Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Information Technology and Electrical Engineering
Department of Electric Power Engineering

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A comparison of the SDP and SDDP method for medium-term scheduling of Meråker hydropower system

Master's thesis in Energy and Environmental Engineering
Supervisor: Gro Klæboe
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Abstract

In society today, more renewable energy sources are being introduced as a consequence of a desire to reduce climate emissions. As several of these renewables cannot be controlled, such as wind power which is dependent on the wind to produce, it is important that hydropower with stored water is utilized in the best possible way. For the hydropower producers, this is done by placing a water value on the stored water, which gives an estimate of the price that can be obtained for the electricity produced from the water in the future.

Setting the water values is done using a medium-term hydropower scheduling model, normally built using either stochastic dynamic programming (SDP) or stochastic dynamic dual programming (SDDP). In this master thesis, the SDP and SDDP methods are compared to see which of them gives the best usage of the water in the test case. The best usage is defined by the amount of income and spillage in the two models and the fulfillment of the minimum discharge constraint in the test case.

The results from this case study show that the SDDP model has the lowest spillage and the largest income and total value. Total value is the sum of income and the value of the end reservoir. The results also show that the two models equally fulfill the minimum discharge constraint. Because of this and the higher income and lower spillage in SDDP compared to SDP, it can be concluded that the SDDP model gives the best usage of the water in the test system.

Sammendrag

I dagens samfunn blir flere fornybare energikilder innført som en konsekvens av et ønske om å redusere klimautslippene. Ettersom flere av disse ikke kan kontrolleres, for eksempel er man avhenging av vind for å produsere vindkraft, er det viktig for samfunnet at vannmagasinene operereres på best mulig måte. For vannkraftprodusentene gjøres dette ved at en vannverdi settes på det magasinerte vannet. Vannverdien gir et estimat på hvilken pris man kan oppnå for elektrisiteten man produserer av vannet i magasinet.

For å bestemme vannverdien brukes en mellomlang vannkraftmodel, gjerne bygget på enten stokastisk dynamisk programmering (SDP) eller stokastisk dynamisk dual programmering (SDDP). I denne masteroppgaven bli SDP og SDDP metodene sammenlignet for å se hvilken av de som gir den beste bruken av vannet i testsystemet. Den beste bruken blir definert av mengden tapt vann og inntekt i de to modellene og oppfyllelsen av minstevannføringskravet i testsystemet.

Resultatene fra denne oppgaven viser at SDDP modellen har lavest tapt vann og størst inntekt og total verdi. Total verdi er summen av inntekt og verdien av sluttmagasin. Resultatene viser også at de to modellene oppfyller minstevannføringskravet likt. På grunn av dette og den høyere inntekten og lavere tapt vann i SDDP sammenlignet med SDP, kan det bli konkludert med at SDDP modellen gir den beste bruken av vannet i testsystemet.

Preface

This master's thesis ends the master's study in Energy and Environmental Engineering at the Norwegian University of Science and Technology (NTNU).

The thesis is written in collaboration with Nord-Trøndelag Elektrisitetsverk (NTE) [1]. I would like to thank NTE for the collaboration and the opportunity to use data they have gathered in this thesis. I would also thank them for the opportunity to sit at their office in Steinkjer once a week to work on the master thesis.

I would also like to thank my supervisor Gro Klæboe and co-supervisor Vivane Aubin for all the help and the regular meetings we have had throughout the year.

Trondheim, June 2023
Karen Margrete Husby

Abbreviations

EER	Equivalent energy representation
EMPS	Multi-Area Power-market Simulator
EOPS	One-area Power-market Simulator
LP	Linear Programming
MAD	SINTEF project to develop methods for aggregation and disaggregation
MIP	Mixed integer programming
NTE	Nord-Trøndelag Elektrisitetsverk
NVE	Norwegian of Water Resources and Energy Directorate
SDDP	Stochastic Dynamic Dual Programming
SDP	Stochastic Dynamic Programming
SINTEF	Norwegian research institute
SOS2	Special Ordered Set of Type Two
SSDP	Sampling Stochastic Dynamic Programming

Nomenclature

Index sets

I	Set of iterations i
P	Set of production plants p
R	Set of reservoirs r
S	Set of scenarios s for price and inflow
T	Set of weeks t (stages)
K	Set of discrete reservoir volumes k (states)
V	Set of volume scenarios v from forward loop in SDDP

Parameters

α_t^s	Future income for time step t, given scenario s
$c_{m^3/s \ to \ Mm^3}$	Conversion factor from m^3/s to Mm^3
ϵ	Accepted error in water value
E_r	Energy equivalent for each reservoir r
E_r^{tot}	Total energy equivalent for each reservoir r
$F_t(v_t)$	Future income for time step t, given reservoir level v_t
i_w	Inflow in week w for the whole watercourse
$i_{agg,w}$	Aggregated inflow in week w
i_{annual}^r	Mean annual inflow to reservoir r
i_t^s	Inflow in time step t, given scenario s
$i_{t,\ natural}^r$	Natural inflow in time step t for reservoir r
K_k	Discrete volume section
k^{high}	Parameter for setting initial future value for the high scenario
k^{mean}	Parameter for setting initial future value for the mean scenario
k^{low}	Parameter for setting initial future value for the low scenario
λ_t^s	Power price in time step t, given scenario s

λ_{mw}	Penalty for missing water
λ_{mean}	Mean price in the price scenarios
μ^i	Mean inflow
m^{high}	Parameter for setting initial cuts for the high scenario
m^{mean}	Parameter for setting initial cuts for the mean scenario
m^{low}	Parameter for setting initial cuts for the low scenario
P_{max}	Maximum production capacity for the aggregated plant
P^p_{max}	Maximum production capacity for each production plant p
$P^{SamløpFunna}_{min}$	Minimum aggregated production to fulfill the minimum discharge
Q^r_{max}	Maximum discharge from reservoir r
$Q^{Samløp\ Funna}_{min}$	Minimum discharge at Samløp Funna
ρ_s	Probability of scenario s
ρ_v	Probability of volume scenario s
R^r	Degree of regulation for reservoir r
s_r	Share of the total inflow to reservoir r
σ^i	Standard deviation in inflow
v^{init}_t	Initial volume for time step t
V_{max}	Maximum reservoir volume for the aggregated reservoir
V^r_{max}	Maximum reservoir volume for reservoir r
w^s_t	Water value for time step t, given scenario s
W^k_t	Aggregated water value for time step t and discrete volume k
$W_t(v_t)$	Water value in for time step t, given reservoir volume v_t

Variables

$\alpha(v_t)$	Future income for the reservoir level v_t.
$\delta_t^v(i)$	Future value for reservoir r in time step t and iteration i
$\delta_{t=T}^{high}$	Initial future value for high scenario in time step $t = T$
$\delta_{t=T}^{mean}$	Initial future value for mean scenario in time step $t = T$
$\delta_{t=T}^{low}$	Initial future value for low scenario in time step $t = T$
γ_k	SOS2 variable between 0 and 1 for the future income restriction in the SDP model for each discrete reservoir level k.
i_t	Inflow in time step t
i_t^r	Inflow in time step t to reservoir r
$\mu_{r,k}$	Variable between 0 and 1 for the future income restriction in the disaggregation model
mw_t^r	Missing water in time step t to reservoir r
$\phi_t^{r,v}(i)$	Cut for reservoir r in time step t and iteration i
$\phi_{t=T}^{high}$	Initial cut for high scenario in time step $t = T$
$\phi_{t=T}^{mean}$	Initial cut for mean scenario in time step $t = T$
$\phi_{t=T}^{low}$	Initial cut for low scenario in time step $t = T$
p_t	Aggregated production in time step t
p_t^p	Production in time step t in production plant p
q_t^r	Discharge in time step t from reservoir r
r_t^v	Revenue including the penalty cost in time step t for volume scenario v
s_t	Aggregated spillage in time step t
s_t^r	Spillage in time step t from reservoir r
$\hat{\sigma}$	Standard deviation of objective function value
v_t	Reservoir volume in time step t
v_t^r	Reservoir volume in time step t in reservoir r
$v_t^{r,v}$	Reservoir volume in reservoir r for time step t and volume scenario v

$w_{r,k}$	Water value for reservoir r and discrete volume k
$w_t^{s,r}$	Reservoir volume in reservoir r for time step t and scenario s
$W_{i,t}$	Water value in iteration i and time step t
z	Objective function value
$\underline{z}$	Lower bound
$\hat{z}$	Mean objective function value
z_t^i	Inflow in time step t without impact of seasonal variations
z^v	Sum of revenue in the planning horizon

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1 Introduction

Section 1.1 and 1.2 are reused from Section 1.1 and 1.2 in [2].

1.1 Meråker Hydropower System

The water course investigated in this thesis is Meråker hydropower system. The hydropower system consist of 5 reservoirs and 3 plants and is one of the water courses owned by NTE. In addition there are several creek inlets in the water course as well. There is also a measure point for discharge downstream two of the power plants, that measure the sum of discharge from these two plants. The measure point is motivated by the minimum discharge constraint in the downstream river. The system is presented in figure 1.

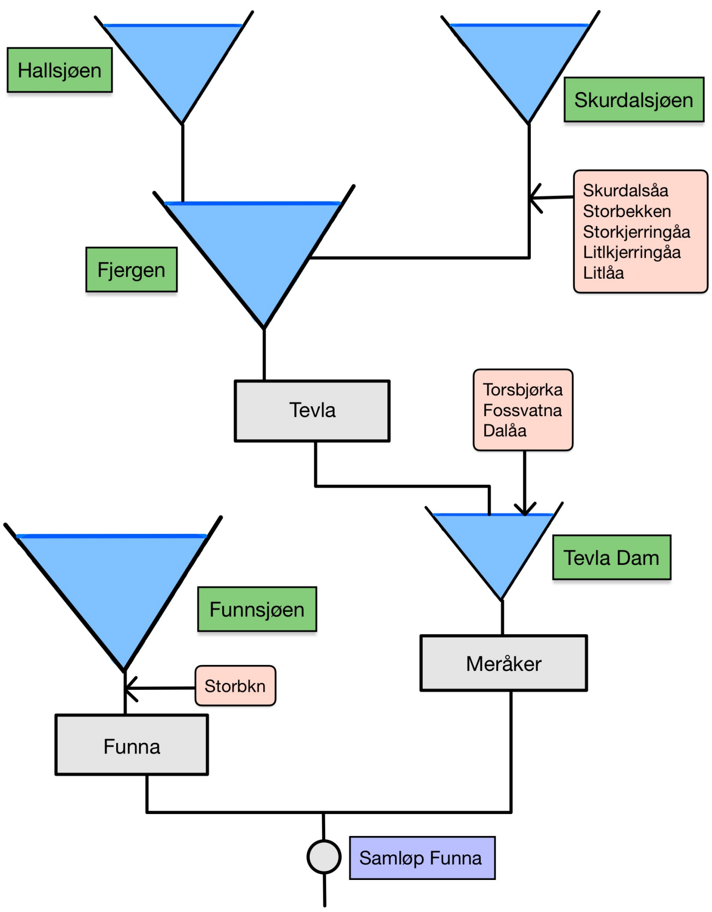

Figure 1: Meråker Hydropower System

The grey boxes are power plants, the blue triangles are reservoirs and the green boxes are the name of the reservoirs. The light red boxes are creek inlets, the grey circle is the measure point for discharge and the purple box is the name of the measure point.

