Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Department of Economics

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Trend Inflation and Monetary Policy Trade-offs in a Small Open Economy

A New Keynesian Dynamic Stochastic General Equilibrium Approach

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Supervisor: Joakim Blix Prestmo
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Abstract

In the first period of high inflation since the financial crisis of 2008, it is becoming clear that trade-offs in monetary policy make it hard to find the right balance between stabilising the price level and maintaining as much economic activity as possible. This thesis examines the effect high inflation periods have on the central banks ability to conduct monetary policy within a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework. The resulting analysis show several mechanisms through which higher trend inflation can lead to trade-offs and loss of flexibility in the conduct of monetary policy. To empirically validate these mechanisms, Bayesian estimation techniques are used to establish a connection between the theoretical framework and Norwegian data.

The model combines Gali and Monacelli (2005) adapted to look at trend inflation à la Ascari and Sbordone (2014), providing a unique perspective on the interplay between trend inflation and open economy dynamics. The result is a clear argument for why the Norwegian central bank must keep today's high inflation at bay so that high inflation expectations do not form. It also shows the loss of monetary policy flexibility if it does.

Sammendrag

I den første perioden siden finanskrisen i 2008 med høy inflasjon er det tydelig at avveininger i pengepolitikken gjør det vanskelig å finne en god balanse mellom å stabilisere prisnivået og å opprettholde så høy økonomisk aktivitet som mulig. Denne masteroppgaven undersøker hva slags effekt perioder med høy inflasjon har på sentralbankens evne til å føre pengepolitikk i en Dynamisk Stokastisk Generell Likevekt (DSGE) model. Den resulterende analysen viser flere mekanismer som indikerer at høyere trendinflasjon kan føre til avveininger og tap av fleksibilitet i pengepolitikken. For å empirisk validere disse mekanismene, brukes Bayesiansk estimering for å etablere en sammenheng mellom den teoretiske modelen og norsk data.

Modellen tar inspirasjon fra Gali og Monacelli (2005), tilpasset for å se på trendinflasjon i tråd med Ascari og Sbordone (2014), og gir et unikt perspektiv på trendinflasjon i en liten åpen økonomi. Resultatet er et tydelig argument for hvorfor Norges Bank må holde dagens høye inflasjon i sjakk, slik at høye inflasjonsforventninger ikke dannes. Den viser også tapet av fleksibilitet i pengepolitikken dersom de dannes.

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Contents

1 Introduction

In the face of the first period of high inflation since the financial crisis of 2008, it is becoming clear that high inflation can lead to trade-offs in the conduct of monetary policy. A mixture of post-covid supply chain frictions and Ukraine war supply disturbances has led to a rapid increase in consumer prices, creating a need for central banks to increase the policy rate to dampen the price growth (Norges Bank, 2023). As is well known in the literature, inflation targeting regimes tend to exacerbate negative supply-side shocks due to the need to increase the interest rate to stabilise the price level when the economy experiences a negative supply shock. The rate increase enlarges the negative shock to the economy, further destabilising production (Rødseth, 2000; Røisland & Torvik, 2004). This dynamic effectively creates a *trade-off* in monetary policy. On the one side, the central bank has to raise rates to reduce inflation and stabilise inflation expectations. On the other, it must ensure that the rate increase is not too costly for households and firms.

Within this context, this thesis will explore the effects of higher trend inflation on monetary policy. By incorporating trend inflation à la Ascari and Sbordone (2014) into the small open economy model of Gali and Monacelli (2005) and employing Bayesian estimation techniques, this research aims to address the following research question:

> *How can the effect of trend inflation on monetary policy be explored within a New Keynesian Dynamic Stochastic General Equilibrium model, and what are the implications for Norwegian monetary policy?*

The results show that higher trend inflation gives rise to more volatility in inflation and output due to changes in the firms' price-setting behaviour, ultimately leading to trade-offs in monetary policy much like what we see today. The consequence of the increased volatility is that the central bank loses its flexibility to respond to fluctuations in production in favour of only being able to stabilise inflation. The result is a clear argument for why the Norwegian central bank must keep today's high inflation at bay so that high inflation expectations do not form. It also shows the loss of monetary policy flexibility if it does.

Closest in spirit to the approach taken in this thesis is that of Yılmaz and Tunc (2022), who combine the same model frameworks to look at the effects of trend inflation in a small open economy. However, they fail to combine the frameworks in a way that allows for the interplay of openness and trend inflation by assuming that the slope of the Phillips curve is independent of the degree of openness and instead opt to examine the effect of higher trend inflation on exchange rate persistence. This thesis thus builds upon their contribution by fully specifying the relationship between trend inflation and openness and estimating the model parameters using Bayesian techniques. This specification provides the new implication that a more open economy is shielded from some of the adverse effects of trend inflation.

A more open economy is shielded from some of the adverse effects of trend inflation because the openness parameter enters directly into the slope of the New Keynesian Phillips Curve, *flattening* the relationship between output and inflation. Interestingly, this implication runs contrary to the argument in Ascari and Sbordone (2014), who points to a decoupling of inflation and production as one of the costs of trend inflation. In the following model specification, it seems more likely that the size of a trade-offs between inflation and production is a costly consequence of trend inflation, not the decoupling itself.

The rest of the thesis is organised as follows. After this introduction, section 2 provides a literature review of relevant approaches to the one taken in this thesis, followed by a brief reminder on trend inflation. Section 3 presents the non-linear DSGE model to be used, that consequently log-linearised in section 4. The main implications of the model are then discussed in section 5, before the implications are tested using Bayesian techniques in section 6. There is then a discussion of the results of the thesis and their resulting policy implications in section 7.

2 Background

2.1 Relevant literature

What we now know as New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models[1] has its roots in the seminal work of Kydland and Prescott (1982) and Prescott (1986) who started the development of the vast Real Business Cycle (RBC) literature. The RBC models were a crucial stepping stone in the development of the later DSGE models, as they laid down many of the central building blocks and assumptions that became essential parts of the New Keynesian DSGE framework. Most notable was the inclusion of micro-founded decision functions for both firms and households, together with an assumption of rational expectations. However, despite their academic significance, these models were highly stylised and fell short of providing a convincing explanation for observed macroeconomic data.

To address this limitation, researchers began incorporating additional frictions into the models, such as Dixit and Stiglitz (1977) monopolistic competition and Calvo (1983) price stickiness. These extensions gave way to the development of the "canonical" New Keynesian model, exemplified by the works of economists like Yun (1996) and Clarida et al. (1999). These models blended the insights from RBC models with frictions and nominal rigidities, allowing for a comprehensive analysis of the economy more aligned with empirical observations.

These New Keynesian models have gained widespread popularity for two reasons. Firstly, they demonstrated an improved ability to capture the observed business cycle data compared to their RBC predecessors. Secondly, they introduced the role of monetary policy in smoothing the impact of transitory shocks in the short term (Christiano et al., 2018). This combination of factors made them both academically interesting *and* established them as workhorses for monetary policy analysis, currently utilised by central banks worldwide (Adolfson et al., 2013; Brubakk et al., 2006; Christoffel et al., 2008)

In parallel to the development of the closed economy New Keynesian models, there was

[1] Often referred to as New Keynesian models in this thesis.

early work on integrating shocks and frictions into the open economy setting as part of the New Open Economy Macroeconomics (NOEM) literature (Lane, 2001). The NOEM literature explores how various economic factors, such as trade, capital flows, and exchange rates, interact with monetary and fiscal policies to shape macroeconomic outcomes in an open economy context. Early contributions were that of Svensson and Wijnbergen (1989) and Obstfeld and Rogoff (1995), who combine global dynamics and frictions in analysing monetary policy. Building upon the NOEM literature, open economy dynamics was later adopted into the New Keynesian literature by the likes of Monacelli et al. (1999) and Gali, Monacelli, et al. (2000), who examine the interplay between the exchange rate and optimal monetary policy.

