Economics students' perception of effective teaching of mathematics at the tertiary level

F. Radmehr, M. Turgut, *Norwegian University of Science and Technology*

ABSTRACT: a growing body of literature in mathematics education focuses on how teaching and learning mathematics could be improved in service mathematics courses of different STEM major programs. Service mathematics courses are part of economics programs in many countries due to the importance of mathematics in this field. In this qualitative study, we focus on norwegian economics students' perceptions of the effective teaching of (service) mathematical courses in their study program. Through a phenomenology study, six economics students participated in one-to-one semi-structured interviews where among other things, their perceptions of effective teaching of mathematics were explored. This study also benefits from the theory of commognition as the theoretical framework to analyse students' perception of effective mathematics teaching at the university level. We identified six strands for effective mathematics teaching in economics programs, such as using more exemplary and authentic tasks during lectures to ease students' participation and providing more opportunities for students to participate in the discourse about new mathematical objects with adept participants (e.g., lecturers and teacher assistants), both in lectures and in tutorials. We hope this study encourage more collaboration between tertiary mathematics educators and mathematics lecturers to find ways to improve teaching mathematics to economics students.

1 INTRODUCTION

Several studies investigated university students' perceptions of teaching and learning at higher education (e.g., Choi, 2021; Radmehr et al., 2020). Past research highlighted that we could obtain a more comprehensive understanding of how the quality of teaching and learning can be improved if we position students as pedagogical consultants (Cook-Sather, 2011). Ramsden (2003) pointed out: "...we cannot teach better unless we are able to see what we are doing from their [students] point of view" (p. 84). This research continues this line of research by exploring how Norwegian economics students experience teaching and learning mathematics at the university level. This cohort of students is selected as our literature search indicates that only a few studies have focused on teaching and learning mathematics to economics students, suggesting this cohort of university students was overlooked in tertiary mathematics education research.

2 MATHEMATICS IN ECONOMICS

Educators discussed the importance of mathematics in economics and the possible relationship between these disciplines (Anthony & Biggs, 2012; Katzner, 2003). For instance, Katzner (2003) pointed out four main reasons why mathematics has become so important in economics, including mathematics helps with achieving "scientific respectability … [and] assure security with respect to claims of truth" (p. 561) in economics. Furthermore, Anthony and Biggs (2012) highlighted that mathematics is significant for economics to express and communicate ideas and describe economic situations leading mathematical modelling to be an indispensable tool in economics. In addition, solid knowledge of mathematics is a prerequisite for economics students to understand state of the art in this field (Werner & Sotskov, 2006). Those few studies that investigated teaching and learning mathematics to economics students focused on teaching and learning of calculus (e.g., Feudel & Biehler, 2021a, 2021b; Goodman, 2010). For instance, Feudel and Biehler (2021a, 2021b) explored how economics students interpret the derivative in the context of marginal cost, a common economics interpretation of the derivative. They found that economics students struggle to use what they learned in their calculus courses when working in this context.

3 COMMOGNITION

The theory of commognition has attracted the attention of several tertiary mathematics educators and has been used as an analytical tool to explore ways in which teaching and learning of mathematics at the university level could be improved (e.g., Viirman & Nardi, 2019). In this theory, mathematics is considered a product of human discourse where discourse is a type of communication distinguishable by four main characteristics: word use (e.g., differentiable), visual mediators (e.g., the derivative symbol), routines (e.g., how to find the derivative of trigonometric functions), and endorsed narratives (e.g., the derivative of the sum of two functions is the sum of the derivatives of the functions) (Sfard, 2008). In this theory, three mathematical routines have been discussed: rituals (process-oriented routine), deeds (practical product-oriented routine), and explorations (discursive product-oriented routine) (Lavie et al., 2019), where rituals and deeds are predecessors of explorations (Sfard, 2008). Mathematical routines to be truly useful "must evolve into full-fledged explorations" (Sfard, 2008, p. 167); however, previous studies reported that transitioning from ritual to exploration is a gradual and slow process, and it does not happen for many students in school (Sfard, 2017). Such transformation is called *de-ritualization*, and a number of characteristics have been identified for it, including *flexibility* (the ability to perform a task in more than one way), *bondedness* (all the steps taken when performing a task is necessary to achieve the results, and no redundancy actions have been made by the discursant), *applicability* (the ability to use the procedure in several contexts), *agentivity* (the ability to make independent decisions while performing a task without receiving guidance from others), *objectification* (the ability to tell a narrative on mathematical objects using the properties of the objects without relying on previous relevant experience with concrete objects), and *substantiability* (the ability to justify actions such that is acceptable by adept discourse participants) (Lavie et al., 2019; Sfard, 2008).

