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Statistical analysis and modeling of decision-making among individuals with autistic and psychotic traits in the bunny task

Master's thesis in Applied Physics and Mathematics
Supervisor: Håkon Tjelmeland
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Preface

With this master's thesis, I am wrapping up my final year in the Applied Physics and Mathematics master's degree programme at the Norwegian University of Science and Technology. Through the main profile of industrial mathematics, I have specialized in statistics. My cross-disciplinary interests make statistics particularly exciting for me as it can be used in many fields of study. In this thesis I have used statistical analysis and modeling to analyze a problem relevant to the field of psychology, and I have studied decision-making in people with autistic and psychotic traits.

I am grateful to my supervisor, Håkon Tjelmeland, for his guidance and support throughout my work with the master´s thesis. His competence in the field of statistics has proven invaluable, assisting in both the computations and the assumptions made in my models. His guidance in structuring the thesis and his constructive feedback have been instrumental in improving the quality of the work.

Furthermore, my gratitude extends to Gerit Pfuhl for her assistance in defining the problem of interest in this thesis. The problem was both challenging and interesting, and her expertise in psychology deepened my understanding of the issue and its relevance in the field. I would also like to thank Rebekka Solvik Lisøy and Robert Biegler for providing me with the data from the bunny task and relevant previous work done within the research field.

Trondheim

Abstract

This thesis examines the inclination to report changes among individuals with autistic traits, psychotic traits, or both. The study investigates whether these individuals have a propensity to report changes when there is less substantial evidence for such changes.

We consider a change-point detection task called the "bunny task". Participants of this task were asked to report when they perceived changes to underlying properties. A Bayesian observer model is constructed to calculate change probabilities, which are probabilities of a change happening at each instance a participant makes a decision in the bunny task. Autistic and psychotic traits in the participants are assessed using an abbreviated version of the Autism-Spectrum Quotient (AQ) and the positive subscale of Community Assessment of Psychic Experiences (CAPEp) scores, respectively. Two analytical approaches are employed. The first is a group-based comparison based on AQ and CAPEp cut-off scores, and the second is analyzing a generalized mixed model with AQ and CAPEp scores as covariates.

In our group-based analysis, we discovered significantly higher change probabilities when a change was reported in individuals with AQ scores above and CAPEp scores below the cut-off, contradicting our initial hypothesis. Through fitting logit models to each participant, we found a significant correlation between the choice of reporting change or not and the corresponding change log odds. The change log odds effects were significantly lower in groups with high CAPEp scores than in the control group. Furthermore, high CAPEp score groups tended to report changes more when change probabilities were low. However, the group consisting of people with AQ above and CAPEp below the cut-off scores was found to generally underreport changes according to the fitted models.

In the generalized mixed model analysis, increasing AQ and CAPEp scores were associated with a decreased effect of change log odds. Higher CAPEp scores correlated with a higher tendency to infer changes at low change probabilities while elevating AQ scores did not reflect a similar trend.

These findings suggest that individuals with psychotic traits are more likely to infer changes with less evidence, unlike those with autistic traits who appeared to underreport changes.

Sammendrag

Denne masteroppgaven undersøker tilbøyeligheten til å rapportere endringer blant individer med autistiske trekk, psykotiske trekk, eller begge deler. Studien undersøker om disse individene har en tendens til å rapportere endringer når det er mindre tydelige indikasjoner på slike endringer.

En oppgave kalt "kaninoppgaven" blir brukt for å vurdere beslutningstakingen til folk med autistiske og psykotiske trekk. Deltakerne i denne oppgaven ble bedt om å rapportere når de oppfattet endringer i underliggende variabler. Vi konstruerer en bayesiansk modell for å beregne sannsynlighetene for at en endring har forekommet hver gang en deltaker tar en avgjørelse i kaninoppgaven. Autistiske og psykotiske trekk hos deltakerne blir vurdert ved hjelp av henholdsvis en forkortet versjon av Autism-Spectrum Quotient (AQ) og den positive subskalaen av Community Assessment of Psychic Experiences (CAPEp). To analytiske tilnærminger ble benyttet. Den første tilnærmingen innebærer å dele inn deltakerne i grupper basert på AQ- og CAPEp-verdiene, og den andre var å analysere en generalisert mikset modell med AQ- og CAPEp-scorer som kovariater.

I vår gruppebaserte analyse oppdaget vi betydelig høyere sannsynlighet for endring de gangene en endring ble rapportert hos individer med høye AQ-verdier og lave CAPEp-verdier, som motstrider vår opprinnelige hypotese. Ved å tilpasse logitmodeller for hver deltaker, fant vi en betydelig korrelasjon mellom valget om å rapportere en endring eller ikke og de tilhørende log odds for at en endring hadde skjedd. Effektene av log odds var betydelig lavere i grupper med høye CAPEp-verdier enn i kontrollgruppen. Videre hadde grupper med høy CAPEp-verdier en tendens til å rapportere endringer hyppigere når sannsynligheten for endring var lave. Imidlertid ble gruppen bestående av mennesker med høye AQ-verdier og lave CAPEp-verdier generelt estimert til å under-rapportere endringer ifølge til de tilpassede modellene.

I analysen av den generaliserte miksede modellen var økte AQ- og CAPEp-score forbundet med en redusert effekt av log odds for endring. Høyere CAPEp-score korrelerte med en høyere tendens til å tolke endringer ved lave sannsynligheter for endring, mens en økt AQ-score ikke viste en lignende trend.

Disse funnene antyder at individer med psykotiske trekk er mer tilbøyelige til å tolke endringer når det var lite som tydet på at en endring hadde skjedd ifølge den bayesianske modellen, i motsetning til de med autistiske trekk som generelt underrapportere endringer.

Contents

1 | Introduction

When people make decisions in everyday life, they often base them on learned experience, and this learned experience is constantly informed by observations made in the decision-making environment. Usually, incomplete information available to the decision maker results in uncertainty and variability in the observations. This variability can occur even in a stable and persistent environment. In this type of environment, the underlying statistical relationships are stable, but there is still variability in the observations. For example, consider observations from a normal distribution with a certain mean and variance. Observations drawn from this distribution will vary even though all observations are from the same distribution. Sometimes, observations from this distribution could be outliers and lie in another area than the observations from the same distribution. In that case, no fundamental change has happened, and the observation is from the same distribution as the previous observations. Other times, this unexpected observation may be due to a fundamental change in the environment, for example, that the observation is from another distribution than the previous observations. This could be a normal distribution with a different mean. Observations from this new distribution could also be within the same area as the previous observations from the first distribution, so even though the observation is not unexpected, there could be a fundamental change in the environment. In such a dynamic environment with natural randomness and where changes to underlying variables can occur, it could be challenging to have accurate beliefs of the variables relevant to the decision-making. In particular, it can be hard to distinguish between changes due to natural variability and actual changes to the underlying properties in the environment. When a fundamental change has happened, previous observations can also give imprecise information about the current state.

A topic of interest in psychology is investigating individuals' tendency to infer change in such environments. Some people might need more substantial evidence to be convinced that a change has happened. Others might be more easily convinced that a change has occurred without as large deviations from previous observations. The tendency to infer changes more frequently has been linked to different personality traits. Psychotic and autistic personality traits

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are of particular interest in this context. It has been theorized that people with psychotic traits, autistic traits, or both differ in how they infer change compared to the general populace.

People with psychotic traits perceive their surroundings differently from others. These traits are found in individuals who have psychotic disorders or who experience psychosis-like symptoms. Psychotic disorders can include schizophrenia, delusional disorders, or toxic psychosis. According to Lieberman and First [2018], a symptom that is common for all psychotic disorders is delusions. According to Kiran and Chaudhury [2009], delusions are defined as beliefs that are firmly held even though evidence contradicting the beliefs is presented. Another common symptom of psychotic disorders is hallucinations, meaning experiencing sensations such as hearing, seeing, or tasting things that are not real. According to Capra et al. [2013], psychotic traits can also include grandiosity and bizarre experiences.

The two main symptoms that characterize autism spectrum disorder (ASD) are social communication impairments and repetitive behavior. Social communication impairments vary between different people with ASD. According to Lord et al. [2018], these impairments can include struggles with conversations, deviations in non-verbal communication skills like eye and body contact, and difficulties with social relationships. The symptom of repetitive behavior can refer to repetitive movements, speech, and use of objects. In addition, it can refer to difficulties in deviating from certain routines, including verbal and nonverbal behavioral routines. Other typical repetitive behaviors include fixation on specific topics and unusually intense narrow interests. Furthermore, people with ASD can experience both sensory overload and sensory sensitivity.

It has been theorized that the autistic symptoms sensory overload, sensory sensitivity, and difficulties handling changing environments can be attributed to a tendency to overestimate prediction errors [Van de Cruys et al., 2014]. This overestimation may cause relatively minor deviations from expected outcomes to be interpreted as more substantial outliers. This tendency is expected to lead to inferring changes to underlying properties more frequently than an average person. This tendency is also proposed for people with psychotic traits [Fletcher and Frith, 2009]. Individuals with psychotic traits are often prone to perceiving changes in relatively stable environments, potentially leading to symptoms of delusions and hallucinations. There is some evidence that people with sub-tendencies to psychosis are related to more frequently inferring

changes in the environment [Kreis et al., 2020]. Results from studies of individuals with a risk for psychosis indicate similar tendencies [Cole et al., 2020]. However, these findings are contested as other studies have produced contradicting results [Powers et al., 2017]. Similarly, there exist studies that indicate that people with autistic traits tend to infer environmental changes more easily [Karvelis et al., 2018], [Nassar and Troiani, 2021]. These results are also mixed as other studies find no link between autistic traits and more frequent reporting of changes [Goris et al., 2021].

Differences in how people infer changes can be investigated through controlled experiments where people are asked to decide whether a change has happened based on their observations. This thesis considers a particular task called "the bunny task". This is a digital change-point detection task which includes both variability in the observations and changes to the underlying properties. The task of the participants are to differentiate between noise and fundamental changes. The bunny task is similar to the example already mentioned at the beginning of this section, where observations are drawn from a normal distribution. The participants of the experiment observe one bunny at a time. The bunny walks from the center of a circle in a particular direction. The direction in which the bunny walks in is drawn from a normal distribution. After a certain number of observations from one normal distribution, a certain number of observations are drawn from another normal distribution where the mean is changed and the variance is the same. Bunnies with walking directions drawn from the new distribution are considered to be part of a new family. After a certain number of observations from this family, walking directions drawn from a new normal distribution with a new mean value and the same variance appear. This repeats until all observations from all families are made. How much the mean value changes from the previous family to a new family is the same each time the family changes. Each time a new bunny is observed, the task participants have to classify the bunny into the family they think it is part of. As each new bunny is observed, the participants must decide if it belongs to a new family, as compared to the family the preceding bunnies were classified into. These decisions of whether a change had occurred or are of particular interest when assessing the participants' tendency to infer changes, and we refer to them as change choices. The bunny task is described in more detail in Section 2.1.

1.1 The problem of interest

Professors at NTNU, Gerit Pfuhl and Robert Biegler, have previously worked with and analyzed the bunny task, and the problem this thesis addresses has been formulated in collaboration with them. The objective of this thesis is to investigate whether individuals with autistic and/or psychotic traits exhibit different tendencies in inferring changes as compared to the general populace. We plan to analyze decisions made during the bunny task, utilizing statistical analyses and modeling to examine the relationship between these traits and the tendency to infer change.

We are generally interested in any findings that could shed light on how individuals with autistic and/or psychotic traits infer changes. Part of our analysis involves examining any general differences in decision-making between these individuals and healthy controls. However, the main hypothesis we intend to investigate is that people with psychotic and/or autistic traits have a propensity to infer changes, and in this thesis we want to investigate whether people with these traits tend to report changes when there is less substantial evidence for a change.

After completing the Bunny Task, participants take a personality test measuring autistic and psychotic traits. Autistic traits are measured by a shortened version of the Autism Spectrum Quotient called AQ-shorts. Psychotic traits were measured by the positive subscale of the Community Assessment of Psychic Experiences scale(CAPEp). Every participant ends up with an AQ score and a CAPEp score. Using these scores, we sort participants into groups. Those scoring below specific cut-off values for AQ and CAPEp form the control group. Moreover, we have three study groups of people who score above the cut-off for either AQ, CAPEp, or both.

We want to explore under what conditions these study groups report changes in the Bunny Task. To do this, we aim to construct a Bayesian observer model. This model aims to estimate the probabilities of different outcomes in the Bunny Task based on what the participant observes each time they make a decision. We are particularly interested in what we refer to as change probabilities, which are the probabilities that each bunny belongs to a new family based on the observations made at the classification point. These probabilities act as a measure to evaluate the degree of evidence that indicates a change has taken place. Recall that the change choices were whether changes of families were reported or not during the bunny task. Each change

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choice corresponds to a change probability, which represents the probability that a change truly happened when they made that particular choice.

We plan to analyze these change probabilities when participants report a family change. If we find that the change probabilities are lower for the study groups, it could suggest that they report changes with less evidence. Of course, this conclusion assumes that our Bayesian observer model accurately represents the participants' perspective on the Bunny Task.

We first want to visually analyze the distribution of change probabilities when a change is reported. Then, we will do some hypothesis tests using permutation tests to see if the differences we find are statistically significant.

Additionally, we plan to do modeling with the change choices as responses. Our plan includes fitting a logit model for each participant to examine the correlative relationship between the change choices and their corresponding change probabilities. This approach will aid in identifying any potential differences between the control group and the study groups with elevated AQ and/or CAPEp scores. Moreover, we want to analyze what the fitted model implies about our primary hypothesis. We, therefore, investigate whether there is any evidence that the study groups infer changes with less evidence.

The second part of our analysis uses CAPEp and AQ scores as continuous explanatory variables. We want to see if there is a relationship between these scores and the choices participants make for different change probabilities. This will be done by fitting a binary random intercept logit model with a random intercept for each participant. This analysis will help us see if higher AQ and CAPEp scores generally relate to differences in decision-making. We also examine what the results tell us about the main hypothesis.

2 | The bunny task

The bunny task was designed to investigate how much a change in the environment is needed before the participants report a change. In this section, we explain the bunny task in more detail. We also describe how the observations in the task were generated. Furthermore, we delve into the personality test taken by the participants and the methods by which autistic and psychotic traits are measured. At last, we describe the data consisting of the observations, decisions, and personality test results for all bunny task participants.

2.1 The task

The bunny task consists of two main blocks. The participants see a circle with a black dot in its center on the screen. This circle can be seen in Figure 2.1. One bunny appears at the time, moving from the center of the circle in a straight line to the circle line. The participants are given instructions before the task begins. They are informed that they will see bunnies move from the center of the circle in a specific direction to find food. They are also told that bunnies come from different families. Bunnies from the same family will move from the center of the circle with the same mean walking direction, but the actual walking directions from the different bunnies in the same family are spread randomly around the mean. The exact distribution these walking directions are drawn from is unknown to the participants. The participants can see how the bunnies from the different families are spread around the same mean in Figure 2.2. This figure shows the demonstration of the bunny task before the actual task begins. In the actual task, the bunnies will appear one at a time, and the history of the bunnies is not shown. In Figure 2.2, one color corresponds to one family, and the dotted line shows the mean of the family. The participants are informed that one family of bunnies leaves the center at the time, which means that before a bunny from a new family appears, all the bunnies from the previous family appear. This means the task starts with a certain number of bunnies from the first family appearing, one at a time. After the last bunny from the first family has appeared, the first bunny from the second family appears, and after that, a bunny from the first family will never appear again. After
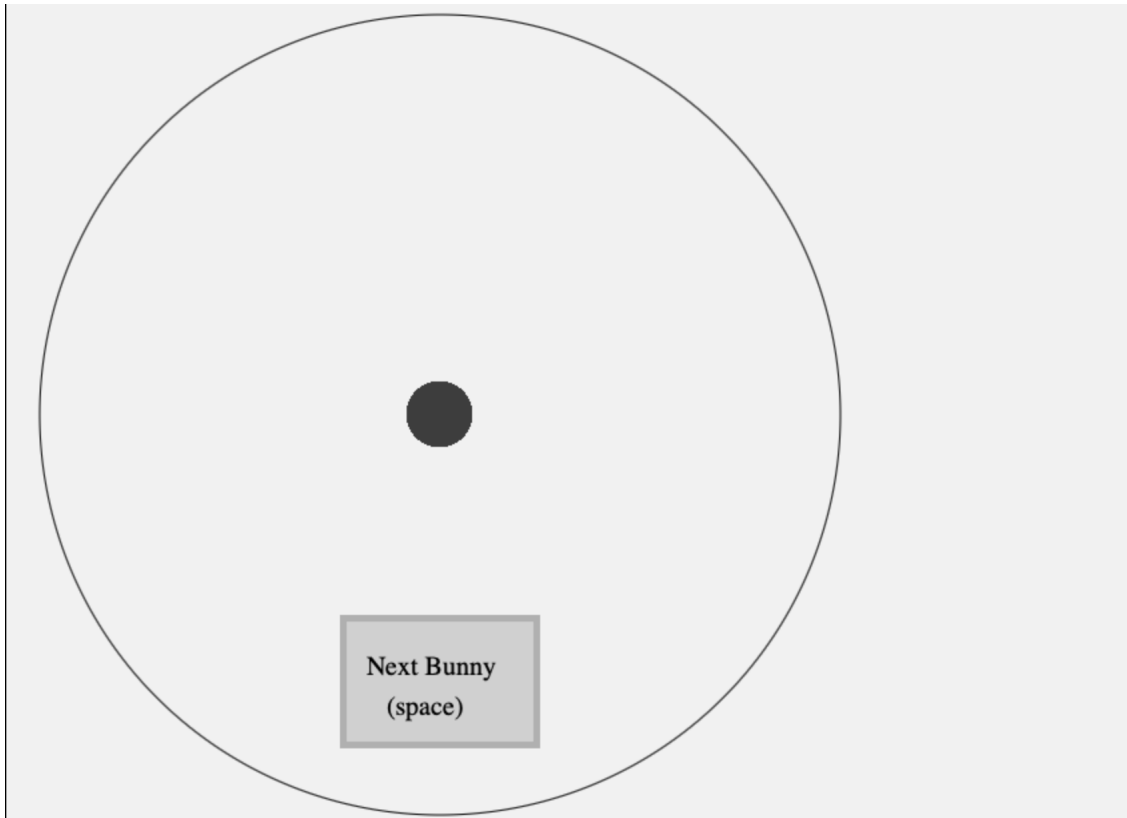
6
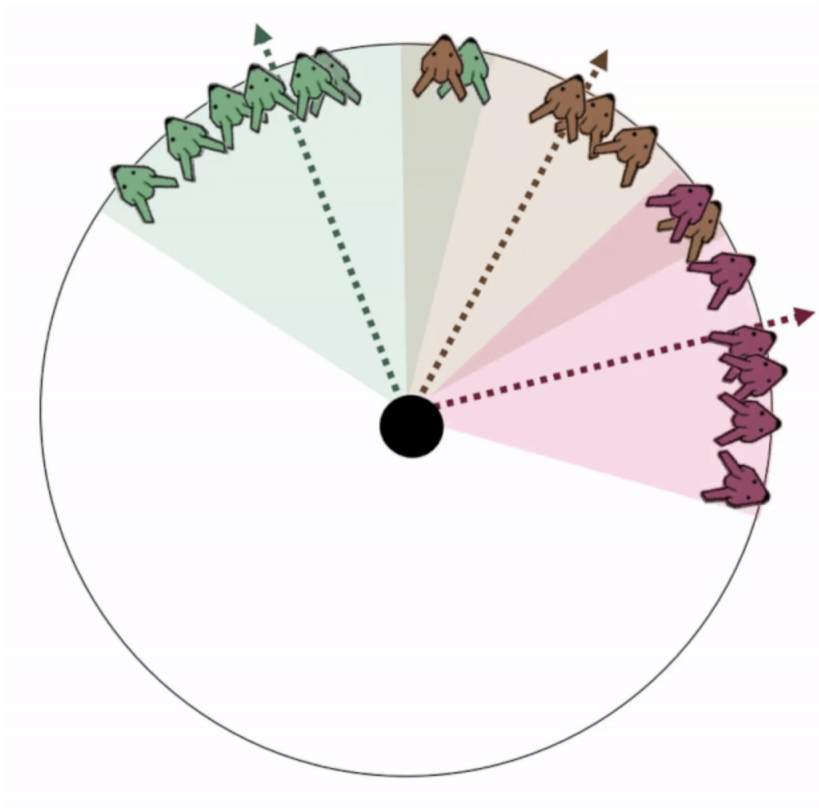
Figure 2.1: The starting screen of the bunny task.

Figure 2.2: Demonstration of the bunny task.

all the bunnies from the second family have appeared, all the bunnies from the third family will appear. This continues until all the bunnies have left the center of the circle. The participants are told that the change of mean walking direction from one family to the next is always in the same direction, but they are not informed exactly how much the mean changes between each family. If we again look at Figure 2.2, the mean of the first family is the dotted green line, the mean of the second is the brown dotted line, and the third has the color red. This shows how the mean changes in the same direction. They are also informed that the number of bunnies in each family can vary so that the bunnies can not be classified into different families by just counting them. The task of the participants is to classify the bunnies into families. The first observation is guaranteed to be from the first family, family 1, so that is the only option. The second bunny could still be from family 1, but now it is also possible that the bunny is from family 2, so they have those two options. They answer by pressing the number of the family they think the bunny comes from on the computer. After having classified bunnies into families for a time, they could reach family number 10 and above, that is, family numbers with more than one digit. Then, they only have to press the last digit of the number. If they, for example, think the bunny belongs to family number 12, they only answer with the last digit, which is 2. After getting the instruction and before starting on the actual task, the participants have to answer two control questions to demonstrate that they have understood that the families can come in different sizes and that they only respond with the last digit if they think the bunny is from a family number with more than one digit. They are also informed that it is possible to change their mind during the task and classify a bunny into the family the preceding bunny was classified into. If you, for example, classified the last bunny into family 3, you can classify the next bunny into family 2. This option is, however, only available if you have classified 3 or less into the same family in a row. If more than 3 bunnies have been classified into the same family in a row, you will only have two options, the latest bunny is either a part of the same family as the previous or the bunny is part of the next family.

To get a better sense of how the participants do the task, we consider different examples that show how the bunnies appear on the screen, the different options they are presented with, and a couple of different scenarios that can play out during the task. Recall the starting screen in Figure 2.1. To see the direction of the first bunny, the participant has to press space on the
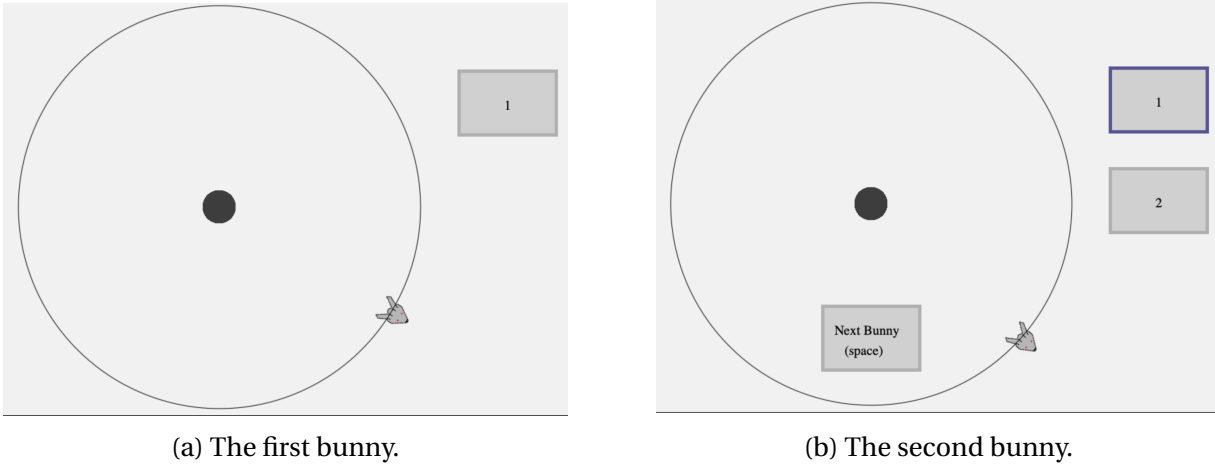
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(a) The first bunny. (b) The second bunny.

Figure 2.3: The first and second bunny in the bunny task.

computer. In Figure 2.3, we see the two first bunnies appearing. The figure to the left shows the first bunny. This bunny walks from the circle center to the line in a straight line. The only option here is family number 1, so they must press 1 and continue with the next bunny. Each time they want a new bunny to appear, they have to press space. The figure to the right in Figure 2.3 shows the second bunny. This bunny could either be from the same family as the first bunny or the next family, family number 2. We also see that family 1 and 2 are the only options on the screen. As earlier stated, the previous observation is not shown, so the participant can not see directly how this observation compares to the previous observation on the screen. However, the previous bunny was just shown, so it is reasonable to assume that the participant sees that the area the new bunny has walked to is very close to the previous one. Therefore the participant also categorizes this bunny into family number 1. This can be seen in the figure as option 1 is marked.

Now, assume that the participant has classified several bunnies into family number 1. Most of the previous bunnies have walked into the same area. A new bunny appears and can be seen to the left in Figure 2.4. The participant thinks this observation significantly differs from the previous observations classified into family 1. Therefore, the participant categorizes this bunny into family 2. The following bunny can be seen to the right in Figure 2.4. The options the participant can choose between now are family 1, 2, or 3. If the bunny is in family 2 it is in the same family as the preceding bunny. If it is in family 3, it comes from another new family with a new mean.

