NTNU

Kunnskap for en bedre verden

Department of Electric Power Engineering

TET4510 - Specialization Project

# Transmission Expansion Planning

*Authors:*
Written by Daniel Holt
Supervised by Salman Zafersnlouei

December 22, 2022

**Abstract**

Transmission expansion planning is an important aspect of the electrical power system, as it involves identifying and implementing the necessary infrastructure to meet the increasing demand for electricity. In this project, a literature study was conducted to review the principles of Transmission Expansion Planning. These principles were then tested using a five-node example with mixed integer linear programming, specifically using the Available Transmission Capacity method. The results and experience of this study serve as a prestudy for a master thesis, which will explore the use of DC power flow as a tool for conducting Transmission Expansion Planning. The aim of the master thesis is to evaluate the effectiveness of using DC power flow as a method for Transmission Expansion Planning, and to identify any potential advantages or limitations of this approach.

# Table of Contents

# List of Figures



# List of Tables

# 1 Introduction

## 1.1 Motivation

The pace of the transition to a zero-emission society has increased in the last couple of years. The total power consumption in Norway in 2021 was 139.5 $TWh$, the highest consumption level registered in Norway [1]. This is a number that will increase significantly over the next 30 years. The Norwegian Transmission System Operator (TSO), Statnett, expect the demand to increase to 220 $TWh$ in 2050. Hence, they have established a new strategy to match the pace of the energy transition [2]. Statnett's main task is to keep the frequency at $50Hz$ in the grid to maintain a stable transmission system that will benefit the socioeconomic.

Transmission expansion planning (TEP) is an essential tool to reach the energy sector's decarbonization target. The European Commission (EC) has outlined a target to reduce greenhouse gas (GHG) emissions by 80-95 % by 2050 compared to the 1990 emissions in order to keep climate change below 2 $°C$ [3]. The energy sector is responsible for more than 75 % of the EU's GHG emissions [4]. This implies a big transition for both the energy producers and the energy consumers in the coming years. The Renewable Energy Directive 2018/2001/EU has established ambitious targets for the EU to reach an energy mix consisting of at least 32 % renewable energy sources (RES), with a clause for possible upwards revision by 2030 [4].

With the increasing penetration of RES in the energy mix, especially offshore wind production, which is located far away from the load centers, it is essential to have a well-planned transmission system in order to reach an efficient operation of the RES. Furthermore, it is vital to have interconnections between national borders that can guarantee efficient transportation and exchange of power, such that the wind blowing in one spot can provide power to areas with deficits [5]. Studies conducted by *European Network of Transmission System Operators for electricity (ENTSOE)* conclude that both international and national grid expansion is essential; due to the increasing volatility in electricity prices and power system balance that comes with the implementation of intermittent RES in the European power system [6][7].

However, several challenges and problems can arise with transmission expansion of the power system. One major issue is the high cost and complexity of constructing new transmission lines and expanding existing ones. This can be a significant financial and logistical challenge, especially in densely populated or environmentally sensitive areas. In addition, there can be challenges associated with obtaining the necessary permits and approvals for transmission expansion projects, as well as opposition from local communities or other stakeholders [2]. Therefore, TEP is strongly needed to develop and implement strategies to upgrade the power system efficiently, especially when considering the presence of uncertainties and the long time scale involved.

The time scale for expansion projects depends on many factors mentioned above. However, an expansion of a long transmission line is estimated to take 10-20 years from submission to implementation [8]. This time scale does not align with the time scale in RES constructions, where RES required 1-3 years to be constructed [9]. This leads to the challenge of accurately predicting the future electricity demand, which can be difficult due to unpredictable factors such as population growth, economic conditions changes, and energy consumption patterns. Additionally, the long timescale of TEP means that any decisions made today must be able to adapt to changing conditions over the next several decades. This requires a high degree of flexibility and the ability to anticipate and respond to future developments in the energy sector. Finally, TEP's high cost and technical complexity require careful planning and coordination with a wide range of stakeholders, including utilities, regulators, and public members.

The social welfare, or the well-being of a society, can be influenced by bottlenecks in the transmission system. These bottlenecks can cause disruptions in the flow of electricity, leading to blackouts and other reliability issues. This can negatively impact individuals and businesses, including loss of income, damage to equipment, and inconvenience. In addition, bottlenecks in the transmission system can also result in higher electricity prices, which can harm the economy as a whole. Transmission expansion planning can help to increase social welfare by addressing these bottlenecks and improving the reliability and efficiency of the transmission system. By expanding the capacity of the transmission grid, more electricity can be transmitted over longer distances, allowing for more efficient use of energy resources and increased access to electricity. This can benefit individuals, businesses, and the economy, leading to improved social welfare.

### 1.2 Project goal

This project will assess literature and methods related to performing TEP and will serve as a pre-study for the master thesis. The thesis will evaluate the performance of expansion planning tools employing direct current (DC) power flow in long transmission lines. Through this project, we aim to learn about the concepts of TEP and understand the importance of conducting such planning in order to maximize social welfare. This project will provide a foundation for the master thesis by familiarizing the relevant literature and methods in the field of TEP. A thorough review of existing literature and methods will provide a strong foundation for conducting a comprehensive evaluation of expansion planning tools that employ DC power flow in long transmission lines in the master thesis.

### 1.3 Research question

In this work, the aim is to see how expansion planning can reduce electricity prices and the operational cost in a power system. This will be conducted by executing an example using transmission expansion planning (TEP) with mixed integer linear programming.

### 1.4 Project structure

Chapter 2 is a literature review of the literature that was used as an inspiration for the project. Chapters 3, 4, and 5 briefly introduce the theory of a power system, the Nordic power market, and transmission expansion planning. Chapter 6 explains the methodology used to solve an example of transmission expansion planning. Chapter 7 presents and discusses the results obtained from the node example introduced in the methodology. Chapter 8 is the conclusion of the project and outlines suggestions for future work.

## 2 Literature review

The literature review for this project has been essential in gaining an understanding of transmission expansion planning. It provides an overview of current research and practices in the field. It helps identify successful methods and approaches used in previous studies and any limitations related to the topic. In conducting the literature review, several research articles on transmission expansion planning were examined. Two articles, in particular, stood out and served as inspiration and motivation for this project. These articles will be discussed in this section.

### 2.1 Nordic hydropower flexibility and transmission expansion to support integration of North European Wind Power

In the specialization course, Electricity markets and energy system planning (TET4565), one of the projects was to conduct a literature study on a relevant topic regarding the specification project. Hence, the research article "Nordic hydropower flexibility and transmission expansion to support integration of North European Wind Power" written by H. Farahmand, S. Jaehnert, T. Aigner, and D. Huertas-Hernando was assessed [10].

This article assesses the challenges that follow with increased offshore wind production in the north and Baltic seas. The integration of wind power will increase the variability of the generation portfolio in North Europe. Sufficient access to flexible power production, such as hydro power, is necessary to maintain balance in the power system. The transmission system is pivotal in harvesting the available wind power located far away from the loads and enabling optimal use of flexible hydropower. Hence, it has been conducted techno-economical analysis in the form of a simulation of three different offshore grid structures with three different onshore grid constraints applied in the proposed structures.

The case study conducted is the projected state of the European power system in 2030. The assumptions for the baseline scenario in the paper are divided into four main assumptions, which deal with the generation mix in 2030, scenario-based wind power production, increase in installed hydropower capacity, and expected grid expansion in 2030. The methodology used in the analysis includes two interrelated models - a market model (EMPS) and a flow-based model (DCOPF). The two models are coupled by first using the market model to obtain variables that will be used as input in the flow-based model to simulate the power flows in the grid and compute the optimal generation dispatch. The methodology is used to solve the case study, consisting of three different cases with different grid topologies presented in Figure 1.

• **Case A:** Original offshore grid, without any connection between Æsgir offshore wind farm in Norway and the other parts of the grid.
• **Case B:** Transmission expansion between Æsgir offshore wind farm and Eemshaven seaport in Netherlands
• **Case C:** Transmission expansion between Æsgir offshore wind farm and Gaia offshore wind farm in Germany.

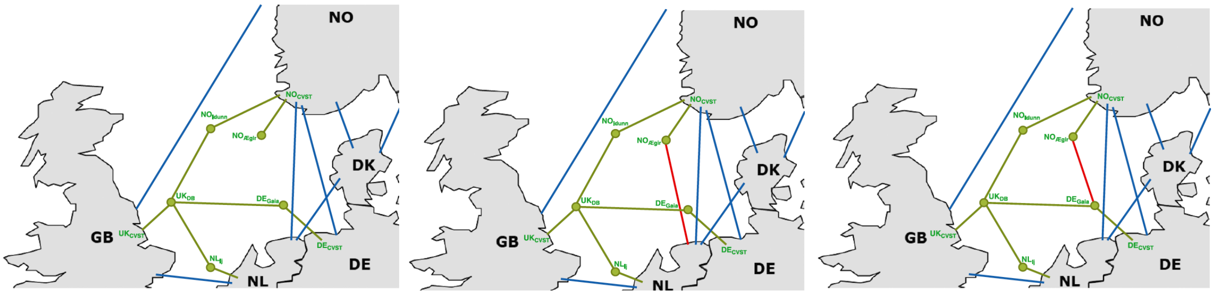


Figure 1: Case A, Case B, Case C in respective order [10]

In order to study the three cases, different scenarios are added to each of the cases. The three scenarios are: No Constraint (NC), Internal Constraint (IC), and Internal Constraint with Expansion (ICE). The purpose is to study the impact of adding the onshore grid constraints on the proposed offshore grid alternatives. Each combination's operation and investment cost is investigated to find the most beneficial alternative. By conducting the analysis, the article concludes that the operational cost in all scenarios decreased with the grid expansion, and the operational savings was positive due to the grid investment.

