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Possible Expansion of Blood Vessels by Means of the Electrostrictive Effect

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Abstract: In cases when it is desirable to transport medication through blood vessels, especially when dealing with brain cancer being confronted with the narrow arteries in the brain, the blood–brain barrier makes medical treatment difficult. There is a need of expanding the diameters of the arteries in order to facilitate the transport of medications. Recent research has pointed to various ways to improve this situation; in particular, the use an ultrasound acting on microbubbles in the blood stream has turned out to be a promising option. Here, a different possibility of enlarging the diameters of arteries is discussed, namely to exploit the electrostrictive pressure produced by internal strong, ultrashort and repetitive laser pulses. Each pulse will at first give rise to inward-directed optical forces, and once the pulse terminates, there will be a hydrodynamical bouncing flow in the outward radial direction, giving an outward impulse to the vessel wall. In the absence of friction, a symmetric oscillation picture emerges. Clearly, a supply of repetitive pulses will be needed (at a parametric resonance) to make the effect appreciable. The effect has, to our knowledge, not been discussed before. We give an approximate optical and hydrodynamical theory of it. The calculations indicate promising results for the wall pressure, although experimental work is desirable to demonstrate whether the idea can be useful in practice. Our calculation is made from a general physical perspective that is not necessarily linked to medical applications.

Keywords: electrostriction; optofluidics; medical technology; brain cancer

1. Introduction

As is known, symmetry properties often play a dominant role in fluid mechanical systems—cf., for instance, the collection of papers discussing such themes in [1]. Here, we will consider a situation which combines classical laser physics with hydromechanics: Assume that a long liquid-filled circular tube is exposed to one or more longitudinal short laser pulses. If the fluid is compressive, each pulse will give rise to a radial electrostrictive force in the inward direction. When the pulse has left, the fluid rebounds, producing in turn an outward hydrodynamic pressure on the wall. Such a force may be of physical interest. If the fluid is nonviscous, there results a stationary symmetric oscillatory wave situation.

The physical motivation for the present study actually came from a well known problem in medical technology. The human brain has a filter that protects it as well as the rest of our nervous system from foreign elements that can damage the tissue. This is the blood–brain barrier, and is normally a useful and vital organ. A problem arises, however, in the case of diseases when one wants to make medication penetrate the barrier. This filter problem is accentuated in the case of brain tumours. Recent research has indicated that there are promising measures that can be taken to overcome the problem, such as to inject microbubbles into the blood, and thereafter expose the brain to a focused ultrasound; cf., for instance, Refs. [2,3]. This "Acoustic Cluster Therapy" (ACT), as it is called, is an ingenious method; the ultrasonic waves transfer energy to the microbubbles, which in turn merge into a single large microbubble, effectively opening the blood–brain barrier.

Some general remarks: the blood–brain barrier (BBB), as a semi-permeable membranous barrier, is located at the interface between the blood and the cerebral tissue. It is

composed of a complex system of endothelial and perivascular mast cells, and is responsible for controlling the exchanges between two compartments, allowing only certain molecules or ions to pass through. In the process of drug delivery, only lipid-soluble molecules with a low weight and of a positive charge can cross the BBB. Other molecules require carrier-mediated transport systems. Some strategies consist of the injection of therapeutic proteins directly into the cerebrospinal fluid. A promising approach seems to be the use of lipid- and polymer-based nanosized particles that assure a controlled release of their cargo by protecting loaded drugs from being metabolized. A useful review article on crossing the blood–brain barrier can be found in Ref. [4].

Among other methods, we mention how a magnetically driven soft continuum micro-robot can be used for intravascular operations in a microscale [5]. Whereas conventional continuum robots have the miniaturization challenge, this paper reviews a microscale soft continuum robot with steering and locomotion capabilities based on magnetic field actuation. In Ref. [6], a new approach is discussed about how the combination of a FET biosensor with a rolled-up microtube can be used to develop a microfluidic diagnostic biosensor. Finally, in Ref. [7], the BBB is discussed in connection with patients having Alzheimer's disease; the use of MR-guided focused ultrasound turned out to be a promising avenue experimentally.

