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## Original Article

# Fatigue strength assessment of riveted railway bridge details based on regression analyses combined with probabilistic models



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## ABSTRACT

With the increasing attention on structural reliability and integrity, the probabilistic fatigue behaviour of riveted joints is drawn more attention, more specifically in the influence that this may have on the fatigue damage accumulation evaluation of structural components. For that, in this paper, the parameters of S–N curves for two riveted joints were obtained using the least-squares regression (LR) and the orthogonal regression (OR) methods, respectively. The results showed that the fitted slope B from the OR method is larger than the one from the LR method of all specimens. The probabilistic fatigue life of the riveted joints is obtained by Castillo & Fernández-Canteli (CFC) method and stochastic analysis using Latin hypercube sampling strategies. Among six different probabilistic functions, the stochastic analysis with the Gumbel distribution contributed to the largest fatigue strength with 95% and 99% confidence levels while the stochastic analysis with the Weibull distribution led to the smallest fatigue strength with 95% and 99% confidence levels. The effects of regression methods on the probabilistic fatigue life are not obvious, however, for the levels of stress range of the high-cycle regime, the fatigue life is substantially different when the comparison is made between the curves obtained by different approaches, which will have implications in the assessment of the fatigue damage accumulation of structural joints operating in this fatigue regimes. The fatigue strength with 95% and 99% confidence levels obtained using constant exponent are larger than when employing the varied exponent. The probabilistic fatigue life with stochastic analysis using constant exponent is closer to the CFC model than by using varied constant.

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## 1. Introduction

After a long period of service, fatigue crack initiation [1,2] and propagation [3,4] are the main concerns for the old riveted steel bridges [5–11], especially the load and amount of vehicles increased a lot compared with the previous design specification. The fatigue strength of riveted joints plays an important role during the fatigue performance evaluation of such type of steel bridges.

Ensuring the safety of riveted joints against fatigue consists of developing two research themes, one related to the estimation of the current damage in the structure and the other related to the estimation of the remaining life. The fatigue behaviour of riveted joints was investigated through both experiments and numerical analysis [12–17] for better prediction of the fatigue life of such old riveted steel bridges.

The fatigue performance prediction of old riveted joints could be very complex. In terms of the design specification of EC3 [18], Class 71 S–N curve is recommended by Kühn et al. [19] to assess the fatigue behaviour of riveted joints. But further investigations for more detailed categorization of riveted joints are recommended by several works [20–32] because Class 71 category of EC3 [18] generally contributes to conservative fatigue life predictions.

With the increasing attention on structural reliability and integrity, the probabilistic fatigue behaviour of riveted joints has called more attention. The fatigue life considering multiple uncertainty source effects are essential for the determination of the riveted details. Correia [33,34] proposed a probabilistic S–N field based on Castillo & Fernández-Canteli (CFC) model [35] using the Weibull distribution function and UniGrow approach for the notched details. Reliable Wöhler curves based on the Stüssi model and the Weibull distribution function were suggested by Caiza et al. [15] to strength the modelling of structural details. Zhu et al. [36] investigated the fatigue life distribution of notched specimens based on the Weibull model and critical distance theory. Besides, the stochastic analysis allows us to obtain the probabilistic fatigue behaviour of riveted joints based on meta modelling [37]. A comparison between Castillo & Fernández-Canteli (CFC) method [35] and stochastic meta modelling is expected to provide an initial impression for the analysis of probabilistic fatigue life using different methods.

In this paper, the parameters of S–N curves for two riveted joints made of S235JR structural steel (“single line double shear riveted” – SLDSR and “double line double shear riveted” – DLDSR) were obtained using both the least-squares regression and the orthogonal regression methods, where the root-mean square errors (RMSE) are calculated. The probabilistic fatigue life fields were firstly obtained based on Castillo & Fernández-Canteli (CFC) method [35] using experimental data. After that, the stochastic analysis using Latin hypercube sampling strategies and different probability functions were conducted to obtain probabilistic fatigue life fields. The probabilistic fatigue behaviour obtained by these two methods were compared and discussed.

### Nomenclature

|                         |   |
|-------------------------|---|
| A                       | Regression parameter ( $\log(\Delta\sigma)$ – $\log(N)$ scale)  |
| Ave                     | Average value   |
| B                       | Regression parameter (negative inverse slope, m); Threshold parameter for the lifetime (CFC method)     |
| C                       | Intercept of N-axis by S–N curve (Material constant); Threshold parameter for stress range (CFC method) |
| CFC                     | Castillo & Fernández-Canteli model  |
| DLDSR                   | double line double shear riveted  |
| F <sub>max</sub>        | Maximum force   |
| F <sub>min</sub>        | Minimum force   |
| LR                      | Least-squares regression  |
| m                       | Negative inverse slope of S–N curve (material constant)   |
| N                       | Number of cycles to failure   |
| N <sub>f</sub>          | Fatigue life  |
| OR                      | Orthogonal regression   |
| PDF                     | probability density function  |
| P <sub>f</sub>          | Failure probability   |
| R                       | Load ratio  |
| Std                     | Standard deviation  |
| SLDSR                   | Single line double shear riveted  |
| V                       | Normalized variable (CFC method)  |
| $\Delta F$              | Force range   |
| $\Delta \sigma$         | Stress range  |
| $\Delta \sigma_{total}$ | Stress range of the gross cross-section   |
| $\Delta \sigma_{net}$   | Stress range of the net cross-section   |
| $\beta$                 | Shape parameter of Weibull model  |
| $\delta$                | Scale parameter   |
| $\lambda$               | Threshold parameter for normalized variable V   |

## 2. Experimental results

The riveted joints of the Trezói Bridge were made of St37 steel grade, close to the current S235JR structural steel grade, according to the European standards. Consequently, the specimens of the riveted joints under consideration – “single line double shear riveted” (SLDSR) and (double line double shear riveted) (DLDSR) – were manufactured using two different materials, namely the S235JR structural steel for the plates, beams, and angles, and the rivet material. According to Silva et al. [38], the microstructure of the riveted material reveals to have a smaller grain size when compared with the current S235JR structural steel.

In this section, the microstructural analysis and mechanical properties of the S235JR structural steel are presented. Besides, the fatigue resistance results of the SLDSR and DLDSR riveted joints are detailed, where the mean S–N curves by applying linear regression methods (Least-squares Regression – LR, Orthogonal Regression – OR) are estimated. From the application of the linear regression methods to the fatigue resistance results of the riveted joints under consideration, the root-mean-square errors (RMSE) are calculated.

**Table 1 – Chemical composition of the S235JR structural steel used in the riveted joints (Wt., %).**

| Steel Grade | C%    | Mn%   | Si%   | P%    | S%    | N%    | Mo% | B%  |
|-------------|-------|-------|-------|-------|-------|-------|-----|-----|
|             | max   | max   | max   | max   | max   | max   | max | max |
| S235JR      | 0.190 | 1.500 | 0.030 | 0.045 | 0.045 | 0.014 | –   | –   |

**Table 2 – Tensile properties of the S235JR structural steel used in the riveted joints.**

| Steel Grade | R <sub>p</sub> | R <sub>m</sub> | Elongation@ fracture |
|-------------|----------------|----------------|----------------------|
|             | [MPa]          | [MPa]          | %                    |
| S235JR      | 235            | 360–510        | 26                   |