## 2.2 A framework to determine optimal offshore grid structures for wind power integration and power exchange

The second research article that stood out in the literature review was "A framework to determine optimal offshore grid structures for wind power integration and power exchange", written by T.Tröchter and M.Korpås [11].

This article assesses a framework for finding optimal offshore grid expansions due to the large amount of offshore wind power set to be implemented in the power system. The proposed method involves using a transportation model of the power grid and extending the standard mixed-integer linear programming approach for solving transmission expansion planning problems. This approach considers wind power generation and load fluctuations, making it particularly useful for identifying optimal transnational offshore high-voltage direct current (HVDC) grid structures for integrating large amounts of offshore wind power. The effectiveness of the proposed method is demonstrated through a case study of the North Sea region.

The study in the article utilizes a transportation model that does not include physical constraints. In contrast, the first article uses a linearized power flow equation, which considers physical constraints and better describes the alternating current network. However, the transportation model is considered to be the more appropriate for analyzing a HVDC grid.

An example of the North Sea area in 2030 is presented as a test model to demonstrate the methodology's applicability. The model includes six price areas in the North Sea region: Norway, Denmark, Germany, the Netherlands, Belgium, and the UK. Moreover, the model simulates the system with 400 randomly selected states for wind production to show how well the resulting grid performed. The grid performance was measured by comparing the investment costs with the operational savings of the expanded system.

# 3 How the power system works

The power system can be divided into three main sections: production, transmission, and consumption, which again can be divided into many subsections. However, the aim is to provide the fundamentals of the electrical power system. The production section refers to electricity generation, which can be done through various means, such as coal-fired power plants, hydroelectric dams, or renewable energy sources, such as solar and wind. The transmission section refers to the transportation of electricity from the production plants to the areas where it is needed, typically through a network of high-voltage power lines. Finally, the consumption section refers to the use of electricity by end-users such as households, businesses, and industrial facilities. These three sections are interconnected and rely on each other to ensure a reliable and stable power supply for a given region.

## 3.1 Production

Power production refers to the generation of electricity to meet the energy demand of a particular area. The energy mix, or the portfolio of energy sources used to produce electricity, is an essential factor in power production. In 2021, the energy mix in Europe was heavily reliant on fossil fuels, including coal, gas, and oil, which made up 76% of the energy mix [12]. While these sources are known for their low cost and reliability, they also contribute significantly to GHG emissions. As the cost of $CO_2$ increases, there is a growing push to shift towards cleaner energy sources, such as renewables, like wind and solar, to reduce emissions and address the global climate crisis [13]. However, one challenge with the transition to intermittent power units is that they are not always available and cannot always meet the electricity demand. This means there must be a system to store excess energy or quickly bring other power sources online when demand exceeds the output of intermittent power units. This can be a complex and costly process, but it is necessary to reduce the reliance on fossil fuels and address the urgent need to reduce GHG emissions.

## 3.2 Transmission

The transmission system is responsible for transmitting electrical energy from the power generation sources to the distribution system, which delivers the electricity to end users. The transmission system typically consists of high-voltage transmission lines, towers, transformers, and substations. The high-voltage transmission lines are used to transmit the electricity over long distances, while the transformers step down the voltage to a suitable level for distribution to end users. The substations are facilities where the voltage is transformed from high to low or vice versa. The electricity is either distributed to the distribution system or transmitted to other transmission lines. Overall, the transmission system ensures that electricity is delivered reliably and efficiently to end users.

Lately, there has been much focus on the terms *grid congestion*, and *bottlenecks* in the transmission system due to the high electricity prices in Norway [14][15]. Grid congestion and bottlenecks refer to situations in which the transmission grid cannot effectively transmit electricity from one region to another due to limitations in the infrastructure. In Norway, grid congestion and bottlenecks can occur due to various factors, such as outdated transmission infrastructure, high electricity demand, and the integration of renewable energy sources. These issues can lead to problems such as reduced transmission capacity, increased transmission losses, and higher electricity prices. To address grid congestion and bottlenecks, Statnett has implemented several measures, including expanding and upgrading transmission infrastructure, using advanced control systems and market mechanisms, and developing new transmission technologies. These efforts aim to ensure that the Norwegian transmission grid can effectively and efficiently transmit electricity to meet the needs of consumers and support the transition to a low-carbon energy system [2].

## 3.3 Consumption

Consumers are the end-users of the power produced by the producers and transferred by the transmission/distribution system operators. The end-users are everyone or everything that requires power to function, e.g., households and industries. The electricity demand has been increasing in recent years due to various factors, including population growth, economic development, and the increasing electrification of various sectors [2]. Electrification refers to replacing traditional energy sources, such as fossil fuels, with electricity to power vehicles, buildings, and industrial processes [16]. This trend has been driven by the concern for climate change and the need for a more sustainable energy system. As electrification becomes more widespread, the electricity demand is expected to rise. Meeting this demand while maintaining the reliability and affordability of the electricity supply will be a challenge for power utilities and policymakers.

# 4 Power market description

The power market in Europe is a dynamic and rapidly changing industry with a diverse mix of generation sources and complex regulatory frameworks. Many European countries have adopted policies to support the transition to renewable energy, resulting in a significant increase in wind and solar power use. As a result, power prices in Europe have been volatile, with significant fluctuations in response to changes in supply and demand [2]. Overall, the power market in Europe is transforming, with the emergence of new technologies and business models, as well as increased competition and regulatory uncertainty [17].

Understanding the concept of market equilibrium is essential to simulate the power market and the impact of transmission expansion planning. Market equilibrium is a state in which the supply of electricity and its demand is balanced, which leads to one market price [18]. In an electrical power market, this occurs when the quantity of electricity generated by power plants is equal to the quantity demanded by consumers, and the price of electricity reflects the marginal cost of producing it. In order to simulate a power market and evaluate transmission expansion plans, it is necessary to model the interactions between generators, consumers, and regulators and to consider how these interactions affect the supply and demand for electricity. Making precise models can help identify potential imbalances in the market and evaluate the potential impacts of different transmission expansion scenarios.

## 4.1 Nord Pool

*Nord pool* is a pooling institution that conducts pool trading, also known as central trading. Central trading is a place where all generation bids and consumption offers are placed at the same time. No one knows about others' bids or offers, which makes it anonymous [19]. Nord pool offers market participants different markets to attend. However, day-ahead trading in the elspot market and intraday trading in the elbas market is the most common [20]. Nord pool is owned by Euronext

(66%) and TSO Holding (34%) [21]. The Nord Pool markets are divided into several bidding areas due to varying available transmission capacities and congestion between the bidding areas. Therefore, different area prices are established [22]. The participants in Nord Pool consist of the Nordic countries, the Baltic countries, and some countries in central Europe. The participating countries may be divided into more areas; for instance, Norway is divided into five areas; NO1, NO2, NO3, NO4, and NO5, and Denmark is divided into two areas; DK1 and DK2.

## 4.2 Market clearing

The electricity market is a platform for buyers and sellers to negotiate and agree on electricity transactions. A central market-clearing algorithm, which plays a crucial role in the day-ahead market (a significant portion of the electricity market), is used to maximize social welfare by determining which bids and offers will be accepted. The day-ahead market operates by allowing sellers to bid on the amount of power they can produce at a given price for each hour of the day and allowing buyers to offer a quantity of electricity they need at a given price for each hour of the day. These contracts are then settled the day before, and the deadline for submitting orders is 12:00 CET. The submitted orders will then be matched, so the balance between supply and demand for each hour of the next day is balanced. This will be announced at 12:45 CET, the same day as the clearing process [23].

The cleared market price is called the *system price* and reflects the market equilibrium in a market with no grid constraints included. Thus, the system price is used as a reference for trading in the electricity market [20]. However, in reality, one must consider the grid constraint, which will affect the spot price in each area due to grid congestion. Hence, there will be a unique equilibrium between supply and demand in each area, which depends on the transmission capacity between surpluses and areas with deficits.

