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Stability of asset pricing models at the Oslo Stock Exchange

Master's thesis in Financial Economics

Supervisor: Snorre Lindset

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Preface

This thesis concludes our Master of Science in Financial Economics at the Norwegian University of Science and Technology.

We would like to thank our supervisor, Professor Snorre Lindset, who has provided guidance and valuable feedback during the work on this thesis. We would also like to express our gratitude towards friends and family who have supported us throughout the process.

Trondheim, June 2023

Abstract

This study examines the stability of asset pricing models at the Oslo Stock Exchange, specifically it focuses on the performance of the Capital Asset Pricing Model (CAPM), the Fama-French three-factor model, the Carhart four-factor model, and a five-factor model that incorporates the Pastor-Stambaugh liquidity risk factor. The research employs the Fama-Macbeth two-step procedure and intercepts statistics from Gibbons-Ross-Shanken (GRS) as testing methods. In order to conduct the analysis we construct portfolios based on size, beta, momentum, and liquidity with stocks listed on the Oslo Stock Exchange from January 2000 to December 2020.

The results of this study reveal that the utilized asset pricing models exhibit instability across portfolio sortings. More precisely, the models demonstrate shortcomings in accurately estimating risk premiums, determine how the risk premiums will unfold, and to consistently identify priced risk factors. We find that liquidity should not be included in models along with market and size factors.

Sammendrag

Denne studien undersøker stabiliteten til aktivaprisingsmodeller på Oslo Børs, og fokuserer spesifikt på Kapitalverdimodellen (CAPM), Fama-French tre-faktor modellen, Carhart fire-faktor modellen og en fem-faktor modell med en Pastor-Stambaugh likviditetsrisikofaktor. De anvendte testmetodene i avhandlingen er henholdsvis en Fama-Macbeth to-steps økonometrisk modell og en Gibbons-Ross-Shanken (GRS) skjæringspunkttest. I forkant av testutførelsen har vi konstruert porteføljer bestående av aksjer basert på størrelse, beta, momentum og likviditet på Oslo Børs i perioden januar 2000 til desember 2020.

Funnene ved denne avhandlingen viser at de anvendte prisingsmodellene for finansielle aktiva ikke viser stabilitet på tvers av ulike porteføljesammensetninger. Mer presist, modellene mislykkes i å estimere risikopremier med nøyaktighet, ved å forutsi utviklingen risikopremiene tar, og de ser heller ikke ut til å konsekvent kunne identifisere prisede risikofaktorer. Vi finner at likviditet ikke burde innlemmes i modeller med markeds- og størrelsesfaktorer.

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1 Introduction

Stock prices are known to be influenced by various factors, including macroeconomic conditions, industry trends, and firm-specific characteristics. Understanding how firm-specific characteristics affect stock prices is important for investors and financial analysts, as it can inform investment decisions and portfolio management strategies.

The aim of this research is to investigate the stability of widely applied asset pricing model in explaining cross-sectional returns on the Oslo Stock Exchange (OSE).

The Oslo Stock Exchange operates the only regulated market for securities in Norway, and is among the world-leading within sectors such as energy, shipping, and seafood. The stock exchange became a part of the Euronext concern back in June 2019, giving access to more investors to which the stock exchange expects increasing capital, liquidity, and interest in the years to come. Historically, from the 1980s to the time of writing, the stock exchange has experienced remarkable growth in value (16.5 billion in 1980 to 2.9 trillion in 2020), number of listed stocks (93 listed in 1980 versus 250 listed in 2020), and trading volume (370 million in 1980 and 219 trillion in 2020). Despite this, the OSE continues to be an energy-driven stock exchange (more than 35% of the listings fall into this category). The OSE is mainly represented by a few heavyweights (the biggest five account for about 45% of the total value) (Euronext (n.d.)).

To maneuver through the research we are going to apply some of the better-known asset pricing models with the desire of determining the stability of the factor models at the Oslo Stock Exchange.

Specifically, we apply the Capital Asset Pricing Model (CAPM) as a starting point, as this model is one of the most recognized and widely used among academics. The CAPM, being very simple in its form implies that there are likely to be other models that will better explain asset returns. This leads to the idea of a model with more risk factors that is able to explain market returns with more precision. One of the well-known asset pricing models we put into action is the Fama-French three-factor model, which includes market risk, size, and book-to-market value. Further, we

expand to the Carhart four-factor model in which momentum is incorporated as a risk factor among the three mentioned Fama-French factors.

More advanced, to amplify the results we use a five-factor model accounting for liquidity, in addition to the risk factors included in the Carhart four-factor model.

A large part of our motivation for choosing these asset pricing models and the research question is that the mentioned models are widely applied in finance, much due to their informative nature on common risk factors. Further, the fact that the models are used extensively in finance courses also motivated us to move from theory to practical implementation and the usage of them. Performing analysis of the Norwegian stock market concerning these models is therefore in our opinion both useful and interesting.

All asset pricing models will be commented on and tested by enforcing the Fama-Macbeth two-step method, the Gibbons-Ross-Shanken test, the mean absolute value of alpha (MAVA), and their goodness-of-fit.

The outline of the paper is as follows. In section 2 we present relevant literature in a historical timeline. We then proceed to section 3 where we present the theoretical framework for the employed asset pricing models. In section 4 we present the data used in our analysis, and in section 5 we present the methodology used to derive our results. Towards the end, in section 6 we undertake the analysis and results, and finally in section 7 we conclude our main objective.

2 Literature review

In this section, we provide an overview of already existing and relevant theoretical literature on asset pricing models. Specifically, the models used to define how distinct risk factors affect the excess returns¹ of financial assets.

Stocks have been issued and traded for hundreds of years, dating back to the seventeenth century when the Dutch East India Company allowed the public to invest in its business in 1602. Despite a long history of risk-bearing and risk-sharing there was little to no modeling of the risk-return relationship in capital markets for centuries. It was not until the 1950s the understanding and desire of theoretical and empirical knowledge became a topic of interest (Perold 2004).

A natural starting point for this section is the work of Markowitz (1952) on the trade-off between risk and return in financial markets. Markowitz's work is among the earliest in illustrating how investors could maximize their return for a given level of risk, or the other way around, minimize their risk for a given level of return. Both by choosing an optimal portfolio.

In the following decade, Sharpe, Lintner, and Treynor developed the Capital Asset Pricing Model (CAPM) built on the previous research by Markowitz. The model has since then become well-known and broadly implemented in risk-return analysis. CAPM aims to explain the asset return in a linear relation to the market risk. Despite its popularity among academics, the CAPM has been subject to criticism due to its simple and very general appearance in terms of assumptions. As mentioned, the model has widely been tested and used in various studies, one of them carried out by Michael C. Jensen (1968) on the U.S. stock market find that the CAPM provides a reasonably good explanation of stock returns. Market risk is found to be a significant predictor of asset returns. Likewise, on the Norwegian stock market, Bernt Arne Ødegaard (1994) also find the CAPM to have explanatory power in terms of returns when analyzing data from 1982 until 1991. Yet both studies comment on how CAPM struggles to explain most of the returns. Some modifications of the model were introduced through the work of Robert Merton (1973) with

¹Excess return is the return in excess of the risk-free rate.

his Intertemporal Capital Asset Pricing Model (ICAPM), Robert Lucas (1978) and Douglas Breeden (1979) with their consumption-based CAPM.

In relation to this, there have been developed several asset pricing models based on CAPM. Other models investigate additional risk factors.

Eugene F. Fama and Kenneth R. French argue that the attraction of the CAPM is that it offers powerful and intuitively pleasing predictions about how to measure risk and the relation between expected return and risk. However, the empirical record of the model is poor, and therefore poor enough to invalidate the way it is used in its application (Fama and French 2004). Fama and French (1992) find that when you include other risk factors in combination with the market risk, the market risk turns out to have rather little explanatory power when assessing the asset returns. More specifically, they find that when used in single-factor models, size, E/P, leverage, and book-to-market equity have explanatory power. When in combination, they find that size and book-to-market equity (BE/ME) have a statistically significant relationship with asset returns and describe returns better than the CAPM alone. In order to analyze the risk factors, they divided the assets into portfolios based on size and book-to-market. Their research on risk factors gave birth to the well-known Fama-French three-factor model introduced in Fama and French (1993). Later, Fama and French (2015) proposed the five-factor model, also known as the Fama-French five-factor model, as an extension of their three-factor model. The extended version includes profitability and investment factors. The three-factor model has been used by Næs et al. (2009) on the Oslo Stock Exchange where they found the size and book-to-market to be significant risk factors.

Another extension of the Fama-French three-factor model proposed by Mark M. Carhart. His model add momentum as a risk factor. Intending to illustrate that stocks tend to perform well or poorly in the future based on previous performance. Similar to Fama and French, Carhart constructed portfolios based on specific characteristics to examine the performance and risk associated with different factors. In various studies on the U.S. market returns it shows to provide a better explanation of the variation in stock returns than CAPM and the Fama-French three-factor model (Carhart 1997). Jegadeesh and Titman (1993) show how momentum persists

in U.S. market returns even after controlling for the three risk factors market, size, and book-to-market. However on the Norwegian market Næs et al. (2009) find little support for any momentum effect.

Næs et al. (2009) finds the presence of liquidity as a risk factor along with market risk and size on the Oslo Stock Exchange. The same is reported by Pastor and Stambaugh (2003) when analyzing a broad range of potential risk factors for stock returns in the U.S. market, including the Fama-French three-factor model and several additional variables. They find that the Fama-French three-factor model is important in explaining asset returns. However, they also argue that there are other variables explaining a significant portion of the variation in stock returns. Specifically, they identified liquidity and captured the tendency of more liquid stocks to outperform less liquid stocks. Their study is a significant contribution to the literature on asset pricing.

3 Theoretical framework

In this section, we briefly go through the efficient market theory as it is an important and necessary concept in order to acknowledge asset pricing models in research. We will then present the employed asset pricing models and their respective appearance in terms of equation and theory. Thereafter, under each model we will provide a subsection containing a more profound description and overview of the risk factors that are specific to said asset pricing model.

3.1 Efficient markets

The efficient market hypothesis (EMH) is a key concept and cornerstone in modern finance theory. EMH states that the asset prices fully reflect all publicly available information and respond quickly to new information, making it impossible to beat the market. According to EMH, the market price is considered the true value of a said asset, and investors cannot purchase undervalued stocks or sell stocks for inflated prices. This means that the only way to obtain a return in excess of the market

is through purchasing riskier investments. The assumption of efficient markets is important for asset pricing models in several ways

The asset pricing models whose goal is to describe the relationship between asset returns and systematic risk rely on the idea of efficient markets. It is crucial for the asset pricing models that the asset prices reflect all information to be able to capture systematic risk associated with an asset and create reliable estimates. The EMH is in line with risk and return being inherently linked in asset pricing.

Noteworthy, the assumptions of market efficiency influence the behavior and strategies of investors. In efficient markets, investors often adopt passive strategies, aiming to replicate the market performance rather than outperform it. Constructing well-diversified portfolios is therefore essential when assessing asset pricing models within the framework of the efficient market hypothesis theory (Bodie et al. 2021).

3.2 Risk factors

Risk factors are variables or characteristics that are believed (or found) to impact the returns of an asset or a group of assets. They capture systematic risks associated with an investment and help explain the returns of assets. There exist several widely recognized risk factors, among them are the ones used in this thesis; market, size, book-to-market, momentum, and liquidity which will be present in the following parts. Risk factors play a central role in this research paper because when they are demonstrated to possess explanatory power in the overall market, they will contribute to enhancing the accuracy of estimates produced by the asset pricing models.

In order to portray the impact of risk factors on investment returns Figure 1 shows the potential historical return associated with an investment in the risk factors *SMB* (size), *HML* (book-to-market), *LIQ* (liquidity), and *PR1YR* (momentum).