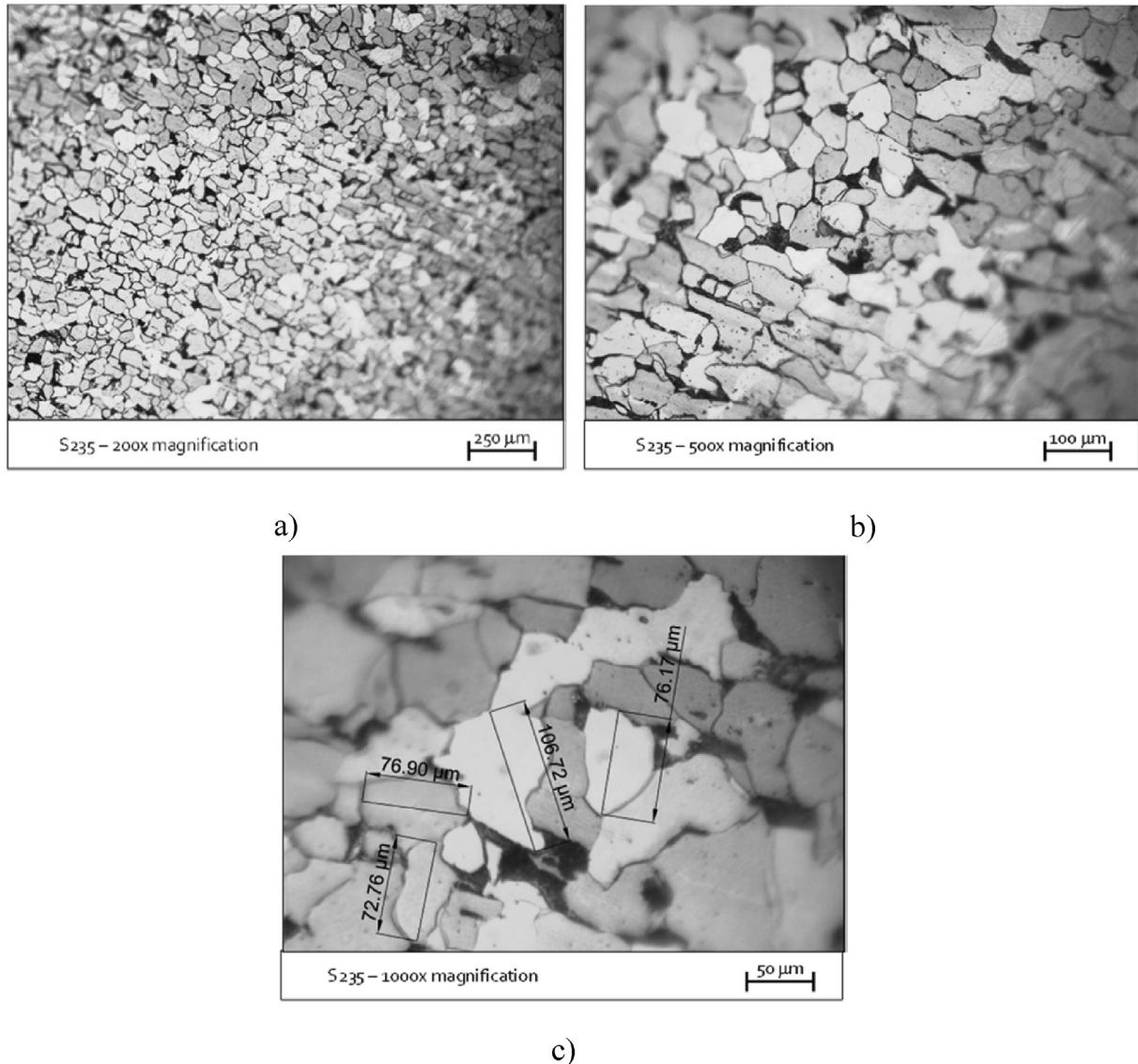
2.1. Microstructural analysis and mechanical properties

Tables 1 and 2 show the chemical composition and tensile properties for the S235JR structural steel used in the SLDSR and DLDSR riveted joints under consideration, respectively.

Fig. 1 plots the observed microstructure of the S235JR structural steel by using optical microscopy. In this study, magnifications of 200 ×, 500 ×, and 1000 × were considered. A global analysis of the micrograph of the S235JR structural steel allowed to observe a typical microstructure of a carbon structural steel, namely an equilibrium microstructure of perlite (dark zone) and ferrite (white zone) is observed [38].

2.2. Fatigue resistance results of riveted joints

A total of 19 specimens [38], including 10 SLDSR specimens and 9 DLDSR specimens, were tested under fatigue loading.



**Fig. 1 – Microstructure of S235JR structural steel: a) magnification of 200; b) magnification of 500; c) magnification of 1000.**

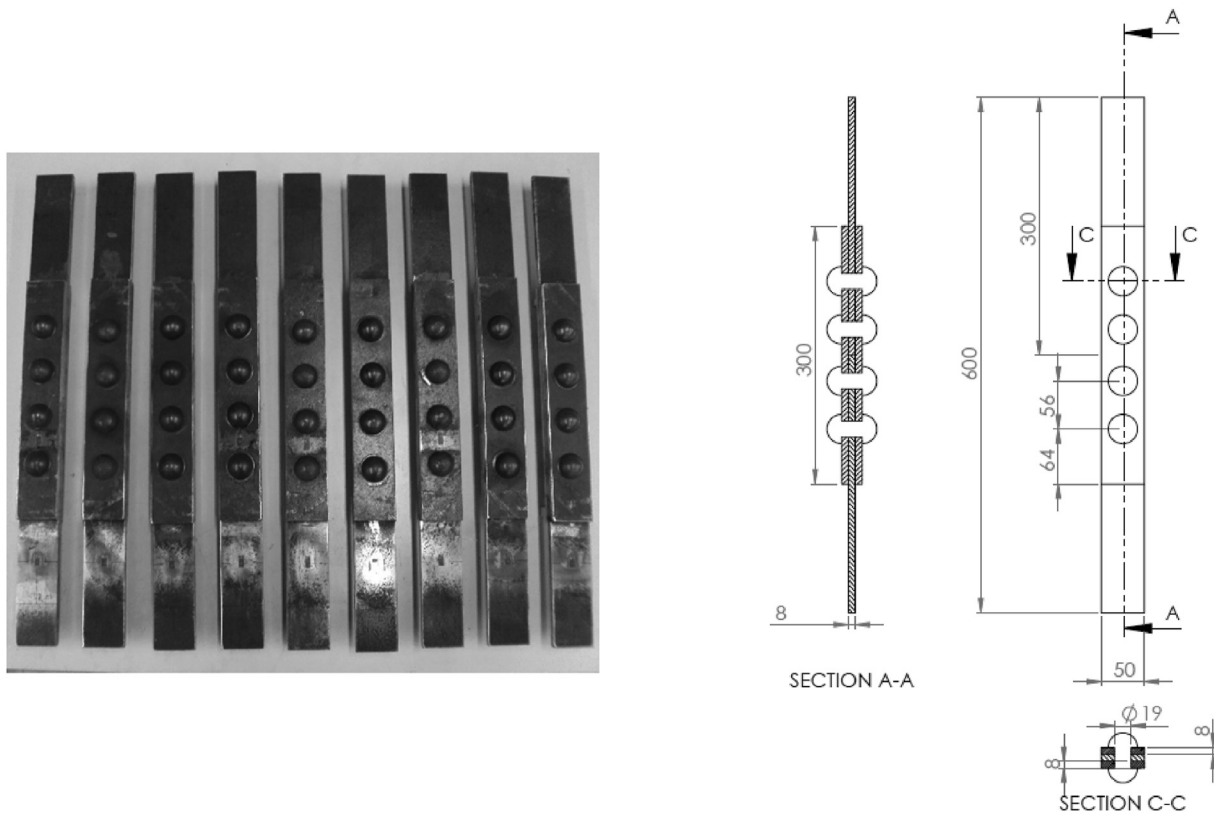


Fig. 2 – Single line double shear riveted (SLDSR) specimens [38].

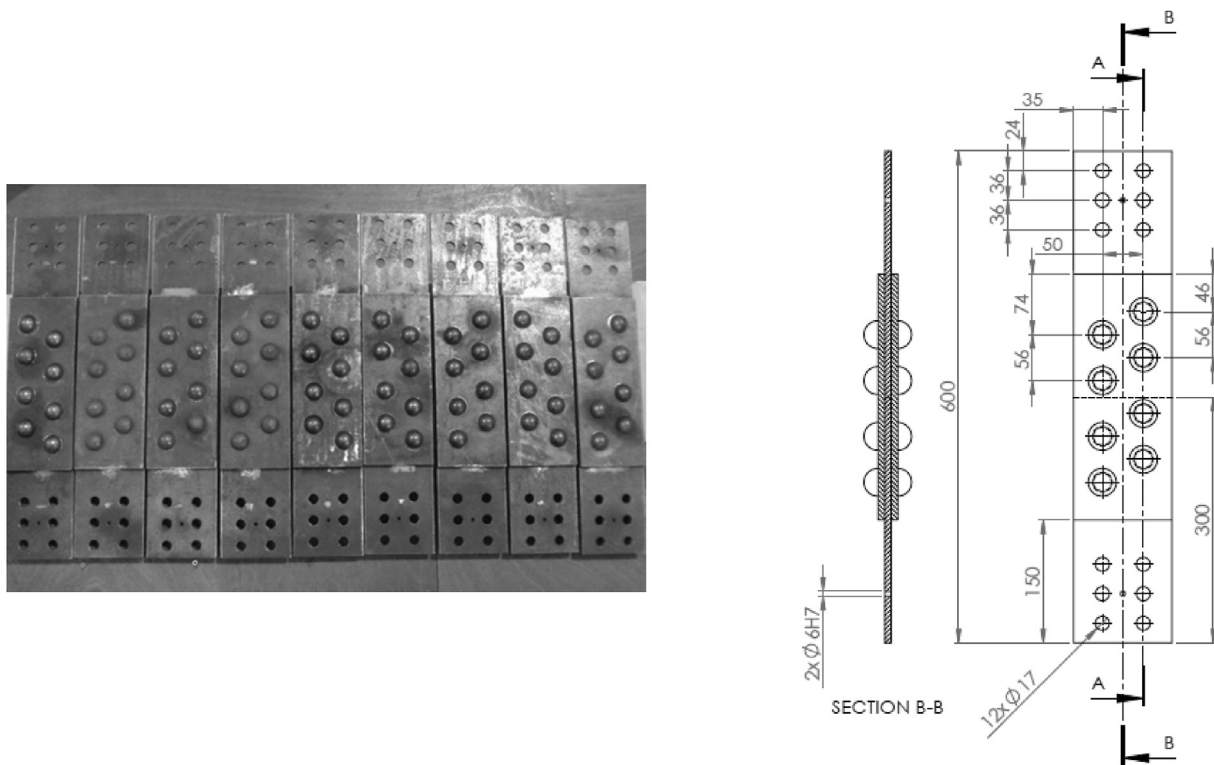


Fig. 3 – Double line double shear riveted (DLDSR) specimens [38].