While significant research has been conducted on the impact of trend inflation in closed economy staggered price setting models, such as (Amano et al., 2007; Ascari, 2004; King & Wolman, 1996), relatively few contributions have focused on analysing the effects of trend inflation in small open economies. There are some notable exceptions in the literature, however. Particularly relevant for the approach taken in this thesis is the work done by Yılmaz and Tunc (2022), Zhang and Dai (2020), and Zhao (2022) who examine spillovers of trend inflation between countries, and its implications for exchange rate dynamics. These contributions shed light on the interplay between trend inflation, international linkages, and exchange rate movements in open economy settings and serve as a natural starting point when examining trend inflation in a small open economy.

Bayesian estimation has been an increasingly popular approach to bringing data to New Keynesian models. Contributions of particular significance in showing how data can be brought to DSGE models are that of Smets and Wouters (2003), who perform full information estimation of a closed economy DSGE model, and Lubik and Schorfheide (2005), who bring an extended version of the small open economy framework of Gali and Monacelli (2005) to data. [2] Particularly useful for the approach taken in this thesis has been An and Schorfheide (2007) and Griffoli (2010), who both provide comprehensive overviews of Bayesian estimation of New Keynesian DSGE models.

[2] Bayesian estimation is often referred to as "full information estimation".

2.2 Trend inflation and inflation targeting

Given the extensive theoretical and empirical literature on inflation, this background chapter selectively focuses on key aspects of inflation that are relevant to the analysis of higher trend inflation. The most important assumption implicitly made in the theoretical model is that inflation can be decomposed into two components. The first component is short-term inflation, reflecting transitory shifts in response to external shocks, price-setting behaviour, or monetary policy. The second component is long-term inflation, which, within the New Keynesian framework, represents the level of inflation aligned with a frictionless steady state.[3] The distinction between short-run and trend inflation can be illustrated using data on the consumer price index.

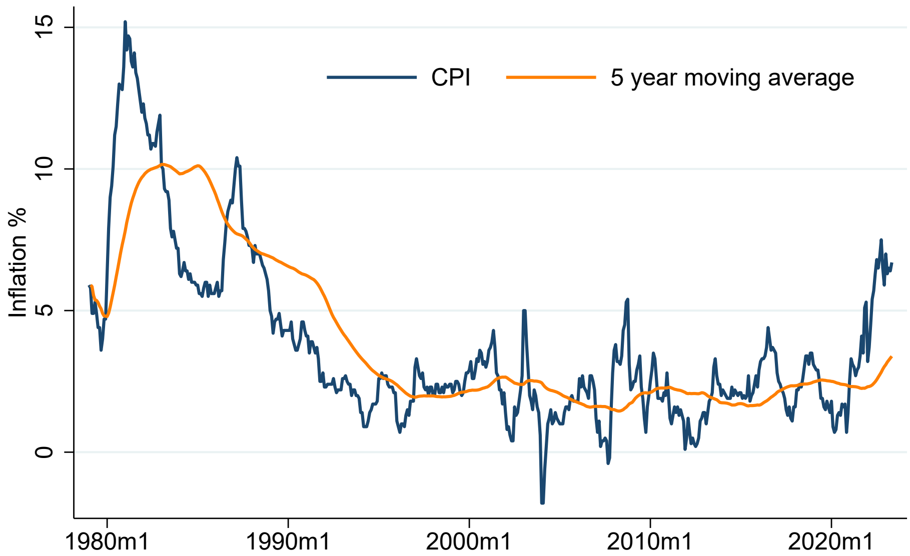

Figure 1: Long and short term inflation

Collected from The yearly inflation rate is in blue, and its five-year moving average is in orange. The plot serves as a way to shape our thinking about long and short-term inflation but is not an accurate estimation of trend inflation.

As modelled in this thesis, trend inflation can be understood as the orange line in the above graph, here assumed to be captured by a five-year rolling average. In contrast to the blue line representing short-term inflation, trend inflation is considered more sluggish and shifts only in response to larger economic trends.

[3]The steady-state can be understood as the saddle point at which the economy rests in the absence of exogenous shocks.

Short-term fluctuations in inflation are a natural consequence of short-term fluctuations in output. The logic behind such a relationship between inflation and production is often conceptualised through the Phillips curve, which postulates a simple relationship between the unemployment rate and inflation. The idea behind the curve is straightforward. When there is excess demand for labour, wages will be bid up by firms fighting to attract workers, leading to inflation (Phillips, 1958). The existence of such a relationship in the real world has been extensively discussed and examined in the literature, (Bergholt et al., 2023), but either way, it is a useful abstraction in macroeconomic modelling. Although not precisely analogous to the original Phillips curve, a similar relationship is typically used in the New Keynesian literature, where short-term inflation is assumed to be a function of output relative to natural output.

Inflation-targeting regimes use the output-inflation relationship to keep the growth in the price level at the target. By deliberately shifting the policy rate to influence the interest rate at which commercial banks can lend from the central bank, the central bank can influence the level of activity in the economy and, by proxy, the level of inflation (Røisland & Sveen, 2018). As output and inflation tend to move in the same direction, inflation targeting will often be able to stabilise both production *and* inflation at the same time, coined the Divine Coincidence in the New Keynesian literature (Alves, 2014). However, if inflation and output move in different directions, stabilising inflation might be more costly because the interest rate change will lead to larger destabilisation of the production gap in order to stabilise the inflation gap.

An implicit aspect of inflation targeting regimes is that the central bank effectively attempts to keep long-run trend inflation in line with the policy target. Long-run trend inflation is slightly harder to grasp than short-term inflation as it is not necessarily tied to fluctuations in the real economy today but is a function of peoples *expectations* of the economy in the future (Bernanke et al., 2007; Mishkin et al., 2007). Suppose households and firms expect high inflation in the future. In that case, they will hedge against the possibility of ending up with low real wages or prices in such a way that the expected inflation manifests into real inflation.

To achieve stable inflation at the target, the central bank needs to 'anchor' inflation expectations to the policy target by consistently responding to deviations from the target and being transparent about the future direction of the policy rate. Anchoring can be achieved by responding adequately and consistently to deviations from the target, in tandem with being transparent about the future direction of the policy rate. For example, a believable promise to keep rates high in the face of high inflation can help create stable expectations about future economic conditions, ultimately leading to inflation returning to trend (Friedman, 1968; Woodford, 2003; Yetman, 2017). A history of fulfilled promises can, in turn, create predictability of the central bank's commitment to stability, making long-term inflation expectations more stable.