## 4.3 Optimal power flow

Optimal power flow (OPF) is an important optimization problem in the operation of an electrical power system. It involves finding the optimal operating point of the system in terms of the power flows through transmission lines and the generation and consumption of electricity. The goal of OPF is to minimize the overall cost of operating the system, while maintaining the necessary level of production in order to balance the demand. This can be achieved by adjusting the power flows and generation/consumption levels to minimize the use of expensive resources, such as high-cost generators, and maximize the use of cheaper resources, such as renewable energy sources. OPF is typically solved using mathematical optimization algorithms, considering various constraints and objectives, such as transmission capacity limits, voltage limits, fuel, and emission costs. The solution to the OPF problem is critical for the efficient and reliable operation of the power system and has significant impacts on the cost of electricity for consumers [18].

There are various approaches for modeling the power system to find the optimal operating configuration. The complexity of these models can vary based on the assumptions made. Models that consider more assumptions may be less accurate, however, sufficient for certain analysis goals. Ultimately, the choice of modeling method will depend on the purpose of the analysis and the trade-off between precision and simplicity.

### 4.3.1 Available transfer capacity

The Available Transfer Capacity (ATC) is a transportation model (non-electric model) used by Nord Pool and TSOs to calculate power flows between two interconnected areas. It represents the remaining transfer capacity after taking into account the already occupied portion (NTF) of the Net Transfer Capacity (NTC). The NTC is calculated by subtracting the Transmission Reliability Margin (TRM) from the Total Transfer Capacity (TTC). The TTC is the maximum transmission capacity of an interconnector, which is limited by factors such as thermal limits, voltage limits,

and stability limits. The TRM accounts for uncertainties in the power flow forecast. All these factors are calculated by the TSOs [24][25].
Thus, ATC can be expressed as shown below:

$$NTC = TTC - TRM \tag{1}$$

$$ATC = NTC - NTF \tag{2}$$

The ATC approach in an optimization model has fewer constraints than an electrical model, resulting in more equal prices in a system [26]. An ATC approach may be formulated as shown below, which will be discussed later in the project.

$$\begin{aligned} \min \quad & f(P) \\ \text{subject to:} \quad & P - D = I \times FL \\ & FL_{min} \leq FL \leq FL_{max} \\ & P_{min} \leq P \leq P_{max} \end{aligned} \tag{3}$$

### 4.3.2 DC optimal power flow

An AC model accurately represents the grid as an AC grid, but it requires extensive calculations and multiple iterations to solve. Additionally, an AC optimal power flow model is a non-convex linear problem, meaning linear programming cannot solve it. However, DC power flow (DCPF) is a simplified way to solve a load flow problem where no iterations are required. This is because DCPF describes an AC grid with DC-like approximation [18]. The approximations that are made, make DCPF into a linear model, which is well suited to be solved with linear programming [26].

In DCPF, one only considers the active power flow, presented in Equation 4. $Y_{ij}$ is the admittance matrix in the system, which describes the admittance in the line between two nodes. At the same time, $\delta$ and $\theta$ is the angle associated with the voltage and the admittance, respectively. Note that the admittance matrix can be formulated differently depending on the distance of the transmission line. This theory will not be discussed in this work but is essential when employing DCOPF as a transmission expansion tool in the master's thesis.

$$P_i = |V_i| \sum |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{4}$$

Three assumptions make it possible to describe an AC grid with DC-like approximations. The proof is described below and is obtained from the syllabus in TET4115 - Power System Analysis [27]:

1. All voltage magnitudes are close to 1 pu.
2. Transmission line resistances are much smaller than reactances.
3. Voltage phase angle differences across the lines are small (less than 10°).

Assumption 1 gives us:

$$P_i = \sum |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j), \quad \text{since} \quad |V_i| = |V_j| = 1 \tag{5}$$

Assumption 2 gives us:

$$|Y_{ij}| = b_{ij}; \quad \theta_{ij} = 90° \tag{6}$$

Combining (5) and (6) we obtain:

$$P_i = \sum b_{ij} \sin(\delta_i - \delta_j), \quad \text{with help from trigonometric identities} \tag{7}$$

Finally we obtain a simplified expression for active power with assumption 3. As $\delta_{ij} < 10°$, we can assume $\sin(\delta_{ij}) = \delta_{ij}$.

$$P_i = \sum b_{ij}(\delta_i - \delta_j), \quad \text{where} \quad i \neq j \tag{8}$$

The non-linear power flow equation (4) is now transformed into the linear power flow equation (8) only by applying the three assumptions. Thus, it is possible to conduct linear programming with the DCPF method. This is called DC optimal power flow (DCOPF) and may be formulated as shown below:

$$\begin{aligned} \min \quad & f(P) \\ \text{subject to:} \quad & g(U,P) = 0 \\ & h(U) \leq 0 \\ & P - P_{max} \leq 0 \end{aligned} \tag{9}$$

Where $f(P)$ is the cost function for the generation, $g(U,P)$ is the energy balance in the system, $h(U)$ is the line capacity constraint, and the last constraint is the generation capacity constraint. The advantage of DCOPF is that the constraint represents the physical limits in the system, which makes the calculations more precise [26].

# 5 Transmission expansion planning

Transmission expansion planning in electrical power systems involves identifying the optimal configuration of transmission lines and other infrastructure to meet the current and future demand for electricity in a given region. This process typically involves analyzing data on existing transmission lines, projected electricity demand, and potential generation sources and using mathematical optimization techniques to determine the most cost-effective and efficient expansions [10][11]. This work uses mixed integer programming to conduct transmission expansion planning and cost-benefit analysis to evaluate if the investment is favorable.

## 5.1 MILP

Mixed Integer Programming (MILP) is a mathematical optimization technique that involves finding the optimal solution to a problem with a combination of continuous and discrete variables. The goal of MILP is to maximize or minimize an objective function subject to a set of constraints. These constraints may include limits on the values of the variables, as well as relationships between the variables. MILP solvers use algorithms such as branch and bound or cutting planes to search for the optimal solution to a problem. In addition to continuous and discrete variables, MILP can also handle logical constraints, which can be used to represent complex real-world situations. Overall, MILP is a powerful tool for optimization and decision-making in a wide range of applications [11] [28].

## 5.2 Cost benefit analysis

Cost-benefit analysis (CBA) is a method used to evaluate the economic feasibility of a project or decision by comparing the costs incurred to the benefits gained. It involves identifying all the costs and benefits associated with a project and estimating their monetary values. The costs can include the initial investment, operational expenses, and ongoing maintenance costs. The benefits can include direct financial benefits, such as revenues or savings, and indirect benefits, such as environmental or social impacts. CBA allows decision-makers to compare the costs and benefits of different options and choose the one that provides the most significant net benefit (benefits minus costs). CBA is a helpful tool for making informed decisions, as it allows for a systematic and quantitative comparison of different options. It is important to note that CBA can be subjective, as it relies on estimates and assumptions about the costs and benefits of a project [29].

# 6 Methodology

## 6.1 Nodal example

In this section, we will delve into the process of conducting a TEP study using Mixed Integer Programming (MILP). To illustrate the concepts and principles of TEP, we will use a nodal example based on the internal transmission grid in Norway, which is divided into five price areas: NO4, NO3, NO2, NO1, and NO5. This is also illustrated in Figure 2. It is important to note that the problem presented in this example is purely hypothetical and contains many assumptions that are not based on the actual transmission grid. The assumptions will be discussed in section 6.4. Despite this, the example is still helpful for demonstrating the principles of TEP and MILP.

In this example, Python and the optimization language Pyomo are utilized to conduct the TEP study rather than relying on commercial software. This decision was made to illustrate the principles of TEP and MILP without using proprietary tools. Using open-source tools such as Python and Pyomo allows us to control the optimization process and gain a deeper understanding of the solution's development. Later in this section, a detailed explanation of the process of using Python and Pyomo to solve TEP problems will be provided.

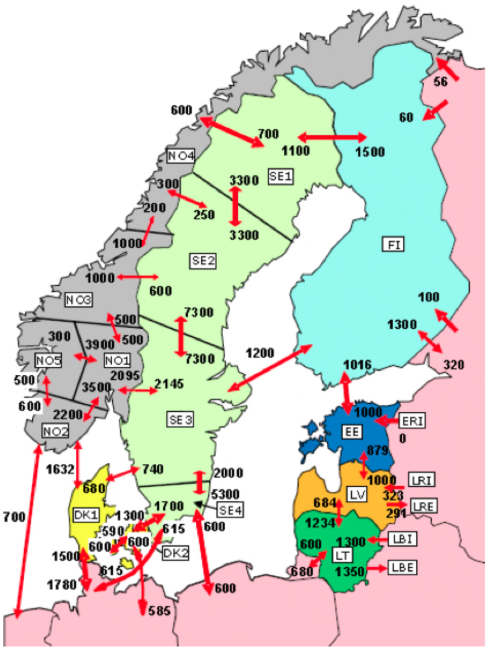


Figure 2: Illustration of NTC between the five price areas in the Nordic and Baltic grid, entsoe(2014)[30]

## 6.2 Market clearing with ATC approach

The theoretical principles of market clearing are described in section 4.2, and a mathematical model for the nodal example is described below, based on the theory. When constructing the mathematical formulation for the optimization problem, a declaration of sets and indices, parameters, variables, objective function, and constraints must be made based on the given information. There are, in total, three sets N, G, and L, with three corresponding indices n, g, and l. This makes it easier to differentiate the nodes, generators, and lines in the problem, in addition to making the mathematical description shorter.