<table>
<tr><td>(a) First reported change.</td><td>(b) The participant change their mind.</td></tr>
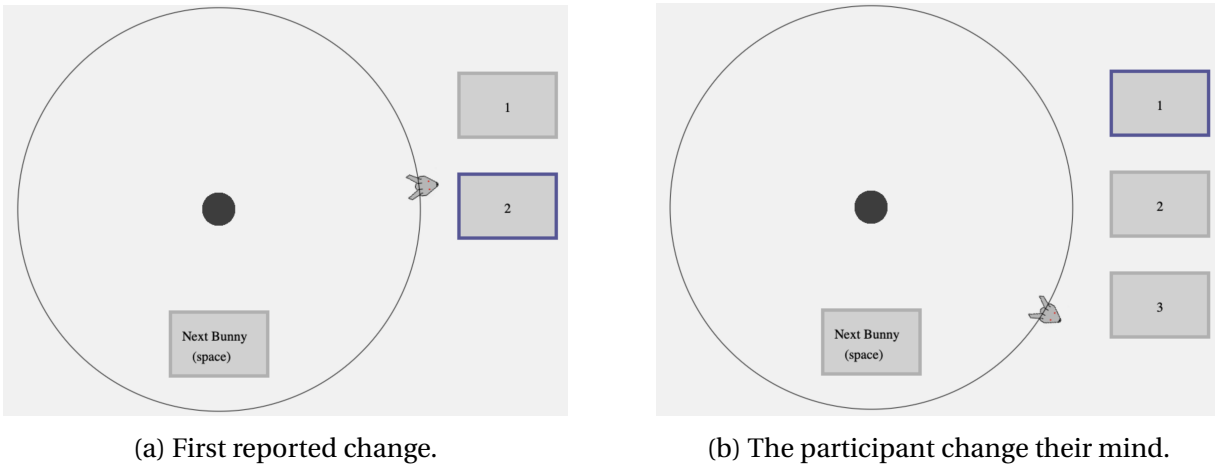</table>

Figure 2.4: The participant reports a change and then change their mind when observing the next bunny.

Family 1 is also an option since only one observation has been classified into family 2 in a row, which is below the threshold of 3. The participant notices that the bunny walks into the same area as the ones classified into family 1. Therefore, the participant wants to change their mind and classify this new observation as part of family 1. This means that they change their mind about the previous bunny and think this is also a part of family 1. This is due to the fact that all the bunnies from a family have to appear before bunnies from the next family appear.

After a couple more bunnies are classified into family 1, a new bunny is shown, and can be seen to the left in Figure 2.5. This bunny walks in a significantly different direction than the previous bunnies, and again the participant reports a change to family 2. The next bunnies who appear after this report of change walk approximately into the same area. Therefore, the participant sticks to classifying the bunnies into family 2 for over 3 times in a row. Now, the only two options for the participant are family 2 or 3, and this will be the case until a new change is reported. This scenario is shown to the right in Figure 2.5.

The task continues until all bunnies have been classified into families. The only restriction on how often a change can be reported is that at least one bunny must be classified into each family. If the participant, for example, reports a change in each observation, the family number of the last observation is the same as the observation number plus one.
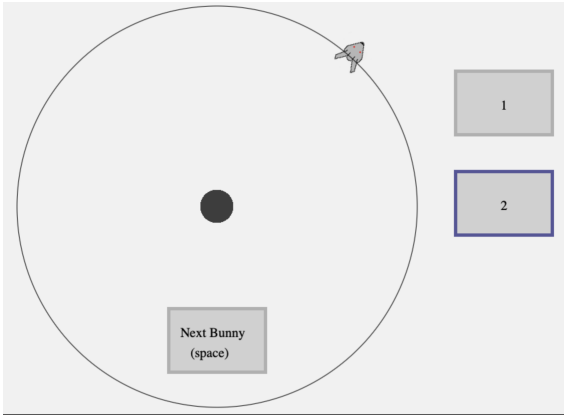
(a) A bunny is classified into family 2.
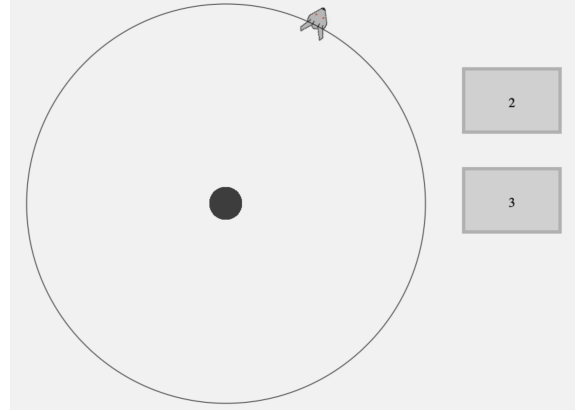
(b) Example where family 2 and 3 are the only options.

Figure 2.5: The participant classifies more than 3 bunnies into family 2.

2.2 Generating walking directions and family sizes.

Let $d = (d_1, \ldots, d_{no})$ be the walking directions of all the bunnies in both blocks of the bunny task. The walking direction d_i denotes the walking direction of bunny number i. Walking directions are given in the number of degrees the bunny walks in from the center of the circle. The directions of the bunnies are randomly and independently drawn from normal distributions. Let us first consider the walking directions drawn from the first family. Let μ be the mean walking direction of the first family. To generate one walking direction, we start by drawing z_i from a standard normal distribution with mean 0 and variance 1. This is turned into a walking direction by $d_i = \mu + \sigma z_i$. This means that to generate d_i by sampling z_i from a normal distribution and transforming it into d_i is equivalent to generating a sample from a normal distribution with mean μ and variance σ^2.

The standard deviation is $\sigma = 20$, and is constant under the whole experiment and the same for each participant. The mean change between one family and the next is the same each time, and we denote this change as Δ. To transform a generated sample from a normal distribution to the walking direction from family number j, the following formula is used

$$d_i = \mu + (j - 1) \cdot \Delta + \sigma z_i, \tag{2.1}$$

for $j = 1, \ldots$. This means that $d_i \sim N(\mu + (j-1) \cdot \Delta, \sigma^2)$. Before the participants begin the two main blocks of the bunny task, they do the task one round in advance. This is called the titration block, and is done to determine the fixed number of degrees the mean direction changes with from one family to the next for one participant. After the titration block, a titration threshold is decided for the participant, and the value of this threshold is adapted to the decisions made by the participants in the titration block. The method used was the adaptive staircase procedure best (PEST) [Lieberman and Pentland, 1982]. The threshold was decided by taking the average of the last three estimates of PEST. This titration threshold is unknown for the participants, and is equal to the absolute value of the mean change $|\Delta|$. The mean change of the participants is either clockwise or counterclockwise. If the change is clockwise, we have $\Delta = |\Delta|$. If the change is counter-clockwise, we have that the mean change is given by $\Delta = -|\Delta|$.

After the titration block, the participants go through the two main blocks. In one block, the mean family size is higher than in the titration round, and this is referred to as the low volatility block. In the other block, the high volatility block, the mean family size is lower than the mean in the titration block. The number of bunnies in each block was the same for each participant. However, the order of the low and high volatility blocks varied among the participants. There were a total of 25 families in each block, meaning there are 24 points where a change occurs.

The number of bunnies in each family is drawn from truncated log-normal distributions. In Table 2.1, we see a summary of the different blocks. In the low volatility block, there are, in total

Table 2.1: The family mean, minimum and maximum family sizes in each block of the bunny task.

Block	Mean family size	Minimum	Maximum	Total bunnies
Titration block	9.32	6	14	233
High volatility block	7.08	5	12	177
Low volatility block	12.52	7	19	313

313 bunnies for all participants. Thus, to generate the walking directions in this block, first 313 values from a standard normal distribution is drawn. In the high volatility block, there were in total 177 bunnies. The total number of bunnies in the two main blocks is therefore $n_o = 490$.

2.3 AQ-short and CAPEp

As mentioned earlier, after the bunny task is completed, the participants do a personality test to assess to what extent they have autistic and psychotic traits. A condensed version of the Autism Spectrum Quotient called the AQ-shorts [Hoekstra et al., 2011] was used to assess the degree of autistic traits in the participants. The AQ is a way to measure autistic traits by self-reporting. The full version of the test contains 50 items, and you get a score between 0 and 50 based on the endorsement of each item. According to [Hoekstra et al., 2011], several studies have shown that AQ is reliable for assessing autistic traits. This includes studies in Japan [Wakabayashi et al., 2006], the UK [Baron-Cohen et al., 2001], and the Netherlands [Hoekstra et al., 2008], which have shown that the AQ scores in people with ASC are significantly higher than in the general population. Furthermore, in [Woodbury-Smith et al., 2005], it was found that using AQ scores is a reliable measurement to predict the diagnosis of AS. However, in a larger study, like the study considered in this thesis, the 50 items can be too lengthy. Therefore, the AQ-short, which consists of 28 items, is used. In [Hoekstra et al., 2011], AQ-short was found to retain the most important structure of AQ and be a reliable way to measure autistic traits. In each of the 28 items used in this study, the participants were asked to rate their agreement from 1 = Definitely agree to 4 = Definitely disagree. Each item can contribute between 1 and 4 points to the total AQ score, and the maximum score is 112.

The degree of psychotic traits in the participants was measured by using the Community Assessment of Psychic Experiences scale (CAPEp) [Stefanis et al., 2002]. This measurement has been found to be a reliable way to measure psychotic traits [Núñez et al., 2021]. The participants are presented with items where they report the frequency of psychotic-like experiences on a scale from 1 = Never to 4 = Nearly always. The CAPEp has 5 sub-scales, which include hallucinations, magical thinking, grandiosity, paranoia, and bizarre experiences. After the test, the participants received a score in each category and an overall score for CAPEp. The overall score is what we use in further analysis in this thesis. The test consists of a total of 20 items, and as each item can contribute up to 4 points to the total score, the maximum score is 80. Three control questions were added to identify people who provided inaccurate or false information about their experiences. The control question included common misconceptions about psychosis, like whether

they had experienced being a historically famous person. Answering yes to such a question may indicate that they are lying since this is not an actual symptom of psychosis.

Remember, as a part of the analysis, we plan to categorize individuals into groups based on their AQ and CAPEp scores and compare the decisions between them. These cut-off points are based on cut-off scores previously used in similar work. However, it is important to note that cut-off scores have mainly been used to create visual representations and not in the analysis itself in previous work with the bunny task. Grouping the individuals in this way, called dichotomizing, has its drawbacks. One is that the results from this analysis are sensitive to where the cut-off scores are set. Furthermore, we end up with people who have substantially different scores in the same group, or people with very similar scores in the same group, depending on which side of the cut-off they fall. Despite this, we think it is worthwhile to do this analysis as it could give us insight into whether these cut-off scores are reasonable in this context. Furthermore, recall that we are going to do an analysis where AQ and CAPEp are considered explanatory variables, meaning we are investigating how they might influence the participants' choices of inferring change in the bunny task. We are interested to see if the findings from the group-based and variable-based approaches align. Both finding that the results from the different analyses align or do not align are interesting results. For instance, we might find that those in the group with CAPEp scores above the cut-off tend to report changes with less evidence compared to the control group with both CAPEp and AQ scores below the cut-off. Yet, we might not find an overarching relationship between elevated CAPEp scores and the tendency to infer change based on less evidence. This would mean that the difference is not detectable when considering CAPEp scores broadly but becomes apparent when focusing on the group with higher CAPEp scores. Hence, we perform both a group-oriented analysis and an investigation using CAPEp and AQ as explanatory variables, as both methods can uncover unique insights into how people with autistic and psychotic traits make decisions.

A cut-off score of 65 was suggested by Hoekstra et al. [2011] specifically for the 28-item AQ-short. This cut-off score was found to have a sensitivity of 97% and a specificity of 82% for distinguishing individuals with AS from controls. This means that 97% of the people with AS were correctly classified as such, and 82% of the controls were correctly classified as such using 65 as a cut-off

score. This cut-off score has also previously been used as a cut-off score in previous work with the Bunny Task. The group of people with an AQ score higher than 65 is referred to as H-AQ, while those with an AQ score less than or equal to 65 is referred to as L-AQ. Regarding CAPEp, a cut-off score of 1.47 was proposed by Bukenaite et al. [2017] for the average CAPE-P15 score, which is a CAPEp score determined by 15 items. Each item can contribute between 1 and 4 points to the total score, and a score of 1.47 means that the total score divided by the number of valid answers is 1.47. This cut-off score showed a sensitivity of 77% and a specificity of 58% for detecting people at ultra-high risk for psychosis (UHR). Previously, an average score of 1.6 has been employed for the bunny task, which is the cut-off used in this thesis. An average score of 1.6 corresponds to a total score of $1.6 \cdot 20 = 32$. The group of people with an average CAPEp score below or equal to 32 is called the L-CAPEp group, and the group of people with a score over 32 is referred to as H-CAPEp.

From these scores, we can divide the participants into 4 mutually exclusive groups. The group of healthy controls is referred to as L-CAPEp & L-AQ group since these people have AQ and CAPE scores below or equal to the cut-off. The group of people with an CAPEp score over the cut-off and an AQ below or equal to the cut-off is referred to as the H-CAPEp & L-AQ. The group of people with a CAPEp below the cut-off and an AQ above the cut-off is called the the L-CAPEp & H-AQ group. The group with both AQ and CAPEp above the cut-off is referred to as the H-CAPEp & H-AQ group. Further in this thesis, we refer to groups that are not the control group as the study groups.

2.4 Data from the bunny task and the personality test

Participants of the bunny task were recruited online. Only adults were allowed to participate and there were a total of 445 participants. Out of these participants, 106 were excluded, leaving 339 to be the final sample. The reason for the exclusion of certain individuals was to avoid including people who were not engaged with the task. Some people were excluded since they had a threshold greater than four times the standard deviation $\sigma = 20$, and thus a threshold over 80. Other people were also excluded because of having a threshold lower than 10. Two people were also excluded because they scored high on the three CAPEp control questions. There were

Table 2.2: Relevant variables for one participant.

Variable	Description
familyNumber	The actual family number
bunnyNumber	The bunny number within the family
familyNumberIndicated	Which family number did the participant indicate

165 females, 167 males, and 7 identifying as non-binary in the final sample. The average age was 28.9($SD = 10.10$), where the youngest was 18 and the oldest was 70.

The different choices and observations the participants made during the bunny tasks were collected in CSV-files with one file for each participant. The relevant variables are explained in Table 2.2. In Table 2.3, an example with the 16 first rows for one participant can be seen. The participant has $\Delta = 73.67$ as the titration threshold. One row represents one bunny in the data. The variable familyNumber tells the actual families each bunny was drawn from. The variable familyNumberIndicated is which family the participant categorized the bunnies into. Note that the first family is encoded as family 0 for the variable familyNumber, and the first family of the variable familyNumberIndicated is family 1. Further in this thesis, we refer to the first family as family 1. The 12 first bunnies are from the first family, then the next four bunnies included are from the second family. We can see that this particular participant reports a change after observing the fourth bunny and then change their mind in the following observation. The same happens in the ninth observation, where the participant reports a change before returning to the first family in the next observation. After reporting a change in the 13th observation, the participant sticks with this family for over three bunnies in a row, so it is impossible to return to the previous family. We notice that the participant starts to consistently report that the bunnies are from the second family when the bunnies from the second family actually begin to appear. In this case, the titration threshold is pretty big and just a little smaller than the exclusion criteria of 80, so there is a pretty big jump in the observed directions from before the change happens to after the change has happened. Other relevant variables for the analysis are the CAPEp - and AQ scores from the personality tests of the different participants. In addition, the titration threshold $|\Delta|$ and which direction it changes are relevant variables. The variables are summarized in Table 2.4. The data for all the participants are collected in one file, which we can see a subset of in Table 2.5. There are two rows for each participant that represent the two main blocks of the

Table 2.3: Relevant data from one participant with $\Delta = -73.67$.

	familyNumber	bunnyNumber	action	familyNumberIndicated	msAfterSpawn	bunnyDirection
1	0	0	Top family clicked.	1	1200	30.53677004
2	0	1	Top family clicked.	1	2711	23.46987944
3	0	2	Top family clicked.	1	927	33.40000012
4	0	3	Middle family clicked.	2	991	-29.7336714
5	0	4	Top family clicked.	1	975	15.26026458
6	0	5	Top family clicked.	1	968	4.35252922
7	0	6	Top family clicked.	1	951	48.85569728
8	0	7	Top family clicked.	1	871	24.23896808
9	0	8	Middle family clicked.	2	1143	-3.0848377
10	0	9	Top family clicked.	1	1328	8.99286
11	0	10	Top family clicked.	1	976	62.29008928
12	0	11	Top family clicked.	1	903	46.38184182
13	1	0	Middle family clicked.	2	839	-48.6721877666667
14	1	1	Middle family clicked.	2	887	-24.3909810666667
15	1	2	Middle family clicked.	2	895	-55.7534918866667
16	1	3	Top family clicked.	2	848	-40.0635098466667
17	1	4	Top family clicked.	2	911	-20.1972492066667
18	1	5	Top family clicked.	2	1023	-10.0725231466667
19	1	6	Top family clicked.	2	951	-71.8384326866667
20	1	7	Top family clicked.	2	863	-14.7965877866667
21	1	8	Top family clicked.	2	887	-64.4296178066667
22	1	9	Top family clicked.	2	871	-62.7357629266667
23	1	10	Top family clicked.	2	847	-43.9764413266667
24	1	11	Top family clicked.	2	919	-37.7863015866667
25	2	0	Middle family clicked.	3	855	-104.142538773333

Table 2.4: Relevant variables for the participants.

Variable	Description
ID	The identification of the participant
CAPEpositive	The CAPEp score from the personality test
AQ	The AQ score from the personality test
titrationTreshold	The absolute value of the number of degrees the mean changes with decided by the titration block
directionchanges	The direction in which the mean changes with. Either counter-clockwise or clockwise
volatilityorder	Whether the high volatility or low volatility block comes first.

Table 2.5: Data from the different participants.

	ID	CAPE_positive	AQ	titrationThreshold	direction_changes	volatility_order
1	24387	25	52	19	Counter-clockwise	High Volatility First
2	24387	25	52	19	Counter-clockwise	High Volatility First
3	24389	27	74	48	Clockwise	High Volatility First
4	24389	27	74	48	Clockwise	High Volatility First
5	24390	26	64	28	Counter-clockwise	Low Volatility First
6	24390	26	64	28	Counter-clockwise	Low Volatility First
7	24430	29	58	78.66666667	Counter-clockwise	Low Volatility First
8	24430	29	58	78.66666667	Counter-clockwise	Low Volatility First
9	24435	37	64	44	Counter-clockwise	High Volatility First
10	24435	37	64	44	Counter-clockwise	High Volatility First

bunny task.

The empirical distribution of AQ and CAPEp scores can be seen in Figure 2.6. In the figure, the mean score is plotted as a red vertical line, while the cut-off scores are plotted as vertical blue lines. The AQ scores range from a minimum of 43 to a maximum of 93, with an average score of 64.09. Similarly, the CAPEp scores extend from a low of 20 to a high of 58, averaging at 29.68. As discussed earlier, different groups are decided based on AQ scores and CAPEp scores. In Table 2.6, the sizes of the different groups can be seen.

(a) Distribution of AQ scores.

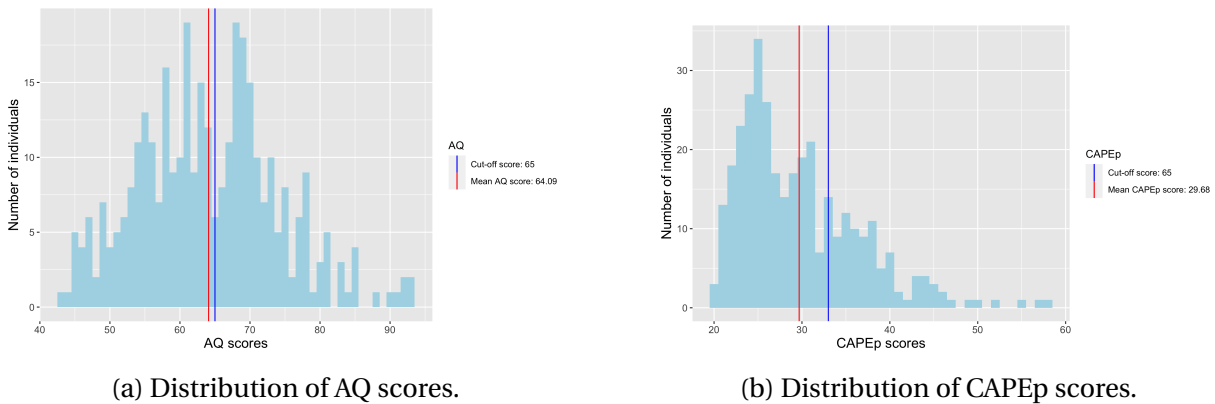(b) Distribution of CAPEp scores.

Figure 2.6: Distribution of AQ and CAPEp scores with corresponding means and cut-off scores.

Table 2.6: The number of individuals in the different groups.

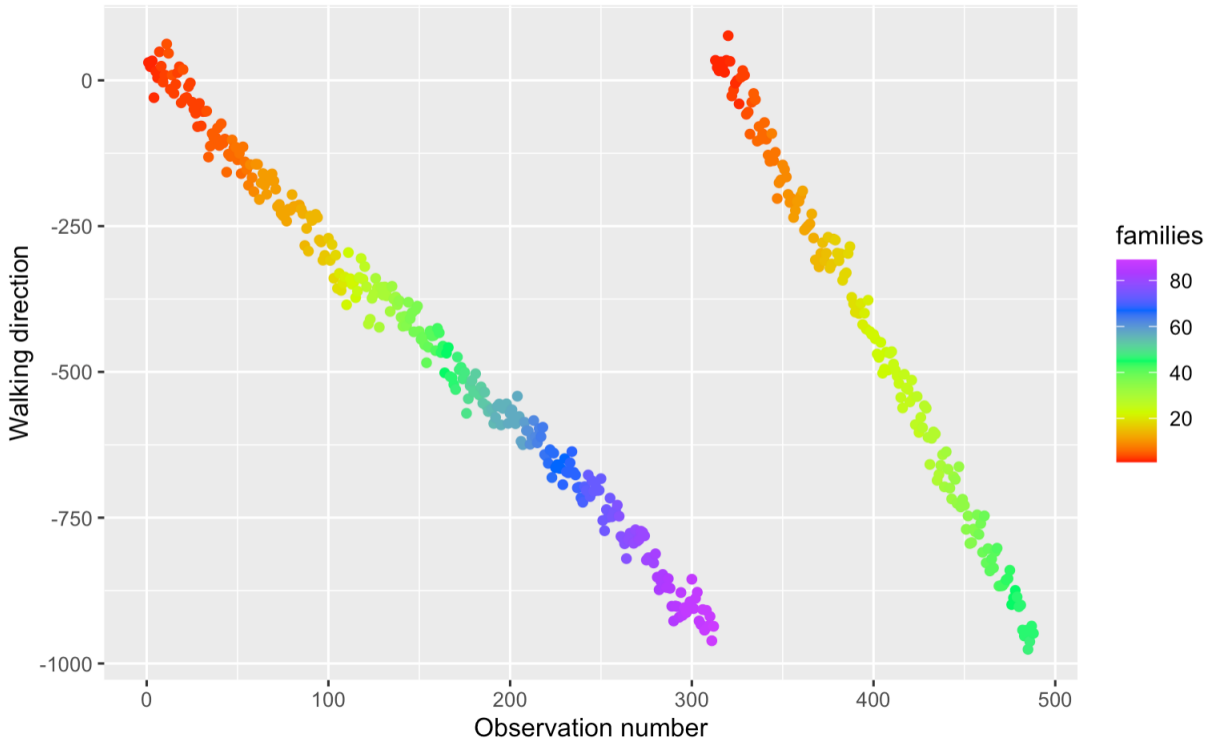|	L-AQ	H-AQ	Sum
L-CAPEp	L-CAPEp & L-AQ: 140	L-CAPEp & H-AQ: 99	239
H-CAPEp	H-CAPEp & L-AQ: 47	H-CAPEp & H-AQ: 53	100
Sum	187	152	339

Figure 2.7: Participant with 89 reported family changes in the first block and 45 changes reported in block two.

How people solve the task varies a lot. Some people tend to report changes more often and others tend to report fewer changes. Figure 2.7 shows an example of a participant who reports many changes. Along the x-axis are the observations numbers. There are in total 490 observations in the two blocks, and thus the observation number ranges from 1 to 490. The y-axis is the degrees the bunnies walk in. Each point is a walking direction, and they are ordered the same way as they appear on the screen. At observation number 314, the second round starts. The participant reports 88 changes in the first block, while 45 changes are reported in the second block. Recall that there are 25 families in both blocks and therefore 24 changes. Thus, this participant overreports changes. The opposite can be seen in Figure 2.8. Here, the participant reports 6 changes in the first block and 5 changes in the second block, which are few reported changes
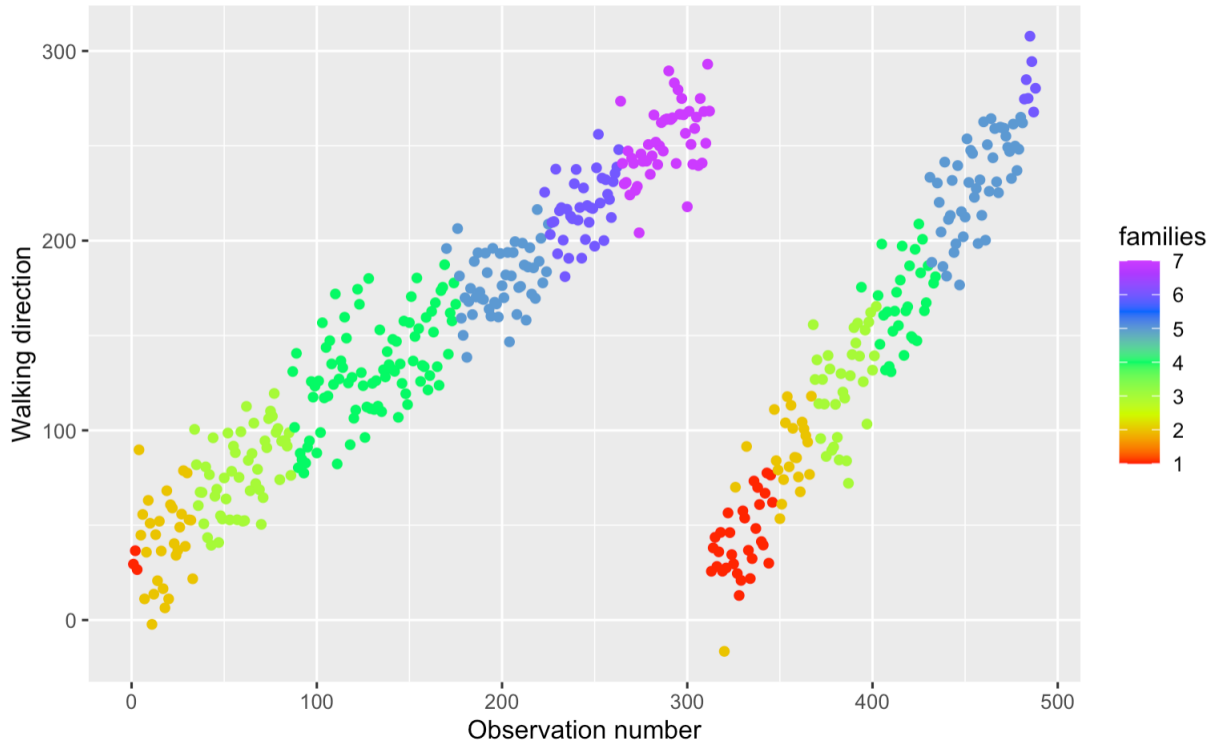
Figure 2.8: Participant with 6 reported family changes in block 1 and 5 reported changes in block 2.

compared to the actual number.

