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A Space Reduction Heuristic for Thermal Unit Commitment Considering Ramp Constraints and Large-Scale Generation Systems

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Abstract: This paper expands the research around a recently proposed method to reduce the search space region for thermal unit commitment problems. The importance of such techniques comes from the combinatorial explosion regarding the variables of the problem when there are a large quantity of generating units in the system. The proposed heuristic approach utilizes sensitivity indices to gather information about the system and fix many of the binary decision variables over the planning horizon. This work further explores the method by demonstrating its effectiveness in large-scale systems subjected to ramp constraints. Despite the significantly increased complexity, the results of this paper indicate that the method can achieve high quality solutions notably faster than other approaches from the literature.

Keywords: thermal unit commitment; combinatorial optimization; search space reduction; heuristic

Citation: de Oliveira, L.M.; da Silva Junior, I.C.; Abritta, R. A Space Reduction Heuristic for Thermal Unit Commitment Considering Ramp Constraints and Large-Scale Generation Systems. *Energies* **2023**, *16*, 5370. https://doi.org/10.3390/en16145370

Academic Editor: Alan Brent

Received: 25 May 2023
Revised: 28 June 2023
Accepted: 1 July 2023
Published: 14 July 2023

1. Introduction

The short-term Unit Commitment (UC) problem is among the most relevant problems in the electrical energy sector, as it targets the planning and operation of generating units, varying from a few hours to a day [1]. In general, UC can be described according to two main stages: the operation decisions regarding the generating units, and the economic dispatch [2]. The former aims to determine which generating units must operate and supply power during each hour of the planning horizon, while the latter has the goal of minimizing operational costs by optimally indicating how much power each operating unit should generate [3,4] in order to attend to the power and reserve demands. UC problems can relate to different planning horizons and time discretizations. This paper addresses applications regarding a short-term 24 h planning window with hourly discretizations. When all generating units are thermoelectric, UC is referred to as Thermal Unit Commitment (TUC). Solving the TUC problem is a complex task due to the large number of binary variables [5] which represent the operation decisions concerning each thermoelectric unit. Furthermore, the problem under investigation is subjected to time-coupling constraints that further increase its complexity. The UC problem can be represented in different manners, as shown in the review presented by Montero et al. [6].

The combinatorial explosion regarding the decision-making process implies a significant mathematical challenge to operational planning. Each unit added to an existing system greatly increases the number of operational possibilities. Large-scale systems, e.g., 40 to 100 units, have large solving difficulty due to the number of variables [7].

Considering ramp constraints brings the solution closer to real operation conditions by including the thermoelectric characteristics of the generators [8]. However, this makes the problem more challenging to solve [9]. Montero et al. [6] indicated that researchers occasionally neglect ramp constraints.

The TUC problem has been addressed by several techniques designed for combinatorial formulations. It is well known that an exhaustive search would be infeasible due to computational burden issues [10]. Among the early methods to solve TUC, Branch and Bound [11] and Dynamic Programming [12] should be highlighted. However, these approaches tend to take a long time to retrieve the solution, and can exclude the global optimum early in the solving process [13]. Priority lists are based on heuristic information regarding the characteristics of the generating units. They are widely utilized due to their simple implementation [14,15]. Metaheuristics are able to provide good results faster than other methods. However, they have the disadvantage of a large number of tuning parameters, and are prone to local stagnation. In addition, their good performance usually depends on a feasible initial solution [16–18].

In previous research [19], we presented a method of reducing the search space of the TUC problem. The proposed constructive heuristic based on sensitivity indices exploits system information to reduce the number of decision variables. This work extends our previous study, and makes two new main contributions to the TUC literature:

- An adaptation of the method from [19] is presented to significantly reduce the search space for the addressed TUC problem considering ramp constraints. Despite the space reduction, the methodology preserves high quality solutions and enables feasibility for cases in which the non-reduced problem does not converge within the established time limit. Montero et al. [6] showed that 50% of the reviewed publications neglected ramp constraints. This a fact increases the value of the results provided in this paper towards advancing the state of the art.
- Based on [6], it has been verified that 48% of the reported papers address systems with more than 50 generating units. In this paper, systems with 60, 80, and 100 units are within the scope of the study. Thus, the value of the proposed method is shown for large systems, which are usually neglected in the specialized literature.

It should be emphasized that the method proposed in [19] has proven able to greatly reduce the search space for small-scale TUC problems. The results from [19] motivated the study presented herein concerning larger systems subjected to ramp constraints.

Beyond this introduction, Section 2 provides the TUC problem formulation for this study, Section 3 summarizes the proposed approach according to previous research, Section 4 explores case studies by expanding the application to ramp-constrained and significantly larger systems compared to the previous research, and Section 5 concludes the work and mentions future studies.

2. Thermal Unit Commitment Problem

Different approaches exist to formulate the TUC problem [20–22]. In this work, the problem is provided by a mixed-integer quadratic programming formulation, as in [23]. A variety of constraints are considered, aiming at obtaining solutions that better mimic real operation, including minimum and maximum up time, cold and hot startup, and ramp limitations.

The objective function seeks to minimize the operational costs (C). These expenses are provided by the fuel consumption costs plus the startup costs, as in Equation (1). This goal is broadly utilized in TUC studies. However, adaptations including other goals and cost factors can be carried out according to the needs of the implementation. As a further remark, Equation (1) models the startup cost of a generating unit according to the period during which the unit has been inactive [24]. This factor changes the temperature conditions of the unit, thereby impacting the startup expenditure:

$$\text{minimize } C = \sum_{t=1}^{T} \sum_{i=1}^{NG} a_i u_{i_t} + b_i Pg_{i_t} + c_i Pg_{i_t}^2 + s_{i_t}^{cold} \cdot s_{i_{cost}}^{cold} + s_{i_t}^{hot} \cdot s_{i_{cost}}^{hot} \quad (1)$$

where T is the quantity of discretizations in the planning horizon; NG is the amount of generating units; a_i, b_i, and c_i are the fuel cost coefficients; u_{i_t} are the binary ON/OFF states of the units; Pg_{i_t} is the power output; $s_{i_t}^{cold}$ and $s_{i_t}^{hot}$ indicate cold and hot startups, respectively; and $s_{i_{cost}}^{cold}$ and $s_{i_{cost}}^{hot}$ are the costs for cold and hot startups, respectively.

As mentioned before, many characteristics constrain the TUC solving process. As in any electrical power system, power balance is mandatory. In other words, the power supply has to meet the demand, as represented by Equation (2):

$$\sum_{i=1}^{NG} Pg_{i_t} = L_t, \forall t \in T \quad (2)$$

where L_t is the power demand (MW) at time t.

