNOK, while the model using change in the oil futures as a threshold variable performs best for SEKNOK. However, although the best-performing model varies between exchange rates, both regime-shifting models outperform their linear counterparts for these three exchange rates. One of the most important findings is that financial assets are more determining in times of high market risk aversion, which is in line with recent media statements from, e.g., Ingvild Borgen[15] and Ida Wolden Bache[16]. Furthermore, oil price changes are more determining for the exchange rates when the changes are below a certain threshold. Neither of these models outperforms the linear ARDL for USDNOK. However, we still believe that a threshold model can improve performance for USDNOK, and exploring other threshold variables and values might be beneficial.

[14] NRK article (Olsson 2023)
[15] DNB article (Furuseth 2022)
[16] NRK article (Skårdalsmo & Hjellen 2023)

6 Conclusion

In conclusion, this thesis contributes to a better understanding of the behavior of the Norwegian krone, with a particular focus on the role of market sentiment and nonlinearity in explaining exchange rate movements. By incorporating methodologies from cutting-edge research and introducing a sentiment index, we gain deeper insights into the dynamics influencing the return in NOK exchange rates. Additionally, we contribute to the existing literature by developing nonlinear models that capture asymmetric responses to changes in predictors beyond traditional monetary fundamentals. Specifically, we consider factors such as oil futures, financial markets, and global economic risk levels. The results of this study provide valuable insights that can enrich current exchange rate models and enhance predictions of future currency movements.

Our research supports the hypothesis that investor sentiment and social mood play a significant role in explaining the fluctuations of the Norwegian exchange rates. Incorporating sentiment indices into the IS structural models of EURNOK, USDNOK, and SEKNOK significantly enhances their explanatory power. Our study also provides consistent evidence of a nonlinear relationship between several predictor variables and the NOK exchange rates, particularly the oil futures price. These results have important implications for the outlook of the Norwegian krone. As the NOK exchange rates react stronger to a fall than a rise in the oil futures, we foresee a continued depreciation of the krone, potentially going even lower than the current levels. This nonlinearity in oil prices is found across all exchange rates studied, except for USDNOK, which demonstrates significant asymmetry against the S&P500. For the USDNOK, a rise in the S&P500 affects the NOK exchange rate more than a price fall, suggesting that the krone is likely to strengthen against the USD in the future.

We believe that the asymmetric behavior of the NOK can be partly explained by its sensitivity to market risk and volatility, as further findings reveal a nonlinear relationship between economic uncertainty and the exchange rate. We observe - in line with statements from the central bank of Norway[17] - that financial markets and oil prices behave differently during periods of macroeconomic unrest. We suggest utilizing other proxies for market risk and volatility for the USDNOK, which does not exhibit the same sensitivity to market risk aversion. It is also worth noting that the sentiment indices consistently retain their significance in all nonlinear regression models, thereby confirming their potential value in exchange rate modeling.

While the IS analysis yields promising results, the OOS results do not exhibit the same consistency across all exchange rates. This confirms that a good IS fit does not necessarily guarantee an accurate OOS forecast. Nevertheless, the forecasting models including asymmetry outperform their linear counterparts for USDNOK, CADNOK and SEKNOK. Furthermore, including sentiment indices in the structural models consistently enhances the forecast accuracy for EURNOK. Consequently, both nonlinearity and the inclusion of market sentiment could prove advantageous in the context of forecasting.

Although our methods offer several benefits, making them suitable for this analysis, it is important to acknowledge certain limitations that might impact the results. One potential constraint of our study is the volume of data, as we must restrict the number of variables to prevent overfitting. Another possible limitation is that both the ARDL ECM and NARDL ECM are univariate models. Utilizing a multivariate approach, such as vector autoregressive (VAR) and vector error correction (VECM) models, could lead

[17] Press conference (Bache 2023)

to richer findings by capturing the interplay between the data and several error correction mechanisms.