In Figure 2.9, the distribution of the number of changes reported by the participants can be seen. Both blocks are included. The average number of reported changes for both blocks is 17.9. This is below the actual number of changes. Thus, on average, people tend to underreport changes. The figure also shows some outliers in which people overreport a lot.

We want to check whether we see any visual differences in the number of reported changes between the different groups based on AQ and CAPEp scores. We plot the empirical distribution of the reported changes in both blocks for each study group, comparing them to the control group, L-CAPEp & L-AQ. Together with the distribution we plot the mean and median reported changes as vertical lines, where the blue lines corresponds to the control group and the red lines corresponds to the study group. We start by comparing the control group with the L-CAPEp & H-AQ group. The empirical distribution of these two groups can be seen in Figure 2.10. From this plot, it seems like the individuals in the control group tend to have a higher number of reported changes. The mean number of reported changes is 18.4 for the control group compared
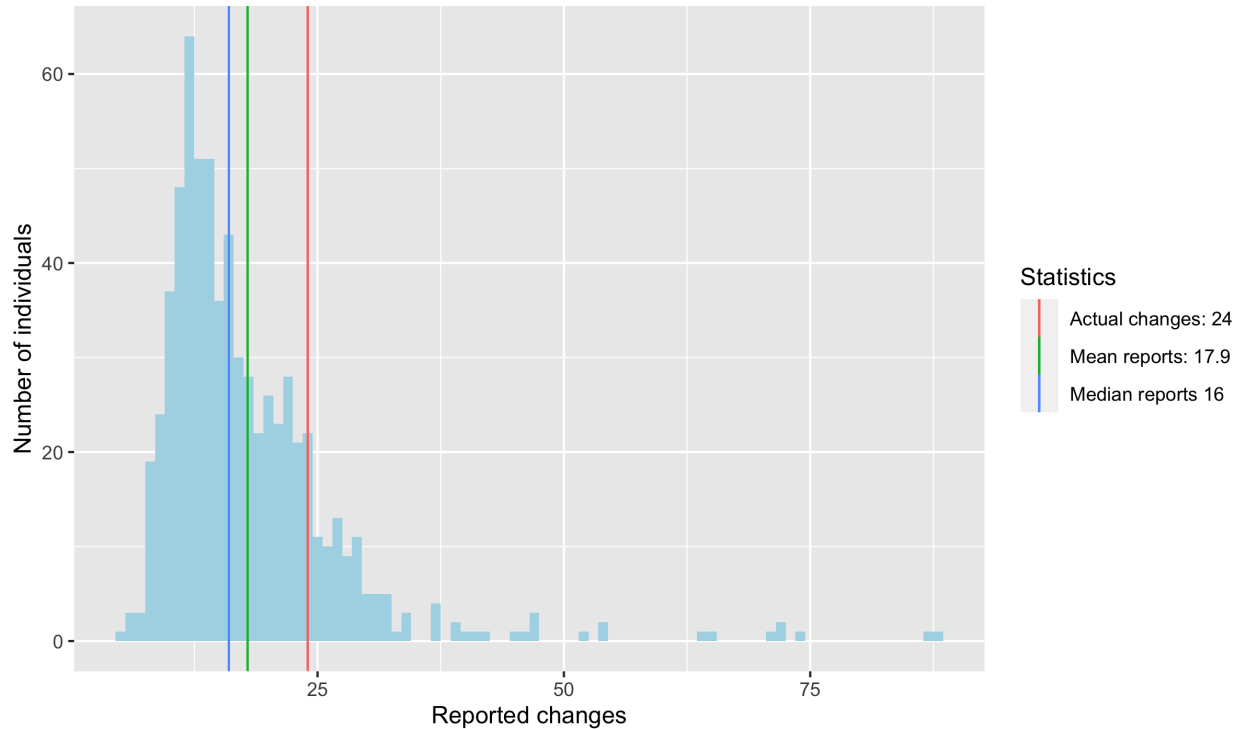
Figure 2.9: Number of reported family changes for all the participants in both blocks.

to 16.1 for the L-CAPEp & H-AQ group. The median is 14 for the control group and 16 for L-CAPEP & H-AQ. Now, let us consider the difference between the H-CAPEp & L-AQ group and the control group. The distribution of the reported changes for these two groups can be seen in Figure 2.11. We see no clear differences between these two groups. The H-CAPEp & L-CAPEp group has slightly higher mean reported changes. The median reported changes is 16 for the control group and 15 for the H-CAPEp & L-CAPEp group.

Finally, we look at the differences between the group with a high degree of both psychotic and autistic traits, the H-CAPEp & H-AQ group. In Figure 2.12, we can see the distribution of reported changes for this group compared to the control group. We see no apparent visual differences between those two groups by looking at the plot. However, the mean and median number of reported changes is notably higher for the H-CAPEp & H-AQ group.

These plots tell something about the general tendency of reporting changes by individuals in the different groups. In further analysis, we are interested in under what conditions these reported changes happened. In particular, we are interested in the probability that a change had occurred
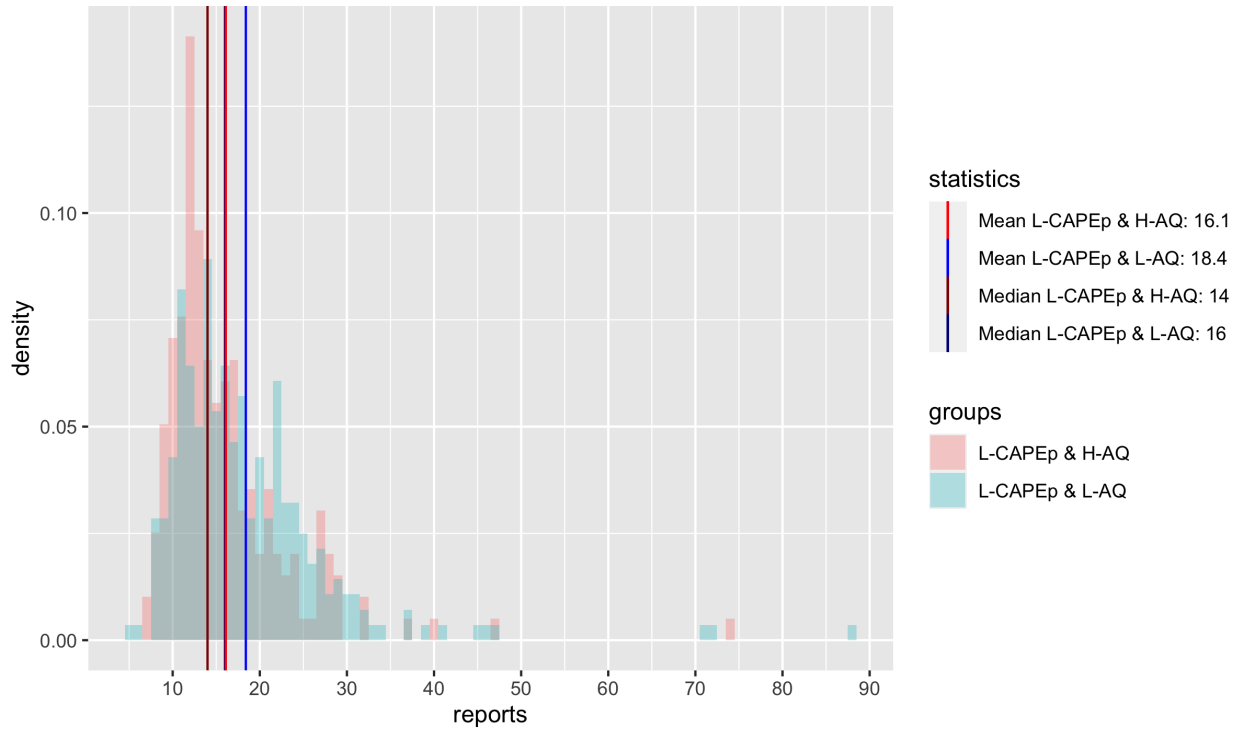
Figure 2.10: Reported changes of the control group and the L-CAPEp & H-AQ group.
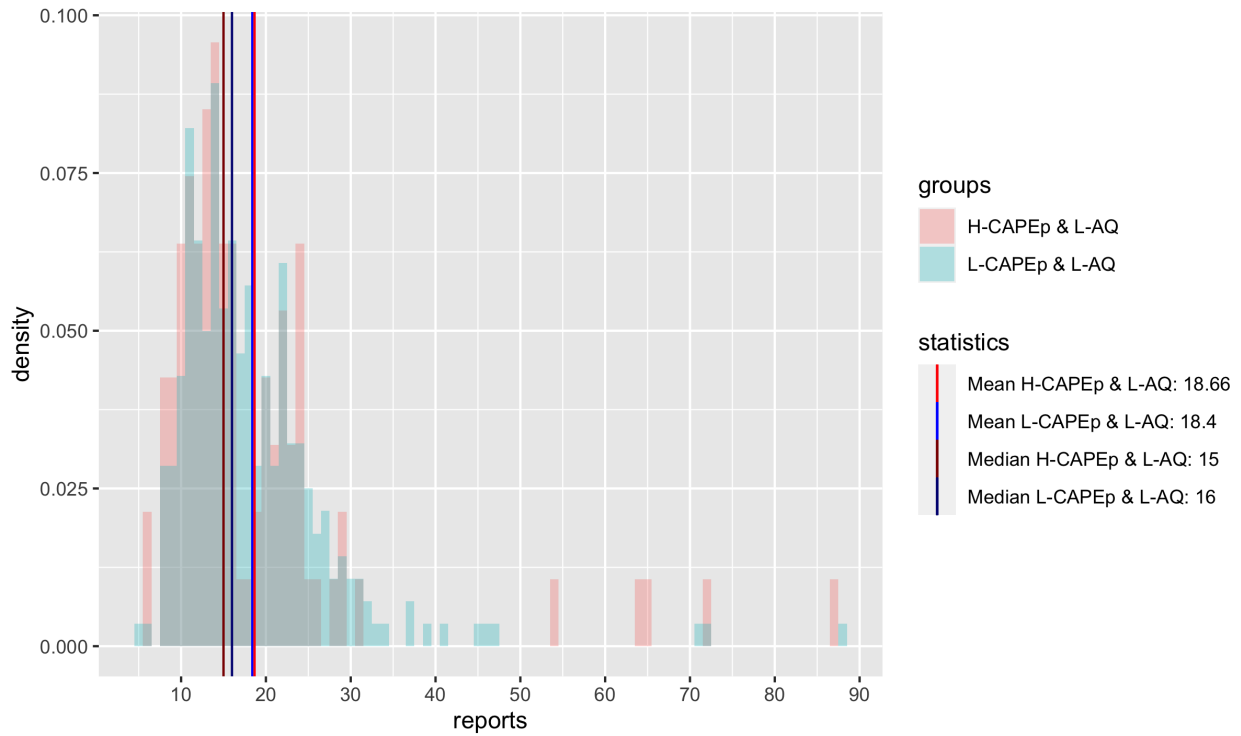

Figure 2.11: Reported changes of the control group and the H-CAPEp & L-AQ group.
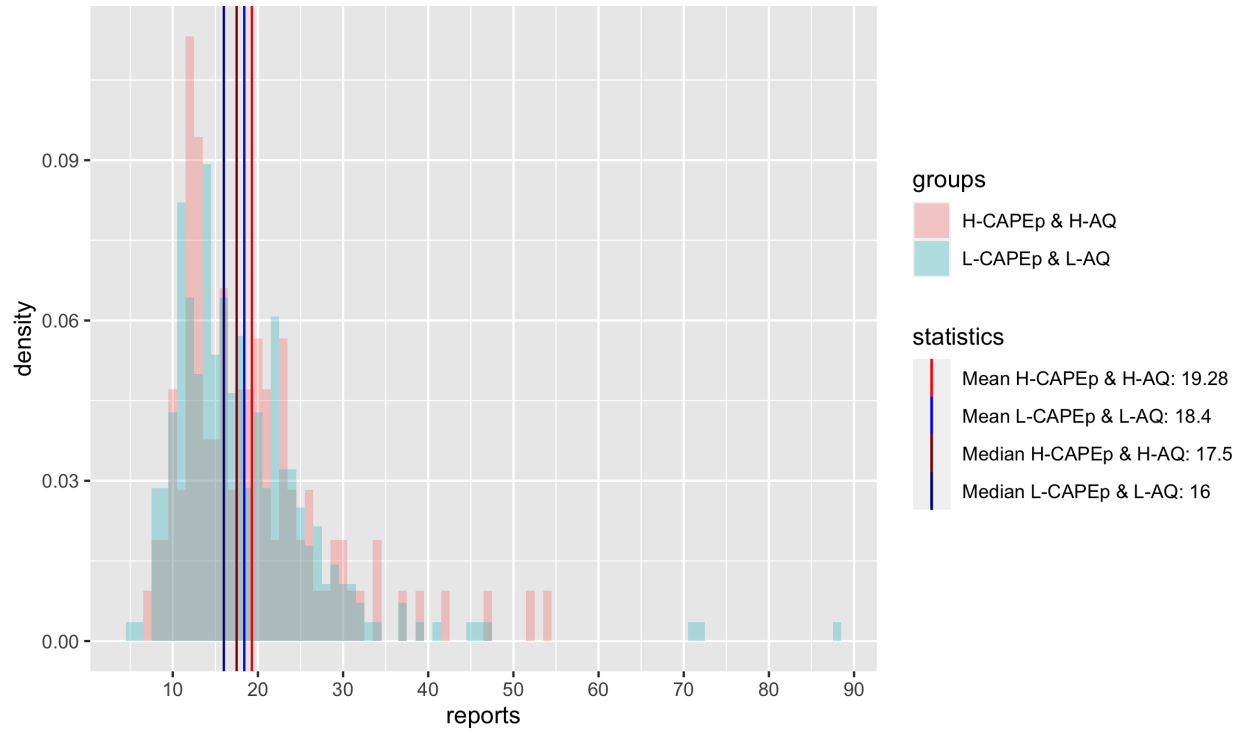
Figure 2.12: Reported changes of the control group and the H-CAPEp & H-AQ group.

at the times they reported changes. This is discussed further in Section 5.

3 | Theory

This section describes the statistical and probability theory needed for the analysis we outlined in the introduction. Recall that we want to construct a Bayesian observer model for finding the probability that the last observed bunny is part of the different possible families it can be a part of, given the observations made. To construct the Bayesian model, the law of total probability and Bayes´ theorem are central, and they are explained in this section. Furthermore, the basic concepts and the philosophy of Bayesian statistics are included in this section. We consider an example related to the bunny task to demonstrate how Bayesian inference is done. The geometric and negative binomial distributions are assumed for two of the model variables in the Bayesian model, and these distributions are explained in this section. We aim to use the probabilities derived from the Bayesian model to analyze the choices made by the participants. At each point a new bunny appears, we consider the odds of this bunny being part of a new family given the observations made. These are the odds we previously referred to as change odds. The participants have to decide whether the bunny is part of a new family, the same family as the previous bunny, or, in some cases, whether the bunny is from the family preceding the family of the previous bunny. We consider this choice as binary, where the choice is either to report a change of family or not, and we estimate the relationship between this binary change choice and the change log odds at the point the decision was made. We fit a logit model with the binary change choices as responses and the corresponding change log odds as covariates. The logit model is a variant of the generalized linear model (glm), and we give a general explanation of the glm in this section. We also show how the logit model is a variant of the glm. We want to investigate whether significant differences exist between the study and control groups given in Table 2.6. We do permutation tests to do hypothesis testing, and in this section, we explain permutation tests for one-sided hypotheses.

As previously stated, we also want to fit a binary random intercept logit model with the AQ and CAPEp scores in addition to change log odds as covariates. This model is a type of generalized linear mixed model (glmm), and this model type is explained in this section.

3.1   Law of total probability

The formulation of the law of total probability is based on how it is described by Härdle et al. [2015]. An understanding of conditional probabilities is needed for formulating the law of total probability. Let B and C be two events on the sample space S. The conditional probability of B given C is defined as

$$P(B|C) = \frac{P(B \cap C)}{P(C)},\tag{3.1}$$

which means that we have

$$P(B \cap C) = P(B|C)P(C).$$

This is called the multiplication rule and can according to Härdle et al. [2015] be generalized for the events $A_1, \ldots, A_n$ by

$$P(A_1 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_2 \cap A_1) \cdot \ldots \cdot P(A_n|A_1 \cap \ldots, A_{n-1}).$$

Assume that we want to find the probability of an event $B \subseteq S$. Furthermore, let us assume that $P(B|A_i)$ and $P(A_i)$ is known for $i = 1, \ldots, n$, and that $A_1, \cdots \cap A_n$ is a partition of S defined by

$$A_i \cap A_j = \emptyset,$$
$$A_1 \cup A_2 \cup \cdots \cup A_n = S.\tag{3.2}$$

The law of total probability states that the probability of the event B is given by

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i) = \sum_{i=1}^{n} P(B|A_i)P(A_i).\tag{3.3}$$

A variant of the law of total probability can also be found for conditional probabilities. Assume that we want to find the conditional probability of the event B given the event C. Let us consider

$$P(B \cap C) = \sum_{i=1}^{n} P(B \cap A_i \cap C).\tag{3.4}$$

By using the product rule for three variables, we get

$$P(B \cap A_i \cap C) = P(C) \cdot P(A_i|C) \cdot P(B|A_i \cap C).$$

Inserting this into (3.4) yields

$$P(B \cap C) = \sum_{i=1}^{n} P(C) \cdot P(A_i|C) \cdot P(B|A_i \cap C)$$

Furthermore, we insert this into (3.1) and get

$$P(B|C) = \frac{\sum_{i=1}^{n} P(C) \cdot P(A_i|C) \cdot P(B|A_i \cap C)}{P(C)} = \sum_{i=1}^{n} P(B|A_i \cap C) \cdot P(A_i|C).$$

The law of total probability can also be formulated when conditioning on events generated by continuous variables. Again, let us assume we want to find the probability of an event B. Let X be a continuous random variable with a density function f_X. According to [Baclawski, 2008], the probability of the event B can be found by

$$P(B) = \int_{-\infty}^{\infty} P(B|X = x) f_X(x) dx. \tag{3.5}$$

Equivalently to when we condition on events generated by discrete variables, we can find the conditional probability of event B given the event C by condition on an event generated by a continuous variable X by the formula

$$P(B|C) = \int_{-\infty}^{\infty} P(B|X = x, C) f(x|C) dx. \tag{3.6}$$

3.2 Bayes´ theorem

Let as in (3.2), the events $A_1, \dots, A_n$ be a partition of the sample space S, and B be an events in this sample space. Bayes´ theorem is useful for finding the conditional probability of an event A_j for $1 \le j \le n$ given B, when the probabilities $P(B|A_i)$ and $P(A_i)$ are known for $i = 1, \dots, n$. The

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theorem states that this probability is given by

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{P(B)}. \tag{3.7}$$

By using the expression for the probability $P(B)$ given in (3.3), we get

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}. \tag{3.8}$$

Let us assume that θ and X are discrete or continuous random variables. A more general formulation of Bayes´ theorem is

$$f(\theta|x) = \frac{f(x|\theta)f(\theta)}{f(x)}. \tag{3.9}$$

The distribution $f(x)$ can be found by the law of total probability. The reason why the notation $f(.)$ is used here is that the formula is valid for not only events but for continuous and discrete random variables. If θ is a continuous random variable, we can use the law of total probability in the continuous case as in (3.5). This yields

$$f(x) = \int_{-\infty}^{\infty} f(x|\theta)f(\theta)d\theta. \tag{3.10}$$

3.3 Bayesian statistics

Bayesian statistics is a field within statistics where a probability of an event expresses the degree of belief in the event occurring. The probabilities are, therefore, subjective beliefs contrary to the frequentist interpretation of probabilities, where probabilities are interpreted as limits of relative frequencies [Härdle et al., 2015].

According to Givens and Hoeting [2012], in Bayesian inference, the parameter of interest are considered random variables. Assume that x denotes observed data which is a realization of X. Let us assume that θ is a parameter of the distribution of X. The probability $f(\theta)$ in (3.9) is interpreted as a prior distribution and represents the prior knowledge about of the parameter before any data is observed. According to [Givens and Hoeting, 2012], this distribution may be chosen based on previous experiments and data. It also may represent the subjective beliefs of a per-

son before making observations, or it could be selected so that it has a limited effect on the end result. We want to make a Bayesian observer model for the bunny task from the participants´ viewpoint. Thus, we want the a priori assumptions and prior distributions to be representative of the a priori beliefs of the participants. These beliefs are based on the information they receive before the task and the experience they have with the task beforehand. Sometimes, assessing which a priori assumptions are reasonable is difficult. In that case, it might be beneficial to use prior distributions that do not have too much influence on the end result. The posterior distribution denoted $f(\theta|x)$ in (3.9), is the main interest in Bayesian statistics. This distribution represents the updated belief of the variable θ after observing the data x. The main driving factor of the updating of beliefs is the joint distribution $f(x|\theta)$. This function changes and updates the posterior distribution through Bayes´ theorem as more data is observed.

Sometimes, it is helpful to use what is called an improper prior. Assume that Θ denotes the parameter space of θ. According to an Robert et al. [2007] an improper prior of θ can be defined as

$$\int_{\Theta} f(\theta)d\theta = +\infty.$$

Improper priors can be used as long as the posterior ends up being a proper distribution. An example of an instance where improper priors can be useful is when we have limited knowledge of the parameter we want to put a prior on, and it is hard to find a proper distribution that represents this a priori knowledge. An improper prior can also be chosen to limit the influence the priors have on the posterior. One example of an improper prior is a uniform distribution on the whole real line. To show how we can do Bayesian inference using this as the prior distribution, we can consider the mean walking direction μ of the first family in the bunny task. A uniform prior for μ on the real line is given by

$$f(\mu) = c \quad \text{for } \mu \in (-\infty, \infty). \tag{3.11}$$

Let us assume that we have observed n walking directions, $x = (x_1, \ldots, x_n)$. In reality, we do not know how many times a change of family has occurred, but in this example, we assume that we know that the n observations are from the first family. We want to do Bayesian inference on μ using the observed data and the prior distribution in (3.11). Assume that each walking direction

is drawn from

$$f_1(x_i|\mu) = N(\mu, \sigma^2),$$

where $\sigma = 20$ is assumed known. The joint distribution $f(x|\mu)$ is then given by

$$f(x|\mu) = \prod_{i=1}^{n} f_1(x_i|\mu).$$

What we are interested in is the posterior distribution $f(\mu|x)$. Bayes´ theorem yields

$$f(\mu|x) = \frac{f(x|\mu)f(\mu)}{\int f(x|\mu)f(\mu)d\mu} = \frac{f(x|\mu) \cdot c}{\int f(x|\mu) \cdot c d\mu} = \frac{f(x|\mu)}{\int f(x|\mu)d\mu}.$$

As we see here, the constant c is canceled out. By inserting the expression for a normal distribution, we get

$$f(\mu|x) = \frac{\prod_{i=1}^{n} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right)}{\int \prod_{i=1}^{n} \exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right)d\mu} = \frac{\exp\left(-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + n\mu^2\right)\right)}{\int \exp\left(-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + n\mu^2\right)\right)d\mu} \tag{3.12}$$

Furthermore, we want to solve the integral in the denominator,

$$\int_{-\infty}^{\infty} \exp\left(-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + n\mu^2\right)\right)d\mu = \sqrt{\frac{2\pi\sigma^2}{n}} \exp\left(\frac{\left(\sum_{i=1}^{n} x_i\right)^2 - n \cdot \sum_{i=1}^{n} x_i^2}{2 \cdot n \cdot \sigma^2}\right)$$

By inserting this into (3.12), we get

$$f(\mu|x) = \sqrt{\frac{n}{2\pi\sigma^2}} \cdot \frac{\exp\left(-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + n\mu^2\right)\right)}{\exp\left(\frac{\left(\sum_{i=1}^{n} x_i\right)^2 - n \cdot \sum_{i=1}^{n} x_i^2}{2 \cdot n \cdot \sigma^2}\right)}$$

$$= \sqrt{\frac{n}{2\pi\sigma^2}} \exp\left(-\frac{n}{2\sigma^2}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2}{n}\mu\sum_{i=1}^{n} x_i + \mu^2 + \frac{1}{n^2}\left(\sum_{i=1}^{n} x_i\right)^2 - \frac{1}{n} \cdot \sum_{i=1}^{n} x_i^2\right)\right)$$

$$= \frac{1}{\sqrt{2\pi}}\sqrt{\frac{n}{\sigma^2}} \cdot \exp\left(-\frac{1}{2}\frac{n}{\sigma^2}\left(\mu - \frac{1}{n}\sum_{i=1}^{n} x_i\right)^2\right).$$

This means that the posterior distribution of μ is

$$\mu|x \sim N\left(\frac{1}{n}\sum_{i=1}^{n} x_i, \frac{\sigma^2}{n}\right).$$

In other words, the posterior distribution of the mean walking direction of the family is a normal distribution. Its expected value is the mean of the observed directions, and its variance is the variance of the distribution the walking directions are assumed drawn from divided by the number of observations. The prior does not have a big influence on the final inference. After just a few observations, we get a good impression of where the mean is. Even if the participants have some prior expectation about the first mean walking direction, using a more informative prior for the mean is unlikely to alter the final inference substantially. In addition, it is hard to find a distribution that is a good representation of all the participants, as they might have different prior beliefs about the mean of the first family.