In this paper, we will discuss a different approach to the enlargement problem. As already mentioned, to our knowledge it has not been considered before. Assume that an ultrashort strong laser pulse is sent longitudinally in the z direction through the blood vessel. As blood is a compressible liquid, there will be electrostrictive radial forces compressing the liquid inward, as long as the pulse lasts. From a hydrodynamic point of view, this compressive event can be regarded to be instantaneous. When the pulse has left, the hydrodynamic response causes the fluid to move radially inwards, and build up a maximum pressure around the symmetry axis. Assuming as an example that the pulse has a strong power of 3 kW, the excess hydrodynamic pressure on the axis will be quite large, about one atmosphere. After this, a hydrodynamic rebound occurs, causing the fluid to move radially outwards and transfer an outward momentum to the vessel wall. The liquid then flows back, and becomes compressed near the centerline $r = 0$ again. At first, we will neglect the shear viscosity, so that a hydrodynamic stationary wave pattern becomes established. To make the effect big enough in practice, it is clear that a large number N of successive laser pulses per second will be needed. This is actually an example of parametric resonance: every new pulse is imaged to be sent through the system during the phase when the flow is running inwards. If one, as the next point, takes into account viscosity, it turns out that the viscous damping will not have time to make any appreciable influence on the first rapid hydrodynamic oscillations at all. For longer time spans, the viscosity will naturally be important.

Note: we have here assumed that the compressed fluid is temporarily prevented from leaking out in the longitudinal direction. That is, there must be an effective blockade against longitudinal fluid motion at the instant just after the onset of the pulse. This seems to be a reasonable assumption for these short times, in view if the many sharp turns of the artery system. For longer times, of course, the fluid is free to move longitudinally.

The method described is conceptually simple, and our calculations below indicate that the electrostrictive forces may give rise to an effect of noticeable magnitude. To describe the physics of the opto-electromagnetic process accurately, would, however, seem difficult. To obtain a realistic knowledge about the usefulness of the method in practice, one needs experimental investigations, preferably at larger dimensions than those envisaged here.

For readers interested in further works about the electrostrictive effect considered from various perspectives, we list a number of papers (not meant to be exhaustive): Refs. [8–24].

2. The Electrostrictive Force Density

The basic expression for the electrostrictive electromagnetic force density is [25,26]

$$\mathbf{f} = \frac{1}{2}\varepsilon_0 \nabla \left(E^2 \rho_m \frac{d\varepsilon}{d\rho_m} \right), \tag{1}$$

where $\varepsilon = n^2$ with n the refractive index, and ρ_m is the density of the fluid. We assume the medium to be nonmagnetic. The electric field E refers to the rms-value. Strictly speaking, the equation above is referred to as a nonpolar medium, so that ε is a function of ρ_m only, but the equation is also approximately valid for many other media. We will, moreover, make use of the Clausius–Mossotti relation. This implies that if we, in general, write the force density as

$$\mathbf{f} = \nabla\chi, \tag{2}$$

we obtain the following explicit expression for the scalar χ,

$$\chi = \frac{1}{6}\varepsilon_0 E^2(\varepsilon - 1)(\varepsilon + 2). \tag{3}$$

Consider now the intensity of the incident laser pulse, which is taken to be constant during the time t_0 the pulse lasts. With cylindrical coordinates $\mathbf{r} = (\rho, \theta, z)$, we shall assume the form

$$E^2(\mathbf{r}, t) = E^2(\mathbf{r})T(t), \tag{4}$$

where $T(t)$ is the temporal function of the pulse,

$$T(t) = \Theta(t) - \Theta(t - t_0), \tag{5}$$

$\Theta(t)$ being the step function, equal to +1 when $t > 0$ and 0 when $t < 0$. For the spatial variation of the quadratic electric field, we adopt the Gaussian form

$$E^2(\mathbf{r}) = \frac{2P}{\pi\varepsilon_0 n c w^2(z)} \exp\left[-\frac{2\rho^2}{w^2(z)} \right], \tag{6}$$

where P is the power of the pulse, and $w(z)$ is the beam radius

$$w(z) = w_0\sqrt{1 + z^2/l_R^2}, \tag{7}$$

with w_0 being the minimum radius at the waist. The length $l_R = \pi w_0^2/\lambda$ is called the Rayleigh length, where λ is the wavelength in the medium. We will focus on the region near the center of the beam (taken to be $z = 0$), assuming $z/l_R \ll 1$, so that $w \approx w_0$. It means neglect of the convergence of the beam, and complies with the restriction $\lambda/w_0 \ll 1$.