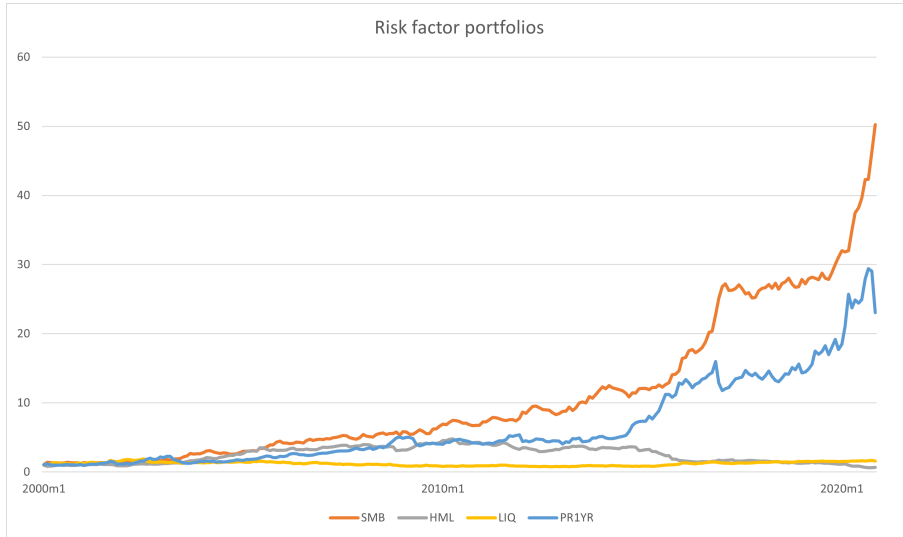


Figure 1: Displays the return of a unit investment in the risk factor portfolios: *SMB*, *HML*, *LIQ*, and *PR1YR*.

3.3 CAPM

One of the most widely used models for pricing financial assets is the Capital Asset Pricing Model (CAPM) developed by Jack Treynor (1962), William F. Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) in the early 1960s. The CAPM is an equilibrium model that describes the relationship between risk and return for individual securities. The premise of the model is that all investors are risk-averse by nature, meaning that they require higher expected return to take on additional risk. Beyond this, investors have the same time period to evaluate information and there is unlimited capital to lend at the risk-free rate of return (Sharpe 1964). The model is specified in an unconditional framework as

$$E[r_i] = r_f + \beta_i(E[r_M] - r_f), \quad (1)$$

where $E[r_i]$ is the expected return of asset i , r_f is the risk-free rate, $E[r_M]$ is the expected return of the market portfolio. Here, $\beta_i = \frac{\sigma(r_i, r_M)}{\sigma^2(r_M)}$ is the sensitivity of return in asset i to the market return M . The numerator denotes the covariance between asset returns and market returns, and the denominator denotes the variance of market returns.

The empirically tested market model derived from CAPM is

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + \epsilon_{i,t}. \quad (2)$$

Evaluating the model now entails using historical values of asset returns, therefore we remove $r_{i,t}$'s notation of expected return and it is interpreted as the return of asset i at time t . Market return, denoted by $r_{M,t}$ is the return of the market portfolio at time t , $r_{f,t}$ is the risk-free rate at time t , α_i is the intercept, and $\epsilon_{i,t}$ is the error term. The β_i can be derived as in equation (1), however, it will be retrieved through time-series regression. More on this in section (5).

3.3.1 Market

The market risk factor was pointed out along with the CAPM in the 1960s. Market risk is a systematic risk, and it refers to the risk affecting the prices of all securities associated with the market. It is driven by various macroeconomic factors, such as geopolitical events, changes in interest rates, and other macro trends. The market risk cannot be diversified away, and investors therefore demand compensation for bearing this risk.

The market risk factor is the excess return of market returns, and is given by

$$\text{Market risk} = (r_{M,t} - r_{f,t}). \quad (3)$$

3.4 Fama-French three-factor model

One of the proposed extended models to the CAPM is the Fama and French three-factor model. In addition to market risk, it contains the size and the book-to-market risk factors. In an article published in the Journal of Finance in 1993 they show that an empirically motivated three-factor model has better explanatory power than the CAPM alone on the U.S. stock market (Fama and French 1993). The Fama-French three-factor model is expressed by

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,1}(r_{M,t} - r_{f,t}) + \beta_{i,2}SMB_t + \beta_{i,3}HML_t + \epsilon_{i,t}. \quad (4)$$

The extension of the CAPM includes two additional risk factors, the other variables are interpreted as in equation (2). *SMB* is a long-short portfolio² referred to as small minus big, and *HML* is a long-short portfolio referred to as high minus low. Following, we provide a more detailed description of the additional risk factors.

The calculations of both *SMB* and *HML* are derived from first dividing into three portfolios sorted on book-to-market, separating between high, median, and low (*H*, *M*, and *L*), and then secondly divided into two size classifications within each book-to-market portfolio, as small and big (*S* and *B*). This results in the combinations *SH*, *SM*, *SL*, *BH*, *BM*, and *BL*.

3.4.1 Size

The first to document the size effect is Rolf W. Banz in 1981 where he find that the size effect is not linear in the market value. The main observation is that smaller firms, on average, have greater risk-adjusted returns than larger firms on the U.S. stock market. Research on the size effect proved it had been present in the U.S. for the last forty years and pointed towards evidence of the Capital Asset Pricing Model being misspecified. The size risk factor is found to mainly affect smaller firms, while there are little differences in return between larger firms. The research comments on how there is not any theoretical foundation for such an effect and that it, potentially, could be a proxy for other true but unknown risk factors correlated with size (Banz 1981).

To isolate returns related to size (*SMB*), Fama and French categorize stocks into different size groups based on market capitalization. Then, they sort the stocks within each size group based on their book-to-market ratio. By constructing portfolios that combine these size and book-to-market categories, they isolate the specific

²Note that long-short portfolio is mainly a notational preference to us, the risk factors are, in short, the average of a weighted average of stocks minus the weighted average of other stocks (excluding the market risk).

size-related effect.

The *SMB* portfolio is meant to mimic returns associated with size, it is derived as

$$SMB = \frac{1}{3} \cdot (SL + SM + SH) - \frac{1}{3} \cdot (BL + BM + BH). \quad (5)$$

For instance, the *SL* portfolio contains stocks with a small market capitalization (S) and low book-to-market (L) ratio. Similarly, the *BH* portfolio contains stocks with a large market capitalization (B) that is also in the high book-to-market (H) ratio group (Fama and French 1993).

3.4.2 Book-to-market value

The book-to-market value is the ratio of the book value of equity, BE, to its market value, ME. The book value represents firm value as reported in its financial statement, while market value reflects the firm value as perceived by investors in the stock market. Fama and French (1992) find that the book-to-market ratio, BE/ME, is a priced risk factor in the U.S. market. There are several other studies supporting such discoveries, for instance, Rosenberg, Reid, and Lanstein conclude with a significant book-to-market effect in the U.S. stock market. (Rosenberg et al. 1985). Likewise, Chan, Hamao and Lakonishok (1991) infer that the book-to-market has a direct relationship with asset returns in Japan (Chan et al. 1991).

In order to isolate effects related to the book-to-market ratio, Fama and French follow a similar approach as with *SMB*. They categorize stocks into different size groups based on market capitalization, and then sort the stocks within each size group based on the book-to-market ratio. In that manner, they are able to isolate the specific return associated with the book-to-market effect.

The *HML* portfolio aims to reflect the risk factor in returns related to the book-to-market ratio and is derived by

$$HML = \frac{1}{2} \cdot (SH + BH) - \frac{1}{2} \cdot (SL + BL), \quad (6)$$

where the variables are defined as in equation (5). A more detailed description of the *SMB* and *HML* are found in Fama and French (1993).

3.5 Carhart four-factor model

Later on, Mark M. Carhart published in The Journal of Finance his four-factor model constructed using Fama-French three-factor model appending an additional risk factor, momentum. Carhart finds that momentum is a risk factor that with favor should be included in asset pricing models to strengthen its explanatory power when investigating mutual funds return (Carhart 1997). He found that mutual funds with high loading on size, book-to-market, and momentum tend to outperform those with low loadings, even after adjusting for the expense ratios and turnover rates. Carhart's four-factor model is given by

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,1}(r_{M,t} - r_{f,t}) + \beta_{i,2}SMB_t + \beta_{i,3}HML_t + \beta_{i,4}PR1YR_t + \epsilon_{i,t}. \quad (7)$$

The variables are interpreted as in equation (2) and (4), and the *PR1YR* is a long-short portfolio mimicking the momentum risk factor.

3.5.1 Momentum

The momentum risk factor was presented by Jegadeesh and Titman in 1993, their article revealed that stocks on an upward-moving trend tend to keep move in the same direction and the other way around for stocks on a downward-moving trend. They find that a strategy based on buying stocks that have performed well in the past and selling stocks that have performed poorly in the past gives significant positive returns when having a three to twelve-month holding period. They determine that observed profitability does not come from systematic risk nor delayed stock price reactions to common factors (Jegadeesh and Titman 1993). The Carhart factor, *PR1YR*, is constructed by sorting companies into three portfolios (top 30%, median 40%, and bottom 30%) at the end of each month based on their return in the

previous eleven months. Thereafter the portfolios are held constant throughout the month and the PR1YR is calculated as the difference in return between the top and the bottom portfolio (Carhart 1997). It can also be interpreted as the return on a long-short portfolio, where you buy top 30% and sell bottom 30%.

3.6 Five-factor model

Motivated by the findings of Næs et al. (2009) on liquidity effects on the Oslo Stock Exchange, we extend Carhart's four-factor model with Pastor and Stambaugh's liquidity risk factor. The model is specified as

$$\begin{aligned}
 r_{i,t} - r_{f,t} = & \alpha_i + \beta_{i,1}(r_{M,t} - r_{f,t}) + \beta_{i,2}SMB_t + \beta_{i,3}HML_t \\
 & + \beta_{i,4}PR1YR_t + \beta_{i,5}LIQ_t + \epsilon_{i,t},
 \end{aligned}
 \tag{8}$$

where *LIQ* is a long-short portfolio representing the liquidity risk factor. Other variables are interpreted as in the previous asset pricing models.

3.6.1 Liquidity

A fourth characteristic often related to CAPM anomalies³ is liquidity. This risk factor is understood as the bid-ask spread, transaction costs, or trading volume.

Discussed by Amihud and Mendelson in their publication of 1986, liquidity is found to have an impact on asset prices. The analysis is carried out on investors having different expected holding periods and trading with different relative spreads. The applied model for their study suggest increasing expected return as the bid-ask spread increases (Amihud and Mendelson 1986). Nevertheless, other studies such as Acharya and Pedersen (2005), Liu (2006), and Sadka (2006) suggest that liquidity is an explanatory variable for size, book-to-market, and momentum. Another drawback for liquidity as a risk factor is that it has several dimensions, for instance, how

³Patterns in average stock returns not explained by the Capital Asset Pricing Model are typically called anomalies (Fama and French 1996).

much it costs to trade, how fast one can trade, and how much one can trade (Berge et al. 2009).

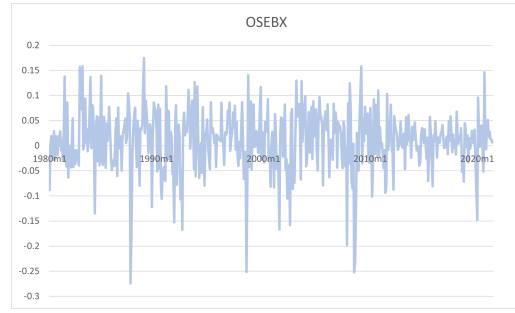
The Pastor and Stambaugh liquidity factor is computed on a monthly basis. The risk factor is created by sorting stocks into three portfolios based on liquidity (trade volume); low (30%), median (40%), and high (30%). The risk factor is then constructed by creating a portfolio where you hold a long position in the low 30% and a short position in the high 30% (Pastor and Stambaugh 2003).

4 Data

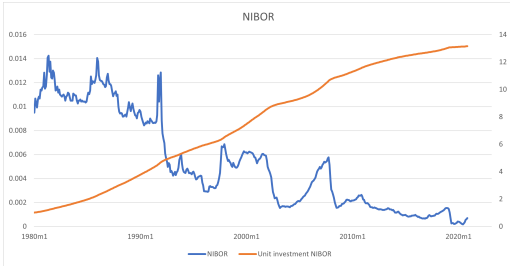
This study is based on a quantitative research design, which involves the use of numerical data to derive conclusions. During our research we conduct an empirical analysis employing a dataset containing monthly information about the stocks at the Oslo Stock Exchange from 1980 until 2020. The dataset is collected from Titlon which is a database operated by the University of Tromsø. Titlon has financial data on stocks, indices, bonds, funds, and derivatives from all exchanges at Euronext and with data from the Oslo Stock Exchange back to 1980 (University of Tromsø 2023). Apart from Titlon, we gathered the Norwegian Interbank Offered Rate (NIBOR) from the Federal Reserve Bank of St. Louis (2023) to implement as the proxy for the risk-free interest rate. We use the Oslo Stock Exchange Benchmark Index as the proxy for the market (see figure 2a & 2b), data collected on the OSEBX is retrieved from Eikon (2023). In addition, we employ risk factors calculated by Ødegaard (2023a), the risk factors are monthly observations on long-short portfolios replicating the Fama-French risk factors, *SMB* and *HML*. The data has been analyzed using the statistical software package Stata. We discuss and comment on alteration, moderation, and processing of the dataset in section 4.1. In Figure 2 we present the development of OSEBX and its volatility, the risk-free interest rate and its return, and the number of listed companies in our dataset each month.