The parameters are fixed in the model and are used to describe information that is decided outside the model, while the decision variables are decided based on the constraints and objective function. $P_{n,g}^{max}$ describes the max generation capacity for generator g at node n and is used to constrain the decision variable $p_{n,g}$. The parameter for demand is defined as the total demand at node n, $D_n$. $c_{n,g}$ is the parameter for the cost related to the corresponding generators in each node, while $c^{shed}$ is the cost associated with the decision variable for the load shedding in node n, $p_n^{shed}$. Load shedding is the practice of interrupting the supply of electricity to specific areas to reduce the demand on the power grid and prevent it from overloading and collapsing. In other words, it is used to balance the supply and demand of electricity on the grid when the power capacity is low or if there are bottlenecks in the transmission system.

The parameters for max and min flow in a transmission line are $FL_l^{max}$ and $FL_l^{min}$, where the max flow is the flow in the line that goes from node $i$ to $j$ while the min flow is interpreted as the reverse flow which is defined as the flow from node $j$ to node $i$. These parameters are used to constrain the decision variable for the flow between the nodes, $fl_l$. Lastly, the connection matrix, $I_{n,l}$, is a matrix that describes the connection between the nodes with 1, -1, and 0. The matrix is defined as $[N_{nodes} \times L_{lines}]$. Equation 23 is the connectivity matrix in the node example, where 1 represents the connection from node $i$ to $j$, -1 represents the reverse connection from node $j$ to $i$ and 0 when there is no connection between nodes.

The aim of the objective function (10) is to minimize production costs by finding the optimal dispatch for production, shedding, and flow between the nodes while satisfying the constraints (11)-(14). Constraint (11) is an equality constraint, also known as the energy balance in the system, which means the power production and demand must be perfectly balanced at any time for all nodes. The node example has five energy balance constraints, one for each node. The dual value of each energy balance constraint is interpreted as the nodal price in the corresponding node. Constraint (12) is the flow constraint, which constraints the flow in line l, $fl_l$ to be bigger or equal to $FL_l^{min}$ and less or equal to $FL_l^{max}$, this yields for all lines in the set of $L$. Constraint (13), ensures that all the generators in each node do not exceed the maximum capacity. The last constraint (14) defines the variables for production and shedding as positive real numbers, while the variable for flow is defined for all real numbers.

**Sets & indices**

- N - Set of Nodes, index n
- G - Set of generators, index g
- L - Set of transmission lines, index l

**Parameters**

| | |
|---|---|
| $P_{n,g}^{max}$ | Max generation capacity for generator g at node n |
| $c_{n,g}$ | Generation cost at generator g at node n |
| $D_n$ | Total load demand at node n |
| $c^{shed}$ | Cost for load shedded |
| $FL_l^{max}$ | Max capacity in line l |
| $FL_l^{min}$ | Min capacity in line l, may be interpreted as reverse flow in line l |
| $I_{n,l}$ | Connectivity matrix between the nodes in n with the lines in l |

**Variables**

| | |
|---|---|
| $p_{n,g}$ | Generation for generator g at node n |
| $p_n^{shed}$ | Total load shedding at node n |
| $fl_l$ | Power transmitted in line l |

**Optimization problem**

$$\text{Minimize} \quad \sum_{n\in N}\sum_{g\in G} p_{n,g}\cdot c_{n,g} + p_n^{shed}\cdot c^{shed} \tag{10}$$

Subject to

$$\sum_{g\in G} p_{n,g} + p_n^{shed} - D_n = \sum_{l\in L} I_{n,l}\times fl_l \quad \forall n \in N \tag{11}$$

$$FL_l^{min} \leq fl_l \leq FL_l^{max} \quad \forall l \in L \tag{12}$$

$$p_{n,g} \leq P_{n,g}^{max} \quad \forall n \in N, \forall g \in G \tag{13}$$

$$p_{n,g}, p_n^{shed} \in \mathbb{R}^+, \quad fl_l \in \mathbb{R} \tag{14}$$

## 6.3 Conducting Transmission expansion planning with MILP

The current model for the nodal example determines the optimal dispatch of power production and flow in the existing transmission system. However, in order to allow for the expansion of the grid, the model must be updated to include the ability to add or upgrade interconnectors. This can be done by adding a variable, $e_l^{cap}$, to represent the expansion of interconnector capacities. This variable will be added to the existing capacities, $FL_l^{max}$ and $FL_l^{min}$, to allow for expansion in the grid. The model for TEP-problem is defined as a MILP due to the existing continuous variables described in section 6.2 and the new integer variable, $y_l^{line}$, which represents the number of new lines.

There are enormous costs associated with the expansion of the grid. These costs can be divided into fixed costs, $FC_l$, and variable costs, $VC_l$. The fixed costs are expenses that are independent of the capacity of the transmission line. These costs are typically related to planning costs, trench and laying costs, switch gears and circuit breakers, and transportation of materials [11]. The fixed costs only depend on the number of lines implemented in the system. Variable costs, on the other hand, are expenses that vary with factors such as expansion capacities, $e_l^{cap}$, and the length of the line, which is denoted as $l_l$ in the model. The upper limit for capacity expansion is denoted as $E_l^{max}$. If the variable for expansion exceeds this limit, a new line will be added. In general, fixed costs are typically higher in the initial stages of transmission expansion, while variable costs tend to increase as the expansion progresses. It is important for transmission system operators to carefully manage both fixed and variable costs in order to ensure that the expansion is financially favorable.

The objective function (16) consists of two parts. The first part describes the production costs in the system, while the second part describes the investment cost related to the expanded capacities and lines. Unlike the objective function described in section 6.2, the TEP problem's objective function is multiplied by the factors $t$ and $a$. Since the system's operational costs are given for each hour through the system's lifetime, and the investment cost is a one-time expense, they cannot be compared directly. In order to compare the costs, it is solved by multiplying the hourly operational cost with $t$ to obtain the annual operational cost of the system. The purpose of $a$ is to compare the operational savings accumulated throughout the lifetime of the grid project [10]. The factor $a$ depends on the discount rate, $r$, and the lifetime of the project, $n$, and is mathematically formulated in Equation 15. In this model, the discount rate is assumed to be 5%, and the lifetime of the project is assumed to be 30 years. With this formulation, it is assumed that the operational costs for all 30 years are the same.

$$a = \sum_{n=1}^{30} \frac{1}{(1+0.05)^n} = \frac{1 - \frac{1}{(1+0.05)^{30}}}{0.05} = 15.3725 \tag{15}$$

The second part of the objective function is related to the investment cost depending on how much the expansion and how many lines are needed in order to minimize the total investment cost. As mentioned above, there is an upper limit, $E_l^{max}$, for the expansion. Constraint (19) is formulated such that if the variable for expansion, $e_l^{cap}$, exceeds the upper limit, the integer variable $y_l^{line}$ will increase until the bigger or equal constraint is satisfied. The investment cost function behaves like a stair-step function due to constraint (19). This means the investment cost will gradually increase until it makes a step, due to a new fixed cost being added to the function. Note that there is no shedding in this model since the aim is to optimize the grid so that no bottlenecks create shedding. Lastly, the constraint for flow (5) is adjusted by adding the expansion variable to the limits.

**Sets & indices**

- N - Set of Nodes, index n
- G - Set of generators, index g
- L - Set of transmission lines, index l

**Parameters**

| | |
|---|---|
| $P_{n,g}^{max}$ | Max generation capacity for generator g at node n |
| $c_{n,g}$ | Generation cost at generator g at node n |
| $D_n$ | Total load demand at node n |
| $FL_l^{max}$ | Max capacity in line l |
| $FL_l^{min}$ | Min capacity in line l, may be interpreted as reverse flow in line l |
| $I_{n,l}$ | Connectivity matrix between the nodes in n with the lines in l |
| $FC_l$ | Fixed cost associated with expansion of line l |
| $VC_l$ | Variable cost associated with expansion of line l |
| $l_l$ | Length of line l |
| $E_l^{max}$ | Max transmission expansion in line l |
| $a$ | The present value of a series of payments made at the end of each period in the planning horizon |
| $t$ | Number of hours during a year, t =8760 |

**Variables**

| | |
|---|---|
| $p_{n,g}$ | Generation for generator g at node n |
| $p_n^{shed}$ | Total load shedding at node n |
| $fl_l$ | power transmitted in line l |
| $e_l^{cap}$ | New interconnector capacity after expansion in line l |
| $y_l^{line}$ | Number of new cables/lines built next to the existing line l |