Additionally, inflation targeting is influenced by the interplay between domestic monetary policy and the global economy. The global economy impacts the monetary policy of open economies through various channels; here we will focus on two. The first is that domestic households consume a mix of domestic and foreign goods, tying the domestic price level to the price of imported goods. An increase in the price of foreign goods called imported inflation, will necessitate a response on the behalf of the central bank (Ciccarelli & Mojon, 2010). The second is that global factors influence the neutral interest rate for monetary policy. As the domestic economy becomes more integrated with the global economy, the neutral interest rate becomes more dependent on global developments rather than domestic factors. This, in turn, influences the level at which the central bank has to set the policy rate.

The DSGE model presented in the next section will incorporate short- and long-term inflation through approximation around a steady state characterised by trend inflation. This will allow us to perform a counterfactual analysis of the effects higher trend inflation has on both economic stability *and* inflation expectations in the context of an inflation-targeting small open economy. The interplay between openness and trend inflation is also examined, providing theoretical evidence that increased globalisation leads to stability as long as the global economy is stable.

3 The DSGE model

The model presented in this section is based on the small open economy model described in chapter 8 of *Monetary Policy, Inflation, and the Business Cycle* by Galí (2015), adapted to look at trend inflation à la Ascari and Sbordone (2014). This specification enables us to analyse the reciprocal relationship between the openness of the economy and the dynamics of trend inflation. The novel contribution of this thesis is found when linearising the intermediate firm optimal pricing decision, given that the open economy marginal costs are integrated into the pricing decision. This yields a Phillips curve sensitive to trend inflation *and* the degree of openness.

A continuum of utility-maximising representative households populates the demand side of the model, each representing a country in the world economy. For convenience, it is assumed that utility is identical across the world, making it so that we, in reality, will be investigating a two-country model. The supply side of the model consists of two types of firms. The first is a perfectly competitive final producer firm; the second is a continuum of intermediate goods firms. The behaviour of intermediate producers, driven by profit optimisation in a sticky prices environment, plays a crucial role in shaping the inflation dynamics of the model.

Once the demand and supply sides of the economy are introduced, they are log linearised and Taylor approximated. By combining the resulting linear expressions, we can derive the relationships that characterise market clearing and aggregate inflation dynamics. The result is a standard New Keynesian IS curve and a New Keynesian Phillips curve that also captures the effects of higher trend inflation on inflation dynamics. The latter collapses to the standard New Keynesian Phillips curve when trend inflation is assumed to be zero, yielding a replica of the model derived by Gali and Monacelli (2005).

Deriving a New Keynesian model requires a lot of algebraic manipulation, and so, for the most part, only the most essential expressions are presented in the text. More thorough derivation of the model equations can be found in Appendix A and Appendix B. A more thorough derivation still can be found in the brilliant lecture notes of Bergholt (2012).

3.1 Demand in a small open economy

We consider a representative household that aims to maximise its expected lifetime utility over its infinite lifespan. We further assume that utility is an increasing function of consumption C_t and a decreasing function of labour hours worked N_t

$$E_0 \sum_{t=0}^{\infty} \beta U_t(C_t, N_t; Z_t), \tag{1}$$

where Z_t is a period t preference shifter, and β is the discount factor of future consumption. In a small open economy, households have the option to consume either domestic goods $C_{H,t}$ or foreign goods $C_{F,t}$. Total consumption can then be described as

$$C_t \equiv \left((1-\upsilon)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \upsilon^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $\upsilon \in [0,1]$ is the domestic economy's degree of openness to the world economy, and $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. As the economy becomes more open, the consumption of the domestic country is weighted towards foreign goods. A higher elasticity of substitution between home and foreign goods increases the weight of openness in determining consumption allocation. We can interpret $C_{H,t}$ and $C_{F,t}$ as being final consumer goods, consisting of an infinite amount of intermediate goods $C_{H,t}(i)$ and $C_{F_t}(i)$, where $i \in [0,1]$. The variety consumed can be described by a constant elasticity of substitution (CES) function. The consumption index of domestic goods is then given by

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $C_{H,t}(i)$ denotes the quantity of home variety (i) consumed by the representative household in period t. Parameter $\epsilon > 1$ is the elasticity of substitution between varieties produced in any country j.[4] The consumption index of all imported goods is given by

[4]A higher elasticity of substitution between intermediate goods will give less market power to firms

$$C_{F,t} \equiv \left(\int_0^1 C_{j,t}(j)^{\frac{\epsilon_f - 1}{\epsilon_f}} dj \right)^{\frac{\epsilon_f}{\epsilon_f - 1}},$$

where $C_{F,t}$ is a consumption index of varieties consumed from country j, and parameter $\epsilon_f > 1$ is the elasticity of substitution between goods imported from different countries in the world economy. Lastly, imports can be further indexed by the variety i imported from country j

$$C_t^j \equiv \left(\int_0^1 C_{j,t}(i)^{\frac{\epsilon - 1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}}.$$

Finding the utility maximising consumption allocation can be broken down into two steps. The first is choosing how much domestic goods $C_{H,t}$ and imported goods $C_{F,t}$ the representative household wants to consume in total. Once the level of goods is decided, they have to choose what variety of goods they want to consume from each country j, represented by $C_{j,t}$. For a given level of expenditures, it is shown in Appendix A that the optimal consumption allocation across all varieties of intermediate goods from the domestic and foreign countries yields the following demand functions

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \quad ; \quad C_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\epsilon} C_{j,t}, \tag{3}$$

where demand for domestic and foreign intermediate good i is a function of its price relative to the price level, and total consumption of that type of good. [5] In a similar fashion we can also show that the optimal basket of import consumption from country j is

$$C_{j,t} = \left(\frac{P_{j,t}}{P_{F,t}} \right)^{-\epsilon_f} C_{F,t}. \tag{4}$$

It is shown in Appendix A that the above demand functions also provide us with a natural

and thus lower markups. Consumption of any intermediate good will also be more sensitive to the price of that intermediate good.

[5] We can see that as the elasticity of substitution between varieties ϵ increases the variety of goods consumed increases.

price index, giving us an aggregate consumer index (CPI) defined by

$$P_t \equiv ((1-\upsilon)P_{H,t}^{1-\eta} + \upsilon P_{F,t}^{1-\eta})^{\frac{1}{1-\eta}} \tag{5}$$

where the aggregate price level P_t is a function of the domestic and foreign price levels, weighed by the degree of openness to the world economy. Combining these definitions, we arrive at the following demand functions for domestic and foreign goods

$$C_{H,t} = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C_{F,t} = \upsilon\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t. \tag{6}$$

Now that we have derived the optimal allocation between all varieties i, we need to find the optimal level of total consumption. In each period the representative household has to decide how much they want to work, how much they want to consume, and how much they want to save for the next period. Obviously, they cannot consume more in any given period than the income they receive from working and previous period savings.[6] Such a condition can be represented by a fairly simple budget constraint of the form

$$P_t C_t + E_t\{Q_{t,t+1} D_{t,t+1}\} \leq D_t + W_t N_t, \tag{7}$$

where it is shown in Appendix A how the constraint aggregate according to $\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 \left(\int_0^1 P_{j,t}(i)C_{j,t}(i)di\right) dj = P_t C_t$. The interpretation of the constraint is that the value of consumption, $C_t P_t$, and one period ahead savings, $E_t\{Q_{t,t+1}D_{t,t+1}\}$, must be smaller or equal to the value of the previous period savings, D_t, and labour hours supplied, $W_t N_t$. We define $Q_{t,t+1}$ as the stochastic discount factor for one period ahead savings and W_t as the nominal wage. The optimal level of consumption and savings can be found by maximising utility (1) subject to the budget constraint (7), and so we specify the utility function as

[6]We assume a no Ponzi Scheme condition so that $\lim_{T\to\infty} E_T \left\{\Lambda_{t,T}\frac{B_T}{P_T}\right\} \geq 0$.