In addition to the capacity on generation and the reservoirs, there are two main restrictions in the watercourse that can cause challenges. These restrictions are a minimum discharge constraint at Samlop Funna, that is a restriction on the sum of discharge downstream both Funna and Meråker power plants. Second a state-dependent constraint at Fjergen reservoir. The state-dependent constraint state that the discharge from Fjergen has to be stopped while the reservoir level is below 512 meters from the first of May, or latest the beginning of the spring flood and until the first of August.

1.2 Nord-Trondelag Elektrisitetsverk (NTE)

NTE are considering replacing the medium-term hydropower scheduling model they are currently using, which is a model that is based on SDP to calculate water values. One of the other options is a model using SDDP, and there is also a third option that uses a scenario tree to calculate the water values.

The advantages with the SDP model they are currently using are the fast computation time, and the possibility to implement state-dependent restrictions. The disadvantage is that in a SDP model the watercourse has to be aggregated, and therefore might lose some details of the hydropower system or overestimate the flexibility in the system.

The SDDP model, on the other hand, does not aggregate the hydropower system, but solves the problem with representation of all reservoirs and plants. The computation time of the SDDP model is, however, somewhat higher. In addition, the SDDP model does not handle state-dependent restrictions as well as the SDP model.

1.3 Research question

In [2] a SDP model for the Meråker hydropower system is investigated, and it is there argued that the aggregated model of the watercourse overestimates the flexibility in the system and the ability to hold inflow. This leads to an overestimation of the income from the watercourse. To get more correct water values and expectations of income, a SDDP model for the watercourse can be further investigated. This means building a SDDP model for the watercourse and compare the result from this model with the result for the SDP model developed in [2].

This master thesis looks closer at the SDDP model for one of NTE's water courses, the Meråker hydropower system, and the aim is to evaluate if the SDDP model delivers better production plans and more income for the producer than the aggregated SDP model. More specific the research question can be formulated as:

- Which of the two methods SDP and SDDP for a medium-term hydropower scheduling problem provides the best usage of the water in Meråker hydropower system?

To figure out which of the models that give the best usage, the amount of income will be investigated in addition to the amount of spillage. The income represents the revenue for the power producer, while the spillage represents the water that is lost and that could have contributed to an even larger income. In addition, the reservoir trajectories and the fulfillment of the minimum discharge constraint will be presented.

1.4 Contribution

Both SDP and SDDP are methods that are used for hydropower scheduling for many years, both in Norway and else in the world where there exist hydropower. A lot of companies use one of these techniques in their daily or weekly hydropower planning to calculate water values or production plans for several months or up to a few years. Because of this, there exists a good amount of research on both methods.

[3] and [4] studies SDP in long-term hydropower scheduling for larger hydropower systems using aggregation and disaggregation. [3] consider an aggregation-disaggregation of the Lower Caroni hydropower system in Venezuela both in time and space, while [4] presents the method that has been implemented for Hydro Quebec's hydro system.

As examples of applications of SDDP are [5], [6] and [7]. [5] studies the trade-offs and risks associated with a large hydropower system with flow requirements and inflow modeling. In [6] the importance of detailed hydropower scheduling modeling when including sales of capacity is investigated, and in [7] a combined method using both SDP and SDDP is presented to optimal schedule a hydropower system considering both markets for energy and reserve capacity.

Since there is no need for aggregation in SDDP, the claim is that the method gives better and more precise results for larger hydropower systems [7]. Despite this, there is not much research comparing the two methods to my knowledge. Especially on watercourses that have state-dependent and minimum discharge constraints.

[8] is an example of an article comparing SDP and SDDP. The paper studies a medium-term hydro problem and is taking into consideration risk-aware operation, short-term production flexibility and provision of spinning reserves. The article shows that both methods produce similar results for a relatively small test system. Since the test system is relatively simple the disadvantages with aggregation and disaggregation in the SDP method do not appear.

In this master thesis, the aim is to compare the two methods SDP and SDDP for hydropower scheduling on a multi-reservoir test system. The main contribution of this thesis is the comparison of the two methods SDP and SDDP for a medium-term hydropower scheduling problem for a test system, the Meråker hydropower system, to see which of them provides the best solution.

2 Theory

Section 2.1 and 2.2 are reused from Section 2.1 and 2.2 in [2].

2.1 Introduction to Hydropower Scheduling

Hydropower scheduling is an optimization problem, where the objective is to maximize the expected profit over the planning horizon with the given weather and price forecast while satisfying all relevant constraints [9]. The constraints that have to be satisfied are typically constraints on generation, reservoir balance, minimum discharge and other environmental constraints. In a hydropower system, water can be seen as a free resource, and the variable cost of operating a hydropower plant is very low. On the other side, the availability of the water is limited and the future availability is uncertain and dependent on the inflow. The inflow again is dependent on the temperature, the amount of rain and the amount of snow storage. This gives a dilemma; generating one unit of electricity today limits the opportunity to generate electricity tomorrow. Also if you do not generate electricity today the risk of spillage tomorrow might increase if inflow is high. Out of this the term opportunity cost or expected marginal value of the hydro is defined [10]. In this thesis, the expected marginal value of one extra storage unit in a reservoir is referred to as the water value.

When deciding for which hydropower scheduling model to use we distinguish between short, medium and long-term scheduling models. Short-term scheduling has a time horizon of 1-2 weeks normally and the weather forecast is assumed to be "perfect". This is therefore a deterministic problem, often solved by using a mixed integer problem (MIP) and a more detailed physical system description [11]. For the medium and long-term models, the weather is an uncertain factor, and the planning horizon is respectively 1-5 years and longer than that. This gives a stochastic problem and is usually solved by using stochastic dynamic programming (SDP) or stochastic dynamic dual programming (SDDP), and often also in combination with aggregation/disaggregation, especially for larger systems. This makes a rather coarse description of the technical system.

In a medium or long-term scheduling problem, the uncertainty in inflow and price is handled by creating inflow and/or price scenarios. The inflow scenarios are often generated based on historical time series for inflow for the particular hydropower system[12]. For a medium-term model, these time series will often have weekly resolution and a duration of 30-80 years. The price scenarios are instead often generated by simulating the price when the historical weather is applied to the current power system.

2.2 Stochastic Dynamic Programming (SDP)

SDP has been the main procedure for calculating water values in a medium-term hydropower scheduling problem for many years, and to some extent still is. SDP is a stochastic optimization method with a dynamic structure, meaning that the problem can be divided into a number of sequential stages. For each stage, typically a week, there are defined a set of states. In a hydropower scheduling problem these states are typically reservoir levels of 100 %, 90 %, ..., 0 % [13]. The storage level, initial state, at the beginning of the first stage is assumed to be known.

The structure of the problem is formulated to start with the last stage, and for each stage, state and scenario for inflow and price the one-stage problem is solved. The structure of the problem is shown in figure 2, where the box for each week solves one-stage problems for all scenarios and states. The expected future income function is assumed to be zero in the last stage of the first iteration. The one-stage problem is in most cases a LP problem and calculates the expected future income and the water values for the given state and scenario.

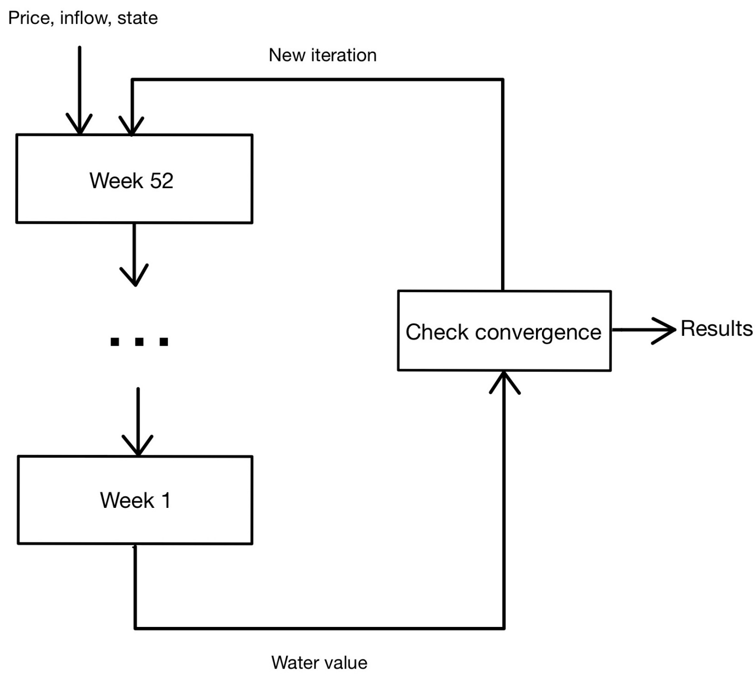

Figure 2: Structure of SDP problem

The expected future income for a state is the mean value over the different scenarios. Out of this, the expected future income function is created for each stage dependent of each state. This is seen in figure 3 where the calculation of the future income function for stage T-1 is shown. For each state the model gives a future income value, which again is used to calculate the future income function. When calculating the next stage the future income function is used to get a future value of the stored water using interpolation over the end reservoir level of that stage. The process is then repeated for all selected stages and the objective is to maximize income [13].

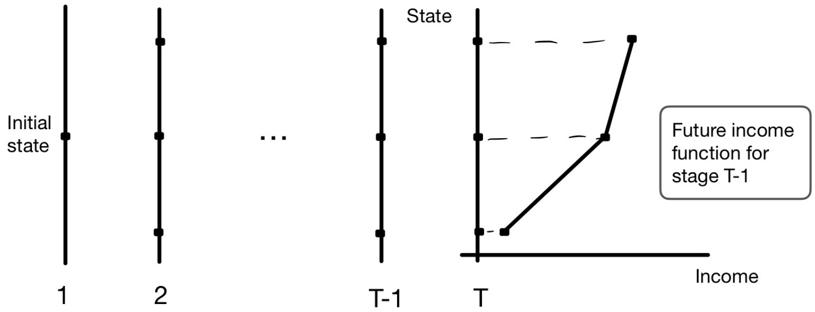

Figure 3: Calculation of Future Income Function for Stage T-1

When all stages are calculated the water values at the start and end are compared to see if the convergence criteria is reached or not. If not the algorithm is repeated starting at the last stage, now with the future income function from the first stage. This algorithm continues until the convergence criteria is reached. In the end, the water values of the reservoir for each stage and state are returned.

A disadvantage of the SDP method is that the computation times will increase drastically when the system consists of more than one reservoir and can become insoluble when considering a multi-reservoir system. This is because the SDP recursion requires the enumeration of all combinations of initial storage values and inflows [13]. Meaning that the number of states increases exponentially with the number of reservoirs in the system.

The SDP algorithm is, therefore, best suited for a system with relatively few reservoirs, or else the system will suffer from extreme computation times when considering realistic multi-reservoir systems [14]. To avoid the curse of dimensionality, the multi-reservoir system can be aggregated to an equivalent energy representation (EER) [15] instead. The inflow, generation capacity and storage capacity are then aggregated to an EER using the energy equivalent, that is a conversion factor from m^3 to kWh based on the efficiency of the system.