Nevertheless, the above findings offer a stepping-stone for advancing the field of foreign exchange prediction, especially for the NOK. It demonstrates the need for consideration of factors beyond traditional financial fundamentals, as well as acknowledging the asymmetrical effects of these variables. In doing so, it addresses a vital gap in understanding the NOK's exchange rates and offers valuable insights for policymakers, economists, and financial experts. This study paves the way for a multitude of new research avenues, of which a few are mentioned below.

First, conducting a deep-dive study into the sentiment indices could be interesting, by further testing or tweaking our variable construction method. This could entail the creation of more focused, specialized sentiment indices. Secondly, one could overcome the limited data volume is to increase data frequency. This is achievable with variables like Brent oil futures, stock market prices, and our risk aversion index. However, extracting Google Trends data at a high frequency for large vocabularies is challenging. Therefore, the challenge of different data granularities could be mitigated by converting the model into a Mixed Data Sampling (MIDAS) model.

Finally, it would be attractive to look at the combination of a NARDL bound testing approach and a TARDL to draw on the strengths discovered with each method. As per the authors' knowledge, studies have yet to combine these two approaches in a model for the NOK exchange rate. Given the complexity of such a model, one might rather be tempted to develop machine learning techniques. However, due to their robustness, interpretability, and alignment with economic theory, there are compelling reasons to stick to advanced econometric models, such as a TARDL bounds test. TARDL bounds test explicitly incorporates economic theory into the model, allowing for understanding long-run equilibrium relationships and short-run dynamics. It also provides the ability to interpret and understand the underlying mechanisms and relationships between variables. Similar to our NARDL model, the clarity provided by the TARDL bounds test could be of substantial value to a wide range of stakeholders - be it businesses, investors, governments, or policymakers.

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Appendix

A Data Sources

Bloomberg Terminal (2023) OSEBX

Bloomberg Terminal (2023) M1 Norway

Bloomberg Terminal (2023) M1 Sweden

Eurostat (2023). Harmonized consumer price index Sweden
`https://ec.europa.eu/eurostat/databrowser/view/PRC_HICP_MIDX__custom_4873221/default/table?lang=en`

Federal Reserve Economic Data (2023). Norwegian Interbank Offered Rate 3 Months
`https://fred.stlouisfed.org/series/IR3TIB01NOM156N`

Federal Reserve Economic Data (2023). European Interbank Offered Rate 3 Months
`https://fred.stlouisfed.org/series/IR3TIB01EZM156N`

Federal Reserve Economic Data (2023). US Interbank Offered Rate 3 Months
`https://fred.stlouisfed.org/series/IR3TIB01USM156N`

Federal Reserve Economic Data (2023). Canadian Interbank Offered Rate 3 Months
`https://fred.stlouisfed.org/series/IR3TIB01CAM156N`

Federal Reserve Economic Data (2023). Swedish Interbank Offered Rate 3 Months
`https://fred.stlouisfed.org/series/IR3TIB01SEM156N`

Federal Reserve Economic Data (2023). Harmonized consumer price index Norway
`https://fred.stlouisfed.org/series/CP0000NOM086NEST`

Federal Reserve Economic Data (2023). Harmonized consumer price index Europe
`https://fred.stlouisfed.org/series/CP0000EZ19M086NEST`

Federal Reserve Economic Data (2023). Harmonized consumer price index USA
`https://fred.stlouisfed.org/series/CP0000USM086NEST`

Federal Reserve Economic Data (2023). Harmonized consumer price index Canada
`https://fred.stlouisfed.org/series/CPALCY01CAM661N`

Investing.com (2023). EURNOK
`https://www.investing.com/currencies/eur-nok-historical-data`

Investing.com (2023). USDNOK
`https://www.investing.com/currencies/usd-nok-historical-data`

Investing.com (2023). CADNOK
`https://www.investing.com/currencies/cad-nok-historical-data`

Investing.com (2023). SEKNOK
`https://www.investing.com/currencies/sek-nok-historical-data`

Investing.com (2023) S&P500
`https://www.investing.com/indices/us-spx-500-historical-data`