Power dispatch problems usually include the requirement of a spinning reserve attendance, as in Equation (3). This reserve serves as a readily available power generation. It can help the overall system in case of unexpected events [25], such as a power demand value that surpasses what has been forecast:

$$\sum_{i=1}^{NG} u_{i_t}\overline{Pg}_i \geq L_t + sr_t, \forall t \in T \quad (3)$$

where sr_t is the predetermined spinning reserve (MW).

The operational characteristics of the units impose several constraints on the problem. The individual power generation of the units is bounded, as indicated by Equation (4). Upon startup/shutdown, a unit must remain on/off according to its minimum up/down time, as shown by Equations (5) and (6):

$$u_{i_t}\underline{Pg}_i \leq Pg_{i_t} \leq u_{i_t}\overline{Pg}_i, \forall t \in T, \forall i \in NG \quad (4)$$

$$\sum_{w=t-MUT_i+1}^{t} x_{i_w} \leq u_{i_t}, \forall t \in T, \forall i \in NG \quad (5)$$

$$\sum_{w=t-MDT_i+1}^{t} y_{i_w} \leq 1 - u_{i_t}, \forall t \in T, \forall i \in NG \quad (6)$$

where $\overline{Pg}_i$ and $\underline{Pg}_i$ are the maximum and minimum generation of unit i, respectively, and MUT_i and MDT_i represent the minimum up and down times, respectively. In other words, they relate to the least amount of time the unit must remain online/offline until it can go inactive/active again. In addition, x_{i_t} and y_{i_t} are binary variables indicating start-up and shut-down occurrences, respectively, while w is an auxiliary index responsible for representing the relevant periods.

In this formulation, the startup and shutdown occurrences require the auxiliary variables x and y. The former indicates the startup of a unit, whereas the latter represents its shutdown. Equation (7) establishes the temporal relation between the on/off variables and the startup/shutdown indications. Equation (8) prevents a unit from simultaneously starting and stopping:

$$u_{i_t} - u_{i_{t-1}} = x_{i_t} - y_{i_t}, \forall t \in T, \forall i \in NG \quad (7)$$

$$x_{i_t} + y_{i_t} \leq 1, \forall t \in T, \forall i \in NG \quad (8)$$

Equation (9) determines whether a startup is cold or hot. Equation (10) indicates if the startup cost should be taken as cold or hot, ensuring that the objective function is adequately quantified:

$$s_{i_t}^{cold} + s_{i_t}^{hot} = x_{i_t}, \forall t \in T, \forall i \in NG \quad (9)$$

$$u_{i_t} - \sum_{w=t-t_{csu_i}-MDT_i-1}^{t-1} u_{i_w} = s_{i_t}^{cold}, \forall t \in T, \forall i \in NG \quad (10)$$

where t_{csu_i} is the number of hours (after MDT) in which unit i has a hot start if activated again.

Finally, the following description regards the ramp constraints. These restrictions impose limitations on how much the power output of a unit can increase or decrease once it is in operation. If a unit is not operating and is about to be activated, it is not subjected to the ramp constraint. In other words, it can be set to any power value within its operational range. This is an important aspect that must be captured by the constraints. Equations (11) and (12) describe the up and down ramping limitations, respectively. The parameters U_i^{up} and U_i^{down} denote how much the generation of unit i can increase or decrease, respectively, if the unit was active in the previous period. Conversely, if the unit was off, ϵ ensures that the unit can be turned on and set to any power value in its generation range. It should be highlighted that ϵ must be greater than or equal to the maximum power output among all units. In cases in which a unit goes from an OFF to an ON state, Equation (4) is responsible for bounding the generation from above.

$$Pg_{i_t} - Pg_{i_{t-1}} \leq u_{i_{t-1}} \cdot U_i^{up} + \left(1 - u_{i_{t-1}}\right) \cdot \epsilon, \forall t \in T, \forall i \in NG \quad (11)$$

$$Pg_{i_{t-1}} - Pg_{i_t} \leq u_{i_t} \cdot U_i^{down} + \left(1 - u_{i_t}\right) \cdot \epsilon, \forall t \in T, \forall i \in NG \quad (12)$$

3. Summary of Previous Work

In [19], we proposed a constructive heuristic method to reduce the search space in TUC problems. The method applies a workflow to generate several hybrid priority lists (HPLs). These HPLs are combinations of pre-established lists obtained via consolidated metrics, such as the full load average production cost [26], the production marginal cost [27], and the Lagrange sensitivity [28]. Each HPL can be used to determine the operation decisions of a TUC problem, followed by economic dispatch optimization. However, the method in [19] consists of grouping all HPLs to obtain the so-called Relevance Matrix (RM).

In essence, each HPL is a matrix of zeros and ones that indicates the status of the units over the planing horizon. The RM comes from the sum of all HPLs, granting activation percentages regarding the units at each period. For instance, if at a certain hour a unit was given as active in 800 out of 1000 HPLs, the method states that this unit had a relevance of 80% to operate at this hour. Readers who wish to understand the method in greater detail are invited to read the third section of [19], which is openly available. This work does not present the method in its entirety, as it has not been modified. As mentioned before, the main contribution of the present paper comes from demonstrating the feasibility and effectiveness of our previous method in larger-scale systems including ramp constraints. Apart from not considering ramp limitations, the largest system in [19] had 40 units. The present paper, on the other hand, applies the method for systems with up to 100 units.

The method considers three relevance levels for RM: high (α), in which units were ON at a certain hour in all HPLs; low (β), in which units were ON at a certain hour in 10% or less of the HPLs; and none(γ), in which units were always OFF at a certain hour. The 10% criterion for low relevance classification was decided empirically. After several trials, this value showed a good balance between the number of the number of fixed variables (which impacts the computational burden) and quality of solutions.

Considering the aforementioned relevance levels, the method fixes several of the ON/OFF decision variables; in other words, RM indicates their operation. Therefore, these variables are no longer for the optimization to quantify. The procedure to fix part of the variables follows Equation (13). If the relevance of a unit at a given time does not relate to the procedure, its decision comes from the optimization of Equations (1)–(12).

$$u_{i_t} = \begin{cases} 1, \forall\, \alpha \\ 0, \forall\, \beta \text{ and } \gamma \end{cases} \tag{13}$$

Table 1 exemplifies the utilization of RM. In this table, the α, β, and γ occurrences are colored as ( ), ( ), and ( ), respectively. This color scheme is utilized in the other tables as well. In Table 1, 18 out of the 24 ON/OFF decision variables are decided by RM. The reduced optimization problem has 2^6 ON/OFF combinations to explore, whereas the original problem needed to search 2^{24} combinations. This example illustrates the extent to which our method can reduce the solution region. Despite being merely a demonstration, this example highlights that the previous work [19] reports reductions in the number of ON/OFF decision variables of around 80%.