(a) OSEBX



(b) Volatility OSEBX



(c) NIBOR



(d) Number of listed companies

Figure 2: Graph (a) illustrates the performance of the OSEBX since January 1980 to November 2020. Indicating a growing trend with some notable recessions corresponding to international events, such as the 2008 financial crisis. Graph (b) showcases the volatility of the OSEBX, depicting periods of fluctuations and varying levels of market stability. In (c) the risk-free interest rate, NIBOR, is illustrated. To give an idea of how its return has been, there is also presented a unit investment in 1980 and how it has developed from 1980 through 2020. (d) plots the number of listed companies at the Oslo Stock Exchange at the end of each month from 1980 to 2020.

4.1 Data processing

While evaluating asset pricing models, we are interested in a general model that is able to explain the excess returns of stocks at the Oslo Stock Exchange. Unfortunately, due to market anomalies⁴, extreme outliers, and volatile stocks, it is difficult to create a comprehensive model that effectively explains excess return.

⁴Market anomalies are unusual occurrence or abnormality in smooth pattern of stock market Latif et al. (2011).

Mentioned issues could possibly generate large skewness in the returns and make it challenging for the models to give accurate results for the entire market. Our objective is to examine the stability of asset pricing models, in order to do so, we depend on observations that are to a minimum extent affected by anomalies or other biases to give a meaningful analysis, discussion, and results in our research. It is therefore favorable to remove and cleanse the data to create a dataset that is closer to a representative and stable sample for the overall market.

Although the raw files provided by Titlon are of high quality, there is still a need to process the data to ensure accuracy and reliability. The data was processed and cleaned using the statistical software Stata. The cleaning process involved removing missing or incomplete data and correcting any errors or inconsistencies.

There are several companies with undefined industry classifications, missing values, and repeated observations. One of the first things we did was to go through each stock with missing sector classification and attributed an industry according to their classification under the Global Industry Classification Standard (Intelligence 2018).

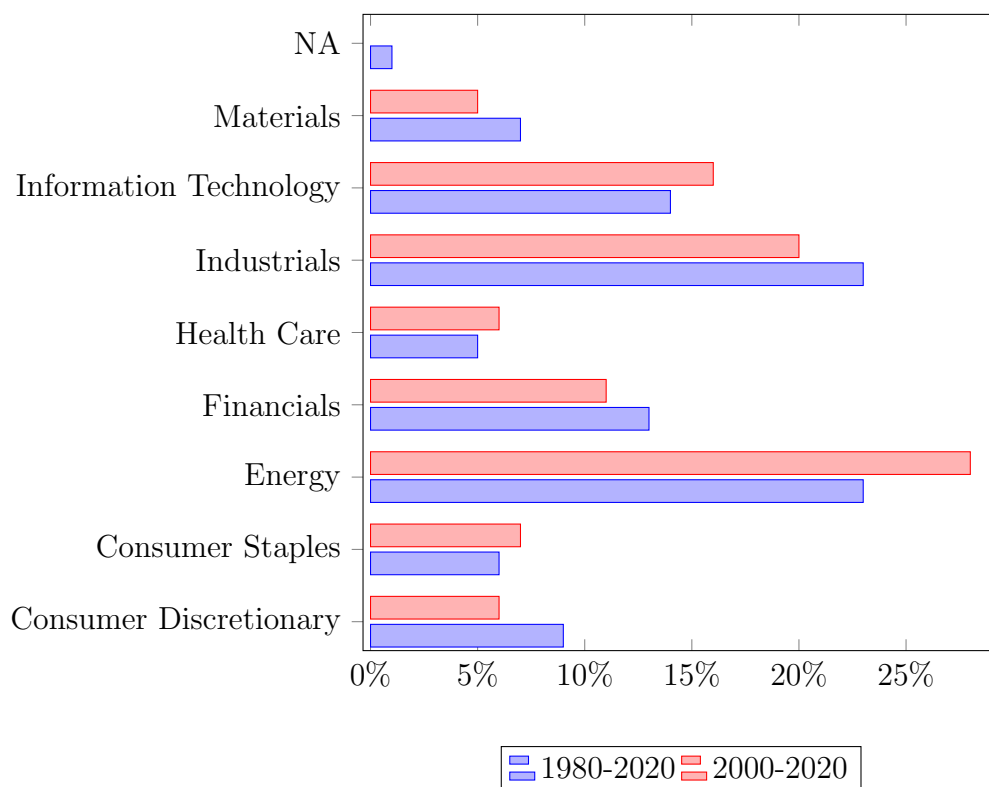


Figure 3: Displays the industry composition in our dataset after attributing industry to the ones missing.

Due to the Oslo Stock Exchange not having a diversified industry composition, stocks in industries comprised of a few companies were attributed an industry classification substitute (for example, Telenor being in the Telecommunication industry, is attributed to the Information Technology industry). We also remove repeated observations and missing values.

Furthermore, we remove firms with small market capitalization. More detailed, firms during our twenty-year sample period with less than 1 million NOK in market capitalization are removed. If they happen to increase or decrease above/below the set value, they will be included or excluded accordingly, as recommended by Ødegaard (2021). It is common knowledge that small-cap stocks tend to be more volatile than large-cap stocks and create large fluctuations. Apart from that, observed outliers in returns are removed to further increase the precision of our model in describing the general market. We define outliers as returns outside the 1st and 99th percentile. When doing so we get rid of the biggest fluctuations that do not reflect how normal returns appear at Oslo Stock Exchange. Well aware that such extreme returns do happen and are part of financial markets, it will not do us any justice in including them as they are not representative of the overall market.

After processing the dataset in terms of correcting missing values, removing outliers, and removing stocks with small market capitalization, the final dataset consists of monthly observations on 615 stocks in our sample period from January 2000 to November 2020.

5 Methodology

In the previous section, we presented the final dataset which we will use to derive results through our methodology. In this section, we will first present the variables we have computed and comment on active choices we have undertaken concerning the method. Thereafter, we will provide an in-depth explanation of our portfolio construction. Following, we introduce our methods for performance testing; the Fama-Macbeth two-step approach, the Gibbons-Ross-Shanken test, and goodness-of-fit.

5.1 Variables

In addition to the data we collected and processed, we compute a set of variables from the final dataset. We comment briefly on decisions we have made regarding the usage of adjusted prices and whether to use simple or logarithmic returns. Apart from that, we show how we constructed the momentum risk factor.

5.1.1 Simple return versus logarithmic return

The decision between simple and logarithmic returns was discussed to a large extent. Both calculations are viable options when studying asset returns. Simple return is calculated as

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \quad (9)$$

and logarithmic return is calculated as

$$r_t = \ln \left(\frac{P_t}{P_{t-1}} \right). \quad (10)$$

There is a distinction between the two methods, and the usages of the methods vary. However, the choice of using simple returns can be partially attributed to Bernt Arne Ødegaard who uses simple return calculations for his risk factors, *SMB*

and *HML*, as well as for his benchmark indexes. Also, logarithmic returns produce a lower total cumulative return than simple returns. We proceed to employ the simple return calculation for our analysis.

Further, we calculate the excess return as

$$R_t = r_t - r_{f,t-1}. \quad (11)$$

Where R_t is the excess return, r_t is the simple return derived from equation (9), and $r_{f,t-1}$ is the risk-free interest rate. We subtract $r_{f,t-1}$ and not $r_{f,t}$ because $r_{f,t}$ is reported at the beginning of each period, whereas r_t is reported at the end of each period.

5.1.2 Adjusted prices

In our analysis, we will use adjusted prices. In finance, the use of adjusted prices is important as it helps to provide a more accurate picture of the investment performance over time. While doing so, certain events or factors affecting the original price are accounted for. Examples are stock splits, dividends, and inflation to name a few.

5.1.3 Momentum risk factor

Since we employ the Carhart (1997) four-factor model in our analysis we calculate the *PR1YR* risk factor. In order to do so we follow the methodology detailed in his paper. We calculate each stock's 11-month return on a rolling basis. In this sense rolling basis means that we calculate the return of an asset from January to December, then February to January, and so on. We first calculate individual asset's 11-month returns as

$$r_{i,t}^{11m} = \frac{P_{t-1} - P_{t-12}}{P_{t-12}}. \quad (12)$$

The assets are then sorted into three portfolios based on their 11-month return; the

top 30%, median 40%, and the bottom 30%. The portfolios are updated monthly and are formed as equally-weighted portfolios. The *PR1YR* risk factor is then constructed as the difference in returns between the top and the bottom portfolios, in the same manner as a long-short portfolio trading strategy.

5.2 Portfolio construction

In this subsection, we explain our methodology for constructing our own portfolios and justify our choices. In section 2, we see how researchers use portfolio returns instead of individual asset returns when applying the asset pricing models. Likewise, we use portfolios to test the asset pricing models. The models are presented in section 3 and we aim to examine how the models perform based on different sorting characteristics for portfolios. Following, to clarify the choice of portfolio construction we comment on certain aspects.

When running the regressions in the Fama-Macbeth two-step approach it may suffer from an error-in-variable problem (EIV) if regressed with individual asset returns. The problem arises when the explanatory variables are measured with error, resulting in biased estimates and underestimated standard errors (Shanken 1992). Fama and MacBeth (1973) tackle the problem by grouping stocks into portfolios. They argue that regressing portfolio returns, rather than individual asset returns, could yield more precise coefficient estimates. That is, if the errors in $\hat{\beta}_i$ are substantially less than perfectly positively correlated, $\hat{\beta}_p$ can be more precise estimates of true β_i 's than $\hat{\beta}_i$. A more intuitive understanding, by creating portfolios we eliminate idiosyncratic risk from individual assets, in turn, coefficient estimates will be more precise in describing the systematic risk-return relationship between excess returns and risk factors.

However, constructing portfolios (to rectify the EIV problem) instead of using individual returns may lead to a loss of information in the risk-return relationship. By aggregating individual assets into portfolios, some detailed information about specific assets may be diluted or overlooked. In order to reduce the information loss, Fama and MacBeth (1973) construct portfolios based on ranked values of a

specific variable, beta coefficients in their case. This approach helps avoid excessive information loss by ensuring that portfolios include a range of assets with different characteristics.

Hence, we construct portfolios. Due to the dataset not providing all stocks listed on the Oslo Stock Exchange, double- and triple-sorting⁵ stocks into portfolios as done by Ødegaard (n.d.) will not offer sufficient diversification benefits or meaningful insights in our case. We will therefore use a simple-sorting method, which can be defined as creating X amount of portfolios based on a single characteristic.

For a given number of assets, the portfolios could be

(A) Numerous. Strengthens the possibility of identifying patterns and trends, but results in poorly diversified portfolios. The EIV-problem discussed above will likely occur.

(B) Few. Gives more diversified portfolios catching features from the market, but could struggle to identify patterns and trends. May suffer from the commented information loss.

Motivated by the objective of capturing the overall market and given the relatively few stocks in our dataset, it was essential to strike a balance between diversification and the ability to effectively describe the market dynamics. To establish a reasonable level of diversification, we presume that a portfolio consisting of 10-15 stocks can be said to be fairly well-diversified on the OSE, as proposed by Ødegaard (2018). To ensure we have a sufficient number and diversification of portfolios we choose to use a sample period from January 2000 to December 2020 (see figure 2d).

Depending on the sorting characteristic, we end up with between eleven and seventeen portfolios counting no less than 10 stocks in each observation throughout the sample period.

⁵Double-sorting is to categorize assets based on two characteristics, allowing for a more detailed analysis of the relationship between factors and expected returns. An example of this is, first sorting stocks into portfolios based on industry, second sorting stocks within industry portfolios on market capitalization.