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for} \quad \sigma \neq 1 \\ \left(logC_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & \text{for} \quad \sigma = 1 \end{cases},$$

where σ and φ determine the curvature of the utility gained from consumption and disutility gained from working, respectively.[7] Utility maximum for the representative household, given the resource constraint above is given by the following optimality conditions

$$Q_{t,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t}\right) \frac{P_t}{P_{t+1}} \right], \tag{8}$$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}. \tag{9}$$

The first equation is the inter-temporal optimality condition, representing optimal consumption allocation across time as a function curvature of the utility function in each period σ, future expected inflation and preferences. A higher value yields a faster-diminishing utility function making it optimal with a flatter consumption path across time. Optimal consumption in a given period also increases in the expected future price level P_{t+1} and a positive shift in preferences Z_t. The second equation is the intra-temporal optimality condition, representing the optimal choice between labour and leisure in a given period. The labour-leisure choice can be interpreted as the households labour supply schedule, which depends on the curvature of the utility function σ, but also the curvature of the disutility of labour, φ, and the real wage $\frac{W_t}{P_t}$.

3.2 Identities of the open economy

This thesis section will explore the dynamics that tie the domestic and foreign economies together. Two assumptions are made from the onset. The first is that the domestic economy is assumed to be infinitesimally small compared to the world economy and so does not affect world supply or demand. The second is that we assume perfect symmetry

[7] Higher values of σ and φ yields a faster declining utility function and sharper rising disutility function .

across all foreign countries, making it so that we, in reality, are examining a two-country world consisting of the domestic economy and the exogenous foreign economy. These assumptions are made so that the resulting model is easy to solve and understand, yielding a framework that broadly characterises the dynamics of an open economy but necessarily lacks many of the nuances of the real world.

3.2.1 The terms of trade

As usual, the terms of trade is defined as the price of foreign goods in terms of domestic goods

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}, \tag{10}$$

and can be interpreted as a natural measure of the domestic economy's competitiveness. A lower domestic price level relative to the foreign price level (higher S_t) makes the domestic economy more competitive on the world market, boosting net exports.

3.2.2 The exchange rate and law of one price

The exchange rate is the price of one currency relative to another. We assume that the law of one price holds for all traded goods so that $P_{F,t} = \wp_t P_t^*$, where $P_{F,t}$ is the price of a traded good in domestic currency, P^* is the world price of that same good and $\wp_t$ is the exchange rate. We further define the real exchange rate as the ratio of the world and domestic CPIs,

$$\xi \equiv \frac{P_{F,t}}{P_t}, \tag{11}$$

both expressed in domestic currency.

3.2.3 International risk sharing

We know that consumption in the small open economy must be connected with the output of the world economy. By assuming that the utility function across all countries in the world is identical to the domestic utility function, we can infer that a condition identical to (8) must also hold for foreign households. This gives us the following optimality condition for the world economy

$$Q_{t,t+1} = \beta E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \left(\frac{P^*_t}{P^*_{t+1}} \right) \left(\frac{\wp_t}{\wp_{t+1}} \right) \right], \tag{12}$$

where the assumption of symmetrical utility makes it so that the price of one period ahead savings $Q_{t,t+1}$ is the same across the world. The identical prices on savings follow from an assumption of a complete set of internationally traded securities so that all households have the same opportunity to store their wealth. When the curvature of the utility functions is assumed to be identical, we know that the whole world will make the same decisions about savings, giving us the same price of savings.[8] By combining the Euler equation (8), the terms of trade (10), the real exchange rate (11), and foreign utility (12), it is shown in Appendix A that we get

$$C_t = \vartheta C^*_t Z_t^{\frac{1}{\sigma}} \wp_t^{\frac{1}{\sigma}}, \tag{13}$$

which links domestic and foreign consumption. The constant ϑ captures differences in initial net asset positions influencing consumption. We assume symmetrical initial asset positions so that $\vartheta = 1$, making it so that domestic consumption relative to foreign consumption is symmetrical unless there is a shift in domestic preferences or the real exchange rate.

[8] We have also assumed that only the domestic household experience preference shifts, so that the world utility function does not include Z_t

3.3 Supply in a small open economy

We assume that there are two types of firms in the supply side of the economy. The first is a perfectly competitive final goods producer that aggregates intermediate goods into final consumer goods via constant elasticity of substitution (CES) aggregation technology

$$Y_t = \left(Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} . \tag{14}$$

The second type of firm is a continuum of intermediate goods producers, who each produces a differentiated intermediate good, $(i) \in [0, 1]$, using an economy-wide production technology A_t. Intermediate firm production is described by a simple Cobb-Douglas production function

$$Y_t(i) = A_t N_t(i), \tag{15}$$

where $N_t(i)$ is the labour hours used in production of good (i). By rearranging the production function, we can get demand for labour for a given level of intermediate good production

$$N_t^d(i) = \left(\frac{Y_t(i)}{A_t} \right), \tag{16}$$

where the labour demanded for the production of good (i) is a function of total output over productivity. Lastly, it can be shown that cost minimisation leads to a description of real marginal costs as a function of real wage times the marginal productivity of labour input, given by

$$MC_t^r(i) = \frac{W_t}{P_t} \frac{1}{A_t}, \tag{17}$$

where W_t is the nominal wage, and $\frac{1}{A_t}$ is the marginal productivity of labour. The assumption of constant returns to scale makes increasing production equally costly for all intermediate firms.

Intermediate goods producers operate in a market characterised by monopolistic competition, where their primary goal is to maximise profits based on their market power. This behaviour, coupled with the assumption of staggered price setting as described by Calvo (1983), introduces the main source of friction in this framework. According to Calvo pricing, only a fraction of firms, represented by $(1-\theta)$, can adjust their prices in a specific period. The remaining firms, denoted as θ, are bound to maintain the prices they set in the previous period. Importantly, the ability of a firm to readjust its price is unrelated to the length of time since its last opportunity to do so. Given these assumptions, the home domestic price level can be expressed as a weighted sum of the previous period price level, $P_{H,t-1}$, and the current period optimal reset price level, $\tilde{P}_{H,t}$, weighted by the probability that firms will be stuck with their previous period price, θ

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\epsilon} + (1-\theta)\tilde{P}_{H,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{18}$$

From this expression we can start to get a sense for the source of inflation in this model. If there is a difference between the last period price level, $P_{H,t-1}$, and the current optimal reset price, $\tilde{P}_t$, there will be growth in the domestic price level $P_{H,t}$.[9] Dividing both sides by $P_{H,t-1}$ in the above expression yields an expression for the domestic inflation rate, $\Pi_{H,t}$

$$\Pi_{H,t}^{1-\epsilon} = \theta + (1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_{H,t-1}}\right)^{1-\epsilon}, \tag{19}$$

where we see that aggregate inflation is strictly decreasing in θ, but crucially, increasing in the difference between the optimal and last period price. The price stickiness parameter is time invariant and thus constant, so it is then apparent that the only driver of domestic inflation comes from the fact that intermediate firms chose to set their prices higher than the previous price level. To understand how inflation occurs in this model we therefore

[9] This follows from an assumption that all resetting firms will chose the same price, so that $P_{H,t}(i) = \tilde{P}_{H,t}$.

has to examine why it is optimal for intermediate firms to set their prices higher than the previous period price level.