**Optimization problem**

$$\text{Minimize} \quad \sum_{n \in N} \sum_{g \in G} p_{n,g} \cdot c_{n,g} \cdot a \cdot t + \sum_{l \in L} e_l^{cap} \cdot l_l \cdot VC_l + y_l^{line} \cdot FC_l \tag{16}$$

Subject to

$$\sum_{g \in G} p_{n,g} - D_n = \sum_{l \in L} I_{n,l} \times fl_l \quad \forall n \in N \tag{17}$$

$$FL_l^{min} - e_l^{cap} \leq fl_l \leq FL_l^{max} + e_l^{cap} \quad \forall l \in L \tag{18}$$

$$e_l^{cap} \leq E_l^{max} \cdot y_l^{line} \quad \forall l \in L \tag{19}$$

$$p_{n,g} \leq P_{n,g}^{max} \quad \forall n \in N, \forall g \in G \tag{20}$$

$$p_{n,g}, p_n^{shed} \in \mathbb{R}^+, \quad fl_l \in \mathbb{R} \tag{21}$$

$$y_l \in \mathbb{Z}^+ \tag{22}$$

## 6.4 Input data and assumptions

This section will discuss the input data and assumptions for the node example. The market clearing and transmission expansion planning models are not scenario- or time-based. This means the model only evaluates a snapshot of the system at one time and only evaluates the given scenario of input data. To conduct TEP, in reality, requires a complex analysis over time with many different scenarios to obtain a precise forecast of the system's behavior such that one can map the bottlenecks in the system. However, in this example, one may assume that the input data are the forecasted data based on an analysis of the system. Further, it is assumed that the demand is completely inelastic, which means consumers' willingness to pay is high. On the other hand, the supply offers its maximum capacity in megawatts, and each generator has a constant marginal production cost. Moreover, the generators' efficiency is not considered at part loads.

The data in Table 2 for the generators are based on arbitrary numbers that are inspired by numbers from the literature review [11][10]. The transmission capacities and connectivity between nodes are based on Figure 2, which illustrates the Nordic transmission system. Since the node example only includes only the Norwegian transmission system, Figure 3 is provided with the directional capacities added to the lines between the nodes to illustrate the applicable system. This is also shown in Table 1, along with the connectivity matrix $I$ given in Equation 23. As previously mentioned, $I$ is made up of -1, 0, and 1, which indicate the direction of the lines and the connections between nodes. Table 3 lists the prices and lengths for expanding the lines, and these numbers are also inspired by those used in the literature review.

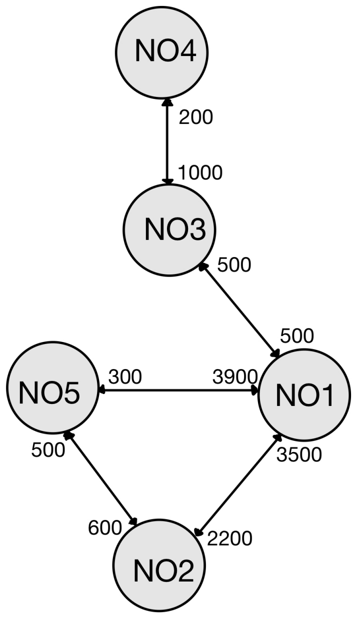


Figure 3: Illustration of NTC between the five price areas in Norway based on values from Figure 2

Table 1: Existing transmission capacities between the areas

| Transmission line | From i | To j | Line Capcity$_{ij}[MW]$ | Line Capacity$_{ji}[MW]$ | Length [km] |
|---|---|---|---|---|---|
| $L_{43}$ | NO4 | NO3 | 1000 | 200 | 1000 |
| $L_{35}$ | NO3 | NO5 | 0 | 0 | 300 |
| $L_{31}$ | NO3 | NO1 | 500 | 500 | 300 |
| $L_{51}$ | NO5 | NO1 | 3900 | 300 | 400 |
| $L_{52}$ | NO5 | NO2 | 600 | 500 | 300 |
| $L_{21}$ | NO2 | NO1 | 3500 | 2200 | 350 |

$$I = N \text{ Nodes} \begin{array}{cc} & L \text{ lines} \\ & \begin{array}{cccccc} L_{43} & L_{35} & L_{31} & L_{51} & L_{52} & L_{21} \end{array} \\ \begin{array}{c} NO4 \\ NO3 \\ NO5 \\ NO1 \\ NO2 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \tag{23}$$

Table 2: Max generation capacity and costs associated with generator units in each area, in addition to the total demand in each area. For all generators, it is assumed that the minimum generation is zero.

| Area | Generator ID | Max production capacity [MW] | Marginal cost [$\frac{EUR}{MWh}$] | Demand [MW] |
|---|---|---|---|---|
| NO4 | $G_{41}$ | 1100 | 12 | - |
| NO4 | $G_{42}$ | 700 | 30 | - |
| NO4 | $G_{43}$ | 800 | 35 | 2800 |
| NO4 | $G_{44}$ | 900 | 40 | - |
| NO4 | $G_{45}$ | 1000 | 45 | - |
| NO3 | $G_{31}$ | 200 | 12 | - |
| NO3 | $G_{32}$ | 400 | 35 | - |
| NO3 | $G_{33}$ | 300 | 45 | 3400 |
| NO3 | $G_{34}$ | 1000 | 50 | - |
| NO3 | $G_{35}$ | 1100 | 55 | - |
| NO5 | $G_{51}$ | 300 | 25 | - |
| NO5 | $G_{52}$ | 350 | 45 | - |
| NO5 | $G_{53}$ | 450 | 55 | 3000 |
| NO5 | $G_{54}$ | 900 | 60 | - |
| NO5 | $G_{55}$ | 1000 | 1000 | - |
| NO1 | $G_{11}$ | 1000 | 25 | - |
| NO1 | $G_{12}$ | 900 | 50 | - |
| NO1 | $G_{13}$ | 2100 | 60 | 8000 |
| NO1 | $G_{14}$ | 1000 | 85 | - |
| NO1 | $G_{15}$ | 1000 | 120 | - |
| NO2 | $G_{21}$ | 2500 | 25 | - |
| NO2 | $G_{22}$ | 2000 | 50 | - |
| NO2 | $G_{23}$ | 1500 | 70 | 6800 |
| NO2 | $G_{24}$ | 1000 | 90 | - |
| NO2 | $G_{25}$ | 1000 | 150 | - |
| Shedding | $G_{shedding}$ | - | 2000 | - |
| Total | - | 24 500 | - | 24 000 |

Table 3: Costs associated with the expansion of each line. These are arbitrary values, however, inspired from numbers used in the articles from the literature review [10][11] .

| Transmission line | Max expansion capacity per line [MW] | Variable cost [$\frac{EUR}{MW \cdot km}$] | Fixed cost [$\frac{EUR}{line}$] |
|---|---|---|---|
| $L_{43}$ | 300 | $1.27 \cdot 10^3$ | $6.82 \cdot 10^7$ |
| $L_{35}$ | 300 | $1.27 \cdot 10^3$ | $6.82 \cdot 10^7$ |
| $L_{31}$ | 300 | $1.27 \cdot 10^3$ | $6.82 \cdot 10^7$ |
| $L_{51}$ | 300 | $1.27 \cdot 10^3$ | $6.82 \cdot 10^7$ |
| $L_{52}$ | 300 | $1.27 \cdot 10^3$ | $6.82 \cdot 10^7$ |
| $L_{21}$ | 300 | $1.27 \cdot 10^3$ | $6.82 \cdot 10^7$ |

## 6.5 Python and pyomo

The presented models are made in Python with Pyomo, a powerful open-source optimization modeling language with a diverse set of optimization capabilities [31]. Since Pyomo is a python-based programming language, it has several advantages. Firstly, Python is easy to learn and has a large and active community of developers, which makes it easier to find help and resources when working with it. Pyomo, on the other hand, allows users to build complex optimization models concisely and intuitively, making it a valuable tool for various applications such as supply chain optimization, energy system modeling, and financial analysis. Additionally, Python and Pyomo have a range of libraries and tools that can be used to extend their functionality and make it easier to work with large-scale optimization problems. Overall, the combination of Python and Pyomo offers a powerful and flexible platform for solving optimization problems. *Gurobi* is the solver that has been used to solve optimization problems; however, it requires an academic license since it is a commercial solver [32]. One of the main drawbacks of Pyomo as an optimization tool is that it can be challenging to retrieve the dual values, especially in mixed-integer linear programming problems. This can make it challenging to use Pyomo to solve large MILP problems. Using optimization techniques such as decomposition techniques or heuristics may be necessary. These techniques can help to decompose the MILP problem into smaller subproblems that are easier to solve and to identify suitable solutions quickly, even if they are not necessarily optimal.

An example of a two-node system was created and tested to verify the accuracy of the mathematical optimization models formulated in Pyomo. Through trial and error, the Pyomo code produced the expected results for each test conducted on the two-node system. Once it was determined that the code was functioning correctly for the two-node example, it was ready to solve for the values in a five-node system.