The approximate form of the radially dependent part $\chi(\rho)$ of the scalar $\chi(\rho, t)$, thus becomes

$$\chi(\rho) = \chi(0) \exp\left[-\frac{2\rho^2}{w_0^2} \right], \tag{8}$$

where the value on the centerline

$$\chi(0) = \frac{(\varepsilon - 1)(\varepsilon + 2)P}{3\pi n c w_0^2} \tag{9}$$

is a constant. The full, time-dependent, electrostrictive force density is

$$\mathbf{f}(\rho, t) = \chi(0)T(t)\frac{d}{d\rho}\left[e^{-2\rho^2/w_0^2} \right]\hat{\mathbf{æ}}. \tag{10}$$

The expression is negative, corresponding to a compressive radial force. The force is constant in time, as long as the pulse lasts.

3. Hydrodynamic Analysis of the Fluid Motion—The Build-Up Phase

There are clearly two widely separated time scales in this problem. The electrodynamic scale is determined by the light velocity c (for definiteness taken in vacuum), while the hydrodynamic scale is determined by the sound velocity u. Hydrodynamically, the pulse of duration t_0 can be regarded as an instantaneous flash, transferring an impulse to the fluid. We will first write down the general governing equations for the fluid motion, neglecting the viscosity. Let $\mathbf{V}$ be the fluid velocity, and start from the equation of continuity,

$$\partial_t \rho_m + \nabla \cdot (\rho_m \mathbf{V}) = 0. \tag{11}$$

A linear expansion to the first order, $\rho_m = \rho_{m0} + \rho'_m$, $p = p_0 + p'$, with $\rho'_m \ll \rho_{m0}$, $p' \ll p_0$, then yields the linearized version

$$\partial_t \rho'_m + \rho_{m0} \nabla \cdot \mathbf{V} = 0. \tag{12}$$

Next, consider the Euler equation when the nonlinear convective term is neglected,

$$\rho_m \partial_t \mathbf{V} = -\nabla p + \nabla \chi. \tag{13}$$

In a linearized form, it reads

$$\rho_{m0} \partial_t \mathbf{V} = -\nabla p' + \nabla \chi. \tag{14}$$

From this the sound velocity, u follows naturally via the equation

$$p' = \left(\frac{\partial p}{\partial \rho_{m0}} \right)_S \rho' = u^2 \rho'. \tag{15}$$

This equation is used to replace the ρ'_m term in the continuity Equation (12). [A note on the time scales: after imposition of the pulse, the time required to reach the state of maximum pressure is of order w_0/u. With w_0, a few μm and $u = 1500 \text{ ms}^{-1}$, the building-up time becomes of order ns. This short time makes it natural to apply the subscript S (constant entropy) in Equation (15)].

As the fluid is so far assumed nonviscous, we can introduce the velocity potential Φ,

$$\mathbf{V} = \nabla \Phi. \tag{16}$$

Introducing this into the Euler Equation (13), we obtain, after the removal of the nabla operator,

$$p' = -\rho_{m0} \partial_t \Phi + \chi. \tag{17}$$

Differentiating this with respect to t and comparing it with the continuity equation, we obtain the general governing equation for Φ,

$$\nabla^2 \Phi(\rho, t) - \frac{1}{u^2} \partial_t^2 \Phi(\rho, t) = -\frac{1}{\rho_m u^2} \partial_t \chi(\rho, t), \tag{18}$$

in which we have replaced ρ_{m0} with ρ_m.