3.4 Geometric and binomial distribution

Consider Bernoulli trials with a probability p for success at each trial. A Bernoulli trial can, according to Casella and Berger [2021], be written as

$$
X_i = \begin{cases} 1 & \text{with probability } p, \\ 0 & \text{with probability } 1-p. \end{cases} \tag{3.13}
$$

Let R denote the number of these Bernoulli trials needed for one success. The probability of no success in a trial is $1-p$. If there have been r Bernoulli trials when a change occurs, there have been $r-1$ trials before the change. Since the Bernoulli trials are independent, the probability that r Bernoulli trials are needed for one success is given by

$$
P(R = r) = (1-p)^{r-1}p. \tag{3.14}
$$

To give an example, we can consider the bunny task where the different bunny families had different sizes. Let $x = (x_1, \ldots, x_n)$ denote the bunny directions, where x_i is observation number i. Assume that $j-1$ changes have occurred and x_n is thus a part of family j. Let the variables $R_1, \ldots, R_{j-1}$ be at which observation numbers the family changes happen, which means that if $R_i = r_i$ it follows that x_{r_i} is the first observed direction of family $i+1$. A size of a bunny family is the number of bunnies appearing from the first bunny of that family until the last bunny

from that family. The family size of family number i is therefore $R_i - R_{i-1}$. In reality, the family sizes were drawn from truncated log-normal distributions. However, this is not known to the participants. Instead, as each bunny appears, we assume that there is a probability p that the bunny is from a new family compared to the previous bunny and, therefore, a probability $1 - p$ that the bunny is in the same family as the previous bunny. Therefore, each time a new bunny appear is seen as a Bernoulli trial where there are two possible outcomes, a family change or not. We start by considering the observation number of the first change, R_1. There is at least one bunny from each family, so the first bunny has to be part of the first family, and therefore we have $R_1 > 1$. In (3.14), it is assumed that a success can happen also in the first trial. Since a change can not happen in the first observation in the bunny task, the probability of R observations before the first change occurs is the same as the joint probability of no changes in observation numbers $2, \ldots, r_1 - 1$ before a change occurs in observation number r_1. Thus, the probability that the first change occurs in r_1 is given by

$$P(R_1 = r_1) = (1 - p)^{r_1 - 2} p. \tag{3.15}$$

If $R_{i-1} = r_{i-1}$ is given, we know where the change to family i happened. Then, the variables $R_1, \ldots, R_{i-2}$ do not provide more information about where the change to family $i + 1$ occurs. Thus, we have

$$P(R_i = r_i | R_1 = r_1, \ldots, R_{i-1} = r_{i-1}) = P(R_i = r_i | R_{i-1} = r_{i-1}).$$

The probability $P(R_i = r_i | R_{i-1} = r_{i-1})$ is the conditional probability that observation number r_i is the first observation drawn from family $i + 1$ given that observation number r_{j-1} is the first observation number from family i. This equals the joint probability of no changes occurring between observation number $r_{i-1} + 1$ to $r_i - 1$ and a change occurring in r_i. Again, because of the independence of these events, this probability is given by

$$P(R_i = r_i | R_{i-1} = r_{i-1}) = (1 - p)^{r_i - r_{i-i} - 1} p. \tag{3.16}$$

Also related to Bernoulli trials is the binomial distribution. Let the sequence $X_1, \ldots, X_n$ be a sequence of Bernoulli trials where each X_i is given by (3.13). Let Y be the number of successes

in this sequence which can be written as

$$Y = \sum_{i=1}^{n} X_i.$$

According to Casella and Berger [2021], Y then has a binomial distribution given by

$$P(Y = y|n, p) = \binom{n}{y} p^y (1-p)^{n-y}, y = 0, 1, 2, \ldots, n. \tag{3.17}$$

Let us again consider the bunny task. Let T_n denote which family the last observation x_n is a part of. If x_n is part of family j it follows that $j-1$ changes have happened up to observation x_n. Hence, the probability of T_n being part of family j is the same as the probability of $j-1$ changes up to x_n. As each bunny appears, there is a probability p of change except at the first observation, which is guaranteed to be part of the first family. Therefore the probability of x_n being of part of family j is equivalent to the probability of $j-1$ successes after $n-1$ Bernoulli trials and is therefore given by

$$P(T_n = j) = \binom{n-1}{j-1} p^{j-1} (1-p)^{n-j}, j = 1, 2, \ldots, n. \tag{3.18}$$

3.5 Generalized linear models

Consider the ungrouped data $(y_i, x_{i1}, \ldots, x_{ik})$ for $i = 1, \ldots, n$, where $(y_1, \ldots, y_n)$ are responses and $x_i = (x_{i1}, \ldots, x_{ik})^T$ is a vector of covariates corresponding to y_i. Let Y_i denote the random variable of which we consider y_i to be a realization. In a linear model, it is assumed that the responses have a normal distribution. The generalized linear model is a generalization of the linear model where other distributions, in addition to the normal distribution, can be assumed for the responses [Fahrmeir et al., 2013]. There are some distributional assumptions for the (generalized linear model) glm. As in a linear model, the first assumption is that for a given covariate vectors $x_i = (1, x_{i1}, \ldots, x_{ik})$ for $i = 1, \ldots, n$, the responses are conditionally independent. According to Fahrmeir et al. [2013], the second assumption is that the distribution of the response variables Y_i for $i = 1, \ldots, n$ must be part of an exponential family. If the response y_i is

part of an exponential distribution, we can write the distribution in the form

$$f(y_i|\theta_i) = \exp\left(\frac{y_i\theta_i - b(\theta_i)}{\phi} + c(y_i, \phi)\right), \tag{3.19}$$

where θ_i is regarded as the natural parameter and ϕ denotes a common dispersion parameter which is independent of i. In addition, we have

$$E(Y_i) = \mu_i = b'(\theta_i), \quad \mathrm{Var}(Y_i) = \sigma_i^2 = \phi b''(\theta_i). \tag{3.20}$$

A structural assumption of the glm is that the mean μ_i is linked to a linear predictor $\eta_i = x_i^T \beta$ through the relations

$$\mu_i = h(\eta_i), \tag{3.21}$$

$$\eta_i = g(\mu_i), \tag{3.22}$$

where h is referred to as a response function and g is the link function which is the inverse of h. What link function is suitable depends on the distribution of the response. According to [Fahrmeir et al., 2013] every distribution belonging to the exponential family has something called the canonical link function, and this is given by $g(\mu_i) = \theta_i = \eta_i$.

An example of when a generalized linear model can be used is when the response is binary. Assume that the responses y_i for $i = 1, \ldots, n$ are binary and that we have one covariate denoted by x_{1i}, which is the corresponding covariate to the response y_i. The response variable Y_i is the random variable from which y_i is generated, and we assume this to be Bernoulli distributed and is therefore given by

$$Y_i = \begin{cases} 1 & \text{with probability } \pi, \\ 0 & \text{with probability } 1 - \pi. \end{cases} \tag{3.23}$$

The probability mass function is given by

$$f(Y_i = y_i) = \pi_i^{y_i}(1 - \pi_i)^{1-y_i}. \tag{3.24}$$

We want to rewrite this equation into the same form as in (3.19). First, take the logarithm of both sides of the expression above and get

$$\log(f(Y_i = y_i)) = y_i \log(\pi_i) + (1 - y_i)\log(1 - \pi_i) = y_i(\log(\pi_i) - \log(1 - \pi_i)) + \log(1 - \pi_i)$$
$$= y_i \log\left(\frac{\pi_i}{1 - \pi_i}\right) + \log(1 - \pi_i).$$

This means that we can write the probability mass function as

$$f(Y_i = y_i) = \exp\left(y_i \log\left(\frac{\pi_i}{1 - \pi_i}\right) + \log(1 - \pi_i)\right). \tag{3.25}$$

By letting $\theta_i = \log\left(\dfrac{\pi_i}{1 - \pi_i}\right)$, we have $\pi_i = \dfrac{\exp(\theta_i)}{1 + \exp(\theta_i)}$. This yields

$$\log(1 - \pi_i) = \log\left(1 - \frac{\exp(\theta_i)}{1 + \exp(\theta_i)}\right) = \log\left(\frac{1 + \exp(\theta_i) - \exp(\theta_i)}{1 + \exp(\theta_i)}\right)$$
$$= \log\left(\frac{1}{1 + \exp(\theta_i)}\right) = -\log(1 + \exp(\theta_i)).$$

Inserting the final expression above into (3.25) yields

$$f(Y_i = y_i|\theta_i) = \exp\left(y_i\theta - \log(1 + \exp(\theta_i))\right). \tag{3.26}$$

Thus, by letting $b(\theta_i) = \log(1 + \exp(\theta_i))$, $\phi = 1$ and $c = 0$, the density is in the same form as in the general expression of an exponential distribution in (3.19). To confirm this, we can check whether the relations in (3.20) hold. First, we consider the mean. Recall that $\theta_i = \log\left(\dfrac{\pi_i}{1 - \pi_i}\right)$. We get that

$$\mu_i = E[Y_i] = b'(\theta_i) = \frac{\exp(\theta_i)}{1 + \exp(\theta_i)} = \frac{\frac{\pi_i}{1 - \pi_i}}{\frac{1}{1 - \pi_i}} = \pi_i,$$

which is the result we expect for a Bernoulli distribution. Furthermore, we have

$$\text{Var}[Y_i] = \phi b''(\theta_i) = \frac{\exp(\theta_i)}{(1 + \exp(\theta_i))^2} = \frac{\frac{\pi_i}{1 - \pi_i}}{\left(\frac{1}{1 - \pi_i}\right)^2} = \pi_i(1 - \pi_i).$$

This is also what we expected. We want to relate the mean $\mu_i = \pi_i$ to a linear predictor $\eta_i = \beta_0 + \beta_1 x_{1i}$. Using the canonical link-function leads to

$$\theta_i = \eta_i \implies \log\left(\frac{\pi_i}{1 - \pi_i}\right) = \beta_0 + \beta_1 x_{1i}. \tag{3.27}$$

Here, the logit link function is used, and a model that uses this link function and assumes a Bernoulli distribution for the response is often referred to as the logit model [Fahrmeir et al., 2013]. We get that the odds of success, $Y_i = 1$, is given by

$$\frac{\pi_i}{1 - \pi_i} = \exp(\beta_0) \cdot \exp(\beta_1 x_{1i}),$$

which means that changing the covariate x_{1i} with one unit changes the odds of success by a factor of $\exp(\beta_1)$. Another common link-function for a binary response is the probit link-function. Let Φ be the normal cumulative distribution function. Then, the probit link-function is given by

$$\pi_i = \Phi(\eta_i). \tag{3.28}$$

3.6   Generalized linear mixed models

The description of the generalized linear mixed models is based on the description given in [Fahrmeir et al., 2013]. Recall that throughout the bunny task, the participants observe 490 bunnies, and they, therefore, have to make 490 decisions about which family each bunny belongs to. Thus, when considering the data sets containing the decisions made by the participants during the bunny task, we have repeated measures for each individual. Let us assume that for individuals $t = 1, \ldots, N$, the data is given by

$$(y_{t1}, \ldots, y_{ti}, \ldots, y_{tn}, \ldots, x_{t1}, \ldots, x_{ti}, \ldots, x_{tn}),$$

where n is the number of observations for each individual. Here, y_{ti} denotes observation number i for individual t. A key component of generalized linear mixed models (glmm), are random

effects. Corresponding to an individual t, we have a random effect γ_t given by

$$\gamma_i \sim N(0, Q), \tag{3.29}$$

where Q is a positive definite covariance matrix. We assume the responses y_{ti} to be conditionally independent given γ_t for $t = 1, \ldots, N$ and $i = 1, \ldots, n$. Furthermore, we assume that the conditional distribution $f(y_i|\gamma_t)$ is part of an exponential family. Recall that in a glm, the linear predictor is given by $\eta_i = x_i^T \beta$. In a generalized linear mixed model (glmm), the linear predictor also includes the random effect γ_t and is given by

$$\eta_{ti} = x_{ti}^T \beta + u_{ti}^T \gamma_t, \quad t = 1, \ldots, N, \quad i = 1, \ldots, n, \tag{3.30}$$

where $x_{ti} = (x_{ti1}, \ldots, x_{tij}, \ldots, x_{tik})^T$ is a covariate vector. The vector u_{ti} is, according to Fahrmeir et al. [2013], often a subvector of the covariate vector. In this thesis, we only consider the special case where $u_{ti} = 1$, for $t = 1, \ldots, N$ and $i = 1, \ldots, n$, and the linear predictor is then given by

$$\eta_{ti} = x_{ti}^T \beta + \gamma_t, \quad t = 1, \ldots, N, \quad i = 1, \ldots, n. \tag{3.31}$$

When this is the linear predictor, the model is called a random intercept model. The conditional mean $\mu_{ti} = E(y_{ti}|\gamma_t)$ is linked to η_{ti} through $\mu_{ti} = h(\eta_{ti})$ or $\eta_{ti} = g(\mu_{ti})$, where g is a link function.

Let us again consider an example where the response is binary. We assume that we have a linear predictor given by (3.31), and that $\gamma_t \sim N(0, \sigma_\gamma^2)$. The conditional distribution $f(y_{ti}|\gamma_t)$ is assumed to be a Bernoulli distribution. The mean is given by

$$\mu_{ti} = E[y_{ti}|\gamma_t] = P(y_{ti} = 1|\gamma_t) = \pi_{ti},$$

and is linked to the linear predictor through

$$\log\left(\frac{\pi_{ti}}{1 - \pi_{ti}}\right) = x_{ti}^T \beta + \gamma_t.$$

Thus, we have

$$\frac{\pi_{ti}}{1-\pi_{ti}} = \exp\left(x_{ti}^T\beta + \gamma_t\right),$$

which means that for a specific individual, changing a covariate x_{tij} with s changes the odds that $Y_{ij} = 1$ with a factor of $\exp(\beta_j \cdot s)$. The corresponding conditional probability given an individual t is

$$\pi_{ti} = P(y_{ti}|\gamma_t) = \frac{1}{1 + \exp\left(-x_{ti}^T\beta - \gamma_i\right)} \tag{3.32}$$

This probability is conditional for one specific individual. We are also interested in the marginal probability $P(Y_{ij} = 1)$, which is the probability of a success $Y_{ij} = 1$ averaged over all individuals $t = 1, \dots, N$. By using the law of total probability, we get

$$P(Y_{ti} = 1) = \int_{-\infty}^{\infty} P(Y_{ti} = 1|\gamma_t) f(\gamma_t) d\gamma_t.$$

According to [Agresti, 2013], this probability can be approximated by

$$P(Y_{ti} = 1) \approx \frac{1}{1 + \exp\left(\frac{-x_{ti}^T\beta}{\sqrt{1+0.6\sigma_w^2}}\right)}, \tag{3.33}$$

or equivalently, the odds can be approximated by

$$\frac{P(Y_{ti} = 1)}{1 - P(Y_{ti} = 1)} \approx \exp\left(\frac{x_{ti}^T\beta}{\sqrt{1+0.6\sigma_w^2}}\right). \tag{3.34}$$

Thus, marginally, changing the covariate x_{tij} by one unit changes the odds of $Y_{ij} = 1$ with a factor of $\exp\left(\beta_j / \sqrt{(1+0.6\sigma_w^2)}\right)$.

3.7   Permutation test for one-sided hypothesis tests

As stated in Givens and Hoeting [2012], a permutation test is a statistical inference method that involves conducting "experiments" on the observations within a data set. What makes the permutation test advantageous is that it does not rely on any distributional assumptions or assumptions of independence and can therefore be applied in many different contexts. In this

thesis, we want to use a permutation test to test whether there are significant differences between two groups, where one group is the control group L-AQ & L-CAPEp and the other group is one of the study groups in Table 2.6. In this section, we refer to the two groups we want to test differences between as groups 1 and 2, where either group 1 or 2 could be the control group depending on the context. Let $w = (w_1, \ldots, w_{n_1})$ be outcomes from group 1 and let $z = (z_1, \ldots, z_{n_2})$ be outcomes from group 2. Let $x = (w, z)$ denote all the outcomes. The outcomes x are either change probabilities or regression parameters in this thesis. Let F_w be the cumulative distribution of the outcomes w_i for $i = 1, \ldots, n_1$, and let F_z be the cumulative distribution of the outcomes z_i for $i = 1, \ldots, n_2$.

We want to use a permutation test to do a one-sided hypothesis test. The null hypothesis is

$$H_0: \ F_w = F_z. \tag{3.35}$$

The alternative hypothesis is that the outcomes from group 1 tend to be larger than those of group 2, and can be written as

$$H_1: \ F_w(z) > F_z(z) \forall z. \tag{3.36}$$

We need to decide on a test statistic that can capture the difference between the two groups to do the hypothesis testing. Let $s = h(w, s)$ be the test statistic. In our case, we consider the difference in mean or median between the groups as test statistics. If we consider the difference in mean, we have $h(w, s) = \bar{w} - \bar{s}$ where $\bar{w}$ is the mean out the outcomes of group 1 and $\bar{z}$ is the mean of the outcomes of group 2. If we consider the difference in mean, we have $h(w, z) = \tilde{w} - \tilde{z}$ where $\tilde{w}$ is the median of the outcomes of group 1 and $\tilde{z}$ is the median of outcomes of group 2. Let s_0 denote the observed statistic with the original group labels. We consider s_0 as a realization of the random variable S. To test the significance of our hypothesis, we estimate the p-value. The p-value is defined as the "probability of observing a test-statistic at least as extreme as the one actually observed" provided that H_0 is true [Aschwanden, 2015]. The alternative hypothesis is that the outcomes w tend to be higher than the outcomes z. In this case, observing an equal or more extreme value than the one actually observed means observing something larger or equal

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to the observed value s_0. Thus, the p-value is given by

$$p = P(S \geq s | H_0 \text{ is true}).\tag{3.37}$$

Under the null hypothesis, all the observations x are drawn from the same distribution. Thus, the control and patient labels are irrelevant since they do not influence the outcome. Therefore, assuming the null hypothesis is true, we could randomly shuffle the labels among the outcomes, and the joint null distribution would be the same. To simulate the distribution of S under the null hypothesis, we randomly shuffle the labels of the outcomes x in B iterations. A new group 1 and 2 is randomly chosen at each iteration. The size of one group is the same at each iteration. When dividing into two groups, we divide the individuals and not the outcomes into the groups. However, if we have one outcome for each individual, dividing the individuals into two groups is equivalent to dividing the outcomes into the groups. We can divide the outcomes into groups when we compare the intercept or effect coefficients between the groups. However, we have multiple change probabilities for each individual. When dividing these probabilities into groups, we randomly place individuals into two groups. The corresponding probabilities of one individual are placed into the same group as the individual. Let w_b denote the outcomes of the randomly selected group 1 at iteration b and let z_b be the randomly selected group 2 at the same iteration. At iteration b, we calculate an observed statistic $s_b = h(w_b, s_b)$ based on the new groups. We, therefore, have statistics $s_1, \ldots, s_B$, one for each permutation of group labels. These statistics together make up an estimation of the distribution of S under the null hypothesis. To estimate the p-value, we divide the number of statistics among $s_1, \ldots, s_B$ that are equal to or larger than the original statistic s_0. To write this mathematically, we first define an indicator function given by

$$I(s_b) = \begin{cases} 1 & \text{if } s_b \geq s_0, \\ 0 & \text{otherwise.} \end{cases}\tag{3.38}$$

Furthermore, the estimated p-value $\tilde{p}$ is

$$\tilde{p} = \frac{\sum_{b=1}^{B} I(s_b) + 1}{B + 1}.\tag{3.39}$$

To reject the null-hypothesis, the estimated p-value $\tilde{p}$ needs to be larger than a selected significance level of α. We use $\alpha = 0.05$ throughout this thesis.

4 | Bayesian observer model for the bunny task

During the bunny task, the participants must decide which family the newest observation is part of. We aim to construct a Bayesian observer model to calculate the probability that the most recently observed bunny belongs to the potential families it can be a part of, given the observed walking directions so far. In this section, we state the assumptions in the Bayesian model and derive the model itself. Additionally, we provide an example of applying the model to actual observations. The goal is to use these probabilities to analyze the choices made by the participants. At each instance, when the participants observe a new walking direction of a bunny, we consider the probability of it belonging to a new family compared to the family the previous bunny was classified into. These probabilities are the ones referred to as change probabilities in the introduction. The actual change probabilities calculated from the observation of the participants will be analyzed in Section 5. The corresponding log odds to the change probabilities, referred to as change log odds, will be used as covariates in the logit model we consider in Section 5.2. The change log odds are also utilized as covariates in the modeling in Section 6.

We aim to make the probabilities in the Bayesian observer model representative of the participants' beliefs, and we make assumptions based on the information they receive before the task. The model needs to consider the number of changes reported by the participants. To illustrate this, consider the example in Figure 4.1. Here, we can see 23 observations drawn from the three first families. The mean of each family is drawn as straight horizontal lines of different colors. The first family is a normal distribution with a mean value of 0, and the change of mean from one family to the next is here 20, so the second family has a mean value of 20 and the third one has a mean of 40. There have been 2 changes, but let us assume that the participant, unaware of this, has reported 5 changes and more than three into family 6 in a row. The possible options for families depend on the reported changes, and in this example, the possible options are family 6 and family 7. Since the participants do not have a complete overview of all the observed bunnies, it can be challenging to realize that too many changes have been reported. Furthermore, the participant cannot go back to a previous family at this point. In this scenario, the model has the potential to provide probabilities that may be misleading and representative of
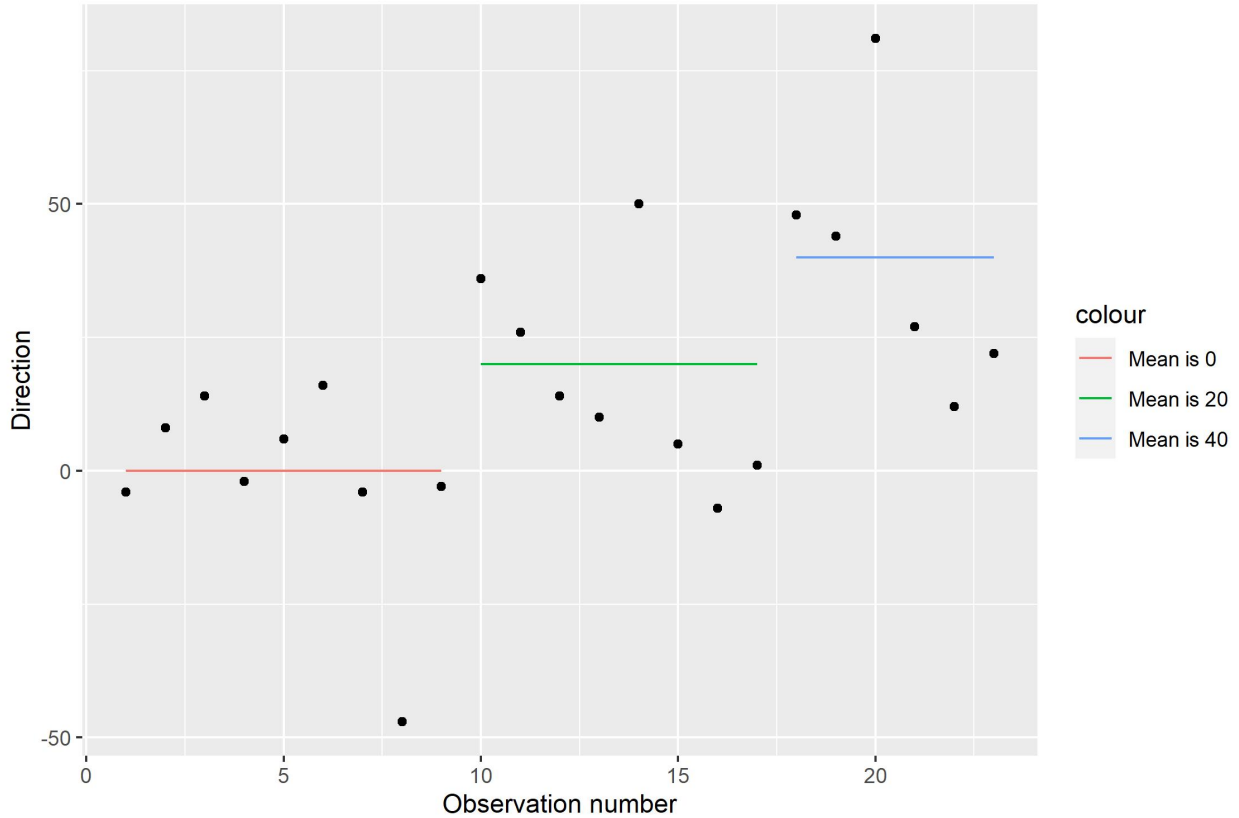
Figure 4.1: Example of observed walking directions in the bunny task.

the participant's beliefs. Let us, for example, assume that the model takes high probabilities for families around where the actual family is, which in this example means that the model would assign high probabilities to families around family 3. Even if the participant stops overreporting changes and starts making decisions more aligned with where the actual changes occur, the reported families will still be out of sync with the families the model gives the highest probabilities to because of the initial decisions. As a result, the model does not reward the participant significantly for making better decisions later. Similar issues may arise if a participant consistently underreports changes and becomes misaligned with the families the model predicts as most likely. Thus, a model disregarding the number of changes reported by participants would not accurately represent their beliefs in these cases.

To explain how we incorporate the participant's previous decisions into our model, let us consider the observed walking directions $d = (d_1, \ldots, d_{n_b})$, which are all the directions observed after bunny number n_b appears. At this point, the participant needs to determine which family

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bunny number n_b should be classified into. Recall that the participant only has two or three options to choose between. Let us consider the first family that is a possible option on the screen at this point. Suppose the participant has two families to choose between. In that case, there have been at least three consecutive bunnies preceding bunny number n_b that were classified into the same family, and this family is the first option on the screen. The first option is therefore the family bunny number $n_b - 1$ was placed into by the participant. However, when less than three bunnies have been classified into the same family in a row, there are three options, and the first available option is the family preceding the one bunny number $n_b - 1$ was classified into. Now, let us assume that bunny number r with walking direction d_r was the first bunny the participant classified into the first available family option displayed on the screen. To incorporate the participant's choices in the model, we only include the observations $d_r, \ldots, d_{n_b}$, and assume that d_r is actually part of the family it was classified into. These observations start with the first one classified into the first family displayed on the screen up to and including observation number n_b. The assumption that d_r is actually from the family it was classified into ensures that we take the participant's choices into consideration, and if the participant gets out of sync, this will not necessarily manifest itself further into the task. Furthermore, since the participants only see one bunny at a time and do not have a full overview of previous observations, it is reasonable to assume that not all previous observations are taken into consideration when determining the family the bunny is part of.

