Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Engineering
Department of Civil and Environmental Engineering

Knut-Egil Opseth

Optimization of Ground Investigations

Using Random Fields and Value of Information analysis

Master's thesis in Civil and Environmental Engineering
Supervisor: Ivan Depina
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Preface

The master thesis was written spring 2023. The report is the final work of a 5-year masters degree in civil and environmental engineering at the Norwegian University of Science and Engineering (NTNU). In this thesis, a method of optimizing ground investigations by utilizing random fields and value of information analysis, is created and assessed. The idea behind this study was raised by Dr. Ivan Depina. The report is written by student Knut-Egil Opseth.

Trondheim 2023

Acknowledgement

I would like to express my gratitude to all individuals who have supported me in completing this thesis. I want to thank my supervisor Dr. Ivan Depina, for the guidance, support and expertise. The feedback and encouragement has been fundamental for this study. I would also like to acknowledge the help provided by the other faculty members Espen Andersen Torsæter and Sigurdur Mar Valsson for their help with obtaining data on costs of ground investigations, as well as insightful discussions on the topics of this study. I am very gratefull for the motivation and assistance provided by my familiy and friends. I want to thank Lars Morten Opseth for taking his time to give feedback and interesting discussion on the thesis. Finally, I would like to express my appreciation to Helene Simensen Bergane and Elvira Opseth Bergane who have supported me in ways both seen and unseen. Your encouragement, patience, and belief in my abilities have been a constant source of motivation and inspiration.

Knut-Egil Opseth
Trondheim June 2023

Summary

Ground investigations are fundamental for obtaining a good understating of soil properties in a geotechnical analysis. They provide the necessary information to select the right type of geotechnical analyses, perform reliable calculations, and make safe design evaluations. Physical properties of the ground are complex and often to a certain extent unknown at the start of a project, and this leads to many uncertainties. Because of this, the planning of investigation placement requires detailed considerations. There are many occasions where the performed investigation results in too little information. Conversely, sometime investigations are conducted without detailed planning and do not provide new relevant information. A tool that can increase efficiency and optimize the process, would result in substantial time- and resource savings.

In this thesis, an attempt has been made to create a program that can optimize the placement and combination of ground investigation for an undrained slope stability problem. In this model the spatial variability and lack of knowledge about soil properties are modelled with random fields. The ground investigations are implemented to update the knowledge on soil properties in random fields by utilizing the principle of conditional random fields. To assess the effect of adding an investigation, the investigations impact on the probability of failure is used with the cost of failure to quantify the information in a value of information decision analysis. The different types of soil investigations are compared to each other and the most cost-effective option in terms of information value, after subtracting the investigation cost, is chosen. This process is repeated for a pre-determined number of investigations and the most profitable amount within this range is presented, along with their type and location.

The assessment of the model was done using a set of tests. Results from the tests show that the correlation lengths have a large impact on the optimal number of investigations, as well as their placement. By increasing the correlation lengths, the optimal number of investigations decrease. However, one exception to this was discovered. When the correlation lengths were small, the model stopped learning more information after a set of investigations. Leading to a lower optimal number than expected. Variation of standard deviation was also tested, and it showed that increasing the standard deviation led to each investigation providing more value in the form of information. Increasing cost of information also proved that implementing a larger risk provides a necessity of more investigations.

Results received from the methodology seems reasonable, in the fact that they place ground investigations spread along the shear circles. In addition to this, Cone penetration test (CPTU) is prioritized in the test with a larger horizontal correlation length. The laboratory tests are usually implemented after two to three CPTU have been added. The tests on the method are based on a simplified case of undrained slope stability, because of the model only being able to consider a small set of variables. This leads to limited usability, and only parts of the information each ground investigation provides is considered. The methodology shows a lot of potential, and through further development the problems encountered in this thesis can be handled

Sammendrag

Grunnundersøkelser er fundamentale for å oppnå en god forståelse av løsmassenes egenskaper i en geoteknisk analyse. Ved å se på grunnundersøkelser vil en kunne få den nødvendige informasjonen en trenger for å velge riktig geoteknisk analysemetode, noe som videre kan bidra til pålitelige beregninger og muligheten til å planlegge trygge løsninger. Løsmassenes fysiske egenskaper er komplekse, og til en viss grad ukjente, ved starten av et prosjekt. Dette medfører mange usikkerheter, noe som gjør at gjennomtenkte vurderinger er en nødvendighet ved planleggingen av grunnundersøkelser. Det finnes flere tilfeller hvor gjennomførte undersøkelser har ført til for lite informasjon. I tillegg til dette kan en også se på nye utførte undersøkelser, med liten grad av detaljstyrt planlegging, som videre ikke har tilført ny informasjon. Å kunne ha tilgang på et verktøy med hensikt å effektivisere og optimalisere grunnundersøkelser, vil kunne resultere i betydelige kutt i ressurser og tidsbruk.

I denne oppgaven er det gjort et forsøk på å lage et program som kan bidra til å optimalisere plassering og kombinasjon av grunnundersøkelser for et udrenert skråningsstabilitetsproblem. Mangel på oversikt over løsmassenes egenskaper og deres romlige variasjon er modellert med tilfeldige felt. Grunnundersøkelser er implementert ved å oppdatere kunnskapen om løsmassenes egenskaper i tilfeldige felt. Dette er gjort med utgangspunkt i prinsippet om betingede tilfeldige felt. For å vurdere effekten av å legge til en grunnundersøkelse er dens påvirkning på skråningens bruddsannsynlighet multiplisert med kostnaden ved et brudd. På denne måten kan man kvantifisere tilegnet informasjon til å ta valg med en informasjonsverdianalyse. Videre blir ulike typer grunnundersøkelser sammenlignet med utgangspunkt i hvilken undersøkelse som er mest kostnadseffektiv i form av informasjonsverdi minus kostnad av undersøkelse. Dette resulterer videre i at den mest gunstige undersøkelsen blir valgt, før prosessen gjentas for et forhåndsbestemt antall undersøkelser, og modellen gjengir optimalt antall, type og lokasjon.

Metoden i oppgaven ble vurdert ved å utføre et sett med tester. Resultatene viser at korrelasjonslengdene har hatt stor innvirkning på det gunstige antallet undersøkelser, og deres plassering. Med økende korrelasjonslengder avtar den optimale mengden undersøkelser. Ved lave korrelasjonslengder avtar mengden informasjon man lærer ved hver grunnun-

dersøkelse hyppig. Hvilket betyr at testene med de laveste korrelasjonslengdene får et optimalt antall undersøkelser som er lavere enn for større korrelasjonslengder. I tillegg har variasjon i standardavviket til udrenert skjærstyrke blitt testet, noe som viste at en økning i standardavvik medfører at hver grunnundersøkelse tilfører mer verdi i form av informasjon. Flere kostnader ved brudd ble vurdert i oppgaven. Kostnaden ved brudd ser ut til å påvirke modellen ved at økte kostnader vil være gunstig med en økt mengde grunnundersøkelser.

Resultatene som er kommet frem ved hjelp av metoden virker fornuftige, ettersom plasseringene blir jevnt fordelt utover skjærsirklene. I tillegg til dette blir CPTU prioritert og laboratorieundersøkelser blir lagt til etter to til tre CPTU har blitt implementert. Testingen av metoden er utført på et forenklet skråningstilfelle, noe som skylles at modellen har et lavt antall parametere den kan vurdere. Dette resulterer videre i at bruksområdet blir begrenset, og at kun deler av informasjonen grunnundersøkelsene tilbyr blir vurdert. Modellen viser stort potensiale, og utfordringene som ble oppdaget i denne oppgaven kan bli håndtert ved videre utvikling.

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Chapter 1

Introduction

1.1 Background

The physical properties of the ground are complex, as they have been shaped by millions of years of physical and chemical processes. Consequently, geotechnical engineers face significant material variability, resulting in substantial uncertainties that must be minimized to ensure the quality of their analysis.

To accomplish this, geotechnical practice typically relies on site investigations to gather the necessary information for decision-making and evaluation. Standard analysis methods often use characteristic values for each layer, based on local averages, which may lead to conservative results due to the soil's spatial variability being overlooked. While mapping the entire site would produce the most accurate analysis, it is prohibitively expensive.

Due to the high costs and time involved in ground investigations, selecting the optimal location to obtain the most information from each investigation is critical. The location selection process relies on engineering judgement based on experience and empiricism, leaving room for uncertainty quantification to enhance the decision-making process and optimize investigations.

1.2 Objective Definition

The aim of this master thesis is to develop and assess a method for optimising ground investigations for a slope stability problem. Specifically, the approach will enable the identification of the optimal location of four different types of investigations, while also providing insight into the optimal number and combination of investigations. The effectiveness of this approach will be evaluated to determine its potential value as a decision-making tool. By improving the efficiency of ground investigations, this method has the potential to reduce costs and time required for site characterisation, ultimately benefiting the preliminary stage of geotechnical engineering.

1.3 Limitations

This study is limited to only evaluate variation in the undrained shear strength in soil. This leads to the geotechnical analysis implemented being limited to an undrained slope stability analysis. The selection of this specific parameter has also influenced the choice of ground investigation method. The chosen ground investigations are the cone penetration test, vane shear test, unconfined compression test, and triaxial test. While additional investigation methods exist to explore the undrained shear strength parameter of soil, this thesis is confined to these four tests.

1.4 Problem Approach

The proposed methodology presented in this thesis involves the utilization of random fields and value of information analysis to optimize the number and locations of geotechnical investigations. The approach starts by developing an unconditional random field that simulates the spatial variability of ground conditions based on prior knowledge. The prior knowledge is defined based on preliminary knowledge on site conditions (e.g., nearby projects, preliminary soil investigations). The prior knowledge is defined in a probabilistic way through the parameters of a random field model. Following the definition of the prior knowledge, the effects of collecting additional soil investigations are modelled with conditional random fields.

To quantify the benefit of information derived from soil investigations, the proposed method applies a Value of Information (VoI) analysis. VoI analysis aims to determine the optimal location and the optimal type of the next ground investigation, which is the ground investigation that leads to the largest reduction in probability of failure relative to its cost. Additionally, VoI is conducted in a sequential approach to determine the optimal quantity of investigations needed.

In order to assess the performance of the VoI method, it is tested by varying prior knowledge and cost of failure. The method is implemented on an undrained slope stability problem.

To summarize, the main goal of this thesis is to create and assess a Value of Information methodology that can optimize type, placement and number of ground investigations. In pursuit of this objective, a series of specific goals have been set, and are listed in the following bullet points.

- A literature study was performed to obtain the necessary background information about the topics of soil variability, ground investigations, probabilistic slope stability and value of information analysis.
- A cost estimation method was developed, in order to accurately represent the ground investigations
- The third specific goal was to expand a Value of Information methodology to account for multiple ground investigations.
- The performance of the developed methodology was evaluated by varying the factors, cost of failure, standard deviation, and correlation lengths.

1.5 Thesis Structure

- **Chapter 2** *provides a theoretical base on the ground investigations that will be considered.*
- **Chapter 3** *describes uncertainties in soil parameters and the random field methodology of modelling the inherent variability of soil parameters.*
- **Chapter 4** *describes the direct method and how it is used for the probabilistic analysis of the factor of safety in this thesis.*
- **Chapter 5** *considers the prior knowledge in the value of information analysis and derives a value of information decision analysis and how to implement additional value of information.*
- **Chapter 6** *describes how costs of ground investigations are retrieved and provides an overview of the cost that are associated with the selected ground investigations.*
- **Chapter 7** *describes the composition of the model, and how the different aspects of the problem is implemented.*
- **Chapter 8** *showcases the tests that are preformed and their corresponding results.*
- **Chapter 9** *discusses the optimal location and amounts from the test results, in addition to how the model performs.*
- **Chapter 10** *provides a summary and a conclusion, along with recommendation for further development of the model.*

Chapter 2

Ground Investigations

Comparison and evaluation of ground/soil investigations is the primary concern of this thesis, making it important to introduce the principle of the considered types. Ground investigations are the central source of information in geotechnical engineering. They are used to find the depth to bedrock, finding the soil layering, determining soil strength parameters, and analysing soil contamination (Statens vegvesen 2015 p. 33). In this thesis ground investigations that can interpret soil strength parameters will be discussed, more specifically undrained shear strength. According to Statens vegvesen 2015 there are multiple investigation methods that can provide strength parameters, but some of them like the total sounding are more useful in determining soil layering and relative strength. The model created for this thesis evaluates four types of ground investigations that can determine undrained shear strength. These are cone penetration test (CPTU), Vane shear test (VST), unconfined compression test (UC) and a triaxial test (TC).

2.1 Cone Penetration Test

The cone penetration test (CPT) is an in-situ soil investigation that is used to classify soil and strength parameters. This test is executed by pushing a cone penetrometer, displayed in Figure 2.1 into the ground which record data at set intervals. The cone penetrometer is made of cone tip, a cylindrical expansion, and bar system that is used to drive the tip (NTNU geotechnical division 2017 p. 37). The interval of recording is approximately every 2 cm, which leads to a nearly continuous measurement over a depth. It records tip resistance q_c and side friction f_s. In addition to this it can be equipped with a pore pressure sensor to measure the pore pressure. The test is then referred to as a CPTU.

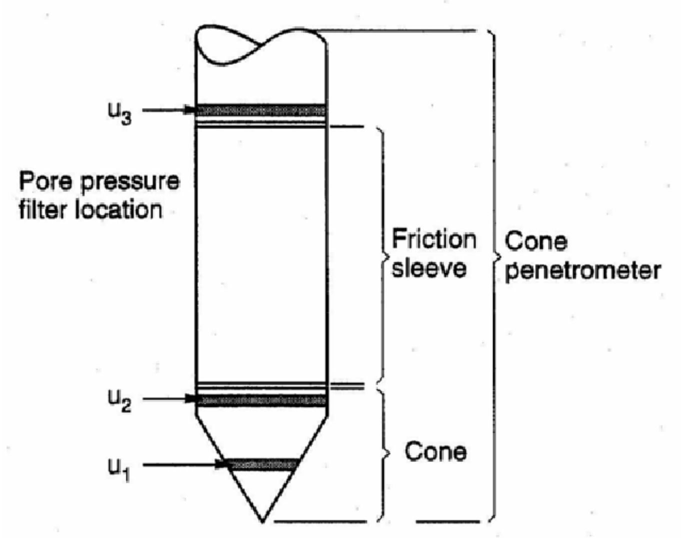


Figure 2.1: Cone penetrometer used in CPTU (Powell and T. A. Lunne 2005)

Due to the geometry of the cone penetrometer, pore pressure can build up by the joint between the cone and cylinder. This will create an unbalanced force, and the pore pressure must be considered in order to accurately determine the tip resistance. The corrected tip resistance q_t is given by equation 2.1 (NTNU geotechnical division 2017 p. 37).

$$q_t = q_c + (1 - a)u_2 \tag{2.1}$$

The pore pressure at the joint is represented by u_2, also shown in Figure 2.1. The factor a is the relationship between the area of the load cells cross section, and the projected area of the cone.

The undrained shear strength s_u can be determined from the corrected tip resistance q_t. The relationship between the two factors is given by Equation 2.2 (NTNU geotechnical division 2017 p. 68).

$$s_u = \frac{q_t - \sigma_{v0}}{N_k} \tag{2.2}$$

2.2 Vane Shear Test

The vane shear test is an in-situ soil investigation method used to measure undrained shear strength and sensitivity. This method is most suitable for fine-grained sediments such as clay and silt (Statens vegvesen 2015 p. 32). To conduct the test, a rod with a vane tip is driven to the desired depth, where the vane is slowly rotated at a rate of 6 to 12 degrees per minute (Geotechdata.info 2023). Torque measurements are taken at regular intervals until the maximum torque is reached, at which point the soil fails in shear. The remolded shear strength can then be determined, and sensitivity can be calculated (Geotechdata.info 2023). The test is typically conducted in intervals of 0.5 to 1 meter (NTNU geotechnical division 2017 p. 62).

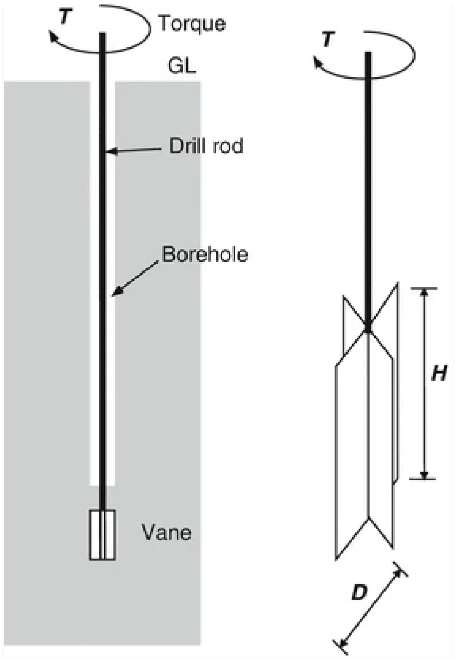


Figure 2.2: Typical geometry of the vane shear test (Ameratunga et al. 2016)

Undrained shear strength is calculated from the measured torque using Equation 2.3, where T is the torque and D is the vane's diameter (Norsk Geoteknisk Forening 1981 p. 3). This equation is based on the assumption that the height of the vane, usually equals 2 × width.

$$s_u = \frac{6T}{7\pi D^3} \tag{2.3}$$

The VST has some limitations in measuring high and low values of $\frac{s_u}{\sigma'}$, where σ' represents effective stresses (NTNU geotechnical division 2017 p. 63). This source of error can be corrected by a correction factor determined by laboratory results, but this correction factor has not been implemented in the methodology of this thesis.

2.3 Unconfined compression test

The unconfined compression test, also known as a uniaxial test, is a laboratory test, which is usually included in a set of standard testing procedures for test sample cylinders (Norsk Geoteknisk Forening 2020 p. 19). The test is usually performed on a Ø54 mm test sample (NTNU geotechnical division 2017 p. 159). The test is executed by applying an axial load at a constant strain rate. The undrained shear strength is measured as the shear-value reached when the test sample fails. It is calculated through equation 2.4 (NTNU geotechnical division 2017 p. 160).

$$s_u = \tau_{\text{max}} = \frac{\sigma_{1,\text{ failure}}}{2} = \frac{P_{\text{failure}}(1-\epsilon)}{2A_0} \tag{2.4}$$

The unconfined compression test is classified as an uncertain test method, and it is not recommended to rely solely on it for evaluating undrained shear strength (s_u) (Statens vegvesen 2015 p.110). The uncertainties associated with it stem from it not being able to account for the anisotropy of the soil's shear strength. However, when analysing a homogenous soil, it is generally considered to produce reasonable results (NTNU geotechnical division 2017 p. 161).

2.4 Triaxial Test

The triaxial test is a laboratory test commonly utilized to determine various soil parameters. Similar to the unconfined compression test, the triaxial test involves applying an axial load. Additionally, an external water pressure is applied as radial load to simulate in-situ conditions on the test sample (Statens vegvesen 2015 p. 93). The triaxial test records the applied loads, strain, and water leaving the soil. Various types of triaxial tests exists and are based on the conditions that the test sample is subjected to. The consolidated undrained test (CU) is the preferred triaxial test for determining the soil's undrained shear strength. There are multiple ways to interpret the undrained shear strength from a triaxial test, and a common method is by using a plot of mobilized shear stress over strain (Statens vegvesen 2015 p. 106). In this plot s_u is determined in the area of 1 - 5 % strain(Statens vegvesen 2015 p. 106). The triaxial test is the preferred test method for measuring s_u (NIFS-prosjektet 2014 p. 27).

Chapter 3

Soil Variability

3.1 Uncertainties in Soil Parameters

The measurement of soil variability has often been carried out by considering the total variability. However, this approach has certain limitations, as it restricts the usefulness of the data to specific types of analysis due to the complex nature of geotechnical uncertainty. In his work "On evaluation of static soil properties" (1992), Kulhawy introduced a novel method for dividing the uncertainty in soil property estimates into multiple components, as illustrated in Figure 3.1 (Phoon and Kulhawy 1999a p. 2).