# 7 Results and discussion

The results of the node example, including a comparison of the operational costs before and after the transmission expansion planning, are presented in this section. These results were obtained using the input data, models, and assumptions described in section 6.

## 7.1 Market clearing with ATC approach

The ATC approach is a widely used method for representing the results of a market clearing in a power system. The ATC approach allows power system operators to determine the maximum amount of power that can be transmitted through the transmission system at a given time, considering technical and operational constraints. This information is used to allocate transmission capacity to different users and ensure the power system's safe and reliable operation. In this subsection, the results of the market clearing in the baseline system will be presented and discussed.

Table 4: Area price and generation for each node after conducting ATC clearing on the grid before expansion.

| Area | Area price $[\frac{EUR}{MWh}]$ | Total production [MW] | Max capacity [MW] | Total Demand [MW] |
|---|---|---|---|---|
| NO4 | 45 | 3800 | 4500 | 2800 |
| NO3 | 55 | 2900 | 3000 | 3400 |
| NO5 | 2000 | 3000 | 3000 | 3000 |
| NO1 | 2000 | 6000 | 6000 | 8000 |
| NO2 | 2000 | 8000 | 8000 | 6800 |
| Total | - | 23 700 | 24 500 | 24 000 |

Table 4 presents the area price and generation compared with the demand for each area after conducting the market clearing. As one can observe in the table, the area price for NO5, NO1, and NO2 are $2000\frac{EUR}{MWh}$, which is almost 40 times higher than the price of NO4 and NO3. This high area price reflects the shedding cost for the system that has been set to be $2000\frac{EUR}{MWh}$. The total power production in the system is 23 700 $MW$, 300 $MW$ less than the total demand. Hence, load shedding occurs in order to balance supply and demand. One can ask why load shedding occurs when the system has sufficient production capacity. In Table 6, the production rate for each generator in each area is presented. When comparing the production rate with the production capacity shown in Table 2, one can observe the generator, $G_{45}$ and $G_{35}$ are not producing at their maximum in order to supply the demand. The mentioned generators have cheaper production costs than many other generators; nevertheless, they are not producing when there are deficits in the system. This is due to the internal bottlenecks in the grid system, which is illustrated in Figure 4. There is congestion between NO4 and NO3 and between NO3 and NO1. This congestion creates a bottleneck effect which means the surplus power in the north cannot reach the deficit area in the south.

NO1, NO2, and NO3 have no congestion in the lines that connect them, which means they will share the surplus energy with the deficit areas. However, in this case, there is a total deficit of 800 $MW$ for the three nodes; hence they depend on power transmission from NO3. Since $L_{31}$ maximum can transfer 500 $MW$, there will be a deficit of 300 $MW$ for the three nodes. The numbers for the power flow are also listed in Table 5, where the *Dual* is included. The dual value may be interpreted as the shadow price of expanding the capacity, which means if the capacity of one line increases by 1 $MW$, the objective function will decrease with the dual value. By observing the dual values, it is fair to claim that the bottleneck in the system occurs between NO3 and all the areas below. The dual value for $L_{31}$ and $L_{35}$ are 1945 $\frac{EUR}{MW}$, which means for each $MW$ capacity that is added in the bottleneck will decrease the objective with 1945 $EUR$. Note that $L_{35}$ is defined as a line with zero capacity, hence the dual value associated with the line. The dual value associated with $L_{43}$ has the value of 10 $\frac{EUR}{MWh}$ since the marginal generator in NO4 has the production cost of 45 $\frac{EUR}{MWh}$ and the marginal generator in NO3 has the production cost of 55 $\frac{EUR}{MWh}$. Increasing the capacity of $L_{43}$ with 1 $MW$ will reduce the production of the marginal generator in NO3 by 1 $MW$ and increase the marginal generator in NO4 with 1 MW, which means the total operational cost of the system decreases by 10 $EUR$.

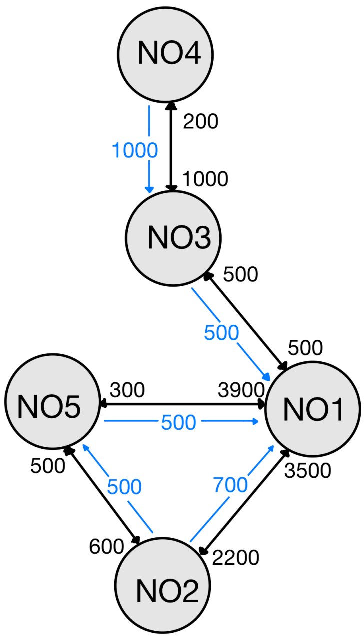


Figure 4: Illustration of flow results (blue font) after ATC clearing.

Table 5: Flow results for each interconnector after conducting ATC clearing

| Transmission line | From i | To j | Flow [MW] | Max flow [MW] | Dual [$\frac{EUR}{MWh}$] |
|---|---|---|---|---|---|
| $L_{43}$ | NO4 | NO3 | 1000 | 1000 | 10 |
| $L_{35}$ | NO3 | NO5 | 0 | 0 | 1945 |
| $L_{31}$ | NO3 | NO1 | 500 | 500 | 1945 |
| $L_{51}$ | NO5 | NO1 | 500 | 3900 | 0 |
| $L_{25}$ | NO2 | NO5 | 500 | 500 | 0 |
| $L_{21}$ | NO2 | NO1 | 700 | 3500 | 0 |

Table 6 shows the production rate, the operational cost for each generator, and the shed cost for one hour. The total operational cost of the system for each hour is 1 957 100 $\frac{EUR}{h}$. In order to get the accumulated operational cost of the system, one must multiply the cost with the factors $t$ and $a$, described in section 6.3. Thus, the accumulated cost of the system for the next 30 years is $2.64 \cdot 10^{11} EUR$, given that the system's parameters are constant.

Table 6: Generated quantity and operational costs for each generator after conducting ATC clearing.

| Area | Generator ID | Production rate [MW] | Marginal cost [$\frac{EUR}{MWh}$] | Operational costs [$\frac{EUR}{h}$] |
|---|---|---|---|---|
| NO4 | $G_{41}$ | 1100 | 12 | 13 200 |
| NO4 | $G_{42}$ | 700 | 30 | 21 000 |
| NO4 | $G_{43}$ | 800 | 35 | 28 000 |
| NO4 | $G_{44}$ | 900 | 40 | 36 000 |
| NO4 | $G_{45}$ | 300 | 45 | 13 500 |
| NO3 | $G_{31}$ | 200 | 12 | 2400 |
| NO3 | $G_{32}$ | 400 | 35 | 14 000 |
| NO3 | $G_{33}$ | 300 | 45 | 13 500 |
| NO3 | $G_{34}$ | 1000 | 50 | 50 000 |
| NO3 | $G_{35}$ | 1000 | 55 | 55 000 |
| NO5 | $G_{51}$ | 300 | 25 | 7500 |
| NO5 | $G_{52}$ | 350 | 45 | 15 750 |
| NO5 | $G_{53}$ | 450 | 55 | 24 750 |
| NO5 | $G_{54}$ | 900 | 60 | 54 000 |
| NO5 | $G_{55}$ | 1000 | 100 | 100 000 |
| NO1 | $G_{11}$ | 1000 | 25 | 25 000 |
| NO1 | $G_{12}$ | 900 | 50 | 45 000 |
| NO1 | $G_{13}$ | 2100 | 60 | 126 000 |
| NO1 | $G_{14}$ | 1000 | 85 | 85 000 |
| NO1 | $G_{15}$ | 1000 | 120 | 120 000 |
| NO2 | $G_{21}$ | 2500 | 25 | 62 500 |
| NO2 | $G_{22}$ | 2000 | 50 | 100 000 |
| NO2 | $G_{23}$ | 1500 | 70 | 105 000 |
| NO2 | $G_{24}$ | 1000 | 90 | 90 000 |
| NO2 | $G_{25}$ | 1000 | 150 | 150 000 |
| NO1 | $G_{shed}$ | 300 | 2000 | 600 000 |
| Total | - | 24 000 | - | 1 957 100 |

## 7.2 Transmission expansion planning

The tables and figures presented in this section are the results of conducting TEP on the node example. After that, a similar ATC approach was conducted on the expanded system to compare the results from the old system in section 7.1. Table 7 presents the results from the investment part of the objective function (16). Only two interconnectors are getting expanded, which are $L_{43}$ and $L_{35}$. However, the solution suggests building three lines in each connection in order to ensure adequate capacity between the nodes. The Figure 5 illustrates the new interconnectors and capacities, shown in red. The blue lines illustrate the new power flow with the new interconnectors and capacities implemented in the system. These numbers are also listed in Table 8. The investment cost related to each line considers the variable cost that depends on the expansion capacity needed, the length of the line, and the fixed cost related to the investment of each line. The total cost of the investment of six new lines with a total expansion of 1300 $MW$ is $1.603 \cdot 10^9$ $EUR$. If the system's expansion pays off, the accumulated savings from the operational costs must be larger than the total investment cost.