Now consider the case where the time function $T(t)$ has the form of Equation (5), from which we derive

$$\partial_t \chi(\rho, t) = \chi(\rho)[\delta(t) - \delta(t - t_0)]. \tag{19}$$

Equation (18) can be solved using Green function methods. We shall do that here, although, as a simplifying element, we shall assume that there is an infinite fluid around the laser beam. To include the finite width of the blood vessel would be quite complicated,

and not justified by the approximate calculation here given. Our simplified equation for the Green function becomes thus

$$(\nabla^2 - u^{-2}\partial_t^2)G(\mathbf{r}, \mathbf{r}', t, t') = \delta(\mathbf{r}' - \mathbf{r})\delta(t' - t), \tag{20}$$

which has the solution

$$G(\mathbf{r}, \mathbf{r}', t, t') = -\frac{1}{4\pi|\mathbf{r}' - \mathbf{r}|}\delta\left(t' - t + \frac{|\mathbf{r}' - \mathbf{r}|}{u}\right). \tag{21}$$

Then, the velocity potential can be written as

$$\Phi(\rho, t) = -\frac{1}{\rho_m u^2}\int_{-\infty}^{t} dt' \int d^3 r' G(\mathbf{r}, \mathbf{r}', t, t')\partial_t \chi(r', t'). \tag{22}$$

It is now convenient to introduce a nondimensional length ξ and a nondimensional time τ,

$$\xi = \frac{\sqrt{2}\rho}{w_0}, \quad \tau = \frac{\sqrt{2}\,ut}{w_0}. \tag{23}$$

Under the present conditions, they are both of order unity. Then, the velocity potential can be written as [20]

$$\Phi(\rho, t) = \frac{\chi(0)w_0}{\sqrt{2}\,u\rho_m}\left[\tilde{\Phi}(\xi, t) - \tilde{\Phi}(\xi, \tau - \tau_0)\right], \tag{24}$$

$$\tilde{\Phi}(\xi, \tau) = \Theta(\tau)e^{-\xi^2 - \tau^2}\int_0^\tau dx I_0(2\xi\sqrt{\tau^2 - x^2})e^{x^2}, \tag{25}$$

where I_0 is the modified Bessel function of the first kind of order zero. The pressure becomes

$$p(\xi, \tau) = \chi(0)[P(\xi, \tau) - P(\xi, \tau - \tau_0)], \tag{26}$$

$$P(\xi, \tau) = 2\tau e^{-\xi^2}\Theta(\tau)\int_0^\tau \frac{x dx e^{-x^2}}{\sqrt{\tau^2 - x^2}}\left[I_0(2\xi x) - \frac{\xi}{x}I_1(2\xi x)\right]. \tag{27}$$

Of special interest is the build-up of pressure on the symmetry axis, $\xi = 0$. Then, the formulas above reduce to

$$\tilde{\Phi}(0, \tau) = \Theta(\tau)F(\tau), \tag{28}$$

where

$$F(x) = e^{-x^2}\int_0^x e^{t^2}dt \tag{29}$$

is the Dawson's integral, and

$$p(0, \tau) = 2\chi(0)[\Theta(\tau)\tau F(\tau) - \Theta(\tau - \tau_0)(\tau - \tau_0)F(\tau - \tau_0)]. \tag{30}$$

The numerically calculated Fig. 4 in Ref. [20] shows for the typical input data how the pressure nicely builds on the axis, almost linearly, until a maximum, somewhat in excess of $\chi(0)$, is reached around the time it takes for sound to traverse the laser beam. For practical purposes, we may say that

$$p_{\max} = \alpha\chi(0), \quad \alpha \approx 1.25. \tag{31}$$

After this maximum, the pressure decreases slowly as long as the pulse lasts, and for $\tau > \tau_0$, it decreases quickly.

Our next task will be to calculate the elastic energy in the fluid, imparted by the pulse. Only the first build-up phase will then be of interest. We take the pulse to start at $t = 0$, and follow it until a maximum pressure is established. We start from the velocity potential given in Equation (25), but make the simplifying assumption that the argument $2\xi\sqrt{\tau^2 - x^2}$ in I_0 is small. As this Bessel function increases only slowly for small arguments,

$I_0(z) \approx 1 + \frac{1}{4}z^2$, this restriction is not very strong. In the lower limit $x = 0$, the argument is $2\xi\tau$, whereas in the upper limit $x = \tau$, the argument is zero. One may estimate that the main contribution to the integral occurs when $ut \sim w_0/2$, i.e., $\xi \sim 1/\sqrt{2}$, and at the position $\rho \sim w_0/2$, i.e., $\xi \sim 1/\sqrt{2}$, what corresponds to $2\xi\tau \sim 1$ at most. To the present accuracy, the approximation is acceptable.

