The direct topographical correction in gravimetric Geoid determination by the Stokes-Helmert method

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Abstract. The direct topographical correction is composed of both local effects and long-wavelength contributions. This implies that the classical integral formula for determining the direct effect may have some numerical problems in representing these different signals. On the other hand, a representation by a set of harmonic coefficients of the topography to, say, degree and order 360 will omit significant short-wavelength signals. A new formula is derived by combining the classical formula and a set of spherical harmonics. Finally, the results of this solution are compared with the Moritz topographical correction in a test area.

Keywords: Direct effect – Helmert condensation – Spherical harmonics – Geoid

1 Introduction

The geoid is frequently determined from gravity data by the well-known Stokes' formula. This formula is the solution of an exterior-type boundary value problem, implying that masses exterior to the geoid are not permitted in the formulation. This is achieved mathematically by removing the external masses or shifting them inside the geoid (direct effect). The masses are then restored after applying Stokes' integral (indirect effect).

Recognizing that a valid solution to geoid determination would occur only if there were no masses outside the geoid, Helmert suggested that the masses outside the geoid be condensed as a surface layer at sea level in a spherical approximation of the geoid. A discussion of some attributes of Helmert's second method of condensation may be found in Heiskanen and Moritz (1967), Wichiencharoen (1982), Martinec et al. (1993) and Vanicek et al. (1995).

Sjöberg (1994) suggested a spherical harmonic approach to derive the topographical corrections. This approach has been implemented by Sjöberg (1995a, b, 1996a, b, c) to the second power of elevation H and by Nahavandchi and Sjöberg (1998) to the third power of elevation H . The direct effect is derived at the surface of the Earth.

Two different formulas for the remove-restore problem were presented by Moritz (1980) and Vanicek and Kleusberg (1987). Moritz (1968, 1980) examined the role of the topography to show a relationship between Helmert's condensation reduction and the approximate solution of the Molodenskii boundary value problem. He derived the direct effect referred to the geoid. In Vanicek and Kleusberg (1987), the classical boundary value problem was restated and the solution was reformulated. This reformulation led to the derivation of expressions for corrections to free-air gravity anomalies due to the presence of masses above the geoid, i.e., the direct effect referred to the Earth's surface. This means that the gravity anomalies corrected with their formula need a downward-continuation correction to be used in Stokes' integral. These two classical formulas are limited to the second power of elevation H and suffer from planar approximation. Wang and Rapp (1990) compared the direct topographical effect in Moritz's, and Vanicek and Kleusberg's approaches. They discovered a significant difference between these two methods. The difference was explained later by Martinec et al. (1993) as being due to the fact that while Vanicek and Kleusberg's results refer to the Earth's surface, Moritz's results refer to the geoid.

A recent description of the Stokes-Helmert method for geoid determination was given by Vanicek and Martinec (1994). The specific problem on determining the direct effect was treated by Martinec and Vanicek (1994), who pointed out that the classical formula may be severely biased because of the planar approximation in the derivations.

In this paper, we will start to compare the Vanicek and Kleusberg formula, which is based on a planar approximation, with those based on a spherical harmonic approach (a difference between planar and spherical models). A compromise between these two methods is derived. The gravity anomalies corrected with this combined formula are then downward-continued to the geoid by the Poisson integral. Finally, these downward-continued gravity anomalies are compared with those corrected with the Moritz formula for topographical correction.

2 Direct topographical correction in Stokes' formula

Nahavandchi and Sjöberg (1998) have derived a spherical model for the direct effect on gravity and the geoid to the third power of elevation, H. The direct topographical effect on gravity at the topographical surface of the Earth, point P , can be evaluated from Nahavandchi and Sjöberg [1998, Eq. (20)]

$$\delta \mathrm{A}(H_P) \doteq -\frac{\pi\mu}{2R}\left[5H_P^2 + 3\overline{H_P^2} + 2\sum_{n,m} n(H^2)_{nm}Y_{nm}(P)\right]$$
$$+\frac{\pi\mu}{2R^2}\left[\frac{28}{3}H_P^3 + \frac{9}{2}\overline{H_P^2}H_P - \frac{1}{2}\overline{H_P^3} + H_P\sum_{n,m} n(2n+9)(H^2)_{nm}Y_{nm}(P) - \frac{1}{3}\sum_{n,m} n(2n+7)(H^3)_{nm}Y_{nm}(P)\right] \quad (1)$$