In section section 7.1, Table 5 presented the dual values of the lines. The dual values for $L_{35}$ and $L_{31}$ were 1945 $\frac{EUR}{MWh}$, and it was anticipated that these lines would be expanded. However, after solving the TEP problem as a MILP problem, the expanded lines turned out to be $L_{41}$ and $L_{35}$.

By expanding Line $L_{43}$, the system can use the inexpensive energy produced in NO4. Additionally, by installing an interconnector between NO3 and NO5, any excess production from NO4 and NO3 can be directed to the areas in the south that are experiencing a deficit. This helps to balance the supply and demand of electricity in the system. It should be noted that the dual values for $L_{35}$ and $L_{31}$ have now changed to 95 $\frac{EUR}{MWh}$, while the dual value for $L_{43}$ remains at 10 $\frac{EUR}{MWh}$. However, no more production capacity is available to reduce operational costs by increasing transmission capacity by one $MW$. Thus, given the constraints, the system is optimized as far as possible.

Table 7: New lines implemented and expanded capacity in the grid after conducting TEP.

| Transmission line | From i | To j | Expansion [MW] | # New lines | Investment Cost [EUR] |
|---|---|---|---|---|---|
| $L_{43}$ | NO4 | NO3 | 700 | 3 | $1093.6 \cdot 10^6$ |
| $L_{35}$ | NO3 | NO5 | 800 | 3 | $509.4 \cdot 10^6$ |
| Total | - | - | 1300 | 6 | $1603 \cdot 10^6$ |

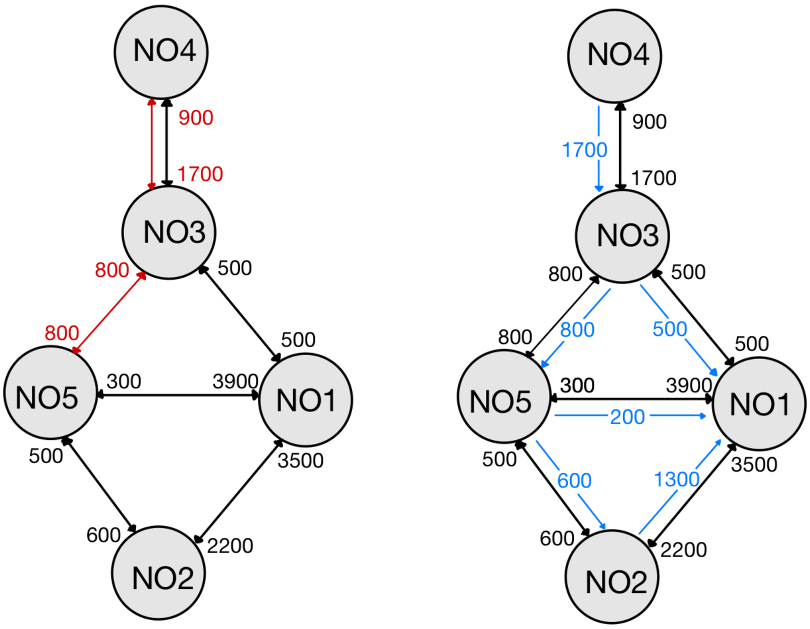


Figure 5: Illustration of the new transmission lines that are implemented in the system and the flow after conducting TEP

Table 8: Flow results for each connector after conducting TEP.

| Transmission line | From i | To j | Flow [MW] | Max flow [MW] | Dual [$\frac{EUR}{MWh}$] |
|---|---|---|---|---|---|
| $L_{43}$ | NO4 | NO3 | 1700 | 1700 | 10 |
| $L_{35}$ | NO3 | NO5 | 800 | 800 | 95 |
| $L_{31}$ | NO3 | NO1 | 500 | 500 | 95 |
| $L_{51}$ | NO5 | NO1 | 200 | 3900 | 0 |
| $L_{52}$ | NO5 | NO2 | 600 | 600 | 0 |
| $L_{21}$ | NO2 | NO1 | 1300 | 3500 | 0 |

So far, the system has been optimized and expanded to minimize the operational cost for the next 30 years while considering the discount rate of 5%. Thus, to compare the operational cost, one must conduct a market clearing of the new system. The results for each area are presented in Table 9, while the results for the generator dispatch are presented in Table 10. One can observe that there is no longer a mismatch between total production and total demand for the areas, which means there is no shedding in the system. Moreover, the area price for NO5, NO1, and NO2 is set to be 150 $\frac{EUR}{MWh}$. In Table 10, one can observe that $G_{45}$ and $G_{35}$ have increased the production to their maximum, while $G_{25}$ has decreased its production. However, $G_{25}$ is still the marginal generator in NO5, NO1 and NO2, thus the area price of 150 $\frac{EUR}{MWh}$. The total operational cost for the system has now decreased to 1,310,100 $\frac{EUR}{h}$ due to the elimination of load shedding and the optimization of production dispatch. This means that the accumulated cost for the system over 30 years is $1.78 \cdot 10^{11}$ $EUR$, resulting in a total savings of $0.86 \cdot 10^{11}$ $EUR$ from the expansion of the transmission system over the same period. This result strongly suggests that investing in expanding the grid is financially beneficial, as the accumulated savings are greater than the cost of expansion.

Table 9: Area price and generation for each node after conducting TEP

| Area | Area price [$\frac{EUR}{MWh}$] | Total production [MW] | Max production [MW] | Total Demand [MW] |
|---|---|---|---|---|
| NO4 | 45 | 4500 | 4500 | 2800 |
| NO3 | 55 | 3000 | 3000 | 3400 |
| NO5 | 150 | 3000 | 3000 | 3000 |
| NO1 | 150 | 6000 | 6000 | 8000 |
| NO2 | 150 | 7500 | 8000 | 6800 |
| Total | - | 24 000 | 24 500 | 24 000 |

Table 10: Generated quantity and operational costs for each generator after conducting TEP

| Area | Generator ID | Production rate [MW] | Marginal cost [$\frac{EUR}{MWh}$] | Operational costs [$\frac{EUR}{h}$] |
|---|---|---|---|---|
| NO4 | $G_{41}$ | 1100 | 12 | 13 200 |
| NO4 | $G_{42}$ | 700 | 30 | 21 000 |
| NO4 | $G_{43}$ | 800 | 35 | 28 000 |
| NO4 | $G_{44}$ | 900 | 40 | 36 000 |
| NO4 | $G_{45}$ | 1000 | 45 | 45 000 |
| NO3 | $G_{31}$ | 200 | 12 | 2400 |
| NO3 | $G_{32}$ | 400 | 35 | 14 000 |
| NO3 | $G_{33}$ | 300 | 45 | 13 500 |
| NO3 | $G_{34}$ | 1000 | 50 | 50 000 |
| NO3 | $G_{35}$ | 1100 | 55 | 60 500 |
| NO5 | $G_{51}$ | 300 | 25 | 7500 |
| NO5 | $G_{52}$ | 350 | 45 | 15 750 |
| NO5 | $G_{53}$ | 450 | 55 | 24 750 |
| NO5 | $G_{54}$ | 900 | 60 | 54 000 |
| NO5 | $G_{55}$ | 1000 | 100 | 100 000 |
| NO1 | $G_{11}$ | 1000 | 25 | 25 000 |
| NO1 | $G_{12}$ | 900 | 50 | 45 000 |
| NO1 | $G_{13}$ | 2100 | 60 | 126 000 |
| NO1 | $G_{14}$ | 1000 | 85 | 85 000 |
| NO1 | $G_{15}$ | 1000 | 120 | 120 000 |
| NO2 | $G_{21}$ | 2500 | 25 | 62 500 |
| NO2 | $G_{22}$ | 2000 | 50 | 100 000 |
| NO2 | $G_{23}$ | 1500 | 70 | 105 000 |
| NO2 | $G_{24}$ | 1000 | 90 | 90 000 |
| NO2 | $G_{25}$ | 500 | 150 | 75 000 |
| NO1 | $G_{shed}$ | 0 | 2000 | 0 |
| Total | - | 24 000 | - | 1 319 100 |

# 8 Conclusion and Further work

This work is a prestudy to the master thesis, consisting of a literature review and methods related to performing the principles of transmission expansion planning. This study aims to provide a solid foundation of knowledge and understanding of the key concepts and methods that will be used in the master thesis. It has described some of the main features of the power system and the market clearing, which decides the area prices. This is a crucial aspect of the study, as the market-clearing process plays a central role in determining the optimal expansion of the grid structure. Furthermore, this work has presented optimization problems that can be established with mixed integer linear programming in order to determine an optimal expansion of the grid structure. By coding and solving the problem in Python and Pyomo, a deeper understanding of the topic will be valuable in further work in the master thesis.