3.3.1 Optimal price setting

In response to the possibility of being unable to reoptimise their prices, firms will set their prices *higher* than the economy average to hedge themselves against a situation where they have to supply their good at a lower than preferred profit. This is reflected in the optimisation problem of a firm resetting their price, given by

$$E_t \sum_{j=0}^{\infty} \theta^j E_t[Q_{t,t+1}(\tilde{P}_{H,t} Y_{t+j,t}(i) - TC_{t+j} Y_{t+j}(i))], \tag{20}$$

where we see that firms set their prices to maximise profits, given the possibility of being stuck with that price for several periods. We assume that $\tilde{P}_t$ is the newly set price of a firm that resets its price in period j, and $Y_{t+j,t}(i)$ is the demand for the intermediate good i in $t+j$, from a firm that last reset its price in period t. In appendix A, it is shown that the firm's optimisation problem yields [10] [11]

$$\frac{\tilde{P}_{H,t}}{P_{H,t}} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j C_{t+k}^{1-\sigma} \Pi_{H,t,t+j}^{\epsilon} MC_{t+j,t}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j C_{t+k}^{1-\sigma} \Pi_{H,t,t+j}^{\epsilon-1}}. \tag{21}$$

The above expression is divided by the domestic price level so that it can be interpreted as a weighted sum of future marginal costs. The expression's numerator can be understood as the discounted present value of marginal costs. The optimal reset price is naturally increasing in future expected marginal costs, because firms want to hedge against being stuck with high costs and a low price. The expression's denominator can be understood as the discounted present value of marginal revenues. If firms expect future revenue to be high they do not want to loose competition to other firms and so choose to set their prices lower. To ease notation we define $\frac{\tilde{P}_{H,t}}{P_{H,t}} \equiv \overline{P}_{H,t}$, and rewrite the optimality condition as

[10] If $\theta = 0$, we see that the following term collapses to the desired markup of the firm $\frac{\epsilon}{\epsilon-1}$

[11] Subject to a demand schedule for intermediate input goods given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$.

$$\overline{P}_{H,t} = \frac{\epsilon}{\epsilon - 1}\frac{\psi_t}{\phi_t}, \tag{22}$$

where ψ and ϕ correspond to the numerator and denominator of (21). The infinite sums can both be expressed recursively as

$$\psi_t \equiv MC_t Y_t^{1-\sigma} + \theta\beta E_t\{\Pi^{\epsilon}_{H,t+1}\psi_{t+1}\}; \tag{23}$$

$$\phi_t \equiv Y_t^{1-\sigma} + \theta\beta E_t\{\Pi^{\epsilon-1}_{t+1}\phi_{H,t+1}\}, \tag{24}$$

with the same interpretations as their infinite sum counterparts. By breaking down the optimal price setting condition in this way it becomes even more clear that changes in marginal costs, MC_t^r, future expected marginal costs, ψ_{t+1} and future expected inflation, $\Pi_{H,t+1}$ are the main drivers behind firms choosing to set their prices higher than than previous period prices, giving rise to inflation. To further understand how such behaviour can be a source of inefficiency, we must examine how price dispersion, understood as the degree to which different firms sell goods at different prices, can be understood as a de facto productivity shifter.

3.3.2 Price dispersion

Price dispersion is the degree to which intermediate goods firms end up supplying their goods at different prices. Given the demand for intermediate goods, $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$, it is evident that intermediate goods firms with a lower than the average price will face higher demand. From equation (16), we can see that the only way the intermediate goods producers can meet this demand is by employing more workers, creating an unnatural production level. To find an expression for price dispersion, we first need to find the aggregate demand for labour, given by

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right) di.$$

By inserting the demand for domestically produced intermediate good (i), we get

$$N_t = \left(\frac{Y_t}{A_t}\right)\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}.$$

Defining aggregate price dispersion as $x_t = \int_0^1 (\frac{P_t(i)}{P_t})^{-\epsilon} di$ and inserting it into the above expression gives us

$$N_t = \left(\frac{Y_t}{A_t}\right) x_t \quad \Rightarrow \quad Y_t = \frac{A_t}{x_t} N_t, \tag{25}$$

which makes it evident that an increase in price dispersion leads to a decrease in aggregate production. Usually, price dispersion is negligible up to a first-order approximation and so of second-order importance in the typical New Keynesian DSGE model. When approximating the model around a steady state characterised by trend inflation, however, price dispersion appears as a problem and serve as a natural measure of the costs of trend inflation. Following Schmitt-Grohé and Uribe (2007), who studies optimal monetary policy in the presence of nominal rigidities, we can solve our definition of x_t forward to yield

$$x_t = (1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\epsilon} + \theta \pi_t^{\epsilon} s_{t-1}. \tag{26}$$

For an expression that is even easier to interpret, we insert for the optimal reset price (21) and evaluate the resulting expression in steady-state

$$x = \frac{1-\theta}{1-\theta\bar{\pi}^{\epsilon}}\left(\frac{1-\theta\bar{\pi}^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{27}$$

From this expression, two interesting observations can be made. The first is that in the absence of trend inflation, $\bar{\pi}$, price dispersion converges to unity, as is typical in New Keynesian models. The second is that price dispersion is an increasing function of sticky prices θ, the elasticity of demand ϵ, and trend inflation $\bar{\pi}$, which in turn result in inefficient allocation of labour, and thus lower aggregate productivity.

4 The linearised model

This section of the thesis will begin by providing an overview of log-linear approximation, followed by log-linearisation of the model around a steady state characterised by trend inflation. The resulting expressions are presented in the same order as their non-linear counterparts and then combined to describe the driving forces of the model dynamics, namely the New Keynesian Phillips curve and the dynamic IS curve. The model description is then completed with a description of monetary policy.

4.1 Logarithmic linearisation and Taylor approximation

Log-linearisation is an analytical technique commonly used in macroeconomic modelling to study the dynamics of a system around a steady-state equilibrium. Systems of difference equations often do not have closed-form solutions and therefore need some approximation strategy to be interpreted. Log-linearising around a particular point is one strategy where one takes the natural logarithm of the non-linear differential equations and then evaluates the linear approximation's dynamics around a chosen point, often referred to as the steady state. The expressions are then manipulated to be interpreted as approximate percentage deviations from said steady state.

Choosing a specific point as the steady state and then analysing the system's dynamics around that point has both advantages and disadvantages. One of the main drawbacks is that this approach restricts the analysis of the system to the state space near the chosen steady state, as the accuracy of the approximation diminishes when examining dynamics further away from the steady state. Furthermore, log-linearisation may result in a poor approximation of the system dynamics when the relationship between variables exhibits a high degree of nonlinearity.

On the positive side, log-linearisation offers an abstraction that facilitates a highly intuitive interpretation of the model and significantly reduces the computational power required to simulate the model. Furthermore, the economy is a relatively stable system in the long term, corresponding to the theoretical idea of a steady state. Therefore one could argue that it is most interesting to examine what happens in the state space in the vicinity of the steady state.

