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Data-Driven Optimization of Multiple Gas-Lifted Well by Using Extremum Seeking Control

Master’s thesis in Petroleum Engineering
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Abstract

This thesis explores the application of Extremum Seeking Control (ESC) for optimizing gas lift operations in the oil and gas industry. Gas lift is a widely used artificial lift method that enhances the production rate of oil wells by injecting gas into the wellbore to reduce hydrostatic pressure and increase fluid flow. However, at a certain point, the more gas injected into the well, the less oil will get, in which the well will be dominated by frictional pressure loss instead of hydrostatic pressure loss.

This research aims to develop an optimization framework using extremum-seeking control algorithms to continuously adapt gas lift parameters in real-time and maximize well productivity in multi-wells. Extremum-seeking control is a feedback-based optimization technique that iteratively adjusts system inputs to drive the system toward an optimal operating point. Integrating this approach into gas lift systems makes achieving operational efficiency and minimizing costly interventions possible.

The thesis begins with a comprehensive literature review of gas lift optimization strategies, highlighting the limitations and potential benefits of extremum-seeking control in this context. Theoretical foundations of extremum seeking control, including optimization algorithms and system identification techniques, are also presented.

The research then focuses on implementing extremum-seeking control methods for gas lift optimization. A dynamic model of the gas lift system is developed to capture the complex interactions between the well's parameters, production dynamics, and controller inputs. Various optimization algorithms for multi-wells, such as perturbation-based and model-based methods, are tested and evaluated to determine their effectiveness in improving gas lift performance.

Extensive simulations use multiphase software, OLGA, to validate the proposed optimization framework. The results are compared with conventional gas lift strategies, demonstrating the superior performance of the extremum-seeking control for optimizing multi-well.

In conclusion, this thesis contributes to gas lift optimization by demonstrating the effectiveness of extremum-seeking control in enhancing oil production and reducing operational costs for multiple wells. The findings offer valuable insights for oil and gas industry professionals seeking to improve gas lift performance through advanced control techniques. Future research directions are suggested to enhance further the applicability and scalability of extremum-seeking control in the context of gas lift optimization.

Sammendrag

Denne oppgaven utforsker anvendelsen av Extremum Seeking Control (ESC) for å optimalisere gassløfteoperasjoner i olje- og gassindustrien. Gassløft er en mye brukt kunstig løftemetode som øker produksjonshastigheten til oljebrønner ved å injisere gass i brønnhullet for å redusere hydrostatisk trykk og øke væskestrømmen. Men på et visst tidspunkt, jo mer gass som injiseres i brønnen, jo mindre olje vil det komme inn, der brønnen vil bli dominert av friksjonstrykktap i stedet for hydrostatisk trykktap.
Denne forskningen tar sikte på å utvikle et optimaliseringsrammeverk ved å bruke ekstremumsøkende kontrollalgoritmer for kontinuerlig å tilpasse gassløftparametere i sanntid og maksimere brønnproduktiviteten i multibrønner. Ekstremsøkende kontroll er en tilbakemeldingsbasert optimaliseringsteknikk som iterativt justerer systeminndata for å drive systemet mot et optimalt driftspunkt. Integrering av denne tilnærmingen i gassløftsystemer gjør det mulig å oppnå driftseffektivitet og minimere kostbare inngrep.
Oppgaven starter med en omfattende litteraturgjennomgang av gassløftoptimaliseringsstrategier, som fremhever begrensningene og potensielle fordelene ved ekstremumsøkende kontroll i denne sammenhengen. Teoretisk grunnlag for ekstremumsøkende kontroll, inkludert optimaliseringsalgoritmer og systemidentifikasjonsteknikker, presenteres også.
Forskningen fokuserer deretter på å implementere ekstremumsøkende kontrollmetoder for gassløftoptimalisering. En dynamisk modell av gassløftesystemet er utviklet for å fange opp de komplekse interaksjonene mellom brønnens parametere, produksjonsdynamikk og kontrollinndata. Ulike optimaliseringsalgoritmer for multibrønner, for eksempel forstyrrelsesbaserte og modellbaserte metoder, blir testet og evaluert for å bestemme effektiviteten deres for å forbedre gassløftytelsen.
Omfattende simuleringer bruker flerfaseprogramvare, OLGA, for å validere det foreslåtte optimaliseringsrammeverket. Resultatene sammenlignes med konvensjonelle gassløftstrategier, og demonstrerer den overlegne ytelsen til den ekstremumsøkende kontrollen for å optimalisere multibrønn.
Avslutningsvis bidrar denne oppgaven til gassløftoptimalisering ved å demonstrere effektiviteten av ekstremumsøkende kontroll for å øke oljeproduksjonen og redusere driftskostnadene for flere brønner. Funnene gir verdifull innsikt for fagfolk i olje- og gassindustrien som ønsker å forbedre ytelsen til gassløft gjennom

avanserte kontrollteknikker. Fremtidige forskningsretninger foreslås for ytterligere å forbedre anvendeligheten og skalerbarheten til ekstremumsøkende kontroll i sammenheng med gassløftoptimalisering.

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Acronyms

BHP Borehole Pressure. 7, 50

ESC Extremum Seeking Control. i, iii, vi–viii, 2, 9, 13, 14, 16–19, 21, 25, 28, 35, 36, 38, 49, 50, 57, 60

GLPC Gas Lift Performance Curve. vii, viii, 9, 10, 14, 16, 20, 21, 26–28, 48, 49, 51

GLV Gas Lift Valve. 8

GOR Gas Oil Ratio. 9

HPF High Pass Filter. 13–15, 18

IPR Inflow Performance Relationship. 11

LPF Low Pass Filter. 13–15, 18

MIMO Multi Input Multi Output. 12

MPC Model Predictive Control. 12, 13, 16, 50

OCI Operating Cash Income. 10

PI Productivity Index. 22–24, 36, 51

PVT Pressure-Volume-Temperature. 9

SISO Single Input Single Output. 13

SPM Side Pocket Mandrel. 8

TIC Tubing Intake Curve. 11

TPR Tubing Performance Relationship. 11

VLP Vertical Lift Performance. 11

WC Water Cut. 7

WHP Well Head Pressure. 5, 50

Chapter 1

Introduction

As a part of petroleum production, production optimization is the key to increasing the production of the well with lower performance than the initial state. In most oil well production, initially, the reservoir pressure has enough pressure to lift reservoir fluid to the surface. At some point, the well pressure will decrease, and insufficient pressure is insufficient to transport the fluid to the surface due to reservoir depletion. Therefore, the bottom hole pressure must be decreased to maintain the well's production rate.

The artificial lift was introduced many years ago as a solution for lowering the bottom hole pressure of the well and consequently increasing the production rate. Gas lift injection is one of the artificial lifts mostly used in oilfields, in which the natural gas is injected into the tubing thru the annulus to decrease the hydrostatic pressure in the well. However, increasing the amount of injected gas in the well will increase the frictional pressure, as an effect of the interaction between gas and the surface of the tubing. Therefore, this needs to be considered for optimizing oil production.

Gas lift optimization is usually conducted by using a model-based approach. However, there are some uncertainties in modeling, and contains errors due to some parameters that have been known to change during the time and unknown parameters that need to be addressed. Therefore, the model must be updated and verified during the production time. Updating the model is a repetitive job that needs to be solved as soon as possible.

An alternative approach besides model-based optimization is known as model-free optimization. Model-free optimization means the optimization process doesn't need any model and only requires feedback data. Extremum-seeking control falls under the category of model-free optimization, where its objective is to search and locate the extremum without relying on a predefined model. Extremum-seeking control is also an adaptive control method, utilizing gradient estimation to optimize performance. Many papers have been published on extremum-seeking control applications for gas-lift injection optimization, but none of the simulations run in a dynamic model and multiphase simulator. Therefore, this thesis can be useful to prove the theory by using the multiphase simulator and dynamic model, which is

close to the real condition in the oilfield.
This thesis is structured as follows. Chapter 2 explains the basic theory of gas-lift optimization and Extremum Seeking Control (ESC). Chapter 3 describes the models and controllers used in this thesis, ESC implementation for multi-well and fictitious well. Chapter 4 describes the result of implementing Extremum Seeking Control (ESC) in OLGA with Matlab as a controller for a single well and multi wells. Chapter 5 discusses the simulation results and compares ESC implementation in both models and recommendations for future work. Chapter 6 This chapter will summarize the key findings of the study.

Chapter 2

Background

2.1 Petroleum Production System

This section will provide a concise overview of petroleum production and the various components comprising a petroleum production system. In general, the production system refers to the overall system responsible for moving fluids from the subsurface reservoir to the surface, treating and processing them, and making them ready for storage and transport.

The basic elements of the production system include the [1]:

- Reservoir
- Wellbore and Wellhead
- Pipeline and Processing Equipment
- Artificial lift equipment

As illustrated in Figure 2.1.

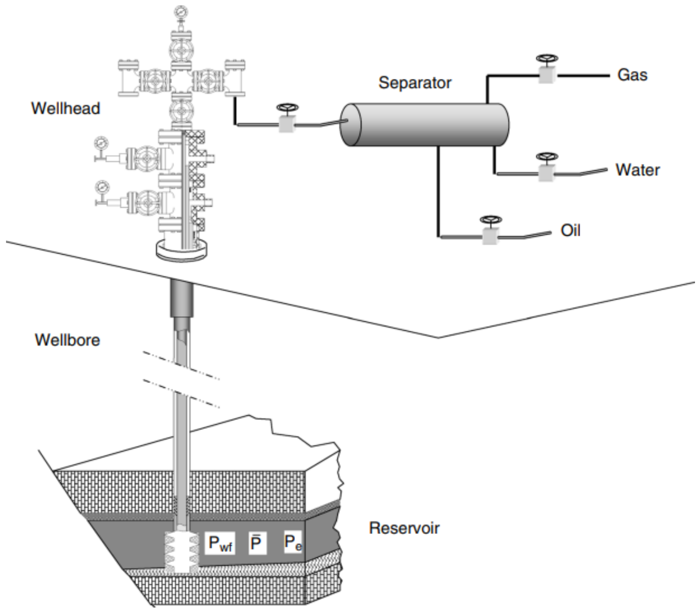


Figure 2.1: Petroleum production system (Buyon Guo, 2007)[2]

Reservoir

In petroleum engineering, a reservoir is an underground formation with permeable and porous characteristics, containing a distinct accumulation of hydrocarbons confined by impermeable rock or water barriers.

Reservoir pressure is normally higher than wellbore pressure, allowing the fluid to migrate from the reservoir to the wellbore naturally. Various factors, including the wellbore pressure, properties of the rock, average reservoir pressure, characteristics of the fluids, constraints on flow near the wellbore, and the drainage area's size and configuration influence the formation's ability to facilitate fluid flow. The reservoir's ability will be naturally reduced with time as fluids are drained from the reservoir, the average pressure declines, and the distribution and saturation of fluids in the reservoir change.
As production continues, fluid migrates over increasing distances. Generally, the initial disturbance takes a long time to affect the fluid far from the wellbore. This time-dependent transmission of pressure response is known as *transient production*, characterized by an unstable inflow performance of the well.

In regards to transient production, the wells experience three distinct production periods [3]:

1. *Infinite-Acting Period*: This period starts at the commencement of production and continues until the propagating pressure disturbance reaches the nearest no-flow or constant-pressure boundary.

2. *Transition Period*: This period begins when the pressure disturbance reaches the nearest boundary and concludes when it reaches the furthest outer boundary. The interpretation of wellbore pressure or rate response during the transition period is challenging due to the unknown geometry of the outer boundary. However, in practical terms, the transition period is usually brief unless the boundary is highly asymmetrical
3. *Pseudosteady-State Period*: This period starts after the transition period when the pressure disturbance reaches the furthest no-flow boundary from the wellbore, and the entire drainage area begins contributing to production. Wellbore conditions, such as rate and pressure, tend to stabilize during the pseudo steady-state, wherein the pressure decreases uniformly across the entire reservoir.

Therefore, the investigation to get optimum oil production needs to be conducted during the lifetime of the well since the transient production is time-dependent.

Wellbore and Wellhead

The wellbore plays a crucial role in the oil industry as it is a conduit for extracting hydrocarbons from the subsurface to the surface. The construction of a wellbore involves several key components. The first is the drilling process, where specialized drilling equipment drills a hole into the subsurface. This hole serves as the initial pathway for accessing the hydrocarbon-bearing formations. Once the drilling is complete, steel casing is inserted into the wellbore to provide structural integrity and prevent the collapse of the formation.

Various additional components are installed within the wellbore to ensure efficient operations. For example, the packer seals the annular space between the casing and production tubing in a gas-lifted well to eliminate casing heading as stated in paper (Gilbert,1954)[4]. Therefore, the investigation and maintaining well integrity is essential during the lifespan of oil production.

Perforations are made in the casing at specific intervals to increase hydrocarbon production. These perforations allow for the entry of reservoir fluids into the wellbore, enabling their subsequent extraction. Furthermore, production tubing is placed within the wellbore to provide a conduit for fluid transport. The flow in the tubing will be affected by pressure loss (or gain) due to friction, gravity, and acceleration. When calculating friction loss, important factors include tubing diameter, fluid density, flow rate, and viscosity. In the case of a gas-lifted well, the flow rate and Well Head Pressure (WHP) are regulated using the choke valve on the top of the well and casing valve where the gas was injected thru the annulus as illustrated in Figure 2.2.

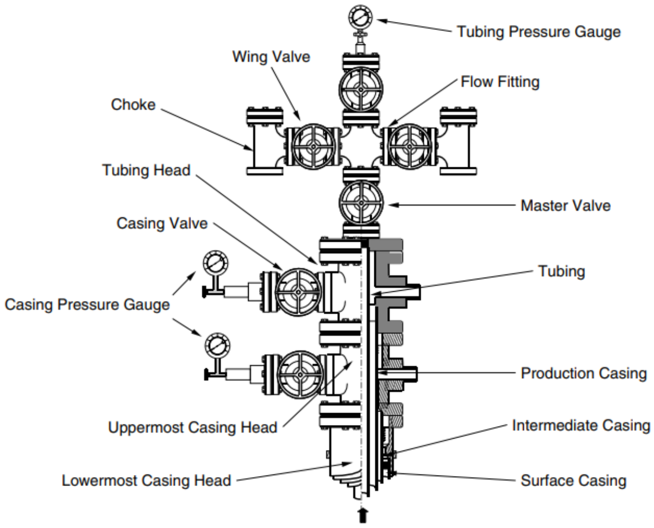


Figure 2.2: Sketch of wellhead (Buyon Guo, 2007)[2]

In addition, since gas, water, and oil can be produced simultaneously in the tubing, a multiphase flow model is necessary to estimate pressure loss. This often involves complex and time-consuming calculations, as many fluid parameters vary with changes in pressure and temperature.

2.1.1 Pipeline and processing equipment

After the fluids reach the surface, the fluids will be transported thru the pipeline to the separator. Therefore, the pressure on the wellhead must be able to move the fluid flowing thru the pipeline. If the pressure is insufficient to transport the fluid, some additional energy must be considered to be installed either between the separator and wellhead or increasing the pressure on the wellhead
Flow assurance is crucial because various factors can hinder the smooth flow of hydrocarbons within production systems, f.e, hydrates, wax and paraffin deposition, corrosion, slugging and liquid accumulation, and scale formation. By implementing effective flow assurance practices, operators can optimize production rates, minimize downtime, reduce operational risks, and ensure reliable and efficient transportation of hydrocarbons.
In the separator, reservoir fluids will be separated in terms of water, oil, and gas. The surplus gas produced and will not sell to the sale point can be utilized as a gas supply for a gas-lifted well.

2.1.2 Artificial lift equipment

As the period elapses, the reservoir pressure will be depleted, and when the reservoir pressure is insufficient to sustain the oil flow to the surface at desired rates, the natural flow must be aided by artificial lift. Artificial lift is a method to lower the producing Borehole Pressure (BHP) on the formation to obtain a higher production rate from the well.[5], lowering the bottom hole pressure means increasing the drawdown. The major forms of artificial lift are:

- Sucker-rod (beam) pumping
- Electrical Submersible Pump (ESP)
- Gas lift and intermittent gas lift
- Reciprocating and jet hydraulic pumping systems
- Plunger lift
- Progressive Cavity Pumps (PCP)

In this thesis, the artificial equipment that will be discussed is a continuous gas lift as an artificial lift. Therefore, the artificial lift that will be explained in this thesis is limited to the gas lift well and will be discussed in section 2.2.

2.2 Gas Lift Concept

Gas lift is a technology to produce oil and gas from wells that have insufficient pressure to flow the reservoir fluid at a desired rate to the surface. Gas is injected into the tubing close to the bottom of the well and mixed with the fluid from the reservoir. The gas lifts the fluid, reduces the fluid's density, and reduces the hydrostatic pressure in the tubing. If the total pressure drop along the tubing mainly due to its gravity component, a lower wellbore flowing pressure can be achieved. Therefore, the drawdown will be higher, and the production rate increase.

The gas lift can be applied to dead and naturally flowing wells. Dead wells mean water has dominated the volume of the well, high Water Cut (WC). Subsequently, there is no flow from the reservoir to the wellbore since the pressure of the wellbore and reservoir are almost equal. Natural flowing means the reservoir pressure is still higher than the wellbore pressure, the reservoir pressure is able to lift the fluids to the surface, but the production rate target is inadequate since the drawdown is not significant.

2.2.1 Gas Lift Production System

Some variables need to be considered in the gas lift production system, as the gas from the wellbore is injected back into the well and related to the cost. Therefore, in cases of multi-well, the storage and amount of gas supply are concerns in the gas-lift production system. The challenge must be overcome by maximizing the produced oil by utilizing the existing gas supply. The Gas Lift Valve (GLV) specification, as well as annulus and tubing pressure, are the critical parameters for the gas lift system to run optimally. As mentioned by (Lake, 1927)[6], a gas lift must be equipped with a minimum amount of gas to produce a maximum amount of oil. He also noted that oil production by gas lift could be controlled by changing gas volumes, injection depth, wellhead pressure, and tubing size.