The addition theorem for spherical harmonics yields

$$P_n(t) = \frac{1}{2n+1}\sum_{m=-n}^{n} Y_{nm}(Q)Y_{nm}(Q') \quad (2)$$

where $P_n(t)$ is Legendre's polynomial of order n, $t = \cos\psi$, ψ is the geocentric angle between the computation point P and the running point, and Y_{nm} are fully normalized spherical harmonics obeying

$$\frac{1}{4\pi}\int\int_\sigma Y_{nm}Y_{n'm'}\,d\sigma = \begin{cases} 1 & \text{if } n = n' \text{ and } m = m' \\ 0 & \text{Otherwise} \end{cases} \quad (3)$$

and

$$(H^\upsilon)_{nm} = \frac{1}{4\pi}\iint_\sigma H_P^\upsilon Y_{nm}\,d\sigma, \qquad \upsilon = 2,3,\dots. \quad (4)$$

$$H_P^\upsilon = \sum_{n,m}(H^\upsilon)_{nm}\,Y_{nm}(P) = \sum_{n=0}^{\infty} H_n^\upsilon(p) \quad (5)$$

$$\overline{H_P^{\upsilon}} = \sum_{n,m} \frac{1}{2n+1} (H^{\upsilon})_{nm} Y_{nm}(P) = \sum_{n=0}^{\infty} \frac{1}{2n+1} H_n^{\upsilon}(p) \tag{6}$$

Here, P is a point on the topographical surface. It should be mentioned here that all the series in Eq. (1) are truncated in maximum degree and order 360, and Nahavandchi and Sjöberg (1998) have shown that to this degree and order all the series are convergent. Nahavandchi and Sjöberg (1998) also showed that the dominant part of the power series in Eq. (1) is the second power of elevation H. The contribution from the harmonic expansion series H^3 is smaller than 9 cm everywhere. In order to be sure of the convergence of Eq. (1), our preliminary computations show that the contributions from H^4 and H^5 can safely be neglected (see also Nahavandchi 1998).

This harmonic presentation of the direct topographical effect is limited to the third power of elevation H and is very simple to compute. It is free from the problems encountered in classical integral formulas, e.g. the singularity of the integration kernels and planar approximation. However, the harmonic expansion series of H^2 and H^3 will include only the long wavelengths. In order to incorporate all significant contributions from both short and long wavelengths, an expansion of spherical representation of H^2 and H^3 to very high degrees is necessary, which is practically difficult and ruins the simplicity of this method.

The classical integral formula for direct effect de- termination at point P, on the surface of the Earth, can be approximated as (see Vanicek and Kleusberg 1987)

$$\delta A(H_P)^{\text{classic}} = \frac{\mu R^2}{2} \iint_{\sigma} \frac{H^2 - H_P^2}{\ell_0^3} d\sigma \tag{7}$$

where $\mu = G\rho_0, G$ being the universal gravitational constant and ρ_0 the density of topography assumed to be constant(2.67 g cm^{-3}); H, H_P = orthometric heights of the running and computation points; $\ell_0 = \sqrt{2(1-\cos\psi)} = 2R\sin\frac{\psi}{2}$; R= mean Earth radius; and σ = the unit sphere.

This formula was derived from a planar model taking into consideration only the far-zone effect where $\ell_0 \gg H$, and the effect of the near zone is missing. As we will show later [see Eq. (14)], another term which cannot be derived from a planar model is also missing in Eq. (7). This term, which represents a correction for the sphericity of the geoid, has also been derived (called Bouguer shell effect) in Martinec and Vanicek (1994). It should also be mentioned that the accuracy of power series used in the integration is limited to the second-order terms in height. The classic integral formulas are not practical for numerical computations, as they require a global integration to include the long-wavelength information. Thus, a compromise may be in order.

Equation (1) can be reformulated as an integral similar to Eq. (7). In order to achieve this, we first re-write Eq. (1) to the second power of H as follows:

$$\delta A(H_P) = -\frac{\pi\mu}{2R}\left[5H_P^2 + \sum_{n=0}^{\infty} \frac{3}{2n+1} H_n^2(P) + \sum_{n=0}^{\infty} n\, H_n^2(P)\right] \tag{8}$$

Inserting

$$H_n^2(P) = \frac{2n+1}{4\pi} \iint_{\sigma} H^2 P_n(\cos\psi)\, d\sigma \tag{9}$$

and considering that

$$\sum_{n=0}^{\infty} P_n(\cos\psi) = \frac{R}{\ell_0} \tag{10}$$

and (Heiskanen and Moritz 1967, p. 39)

$$-\frac{1}{R}\sum_{n=0}^{\infty} n H_n^2(P) = \frac{R^2}{2\pi}\iint_\sigma \frac{H^2-H_P^2}{\ell_0^3}\,d\sigma \tag{11}$$

we arrive at

$$\delta A(H_P) = -\frac{\pi\mu}{2R}\left[5H_P^2 + \frac{3R}{4\pi}\iint_\sigma \frac{H^2}{\ell_0}\,d\sigma - \frac{R^3}{\pi}\iint_\sigma \frac{H_P^2-H^2}{\ell_0^3}\,d\sigma\right] \tag{12}$$