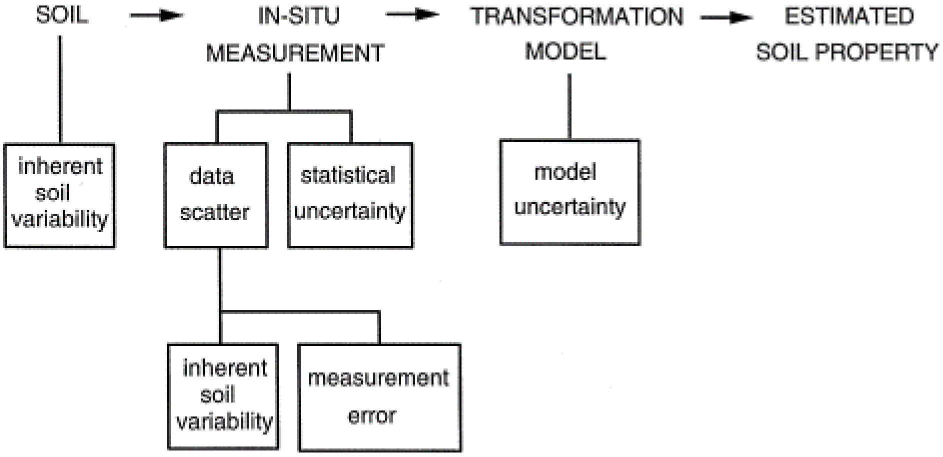


Figure 3.1: Uncertainties in soil properties

This methodology recognizes that design parameters of soil are subject to more than one source of uncertainty. Of relevance to this thesis is the section about in-situ measurement, which is further subdivided into three distinct sources of uncertainty. The first source, referred to as inherent variability, represents the natural variability resulting from geological processes that modify the soil. The second source, measurement error, captures deviations from equipment, random test effects, and operational differences. The third and final source, statistical uncertainties, arises from limited amounts of available information (Phoon and Kulhawy 1999a p. 3).

3.1.1 Soil Variability Modelling

Soil is a complex material with variations in its properties both in the vertical and horizontal directions. To capture and model the spatial variability of soil, the random fields method could be employed. This methodology is elaborately discussed in Section 3.2. However, to apply this method effectively, statistical information regarding the desired soil properties must be provided. This information is based on mean values, standard deviations, and correlation distances. In geotechnical practice, the uncertainties in soil properties are commonly expressed using coefficients of variation (COV). Mathematically this is described as in Equation 3.1 (Taylor 2023).

$$\text{COV} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100\% \tag{3.1}$$

By looking at the spatial variation as a function of a smoothly varying trend $t(z)$ and a fluctuating component $w(z)$: $\xi = t(z) + w(z)$, the standard deviation and COV of inherent soil variability can interpreted as in Equation 3.2 and Equation 3.3 (Phoon and Kulhawy 1999a p. 2).

$$\text{SD}_\text{w} = \sqrt{\frac{1}{n-1} \cdot \Sigma[w(z)^2]} \tag{3.2}$$

$$\text{COV}_\text{w} = \frac{\text{SD}_\text{w}}{t} \tag{3.3}$$

3.1.2 Modelling Measurement Error

To obtain geotechnical design properties, measurements are required. This will as previously stated lead to some sort of measurement error. Lumb 1971 described the total variability of a measured property as shown in Equation 3.4.

$$\mathrm{E_m}(z) = \mathrm{E}(z) + e(z) \tag{3.4}$$

In this equation $\mathrm{E_m}(z)$ is measured property, $\mathrm{E}(z)$ the in-situ property and e the measurement error. The in-situ property can be defined as the actual value in addition to the inherent soil variability, expanding the equation as in Equation 3.5 (Phoon and Kulhawy 1999a p. 8).

$$\mathrm{E_m}(z) = \mathrm{E}(z) + e(z) + w(z) \tag{3.5}$$

Orchant et al. 1988 describes w and e as uncorrelated as they are derived from unrelated sources. The inherent soil variability can be modelled using random fields. The measurement error can be modelled by empirically estimated coefficient of variation. This is possible by introducing measurement error as a factor ϵ with a unit mean $\mu = 1$ and a belonging coefficient of variation, the measurement error can then be interpreted as Equation 3.6.

$$\mathrm{E_m}(z) = (\mathrm{E}(z) + w(z)) \cdot \epsilon \tag{3.6}$$

3.1.3 Undrained Shear Strength

The undrained shear strength (s_u) is a critical parameter that characterizes the maximum shear resistance of a soil without experiencing volume change or drainage. It is used for undrained stability analysis, also known as short-term stability analysis. Numerous methods are available for measuring the undrained shear strength of soils. In this thesis CPTU, vane shear test, unconfined compression test and triaxial test are analysed.

In their paper titled ”Characterization of Geotechnical Variability,” Phoon and Kulhawy discusses the inherent variability and measurement errors that arise when using these testing methods. These sources of uncertainty can lead to significant variations in the estimated value of the undrained shear strength. Therefore, it is important to understand

and account for these uncertainties in the interpretation of the test results. Their results for the mentioned test types are presented in table 3.1 and table 3.2.

Test type	**Property**	**COV range**	**COV mean**
Triaxial test	s_u	18 - 42	32
Uniaxial test	s_u	6 - 56	33
CPTu	q_t	2 - 17	8
Vane shear test	s_u	4 - 44	24

Table 3.1: Summarized inherent variability from Phoon and Kulhawy 1999a

Test type	**Property**	**COV range**	**COV mean**
Triaxial test	s_u	8 - 38	19
Uniaxial test	s_u	21 - 57	-
CPTu	q_t	5 - 15	7 -12
Vane shear test	s_u	10 - 20	14

Table 3.2: Summarized measurement error from Phoon and Kulhawy 1999a

They have also analysed fluctuations of undrained shear strength in x and y direction. The fluctuation distance is also called correlation lengths and is shown in table 3.3. The correlation lengths indicates the range of which a point correlates in a field, and in this thesis they are often represented with the variables θ_x for the horizontal fluctuation, and θ_z for the vertical fluctuation. In this thesis they are interpreted with the unit meter.

Property	**Soil type**	**Fluctuation scale range [m]**	**Fluctuation scale mean [m]**
Vertical s_u	Clay	2,0 - 6,2	3,8
Horisontal s_u	Clay	46,0 - 60,0	50,7

Table 3.3: Scale of fluctuation from Phoon and Kulhawy 1999a

3.1.4 Vane Shear Test

In the vane shear test a torque is measured and the undrained shear strength is calculated as a result of the maximum torque value. Phoon and Kulhawy 1999b states that a correction factor is needed in order to account for strain-rate effects and soil anisotropy. This is done by expanding equation 3.5 by adding a factor ϵ. The COV of the design property can then be interpreted as equation 3.8

$$\mathrm{COV}_d^2 = \mathrm{COV}_w^2 + \mathrm{COV}_e^2 + \mathrm{COV}_\epsilon^2 \tag{3.7}$$

In this model the inherent soil variability is modelled by random fields, which leads to COV_w^2 being removed from the error consideration. This leaves the COV of the vane shear test as follows.

$$\mathrm{COV}_d^2 = \mathrm{COV}_e^2 + \mathrm{COV}_\epsilon^2 \tag{3.8}$$

3.1.5 Cone Penetration Test

The cone penetration test (CPTU) is a unique ground investigation method used in this thesis, as it measures the tip resistance and not the undrained shear strength directly, unlike the other three methods. The undrained shear strength is derived from the corrected cone tip resistance $\mathrm{q_t}$ using the equation described in Equation 3.9 (T. Lunne et al. 1997):

$$\mathrm{s_u} = \frac{\mathrm{q_t} - \sigma_{\mathrm{v0}}}{\mathrm{N_k}} \tag{3.9}$$

In this equation, $\mathrm{q_t}$, σ_{v0}, and $\mathrm{N_k}$ are parameters that contribute to the uncertainty associated with the $\mathrm{s_u}$ design parameter. This thesis considers only the variation in $\mathrm{q_t}$ and $\mathrm{N_k}$, as σ_{v0} is dependent on unit weight and water level, which are not included in this study. The $\mathrm{N_k}$ factor is determined empirically from test sites, and its accuracy can be improved by employing a suitable test method, such as a triaxial test. Depina 2016 describes a probabilistic derivation of $\mathrm{s_u}$ from $\mathrm{q_t}$, with the same assumptions as described, Equation 3.10

$$\mathrm{s_u} = \frac{\epsilon_{\mathrm{e}}\mathrm{q_t} - \sigma_{\mathrm{v0}}}{\mathrm{N_k}} \tag{3.10}$$

Here, ϵ_{e} is a lognormally distributed random variable with a unit mean and a coefficient of variation as described in Table 3.2. Additionally, both $\mathrm{N_k}$ and $\mathrm{q_t}$ are assumed to be lognormally distributed. These assumptions are important to implement the variability associated with the measurement errors of the cone penetration test and the empirical determination of $\mathrm{N_k}$.

3.2 Random Fields

A random field is a mathematical method of modelling spatial variation. It is a model that can assign random values to each point in a domain from one to multiple dimensions (Opseth 2022 p. 2). This methodology is a possibility under the circumstance that spatial variation is a result of a random process (Baecher and Christian 2003 p. 243). Baecher and Christian 2003 defines a random field as a joint probability function which describes the simultaneous variation of variables in a domain, and they can be represented with mean values and variation, as in equation 3.11.

$$\mathrm{E}[z(x)] = \mu(x), \qquad \mathrm{Var}[z(x)] = \Sigma(x) = \mathrm{COV}[z_i(x), z_j(x)] \tag{3.11}$$

Σ_z denotes a covariance matrix, while x refers to the spatial position and z is a vector with properties. To represent a random field with just a mean vector and a correlation matrix certain simplification must be made, which include the use of Gaussian random fields (Opseth 2022 p. 2).

3.2.1 Gaussian Random Fields

By utilizing Gaussian random fields, a set of simplifications are made available. These random fields are often used to model spatial processes (Hristopulos 2020 p. 245). Gaussian random fields have a wide range of applications in nature, because of the central limit theorem (Hristopulos 2020 p. 245). Baecher and Christian 2003 have listed four advantageous properties of Gaussian random fields.

- Gaussian random field are completely characterized by the first and second order moments, which in this case is the mean vector and autocorrelation matrix.
- All subset variables are jointly Gaussian.

- The conditional probability of two vectors are also Gaussian distributed
- When two variables are bi-variate gaussian and their covariance is zero, they are independent

In this thesis the random fields are modelled as multivariate Gaussian random fields with mean and a covariance matrix. Their probability density distribution is defined as equation 3.12 (Baecher and Christian 2003 p. 249).

$$(2\pi)^{-\frac{n}{2}}|\Sigma|^{-\frac{1}{2}}\text{exp}\left\{-\frac{1}{2}(\mathbf{z}-\mu)'\Sigma^{-}1(\mathbf{z}-\mu)\right\} \tag{3.12}$$

3.2.2 Autocorrelation and Autocovariance Functions

A regular approach of defining a random field is by a probability density function, which in this case is a Gaussian probability function, and a covariance or autocovariance function (C) (Phoon and Ching 2017 p. 561). Baecher and Christian 2003 have defined a wide set of autocovariance functions, and two of these are utilized to model soil variability and measurement error. For two-dimensional gaussian random field the covariance function can be given by equation 3.13 (Phoon and Ching 2017 p. 562). In this function the correlation lengths are represented by θ_x, and θ_z and the variation of the field by σ^2 (Opseth 2022 p. 5).

$$C([x_1, z_1], [x_2, z_2]) = \sigma^2 \cdot \text{exp}\left\{-2\sqrt{\left(\frac{x_1 - x_2}{\theta_x}\right)^2 + \left(\frac{z_1 - z_2}{\theta_z}\right)^2}\right\} \tag{3.13}$$

The covariance function has two properties which are well fit for the further modelling. It is a symmetric function meaning that $C(\delta)$ = $C(\text{-}\delta)$, where δ symbolises distance (Opseth 2022 p. 5). The second property is the limit values where $C(0)$ = σ^2 and $\lim_{|\delta|\to\infty} \frac{C(\delta)}{|\delta|^{-(n-1)/2}} = 0$, which indicate that the similarities in parameters will decrease with distance (Opseth 2022 p. 5).

In this thesis an autocorrelation function is used to define the random field. This is the covariance function divided by variation (Phoon and Ching 2017 p. 561). This is beneficial for the algorithms created as the variation can be added in a later state of the programming sequence (Opseth 2022 p.5).

3.2.3 Correlation Matrix Decomposition

In this thesis the Cholesky decomposition is used to model random variations in the random field. This is a method that decomposes a given matrix into a lower triangular matrix and its conjugate transpose (Parker 2017 p. 152). This decomposition can be performed when the original matrix is positively definite, a Hermitian matrix, or only containing real eigenvalues that are greater than zero (Parker 2017 p. 152). The decomposition can be expressed as in equation 3.14. In this equation C denotes the correlation matrix, while L is the corresponding lower tirangular matrix (Opseth 2022 p. 5). The autocovariance function described in subsection 3.2.3 is both non-negatively defined and symmetric, which allows for a valid Cholesky decomposition.

$$C = LL^T \tag{3.14}$$

Covariance matrix decomposition is a direct method of creating a homogeneous random field when using a covariance structure with discrete points in the field (Fenton and Griffiths 2008 p. 216). The lower triangular matrix from a Cholesky decomposition can be multiplied with a vector of mean zero unit-variance Gaussian random variables (U), to obtain a matrix of correlated variation values (Z_v) (Fenton and Griffiths 2008 p. 217). A realisation of a random field (Z) is then created as the sum of the mean vector and this matrix, see equation 3.15.

$$Z = \mu + Z_v = \mu + [L \cdot U] \tag{3.15}$$

The simplicity of this method makes it easy to implement, but according to Fenton and Griffiths 2008 it is only useful for small fields. Working with larger fields they will lead to round off error and be time consuming.

3.2.4 Conditional Random Fields

Random fields that includes certain known values are referred to as conditional random fields (Fenton and Griffiths 2008 p. 234). Conditional random fields make it possible to use measured deterministic values to characterise variation in other locations of the field

(Fenton and Griffiths 2008 p. 91). This principle can be used to model an observation in the modelled soil, in the form of a ground investigation. Conditional random field can be said to consist of three parts. The first being an unconditional simulation of the random field (Z_i), the second being the best estimate based on measured values (Z_c) and the third being the based estimate based on unconditional values (Z_u). These represent the random field as in equation 3.16 (Fenton and Griffiths 2008 p. 234).

$$Z = Z_i + [Z_c - Z_u] \tag{3.16}$$

For the modelling process in this thesis, it is favourable to express the conditional random fields by using a conditional mean and a conditional covariance matrix. For a Gaussian random field, the mean and variance of a one-dimensional random field can be expressed as in equation 3.17, where ρ is a correlation coefficient and o is the observation (Fenton and Griffiths 2008 p. 96).

$$E = \mu + (o - \mu)\rho, \qquad Var = \sigma^2(1 - \rho^2) \tag{3.17}$$

Expanding these definitions to a two-dimensional random field, results in equation 3.18.

$$E[Z|o] = \mu + \Sigma_{Z|o}(o - \mu), \qquad Var[Z|o] = \Sigma - \Sigma_{Z|o}\Sigma_o \tag{3.18}$$

This mean vector and covariance matrix can then be used to create realisations of conditional random fields as described in section 3.2.3.

Chapter 4

Slope Stability

Slope stability is a critical aspect of geotechnical engineering and involves the evaluation of different types of cuts, fillings, and dams, as well as the safety against landslides. Various types of slope stability analyses are available, which assess rotational, translational, or irregular failure mechanisms (Salunkhe et al. 2016 p. 528). These analysis methods are often categorized as drained or undrained . A drained analysis is based on the friction angle and cohesion of the soil materials and assumes that excess pore water will fully dissipate over time. This assumption makes the drained analysis a long-term analysis. The undrained analysis assumes that water will not immediately leave the material, and both soil material and water can be interpreted as a homogenous material.

Slope stability analysis primarily involves determining a factor of safety (F), which is the value by which the shear strength of a material must be divided to reach the point of failure. To determine if a slope is safe, limit equilibrium is assessed based on driving and resisting forces and moments acting on the assumed failure surface (Salunkhe et al. 2016 p. 530). Several methods are available to determine slope stability, and in this thesis the direct method is used.

4.1 The Direct Method

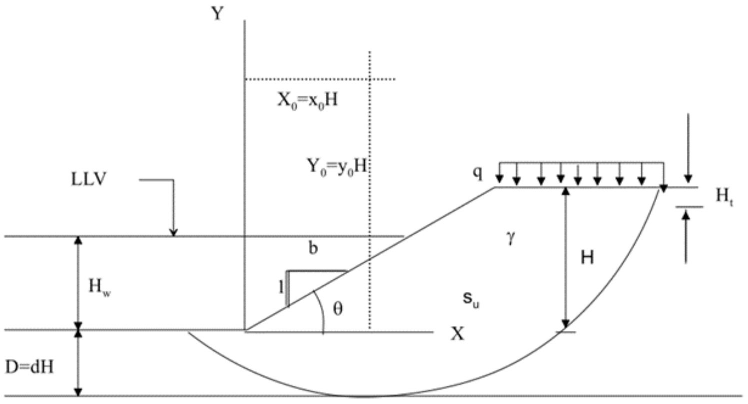


Figure 4.1: Figure of parameters in direct method (Tsegaye 2019)

The direct method for undrained slope stability defines the factor of safety as Equation 4.1 (Tsegaye 2019 p. 3).

$$\mathrm{F} = \frac{N_0 \cdot \mathrm{s}_u}{\mathrm{P}_d} \tag{4.1}$$

In this equation, N_0 denotes a parameter that is obtained from the graph depicted in Figure 4.2. The factor of safety depends on the slope angle and a factor d, which is defined as the ratio of depth to hard bedding D and the slope height H. Moreover, s_u represents the average undrained shear strength along the shear surface. The driving forces acting on the slope are captured by P_d, which is evaluated using Equation (4.2) (Tsegaye 2019 p. 3).

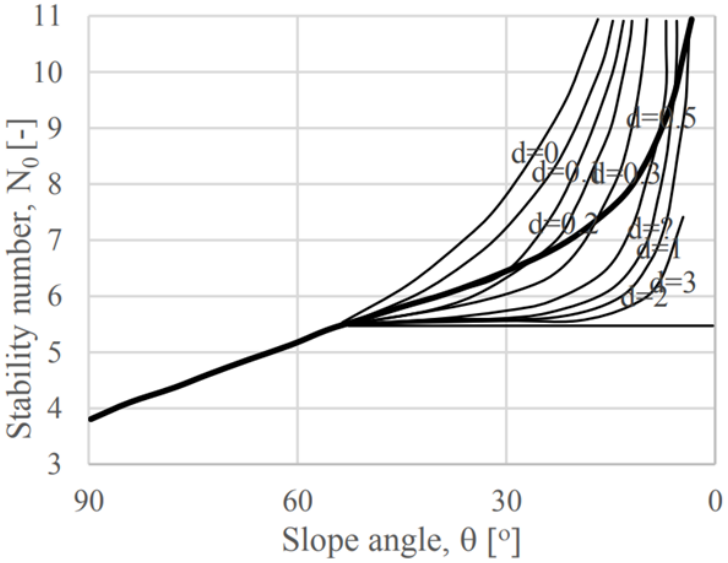


Figure 4.2: Undrained stability number N_0 over slope angle (Tsegaye 2019)

$$\mathrm{P}_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} \tag{4.2}$$

In this equation, γ and γ_w refer to the saturated unit weight of the soil and the unit weight of water, respectively. The water level is represented by H_w. The correction factors of surcharge (q), water content (w), and tension cracks (t) are incorporated through the variables μ_q, μ_w, and μ_t, respectively.