The SDP problem will then first be solved for the aggregated system to calculate the aggregated water values. Then a disaggregation algorithm will disaggregate the water values and generate production plans for each reservoir and plant. This is a forward iteration, starting at week 1 with the initial reservoir levels and the water values from the SDP problem as inputs.

2.3 Stochastic Dynamic Dual Programming (SDDP)

Due to the need of aggregation in the SDP method, the SDDP approach is an alternative for medium- and long-term scheduling problems. The SDDP algorithm is a sampling-based variant of multi-stage Benders decomposition and does not need to fully discretize the state variables. This reduces the solution time and therefore allows a more detailed formulation of the hydropower scheduling problem [14]. The drawback of the SDDP method is that it cannot handle non-convexities, which might appear when adding a state-dependent constraint. A state-dependent constraint is a constraint that is dependent on the state of the system.

The SDDP algorithm is based on dual values and does not have a direct estimation of the future value, as in SDP. Instead, the SDDP algorithm is an approximation using dual values. This is shown in figure 4 by looking at the calculation of the piecewise future cost function for stage T-1 [13] for a minimisation problem. Each cut is determined by the expected cost and the dual values which give the slope. Together all cuts for a given stage make the piecewise future cost surface.

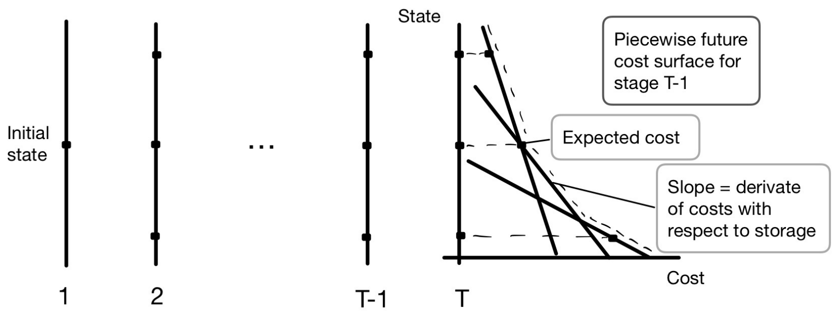

Figure 4: Calculation of Piecewise Future Cost Function for Stage T-1

The structure of the method is that it has a backward and a forward loop, and the method iterates between these two loops until convergence is reached, as shown in figure 5. The backward loop iterates over all price and inflow scenarios, volume scenarios and stages, beginning with the last. For each of these, it solves a one-stage problem with input price, inflow, a start volume from the volume scenarios and cuts generated from earlier iterations. The one-stage problem in the backward loop then returns the water values and the objective function value. These again are used to calculate the cuts for the next iteration and the lower bound in a minimizing problem[13]. In most commercial SDDP models the inflow and/or the price are sampled from a statistical model in the backward loop, and sometimes also in the forward loop as well [16].

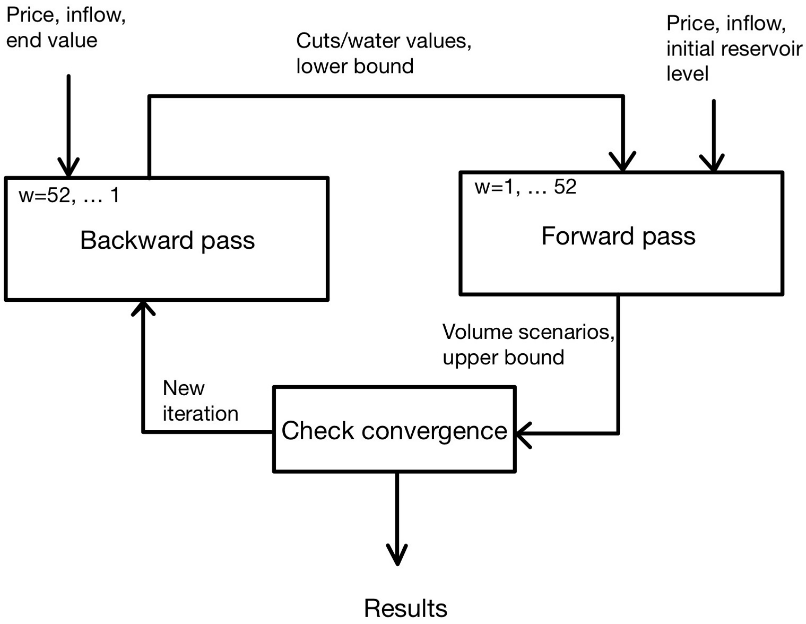

Figure 5: Structure of SDDP problem

The forward loop iterates over a random set of scenarios and all stages, starting at the first. The random set of scenarios is called the volume scenarios. The reason for not using the whole set is to keep the computation time low. For each stage and scenario, it solves a one-stage problem with input price, inflow, start volume and cuts generated from earlier iterations. The start volume is the actual start volume for the first week, afterwards the start volume is equal to the end volume in the previous week. The one-stage problem in the forward loop returns the end volumes and the income. The end volumes are used as input to the backward loop for the different volume scenarios, while the income is used to calculate the confidence interval of the upper bound. The confidence interval consists of the mean revenue over the volume scenarios plus/minus 1.96 times the standard deviation [13].

The method converges when the lower bound is inside the confidence interval of the upper bound. One problem with this is that if the standard deviation is high, you get a very large confidence interval, which again can turn to fast convergence. Other convergence criterias that can be used are a minimum or maximum number of iterations or a limit for the weighted deviation in all reservoirs in all weeks for all simulated scenarios [17]. Some models also use the stabilisation of the lower bound for a minimisation problem and the upper bound for a maximisation problem as a convergence criteria as well [15], [18].

2.3.1 End value setting in SDDP

In the SDDP model, the end value setting is important and can influence the results, especially when running only for 1 year. At the end of the planning horizon, the reservoirs will still have some water stored and hence a value. This value is reflected in the initial cuts for the final stage. If this value is set too high, the model will save more water than convenient. Opposite, if the end value is set too low the model will use more water, and for the next year, you will have less water to produce from. When running for more years this end value will have less influence on what is happening in the first year. This will be a trade-off between running time and accuracy in results.

2.4 Autocorrelation

In a hydropower problem both the price and the inflow in one week are normally strongly correlated to the price and inflow in the previous week [16]. This means that if the price or the inflow is high in one week it is more likely high in the next week and opposite. In a complex hydropower model, the autocorrelation in price and inflow are taken into account, meaning that the model will give a higher probability to scenarios close to the previous week than others. While a model without autocorrelation taking into account will produce more if the price gets high, a model with autocorrelation might not produce that much since it knows it is a high probability the price is high also next week.

Taking the autocorrelation in inflow into account is especially important in a SDDP problem, where you will not visit all states compared to the SDP. As presented in section 2.2, in SDP the states visited are from 0% up to 100% of the reservoir maximum volume for all stages. While in the SDDP method, the states that are visited are determined by the forward loop and the randomly picked scenarios in that loop. To guarantee that more of the solution space is visited in SDDP, a inflow model often are used to generate the inflow scenarios, instead of using the historical inflow series. This is done in [16], where the inflow model generates the inflow scenarios by taking into account the autocorrelation and adding some random noise to ensure that more states are visited.

The autocorrelation in price is not possible to handle in SDDP due to giving a non-convex objective function. In the SINTEF model called ProdRisk, this is dealt with in an outer SDP loop to retain the convexity in the problem [16].

3 Literature Review

This chapter presents an overview of existing research comparing the two methods SDP and SDDP for medium-term hydropower scheduling. The two methods have been used for hydropower scheduling for decades, but there does not exist that much research on comparing those two methods although.

One of the articles comparing the two methods is [8]. In the article the SDDP algorithm is used to solve a medium-term hydropower optimization while considering risk-averse operation, provision of spinning reserves and short-term production flexibility. The SDDP algorithm used is compared to a SDP algorithm to see which one of the two methods gives the best result. The hydropower system considered in this article is quite small with one seasonal storage and one daily storage with two turbines and pumps between them, and two turbines downstream the daily storage as well. The authors of the article conclude that for this small reservoir both SDP and SDDP give similar results, but that there is a need for research on more complicated systems where SDP would not be applicable.

Article [19] also presents a comparison of the two methods to schedule energy storage units, such as batteries, providing multiple services. The article shows that SDDP outperforms SDP when storage efficiency is high, but when short computation times are required SDP may be preferred. Storage efficiency is the efficiency of charging and discharging the energy storage unit. Since the article looks at energy storage and not hydro problems the result is not directly transferable, but the problems have large similarities and therefore give good insight to the performance of the two methods.

[7] presents a method for optimal scheduling of hydropower systems using a combination of SDP and SDDP. This is the same methodology that is used in the SINTEF model called ProdRisk. The method is applied in a case study for a Norwegian watercourse to see the changes in water values when going from an energy market to an energy and reserve capacity market. The method that combines the SDP and SDDP algorithms uses the benefits of each algorithm to create a better methodology to solve hydropower problems. This is for example that the price forecasts are treated in the SDP, while reservoir levels and inflows are continuously approximated in the SDDP [7].

Based on the findings in [8], it is in this thesis also expected similar results for the SDP and SDDP models. This is due to the watercourse studied consisting of only a few reservoirs and plants, similar to the watercourse studied in [8]. In addition, the watercourse studied in this thesis includes a minimum discharge constraint which the watercourse in [8] does not have. In return the watercourse in [8] has some constraints for spinning reserves that this model does not have.

4 Presentation of the Case

In this section, a presentation of the case is done. First, the details of the Meråker hydropower system are presented. This information is gathered from NTE. Next, a presentation of how the price and inflow scenarios are generated, and lastly a presentation of the main restrictions in the system are done. Section 4.1, first part of 4.2 and 4.3 are reused from Section 4.1, 4.2 and 4.3 in [2].

4.1 Details of the Meråker Hydropower System

Meråker hydropower system consists of three power plants; Tevla, Meråker and Funna. Tevla and Meråker are two cascaded power plants connected to Funna power plant only through a minimum discharge of 9.5 m^3/s constraint in the upper part of Stjørdalselva. Tevla power plant has, in addition to two generators, two pumps. These pumps were originally built to reduce the spring flood, but are now both used for seasonal and day-night pumping. The installed capacities and head for each power plant are presented in table 1. To simplify the problem, the pumps are not included in this master's thesis case study.

Table 1: Power Plant Information

Power Plant	**G1** [MW]	**G2** [MW]	**P1** [MW]	**P2** [MW]	**Head** [m]
Tevla	20.0	20.0	21.0	21.0	164.5
Meråker	61.0	26.0	-	-	263.7
Funna	21.5	-	-	-	336.2

The reservoirs in the system are Skurdalssjøen, Hallsjøen, Fjergen, Tevla Dam and Funnsjøen. Fjergen, Tevla Dam and Funnsjøen delivers water to the power plants Tevla, Meråker and Funna respectively, while Skurdalssjøen and Hallsjøen do not have any power plant on their own, but delivers water to Fjergen. The regulating heights and volume of the reservoirs are presented in table 2.