As illustrated in the Figure 2.3, the compressor is routing back the gas lift from the surface to the well's annulus. The gas enters the tubing through a Gas Lift Valve (GLV) seated in Side Pocket Mandrel (SPM). A gas lift well completion typically has several SPM installed in different depths. The offloading valve is usually used during the production start to remove the fluid at the shallow depth since the pressure at the bottom is too high for Gas Lift Valve (GLV) to lift the fluid column in the tubing and potentially damage the valve since the GLV is a checked valve that will keep open in a certain different pressure.

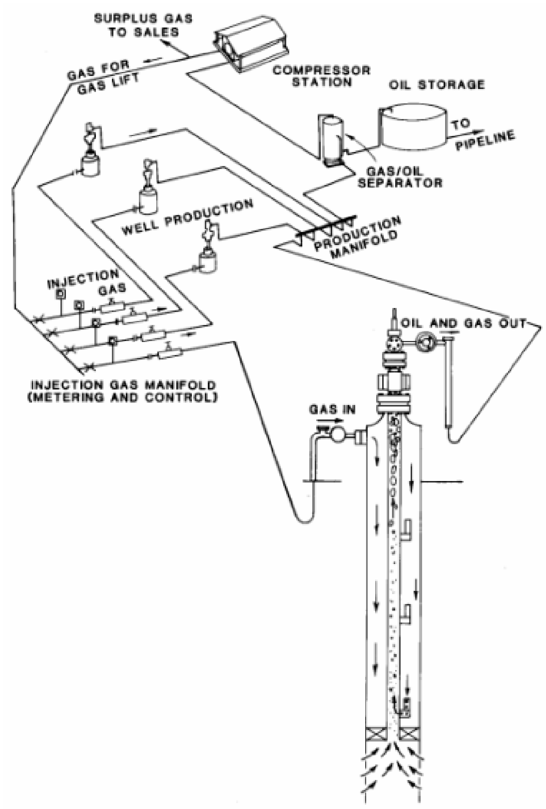


Figure 2.3: Gas lift system schematic - Courtesy American Petroleum Institute

2.2.2 Gas Lift Performance Curve

As briefly explained above, injecting gas into the tubing lowers the fluid density inside the tubing, then the hydrostatic pressure will decrease, and the fluids can flow to the surface. However, the frictional pressure drop increases while the fluid flows to the surface due to higher velocity in the tubing and slip effects (gas and liquids flow at different speeds). Subsequently, the more gas injected into the well, the more frictional pressure drop will dominate the total pressure drop. Therefore, there will be a specific amount of gas lift injection rate that gives an optimum production rate value. Plotting the gas injection rate versus the produced oil rate will give a curve known as Extremum Seeking Control (ESC), as illustrated in Figure 2.4.

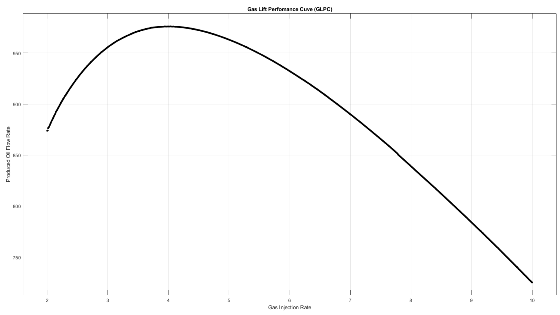


Figure 2.4: Gas lift Performance Curve created in Matlab

Changes in reservoir conditions with depletion (water cut, Pressure-Volume-Temperature (PVT) properties, Gas Oil Ratio (GOR), temperature and pressure of the production fluid) imply that the gas-lift characteristics curves also change, as it will affect change in the wellhead pressure.

2.3 Gas Lift Performance Curve Determination

Typically there are two ways to generate GLPC, numerically by simulations f.e nodal analysis, and field data by measuring the gas injection rate and produced oil rate f.e production test. However, to determine the curve, typically, the curve was determined by interpolating the group of data. Bergeron et al. (1999) [7] proposed an automated continuous gas lift control system by using proportional-plusintegral-plus-derivative (PID) control. The set point was gathered from the "step-rate" well test data that need to be updated routinely and transferred to a mathematical equation using a curve/fit routine. To create a GLPC function, (Alarcón et.al, 2002)[8] used second-degree polynomial that has been used traditionally to fit the field data and the unknown coefficient are obtained data by using the least square technique. Camponagara et.al (2010)[9] presented using a mixed-integer linear program (MILP) by piecewise-linearizing the GLPC at a given set testpoint for gas allocation as illustrated,

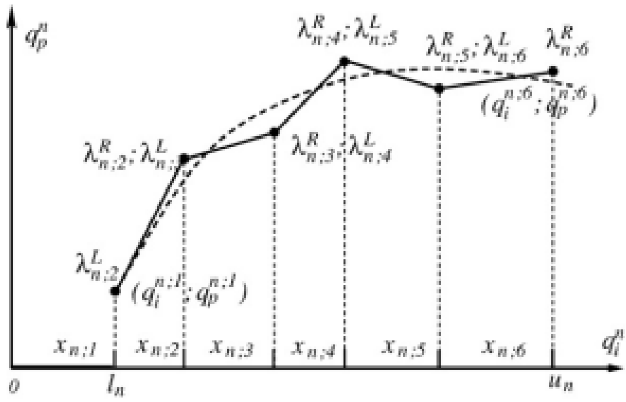


Figure 2.5: Piecewise Linearization in production rate (q_p^n) and gas-lift rate (q_l^n) Camponagara et.al, (2010)

From Figure 2.5, it can be seen that point number 5 is below points 4 and 6. This doesn't match with the theory of GLPC. However, it could happen when considering the complexity and dynamic behavior of the real system or the model trying to capture the behavior.

2.4 Gas Lift Optimization

In terms of optimization in the gas lift well does not always mean maximum production oil rate. Since the high cost of gas compression and separation equipment needed to separate large gas quantities must be considered, the maximum oil rate is not economically optimum. Therefore the Operating Cash Income (OCI) is introduced in gas-lift optimization.
By considering OCI, the revenue of produced oil, and the cost of gas injected into the well, the operator may express the gas lift curves as Figure 2.6. In general, there are one of two cases to select the optimal operating conditions for cost-revenue analysis:

1. A single well or multi wells in a facility has an unlimited injection gas supply.
2. A limited gas must be distributed among the wells.

In the case of an unlimited gas supply and for a given depletion stage, the most profitable rate happens at a unit slope in Figure 2.6. This point represents the condition when the equilibrium point between the revenue from produced oil rate and the cost of gas injection expenses. This point is known as *maximum daily OCI*. Up to this point, increasing gas injection yields a gain in profit; above this point, the profits start to be negative. For the cases of limited gas supply, a study about gas allocation needs to be conducted; see Golan (1991) [3].

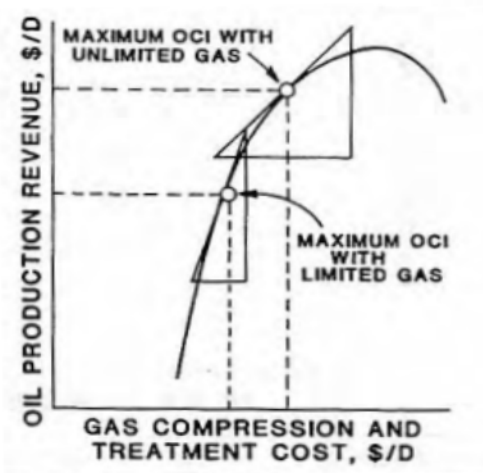


Figure 2.6: Production revenue vs Cost curve

However, as choosing the optimal profit point depends on the operator and complex, maximizing the oil rate is typically used as a term for optimization and will be the objective of this thesis.

2.4.1 Offline Optimization

In offline optimization, the process is conducted based on historical or pre-recorded data rather than continuously updating in real-time. The optimization decisions and adjustments are made non-real-time, typically during scheduled intervals or offline analysis.

Nodal Analysis

Nodal Analysis is the technique for optimizing the oil and gas production system by analyzing the system's performance composed of interacting components. (Beggs, 1991) [10] The changes in one component are important for the system. Nodal analysis is the most used technique for production optimization in the petroleum industry. The location of nodes in the production system can be a reservoir, wellbore, wellhead, etc. Typically the way to optimize the production with nodal analysis is by comparing the inflow and outflow, in this case Inflow Performance Relationship (IPR) and Tubing Performance Relationship (TPR). TPR is known by several names, f.e Vertical Lift Performance (VLP), Tubing Intake Curve (TIC).

The different operating points can be obtained by finding the intersection between those curves on various gas lift rates. Using interpolation for each gas lift rate corresponding to produced oil rate, a gas lift performance curve can be created, and an optimum point can be found.

Further investigation regarding production optimization with PROSPER has been conducted by Shedid and Yakoot (2016) mentioned that injected gas com-

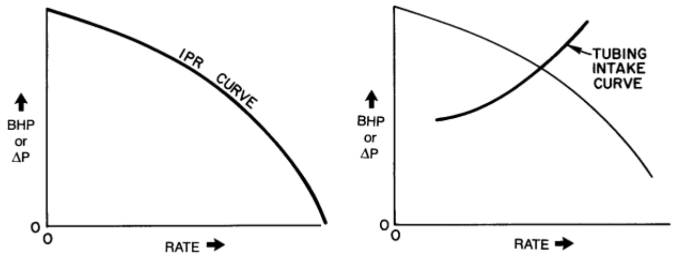


Figure 2.7: IPR vs TPR [11]

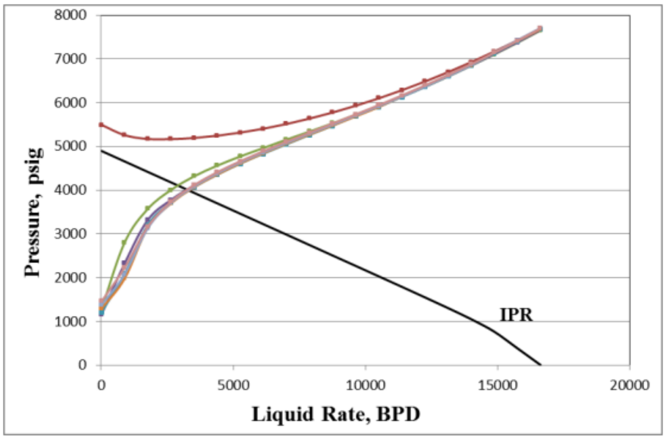


Figure 2.8: IPR vs VLP with different gas rate [12]

position, water-cut, and wellhead pressure significantly impact oil production, and tubing roughness has little impact and can be neglected. Since those parameters keep changing with the production time, the production engineer must routinely update the model. The inaccurate model and data will lead to the sub-optimal or far from the optimum point. This work takes a lot of effort as the model needs to be updated over time, and possibly the solution is found after the condition of the well has changed. This will be a drawback for using nodal analysis for optimization since the optimum point depends on the model and pre-recorded data.

2.4.2 Real-Time Optimization

Real-Time Optimization is a type of closed-loop process control that attempts to optimize process performance online in real-time.[13]

Model Predictive Control (MPC)

The most common process control in the oil industry nowadays is MPC system, since MPC is suitable for constrained Multi Input Multi Output (MIMO), in which multivariable inputs control the outputs simultaneously by considering all the interactions between system variables. Therefore, MPC needs an accurate dynamic model, in which the model and current measurement can be used to predict output values. Afterward, the appropriate changes in the input variables can be calculated based on both prediction and measurements.

However, there are typical uncertainty in oil and gas production networks as explained in [14],

1. Model uncertainty - the model may have a structure that makes it impossible to be modeled or lacks knowledge.
2. Parametric uncertainty - the data may have insufficient excitation to determine the parameters of the model uniquely
3. Measurement error - the difference between actual and sensor readings, f.e incorrect calibration of measurement equipment.

By addressing some uncertainties above, designing an accurate model for MPC can be challenging.

Extremum Seeking Control (ESC)

An alternative approach to finding a real-time optimization is Extremum Seeking Control (ESC). Extremum-seeking is an adaptive control method where perturbation is applied to optimize the performance of a steady-state system. The main advantage of extremum-seeking control compared to other real-time optimizers is that no plant model is required. This enables extremum-seeking control to optimize the performance of complex systems where the model is not known accurately. The ESC aims to find the operating-set points that maximize or minimize an objective function and are suitable for use in non-linear functions with a local minimum or maximum.

2.4.3 Classical Extremum Seeking Control (ESC)

In this thesis, the ESC applied for the optimization problem is classical ESC or perturbation-based ESC. It finds the extremum by estimating the plant's input-output gradient by applying a perturbation to the plant. The algorithm aims to make the input-output gradient as small as possible; see [15]. From work presented by [16], the schematic of perturbation-based Extremum Seeking Control (ESC) is illustrated in **??**.

The model is a Single Input Single Output (SISO) non-linear, the input is θ with the output function is $y = h(x)$. It is a response of θ; it can be written as $f(\theta)$. This can be shown by a Taylor expansion of $f(\theta)$. The signal will go through a High Pass Filter (HPF) and Low Pass Filter (LPF). Those two filters aim to extract the objective function's gradient and integrate the result into $(\hat{\theta})$.The loop continuously works until the error between θ and $\hat{theta}$ converges to zero, and the output function reaches the near-optimal solution.

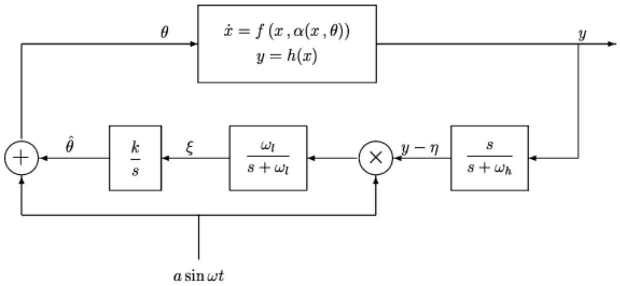


Figure 2.9: Peturbation-based ESC scheme [16]

The following is a short explanation of the components of the extremum-seeking control system.

Dither Signal

The dither signal determined the convergence speed in ESC. The works have been done by (Nešić, 2009)[17] by comparing sine, square, and triangle waves. It proved that the square wave produces the fastest convergence among all signals with the same amplitude and frequency. However, considering the transition movement of opening choke in the petroleum industry, this thesis chooses a sine wave as a dither signal, simplifying the choke opening and fluid flow model. In addition, the dither signal has an amplitude and phase that can cause unwanted fluctuation. In the multi-well system, each well will amplify the total fluctuation. Therefore, the phase for each well needs to be coordinated to reach a stable system.[18]

Plant

The plant refers to the system controlled by the ESC. In terms of optimizing the gas-lift well production, the plant is the well itself. The objective function is GLPC of the well, which maps the static relationship between the oil production and gas lift injection rate.[19]. Haring (2016)[20] describes the variety of plant models that are suitable for ESC application.

Estimator

The estimator is the process where the gradient is estimated, and also often referred to an observer since the estimator will predict the trajectory of the output gradient.
As illustrated in Figure 2.9, two components that are commonly used to estimate the gradient, HPF and LPF. The first filter that the output from the plant,y, needs to pass through is HPF. In general, HPF is used let the frequency that is higher than the cut-off frequency (f_c). The signal is demodulated by multiplying with

perturbation frequency before sending to LPF. On the other hand, LPF works in the opposite way as HPF. A LPF works by letting the frequency lower than the cut-off frequency (f_c). The main objective of those filters is to remove other parts from the gradient. Therefore, HPF,LPF, and the frequency of perturbation, f, must be adjusted accordingly. An illustration of how the cut-off frequency works for both HPF and LPF can be seen in Figure 2.10.

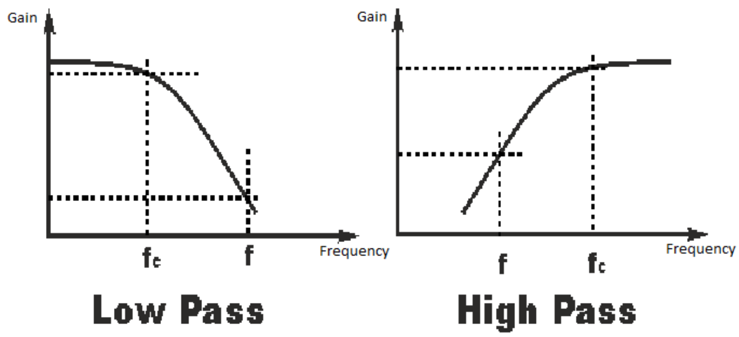


Figure 2.10: Low Pass and High Pass Filter response

Optimizer

The optimizer refers to where the gradient estimation from the estimator generates the new input value for the objective function in the plant. Optimizer is typically an integrator and the gain. The purpose of gain is to amplify the estimated gradient before the perturbation frequency modulates the signal to the plant. This is used to speed up the convergence of the controller, but the drawback of using high integrator gain is overshooting the optimum value or high fluctuation at the beginning of the optimization. Therefore, selecting an integrator gain is crucial.

2.4.4 Time scales

In a realistic plant model, a transient behavior will be happened before reaching a steady-state value. Hence, tuning the controller not to react before the transient period is over is essential. Therefore, the tuning parameter should be chosen to obtain accurate gradient estimates with the observer so that the extremum-seeking controller exhibits three-time scales [21]:

- fast - the perturbations;
- medium - the observer;

- slow - the dynamic optimizer;

For any fixed perturbation amplitude, the perturbation frequency (η), the tuning parameter (λ) of the observer, and the tuning parameter (ν) of the dynamic optimizer need to follow appropriately as,

$$\nu \gg \lambda \gg \eta \tag{2.1}$$

2.5 ESC for Gas-Lift Optimization

There have been developments in optimization approaches that do not require solving a numerical optimization problem. Instead, the optimal operation is achieved via feedback control, f.e MPC. However, this optimization approaches rely on complex physical models, either online or offline, that is sometimes difficult to be modeled due to a lack of knowledge and use simplification to operate, leading to uncertainties, as described in subsection 2.4.2. In regards to the issue of model uncertainties, data-driven optimization, which is ESC, becomes an alternative method for gas-lift optimization. (Khrisnamoorthy, 2019) [22]
In (Pavlov, 2017)[21], stated that ESC is suitable for production curves optimization since the fluid composition (the ratio of oil, water, and gas flowing from the reservoir) changes over time, and the GLPC is concave (Rashid, 2020) [23] and has a unique has unique optimum point.