In view of the fact that

$$\frac{R}{4\pi}\iint_\sigma \frac{d\sigma}{\ell_0} = 1 \tag{13}$$

we finally obtain

$$\delta A(H_P)^{\text{new}} = -\frac{4\pi\mu}{R}H_P^2 - \frac{3\mu}{8}\iint_\sigma \frac{H^2-H_P^2}{\ell_0}\,d\sigma + \frac{\mu R^2}{2}\iint_\sigma \frac{H_P^2-H^2}{\ell_0^3}\,d\sigma \tag{14}$$

Comparing the classical formula of Eq. (7) with the new one of Eq. (14), we obtain the difference

$$\Delta\delta A(H_P) = \delta A(H_P)^{\text{classic}} - \delta A(H_P)^{\text{new}} = -\frac{4\pi\mu}{R}H_P^2 - \frac{3\mu}{8}\iint_\sigma \frac{H^2-H_P^2}{\ell_0}\,d\sigma \tag{15}$$

or, in view Eq. (13)

$$\Delta\delta A(H_P) = -\frac{5\pi\mu}{2R}H_P^2 - \frac{3\mu}{8}\iint_\sigma \frac{H^2}{\ell_0}\,d\sigma \tag{16}$$

or, in spectral form

$$\Delta\delta A(H_P) = -\frac{5\pi\mu}{2R}H_P^2 - \frac{3\pi\mu}{2R}\overline{H_P^2} \tag{17}$$

This difference is significant. The first term on the right-hand side of Eq. (17) may reach as much as 0.36 mGal for H = 4 km, which cannot be neglected when a precise geoid is to be determined. It is also evident that the second term in Eq. (17) cannot be neglected either. For a smooth topography, this term can be approximated by

$$-\frac{3\mu}{8}\iint_\sigma \frac{H^2}{\ell_0}\,d\sigma = -\frac{3\pi\mu}{2R^2}H_P^2 s_0$$

where s_0 is the maximum polar radius. For example, with s_0 = 555 km (corresponding to a geocentric radius of about 5°) and H_P = 6 km it ranges to 0.04 mGal, which cannot be neglected for a precise geoid determination. It should be noted that there might be some other topographic reduction errors in high elevations that could infer that the difference in Eq. (17) is insignificant. They are not under investigation in this study. In Eq. (14), the effect of bending the Bouguer plate into the Bouguer shell (first term on the right-hand side) and some long-wavelength contributions (second term on the right-hand side) are present. However, the problem with this formula is the third term, which only considers the far-zone contributions, where $\ell_0 \gg H$. It has to be modified in some way to consider both the far- and near-zone effects (see below).

Equation (17) shows that there are some long-wavelength differences of power H^2 between the classical and the new formulas. The most likely explanation of this difference is that the classical method suffers from the planar approximation. Hence $\Delta\delta A(H_P)$ above can be regarded as a correction to the classical method, which leads to the formula

$$\delta A(H_P)^{\text{new}} = \delta A(H_P)^{\text{classic}} - \Delta\delta A(H_P) \tag{18}$$

In order to modify Eq. (14) to consider both the far- and near-zone effects, we rewrite Eq. (1) for a point P at the topographical surface only to the second power of H, resulting in

$$\delta A^*(H_P) = -\frac{2\pi\mu}{R}\sum_{n,m}\left(\frac{R}{r_P}\right)^{n+1}\frac{(n+2)(n+1)}{2n+1}(H^2)_{nm}Y_{nm}(P) \tag{19}$$

Equation (19), similar to Eq. (1), can be rewritten as a surface integral (see also Sjöberg 1998)

$$\delta A^*(H_P)^{\text{new}} = -\frac{4\pi\mu}{R}H_P^2 - \frac{3\mu}{8}\iint_\sigma \frac{H^2-H_P^2}{\ell_0}d\sigma + \frac{\mu R^2}{2}\iint_\sigma \frac{H_P^2-H^2}{\ell^3}\left(1-\frac{3H_P^2}{\ell^2}\right)d\sigma \tag{20}$$

where $\ell = \sqrt{r_P^2 + r^2 - 2r_P r\cos\psi)}$, and $r_P = R + H_P$. As it can be seen from the above equation, the first two terms are the same as those in Eq. (14). The third term uses ℓ instead of ℓ_0 and also an additional term $-\frac{\mu R^2}{2}\iint_\sigma \frac{H_P^2-H^2}{\ell^3}\frac{3H_P^2}{\ell^2}d\sigma$ is present. These differences with Eq. (14) take into consideration both the far- and near-zone effects. Rewriting Eq. (20), therefore, Eq. (14) is modified to

$$\delta A^*(H_P)^{\text{new}} = -\frac{5\pi\mu}{2R}H_P^2 - \frac{3\pi\mu}{2R}\overline{H_P^2} + \frac{\mu R^2}{2}\iint_\sigma \frac{H_P^2-H^2}{\ell^3}\left(1-\frac{3H_P^2}{\ell^2}\right)d\sigma \tag{21}$$