Table 1. Example of RM.

Hour	U1	U2	U3	U4
1	100	98	2	0
2	100	100	9	91
3	100	100	98	2
4	100	100	100	100
5	100	100	23	77
6	100	92	8	0

4. Results and Discussions

The ten-unit system from [24] was used as an initial test for the proposed method. This system uses a planning horizon of 24 h. Table 2 provides the technical data on the system. The power demands are found in [24]. The spinning reserve is taken as 10% of the load, as in [29]. A total of 1000 HPLs were created, according to the procedure from [19]. Figure 1 shows the boxplot regarding the solutions for each one of the HPLs separately. The lowest cost found is equal to USD 564,795. The median of all solutions is equal to USD 569,327. The literature reports an optimal cost of USD 563,937. It can be observed that the process of generating HPLs was able to achieve solutions that are close to the optimal.

The approach proposed in this paper aims to reduce the search space while keeping good quality solutions and alleviating the computational burden. Considering that the HPLs reduce the search space by deciding the ON/OFF variables, Figure 1 provides an idea of the quality of solutions after fixing the binary decisions. However, the true potential of the method is yet to be shown. Combined with RM, an academic license of the commercial solver Mosek [30] is utilized to demonstrate the computational benefits that can be obtained through this approach.

Table 2. Data on the ten-unit system.

Unit	1	2	3	4	5	6	7	8	9	10
a	1000	970	700	680	450	370	480	660	665	670
b	16.19	17.26	16.6	16.5	19.7	22.26	27.74	25.92	27.27	27.79
c	0.00048	0.00031	0.002	0.00211	0.00398	0.00712	0.00079	0.00413	0.00222	0.00173
Pg_i^{max}	455	455	130	130	162	80	85	55	55	55
Pg_i^{min}	150	150	20	20	25	20	25	10	10	10
MUT_i	8	8	5	5	6	3	3	1	1	1
MDT_i	8	8	5	5	6	3	3	1	1	1
s_i^{hot}	4500	5000	550	560	900	170	260	30	30	30
s_i^{cold}	9000	10,000	1100	1120	1800	340	520	60	60	60
t_{csu_i}	5	5	4	4	4	2	2	0	0	0
IC	8	8	−5	−5	−6	−3	−3	−1	−1	−1

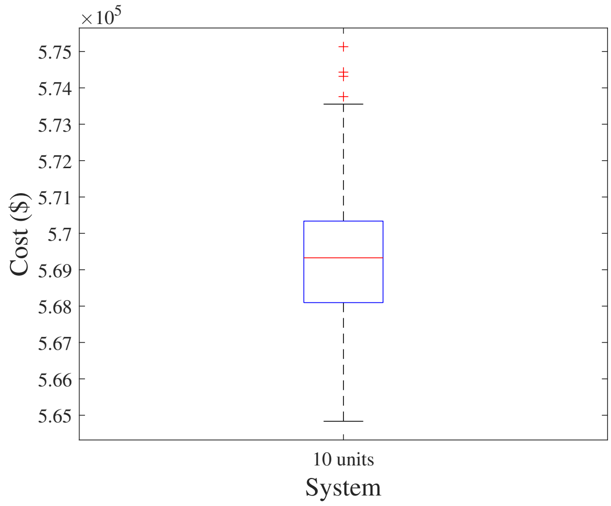

Figure 1. Cost dispersion for the ten-unit system.

As shown in the example of Table 1, the proposed RM does not decide all ON/OFF decisions. Therefore, Mosek is responsible for optimizing the remaining binary variables, in addition to solving the economic dispatch of the generating units. As described below, five cases are considered regarding the search space reduction. Table 3 presents the reduction magnitude of each case for the ten-unit system in Table 2. Table 3 was obtained according to the procedure summarized in Section 3. In other words, 1000 HPLs were created to define the relevance levels.

1. No reduction; Mosek solves the full problem.
2. RM fixes decisions only for α occurrences.
3. RM fixes decisions only for γ occurrences.
4. RM fixes decisions only for β occurrences (which include γ).
5. RM fixes decisions for α, β, and γ occurrences.

Table 3. Reduction alternatives and their impacts.

Total Amount of Binary Variables	Case	Number of Fixed Variables	Reduction (%)
240	1	0	0
	2	104	43.33
	3	54	22.5
	4	89	37.08
	5	193	80.42

Considering the meaning of cases 2 and 3, Table 3 shows that 104 occurrences always participate in the power demand attendance. In opposition, 54 occurrences never take part. For clarification, an occurrence refers to a certain unit operating at a certain hour. After reducing the search space, Mosek solves the remaining problem. An optimality gap of up to 0.5% was allowed. This gap is provided by the percentage difference between the objective function evaluation of a feasible solution and the evaluation of the relaxed problem regarding the binary variables [30]. Thus, as long as the gap is greater than 0.5%, Mosek's Branch and Bound keeps exploring the nodes in the search for an acceptable feasible solution. Table 4 presents the optimal cost, number of explored branches (NB), and solving time (ST) for the five cases.

Table 4. Optimization results with RM space reduction — ten-unit system.

Gap	Case	*C* ($)	NB	ST (s)
0 %	1	563,937.68	5843	72.45
	2	563,937.68	1277	11.80
	3	563,937.68	944	6.49
	4	563,937.68	36	0.58
	5	563,937.68	23	0.58
0.05 %	1	563,937.68	5693	67.5
	2	563,937.68	1020	9.80
	3	563,937.68	942	6.73
	4	563,978.82	24	0.42
	5	563,978.82	8	0.45
0.25 %	1	563,989.99	170	3.13
	2	564,189.16	154	2.14
	3	563,989.85	258	2.08
	4	564,916.16	8	0.39
	5	564,218.91	0	0.38
0.5 %	1	564,241.32	98	1.94
	2	565,486.43	6	0.72
	3	564,444.31	98	1.38
	4	564,916.15	0	0.25
	5	564,218.91	0	0.09

Table 4 shows that the optimizer required around 72 seconds to find the global solution when the proposed method was not applied (null gap, case 1). For all reduction cases and gap possibilities, ST is reduced significantly compared to case 1. It is worth highlighting that all simulations utilized the same hardware. Therefore, the reductions to ST are all due the proposed method. Regarding the cases with a gap of 0%, Table 4 indicates that the global solution was achieved even in case 5, which is the one with the most significantly reduced search region. This is an important observation, as it shows that for this system the proposed approach is able to fix binary variables in such a way that the global optimum was not compromised.