To understand how we can approximate difference equations to interpret them as percentage deviations around a steady state, we follow the helpful guide of Zietz (2006). First, we define hatted lowercase variables as log deviations from steady state

$$\hat{z}_t \approx ln(Z_t) - ln(Z),$$

where $\hat{z}$ represents percentage deviation from steady state, and Z without a subscript represents the time-invariant steady state. By definition, both sides of the above expression can be rewritten as

$$ln\left(\frac{Z_t}{Z}\right) = ln\left(1 + \frac{Z_t - Z}{Z}\right),$$

where 1 and $\frac{z}{z}$ cancel each other out, and $\hat{z}_t \equiv \frac{Z_t}{Z}$. To show why a first-order Taylor approximation of a linear expression can be interpreted as percentage deviations from the steady state we approximate the right-hand side of the above expression using the formula for Taylor expansion about a particular point z, where z belongs to the set of possible values of z_t

$$f(z_t) = f(z) + \frac{f'(z)}{1!}(z_t - z) + \frac{f''(z)}{2!}(z_t - z)^2 + \frac{f'''(z)}{3!}(z_t - z)^3 + ...,$$

where the resulting approximation yields an expression that can be interpreted as a percentage deviation from steady state

$$ln\left(1 + \frac{z_t - z}{z}\right) \approx ln1 + \frac{1}{z}(z_t - z) \approx \frac{z_t - z}{z} \approx \tilde{z}_t.$$

This shows us that Taylor approximation is a convenient way to linearise a system of difference equations, yielding highly interpretable expressions. Unfortunately, such an approximation quickly becomes inaccurate for large deviations from steady state, so linearised DSGE models are only useful when examining relatively small shocks to the economy.

4.2 Linearising the household optimality conditions

4.2.1 Labour-leisure

Logarithmic transformation of the intratemporal optimality condition (9) yields

$$w_t - p_t = \sigma c_t + \varphi n_t, \tag{28}$$

were lowercase letters denote the natural logarithm of their uppercase counterpart so that $w_t = ln(W_t)$. Holding the utility derived from consumption constant, we see that the optimal labour supply schedule determines the quantity of labour supplied as an increasing function of the real wage. The inverse Frish labour supply elasticity φ determines the willingness to work more when the real wage increase.

4.2.2 Euler equation

It is shown in Appendix B that the linear approximation of the Euler equation (8) yields

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho), \tag{29}$$

where period t consumption decreases in the nominal interest rate, increases in expected future inflation and increases in the discount factor. The inter-temporal elasticity of substitution σ, determines the strength of these effects, where a higher value gives flatter consumption across time.

4.3 Linearising the open economy identities

4.3.1 Terms of trade

The logarithmic transformation of equation (10) gives us the linearised terms of trade

$$s_t = p_{F,t} - p_{H,t}, \tag{30}$$

with the same interpretation as its non-linear counterpart.

4.3.2 Consumer price index

By making use of the linearised terms of trade and assuming a steady state characterised by symmetrical prices so that $P_F = P_H = P$, yields the log-linearised consumer price index

$$p_t = p_{H,t} + \upsilon s_t, \tag{31}$$

where the consumer price index is a weighted sum of the domestic price level and the terms of trade, weighed by the degree of openness υ. By assuming domestic inflation is given by $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$, we can infer that CPI inflation must be given by[12]

$$\pi_t = \pi_{H,t} + \upsilon E_t\{\Delta s_t\}, \tag{32}$$

showing that CPI inflation is the sum of domestic inflation and the period difference in terms of trade, weighted by the openness parameter.

[12] Given the definition of inflation $\Pi_t = \frac{P_{H,t}}{P_{H,t-1}}$

4.3.3 The real exchange rate

Logarithmic transformation of the real exchange rate (11) gives us

$$q_t = p_{F,t} - p_t, \tag{33}$$

that when combined with the linearised terms of trade (30), gives us

$$q_t = (1 - \upsilon)s_t. \tag{34}$$

In a fully open economy, the exchange rate is symmetrical and so takes the value of 0. When the domestic economy is only partially open to the world economy, we allow for transitory differences between the domestic and foreign price levels, giving us a non-zero real exchange rate.

4.3.4 International risk sharing

By assuming that the home economy is small compared to the world market, we can infer that $c_t^* = y_t^*$ for all t, independent of the home economy.[13] We also assume symmetric initial net asset positions so that $\vartheta = 1$. Linearising the link between domestic and foreign consumption (13), then gives us

$$c_t = y_t^* + \left(\frac{1-\upsilon}{\sigma}\right) s_t + \frac{1}{\sigma} z_t, \tag{35}$$

showing that home consumption is linked to foreign output, the terms of trade and domestic preferences. Shocks to trade terms and domestic preferences will yield higher domestic consumption.

[13] We use the fact that $c_t^* = y_t^*$ in the linearisation above.

4.4 Deriving the New Keynesian Phillips Curve

This part of the thesis presents the log-linear approximation of the supply side of the economy. At this stage, the impacts of trend inflation will become apparent as we approximate the optimality condition of intermediate firms around a steady state characterised by trend inflation, denoted as $\bar{\pi}$. In the canonical New Keynesian model, a zero inflation steady state is commonly assumed, resulting in the loss of many nuances in the supply-side equations. Consequently, this simplification leads to a straightforward expression that determines the supply's law of motion but sacrifices a more detailed understanding of how firms adapt to an inflationary environment. In reality, any inflation targeting regime will always be characterised by some positive trend inflation, creating a clear motivation to break with the usual assumption of zero-trend inflation when examining inflation dynamics.

4.4.1 Linearising the firm optimal price setting function

To log-linearise the intermediate firm optimal pricing condition, we first take the logarithmic transformation (21)

$$\bar{p}_t(i) = \hat{\psi}_t - \hat{\phi}_t, \tag{36}$$

allowing for the convenient separation of the present value of marginal costs $\hat{\psi}_t$ and the present value of marginal profits $\hat{\phi}_t$ in determining the optimal reset price. They are separately dealt with by approximating them around a steady state characterised by trend inflation, yielding after some algebra

$$\hat{\psi}_t = [1 - \beta\theta\bar{\pi}^{\epsilon}]\,[\hat{mc} + (1-\sigma)\hat{y}] + [\theta\beta\epsilon\bar{\pi}^{\epsilon}]\,E_t\{\hat{\pi}_{H,t+1}\} + [\theta\beta\bar{\pi}^{\epsilon}]\,E_t\{\hat{\psi}_{t+1}\}; \tag{37}$$

$$\hat{\phi}_t = \left[1 - \beta\theta\bar{\pi}^{1-\epsilon}\right](1-\sigma)\hat{y}_t + \left[\theta\beta\bar{\pi}^{\epsilon-1}\right]\left[(\epsilon-1)E_t\{\hat{\pi}_{H,t+1}\} + E_t\{\hat{\phi}_{t+1}\}\right]. \tag{38}$$

From these equations, it is clear that higher trend inflation makes the firms more *forward-looking* when resetting their prices as the present value of marginal costs and profits depend less on the economic conditions of today, $\hat{MC}$ and $\hat{y}$, and more on future inflation, costs and profits, $E_t\{\hat{\pi}_{t+1}\}$, $E_t\{\hat{\psi}_{t+1}\}$ and $E_t\{\hat{\phi}_{t+1}\}$. In a high-inflation environment, the price setting of intermediate firms is thus less sensitive to the real economy and more sensitive to expected future conditions. We can think of this as a response to markups eroding faster in a high-trend inflation environment, creating a need to hedge against future inflation when resetting prices. To find the log-linearised expression for the law of motion of the general price level, we linearise (18)

$$\bar{p}_{H,t} = \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_{H,t}, \tag{39}$$

where it is clear that the domestic price level increases with period t inflation, amplified by the level of trend inflation, price stickiness and elasticity of substitution.