In selecting the sorting variables, our initial intention was to sort portfolios based on industries. However, upon closer examination of the sector distribution on the Oslo Stock Exchange (see figure 3), it became clear to us that relying on sectors to create diversified portfolios would not yield reliable insights. The reason behind this is that two sectors alone account for over 50% of the total stocks. Furthermore, the insight from Fama and MacBeth (1973) regarding the use of ranked values when constructing portfolios is useful. We choose to sort portfolios based on the risk factors included in our asset pricing models (size, beta, momentum, and liquidity). Note that due to the lack of accounting data in the dataset, we will not create portfolios based on book-to-market ratio.

Another important aspect when constructing portfolios is how the individual assets are assigned weights in their respective portfolios. The two options we consider are:

(A) Value-weighted. Assigns weight regarding the asset's market capitalization and gives the larger assets greater influence, yet potentially increasing risk and reducing diversification. A value-weighted portfolio return is derived by

$$R_{p,t} = \sum_{i=1}^N w_{i,t} R_{i,t}, \quad (13)$$

where $R_{p,t}$ is the excess return on the portfolio, $R_{i,t}$ is the excess return for individual assets, and N is the total number of shares included in portfolio p at time t . The weight ($w_{i,t}$) is assigned to each asset, and $w_{i,t}$ is determined by

$$w_{i,t} = \frac{\text{Market Capitalization}_{i,t}}{\sum_{i=1}^N \text{Market Capitalization}_{i,t}}. \quad (14)$$

The numerator denotes the market capitalization for asset i in period t and the denominator denotes the total market capitalization for portfolio p in period t .

(B) Equally-weighted. Promotes diversification by giving all assets equal weight, but potentially overlooks the impact of assets with larger market capitalization. An equally-weighted portfolio return is derived by

$$R_{p,t} = \frac{1}{N} \sum_{i=1}^N R_{i,t}, \quad (15)$$

where the variables are interpreted as in equation (13).

One compelling argument for choosing equally-weighted portfolios is that they promote diversification across assets. By assigning equal weight to each asset, we ensure that each asset has an equal impact on the portfolio performance. This approach prevents the dominance of a few large-cap stocks, which can skew the portfolio returns and expose it to higher levels of risk. In essence, equal weighting allows for a more balanced representation of the entire portfolio. Therefore, we use equally-weighted portfolios in our analysis.

5.2.1 Size-sorted portfolios

We create 17 portfolios based on ranked values of market capitalization, from low to high. The portfolios have a mean of 12.42 stocks included at each observation, with a maximum of 16 and a minimum of 10.

5.2.2 Beta-sorted portfolios

We calculate the market betas of each stock on a rolling basis of two years through Ordinary Least Squares⁶ (OLS) time-series regressions. The beta estimations run from January 1998 to the end of November 2020. When betas are calculated we remove observations prior to January 2000. In doing so we ensure that we have beta estimations from the start of our main sample period, that is from January 2000 to November 2020.

Furthermore, stocks with less than 24 monthly observations are not included in the portfolios. Also, the stocks' first 23 observations are not included when sorting portfolios, thereby losing a significant amount of observations. Due to this, we reduce the number of portfolios in order to achieve portfolios that are diversified

⁶Ordinary Least Squares is a method for finding a best-fit line, given a set of data points.

enough to remove the EIV-problem. In turn, this raises the question if we have enough portfolios to properly capture the market. Some research seems to have focused on achieving more portfolios rather than diversified portfolios, however, we focus on achieving diversified portfolios. We sort stocks into 11 portfolios based on low to high beta, with a mean of 13.48 stocks included each month, with a maximum of 17 and a minimum of 10.

5.2.3 Momentum-sorted portfolios

We calculate the 11-month return for each stock, in the same manner as Carhart's *PR1YR* risk factor. We create 17 well-diversified portfolios with a mean of 11.36 stocks included each month, with a minimum of 10 and a maximum of 14. Similarly to the beta-sorted portfolios, we start the 11-month return calculations in January 1999 and end in December 1999, the simple return for this period is then used for the period January 2000, and so on. Using the "warm-up" period allows us to keep observations from the start of our sample period.

5.2.4 Liquidity-sorted portfolios

We create 17 portfolios based on the number of trades per month, from least liquid to most liquid. The portfolios have a mean of 12.42 stocks, with a minimum of 10 and a maximum of 16.

5.3 Performance testing

We will now reveal the methods used when evaluating the performance of the asset pricing models. During the entirety of this academic paper, we utilize the Fama and Macbeth two-step regression method when validating chosen asset pricing models. The method is developed by Fama and MacBeth (1973), and has since then proven to be a practical way to measure how well risk factors explain asset returns. To further evaluate the performance of the factor models, we will conduct GRS-tests. In 1989, Michael R. Gibbons, Stephen A. Ross, and Jay Shanken developed a test

based on the principle that if a particular asset pricing model is correctly specified, there will be no correlation between the returns and the residuals. The key difference between the tests is that the Fama-Macbeth approach validates the significance of individual risk premium coefficients for each asset pricing model whereas the GRS-test examines the joint significance of all the risk factors in a model. We will also implement two supplementary components, the mean absolute variable alpha (MAVA) and the \bar{R} -squared⁷ as part of our methodology for performance testing.

5.3.1 Fama-Macbeth two-step approach

The Fama-Macbeth two-step regression approach does, as indicated by the name, follow a two-step approach. We will begin by explaining the steps intuitively before visualizing them. In short, the first step estimates the risk factor coefficients via time-series regressions which will be used as factor loadings⁸ in step two. The second step estimates the risk premium attributed from the risk factors.

The first step entails running Ordinary Least Squares time-series regressions, where we obtain the coefficient estimates for each risk factor. The coefficients are assumed to be constant throughout the entire sample period⁹. To estimate the coefficients when having p portfolios and m risk factors, we regress the equations accordingly

$$\begin{aligned}
 R_{1,t} &= \alpha_1 + \beta_{1,F_1}F_{1,t} + \beta_{1,F_2}F_{2,t} + \dots + \beta_{1,F_m}F_{m,t} + \epsilon_{1,t} \\
 R_{2,t} &= \alpha_2 + \beta_{2,F_1}F_{1,t} + \beta_{2,F_2}F_{2,t} + \dots + \beta_{2,F_m}F_{m,t} + \epsilon_{2,t} \\
 &\vdots \\
 R_{p,t} &= \alpha_p + \beta_{p,F_1}F_{1,t} + \beta_{p,F_2}F_{2,t} + \dots + \beta_{p,F_m}F_{m,t} + \epsilon_{p,t}.
 \end{aligned} \tag{16}$$

Where $R_{p,t}$ is the excess return on portfolio p , α_p is the intercept, $F_{m,t}$ is the independent risk factor, and t denotes the time. All regressions have the same independent risk factors to enable examination of the exposure on the portfolio returns

⁷Adjusted R^2 . Defined in equation 23.

⁸Coefficients estimated in the time-series regression, used as dependent variables for the cross-sectional regressions.

⁹Assuming that coefficients are constant entails that excess return attributed from risk factors will not be subject to structural changes in the period.

from the set of risk factors.

The second step involves cross-sectional regressions where we regress all portfolio returns for each period t on the estimated coefficients ($\hat{\beta}_{p,F_m}$) from the first step, also referred to as factor loadings. In turn, this estimates the risk premium attributed from each risk factor. The cross-sectional regressions are carried out as

$$\begin{aligned}
R_{p,1} &= \gamma_{1,0} + \gamma_{1,1}\hat{\beta}_{p,F_1} + \gamma_{1,2}\hat{\beta}_{p,F_2} + \dots + \gamma_{1,m}\hat{\beta}_{p,F_m} + \epsilon_{p,1} \\
R_{p,2} &= \gamma_{2,0} + \gamma_{2,1}\hat{\beta}_{p,F_1} + \gamma_{2,2}\hat{\beta}_{p,F_2} + \dots + \gamma_{2,m}\hat{\beta}_{p,F_m} + \epsilon_{p,2} \\
&\vdots \\
R_{p,T} &= \gamma_{T,0} + \gamma_{T,1}\hat{\beta}_{p,F_1} + \gamma_{T,2}\hat{\beta}_{p,F_2} + \dots + \gamma_{T,m}\hat{\beta}_{p,F_m} + \epsilon_{p,T}.
\end{aligned} \tag{17}$$

Where $\gamma_{T,0}$ is the intercept, $\gamma_{T,m}$ is the estimated risk premium of a unit exposure to risk factor F_m .

We then proceed to derive the test statistics with the average of estimated risk premiums ($\bar{\hat{\gamma}}_j$). The hypothesis being tested is $\bar{\hat{\gamma}}_j = 0$. If the null hypothesis cannot be rejected, the risk premium is significant. This entails both the intercept and risk factor coefficients.

The test statistic is given by

$$t(\bar{\hat{\gamma}}_j) = \frac{\bar{\hat{\gamma}}_j}{s(\hat{\gamma}_j)/\sqrt{n}}. \tag{18}$$

Where $s(\hat{\gamma}_j)$ is the standard deviation of the estimated risk premium and n denotes the number of months in the sample, which as well is used to compute ($\bar{\hat{\gamma}}_j$) and $s(\bar{\hat{\gamma}}_j)$ (Fama and MacBeth 1973).

The test statistic is compared to a critical value in the t -distribution to determine the significance of the parameter.

5.3.2 GRS

The GRS-test is a multivariate test approach to assess the performance of an asset pricing model. Specifically, it is designed to evaluate whether a given asset pricing model is capable of explaining cross-sectional variation in excess returns. More intuitively, the capability of the model to produce a low and insignificant intercept, α_p . The null hypothesis is $H_0 : \alpha_p = 0 \quad \forall \quad p$, and is tested by the GRS statistic and compared to a F -distribution. To not reject the null hypothesis, and with that the efficiency of the asset pricing model, all α_p must be jointly equal to zero. In cases where we cannot reject the null hypothesis, it indicates that the model is correctly specified and is capable of explaining the variation in excess returns (Gibbons et al. 1989).

The approach initially runs OLS time-series regressions to compute the intercept for each portfolio. We test each asset pricing model under its respective sorting characteristic separately.

$$\begin{aligned} R_{1,t} &= \alpha_1 + \beta_1 R_{1,t} + \dots + \beta_i R_{i,t} + \epsilon_{1,t} \\ R_{2,t} &= \alpha_2 + \beta_1 R_{1,t} + \dots + \beta_i R_{i,t} + \epsilon_{2,t} \\ &\vdots \\ R_{p,t} &= \alpha_p + \beta_1 R_{1,t} + \dots + \beta_i R_{i,t} + \epsilon_{p,t}. \end{aligned} \tag{19}$$

Where $R_{p,t}$ is the excess return of portfolio p in period t . The α_p is the intercept. $R_{i,t}$ is the return on the risk factor portfolio and $\epsilon_{p,t}$ is the error term. β_i is the estimated coefficient of the risk factor.

The null hypothesis is

$$\begin{aligned} H_0 : \alpha_1 &= 0 \\ H_0 : \alpha_2 &= 0 \\ &\vdots \\ H_0 : \alpha_p &= 0, \end{aligned} \tag{20}$$

and the GRS statistics (J_i) is defined by Gibbons et al. (1989) and expressed by Ødegaard (2023b) in the following manner

$$J_i = \frac{(T - N - K)}{N} \cdot \left[1 + \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2} \right]^{-1} \cdot \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{N, T-N-K}. \quad (21)$$

Here T denotes the number of periods in the time-series regressions, N denotes the number of portfolios, and K denotes the number of independent variables included in the regression (risk factors). Further, $\hat{\mu}_m^2$ and $\hat{\sigma}_m^2$ are estimations of the squared mean and variance in excess return, measuring the portfolio performance. $\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}$ is the Sharpe ratio, giving the average excess return per unit of risk in said portfolio (Ødegaard 2023b).

The test statistic, J_i , is then compared to its corresponding critical value found by an F -distribution, $F_{N, T-N-K}$.

5.4 Goodness-of-fit

Goodness-of-fit statistics are used to evaluate how well a regression model fits the data and explains variations in the dependent variable. One common measure is R -squared (R^2), which represents the square of the correlation coefficient between the dependent variable and the corresponding fitted values from the model. R -squared ranges between 0 and 1, with higher values indicating a better fit.