**Table 3 – Fatigue test results of SLDSR specimens [38].**

| Specimens              | Load R-ratio | F <sub>max</sub> [kN] | F <sub>min</sub> [kN] | ΔF [kN] | Δσ <sub>total</sub> [MPa] | Δσ <sub>net</sub> [MPa] | N <sub>f</sub>      |
|------------------------|--------------|-----------------------|-----------------------|---------|---------------------------|-------------------------|---------------------|
| SLDSR -01 <sup>a</sup> | 0.01         | 50.00                 | 0.50                  | 49.5    | 122.6                     | 198.0                   | 391735 <sup>a</sup> |
| SLDSR -02              | 0.01         | 95.95                 | 0.96                  | 94.99   | 232.8                     | 375.7                   | 28764               |
| SLDSR -03              | 0.01         | 95.00                 | 0.95                  | 94.05   | 234.9                     | 379.6                   | 18631               |
| SLDSR -04              | 0.01         | 80.00                 | 0.80                  | 79.20   | 194.8                     | 314.2                   | 361858              |
| SLDSR -05              | 0.01         | 65.00                 | 0.65                  | 64.35   | 160.2                     | 258.9                   | 807561              |
| SLDSR -06              | 0.01         | 80.00                 | 0.80                  | 79.20   | 198.1                     | 320.2                   | 47843               |
| SLDSR -07              | 0.01         | 60.00                 | 0.60                  | 59.40   | 147.7                     | 238.3                   | 1130000             |
| SLDSR -08              | 0.01         | 80.00                 | 0.80                  | 79.20   | 196.7                     | 317.7                   | 127102              |
| SLDSR -09              | 0.01         | 70.00                 | 0.70                  | 69.30   | 171.9                     | 277.8                   | 868598              |
| SLDSR -10              | 0.01         | 95.95                 | 0.96                  | 94.99   | 235.3                     | 379.9                   | 77904               |

<sup>a</sup> Run-out (excluded during fitting).

**Table 4 – Fatigue test results of DLDSR specimens [38].**

| Specimens | R    | F <sub>max</sub> [kN] | F <sub>min</sub> [kN] | ΔF [kN] | Δσ <sub>total</sub> [MPa] | Δσ <sub>net</sub> [MPa] | N <sub>f</sub> |
|-----------|------|-----------------------|-----------------------|---------|---------------------------|-------------------------|----------------|
| DLDSR -01 | 0.10 | 160.00                | 16                    | 144     | 148.7                     | 176.9                   | 660121         |
| DLDSR -02 | 0.10 | 180.00                | 18                    | 162     | 167.1                     | 199.3                   | 455794         |
| DLDSR -03 | 0.10 | 100.00                | 10                    | 90      | 92.8                      | 110.4                   | 3125148        |
| DLDSR -04 | 0.10 | 120.00                | 12                    | 108     | 112.4                     | 133.9                   | 1633734        |
| DLDSR -05 | 0.10 | 120.00                | 12                    | 108     | 112.4                     | 133.9                   | 1146928        |
| DLDSR -06 | 0.10 | 200.00                | 20                    | 180     | 187.1                     | 222.7                   | 208727         |
| DLDSR -07 | 0.10 | 120.00                | 12                    | 108     | 112.3                     | 133.7                   | 1633734        |
| DLDSR -08 | 0.10 | 180.00                | 18                    | 162     | 167.2                     | 198.8                   | 309294         |
| DLDSR -09 | 0.10 | 160.00                | 16                    | 144     | 148.6                     | 176.6                   | 662938         |

The photos and geometry of SLDSR and DLDSR specimens are shown in Figs. 2 and 3, respectively.

The SLDSR specimens were tested in an INSTRON 8801 servo-hydraulic machine (capacity of 100 kN) using a load R-ratio of 0.01. The DLDSR specimens were tested by using an MTS servo-hydraulic machine (capacity of 250 kN), under a load R-ratio of 0.1.

The fatigue test results of SLDSR and DLDSR specimens are listed in Tables 3 and 4, including the load ratio R, maximum force F<sub>max</sub> and minimum force F<sub>min</sub>, the stress range of the net cross-section Δσ<sub>net</sub>, and fatigue life N<sub>f</sub>. More details about experimental tests can be found in Silva et al. [38].

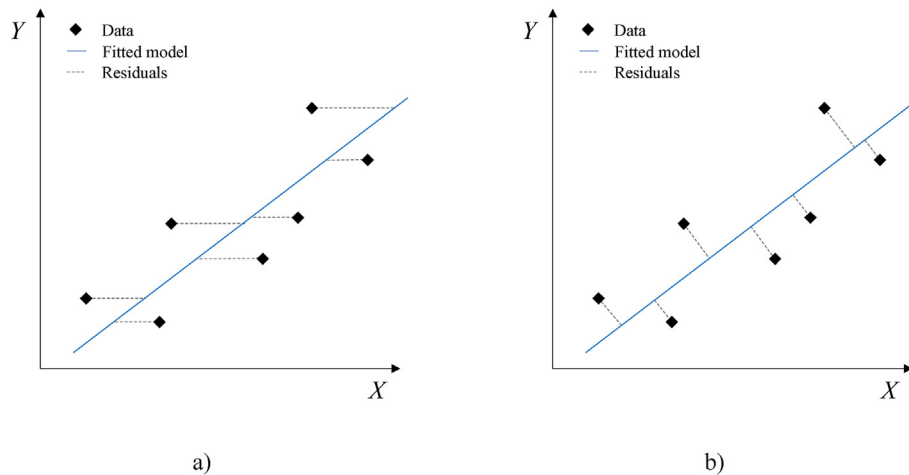
According to design codes, the fatigue strength curves, called S–N or Wöhler curves, of materials and/or structural details are given by:

$$\Delta\sigma^m N = C \tag{1}$$

where: Δσ is the stress range; N is the number of cycles to failure; C and m are material constants. Alternatively, the S–N curve [39], expressed in Eq. (2), is used to describe the relationship between stress range, Δσ, and fatigue life, N:

$$\Delta\sigma = AN^B \tag{2}$$

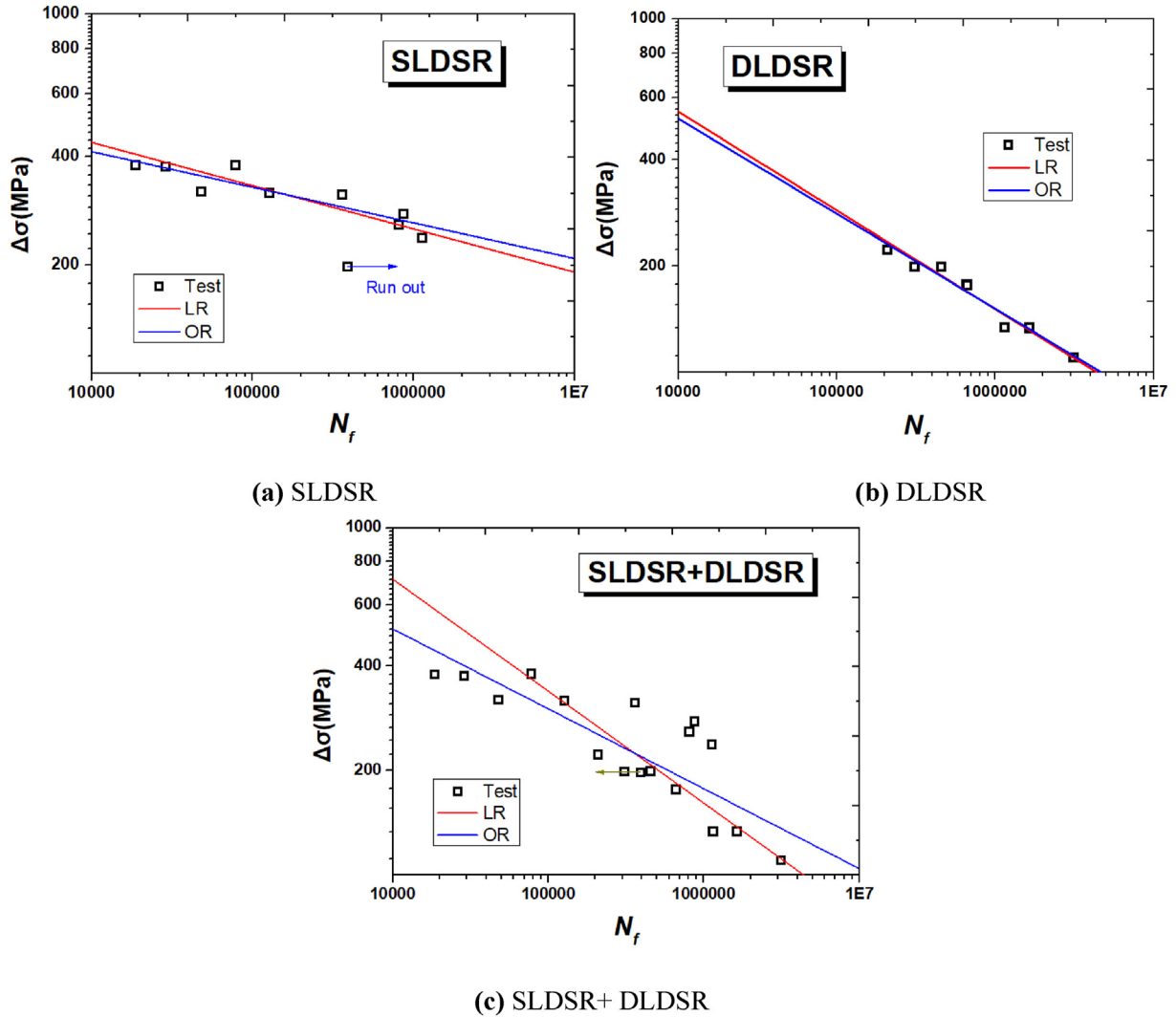
or



**Fig. 4 – Linear regression methods: a) least-squares regression (LR); b) orthogonal regression (OR).**

**Table 5 – The fitted material parameters of the S–N curve.**

| Specimens     | Least-squares Regression (LR) |        |        |        |             | Orthogonal Regression (OR) |        |         |        |             |
|---------------|-------------------------------|--------|--------|--------|-------------|----------------------------|--------|---------|--------|-------------|
|               | A                             |        | B      |        | RMSE (LogN) | A                          |        | B       |        | RMSE (LogN) |
|               | Ave.                          | Std.   | Ave.   | Std.   |             | Ave.                       | Std.   | Ave.    | Std.   |             |
| SLDSR         | 25.9414                       | 3.6447 | 8.3040 | 1.4591 | 0.2699      | 30.3695                    | 4.0109 | 10.0774 | 1.6057 | 0.2971      |
| DLDSR         | 13.7938                       | 0.6148 | 3.5772 | 0.2783 | 0.0721      | 14.1049                    | 0.6260 | 3.7181  | 0.2834 | 0.0734      |
| SLDSR + DLDSR | 12.8460                       | 1.1997 | 3.1008 | 0.5088 | 0.3421      | 15.7704                    | 1.4059 | 4.3442  | 0.5962 | 0.4009      |



**Fig. 5 – Comparisons between fitted S–N curves and experimental data (Stress range,  $\Delta\sigma$ , vs. number of cycles,  $N_f$ ).**

**Table 6 – Material parameters of CFC models.**

| Specimens     | B   | C   | $N_0$              | $\Delta\sigma_0$   | $\lambda$ | $\delta$ | $\beta$ |
|---------------|-----|-----|--------------------|--------------------|-----------|----------|---------|
| SLDSR         | 9.2 | 5.3 | $1.58 \times 10^9$ | $2.00 \times 10^5$ | 0.00      | 1.26     | 3.10    |
| DLDSR         | 0.0 | 1.2 | 1.0                | 15.8               | 43.73     | 9.12     | 15.25   |
| SLDSR + DLDSR | 0.0 | 1.0 | 1.0                | 10.0               | 46.33     | 11.25    | 2.80    |

$$\log(N) = A - B \log(\Delta\sigma) \quad (3)$$

where:  $A$  and  $B$  are regression parameters estimated by applying linear regression methods. These  $A$  and  $B$  regression parameters can be related to the  $C$  and  $m$  material and/or structural detail parameters of the design codes curves as follows:

$$C = 10^A \quad (4)$$

$$m = -B \quad (5)$$

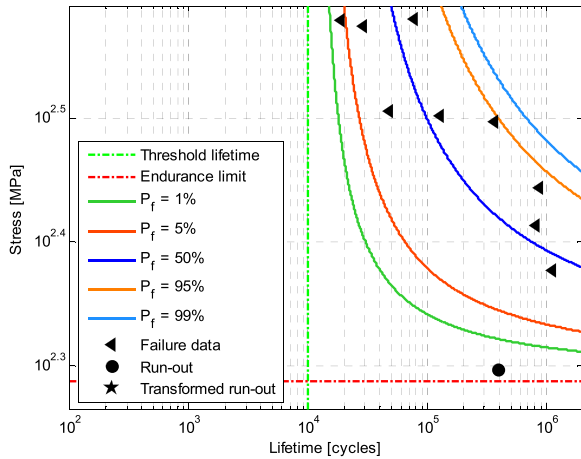
The material parameters  $A$  and  $B$  are fitted by the least-squares regression (LR) [40] and the orthogonal regression (OR) [41] using the linear Equation (3). The LR method (Fig. 4a) aims to approximate the solution by minimizing the sum of the squares of the residuals between every single data point and the regression line. The error in the independent variable is negligible of the LR method. The sum of the squares of the LR method is to be calculated in Equation (6). While the OR

method (Fig. 4b) assumes that both independent and dependent variables may be subject to error and the sum of squares is to be calculated in Equation (7).

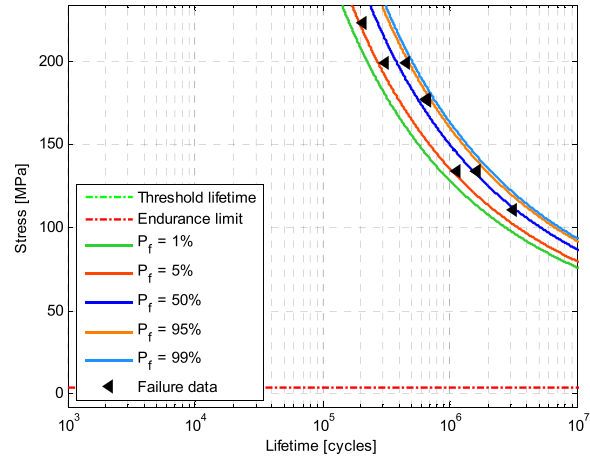
$$E_{LR} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n (y_i - B_{LR}x_i - A_{LR})^2 \quad (6)$$

$$E_{OR} = \sum_{i=1}^n \frac{(y_i - B_{OR}x_i - A_{OR})^2}{B_{OR}^2 + 1} \quad (7)$$

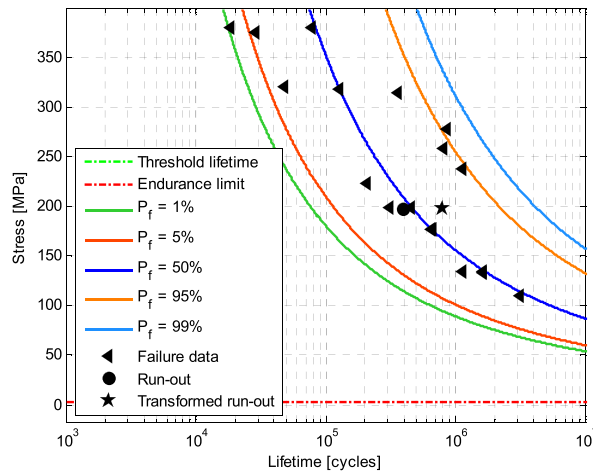
In Table 5, the fitted material parameters are summarized. The comparisons between the experimental observations and fitted  $S-N$  curves are shown in Fig. 5. The results showed that the fitted slope  $B$  from the OR method is larger than the result from the LR method for both specimen series. In the relatively lower-stress range (medium- and high-cycle regimes), the parameters from the OR method predicted the longer fatigue lives than that from the LR method, detailed as  $0 < \Delta\sigma \leq 305$  MPa for SLDSR specimen,  $0 < \Delta\sigma \leq 160$  MPa for DLDSR



(a) SLDSR



(b) DLDSR



(c) SLDSR+ DLDSR

Fig. 6 – P–S–N fields for different riveted details test data.

specimen, and  $0 < \Delta\sigma \leq 223$  MPa for “SLDSR + DLDSR” results. In the upper-stress range (low- and medium-cycle regimes), the parameters from the OR method predicted the shorter fatigue lives than that from the LR method, detailed as  $\Delta\sigma > 305$  MPa for SLDSR specimen,  $\Delta\sigma > 160$  MPa for DLDSR specimen, and  $\Delta\sigma > 223$  MPa for “SLDSR + DLDSR” results.