In commognition, two types of learning have also been discussed: object-level and meta-level. Object-level learning refers to "extending the existing sets of endorsed narratives about already constructed mathematical objects" (Sfard, 2020, p. 98). On the contrary, meta-level learning is "changes in meta-rules of the discourse" (Sfard, 2020, p. 98), usually making the new discourse incommensurable with its predecessor. As a consequence, a number of previously endorsed narratives become unacceptable, and some are subsumed in the new discourse (Sfard, 2020). Lecturers have an important role in meta-level learning. While object-level learning theoretically could happen without the direct help of a lecturer (or an adept participant of the discourse), meta-level learning requires such interactions (Sfard, 2017, 2020). Sfard (2020) highlighted: "This type of learning cannot be motivated or guided by the learner's own genuine interest in the outcome. For the student, the only way to enter the discourse is to imitate teacher's expert performances" (p. 99). Such participation is called ritual participation, and it is inevitable in the early stages of meta-level learning and also when learning new mathematical objects (Lavie et al., 2019; Sfard, 2017). Lectures could facilitate the de-ritualization "by demonstrating the type of explorative discourse they would like their students to develop [and] explicitly encourage the desired kind of discourse by appropriate pedagogical moves" (Sfard, 2017, p. 44).

4 METHODOLOGY

The qualitative research reported here is a phenomenology study (Creswell & Poth, 2018), where we discuss how six Norwegian economics students perceived the teaching of mathematics in their economics program. The economic students participated in one-to-one semi-structured interviews with the first author, where their perceptions of teaching and learning mathematics at the university level were explored. These economic students have completed their mandatory mathematics services courses as part of their economics programs at the time of the interview and therefore could contribute to the aim of the study. The majority of the interviews lasted for 30 minutes and were conducted in English, not the students' first language. An exemplary interview question was: In your opinion, what is an example of effective teaching of mathematics at the university level?

The interviews were audio recorded and transcribed. Then a thematic analysis was used to analyse the collected data. The analysis was inductive in nature; however, the emerging themes were reported in terms of different aspects of commognition (when it was feasible). We also considered several criteria of excellence in qualitative research that was highlighted in the literature (see Tracy, 2010). First, we chose a *worthy topic* that was overlooked in past research to make a *significant contribution* to the tertiary mathematics education literature. We also provided several quotes in the results section to improve the *credibility* of the findings. Furthermore, the Norwegian data protection agency (NSD) approved this study, which monitors the *ethical* issues concerning collecting personal data and the General Data Protection Regulations (GDPR).

5 RESULTS

An analysis of six interviews revealed six main strands for effective mathematics teaching in economics. Table 1 overviews the main strands and associated tenets.

Table 1. An overview of the identified strands and associated tenets

Strand	The main tenets
Lecturer participation when students learn a new mathematical object	• Direct step-by-step instruction followed with basic examples solved by the lecturer at the beginning of students' participation in a discourse about a new mathematical object. • Students are given opportunities to solve (short) tasks in the lecture to ease their participation; however, those tasks are then solved by the lecturer in the same session.
The context of exemplary tasks	• Using practical examples addressing why students learn the topic and how they can use it in economics. • Using authentic tasks.
Lecturer preparedness and use of resources	• The lecturer needs to be well-prepared and upload everything to online platforms before teaching. • Using blended learning resources (e.g., videos, worksheets), not just using PowerPoints when teaching. • Using consistent terminologies in the resources, especially when using keywords and visual mediators. • Inviting economists to participate in the university discourse and discuss how mathematics could be used in the field as an outsider adept participant.
Student learning and routine development	• The lecturer needs to consider students' learning - they initially participate in the discourse ritually. • The pace of lecturer participation in the discourse should not be fast.
Lecturer & teacher-assistant (TA) participation in the tutorials	• Visiting groups to ensure student learning and facilitating the de-ritualization process. • Solving the challenging tasks for students in front of all groups. • Lecturers and TAs should have high societal and mathematical competence
The length of discourse (teaching)	• Shorter but more frequent teaching episodes (e.g., two times two-hour teaching in a week instead of one four-hour session)