The R -squared can be derived in multiple ways, following we outline the formulations given in Brooks (2019, p. 228-229),

$$R^2 = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}. \quad (22)$$

Where TSS is the total sum of squares, ESS is the explained sum of squares, and RSS is the residual sum of squares. R^2 will always take a value between zero and one when there is a constant term in the regression.

Another measure, the adjusted R -squared, takes into account the number of explanatory variables in the model and adjusts R -squared accordingly. It helps address the issue of adding more variables that may artificially inflate R -squared. The adjusted R -squared can be used as a criterion for model selection, considering both the fit of

the model and the number of variables. The adjusted R -squared is expressed as \bar{R}^2 and derived by

$$\bar{R}^2 = 1 - \left[\frac{T-1}{T-k} (1 - R^2) \right]. \quad (23)$$

Where T refers to the total number of observations or data points in the regression analysis, and k refers to the number of predictors or independent variables included in the regression model, excluding the constant term. The R^2 is derived in equation (22). Contrary to its counterpart, the adjusted R -squared can be negative, and if this is the case, the model is a very poor fit for the data.

It is important to note that R -squared and \bar{R} -squared provide an indication of how well the model fits the data but do not reveal how well the model represents the true relationship between the variables, as the true relationship is typically unknown. These statistics serve as useful tools for assessing the fit of the regression model. We will use \bar{R} -squared to compare models within the sorting characteristics. Lewellen et al. (2009) argue that cross-sectional asset pricing models often obtain high and misleading estimates of R -squared. They find that a \bar{R} -squared should be as high as 44% for a one-factor model, 62% for a three-factor model, and 69% for a five-factor model in order to be statistically significant. Furthermore, although R -squared cannot be used to directly compare models with different dependent variables, we will use it as a "weak indicator"¹⁰ of stability. That is because we have different portfolios comprised of the same assets, the risk factors employed in the cross-sectional regressions should then be able to explain the same level of variation in returns.

6 Analysis and results

In this section, we present and discuss the results derived through our methodology. First, we present and comment on the results of the second-step regressions for each portfolio sorting characteristic. Second, the obtained R -squared and \bar{R} -squared from

¹⁰Supportive indicator in decision making, not used to draw conclusions.

the second step regressions will undergo a goodness-of-fit discussion. There will also be undertaken an intercept analysis of the first-step time-series regression to discuss if the models are specified correctly and contain all priced risk factors. The intercept analysis is based on the GRS statistics and its associated p-values, as well as the average absolute mean of alpha. Further, the stability of the asset pricing models will be discussed, and compared across the different portfolio sorting characteristics: size, beta, momentum, and liquidity.

6.1 Results of second-step Fama-Macbeth regressions

6.1.1 Size

Table 1

Results of cross-sectional regressions conducted on seventeen size-sorted portfolios.

Fama-Macbeth second step								
Model (size sorted)	Intercept	Market	SMB	HML	PR1YR	LIQ	R^2	Adj. R^2
CAPM	-.060*** (.0054) -10.98	.083*** (.0070) 11.85					.21	.15
Fama-French (3)	-.071*** (.0070) -10.07	.092*** (.0084) 10.99	.011* (.0065) 1.67	.024** (.0098) 2.39			.34	.18
Carhart (4)	-.064*** (.0076) -8.43	.084*** (.0089) 9.47	.015** (.0071) 2.08	.014 (.0114) 1.24	-.026 (.0148) -1.46		.40	.19
Factor model (5)	-.056*** (.0075) -7.49	.077*** (.0088) 8.75	.038*** (.0091) 4.19	-.048*** (.0173) -2.77	.022 (.0179) 1.21	.055*** (.0153) 3.57	.46	.22

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R -squared and \bar{R} -squared, respectively. Asterisks denote the variables significance level;

(1%)***, (5%)**, and (10%)*.

In the size-sorted portfolios, the intercept is significant at the 1% level in all models. A significant intercept indicates that a model is poorly specified and that there are

other risk factors that are priced¹¹, but not included in the model. Either way, you will favor a model with a lower intercept value as it means that the risk factors included, explain more of portfolio returns¹². The intercept values are between -5.6% and -7.1%. Implying that mentioned proportion of returns are priced in, but not included as risk factors in the models. The significant intercept directly violates the Capital Asset Pricing Model theory which states that market risk is the only priced risk factor.

The market risk factor is significant at the 1% level with positive and high estimates of risk premium throughout the asset pricing models. The *SMB* risk factor is increasing in its degree of significance and risk premium as we include additional risk factors. *HML* varies in its estimate of risk premium, moving from positive at 2.4% in the three-factor model to negative at -4.8% in the five-factor model. While being significant in mentioned models at 5% and 1%, respectively. *PR1YR* is neither statistically significant in the four-factor nor the five-factor model. Introducing the Pastor-Stambaugh liquidity risk factor (*LIQ*) we observe a positive and significant risk premium of 5.5%.

¹¹Priced refers to a significant risk factor in the market

¹²Note that henceforth the term returns refers to excess returns.

6.1.2 Beta

Table 2

Results of cross-sectional regressions conducted on eleven beta-sorted portfolios.

Fama-Macbeth second step								
Model (beta sorted)	Intercept	Market	SMB	HML	PR1YR	LIQ	R^2	Adj. R^2
CAPM	.006*** (.0018) 3.42	-.003 (.0043) -.71					.40	.33
Fama-French (3)	.007** (.0034) 2.12	-.001 (.0049) -.25	-.010 (.0134) -.73	.005 (.0204) .26			.54	.34
Carhart (4)	.005 (.003) 1.42	.006 (.0054) 1.18	-.004 (.0131) -.03	-.017 (.0199) -.87	.042*** (.0127) 3.28		.63	.39
Factor model (5)	.001 (.0037) .18	.010* (.0057) 1.73	-.009 (.0148) .63	-.016 (.0199) -.79	.062*** (.0160) 3.98	-.059** (.0254) -2.30	.70	.40

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R-squared and adjusted R-squared, respectively. Asterisks denote the variables significance level;

(1%)***, (5%)**, and (10%)*.

In the beta-sorted portfolios, the intercept is significant in both the single-factor and three-factor models at 1% and 5%, respectively. Indicating that there are other risk factors that are priced, but not included in the models. As we extend with additional risk factors, the intercept becomes insignificant, indicating that the models include the priced risk factors. The intercept value varies as the models are extended with additional risk factors, but is positive in all models.

The market risk factor is only significant for the five-factor model at the 10% level with an estimate of 1%. The *SMB* and *HML* risk factors are not comment-worthy as they are insignificant for all models. When assessing the momentum risk factor (*PR1YR*) included in the Carhart four-factor model and the five-factor model, it is highly significant at the 1% level with a risk premium of 4.2% and 6.2%. The

liquidity risk factor is significant at the 5% level with a value of -5.9%.

6.1.3 Momentum

Table 3

Results of cross-sectional regressions conducted on seventeen momentum-sorted portfolios.

Fama-Macbeth second step								
Model (momentum sorted)	Intercept	Market	SMB	HML	PR1YR	LIQ	R^2	Adj. R^2
CAPM	.044*** (.0060) 7.32	-.056*** (.0098) -5.70					.17	.11
Fama-French (3)	.048*** (.0080) 5.89	-.057*** (.0148) -3.84	-.001 (.0103) -.09	-.026* (.0140) -1.87			.34	.18
Carhart (4)	.014* (.0080) 1.85	-.020 (.0143) -1.43	.010 (.0110) .87	.023 (.0143) 1.63	.034*** (.0052) 6.50		.40	.19
Factor model (5)	.014* (.0078) 1.86	-.020 (.0143) -1.43	.010 (.0105) 0.91	.022 (.0157) 1.38	.034*** (.0052) 6.50	.009 (.0114) 0.77	.46	.22

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R-squared and adjusted

R-squared, respectively. Asterisks denote the variables significance level;

(1%)***, (5%)**, and (10%)*.

The intercepts for momentum-sorted portfolios are significant throughout the models, albeit decreasing in significance from the 1% level for the single-factor and three-factor model to a 10% level for the four-factor and five-factor models. The intercept estimates drops by approximately 0.03 when extending from the single- and three-factor model to the four and five-factor model. This indicates that the included risk factor is able to explain more of the variance in the portfolio returns, and that it is a priced risk factor. Ideally, we aim for low, if not insignificant intercept values, indicating that the models include all priced risk factors.

The market risk premium is significant in the CAPM and the three-factor model with estimates of -5.6% and -5.7%. When proceeding to the four-factor model and

the five-factor model, market risk is insignificant with lower values. Again, the *PR1YR* appears to be a priced risk factor. The *SMB* is not significant for any of the tested models. When considering the *HML*, the significance is only found at a 10% level with an estimate of -2.6% in the Fama-French three-factor model. In the Carhart four-factor and the five-factor models, the momentum risk factor is significant at a 1% level with 3.4% for both models including said risk factor. Liquidity is insignificant.

6.1.4 Liquidity

Table 4

Results of cross-sectional regressions conducted on seventeen liquidity-sorted portfolios.

Fama-Macbeth second step regression results								
Model (liquidity sorted)	Intercept	Market	SMB	HML	PR1YR	LIQ	R^2	Adj. R^2
CAPM	-.004** (.0021) -2.11	.009* (.0047) 1.91					.23	.18
Fama-French (3)	-.003 (.0019) -1.29	.015* (.0065) 2.38	-.021 (.0137) -1.53	.007 (.0121) .56			.35	.20
Carhart (4)	-.003 (.0020) -1.39	.015** (.0065) 2.23	-.022 (.0138) -1.59	-.008 (.0123) .69	-.120 (-.0028) -.94		.41	.21
Five-factor	-.003 (.0020) -1.44	.016** (.0072) 2.27	-.025* (.0149) -1.70	.010 (.0124) .81	-.009 (.0140) -.65	.001 (.0095) .13	.47	.23

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R-squared and adjusted R-squared, respectively. Asterisks denote the variables significance level; (1%)***, (5%)**, and (10%)*.

The intercept is significant at a 5% level for the CAPM with a value of -0.4%. For the rest of the models, it is not significant. This indicates that the CAPM is not capturing all the priced risks with its single risk factor. For the rest of the models, the insignificant intercept suggests stronger explanatory power than the CAPM.

When examining the market risk, it is significant at a 10% level for the single-factor and three-factor model with risk premiums of 0.9% and 1.5%. For the four-factor and five-factor models, the significance level has increased to a 5% level. Having a slight increase in its value to 1.5% and 1.6%. The Fama-French risk factor, *SMB*, does not appear significant until the five-factor model with a value of -2.5%. Whereas the *HML* is not significant at any level. The momentum and the liquidity are insignificant, indicating that they are not priced risk factors.

6.2 Goodness-of-fit

Tables 1 through 4 report the second-step regression results with their corresponding measures of goodness-of-fit. Following, we present and comment on the results of the R -squared (R^2) and the adjusted R -squared (\bar{R}^2).

For the size-sorted portfolios, there is a steadily increasing R -squared, as we would expect. The CAPM reports an R -squared of 0.21, whereas the five-factor model reports an R -squared of 0.46. The Fama-French three-factor and Carhart's four-factor model report an R -squared of 0.34 and 0.40, respectively. Indicating that there is a significant amount of the excess returns which is explained by the included variables.

Due to the fact that the value of R -squared will always increase when including additional explanatory variables we employ the \bar{R} -squared. By doing so, we get an indication of whether the added variables contribute significantly to explaining excess returns. A significant contribution can be defined in various ways, however, we define a significant contribution as a 0.01 increase in the value of \bar{R} -squared from the former model. Throughout all the models we find that the \bar{R} -squared is increasing when introducing additional variables. In the beta-sorted portfolios, we have the highest values of R -squared and \bar{R} -squared, ranging from 0.40 to 0.70, and 0.33 to 0.40, respectively. Again, it should not come as a surprise that the five-factor model has the best measure of goodness-of-fit. The momentum-sorted portfolios have the lowest R -squared and \bar{R} -squared across all portfolio sorts. Where the \bar{R} -squared ranges from 0.11 to 0.22. In the last portfolio sorting characteristic, liquidity, the

values of R -squared are quite similar to that of size.

In conclusion, we find that the \bar{R} -squared across all models, within their respective characteristic portfolios are increasing as we introduce additional variables. Indicating that employing these risk factors does explain more of the variation in the portfolio returns. Excluding the beta-sorted portfolios, we find that the variation explained by the models across different sorting characteristics is fairly similar. However, with regard to the findings of Lewellen et al. (2009) we cannot conclude that any of the models are a good fit.