As a highlight, Figure 1 shows that none of the 1000 HPLs was individually capable of providing the optimal cost of USD 563,937.68 after optimization of the economic dispatch. However, as shown in Table 4, combining the HPLs to create RM could achieve global optimality.

For non-null gap values, it can be observed that tightening the search space can affect the obtained solution concerning the global optimum. However, this is related to both the space reduction and the gap itself. For the the cases with a gap of 0.05%, it can be noted that the best solution was not achieved by the case with the most significant reduction. It is worth emphasizing that the final solution for non-null gap binary optimization via branch-and-bound algorithms always depends on how the binary search is performed. The most important result to be highlighted here is that the proposed approach is capable of greatly decreasing the computational burden while keeping high quality solutions. Apart from the results presented for the ten-unit system, the following subsections explore studies for systems with up to 100 units planned over 24 h. Such systems are obtained by replicating the data from Table 2, as in [24].

4.1. Large-Scale Systems without Ramp Constraints

As a thermoelectric system becomes larger, the number of combinations for the binary ON/OFF decisions grows exponentially. For 100 units operating over 24 h, there are 2400 variables, for a total of 2^{2400} ON/OFF combinations. This combinatorial explosion tends to make classical optimization approaches not viable. Therefore, heuristics can play an important role in reducing the number of combinations to be evaluated.

In order to validate the method brought in this paper, RM was put to the test in systems with 20, 40, 60, 80, and 100 units. These systems were obtained by duplicating the data from Table 2. Figure 2 shows the cost dispersion for these systems. Table 5 provides the median and minimum costs. Analogously to analysis of the the ten-unit system, these results were obtained prior to applying RM.

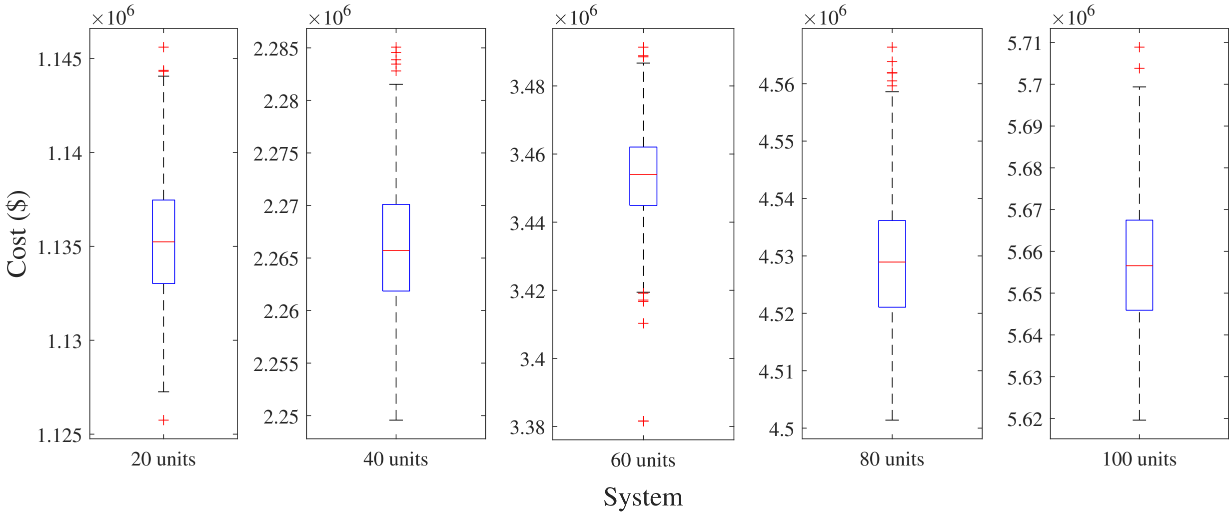

Figure 2. Cost dispersion for the larger systems.

Table 5. Operation costs regarding the 1000 HPLs.

System	20 Units	20 Units	40 Units	60 Units	80 Units
Minimum cost	$1,125,747	$2,249,574	$3,381,603	$4,501,392	$5,619,591
Median cost	$1,135,242	$2,265,708	$3,454,001	$4,528,932	$5,656,563

For each of the five systems, the RM was obtained in order for the Mosek optimization to take place. The same gap possibilities from Table 4 were utilized. Certain simulations were quite long due to the size of the system. Computations that reached 3 h without convergence were interrupted. The following tables tag these cases with a "*". The results are summarized in Table 6.

As seen in Table 6, several small gap cases were interrupted due to reaching the 3 h limit. Only the twenty-unit system converged (for certain cases) with a null gap. In this system, case 5 increased the cost by 0.087% compared to case 2. However, the solving time decreased by 99.57%. Similar trends can be observed for the other gap possibilities and for the other systems. With a gap of 0.05%, only the twenty-unit and forty-unit systems converged. The latter was only solved within a viable time in case 5. The process took almost 1 h. This exemplifies how the computational effort increases as the system becomes larger. For a gap of 0.25%, cases 1 and 3 did not converge for systems with forty or more units. As a reminder, case 1 applied no reduction, whereas case 3 had a small impact as it only fixed occurrences where the units were switched off in all HPLs (see Table 3). With the more relaxed gap of 0.5%, all cases converged. It is interesting to note that case 2 often presents a reasonable solving time despite the system being larger than in case 5. For instance, in cases 2 and 5 for the 100-unit system with a gap of 0.5%, it was expected that the solution of case 2 would have better quality than case 5, as its search space reduction was less intense. It can be seen that the solving time of case 2 is significantly larger than that of case 5. However, around 5 minutes is an acceptable time when it comes to daily planning. This indicates that the best way of utilizing such an heuristic depends on how the user balances the solving time versus the solution quality.

4.2. Large-Scale Systems with Ramp Constraints

To verify the applicability of the proposed method to systems subjected to ramp constraints, the same cases of the previous subsection, i.e., based on [24], were simulated, including Equations (11) and (12). The parameters U_i^{up} and U_i^{down} for each unit were both taken as equal to 20% of the maximum power output of the unit [31]. Ramp constraints model the behavior where the power generation of a unit cannot reach its maximum capacity in an one hour interval. Adding new constraints to an optimization problem tends to make it computationally heavier. Therefore, heuristics that aim to reduce the search region become even more valuable. Table 7 summarizes the results for this part of the research.