Investing.com (2023) ICE Brent Crude Future (LCOc1)
https://www.investing.com/commodities/brent-oil-historical-data

Investing.com (2023) VIX
https://www.investing.com/indices/volatility-s-p-500-historical-data

Nancy R. Xu (2022). Risk aversion index
https://www.nancyxu.net/risk-aversion-index

OECD data (2023). Industrial production Norway
https://data.oecd.org/industry/industrial-production.htm

OECD data (2023). Industrial production Europe
https://data.oecd.org/industry/industrial-production.htm

OECD data (2023). Industrial production USA
https://data.oecd.org/industry/industrial-production.htm

OECD data (2023). Industrial production Sweden
https://data.oecd.org/industry/industrial-production.htm

OECD data (2023) M1 Europe
https://data.oecd.org/money/narrow-money-m1.htm

OECD data (2023) M1 USA
https://data.oecd.org/money/narrow-money-m1.htm

OECD data (2023) M1 Canada
https://data.oecd.org/money/narrow-money-m1.htm

B Unit Root Tests of Monetary Fundamentals

Country	Variable	ADF		PP		KPSS	
		level	1st diff	level	1st diff	level	1st diff
Euro Area	Interest rate diff	-2.95	-5.15***	-12.82	-120.25***	1.26	0.11***
	Price diff	-1.89	-6.23***	-6.80	-251.56***	3.45	0.18***
	Industrial production diff	-3.50**	-	-33.33***	-	2.13	0.04***
	M1 diff	-2.21	-5.54***	-8.95	-110.65***	2.81	0.06***
US	Interest rate diff	-2.70	-4.03***	-4.70	-213.86***	0.73	0.21***
	Price diff	-2.53	-7.80***	-11.15	-199.71***	2.25	0.10***
	Industrial production diff	-2.02	-6.93***	-17.69	-210.77***	2.47	0.07***
	M1 diff	-1.69	-5.58***	-5.70	-181.18***	0.97	0.22***
Canada	Interest rate diff	-2.82	-4.07***	-6.96	-128.49***	0.97	0.22***
	Price diff	-2.38	-6.70***	-14.83	-245.59***	3.62	0.06***
	Industrial production diff	-2.32	-6.85***	-20.96*	-	3.22	0.05***
	M1 diff	-2.00	-5.19***	-6.36	-106.96***	1.18	0.08***
Sweden	Interest rate diff	-3.01	-5.72***	-15.62	-135.87***	0.22***	-
	Price diff	-2.01	-6.24***	-7.82	-285.40***	4.01	0.11***
	Industrial production diff	-3.30*	-	-36.12***	-	1.66	0.02***
	M1 diff	-2.32	-5.55***	-9.23	-115.4***	1.90	0.06***

Table 13: Unit Root Tests of Monetary Fundamentals

The table summarizes the result of the three unit root tests ADF, PP and KPSS for all variables included in the models. *, ** and *** indicate a significance level of 10, 5 and 1 percent, respectively.