Table 2: Reservoir Information

Reservoir	**HRV [m]**	**LRV [m]**	**Regulating height [m]**	**Volume** $[Mm^3]$
Skurdalssjø	694.25	687.75	6.5	12.87
Hallsjø	613.00	605.80	7.2	25.00
Fjergen	514.00	498.00	16.0	207.00
Tevla Dam	358.50	350	8.5	4.50
Funnsjøen	442.00	430.50	11.5	64.00

The gates at Skurdalsjøen and Hallsjøen are normally closed before the beginning of the spring flood and held closed through the summer and autumn. Then in the early winter, the gates are gradually opened, and at the beginning of the next year, they are fully opened. This means that the inflow to Fjergen from these reservoirs mostly happens during the winter. In the summer the only potential inflow is if the reservoirs are full and the spillage will then flow to Fjergen.

In this watercourse, there are several catchments that give inflow to the reservoirs. In table 3 each catchment is listed, as well as information about which reservoir the water flows to, the area of the catchment and the yearly average inflow.

Table 3: Inflow Information

Inflows	**Flows to Reservoir**	**Area of Catchment** $[km^2]$	**Yearly Inflow** $[Mm^3]$
Hallsjøen	Hallsjøen	33.4	47.3
Skurdalsjøen	Skurdalsjøen	24.5	34.7
Skurdalsåa	Fjergen	4	5.7
Storbekken	Fjergen	4.2	6
Storkjerringåa	Fjergen	11.8	16.1
Litlkjerringåa	Fjergen	13.1	18.6
Litlåa	Fjergen	12.2	16.4
Fjergen	Fjergen	169.7	115.7
Torsbjørka	Tevla Dam	69.9	83.9
Fossvatna	Tevla Dam	28	33.4
Dalåa	Tevla Dam	161.8	193.9
Tevla Dam	Tevla Dam	19.2	14.5
Funnsjøen	Funnsjøen	80.1	41
Storbekken	Funnsjøen	3.4	4.8

The degree of regulation is a term used in hydropower to see the relationship between the reservoir volume and the mean annual inflow to the catchment. Equation 1 shows the formula for calculating the degree of regulation, R^r, where V^r_{Max} is the reservoir volume and i^r_{annual} is the mean annual inflow. In table 4 the degree of regulation for the two main reservoirs, Fjergen and Funnsjøen are shown.

$$R^r = \frac{V^r_{Max}}{i^r_{annual}} \tag{1}$$

Table 4: Degree of Regulation

Reservoir	**Reservoir Volume** $[Mm^3]$	**Mean Annual Inflow** $[Mm^3]$	**Degree of Regulation**
Fjergen	207	178.5	1.16
Funnsjøen	64	45.8	1.40

4.2 Price and Inflow Scenarios

The inflow scenarios used in this case are generated from historical time series for inflow for each of the years from 1930 to 2000. The time series is based on a water mark series from that area collected from NVE and then scaled to each reservoir and creek inlet. Since the reservoirs and creek inlets in this watercourse are located inside a small geographical area, the method would only give minor or negligible faults. In the historical data, the inflows are sampled with an average value for each day, each year. Through Python code, these series are converted to weekly resolutions for the given years.

The price scenarios are generated using a NTE model, where the intention is to maintain the correlation between price and inflow, like in the EMPS scenarios. The price scenarios have a weekly resolution for four price sections for the years from 1930 to 2000. In a week the different sections are having respectively 33, 57, 30 and 48 hours. Out of this, a Python code is developed to create a time series with one average price for each week for the desired years, based on the number of hours for each price section. This gives in total 70 scenarios for both price and inflow that are used in the SDP and SDDP algorithms for this master thesis.

In figure 6 and 7 the autocorrelation lag plot for the price and inflow series are shown. To calculate the autocorrelation for the inflow series without the impact of seasonal variation, equation 2 [16] is used, where i_t is the inflow for timestep t, μ^i is the mean inflow and σ^i is the standard deviation in inflow. The figures show that both the inflow and the price have a high degree of autocorrelation. The horizontal line in the autocorrelation lag plot for the price series, comes from the transition from one scenario year to the next.

$$z_t^i = \frac{i_t - \mu^i}{\sigma^i} \tag{2}$$

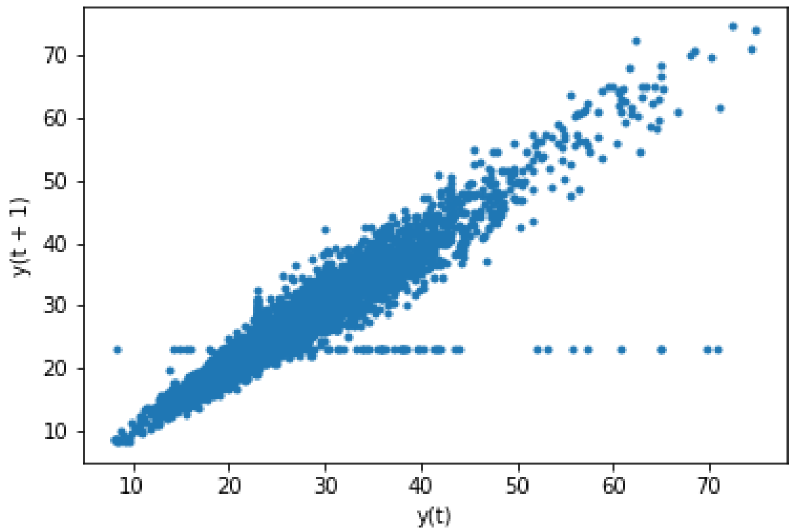

Figure 6: Autocorrelation Lag Plot for Price Series

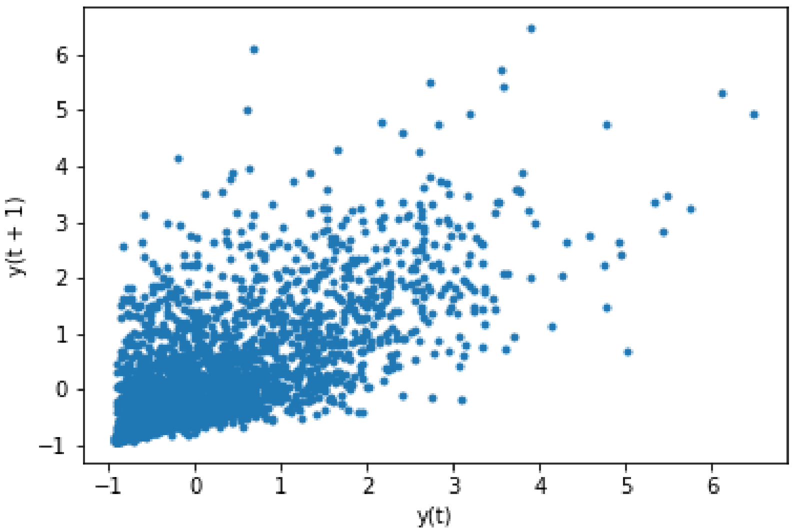

Figure 7: Autocorrelation Lag Plot for Inflow Series

In figure 8 and 9 the partial autocorrelation plot for the price and inflow series are shown. They show that for the price in the first lag, the autocorrelation factor is around 0.9, while for the inflow the autocorrelation factor is around 0.7. This again shows that both price and inflow are highly autocorrelated. This means that if the price is high today, it is a large probability that the price is high tomorrow also. The same applies to the opposite and for inflow. A model that has not taken autocorrelation into account does not see this and will therefore produce more today than a model that has taken autocorrelation into account.

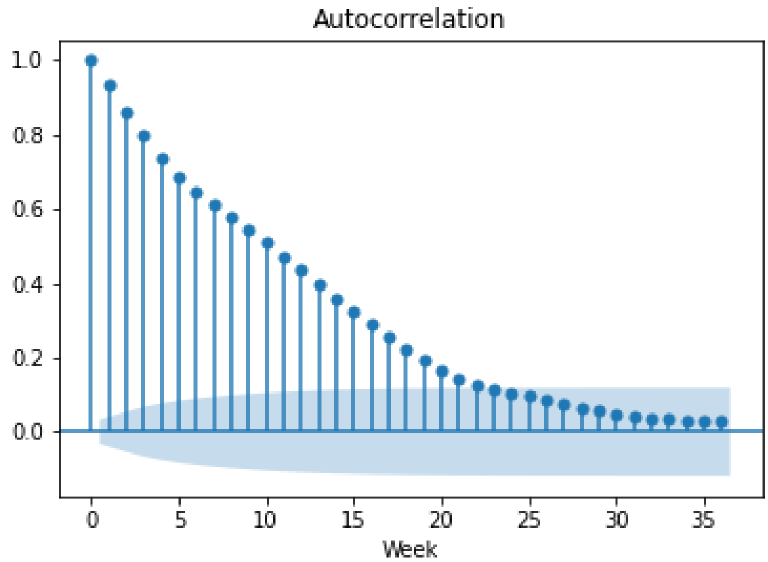

Figure 8: Partial Autocorrelation Plot for Price Series

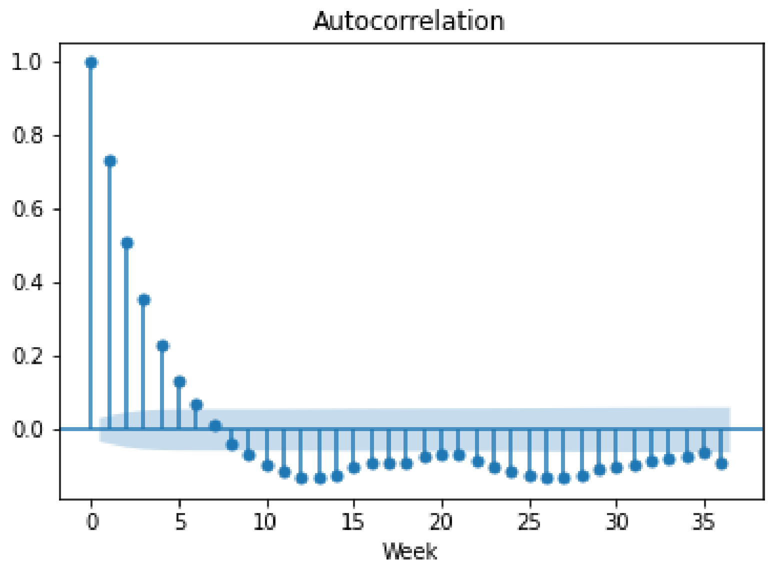

Figure 9: Partial Autocorrelation Plot for Inflow Series

4.3 Main Restrictions of the Water Course

One of the main restrictions in the Meråker water course is the minimum discharge at Samløp Funna. This is a measuring point downstream of both Meråker and Funna power stations. The discharge at this point has to be larger than 9.5 m^3/s at all times. The discharge is determined by the production at Meråker and Funna, in addition to the spillage from Fjergen, Funnsjøen and Tevla Dam.

To keep Fjergen over a certain minimum reservoir level during summer, a state-dependent environmental summer restriction is applied. The constraint states that the discharge from Fjergen has to be zero as long as the water level in Fjergen is below 512 meters. This yields from the first of May or at the latest the beginning of the spring flood to the first of August.

These constraints are stated in the concession for the watercourse, given by the regulator NVE [20]. In this master thesis, only the minimum discharge will be simulated in the algorithms. This is due to difficulties with state-dependent constraints in the SDDP algorithm due to non-convexity, and not enough time to look into it.

Some previous and ongoing work that has looked into handling state-dependent constraints in the SDDP algorithm is the HydroCen project at SINTEF [21]. In [16] a state-dependent maximum discharge constraint is presented and how it is possible to include it in a SDDP algorithm.