Compared to the simulations without ramping constraints (Tables 4 and 6), the following simulations did not converge within the established time limit: gap 0, twenty units, case 2; gap 0.25%, twenty units, cases 1 and 3; gap 0.05%, forty units, case 5; gap 0.25%, forty, sixty, and eighty units, case 2. These are consequences of the additional layer of complexity added by the ramp constraints. In general, similar conclusions to the previous subsection can be drawn regarding the tradeoffs between the solving time and optimality gap. Overall, the most important indication is that again the proposed approach is shown to greatly reduce the computational burden, although the quality of the solutions is hardly affected.

Table 6. Optimization results with RM space reduction applied to large systems.

Gap	Case	20 Units			40 Units			60 Units			80 Units			100 Units		
		C ($)	NB	ST (s)	*C* ($)	NB	ST (s)	*C* ($)	NB	ST (s)	*C* ($)	NB	ST (s)	*C* ($)	NB	ST (s)
0%	1	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	2	1,123,297	165,418	3367.5	*	*	*	*	*	*	*	*	*	*	*	*
	3	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	4	1,124,274	11,621	79.1	*	*	*	*	*	*	*	*	*	*	*	*
	5	1,124,274	2649	14.5	*	*	*	*	*	*	*	*	*	*	*	*
0.05%	1	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	2	1,123,300	21,979	426.7	*	*	*	*	*	*	*	*	*	*	*	*
	3	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	4	1,124,274	10,651	64.5	*	*	*	*	*	*	*	*	*	*	*	*
	5	1,124,274	2224	12.9	2,246,107	266,720	3299.1	*	*	*	*	*	*	*	*	*
0.25%	1	1,123,464	23,389	799.4	*	*	*	*	*	*	*	*	*	*	*	*
	2	1,123,783	97	4.81	2,242,965	10,106	813.8	3,366,371	21,222	2687.6	4,481,489	13,153	3108.5	*	*	*
	3	1,123,893	28,230	755.3	*	*	*	*	*	*	*	*	*	*	*	*
	4	1,124,527	98	1.5	2,247,521	426	9.8	3,371,769	1616	363.3	4,489,471	453	45.7	5,604,355	6915	495.5
	5	1,125,074	0	0.4	2,246,575	194	4.7	3,370,974	356	30.8	4,490,665	380	32.1	5,605,442	803	47.9
0.5%	1	1,125,770	466	16.1	2,247,391	879	34.8	3,364,026	585	171.2	4,488,501	671	327.2	5,607,777	742	309.4
	2	1,123,783	97	3.9	2,244,488	539	16.9	3,373,450	0	5.9	4,484,954	489	112.4	5,602,995	739	263.6
	3	1,125,265	192	7.1	2,246,949	788	23.8	3,371,815	621	131.3	4,484,159	597	116.8	5,611,380	686	303.4
	4	1,127,305	0	0.4	2,252,604	0	0.4	3,376,798	531	80.2	4,494,195	0	0.9	5,610,948	0	2.5
	5	1,127,290	0	0.2	2,251,593	0	0.2	3,373,512	0	2.91	4,496,674	0	0.9	5,609,350	0	1.6

Table 7. Optimization results with RM space reduction applied to large systems considering ramp constraints.

		10 Units			20 Units			40 Units			60 Units			80 Units			100 Units		
Gap	**Case**	***C* ($)**	**NB**	**ST (s)**	***C* ($)**	**NB**	**ST (s)**	***C* ($)**	**NB**	**ST (s)**	***C* ($)**	**NB**	**ST (s)**	***C* ($)**	**NB**	**ST (s)**	***C* ($)**	**NB**	**ST (s)**
0%	1	565,186	28,732	434.3	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	2	565,186	1555	19	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	3	565,186	3559	26.8	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	4	565,186	71	0.6	1,126,541	25,551	310.6	*	*	*	*	*	*	*	*	*	*	*	*
	5	565,186	39	0.4	1,126,541	4607	37.7	*	*	*	*	*	*	*	*	*	*	*	*
0.05%	1	565,186	28,572	387.4	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	2	565,186	1258	14.3	1,125,576	73,900	1890.1	*	*	*	*	*	*	*	*	*	*	*	*
	3	565,186	3559	22.6	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	4	565,186	71	0.56	1,126,541	24,273	212.5	*	*	*	*	*	*	*	*	*	*	*	*
	5	565,227	28	0.44	1,126,541	3225	22.3	*	*	*	*	*	*	*	*	*	*	*	*
0.25%	1	565,227	8485	100	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	2	565,518	238	3.1	1,126,160	1322	45.1	*	*	*	*	*	*	*	*	*	*	*	*
	3	565,254	846	5.4	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	4	565,421	0	0.2	1,126,985	303	5.3	2,251,372	16,197	600	3,376,457	3656	276.3	4,496,094	714	65.8	5,612,040	1791	251.2
	5	565,421	0	0.1	1,126,605	182	2.8	2,251,818	234	8.8	3,376,664	338	39.6	4,498,484	839	58.0	5,614,517	1218	113.3
0.50%	1	565,829	98	2.3	1,127,222	272	11.2	2,248,551	418	52.2	3,370,788	716	116.8	4,495,848	852	510.4	5,615,392	1141	350.9
	2	565,711	98	2.1	1,126,928	192	7.5	2,248,029	291	24.6	3,372,529	405	52.4	4,490,705	544	103.3	5,612,895	675	166.1
	3	565,509	178	1.8	1,127,115	369	12.8	2,249,392	353	25.8	3,374,612	585	61.3	4,492,980	729	271.1	5,614,996	736	288.9
	4	565,421	0	0.1	1,128,091	98	1.9	2,255,748	0	1.0	3,379,952	504	42	4,506,255	0	3.8	5,622,632	0	3.1
	5	565,421	0	0.1	1,128,047	0	0.1	2,256,506	0	1.1	3,376,664	338	31.4	4,502,989	0	1.2	5,618,623	0	1.9

4.3. Comparisons to Other Publications

To validate the proposed approach, this subsection presents comparisons to the TUC literature regarding the optimal cost and processing time. For all executions of the proposed heuristic, the simulation duration is based on a Core i5 1.19 Ghz, 8 GB de RAM computer. Tables 8 and 9 provide comparisons without and with the ramp constraints, respectively. In these tables, the lowest cost for each system is presented in bold. The values of C for the proposed approach were retrieved from Tables 4, 6 and 7. They were chosen according to what we believe represents a good balance between cost and solving time. Thus, these cost values are not necessarily the smallest ones among all executed simulations.

Table 8. Literature comparison for the TUC problem without ramp constraints.