6.3 Intercept analysis

The intercept analysis is based on Table 5 which reports the statistics for the time-series intercept analysis. The table provides the mean absolute value of alpha (MAVA), $\bar{\alpha}$. In cases where the MAVA is equal to zero, there do not exist any missing priced risk factors. The GRS statistic is the test statistic of a joint hypothesis test of significance on the intercepts in the time-series portfolio regressions ($H_0 : \alpha_p = 0 \quad \forall \quad p$). The p-value is the corresponding probability value of the GRS statistic, a high p-value indicates that the model is well specified. Following, we present the results of the MAVA and GRS-tests, with an emphasis on the GRS statistic. We will also highlight which model performs the best within each sorting and overall.

Table 5

Statistics for time-series intercept analysis

Model (size sorted)	$\bar{\alpha}$	GRS (J)	Prob. (GRS)	Model (momentum sorted)	$\bar{\alpha}$	GRS (J)	Prob. (GRS)
CAPM	-0.001733 (.002628)	15.26	.000000	CAPM	-0.001149 (.002740)	5.39	.000000
Fama-French (3)	-0.007289 (.002537)	17.45	.000000	Fama-French (3)	-0.006516 (.002695)	6.56	.000000
Carhart (4)	-0.005584 (.002694)	13.49	.000000	Carhart (4)	-0.004679 (.002724)	4.19	.000000
Five-factor	-0.005577 (.002696)	13.41	.000000	Five-factor	-0.004684 (.002496)	4.15	.000000

Model (beta sorted)	$\bar{\alpha}$	GRS (J)	Prob. (GRS)	Model (liquidity sorted)	$\bar{\alpha}$	GRS (J)	Prob. (GRS)
CAPM	-0.000234 (.002414)	3.15	.000528	CAPM	-0.001833 (.002602)	1.79	.029808
Fama-French (3)	-0.005112 (.002367)	4.62	.000002	Fama-French (3)	-0.007458 (.002528)	3.82	.000001
Carhart (4)	-0.003669 (.002492)	2.29	.011213	Carhart (4)	-0.005765 (.002694)	4.19	.000000
Five-factor	-0.003699 (.002496)	2.32	.009949	Five-factor	-0.005748 (.002691)	3.05	.000079

The second column reports the mean absolute value of alpha in the time-series regression. The third column is the Gibbons et al. 1989 test statistic, and the fourth column reports its corresponding p-value.

For the size-sorted portfolios, the first-step regression intercepts cannot produce a MAVA equal to zero, however, the lowest value is reported for the Capital Asset Pricing Model. The GRS statistics are very high, ranging from 13.41 to 17.45, and the corresponding probability values are equal to zero. Through the GRS statistic, we conclude that none of the models include all the priced risk factors, but that models with additional risk factors are capable of explaining more variation in portfolio returns.

Moving on, for the beta-sorted portfolios the smallest mean value of alpha is found for the CAPM. The GRS statistics are much lower for all the models compared to the size portfolios, as the highest GRS statistic is 4.62 in the Fama-French three-factor model. The lowest statistic is found for the Carhart four-factor model, which is marginally better than the five-factor model.

For the momentum-sorted portfolios, the GRS statistic is, again, much lower than for the size-sorted portfolios, but higher than the beta-sorted portfolios. The lowest

GRS statistic is reported for the five-factor model, however, the null hypothesis is rejected for all models.

In the last sorting, liquidity, we find that the GRS statistic is by far the lowest for the Capital Asset Pricing Model, with a p-value of approximately 3%, but we still reject the null hypothesis.

In conclusion, we find that there are no models across our employed sorting characteristics that reject the null hypothesis of insignificant intercept in the first-step regressions. Therefore, all models are missing priced risk factors. Surprisingly, and contrary to other portfolio sorting characteristics, we find that the CAPM under liquidity-sorted portfolios yields the lowest GRS statistic and therefore based on the GRS statistics can be said to be the best model. However, overall we find that models with more risk factors included are better.

6.3.1 Economic intuition and significance of the risk factors

In this section, we take a brief detour from our main topic and give an economic intuition pertaining to results from the first- and second-step of the Fama-Macbeth approach. We also investigate the significance of employed risk factors. The second-step regression results in Tables 1 to 4 display the risk premium estimates for a unit exposure to the risk factors. However, the economic impact each risk factor attributes to the stocks' excess return can be interpreted from Tables 7 to 10, which show the estimated coefficients in the first-step time-series regressions. In this discussion, we will mainly focus on the mean of the estimated coefficients ($\bar{\hat{\beta}}_p$)¹³ across portfolios. With an emphasis on the results from the five-factor model (as it includes all risk factors).

In Tables 7 to 10 we observe that the means of the risk factors across sorting characteristics are very similar. Approximately 0.7 for market risk, 0.3 for *SMB*, 0.11 for *HML*, -0.07 for *PR1YR*, and 0 for *LIQ*. To visualize, we consider the mean of factor loadings and the five-factor model under size-sorted portfolios. The mean of

¹³ $\bar{\hat{\beta}}_p = \frac{1}{N} \sum_{p=1}^N \hat{\beta}_p$. Where p is portfolio p and N is the number of portfolios

excess portfolio returns can then be expressed as

$$\bar{R}_{p,t} = -5.6\% + 7.7\% \cdot 0.697 + 3.8\% \cdot 0.31 - 4.8\% \cdot 0.109 + 2.2\% \cdot (-0.072) + 5.5\% \cdot (-0.003),$$

giving an average monthly excess return of

$$\bar{R}_{p,t} = 0.002468 \approx 0.25\%.$$

Furthermore, we find that *LIQ* has very low coefficients, with estimates ranging from -0.006 to 0.012, and is likely to be statistically insignificant. Indicating that in reality, liquidity risk is not a priced risk factor at the Oslo Stock Exchange, and thus should not be included as a risk factor in asset pricing models (at OSE). We also find that *HML* and *PR1YR* have a very low mean, ranging from 0.105 to 0.109 and -0.077 to -0.061, respectively. Further, to determine if risk factors are truly priced, and with that should be included in asset pricing models, we run a series of regressions on each sorting characteristic. More precisely, we run a series of panel regressions¹⁴, and time-series regressions on each portfolio (these include all five risk factors). For the panel regressions, we find that *LIQ* is not significant in any, and in the time-series regressions we find that *LIQ* is only significant in 8 out of 62 portfolios¹⁵.

Further, we find that *LIQ* is significant when regressed on excess portfolio returns alone, insignificant when we additionally include the market risk factor and *SMB*, but significant when regressed with market risk and *SMB* separately. These results imply that market- and size risk capture risk related to liquidity, indicating that the model¹⁶ is misspecified (omitted variables). To further evaluate the relationship between market, size, and liquidity risk we turn to their correlation. Table 6 displays the correlation matrix of the independent risk factors used in the time-series regression, it also includes a test of multicollinearity¹⁷ on the risk factors.

¹⁴Specifically, we use these panel techniques: pooled OLS, fixed effects, and random effects.

Outlined in Brooks (2019, p. 625-639)

¹⁵Where it is significant at the 10% level five times, 5% level one time, and 1% two times.

¹⁶ $R_{p,t} = \alpha_p + \beta_1 LIQ_t + \varepsilon_{p,t}$

¹⁷Multicollinearity occurs when independent variables in a regression model are highly correlated.

Table 6

Correlation matrix

	Market	SMB	HML	PR1YR	LIQ	VIF
Market	1					1.21
SMB	.0338	1				1.14
HML	.0844	-.1705	1			1.06
PR1YR	-.1602	.0398	.0513	1		1.31
LIQ	-.3869	-.2818	-.0926	.0515	1	1.03

Displays the correlation of the independent variables used in the first-step time-series regressions and their variance inflation factor.

Firstly, we find that *Market* and *SMB* have a correlation with *LIQ* of -0.38 and -0.28, respectively. Further, the variance inflation factor¹⁸ is sufficiently low, indicating that the correlation between risk factors does not induce multicollinearity. Overall, this leads to the conclusion that *LIQ* are not highly correlated with any risk factors, only moderately with *Market* and *SMB*, and that *Market* and *SMB* sufficiently capture risk associated with liquidity risk. Also, liquidity risk is not found to be a priced risk factor in the test regressions, and therefore should not be included in the cross-sectional regressions as an extension of the Carhart four-factor model. Identical procedures do not yield similar results for the other risk factors. In the following section, we will refrain from any further discussion regarding the five-factor model.

¹⁸ $VIF = \frac{1}{1-R_m^2}$, where R_m^2 is the value of R^2 from an auxiliary regression of the explanatory variable m on an intercept plus the other explanatory variables (Brooks 2019, p. 294).

6.4 Stability discussion

In this section, we comment on and discuss the stability of the models. A stable model has the capability of explaining returns independent of the portfolio sorting characteristics. That is, it will produce the same intercept, estimates, and R -squared independent of the portfolio sorting characteristics. We will discuss each model and its stability following the results given in Tables 1 to 4 for the size-, beta-, momentum-, and liquidity-sorted portfolios.

Following, we refer to the respective sorting characteristic as S (size), B (beta), M (momentum), and L (liquidity).

The second-step regressions for the Capital Asset Pricing Model have significant intercepts ranging from -6% (S) to 4.4% (M), it does not showcase any stability with values being both negative and positive. The market risk factor has a large dispersion¹⁹, with estimates ranging from -5.6% to 8.3%, where it is significant for all portfolio sortings, except beta. Similar to the intercept, the market risk factor fluctuates between negative and positive estimates, not exhibiting stability. The R -squared ranges from 0.17 to 0.40, indicating instability in the model's ability to consistently explain the variation in returns across different portfolio sorting criteria.

The Fama-French three-factor model has intercepts ranging from -7.1% (S) to 4.8% (M), where they are significant in all, except the liquidity-sorted portfolios. The market risk factor has premiums from -5.7% (B) to 9.2% (S), where it is significant for all sortings, except beta. The SMB risk factor ranges from -2.1% (L) to 1.1% (S) and is only significant for size-sorted portfolios. HML ranges from -2.6% (M) to 2.4% (S), and is significant for momentum- and size-sorted portfolios. Based on this we cannot conclude that the Fama-French three-factor model exhibits stability across different compositions of the market portfolio. The model fails to precisely estimate the risk premiums. Further, the model's R -squared are 0.34 (S), 0.54 (B), 0.34 (M), and 0.35 (L). Excluding beta-sorted portfolios we find that the model showcases more stability in explaining the variation of returns than with its risk premium estimates.

¹⁹Dispersion is the difference between the highest and lowest value.

For the Carhart four-factor model, the intercept varies between being positive and negative, it ranges from -6.4% (S) to 1.4% (M), where it is significant in these portfolio sortings as well. The market risk factor is significant in size- and liquidity-sorted portfolios, with estimates of 1.5% and 8.4%, respectively. The *SMB* risk factor has estimates of 1.5% (S), -0.4% (B), 1% (M), and -2.2% (L), only significant for size. *HML* is not significant under any portfolios. The *PR1YR* has a dispersion of 0.16 in the four-factor model, which is also the greatest dispersion of all risk factors. Again, when excluding beta-sorted portfolios we find that the *R*-squared is practically identical across the sortings

Throughout the models and the different sorts of portfolios, we can conclude that the asset pricing models do not showcase any stability. This observation aligns with what Blanco (2012) highlighted about how the results vary depending on how the portfolios are formed. In our analysis, we study equally-weighted portfolios comprised of stocks listed on the Oslo Stock Exchange. The models do not indicate stability, with that, they are not able to provide precise estimates of the risk premium associated with each risk factor.

To further enhance the stability analysis of asset pricing models at the Oslo Stock Exchange we include additional tests on different portfolios. In Tables 11 to 19 we present the results for the first- and second-step regressions for another set of portfolios without "penny stocks"²⁰. Portfolios are created on the same methodology outlined in section 5.2, however, we also remove stocks with a close price below 10 NOK. Ødegaard (2021) suggest removing penny stocks because they are not representative of the returns at the OSE and therefore not meaningful to include for empirical asset pricing investigations. We find that the capital asset pricing model now yields more stable estimates, $\hat{\gamma}_i$, for size- and liquidity-sorted portfolios. In other words, the dispersion is low, at only 0.003. Although it is interesting, it is also to be expected since companies with a high market capitalization tend to have high trading volumes. Interestingly, we did not observe this in Tables 1 and 4, possibly due to the penny stocks' variance in returns, making it more difficult for the models to produce precise estimates. Overall, the new set of portfolios does not accredit

²⁰Penny stocks are low-valued stocks, both in terms of market capitalization and price

asset pricing models any stability.