4.4.2 The Phillips-curve in terms of marginal costs

Now that we have a linear expression of the optimal pricing decision, we rearrange it to give us a relationship between the marginal cost and inflation gaps. We start by substituting (36) into (39), giving us an expression for $\hat{\phi}_t$

$$\hat{\phi}_t = \hat{\psi}_t - \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_{H,t}, \tag{40}$$

where it is clear that higher trend inflation, and consequently higher period t inflation, erodes the future expected profits of the firm. To get a simple relationship between the marginal cost and inflation, we insert the above expression into (38), eliminating $\hat{\phi}_t$ and $\hat{\phi}_{t+1}$, giving us

$$
\begin{aligned}
\hat{\psi}_t &= \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_t + \left[1-\beta\theta\bar{\pi}^{1-\epsilon}\right](1-\sigma)\hat{Y}_t \\
&+ \left[\theta\beta\bar{\pi}^{\epsilon-1}\right]\left[E_t\{\hat{\psi}_{t+1}\} - \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}E_t\{\hat{\pi}_{H,t+1}\} + (\epsilon-1)E_t\{\hat{\pi}_{H,t+1}\}\right].
\end{aligned} \tag{41}
$$

Substituting this expression into (37) and rearranging gives us an expression for period t inflation as a function of the marginal cost gap, future inflation gap, production gap and future marginal costs

$$
\begin{aligned}
\hat{\pi}_t &= \frac{(1-\beta\theta\bar{\pi}^{\epsilon-1})(1-\theta\bar{\pi}^{\epsilon-1})}{\theta\bar{\pi}^{\epsilon-1}}\hat{mc}_t \\
&+ \beta\left[1+\epsilon(\bar{\pi}-1)(1-\theta\bar{\pi}^{\epsilon-1})\right]E_t\{\hat{\pi}_{H,t+1}\} \\
&+ \beta[1-\bar{\pi}][1-\theta\bar{\pi}^{\epsilon-1}][(1-\sigma)\hat{y}_t - E_t\{\hat{\psi}_{t+1}\}].
\end{aligned} \tag{42}
$$

An interesting observation from the above equations is that firms become more forward-looking in their price setting in the face of higher trend inflation because the link between period t inflation and the marginal cost gap weakens when trend inflation is higher. Conversely, the link between period t inflation and expected future inflation becomes stronger when there is higher trend inflation.

4.4.3 The Phillips curve

Having established the connection between the marginal cost gap and the domestic inflation gap, we can proceed by finding the relationship between the marginal cost gap and the production gap. By substituting this relationship into equation (42), we obtain the Phillips curve. To establish the connection between marginal costs and output, we begin by analysing the log-linear expression for real marginal costs (17)

$$
mc_t^r = w_t - p_{H,t} - a_t.
$$

Through the manipulations outlined in Appendix B, this expression yields a relationship between marginal costs, price dispersion, domestic production, foreign production, and the level of technology.

$$mc_t = \varphi x_t + (\sigma_\upsilon + \varphi)y_t + (\sigma - \sigma_\upsilon)y^* - (1-\varphi)a_t. \tag{43}$$

From this expression, we can observe that the link between marginal costs and domestic output is affected by the household labour supply elasticity and the terms of trade. The latter effect is captured through $\sigma_\upsilon \equiv \frac{\sigma}{(1-\upsilon)*\Theta}$, where $\Theta \equiv \sigma\eta + (1-\upsilon)(\sigma\eta - 1)$ is a parameter consisting of the degree of openness and the substitutability between home and foreign goods. When $\sigma\eta > 1$, expanding the openness of the economy diminishes the impact of domestic output on marginal costs while augmenting the influence of world output. To find a relationship between the marginal cost gap and the production gap, we assume that y^n is the level of production that is consistent with a frictionless markup, giving us

$$mc^n = \varphi x^n + (\sigma_\upsilon + \varphi)y^n + (\sigma - \sigma_\upsilon)y^* - (1-\varphi)a_t. \tag{44}$$

The production gap is defined as the difference between period t production and the natural production level, given by $\hat{y_t} = y_t - y^n$. Similarly, the marginal cost gap is the logarithmic difference between the period marginal cost and the natural marginal costs, denoted as $\hat{mc_t} = mc_t - mc^n$. Subtracting(43) from (44), we establish the following relationship between the two.

$$\hat{mc_t} = \varphi\hat{x}_t + (\sigma_\upsilon + \varphi)\hat{y}, \tag{45}$$

where we have defined the price dispersion gap as $\hat{x}_t \equiv x_t - x$, yielding a relationship between the output gap and marginal cots gap. Inserting that expression into (42) gives us the New Keynesian Phillips curve

$$\pi_{H,t} = \kappa_v(\bar{\pi})\hat{y}_t + \lambda(\bar{\pi})\varphi\hat{x}_t + b_1(\bar{\pi})E_t\{\pi_{H,t+1}\} + b_2(\bar{\pi})\left[(1-\sigma)\hat{y}_t - E_t\{\psi_{t+1}\}\right], \tag{46}$$

where we have defined $\kappa_v(\bar{\pi}) \equiv \lambda(\bar{\pi})(\varphi + \sigma_v)$, $\lambda(\bar{\pi}) = \frac{(1-\beta\theta\bar{\pi}^{\epsilon-1})(1-\theta\bar{\pi}^{\epsilon-1})}{\theta\bar{\pi}^{\epsilon-1}}$, $b_1 \equiv \beta[1+\epsilon(\bar{\pi}-1)(1-\theta\bar{\pi}^{\epsilon-1})$ and $b_2 \equiv \beta[1-\bar{\pi}][1-\theta\bar{\pi}^{\epsilon-1}]$ to ease notation. New to this particular formulation of the Phillips curve as compared to its marginal cost equivalent is the relationship between the output and inflation gaps, which go through the marginal costs gap. The strength of the relationship is governed by κ_v, which is decreasing in both the economy's openness and the trend inflation level, *flattening* the Phillips curve.

To fully understand the relationship between optimal price-setting behaviour and inflation, we must examine how trend inflation shifts the firm's time perspective when setting prices. In an environment characterised by high inflation, we will demonstrate that firms tend to adopt a more forward-looking approach in their price-setting decisions. This shift occurs because maintaining the same price over a period of time becomes costlier due to more substantial discrepancies between the firm's price and the general price level, all else being equal. To capture this dynamic, we insert (45) into (37), yielding

$$\hat{\psi}_t = [1-\theta\beta\bar{\pi}^{\epsilon}]\left[\varphi\hat{x}_t + (\sigma_v+\varphi)(1-\sigma)\hat{y}_t\right] + [\theta\beta\bar{\pi}^{\epsilon}]\, E_t\{\hat{\psi}_{t+1}\} + [\theta\beta\epsilon\bar{\pi}^{\epsilon}]\, E_t\{\hat{\pi}_{H,t+1}\}, \tag{47}$$

where we again observe that higher trend inflation changes optimal price-setting behaviour by making firms look further into the future when setting prices. In the later sections, we will see that this behaviour change will make the dynamic system more unstable because it reduces the transmission of higher interest rates to the firm's price-setting decision.