Martinec and Vanicek (1994) divided the integration area (σ) into a near zone (σ_1) and a far zone (σ_1), resulting in

$$\delta A(H_P)^{\text{MV}} = +\frac{\mu R^2}{2}\iint_{\sigma_1}\frac{H_P^2-H^2}{\ell^3}\left(1-\frac{3H_P^2}{\ell^2}\right)d\sigma + \frac{\mu R^2}{2}\iint_{\sigma_2}\frac{H_P^2-H^2}{\ell^3}\left(1-3\sin^2\frac{\psi}{2}\right)d\sigma \tag{22}$$

which differs from Eq. (21) by

$$-\frac{3\mu}{8}\iint_{\sigma_1}\frac{H^2-H_P^2}{\ell_0}d\sigma - \frac{3\mu R^2}{2}\iint_{\sigma_2}\frac{(H_P^2-H^2)H_P^2}{\ell^5}d\sigma \tag{23}$$

This difference has been evaluated in a test area in the north-west of Sweden with a height variation between 354 and 1147 m. The maximum difference for the maximum height elevation of H = 1147 m has reached 2.31 µGal. The difference between the two methods is acceptable for a precise geoid determination in our test area. However, it should be tested in different test areas.

3 Downward continuation of gravity anomalies by Poisson's integral

The new formula of Eq. (21) refers the gravity anomalies to a surface with elevation H (Earth's surface) and is free of the downward-continuation of gravity anomalies from the surface point to the geoid. The gravity anomalies corrected by this formula thus cannot be used in Stokes' formula. The downward continuation of these topographical corrected gravity anomalies must
first be carried out. Hence, we write

$$\Delta g^{\mathrm{obs}} + \delta A^{*}(H_P)^{\mathrm{new}} = f(\Delta g^{*}) \tag{24}$$

where Δg^{*} is the gravity anomaly on the geoid (the one which is supposed to be used in Stokes' formula), Δg^{obs} is the gravity anomaly coming from the gravity observations and function f is easily expressed (including the spherical harmonics of degrees zero and one) by the Poisson integral as (Kellogg 1929; MacMillan 1930)

$$\Delta g = \frac{t^2(1-t^2)}{4\pi} \iint_{\sigma} \frac{\Delta g^{*}}{D^3} d\sigma \tag{25}$$

where

$$\Delta g = \Delta g^{\mathrm{obs}} + \delta A(H_P)^{\mathrm{new}}$$

$t = R/r$ and $D = \sqrt{1 - 2t\cos\psi + t^2}$. In this equation, the spherical approximation has been used. Equation (25) can be solved in different ways; for example by a linear approximation as

$$\Delta g_P^{*} = \Delta g_P - \frac{\partial \Delta g}{\partial H_P} H_P \tag{26}$$

This linear approximation makes sense if the higher orders can be neglected, i.e. if the Taylor series converges very rapidly.

Vanicek et al. (1996) proposed an iterative process to solve the integral of Eq. (25), which is more accurate than the linear approximation of Eq. (26). Fortunately, the Poisson's integration kernel vanishes quickly with increasing distance from the computation point P. This means that it is enough to integrate Eq. (25) over a small area σ_0 around the computation point P, instead of the whole Earth (σ). However, limiting the area of integration to σ_0 causes an error which is here called the truncation error. We have tested different radii of integration and found out that a radius of integration $\psi_0 = 1^{\circ}$gives a truncation error of about 0.3 mGal (see also Vanicek et al. 1996; Nahavandchi 1998). In order to achieve accurate results for the downward-continuation correction, Poisson's kernel is also modified by minimizing the upper limit of the truncation error (Molodenskii et al. 1960; Sjöberg 1984; Vanicek and Sjöberg 1991). Describing Poisson's kernel by $K(H, \psi)$, the modified Poisson kernel is expressed as

$$K^m(H,\psi,\psi_0) = K(H,\psi) - \sum_{n=0}^{L} \frac{2n+1}{2} s_n(H,\psi_0) P_n(\cos\psi) \tag{27}$$

where $s_n(r,R,\psi_0)$ are the unknown coefficients to be computed from the following system of equations (see Vanicek and Kleusberg 1987):

$$\sum_{n=0}^{L} \frac{2n+1}{2} s_n(H,\psi_0) e_{in}(\psi_0) = Q_i(H,\psi_0); \quad i = 0,1,\dots,L \tag{28}$$

where

$$e_{in}(\psi_0) = \int_{\psi_0}^{\pi} P_i(\cos\psi)\, P_n(\cos\psi) \sin\psi\, d\psi \tag{29}$$

and

$$Q_n(H,\psi_0) = \int_{\psi_0}^{\pi} K(H,\psi)\, P_n(\cos\psi) \sin\psi\, d\psi \tag{30}$$

We have selected L = 20 in our computations.