2.5.1 Literature

Peixoto et al. (2017) [24] addressed the design of a perturbation-based ESC for gas lifted wells optimization. A simple non-linear dynamic model is proposed, and the essential dynamics of Eikrem's model, i.e., the transient behavior and optimal steady state GLPC. Pavlov et.al[21] presented a distributed extremum seeking for multi wells since in a production facility, multiple gas-lift wells need to be optimized, and the total available gas is usually limited. The algorithm is based on the "synchronization" of the production performance gradient for all individual wells. A fictitious well is introduced and handled when the optimum can lie on the boundary of the constraints. The ESC controller is based on Krstić (2000) [16] as described in Figure 2.9. The system (in a closed loop via state feedback) can behave approximately as static by assuming that the perturbation frequency is slow compared to the system's time constant.

Chapter 3

Methodology

3.1 Controller

3.1.1 Matlab and Simulink

The investigation into optimizing gas-lifted oil production using Extremum Seeking Control (ESC) was initiated based on the work conducted by Pavlov et al. (2017)[21]. The approach employed in this study involved simulations and focused on two key aspects: developing the ESC scheme controller and constructing the plant model. The controller was built using Matlab and Simulink, with Matlab being a programming language and environment developed by MathWorks and Simulink serving as a block diagram environment for Model-Based Design within Matlab. Simulink adopts a graphical programming approach, utilizing blocks for various operations. The controllers utilized in this thesis were created using a combination of Simulink and Matlab. The main controller was developed in Simulink, while the controller's parameters were defined in Matlab. Although it was possible to create the controller solely in Matlab, Simulink was chosen based on personal preference. Matlab was preferred for defining parameters, as it generally proves to be easier, particularly when the system involves complex matrices that can be challenging to define in Simulink.

3.1.2 Classic Extremum Seeking-Control

Assuming that u_0 represents a known or measured input value, and $f(u_0)$ represents the plant's response to u_0, in the context of optimizing the gas lift rate, u_0 would correspond to the gas lift rate itself, while $f(u_0)$ would represent the oil rate. In this scenario, the main goal of the controller would be to determine the subsequent value u_1 so that $f(u_1)$ increases to the extremum point.

$$u = u_0 + asin(\omega t + \phi) \tag{3.1}$$

The function $f(u)$ can be written in the form of Taylor expansion as,

$$f(u) = f(u_0) + f'(u_0)(u - u_0) + f''(u_0)\frac{(u - u_0)^2}{2!} + ... \tag{3.2}$$

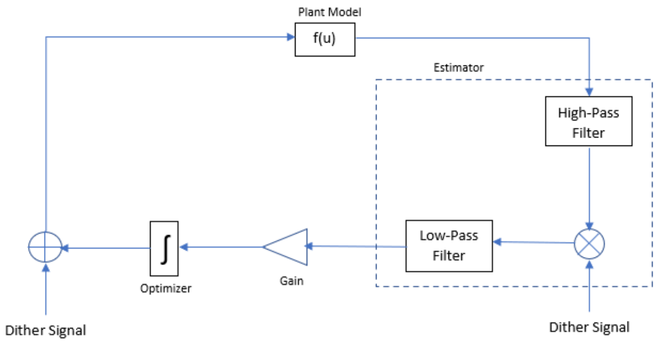


Figure 3.1: Extremum Seeking Control (ESC) schematic

By substitute the $u-u_0$ in Equation 3.1, to the Equation 3.2. Therefore, the form will be,

$$f(u_o + asin(\omega t + \phi)) = f(u_0) + f'(u_0)asin(\omega t + \phi) + f''(u_0)\frac{asin^2(\omega t + \phi)}{2!} + ... \tag{3.3}$$

By considering an approximation $f(u)$ by a first-order Taylor expansion, the equation can be approximated as,

$$f(u) = f(u_0) + \nabla f(u_0)asin(\omega t + \phi) \tag{3.4}$$

.

The washout(high-pass) filter. The High Pass Filter (HPF) applied to the output to separate the reminder $\nabla f(u_0)asin\omega t$from $f(u_0)$. The next signal is then "demodulated" by multiplication with $asin\omega t$. Therefore the equation of remainder will be,

$$\nabla f(u_0)a^2 sin^2(\omega t + \phi) \tag{3.5}$$

Applying trigonometric identity $2sin^2(\omega t + \phi = 1 - cos2(\omega t + \phi)$, the Equation 3.5, can be written as:

$$\nabla f(u_0)\frac{1 - cos2(\omega t + \phi)}{2} \tag{3.6}$$

Afterward, the signal enters the Low Pass Filter (LPF). The LPF will filter out the cosine in Equation 3.6. The output of LPF is the gradient, which gives a trajectory for the next value. The gradient further goes to the integrator gain. The gain, k, will increase the speed but create an overshoot to the optimization, and the integrator will be used as an optimizer and obtain the new value, the new u_0. Then, the new u_0 will be perturbed, and the closed loop will be completed when u is approximately equal to u_0, meaning that the gradient is zero. The closed-loop of this process is described in Figure 3.2.

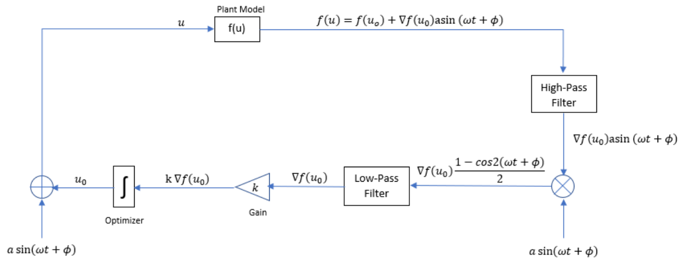


Figure 3.2: Detailed controller Extremum Seeking Control (ESC)

3.2 Gas Injection Rate without Constraints

As mentioned before, Extremum Seeking Control (ESC) doesn't need any model, and the amount of required gas to optimize the multi-well is unknown. In case there is no constraint among the multi-well, each well gradient moves by following its gradient to reach an optimum point

$$\nabla f(u_0)_i \neq \nabla f(u_0)_j, \forall_i, j = 1, ..., N. \tag{3.7}$$

, and production optimization will not be a problem.
However, the problem will arise when enough gas is unavailable to optimize each well's production. Therefore gas needs to be allocated to optimize the production for each well, and synchronization-based optimizer and the gas injection rate subject to constraints, which will be explained in the next section, need to be applied to optimize the production using limited available gas.

3.3 Synchronization-Based Optimizer

For the multi wells, the optimizer is modified to synchronize the gradient estimates, u^*, for each well. The optimization problem can be solved if and only if

$$\frac{\partial \hat{f}_i}{\partial u_i}(u_i^*) = \frac{\partial \hat{f}_j}{\partial u_j}(u_j^*), \forall_i, j = 1, ..., N+1. \tag{3.8}$$

Based on this equation, the input to the integrator will be,

$$\dot{u}_i = \Sigma \gamma_{i,j}(\frac{\partial \hat{f}_i}{\partial u_i}(u_i) - \frac{\partial \hat{f}_j}{\partial u_j}(u_j)), \forall_i, j = 1, ..., N+1. \tag{3.9}$$

where, $\gamma_{i,j} = \gamma_{j,i} \geq 0$ are synchronization gains, and u^* is updated gradient estimate.

In the matrix form for case 3 wells, the (3.9) can be written as

$$\begin{bmatrix} \dot{u_1} \\ \dot{u_2} \\ \dot{u_3} \end{bmatrix} = \begin{bmatrix} \gamma_{1,2} + \gamma_{1,3} & -\gamma_{1,2} & -\gamma_{1,3} \\ -\gamma_{2,1} & \gamma_{2,1} + \gamma_{2,3} & -\gamma_{2,3} \\ -\gamma_{3,1} & -\gamma_{3,2} & \gamma_{3,1} + \gamma_{3,2} \end{bmatrix} \begin{bmatrix} \frac{df_1}{du_1} \\ \frac{df_2}{du_2} \\ \frac{df_3}{du_3} \end{bmatrix} \tag{3.10}$$

By referring to Equation 3.9, the updated gradients estimates are equal to zero ($\dot{u}_i$) when the gradients ($\frac{df_i}{du_i}$) are equal to each other ("synchronized"). When the gradient is unequal, the Equation 3.9 will steer the component of u towards gradient synchronization.

3.4 Gas Injection Rate Subject to Constraints

This section is based on the work presented in Pavlov et al. (2017) [21]. It has been known that the gas injection for the gas lift well is determined by the GLPC as,

$$q_i = f_i(u_i) \tag{3.11}$$

, where the gas injection rates are subject to constraints for each well

$$0 \leq {u_i}^{min} \leq u_i \leq {u_i}^{max}, i = 1, ..., N. \tag{3.12}$$

The total gas injection rate subject to constraints,

$$\Sigma_{i=1}^{N} u_i \leq U^{max} \tag{3.13}$$

The optimization problem is to maximize the total production of the wells

$$\Sigma_{i=1}^{N} f(u_i) \rightarrow max \tag{3.14}$$

Before solving the problem, the formulation is modified by introducing the fictitious input u_{N+1} which denotes the available gas rate not injected to the wells: $u_{N+1} = U^{max}$ - $\Sigma_{i=1}^{N} u_i$. Therefore, it will be either zero or upper limit. The corresponding fictitious function $f_{N+1}(U_{N+1})$ is set to zero. Then equation Equation 3.14 is become,

$$\begin{aligned} \Sigma_{i=1}^{N+1} f(u_i) = U^{max} \\ {u_i}^{min} \leq u_i \leq {u_i}^{max}, i = 1, ..., N+1 \end{aligned} \tag{3.15}$$

The reason for modification from the inequality Equation 3.13 to equality Equation 3.15, can be found in detail in Pavlov et,al (2017).

The modified problem formulation can be handled when the optimum lie on the boundary of the constraint. The boundary function is,

$$\hat{f}_i(u_i) = \begin{cases} f_i(u_i) + \mu ln(u_i - {u_i}^{min}) & \\ +\mu ln({u_i}^{max} - u_i), & if\, u_i \in ({u_i}^{min}, {u_i}^{max}) \\ -\infty & otherwise \end{cases} \tag{3.16}$$

The details of how this formulation is applied in Matlab Simulink can be found in **??**.

3.5 Static Model

The three static models were created by simulating three different wells in OLGA to get GLPC. For clarity, that ESC doesn't need any model to find the optimum point, the GLPC model is used as a comparison for a dynamic OLGA model. Slowly ramping up the amount of injection gas in Simulink ensures the response in OLGA is stable. As mentioned in OLGA that the oil production rate is not stable below the gas rate of 1.5 kg/s; the simulation was started from 2 kg/s to 10 kg/s with a slope of 0.01. Matlab Curve Fitter is used in the process of making the equation. Fit type polynomial degree 4 is used to make the formula of Gas Lift Performance Curve (GLPC) instead of quadratic function, since for polynomial interpolation, the error decrease as the order increases but only at a certain point.[25], and through selecting the polynomial degree in Matlab, the best fit for the tabulated data is polynomial degree 4.

3.6 Dynamic Model

Previous research on Extremum Seeking Control (ESC) application in gas-lifted wells has primarily utilized a static plant model known as a GLPC function. However, these models do not consider the dynamic behavior of multiphase flow specific to gas-lifted wells. When adjusting the gas lift rate, a transient period occurs before the well stabilizes. The perturbation frequency must be slower than the system dynamics to ensure a stable signal response. Therefore, the logical next step is to incorporate a dynamic plant model. Developing a dynamic multiphase flow model can be complex and rigorous, often requiring simplifications such as assuming a two-phase flow where the liquid phase represents a pseudo-phase of oil and water. The simulations used the OLGA software to ensure a realistic and validated model. In this thesis, Matlab time and Simulink time are independent. The time scales in OLGA are not representing the time in Matlab. Convergence time is not the main objective of this thesis; therefore, simulator mode is chosen in OLGA instead of external mode.

3.6.1 OLGA

OLGA is a sophisticated software tool developed by Schlumberger, a leading oilfield services company. It is specifically designed for dynamic modeling and simulation of multiphase flow in oil and gas production systems. OLGA is widely used in the oil and gas industry to optimize offshore and onshore production facilities' design, operation, and troubleshooting.

The OLGA, dynamic multiphase flow simulator, can model time-dependent behaviors, and transient flow, to optimize production potential. Incorporating transient modeling is crucial for conducting feasibility studies and designing field development plans. Dynamic simulation plays a vital role in deepwater operations and is extensively utilized in offshore and onshore projects to examine transient behaviors in pipelines and wellbores.

By employing the OLGA simulator for transient simulation, steady-state analyses gain an additional dimension as it predicts system dynamics. These dynamics encompass flow rate changes, fluid composition, temperature, solids deposition, and operational modifications.

Whether it pertains to wellbore dynamics for various well completions or pipeline systems equipped with various process equipment, the OLGA simulator offers precise predictions of critical operational conditions involving transient flow.

The OLGA used in this thesis is version 2020.2.0. Three different wells were created. The three vertical dummy well is based on a pre-existing case in OLGA with varying Productivity Index (PI)

Well Schematic

The chosen well for the study was based on the "Gas lift well casing heading" case, initially designed to simulate casing heading and unstable flow. However, as mentioned on the OLGA cases page, it is also suitable for experimenting with production parameters, such as gas lift rate, to optimize production.

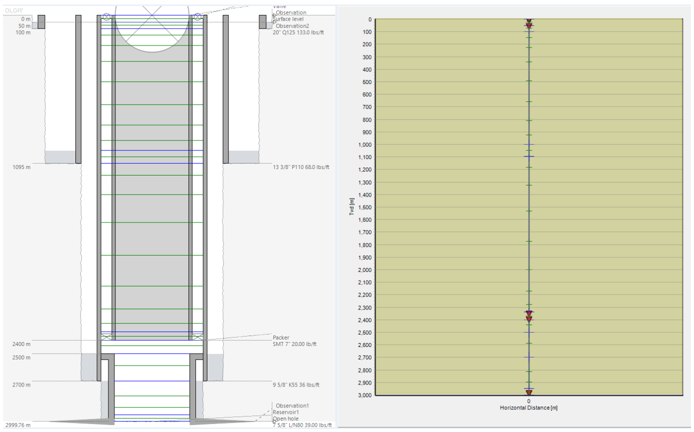


Figure 3.3: Well schematic and survey

Figure 3.3 provides the well schematic and survey details. The well has a length of 3000 meters with an open hole at the bottom. The tubing extends to 2400 meters, and a packer is installed to isolate the annulus from the wellbore. A production choke is positioned at the top of the well, while a valve is placed at the top of the annulus to regulate inflow. At 2350 meters, a tubing rupture is the connection point between the gas injected into the annulus and the tubing. Alternatively, a gas lift valve could be used, but specific valve specifications would need to be known. The main distinction between a tubing rupture and a valve is the excess pressure drop across the valve, and it requires more detailed specifications, such as valve characteristics, although it was considered negligible for this thesis. The flow in the annulus can only be simulated using the "OLGA Well Module," employed for the simulations.

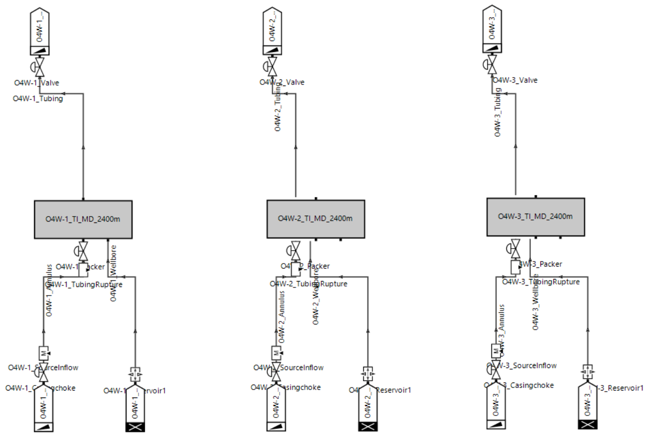


Figure 3.4: OLGA network model for 3 wells

Figure 3.4 shows the OLGA network used in this thesis. The bottom and top annulus is the pressure node, the valve in the annulus is closed, and the top of the tubing is open. The leak is installed at the top of the closed valve in the annulus, representing the packer. The model has used a closed node at the bottom of the reservoir, meaning there is no flow to the bottom. The mass flow source is installed at the top of the annulus, representing the gas injection valve in the oil well production.

Reservoir Model

As mentioned in OLGA, the reservoir inflow model is linear, meaning the assumption that the flow of fluids from a reservoir into a wellbore or production system follows linear behavior. In this context, a linear reservoir inflow model assumes that the rate of fluid production (inflow) from the reservoir is directly proportional to the pressure difference between the reservoir and the wellbore, by following.

$$Q_p = \Delta P * PI \tag{3.17}$$

where Q_p is volumetric flow rate produced, ΔP is differential pressure, and PI is Productivity Index,

This assumption is often used in simplified reservoir models where the reservoir characteristics, such as permeability and fluid properties, are assumed to be homogeneous and constant throughout the reservoir.

Under this linear assumption, the inflow rate can be calculated using Darcy's law, which describes fluid flow through porous media. Darcy's law states that the flow rate is proportional to the reservoir rock's pressure gradient and permeability.

$$\Delta P = P_R - P_{wf} \tag{3.18}$$

where P_R is reservoir pressure, and P_{wf} is borehole pressure.

This thesis selects the Productivity Index (PI) as per Table 3.1. Hence, the maximum or minimum point of the volume of oil being produced and the rate at which gas is generated varies for each well.

Table 3.1: Productivity Index (PI) for reservoir model

Well	Productivity Index (PI)
O4W-1	14.9
O4W-2	5
O4W-3	18

The selection of fluid properties involved using one of the PVT cases provided by OLGA, specifically the "oilsample" PVT data, which was employed for all OLGA simulations.

3.6.2 Connecting Simulink with OLGA as an OPC Server

An OPC server is a software component that facilitates communication between various applications and systems within industrial automation and control.