C Asymmetries in Oil Futures and S&P500 for CADNOK & SEKNOK

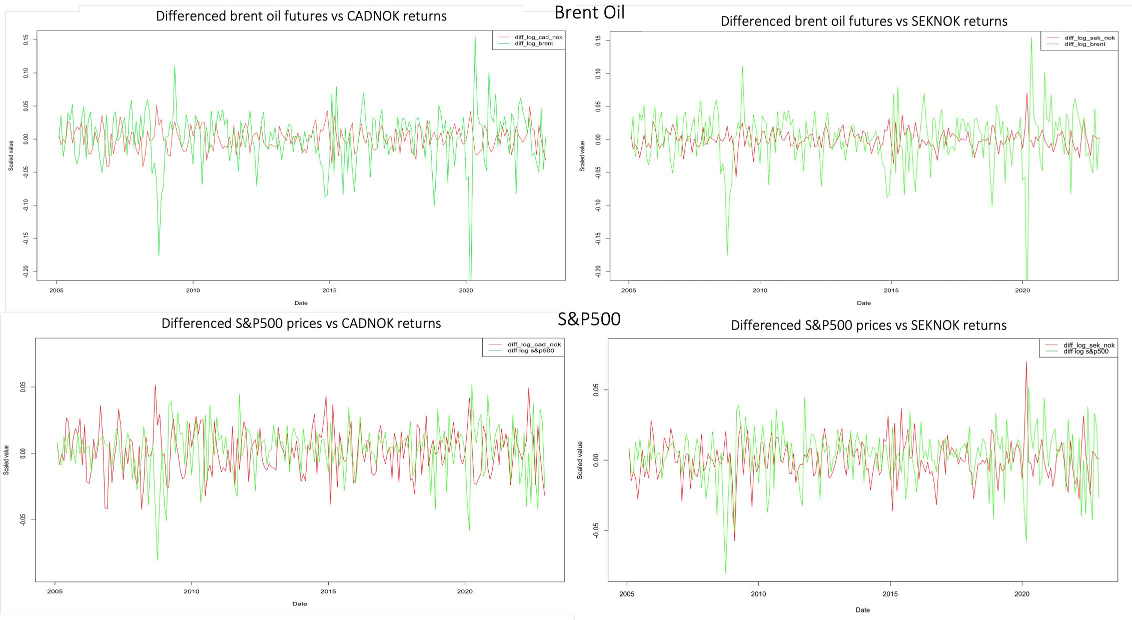

Figure 3: Asymmetries in oil futures and S&P500 prices.

Plots showing the timeseries of the exchange rate returns in red (CADNOK and SEKNOK) and the differenced brent futures or S&P500 (in green) to assess whether up and down movements in the latters are correlated with asymmetric behaviors in the movement of the former.