Ref.	Year	10 Units		20 Units		40 Units		60 Units		80 Units		100 Units	
		C ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)
[24]	1996	565,825	221	1,126,243	733	2,251,911	2697	3,376,625	5840	4,504,933	10,036	5,627,437	15,733
[32]	2011	570,006	18.34	1,139,005	65.87	2,277,396	317.3	3,420,438	572.3	4,554,346	1069	5,706,201	1735
[33]	2014	**563,937**	-	1,123,996	-	2,246,445	-	3,364,665	-	4,488,039	-	5,607,838	-
[34]	2016	563,938	1.3	1,123,587	4	2,243,688	26.7	3,362,951	39.3	4,484,497	90.1	5,602,364	144.2
[14]	2017	564,835	0.154	1,126,231	0.169	2,250,405	0.194	3,370,832	0.215	4,495,873	0.246	5,616,303	0.275
[35]	2017	563,977	-	1,124,056	-	2,243,721	-	3,362,485	-	4,484,974	-	5,605,622	-
[36]	2018	**563,937**	28.19	**1,123,297**	42.64	**2,242,957**	115.5	**3,361,298**	176.2	**4,481,770**	310	**5,601,726**	381
[37]	2019	565,807	231.3	-	-	-	-	-	-	-	-	-	-
[38]	2019	**563,937**	204.3	1,124,931	427.8	2,251,891	2688	-	-	-	-	-	-
[39]	2020	564,810	20.47	-	-	-	-						
[40]	2020	**563,937**	-	1,124,389	-	2,246,837	-	3,367,348	-	4,491,179	-	5,611,494	-
[41]	2021	563,977	0.18	1,124,926	0.23	2,248,413	0.32	3,368,150	0.45	4,492,188	0.61	5,611,886	0.76
[42]	2021	563,977	-	1,123,311	-	-	-	-	-	-	-	-	-
[43]	2021	563,978	17.78	1,123,825	26.22	2,247,165	43.27	3,369,731	62.33	4,493,825	76.62	5,618,038	96.64
[44]	2015	563,938	14.76	**1,123,297**	36.96	2,243,996	86.52	3,364,076	188.2	4,486,528	323	5,605,748	452
[45]	2022	**563,937**	40.44	1,124,389	-	2,246,837	-	3,367,348	-	4,491,179	-	5,611,494	-
Proposed		**563,937**	0.58	1,123,783	3.9	2,244,488	16.9	3,373,512	2.91	4,484,159	116.8	5,602,995	263.6

Table 9. Literature comparison for the TUC problem with ramp constraints.

Ref.	Year	10 Units		20 Units		40 Units		60 Units		80 Units		100 Units	
		C ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)	*C* ($)	ST (s)
[46]	2004	566,404	7.4	1,127,244	22.4	2,254,123	58.3	3,378,108	117.3	4,498,943	176	5,630,838	242.5
[31]	2006	565,988	3.35	1,127,955	16.8	2,252,125	88.28	-	-	4,501,156	405	5,624,301	696.4
[44]	2015	565,671	35.49	1,126,634	87.36	2,252,544	212.9	3,375,368	396	4,500,742	614.7	5,628,853	1007
[34]	2016	565,723	2.4	1,128,273	14.6	2,251,134	33.7	3,376,480	88.5	4,501,641	123	5,625,921	210.9
[36]	2018	565,195	41.56	**1,125,667**	65.8	**2,247,774**	157.6	**3,369,320**	238.2	4,494,574	404.2	5,616,689	485.3
[47]	2020	565,671	21.71	1,126,461	59.74	2,251,398	157.8	3,373,241	244.9	4,498,081	382.5	**5,609,984**	597.5
[48]	2020	565,527	-	1,126,702	-	2,249,413	-	3,370,854	-	4,495,995	-	5,618,169	-
Proposed		**565,186**	0.4	1,126,541	22.3	2,248,029	24.6	3,376,664	31.4	**4,490,705**	103.3	5,614,517	113.3

For the simulations without ramp constraints (Table 8), the proposed RM method was the fastest to achieve the global optimum for the ten-unit system compared to the papers that reported simulation duration. For the twenty-, forty-, and sixty-unit systems, the cost values from RM were only slightly higher than the best ones in the analyzed literature. However, the simulation lengths were significantly shorter. For the 80- and 100-unit systems, the gains in computational burden were relevant, despite not being as prominent as in the other systems. As a remark regarding the two larger systems, it can be seen in Table 6 that the proposed approach was able to find costs of 4,496,674 and 5,609,350 in 0.9 and 1.6 s, respectively. On the one hand, these values are around 0.35% greater than the lowest costs reported in the literature. On the other hand, the processing times decreased by more than 99%. This indicates how powerful the proposed RM method can be in reaching good quality solutions for the TUC problem in incredibly short time periods.

When the ramp constraints are included (Table 9), RM outperformed the other papers both in terms of cost and time concerning the ten- and eighty-unit systems. For all other systems, the proposed method converged with slightly higher cost, though in a shorter period of time. Analogously to the studies without ramp constraints, Table 7 reveals that RM can find solutions for the 80- and 100-unit systems faster than the values presented in Table 9. The method does this without imposing a significant increment to the cost values.

It is worth noting that in Tables 8 and 9 the proposed approach provides values from different reduction cases and optimality gaps. In other words, the proposed heuristic should be tested for different cases and gaps before being applied to real problems. Depending on the system and constraints in place, a different case may be the most promising one.

5. Conclusions

This paper further explores a search region reduction method proposed by the authors for the thermal unit commitment problem. As a sequence to the previous research, the method was applied to larger systems with up to 100 generating units. Furthermore, ramp constraints were included as an additional layer of complexity. The proposed approach has been shown to greatly reduce the computational burden with a minor overall impact on the obtained cost. For certain cases, the method was able to outperform the investigated alternatives from the literature in terms of the final cost as well.

The implementation of the proposed method is relatively simple. In addition, although Mosek was used to solve the economic dispatch and operation decisions that were not fixed by RM, many other optimization techniques could be applied. It should be emphasized that the method can also be used to generate initial solutions for other optimization techniques.

The findings of this paper demonstrate that the proposed method has the potential to be a solid contribution to the solving processes of large thermal systems. In the investigations carried out in this work, RM reduced the computational burden by up to 99.2% in certain cases compared to the convergence time required when the method was not utilized. Adding network constraints to the formulation is the main factor in expanding the scope of future research.

Author Contributions: Conceptualization, I.C.d.S.J. and L.M.d.O.; methodology, I.C.d.S.J. and L.M.d.O.; software, L.M.d.O. and R.A.; validation, L.M.d.O.; formal analysis, I.C.d.S.J., L.M.d.O. and R.A.; investigation, L.M.d.O.; resources, I.C.d.S.J. and L.M.d.O.; data curation, L.M.d.O.; writing—original draft preparation, L.M.d.O.; writing—review and editing, I.C.d.S.J., L.M.d.O. and R.A.; visualization, L.M.d.O. and R.A.; supervision, I.C.d.S.J.; project administration, I.C.d.S.J.; funding acquisition, I.C.d.S.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by FAPEMIG, grant number PPM-00184-17.