4.2 Probabilistic Approach

Design parameters used in geotechnical engineering must be obtained through ground investigations. However, the spatial variability of soil introduces considerable uncertainty in these parameters. Statistical evaluations should be performed to assess the possible variation, enabling determination of the probability of failure. One way of assessing the probability of failure of a slope is to use the Monte Carlo method, which is possible to incorporate with the direct method. To initiate this method the uncertainties associated with the problem must be evaluated and described as random variables. The problem is then defined by an assessment criterion called a performance function $g(X)$, which when considering a factor of safety F can be defined as in Equation 4.3.

$$g(X) = \mathrm{F}–1 \tag{4.3}$$

The output of the Monte Carlo simulation is a set of observations, because of multiple calculations of $g(X)$. The system performance of a slope can be defined as a binary variable Y based on $g(X)$, which equal 1 when the system fails and 0 otherwise (Hu et al. 2020 p. 2).

$$Y = \begin{cases} 0, & g(X) > 0 \\ 1, & g(X) \leq 0 \end{cases} \tag{4.4}$$

If the random input variables that represents the uncertainties in the system then is defined as $f_X(x)$, the probability of failure P_f can be mathematically describes as Equation 4.5 (Hu et al. 2020 p. 2).

$$P_f = \int_{g(x)\leq 0} f_X(x)dx \tag{4.5}$$

By defining the desired number of simulations as N_s and number of failed samples as N_f. It is possible to estimate the failure probability with Monte Carlo method as Equation 4.6 (Hu et al. 2020 p. 4).

$$\tilde{P}_f = \frac{1}{N_s}\sum_{i=1}^{N_s} Y(x_i) = \frac{N_f}{N_s} \tag{4.6}$$

The direct method for undrained stability analysis consists of multiple variables that contains uncertainty. This thesis has been limited to only considering the undrained shear strength s_u as a random variable in the direct method.

Chapter 5

Value of Information

The Value of Information (VoI) analysis serves as a decision-making tool that quantifies the potential benefits of collecting additional information. This methodology centers on the impact that the acquisition of new data may have on potential decisions (Gilbert and Habibi 2017 p. 1). The VoI process necessitates the identification of a decision problem, as well as criteria for decision options. Once the problem is clearly delineated, the currently known information must be assessed, also referred to as prior knowledge. The process then involves identifying sources of uncertainty to allow for the quantification of the effect of collected information in terms of reducing or increasing uncertainty. The outcomes of VoI analysis can then be utilized in a cost-benefit assessment, enabling a final informed decision. As Gilbert and Habibi 2017 have noted, there is potential for enhancing the decision-making process with respect to the type and quantity of data in the field of geotechnics. Various situations arise where investigations may be either inadequate or excessive.

5.1 Prior Knowledge

Defining prior knowledge is interpreting existing information about a site-specific area, in order to get an idea about the soil classification, their parameters and the soil layering. Cao et al. 2016 Split site characterization into six different steps, where the two first are a desk study and a site reconnaissance. These steps can be said to represent gathering prior knowledge. A way of representing the prior knowledge of soil parameters is to define them with statistical model parameters, more specifically a mean, standard deviation, and a scale of fluctuation (Cao et al. 2016 p. 2). Cao et al. 2012 stated that it is important to not interpret prior knowledge as perfect information as it contains propagation errors, due to former measurement errors and inherent soil variability. Uncertainties such as these make it difficult to quantify prior knowledge through distributions, and complexity rises with acquired information (Cao et al. 2016 p. 2).

5.1.1 Sources of Prior Knowledge

Prior knowledge can be separated in two categories defined as informative and non-informative. Non-informative prior knowledge refers to information that does not have a significant impact on the posterior distribution. An example of this could be characteristic values of soil parameters. Informative prior knowledge then refers to the information that has a significant impact on the former distribution (Gelman et al. 2021 (Bayesian Data Analysis)). In geotechnical context this could be that an adequate number of former site investigations is performed (Cao et al. 2016 p. 4). Both these types of prior knowledge could give information about the actual parameter distribution. If the non-informative information is combined with engineering expertise and site reconnaissance, it could reduce the range of characteristic values considerably (ibid). Table 5.1 shows a summary of sources to prior knowledge geology and geotechnics.

Table 5.1: Sources on prior knowledge on geology, geotechnical problems and properties, and groundwater conditions (Cao et al. 2016 p. 2)

Type	Source
Geology	Geological maps
	Geological reports
	Geological publications
	Regional guides
	Air photographs
	Soil survey maps and records
Geotechnical problems and properties	Geotechnical reports
	Academic journals
	Previous ground investigation reports
Groundwater conditions	Topographical maps
	Air photographs
	Well records
	Previous ground investigations reports

5.1.2 Estimates and Distributions of Prior Knowledge

According to Cao et al. 2016, when estimating a model parameter based on prior knowledge, uncertainties are inherent, and the result is not deterministic. The authors propose a method for estimating non-informative prior knowledge, which involves modelling the prior information as a uniform distribution with upper and lower bounds, denoted by $\theta_{\max}$ and $\theta_{\min}$, respectively. This distribution is given by Equation (5.1), where $P(\theta_i)$ is the probability density function of the parameter θ_i.

$$P(\theta_i) = \begin{cases} \frac{1}{\theta_{i\,\max} - \theta_{i\,\min}} \\ 0 \end{cases} \tag{5.1}$$

The boundaries of soil parameters are often well-established in the literature, as exemplified in Kulhawy's work "On evaluation of static soil properties". Prior estimates of means and coefficients of variation for undrained shear strength can also be found in Chapter 3 of the same work. However, additional information, such as engineering knowledge, can be incorporated to further refine the range of parameter values. Such knowledge is also necessary to assess the impact of informative data on the resulting distributions.

5.2 Decision Analysis

The method is based on clearly defined decisions with possible outcomes. This section will explain the framework of a value of information decision analysis used in this thesis, which is based on an approach by Hu et al. 2020. It involves accounting for uncertainty in the form of consequences, denoted as C, associated with the decision at hand. This uncertainty can be expressed through a probability distribution, represented as $p_c(C)$. Expected benefits resulting from potential decisions can then be denoted as a variable $E(C)$ (Gilbert and Habibi 2017 p. 2).

$$E(C) = \sum c \times p_c(c) \tag{5.2}$$

By evaluating $E(C)$, the optimal decision could be retrieved from its maximum value as shown in equation 5.3.

$$E(\text{optimal } C) = \max[E(C_0), \ldots, E(C_n)] \tag{5.3}$$

This methodology can be adapted to address a slope stability issue, where potential decisions involve measures that could enhance slope stability or minimize uncertainty through ground investigations, both of which may decrease the likelihood of failure. Such measures can be defined as d, while a binary variable Y can account for the failure state. A geotechnical model's system response can be articulated through a performance function, $g(r, d)$, which incorporates prior knowledge of soil properties, denoted as $f(r)$, and the decision at hand, d. The function can be utilized to represent Y as in figure 5.4.

$$Y = \begin{cases} 0, & g(r, d) > 0 \\ 1, & g(r, d) \leq 0 \end{cases} \tag{5.4}$$

Costs associated with slope failure, c_f, and engineering measures, $c_e(d)$, can be incorporated into this model. A utility function, denoted as u, which accounts for the costs in the context of system performance, can be created (Hu et al. 2020 p. 2). This function will depend on the failure state Y and the decision d, thus defined as $u(Y, d)$. If the slope fails ($Y = 1$), the consequence will be $c_e + c_f$, while if it is safe ($Y = 0$), the consequence will be c_e.

$$u(Y, d) = \begin{cases} -c_e(d), & \text{if } Y = 0 \iff g(r, d) > 0 \\ -(c_e(d) + c_f), & \text{if } Y = 1 \iff g(r, d) \leq 0 \end{cases} \tag{5.5}$$

The initial probability of failure can be implemented to the decision-making process through the definition of the performance function, $g(r, d)$. Specifically, the failure probability $p_f = P(Y = 1|d = 0) = \int_{g(r,d)\leq 0} f(r)dr$ can be calculated (Hu et al. 2020 p. 2). Expected benefits from a decision can be determined by taking the expectation of the utility function (Hu et al. 2020 p. 2).

$$E[u(Y, d)] = u(0, d)(1 - p_{f,0}) + u(1, d)p_{f,0} = -[c_e(d) + c_f p_{f,0}] \tag{5.6}$$

To evaluate multiple decisions, such as various ground investigation options, it is necessary to introduce a tool for additional value of information.

5.3 Value of additional information

To conduct a comprehensive geotechnical analysis, it is necessary to carry out multiple ground investigations. To establish the soil layering in two dimensions, a minimum of two soundings is necessary, while a minimum of three soundings is required for establishing soil layering in three dimensions. These criteria represent the lower limits of ground investigations. To determine how additional ground investigations can improve the accuracy of soil parameters, it is essential to incorporate the possibility of acquiring additional information into the decision model using Bayes' theorem and the law of total probability, that are shown in equation 5.7.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \qquad P(B) = \sum_n P(B|A_n)P(A_n) \tag{5.7}$$

The updated decision model considers the new consequence given additional information as $P(C|I)$, C being the decision consequence and I being information. The law of total probability is then used to determine $P(I)$ as $\sum_n P(I|C_n)P(C_n)$ (Hu et al. 2020 p. 2).

Prior knowledge is represented as $f(r|O)$, where O represents an observation. This allows the conditional probability, equation 5.8, to be defined for conditional Gaussian random fields, which utilize multivariate normal distributions, resulting in analytical solutions for Bayesian updating (Hu et al. 2020 p. 2).

$$f(r|O) = \frac{f(O|r)f(r)}{\int(f(O|r)f(r)dr)} \tag{5.8}$$

Furthermore, by defining $p_{f,d}$ as $P(Y = 1|O, d)$, the expected utility function in equation 5.6 can be modified to reflect the updated distribution of Y given O, as shown in equation 5.9 (Hu et al. 2020 p. 2).

$$E[u(Y, d)] = u(0, d)(1 - p_{f,0}) + u(1, d)p_{f,0} = -[c_e(d) + c_f p_{f,0}] \tag{5.9}$$

In summary, incorporating the possibility of acquiring additional information using Bayesian methods and the law of total probability allows for the determination of the impact of additional ground investigations on the accuracy of soil parameters.

Chapter 6

Cost Estimates

The estimation of costs for ground investigations poses a challenge, as it is influenced by multiple factors. Typically, such investigations are contracted to the company offering the most competitive bid, unless it is an internal job. Because of this, obtaining relevant cost information can be difficult since it is often concealed within private bidding documents. Tenders on ground investigations are often created, considering the entire project. This means that tenders may be structured to reduce costs of ground investigation by assuming fewer consulting hours. While various practices for determining costs exist, the Norwegian Geotechnical Society (NGF) has published an indicative Excel spreadsheet that can aid in the preparation of bidding documents. In this current study, a reasonable offer was obtained with the assistance of Espen Andersen Torsæter and Sigurdur Mar Valsson from the Norwegian Public Roads Administration. Due to the competitive nature of the bidding process, the identity of the offering company has been withheld. The pertinent values for the ground investigations considered in this study are presented in the tables provided in this chapter, while the complete Excel sheet can be found in the appendix.

Due to the complexity of ground investigation costs, simplifications has been made in order to implement the costs to the methodology. These simplifications include the assumption of equipment not being destroyed. In addition to this no further measures are added then what is needed to obtain the undrained shear strength parameters from the test methods.

Table 6.1: Specific costs for preparation and general work

Description	**Unit**	**Price [NOK]**
Notification and utility detection		
General	RS	25 000
Pr. bore point	pcs	100
Main Rigging		
Rigging of drilling rig on land	RS	50 000
Rigging of drilling rig on fleet/vessel	RS	160 000
Land clearing	hour	980
Movement between boreholes	pcs	3 000
Rigging of core drilling rig/equipment	RS	100 000
Measuring equipment		
General rigging of measuring equipment	RS	5000
Pr. borehole	pcs	100

Table 6.2: Specific costs of vane shear test

Description	**Unit**	**Price [NOK]**
Movement and rigging per bore point	pcs	10 000
Measurement of undisturbed and remoulded shear strength	pcs	-
pr. level to 10 m	pcs	265
pr. level from 10 m	pcs	265
Penetration in soil (entire depth interval)	m	625

Table 6.3: Specific cost of routine test on cylinder samples with clay/clayey materials

Description	**Unit**	**Price [NOK]**
Pr. cylinder Ø54 mm	pcs	1 600

Table 6.4: Specific cost of retrieving Ø54 mm test cylinder samples

Description	**Unit**	**Price [NOK]**
Movement and rigging on land pr. bore point	pcs	
Sample collection to 10 m in soil	pcs	2 500
Sample collection 10 - 20 m in soil	pcs	3 000
Sample collection 20 - 30 m in soil	pcs	3 000
Sample collection 30 - 40 m in soil	pcs	5 000
Penetration in soil (entire depth interval)	m	0
Surcharge for test from fleet/vessel, pr. test	pcs	1 000
Wait	hour	0
Loss of test cylinder Ø54 mm	pcs	1 000

Table 6.5: Specific cost of routine test on cylinder samples with clay/clayey materials

Description	**Unit**	**Price [NOK]**
Pr. cylinder Ø54 mm	pcs	1 600

Table 6.6: Specific costs of the consolidated undrained triaxial test

Description	**Unit**	**Price [NOK]**
Consolidated undrained triaxial test	pcs	6 820
Surcharge difficult/wet samples	pcs	940
Surcharge side trimming	pcs	940
Surcharge stamp of sand sample	pcs	940
Surcharge measurement of Gmax	pcs	1 210
Surcharge permeability measurement when testing	pcs	1 200

Table 6.7: Specific costs of CPTU

Description	**Unit**	**Price [NOK]**
Movement and rigging on land pr. bore point	pcs	8 000
Cone penetraion test in soil to 20 m	m	0
Cone penetraion test in soil from 20 m	m	100
Pore pressure dissipation pr. measurement	pcs	450
Pore pressure dissipation pr. time unit	hour	450
Surcharge resistivity measurement (RCPTU)	pcs	1 500
Surcharge measurment of shear wave velocity (SCPTU)	pcs	1 500
Loss of cone	pcs	2 000
Loss of friction sleeve	pcs	2 000
Loss of transition section	pcs	30 000

Chapter 7

Model Description

The present thesis focuses on a methodology for determining the optimal location and number of ground investigations, and is based on a process designed by Dr. Depina. The methodology is outlined in the flow chart depicted in Figure 7.1. Enhancements have been introduced to expand the possibilities of the methodology. The trial analysis has been refined to incorporate four distinct types of ground investigations, cone penetration test, vane shear test, unconfined compression test, and triaxial compression test. Realistic cost estimates for each of these investigation types have been incorporated. Additionally, the code structure has been written in an object-oriented style, which makes the methodology more adaptable to diverse analytical contexts.

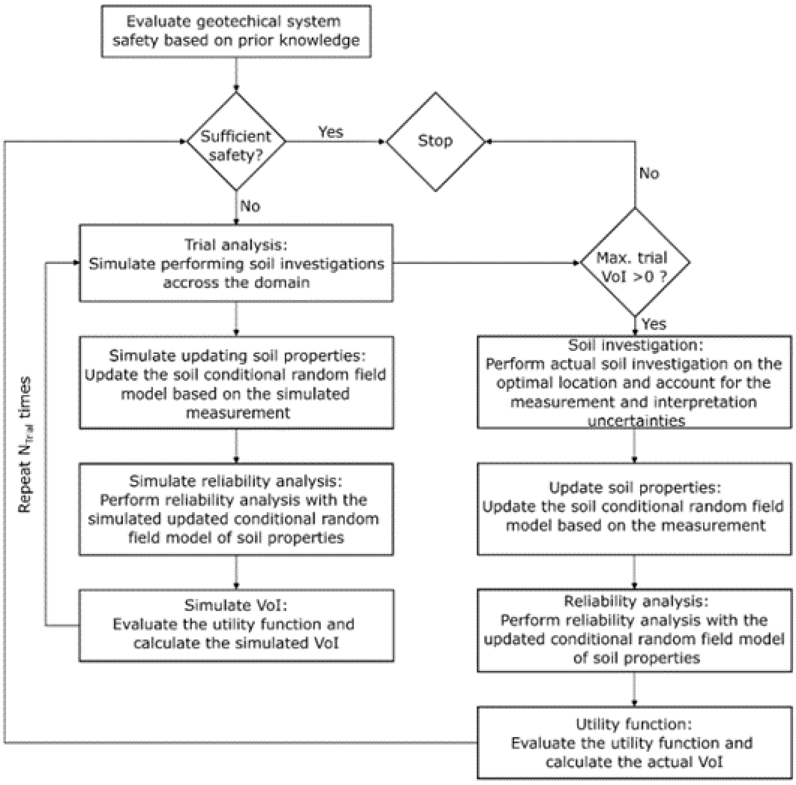


Figure 7.1: Implementation of the value of information analysis by Dr. Depina

7.1 Code Architecture

The method is created using the Python programming language, together with the software packages detailed in the table 7.1. The project is structured around eight classes, as illustrated in the UML diagram displayed in figure 7.2. The primary class, known as ”Trial_analysis”, serves as a parent class that collects all the relevant data and performs the necessary ground investigation analyses.

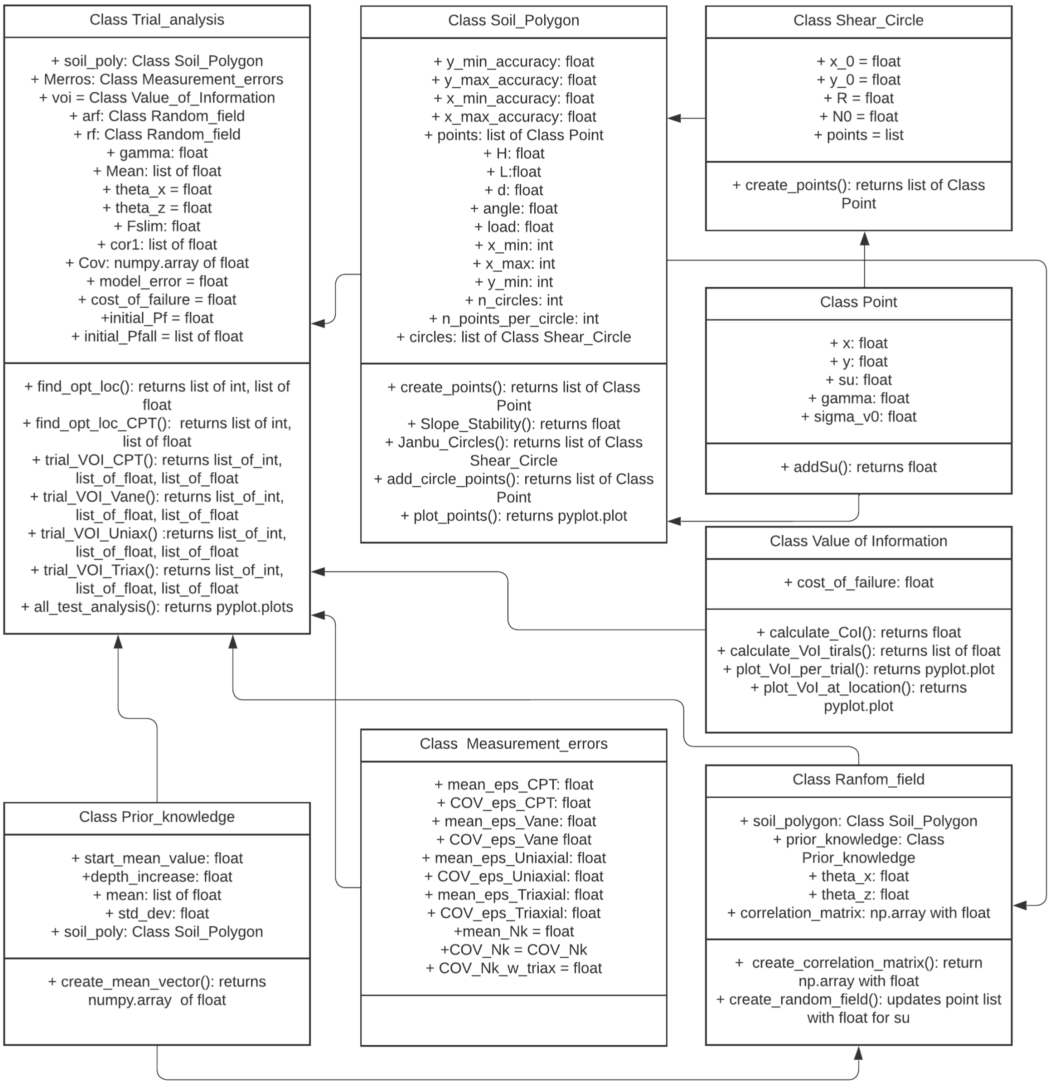


Figure 7.2: Class structure in python project

Additionally, the ”Soil_Polygon” class manages the discretization of the domain utilizing the ”Point” class. Calculating of the factor of safety is done through the ”Shear_Circle” class. The ”Value_of_Information” class is responsible for calculating the cost of information and value of information. The ”Prior_knowledge” class generates a mean value vector for the entire domain and stores the corresponding standard deviation. The ”Random_field” class utilizes the soil polygon and prior knowledge to produce the initial realization of soil variability for the domain. The ”Measurement_error” class stores the different error types associated with each ground investigation. The code can be provided in its entirety by contacting the author of this thesis.