Let us again consider the example in Figure 4.1. We assume that a participant correctly classified all observations in this example. Thus, the horizontal lines in the figure also show into which families this participant classified the observations. Before each time one of the first 10 bunnies was classified into a family by the participant, the first option displayed on the screen was family 1, same as to the left in Figure 2.4. Thus, when finding probabilities for the different outcomes in these instances, all observations were included. After observing bunny number 11 or 12, the participant has three possible family options: family 1, family 2, or family 3 as shown to the right in Figure 2.4. Hence, in these scenarios, family 1 is the first option, and all observations until bunny number 11 or 12 are considered when determining the probabilities of them belonging to the various potential families. However, between observation numbers 13 and 18, the participant can only choose between family numbers 2 and 3, and family 1 is no longer an

option. In these instances, we only include observations starting from observation number 10 until the last, and we assume that observation number 10 actually is a part of family 2.

4.1 The Model

We use assumptions discussed in Section 3 to construct the Bayesian observer model of the bunny task problem. The mean of the first family is denoted μ, and we assume an improper uniform prior distribution on the whole real as in (3.11) for this variable. Furthermore, as in Section 3.4, the observation numbers of the first bunnies in each family are denoted $R_1, \ldots, R_{j-1}$ for family $2, \ldots, j$. The distribution of $P(R_1 = r_1)$ is assumed to be geometrically distributed and given by (3.15). Furthermore, we assume that $P(R_i = r_i | R_{i-1} = r_{i-1})$ is a geometric distribution given by (3.16). Recall that we only include certain observations in the Bayesian observer model. We denoted the observations included after observing d_{n_b} as $(d_r, \ldots, d_{n_b})$. When we describe the model in this section, we aim to establish suitable and consistent notation, and we denote $x = (x_1, \ldots, x_n)$ as the observations included at a given point when x_n is the last observed direction. This means that x_1 is the first observation from the first family appearing as a possible option, and this observation is assumed to be correctly classified. The family of x_1 could be many different families depending on how many changes the participant has reported. Again, since we want to describe the model generally for all cases, we refer to the family of x_1 as family 1 here. We let the variable T_n be a variable denoting the family number of x_n given that the family of x_1 is family 1. In other words, if k_n changes occur between x_1 and x_n, we have $T_n = k_n + 1$. The probability we are interested in is $P(T_n = j | x)$. That is, the probability that x_n is a part of family j given the observations x. We assume the prior distribution of T_n to be a binomial distribution given by (3.18).

During the titration process, the participants get a sense of how much the mean value changes between each family. Each participant receives a titration threshold. Remember that this threshold is the same as the absolute value of how much the mean value changes between families. The mean either changes clockwise or counter-clockwise for each participant in both blocks. If the change direction is clockwise, the shift in mean denoted Δ is the same as the titration threshold. The mean change Δ is the negative titration threshold if the change direction is counter-

clockwise. Since the titration threshold is determined by how the participants do the task in the titration block, and they get somewhat of a sense of where this threshold lies, we assume that Δ is known.

Now, let us consider the distributions the walking direction of the bunnies is drawn from. Let f_k denote the probability density function from which bunnies from family number k are drawn. All the families have the same standard deviation, which we assume to be known and denoted by σ. The mean walking direction of the first family is μ, and the mean value changes with Δ each time the family change. We assume that each observation x_i is independently drawn from a normal distribution where

$$f_k(x_i|\mu) = N(\mu + (k-1) \cdot \Delta, \sigma^2) \ \text{ for } k = 1, \ldots, j, \tag{4.1}$$

and thus

$$f_k(x_i|\mu) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{(x_i - \mu - (k-1) \cdot \Delta)^2}{\sigma^2}\right). \tag{4.2}$$

Again, recall that $R_1, \ldots, R_{j-1}$ denote the first observation number of each family from family $2, \ldots, j$. If $R_1 = r_1, \ldots, R_j = r_j$ and μ are given, we have that observation 1 until $r_1 - 1$ are drawn from family 1. Furthermore, in general, observation numbers r_{i-1} to $r_i - 1$ are drawn from family i, while observation numbers r_{j-1} to the last observation, observation number n, are drawn from family j. Thus, we have

$$f(x|\mu, R_1 = r_1, \ldots, R_j = r_j, T_n = j) = \prod_{i=1}^{r_1 - 1} f_1(x_i|\mu) \cdot \prod_{i=r_1}^{r_2 - 1} f_2(x_i|\mu) \cdot \ldots \cdot \prod_{i=r_{j-2}}^{r_{j-1} - 1} f_{j-1}(x_i|\mu) \cdot \prod_{i=r_{j-1}}^{n} f_j(x_i|\mu).$$
$$\tag{4.3}$$

In general, we can write this as

$$f(x|\mu, R_1 = r_1, \ldots, R_{j-1} = r_{j-1}, T_n = j) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \exp\left(-\frac{1}{2\sigma^2} \cdot (n\mu^2 + b_j(r)\mu + c_j(r))\right), \tag{4.4}$$

where

$$b_j(r) = \sum_{i=1}^{j-1} -2r_i\Delta + (2\cdot(j-1)\cdot\Delta)(n+1) - 2\cdot\sum_{i=1}^{n} x_i$$

$$c_j(r) = \Delta^2 \cdot \left(\sum_{i=1}^{j-1} -r_i\cdot(2i-1) + n(j-1)^2 + (j-1)^2\right) + \sum_{s=1}^{j-2}(-2s)\cdot\Delta\sum_{i=r_s}^{r_{s+1}-1} x_i - 2\cdot(j-1)\cdot\Delta\sum_{i=r_{j-1}}^{n} x_i + \sum_{i=1}^{n} x_i^2,$$

$$(4.5)$$

where we define a sum $\sum_{r=s}^{t} u_r$ equal to zero for $t < s$.

We want to find the probability of the event $T_n = j$ given the observations x. In theory, observation x_n could be a part of family $1,\ldots,n$. Bayes´ theorem yields

$$P(T_n = j|x) = \frac{f(x|T_n = j)P(T_n = j)}{\sum_{i=1}^{n} f(x|T_n = i)P(T_n = i)}. \qquad (4.6)$$

Thus, we must find an expression of $f(x|T_n = j)$ for $j = 1,\ldots,n$. We use the law of total probability by integrating over the possible values of μ and summing over the possible values of $R_1,\ldots,R_{j-1}$. By letting $R = (R_1,\ldots,R_{j-1})$ and $r = (r_1,\ldots,r_{j-1})$, the joint conditional distribution can be expressed as

$$f(x|T_n = j) = \sum_{r_1=2}^{n-j+2} \cdots \sum_{r_{j-1}=r_{j-2}+1}^{n} \int_{-\infty}^{\infty} f(x|R = r, \mu, T_n = j)f(R = r, \mu|T_n = j)d\mu. \qquad (4.7)$$

Given that we know the family of the last observation, knowing the mean of the first family μ does not provide more information about where the changes R occurred without the observation x given. This means that we have $f(R = r|T_n = j, \mu) = f(R = r|T_n = j)$. From this we get

$$f(R = r, \mu|T_n = j) = P(R = r|T_n = j, \mu)f(\mu|T_n = j) = P(R = r|T_n = j)f(\mu|T_n = j).$$

Furthermore, we assume $f(\mu|T_n = j) = f(\mu) = c$. Therefore, we have

$$
\begin{aligned}
f(x|T_n = j) &= \sum_{r_1=2}^{n-j+2} \cdots \sum_{r_{j-1}=r_{j-2}+1}^{n} \int_{-\infty}^{\infty} f(x|R = r, \mu, T_n = j) P(R = r|T_n = j) \cdot c d\mu \\
&= c \cdot \sum_{r_1=2}^{n-j+2} \cdots \sum_{r_{j-1}=r_{j-2}+1}^{n} P(R = r|T_n = j) \int_{-\infty}^{\infty} f(x|R = r, \mu, T_n = j) d\mu.
\end{aligned}
\tag{4.8}
$$

Thus, we need to find the integral

$$
\int_{-\infty}^{\infty} f(x|R_1 = r_1, \ldots, R_{j-1} = r_{j-1}, \mu, T_n = j) d\mu.
\tag{4.9}
$$

By inserting (4.4) for the integrand, we get

$$
\begin{aligned}
\int_{-\infty}^{\infty} f(x|\mu, R_1 = r_1, \ldots, R_{j-1} = r_{j-1}, T_n = j) d\mu &= \int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left(-\frac{1}{2\sigma^2} \cdot (n\mu^2 + b_j(r)\mu + c_j(r))\right) d\mu \\
&= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \cdot \sqrt{\frac{2\pi\sigma^2}{n}} \exp\left(\frac{b_j(r)^2 - 4n c_j(r)}{8n\sigma^2}\right).
\end{aligned}
\tag{4.10}
$$

To finish the derivation of $f(x|T_n = j)$ in (4.8) we need to find $P(R = r|T_n = j) = P(R_1 = r_1, \ldots, R_j = r_j|T_n = j)$. This is the joint conditional probability that the changes occur in $r_1, \ldots, r_{j-1}$ given that x_n is a part of family j. Bayes' theorem yields

$$
P(R_1 = r_1, \ldots, R_{j-1} = r_{j-1}|T_n = j) = \frac{P(T_n = j|R_1 = r_1, \ldots, R_{j-1} = r_{j-1}) P(R_1 = r_1, \ldots, R_{j-1} = r_{j-1})}{P(T_n = j)}.
\tag{4.11}
$$

We start by considering the probability $P(T_n = j|R_1 = r_1, \ldots, R_{j-1} = r_{j-1})$ in the numerator in (4.11). This is the conditional probability that observation x_n is part of family number j given that changes occur at $r_1, \ldots r_{j-1}$. If x_n is a part of family j, it means that no changes occur between observation number $r_{j-1}+1$ to n. Recall that there is a probability of $(1-p)$ for no change in each observation. Thus, we have

$$
P(T_n = j|R_1 = r_1, \ldots, R_{j-1} = r_{j-1}) = (1-p)^{n-r_{j-1}}.
$$

The other probability $P(R_1 = r_1, \ldots, R_{j-1} = r_{j-1})$ in the numerator in (4.11) is found by using the

multiplication rule. We have that,

$$P(R_1 = r_1,\ldots,R_{j-1} = r_{j-1}) = P(R_1 = r_1) \cdot P(R_2 = r_2 | R_1 = r_1) \cdot \ldots \cdot P(R_j = r_j | R_1 = r_1,\ldots,R_{j-1} = r_{j-1}).$$

(4.12)

As previously mentioned, the probability $P(R_1 = r_1)$ is geometrically distributed and is given by (3.15), and that $P(R_i = r_i | R_1 = r_1,\ldots,R_{i-1} = r_{i-1})$ for $i = 2,\ldots,j-1$ is also a geometric distribution given by (3.16). By inserting these expressions into (4.12), we have that

$$P(R_1 = r_1,\ldots,R_{j-1} = r_{j-1}) = (1-p)^{r_{j-1}-j} p^{j-1}.$$

Now, recall that the denominator in (4.11), $P(T_n = j)$, has a binomial distribution which is given by (3.18). By inserting this expression and the expression of $P(R_1 = r_1,\ldots,R_{j-1} = r_{j-1})$ and $P(T_n = j | R_1 = r_1,\ldots,R_{j-1} = r_{j-1})$ into (4.11), we get

$$P(R_1 = r_1,\ldots,R_j = r_j | T_n = j) = \frac{(1-p)^{n-r_{j-1}}(1-p)^{r_{j-1}-j} \cdot p^{j-1}}{\binom{n-1}{j-1} p^{j-1}(1-p)^{n-j}} = \frac{1}{\binom{n-1}{j-1}}.$$

(4.13)

Now, we have derived everything needed for finding $f(x|T_n = j)$. We insert the expression of the integral in (4.10) and the expression of $P(R_1 = r_1,\ldots,R_j = r_j | T_n = j)$ in (4.13) into (4.8), and we get

$$\begin{aligned}
f(x|T_n = j) &= \sum_{r_1=2}^{n-j+2} \cdots \sum_{r_{j-1}=r_{j-2}+1}^{n} \int_{-\infty}^{\infty} f(x|R = r, \mu, T_n = j) P(R = r | T_n = j) f(\mu | T_n = j) d\mu \\
&= \sum_{r_1=2}^{n-j+2} \cdots \sum_{r_{j-1}=r_{j-2}+1}^{n} \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \cdot \sqrt{\frac{2\pi\sigma^2}{n}} \exp\left(\frac{(b_j(r))^2 - 4nc_j(r)}{8 \cdot n \cdot \sigma^2}\right) \cdot \frac{c}{\binom{n-1}{j}}.
\end{aligned}$$

(4.14)

All components needed to find the probability of interest $P(T_n|x)$ given in (4.6) are in place. The expression above and the binomial distribution $P(T_n = j)$ given in (3.18) are inserted into the numerator of the expression and are also used to find the sum $\sum_{i=1}^{n} P(T_n = i|x)$ in the denominator. To demonstrate how these probabilities are found based on real bunny directions, we consider an example.

Example 4.1.1 *We again consider the bunny directions given in the example in Figure 4.1. First,*

we look at the first three observations from this example, which are $x = (x_1, x_2, x_3) = (-4, 8, 14)$.
After three observations, there are three possibilities. Either there have been no changes, meaning
that x_1 is part of family 1. If one change has occurred, observation x_3 is part of family 2, and if
two changes have happened, x_3 comes from family 3. The probabilities we are interested in are
therefore $P(T_3 = j)$ for $j = 1, 2, 3$, which are the probabilities that x_3 is part of family 1, 2 or 3 given
all observations so far, x.

As earlier stated, a model assumption is that the standard deviation $\sigma = 20$ and the mean change
$\Delta = 20$ are known. Recall that family 1 is assumed to have an unknown mean walking direction
of μ, family 2 a mean of $\mu + \Delta$ and family 3 a mean value of $\mu + 2\Delta$. By using the general expression
of $P(T_n = j|x)$ in (4.6) for $n = 3$, we get

$$P(T_3 = j|x) = \frac{f(x|T_3 = j)P(T_3 = j)}{f(x|T_3 = 1)P(T_3 = 1) + f(x|T_3 = 2)P(T_3 = 2) + f(x|T_3 = 3)P(T_3 = 3)} \qquad (4.15)$$

for $j = 1, 2, 3$. First, we consider the expressions of $f(x|T_3 = j)$ given by (4.14), where $b_j(r)$ and
$c_j(r)$ given in (4.5) are needed. We start by looking at $f(x|T_3 = 1)$. We use the values of $\Delta = 20$,
$\sigma = 20$ and $x = (-4, 8, 14)$ which yields

$$b_1 = -2 \cdot \sum_{i=1}^{n} x_i = -36,$$

$$c_1 = \sum_{i=1}^{n} x_i^2 = 276.$$

By inserting these values into (4.14), we get

$$f(x|T_3 = 1) = c \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \cdot \sqrt{\frac{2\pi\sigma^2}{n}} \exp\left(\frac{b_j^2 - 4a_j c_j}{8 \cdot n \cdot \sigma^2}\right) = c \cdot \left(\frac{1}{\sqrt{2 \cdot \pi \cdot 20^2}}\right)^3 \cdot \sqrt{\frac{2 \cdot \pi \cdot 20^2}{3}} \exp\left(\frac{36^2 - 4 \cdot 3 \cdot 276}{8 \cdot 3 \cdot 20^2}\right)$$

$$= \frac{c}{2\pi \cdot 400 \cdot \sqrt{3}} \exp\left(-\frac{21}{100}\right).$$

Now that we have found $f(x|T_3 = 1)$, the next step is to find $f(x|T_3 = 2)$. We start by considering

$b_2(r_1)$ and $c_2(r_1)$ which is found by inserting $j = 2$ into (4.5), and this yields

$$b_2(r_1) = -40r_1 + 124,$$

$$c_2(r_1) = 1876 - 400r_1 - 40 \cdot \sum_{i=r_1}^{n} x_i. \tag{4.16}$$

By inserting these into the expression of $f(x|T_n = 2)$ given in (4.14), we get

$$
\begin{aligned}
f(x|T_n = 2) &= \sum_{r_1=2}^{n} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right)^n \cdot \sqrt{\frac{2\pi\sigma^2}{n}} \exp\left(\frac{b_2(r_1)^2 - 4nc_2(r_1)}{8 \cdot n \cdot \sigma^2} \right) \cdot \frac{c}{n-1} \\
&= \sum_{r_1=2}^{3} \frac{c}{2\pi \cdot 400 \cdot \sqrt{3} \cdot 2} \exp\left(\frac{b_2(r_1)^2 - 12c_2(r_1)}{9600} \right).
\end{aligned}
\tag{4.17}
$$

To find a numerical value for the expression above, we need to find $b_2(r_1)$ and $c_2(r_1)$ for $r_1 = 2, 3$. We start by considering $r_1 = 2$ and insert this into (4.16) which yields $b_2(2) = 44$ and $c_2(2) = 196$, $b_2(3) = 4$ and $c_2(3) = 116$. By inserting these values into (4.17), we get

$$
\begin{aligned}
f(x|T_3 = 2) &= \frac{c}{2\pi \cdot 800 \cdot \sqrt{3}} \cdot \left(\exp\left(\frac{(44)^2 - 12 \cdot 196}{9600} \right) + \exp\left(\frac{(4)^2 - 12 \cdot 116}{9600} \right) \right) \\
&= \frac{c}{2\pi \cdot 800 \cdot \sqrt{3}} \left(\exp\left(-\frac{13}{300} \right) + \exp\left(-\frac{43}{300} \right) \right).
\end{aligned}
$$

At last, we consider $f(x|T_3 = 3)$. When $T_3 = 3$, there have been two changes in the three first observations and since the first change can not happen in the first observation, we have that the first change happens in $r_1 = 2$ and the second change happens in $r_2 = 3$. We find $f(x|T_n = 3)$ by inserting $j = 3$ into (4.14), and this yields

$$f(x|T_n = 3) = \frac{c}{\binom{n-1}{j-1}} \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right)^n \cdot \sqrt{\frac{2\pi\sigma^2}{n}} \exp\left(\frac{(b_3(2,3))^2 - 4 \cdot nc_3(2,3)}{8 \cdot n \cdot \sigma^2} \right). \tag{4.18}$$

We need to find $b_j(2,3)$ and $c_j(2,3)$. By inserting $j = 3$ and $\Delta = 20$, $\sigma = 20$ into (4.5), we get $b_3(2,3) = 84$ and $c_3(2,3) = 836$, and inserting these values and the values of $\sigma = 20$ and $\Delta = 20$ into (4.18) yields

$$f(x|T_n = 3) = \frac{c}{2\pi 20^2 \sqrt{3}} \exp\left(\frac{84^2 - 4 \cdot 3 \cdot 836}{8 \cdot 20^2} \right) = \frac{c}{2 \cdot \pi \cdot 20^2 \cdot \sqrt{3}} \exp\left(-\frac{31}{100} \right). \tag{4.19}$$

The last step for finding $P(T_3 = j|x)$ for $j = 1, 2, 3$ is to find the prior distributions $P(T_3 = j)$ for $j = 1, 2, 3$. We insert these values of j and $n = 3$ into the binomial distribution given in (3.18) which yields

$$P(T_n = 1) = (1 - p)^2,$$
$$P(T_n = 2) = 2 \cdot p \cdot (1 - p),$$
$$P(T_n = 3) = p^2.$$

A value of p has to be determined. Keep in mind that the family sizes are assumed to be geometrically distributed, and the mean of a geometric distribution with success probability p is equal to $1/p$. In this thesis, we only consider the results of the two main blocks of the bunny task. Before these blocks, the participants form an impression of the sizes of the families in the titration block. As reported in Table 2.1, the average family size in this block is 9.32. Thus, we use a value of $p = 1/9.32$. We insert the values we found for $f(x|T_n = j)$ for $j = 1, 2, 3$, into (4.15) and get

$$P(T_n = 1|x) = 0.779$$
$$P(T_n = 2|x) = 0.211$$
$$P(T_n = 3|x) = 0.011.$$

After 3 observations, the probability that the last bunny is part of family 1 and, therefore, that no changes have happened is 0.779 according to the Bayesian observer model. The probability of 1 change and the that the last bunny is part of family 2 is 0.211. The probability of 2 changes is 0.011. Since we only have made 3 observations at this point, it is not surprising that the it is most likely that no changes have happened so far. The prior probability that one change has happened is $P(T_3 = 2) = 2 \cdot p \cdot (1 - p) = 2 \cdot (1/9.32) \cdot (1 - 1/9.32) = 0.192$, which is lower than the posterior probability $P(T_3 = 2|x) = 0.211$. This increase in probability can be explained by the increase of walking directions from -4 to 14, which leads to that $f(x|T_3 = 2) > f(x|T_3 = 1)$. We look at how the probabilities develop as more bunnies are observed. Figure 4.1 shows that the first change occurs at observation number 10. In Figure 4.2, we can see a plot of the 9 first observations and the probabilities that observation 9 is part of family $1, 2$ and later families. According to the Bayesian
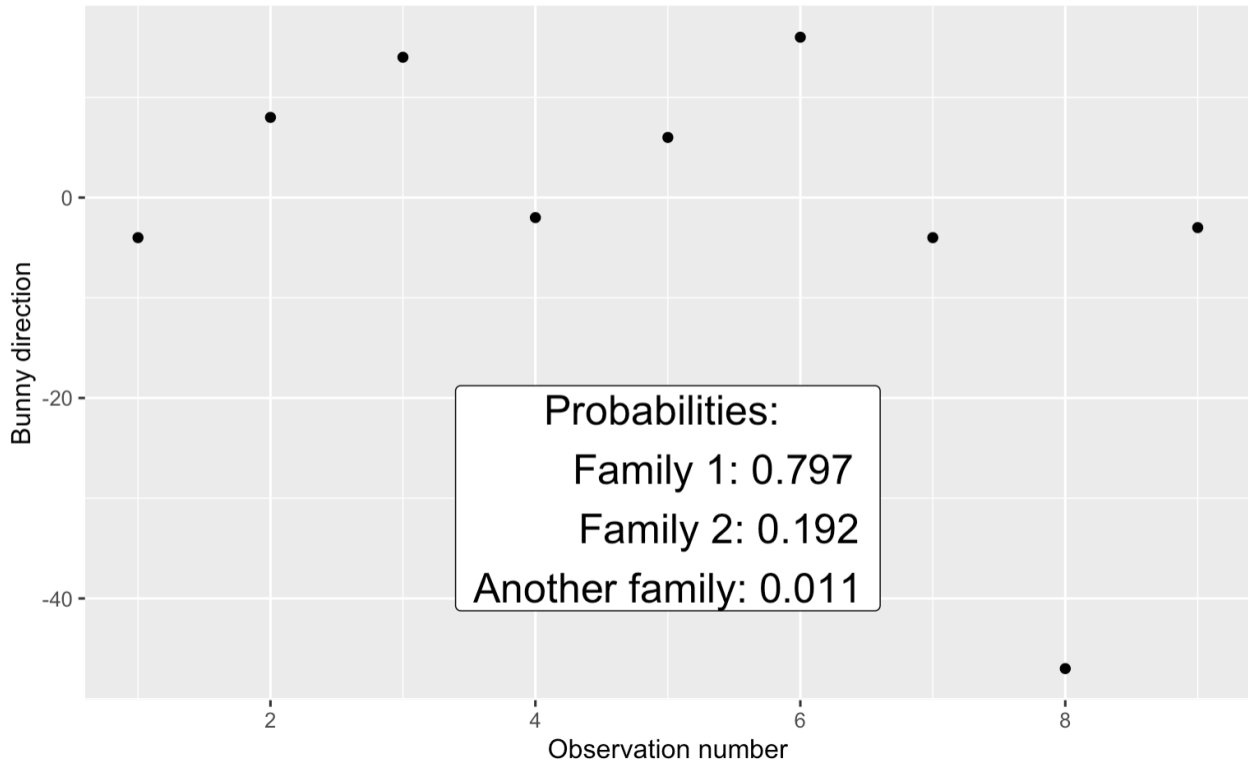
Figure 4.2: Example of the 9 first walking directions just before a change occur.

model, it is most likely that no changes have occurred at this point as the probability of family 1 *is* 0.797.

The next observation is part of a new family, and we also see a jump in the bunny direction as the observation has a higher walking direction compared to the previous observations. In Figure 4.3, we can see the probabilities that the last bunny is part of the different possible families after this last observation is included. The family the last observation is most likely to be part of, according to the Bayesian observer model, is family 1. *However, it is more likely than not that a change has happened at this point since the probability of the last observation being part of a family higher than* 1 *is* 0.432 + 0.087 = 0.519.