As we are integrating the Poisson kernel over a small area σ_0 around the computation point, the contribution $T_g(P)$ of the rest of the world must be evaluated. Considering the smallness of this contribution after $\psi_0 = 1^\circ$, it can be evaluated from a global gravity model (Vanicek et al. 1996) as

$$T_g(P) = \frac{R\gamma}{2r} \sum_{n=2}^{\infty} \sum_{m=-n}^{n} (n-1)\, \bar{Q}_n(H,\psi_0) T_{nm} Y_{nm}(P) \tag{31}$$

where

$$\bar{Q}_n(H,\psi_0) = \int_{\psi_0}^{\pi} K^m(H,\psi,\psi_0)\, P_n(\cos\psi) \sin\psi\, d\psi \tag{32}$$

γ is the normal gravity and T_{nm} are the potential coefficients taken from a global gravity model. The modified Poisson kernel K^m in a spectral form is

$$K^M(H,\psi,\psi_0) = \sum_{n=0}^{\infty} \frac{2n+1}{2} \bar{Q}_n(H,\psi_0) P_n(\cos\psi) \tag{33}$$

The low-degree harmonics Δg_L (L = 1, 20) are also subtracted from the gravity anomalies Δg at the surface of the Earth, resulting in Δg^L, which is the high-frequency part of the gravity anomalies on the topography (see also Vanicek et al. 1996). Δg_L is computed from the EGM96 global model (Lemoine et al. 1997). This long-wavelength part is downward continued, separately. Finally, the contributions from the (downward-continued) long-wavelength part and truncation error are added to the short-wavelength part of the gravity anomaly which is downward continued by the iterative procedures.

The iterative process begins with (see also Vanicek al. 1996)

$$q_i^{k+1} = q_i^k - \frac{R}{4\pi(R+H_i)} \sum_j K_{ij}^m q_j^k \tag{34}$$

for the *i*th and *j*th cells, and the summation is taken over all the cells contained within the integration cap of radius ψ_0. The initial values are

$$q_i^0 = \Delta g_i - T_g(P) - \Delta g_L = \Delta g_i^L - T_g(P) \tag{35}$$

where

$$q = \Delta g - \frac{t^2(1-t^2)}{4\pi} \iint_\sigma \frac{\Delta g^*}{D^3} d\sigma \tag{36}$$

Once all the individual q_i^k are calculated, we can obtain the final gravity anomalies Δg^* and the downward continuation of gravity anomalies, $D\Delta g_i^*$, as

$$\Delta g_i^* = \sum_{l=0} q_i^{(l)} \tag{37}$$

and

$$D\Delta g_i^* = \sum_{l=1} q_i^{(l)} - T_g(P) - \Delta g_L \tag{38}$$

We end up with gravity anomalies, Δg_i^*, which are downward continued to the geoid and can be used in Stokes' formula.

Moritz (1980) had derived a different correction term to be applied to the gravity anomalies due to the topography, as

$$C = \frac{\mu R^2}{2} \iint_{\sigma_0} H - H_P)\ell_0^{-3} d\sigma \tag{39}$$

The topographical correction *C* is applied to the anomalies at points on the geoid. In order to derive this formula for topographical correction, Moritz (1980) assumed that the gravity anomalies in a downward continuation integral were linearly proportional to topographical height according to the so-called Pellinen approximation. Hence, the resulting Moritz topographical correction includes the effect of the downward continuation of gravity anomalies. This effect is, however, described somehow approximately since the linear relationship between gravity anomalies and topographical heights describes the reality only approximately (see e.g. Heiskanen and Moritz 1967).

Now we are in the position to compare our new formula (including downward-continuation correction) for topographical effect with that of Moritz.

4 Numerical investigations

A test area of $1^\circ \times 1^\circ$ is chosen. This area is located in the north-west of Sweden and limited by latitudes 62 and 63°N, and longitudes 13 and 14°E. The topography in this area, depicted in Fig. 1, varies from 354 to 1147 m.

The height coefficients $(H^2)_{nm}$ are determined from Eqs. (4) and (5). For this, a $30' \times 30'$ digital terrain model (DTM) is generated using the GETECH $5' \times 5'$ DTM (GETECH, 1995a). This $30' \times 30'$ DTM is averaged using area weighting. Since the interest is in continental elevation coefficients and we are trying to evaluate the effect of the masses above the geoid, the heights below sea level are all set to zero. The spherical harmonic coefficients are computed to degree and order

360. The parameter $\mu = G\rho_0$ is evaluated using $G = 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$ and $\rho_0 = 2670$ kg m^{-3}. R = 6371 km and $\gamma = 981$ Gal are also used in computations. In the integral equations a $2.5' \times 2.5'$ (GETECH, 1995b) DTM is used. It should be mentioned that this DTM is not adequate for computing the topographical correction in practice. Denser DTM is in order. In order to avoid leakage, height data are extended to 6° from the computation point.