D Complete Vocabularies for Google Trends

Language	Respective Vocabulary
English	Inflation, Prices, CPI, Cheap, Interest rate, Electricity, Expensive, Deflation, Quantitative easing, Conversion rates, Exchange rate, Euro nok, Euro dollar, Dollar nok, Buy, Spend, Stocks, Save, Donate, Job, Foreclosure, Vacation, Layoff, Invest, Economy, Shopping, Loan, Debt, Budget, Economic recovery, Upturn, Bear market, Bull market, Easy money, Import, Export, Patent, Intellectual property, Luxury, Gift, Shortage, Bet, Gamble, Deferred payment, Insurance, Social support, Social benefits, GDP, Privileged, Pollution, Co2 emissions, Bankruptcy, Cash, Credit, ATM, Withdrawal, Liquidation, Risk, Instability, Economic bubble, Economic Depression, Protectionism, Financial crisis, Oil and natural gas, Oil, Oil price, Crude oil, Brent oil, Natural gas, LNG, Oil futures, S&P, NYSE, Nasdaq, Euronext, London Stock Exchange, Deutsche Börse, Stock exchange, ECB, Federal Reserve, Norwegian oil fund, Norwegian pension fund, Oslo Stock Exchange, FTSE, MSCI, Oljefondet
Norwegian	Inflasjon, Priser, KPI, Billig , Rente , Elektrisitet, Deflasjon, Kvantitative lettelser, styringsrente, valutakurs, dyrt, Kjøpe, Bruke, Spare, Aksjer, Donere, Ferie, Jobb, Permittering, Investere, Økonomi, Lån, Gjeld, Budsjett, Økonomisk oppgang, Opptur, bjørnemarked, oksemarked, lettjente penger, Patentere, Immaterielle rettigheter, Luksus, Gave, Knapphet, Forsikring, Sosial støtte, Sosiale ytelser, BNP, Privilegert, Forurensing, Co2 utslipp, Konkurs, Penger, Kreditt, minibank, Uttak, Likvidering, risiko, ustabilitet, Økonomisk boble, Økonomisk depresjon, Proteksjonisme, finanskrise, oljepris, olje, naturgas, Oslo børs
French	prix, IPC, Bon marché, Taux d'intérêt, Électricité, Cher, Déflation, Assouplissement quantitatif, Acheter, Dépenser, Sauver, Actions,Faire un don, Vacances, Emploi, Forclusion, Licenciement, Investir, Économie, Shopping, Prêt, Dette, Budget, Reprise économique, Amélioration, marché baissier, marché haussier, argent facile, Importer, Exporter, Brevet, Propriété intellectuelle, Taux de change, Luxe, Cadeau, Pénurie, Pari, Paiement différé, Assurance, Aide sociale, Bénéfices sociaux, PIB, PNB, Privilégié, Emissions de CO2, Faillite, Argent liquide, Crédit, guichet automatique, Retrait, Liquidation, Risque, instabilité, Bulle économique, Dépression économique, Protectionnisme, Crise financière, GNL
German	Preise, VPI, Billig, Zinssatz, Elektrizität, Teuer, Quantitative Lockerung, Besorgen, Ausgeben, Sparen, Aktien, Spenden, Urlaub, Arbeit, Zwangsvollstreckung, Entlassen, Investieren, Wirtschaft, Einkaufen, Darlehen, Schulden, Budget, Erholung, Aufschwung, Baisse, Hausse, leichtes Geld, Importieren, Geistigen Eigentums, Tauschrate, Luxus, Geschenk, Mangel, Wette, Zocken, Zahlungsaufschub, Versicherung, Sozialhilfe, Soziale Vorteile, BIP, Privilegiert, Verschmutzung, CO2 Emissionen, Konkurs, Kasse, Kredit, Geldautomat, Rückzug, Liquidation, Risiko, Instabilität, Wirtschaftsblase, Wirtschaftskrise, Protektionismus, Finanzkrise
Swedish	Priser, KPI, Billig, Ränta, Elektricitet, Dyrt, Kvantitativ lättnad, Inköp, Använda, Spara, Donera, Semester, Jobb, Avskärmning, Permittering, Investera, Ekonomi, Lån, Skuld, Ekonomisk återhämtning, Uppgång, björnmarknad, tjurmarknad, enkla pengar, Immateriella rättigheter, Växlingskurs, Lyx, Gåva, Brist, Uppskjuten betalning, Försäkring, Socialt stöd, Sociala fördelar, BNP, Privilegierad, Förorening, Co2 utsläpp, Bankrutt, Pengar, Kreditera, bankomat, Uttag, instabilitet, Ekonomisk bubbla, Ekonomisk depression, Protektionism, finanskris, Stockholmsbörsen, OMX
English (for Canada)	Toronto Stock Exchange, TSX, Bourse de Toronto

Table 14: Vocabulary of search words
Complete vocabularies for all languages besides English

E Comparison Measures

For the MSPE calculation, we calculate the forecast error of each forecast, $pe_i = y_t - \hat{y}_t$ and define the squared prediction error as $spe_i = (y_t - \hat{y}_t)^2$. We can use this to describe the MSPE as:

$$MSPE = \frac{1}{k}\sum_{i=1}^{k} spe_i \tag{18}$$

Clark & West (2006) proposed a statistical significance test for the equal predictability of a structural model and a martingale difference model. We start by defining a loss differential function, d_t, as the difference between the spe_i of the structural model i, and that of the benchmark null, spe_b. This gives $d_t = spe_b - spe_i$. The null hypothesis of equal predictive performance is

$$H_0 = E[d_t] = E[spe_b - spe_i] = 0 \tag{19}$$

We use the following adjusted loss differential, as suggested by Clark & West (2006):

$$\tilde{d} = spe_b - spe_i - adj = spe_b - spe_i - (\hat{y}^b - \hat{y}^i)^2, \tag{20}$$

where $\hat{y}^b$ is the prediction of the benchmark null and $\hat{y}^i$ is that of the econometric model. The adjusted test statistic, $\tilde{d}$ is distributed normally with zero mean. When testing the forecast accuracy, the CW test statistic takes the following form:

$$CW = \frac{\tilde{d}}{a\hat{var}(\tilde{d})^{1/2}} \tag{21}$$

Statistically significant positive CW test statistics indicate that model i performs better than the benchmark model b. The null is rejected at a 10% level if the statistic is greater than 1.282 or at a 5% level if it is greater than 1.645 (Clark & West 2007).