4.2   Change probabilities and odds

As stated earlier, we want to use the Bayesian observer model to calculate probabilities based on real observations made by the participants. In Section 2.2, we denoted all observed bunny direc-
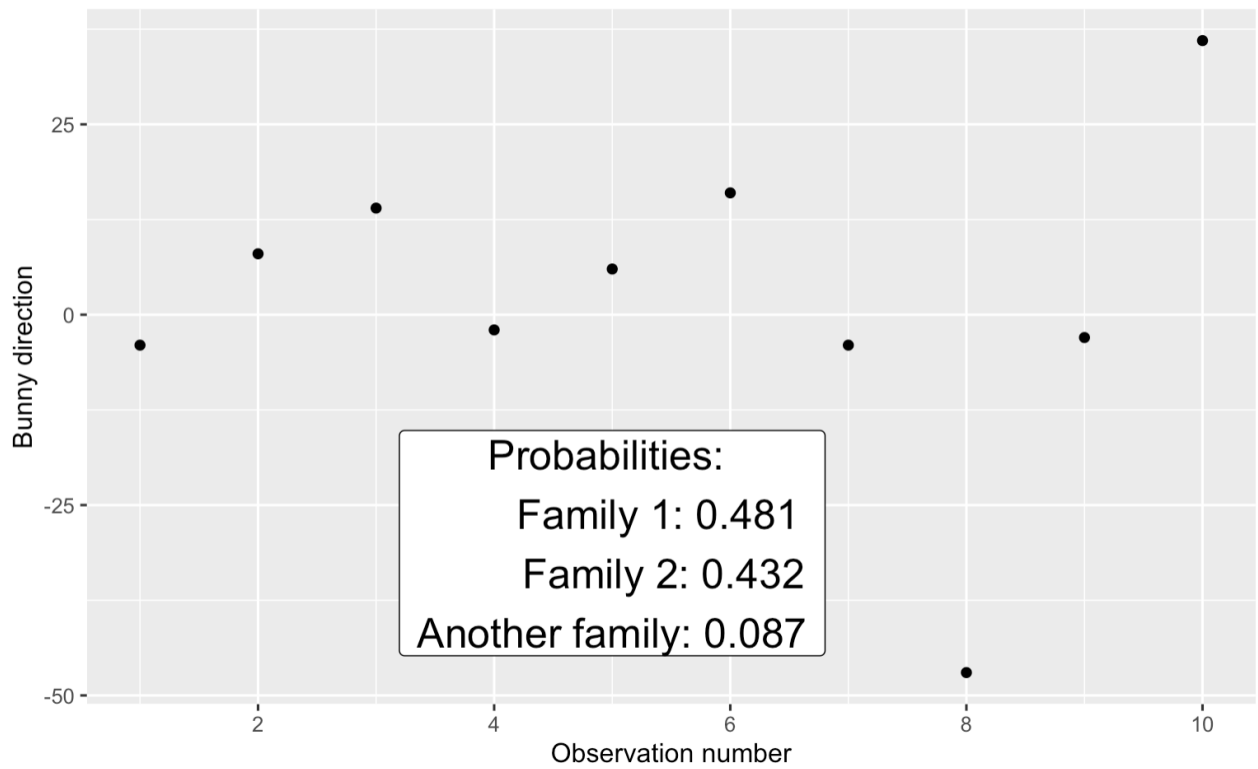
Figure 4.3: Example of the 10 first observed walking directions where observation 10 is the first from family 2.

tions in both the low volatility and high volatility block combined by $d_1, \ldots, d_{n_o}$, where $n_o = 490$ for all participants. The first bunny in each block is guaranteed to be part of the first family in each block, and the participants have to place these two bunnies into family 1. The first family decision the participants make in each block is therefore of no interest, and there are therefore $n = 488$ decisions we are interested in. Let $d_1, \ldots, d_n$ represent the walking directions of the 488 bunnies we are interested in examining in terms of their placement by the participants. Recall that we do not include all observations in the Bayesian observer model; only the observations categorized into the first possible family on the screen until the last observed bunny are included. In Section 4.1, we described how we found probabilities $P(T_n = j|x)$ for $j = 1, \ldots, n$ in general, where x was the general denotation of the observations included in the model. Now, let $\mathbf{x}_i = (x_{i1}, \ldots, x_{in_i})$ denote the observations included in the model when d_i has just been observed, where n_i is the number of observations included at that point. Remember that in the Bayesian observer model, we defined family 1 at any given point in the task as the first possible family option displayed on the screen. At the point where d_i is observed, the family x_{i1} was classified into, is considered family 1. Let $z_1, \ldots z_n$ be the choices the participant makes between the possible displayed options at each time they make the decision, where $z_i = 1$ if the first family displayed on the screen is chosen, $z_i = 2$ if the second family is chosen and $z_i = 3$ if the eventual third option is chosen. We let T_i be a variable denoting which family d_i is part of given that x_{i1} is part of family 1. In this thesis we focus on people´s tendency to assess changes. The probability we are mainly interested in is therefore the probability that the most recent observation x_{in_i} is part of a new family compared to the one x_{in_i-1} was classified into. This probability is what we refer to as change probabilities. For one participant, we have change probabilities

$$p_i = P(T_i > z_{i-1}|\mathbf{x}_i), \text{ for } i = 1, \ldots, n. \tag{4.20}$$

These change probabilities have corresponding change odds given by

$$\rho_i = \frac{p_i}{1 - p_i} \text{ for } i = 1, \ldots, n. \tag{4.21}$$

Let us again consider the example given in Figure 4.2, where 9 bunnies have been observed. We assume that the participant has placed the 8 first bunnies into family 1. Therefore the options

the participant can choose between are family 1 and family 2 at this point. The probability that bunny number 9 is part of family 2 or above, and thus the change probability that a change has occurred is $p_8 = 0.192 + 0.011 = 0.203$. The corresponding change odds is $\rho_8 = 0.255$. After 10 observations in Figure 4.3, the corresponding change probability is $p_9 = 0.432 + 0.087 = 0.519$, and the change odds thus $\rho_9 = (0.519)/0.481 = 1.079$.

5 | Group-based analysis

As stated previously, we want to use the probabilities calculated from the Bayesian observer model to analyze the decisions made by the participants. Based on the observations the participants made, which we can see a subsection of for one participant in Table 2.3, we calculate change probabilities $p_1, \ldots p_{488}$ for each participant. Recall that the central hypothesis we want to investigate is whether people with autistic or psychotic traits tend to need less evidence before inferring changes. In this thesis, we consider the change probabilities as a measurement of the level of evidence that a change has occurred. The change probabilities are specifically analyzed when the participants report a change. Suppose these probabilities give a good representation of the participants' perception of the bunny task and the hypothesis is true. In that case, it is expected that people with autistic or psychotic traits need lower change probabilities when reporting changes. We, therefore, start by investigating whether there are any visual differences between the groups in Table 2.6. We discover that the mean and median change probability when reporting change is larger for the H-AQ & L-CAPEp group compared to the control group L-CAPEp & L-AQ. We test whether the L-CAPEp & H-AQ group tends to have higher change probabilities when reporting changes than the control group. This is done by performing a permutation test for a one-sided hypothesis to see if the p-value is over a $\alpha = 0.05$ significance level. Furthermore, we are interested in looking at the distribution of the change probabilities when participants do not infer changes. These probabilities help understand the results from the modeling presented in Section 5.3.

Recall that in addition to testing the hypothesis of whether people with autistic and psychotic traits need less deviation from expected results to infer changes, we are also interested in investigating whether we find other differences in how people infer changes. These possible differences provide interesting insight into how the groups make decisions. We consider a logit model where the choices the participants make of whether they infer change or not at each point they make a decision are considered responses. We want to estimate the relationship between the response variable and the corresponding odds for each participant by fitting a logit model with the $\log(\rho_i)$ as a covariate. For each participant, we estimate an intercept coefficient and a coef-

ficient for the effect of the log odds. As we will see, the estimated intercept term of a participant is equal to the estimated log odds that the participant infers a change when there is an equal chance of change and no change according to the model. The estimated effect coefficients are estimates of the correlation between the change log odds and the change choices made by the participants. We compare these estimates between the participants of the different groups in Table 2.6. At last, we also want to investigate whether these estimated coefficients align with our hypothesis. This investigation is done by comparing the estimated average log odds and probability that a change was reported for different values of change probabilities and log odds.

5.1 Group differences in change probabilities

Recall that the participants of the bunny task are given two or three choices into which the latest bunny can be classified. We use these choices as responses in the model described in Section 5. However, to circumvent the problem that the choices are sometimes binary and other times ternary, we simplify these choices into binary responses. Our primary focus in this thesis is understanding how people perceive change. Therefore, we treat reporting a change of family as a separate category. Additionally, we combine the choices of classifying the latest bunny into the same family or the previous family into one category. Let us denote these binary choices of one participant as $y_1, \ldots, y_n$ for $n = 488$. Let d_i be the last observed bunny direction before the choice y_i is made. The event $y_i = 1$ means that observation d_i was categorized into a new family, and $y_i = 0$ means that observation d_i was classified into the same or a previous family. As in Section 4.2, $z_1, \ldots, z_n$ denote which of the family options on the screen the bunnies were classified into. Mathematically, we define the choices y_i of one participant as

$$y_i = \begin{cases} 1 & \text{if } z_i > z_{i-1}, \\ 0 & \text{if } z_i \leq z_{i-1}, \end{cases} \tag{5.1}$$

for $i = 1, \ldots, n$. We refer to $y_1, \ldots, y_n$ as change choices. We aim to investigate whether significant differences exist between the groups in Table 2.6. We consider the change probability p_i when the participants answer that a change has occurred. That is, probabilities p_i for i when
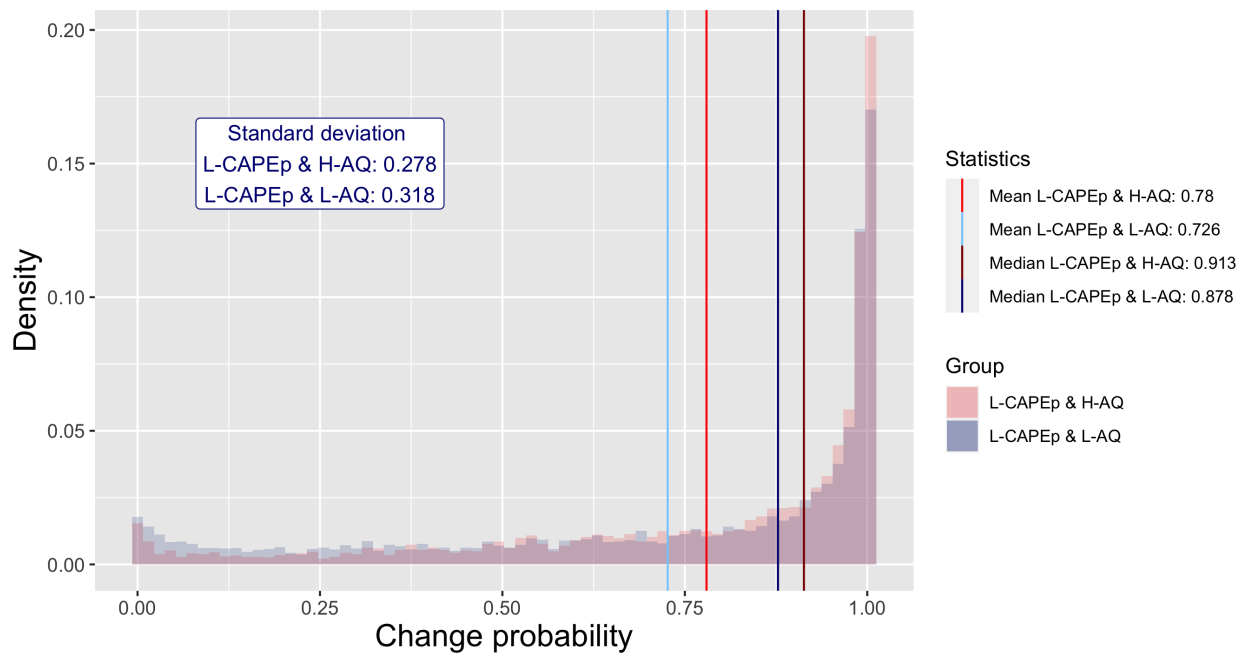
Figure 5.1: Comparing the distribution of change probabilities when changes were reported, between the L-CAPEp & H-AQ group and the control group.

$y_i = 1$. These probabilities tell how likely, according to the Bayesian observer model, a change has happened when the participants report changes. Keep in mind that if the model gives a good representation of the participants' beliefs and the hypothesis is correct, we would suspect that people with autistic and psychotic traits tend to have lower change probabilities when reporting a change, since a lower probability, in this case, indicates less evidence for a change when reporting a change.

We first compare the control group, L-AQ & L-CAPEp, with the H-AQ & L-CAPEp group. That is, we compare the change probabilities between the control group with low scores of autistic and psychotic traits with those with high scores of autistic traits and a low score of psychotic traits. In Figure 5.1, we see the empirical distribution of change probabilities when the participants answer that a change has occurred for the L-AQ & L-CAPEp group and for the H-AQ & L-CAPEp group. In addition, the mean and median for both groups are indicated by vertical lines. Visually, it is not easy to see significant differences between the two groups. However, a slightly higher fraction of people from the L-AQ & L-CAPEp group have low change probabilities, and a slightly higher fraction of the H-AQ & L-CAPEp group have high change probabilities. Furthermore,
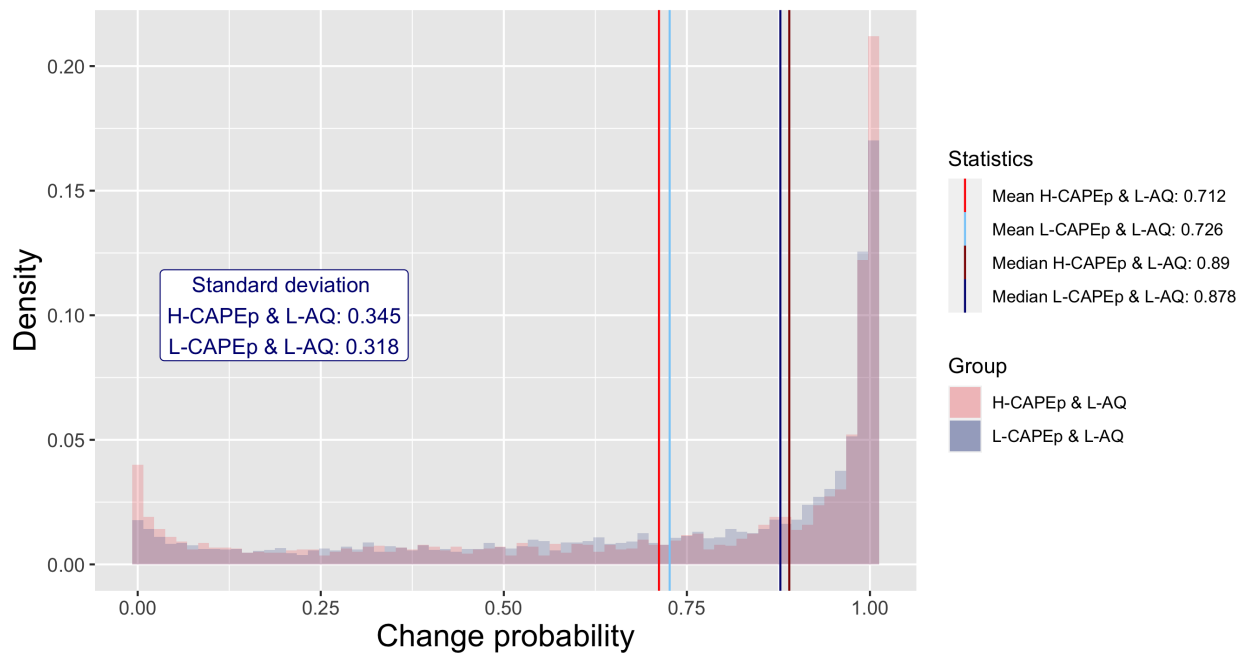
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Figure 5.2: Comparing the distribution of change probabilities when changes were reported, between the H-CAPEp & L-AQ group and the control group.

the mean and median change probability is higher for H-AQ & L-CAPEp. Note that this is the opposite result of what we expected according to our hypotheses.

In Figure 5.2, we see the distribution of change probabilities for the control group L-AQ & L-CAPEp and the L-AQ & H-CAPEp group. It is not easy to spot any significant differences between the groups by looking at this plot. There are no apparent differences in mean or median either.

At last, we consider the difference between the L-AQ & L-CAPEp group and the H-AQ & H-CAPEp group. In Figure 5.3, we see the distribution of the change probabilities for H-CAPEp & H-AQ and L-CAPEp & L-AQ together. We see little evidence of any significant differences between the groups from this plot.

Since the focus of this thesis is to look for differences in how people with psychotic or autistic traits infer changes, we do not look for significant differences between the groups in the probability that a change has occurred when the participants do not report changes. However, these probabilities are of interest when considering the results of the logit model in Section 5 where
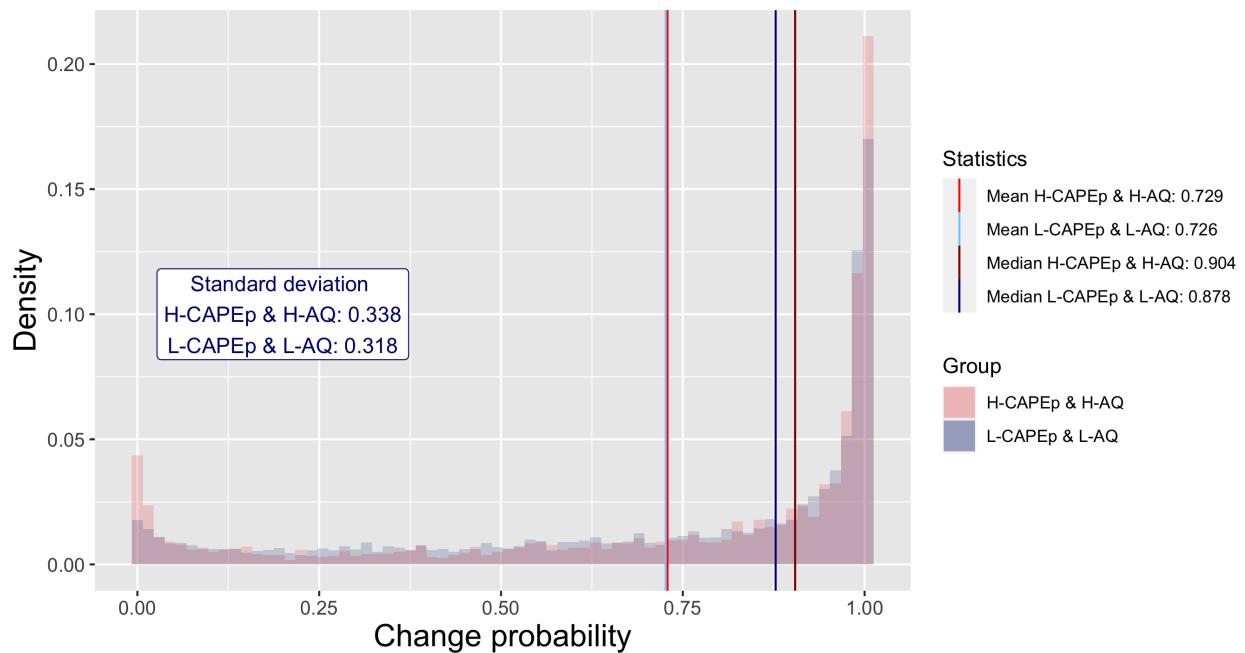
Figure 5.3: Comparing the distribution of change probabilities when changes were reported, between the H-CAPEp & H-AQ group and the control group.

we estimate the relationship between the change choice and the log odds of a change occurring according to the Bayesian observer model. The logit model is fitted to all the data, both where participants report changes and when participants do not report changes. Therefore, it is also interesting to look at the change probabilities when participants do not report changes. In Figure 5.4, we can see the empirical distribution of change probabilities when a change was not reported for all the participants. This plot shows as expected, that the highest concentration of change probabilities is close to 0 when the participants did not report changes. 62.2% of the change probabilities when a change was reported is below 0.5. However, we can see that a considerable proportion of the change probabilities when the participants did not report changes even are pretty close to 1 too. This can partially be explained by the attributes of the Bayesian observer model. Even though, as discussed in Section 4.1, our model intends not to penalize missing out on family changes too much, missing out on family changes can still temporarily influence the probabilities. To explain this, let us consider an example where a participant has observed the walking directions $(-5, 5, 0, 5, 3, 0, 5, 15, 20, 25, 35, 45, 40)$. The mean change is $\Delta = 40$ here. Since the walking directions have increased pretty steadily, and there has not been any sig-
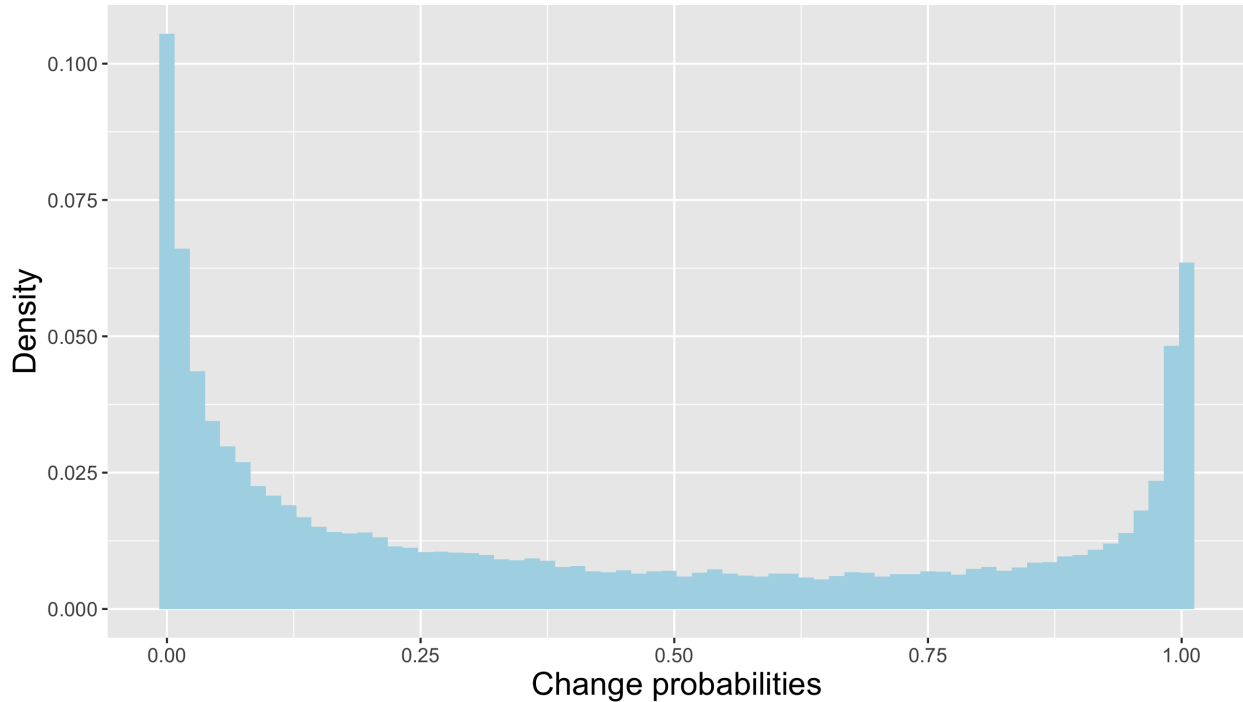
Figure 5.4: Change probabilities when participants did not report changes

nificant spike in walking directions, the participant has classified all the bunnies into family 1. However, according to the model, the probability that the last bunny is part of family 1 is only 0.0767 at this point, and the probability that a change has occurred is, therefore, 0.9233. Let us assume that the walking directions of the next bunnies are 35 and 40 degrees. These observations are neither indicative of a change for the participant. Now, the change probability has increased to 0.971. Because the participant only observes one bunny at a time and has not noticed that the walking directions are considerably higher than the first walking directions, there will be a lot of change probabilities close to 1 in a row. These situations where participants miss out on significantly increased walking directions might explain the high proportion of people not reporting changes even when the change probabilities are high.