5 Methodology and Model Description

This section presents the medium-term hydropower scheduling model for the Meråker watercourse, both the SDP and the SDDP model. The models aim to maximize profit from the power producers' perspective while the constraints in the water course are held. The SDP solution process consists of two main steps. A strategy part, where the water values are calculated for an aggregated watercourse, and a simulation part, where the detailed dispatch of the disaggregated watercourse is done [22]. The SDDP algorithm consists of a forward and backward loop that calculates both the water values and the production plans at the same time. In this thesis the Python optimization package called Pyomo is used to build and run the problems [23]. Section 5.1.1, 5.1.2, 5.1.3 and 5.1.4 are reused from Section 5.1, 5.2, 5.3 and 5.4 in [2].

5.1 SDP model description

Figure 10 shows the SDP model description, where the aggregated power system goes into the SDP model. The output from the SDP model is the water values, which again are fed into the disaggregation model. The disaggregation model gives the production plan for the system.

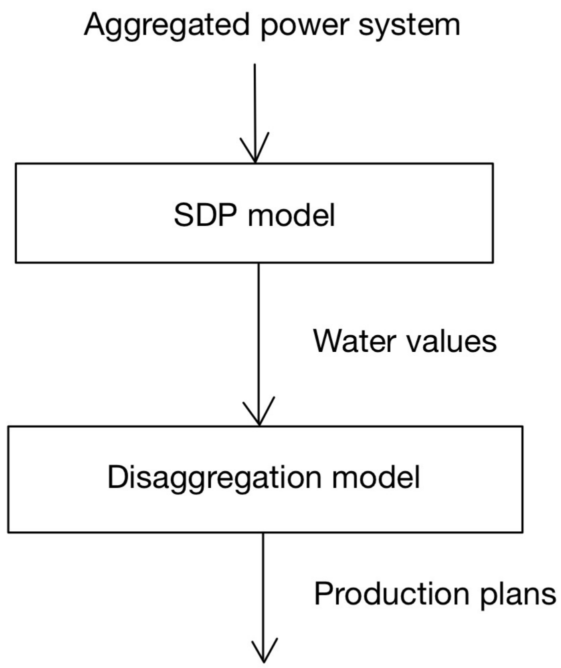

Figure 10: SDP Model Description

5.1.1 Aggregation of the Water Course

The SDP model applies best to a one-reservoir model, and aggregation from a multi-reservoir system to a single equivalent hydro system is necessary. This is done by aggregating all values for inflow, reservoir volume and production into one single value for inflow, reservoir volume and production, given in GWh for this case. To achieve this an energy equivalent for each reservoir is used to convert from volume of inflow and water stored to GWh. This energy equivalent is calculated based on the efficiency of the downstream plants and the average head. The aggregated maximum reservoir volume, V_{max}, is expressed in equation (3) from the sum over all maximum reservoirs volumes, V^r_{max}, times the total energy equivalent, E^{tot}_r.

$$V_{max} = \sum_{r=1}^{R} E^{tot}_r * V^r_{max} \tag{3}$$

The aggregated maximum plant production, P_{max}, is equal to the sum over all production plants maximum, P^p_{max}, shown in equation (4).

$$P_{max} = \sum_{p=1}^{P} P^p_{max} \tag{4}$$

The aggregated inflows are calculated based on a weekly inflow series for the whole watercourse. To convert this to an aggregated inflow, the part that flows to each reservoir in the system has to be known. This can be calculated by using the inflow share, s_r, which is the average yearly inflow for each inlet point divided by the total yearly inflow to the watercourse. The inflow is normally given i m^3/s, so a conversion factor from m^3/s to Mm^3 is also needed, $c_{m^3/s\ to\ Mm^3}$. The aggregated inflow each week, $i_{agg,w}$, is given in equation (5).

$$i_{agg,w} = i_w * c_{m^3/s\ to\ Mm^3} * \sum_{r=1}^{R} s_r * E^{tot}_r \tag{5}$$

In addition to this inflow, there is another inflow to the Fjergen reservoir from Hallsjøen and Skurdalsjøen reservoir, affected by the operation of the gates. These gates are normally closed at the beginning of the spring flood and then opened partly again late in the autumn, and opened fully at the beginning of the year. From this, the inflow from Hallsjøen and Skurdalsjøen are assumed to be zero from week 18-

47. For week 48-52 and week 1-17, the inflow from Hallsjøen and Skurdalsjøen is calculated based on the yearly inflow to the two reservoirs. The weekly inflow from these reservoirs to Fjergen in week 1-17 is twice as high as the weekly inflow in week 48-52, because of the larger gate opening.

5.1.2 Solution Algorithm - SDP

The solution algorithm used to solve the hydropower scheduling problem for Meråker watercourse is described in this subsection. The large and complex problem is solved by solving many smaller sub-problems, the one-stage problems. Each one-stage problem is solved for all scenarios, each discrete reservoir volume and all time steps. After solving the one-stage problem for all scenarios and volumes in week t, the future income function for the next step (t-1) is calculated based on the previous step.

When the problem has been solved for all weeks in the planning horizon the algorithm re-solves, with the future income of week 1 as the end-value of the reservoir in week 52, until the convergence criteria is obtained. Convergence is obtained when the water values in one week are the same for two subsequent iterations. When the SDP model has converged, the calculated water values can be used for a final forward linear simulation in order to obtain detailed production plans for each plant each week and the reservoir lanes for each reservoir.

The scenarios used in this case are scenarios for income and price for each of the years from 1930 to 2000. In total, there are in this thesis used 70 scenarios in the SDP algorithm. These are gathered from NTE.

Algorithm 1 SDP algorithm

```
while W_{i-1,t=1} ≠ W_{i,t=1} ± ε do
    for t ∈ {52, 51, ..., 1} do
        for k ∈ {0, 10, ..., K} do
            for s ∈ {1, ..., S} do
                Solve one-stage problem with inputs λ_t^s, i_t^s, v_{t-1}, F_{t+1}(k),
                that returns ω_t^s and α_t^s (5.1.3)
            end for
            F_t(k) = Σ_{s∈S} (1/ρ_s) * α_t^s
            W_t(k) = Σ_{s∈S} (1/ρ_s) * ω_t^s
        end for
    end for
end while
```

5.1.3 One-stage Problem

In a medium-term hydropower scheduling problem a one-stage problem is solved one time for each scenario, start volume and week, as described in Algorithm 1, to calculate the water values that later can be used to find the production plans. The one-stage problem determines how much of the stored water that is used for production of electricity each week.

If hydro is produced in one week, the opportunity to produce hydro at a later time decreases with the amount of water used. At the same time, the risk of spillage increases with increasing reservoir levels. The optimal decision each week is dependent on electricity price, the amount of inflow, the initial reservoir level and the expected future profit of the stored hydro. In the one-stage problem, these quantities are assumed known. Therefore the one-stage problem is deterministic, meaning that all uncertainties are known.

The one-stage problem consists of one objective function that aims to maximize profit for the producer. In addition, the problem consists of multiple constraints that reflect the limitations of the water course. This can be constraints on reservoir capacity, production capacity, restrictions on discharge, etc. The objective function and constraints are explained in more detail in the following subsections.

SDP Model Formulation

In equation (6) the model formulation for the one-stage problem is presented. The objective function is described in equation (6a). The objective function maximizes the income in the present week in addition to the future income as a function of the end reservoir level. The price is given in EUR/GWh, the production in GWh and the future income in EUR.

$$max\ z = \lambda_t^s * p_t + \alpha(v_t) \tag{6a}$$

$$0 \leq v_t \leq V_{max} \tag{6b}$$

$$0 \leq p_t \leq P_{max} \tag{6c}$$

$$v_t = v_{t-1} - p_t - s_t + i_t^s \qquad (\omega_t^s) \tag{6d}$$

$$\alpha(v_t) = \sum_{k \in V} \gamma_k * F_{t+1}(k) \tag{6e}$$

$$v_t = \sum_{k \in K} \gamma_k * K_k \tag{6f}$$

$$\sum_{k \in V} \gamma_k = 1 \tag{6g}$$

$$p_t + s_t \geq P_{min}^{Samlop\ Funna} \tag{6h}$$

In a hydropower system, there are several physical constraints. The purpose of the physical constraints is to ensure that the physics in the watercourse is held. One of these physical constraints is the maximum reservoir capacity, described in equation (6b). This constraint states that the reservoir volume in one stage has to be higher than zero and lower than the maximum reservoir volume permitted. For the aggregated case the unit of the reservoir volume is GWh.

Another physical constraint is the maximum production capacity. This constraint ensures that the production plants are operated safely and legally. The maximum production constraint is described in equation (6c). This constraint states that the production has to be greater than zero and lower than the upper production limit. For the aggregated case, the unit of production is GWh. In this master's thesis, no efficiency effects nor head effects are included.

For the aggregated watercourse there are one waterway into the system and two waterways out of the system. The inflow, i_t^s, is the water coming into the system and the production p_t is the water converted to electricity. There is assumed to be no bypass discharge in this system, so therefore the second outflow from the system is the spillage, s_t. The spillage is the water that flows out of the system when the stored water exceeds the reservoir capacity. For that reason, the spillage is wasted energy, because it cannot be used for electricity production. The reservoir balance is presented in equation (6d).

The future income in week t is calculated based on an interpolation between the values is the future vector from week t + 1, $F_{t+1}(k)$, and the volume vector with the discrete volumes, V. To do this a so-called SOS2-variable, γ_k, is used. This is a variable that has to be between 0 and 1, and where at most two variables can be non-zero. These variables also have to be adjacent to each other. Equation (6e)-(6g) is the equations used to describe the future income. Since the problem is convex, there is no need to have an extra constraint to ensure that the SOS2-variable behaves as intended in the model.

In equation (6h) a constraint representing the minimum discharge is presented. The constraints state that the sum of the production and spillage has to be larger than a minimum level to fulfill the minimum discharge at Samløp Funna. The minimum discharge is 9.5 m^3/s, but are in the SDP model converted to GWh. This is done by using the energy equivalents for the two upstream plants, Meraker and Funna. These are again weighted by 50 % each, meaning that it is assumed that Samløp Funna will get half of the water to fulfill the restriction from each plant.

5.1.4 Disaggregation Based on Obtained Water Values

After the SDP problem is solved for the aggregated water course and the water values for each week and scenario are calculated, a disaggregation has to be done to find the production plans for each reservoir and plant. This is done by creating restrictions for each reservoir and plant, and the connections between them. In addition, restrictions for minimum discharge are created.

The problem is then solved for every week and scenario, starting at week one and going forward. The input to the problem is the start reservoir in week 1 for all reservoirs. The start reservoirs are found by looking at the average reservoir levels at the beginning of the year for the last 10 years. In addition, the water values from the SDP model are input to the disaggregation model.

Algorithm 2 Disaggregation algorithm

```
for s ∈ {1, ..., S} do
    for t ∈ {1, 2, ..., 52} do
        Solve weekly problem with inputs λ_t^s, i_t^s, v_{t−1}, ω_t^s
                that returns p_t^p, v_t^r and s_t^r
    end for
end for
```

The algorithm for the disaggregation is shown in algorithm 2. The weekly problem returns production, reservoir volumes and spillage for each unit, each scenario and each week. This is used to print results from the model, which is shown in section 6. In the next sub-sections, the objective function and the constraints for the weekly problem are presented.