All students think that the lecturers' participation in the discourse is high and how they arrange student participation in this discourse is important. Students underlined that they prefer direct instruction followed by basic examples at the beginning of their participation in a discourse about a new mathematical object. For example, Emma pointed out that "… the teacher [can] starts from very simple terms … explaining the process behind the formulas and then teaching us how to solve it, why we do it". Students emphasized that short tasks could be given to students to solve in the lecture just after basic examples had been discussed by the lecturer to ease their participation in the discourse. Nora further highlighted what many students prefer when they ritually participate in the discourse: the lecturers should "...give us some examples [task] in the lecture… give us time to think, and then later solve them. So that the students shouldn't look after the answers when they get back home … I need to do some tasks at home still but just cover as much as [possible] I need to do to be able to work on it on my own".

The second main strand is the context of classroom examples/tasks shared in the discourse. Students perceived that careful use of "practical" examples could facilitate the de-ritualisation process, at least through helping students realize the *applicability* of the mathematical objects discussed in the discourse and advance why students learn it. Furthermore, using real data could consolidate their understanding of how they can use mathematical knowledge in practice. For example, Oliver emphasized that "… like get some practical examples from it. What can I use this to? … This is things that I want to see. If I don't know why I'm learning it, no, I don't like math. So, I want to know why we're learning it and what we can use it to".

The third strand is about the lecturer's preparedness and use of resources for teaching to facilitate the de-ritualization process. It is about the effective use of supplementary resources and making them ready before teaching. Resources need to be ready in advance and should not be "drowning", as characterized by Nora, and resources should not be limited to (only) slides. For example, Tobias underlined that it could be more effective the use of short videos in teaching rather than lecturing for 45 minutes. Another student, Magnus, spoke about the mathematical keywords and visual mediators, the use of consistent terminologies among resources. It seems it is difficult for students to realize different realizations (*in the commognition, the term realization is used instead of representation*) of mathematical objects when they have ritual participation in the discourse. Furthermore, Oliver suggested inviting economists as outsider adept participants to show practical examples in the lectures.

The fourth strand is about lecturers considering student learning and routine development in the mathematics discourse, particularly when students participate in the discourse ritually. Tobias discussed moving step-by-step in teaching and said that the lecturers "… should really make that the students really understand what they're doing, not going too fast forward because if they get too fast forward the students really, they just hop out…". He pointed out that the students might feel lost and stupid, and they would not show up again.

The fifth strand is about students' participation in tutorials (also known in Norway as seminar/group work assignments setting) with adept participants of the discourse, i.e., lecturers and TAs. During ritual participation in tutorial sessions, students prefer to be visited by lecturers and TAs from time to time. For example, Frida underlined this by "… like one thing is that you [the lecturer] open up for if you have a question, you can open the door, or like that. Just to step by like if you have some free time because no one asking questions just like going, hey, how are you doing…". The idea of solving the common problem(s) was proposed by Tobias, stating "… if some questions that many students have problem with it. I will probably if it is a big class, I will solve it in the class". And Magnus highlighted that the societal and mathematical competencies of the lecturers and TAs are quite important to have a fruitful discourse in tutorials. If they are competent in these areas, they can imagine the students' needs and could create an open dialogue between students too.

The sixth strand is about the time length of the mathematical discourse. Magnus criticized that it could be more teaching episodes more than once a week, like twice two-hour teaching weekly. So that they can be exposed to more mathematics in a week, which could provide more opportunities for them to have explorative participation in the discourse and go through mathematical ideas in depth.