OLGA is a dynamic multiphase flow simulator in the oil and gas industry. Meanwhile, MATLAB is a programming and numerical computing environment. In this particular thesis, the objective is to establish data exchange and interaction between these two software tools.

To enable seamless communication between MATLAB and OLGA, an OPC server acts as an intermediary, employing the OPC protocol. This setup allows OLGA to share real-time data with MATLAB or exchange information through periodic updates. In practice, MATLAB can retrieve simulation results from OLGA while also being capable of sending control signals or parameters to OLGA. This bidirectional interaction enables MATLAB to influence the simulation behavior or engage in closed-loop simulations with OLGA.

Setting up OLGA as OPC Server

The tutorial video explains how to accomplish this setup and is available on the internet [26]. The server options are defined within an OLGA project's "Case definition" section. It is important to assign unique names to the server, module, and model because the client connecting to the OPC server relies on these names to identify the correct model for connection.

All variables and parameters the client will utilize must be specified in OLGA. The OLGA output data that the client will read can be stored in a "ServerData." The keyword "Expose" is used for variables the client can modify or write to. Additionally, in the Tools Configuration I/O, the variable that would like to be "Expose" must be selected. In the context of gas lift optimization with an Extremum Seeking Control (ESC), the oil rate is the variable for the client to read. In contrast, the gas lift rate is the variable that the controller will modify and input into the OLGA model. Therefore, the variable "MASSFLOW" needs to be "Expose" in both "ServerData" and Tools Configuration I/O.

It should be noted that there is a discrepancy in the standard unit of measurement used in OLGA for trend-line plots and server data. Specifically, when plotting the oil rate in OLGA, the standard unit is Sm3/d (Standard cubic meters per day), while the standard unit for oil rate in server data is Sm3/s (Standard cubic meters per second). Therefore, unit conversion needs to be applied during the simulation in Simulink.

Setting up Simulink as controller and client

To establish a connection between a Simulink model in MATLAB and an OPC server, MATLAB provides an OPC Toolbox that needs to be downloaded. Additionally, it is necessary to install the OPC Foundation Core Components to avoid encountering an error message when attempting to connect to an OPC server.

Once the OPC Toolbox is downloaded, the OPC Foundation Core Components can be installed by executing the command "opcregister('install')" in the MATLAB command line.

The OPC Toolbox offers the capability to both read and write data from and to an OPC server. In Simulink, three different blocks in Matlab are utilized to connect with OLGA: an OPC configuration block, an OPC write block, and an OPC read block.

The OPC configuration block establishes a connection with the OPC server. Ensuring the OPC server runs before utilizing the OPC configuration block is crucial. The controller component retrieves the oil rate from the OPC read block and then sends updated gas lift rates to OLGA using the OPC write block.

Chapter 4

Result

4.1 GLPC Determination

The GLPC for each well is simulated in OLGA by adjusting the slow step rate in Maltab from 1.6 kg/s to 10 kg/s with a step rate of 0.01, as presented in Figure 4.1.

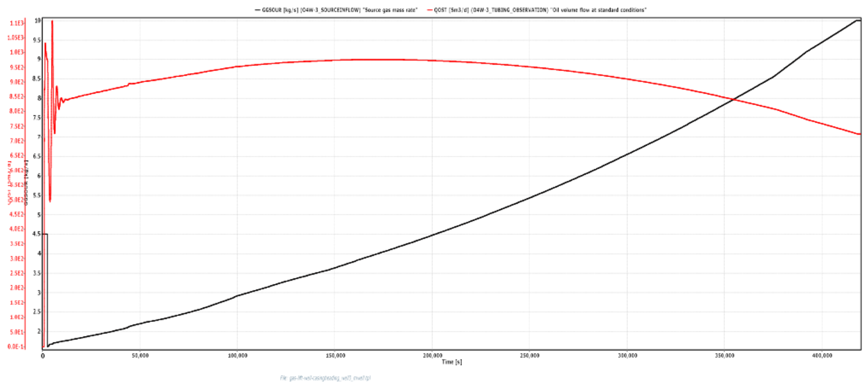


Figure 4.1: Gathering data for GLPC from OLGA

The data from OLGA is tabulated into the Matlab Curve Fitter. The linear-least square fitting is conducted by Curve Fitter in Matlab as follows:

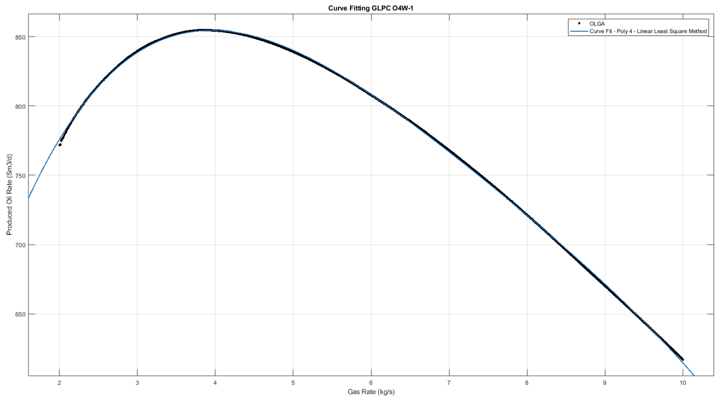


Figure 4.2: GLPC - O4W3

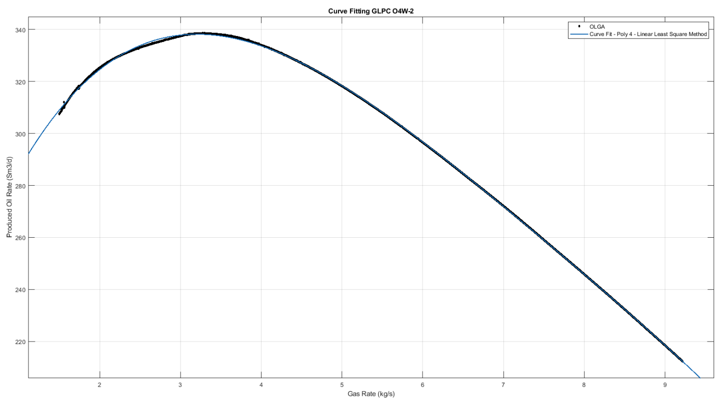


Figure 4.3: GLPC - O4W3

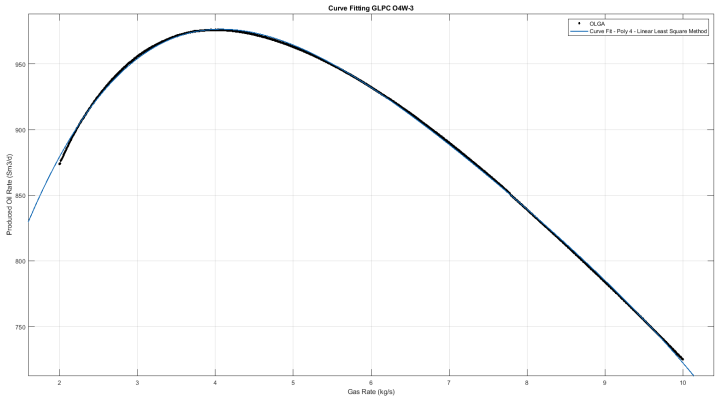


Figure 4.4: GLPC - O4W3

From the simulation and Matlab Curve Fitter, the polynomial degree 4 best fits all the tabulated data in every well. The formula for each well can be found in Table 4.1, in which the units for gas lift is (kg/s) and the oil rate is (Sm^3/d).

Table 4.1: Static model and extremum values

Well Number	Gas lift performance function	Max. Gas lift rate	Max. Oil rate
O4W-1	$-0.1649 * u^4 + 5.057 * u^3 - 59.82 * u^2 + 276.4 * u + 424.4$	3.95	854.3
O4W-2	$-0.06364 * u^4 + 1.942 * u^3 - 22.7 * u^2 + 93.96 * u + 212.9$	3.22	338.1
O4W-3	$-0.1814 * u^4 + 5.575 * u^3 - 66.47 * u^2 + 313.2 * u + 477$	3.97	976.7

4.2 ESC for Single Well

4.2.1 Static Model

The static model is conducted to find the reference point and compare it to the result obtained by applying ESC in the dynamic model, as represented in Matlab model in Appendix A. In this section, the simulation is conducted only for well 04W-3.

The tuning parameters that are used for the static model are shown in Table 4.2

Table 4.2: ESC tuning parameter for Single Well O4W-3 - Static model

Parameter	**Symbol**	**value**	**unit**
Period	P	12	s
Perturbation Frequency	ω	0.5236	rad/s
Perturbation Amplitude	A	0.03	-
High-pass filter cut-off	HPF $= \omega/2$	0.2618	rad/s
Low-pass filter cut off	LPF $= \omega/4$	0.1309	rad/s
Gain	g	0.05	-

Initial Value Below Optimum Point

In this part, the initial value is 0 kg/s.

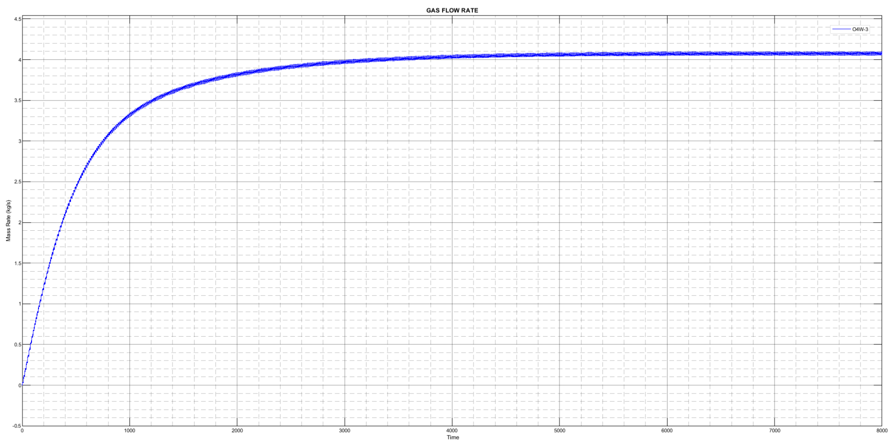


Figure 4.5: Gas injection rate with initial value below optimum point - O4W3 with static model

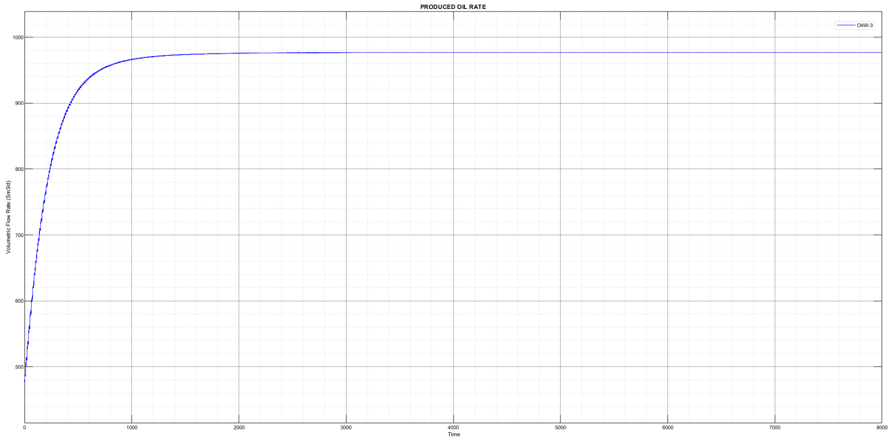


Figure 4.6: Produced oil rate with initial value below optimum point - O4W3 with Static Model

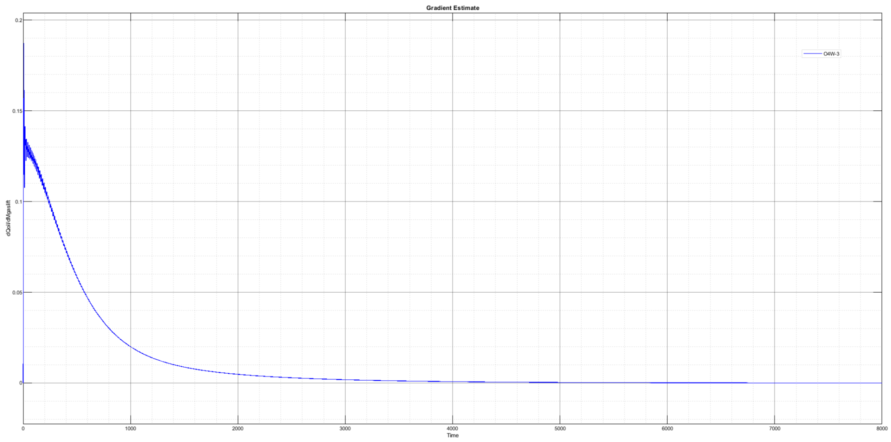


Figure 4.7: Gradient estimate with initial value below optimum point - O4W3 with Static Model

Initial Value Above Optimum Point

In this part, the initial value is 6 kg/s.

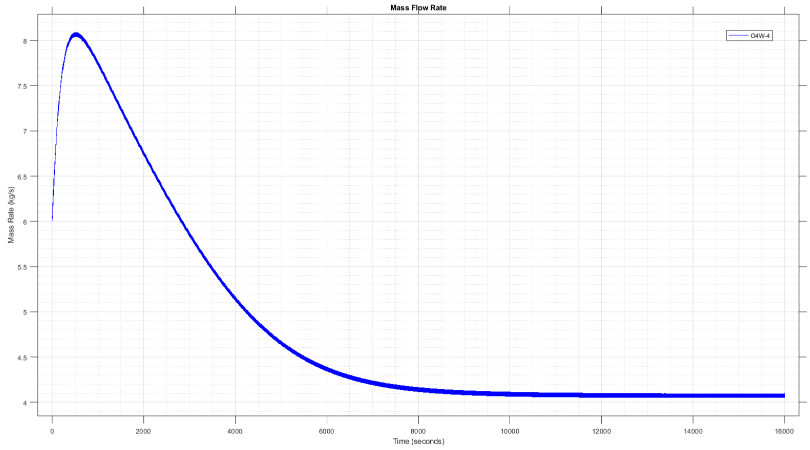


Figure 4.8: Gas injection rate with initial value above optimum point - O4W3 with static model

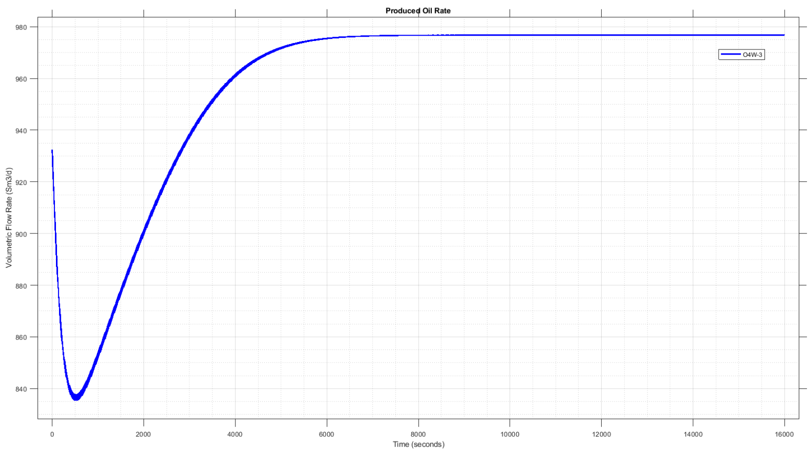


Figure 4.9: Produced oil rate with initial value above optimum point - O4W3 with static model

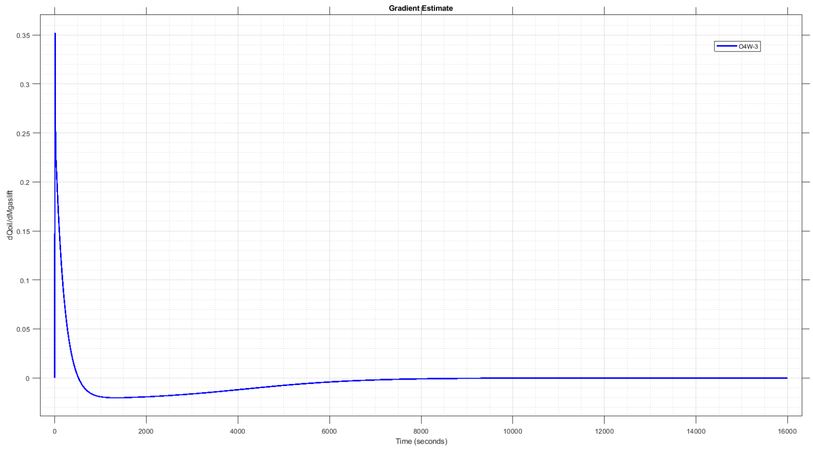


Figure 4.10: Gradient estimate with initial value above optimum point - O4W3 with static model

4.2.2 Dynamic Model

The tuning parameters that are used for the dynamic model are shown in Table 4.3

Table 4.3: ESC parameter for Single Well O4W-3 - Dynamic model

Parameter	**Symbol**	**value**	**unit**
Period	P	120	s
Perturbation Frequency	ω	0.0524	rad/s
Perturbation Amplitude	A	0.03	-
High-pass filter cut-off	HPF $= \omega/2$	0.0262	rad/s
Low-pass filter cut off	LPF $= \omega/20$	0.0026	rad/s
Gain	g	0.006	-

Initial Value Below Optimum Point

The initial value for this part is 3 kg/s since the OLGA model is shown unstable flow below 1.5 kg/s.