7 Conclusion

This research paper examined the performance of various asset pricing models, namely the Capital Asset Pricing Model (CAPM), Fama-French three-factor model, Carhart four-factor model, and a five-factor model including liquidity. Through analysis of the empirical results, several findings have emerged.

Firstly, the evidence suggests that the tested asset pricing models exhibit poor specification. The intercept analysis indicates that the models fail to capture all the priced risk factors in the market. This finding implies that there are additional risk factors influencing asset prices that are not accounted for by the employed models. Consequently, relying solely on these models for asset pricing decisions may lead to mispricing and inaccurate estimations of expected returns. We also find that the Pastor-Stambaugh liquidity risk factor should not be included as an extension of the Carhart four-factor model. Further, we find that the market and size factors capture risk associated with liquidity, and therefore including liquidity in models with these is meaningless.

Secondly, the analysis reveals substantial variations in intercept estimates across different portfolio characteristics and models. This variability signifies instability and inconsistency in the estimated intercepts, further questioning the reliability of the tested models. Such inconsistencies may arise due to model misspecification and limitations in the explanatory power of the selected risk factors. Hence, it is crucial for investors and researchers to exercise caution when utilizing these models and consider the limitations inherent in their construction.

This study underscores the need for further research to identify and incorporate new risk factors into asset pricing models. For instance, factors such as investment and profitability have been identified in the literature as potential contributors to asset pricing. Integrating these factors, among others, into the tested models may provide a more comprehensive framework for asset pricing and improve the explanatory

power of the models.

Further, we comment on some recent research that challenges the traditional understanding of asset pricing models. In 2022, Simon C. Smith and Alan Timmermann published a study on whether risk premium have vanished on the U.S. stock market. Their findings suggest that, over a certain time period, the risk premiums associated with certain risk factors have weakened or even disappeared (Smith and Timmermann 2022). There has also been a growing interest in the application of machine learning algorithms in recent years. The ability of algorithms to analyze vast amounts of data and identify complex patterns offers an advantage over traditional econometric models. One promising area is identifying new risk factors that can explain variation in excess return. For example, a study by Gu et al. (2018) uncovered a new risk factor related to corporate social responsibility on the Chinese stock market by using machine learning. It highlights the potential within this area in the future. These studies, in addition to the one conducted by us, add layers of complexity to the existing understanding of asset pricing and raise questions about the effectiveness of traditional models in capturing evolving market dynamics.

In conclusion, this paper contributes to the existing literature by highlighting the limitations and challenges associated with the tested asset pricing models. The findings emphasize the need for ongoing research and development of more sophisticated models that can capture the complexity and dynamics of asset pricing. By refining and expanding our understanding of asset pricing mechanisms, we can enhance investment decision-making processes, mitigate risk, and improve portfolio performance in the ever-evolving financial markets.

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Appendix

Table 7

Results of time-series regressions conducted on seventeen size-sorted portfolios.

First step Fama-Macbeth regressions. Factor loadings.

Portfolio	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model				
	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ
1	.467	.453	.441	.024	.339	.33	-.014	-.278	.326	.336	-.008	-.277	-.094
2	.552	.527	.367	.199	.464	.307	.178	-.151	.481	.299	.169	-.153	.128
3	.588	.563	.369	.194	.536	.342	.184	-.068	.537	.341	.183	-.068	.015
4	.636	.613	.49	.123	.544	.423	.1	-.168	.554	.418	.095	-.169	.079
5	.672	.65	.478	.121	.643	.471	.119	-.015	.627	.479	.128	-.014	-.13
6	.722	.694	.558	.172	.662	.527	.161	-.076	.657	.53	.164	-.076	-.04
7	.716	.697	.364	.119	.662	.33	.107	-.086	.66	.331	.108	-.086	-.018
8	.718	.704	.364	.061	.67	.331	.05	-.082	.668	.332	.051	-.082	-.015
9	.715	.69	.512	.135	.619	.444	.111	-.172	.624	.441	.108	-.172	.035
10	.767	.742	.455	.166	.691	.407	.149	-.121	.691	.407	.149	-.121	.001
11	.808	.789	.3	.143	.799	.309	.146	.024	.8	.309	.146	.024	.009
12	.902	.877	.31	.222	.871	.304	.22	-.015	.875	.302	.217	-.015	.034
13	.82	.798	.243	.2	.806	.251	.202	.019	.815	.246	.198	.018	.069
14	.804	.79	.249	.102	.777	.237	.098	-.03	.775	.238	.099	-.03	-.019
15	.872	.861	.202	.067	.863	.203	.068	.004	.858	.206	.07	.004	-.037
16	.975	.972	.127	-.01	.965	.12	-.013	-.019	.956	.124	-.009	-.018	-.066
17	.943	.946	-.081	-.01	.952	-.075	-.008	.015	.952	-.075	-.008	.015	.000
<i>Mean</i>	<i>0.746</i>	<i>.727</i>	<i>.338</i>	<i>.119</i>	<i>.698</i>	<i>.309</i>	<i>.109</i>	<i>-.072</i>	<i>.697</i>	<i>.31</i>	<i>.109</i>	<i>-.072</i>	<i>-.003</i>

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on market capitalization from low (1) to high (17).

Table 8

Results of time-series regressions conducted on eleven beta-sorted portfolios.

First step Fama-Macbeth regressions. Factor loadings.

Portfolio	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model				
	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ
1	-.022	-.038	.2	.149	-.027	.211	.153	.027	-.031	.213	.155	.028	-.029
2	.231	.216	.244	.11	.247	.274	.12	.075	.249	.273	.119	.075	.019
3	.335	.321	.198	.121	.327	.204	.123	.015	.329	.204	.123	.015	.011
4	.501	.485	.204	.133	.475	.195	.129	-.024	.481	.192	.126	-.024	.044
5	.641	.621	.255	.162	.625	.259	.164	.009	.627	.258	.163	.009	.012
6	.727	.709	.343	.11	.715	.349	.112	.015	.725	.344	.106	.014	.079
7	.817	.802	.303	.093	.784	.286	.088	-.043	.786	.286	.087	-.043	.014
8	.988	.974	.293	.085	.957	.277	.079	-.041	.962	.274	.076	-.041	.037
9	1.132	1.108	.417	.167	1.069	.379	.153	-.095	1.074	.377	.151	-.095	.036
10	1.247	1.235	.291	.057	1.143	.202	.026	-.223	1.134	.206	.03	-.222	-.064
11	1.726	1.707	.518	.069	1.548	.365	.016	-.385	1.545	.366	.018	-.385	-.024
<i>Mean</i>	<i>.757</i>	<i>.74</i>	<i>.297</i>	<i>.114</i>	<i>.715</i>	<i>.273</i>	<i>.106</i>	<i>-.061</i>	<i>.716</i>	<i>.272</i>	<i>.105</i>	<i>-.061</i>	<i>.012</i>

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on beta from low (1) to high (11).

Table 9

Results of time-series regressions conducted on seventeen momentum-sorted portfolios.

First step Fama-Macbeth regressions. Factor loadings.														
Portfolio	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model					
	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ	
1	.899	.886	.488	.001	.670	.279	-.072	-.524	.666	.281	-.070	-.523	-.029	
2	.945	.910	.665	.212	.703	.464	.142	-.502	.703	.464	.143	-.502	-.001	
3	.838	.813	.579	.120	.665	.436	.071	-.358	.660	.438	.073	-.358	-.039	
4	.808	.787	.372	.147	.668	.257	.107	-.286	.656	.263	.114	-.286	-.096	
5	.858	.833	.398	.190	.690	.260	.142	-.346	.703	.255	.136	-.346	.096	
6	.771	.752	.405	.113	.693	.348	.093	-.144	.689	.349	.095	-.143	-.024	
7	.701	.684	.247	.140	.638	.202	.125	-.112	.638	.202	.125	-.112	.002	
8	.655	.636	.290	.144	.590	.246	.128	-.110	.602	.241	.122	-.111	.091	
9	.693	.675	.188	.162	.652	.166	.154	-.056	.652	.166	.154	-.056	-.000	
10	.663	.642	.230	.201	.651	.239	.204	.024	.665	.233	.197	.023	.105	
11	.689	.675	.170	.128	.680	.175	.130	.012	.684	.173	.127	.012	.033	
12	.605	.595	.198	.065	.629	.230	.076	.082	.629	.230	.076	.082	.002	
13	.629	.618	.172	.089	.652	.205	.101	.083	.655	.204	.099	.083	.023	
14	.573	.558	.152	.140	.630	.221	.164	.173	.625	.223	.166	.174	-.035	
15	.670	.657	.238	.089	.731	.310	.114	.180	.727	.312	.116	.181	-.031	
16	.731	.718	.266	.069	.803	.348	.098	.205	.798	.350	.099	.205	-.034	
17	.866	.850	.497	.025	.999	.642	.075	.362	.996	.644	.077	.363	-.030	
<i>Mean</i>	<i>.741</i>	<i>.723</i>	<i>.327</i>	<i>.120</i>	<i>.691</i>	<i>.296</i>	<i>.109</i>	<i>-.077</i>	<i>.691</i>	<i>.296</i>	<i>.109</i>	<i>-.077</i>	<i>.002</i>	

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on momentum from low (1) to high (17).

Table 10

Results of time-series regressions conducted on seventeen liquidity-sorted portfolios.

First step Fama-Macbeth regressions. Factor loadings.														
Portfolio	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model					
	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ	
1	.287	.279	.093	.079	.259	.075	.073	-.047	.255	.076	.075	-.047	-.031	
2	.396	.384	.229	.065	.35	.196	.054	-.082	.356	.193	.051	-.083	.053	
3	.461	.446	.221	.119	.421	.197	.111	-.06	.425	.196	.109	-.060	.026	
4	.469	.449	.285	.157	.447	.282	.156	-.006	.447	.282	.156	-.006	.003	
5	.567	.551	.300	.098	.530	.281	.091	-.048	.539	.277	.086	-.049	.069	
6	.567	.549	.327	.115	.521	.299	.106	-.068	.525	.297	.104	-.069	.031	
7	.671	.648	.340	.185	.640	.333	.183	-.018	.649	.328	.178	-.019	.069	
8	.622	.606	.335	.085	.558	.288	.068	-.117	.564	.285	.065	-.118	.051	
9	.733	.714	.373	.117	.699	.358	.112	-.037	.690	.362	.117	-.036	-.073	
10	.781	.768	.350	.053	.763	.346	.052	-.011	.761	.347	.053	-.011	-.019	
11	.875	.855	.380	.126	.848	.373	.124	-.018	.846	.374	.125	-.017	-.014	
12	.901	.872	.483	.221	.819	.432	.203	-.128	.837	.423	.194	-.129	.140	
13	.987	.965	.442	.134	.895	.374	.110	-.171	.893	.375	.111	-.171	-.013	
14	1.010	.982	.425	.215	.951	.396	.205	-.074	.923	.409	.219	-.072	-.216	
15	1.089	1.079	.328	.014	1.015	.266	-.008	-.156	1.014	.266	-.008	-.156	-.001	
16	1.127	1.116	.377	.008	1.084	.345	-.003	-.079	1.069	.352	.004	-.078	-.110	
17	1.186	1.155	.526	.214	1.118	.49	.202	-.090	1.109	.494	.207	-.089	-.073	
<i>Mean</i>	<i>.749</i>	<i>.730</i>	<i>.342</i>	<i>.118</i>	<i>.701</i>	<i>.314</i>	<i>.108</i>	<i>-.071</i>	<i>.700</i>	<i>.314</i>	<i>.109</i>	<i>-.071</i>	<i>-.006</i>	

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on liquidity from low (1) to high (17).

Table 11

Results of cross-sectional regressions conducted on seventeen size-sorted portfolios with restriction close <10 .