Package	**Description**
Numpy	Package used for scientific computing
Matplotlib	A plotting library that allows for creation of visualizations in python
Scipy	Additional package to Numpy for scientific computing
Scikit-learn	A machine learning library for data analysis
Openpyxl	library for reading and writing Excel files
Tqdm	library to create progress bars for loops
Time	A python module that provides a variaty of function to work with time related operations
Datetime	A python module that allows for working with dates, times and time zones etc.

Table 7.1: Imported packages in pyhton program

7.2 Modelling of the Slope

The slope is modelled by a discretized domain with defined boundary conditions. The length and height of the slope, as well as minimum and maximum x values (where zero denotes the beginning of the slope incline), and minimum z value, define the slope boundaries. The domain consists of evenly spaced points in the x and z directions, with their density determined by individual factors equation 7.1.

$$\text{Points/m} = \frac{1 \text{ point/m}}{\text{Density factor}} \tag{7.1}$$

Furthermore, the slope model includes a set of n shear circles that are evenly distributed. The circle geometry is obtained using a regression method developed by Dr. Depina, which utilizes the stability charts of the direct method. Each circle is represented by m points along the shear surface, which are used to calculate the average undrained shear strength. This calculation is then used to determine the factor of safety. In each stability calculation, all the surfaces are considered, and the lowest factor of safety is decisive.

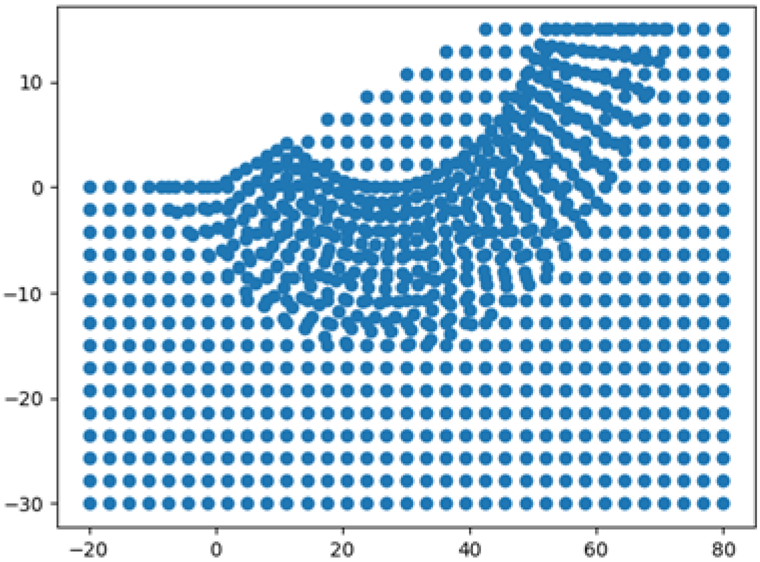


Figure 7.3: Example of discretized domain

Figure 7.3 depicts a discretized domain with an x- and z-density factor of 2, 12 shear circles, and 30 points per circle. The boundary conditions for the figure are a minimum x value of - 20, maximum x value of 80, minimum z value of - 30, and a slope geometry with a length of 40 meters and a height of 15 meters.

7.3 Modelling Ground Investigations

The ground investigations are modelled using conditional random fields, together with individual costs and uncertainties. The costs are implemented through the value of information analysis. Uncertainties in the measurements are described as measurement errors. These are modelled as normally distributed random variables ϵ, with a unit mean and a corresponding coefficient of variation COV that is retrieved from literature. The values are portrayed in Table 3.2, and the mean values of the COV are used. The measurement error is implemented to the random field using a covariance matrix based on the white noise autocorrelation function Equation 7.2 by Beacher and Christian 2003.

$$R_x(\delta) = \begin{cases} 1 & \text{if } \delta = 0 \\ 0 & \text{otherwise} \end{cases} \tag{7.2}$$

In this function δ equals the distance to the measured point. The result is a diagonal matrix with the measurement errors Σ_{error}. The location of measurement is implemented as a binary observation location matrix H, with 1 equaling an observation point. The actual measurement values are retrieved from a realisation of the inherent soil variability, created by a random field. These are represented as an observation value vector O. The conditional mean from implemented ground investigation is then calculated as in Equation 7.3.

$$\mu_{r|O} = \mu + \Sigma H^T [H \Sigma H^T + \Sigma_{\text{error}}]^{-1} (O - \mu H) \tag{7.3}$$

The conditional covariance matrix is calculated from Equation 7.4.

$$\Sigma_{r|O} = \Sigma - \Sigma H^T [H \Sigma H^T + \Sigma_{\text{error}}]^{-1} H \Sigma \tag{7.4}$$

These values are then used in order to create conditional realisations of the random field with ground investigation. Four different types of ground investigations are implemented with their corresponding characteristics.

7.3.1 Vane Shear Test

The vane shear test is simplified to a point observation, although it is usually performed at several depth for each borehole. The coefficient of variation for the measurement error in context of the vane shear test is modelled as a combination of its transformation error (COV_t), and the measurement error (COV_e). This results in COV_e defined as in Equation 7.5.

$$\text{COV}_\epsilon = \sqrt{\text{COV}_t^2 + \text{COV}_e^2} \tag{7.5}$$

Because of this the input variable is implemented as the resulting measurement error $\mu_{\epsilon \text{Vane}} = 1$, $\text{COV}_{\epsilon \text{Vane}} = 0,172$.

7.3.2 Unconfined Compression Test

The unconfined compression test is implemented as a point observation in the slope domain. This test is usually a part of a routine check for a test cylinder, which makes the foundation of its cost estimate. The literature researched for this thesis did not provide a mean value for the COV measurement error. This resulted in a simplification of the given range made for COV_ϵ.

$$\text{COV}_\epsilon = \text{COV}_{\text{min}} + \frac{\text{COV}_{\text{range}}}{2} \tag{7.6}$$

The range of measurement error that was retrieved form literature was $\text{COV}_{\text{min}} = 0,21$ and $\text{COV}_{\text{max}} = 0,57$ leading to the approximated $\text{COV}_\epsilon = 0,39$.

7.3.3 Triaxial Test

Just as the two previous tests the triaxial test is modelled as a point observation. The chosen type of triaxial test for this method is the consolidated undrained (CU), due to it providing the most accurate undrained shear strength parameter. The assigned measurement error $\text{COV}_\epsilon = 0,19$, and is read from Table 3.2. The triaxial test can give results that can be useful for much more than determining the undrained shear strength, although that is its main purpose in this thesis. One relevant feature is that it can be used to determine the N_k factor more accurately, which is used to determine the undrained shear strength from a CPTU. Leading to more accuracy in the CPTU measurements. This principle is implemented into the model as a reduction of uncertainty in the N_k factor used in the CPTU test. The process is done by recalculating former CPTUs with increased accuracy when testing a new triaxial test. When a triaxial test is chosen the N_k factor is reduced leading to the next CPTU tests being more accurate.

7.3.4 Cone Penetration Test

CPTU differs from the other tests in that it considers a nearly continuous line of measurements. The measurements are implemented as a line of points that reaches from top dlope border to bottom border, which is done by creating a row for each observation in the observation matrix. Another relevant difference is the fact that it measures tip resistance q_t instead of undrained shear strength.

$$s_u = \frac{q_t - \sigma_{v0}}{N_k} \tag{7.7}$$

In this model σ_{v0} is interpreted as a deterministic value. The N_k factor is modelled as a lognormally distributed random variable, and the measurement error ϵ is also lognormally distributed. The model of the inherent variability of the soil is made considering the variability of s_u in the soil. Since CPTU measures tip resistance, the realisation values of the random field must be transformed to tip resistance values in order to implement the measuring error ϵ of the CPTU. The measured values represented by $q_t \cdot \epsilon$ is then transformed back using the same equation.

7.4 Modelling Inherent Soil Variability

The inherent soil variability is modelled using realisations of Gaussian random fields. These random fields are based on a mean vector μ of undrained shear strength s_u values, that are determined from prior knowledge. The other parameters in the direct method for undrained slope stability analysis are in this thesis considered to be deterministic. To create correlated values in the random field a correlation matrix must be defined. This matrix is based on the ellipsoidal autocorrelation function, with correlation length defined as θ_x and θ_z.

$$C = \exp\left\{ -2 \cdot \sqrt{\left(\frac{x_1 - x_2}{\theta_x}\right)^2 + \left(\frac{z_1 - z_2}{\theta_z}\right)^2} \right\} \tag{7.8}$$

The result is a symmetric diagonal correlation matrix. In this matrix each row represents one points correlation to the other points in the domain. The standard deviation assumed for the undrained shear strength is then introduced by scaling the correlation matrix.

$$C_{\text{scaled}} = C \cdot \sigma \tag{7.9}$$

Inherent soil variability is then calculated as the dot product of a random normally distributed vector U with size equal to the number of points in the discretised slope domain, and the Cholesky decomposition of the scaled correlation matrix A. Realisations of undrained shear strength across the domain is then calculated as $\vec{\mu} + (U \cdot A)$.

7.4.1 Implementing Prior Knowledge

In this model prior knowledge is represented using a mean value for undrained shear strength accompanied by an assumed standard deviation. The mean value can either be a universal value across the entire domain or it may increase with depth. The mean value is created as a mean vector where each element represents the mean value for a specific point.

$$\vec{\mu} = [\mu_0, \mu_1, \ldots, \mu_n] \tag{7.10}$$

The standard deviation is represented as a single value that is assigned to all positions. The values utilized as prior knowledge are specific to the situation at hand and require an analysis of the available data on a particular test site. The amount of informative information obtained from this analysis determines the extent to which the standard deviation of the undrained shear strength parameter can be reduced.

7.4.2 Updating inherent soil variability

The inclusion of measurements is meant to reduce the uncertainty in the field as it serves to reduce the variation at specific areas across the random field domain. In order to model multiple ground investigations an iterative approach is necessary. This involves accounting for the reduction in variation from the previous observation when determining the placement of the next observation. To achieve this the mean vector and correlation matrix are replaced with the conditional mean and variance of a ground investigation simulation at the previously determined location. This iterative process allows for a more accurate representation of the ground properties and reduces the uncertainty associated with the model predictions.

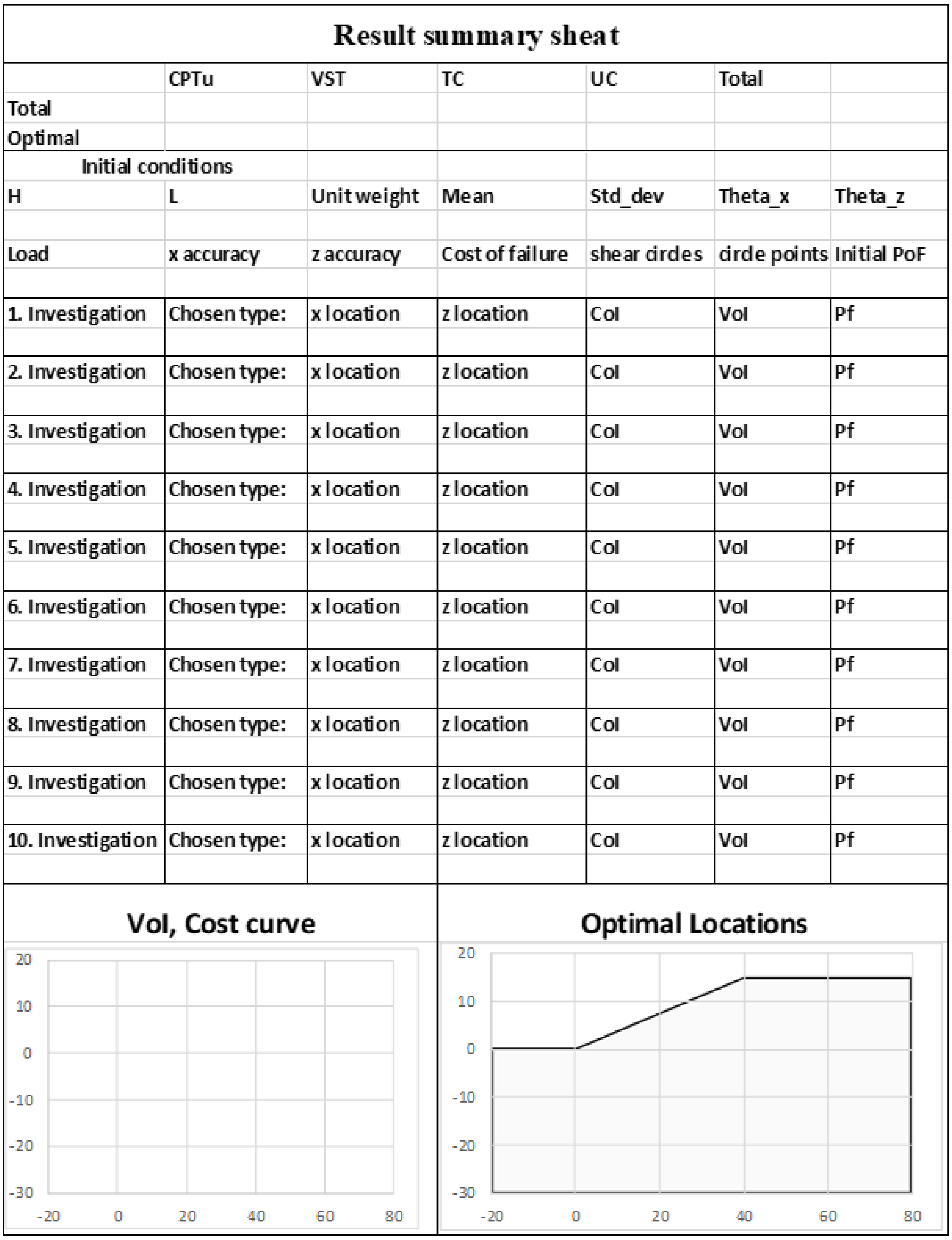
7.5 Implementing Value of Information

The value of information analysis quantifies the impact of each ground investigation which is the foundation of the decision analysis of this model. In this model the expected benefit E is determined from the cost of failure c_f, the probability of failure p_f, and the cost of investigation c_e. The conditional value of information (CVOI) is in this case defined as the difference in E for the initial conditions given prior knowledge and after the simulated ground investigation O (Hu et al. 2020 p. 3). This model uses the average CVOI for a given investigation as the decision variable when comparing locations for investigations. The location with the largest CVOI is then chosen as the optimal test location for that type of investigation.

The next step of the analysis is to compare the different alternatives of ground investigations. When the optimal location for each test has been calculated, the provided CVOI of each test has all associated costs subtracted. This leads to the basis of comparison of the different ground investigations. The tests that has the most profitable income is then chosen. This leads to storage of this investigations result. When the simulation process is finished, the investigation number that provide the largest result of CVOI - cost of investigations is considered the optimal number.

7.6 Output

The program created in python creates results in the form of plots and a summary excel sheet containing test results. The plots are made to showcase the effect of each added ground investigation, as well as the results of adding observations along with the costs that follows. The excel sheet store data in order to control the results, and have a more detailed view of the simulation results. The summary excel sheet is showcased in figure 7.4. It provides a summary of the initial conditions of the simulation, in addition to a result log of each performed analysis. The log includes chosen type of investigation, the location of the investigation, the cost of the information along with the value of information, and the probability of failure after the investigation is added.



Result summary sheat

	CPTu	VST	TC	UC	Total	
Total						
Optimal						
Initial conditions						
H	L	Unit weight	Mean	Std_dev	Theta_x	Theta_z
Load	x accuracy	z accuracy	Cost of failure	shear circles	circle points	Initial PoF
1. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
2. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
3. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
4. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
5. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
6. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
7. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
8. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
9. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf
10. Investigation	Chosen type:	x location	z location	CoI	VoI	Pf

Figure 7.4: Example of the summary sheet used to log the output of a simulation

The program calculates the value of information of an investigation type for each point assigned in the domain. These values are represented in a plot in order to visualise where the significant points can be found, and if the location selection provides a pattern. This plot is showed for both a point observation and a CPTU in figure 7.5 and 7.6. The point observations seems to provide most value at a depth of 5 to 10 meters and between the x coordinates 20 and 40. The CPTU investigation considers the entire depth, which leads to the x coordinate to be most important. According to figure 7.6 the optimal location of the CPTU is somewhere between the middle and bottom of the slope. This covers the same area as the point observations.

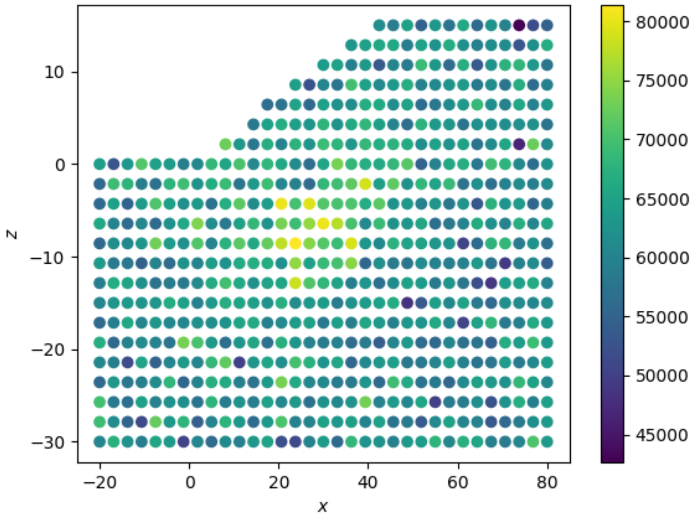


Figure 7.5: Example plot containing a failure probability curve

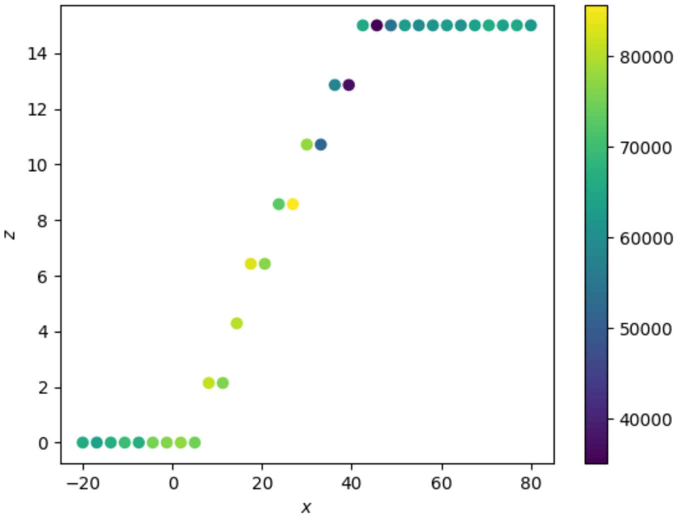


Figure 7.6: Example plot containing a failure probability curve

During the simulation process the program plots the effects of how the added investigation alters the variation and mean in the random field that models soil variability. An example plot of how an added CPTU changes the standard deviation in the correlation matrix is showcased in figure 7.7.

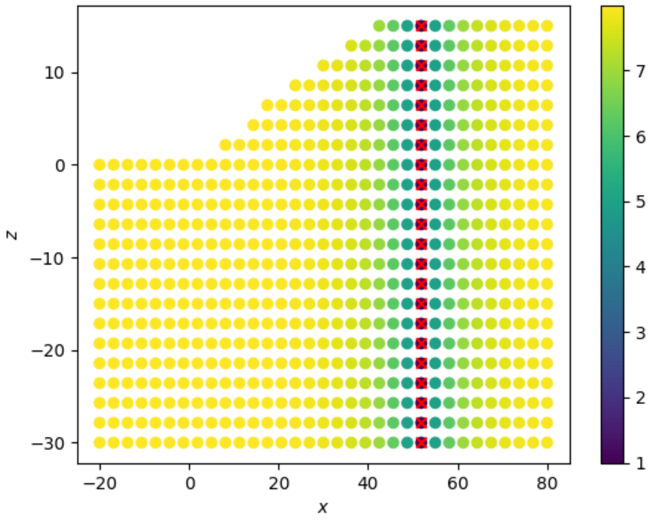


Figure 7.7: Example plot containing a failure probability curve

When the simulation process is finished the program plots the results in form of a failure probability plot, a plot containing the value and cost of information, the last plot shows value of information after the cost of information is subtracted.