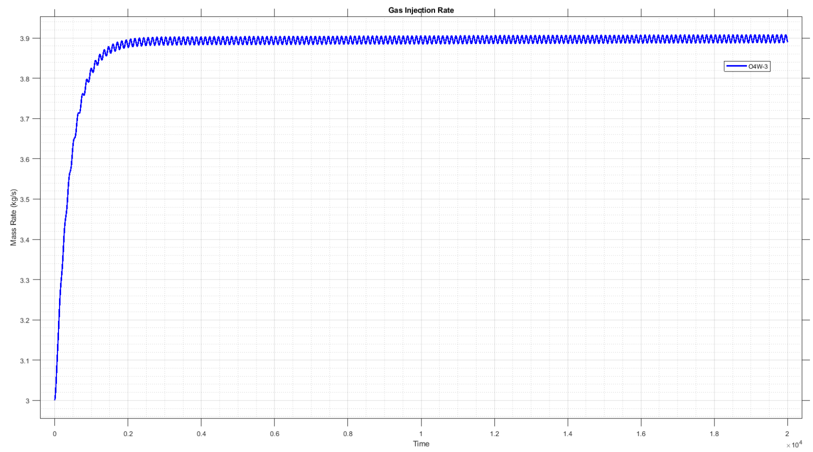


Figure 4.11: Gas injection rate with initial value below optimum point - O4W3 with dynamic model

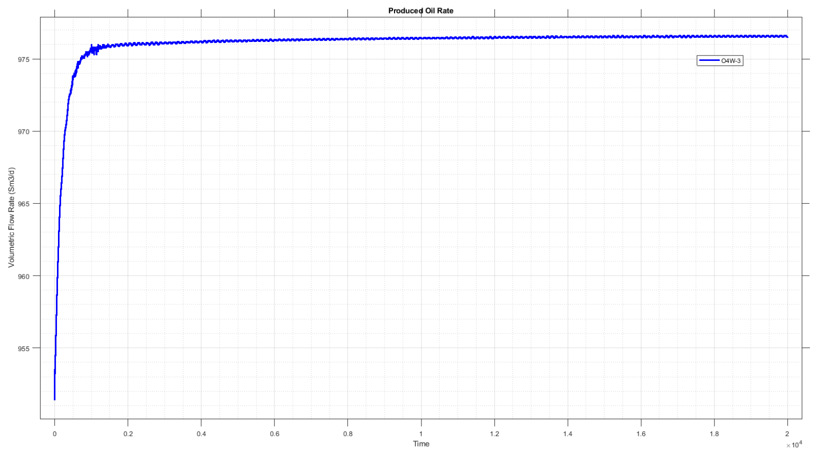


Figure 4.12: Produced oil rate with initial value below optimum point - O4W3 with dynamic model

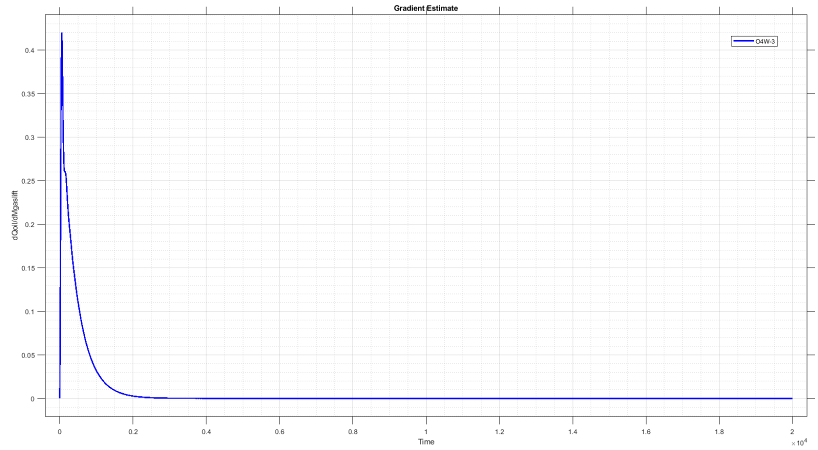


Figure 4.13: Gradient estimate with initial value below optimum point - O4W3 with dynamic model

Initial Value Above Optimum Point

The difference in the tuning parameter for this part is only the gain change to 0.01, and the initial value is 4.5 kg/s.

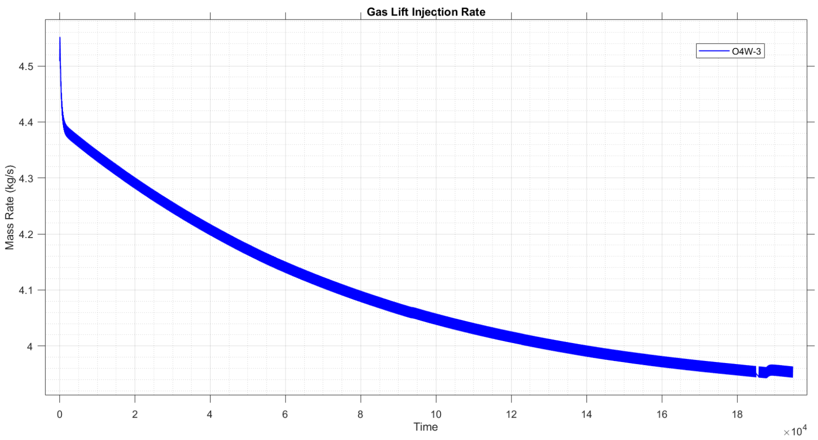


Figure 4.14: Gas injection rate with initial value above optimum point - O4W3 with dynamic model

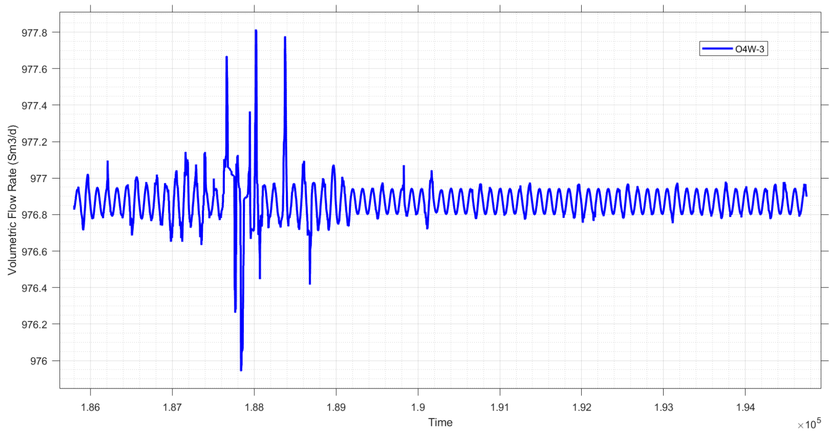


Figure 4.15: Produced oil rate with initial value above optimum point - O4W3 with dynamic model

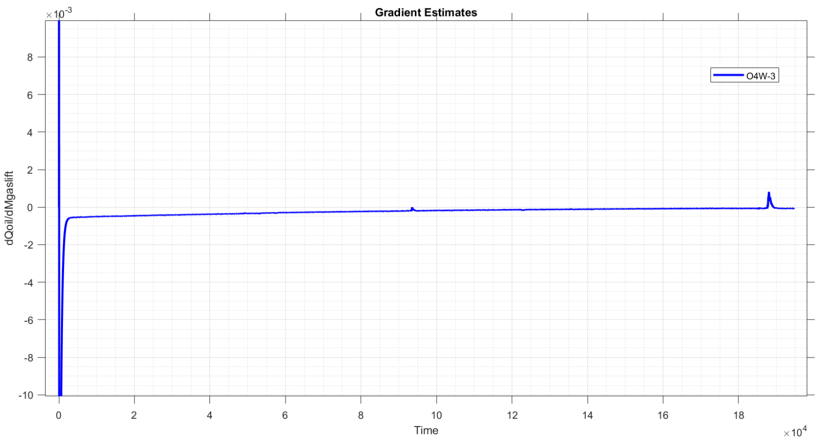


Figure 4.16: Gradient estimate with initial value above optimum point - O4W3 with dynamic model

In cases of approaching the optimum point from above and below, the dynamic and static shows the same transient gradient. The gradient is negative when approaching from above, see Figure 4.10 and Figure 4.16, and positive when approaching from below, see Figure 4.10 and Figure 4.13, meaning that implementing Matlab as a controller and OLGA for a simulator dynamic plant can be implemented for ESC.

The static and dynamic simulation result for a single well is shown in Table 4.4. It is shown that in the static model, ESC achieved the same extremum value of gas lift rate, and in the dynamic model is a slight difference between approaching from above and below of optimum point. However, both models show that the produced oil is the same.

Table 4.4: Result of the static and dynamic model for O4W-3

Model	Initial Value Variation	Extremum value gas lift rate (kg/s)	Extremum value oil rate (Sm3/d)
Static	i = 0 kg/s	4.07	976.7
	I = 6 kg/s	4.07	976.4
Dynamic	i = 3 kg/s	3.89	976.5
	I = 4.5 kg/s	3.95	976.9

4.3 ESC for 3 Wells without Constraint

4.3.1 Static Model

In this part, the ESC utilization by extending up to 3 wells. The wells have different Productivity Index (PI) as illustrated in Table 3.1. Therefore the extremum value is expected to be varied.

Table 4.5: ESC parameter for 3 wells static model

Parameter	**Symbol**	**value**	**unit**
Period	P	12	s
Perturbation Frequency	ω	0.5236	rad/s
Perturbation Amplitude	A	0.03	-
High-pass filter cut-off	HPF = $\omega/2$	0.2618	rad/s
Low-pass filter cut off	LPF = $\omega/4$	0.1309	rad/s
Gain	g	0.05	-
Initial value of gas lift rate	i	0	kg/s
Phase O4W-1	ϕ	0	rad
Phase O4W-2	ϕ	2*pi/3	rad
Phase O4W-3	ϕ	4*pi/3	rad

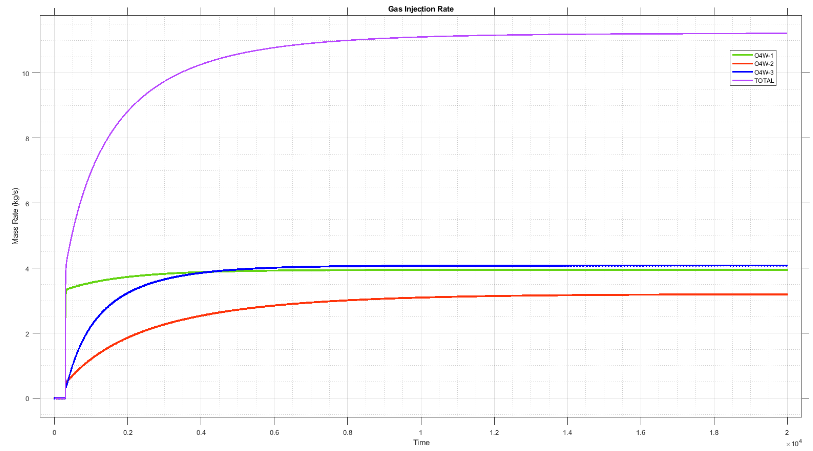


Figure 4.17: Gas injection rate for 3 wells without constraints with static model

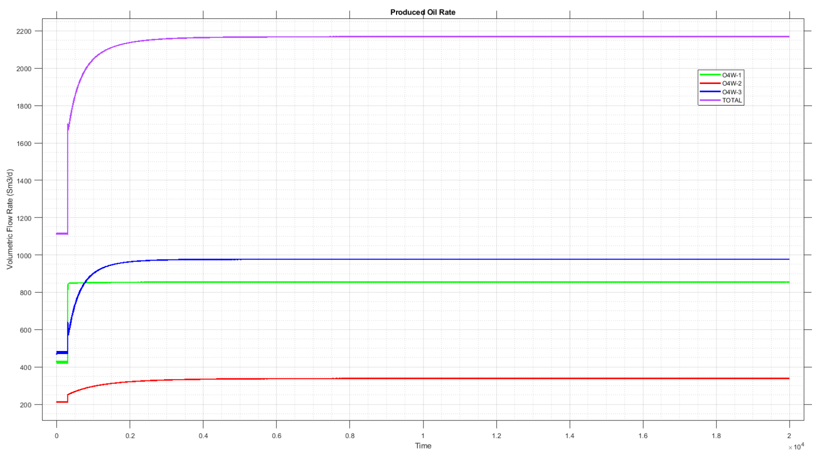


Figure 4.18: Produced oil rate for 3 wells without constraints with static model

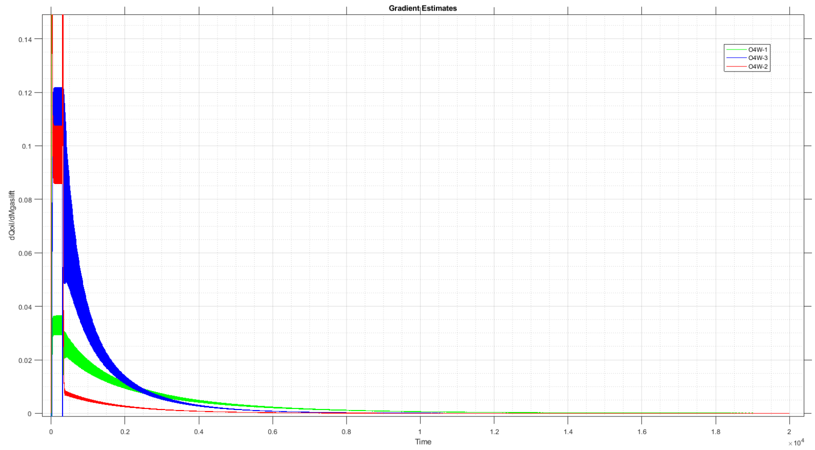


Figure 4.19: Gradient estimate for 3 wells without constraints with static model

4.3.2 Dynamic Model

In this part, the ESC is applied to 3 independent wells without constraint, with the tuning parameter shown in Table 4.6.

Table 4.6: ESC parameter for 3 wells dynamic model

Parameter	**Symbol**	**value**	**unit**
Period	P	120	s
Perturbation Frequency	ω	0.0524	rad/s
Perturbation Amplitude	A	0.01	-
High-pass filter cut-off	$HPF = \omega/2$	0.0262	rad/s
Low-pass filter cut off	$LPF = \omega/20$	0.0026	rad/s
Gain	g	0.006	-
Initial value of gas lift rate	i	3	kg/s
Phase O4W-1, O4W-2, and O4W-3	ϕ	0	rad

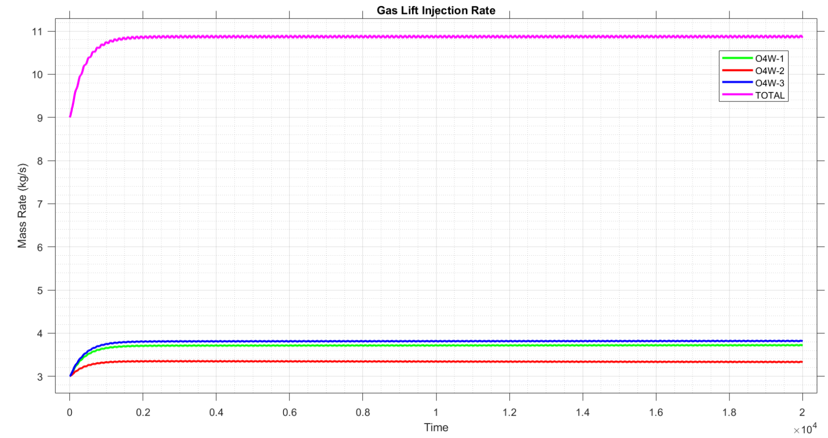


Figure 4.20: Gas injection rate for 3 wells without constraints with dynamic model

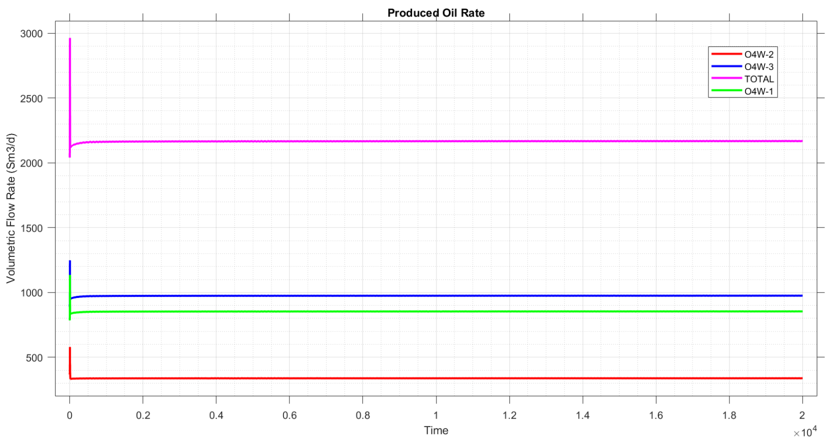


Figure 4.21: Produced oil rate for 3 wells without constraints with dynamic model

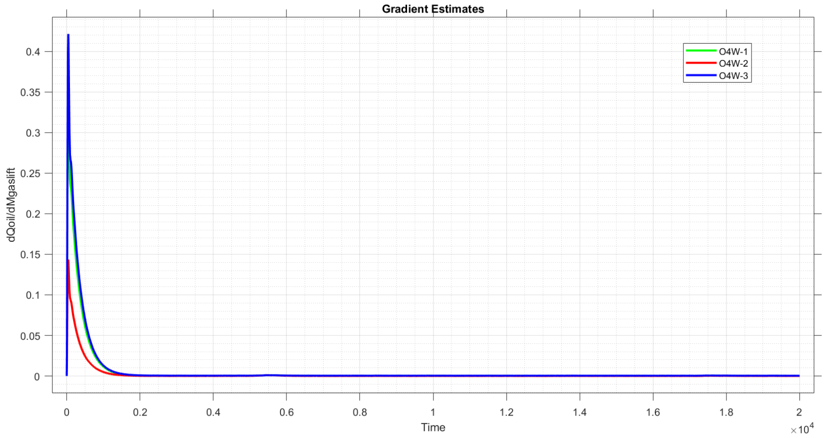


Figure 4.22: Gradient estimate for 3 wells without constraints with dynamic model

In the static model, varying phase of the perturbation signal is applied to reduce the fluctuation of total performance, and the step block is setup up to 300s to active to minimize the fluctuation of the gas injection to zero due to varying phase.
Both static and dynamic reach the optimum point, and a similar shape of the gradient estimate for both models can be seen, which equals zero after going to the positive value. The static and dynamic simulation results are presented in Table 4.7 as follows,

Table 4.7: Result of 3 wells without constraint in static and dynamic model

Model	Well Number	Max.Gas Lift (kg/s)	Max.Oil Rate (Sm3/d)
Static	O4W-1	3.94	854.3
	O4W-2	3.17	338.1
	O4W-3	4.08	976.4
	Total	11.2	2168.8
Dynamic	O4W-1	3.76	854.1
	O4W-2	3.3	338.3
	O4W-3	3.87	976.4
	Total	10.93	2168.8

4.4 Synchronization Based Optimizer

The synchronization gradient is conducted for both static and dynamic models, as both models will be compared in the optimizing three wells with constraint, which will be simulated further. The tuning parameter used for the static model follows Table 4.5 and for the dynamic model, follows Table 4.6. The Matlab Model for synchronization-based optimizer can be seen in Appendix C.