Weekly Problem Formulation

The weekly problem is formulated in equation (7). The objective function for the weekly problem aims to maximize the price times the production for each unit plus the future income minus the penalty cost for missing water. Equation (7a) shows the objective function.

$$max\ z = \sum_{p \in P} \lambda_t^s * p_t^p - \sum_{r \in R} \lambda_{mw} * mw_t^r * E_r + F_{t+1}(v_t) \tag{7a}$$

$$0 \leq v_t^r \leq V_{max}^r \qquad r = r_1, \ldots, R \tag{7b}$$

$$0 \leq p_t^p \leq P_{max}^p \qquad p = p_1, \ldots, P \tag{7c}$$

$$v_t^r = v_{t-1}^r + c_{m^3/s\ to\ Mm^3} * (-q_t^r - s_t^r + i_t^r) \qquad r = r_1, \ldots, R \tag{7d}$$

$$i_t^r = q_t^{r-1} + s_t^{r-1} + i_{t,\ natural}^r \qquad r = r_1, \ldots, R \tag{7e}$$

$$p_t^p = E_r * c_{m^3/s\ to\ Mm^3} * q_t^r \qquad r = Fjergen, Tevla\ Dam, Funnsjøen \tag{7f}$$

$$w_{r,k} = W_t^k * E_r^{tot} \qquad r = r_1, \ldots, R \qquad k = k_1, \ldots, K \tag{7g}$$

$$F_{t+1} \leq \sum_{r \in R} \sum_{k \in K} \mu_{r,k} * w_{r,k} * K_k \tag{7h}$$

$$v_t^r = \sum_{k \in K} \mu_{r,k} * K_k \qquad r = r_1, \ldots, R \tag{7i}$$

$$q_t^r \leq Q_{max}^r \qquad r = Hallsjøen, Skurdalsjøen \qquad t = 1, \ldots, 17 \tag{7j}$$

$$q_t^r \leq 0 \qquad r = Hallsjøen, Skurdalsjøen \qquad t = 18, \ldots, 47 \tag{7k}$$

$$q_t^r \leq 0.5 * Q_{max}^r \qquad r = Hallsjøen, Skurdalsjøen \qquad t = 48, \ldots, 52 \tag{7l}$$

$$q_t^{Funnsjoen} + q_t^{Meråker} + s_t^{Fjergen} + s_t^{Funnsjøen} + s_t^{Tevla\ Dam} \geq Q_{min}^{Samløp\ Funna} \tag{7m}$$

Also in this model, constraints on maximum generation and production are needed. The difference is that the constraint now applies to each reservoir and production plant instead for the aggregated unit. The constraint are presented in equation (7b) and (7c).

A constraint for reservoir balance is also needed in this model. In this model, there is one reservoir balance constraint for each reservoir. Since discharge, spillage and inflow has the unit m^3/s, while the reservoir volume has a unit of Mm^3, a conversion factor, $c_{m^3/s\ to\ Mm^3}$, is added in the equation. The reservoir balance constraint is shown in equation (7d).

To formulate the connections between the different reservoirs, a constraint for inflow and discharge is formulated. These constraints state that the inflow in one reservoir is equal to the discharge and spillage from the upper reservoirs together with the natural inflow to the reservoir. The natural inflow is calculated using a share of the average yearly inflow times the inflow in scenario s for the whole watercourse. The constraint for inflow and discharge is presented in equation (7e).

In this case, all discharge from a reservoir is assumed to be discharge to produce electricity. Therefore, a constraint showing the relationship between discharge and production is needed. Since the discharge has the unit m^3/s and production has the unit GWh the conversion factor from m^3/s to Mm^3 in combination with the local energy equivalent, E_r, has to be used. Equation (7f) shows this constraint. Mark that this constraint only applies for the reservoirs r with a production unit p directly downstream.

From the SDP model a water value for the aggregated water course is calculated. For the disaggregation model, this water value has to be converted to a water value that applies to each reservoir. This is done by using the total energy equivalent for each reservoir. Equation (7g) shows the constraint for converting the water value to apply for each reservoir.

The water value is again used to calculate the future income. Since the water values are used to calculate the future income now, some sort of integral is utilized. The future income is then equal to the sum of the water values times the discrete volume section times the μ-variable over the reservoirs and the discrete volumes. The μ-variable is a variable between 0 and 1, that is 1 up to the present volume, 0 over the present volume, and a value between 0 and 1 at the present volume. Since the problem is convex the variable wants to take the value 1 for the lowest volume first since the water value there is the largest. Equation (7h) and (7i) are the two constraints used to describe the future income and volume.

The discharge from Hallsjøen and Skurdalsjøen reservoirs is determined by the operation of the gates at Hallsjøen and Skurdalsjøen. These are normally closed from the beginning of the spring flood until about the first of December. Then they are opened partly, and then fully opened after Christmas. In this model, the gate opening is set to fully open in week 1 to 17, closed in week 18 to 47 and half-open from week 48 to 52, as an approximation of the normal operation. Equation (7j), (7k) and (7l) describes this relationship.

One additional constraint in the Meråker water course is the minimum discharge at Samløp Funna. This is a measuring point downstream of both Meråker and Funna power stations. The discharge at this point has to be larger than 9.5 m^3/s at all times. The discharge is determined by the production at Meråker and Funna, in addition to the spillage from Fjergen, Funnsjøen and Tevla Dam. The minimum discharge constraint is shown in equation (7m).

5.2 Solution Algorithm - SDDP

The solution algorithm used to solve the hydropower scheduling problem for the Meråker watercourse using SDDP is described in this subsection. Also here the complex problem is solved by solving many smaller sub-problems, but now using both a forward and a backward pass. The solution algorithm is shown in algorithm 3.

Algorithm 3 SDDP algorithm

$\mathbf{v_t^r}$ *initially set to the mean reservoir volume values from NTE's historical data*
Set initial values for $\phi^{\mathbf{r,v}}_{\mathbf{t=T}}$ *and* $\delta^{\mathbf{v}}_{\mathbf{t=T}}$

i=1

while $\underline{z} \notin [\hat{z} - 1.96 * \hat{\sigma}, \hat{z} + 1.96 * \hat{\sigma}]$ & i $\leq I$ **do**

  for $t \in \{T, \ldots, 1\}$ **do**
    for $v \in \{1, 2, \ldots, V\}$ **do**
      for $s \in \{1, 2, \ldots, S\}$ **do**
        Solve backward one-stage problem *with inputs* λ_t^s, i_t^s, $v_t^{r,v}$, $\phi_{t+1}^{r,v}(i)$, $\delta_{t+1}^v(i)$, *that returns* $\omega_t^{s,r,v}$ *and* $z_t^{s,v}$ (9)
      end for
      $\phi_t^{r,v}(i = last) = \sum_{s \in S} \rho_s * \omega_t^{s,r,v}$
      $\delta_t^v(i = last) = \sum_{s \in S} \rho_s * z_t^{s,v} - \sum_{r \in R} \phi_t^{r,v}(i = last) * v_t^{r,v}$
    end for
  end for

  $\underline{z} = \rho_s * \rho_v * \sum_{v \in V} \sum_{s \in S} z_{t=1}^{s,v}$

  Pick V random scenarios
  for $v \in \{1, \ldots, V\}$ **do**
    $v_0^v = v_{init}$
    for $t \in \{1, 2, \ldots, T\}$ **do**
      Solve forward one-stage problem *with inputs* λ_t^v, i_t^v, $v_t^{r,v}$, $\phi_{t+1}^{r,v}$, δ_{t+1}^v, *that returns* r_t^v *and* v_{t+1}^v (9)
    end for
    $z^v = \sum_{t \in T} r_t^v$
  end for

  $\hat{z} = \sum_{v \in V} \rho_v * z^v$ $\hat{\sigma} = (\frac{1}{V-1} * \sum_{v \in V} (z^v - \hat{z})^2)^{\frac{1}{2}}$

  $i = i + 1$

end while

The backward pass optimizes the backward one-stage problem for each week, each volume scenario from the forward pass and each inflow scenario. The backward pass starts at the last week in the planning horizon. The input is the price and the inflow from the current inflow scenario, the start volume for the current week from the forward pass and the future value and cuts from the earlier iterations.

The initial cut and future value for the last week are set to three different values based on the mean price and the mean price times the initial start volume, respectively. The reason for this is to create a more realistic end value that depends on the end reservoir level. In equation 8 the initial cuts, ϕ, and future values, δ, for the three different scenarios can be seen, and in table 5 the choice of values for the parameters in the equation are shown.

$$\delta_{t=T}^{high} = k^{high} * \lambda_{mean} * v_{t=T}^{init} \tag{8a}$$

$$\phi_{t=T}^{high} = m^{high} * \lambda_{mean} \tag{8b}$$

$$\delta_{t=T}^{mean} = k^{mean} * \lambda_{mean} * v_{t=T}^{init} \tag{8c}$$

$$\phi_{t=T}^{mean} = m^{high} * \lambda_{mean} \tag{8d}$$

$$\delta_{t=T}^{low} = k^{low} * \lambda_{mean} * v_{t=T}^{init} \tag{8e}$$

$$\phi_{t=T}^{low} = m^{low} * \lambda_{mean} \tag{8f}$$

Table 5: Value of Parameters for End Value

Parameter for End Value	**Value**
$\mathbf{k^{high}}$	$\frac{6}{5}$
$\mathbf{m^{high}}$	$\frac{1}{2}$
$\mathbf{k^{mean}}$	1
$\mathbf{m^{mean}}$	1
$\mathbf{k^{low}}$	$\frac{2}{5}$
$\mathbf{m^{low}}$	2

The initial volume scenario is equal to a mean reservoir volume slope over the 10 last years, from NTE's historical data. The backward problem returns the water values or cuts for each reservoir and the future value. The average future value in week one for all scenarios and initial reservoir volumes is used to calculate the lower bound.

The forward pass optimizes a forward one-stage problem for each volume scenario and week. The volume scenarios are the randomly picked scenarios that give the initial reservoir volumes for the backward loop. For the forward pass, 40 out of 70 scenarios are randomly picked to represent the uncertainty in the future. If all 70 scenarios are used, the computational time gets significantly high. Because of this, only 40 scenarios are picked to get the best possible solution in a reasonable time.

The input to the forward loop is the price and the inflow from the current scenario, the start reservoir volume for the current week and the cuts and the future values from earlier iterations. The start reservoir volumes in the first week of every iteration are set to an average value for the beginning of the year for the last 10 years. The model returns values for reservoir volumes each week and the objective function value without the future value. The objective function value without the future value is used to calculate the convergence interval.

When the problem has been solved for all weeks in the planning horizon for both the forward and backward pass the algorithm re-solves, with new values for start volumes each week, until the convergence criteria is obtained. Convergence is obtained when the lower bound from the backward pass is inside the confidence interval from the forward pass and the minimum number of iterations is reached. When the SDDP model has converged, the obtained water values and the detailed production plans for each plant each week are returned and plotted.