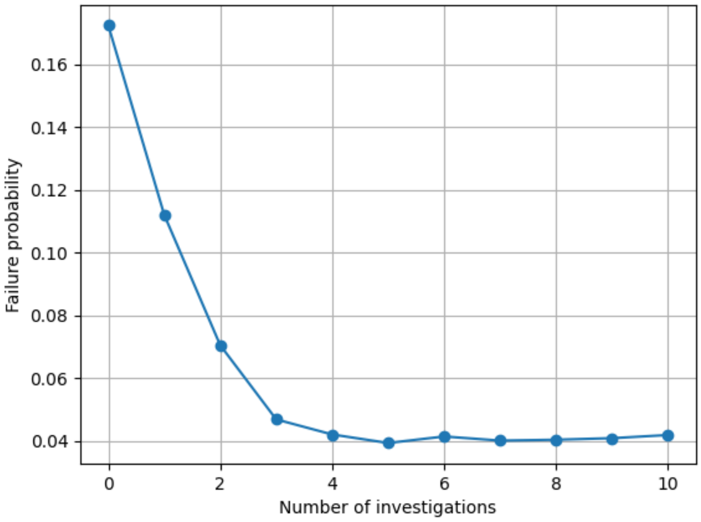


Figure 7.8: Example plot containing a failure probability curve

The failure probability plot as in figure 7.8. This plot shows that the first three investigations have a significant impact on the reduction of failure probability making them more valuable. The investigations after this does not seem to affect the probability of failure.

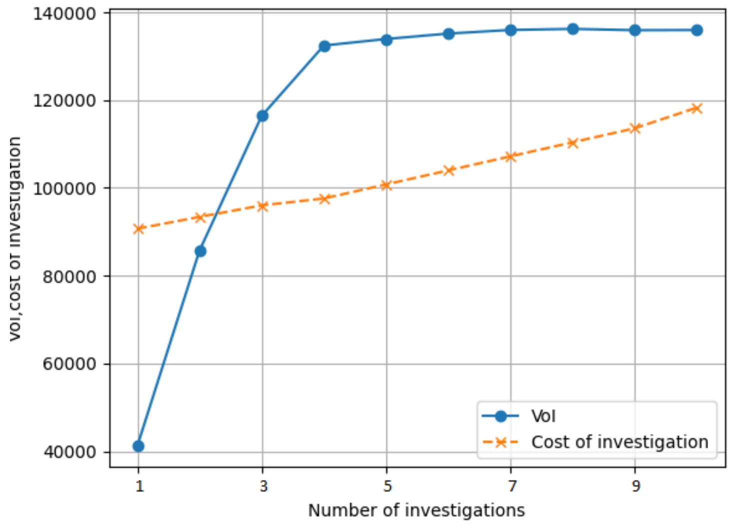


Figure 7.9: Example plot containing a failure probability curve

The next plot figure 7.9 compares the value of information (blue line) and the cost of information (orange line). In this figure the value of information surpasses the cost of information after three investigations are performed, and that it stabilizes after about five investigations. This figure provides an overview of when the investigations are profitable.

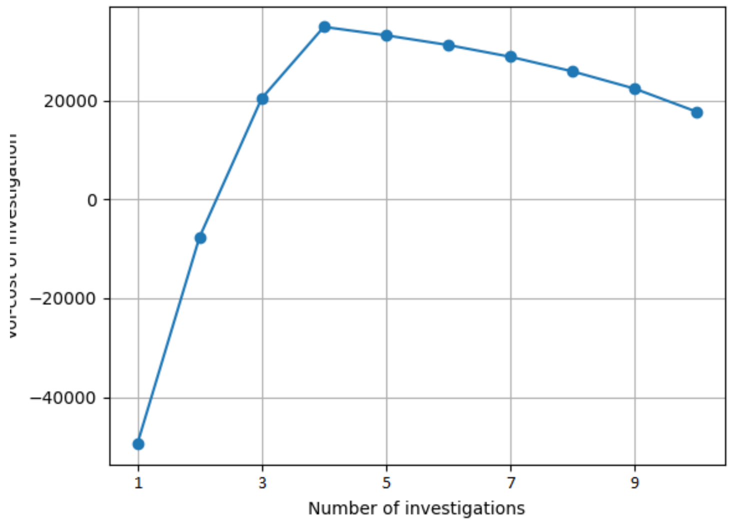


Figure 7.10: Example plot containing a failure probability curve

The value of information - cost of information plot, as in figure 7.10, reveals the optimal amount of investigations. The x value of which the maximum y value of the curve is located can be interpreted as the optimal amount of investigations. In this case that would mean that the optimal number is four.

Chapter 8

Results

8.1 Basis of Simulation

To assess the performance of the implemented approach, a simplified case is being examined. The scenario involves a slope with a load of 100 kN/m on top, which could be a result of infrastructure leading to the failure consequence. The slope in question has a height of 15 meters and a width of 40 meters, resulting in an inclination of 3:8. Multiple variations of both prior knowledge and cost of failure are considered to evaluate their effect on the resulting optimal number of investigations, their position, and the combination of investigations.

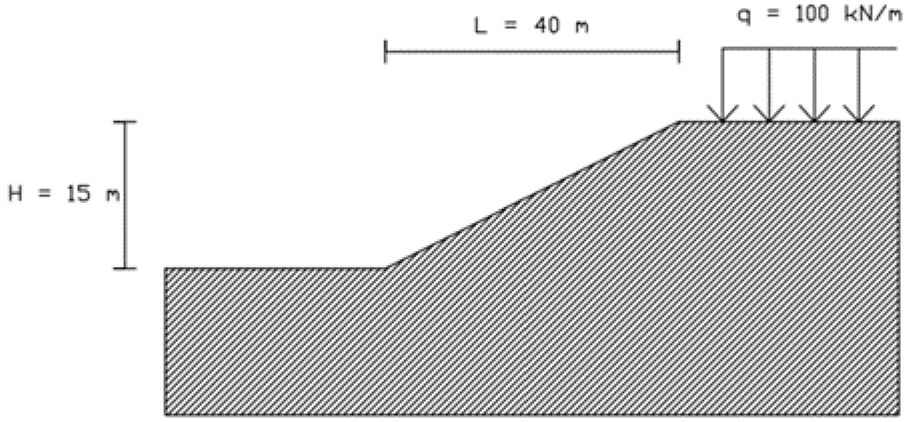


Figure 8.1: Geometry of the slope used in tests

8.1.1 Assumed prior knowledge

The theoretical material that is investigated in this case is presumably a uniformly graded soil, consisting of clay/silty clay. The material is assumed to be medium firm. The bedrock at the test site is assumed to be located at a depth of 30 m. The prior knowledge that are provided in this example could be a result of former total soundings or similar tests executed in the area, in addition to potential geotechnical experience that a geotechnical engineer may have from similar projects. Values describing the test material that are not varied in the tests are provided in table 8.1.

Parameter	**Value**
Unit weight γ	18 [kN/m]
Start mean of undrained shear strength s_u	35 + 2 $\cdot$ depth [kPa]

Table 8.1: Static variables used in tests

8.1.2 Predefined model parameters

The methodology was coded with the possibility of adjusting various parameters to select the detail of the calculations. The predefined values are presented in Table 8.2. The program does require a significant amount of time to perform the analysis, and the time needed seems to increase exponentially with size of the correlation matrix. The values chosen are a result of balancing processing time with accuracy. Resulting in maintaining the requirement of two points per unit of correlation length and keeping the time needed to finish one analysis at approximately eight hours.

Parameter	**Value**
minimum x border	-20
minimum x border	80
minimum y border	-30
x accuracy factor	3
y accuracy factor	2
Number of shear circles	12
Points per shear circle	30
Number of trial investigations	10

Table 8.2: Predefined modelling values

The number of points used in the shear circles, as well as the x and y accuracy factors, are adjusted for small correlation lengths, such that the ratio of points per unit exceeds half of the correlation length. This approach ensures that quality in the result is obtained while maintaining reasonable processing times for the total set of tests.

8.2 Test of Standard Deviation

The amount of informative prior knowledge will have an impact on the uncertainties connected to the soil properties. In this thesis the change is tested by increasing and decreasing the standard deviation of the undrained shear strength. A change in mean value will probably have an effect on the end results. Because of a need to prioritise different tests it is not considered. Increasing the standard deviation will model an increase in uncertainty as it leaves room for more variation in the initial random field. The chosen static values of standard deviation are described in table 8.3. In order to avoid negative values of s_u for $\sigma = 15$, the distribution was truncated with the minimum value of $s_u = 1$ kPa. These tests are modelled with the correlation lengths $\theta_x = 50$ and $\theta_z = 5$ and a $c_f = 5000000$ NOK.

Table 8.3: Variations of standard deviation

Test nr.	Standard deviation σ
1	4
2	8
3	15

The results of the three tests performed indicate notable differences in optimal location, number of investigations and value of information. These test outcomes are presented in Table 8.4, while the corresponding testing locations are illustrated in Figure 8.4. This figure displays the region in which the calculated shear circles are located. All investigations were placed within this area.

Table 8.4: Summary of test results from standard deviation tests

Test nr.	Optimal number of investigations	Investigation types
1	0	0
2	5	3×CPTU, 1×UC, 1×TC
3	4	3×CPTU, 1×UC

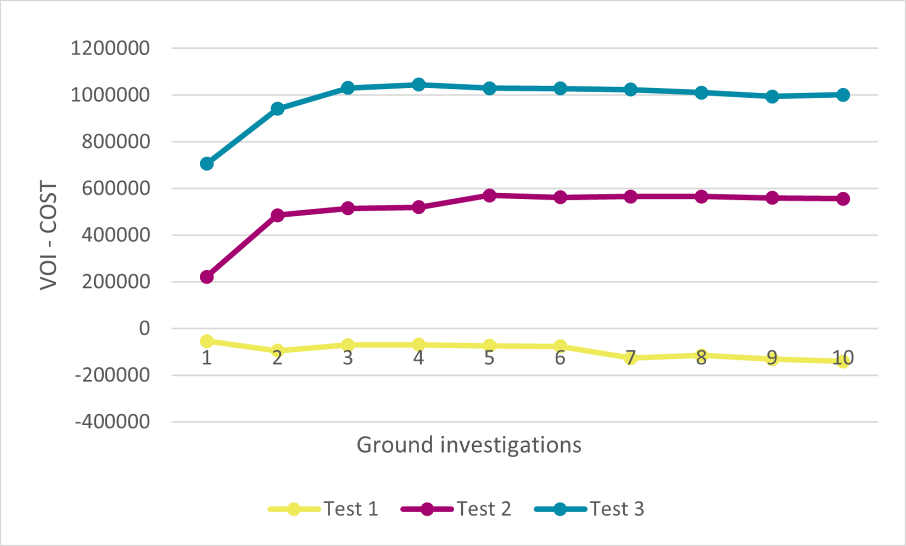


Figure 8.2: Value of information reduced by the cost of investigation over number of ground investigations

The probability of failure of Test 1 starts at a value of 0,072 and reaches a probability of failure of 0,059, but never reaches a situation where the benefit of conducting soil investigations outweighs the cost of investigations. Test 2 starts at a probability of failure equal to 0,18 and reduces to about 0,063 after the first two investigations. From this point on each investigation seems to provide a small decrease in probability, and at 5 it reaches a probability of 0,036 at which point the optimal amount has been reached. In similarity to Test 2, the probability of failure decreases quickly for the first two investigations in Test 3. It starts at 0,37 and reaches 0,14 at the optimal number of four investigations. The large variation in the field, probably leads to the investigations not providing enough information to learn anything profitable beyond four investigations. This could also be a result of multiple small areas along the shear circles still having large variation, and the implemented investigations are not able to cover these.

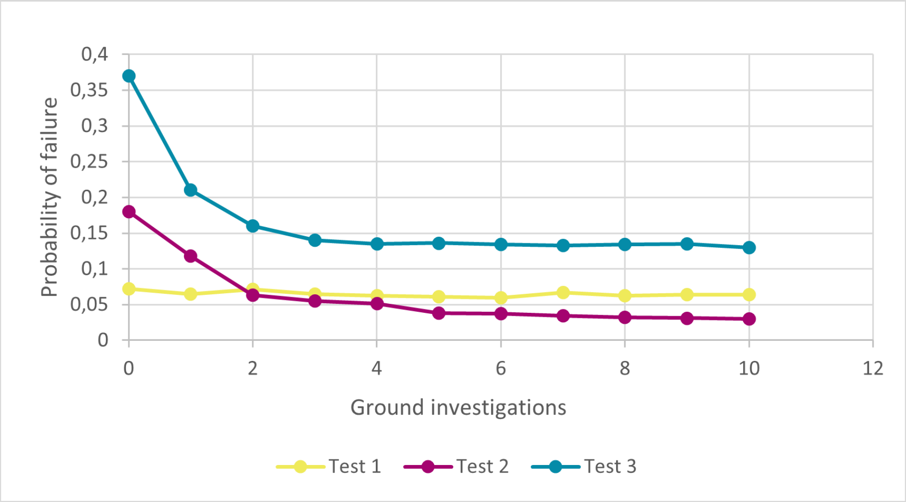


Figure 8.3: Probability of failure over number of investigations

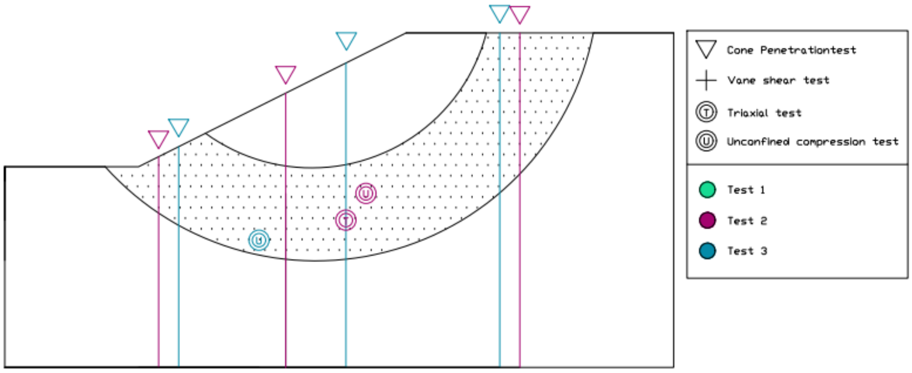


Figure 8.4: Investigation locations for the optimal number of investigations

For Test 2 and Test 3, the executed Cone Penetration Tests (CPTU) were placed in a similar pattern, with one at the bottom and one at the top of the slope situated approximately three meters apart. The final CPTU was placed between the other two investigations for both tests, Test 2 being located slightly to the right of the bottom of the shear circles and a little to the right of the bottom of the shear circles for Test 2. Test 1 never reached a positive result, and the chosen site investigations was a combination of five unconfined compression (UC) tests and five CPTU.

8.3 Test of Correlation Length

In order to assess the impact of correlation lengths on the performance of the model, tests were conducted using varying correlation lengths based on table 3.3. The selection of these correlation lengths were based on minimum and maximum values as well as the mean values corresponding to undrained shear strength. The outcomes of these tests are summarized in Table 8.5, while more detailed results are presented in the subsections of this chapter. The findings reveal a reduction in the optimal number of investigations as the correlation length increases. Low vertical correlation lengths appear to prioritise the utilization of Cone Penetration Testing. The results indicate that a high frequency of CPTU often results in a triaxial test (TC) as opposed to an UC.

Table 8.5: Summary of correlation length tests

Correlation lengths	**Optimal number of GI**	**CPTU**	**VST**	**UC**	**TC**
$\theta_x = 46\ \theta_z = 2,0$	4	2	0	0	2
$\theta_x = 46\ \theta_z = 3,8$	6	3	0	1	2
$\theta_x = 46\ \theta_z = 6,2$	4	2	0	1	1
$\theta_x = 50\ \theta_z = 2,0$	7	6	0	0	1
$\theta_x = 50\ \theta_z = 3,8$	6	2	0	3	1
$\theta_x = 50\ \theta_z = 6,2$	4	2	0	1	1
$\theta_x = 60\ \theta_z = 2,0$	5	4	0	0	1
$\theta_x = 60\ \theta_z = 3,8$	4	3	0	0	1
$\theta_x = 60\ \theta_z = 6,2$	4	3	0	1	0

The following subsections will refer to tests with $\theta_z = 2,0$ as test 1, $\theta_z = 3,8$ as test 2 and $\theta_z = 6,2$ as test 3. In addition to the tests based on values presented by Phoon and Kulhawy 1999a, simulations with correlation lengths exceeding this have been performed.

8.3.1 Horisontal correlation length = 46

All three tests show a large increase value of information - cost of information in the first four ground investigations. Test 2 has a different pattern. The VOI - COST - curve has a sudden increase and the maximum value is found at six investigations. The sudden increase is where the first TC is added.

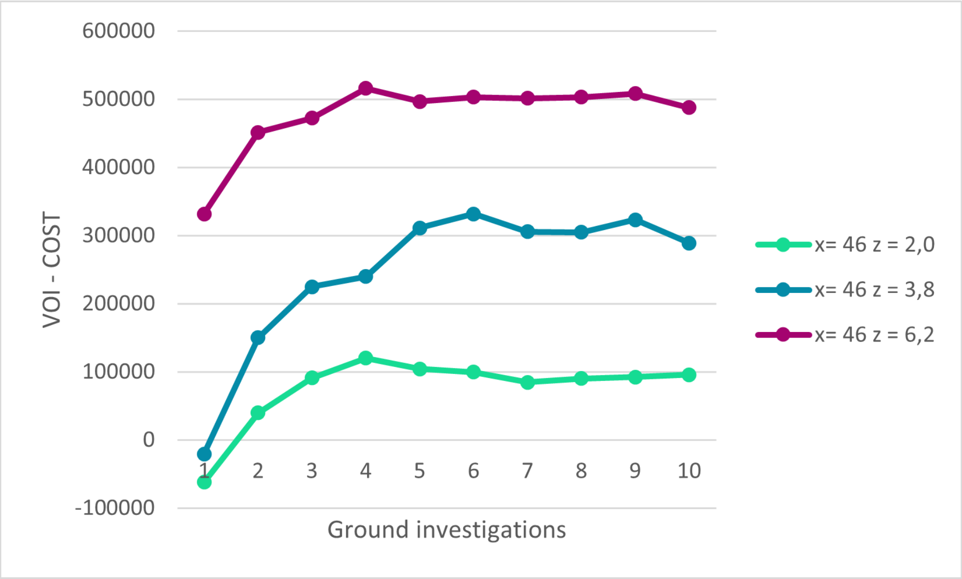


Figure 8.5: Value of information - cost of information per ground investigation implemented

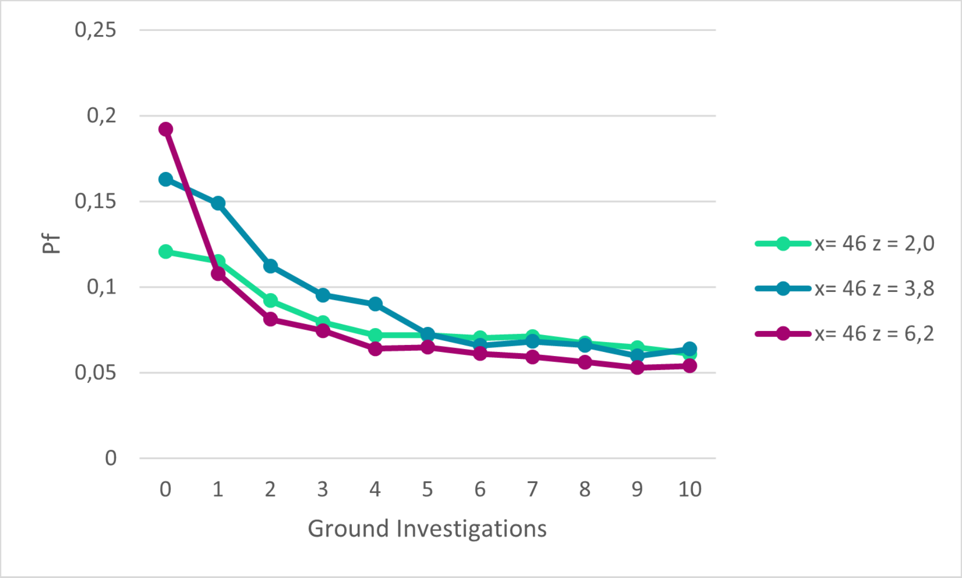


Figure 8.6: Probability of failure after implemented ground investigation

The results show that an increase in horizontal correlation lengths results in a greater difference between the value of the information gathered and the associated costs. This is reasonable as an increase in correlation length leads to less variation in the field following the implementation of an investigation.