The study examined the five-node example with a transportation model (ATC) with many assumptions that do not accurately reflect the behavior of a dynamic power system. As a result, some may argue that the example is misleading due to the use of arbitrary values. It is important to note that cost-benefit analysis cannot simply conclude that an investment is always beneficial. Instead, a detailed forecast of the system's behavior is required, taking into account various scenarios and states in order to determine if an expansion in the grid is necessary. Even if an expansion is deemed necessary, it does not necessarily mean that it will be financially beneficial. A thorough cost analysis must be conducted and compared to determine the proposed solution's financial feasibility. Despite these limitations, the principles of transmission expansion planning were presented, and it can be concluded that the grid expansion increased the capacity between the nodes and reduced operational costs in the system.

Understanding the fundamentals of transmission expansion planning will be used to conduct further research in a master's thesis. This research aims to delve deeper into the topic and assess the effectiveness of using DC power flow in transmission lines for expansion planning. To carry out a more comprehensive study, it will be necessary to review additional literature and gather data from power producers and transmission system operators on the energy market, power production, demand, transmission, and costs. By accumulating data over a certain period, it will be possible to create a large, accurate simulation of the power system that considers uncertainties related to fluctuations in power production and demand.

To improve the accuracy of the transmission system model, an extension to the current methodology can be implemented. One approach is to use a combination of the ATC and DC power flow methods, depending on whether the transmission line is an AC or DC line. For AC lines, DC power flow can be used, while the ATC method can be applied to DC lines. This means that AC/DC and DC/AC converters must be included in the system. Additionally, when utilizing DC power flow, it is important to consider three major line-loading limits: thermal limits, voltage-drop limits, and steady-state stability. These limits typically depend on the length of the transmission line.

The goal is to create a more extensive system simulation with more data to understand better the effects of various expansion scenarios on a larger scale. However, solving a multi-stage optimization problem on this scale can be computationally intensive. Therefore, it may be helpful to use decomposition techniques to solve these optimization problems. Lastly, it would also be interesting to investigate the impact on emissions and how different expansion scenarios affect the overall carbon footprint of the system.

# Bibliography


[1] T. Aanensen, "SSB: Rekordhøyt strømforbruk i fjor," Jun. 2022. [Online]. Available: https://www.ssb.no/energi-og-industri/energi/statistikk/elektrisitet/artikler/rekordhoyt-stromforbruk-i-fjor.

[2] Statnett, *Nettutviklingsplan 2021*, Sep. 2021. [Online]. Available: https://www.statnett.no/globalassets/for-aktorer-i-kraftsystemet/planer-og-analyser/nup-2021/nettutviklingsplan-2021.pdf.

[3] E. commission, "A Roadmap for moving to a competitive low carbon economy in 2050," Aug. 2011. [Online]. Available: https://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=COM:2011:0112:FIN:en:PDF.

[4] E. commission, "Renewable energy targets," Jul. 2022. [Online]. Available: https://energy.ec.europa.eu/topics/renewable-energy/renewable-energy-directive-targets-and-rules/renewable-energy-targets_en.

[5] F. Hulle, J. Tande, K. Uhlen, *et al.*, "Wind. Developing Europe's power market for the large-scale integration of wind power," *Final report*, Jan. 2009. [Online]. Available: https://backend.orbit.dtu.dk/ws/portalfiles/portal/195981748/Fulltext.pdf.

[6] ENTSOE, "SCENARIO OUTLOOK AND ADEQUACY 2014-2030," 2014. [Online]. Available: https://docstore.entsoe.eu/Documents/TYNDP%20documents/TYNDP%202014/141017_SOAF%202014-2030.pdf.

[7] ENTSOE, "Ten-Year Network Development Plan 2014," Jul. 2014. [Online]. Available: https://eepublicdownloads.entsoe.eu/clean-documents/tyndp-documents/TYNDP%202014/140710_Draft_TYNDP%202014_Exec%20Summary_Consultation.pdf.

[8] resmonitor, "GRID REGULATION AND INFRASTRUCTURE," Feb. 2022. [Online]. Available: https://resmonitor.eu/en/at/barriers/1559/.

[9] G. the Grid, "Transmission Expansion Planning," 2015. [Online]. Available: https://greeningthegrid.org/Renewable-Energy-Zones-Toolkit/topics/Transmission%20Expansion%20Planning.

[10] H. Farahmand, S. Jaehnert, T. Aigner, and D. Huertas-Hernando, "Nordic hydropower flexibility and transmission expansion to support integration of North European wind power," *Wind Energy*, vol. 18, no. 6, pp. 1075–1103, 2015. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/we.1749.

[11] T. Trötscher and M. Korpås, "A framework to determine optimal offshore grid structures for wind power integration and power exchange," *Wind Energy*, vol. 14, no. 8, pp. 977–992, 2011. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/we.461.

[12] M. Haddad, "How much of Europe's energy comes from gas?," Sep. 2022. [Online]. Available: https://www.aljazeera.com/news/2022/9/6/infographic-how-much-of-europes-energy-gas.

[13] TradingEconomics, "EU Carbon Permits," 2022. [Online]. Available: https://tradingeconomics.com/commodity/carbon.

[14] Y. Gotaas Bonde, "Statnett vil ha mer flyt av kraft mellom nord og sør. Men det vil ikke Sp og SV. Ap er i tenkeboksen.," Aug. 2022. [Online]. Available: https://www.aftenposten.no/norge/politikk/i/z7vWJw/statnett-vil-ha-mer-flyt-av-kraft-mellom-nord-og-soer-men-det-vil-ikke-sp-og-sv-ap-er-i-tenkeboksen.

[15] A. L. Poppe, "Alle snakker om de urettferdige strømgrensene. Dette er historien bak.," Aug. 2022. [Online]. Available: https://e24.no/olje-og-energi/i/ALj4jq/alle-snakker-om-de-urettferdige-stroemgrensene-dette-er-historien-bak?referer=https%3A%2F%2Fwww.aftenposten.no.

[16] E. commission, "Energy infrastructure priorities for 2020 and beyond - A Blueprint for an integrated European energy network," Nov. 2010. [Online]. Available: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52010DC0677&from=EN.

[17] E. Samseth, F. Stockhausen, X. Veillard, and A. Weiss, "Five trends reshaping European power markets," Oct. 2021. [Online]. Available: https://www.mckinsey.com/industries/electric-power-and-natural-gas/our-insights/five-trends-reshaping-european-power-markets.

[18] I. Wangsteen, *Power System Economics - the Nordic Electricity Market*, 2nd edition. Trondheim: Tapir Academic Press, 2012.

[19] H. Farahmand, *Lecture slides from TET4185, Power Markets: Organization of Electricity Markets*, 2022.

[20] J. Krokeide Kolberg and K. Waage, "Artificial Intelligence and Nord Pool's Intraday Electricity Market Elbas: A Demonstration and Pragmatic Evaluation of Employing Deep Learning for Price Prediction," 2018. [Online]. Available: https://openaccess.nhh.no/nhh-xmlui/bitstream/handle/11250/2560898/masterthesis.PDF?sequence=1&isAllowed=y.

[21] NordPool, "Organisation," 2022. [Online]. Available: https://www.nordpoolgroup.com/en/About-us/organisation/.

[22] NordPool, "Bidding areas," 2022. [Online]. Available: https://www.nordpoolgroup.com/en/the-power-market/Bidding-areas/.

[23] NordPool, "Day-ahead market," 2022. [Online]. Available: https://www.nordpoolgroup.com/en/the-power-market/Day-ahead-market/.

[24] ENTSOE, "Net Transfer Capacity and Available Transfer Capacities in the Internal market of Electricity in Europe," 2000. [Online]. Available: https://eepublicdownloads.entsoe.eu/clean-documents/pre2015/ntc/entsoe_NTCusersInformation.pdf.

[25] S. NEMO and D. B. AG, *Continues Trading Matching Algorithm.* [Online]. Available: https://www.nordpoolgroup.com/49385a/globalassets/download-center/intraday/public-description-of-the-continues-algorithm.pdf.

[26] H. Farahmand, *Lecture slides from TET4185, Power Markets: Nodal prices and OPF*, 2022.

[27] S. Mulukutla, G. Duncan, T. Overbye, and J. Sarma, *Power System Analysis & Design*, 6th edition. Cengage Learning, 2016.

[28] A. Gullhav, *Lecture slides from TIØ4130 Optimization methods with applications: Integer programming*, 2021.

[29] S. Anderson, "What Is Cost-Benefit Analysis, How Is it Used, What Are its Pros and Cons?," Oct. 2022. [Online]. Available: https://www.investopedia.com/terms/c/cost-benefitanalysis.asp.

[30] ENTSOE, *Regional Investment Plan Baltic Sea region*, Jun. 2015. [Online]. Available: https://consultations.entsoe.eu/system-development/regional-investment-plans/user_uploads/regional-investment-plan-2015---rg-bs---for-consultation.pdf.

[31] Pyomo, "About Pyomo," 2022. [Online]. Available: http://www.pyomo.org/about.

[32] "Gurobi optimization," 2022. [Online]. Available: https://www.gurobi.com/solutions/gurobi-optimizer/.