The standard derivation  $S_A$  for material parameter A and  $S_B$  for material parameter B is also calculated by definition of  $y = \log(N)$  and  $x = \log(\Delta\sigma)$  using Equations 8–10. The values of  $S_A$  and  $S_B$  are listed in Table 5. It is noted that the standard derivation obtained from the OR method is slightly larger than that from the LR method.

$$S_A = S \left( \frac{1}{n} + \frac{\bar{x}^2}{\sum x_i^2 - (\sum x_i)^2/n} \right)^{1/2} \tag{8}$$

$$S_B = S \left( \frac{1}{\sum x_i^2 - (\sum x_i)^2/n} \right)^{1/2} \tag{9}$$

$$S = \left( \frac{\sum (y_i - \bar{y}_i)^2}{n - 2} \right)^{1/2} \tag{10}$$

**Table 7 – Parameters estimated of each probability density function under consideration.**

| Name                   | Probability density function   |    |             | SLDSR | DLDSR  | SLDSR + DLDSR |
|------------------------|--|----|-------------|-------|--------|---------------|
| Gaussian distribution  | $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$  | LR | $\mu_A$     | 25.94 | 13.79  | 12.85         |
|                        |  |    | $\sigma_A$  | 3.64  | 0.61   | 1.20          |
|                        |  |    | $\mu_B$     | 8.30  | 3.58   | 3.10          |
|                        |  |    | $\sigma_B$  | 1.46  | 0.28   | 0.51          |
|                        |  |    |             |       |        |               |
|                        |  | OR | $\mu_A$     | 30.37 | 14.10  | 15.77         |
|                        |  |    | $\sigma_A$  | 4.01  | 0.63   | 1.41          |
|                        |  |    | $\mu_B$     | 10.08 | 3.72   | 4.34          |
|                        |  |    | $\sigma_B$  | 1.61  | 0.28   | 0.59          |
|                        |  |    |             |       |        |               |
| Lognormal distribution | $f_X(x) = \frac{1}{\sqrt{2\pi}\zeta x} \exp\left(-\frac{(\ln x - \lambda)^2}{2\zeta^2}\right)$   | LR | $\lambda_A$ | 3.24  | 2.62   | 2.55          |
|                        |  |    | $\zeta_A$   | 1.40  | 0.04   | 0.09          |
|                        |  |    | $\lambda_B$ | 2.10  | 1.27   | 1.12          |
|                        |  |    | $\zeta_B$   | 1.74  | 0.08   | 0.16          |
|                        |  |    |             |       |        |               |
|                        |  | OR | $\lambda_A$ | 3.41  | 2.05   | 2.75          |
|                        |  |    | $\zeta_A$   | 0.13  | 0.04   | 0.09          |
|                        |  |    | $\lambda_B$ | 2.30  | 1.31   | 1.46          |
|                        |  |    | $\zeta_B$   | 1.58  | 0.08   | 0.14          |
|                        |  |    |             |       |        |               |
| Weibull distribution   | $f_X(x) = \begin{cases} \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^\beta}, & \text{if } x \geq 0 \\ 0, & x < 0 \end{cases}$ | LR | $\alpha_A$  | 27.50 | 14.07  | 13.36         |
|                        |  |    | $\beta_A$   | 8.48  | 28.07  | 13.06         |
|                        |  |    | $\alpha_B$  | 8.90  | 3.70   | 3.31          |
|                        |  |    | $\beta_B$   | 6.67  | 15.80  | 7.18          |
|                        |  |    |             |       |        |               |
|                        |  | OR | $\alpha_A$  | 32.06 | 14.38  | 16.38         |
|                        |  |    | $\beta_A$   | 9.06  | 28.19  | 13.71         |
|                        |  |    | $\alpha_B$  | 10.74 | 3.84   | 4.60          |
|                        |  |    | $\beta_B$   | 7.41  | 1.61   | 8.70          |
|                        |  |    |             |       |        |               |
| Gamma distribution     | $f_X(x) = \frac{\lambda^k}{\Gamma(k)} x^{k-1} e^{-\lambda x}$  | LR | $\lambda_A$ | 1.95  | 36.49  | 8.93          |
|                        |  |    | $k_A$       | 50.67 | 503.40 | 114.70        |
|                        |  |    | $\lambda_B$ | 3.90  | 46.19  | 11.98         |
|                        |  |    | $k_B$       | 32.39 | 165.20 | 37.14         |
|                        |  |    |             |       |        |               |
|                        |  | OR | $\lambda_A$ | 1.89  | 35.99  | 7.98          |
|                        |  |    | $k_A$       | 57.33 | 507.70 | 125.80        |
|                        |  |    | $\lambda_B$ | 3.91  | 46.29  | 12.22         |
|                        |  |    | $k_B$       | 39.40 | 172.10 | 53.09         |
|                        |  |    |             |       |        |               |
| Logistic distribution  | $f_X(x) = \frac{e^{-\frac{x-\mu}{s}}}{s \left(1 + e^{-\frac{x-\mu}{s}}\right)^2}$  | LR | $\mu_A$     | 25.94 | 13.79  | 12.85         |
|                        |  |    | $s_A$       | 2.01  | 0.34   | 0.66          |
|                        |  |    | $\mu_B$     | 8.30  | 3.58   | 3.10          |
|                        |  |    | $s_B$       | 0.80  | 0.28   | 0.28          |
|                        |  |    |             |       |        |               |
|                        |  | OR | $\mu_A$     | 30.37 | 14.10  | 15.77         |
|                        |  |    | $s_A$       | 2.21  | 0.35   | 0.78          |
|                        |  |    | $\mu_B$     | 10.08 | 3.72   | 4.34          |
|                        |  |    | $s_B$       | 0.89  | 0.16   | 0.33          |
|                        |  |    |             |       |        |               |
| Gumbel distribution    | $f_X(x) = \frac{1}{\beta} e^{-\frac{x-\mu}{\beta}} - e^{-\frac{x-\mu}{\beta}}$   | LR | $\mu_A$     | 24.30 | 13.52  | 12.31         |
|                        |  |    | $\beta_A$   | 2.84  | 0.48   | 0.94          |
|                        |  |    | $\mu_B$     | 7.65  | 3.45   | 2.87          |
|                        |  |    | $\beta_B$   | 1.14  | 0.22   | 0.40          |
|                        |  |    |             |       |        |               |
|                        |  | OR | $\mu_A$     | 28.56 | 13.82  | 15.14         |
|                        |  |    | $\beta_A$   | 3.13  | 0.49   | 1.09          |
|                        |  |    | $\mu_B$     | 9.36  | 3.59   | 4.08          |
|                        |  |    | $\beta_B$   | 1.25  | 0.22   | 0.46          |
|                        |  |    |             |       |        |               |