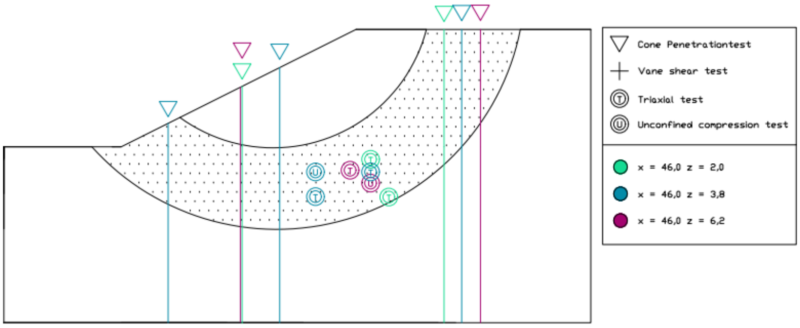


Figure 8.7: Investigation locations with horizontal correlation length = 46

Test 1 returns an optimal combination of four ground investigations. This test uses two CPTU investigations and two TC tests. The CPTU investigations are positioned to cover the middle of the slope and the top. The laboratory tests are all performed in the region between the CPTU.

Test 2 results in three CPTU investigations, two TC tests and one UC test. The CPTU tests are positioned 8 meters into the slope from the toe, in the middle, and 18 meters from the top. The laboratory tests are located between the top and middle CPTU, with one triaxial and uniaxial begin placed at the same x - coordinate but at different depths.

Finally, the last test includes two CPTU investigations, one UC test and one TC test. The CPTU tests are positioned in the middle of the slope and 21 meters to the right of the crest. The laboratory tests are located between the CPTU and are spaced closely together at approximately 3.5 meter intervals.

8.3.2 Horisontal correlation length = 50

Increasing the horisontal correlation length to 50 seems to lower the optimal number of investigations. The curves in figure 8.8 shows a connection between increasing the vertical correlation length and the value of information obtained. All three tests hit a maximum within the range of ten investigations. The two tests with lowest vertical correlation length both seems to stabilize early but starts to increase after five performed investigations. The last curve decreases after a maximum at four investigations, this test has an increase after eight investigations but it does not reach the value achieved at four investigations.

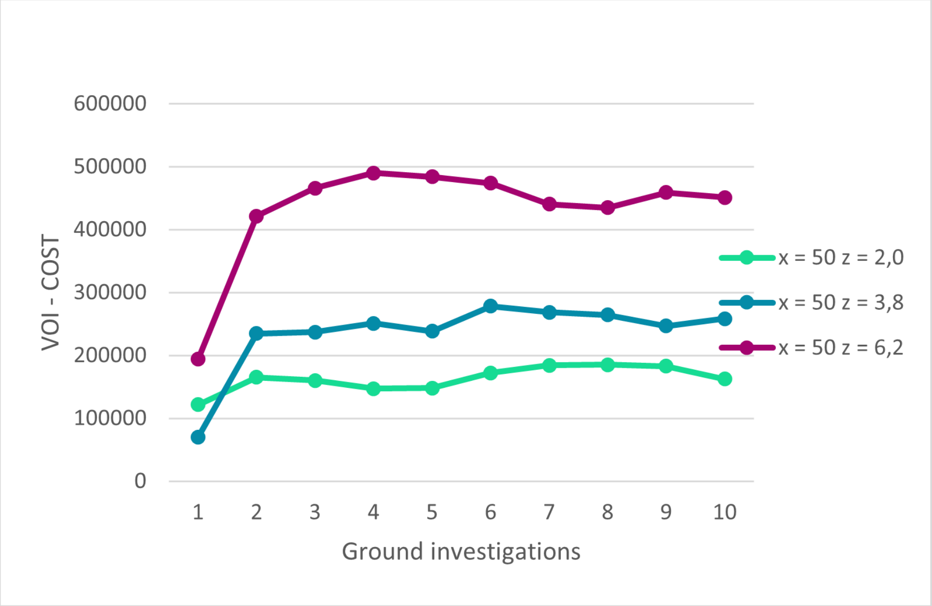


Figure 8.8: Value of information - cost of information per ground investigation implemented

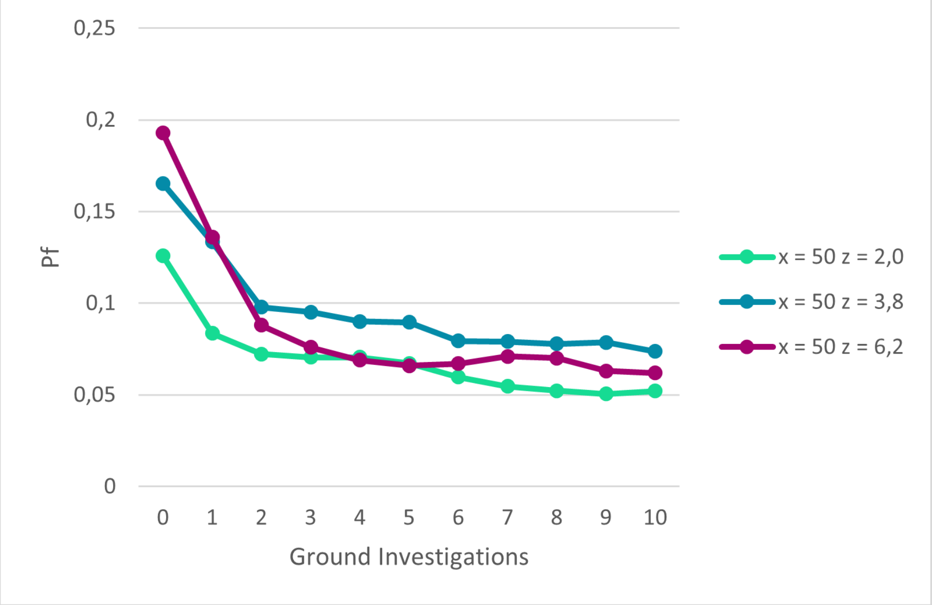


Figure 8.9: Probability of failure after implemented ground investigation

The probability of failure corresponds to the results of figure 8.8, as it shows a big decrease in the beginning and then a smaller rate of decreasing after the first two investigations. The test with largest vertical correlation length is closer to converging than the other two tests which seems to have a evenly distributed decrease after the first two investigations.

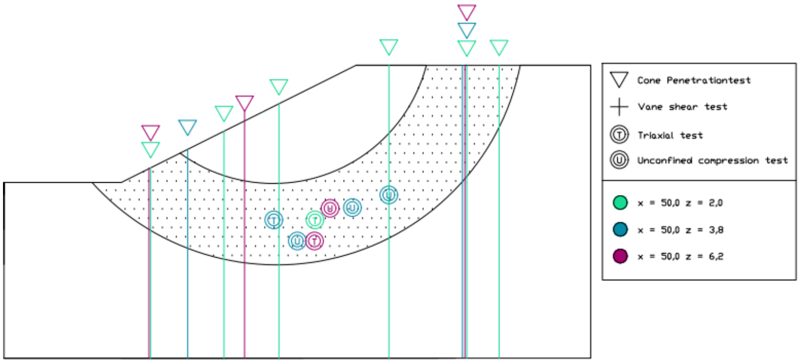


Figure 8.10: Investigation locations with horizontal correlation length = 50

The locations of investigations are more spread in this test, when comparing it to the previous test. Test 1 consists of six CPTU and one TC test. There are three CPTU placed in the slope itself which are evenly spread from 5 to 33 meters into the length of the slope. The last three are located at the top of the slope at distances of 5, 11 and 24 meters from the crest. Between these groups a TC test is placed at a depth of 23 meters.

Test 2 has a combination of three UC tests, two CPTU and one TC test. This shows a significant difference in the amount of CPTU tests in comparison to the test with lower vertical correlation length. The first CPTU is located at 18 meters to the right of the crest while the other is placed 11 meters into the slope length. The laboratory test is spread in both height and depth between CPTU. If the toe of the slope is considered as origo, the coordinates of the UC tests are (30, -10,7), (45,6, -2,1) and (39,4, -4,3) and of the TC test (26,9, -6,4).

The final test with vertical correlation length equal to 6,2 has a result of two CPTU, one UC test and one TC test. The first CPTU is placed at 11 meters to the right of the crest, while the other is located at 11 meters into the slope length. These two locations have also been chosen for vertical correlation length of 2,0.

8.3.3 Horisontal correlation length = 60

Increasing the horizontal correlation length to 60 leads to a reduction in the optimal number of investigations for Test 1 and 2, while the optimal number remains the same for test 3. Figure 8.11 demonstrates a positive relationship between increasing vertical correlation length and the value of information obtained from investigations. The lower vertical bound test reaches its maximum at five investigations, while test 2 reaches its maximum at four investigations. This is the same optimal number as the last test. Test 3 has a different behaviour compared to the other curves, as the first two investigations have a significant impact, whereas following investigations result in marginal increases or decreases in the value of information - cost of information curve.

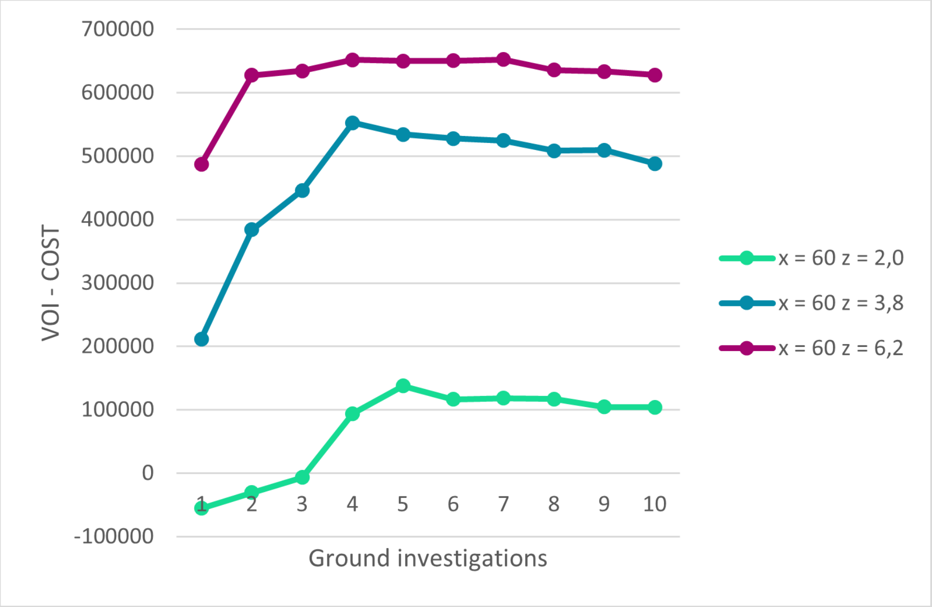


Figure 8.11: Value of information - cost of information per ground investigation implemented

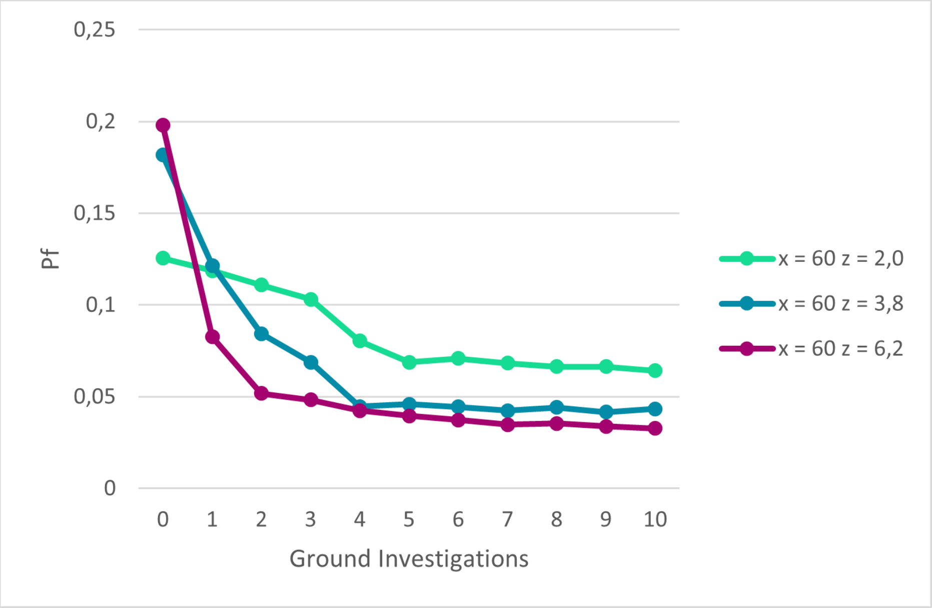


Figure 8.12: Probability of failure after implemented ground investigation

The probability of failure curve supports this finding, as the first two investigations in test 3 show a less steep decrease compared to the other tests. Following the optimal number of investigations, the decrease stabilises for all three tests with only a slight decrease in value of information per added investigation.

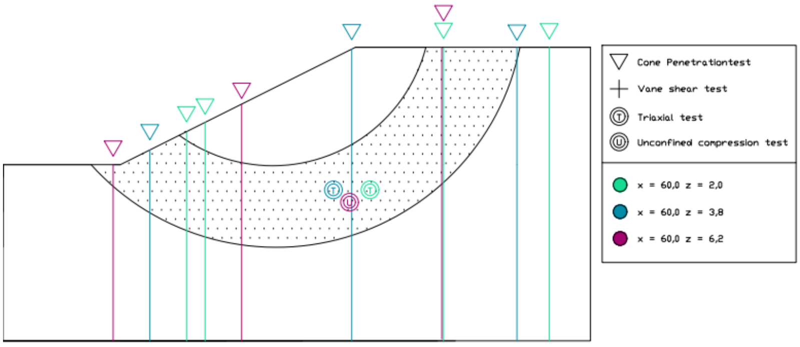


Figure 8.13: Investigation locations with horizontal correlation length = 60

In terms of investigation locations, these three tests display a wider distribution compared to the previous correlation length tests. Test 1 consists of four CPTU and one TC test. These tests with the lowest vertical correlation length tend to have a higher ratio of CPTU compared to the other tests. The results of this test 1 show two CPTU investigations placed with a three-meter interval at depths of 11 and 14 meters into the slope. The third CPTU is located 15 meters behind the crest of the slope. The final CPTU is situated 33.4 meters away from the crest, lying outside the shear circle area. This is the only investigation among the tests with such placement. The TC test is positioned at a depth of 19 meters, one meter behind the slope top.

The second test consists of three CPTU and one TC test. The first CPTU is placed 5 meters into the slope while the second is placed at the top. The final CPTU is found 27 meters behind the top of the slope. The TC test is placed at a depth of 18 meters and four meters to the left of the top of the slope.

The results of the final test consist of three CPTU and one UC test. In this case the model excludes the use of a TC test due to the correlation lengths, implying that the CPTU tests provide sufficient information with the addition of an UC test. The CPTU tests are spread across the slope, with one at the toe, one in the middle and one located 15 meters behind the slopes crest. The UC test is positioned in the largest gap between CPTU tests, nearly directly beneath the top of the slope at a depth of 21.5 meters.

8.3.4 Tests with extensive correlation lengths

In addition to the values deviated from the results of Phoon and Kulhawy 1999a, some more extreme values of correlations lengths have been tested in order to see how it affects the performance of the model. These values contains larger horizontal correlation lengths in addition to a wider range of vertical correlation lengths. The tested values are listed along with a result summary in table 8.6.

Table 8.6: Results from tests with extensive correlation lengths

Correlation length	Optimal number	CPTU	VST	UC	TC
$\theta_x = 5\ \theta_z = 5$	7	2	0	4	1
$\theta_x = 25\ \theta_z = 5$	7	4	0	1	2
$\theta_x = 50\ \theta_z = 5$	5	2	0	2	1
$\theta_x = 75\ \theta_z = 5$	4	2	0	1	1
$\theta_x = 100\ \theta_z = 5$	3	2	0	1	0
$\theta_x = 1000\ \theta_z = 5$	2	2	0	0	0
$\theta_x = 5\ \theta_z = 50$	5	0	0	5	0
$\theta_x = 50\ \theta_z = 50$	3	2	0	1	0
$\theta_x = 100\ \theta_z = 100$	3	1	0	2	0

From table 8.6 it can be read that increasing the horizontal correlation length leads a reduction in the total optimal number of investigations. In addition to this the ratio of CPTU to other investigations also increase with the horizontal correlation length. The last three tests in the table seems to prioritise the use of UC tests which is a result of increasing the vertical correlation length. The same pattern can be seen for $\theta_x = 5$, $\theta_z = 5$, where the horizontal length is considerably reduced.

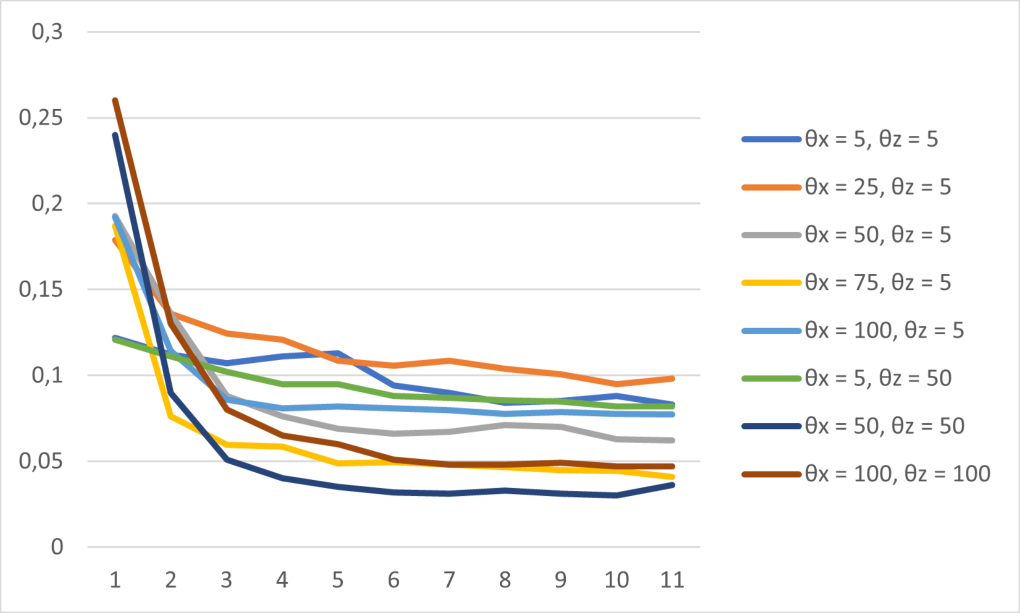


Figure 8.14: Probability of failure over simulated ground investigations

In figure 8.14 the development of the failure probability as a result of simulating ground investigations are shown. This graph shows a trend of large decrease for first 2 - 3 investigations, and after this the trend is to flatten somewhat out. The tests with $\theta_x = 5$ $\theta_z = 5$ and $\theta_x = 5$ $\theta_z = 50$ differs from the trend as they start at a relatively low probability of failure and maintain a less steep decrease for a longer interval. Another trend is the fact that increasing the correlation length seems to increase the initial probability of failure.

8.4 Test of Cost of Information

Estimating the cost of a slope failure is challenging as it may result in not only structural damage but also loss of life. For the purposes of this study the costs will be limited to structural damage incurred on some form of infrastructure. The cost estimation could be a very complex part of the process, as a result of this the costs associated with a slope failure has huge variations. As a consequence of this the evaluation of the impact due to cost of failure (c_f) will be based on a set of three values being 1 000 000 NOK, 5 000 000 NOK and 10 000 000 NOK. These values have been chosen as they could be representative of the costs they could result in if a single house or a road is present in the slope area.