6 DISCUSSION AND CONCLUSIONS

In this paper, we have explored economics students' perception of effective mathematics teaching at the university level, an overlooked area of tertiary mathematics education. We have identified six main strands regarding economic students' perception of effective teaching. For many economics students, effective teaching of mathematics at the university level is characterised by lecturers providing several types of support when students participate in a discourse about a new mathematical object, including providing a variety of resources for students to learn about the new mathematical object(s), providing step-by-step guidance on how to do a particular routine related to the new mathematical object, using consistent keywords and visual mediators during participation with students and also in resources, and providing many opportunities for students to participate in the discourse with adept participants (e.g., lecturers and TAs). As highlighted in the commognition theory (Sfard, 2008, 2017, 2020), students first participate ritually in a discourse about a new mathematical object and the initial stage of meta-level learning, and lecturers and TAs could take several actions to ease students' de-ritualization. For instance, Sfard (2017) noted lecturers could facilitate de-ritualization by demonstrating how students could exploratively participate in the discourse and also explicitly encourage such participation in interaction with students. Furthermore, the findings indicate that economics students would appreciate more authentic and exemplary tasks discussed in the lectures. The exemplary tasks could help students imitate the lecturers' participation (Sfard, 2020), and using authentic tasks could make teaching more interesting for many students, particularly those interested in mathematics learning, just because of its *use-value* (Williams, 2012), as shown by Oliver in the interview. We hope the findings shared here could initiate more collaboration between tertariy mathematics educators and mathematics lecturers to find ways in which teaching and learning of mathematics could be improved for this cohort of university students.

REFERENCES

Anthony, M., & Biggs, N. (2012). *Mathematics for Economics and Finance: Methods and Modelling*. Cambridge University Press.

Choi, L. J. (2021). English as an important but unfair resource: university students' perception of English and English language education in South Korea. *Teaching in Higher Education*, 1–15. https://doi.org/10.1080/13562517.2021.1965572

Cook-Sather, A. (2011). Layered learning: Student consultants deepening classroom and life lessons. *Educational Action Research, 19*(1), 41–57. https://doi.org/10.1080/09650792.2011.547680

Creswell, J. W., & Poth, C. N. (2018). *Qualitative Inquiry and Research Design: Choosing Among Five Approaches* (4th edition). Sage publications.

Feudel, F., & Biehler, R. (2021a). Students' understanding of the derivative concept in the context of mathematics for economics. *Journal für Mathematik-Didaktik,42*(1), 273–305. https://doi.org/10.1007/s13138-020-00174-z

Feudel, F., & Biehler, R. (2021b). Students' understanding of the economic interpretation of the derivative in the context of marginal cost. *International Journal of Research in Undergraduate Mathematics Education*, 1–32. https://doi.org/10.1007/s40753-021-00144-x

Goodman, R. J. B. (2010). Problem-based learning: Merging of economics and mathematics. *Journal of Economics and Finance, 34*(4), 477–483. https://doi.org/10.1007/s12197-010-9154-7

Katzner, D. W. (2003). Why mathematics in economics? *Journal of Post Keynesian Economics, 25*(4), 561–574. https://doi.org/10.1080/01603477.2003.11051373

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics,101*(2), 153–176. https://doi.org/10.1007/s10649-018-9817-4

Radmehr, F., Laban, H. L. W., Overton, J., & Bakker, L. (2020). Student perceptions of effective lecturers: the need to recognise the role of ethnicity and choice of discipline. *Higher Education Research & Development, 39*(2), 302–317. https://doi.org/10.1080/07294360.2019.1674789

Ramsden, P. (2003). *Learning to teach in higher education*. London: Routledge

Sfard, A. (2008). *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*. Cambridge University Press.

Sfard, A. (2017). Ritual for ritual, exploration for exploration or what the learners get is what you get from them in return. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 41–63). London: Routledge.

Sfard, A. (2020). Commognition. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (2nd ed., pp. 95–101). Springer. https://doi.org/10.1007/978-3-030-15789-0

Tracy, S. J. (2010). Qualitative quality: Eight "big-tent" criteria for excellent qualitative research. *Qualitative Inquiry,16*(10), 837–851. https://doi.org/10.1177/1077800410383121

Viirman, O., & Nardi, E. (2019). Negotiating different disciplinary discourses: biology students' ritualised and exploratory participation in mathematical modeling activities. *Educational Studies in Mathematics, 101*(2), 233–252. https://doi.org/10.1007/s10649-018-9861-0

Werner, F., & Sotskov, Y. N. (2006). *Mathematics of Economics and Business*. Taylor & Francis.

Williams, J. (2012). Use and exchange value in mathematics education: Contemporary CHAT meets Bourdieu's sociology. *Educational Studies in Mathematics, 80*(1), 57–72. https://doi.org/10.1007/s10649-011-9362-x