Theil's U-test statistics compare the MSPE of model i to that of a driftless random walk. There exists alternative versions of this test, but the one employed here is

$$TU = \sqrt{\frac{MSPE^i}{MSPE^b}}, \tag{22}$$

where $MSPE^i$ and $MSPE^b$ is the $MSPE$ of model i and the bechmark random walk, respectively. If the value of TU is less than one the model under consideration gives more accurate forecasts than the benchmark.

Lastly, the DOC test gives an indication of whether model i is able to predict the correct direction of change. We define $S(.)$ as a sign function that takes the value of one for positive values and the value of 0 for negative values. We then compute the DOC test statistic as follows

$$DOC = [S(y_{R+1})S(\hat{y}^i_{R+1})] + [1 - S(y_{R+1})][1 - S(\hat{y}^i_{R+1})], \tag{23}$$

where y_{R+1} is the actual return series, while $\hat{y}^i_{R+1}$ is the forecast from model i. The rolling regression window is set to R, for a total of T exchange rate series. This gives us $T - R$ OOS forecasts. Furthermore, we use the following test statistic to test whether the DOC test statistic is bigger than 0.5 or not, which is the benchmark for a random walk

$$DOC\ test = \frac{(D\bar{O}C - 0.5)}{\sqrt{\frac{0.25}{N}}} \tag{24}$$

Here, $D\bar{O}C$ is the percentage of times the model predicts the correct direction of change in the exchange rate series, and N is the total number of forecasts.

F Critical Values F-bounds test

Model	0.050		0.025		0.010	
	Lower value	Upper Value	Lower value	Upper Value	Lower value	Upper Value
ARDL ECM	2.62	3.79	2.96	4.18	3.41	4.68
NARDL ECM	2.45	3.61	2.75	3.99	3.15	4.43

Table 15: Critical Values for F-Bounds Test

The table presents the critical values for the F-bounds test, at a 5, 2.5 and 1 percent level. The reasons that the ARDL and NARDL have different bounds is that the NARDL has one more regressor variable in the model equation.

G Complete results from Sentiment Index Models

	EURNOK		USDNOK		CADNOK		SEKNOK	
PPP model	Base	SI	Base	SI	Base	SI	Base	SI
MSPE	4.18	4.15	6.95	7.02	3.44	3.48	2.69	2.81
ΔMSPE (vs. RW)	-0.08%	-0.85%	+3.97%	+5.02%	+4.21%	+3.23%	+3.33%	-1.08%
ΔMSPE (vs. base)	-	-0.77%	-	+1.01%	-	+0.96%	-	+4.45%
Theil's U	1.000	0.996	1.020	1.025	1.016	1.021	0.995	1.017
CW-stat (vs. RW)	0.036	0.127	-0.105	0.039	-0.049	-0.011	0.118	0.070
CW-stat (vs. base)	-	0.144	-	0.092	-	0.038	-	0.028
DOC-stat	0.432	0.580	0.481	0.481	0.469	0.494	0.556	0.494
UIRP model	Base	SI						
MSPE	4.32	4.25	6.61	6.62	3.34	3.39	2.79	2.90
ΔMSPE (vs. RW)	+3.18%	+1.56%	-1.20%	-1.06%	+0.23%	+1.56%	+2.64%	+6.80%
ΔMSPE (vs. base)	-	-1.58%	-	+0.14%	-	+1.33%	-	+4.06%
Theil's U	1.016	1.008	0.994	0.995	1.001	1.008	1.013	1.033
CW-stat (vs. RW)	0.045	0.0.093	0.112	0.155	0.031	0.044	-0.087	-0.004
CW-stat (vs. base)	-	0.180	-	0.123	-	0.026	-	0.031
DOC-stat	0.469	0.506	0.543	0.543	0.543	0.543	0.420	0.444
Monetary model	Base	SI						
MSPE	4.42	4.33	7.44	7.35	3.35	3.39	2.96	3.08
ΔMSPE (vs. RW)	+5.56%	+3.46%	+11.23%	+9.98%	+0.49%	+1.65%	+9.01%	+13.22%
ΔMSPE (vs. base)	-	-1.99%	-	-1.13%	-	+1.16%	-	+3.86%
Theil's U	1.027	1.017	1.055	1.049	1.002	1.008	1.044	1.064
CW-stat (vs. RW)	0.002	0.080	-0.021	0.040	0.011	0.036	-0.197	-0.099
CW-stat (vs. base)	-	0.168	-	0.143	-	0.032	-	0.031
DOC-stat	0.432	0.432	0.469	0.494	0.506	0.519	0.407	0.494