In order to evaluate the differences caused by cost of failure, the three values have been tested over different sets of correlation lengths. This is done to get a more detailed view on how c_f impacts the selection of ground investigation type. The correlation lengths along with the results are showed in figures 8.15. As the optimal location for all investigation types is a result of the decision requirement (uniform cost of failure × individual probability of failure), raising the cost of failure will not impact the optimal location itself and is not considered in this test analysis.

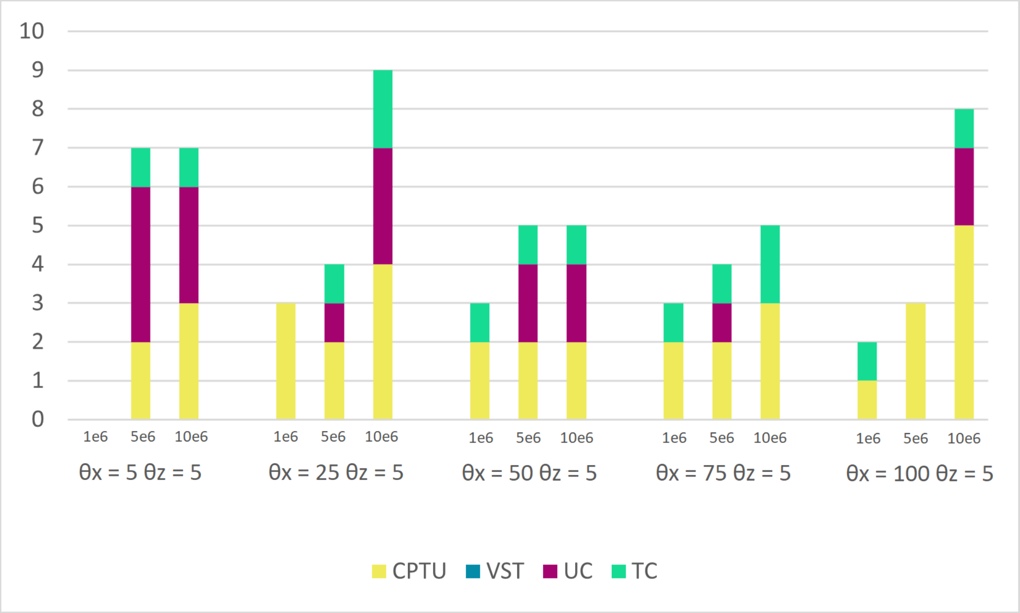


Figure 8.15: Results of simulations with varying cost of failure

The results show that the optimal number of investigations increases with the cost of failure, with one exception observed at $\theta_x = 25$ and $\theta_z = 5$. In this particular test, the same number and type of investigations were optimal for both c_f = 5,000,000 NOK and c_f = 10,000,000 NOK. No obvious differences in the selection of investigation methods based on the cost of failure could be inferred from this test. After these tests, the relationship between the correlation lengths appears to be the decisive factor influencing the choice of investigation type.

An observation worth noting is the fluctuation in the optimal number of investigations. For c_f = 1,000,000 NOK, the initial test recommends zero investigations as the optimal number, followed by three investigations with an optimal number of three, and for the final test it is reduced to two. In the case of c_f = 5,000,000 NOK, the optimal number of investigations varies, with four investigations identified as optimal for θ_x = 25 and 75, and the highest value of seven occurring at θ_x = 5, while the lowest number of three investigations is observed in the final test with the largest correlation length value. The test with c_f = 10,000,000 NOK, the optimal number of investigations is considerably higher for θ_x = 25 and 100, and the test show a greater fluctuation. The lowest optimal number of investigations is observed at θ_x = 50 and 75, while the optimal has larger values in both directions from these.

Chapter 9

Discussion

The scope of this thesis is to expand a ground investigation optimisation model based on Value of Information, to optimize the number, combination, and placement of four different ground investigation types for an undrained slope stability problem. The performance of the model was assessed on a simple slope stability case study. This was made in order to evaluate input in the form of prior knowledge, and output in the form of combination of investigations and their locations.

9.1 Placement of Ground Investigations

A typical approach in geotechnical engineering is to place ground investigations at critical areas of a slope. This would mean covering the top, middle and toe of the slope, as well as ensure that they are at depths covering the assumed failure surfaces. Test results show that the CPTUs tend to be placed at the toe, middle and top of the slope. The top placement is usually the first selected. The laboratory test is often placed between CPTU. This is not usually the case in geotechnical engineering as the samples are often taken close to soundings, due to saving expenses or more accurately measuring the same soil mass with different test methods.

All but one of the performed tests choose locations that are within the area of the predefined shear circles. This is probably because of the use of the direct method as the factor of safety calculation. This method is based on the average undrained shear strength along the shear surface. The model will try to reduce the variation in the entire length of the circle, since this method sees all points along the circle as equally important. This will also force the model to prioritise CPTUs at the top of the slope. The reasoning behind this is the geometry of the shear circles in a slope and is illustrated in figure 9.1. The figure shows that the reduction of variation is affecting a larger part of the arcs belonging to the shear circles when the CPTU is placed at the top of the slope.

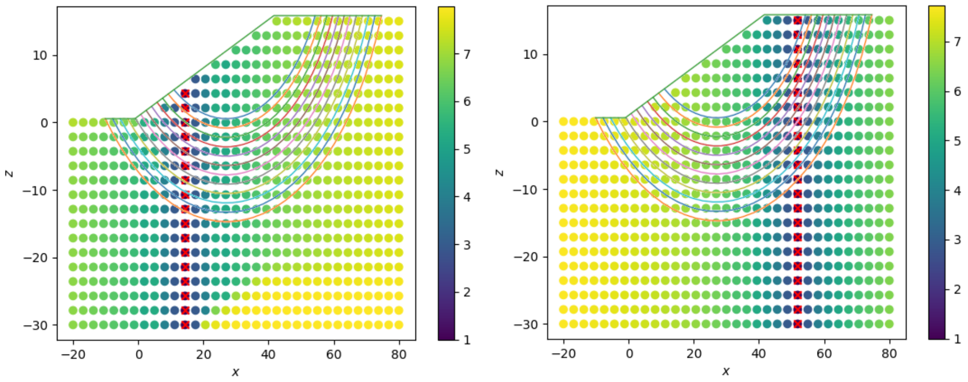


Figure 9.1: Illustration of the impact of two CPTU at different locations in the slope

The tested slope has been modelled in Plaxis using values previously defined and the selected mean values of undrained shear strength. The resulting shear strain distribution is shown in figure 9.2. The distribution of shear strain indicates that the assumed area for the critical failure surface used in the test analysis is reasonable. This is due to the figure showing the critical area within the modelled area of shear circles.

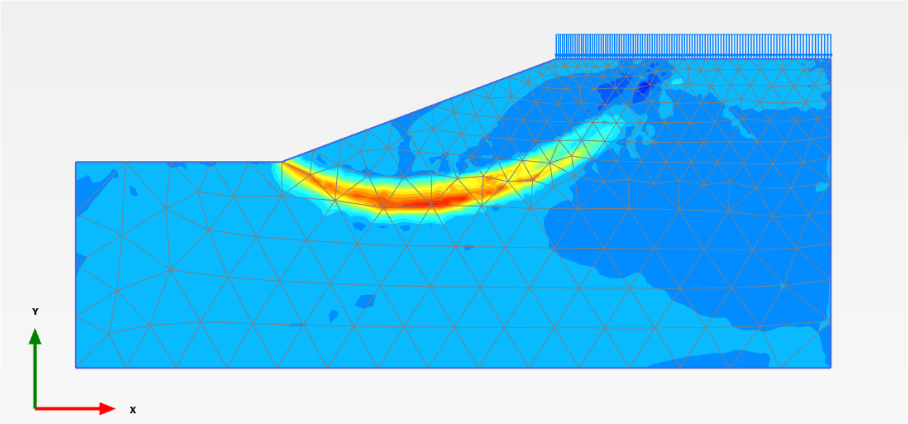


Figure 9.2: Plot of the shear strain distribution through the tested slope modelled in Plaxis 2D, using mean value of undrained shear strength

A change in calculation method could be implemented in order to account for the weighting of locations along the shear surfaces. An attempt was made to implement the lamella method to test the differences but due to complications in the code and limitations in time it was not finished. Another option is to use a FEM-program. This could result in the ability to interpret non-circular shear circles as well, which might lead to different results.

The results tend to show a large set of laboratory tests in the same area. The tested slope consisting of uniform material that makes this seem unnecessary, although the significant increase in undrained shear strength with depth is valuable to confirm. After comparing the test locations based on correlation lengths it becomes clear that the distance between point observations increases when the vertical correlation length is increased. These parameters are also following the terrain which means that investigations can be placed pretty close without intervening much with each other.

In addition to these evaluations, it must be mentioned that the process of developing a bore plan and identifying optimal ground investigation placements is complicated. This model is designed to determine the most profitable locations and combination of ground investigations, but numerous limitations exist with regards to the site investigations. Their placement often necessitates compromises because of various factors. For instance, restrictions imposed by the landowner, or the presence of dense forests or steep terrain can make it impossible to reach an ideal point for investigation using a drilling rig. Infrastructure such as buildings and underground pipes also poses constraints, as there are legal regulations governing the proximity of borings to such elements. There are potential strategies to approach these limitations. One could be to compare predetermined investigation points to ascertain the most favorable option. Another possibility is to restrict the vector containing potential test locations to a predefined range of values within areas suitable for investigations.

9.2 Combinations of Investigations

The results indicate that the preferred combination of ground investigations is heavily dependent on the correlation lengths in the field. When considering the range found by Phoon and Kulhawy 1999a, the preferred approach is a set of CPTU and a triaxial test, plus a range of unconfined compression tests from 0 – 3. Since the slope is made of a single material and only the undrained shear strength parameter is considered, the use on a single triaxial test seems reasonable considering it is combined with CPTU. If drained slope stability had been considered, parameters such as friction angle and attraction would be necessary. This might have led to an increase in the value of information from triaxial tests.

The triaxial test provides a higher level of accuracy compared to the unconfined compression test but is significantly more expensive. The test patterns indicate that a sufficient number of CPTU must be performed to enable the selection of a triaxial test. The inclusion of a triaxial test stands out in the results as it leads to a noticeable improvement in the probability of failure curve. This improvement is likely a result of its enhancement of the previously conducted CPTU. However, the triaxial test alone does not provide enough information to be deemed a superior alternative to the uniaxial test. This is opposed to what is stated in existing literature, where the uniaxial test is seen as inadequate without comparison to other investigative methods (NTNU geotechnical division 2017 p. 161).

The results show two specific situations in which the unconfined compression test is selected. The first scenario occurs when there are large correlation lengths in the vertical direction, making it a more cost-effective alternative to the CPTU. The second scenario is when the variation is reduced across larger areas of the slope. In these cases, the lower cost of the uniaxial test becomes the decisive factor. When the uniaxial tests are implemented this way, the probability of failure curve only exhibits a minor decrease indicating that the majority of uncertainties is already accounted for.

When the horizontal correlation length is larger than the vertical correlation length, CPTU is the dominating investigation method. This is likely due to it providing multiple points in the vertical direction. CPTU is chosen for the first 2 - 3 investigations, before being complemented by other investigation types. The uncertainty created by it not measuring undrained shear strength directly is large enough for the model to implement a triaxial test in most of the cases. In the cases when $\theta_x \geq 100$, the provided information from the CPTU seems to be adequate without triaxial test.

The vane shear test is never chosen as the option. Of the implemented point observations it has the lowest measurement error, but the cost difference compared to both triaxial and uniaxial is too large. This could be due to each vane shear test not containing more than one s_u - measurement. This would have led to more value of information provided in specific areas. Another measurement would only lead to an increase in cost of 296 NOK. After considering this the conclusion has to be that the implementation of VST is inadequate making this test only representable for the evaluation of UC, TC and CPTU.

9.3 Evaluating the Effect of Prior Knowledge

In this thesis the impact of prior knowledge has been tested through variations of standard deviation and correlation lengths. The results show that both these factors have a significant impact on the optimal number and combination of investigations. The placement of investigations is also varying but seems to follow the same principle of placing CPTU in the toe, middle and top of the slope. The point observations are then placed in the largest open area between CPTU, and in the area of the considered circular failure surfaces.

The model does require a certain amount of prior knowledge to calculate a reasonable result. More specifically one should have information about the geological processes in the site area, so that an idea of layering and directions of correlations can be assumed. An assumption of mean and standard deviation of the soil also needs to be provided.

In areas with complex geology containing multiple layers with different characteristics, this would require a solid set of initial ground investigation to establish prior knowledge. The variations from the tested parameters does imply the importance of having a solid foundation of prior knowledge.

9.3.1 Standard deviation

The impact of standard deviation in the modelled inherent soil variability was tested with values of 4, 8 and 15, and the results show that an increase in standard deviation increases value of each investigation. This because an observation reduces the variation of the area around it, and an increase in standard deviation leads to a larger variation throughout the random field. The result seems reasonable as a low standard deviation in prior knowledge would be a result of more previous data in form of site investigation of similar, which then again means less new investigation are necessary.

The first test performed on the standard deviation is supposed to simulate a scenario in which the soil parameters are relatively well-known. In this simulation, when the standard deviation is set at four, the model determines that conducting further investigations is unnecessary when the cost of failure amounts to 5,000,000 NOK. This result would probably stay the same for all costs of failure as the failure probability do not seem to fluctuate much. This choice is based on the fact that the failure probability begins at a low value rendering the value of information decision criterion consistently negative. This assumption appears reasonable given the intended information known on the soil parameters. The model appears to select uniaxial tests or CPTU at random positions, deviating from the typical pattern. This behaviour may be a result of random fluctuations in the random field realisations.

The second test with a standard deviation of eight, reaches an optimal amount of ground investigations of five. This test seems to converge to a somewhat similar value as the the previous test, but takes a small leap in probability of failure after the triaxial test is implemented as observation number 5. This could also be the cause of the difference, as the triaxial test is not considered for test 1. The investigations of this simulation is placed with a larger distance between them when compared to the third test. The reasoning for this phenomenon might be that the interference of site investigations will lead to a reduction in variance in the area between them.

The final standard deviation test simulates a scenario where limited information is available, resulting in a large standard deviation. As expected, this test has a considerably higher initial probability of failure. Surprisingly, it yields a lower optimal number of investigations compared to the second test, contrary to the expected outcome. The rate of decrease in the probability of failure appears to slow down at a probability of failure around 0.14, which is approximately 0.08 more than test 1. This could be due to certain points along the shear surface still having a significant variation and the performed tests not providing coverage over a wide enough area to eliminate these uncertainties. The number of iterations conducted is insufficient to determine whether the tests will continue to yield a minor decrease in the probability of failure or if they will converge before reaching a similar value as the previous tests.

9.3.2 Correlation length

Correlation lengths are probably the most difficult parameter to estimate in the implemented approach. Correlation length is also one of the most impactful on the simulation process. The correlation length tests show that increasing the horizontal correlation length from 46 to 60 meters, seems to result in a decrease of optimal amount of investigations. This conclusion is easier to interpret when looking at a larger range of vertical correlation lengths, as shown in figure 9.3. Increasing θ_z also seems to lead to an increase in initial probability of failure. When the correlation lengths increases it leads to higher likelihood of larger Weak zones (i.e., low shear strength values) in the domain and will lead to an increase in the initial probability of failure.

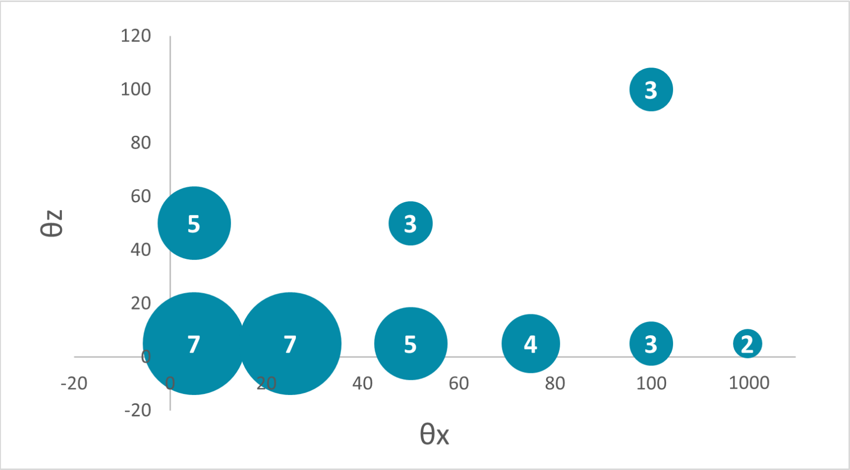


Figure 9.3: Optimal number of investigations considering an increase in horizontal correlation length

The model handles larger horizontal lengths by implementing a larger ratio of CPTUs. It seems to be a reasonable result as this test will provide information targeting a larger area than point observations. When increasing the vertical correlation length, the difference in provided information reduces. Since the uniaxial test is cheaper it will be the preferred option when the vertical correlation length becomes large enough.

The vertical correlation lengths of 2,0, 3,8 and 6,2 were tested for three variations of horizontal correlation lengths, with the first one being 46,0. This test provided a surprising result in the fact that the lowest and highest values concluded with the same optimal number of four investigations. This could be a result of the range of θ_z being to small to create large differences which leads to the test with low correlation lengths to not provide any profitable information from this point. A result of this is a problem in the models consideration of profit and safety, as some scenarios will prioritise safety although it is not profitable. The results from the three other tests does indicate a connection between this increase and reduction in investigations.

The problem of modelling this parameter is the uncertainties associated with it. There is currently no representative database of characteristic values for Norwegian soils on these parameters. They will also vary much depending on the geological processes that has occurred in the considered site.

9.4 Evaluating the Effect of Cost of Failure

The tests executed on failure costs reveal that increasing the cost of failure has a positive correlation with the optimal number of site investigations. This relationship can be considered logical as a higher consequence of failure necessitates a greater need for certainty in the results and a desire to mitigate the risk of failure. Surprisingly, the cost of failure does not appear to influence the selection of investigation type. It was initially expected that a lower cost of failure would lead to the preference for cheaper options in ground investigations as they would provide at least some information at a lower cost. This somewhat unexpected result could be due to the relatively small cost differences among the various ground investigation options, leading to the reduction in variation being the decisive factor.

When considering the different cost alternatives associated with failure, it is possible to calculate the minimum required reduction in the probability of failure needed to obtain an increase in value of information relative to its cost. Including an additional investigation leads to costs ranging from 10 000 NOK to 30 000 NOK. For a cost of failure (c_f) of 1 000 000 NOK, a decrease in the probability of failure of 0,01 to 0,03 is necessary. For c_f values of 5 000 000 NOK and 10 000 000 NOK, the required decrease in probability is 0,002 to 0,006 and 0,001 to 0,003.

The Monte Carlo Line Sampling algorithm employed in this study involves 1,000 simulations for Cone Penetration Testing (CPTU) and 100 simulations for point observations. This method is accurate for probabilities up to approximately 0,001. However, as the probabilities decrease to values lower than this, inaccuracies in the probability calculations may then impact the profitability assessment of an investigation. Consequently, the model's accuracy reduces with higher costs of failure. To fix this issue, certain measures can be taken, such as terminating the simulations when the reduction in the probability of failure reaches a minimum value.

Chapter 10

Conclusions and Recommendations

10.1 Conclusions

For this thesis a method of optimizing placement, type and number of ground investigations for an undrained slope stability problem was created and evaluated. The test results show that the optimal number of investigations is between 0 - 9 for a 15 meter tall and 40 meter wide slope, with a uniform material resembling clay or silty clay. The investigations mainly consists of 2 - 3 cone penetrations tests and a triaxial test, followed by a varied number of unconfined compression tests. The order of placement was seen to be placing CPTU in top, middle and then bottom of the slope. After these, laboratory tests are placed to fill the gaps between the prior investigations. Variation in cost of failure (c_f) and prior knowledge in the form of standard deviation and correlation lengths is proven to have a significant effect on the results of the created methodology.

Tests on cost of failure shows that it affects the optimal number of investigations, but they do not indicate a change in chosen investigations. Results show that for for c_f equal to 1 000 000 the chosen number of investigations varies between 0 - 3. For $c_f = 5\ 000\ 000$ the optimal number of investigations is in the range of 3 - 7, and for $c_f = 10\ 000\ 000$ it is between 5 - 9.