We want to investigate the statistical significance of the differences between the groups. As mentioned earlier, we see little visual evidence for differences between the control group and the H-CAPEp & L-AQ group. This is also the case when comparing the control group with the H-CAPEp & H-AQ group. However, the difference between the control group and L-CAPEp & H-AQ looks more significant. We, therefore, do a hypothesis test of the difference between
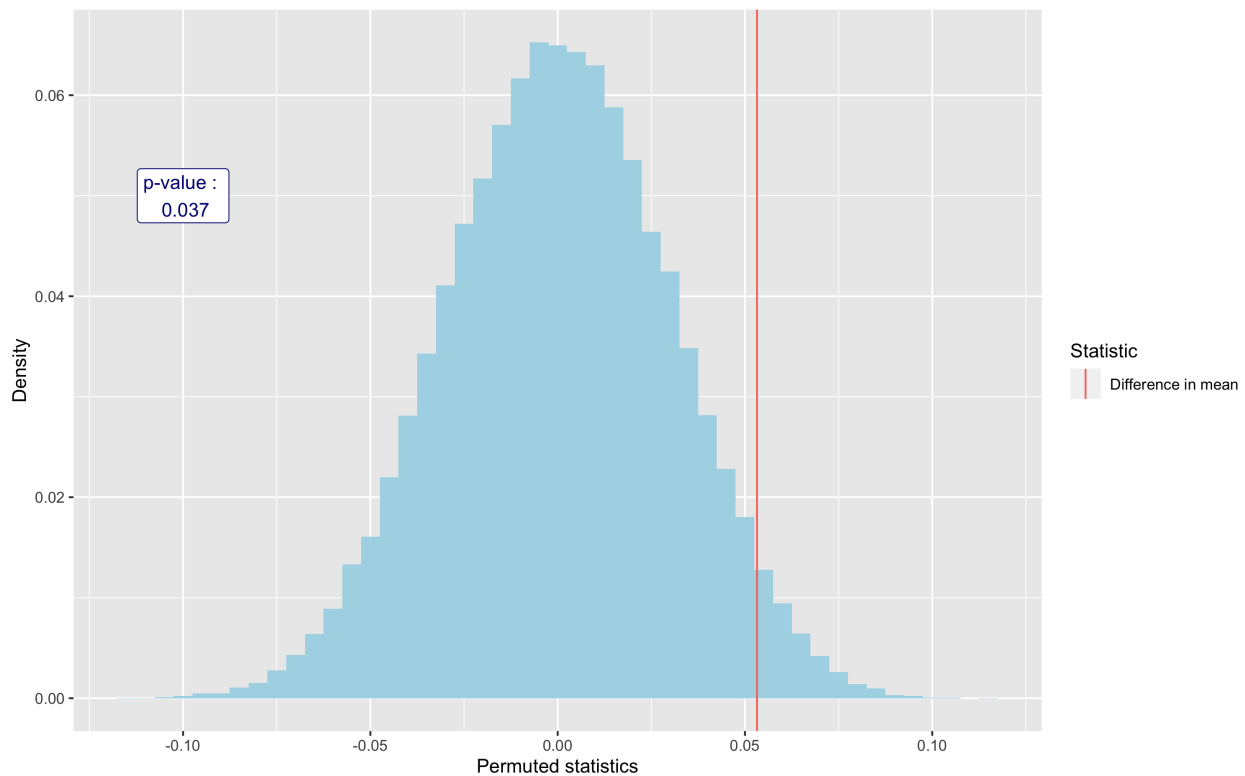
Figure 5.5: Result of 100000 permutations using the difference in mean change probabilities between the L-CAPEp & H-AQ group and the control group for change choices $y_i = 1$ as test-statistic.

these groups. First, we consider the difference in mean between the L-CAPEp & H-AQ and and L-CAPEp & L-AQ groups. We do a permutation test like the one described in Section 3.7. The alternative hypothesis is that the change probabilities tend to be higher for people in the L-CAPEp & H-AQ group. The test statistic we use is first the mean of the change probabilities when reporting change for the L-CAPEp & H-AQ group minus the mean of the control group. We do $B = 100000$ permutations. The results can be seen in Figure 5.5. The difference in the mean is 0.0532, and by using the formula given in (3.39), we estimate the p-value to be 0.037, which is lower than the significance level. Thus, we find the change probabilities to be significantly higher for the L-CAPEp & H-AQ group compared to the control group L-CAPEp & L-AQ when using the difference in mean as a test statistic. Given our hypothesis, this is a surprising result. This contradicts the assessment that people with autistic traits report changes with less evidence. This results together with Figure
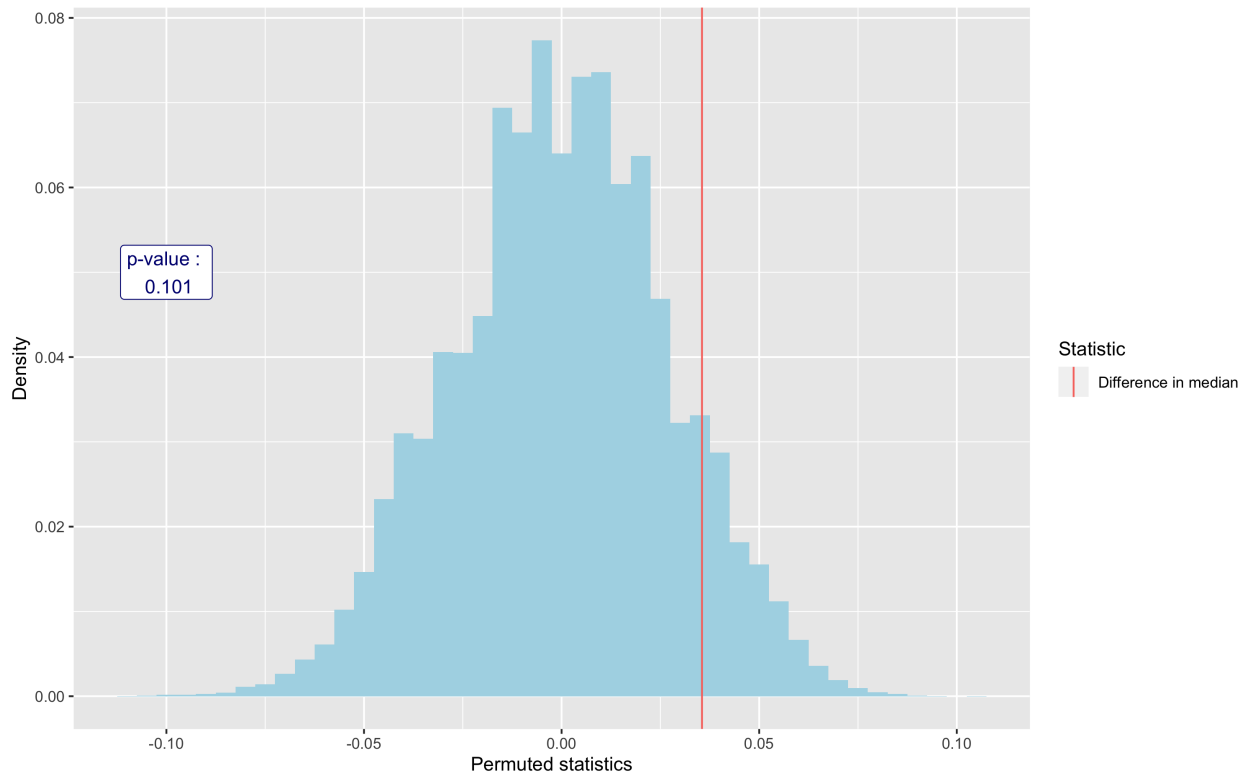
Figure 5.6: Result of 100000 permutations using the difference in median change probabilities between the L-CAPEp & H-AQ group and the control group for change choices $y_i = 1$ as test-statistic.

We also investigate whether there are significant differences when the L-CAPEp & H-AQ group's median minus the control group's median is used as a test statistic. The alternative hypothesis is the same as when using the mean difference: the L-CAPEp & H-AQ group tends to have higher change probabilities when reporting change. By using (3.39), we get that the estimated p-value is 0.102. Thus, the difference between the groups is insignificant when using the difference in median as a test-statistic. In Figure 5.6, we see the simulated density of the difference in median under the null hypothesis. The originally observed difference in median is plotted as a vertical line, and this difference is 0.0354.

5.2   Logit model

Now, we are interested in estimating the relationship between the change choices y_i for $i = 1,\dots,n$ given in (5.1) and the corresponding log odds ρ_i given in (4.21) for each participant. We

aim to employ a logit model to estimate this relationship and use the estimated coefficients from each participant in this model to generally see if there are differences between the study groups and the control group. The model we use is similar to the one considered in the example given in Section 3.5. The responses $y_1, \ldots, y_n$ are considered realizations of the random variables $Y_1, \ldots, Y_n$. The responses are also here assumed to be Bernoulli(π_i)-distributed, and we have

$$\pi_i = P(Y_i = 1) = E[Y_i],$$

which means that π_i is the probability that the participant classifies d_i into a new family. As in (3.27), we use the logit-link. Furthermore, as mentioned, we use the change log odds as covariates and the linear predictor is given by $\eta_i = \alpha + \beta \log(\rho_i)$. This linear predictor is linked to the mean through

$$\alpha + \beta \log(\rho_i) = \log\left(\frac{\pi_i}{1 - \pi_i}\right). \tag{5.2}$$

Thus, according to the model, increasing the log odds of change, $\log(\rho_i)$, with one increases the estimated odds of the participant reporting a change with β. The intercept α is the log-odds that the participant reports a change when the change odds is $\rho_i = 1$, which means there is an equal chance of change and no change. If for example $\alpha = 0$ for a participant, it means that the estimated odds of the participant reporting a change is 1 when the change odds is 1. In other words, there is a 50% chance that the participant reports a change when there is a 50% chance of change according to the Bayesian model.

We fit the model by using the `glm()` function in R [R Core Team, 2022]. This model fits a glm by using the Fisher-scoring algorithm to find the maximum likelihood estimates of the coefficients. We denote the maximum likelihood estimate of α as $\hat{\alpha}$ and the maximum likelihood estimate of β as $\hat{\beta}$.

5.3   Analysis of $\hat{\alpha}$ and $\hat{\beta}$

In Figure 5.7, the distribution of $\hat{\alpha}$ and $\hat{\beta}$ for all the participants can be seen. Recall that if $\hat{\alpha} = 0$, the probability that a participant reports a change is 0.5 when there is a 0.5 probability of change according to the Bayesian observer model. As we see in the plot, the proportion of people with
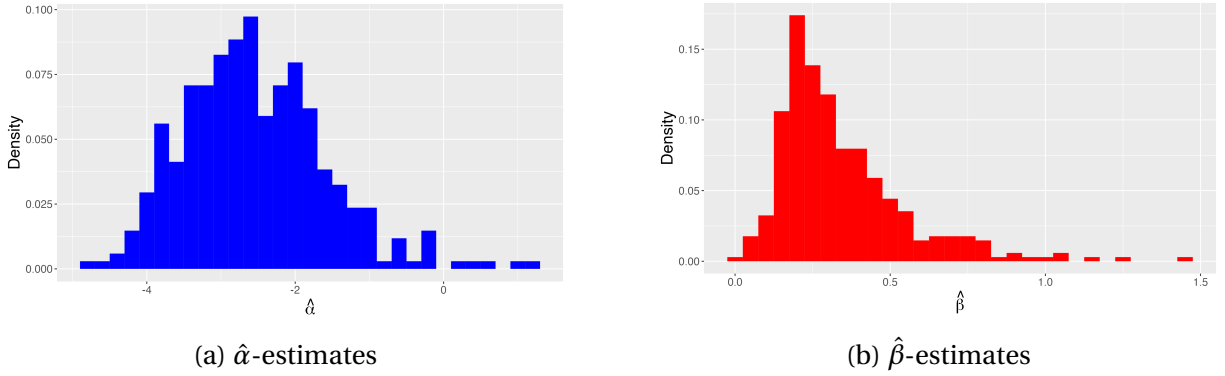
(a) $\hat{\alpha}$-estimates

(b) $\hat{\beta}$-estimates

Figure 5.7: α- and β-estimates for all participants

a $\hat{\alpha}$ over 0 is low. Out of 339 participants, only 5 people have an $\hat{\alpha}$ over 0, meaning people tend to need a higher change probability than 0.5 before reporting a change. The average $\hat{\alpha}$-estimate is -2.547, which means that the average estimate of the odds that a participant reports a change when there is 50% chance of change is $\exp(-2.547) = 0.0783$. This odds corresponds to a probability of 0.0726.

The average $\hat{\beta}$-estimate is 0.342. This means that, on average, the estimated odds that a participant reports a change increases by 0.342 when the log-odds of change increases by 1.

We also want to compare these estimated coefficients between the different study groups and the control group, L-CAPEp & L-AQ. In Figure 5.8, the empirical distributions of the estimates of $\hat{\alpha}$ and $\hat{\beta}$ is compared between the different study groups and the control group. On the left side, the $\hat{\alpha}$−estimates are plotted, and on the right side the $\hat{\beta}$-estimates. For each comparison, the control group is consistently visualized in blue, while the study group is represented in red. The mean and median estimates are displayed as vertical lines, with the mean and median coefficients of the study groups depicted by red and dark red lines respectively. Similarly, the mean and median coefficients of the control group are represented by light blue and dark blue lines respectively. The values of the median and mean of the coefficients for the different groups are presented in Table 5.1.

We start by considering the $\hat{\alpha}$-estimates. We see no clear visual differences between the $\hat{\alpha}$-estimates of the study groups and the control group. However, in Table 5.1, we see that the mean $\hat{\alpha}$ is considerably lower for the L-CAPEp & H-AQ group than the control group. We test whether

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the difference between these two groups is significant by doing a permutation test like the one described in Section 3.7. The alternative hypothesis is that the $\hat{\alpha}$−estimates tend to be higher for the control group. The statistic we consider is the difference in mean between the control group and H-CAPEp & L-AQ, and the observed statistic is thus $s_0 = 0.205$. We do 100000 permutations of group labels. The estimated p-value is 0.0514. Using a significance level of $\alpha = 0.05$, the difference is not considered being statistical significant.

We also notice that the median $\hat{\alpha}$ is substantially larger for the H-CAPEp & H-AQ group than the control group. We again perform a permutation test where the alternative hypothesis is that the $\hat{\alpha}$-estimates tend to be larger for the H-CAPEp & H-AQ group than for the control group. The estimated p-value is 0.179, and the difference between the groups is deemed insignificant. We therefore conclude that there are no significant differences in $\hat{\alpha}$-estimates between the different study groups and the control group.

Now, we consider the $\hat{\beta}$−estimates for the different groups. In Figure 5.8, we see little evidence of difference between L-CAPEp & H-AQ and the control group. The estimates mostly overlap, and difference between the mean and median is small. However, we see that there is a notable difference between the H-CAPEp & L-AQ group and the control group and between the H-CAPEp & H-AQ and the control group. In both instances, the empirical distribution of the $\hat{\beta}$−estimates seem to be shifted a bit to the right compared to the study group, and the $\hat{\beta}$−estimates tend to be higher for the study groups. In Table 5.1 we also note that both the mean and median of the $\hat{\beta}$−estimates are substantially lower for both the H-CAPEp & L-AQ group and the H-CAPEp & H-AQ group compared to the control group. We investigate whether these differences are significant by again performing a permutation tests similar to the previously performed in the section. In these hypothesis tests, the alternative hypothesis is that is that the $\hat{\beta}$−estimates tend to be higher for the control group. The results from the permutation tests are summed up in Table 5.2. All the p-values from the tests are smaller than 0.05 and this indicates that the $\hat{\beta}$−estimates are significantly lower for the study groups, H-CAPEp & L-AQ group and H-CAPEp & H-AQ. These results mean that as the log-odds of change increases, the probability that the participants reports a change tend to be higher for the two groups with a CAPEp-score higher than 65.

(a) $\hat{\alpha}$-estimates

(b) $\hat{beta}$-estimates

(c) $\hat{\alpha}$-estimates

(d) $\hat{\beta}$-estimates

(e) $\hat{\alpha}$-estimates
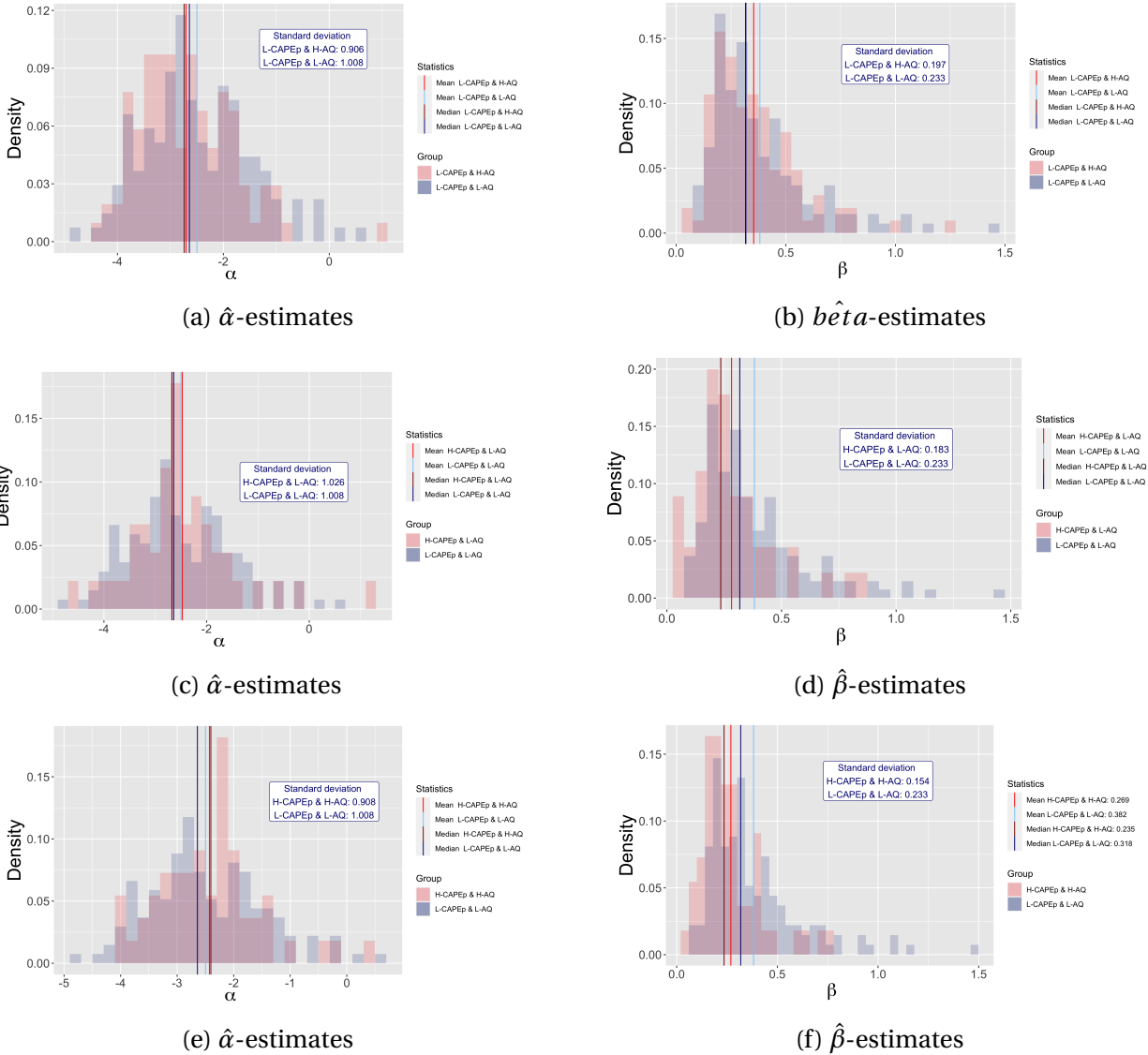
(f) $\hat{\beta}$-estimates

Figure 5.8: Comparison of $\hat{\alpha}$- and $\hat{\beta}$-estimates for all groups.

Table 5.1: Mean and median $\hat{\alpha}$ and $\hat{\beta}$ for the different groups.

Group	Mean $\hat{\alpha}$	Median $\hat{\alpha}$	Mean $\hat{\beta}$	Median $\hat{\beta}$
L-CAPEp & L-AQ	-2.499	-2.641	0.382	0.318
L-CAPEp & H-AQ	-2.704	-2.740	0.353	0.316
H-CAPEp & L-AQ	-2.472	-2.676	0.282	0.235
H-CAPEp & H-AQ	-2.430	-2.404	0.269	0.235

Table 5.2: Results from permutation tests of the difference between $\hat{\beta}$ for the different study groups with high CAPEp scores and the control group.

Study group	Statistic	p-value
H-CAPEp & L-AQ	Difference in mean	0.00272
H-CAPEp & L-AQ	Difference in median	0.00378
H-CAPEp & H-AQ	Difference in median	0.00019
H-CAPEp & H-AQ	Difference in median	0.00156

We are also interested in investigating what these results tell about the main hypothesis. In other words, we are interested in what these estimated coefficients tell us about the different groups' tendency to report changes. Recall that the linear predictor $\alpha + \beta \log(\rho)$ is linked to the probability that a participant reports a change π through the link function in (5.2). There is a linear relationship between the log odds that a participant reports a change and the log odds of change, and we make a plot showing this relationship for the different groups. Along the x-axis, we have the log odds of change between -8 to 8, and along the y-axis, we have the corresponding average estimated log odds that the participants report change for the different groups. The average log-odds that a participant reports a change is found by using the mean value of the $\hat{\alpha}$ and $\hat{\beta}$-estimates for the different groups, which we can see in Table 5.1. The plot is shown in Figure 5.9. Let us start considering the groups with high score of psychotic traits, the H-CAPEp & L-AQ group and the H-CAPEp & H-AQ. These two groups are represented by a green and red line respectively in the plot, and these lines follow each other closely. When comparing to the control group, L-CAPEp & L-AQ, represented by the purple line, we see that the average log-odds that participants in the two H-CAPEp groups is higher for low change log-odds. As we already have discussed, as the $\log(\rho)$ increases, the log-odds that a participant reports a change, $\log(\pi/(1-\pi)$ increases more for the control group. At values of $\log(\rho) > 0.270$ and $\log(\rho) > 0.611$, the average $\log(\pi/(1-\pi)$ is larger for the control group than the H-CAPEp & L-AQ group and the H-CAPEp & H-AQ group respectively. These change log-odds corresponds to change probabilities of 0.567 for the H-CAPEp & L-AQ group and 0.648 for the H-CAPEp & H-AQ group. For lower change probabilities than these, the average probability that people in these two groups report changes is higher for the two groups with high scores of psychotic traits. This indicates that in contexts where there is less evidence of change according to the Bayesian observed model, the groups with high scores of psychotic traits are on average more likely to
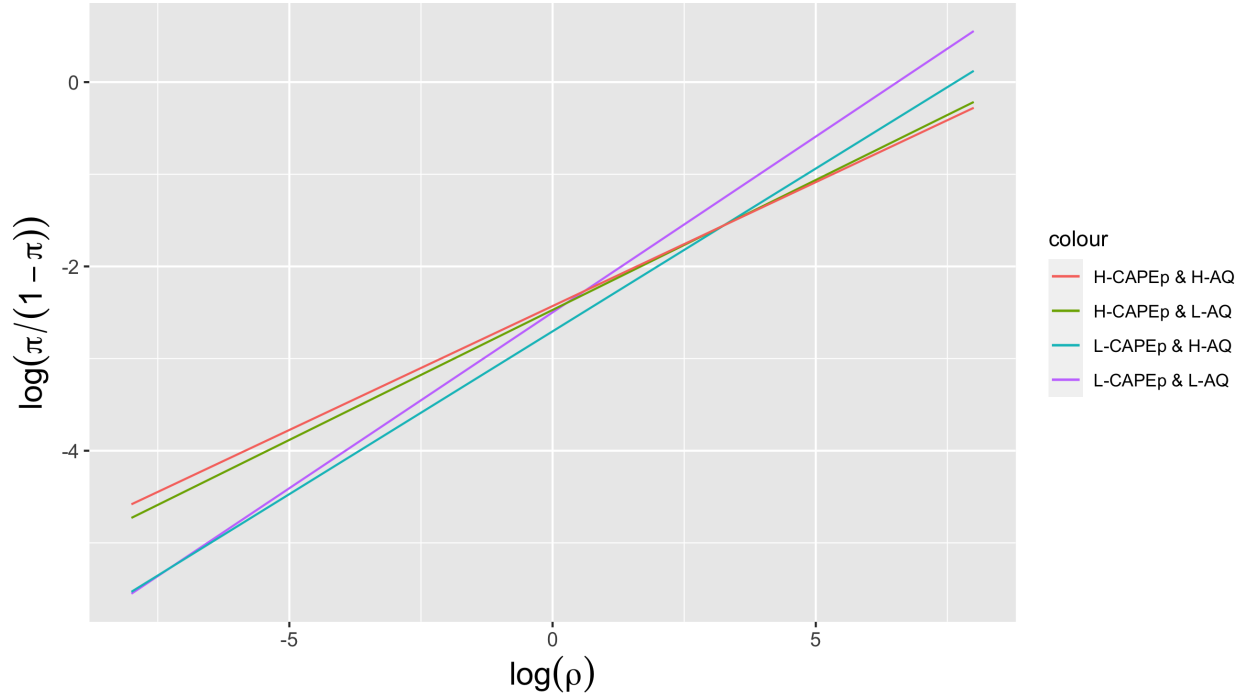
Figure 5.9: The average relationship between the change log odds $\log(\rho)$
and the log odds that a change was reported $\log(\pi/(1-\pi))$

report changes. This is consistent with the main hypothesis which suggests that people with psychotic traits have a tendency to infer changes in more stable contexts with less evidence for change. This is not the case with the L-CAPEp & H-AQ group. For this group the average log odd that a change is reported is expected to be higher for all change log odds over -7.069, which corresponds to change probabilities over 0.00085. Thus, people with a high degree of autistic traits and a low degree of psychotic traits have a lower average tendency to report changes when the change probability is over 0.00085, which includes most cases. In Figure 5.10, the equivalent plot to 5.9 is shown, but along the x-axis, we have change probabilities p_i, and along the y-axis, we have the corresponding average probabilities that changes as reported, $\hat{P}(Y_i|p_i)$. If we compare this plot to the one in
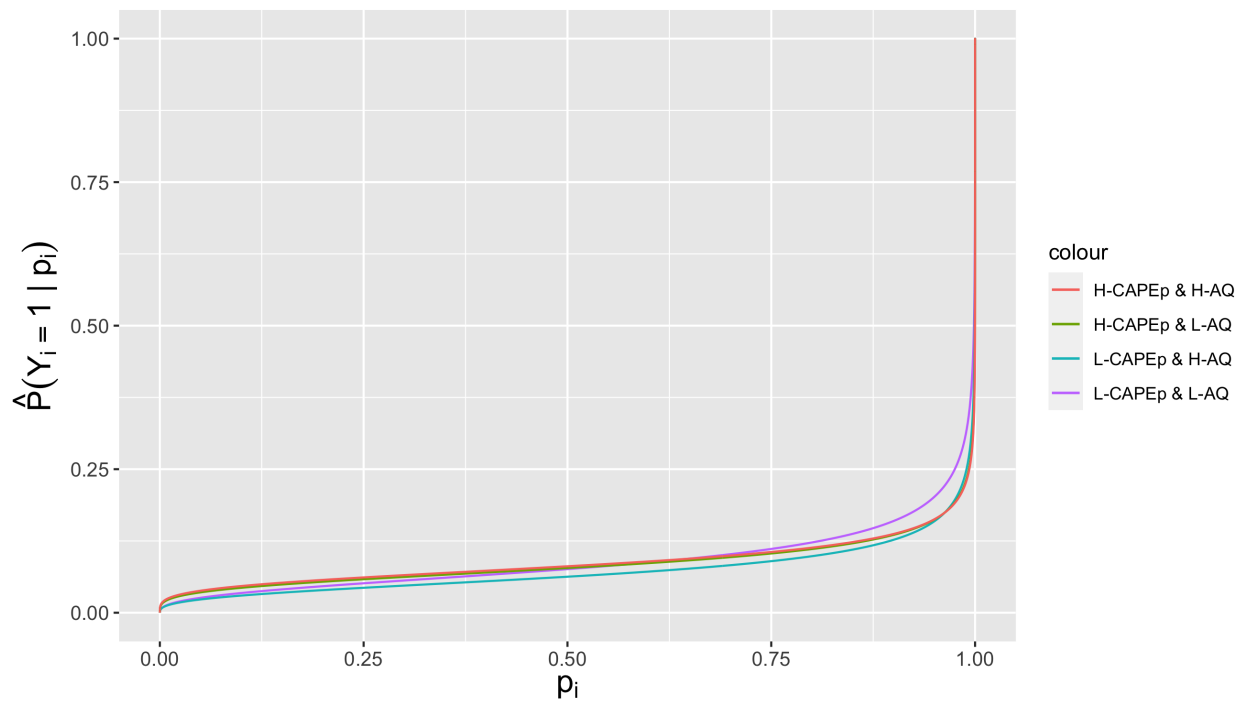
Figure 5.10: The average of the probability that a change was reported, $\hat{P}(Y_i = 1|p_i)$, for change probabilities p_i between 0 and 1

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6 | Modelling using CAPEp and AQ as covariates

In Section 5, we conducted analyses by categorizing participants into groups based on AQ and CAPEp cut-off scores. Through this analysis, we discovered significant differences among the groups. These findings are interesting as they show that there are differences between the groups based on cut-off scores that have previously been used for the bunny task. However, as discussed earlier, a potential problem with this type of analysis is that it could be sensitive to where the cut-off scores are set. In this section, we do not divide people into groups. We are instead interested in the relationship between the choices of the participants and their corresponding CAPEp and AQ scores. In particular, we aim to investigate how increments in AQ or CAPEp score influence participants' inclination to infer changes for various change probabilities.