Data Availability Statement: Not applicable.

Acknowledgments: The authors thank FAPEMIG and the Electrical Engineering Post-Graduation Program of the Federal University of Juiz de Fora for supporting this research.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study, in the collection, analysis, or interpretation of data, in the writing of the manuscript, or in the decision to publish the results.

Abbreviations

The following abbreviations are used in this manuscript:

TUC	Thermal Unit Commitment
RM	Relevance Matrix
HPL	Hybrid Priority List
NB	Number of explored Branches
ST	Solving Time

References

1. Tovar-Ramírez, C.; Fuerte-Esquivel, C.; Mares, A.M.; Sánchez-Garduño, J. A generalized short-term unit commitment approach for analyzing electric power and natural gas integrated systems. *Electr. Power Syst. Res.* **2019**, *172*, 63–76. [CrossRef]
2. Saravanan, B.; Das, S.; Sikri, S.; Kothari, D. A solution to the unit commitment problem—A review. *Front. Energy* **2013**, *7*, 223–236. [CrossRef]
3. Knueven, B.; Ostrowski, J.; Watson, J.P. On mixed-integer programming formulations for the unit commitment problem. *INFORMS J. Comput.* **2020**, *32*, 857–876. [CrossRef]
4. Alam, M.S.; Kumari, M.S. Unit commitment of thermal units in integration with wind and solar energy considering ancillary service management using priority list (IC) based genetic algorithm. In Proceedings of the 2016 International Conference on Innovation and Challenges in Cyber Security (ICICCS-INBUSH), Greater Noida, India, 3–5 February 2016; pp. 277–282. [CrossRef]
5. Wood, A.J.; Wollenberg, B.F.; Sheblé, G.B. *Power Generation, Operation, and Control*; John Wiley & Sons: Hoboken, NJ, USA, 2013.
6. Montero, L.; Bello, A.; Reneses, J. A review on the unit commitment problem: Approaches, techniques, and resolution methods. *Energies* **2022**, *15*, 1296. [CrossRef]
7. Correa-Posada, C.M.; Morales-Espana, G.; Duenas, P.; Sánchez-Martín, P. Dynamic ramping model including intraperiod ramp-rate changes in unit commitment. *IEEE Trans. Sustain. Energy* **2016**, *8*, 43–50. [CrossRef]
8. Wang, C.; Shahidehpour, S. Ramp-rate limits in unit commitment and economic dispatch incorporating rotor fatigue effect. *IEEE Trans. Power Syst.* **1994**, *9*, 1539–1545. [CrossRef]
9. Deka, D.; Datta, D. Optimization of unit commitment problem with ramp-rate constraint and wrap-around scheduling. *Electr. Power Syst. Res.* **2019**, *177*, 105948. [CrossRef]
10. Finardi, E.C.; Silva, E.L.d.; Sagastizábal, C. Solving the unit commitment problem of hydropower plants via Lagrangian relaxation and sequential quadratic programming. *Comput. Appl. Math.* **2005**, *24*, 317–342. [CrossRef]
11. Ostrowski, J.; Anjos, M.F.; Vannelli, A. Modified orbital branching for structured symmetry with an application to unit commitment. *Math. Program.* **2015**, *150*, 99–129. [CrossRef]
12. Singhal, P.K.; Sharma, R.N. Dynamic programming approach for large scale unit commitment problem. In Proceedings of the 2011 International Conference on Communication Systems and Network Technologies, Katra, India, 3–5 June 2011; pp. 714–717. [CrossRef]
13. Diniz, A. Uma Estratégia de Decomposição por Relaxação Lagrangeana para a Otimização da Programação diária da Operação de Sistemas Hidrotérmicos com Modelagem Detalhada da rede Elétrica–aplicação ao Sistema Brasileiro. Ph.D. Thesis, COPPE/UFRJ, Rio de Janeiro, Brazil, 2007.
14. Elsayed, A.M.; Maklad, A.M.; Farrag, S.M. A new priority list unit commitment method for large-scale power systems. In Proceedings of the 2017 Nineteenth International Middle East Power Systems Conference (MEPCON), Cairo, Egypt, 19–21 December 2017; pp. 359–367. [CrossRef]
15. Shahbazitabar, M.; Abdi, H. A novel priority-based stochastic unit commitment considering renewable energy sources and parking lot cooperation. *Energy* **2018**, *161*, 308–324. [CrossRef]
16. Reddy, K.S.; Panwar, L.K.; Panigrahi, B.; Kumar, R. A new binary variant of sine–cosine algorithm: Development and application to solve profit-based unit commitment problem. *Arab. J. Sci. Eng.* **2018**, *43*, 4041–4056. [CrossRef]
17. Reddy K, S.; Panwar, L.; Panigrahi, B.; Kumar, R. Binary whale optimization algorithm: A new metaheuristic approach for profit-based unit commitment problems in competitive electricity markets. *Eng. Optim.* **2019**, *51*, 369–389. [CrossRef]
18. Barati, M.; Farsangi, M.M. Solving unit commitment problem by a binary shuffled frog leaping algorithm. *IET Gener. Transm. Distrib.* **2014**, *8*, 1050–1060. [CrossRef]
19. de Oliveira, L.M.; da Silva Junior, I.C.; Abritta, R. Search Space Reduction for the Thermal Unit Commitment Problem through a Relevance Matrix. *Energies* **2022**, *15*, 7153. [CrossRef]
20. Tuffaha, M.; Gravdahl, J.T. Mixed-integer formulation of unit commitment problem for power systems: Focus on start-up cost. In Proceedings of the IECON 2013—39th Annual Conference of the IEEE Industrial Electronics Society, Vienna, Austria, 10–13 November 2013; pp. 8160–8165. [CrossRef]
21. Tumuluru, V.K.; Huang, Z.; Tsang, D.H. Unit commitment problem: A new formulation and solution method. *Int. J. Electr. Power Energy Syst.* **2014**, *57*, 222–231. [CrossRef]
22. Tejada-Arango, D.A.; Lumbreras, S.; Sánchez-Martín, P.; Ramos, A. Which unit-commitment formulation is best? A comparison framework. *IEEE Trans. Power Syst.* **2019**, *35*, 2926–2936. [CrossRef]
23. Viana, A.; Pedroso, J.P. A new MILP-based approach for unit commitment in power production planning. *Int. J. Electr. Power Energy Syst.* **2013**, *44*, 997–1005. [CrossRef]
24. Kazarlis, S.A.; Bakirtzis, A.; Petridis, V. A genetic algorithm solution to the unit commitment problem. *IEEE Trans. Power Syst.* **1996**, *11*, 83–92. [CrossRef]
25. Abujarad, S.Y.; Mustafa, M.W.; Jamian, J.J. Recent approaches of unit commitment in the presence of intermittent renewable energy resources: A review. *Renew. Sustain. Energy Rev.* **2017**, *70*, 215–223. [CrossRef]
26. Muralidharan, S.; Kumar, V.M.; Baalavignesh, A. Thermal unit commitment using flac guided modified dynamic programming approach. In Proceedings of the 2011 International Conference on Recent advancements in Electrical, Electronics and Control Engineering, Sivakasi, India, 15–17 December 2011, pp. 339–345. [CrossRef]
27. Pang, C.; Chen, H. Optimal short-term thermal unit commitment. *IEEE Trans. Power Appar. Syst.* **1976**, *95*, 1336–1346. [CrossRef]