Static Model

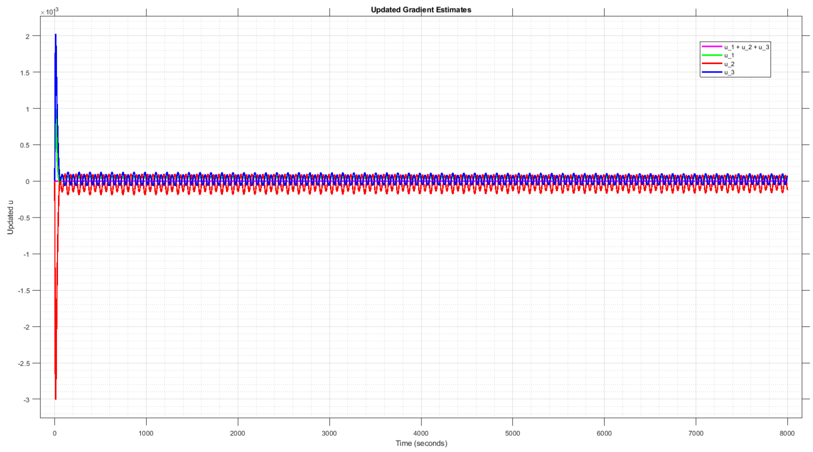


Figure 4.23: Synchronization Gradient in Static Model

Dynamic Model

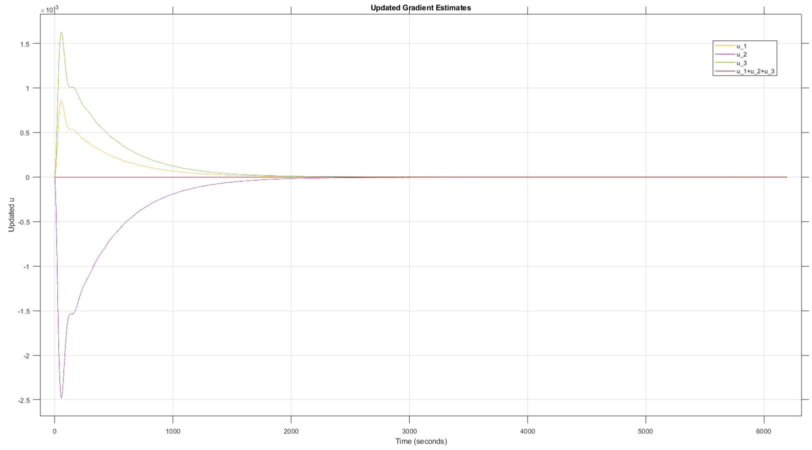


Figure 4.24: Synchronization Gradient in Dynamic Model

The result for the synchronization-based optimizer for static and dynamic satisfied the condition in Equation 3.8.

4.5 ESC for 3 Wells with Constraints

For 3 wells with constraints, the simulation is conducted with cases for sufficient and insufficient available gas injected into the well for both static and dynamic. The Matlab Model for static and dynamic models can be seen in Appendix D.

4.5.1 Static Model

The tuning parameter is shown in Table 4.8 for this part.

Table 4.8: ESC parameter for 3 wells static model

Parameter	**Symbol**	**value**	**unit**
Period	P	12	s
Perturbation Frequency	ω	0.5236	rad/s
Perturbation Amplitude	A	0.03	-
High-pass filter cut-off	HPF $= \omega/2$	0.2618	rad/s
Low-pass filter cut off	LPF $= \omega/4$	0.1309	rad/s
Gain	g	0.05	-
Initial value of gas lift rate	i	1	kg/s
Phase O4W-1	ϕ	4*pi/3	rad
Phase O4W-2	ϕ	2*pi/3	rad
Phase O4W-3	ϕ	0	rad

Sufficient Gas Injection Condition

The available gas in the storage, U_{max}, is 10 kg/s, as the initial value for each well is 1 kg/s so that the total available gas is 13 kg/s, while the required gas to optimize all the well is 11.2 kg/s. Therefore, the total available gas to optimize all the wells is sufficient in this condition.

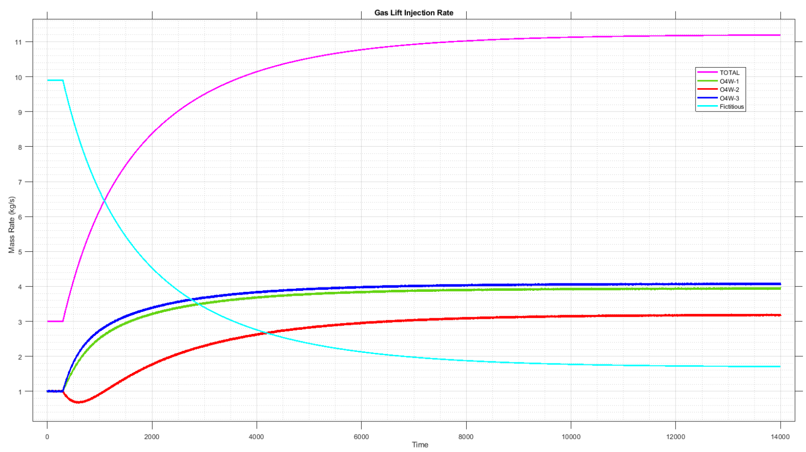


Figure 4.25: Gas injection rate for 3 wells with constraints sufficient gas injection with static model

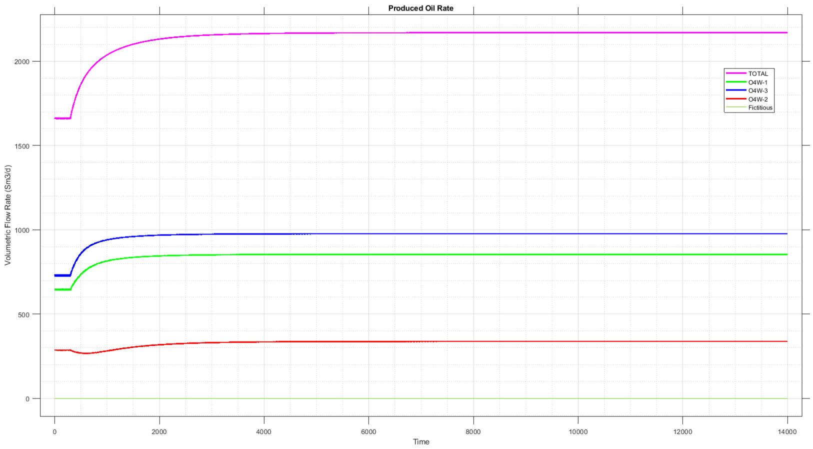


Figure 4.26: Produced oil rate for 3 wells with constraints sufficient gas injection with static model

The result is presented in Table 4.9

Table 4.9: Result multi-well constraint with sufficient available gas - Static Model

Well Number	Gas Lift (kg/s)	Oil Rate(Sm^3/d)
O4W-1	3.95	854.4
O4W-2	3.20	338.1
O4W-3	4.09	976.8
Total	11.24	2169.3

Insufficient Gas Injection Condition

The gas not injected yet, is 6.6 kg/s, as the initial value for each well is 1 kg/s, so the total available gas,U_{max}, will be injected 9.6kg/s, while the required gas to optimize all the well is 11.2 kg/s. Therefore, this condition is called insufficient gas injection, since the total available gas is less than required.

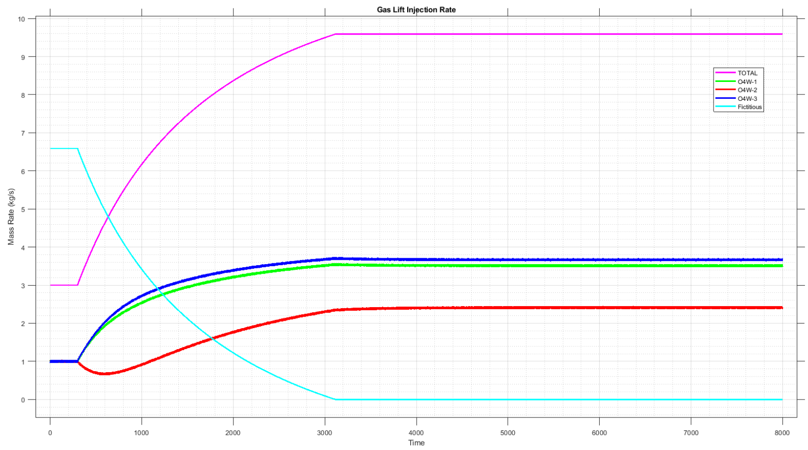


Figure 4.27: Gas injection for rate 3 wells with constraints insufficient gas injection with static model

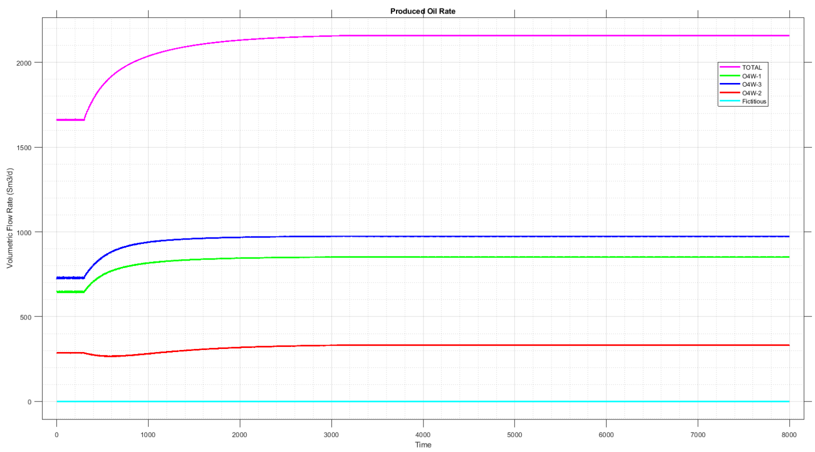


Figure 4.28: Produced oil rate 3 wells with constraints insufficient gas injection with static model

The result is presented in Table 4.10

Table 4.10: Result multi-well constraint with insufficient available gas - Static Model

Well Number	Gas Lift (kg/s)	Oil Rate(Sm^3/d)
O4W-1	3.5	851.6
O4W-2	2.4	332.1
O4W-3	3.7	973.9
Total	9.6	2157.6

4.5.2 Dynamic Model

For the dynamic model, mainly the tuning parameter used for sufficient and insufficient cases follows Table 4.6.

Sufficient Gas Injection Condition

The only different parameter in this sufficient gas injection condition is the phase for all the wells is 0 rad.

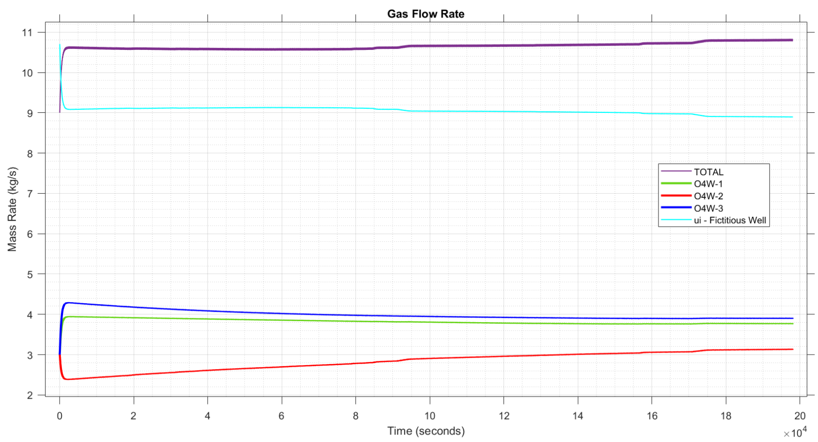


Figure 4.29: Gas rate injection for 3 wells with constraints sufficient gas injection with dynamic model

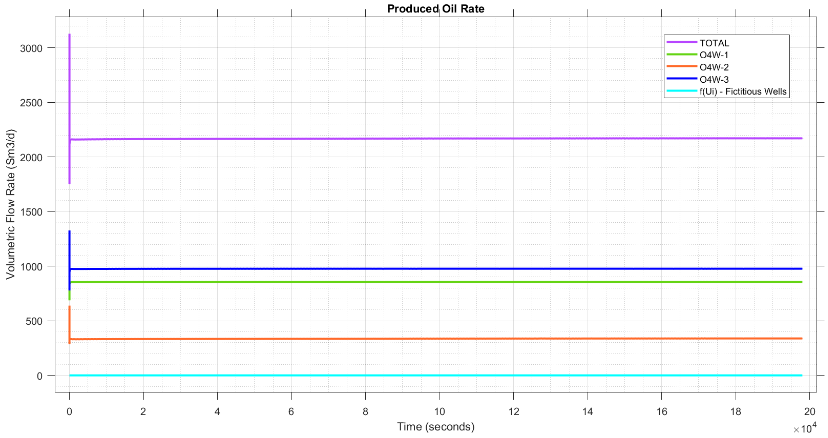


Figure 4.30: Produced oil for 3 wells with constraints sufficient gas injection with dynamic model

The result for sufficient gas injections is presented in Table 4.11. The result shows that the gas lift and oil rate amounts are the same as the 3 wells dynamic system without constraint in Table 4.7. The only difference between them is the gradient transient since the gradient has been synchronized for cases of the gas-lift subject to constraints.

Table 4.11: Result multi-well constraint with sufficient available gas - Dynamic Model

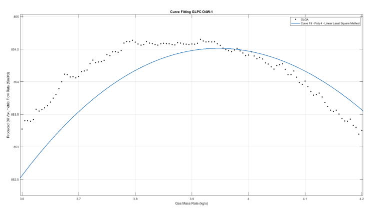
Well Number	Gas Lift (kg/s)	Oil Rate(Sm^3/d)
O4W-1	3.77	855.5
O4W-2	3.14	338.3
O4W-3	3.91	976.9
Total	10.82	2170.7

Chapter 5

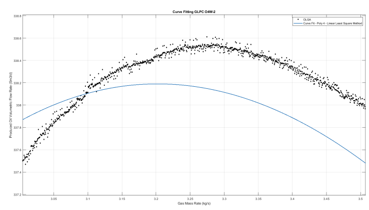
Discussion

5.1 Curve Fitter

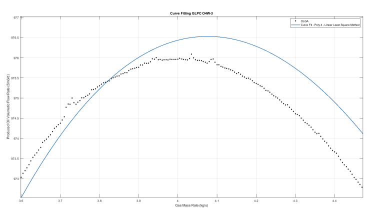
The Curve Fitter is achieved successfully as the tabulated data to create the GLPC mainly fits with the curve. By comparing the static and dynamic model values, there is some slight discrepancy if the curve is zoomed in and the fitting curve is not actually fit the tabulated data, as illustrated in Figure 5.1. This is why the result between the static and dynamic models is not perfectly matched with each other.



(a) O4W-1



(b) O4W-2



(c) O4W-3

Figure 5.1: Showing three cars in different colors horizontally.

5.2 Static Model

The simulation run on the static model shows a good result and looks perfect as per GLPC curve built on this thesis. Since no dynamic system is involved, the tuning parameter can be chosen flexibly. Therefore, the frequency is faster than the dynamic model and gets a simulation result quickly.
Applying the different phases for each dither signal for the three wells is proven to reduce the fluctuation in the system; it can be seen from the total curve fluctuation that is reduced and almost flat line. Synchronizing the gradient and constraining the gas lift with a fictitious well is achieved. It can be seen that the gradient for all of the wells is moving on the same gradient on the gradient estimates curves. Therefore the transient in gas injection rate for cases constraint and non-constraint are different.
Applying the fictitious well for multi-well with constraint is also achieved for insufficient and sufficient available gas cases. In cases of sufficient gas supply, the result value of extremum gas lift and produced oil are precisely the same as that of multi-well without constraint. However, there are some challenges when selecting the solver selection in Matlab for cases of fictitious wells. The solver must be selected in fixed-step since the solver with various variable steps will lead the fictitious well function to an error in Matlab.

5.3 Dynamic Model

The dynamic model run with OLGA contains time-dependent transient behavior. The frequency is chosen arbitrarily with a period 120 s for all dynamic simulations. Due to picking a large period, the simulation can take 55 hours to converge for cases of gas injection rates subject to constraints. It can be seen from the Figure 5.1 that implementation of Extremum Seeking Control (ESC) in a dynamic model. The simulation shows a good result, in which the optimization for multi-well with constraints can be established in OLGA, multiphase software, by using Matlab as a controller. Though there is a slight discrepancy between the static and dynamic results, this thesis's objective is not the dynamic model's accuracy compared to the static model, since ESC doesn't need a model. The phase on the dither signal is also proven to reduce the fluctuation in the system, as can be seen in the simulation, which has a different dither signal, showing less fluctuation in the total rate. However, a block step must be added to prevent the signal from going below an unexpected point at the beginning of the simulation due to phase selection. In Matlab, the gradient calculation was kept going even before the step was activated to give output to the integrator. This algorithm helps the write output in OLGA, which will not be stable if the gas injection is below 1.5 kg/s.

5.4 Recommendation for future work

The investigation of applying ESC in gas lift optimization is relatively new. Most research has been conducted with static wells, especially for multi-well optimization. Applying ESC, by combining Matlab and multiphase dynamic simulator like OLGA, is an excellent step to prove the application of ESC in the more applicable stage. However, a lot of investigation and research needs to be done regarding this topic to be applicable. Some recommendations for future work can be listed as follows,

- The dither signal in this work simplifies opening choke and flow. Determining the response of choke opening and multiphase fluid flow rate is interesting, as this would greatly affect the general application of ESC in the oilfield.
- Combining Extremum Seeking Control (ESC) with existing real-time model-based optimization like MPC. Since ESC is adaptive and model-free optimization, the problem in the well will be difficult to be detected. Therefore, to handle the drawback of ESC, combining the ESC with MPC to handle uncertainties will be interesting to get a more robust optimization controller.
- In a production operation, the oil production is not directly measured on the well. Usually, the total oil rate is measured in the separator, and the contribution from each well is measured based on the most often available data such as WHP, BHP, and temperature. Therefore, research for applying ESC using the most often available data for optimizing multiple wells will be interesting.