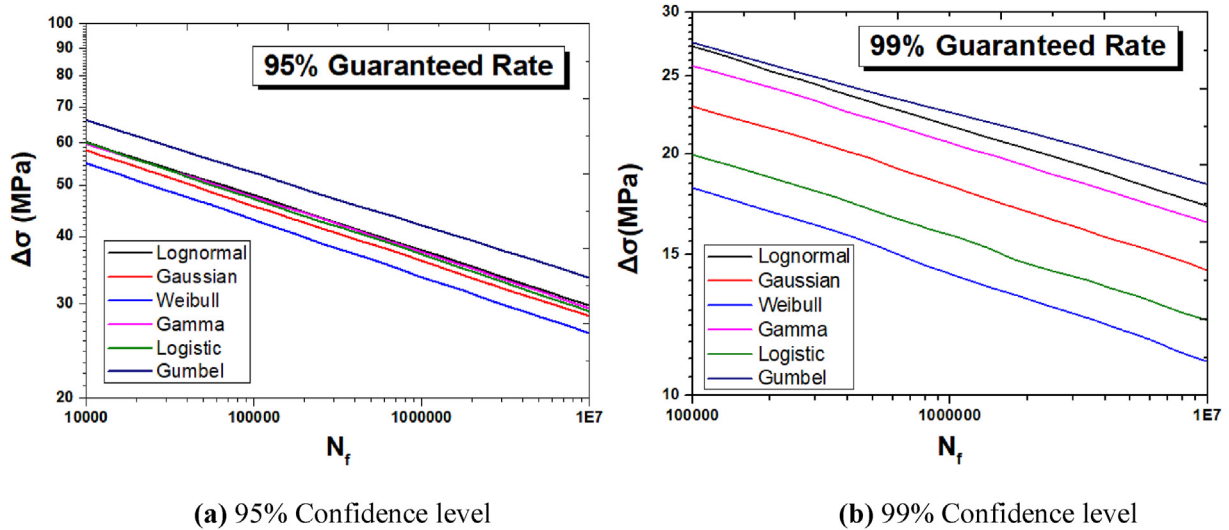


Fig. 7 – S–N curves comparison with 95% and 99% Confidence levels obtained with different probabilistic functions using the standard derivation of the least-squares regression.

Table 8 – The fitted material parameters of the S–N curve with different probabilistic functions using the standard derivation of the least-squares regression.

| Functions |                      | SLDSR   |         | DLDSR   |        | SLDSR + DLDSR |        |
|-----------|----------------------|---------|---------|---------|--------|---------------|--------|
|           |                      | A       | B       | A       | B      | A             | B      |
| Lognormal | 95% confidence level | 21.6588 | 10.1485 | 13.0252 | 3.9188 | 11.4617       | 3.7717 |
|           | 99% confidence level | 20.6714 | 11.3244 | 12.7505 | 4.0869 | 10.9360       | 4.0992 |
| Gaussian  | 95% confidence level | 21.0050 | 9.8791  | 12.9885 | 3.9028 | 11.2503       | 3.6693 |
|           | 99% confidence level | 19.1775 | 10.5856 | 12.6515 | 4.0240 | 10.6007       | 3.8999 |
| Weibull   | 95% confidence level | 20.3990 | 9.6169  | 12.8141 | 3.8143 | 10.9911       | 3.5732 |
|           | 99% confidence level | 17.1774 | 9.9466  | 12.1291 | 3.8301 | 9.7515        | 3.6539 |
| Gamma     | 95% confidence level | 21.4503 | 10.0389 | 13.0264 | 3.9092 | 11.4144       | 3.7368 |
|           | 99% confidence level | 19.9581 | 11.0811 | 12.7358 | 4.0643 | 10.8745       | 4.0640 |
| Logistic  | 95% confidence level | 21.1522 | 9.9110  | 12.9840 | 3.8874 | 11.2683       | 3.6684 |
|           | 99% confidence level | 18.4872 | 10.5598 | 12.6292 | 4.0711 | 10.4211       | 3.9203 |
| Gumbel    | 95% confidence level | 22.5159 | 10.3895 | 13.2417 | 4.0001 | 11.7341       | 3.8388 |
|           | 99% confidence level | 22.2392 | 12.1443 | 13.1700 | 4.3383 | 11.5937       | 4.4508 |

Table 9 – The fitted material parameters of the S–N curve with different probabilistic functions using the standard derivation of the orthogonal regression.

| Functions |                      | SLDSR   |         | DLDSR   |         | SLDSR + DLDSR |        |
|-----------|----------------------|---------|---------|---------|---------|---------------|--------|
|           |                      | A       | B       | A       | B       | A             | B      |
| Lognormal | 95% confidence level | 25.7537 | 12.1197 | 13.3356 | 4.0610  | 14.0943       | 5.0960 |
|           | 99% confidence level | 24.3500 | 13.3060 | 13.0626 | 4.2466  | 13.5625       | 5.5246 |
| Gaussian  | 95% confidence level | 24.9809 | 11.8470 | 13.2991 | 4.0452  | 13.9302       | 5.0353 |
|           | 99% confidence level | 22.7827 | 12.5331 | 12.9596 | 4.1832  | 13.1197       | 5.2771 |
| Weibull   | 95% confidence level | 24.2640 | 11.4933 | 13.1041 | 3.9489  | 13.5559       | 4.8543 |
|           | 99% confidence level | 20.6008 | 11.7408 | 12.4229 | 3.9780  | 12.0849       | 4.9607 |
| Gamma     | 95% confidence level | 25.4870 | 12.0189 | 13.3205 | 4.0577  | 14.0315       | 5.0596 |
|           | 99% confidence level | 23.9104 | 13.1328 | 13.0372 | 4.2156  | 13.4129       | 5.4324 |
| Logistic  | 95% confidence level | 25.1926 | 11.8616 | 13.2986 | 4.0421  | 13.9539       | 4.9884 |
|           | 99% confidence level | 22.0841 | 12.5543 | 12.8603 | 4.1972  | 13.0315       | 5.3303 |
| Gumbel    | 95% confidence level | 26.6035 | 14.4410 | 4.1374  | 13.5354 | 14.4593       | 5.2103 |
|           | 99% confidence level | 26.2648 | 14.4655 | 4.4840  | 13.4743 | 14.3349       | 5.9853 |

The root-mean square errors (RMSE) obtained from the OR and LR methods are presented in Table 5. It allows to verify that the root-mean square errors (RMSE) obtained from the OR method are slightly larger than that from the LR method.

### 3. Castillo & Fernández-Canteli (Cfc) method

Castillo and Fernández-Canteli (CFC) [35] proposed a probabilistic  $\Delta\sigma$ -N relationship based on the Weibull distribution as expressed in Equation (11):

$$P_f(N, \Delta\sigma) = 1 - \exp \left[ - \left( \frac{(\log N - B)(\log \Delta\sigma - C) - \lambda}{\delta} \right)^\beta \right] \quad V = (\log N - B)(\log \Delta\sigma - C) \geq \lambda \quad (11)$$

where:  $P_f$  is the failure probability;  $N$  is the number of cycles to failure;  $\Delta\sigma$  is the stress range;  $B$  is the threshold parameter for

the lifetime;  $C$  is threshold parameter for stress range,  $\lambda$  is threshold parameter for normalized variable  $V$ ;  $\delta$  is the scale parameter;  $\beta$  is the Shape parameter of Weibull model.

The material parameters of the CFC model are fitted based on experimental results for “SLDSR”, “DLDSR” and “SLDSR + DLDSR” test series. The fitted parameters are summarized in Table 6. The S–N curves with different probabilities are presented in Fig. 6. The results showed that the threshold parameter  $B$  and  $C$  of “SLDSR” test series is larger than that from “DLDSR” and “SLDSR + DLDSR” test series while the threshold parameter  $\lambda$  of the normalized variable of “SLDSR” is smaller.

### 4. Stochastic analysis method

In the stochastic analysis, fatigue life can be predicted by the materials parameters of S–N curves and a corresponding probability density function (PDF). This section discussed the probabilistic functions, regression methods, and exponent effects on probabilistic fatigue life.