First, the direct topographical correction is computed with the new formula of Eq. (21) and applied to the gravity anomalies. This formula is limited to the second power of elevation H. Figure 2 depicts the direct topo- graphical correction with the new formula on gravity which ranges from -25.43 to 40.35 mGal with a mean value of -1.35 mGal. It should be mentioned that these corrections are computed at the surface of the Earth and the corrected gravity anomalies cannot be used in Stokes' formula. We therefore investigate the downward continuation of these topographically corrected gravity anomalies by Poisson's integral based on an iterative procedure (see Vanicek et al. 1996). It should be mentioned that downward-continuation procedures are implemented with the point values rather than mean values for the Poisson integral. In order to reduce the effect of leakage of the data coverage for the integration caps, the integration area is increased 6° in each direction, so that the area for which the downward continuation would actually be computed is $13^{\circ} \times 13^{\circ}$. However, to escape from the edge effect (the effect of leakage of the data coverage along the edge of the test area), the original $1^{\circ} \times 1^{\circ}$ test area is used at the end. The prescribed limit of convergence in the iterative process is 10 μGal in Tchebyshev's norm. The potential coefficients used in this study are taken from the EGM96 model.

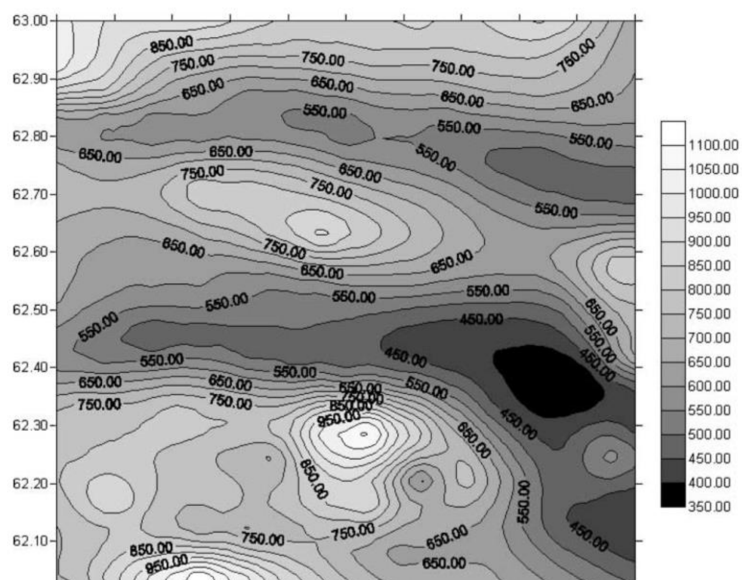

Fig. 1. Presentation of topography in the test area [m]

The truncation error is computed in the test area according to Eq. (31). This error reaches at most 5.6 mm. The effect of truncation error on gravity anomalies ranges from -0.21 to 0.25 mGal. As our gravity anomalies are in discrete $6' \times 10'$ cells, instability of the downward continuation has not posed any problem in our study. The given iterative scheme has converged after 12 iterations. Figure 3 shows the differences between gravity anomalies on the topography and on the geoid. The differences range from -33.65 to 59.56 mGal with a mean value of 3.29 mGal. We are now in the position to compare the gravity anomalies corrected by the new formula (including downward-continuation correction) with gravity anomalies corrected by the Moritz formula [Eq. (39)]. Figure 4 shows the direct topographical effect on gravity using the Moritz formula. It ranges from 0.58 to 19.23 mGal with a mean value of 10.35 mGal.

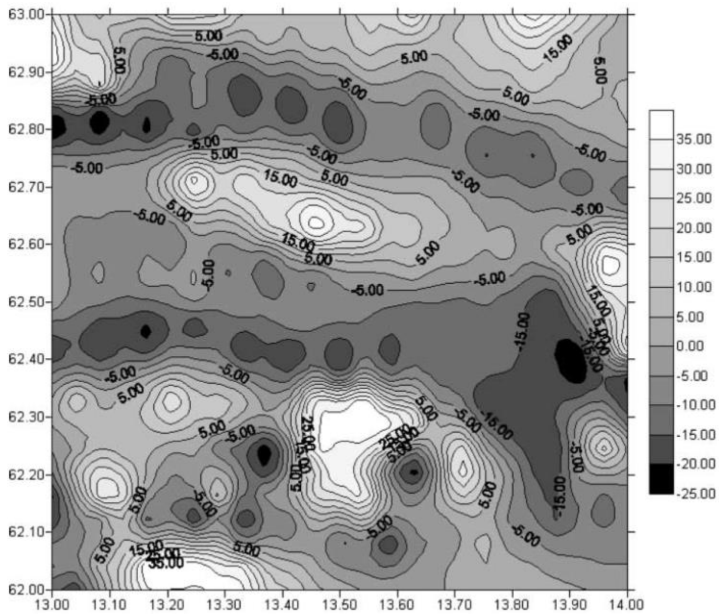

Fig. 2. Direct topographical correction on gravity computed by the new formula. Contour interval 5 mGal