Chapter 6

Conclusion

The following summary of the key finding can be extracted from this thesis:

- Gas-lift is the most common artificial lift method in the petroleum industry. Injecting the gas into the well lowers the tubing production's hydrostatic pressure. Offline optimization, NODAL Analysis, will take time and effort of the production engineer to optimize the gas-lift well. Real-time optimization with a model-based optimization typically has model uncertainties that can lead to production under the optimum point. Extremum-seeking control is an adaptive control and model-free, the gradient estimators drive the input of the plant until the output reaches the optimum point. Therefore, extremum-seeking control is suitable for gas lift optimization, since the GLPC shape is concave and has only one extremum point.
- Optimization synchronization-based optimizer and gas injection subject to constraints method, refer to Pavlov et al. (2017) [21] works, are successfully implemented in the dynamic multiphase simulator, OLGA. Both insufficient and sufficient available gas conditions are simulated in this work. In cases of multiple wells for sufficient available gas, the wells move in the same gradient and can find the optimum point as the multiple wells without constraint.
- Extremum seeking control was applied to static and dynamic plat models. The static model was GLPC function and successfully converged to the optimum point for a case with a single well and multiple well without and with constraints.
- The dynamic model was simulated using the dynamic multiphase simulator, OLGA. Three dummy vertical well was created based on the default template in OLGA with varied Productivity Index (PI). A step-rate test performed in OLGA illustrated that GLPC is concave.
- The curve fitter on Matlab is implemented to find the static plant function. The polynomial degree 4 is the best fit for all the wells. Though there is some discrepancy in the optimum point between the curve fitter and tabulated data from OLGA, accuracy is not the main concern for this work.
- The amplitude in the dither signal is relatively small, but the accumulation

of the dither signal in total performance can be considered to the system. The implementation coordinating phase of the dither signal was successfully implemented in some simulations of this work; refer to the method from Pavlov et al. (2021) [18].

Bibliography

[1] *Production system*, en, Jun. 2017. [Online]. Available: https://petrowiki.spe.org/Production_system (visited on 26/06/2023).

[2] B. Guo, *Petroleum Production Engineering, A Computer-Assisted Approach*, eng. Elsevier Science, 2011, OCLC: 968995194, ISBN: 9780080479958.

[3] M. Golan and C. H. Whitson, *Well performance*, 2nd ed. Englewood Cliffs, N.J: Prentice Hall, 1991, ISBN: 9780139466090.

[4] W. Gilbert, 'Flowing and Gas Lift Well Performance,' English, no. American Petroleum Institute, May 1954. [Online]. Available: http://www.ipt.ntnu.no/~asheim/prodbr/Paper_Gilbert.pdf (visited on 26/06/2023).

[5] *Artificial lift*, en, Apr. 2017. [Online]. Available: https://petrowiki.spe.org/Artificial_lift (visited on 26/06/2023).

[6] F. Lake, 'Relation of the Air-gas Lift to Gas-oil Ratios and Effect on Ultimate Production,' en, *Transactions of the AIME*, vol. 77, no. 01, pp. 173–188, Dec. 1927, ISSN: 0081-1696. DOI: 10.2118/927173-G. [Online]. Available: https://onepetro.org/TRANS/article/77/01/173/161238/Relation-of-the-Air-gas-Lift-to-Gas-oil-Ratios-and (visited on 26/06/2023).

[7] T. Bergeron, A. Cooksey and J. S. Reppel, 'New Automated Continuous Gas-Lift Control System Improves Operational Efficiency,' in *All Days*, Oklahoma City, Oklahoma: SPE, Mar. 1999, SPE–52123–MS. DOI: 10.2118/52123-MS. [Online]. Available: https://onepetro.org/SPEOKOG/proceedings/99MCOS/All-99MCOS/SPE-52123-MS/60309 (visited on 26/06/2023).

[8] G. A. Alarco´n, C. F. Torres and L. E. Go´mez, 'Global Optimization of Gas Allocation to a Group of Wells in Artificial Lift Using Nonlinear Constrained Programming,' en, *Journal of Energy Resources Technology*, vol. 124, no. 4, pp. 262–268, Dec. 2002, ISSN: 0195-0738, 1528-8994. DOI: 10.1115/1.1488172. [Online]. Available: https://asmedigitalcollection.asme.org/energyresources/article/124/4/262/456748/Global-Optimization-of-Gas-Allocation-to-a-Group (visited on 26/06/2023).

[9] E. Camponogara, A. Plucenio, A. F. Teixeira and S. R. Campos, 'An automation system for gas-lifted oil wells: Model identification, control, and optimization,' en, *Journal of Petroleum Science and Engineering*, vol. 70, no. 3-4, pp. 157–167, Feb. 2010, ISSN: 09204105. DOI: 10.1016/j.petrol.2009.11.003. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S092041050900237X (visited on 26/06/2023).

[10] H. D. Beggs, *Production optimization: using NODAL analysis*. Tulsa, Okla: OGCI Publications, 1991, ISBN: 9780930972141.

[11] K. E. Brown and J. F. Lea, 'Nodal Systems Analysis of Oil and Gas Wells,' en, *Journal of Petroleum Technology*, vol. 37, no. 10, pp. 1751–1763, Oct. 1985, ISSN: 0149-2136, 1944-978X. DOI: 10.2118/14714-PA. [Online]. Available: https://onepetro.org/JPT/article/37/10/1751/73380/Nodal-Systems-Analysis-of-Oil-and-Gas-Wells (visited on 26/06/2023).

[12] M. Yakoot and S. Shedid, 'Simulation Study of Technical and Feasible Gas Lift Performance,' English, *Internation Journal of Petroleum Science and Engineering*, vol. 10, pp. 21–44, Sep. 2016. [Online]. Available: https://www.researchgate.net/publication/308062978_Simulation_Study_of_Technical_and_Feasible_Gas_Lift_Performance.

[13] *Real-Time Optimization*, en-US, Mar. 2017. [Online]. Available: https://www.techopedia.com/definition/182/real-time-optimization-rto (visited on 26/06/2023).

[14] D. Krishnamoorthy, B. Foss and S. Skogestad, 'Real-Time Optimization under Uncertainty Applied to a Gas Lifted Well Network,' en, *Processes*, vol. 4, no. 4, p. 52, Dec. 2016, ISSN: 2227-9717. DOI: 10.3390/pr4040052. [Online]. Available: http://www.mdpi.com/2227-9717/4/4/52 (visited on 26/06/2023).

[15] K. B. Ariyur and M. Krstić, *Real time optimization by extremum seeking control*. Hoboken, NJ: Wiley Interscience, 2003, ISBN: 9780471468592.

[16] M. Krstić and H.-H. Wang, 'Stability of extremum seeking feedback for general nonlinear dynamic systems,' en, *Automatica*, vol. 36, no. 4, pp. 595–601, Apr. 2000, ISSN: 00051098. DOI: 10.1016/S0005-1098(99)00183-1. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0005109899001831 (visited on 26/06/2023).

[17] D. Nesic, 'Extremum seeking control: Convergence analysis,' in *2009 European Control Conference (ECC)*, Budapest: IEEE, Aug. 2009, pp. 1702–1715, ISBN: 9783952417393. DOI: 10.23919/ECC.2009.7074649. [Online]. Available: https://ieeexplore.ieee.org/document/7074649/ (visited on 26/06/2023).

[18] M. Haring, S. Fossoy, T. L. Silva and A. Pavlov, 'Nondisturbing Extremum Seeking Control for Multiagent Industrial Systems,' *IEEE Transactions on Automatic Control*, vol. 68, no. 3, pp. 1492–1507, Mar. 2023, ISSN: 0018-9286, 1558-2523, 2334-3303. DOI: 10.1109/TAC.2022.3153228. [Online]. Available: https://ieeexplore.ieee.org/document/9721126/ (visited on 30/06/2023).

[19] D. Krishnamoorthy, A. Pavlov and Q. Li, 'Robust Extremum Seeking Control with application to Gas Lifted Oil Wells,' en, *IFAC-PapersOnLine*, vol. 49, no. 13, pp. 205–210, 2016, ISSN: 24058963. DOI: 10.1016/j.ifacol.2016.07.952. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S240589631631240X (visited on 26/06/2023).

[20] M. A. Haring, 'Extremum-seeking control: Convergence improvements and asymptotic stability,' English, Doctoral theses at NTNU, 2016:167, NTNU, Trondheim, Jun. 2016. [Online]. Available: https://ntnuopen.ntnu.no/ntnu-xmlui/bitstream/handle/11250/2394949/Mark_Haring_PhD.pdf?sequence=1 (visited on 26/06/2023).

[21] A. Pavlov, M. Haring and K. Fjalestad, 'Practical extremum-seeking control for gas-lifted oil production,' in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Melbourne, Australia: IEEE, Dec. 2017, pp. 2102–2107, ISBN: 9781509028733. DOI: 10.1109/CDC.2017.8263957. [Online]. Available: http://ieeexplore.ieee.org/document/8263957/ (visited on 26/06/2023).

[22] D. Krishnamoorthy, J. Ryu and S. Skogestad, 'A Dynamic Extremum Seeking Scheme Applied to Gas Lift Optimization,' en, *IFAC-PapersOnLine*, vol. 52, no. 1, pp. 802–807, 2019, ISSN: 24058963. DOI: 10.1016/j.ifacol.2019.06.160. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S2405896319302472 (visited on 26/06/2023).

[23] K. Rashid, 'Optimal Allocation Procedure for Gas-Lift Optimization,' en, *Industrial & Engineering Chemistry Research*, vol. 49, no. 5, pp. 2286–2294, Mar. 2010, ISSN: 0888-5885, 1520-5045. DOI: 10.1021/ie900867r. [Online]. Available: https://pubs.acs.org/doi/10.1021/ie900867r (visited on 26/06/2023).

[24] A. J. Peixoto, D. Pereira-Dias, A. F. Xaud and A. R. Secchi, 'Modelling and Extremum Seeking Control of Gas Lifted Oil Wells,' en, *IFAC-PapersOnLine*, vol. 48, no. 6, pp. 21–26, 2015, ISSN: 24058963. DOI: 10.1016/j.ifacol.2015.08.004. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S240589631500871X (visited on 26/06/2023).

[25] H. Ausen, 'A Study in MINLP-class Optimization Problems for Simulated Petroleum Production,' Ph.D. dissertation, Norwegian University of Science and Technology, Trondheim, Jun. 2012. [Online]. Available: https://ntnuopen.ntnu.no/ntnu-xmlui/bitstream/handle/11250/260617/565928_FULLTEXT01.pdf?sequence=2&isAllowed=y.

[26] *Part 1 - OLGA OPC Server Configuration*, en. [Online]. Available: https://vimeo.com/57932521 (visited on 26/06/2023).

[27] M. Guay and D. Dochain, 'A proportional-integral extremum-seeking controller design technique,' en, *Automatica*, vol. 77, pp. 61–67, Mar. 2017, ISSN: 00051098. DOI: 10.1016/j.automatica.2016.11.018. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0005109816304538 (visited on 26/06/2023).

[28] D. Dochain, M. Perrier and M. Guay, 'Extremum seeking control and its application to process and reaction systems: A survey,' en, *Mathematics and Computers in Simulation*, vol. 82, no. 3, pp. 369–380, Nov. 2011, ISSN: 03784754. DOI: 10.1016/j.matcom.2010.10.022. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0378475410003290 (visited on 26/06/2023).

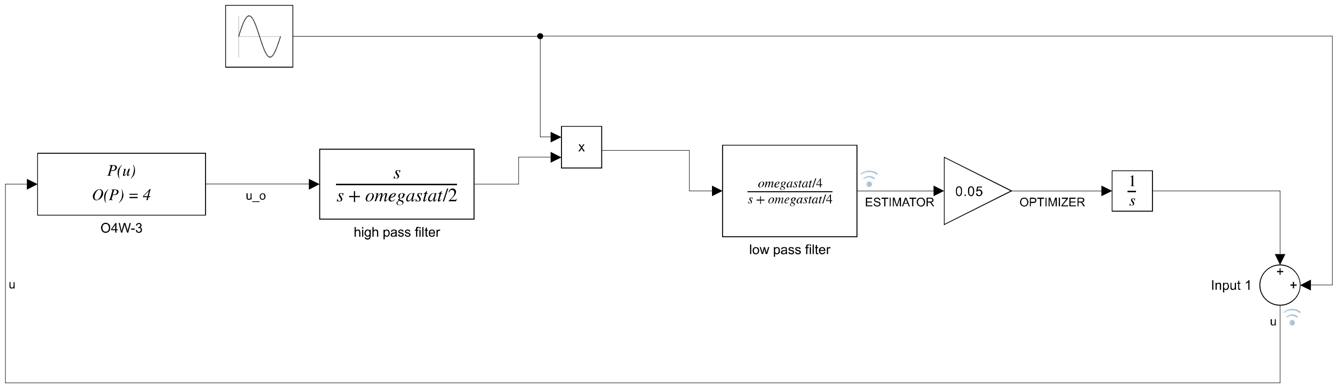
[29] P. Wang, M. Litvak and K. Aziz, 'Optimization of Production Operations in Petroleum Fields,' in *All Days*, San Antonio, Texas: SPE, Sep. 2002, SPE–77658–MS. DOI: 10.2118/77658-MS. [Online]. Available: https://onepetro.org/SPEATCE/proceedings/02ATCE/All-02ATCE/SPE-77658-MS/136047 (visited on 26/06/2023).

[30] S. M. Elgsæter, 'Modeling and Optimizing the Offshore Production of Oil and Gas under Uncertainty,' Ph.D. dissertation.

Appendix A

Matlab Model - Single Well - O4W3 for ESC

esc_static_omega_05236_O4W3_Reference



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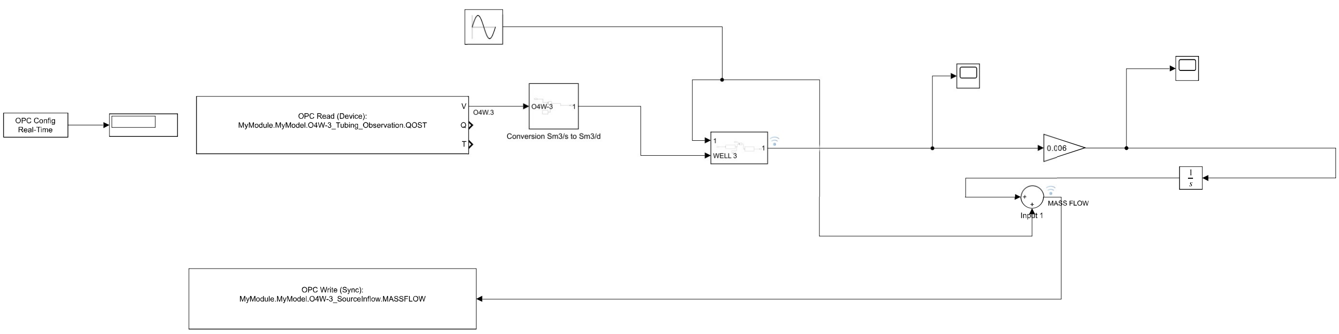
esc_dynamic_omega_005236_O4W3



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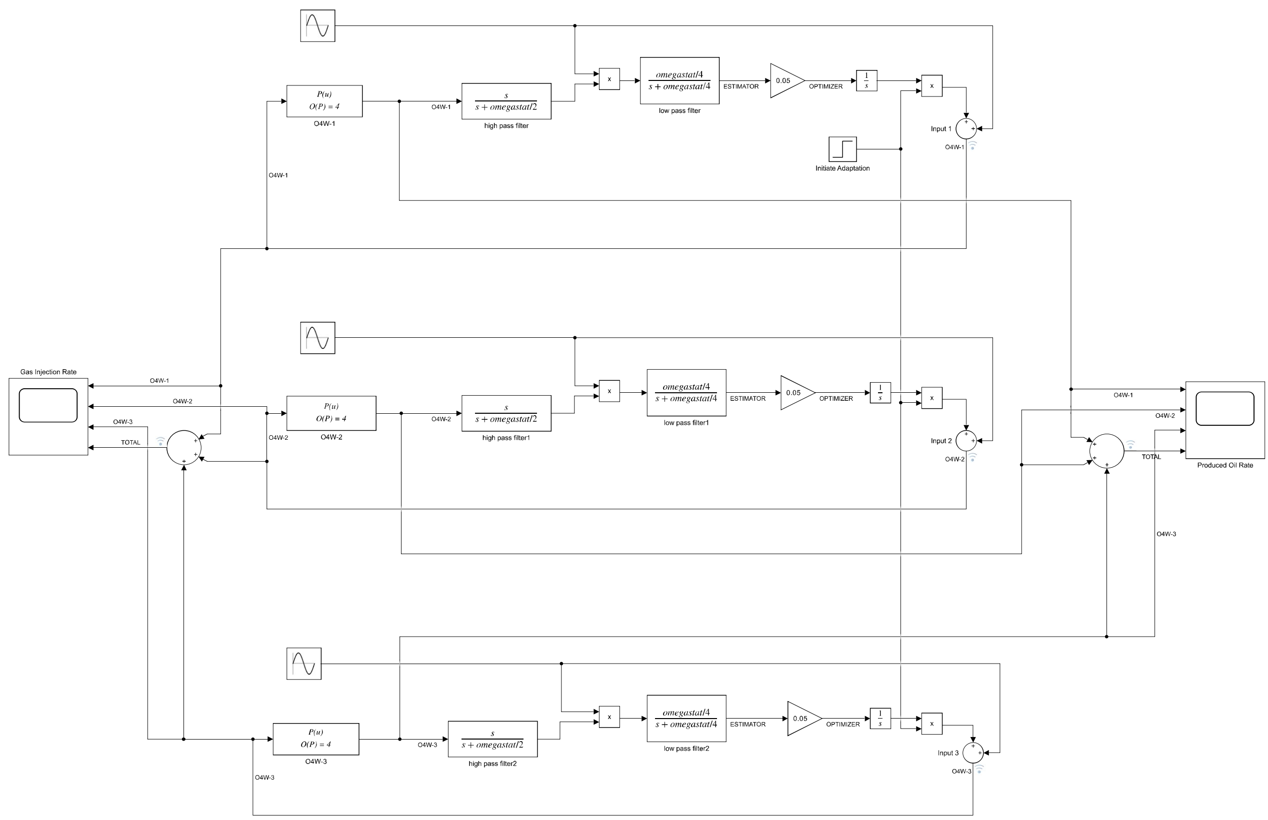
Appendix B