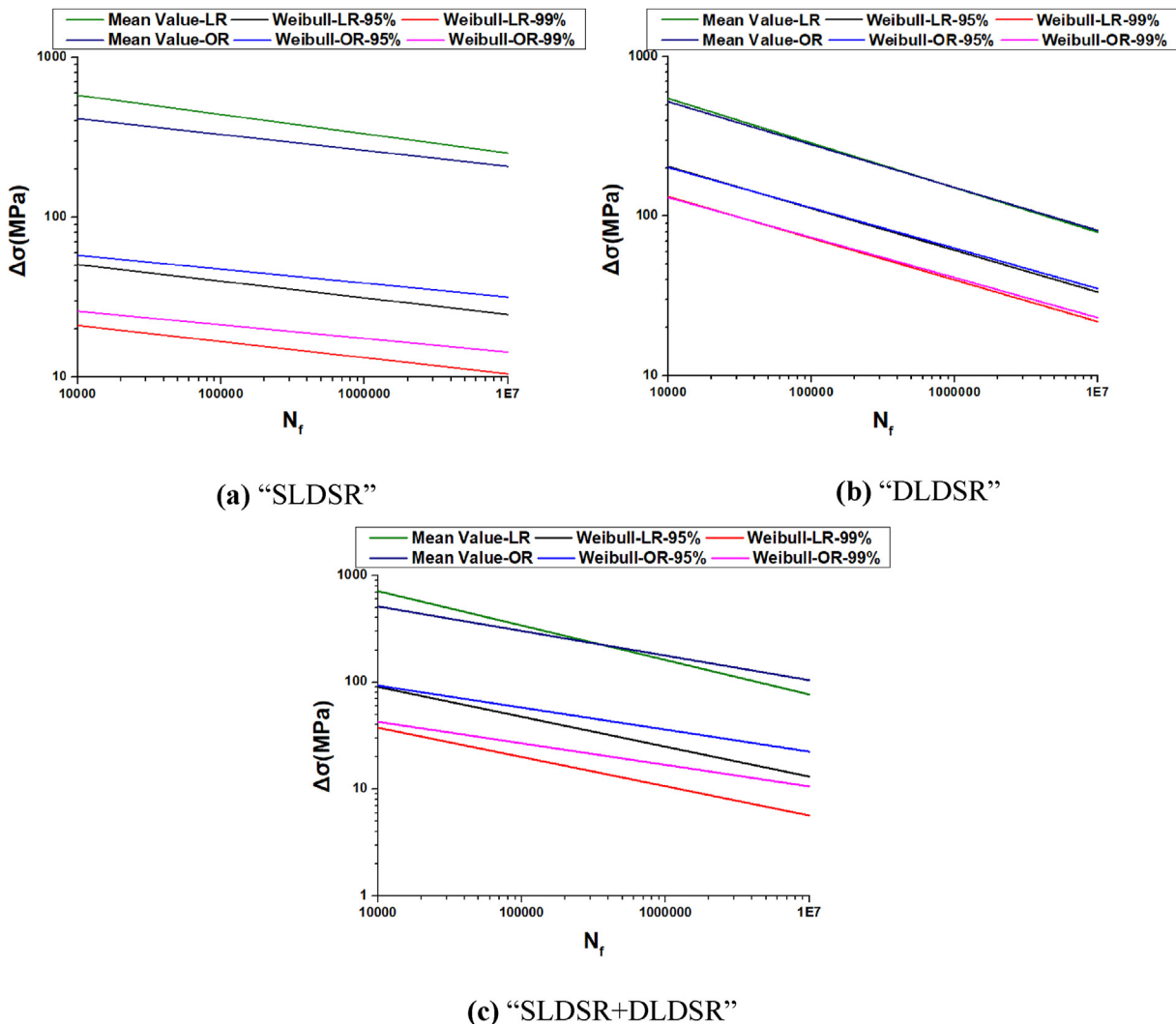


Fig. 8 – Regression method effects on probabilistic fatigue life – Weibull distribution.

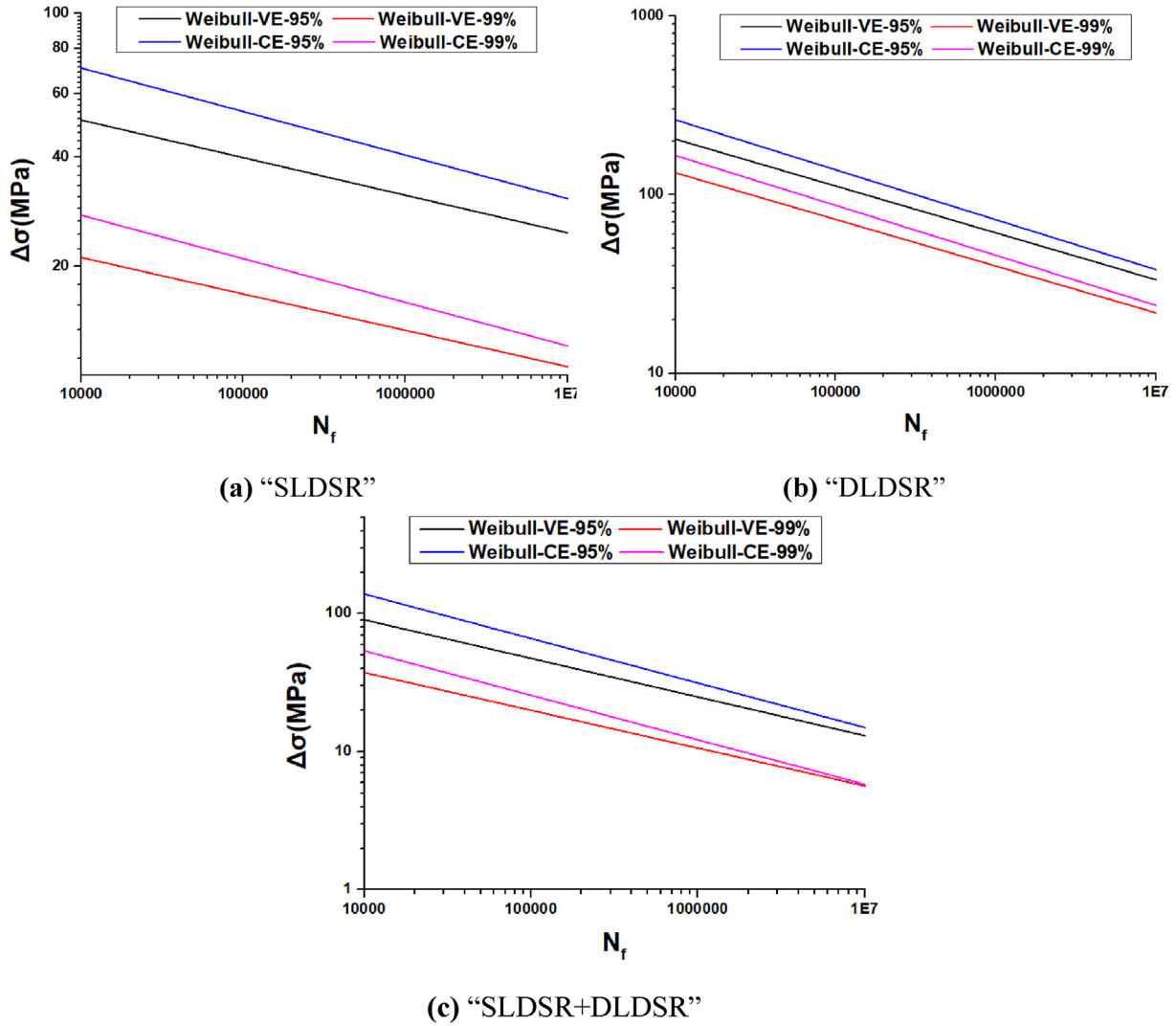


Fig. 9 – Regression method effects on probabilistic fatigue life – Weibull distribution.

4.1. Probabilistic functions effects

Six different probabilistic functions, namely Gaussian distribution, Lognormal distribution, Weibull distribution, Gamma distribution, Logistic distribution, and Gumbel distribution, were used to obtain the probabilistic fatigue life [42]. The Probability density function and its estimated parameters of different distributions are presented in Table 7. A stochastic analysis using Latin hypercube sampling strategies [43] of the S–N curves of the SLDSR test series using the standard derivation of the least-squares regression, for different

probabilistic functions, was used. The S–N curves comparison with 95% and 99% confidence levels obtained with different probabilistic functions using the standard derivation of the least-squares regression are shown in Fig. 7. Noted that the confidence level can be calculated as one minus the failure probability in Equation (10). The results showed that the stochastic analysis with the Gumbel distribution contributed the largest fatigue strength with 95% and 99% confidence level while the stochastic analysis with the Weibull distribution led to the smallest fatigue strength with 95% and 99% confidence level. The material parameters A and B of the S–N curves with

Table 10 – The exponent effects on the probabilistic fatigue life using the standard derivation of the least-squares regression.