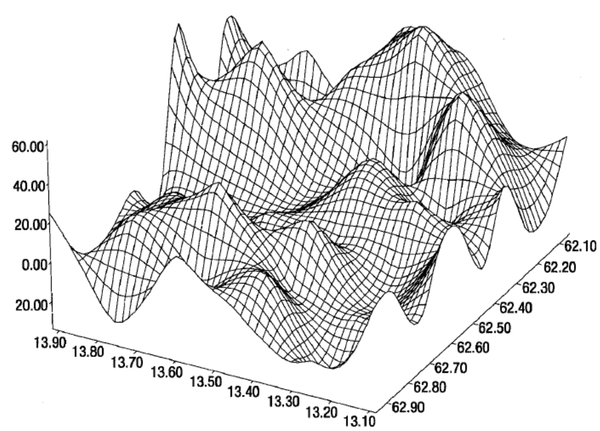

Fig. 3. Differences between topographically corrected gravity anomalies on the topography and on the geoid [mGal]

The direct topographical correction is also computed on the geoid. The statistics of differences on the geoid between the Moritz and new formulas are shown in Table 1. The results show a maximum difference of 7.21 cm with a mean value of 5.43 cm. There may be two reasons for these differences. First, the Moritz integral formula suffers from the planar approximation and only includes the short-wavelength contributions, while both short- and long-wavelength information is included in our formula. Second, the Pellinen approximation is used in the Moritz formula. The new formula for the direct topographical corrections treats the effect of the downward continuation more precisely. Nahavandchi (1998) showed that the difference between an accurate treatment by Poisson's integral and the Pellinen approximation for the downward continuation of gravity anomalies on the geoid reaches 4.28 cm (the test area was the one of the present study).

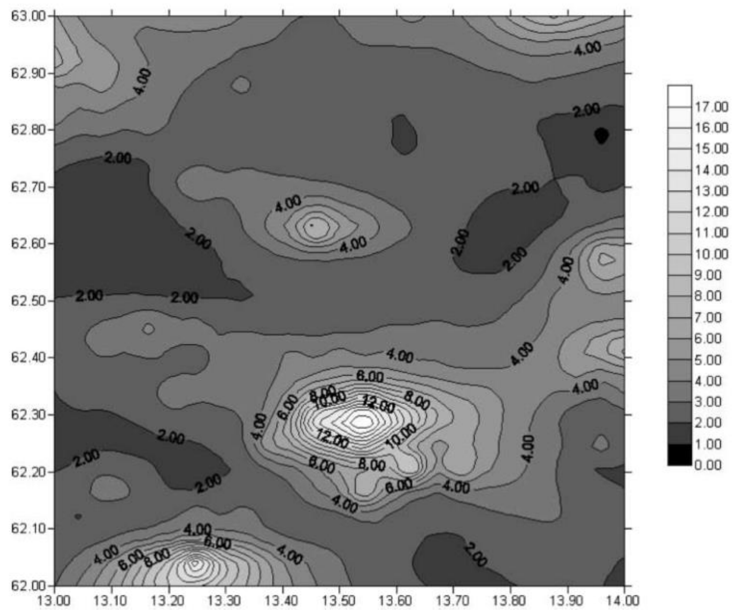

Fig. 4. The direct topographical correction on gravity computed by the Moritz formula. Contour interval 1 mGal

Table 1. Statistics of differences between the topographical correction on the geoid by the new expression and by the Moritz formula (cm)

Min	Max	Ave	SD
2.15	7.21	5.43	3.11

4 Conclusions

The direct topographical effect in gravimetric geoid determination is composed of both local effects and long-wavelength contributions. This implies that most classical formulas may have some numerical problems in representing of these long-wavelength contributions. The classical formula of Eq. (7) requires that the integrated area covers most of the globe to include the long wavelengths, while a pure set of spherical harmonics, Eq. (1), truncated to, say, degree 360, will not contain the local details. We conclude that Eq. (21) may be a suitable compromise between the local contribution [represented by the classical formula of Eq. (7)] and the set of spherical harmonics in Eq. (1). The results of comparison with Moritz topographical correction show some differences at the centimeter level. A mean difference of 5.43 cm is computed in the test area. There may be two reasons for these differences: more precise treatment of the downward continuation correction and the inclusion of the long-wavelength information in the new formula. Finally, it should be stated that our results are approximately the same as those obtained from the Martinec and Vanicek (1994) formula. However, there are significant differences with the Vanicek and Kleusberg (1987) formula.