5.2.1 Forward and Backward One-Stage Problem

In this section, the forward and backward one-stage problems are presented. The forward and backward one-stage problem are the same, but have different inputs, as earlier described. The one-stage problem is shown in equation (9).

$$min\ z = \sum_{p \in P} -\lambda_t^s * p_t^p + \sum_{r \in R} \lambda_{mw} * mw_t^r * E_r + \alpha_{t+1} \tag{9a}$$

$$0 \leq v_t^r \leq V_{max}^r \qquad r \in R \tag{9b}$$

$$0 \leq p_t^p \leq P_{max}^p \qquad p \in P \tag{9c}$$

$$v_t^r = v_{t-1}^{r-1} - q_t^r - s_t^r + i_t^{s,r} + mw_t^r \qquad r \in R \quad (\omega^r) \tag{9d}$$

$$\alpha_{t+1} \geq \sum_{r \in R} \phi_{t+1}^r(i) * v_t^r + \delta_{t+1}(i) \qquad i = 1, \ldots, I \tag{9e}$$

$$q_t^{Funnsjøen} + q_t^{Tevla\ Dam} + s_t^{Fjergen} + s_t^{Funnsjøen} + s_t^{Tevla\ Dam} \geq Q_{min}^{Samløp\ Funna} \tag{9f}$$

The forward and backward one-stage problems have many similarities with the one-stage problem from SDP. Equation (9a) is the objective function, which aims to minimize the negative of the income plus the penalty cost and the future value. This is the same as maximizing the income, as done in the SDP one-stage problem. The constraints for the maximum generation and production are presented in equation (9b) and (9c). The reservoir balance constraint is given in equation (9d). The dual value of this constraint is the water value which gives the cuts for the next iterations.

In equation (9e) the constraint for the future value is shown. This is a bit different from the future income constraint in SDP. The future value in SDDP is limited by the cuts and future values from earlier iterations. This means that for each iteration the one-stage problem has one more constraint. Lastly, the constraint for the minimum discharge is shown in equation (9f).

6 Results

In this section the results from the SDP and SDDP model will be presented and compared. First the reservoir trajectories for the two large reservoirs Fjergen and Funnsjøen will be presented, then the fulfillment of the minimum discharge constraint, the total spillage in the watercourse, the accumulated production and the total income. At the end the water values are presented for the two models.

6.1 Reservoir Trajectories

In this section the reservoir trajectories for Fjergen and Funnsjøen are presented using both SDP and SDDP.

6.1.1 Fjergen

In figure 11 the reservoir trajectories for Fjergen using SDP are shown, and figure 12 shows the reservoir trajectory for Fjergen using SDDP.

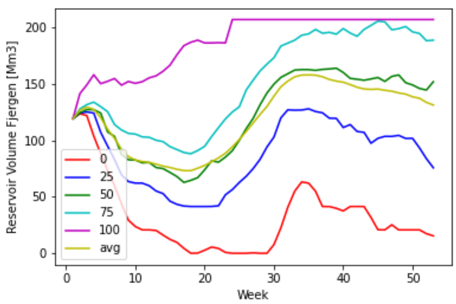

Figure 11: Reservoir Trajectory for Fjergen - SDP

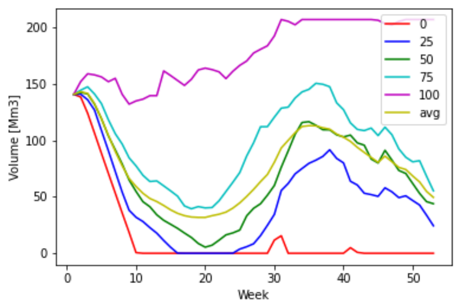

Figure 12: Reservoir Trajectory for Fjergen - SDDP

The main differences in these figures are that when using SDDP the reservoir volume for Fjergen generally are lower, and the two lowest percentiles often gives empty reservoir. Unlike, in SDP where the volume are higher and the reservoir are more often full in the higher percentiles, leading to more spillage.

6.1.2 Funnsjøen

In figure 13 the reservoir trajectories for Funnsjøen using SDP are shown, and figure 14 shows the reservoir trajectory for Funnsjøen using SDDP.

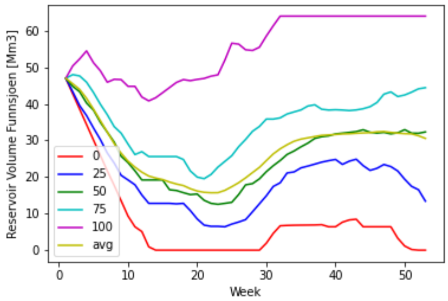

Figure 13: Reservoir Trajectory for Funnsjøen - SDP

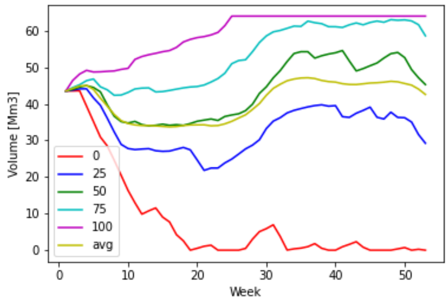

Figure 14: Reservoir Trajectory for Funnsjøen - SDDP

For Funnsjøen the reservoir trajectories from SDP generally gives lower reservoir volumes and more empty reservoir, while SDDP gives generally higher reservoir volumes, more full reservoir and hence more spillage. This is the opposite of what happens for Fjergen.

6.2 Missing water to fulfill the minimum discharge constraint

To see whether the minimum discharge constraint is equally well fulfilled in both models the missing water in the model is presented in table 6. The table shows the total number of scenarios in the two models. Since the results in SDDP are gathered from the forward loop, the total number of scenarios in SDDP is 40 and not 70. The table also shows the number of scenarios that are missing water in one or more weeks and the total amount of missing water over all scenarios.

Table 6: Missing water to fulfill the minimum discharge

Method	**Number of scenarios**	**Scenarios missing water**	**Scenarios missing water** [%]	**Total amount** [Mm^3]
SDP	70	3	4.3	24.91
SDDP	40	3	7.5	24.33

These results show that the number of scenarios missing water in the two models is the same, but the share of scenarios missing water is a bit larger in SDDP. The total amount of missing water in the two models is about the same.

6.3 Total Spillage

In figure 15 the total spillage for Fjergen, Funnsjøen and Tevla Dam using SDP are shown, and figure 16 shows the total spillage for Fjergen, Funnsjøen and Tevla Dam using SDDP. The figures shows that the spillage from the SDP model are generally higher than from the SDDP model most of the planning horizon.

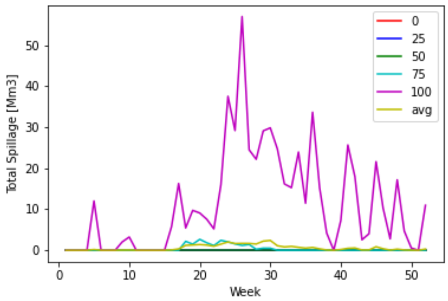

Figure 15: Total Spillage for Fjergen, Funnsjøen and Tevla Dam - SDP

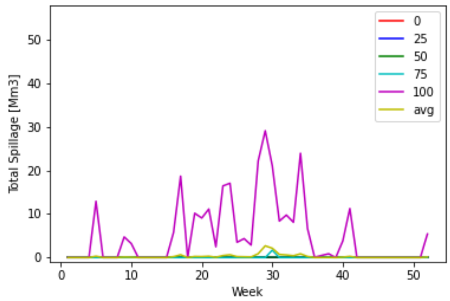

Figure 16: Total Spillage for Fjergen, Funnsjøen and Tevla Dam - SDDP

In table 7 the total accumulated spillage for SDP and SDDP are presented. It shows that all the percentiles and the average are higher when using SDP. This means that when using SDP, the model are not able to save as much of the inflow as the SDDP model does. This leads to lost production opportunities, and possible lower income.

Table 7: Total Accumulated Spillage

Percentile	**0**	**25**	**50**	**75**	**100**	**Average**
Total Spillage $[Mm^3]$ **SDP**	0	10.9	25.3	40.8	134.8	29.8
Total Spillage $[Mm^3]$ **SDDP**	0	1.5	9.2	21.1	57.6	13.0

6.4 Accumulated Production

In figure 17 the total accumulated production for Tevla, Meråker and Funna power stations using SDP are shown, and figure 18 shows the total accumulated production using SDDP. The figures shows that the production are somewhat higher in the SDDP model compared to the SDP model.

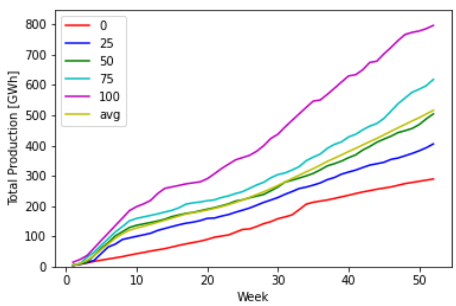

Figure 17: Total Accumulated Production - SDP

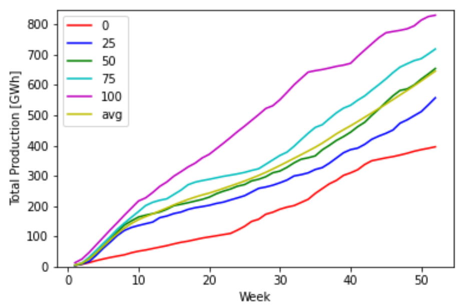

Figure 18: Total Accumulated Production - SDDP

In table 8 the total accumulated production for the planning horizon are shown. The total accumulated production are higher for SDDP than for SDP for all percentiles and for the average. For the average the production are 25 % higher in SDDP compared to SDP. This difference is mostly because of the SDDP model using more of the water in Fjergen. In addition the greater spillage in the SDP model contributes as well. For the average about 90 % of the difference comes from the lower end reservoir in SDDP, while about 10 % comes from higher spillage in the SDP model.

Table 8: Total Accumulated Production

Percentile	**0**	**25**	**50**	**75**	**100**	**Average**
Total Production $[GWh]$ **SDP**	289.4	405.4	504.6	618.5	797.0	516.5
Total Production $[GWh]$ **SDDP**	395.8	557.8	654.5	719.0	830.7	646.1

6.5 Total Income

In table 9 the average values for end reservoir for Fjergen and Funnsjøen for both SDP and SDDP are presented. In addition the average income including the penalty for not having enough water to fulfill the minimum discharge are presented. The penalty is set to 700 000 EUR/GWh. In the end of the table the average total value for both SDP and SDDP are presented. The total value are the accumulated income including the penalty cost plus the value of the end reservoirs. The value of the end reservoirs are calculated using the total energy equivalent to convert from Mm^3 to GWh, and then multiplying with the average price in the price scenarios, which is 28 283.9 EUR/GWh.

Table 9: Average End Reservoir, Income Including Penalty and Total Value

Method	**End Reservoir Fjergen** [Mm^3]	**End Reservoir Funnsjøen** [Mm^3]	**Income [MEUR]**	**Total Value [MEUR]**
SDP	131.3	30.6	16.8	21.3
SDDP	54.8	44.1	19.6	22.2

The table shows that the end reservoir in Fjergen is higher in SDP than in SDDP, while for Funnsjøen the end reservoir is higher in SDDP. The accumulated income including the penalty cost are higher for SDDP. The total value, which is the sum of accumulated income and the value of the end reservoir, is also higher for the average in SDDP. The average total value is 4 % higher in SDDP compared to the SDP model.

In table 10 the total value for all percentiles are presented. The table shows that for all percentiles except the 75-percentile the SDDP model gives higher total value compared to the SDP model.