In this way, we obtain an expression for the velocity potential in which Dawson's integral is the main ingredient,

$$\tilde{\Phi}(\xi, \tau) = e^{-\xi^2}F(\tau), \tag{32}$$

which gives the following expression for the dimensional radial fluid velocity,

$$V(\rho, t) = \frac{d\Phi(\rho, t)}{d\rho} = \frac{\chi(0)}{u\rho_m}F(\tau)\frac{d}{d\xi}e^{-\xi^2}. \tag{33}$$

Similarly, the radial component of the electrostrictive force density becomes

$$f(\rho) = \chi(0)\frac{\sqrt{2}}{w_0}\frac{d}{d\xi}e^{-\xi^2}. \tag{34}$$

It is worth noticing that whereas $V(\rho, t)$ depends on time, $f(\rho)$ does not. The latter behavior is related to the constancy of the pulse as long as it acts.

The produced elastic energy E_{elastic} (per unit length) in the fluid is equal to the work conducted by the inward directed electrostrictive forces,

$$E_{\text{elastic}} = \int 2\pi\rho d\rho f(\rho)V(\rho, t)dt. \tag{35}$$

Making use of Equations (33) and (34), we obtain, after some calculation

$$E_{\text{elastic}} = \frac{\pi w_0^2}{2\rho_m u^2}\chi^2(0)\int_0^{\tau_{\max}}F(\tau)d\tau, \tag{36}$$

where $p_{\max}$ is given in Equation (31).

We now turn to some numerical estimates. Our main interest is the small arteries; as informed in the review article [27], these arteries may have an outer diameter of about 1 mm. With a wall thickness of about 200 μm, this gives an inner radius a of a few hundred μm. For calculational convenience, we will choose the low value 100 μm in the following. This will also enhance the pressure effect. The refractive index for blood is 1.37; the density is 10^3 kg m^{-3} as for water, and for the power P of the pulse, we adopt the same value 3 kW as used by Ashkin and Dziedzic in their classic radiation pressure experiment on water [28] (this value was reported to lie below the region of nonlinearity). For the beam radius at the waist, we choose 2 μm. To summarize,

$$a = 100 \ \mu\text{m}, \quad n = 1.37, \quad P = 3 \text{ kW}, \quad w_0 = 2 \ \mu\text{m}. \tag{37}$$

A remark on the nondimensional time τ: as defined in Equation (23), this parameter will take a convenient value of order unity when $t = w_0/u$, which is the hydrodynamically important scale. This follows directly from $\tau = \sqrt{2}t/(w_0/u) \sim 1$ at that instant.

We can now calculate, from Equation (9),

$$\chi(0) = 4.9 \times 10^5 \text{ Pa}, \tag{38}$$

which is quite a high pressure, at about 5 atmospheres. The elastic energy evaluates to

$$E_{\text{elastic}} = 6.6 \times 10^{-10}\int_0^{\tau_{\max}}F(\tau)d\tau. \tag{39}$$

The function $F(\tau)$ has a maximum of 0.541 at $\tau = 0.924$. The integral (39) is divergent for high arguments, but we have made a simple numerical evaluation of it assuming the upper limit $\tau_{\max} = 2$, which seems to be reasonable. The result was approximately 0.8, and so we finally obtain

$$E_{\text{elastic}} = 5.3 \times 10^{-10} \text{ Jm}^{-1}. \tag{40}$$

This is thus the energy imparted to the fluid from one single pulse.

4. Hydrodynamic Analysis: The Nonviscous Oscillating Phase

We now analyse the fluid motion after the pulse has left. To begin with, defined hereafter as $t = 0$, the fluid is at rest at maximum pressure, and for $t > 0$, it rebounds radially outwards, giving rise to a radial oscillating wave pattern. As long as the viscosity is ignored, the pattern will be stationary. When reflected against the outer wall, the fluid imparts an outward force on it. This is the force we intend to calculate. At first, we consider the influence from one single pulse only.