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References

GETECH (1995a) Global DTM5. Geophysical Exploration Technology (GETECH), University of Leeds
GETECH (1995b) DTM2.5 of Europe. Geophysical Exploration Technology (GETECH), University of Leeds
Heiskanen WA, Moritz H (1967) Physical geodesy. WH Freeman, San Francisco
Kellogg OD (1929) Foundations of potential theory. Springer, Berlin Heidelberg New York (reprinted 1967)
Hagawara Y (1976) A new formula for evaluating the truncation error coefficient. Bull Geod 50: 131-135
Lemoine FG, Smith DE, Kunz L, Smith R, Pavlis EC, Pavlis NK, Klosko SM, Chinn DS, Torrence MH, Williamson RG, Cox CM, Rachlin KE, Wang YM, Kenyon SC, Salman R, Trimmer R, Rapp RH, Nerem RS (1997) The development of the NASA GSFC and NIMA Joint Geopotential Model. In: Segawa J, Fujimoto H, Okubo S (Eds) Gravity, geoid and marine geodesy, vol 117, IAG Symp, pp 461-469. NASA Technical Report NASA/TP-199618-206861
MacMillan WD (1930) The theory of the potential. Dover Publ. Inc, New York (reprint 1958)
Martinec Z, Vanicek P (1994) Direct topographical effect of Helmert's condensation for a spherical approximation of the geoid. Manuscr Geod 19: 257-268
Martinec Z, Matyska C, Grafarend EW, Vanicek P (1993) On the Helmert's 2nd condensation method. Manuscr Geod 18: 417-421
Molodenskii MS, Eremeev VF, Yurkina MI (1960) Methods for study of the external gravitational field and figure of the earth. Office of Technical Services, Department of Commerce, Washington, DC
Moritz H (1968) On the use of the terrain correction in solving Molodenskii's problem. Rep 79, Department of Geodetic Science, The Ohio University, Columbus
Moritz H (1980) Advanced Physical Geodesy. Herbert Wichmann Verlag, Karlsruhe
Nahavandchi H (1998) On some models of downward continuation of mean free-air gravity anomaly. IGeS Bull No. 8: 1-16
Nahavandchi H, Sjöberg LE (1998) Terrain correction to power H^3 in gravimetric geoid determination. J Geod 72: 124-135
Sjöberg LE (1984) Least squares modification of Stokes' and Vening Meinez' formulas by accounting for truncation and potential coefficient errors. Manuscr Geod 9: 209-229
Sjöberg LE (1994) On the total terrain effects in geoid and quasi- geoid determinations using Helmert second condensation method. Rep 36, Division of Geodesy, Royal Institute of Technology, Stockholm
Sjöberg LE (1995a) On the quasigeoid to geoid separation. Manuscr Geod 20: 182-192
Sjöberg LE (1995b) The terrain effect in geoid computation from satellite derived geopotential models. In: European Geophysical Society XX, General Assembly, Hamburg, 3-7 April. Boll Sci Affini LV(4): 385-392
Sjöberg LE (1996a) On the error of analytical continuation in physical Geodesy. J Geod 70: 724-730
Sjöberg LE (1996b) On the downward continuation error at the Earth's surface and the geoid of satellite derived geopotential models. Boll Sci Affini LVIII(3): 215-229
Sjöberg LE (1996c) The total terrain effect in gravimetric geoid determination. In: European Geophysical Society Meeting, The Hague, 6-7 May. Boll Sci Affini LVI(2): 209-222
Sjöberg LE (1998) On the topographical effects by the Stokes- Helmert method of geoid and quasi-geoid determination. J Geod submitted
Vanicek P, Martinec Z (1994) The Stokes-Helmert scheme for the evaluation of a precise geoid. Manuscr Geod 19: 119-128
Vanicek P, Kleusberg A (1987) The Canadian geoid-Stokesian approach. Manuscr Geod 12(2): 86-98
Vanicek P, Sjöberg LE (1991) Reformulation of Stokes theory for higher than second degree reference field and modification of integration kernels. J Geophys Res 96 (B4): 6529-6539
Vanicek P, Najafi M, Martinec Z, Harrie L, Sjöberg LE (1995) Higher order reference field in the generalized Stokes-Helmert scheme for geoid computation. J Geod 70(3): 176-182
Vanicek P, Sun W, Ong P, Martinec Z, Najafi M, Vajda P, ter Horst B (1996) Downward continuation of Helmert's gravity. J Geod 71: 21-34
Wang YM, Rapp RH (1990) Terrain effects on geoid undulation computations. Manuscr Geod 15: 23-29
Wichiencharoen C (1982) The indirect effects on the computation of geoid undulations. Rep 336, Department of Geodetic Science, The Ohio State University, Columbus