Table 16: Results
Table text

H Robustness Check Structural Models

Period	PPP Model		UIRP Model		Monetary Model		
	Without SI	With SI	Without SI	With SI	Without SI	With SI	
2008-2022	Euro area	4.72	4.71	4.90	4.86	4.72	4.70
	US	7.40	7.48	7.14	7.25	8.09	8.06
	Canada	3.56	3.57	3.44	3.46	3.45	3.46
	Sweden	2.81	2.97	2.87	3.04	2.29	3.06
2009-2022	Euro area	5.18	5.14	5.25	5.20	5.29	5.23
	US	8.05	8.05	7.94	7.94	8.77	8.68
	Canada	3.91	3.95	3.76	3.81	3.74	3.79
	Sweden	3.18	3.25	3.27	3.35	3.32	3.41
2010-2022	Euro area	6.22	6.17	6.30	6.26	6.26	6.21
	US	8.99	8.81	8.74	8.56	9.83	9.54
	Canada	3.47	3.57	3.52	3.56	3.56	3.55
	Sweden	3.47	3.57	3.52	3.63	3.55	3.65
2010-2020	Euro area	1.69	1.68	1.81	1.79	1.88	1.85
	US	4.14	4.30	4.55	4.70	3.89	3.99
	Canada	2.56	2.55	2.78	2.74	2.71	2.69
	Sweden	1.69	1.76	1.90	1.97	2.06	2.19

Table 17: Robustness Checks

The table presents a selection of the different time periods tested for comparing the structural models with the index based models. The improvement in MSPE for EURNOK is consistent across all time periods.

I Benchmark ARDL

Variable	EURNOK	USDNOK	CADNOK	SEKNOK
Intercept	5.2e-04	0.0013*	0.0006	3.0e-05
s_{t-1}	7.9e-02	0.049	0.18***	1.4e-01**
SI_t	1.3e-04	0.0008***	0.0005*	4.7e-04***
SI_{t-1}	9.5e-02***	-	0.008***	5.0e-04***
SI_t^*	-9.8e-05	-0.0004	0.0002	2.1e-04
SI_{t-1}^*	-	0.0022***	0.0014**	1.1e-03***
Δobx_t	1.7e-02	-0.026	0.018	1.9e-02
Δobx_{t-1}	-	0.066	-	-4.8e-02*
$\Delta s\&p_t$	-2.6e-02	-0.25***	-0.0093	2.7e-02
$\Delta s\&p_{t-1}$	-	0.089	-	7.2e-02**
Δop_t	-8.9e-02***	-0.16***	-0.057***	-5.9e-02***
Δop_{t-1}	1.8e-02	-	-	-
R^2_{adj}	0.362	0.542	0.192	0.295
AIC	-1544.3	-1346.2	-1454.7	-1571.4
BIC	-1510.6	-1309.2	-1421.0	-1531.0

Table 18: ARDL Results

The table presents the results of the benchmark ARDL model. *, ** and *** indicates the 10, 5 and 1 percent significance levels, respectively.

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