The governing equation for the velocity potential is now Equation (18), with the right hand side equal to zero,

$$\left(\partial_\rho^2 + \frac{1}{\rho} \partial_\rho - \frac{1}{u^2} \partial_t^2 \right) \Phi(\rho, t) = 0. \tag{41}$$

We will write the solution as

$$\Phi(\rho, t) = -A J_0(k\rho) \sin \omega t, \quad k = \omega / u, \tag{42}$$

with J_0 being the ordinary Bessel function of order zero and A a positive constant. The radial velocity, which is positive to begin with, is thus

$$V(\rho, t) = \partial_\rho \Phi(\rho, t) = k A J_1(k\rho) \sin \omega t. \tag{43}$$

The kinematic condition $J_1(ka) = 0$ at the wall (the radial movement of the wall negligible in this context) determines the allowable values of the wave number k. We shall consider the lowest mode only,

$$ka = 3.83, \tag{44}$$

which corresponds to the wavelength $\lambda = 2\pi / k = 1.64\,a$ of the fluid oscillations. The angular frequency becomes

$$\omega = \frac{5.75}{a} \times 10^3 \text{ rad s}^{-1}, \tag{45}$$

which means a frequency of $\omega / 2\pi \approx 10$ MHz when $a = 100$ μm.

We now calculate the kinetic energy E_{kin}, exploiting the useful general property of any linearly varying mechanical system, namely that its total mean energy is evenly divided between the kinetic and potential energy. We thus have simply $E_{\text{kin}} = \frac{1}{2} E_{\text{elastic}}$, where the latter quantity was found in the previous section. We start from the general expression,

$$E_{\text{kin}} = \int_0^a \frac{1}{2} \rho_m V^2 2\pi \rho d\rho = \pi \rho_m A^2 \sin^2 \omega t \int_0^{ka} x J_1^2(x) dx, \tag{46}$$

take the time average over a period, and make use of the following formula [29],

$$\int x J_\mu^2(x) dx = \frac{1}{2} x^2 [J_\mu^2(x) - J_{\mu-1}(x) J_{\mu+1}(x)] \tag{47}$$

to obtain

$$E_{\text{kin}} = \frac{1}{4} \pi \rho_m A^2 (ka)^2 |J_0(ka) J_2(ka)|. \tag{48}$$

Since this is equal to one half of the total energy E_{elastic} given in Equation (36), we have now the opportunity to determine the value of the constant A,

$$A = \frac{w_0 \chi(0)}{\rho_m u k a} \left| \frac{\int_0^{\tau_{\max}} F(\tau) d\tau}{J_0(ka) J_2(ka)} \right|^{1/2}. \tag{49}$$

Inserting $w_0 = 2$ μm, $\chi(0) = 4.9 \times 10^5$ Pa, $J_0(ka) = -0.40$, $J_2(ka) = +0.40$ for $ka = 3.83$, and adopting the same value 0.80 for the τ integral as above, we obtain

$$A = 3.8 \times 10^{-7} \text{ m}^2\text{s}^{-1}. \tag{50}$$

The radial fluid velocity follows from Equation (43). As an example, for an intermediate point $k\rho = 1$, we find $V = (0.64 \sin \omega t) \text{ cm s}^{-1}$. The fluid velocities are moderate. Although the ratio between the angular frequency and the wave number for the fluid wave is fixed by the velocity of sound, $\omega/k = u$, the magnitude of V is much smaller than u because of the weakness of the velocity potential Φ.

It ought to be recalled that, instead of using the expression (48), one might equally well calculate E_{kin} directly from Equation (36),

$$E_{\text{kin}} = \frac{\pi w_0^2}{4\rho_m u^2} \chi^2(0) \int_0^{\tau_{\max}} F(\tau) d\tau. \tag{51}$$

Now, we turn to the pressure. We are concerned only with the dynamic pressure $p' = -\rho_m \partial_t \Phi$. (The physiological range for blood pressure is between 80 and 120 mm Hg, where 1 mm Hg = 133 Pa, and plays, in our context, the role of the background pressure, p_0.) For simplicity, we omit hereafter the prime on p. From Equation (42), we have thus

$$p(\rho, t) = \rho_m A \omega J_0(k\rho) \cos \omega t. \tag{52}$$

As $J_0(ka)$ is negative, the pressure $p(a)$ on the wall becomes positive in the half-period $\pi/2 < \omega t < 3\pi/2$. It is this pressure that we wish to maximize. To make the effect appreciable, it is clear that one has to make use of a succession of short pulses. The maximum case is when one pulse is sent through the system each oscillation period, during the part of the cycle where the fluid is moving inwards, corresponding to the $\pi < \omega t < 2\pi$ above. Then, each new pulse serves to increase the inward velocity. Physically, this is a parametric resonance effect.