The model considered in this section, similar to the logit model described in Section 5.2, also has the binary change choices as responses. However, instead of fitting a model for each participant, we use CAPEp and AQ score as covariates, fitting a single model to all the participant data. The data consists of repeated observations since each participant makes 488 change choices and has 488 corresponding change probabilities. As discussed in Section 3.6, we can include a random intercept for each participant to account for the dependency structure in data with repeated measures for individuals. The model considered in this section is a binary random intercept model, akin to the example discussed in 3.6. The aim of the modeling is to investigate whether using AQ and CAPEp as variables yields results consistent with our findings in Section 5.3, and the fixed effects included are determined accordingly. First, as in the logit model described in Section 5.2, we have a fixed effect of the change log odds. Furthermore, we include the effects of the CAPEp and AQ scores. Our previous analysis showed that the mean of the estimated effect of log odds was significantly lower for groups with high CAPEp scores. Thus, we are interested in whether increasing CAPEp scores correlate with a lower effect of log odds on the change choice. To explore this, we include an interaction term between the CAPEp score and the change log odds in the current model. We also saw that for low change log odds and thus low change probabilities, the probability that the two groups with high CAPEp scores reported changes was higher. Therefore, we investigate whether increasing CAPEp scores is generally associated with

a lower tendency to report changes for low change probabilities. In the previous section, the results for the two groups with high scores of AQ were mixed. The results of the group with high scores of AQ and CAPEp, the H-CAPEp & H-AQ group, were similar to the H-CAPEp & L-AQ group as we see in Figure 5.9. Based on the modeling conducted earlier, we found that when it came to low change probabilities, the likelihood of reporting changes was nearly identical to that of the control group. However, the L-CAPEp & H-AQ group generally had lower estimated probabilities of reporting changes. As the change probability increases, the difference increases. We are interested in how the effect of the log odds changes with an increased score of AQ, and we, therefore, include an interaction term between the AQ score and the change in log odds. We check whether the coefficients in the interaction terms significantly differ from 0, which is done by performing z-tests. We also conduct z-tests to verify the significance of the other estimated coefficients.

We also investigate what the estimated values of the coefficients tell about the tendency of reporting change for the different participants and whether these results align with our central hypothesis. We investigate this by considering the estimated probability that a change was reported for different values of change probability and values of AQ and CAPEp. First, we evaluate the conditional probability that a change was reported for a given individual with a random intercept of zero. Furthermore, we evaluate the marginal probability that a change was reported.

6.1 Binary random intercept logit model

Recall that in Section 5.2, we considered responses $y_1, \ldots, y_n$, for $n = 488$ from one participant, where each change choice y_i is whether the participant reported a change or not for observation number i. We only considered responses for one participant at a time and fitted one model to each participant. However, this section considers all the binary change choices of all $N = 339$ participants together. Let $(y_{11}, \ldots, y_{1n}, \ldots, y_{t1}, \ldots, y_{ti}, \ldots, y_{tn}, \ldots, y_{Nn})$ denote the change choices of all participants where y_{ti} denotes change choice number i of individual t. To each response y_{ti} we have a corresponding change odds ρ_{ti} where ρ_{ti} is change odds number i of participant t. A response y_{ti} is considered a realization of the random variable Y_{ti}. Equivalent to the example of the binary random intercept logit model considered in Section 3.6, the responses are binary.

Let $\gamma_t \sim N(0, \sigma_w^2)$ denote a random effect corresponding to an individual t. All the responses y_{ti} for $t = 1, \ldots, N$ and $i = 1, \ldots, n$ are assumed conditionally independent given the random effects γ_t for $t = 1, \ldots, N$, and $y_{ti}|\gamma_t \sim \text{Bernoulli}(\pi_{ti})$. Let AQ_t and CAPEp_t denote the AQ- and CAPEp score of individual t respectively. In the linear predictor, we include an effect of the AQ and CAPEp score and the log-odds ρ_{ti}. Furthermore, we aim to investigate how the effect of the change log odds ρ_{ti} changes when the AQ and CAPEp score is changed. We include interaction terms between the log odds ρ_{ti} and the AQ score and the CAPEp score. Thus, the linear predictor is given by

$$\eta_{ti} = \beta_0 + \beta_1 \cdot \text{CAPEp}_t + \beta_2 \cdot \text{AQ}_t + \beta_3 \log(\rho_{ti}) + \beta_4 \cdot \text{CAPEp}_t \cdot \log(\rho_{ti}) + \beta_5 \cdot \text{AQ}_t \cdot \log(\rho_{ti}) + \gamma_t. \quad (6.1)$$

The conditional mean $\mu_{ti} = E[Y_{ti} = 1|\gamma_t] = \pi_{ti}$ is linked to the linear predictor through

$$\log\left(\frac{\pi_{ti}}{1 - \pi_{ti}}\right) = \eta_{ti}. \quad (6.2)$$

Let $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)^T$ and $x_{ti} = (1, \text{CAPEp}_t, \text{AQ}_t, \log(\rho_{ti}), \text{CAPEp}_t \cdot \log(\rho_{ti}), \text{AQ}_t \cdot \log(\rho_{ti}))^T$. The marginal odds that a change was reported, $P(Y_{ti} = 1)$, can then be approximated by (3.34).

According to the model, for a given change log odds $\log(\rho_{ti})$ and AQ_t score of an individual t, increasing the CAPEp score with s units, changes the estimated odds that the individual reports a change with a factor of

$$\text{Factor change} = \exp(\beta_1 \cdot s) \cdot \exp(\beta_4 \cdot s \cdot \log(\rho_{ti})). \quad (6.3)$$

The corresponding marginal odds that a change is reported changes with a factor

$$\text{Factor change} = \exp\left(\frac{\hat{\beta}_1 \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right) \cdot \exp\left(\frac{\hat{\beta}_4 \cdot \log(\rho_{ti}) \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right) \quad (6.4)$$

when the CAPEp score changes with s units.

Furthermore, for a given change log odds $\log(\rho_{ti})$ and CAPEp_t score of an individual t, increasing the AQ score with s units, changes the odds that the individual reports a change with a factor

of

$$\text{Factor change} = \exp\left(\beta_2 \cdot s\right) \cdot \exp\left(\beta_5 \log(\rho_{ti}) \cdot s\right). \tag{6.5}$$

For a change of s units of AQ_t, the marginal odds that a change is reported, changes with

$$\text{Factor change} = \exp\left(\frac{\hat{\beta}_2 \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right) \cdot \exp\left(\frac{\hat{\beta}_5 \cdot \log(\rho_{ti}) \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right). \tag{6.6}$$

The model is fitted by using maximum likelihood estimation via the `glmmTMB()`-function from the glmmTMB-library in R [Brooks et al., 2017]. This function fits a glmm using maximum likelihood estimation(MLE) by TMB (Template model builder), while random effects are integrated out using Laplace approximation. The estimates of the fixed effects are denoted $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3,$ $\hat{\beta}_4$ and $\hat{\beta}_5$. The estimate of the random effects' variance is denoted $si\hat{g}ma_w^2$.

We are interested in whether the regression coefficients significantly differ from 0. We check this by doing z-tests. Assume that we are considering the coefficient β_e with a corresponding MLE estimate β_e. The null hypothesis is

$$H_0 : \beta_e = 0,$$

and the alternative hypothesis is

$$H_1 : \beta_e \neq 0.$$

The z score corresponding to β_e is given by

$$z = \frac{\hat{\beta}_e}{\text{SD}(\hat{\beta}_e)}.$$

The observed z-statistic z is considered a realization of the random variable Z, which under the null hypothesis is assumed $N(0,1)$-distributed. The corresponding p-value is $P(Z > |z|)$. With a p-value below 0.05, we consider the effect significant.

Table 6.1: Estimated conditional fixed effects for the binary random intercept logit model.

Conditional fixed effects

Term	Coefficient	Estimate	Std. Error	z value	Pr(>|z|)
Intercept	$\hat{\beta}_0$	−2.48214	0.37254	−6.66	$2.69 \cdot 10^{-11}$
CAPEp	$\hat{\beta}_1$	0.01680	0.00759	2.21	0.0269
AQ	$\hat{\beta}_2$	−0.00818	0.00520	−1.573	0.1157
change log odds	$\hat{\beta}_3$	0.40881	0.01454	28.12	$< 2 \cdot 10^{-16}$
CAPEp:change log odds	$\hat{\beta}_4$	−0.00498	0.00025	−19.67	$< 2 \cdot 10^{-16}$
change log odds:AQ	$\hat{\beta}_5$	−0.00052	0.00021	−2.45	0.0145

Table 6.2: Estimated variance and standard deviation of random effects in the binary random intercept logit model.

Random effects variances

	Groups	Name	Variance $\hat{\sigma}_w^2$	Std.Dev $\hat{\sigma}_w$
1	candidate	Intercept	0.8648	0.92993

6.2 Analysis of the results from the binary intercept model

The estimated regression coefficients and their corresponding standard deviation, z-score, and p-value are presented in Table 6.1. The estimated variance $\hat{\sigma}_w^2$ and standard deviation $\hat{\sigma}_w$ for the random effects can be seen in Table 6.2. We notice that all estimated coefficients except $\hat{\beta}_3$ are significantly different from 0.

We start by analyzing the effect of the log odds for different scores of AQ and CAPEp and compare the results with what we found in Section 5.3. To better understand what a change of unit in AQ and CAPEp score means, recall the empirical distributions of the scores of the participants in Figure 2.6. The AQ scores of the participants lie between 43 and 93 and the CAPEp scores lie between 20 and 58. Changing the scores by one unit is therefore a relatively small change. We also note that a change of one unit of AQ scores is relatively smaller compared to the range of AQ scores than the change of one unit in CAPEp is to the range of CAPEp scores. The negative $\hat{\beta}_4$-estimate means that increasing the score of CAPEp yields a more negative effect of the log odds on the change choice. This is consistent with what we found in Section 5.3 where both groups with a score over the cut-off values for CAPEp had lower estimated effects of the log odds. Furthermore, we saw that the L-CAPEp & H-AQ group had a slightly lower mean effect of the

log odds compared to the control group. However, the difference between the groups was not found to be significant. The negative $\hat{\beta}_5$-estimate tells that increasing the AQ score lowers the estimated effect of the change log odds. However, the $\hat{\beta}_4$-estimate is considerably lower than the $\hat{\beta}_5$-estimate, which means that the log odds effect is expected to decrease more when the CAPEp score is increased compared to when the AQ score in increased with the same number of units.

Recall that, the factor increment of the odds that a change is reported for a given change log odds $\log(\rho_{ti})$ and AQ_t-score of an individual t is given by (6.3) when CAPEp changes by s units. By inserting the estimates of the regression coefficients, we get

$$\text{Factor change} = \exp\left(0.01860 \cdot s\right) \cdot \exp\left(-0.00498 \cdot s \cdot \log(\rho_{ti})\right).$$

The factor change of marginal odds is given by (6.4) and by inserting the relevant estimated coefficients, we get

$$\text{Factor change} = \exp\left(\frac{\hat{\beta}_1 \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right) \cdot \exp\left(\frac{\hat{\beta}_4 \cdot \log(\rho_{ti}) \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right).$$

In Figure 6.1, we plot this factor change of conditional and marginal odds for different change log odds values and an increment of CAPEp score $s = 10$. The figure shows that for change log odds under 3.37, increasing the CAPEp score is expected to increase the odds that a change is reported. This corresponds to a change probability of 0.967. For values above this, increasing the CAPEp score is expected to decrease the probability that a change is reported. This is similar to what we found in Section 5.3. We found that the two groups with high scores of CAPEp had higher estimated odds of reporting a change when the change probabilities were below 0.567 and 0.648 for the H-CAPEp & L-AQ group and H-CAPEp & H-AQ group respectively. The findings in this section suggest that, according to the random intercept model, the tendency of inferring change in people with elevated scores of CAPEp is generally higher. Only when the probability that a change has occurred is over 0.967, CAPEp score has a negative correlation with reporting changes.
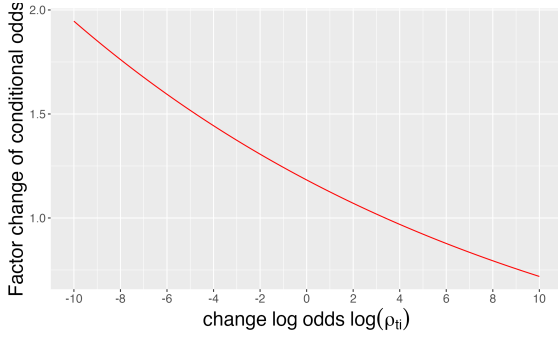
(a) Conditional odds for a given individual
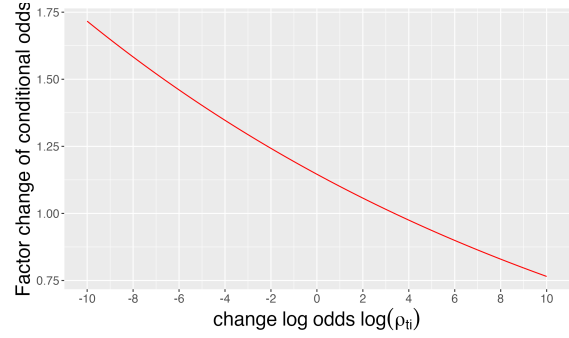
(b) Marginal odds

Figure 6.1: The factor the conditional and marginal odds changes with when the CAPEp increases with 10 unites for different values of change log odds $\log(\rho_{ti})$

Now, we consider an individual with a given CAPEp score and change log odds ρ_{ti}. Recall that changing the AQ score with s unites changes the log-odds that the individual reports a change with a factor of equal to the one in (6.6). Inserting the estimated regression coefficients yields

$$\text{Factor change} = \exp\left(-0.00818 \cdot s\right) \cdot \exp\left(-0.00052 \log(\rho_{ti}) \cdot s\right).$$

The factor change of marginal odds is estimated by

$$\text{Factor change} \approx \exp\left(\frac{-0.00818 \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right) \cdot \exp\left(\frac{-0.00052 \cdot \log(\rho_{ti}) \cdot s}{\sqrt{1 + 0.6 \cdot \sigma_w^2}}\right).$$

The factor change of the conditional and marginal odds when changing AQ by $s = 10$ for different change log odds is shown in Figure 6.2. For all change log odds between -10 and 10, an increase in AQ score is expected to lower the probability that a change is reported. These results are also similar to what we found for the L-CAPEp & H-AQ group in Section 5.3, where for most change log odds, the estimated odds that a change was reported was smaller for the L-CAPEp & H-AQ compared to the control group.

In addition to finding that the interactions effects are significant, the plots of the factor change of conditional and marginal odds suggest that these effects have a practical impact on the results. For example, we see that changing the CAPEp score by 10 is estimated to almost double the odds that a change is reported for very low change probabilities. Let us now consider a change
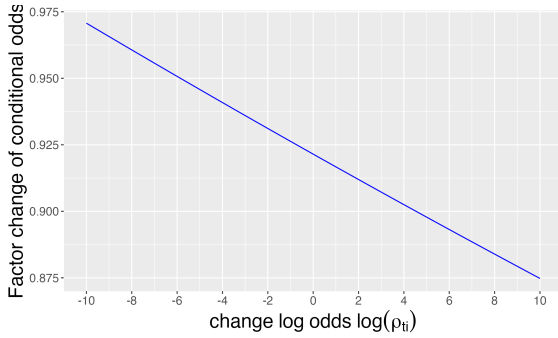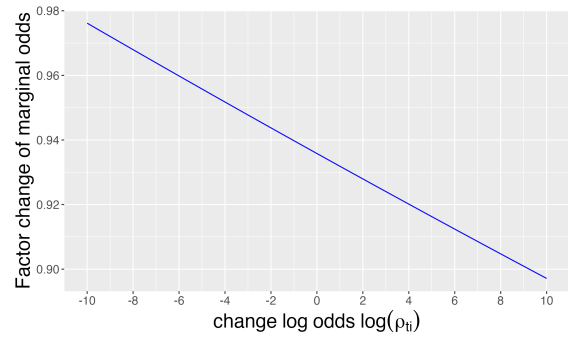
(a) Conditional odds for a given individual   (b) Marginal odds

Figure 6.2: The factor the conditional and marginal odds changes with when the AQ increases with 10 unites for different values of change log odds $\log(\rho_{ti})$

probability of 0.01, which means that the probability for a change is very low according to the Bayesian model. This probability corresponds to a change probability of approximately -2. If we consider a change of CAPEp from the minimum of 20 to the maximum of 58, which is a change of 38, the conditional odds that a change is reported, changes by an estimated factor of $\exp(0.01860 \cdot 38) \cdot \exp(-0.00498 \cdot 38 \cdot (-2)) = 2.96$. This means that the odds has almost tripled according to the model. To further showcase the practical meaning of the results, we consider the probability that a change is reported for different change probabilities p_{ti}, AQ scores and CAPEp scores. First we consider the probability $P(Y_{ti} = 1|\gamma_t = 0)$, which is the probability that a change is reported for a given individual with a mean random effect of γ_t. We can see these conditional probabilities for different CAPEp scores, AQ scores and change probabilities in Table 6.3. This table presents the calculated conditional probabilities that a change is reported, given a range of AQ and CAPEp scores. AQ scores are shown in the first column and range from 50 to 90, while CAPEp scores are displayed in the second column, and ranges from 20 to 50. Each row of the table represents a unique combination of AQ and CAPEp scores.

The rest of the column names indicate the change probabilities, specifically for p_{ti} values of 0.25, 0.5, 0.75, 0.9, and 0.99. The corresponding probabilities that a change is reported, given the specific AQ, CAPEp, and p_{ti} values, are displayed in the cells in the table. The equivalent table for the marginal probabilities $P(Y_{ti} = 1)$ is shown in Table 6.4. These tables show what we have discussed earlier. For change probabilities $p_{ti} \in \{0.25, 0.5, 0.75, 0.9\}$ which are all below 0.967, the probability that a change is reported increases when the CAPEp increases. For $p_{ti} = 0.99$, an

Table 6.3: Conditional probabilities $P(Y_{ti} = 1|\gamma_i)$ for different values of CAPEP, AQ and change probabilities p_{ti}.

		Conditional probabilities				
AQ	**CAPEp**	$p_{ti} =0.25$	$p_{ti}=0.5$	$p_{ti}=0.75$	$p_{ti} = 0.9$	$p_{ti} = 0.99$
50.00	20.00	0.05	0.07	0.10	0.13	0.22
50.00	30.00	0.07	0.08	0.11	0.13	0.21
50.00	40.00	0.08	0.10	0.12	0.14	0.20
50.00	50.00	0.10	0.11	0.13	0.15	0.19
60.00	20.00	0.05	0.07	0.09	0.12	0.20
60.00	30.00	0.06	0.08	0.10	0.12	0.19
60.00	40.00	0.08	0.09	0.11	0.13	0.19
60.00	50.00	0.09	0.11	0.12	0.14	0.18
70.00	20.00	0.05	0.06	0.08	0.11	0.19
70.00	30.00	0.06	0.07	0.09	0.11	0.18
70.00	40.00	0.07	0.08	0.10	0.12	0.17
70.00	50.00	0.09	0.10	0.11	0.13	0.16
80.00	20.00	0.04	0.06	0.08	0.10	0.17
80.00	30.00	0.05	0.07	0.08	0.10	0.16
80.00	40.00	0.07	0.08	0.09	0.11	0.16
80.00	50.00	0.08	0.09	0.12	0.15	
90.00	20.00	0.04	0.05	0.07	0.09	0.16
90.00	30.00	0.05	0.06	0.08	0.10	0.15
90.00	40.00	0.06	0.07	0.09	0.10	0.14
90.00	50.00	0.08	0.08	0.10	0.11	0.13

Table 6.4: Marginal probabilities $P(Y_{ti} = 1)$ for different values of CAPEp, AQ and change probabilities p_{ti}.

Marginal probabilities

AQ	CAPEp	$p_{ti} = 0.25$	$p_{ti} = 0.5$	$p_{ti} = 0.75$	$p_{ti}=0.9$	$p_{ti} = 0.99$
50.00	20.00	0.09	0.11	0.14	0.17	0.27
50.00	30.00	0.10	0.13	0.15	0.18	0.26
50.00	40.00	0.12	0.14	0.16	0.19	0.25
50.00	50.00	0.14	0.16	0.18	0.19	0.24
60.00	20.00	0.08	0.11	0.13	0.16	0.25
60.00	30.00	0.10	0.12	0.14	0.17	0.24
60.00	40.00	0.12	0.13	0.15	0.18	0.23
60.00	50.00	0.14	0.15	0.17	0.18	0.22
70.00	20.00	0.08	0.10	0.12	0.15	0.23
70.00	30.00	0.09	0.11	0.13	0.16	0.22
70.00	40.00	0.11	0.13	0.14	0.16	0.22
70.00	50.00	0.13	0.14	0.16	0.17	0.21
80.00	20.00	0.08	0.09	0.12	0.14	0.22
80.00	30.00	0.09	0.11	0.13	0.15	0.21
80.00	40.00	0.10	0.12	0.14	0.15	0.20
80.00	50.00	0.12	0.13	0.15	0.16	0.19
90.00	20.00	0.07	0.09	0.11	0.13	0.20
90.00	30.00	0.08	0.10	0.12	0.14	0.20
90.00	40.00	0.10	0.11	0.13	0.14	0.19
90.00	50.00	0.12	0.13	0.14	0.15	0.18

increment of CAPEp scores is associated with a decrease of probability that a change is reported according to the model. We also see that for all the change probabilities considered here, an increment of AQ is associated with a decrease of probability that a change is reported. This decrease is higher for high change probabilities.

7 | Closing remarks

In this thesis, we investigated whether individuals with either autistic or psychotic traits, or both, demonstrate a difference in their tendency to infer change in their environment compared to the general population. In particular, we focused on the hypothesis that those with autistic and/or psychotic traits require less evidence before concluding that a fundamental change has occurred.

To test our hypotheses, we used the 'bunny task', a digital task where participants classify bunnies into families based on their walking direction. The means of the walking directions represented the distinct families, and a change of family, that is, a change in mean walking direction, was the subject of interest. The change probabilities calculated from the Bayesian observer model were central to analyzing the participants' decision-making. The change probabilities aimed to quantify the perceived evidence for a change given the observed walking directions when the decision was made.

AQ and CAPEp scores were employed to measure the level of autistic and psychotic traits in the participants, respectively. Our analysis followed two main paths: a group-based comparison using cut-off scores for CAPEp and AQ, and a generalized mixed model treating CAPEp and AQ scores as continuous variables.

For the group-based analysis, participants were divided into groups based on cut-off scores of 65 for AQ and 28 for CAPEp. From this analysis, we found no significant differences between the L-CAPEp & H-AQ group and the control group. We did neither observe any significant differences between the H-CAPEp & H-AQ group and the control group. Interestingly, contrary to our main hypothesis, the L-CAPEp & H-AQ group had significantly higher change probabilities when changes were reported, which does not suggest that people in this group infer changes when the evidence of change is lower than the control group.

Furthermore, in the group-based analysis, we fitted logit models to each participant to assess participants' tendency to report changes in the bunny task. The logit model for a participant included the change log odds from the Bayesian observer model as an explanatory variable.

Groups with high CAPEp scores, the H-CAPEp & L-AQ group, and the H-CAPEP & H-AQ group, showed significantly lower effects from the change log odds, suggesting a less consistent correlation between increased odds and the reporting of change. However, for low change probabilities, the two groups with a high score of CAPEp were more inclined to report changes compared to the control group. People were less likely to report changes according to the fitted model for change probabilities exceeding 0.567 and 0.648 for the H-CAPEp & L-AQ group and H-CAPEP & H-AQ groups respectively. Meanwhile, the L-CAPEp & H-AQ group generally reported changes less frequently for most change probabilities.

The subsequent generalized mixed model(glmm) analysis, where CAPEp and AQ scores were treated as continuous explanatory variables, provided further insights. This model also had the binary change choices as responses, but we fitted one model for all participants with one random intercept corresponding to each individual. We found that, according to the fitted model, increasing the AQ and CAPEp scores were associated with a decreased effect of change log odds, aligning with our findings when we classified participants based on cut-off scores. Furthermore, the estimates from the fitted glmm indicated that with low change probabilities, increased CAPEp scores were related to a higher inclination to infer changes. This finding supported our main hypothesis and matched the results from the group-based analysis, where we saw that the two groups with high scores of CAPEp had significantly lower effects of change log odds. Furthermore, elevated AQ scores did not correlate with a higher tendency to infer changes, a finding consistent with the one in the group-analysis, and does not support the main hypothesis.

Therefore, the findings in this thesis indicate that individuals with psychotic traits tend to infer changes in situations where there is less evidence, a trend not observed in individuals with autistic traits who seemed to underreport changes.

However, there are limitations to the work done in this thesis. The model we used, the Bayesian observer model, assumes a specific perspective on the task, which might not be entirely reflective of how the participants perceive it. For future research, we recommend considering variations of the Bayesian observer model that could provide a more accurate depiction of the participants' points of view. One possibility could be to add uncertainty to the number of degrees the mean changes between families. The participants do not know the number of degrees

the mean changes with, and assuming this is known might be an oversimplification. All participants have a mean change between 10 and 80 degrees, and assuming a uniform continuous prior distribution between 10 and 180 for the mean change can be considered. In further research, considering an alternative Bayesian observer model that accommodates more of the participants' decisions could also be an alternative. Our model calculates the probability of a bunny belonging to a new family, given that the first bunny classified into the first family option on the screen was correctly classified. However, an alternative model could calculate the conditional probability that the latest bunny belongs to a new family under the assumption that all preceding bunnies were correctly classified. This model can potentially be more aligned with the participants' view, as the participant's preceding decisions determine the options displayed on the screen. Consequently, it is plausible that participants make subsequent decisions based on the assumption that their previous classifications were accurate.

Moreover, it could be beneficial to consider statistics focusing more on the lower end of the spectrum of change probabilities, especially since we are primarily interested in participants' decision-making in more stable contexts.

Another potential improvement could be to differentiate between the results from the task's low volatility and high volatility blocks. The family sizes are larger in the low volatility block, and changes occur less frequently. Investigating whether participants with high AQ or CAPEp scores are more inclined to report changes in these more stable environments would be interesting.

To summarize, our findings reveal compelling differences in how individuals with psychotic traits perceive changes in their environment. These results provide an important step towards a better understanding of these traits and suggest areas for future research. However, the lack of a similar pattern in individuals with autistic traits, who instead seem to underreport changes, illustrates the complex nature of these traits and the human decision-making process.

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