28. Silva Junior, I.C.d.; Carneiro Junior, S.; Oliveira, E.J.d.; Pereira, J.L.R.; Garcia, P.A.N.; Marcato, A.L. Determinação da operação de unidades térmicas para o estudo de Unit Commitment através de uma análise de sensibilidade. *Sba Controle Automaç Ao Soc. Bras. De Autom.* **2006**, *17*, 300–311. [CrossRef]
29. Han, W.; Wang, H.h.; Zhang, X.s.; Chen, L. A unit commitment model with implicit reserve constraint based on an improved artificial fish swarm algorithm. *Math. Probl. Eng.* **2013**, *2013*, 912825. [CrossRef]
30. ApS, M. Mosek Documentation: Optimizers. 2019. Available online: https://docs.mosek.com/latest/capi/cont-optimizers.html (accessed on 13 May 2023).
31. Simopoulos, D.N.; Kavatza, S.D.; Vournas, C.D. Unit commitment by an enhanced simulated annealing algorithm. *IEEE Trans. Power Syst.* **2006**, *21*, 68–76. [CrossRef]
32. Wang, B.; Li, Y.; Watada, J. Re-scheduling the unit commitment problem in fuzzy environment. In Proceedings of the 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011), Taipei, Taiwan, 27–30 June 2011; pp. 1090–1095. [CrossRef]
33. Yuan, X.; Ji, B.; Zhang, S.; Tian, H.; Hou, Y. A new approach for unit commitment problem via binary gravitational search algorithm. *Appl. Soft Comput.* **2014**, *22*, 249–260. [CrossRef]
34. Zheng, H.; Jian, J.; Yang, L.; Quan, R. A deterministic method for the unit commitment problem in power systems. *Comput. Oper. Res.* **2016**, *66*, 241–247. [CrossRef]
35. Qin, H.; Wei, H. A quantum-inspired approximate dynamic programming algorithm for unit commitment problems considering wind power. In Proceedings of the 2017 IEEE International Conference on Smart Grid and Smart Cities (ICSGSC), Singapore, 23-26 July 2017; pp. 94–98. [CrossRef]
36. Tsalavoutis, V.A.; Vrionis, C.G.; Tolis, A.I. Optimizing a unit commitment problem using an evolutionary algorithm and a plurality of priority lists. *Oper. Res.* **2021**, *21*, 1–54. [CrossRef]
37. Khunkitti, S.; R. Watson, N.; Chatthaworn, R.; Premrudeepreechacharn, S.; Siritaratiwat, A. An improved DA-PSO optimization approach for unit commitment problem. *Energies* **2019**, *12*, 2335. [CrossRef]
38. Bhadoria, A.; Marwaha, S.; Kamboj, V.K. An optimum forceful generation scheduling and unit commitment of thermal power system using sine cosine algorithm. *Neural Comput. Appl.* **2020**, *32*, 2785–2814. [CrossRef]
39. Bhadoria, A.; Marwaha, S. Moth flame optimizer-based solution approach for unit commitment and generation scheduling problem of electric power system. *J. Comput. Des. Eng.* **2020**, *7*, 668–683. [CrossRef]
40. Zhao, S.; Yang, Z.; Zhu, X.; Wang, Y. A Novel Binary Social Learning Particle Swarm Optimizer for Power System Unit Commitment. In Proceedings of the 2020 IEEE 4th Conference on Energy Internet and Energy System Integration (EI2), Wuhan, China, 30 October–1 November 2020; pp. 1361–1365. [CrossRef]
41. Elsayed, A.M.; Maklad, A.M.; Farrag, S.M. Three-Stage Priority List Unit Commitment for Large-Scale Power Systems Considering Ramp Rate Constraints. *IEEE Can. J. Electr. Comput. Eng.* **2021**, *44*, 329–342. [CrossRef]
42. Sayed, A.; Ebeed, M.; Ali, Z.M.; Abdel-Rahman, A.B.; Ahmed, M.; Abdel Aleem, S.H.; El-Shahat, A.; Rihan, M. A hybrid optimization algorithm for solving of the unit commitment problem considering uncertainty of the load demand. *Energies* **2021**, *14*, 8014. [CrossRef]
43. Kigsirisin, S.; Miyauchi, H. Short-term operational scheduling of unit commitment using binary alternative moth-flame optimization. *IEEE Access* **2021**, *9*, 12267–12281. [CrossRef]
44. Singhal, P.K.; Naresh, R.; Sharma, V. A modified binary artificial bee colony algorithm for ramp rate constrained unit commitment problem. *Int. Trans. Electr. Energy Syst.* **2015**, *25*, 3472–3491. [CrossRef]
45. Zhu, X.; Zhao, S.; Yang, Z.; Zhang, N.; Xu, X. A parallel meta-heuristic method for solving large scale unit commitment considering the integration of new energy sectors. *Energy* **2022**, *238*, 121829. [CrossRef]
46. Damousis, I.G.; Bakirtzis, A.G.; Dokopoulos, P.S. A solution to the unit-commitment problem using integer-coded genetic algorithm. *IEEE Trans. Power Syst.* **2004**, *19*, 1165–1172. [CrossRef]
47. Kumar, V.; Naresh, R. Monarch butterfly optimization-based computational methodology for unit commitment problem. *Electr. Power Compon. Syst.* **2021**, *48*, 2181–2194. [CrossRef]
48. Zhu, Y.; Gao, H. Improved binary artificial fish swarm algorithm and fast constraint processing for large scale unit commitment. *IEEE Access* **2020**, *8*, 152081–152092. [CrossRef]