Ideally, we shall assume that by sending in N pulses per second, the pressure becomes enhanced by a factor N. Moreover, as another approximation, we shall confine ourselves to calculating the root-mean-square (rms) pressure, called $\langle p \rangle_{\text{rms}}$. The mean pressure at the wall becomes, accordingly

$$\langle p(a) \rangle_{\text{rms}} = \frac{N\rho_m}{\sqrt{2}} A\omega |J_0(ka)|. \tag{53}$$

To obtain information about what influence this pressure has on the wall, one has to take into account its Young's modulus. Before touching on this, we shall estimate the influence from viscosity on the oscillating fluid system.

5. Viscous Damping of the Oscillating Fluid

The two viscosity coefficients of the fluid—shear viscosity η and bulk viscosity ζ—cause the oscillating motion to be damped out. For simplicity, we consider the oscillating stationary wave picture from the previous section as our starting point, and will neglect the bulk viscosity, as its magnitude seems to be largely unknown for blood. The measured values of η lie roughly in the interval from 2 to 5×10^{-3} Pa s. For definiteness, we adopt the value

$$\eta = 2 \times 10^{-3} \text{ Pa s}. \tag{54}$$

Consider now the time derivative $\dot{E}_{\text{kin}}$ of the kinetic energy [30],

$$\dot{E}_{\text{kin}} = -\frac{1}{2}\eta \int [\partial_k V_i + \partial_i V_k - \frac{2}{3}\delta_{ik}(\nabla \cdot \mathbf{V})^2]^2 d^3r. \tag{55}$$

We may calculate this expression by first employing Cartesian coordinates, as given in the expression above, and thereafter transfer the result to cylindrical coordinates. If we assume a one-dimensional harmonic motion along the x axis, with velocity $V = V(x,t)$, we obtain, after some manipulations,

$$\dot{E}_{\text{kin}} = -\frac{4}{3}\eta \int (\partial_x V)^2 d^3r. \tag{56}$$

Now replace $(\partial_x V)$ with $\nabla \cdot \mathbf{V} = \frac{1}{\rho}\frac{d}{d\rho}(\rho V)$ in cylindrical coordinates, and insert $V(\rho,t)$ from Equation (43) to calculate

$$\dot{E}_{\text{kin}} = -(\frac{2}{3}\eta)\pi k^2 A^2 \int_0^{ka} \frac{dx}{x}\left[\frac{d}{dx}(xJ_1(x))\right]^2, \tag{57}$$

where we have averaged over one period. Now make use of Eqsuations 10.6(ii) and 10.22.7 in Ref. [29],

$$\left(\frac{1}{x}\frac{d}{dx}\right)^k (x^\nu J_\nu(x)) = x^{\nu-k}J_{\nu-k}(x), \tag{58}$$

$$\int x^{2\mu+1}J_\mu^2(x)dx = \frac{x^{2\mu+2}}{2(2\mu+1)}[J_\mu^2(x) + J_{\mu+1}^2(x)], \quad 2\mu \neq -1, \tag{59}$$

to obtain

$$\dot{E}_{\text{kin}} = -\frac{2}{3}\eta\pi(kA)^2(ka)^2 J_0^2(ka). \tag{60}$$

We want to find how much of the initial kinetic energy E_{kin} becomes dissipated into heat per second. Calling the fraction γ we have, using the expression (48) for E_{kin},

$$\gamma = \left|\frac{\dot{E}_{\text{kin}}}{E_{\text{kin}}}\right| = \frac{8\eta}{3}\frac{k^2}{\rho_m}\left|\frac{J_0(ka)}{J_2(ka)}\right|. \tag{61}$$