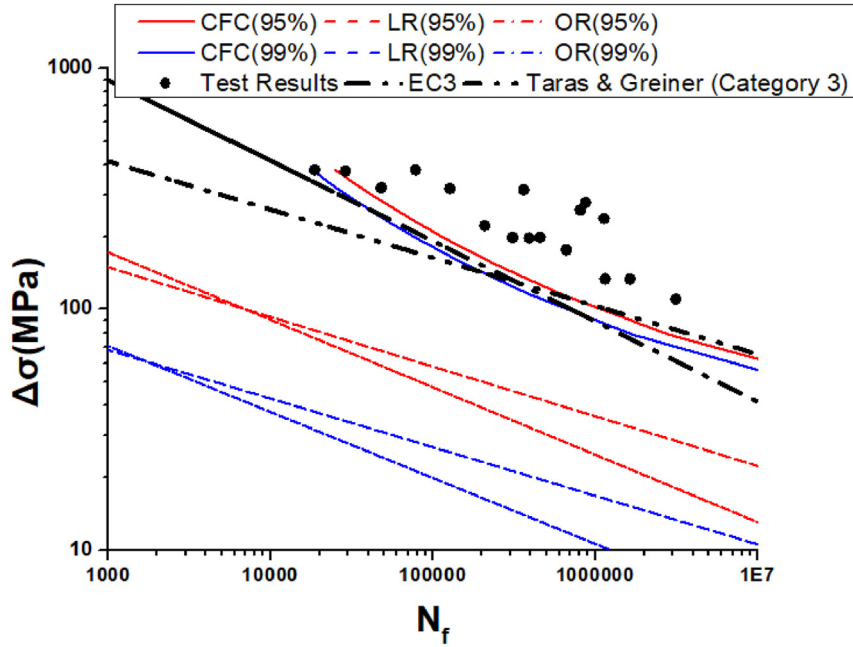
| Functions |          |     | SLDSR   |        | DLDSR   |        | SLDSR + DLDSR |        |
|-----------|----------|-----|---------|--------|---------|--------|---------------|--------|
|           |          |     | A       | B      | A       | B      | A             | B      |
| Weibull   | Constant | 95% | 19.3562 | 8.3040 | 12.6533 | 3.5772 | 10.6457       | 3.1008 |
|           |          | 99% | 15.9741 | 8.3040 | 11.9397 | 3.5772 | 9.3692        | 3.1008 |
|           | Varied   | 95% | 20.3990 | 9.6169 | 12.8141 | 3.8143 | 10.9911       | 3.5732 |
|           |          | 99% | 17.1774 | 9.9466 | 12.1291 | 3.8301 | 9.7515        | 3.6539 |

95% and 99% confidence level is fitted in terms of different test series, listed in Table 8 for the LR method and Table 9 for the OR method.

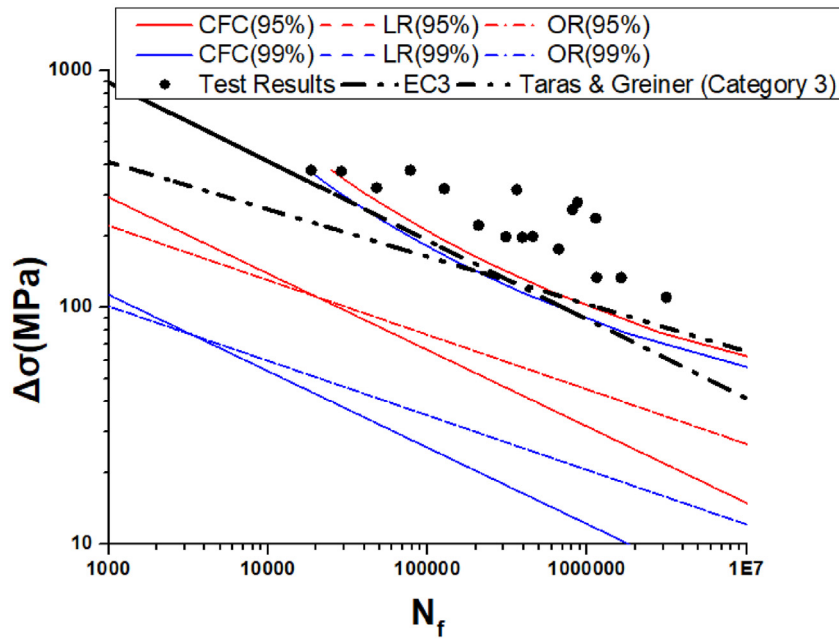
4.2. Regression method effects

The S–N curves with 95% and 99% confidence levels using different standard derivation are compared in Fig. 8a, b and c

for “SLDSR”, “DLDSR” and “SLDSR + DLDSR” test series for both LR and OR methods, respectively, using the Weibull distribution. The trend on the regression methods on the probabilistic fatigue life is not obvious. In terms of the “SLDSR” test series, the fatigue strength with 95% confidence level obtained using a standard derivation of LR methods is smaller than obtained using a standard derivation of LR methods, but the fatigue strength with 99% confidence level obtained using



(a) Varied exponent



(b) Constant exponent

Fig. 10 – Comparison between the CFC model and stochastic analysis using the Weibull distribution.

a standard derivation of LR methods is larger than obtained using a standard derivation of LR methods. In terms of the “DLDSR” test series, the differences of the fatigue strength with 95% and 99% confidence levels obtained using a standard derivation of LR and OR methods are quite small concerning the Weibull distribution. In terms of the “SLDSR + DLDSR” test series, the fatigue strength with 95% and 99% confidence levels obtained using the LR method is smaller than the one obtained using the OR method.

#### 4.3. Exponent effects

To investigate the exponent effects, the parameters  $B$  kept as a constant during stochastic analysis, denoted as “CE”, while the parameters  $B$  in the previous analysis is varied with assumed probabilistic function, denoted as “VE”. The  $S-N$  curves with 95% and 99% confidence levels using varied and constant exponents are compared in Fig. 9 with the lower bound using the Weibull distribution. The results showed that the fatigue strength with 95% and 99% confidence levels obtained using constant exponent is larger than using a variable parameter. The material parameters using the standard derivation of the least-squares regression with a varied and constant exponent are summarized in Table 10.

### 5. Comparison between Cfc and stochastic analysis method

The comparison between the test results, CFC model, stochastic analysis [43], EC3 [18] (class 71  $S-N$  curve), and Category 3 proposed by Taras & Greiner [20,21] is shown in Fig. 10 for Weibull distribution. The results showed that the fatigue strength with both 95% and 99% confidence levels obtained using the stochastic analysis with the Weibull distribution is smaller than that of the CFC model, EC3 (class 71  $S-N$  curve) and  $S-N$  curves of Category 3 proposed by Taras & Greiner. The probabilistic fatigue life with stochastic analysis using constant exponent is closer to the CFC model than that based on variable parameter.

### 6. Conclusions

With growing concern about the structural reliability and integrity, the probabilistic fatigue behaviour of riveted joints has been deserved more attention, more specifically about the influence that this may have on the fatigue damage accumulation evaluation of structural components. The fatigue life considering multiple uncertainty sources' effects is essential for the determination of the riveted details. The following conclusions could be drawn.

- (1) The parameters of  $S-N$  curves for two riveted joints were obtained using the least-squares regression (LR) and the orthogonal regression (OR) methods, respectively, in this paper. The results showed that the fitted

slope  $B$  from the OR method is slightly larger than that from the LR method of all specimens. In the relatively lower-stress range, the parameters from the OR method predicts the larger fatigue lives than from the LR method, detailed as  $0 < \Delta\sigma \leq 305$  MPa for SLDSR specimen,  $0 < \Delta\sigma \leq 160$  MPa for DLDSR specimen, and  $0 < \Delta\sigma \leq 223$  MPa for “SLDSR + DLDSR” results. In the upper-stress range, the parameters from the OR method predict the smaller fatigue life than from the LR method, detailed as  $\Delta\sigma > 305$  MPa for SLDSR specimen,  $\Delta\sigma > 160$  MPa for DLDSR specimen, and  $\Delta\sigma > 223$  MPa for “SLDSR + DLDSR” results. Therefore, in the fatigue damage accumulation assessment of structural components, the design  $S-N$  curves based on the LR or OR methods, more precisely for high-cycle regimes, may estimate substantially different damage values.

- (2) The probabilistic functions, regression methods and exponent effects on the probabilistic fatigue life are discussed. Among six different probabilistic functions, the stochastic analysis with the Gumbel distribution contributed the largest fatigue strength with 95% and 99% confidence levels while the stochastic analysis with the Weibull distribution led to the smallest fatigue strength with 95% and 99% confidence levels. The effects of regression methods on probabilistic fatigue life are not obvious. The fatigue strength with 95% and 99% confidence levels obtained using constant exponent is larger than that resulting from variable exponent.
- (3) The fatigue strength with both 95% and 99% confidence rate obtained using the stochastic analysis with the Weibull distribution is smaller than that provided by the CFC model, EC3 and  $S-N$  curves of Category 3 proposed by Taras & Greiner. The probabilistic fatigue life with stochastic analysis using constant exponent is closer to the CFC model than using variable exponent.

### Credit authorship Contribution statement

José Correia: formal analysis, writing - original draft, validation, supervision, writing - review & editing. António Mourão: formal analysis, investigation, writing - original draft. Haohui Xin: formal analysis, investigation, writing - original draft. Abílio De Jesus: formal analysis, writing - review & editing, validation, supervision. Túlio Bittencourt: formal analysis, validation, supervision, writing - review & editing. Rui Calçada: formal analysis, validation, supervision, writing - review & editing. Filippo Berto: formal analysis, validation, writing - review & editing.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.



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