Table 10: Total Value

Percentile	**0**	**25**	**50**	**75**	**100**
Total Value [MEUR] - SDP	5.9	13.4	21.0	27.8	41.6
Total Value [MEUR] - SDDP	6.1	15.9	21.4	26.4	43.8

6.6 Water Values

In figure 19 the aggregated water values for the different start reservoir levels in SDP are presented. The start reservoir levels that are shown are 0, 10, 20, 50, 80 and 100 % of reservoir capacity. In the lower start reservoir levels, it can be seen a peak in the water value around week 20. This peak corresponds to the beginning of the spring flood. This means that there might be a shortage of water just before the spring flood, which leads to high water values. For the higher start reservoir levels, the water values are a bit higher than the average price in the autumn and winter and lower during summer.

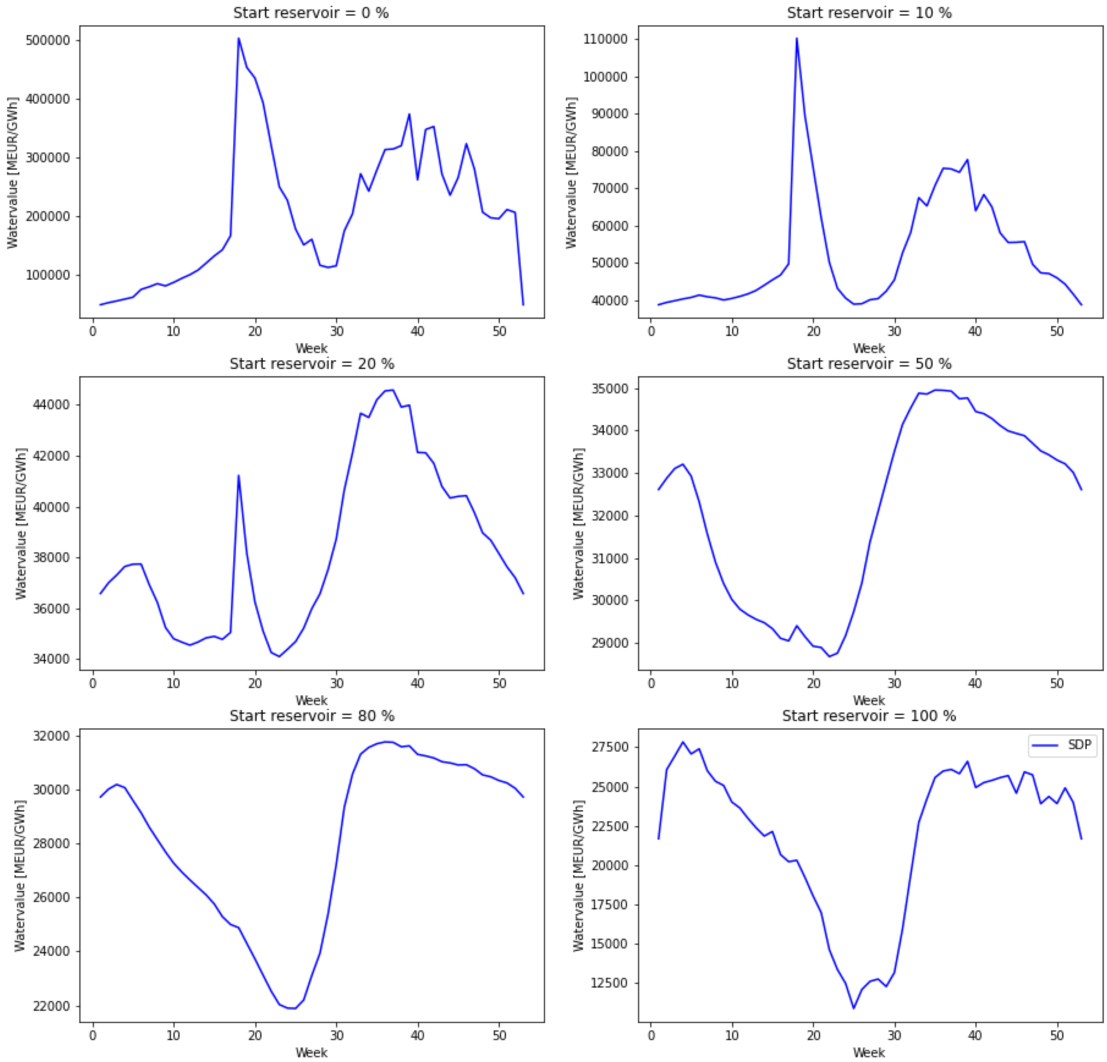

Figure 19: Water Values - SDP

In figure 20 and 22 the water values using SDDP for Fjergen and Funnsjøen are shown respectively. Figure 21 and 23 show the same water values zoomed in. Different from the SDP, the water values from SDDP are presented for each reservoir in percentiles and the average.

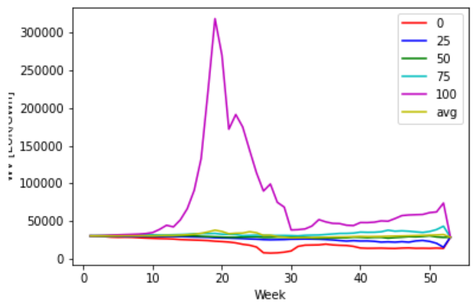

Figure 20: Water Values, Fjergen - SDDP

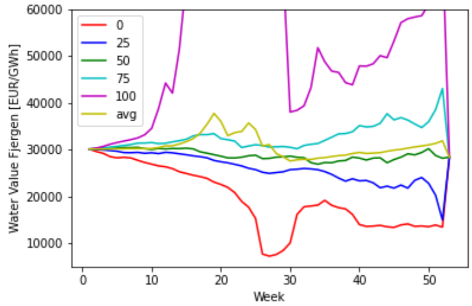

Figure 21: Water Values Zoomed in, Fjergen - SDDP

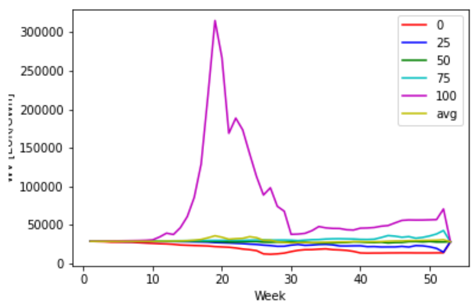

Figure 22: Water Values, Funnsjøen - SDDP

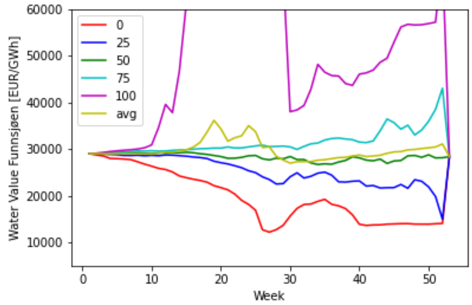

Figure 23: Water Values Zoomed in, Funnsjøen - SDDP

The figures show that the water values in Fjergen and Funnsjøen are somewhat similar. In the highest percentile, the SDDP model also gives a high peak around week 20 like in the SDP model. The level of the peak is between the peak from the 0 and the 10 % start reservoir level in SDP.

In the other percentiles, the water values vary around the average price, without any clear pattern. The reason for this can be the percentiles that do not follow one scenario.

7 Discussion

The results in section 6 have shown that the reservoir volume in Fjergen is lower using the SDDP model, while in Funnsjøen the reservoir volume is higher using SDDP. The reason for this could be that in SDP you have production plans based on the aggregated water values for the whole water course. This means that the SDP model will plan both Funnsjøen and Fjergen based on the aggregated water values. However, in SDDP the model will give individual water values and production plans for each plant and reservoir. Since the degree of regulation is higher in Funnsjøen than in Fjergen, the SDDP model will tend to discharge more from Fjergen than from Funnsjøen to have more available capacity for the inflow.

The results also show lower total spillage when using SDDP compared to when using SDP. The reason for this can again be the more customized production plans. The SDDP model will give lower reservoir volumes where the degree of regulation is lowest giving more space for the inflow in the right places. This will lead to better handling of inflow and less spillage.

Another outcome of this is that the production is also higher in SDDP. This is mostly because of the lower end reservoir level in SDDP, but about 10 % comes from better utilization of inflow. This again gives a higher accumulated income, also when taking the penalty cost into account. Since the end reservoir levels are different in the two models, the total value has been calculated, using the average price to value the water left in the reservoir at the end of the planning horizon. This has shown that the SDDP model gives the highest total value for all percentiles and the average, except the 75-percentile. This can be explained by the better production plans giving better handling of inflow, more production and hence more income, but still leaving some water left in the reservoir.

One of the disadvantages of using the SDDP model is the low reservoir volumes in the lowest percentiles for Fjergen. In large parts of the planning horizon, both the 0 and the 25-percentile give empty reservoir. Despite this difference, the results for the missing water show that the two models are missing water somewhat equally. They are both missing water in only 3 scenarios and the total amount is about 24 Mm^3. The missing water can possibly lead to breaking the minimum discharge, and give a high fee for the power producer. In the worst case, the power producer can lose the right to produce hydropower in that area. The risk of this happening is, as shown, equal in the two methods.

8 Conclusion and Further Work

8.1 Conclusion

This thesis has studied which of the two models, stochastic dynamic programming (SDP) and stochastic dynamic dual programming (SDDP), that provides the best usage of the water for the test case. This has been done by examining the reservoir trajectories, amount of missing water to fulfill the constraints, the total spillage, the production and income, as well as the total value for both models. The total value are the sum of the accumulated income, including the penalty cost for breaking the constraints, and the value of the end reservoirs.

The results has shown that the SDDP model give lower reservoir volumes for Fjergen and higher for Funnsjøen compared to the SDP model. The SDDP model has also given less spillage and higher production and income than the SDP model has. The total value has also been larger for SDDP than SDP for all percentiles except the 75-percentile.

The reason for these results are the better production plans in SDDP that comes from individual watervalues for each reservoir, leading to better handling of inflow. This again gives lower spillage, higher production, income and total value.

One disadvantage with the SDDP model are the low reservoir levels for Fjergen. The lowest percentiles gives empty reservoir during many weeks in the planning horizon. This can lead to not having enough water to fulfill the minimum discharge and possibly high costs for the power producer. Despite this, the results for missing water shows that the number of scenarios missing water and the total amount of missing water are about the same for the two models.

This shows that the SDDP model gives the best usage of the water for the test case, meaning that the production, the income and the total value are higher for the SDDP model, compared to the SDP model. Since both models give about the same amount of missing water, it can be concluded that the SDDP model gives the best results.

8.2 Further Work

This thesis has shown that the SDDP model gives less spillage, higher income and total value than the SDP model does for the test case, and the two models equally fulfill the minimum discharge constraint. Because of this, the thesis can conclude that the SDDP model gives the best usage of the water in the test system.

Since there are done some simplifications in this thesis there is a need for more research on comparing the two models for more complex hydropower systems. This can be systems including pumps and state-dependent constraints.

In addition the autocorrelation in price and inflow is not taken into account in this thesis. Including autocorrelation in income can lead to more states being visited in the SDDP algorithm, and give a more realistic picture of the uncertainty in inflow. Including the autocorrelation in price will possibly lead to better price expectations and production plans, and therefore better operation of the hydropower plants. This can also be further investigated to see if it will influence the problem.

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