Matlab Model - Extended to 3 Wells for ESC without Constraint

esc_static_3wells_wo_contraint



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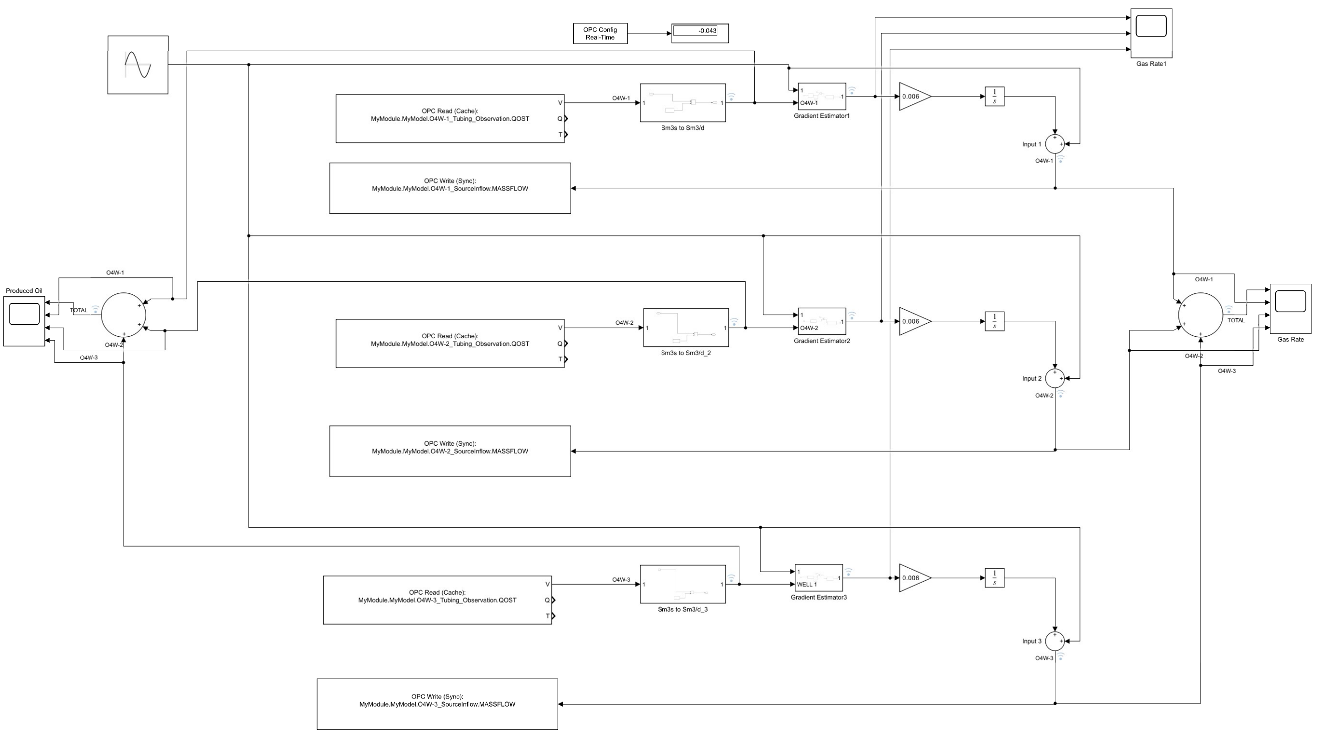
esc_3wells_wo_constraint_sketch



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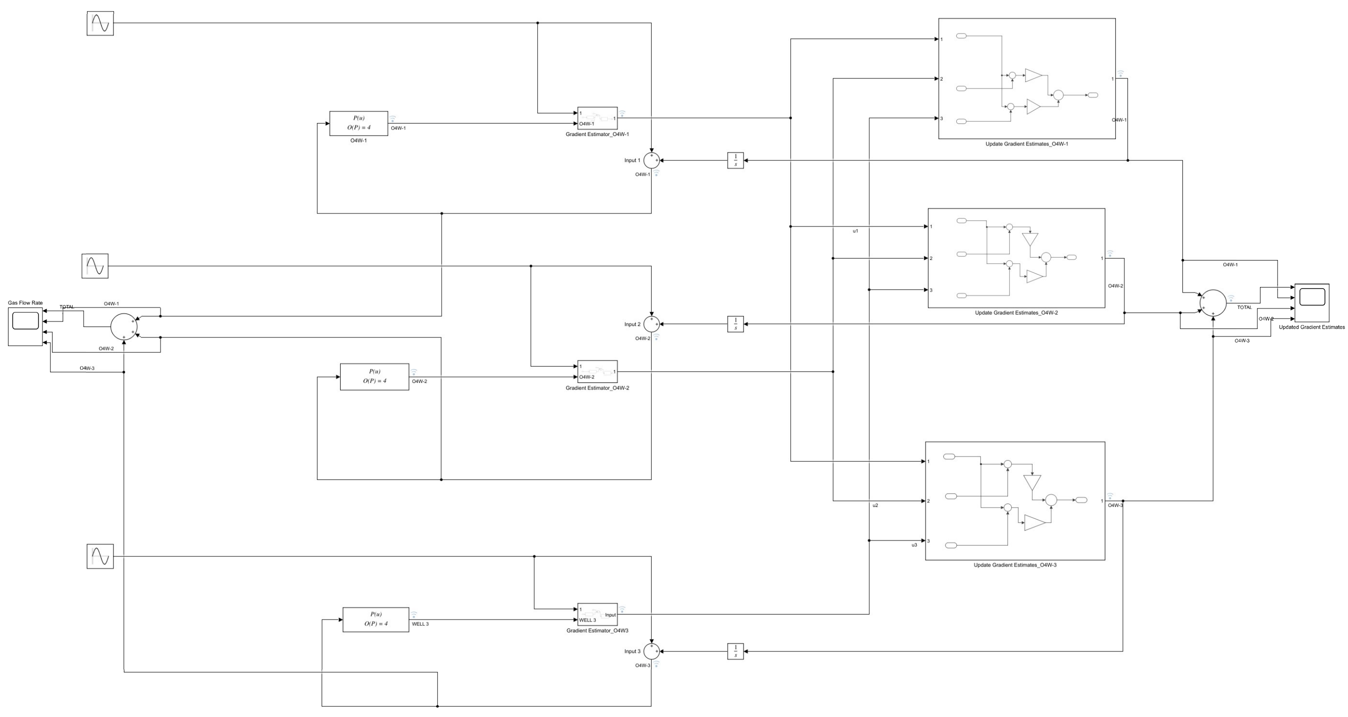
Appendix C

Matlab Model - Synchronization Based Optimizer

esc_static_sync_sketch



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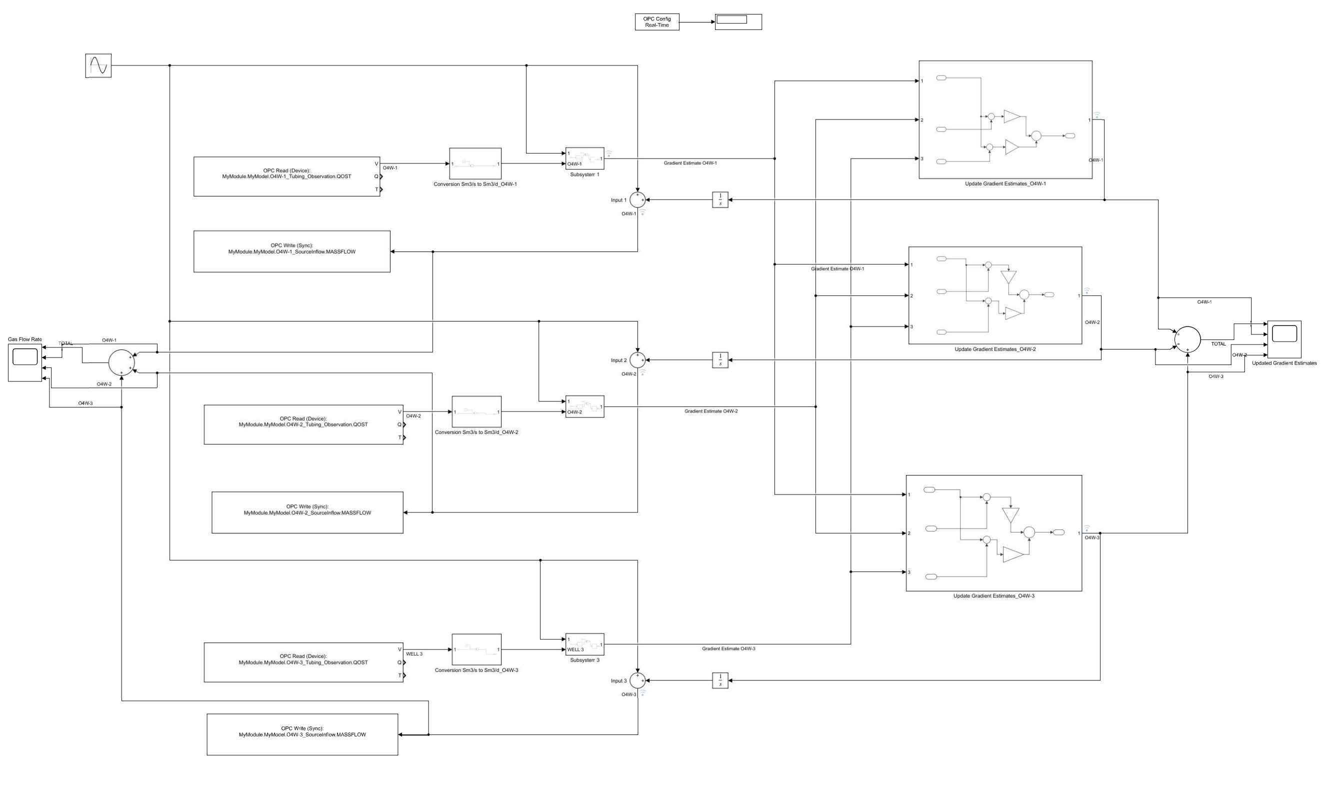
esc_dynamic_sync_sketch



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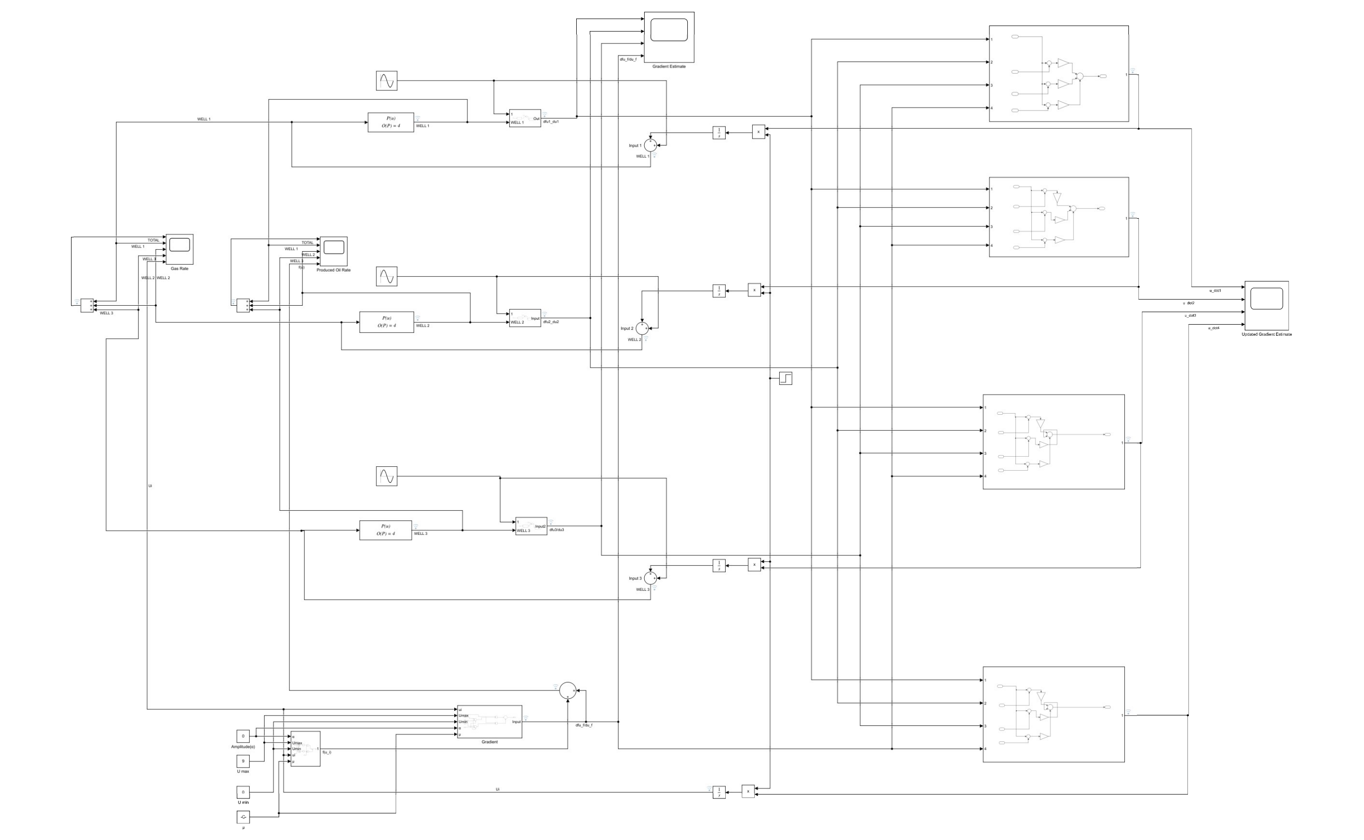
Appendix D

Matlab Model - 3 Wells with Constraint and Fictitious Well

esc_static_3wells_constraint_fictious



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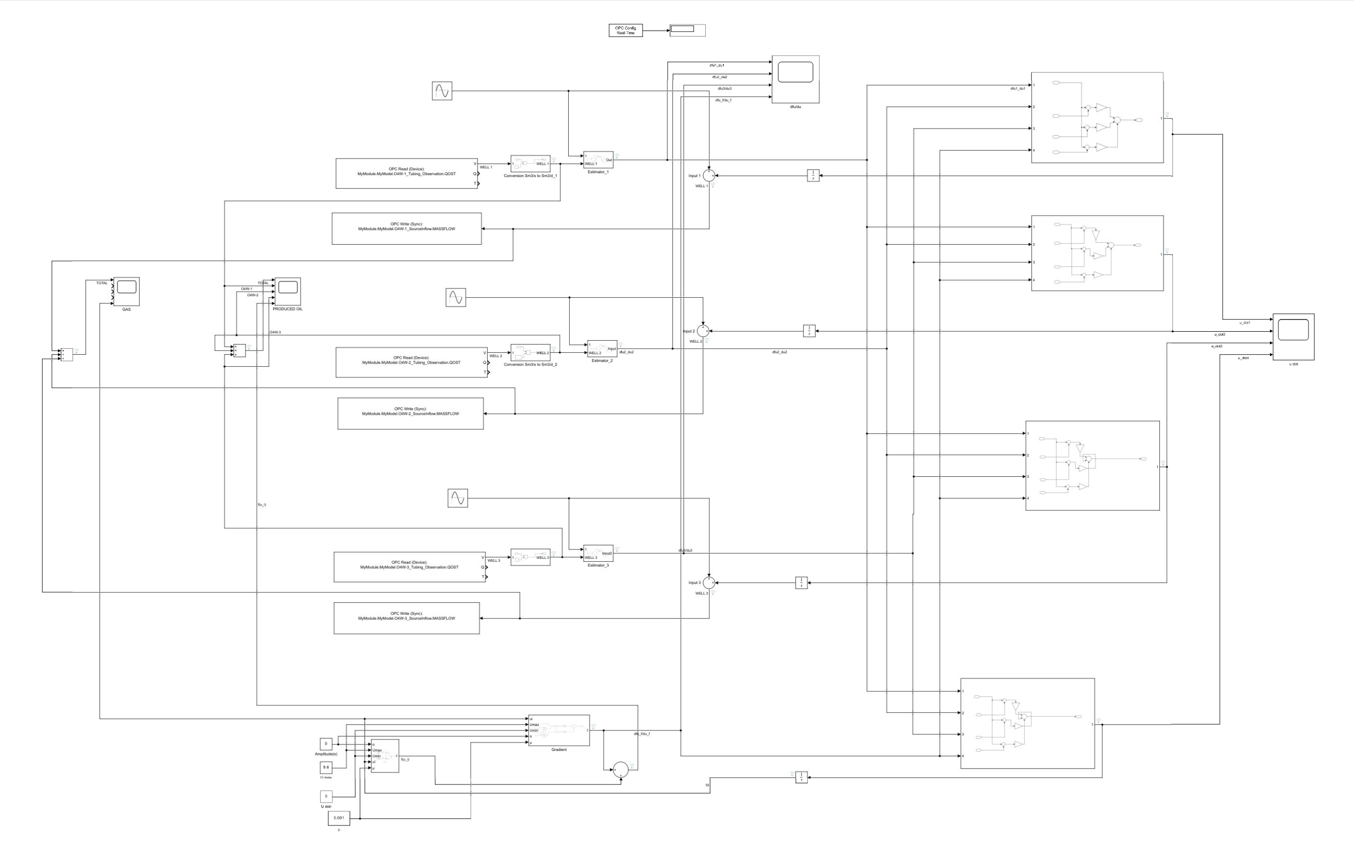
esc_OLGA_MASSFLOW_3wells_constraint_fictious_well



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