A standard deviation of 4, 8 and 15 for the undrained shear strength was tested, and resulted in corresponding optimal amounts of investigations being 0, 5 and 4. The test with the largest standard deviation had a smaller optimal amount than what the test with eight as standard deviation provided, but stagnated at a significantly larger probability of failure. This is likely a result of the investigations not being able to provide further learning when the standard deviation is this large. Larger standard deviation also resulted in the investigations being placed closer together.

A set of 18 variations of correlation lengths was tested, resulting in an optimal number between 2 - 7 investigations. The distance between test locations followed the corresponding correlation length, meaning that an increase in horizontal correlation length would lead to larger horizontal distance between investigations. This also led to fewer investigation being optimal with larger correlation lengths. Correlation lengths proved to have impact on the selection of investigation type as CPTU was preferred to a greater extent when the horizontal correlation length was large compared to the vertical correlation length. In the opposite case, unconfined compression tests were preferred.

The complexity of this process has led to some limitations to the methodology. The random field has only been modelled with the undrained shear strength as a random variable, while more parameters such as the soils unit weight would experience spatial variability. The data required to provide the necessary prior knowledge can prove difficult to obtain, as there is not much data on correlation lengths of soil. This parameter could also be site specific due to the local geological processes. The restriction in the process of creating correlated values for the random fields has led to the correlation lengths not being modelled as random variables. In addition to providing the undrained shear strength of soil, the investigation types considered can provide more advantageous information in geotechnical context. The model is restricted to an undrained slope stability analysis and the unit weight being a constant variable, which leads to no other benefits of the investigations being evaluated. The model decision criteria is based on profit, which is not always the most important aspect. There are scenarios where safety is considered more important, making a larger number of investigations than the model would recommend necessary.

In conclusion this model can be said to show a lot of potential but is limited by the demanding calculations of the Monte Carlo method and the simplicity of the direct method. The model provides a mathematical foundation for the selection of site investigations to assess an undrained slope stability problem, and the tests mostly provides results that could be considered reasonable. Further work is necessary for this model to provide a result for a more realistic case study, and recommendation to expand the modelled is described in the final section.

10.2 Recommendations for Further Work

There are many possible expansions to this methodology in its current state, and this section is written to provide a set of ideas for further development. The method is focused on a very specific geotechnical analysis and for it to be beneficial in a realistic setting it needs to be able to consider a wider range of problems. Recommendations for further work are presented in the following list.

- Analysis of the optimal number of samples, types, and locations can be extended to other types of geotechnical problems (e.g., bearing capacity of shallow and deep foundations). The soil and investigations can be modelled with the same procedure, while using different discretization for the soil domain and different methods of assessing safety of a geotechnical design.
- The computational performance of the implemented approach can be improved. This would lead to an increase the applications potential of the approach to a wider range of problems and improve its accessibility in planning and execution of geotechnial investigations in practice. One possibility is to implement an FOSM reliability method for linear reliability problems instead of the Monte Carlo method, as this will lead to less iterations and in that way less time consuming calculations. The differences in accuracy would have to be tested and compared for both methods, but in simple undrained slope stability problem the differences are not expected to be considerable.
- To obtain a more detailed model on the placements of ground investigations, a different stability calculation method could be implemented. A finite element program such as Plaxis, or a lamelle method might be an alternative.
- To create a model that can compare different soil investigation methods more accurately, the model can be expanded by modelling multiple parameters of a geo-

technical problem that need investigations as random variables (e.g., groundwater level, friction angle). This would also make more investigation methods relevant for this analysis such as the use of piezometer tests or more laboratory tests.

- To provide a more realistic representation of geotechnical problems, the soil stratigraphy model should be extended to 3D and account for layering of soil.

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Appendix A

Basis for the costs of ground investigations

Post nr.	Beskrivelse arbeid	Enhet	Mengde	Enhetspris	SUM
FELTARBEID					
1.	**FORBEREDENDE OG GENERELLE ARBEIDER**				
1.1	**Gravemelding og påvisning**				
1.11	Generelt	RS		25 000	0
1.12	Pr. borpunkt	stk		100	0
50					
1.2	**Varsling av grunneier**				
1.21	Generelt	RS		30 000	0
1.22	Pr. grunneier utsendelse av brev	stk		100	0
1.3	**Varsling - oppmerking og signaler til vanns**	RS	20 000		
1.4	**Midlertidig trafikkavvikling**				
1.41	Generelt, søknad om arbeidsvarsling	RS		25 000	0
1.42	Ekstern varsling, vakt etc etter medgåtte timer	time		1 500	0
1.5	**Hovedrigging**				
1.51	Tilrigging og nedrigging av borerigg på land	RS		50 000	0
1.52	Tillegg for tilrigging og nedrigging av fartøy/flåte	RS		160 000	0
1.53	Markrydding	time		980	0
1.54	Flytting mellom områder/boresteder > 1 km	stk		3 000	0
1.55	Tilrigging av kjerneboringsrigg/utstyr	stk		100 000	0
1.6	**Oppmåling**				
1.61	Generell tilrigging av oppmålingsutstyr	RS		5 000	0
1.62	Pr. borpunkt	stk		100	0
SUM HOVEDPOST 1 Forberedende og generelle arbeider					**0**
2.	**SONDERINGER**				
2.1	**Totalsondering**				
2.11	Forflytning, oppstilling og klargjøring på land pr. borpunkt	stk		7 500	0
2.12	Boring i løsmasser inntil 30 m	m		0	0
2.13	Boring i løsmasser for dybder større enn 30 m	m		1 000	0
2.14	Boring i berg	m		250	0
2.15	Tillegg til 2.11 ved avstand til vannkilder > 100 m, pr borpunkt	stk		1 500	0
2.16	Tap av borestang - totalsondering	stk		2 150	0
2.17	Tap av skjøtetapp - totalsondering	stk		698	0
2.18	Tap av borekrone - totalsondering	stk		1 480	0
2.2	**Dreietrykksondering**				
2.21	Forflytning, oppstilling og klargjøring på land pr. borpunkt	stk		5 000	0
2.22	Boring i løsmasser inntil 30 m	m		0	0
2.23	Boring i løsmasser fra 30 m	m		50	0
2.24	Tap av borestang - dreietrykksondering	stk		2 000	0
2.3	**Bergkontrollboring**				
2.31	Forflytning, oppstilling og klargjøring på land pr. borpunkt	stk		10 000	0
2.32	Boring i løsmasser inntil 30 m	m		0	0
2.33	Boring i løsmasser for dybder større enn 30 m	m		1 000	0
2.34	Boring i berg	m		1 000	0
2.35	Tillegg til 2.31 ved avstand til vannkilder > 100 m, pr borpunkt	stk		1 500	0
2.4	**Enkel sondering**				
2.41	Forflytning, oppstilling og klargjøring pr. borpunkt	stk		20 000	0
2.42	Sondering inntil 10 m	m		0	0
2.43	Sondering for dybder større enn 10 m	m		0	0
SUM HOVEDPOST 2 Sonderinger					**0**
3.	**IN SITU MÅLINGER**				
3.1	**Vingeboring**				
3.11	Forflytning og oppstilling pr. borpunkt	stk		10 000	0
3.12	Måling av uforstyrret og omrørt skjærstyrke pr. nivå inntil 10 m	stk		265	0
3.13	Måling av uforstyrret og omrørt skjærstyrke pr. nivå fra 10 m	stk		265	0
3.14	Penetrering i løsmasser (hele dybdeintervallet)	m		625	0
3.2	**Trykksondering**				
	Kvalitetsklasse angis:_________________________				
3.21	Forflytning, oppstilling og klargjøring pr borpunkt	stk		8 000	0
3.22	Trykksondering i løsmasser inntil 20 m	m		0	0
3.23	Trykksondering i løsmasser fra 20 m	m		100	0

Post nr.	Beskrivelse arbeid	Enhet	Mengde	Enhetspris	SUM
3.24	Poretrykksutjevning pr. måling	stk		450	0
3.25	Poretrykksutjevning pr. tidsenhet	time		450	0
3.26	Tillegg for måling av resistivitet, pr sondering (RCPTU)	stk		1 500	0
3.27	Tillegg for måling av skjærbølgehastighet pr. måling (SCPTU)	stk		1 500	0
3.291	Tap av spiss	stk		2 000	0
3.292	Tap av friksjonshylse	stk		2 000	0
3.393	Tap av overgang	stk		30 000	0
3.3	**Poretrykksmålinger**				
3.31	Forflytning, oppstilling og klargjøring pr. installasjon	stk		5 000	0
	(inkl. avlesning mens borelaget er på stedet)				
3.32	Levering av piezometer - elektrisk	stk		9 000	0
3.33	Tillegg for minne i sensor / lokal logging av målinger	stk		0	0
3.34	Levering av piezometer - hydraulisk	stk		10 000	0
3.35	Levering av standrør/spiss	stk		800	0
3.36	Levering av rør tilpasset til 3.32, 3.34 og 3.35	m		250	0
3.37	Nedpressing/installasjon av piezometer	m		780	0
3.38	Sikring av piezometer inkludert eventuell lås	stk		1 200	0
3.39	Tilleggsavlesninger pr. sensor pr. avlesing	stk		980	0
SUM HOVEDPOST 3 In situ målinger					**0**
4.	**PRØVETAKING (omrørte/forstyrrede prøver)**				
4.2	**Naverboring med maskin**				
4.21	Forflytning, oppstilling og klargjøring pr punkt	stk		1 775	0
4.22	Opptak av prøver inntil 5 m	stk		600	0
4.23	Opptak av prøver inntil 5 m - 10 m	stk		800	0
4.24	Opptak av prøver fra dybder større enn 10 m	m		1 000	0
4.25	Navering	m		450	0
4.26	Tap av naverbor/skovelbor	stk		1 750	0
4.3	**Alternativ prøvetaking av representative prøver**				
	Metode for prøvetaking skal spesifiseres: Moreneprøvetaker				
4.31	Forflytning, oppstilling og klargjøring pr punkt	stk		2 500	0
4.32	Opptak av prøver inntil 10 m	stk		1 500	0
4.33	Opptak av prøver fra 10 m - 20 m	stk		2 500	0
4.34	Opptak av prøver fra 20 m - 30 m	stk		3 500	0
4.35	Opptak av prøver fra 30 m - 40 m	stk		4 500	0
4.36	Penetrering i løsmasser (hele dybdeintervallet)	m		250	0
4.37	Tillegg for prøvetaking fra båt/flåte, pr. prøve	stk		1 000	0
SUM HOVEDPOST 4 Prøvetaking (omrørte/forstyrrede prøver)					**0**
5.1	**Ø54 mm prøvetaking**				
	Kvalitetsklasse angis:___________________________				
5.11	Forflytning, oppstilling og klargjøring på land pr. borpunkt	stk		2 500	0
5.12	Opptak av prøver inntil 10 m i løsmasser	stk		2 500	0
5.13	Opptak av prøver 10 - 20 m i løsmasser	stk		3 000	0
5.14	Opptak av prøver 20 - 30 m i løsmasser	stk		3 000	0
5.15	Opptak av prøver 30 - 40 m i løsmasser	stk		5 000	0
5.16	Penetrering i løsmasser (hele dybdeintervallet)	m		0	0
5.17	Tillegg for prøvetaking fra fartøy/flåte, pr. prøve	stk		1 000	0
5.18	Ventetid før opptak av prøver	time		0	0
5.19	Tap av prøvesylinder Ø54 mm	stk		1 000	0
5.2	**Ø72 - 76 mm prøvetaking**				
	Kvalitetsklasse angis:___________________________				
5.21	Forflytning, oppstilling og klargjøring på land pr. borpunkt	stk		3 500	0
5.22	Opptak av prøver inntil 10 m i løsmasser	stk		2 500	0
5.23	Opptak av prøver 10 - 20 m i løsmasser	stk		3 000	0
5.24	Opptak av prøver 20 - 30 m i løsmasser	stk		6 000	0
5.25	Opptak av prøver 30 - 40 m i løsmasser	stk		6 500	0
5.26	Penetrering i løsmasser (hele dybdeintervallet)	m		500	0
5.27	Tillegg for prøvetaking fra fartøy/flåte, pr. prøve	stk		1 000	0
5.28	Ventetid før opptak av prøver	time		2 000	0
5.19	Tap av prøvesylinder Ø72-Ø76 mm	stk		1 600	0
SUM HOVEDPOST 5 Prøvetaking (uforstyrrede prøver)					**0**
6.	**TILLEGGSARBEIDER OG SPESIELLE FELTFORSØK**				
6.1	**Forboring** (ved boremetoder, post 2 - 5)				
6.11	Forflytning, oppstilling og klargjøring pr. borpunkt	stk		100	0

Post nr.	Beskrivelse arbeid	Enhet	Mengde	Enhetspris	SUM
6.12	Forboring (med naver, sonderingsstenger etc.)	m		500	0
6.2	**Boring av foringsrør** (ved boremetoder, post 2 - 5)				
6.21	Rigging av utstyr til boring med foringsrør, kun ved bruk av brønnboringsrig	RS		10 000	
6.22	Forflytning, oppstilling og klargjøring pr borpunkt	stk		5 200	0
6.23	Boring av foringsrør (inkluderer ikke tapte rør)	m		1 500	0
6.24	Tap/forbruk av foringsrør	m		1 800	0
6.3	**Tillegg ved sjøboring** (ved boremetoder, post 2 - 5)	Uaktuelt			
6.31	Tillegg i oppstilling ved sjøboring, pr. borpunkt	stk			0
6.32	Montering av foringsrør og "boring" i vann, d < 10m	m			0
6.33	Montering av foringsrør og "boring" i vann, d = 10 - 20 m	m			0
6.34	Montering av foringsrør og "boring" i vann, d = 20 - 30 m	m			0
6.35	Montering av foringsrør og "boring" i vann, d = 30 - 40 m	m			0
6.36	Montering av foringsrør og "boring" i vann, d > 40 m	m			0
6.4	**Ulendt terreng pr borpunkt**	stk		7 500	0
6.5	**Transport av prøver for å bevare kvaliteten av prøvene, pr. prøveserie**	Stk		500	0
6.6	**Ventetid for mannskap/utstyr**				
6.61	Ved landboring	time		3 000	0
6.62	Ved sjøboring, også landligge pga dårlig vær	time		5 000	0
6.63	Sikkerhetsvakt tog	time		1 000	0
6.64	Natt tillegg pr. pers	time		500	0
6.7	**Lapping av borhull med kaldasfalt**	stk		1 240	0
SUM HOVEDPOST 6 Tilleggsarbeider og spesielle feltforsøk					**0**
Sum feltarbeid, hovedposter 1 - 6					**0**
	LABORATORIEARBEID (inkl. opptegning/presentasjon)				
10	**KLASSIFISERING**				
10.1	**Jordartsklassifisering av poseprøver**				
	Rutineundersøkelse i henhold til beskrivelsen				
10.11	Pr. prøve	stk		700	0
10.12	Lagring < 3 mnd	stk		100	0
10.2	**Vanninnhold pr. forsøk**	stk		70	0
10.3	**Densitet for jord pr. forsøk**	stk		500	0
10.4	**Korndensitet pr. forsøk**	stk		720	0
10.5	**Konsistensgrenser, flytegrense/plastisitetsgrense**	stk		1 200	0
10.6	**Kornfordelingsanalyser**				
10.61	Tørrsikting > 0,075 mm (0,063 mm)	stk		1 200	0
10.62	Våtsikting > 0,075 mm (0,063 mm)	stk		2 500	0
10.63	Sedimentasjonsanalyse pr. forsøk	stk		2 000	0
10.64	Kombianalyse pr. forsøk	stk		3 200	0
10.7	**Humusinnhold ved glødetap pr. forsøk**	stk		750	0
10.8	**Max/min densitet av sand**	stk		500	0
10.9	**Fotografi av prøve**	stk		100	0
	Fotografi av prøve - alle bilder levert uredigert	stk		1 000	0
10.10	**Konusforsøk på omrørt prøvemateriale**	stk		250	0
SUM HOVEDPOST 10 Klassifisering					**0**
11	**RUTINEUNDERSØKELSER AV PRØVESYLINDRE**				
11.1	**f54, 75 og 95 mm prøver av leire/leirholdige materialer**				
	Rutineundersøkelse i henhold til beskrivelsen				
11.11	Pr. sylinder Ø54 mm	stk		1 600	0
11.12	Pr. sylinder Ø72-76 mm	stk		2 300	0
11.13	Pr blokkprøve, oppdelling og rutine på ett av to nivåer i blokken	stk		4 500	0
11.14	Lagring 3-6 mnd	stk		1 500	0
11.2	**Prøver av sand og siltmaterialer**				
	Rutineundersøkelse i henhold til beskrivelsen				
11.21	Pr. sylinder Ø54 mm	stk		1 600	0
11.22	Pr. sylinder Ø72-76 mm	stk		1 550	0
11.23	Lagring 3-6 mnd	stk		1 500	0

Post nr.	Beskrivelse arbeid	Enhet	Mengde	Enhetspris	SUM
SUM HOVEDPOST 11 Rutineundersøkelser av prøvesylindre					**0**
12	**Kalk/sement innblandingsforsøk**				
12.11	Innblanding av kalk/sement	stk		600	0
12.12	Enaksial test på kalk/sement innblandet leire	stk		400	0
SUM HOVEDPOST 12 Kalk/sement innblandingsforsøk					**0**
13	**TREAKSIALFORSØK**				
13.1	**Statisk konsolidert udrenert treaksialforsøk**				
13.11	Pr. forsøk	stk		6 820	0
13.12	Tillegg for vanskelige/bløte prøver	stk		940	0
13.13	Tillegg for trimming av sidene	stk		940	0
13.14	Tillegg for innstamping av sandprøver	stk		940	0
13.15	Tillegg for måling av Gmax	stk		1 210	0
13.16	Tillegg for permeabilitetsmåling under forsøk	stk		1 200	0
SUM HOVEDPOST 13 Treaksialforsøk					**0**
14	**DIREKTE SKJÆRFORSØK**				
14.1	**Statisk direkte skjærforsøk**				
14.11	Pr. forsøk	stk			0
14.12	Tillegg for direkte innbygging fra sylinder	stk			0
14.13	Tillegg for innstamping av sandprøver	stk			0
SUM HOVEDPOST 14 Direkte skjærforsøk					**0**
15	**ØDOMETERFORSØK**				
15.1	**Trinnvis belastning**				
15.11	Pr. forsøk	stk		4 500	0
15.12	Tillegg for av-/rebelastning	stk		1 500	0
15.13	Tillegg for innbygging fra sylinder - bløt leire etc.	stk		940	0
15.14	Tillegg for permeabilitetsmåling under forsøk	stk		1 200	0
15.2	**Kontinuerlig belastning CRS/CPR-prosedyre**				
15.21	Pr. forsøk	stk		6 000	0
15.22	Tillegg for av-/rebelastning	stk		2 000	0
15.23	Tillegg for innbygging fra sylinder - bløt leire etc	stk		1 000	0
15.24	Tillegg for permeabilitetsmåling under forsøk	stk		1 200	0
15.25	Tillegg for kryptrinn	døgn		2 500	0
SUM HOVEDPOST 15 Ødometerforsøk					**0**
Sum laboratoriearbeid, hovedposter 10 - 15					**0**
C - DATARAPPORTERING, ADMINISTRASJON OG OPPFØLGING					
20.	**DATARAPPORTERING**				
20.1	**Datarapport**				
20.11	Rapportering, fastprisdel	RS		**90 000**	
			Grunnlag	Prosentsats	
20.12	Variabel del - prosentandel av feltundersøkelsene, post 2 - 6.4	%		15 %	0
20.13	Variabel del - prosentandel av laboratorieundersøkelsene, post 10-15	%	0	10 %	0
20.131	Opptegninger av borfiler. NB! kun når det ikke leveres rapport	stk		1 500	0
20.132	PRV-fil lab	stk		1 500	0
SUM HOVEDPOST 20 Datarapportering					**0**
30.	**ADMINISTRASJON/OPPFØLGING - FELT/LAB.**				
30.1	Utarbeidelse av boreplan/kartgrunnlag	time		1 480	0
30.2	Teknisk støttepersonell	time		1 050	0
30.3	Administrasjon og oppfølging av grunnundersøkelser	time		1 480	0
30.4	Geotekniker/Geolog/Miljøgeolog	time		1 570	0
SUM HOVEDPOST 30 Administrasjon/oppfølging					**0**
TOTALSUM					**0**

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