Fama-Macbeth second, close <10 restriction								
Model (size sorted)	Intercept	Market	SMB	HML	PR1YR	LIQ	R^2 fmb	Adj. R^2 fmb
CAPM	-.006* (.0032) -1.85	.022*** (.0051) 4.28					.22	.13
Fama-French (3)	-.004 (.0037) -1.12	.021*** (.0055) 3.72	-.003 (.0060) -.42	.001 (.0096) .13			.44	.16
Carhart (4)	-.004 (.0037) -1.15	.021*** (.0055) 3.76	-.003 (.0063) -.44	.001 (.0099) .11	-.010 (.0184) -.56		.55	.19
Factor model (5)	-.007 (.0047) -1.43	.024*** (.0064) 3.69	.000 (.0069) .01	-.006 (.0113) -.51	-.019 (.0210) -.89	.016 (.0189) .87	.64	.19

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R -squared and \bar{R} -squared, respectively. Asterisks denote the variables significance level;

(1%)***, (5%)**, and (10%)*.

Table 12

Results of cross-sectional regressions conducted on eleven beta-sorted portfolios with restriction close <10 .

Fama-Macbeth second, close <10 restriction								
Model (beta sorted)	Intercept	Market	SMB	HML	PRIYR	LIQ	R^2 fmb	Adj. \bar{R}^2 fmb
CAPM	.008***	-.001					.46	.35
	(.0018)	(.0043)						
	4.55	-.30						
Fama-French (3)	.002	-.003	.033*	.015			.70	.40
	(.0033)	(.0049)	(.0188)	(.0210)				
	.55	-.57	1.77	.71				
Carhart (4)	.002	.002	.018	.008	.011		.81	.45
	(.0032)	(.0058)	(.0199)	(.0212)	(.0192)			
	.56	.42	.91	.39	.58			
Factor model (5)	.003	.001	.013	.010	.004	.027	.91	.47
	(.0036)	(.0062)	(.0235)	(.0221)	(.0221)	(.0487)		
	.83	.13	.54	.47	.17	.56		

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R -squared and \bar{R} -squared, respectively. Asterisks denote the variables significance level;

(1%)***, (5%)**, and (10%)*.

Table 13

Results of cross-sectional regressions conducted on seventeen momentum-sorted portfolios with restriction close <10 .

Fama-Macbeth second, close <10 restriction								
Model (momentum sorted)	Intercept	Market	SMB	HML	PRIYR	LIQ	R^2 fmb	Adj. \bar{R}^2 fmb
CAPM	.005	.007					.23	.13
	(.0048)	(.0083)						
	.96	.78						
Fama-French (3)	.036***	-.052**	.053***	-.042**			.49	.23
	(.0110)	(.0215)	(.0185)	(.0190)				
	3.29	-2.43	2.84	-2.20				
Carhart (4)	.018	-.027	.043**	.001	.008		.59	.27
	(.0109)	(.0207)	(.0179)	(.0203)	(.0054)			
	1.65	-1.28	2.38	.07	1.41			
Factor model (5)	-.000	.009	.005	.043	.009	-.069***	.70	.32
	(.0113)	(.0211)	(.0183)	(.0252)	(.0054)	(.0221)		
	-.02	.41	.26	1.72	1.61	-3.11		

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R -squared and \bar{R} -squared, respectively. Asterisks denote the variables significance level;

(1%)***, (5%)**, and (10%)*.

Table 14

Results of cross-sectional regressions conducted on seventeen liquidity-sorted portfolios with restriction close <10 .

Fama-Macbeth second, close <10 restriction								
Model (liquidity sorted)	Intercept	Market	SMB	HML	PR1YR	LIQ	R^2 fmb	Adj. R^2 fmb
CAPM	-.004* (.0021) -1.82	.019*** (.0045) 4.18					.31	.22
Fama-French (3)	-.002 (.0040) -.56	.015** (.0060) 2.49	.017 (.0150) 1.13	-.032 (.0409) -.77			.52	.27
Carhart (4)	-.003 (.0052) -.60	.0156** (.0066) 2.38	.015 (.0168) .91	-.024 (.0523) -.46	-.016 (.0256) -.63		.60	.28
Factor model (5)	.007 (.0057) 1.19	-.002 (.0088) -.18	.061** (.0249) 2.44	-.111* (.0596) -1.86	.031 (.0278) 1.13	-.061** (.0238) -2.57	.68	.28

The first row in the risk factor cells is the coefficient estimates, the second row is the coefficient standard deviation, and the third row is the coefficients t-statistic. Column 8 and 9 displays the R -squared and \bar{R} -squared, respectively. Asterisks denote the variables significance level; (1%)***, (5%)**, and (10%)*.

Table 15

Statistics for time-series intercept analysis

Model (size sorted)	Mean alpha	$GRS (J)$	Prob. (GRS)	Model (momentum sorted)	Mean alpha	$GRS (J)$	Prob. (GRS)
CAPM	.005713 (.002128)	12.46	.000000	CAPM	.005277 (.002209)	5.35	.000000
Fama-French (3)	.001596 (.002071)	11.73	.000000	Fama-French (3)	.001459 (.002203)	3.57	.000200
Carhart (4)	.002181 (.002219)	9.52	.000000	Carhart (4)	.002160 (.002226)	2.20	.018584
Five-factor	.002173 (.002221)	9.48	.000000	Five-factor	.002138 (.002229)	2.23	.017097

Model (beta sorted)	Mean alpha	$GRS (J)$	Prob. (GRS)	Model (liquidity sorted)	Mean alpha	$GRS (J)$	Prob. (GRS)
CAPM	.003255 (.001953)	3.25	.002566	CAPM	.005758 (.002117)	5.82	.000000
Fama-French (3)	.000239 (.001971)	1.96	.061025	Fama-French (3)	.001605 (.002091)	3.44	.000312
Carhart (4)	.001029 (.002078)	.49	.841253	Carhart (4)	.002204 (.002241)	3.69	.000132
Five-factor	.000953 (.002080)	.47	.853805	Five-factor	.002195 (.002241)	3.94	.000055

The second column reports the mean absolute value of alpha in the time-series regression. The third column is the Gibbons et al. 1989 test statistic, and the fourth column reports its corresponding p-value.

Table 16

Results of time-series regressions conducted on seventeen size-sorted portfolios with restriction close <10 .

First step Fama-Macbeth, close <10 restriction.														
	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model					
Portfolio	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ	
1	.308	.290	.288	.129	.276	.274	.124	-.034	.286	.270	.119	-.035	.079	
2	.513	.494	.335	.126	.473	.314	.119	-.051	.471	.315	.120	-.051	-.016	
3	.621	.612	.348	-.008	.613	.349	-.008	.002	.610	.350	-.006	.002	-.024	
4	.642	.622	.455	.105	.593	.427	.095	-.070	.595	.426	.094	-.071	.016	
5	.712	.696	.273	.115	.654	.232	.101	-.103	.651	.233	.102	-.103	-.018	
6	.835	.810	.304	.218	.805	.298	.216	-.013	.808	.297	.214	-.014	.022	
7	.796	.781	.234	.113	.790	.243	.116	.023	.797	.240	.113	.022	.052	
8	.785	.772	.151	.110	.768	.147	.109	-.010	.768	.147	.109	-.010	-.003	
9	.938	.931	.200	.020	.933	.202	.020	.004	.922	.207	.026	.004	-.085	
10	.967	.971	-.083	-.018	.974	-.079	-.016	.008	.975	-.080	-.017	.008	.008	
<i>Mean</i>	<i>.712</i>	<i>.698</i>	<i>.251</i>	<i>.091</i>	<i>.688</i>	<i>.241</i>	<i>.088</i>	<i>-.025</i>	<i>.688</i>	<i>.241</i>	<i>.087</i>	<i>-.025</i>	<i>.003</i>	

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on market capitalization from low (1) to high (17).

Table 17

Results of time-series regressions conducted on eleven beta-sorted portfolios with restriction close <10 .

First step Fama-Macbeth, close <10 restriction.														
	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model					
Portfolio	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ	
1	.091	.081	.133	.091	.114	.165	.102	.080	.117	.164	.100	.080	.020	
2	.334	.320	.158	.125	.326	.163	.127	.014	.330	.161	.125	.014	.032	
3	.511	.500	.110	.107	.507	.116	.109	.015	.508	.116	.109	.015	.012	
4	.685	.668	.220	.147	.667	.219	.147	-.003	.670	.217	.145	-.003	.029	
5	.869	.856	.208	.101	.852	.204	.100	-.009	.861	.200	.095	-.010	.070	
6	1.084	1.074	.253	.039	1.053	.232	.032	-.051	1.061	.229	.028	-.052	.061	
7	1.471	1.465	.209	.007	1.349	.097	-.032	-.279	1.349	.098	-.031	-.279	-.007	
<i>Mean</i>	<i>.721</i>	<i>.709</i>	<i>.184</i>	<i>.088</i>	<i>.695</i>	<i>.171</i>	<i>.084</i>	<i>-.033</i>	<i>.699</i>	<i>.169</i>	<i>.082</i>	<i>-.034</i>	<i>.031</i>	

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on beta from low (1) to high (11).

Table 18

Results of time-series regressions conducted on seventeen momentum-sorted portfolios with restriction close <10 .

First step Fama-Macbeth, close <10 restriction.														
Portfolio	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model					
	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ	
1	1.021	.995	.609	.125	.810	.429	.062	-.450	.799	.434	.068	-.449	-.083	
2	.854	.839	.352	.069	.744	.260	.037	-.230	.747	.259	.036	-.230	.020	
3	.700	.688	.215	.074	.611	.141	.048	-.186	.613	.140	.047	-.186	.014	
4	.687	.671	.180	.150	.635	.146	.138	-.087	.645	.141	.132	-.087	.078	
5	.592	.579	.164	.115	.558	.145	.108	-.049	.562	.143	.107	-.049	.027	
6	.602	.589	.147	.118	.598	.155	.121	.021	.605	.152	.117	.020	.052	
7	.597	.589	.104	.069	.622	.136	.080	.080	.623	.136	.079	.080	.013	
8	.556	.547	.056	.090	.595	.103	.106	.116	.598	.101	.105	.116	.019	
9	.647	.639	.176	.046	.699	.234	.066	.145	.694	.236	.069	.146	-.042	
10	.756	.748	.320	-.012	.890	.457	.036	.344	.889	.458	.037	.344	-.004	
<i>Mean</i>	<i>.701</i>	<i>.688</i>	<i>.232</i>	<i>.084</i>	<i>.676</i>	<i>.221</i>	<i>.080</i>	<i>-.030</i>	<i>.677</i>	<i>.220</i>	<i>.080</i>	<i>-.030</i>	<i>.009</i>	

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on momentum from low (1) to high (17).

Table 19

Results of time-series regressions conducted on seventeen liquidity-sorted portfolios with restriction close <10 .

First step Fama-Macbeth, close <10 restriction.														
Portfolio	CAPM	Fama-French three-factor model			Carhart four-factor model				Five-factor model					
	Market	Market	SMB	HML	Market	SMB	HML	PR1YR	Market	SMB	HML	PR1YR	LIQ	
1	.293	.284	.103	.086	.276	.096	.083	-.018	.274	.097	.084	-.018	-.015	
2	.418	.406	.203	.085	.371	.17	.073	-.084	.376	.168	.071	-.084	.035	
3	.456	.441	.221	.125	.45	.23	.128	.022	.457	.227	.125	.022	.05	
4	.541	.526	.269	.11	.517	.26	.107	-.022	.525	.256	.103	-.022	.065	
5	.645	.628	.309	.109	.619	.3	.106	-.022	.626	.297	.102	-.023	.053	
6	.718	.701	.296	.12	.694	.29	.118	-.017	.695	.289	.118	-.017	.004	
7	.882	.868	.297	.089	.875	.305	.091	.018	.881	.302	.089	.018	.042	
8	.942	.926	.367	.083	.913	.354	.079	-.031	.913	.354	.079	-.031	-.001	
9	1.079	1.065	.287	.085	1.042	.265	.077	-.055	1.028	.272	.085	-.054	-.113	
10	1.158	1.152	.174	.021	1.133	.156	.015	-.045	1.122	.161	.021	-.044	-.085	
<i>Mean</i>	<i>.713</i>	<i>.700</i>	<i>.253</i>	<i>.091</i>	<i>.689</i>	<i>.243</i>	<i>.088</i>	<i>-.025</i>	<i>.690</i>	<i>.242</i>	<i>.088</i>	<i>-.025</i>	<i>.004</i>	

Shows estimated coefficients from time-series regressions for each asset pricing model used as factor loadings in the cross-sectional regressions. Where the portfolios are ranked on liquidity from low (1) to high (17).