With η as given above, and with $a = 10^{-4}$ m, $ka = 3.83$, this gives

$$\gamma = 7.8 \times 10^3 \text{ s}^{-1}. \tag{62}$$

This large number may at first appear to be a difficulty for the present theory, but here one needs to consider how the numbers appear in the physical process: As observed above, the maximum pressure on the symmetry axis is established a very short time after the onset of the laser pulse, $\tau \sim 1$ or $t \sim 1$ ns. The fluid rebounds immediately, and the return wave propagates with velocity u and reaches the wall in about 100 ns. If we assume as a simple model that the decay of kinetic energy occurs exponentially, $E_{\text{kin}} \propto e^{-\gamma t}$, we observe that the kinetic energy has by then diminished only by the fraction

$$\left|\frac{\Delta E_{\text{kin}}(100 \text{ ns})}{E_{\text{kin}}}\right| \sim 0.1 \text{ \%}. \tag{63}$$

There is thus ample time for the wave to rebound and strike upon the wall, before the viscosity begins to be appreciable. Actually, the viscosity is a helpful factor here, because in the long run it reduces the amplitude of the contracting phases of the wall. This was in fact the physical idea behind the introduction of the rms pressure in Equation (53). Namely, for an ideal nonviscous fluid, a pulsating force on the wall without any damping would have given rise to an oscillating behavior of the wall only, around its equilibrium position, and thus not provide a net outward force.

6. Mean Deformation of the Wall

Now, return to the expression (53) for the mean pressure on the artery wall, having observed that it is roughly justified to omit the influence from viscosity. The maximum case, as mentioned above, is that one pulse is sent in synchronously with the natural oscillations of the fluid. To estimate how much the wall becomes expanded by the repetitive bouncing forces, we must know the approximate value of Young's modulus Y for the arteries. For most condensed bodies, Y is known to lie in the GPa region. In the present case, however, the actual values are considerably smaller, in the MPa region. Taking the internal mammary artery as an example, we may use data from Refs. [27,31] to choose

$$Y = 10 \text{ MPa}. \tag{64}$$

Thus, using the expression (53) for the mean pressure $\langle p(a) \rangle_{\text{rms}}$, we may write

$$\frac{N\rho_m}{\sqrt{2}} A\omega |J_0(ka)| = Y \frac{\Delta a}{a}, \tag{65}$$

where Δa means a small or moderate expansion of the radius a. Insertion of the number given above leads to the relationship

$$N \sim 10^3 \frac{\Delta a}{a}. \tag{66}$$

This is actually a promising number in our context. It means, for instance, that in order to make the radius to expand by a fraction of 0.1, we need a moderate number of about 100 successive laser pulses per second. This is much less than the theoretical optical limit which is one pulse per fluid oscillation frequency, which is in the MHz region, as we have observed (Equation (45)). At least in principle, the effect seems to be appreciable.

7. Summary

In this paper, we have gone through the following items, and can summarize them as follows:

(1) The basic idea behind the above analysis was to investigate whether a rapid succession of short laser pulses sent longitudinally through a long cylindrical tube of radius a is able to expand the radius to some extent, via the electrostrictive effect. The possibility of enlargement is of great interest in medical applications, and for that reason we used in our analysis a typical radius of $a = 100$ μm for a small blood artery. There may evidently be other technological applications also, and in most cases one would find it natural to assume larger dimensions than those used here.

(2) In Section 3, we used Green function methods to describe the build-up of an inner hydrodynamic pressure in the central region, caused by one single laser pulse. The important scale in this initial part of the process is given by the velocity u of sound. It turns out that the pressure is established very quickly, when the nondimensional time parameter τ defined in Equation (23) is of the order unity.

(3) In Section 4, we analyzed the hydrodynamical problem with transverse oscillations in the tube after the laser pulse had left, and derived an expression for the kinetic energy per unit length, as well as the rms pressure on the wall.

(4) The shear viscosity in the fluid was taken into account in Section 5. The effect of the viscosity is to damp the hydrodynamic energy in general, although we found that the rebound of the fluid after the initial compression takes so short a time that the damping effectively does not come into play.

(5) As shown in Equation (66), the number N of successive laser pulses necessary to make an appreciable enlargement of the tube is moderate. So far, the results are promising.

(6) What can be said about future developments of this idea? Obviously, our analysis above is crude, and we should point out that the heating effects have not been

discussed at all. To the author, it seems most natural that the proposed effect could be tested experimentally, preferably under larger geometric dimensions than those assumed in the biomedical case above. It is clear that the effect physically exists; what is unclear is its magnitude in practice.

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