4.4.4 Price dispersion

To assess the impact of higher trend inflation, it is necessary to linearise the expression for price dispersion. We start by linearising equation (26)

$$\hat{x}_t = \left[\frac{-\epsilon(1-\theta)(\bar{p}_i)^{-\epsilon}}{x}\right]\hat{p_{i,t}} + \left[\frac{\epsilon\theta\bar{\pi}^{\epsilon}}{x}\right]\hat{\pi}_{H,t} + \left[\frac{\theta\bar{\pi}^{\epsilon}}{x}\right]\hat{x}_{t-1}, \tag{48}$$

that when inserting for steady-state price dispersion gives us

$$\hat{x} = \left[\frac{\epsilon\theta\bar{\pi}^{\epsilon-1}}{1-\theta\bar{\pi}^{\epsilon-1}}(\bar{\pi}-1)\right]\hat{\pi}_{H,t} + \theta\bar{\pi}^{\epsilon}\hat{x}_{t-1}. \tag{49}$$

We see that price dispersion is increasing in θ, ϵ and $\bar{\pi}$ for the same reasons discussed above. Interestingly we see that dispersion enters into the Phillips curve through its effect on the marginal cost gap and that the Phillips curve enters into the expression for price dispersion. This will create a feedback loop between the two, making inflation more volatile.

4.5 Market clearing

This thesis only briefly outlines the derivation of market clearing leading to the dynamic IS curve. A more detailed derivation can be found in Appendix B, while the complete derivation is available in the Appendix of Gali and Monacelli (2005). In a small open economy, we know that domestic production of intermediate goods must be the sum of domestic and foreign demand for domestic intermediate goods, given by

$$Y_t(i) = C_{H,t}(i) + X_t(i), \tag{50}$$

where $X_t(i)$ is domestic net exports of intermediate good (i). To find the domestic economy's aggregate output, one thus has to aggregate domestic demand and foreign countries' demand for domestic goods. It can be shown that a convenient way to express aggregate net exports is

$$X_t = \upsilon S^{\eta} C_t^*, \tag{51}$$

where aggregate net exports X_t is a function of the openness of the domestic economy, υ, the terms of trade, s_t and aggregate world demand.[14] By substituting the above expression into our description of aggregation technology, as shown in equation (14), we obtain an expression for the aggregate output of the domestic economy

$$Y_t = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \upsilon S_t^{\eta} Y_t^*. \tag{52}$$

The above expression is log-linearised around a frictionless steady state, assuming symmetry across all foreign countries. This log-linearisation results in a simple relationship between domestic output, domestic demand, and world demand

$$y_t = (1-\upsilon)c_t + \upsilon(2-\upsilon)\eta s_t + \upsilon y_t^*. \tag{53}$$

To derive the dynamic Euler equation, we combine the above expression with the linearised Euler equation (29) and our definition of domestic inflation (31), resulting in

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \frac{\upsilon}{\sigma}E_t\{\Delta s_t\} + \frac{1}{\sigma}(1-p_z)z_t, \tag{54}$$

which ties domestic demand to the domestic real interest rate and changes in the terms of trade, weighed by the degree of openness of the domestic economy. Combining (53) with (54), one can find an expression evaluating frictionless production, y^n. Defining the output gap as $\hat{y} = y_t - y_t^n$, we obtain the dynamic IS curve for an open economy

[14] We assume world market clearing so that $C^* = Y^*$.

$$\hat{y} = E_t\{\hat{y}_{t+1}\} - \frac{1}{\sigma_\upsilon}(i_t - E_t\{\hat{\pi}_{H,t+1}\} - r^n), \tag{55}$$

where the output gap is a function of expected future output gap, $E_t\{\hat{y}_{t+1}\}$, the interest rate, i, future expected inflation, $E_t\{\hat{\pi}_{H,t+1}\}$ and the natural rate of interest r^n. A key difference between the open economy IS curve and its closed economy counterpart is that the degree of openness influences the output gap by reducing the responsiveness of domestic output to domestic consumption. The transmission of monetary policy to production is thus lowered in an open economy. The corresponding natural rate of interest is given by

$$r_t^n \equiv \rho - \sigma_\upsilon \Gamma_a(1-\rho_a)a_t + \sigma_\upsilon \Gamma_x E_t\{\Delta x_{t+1}\} + \Psi_* E_t\{\Delta y^*_{t+1}\} + \Psi_z(1-\rho_z)z_t, \tag{56}$$

where $\Gamma_a \equiv \frac{1-\varphi}{\sigma_\upsilon+\varphi}$, $\Gamma_x = \frac{\varphi}{\sigma_\upsilon+\varphi}$ $\Psi_* \equiv \sigma_\upsilon(\upsilon(\Theta-1)+\Gamma_*)$, $\Gamma_* \equiv -\frac{\upsilon(\Theta-1)\sigma_\upsilon}{\sigma_\upsilon+\varphi}$, $\Psi_z \equiv (1-\upsilon)\omega-\sigma_\upsilon\Gamma_z$ and $\Gamma_z \equiv -\frac{\upsilon\Theta\omega}{\sigma_\upsilon+\varphi}$. Some interesting observations can be made about the natural rate of interest. The first is that the degree of openness makes the natural rate of interest depend more on the world economy and less on domestic preferences because $\lim_{\upsilon\to 0}\Psi_* = 0$ and $\lim_{\upsilon\to 0}\Psi_Z = 1$. The second is that price dispersion enters into the natural interest rate much the same way as productivity, reflecting its role as a de facto productivity shifter. The strength of the effect of price dispersion on the natural interest rate is largely determined by φ.

4.6 Monetary policy

To complete the model description, we need a formulation of monetary policy. As standard for much of the literature, we describe monetary policy as an endogenous interest rate rule that has the central bank respond to fluctuations in the production gap, $\hat{y}$, and domestic inflation $\pi_{H,t}$. The weights put on fluctuations of production, and inflation is by ϕ_y and ϕ_π, giving us the following interest rate rule

$$i_t = \phi_\pi \pi_{H,t} + \phi_y \hat{y}_t, \tag{57}$$

where we can interpret ϕ_π and ϕ_y as the percentage change in the policy rate as a response to a percentage deviation of $\pi_{H,t}$ and $\hat{y}_t$. As a baseline, these parameters are calibrated according to the standard Taylor rule.[15]

5 The complete theoretical model and some implications

This section will display the mechanisms through which trend inflation affects the small open economy model. The implications will help us understand the empirical results in section 6. The theoretical model has four main implications that will be highlighted. Firstly, higher trend inflation amplifies fluctuations in the business cycle, resulting in larger variances in production and inflation gaps. Secondly, trend inflation can lead to situations where the divine coincidence does not hold, resulting in prolonged deviations from the steady state due to trade-offs in monetary policy.

The third implication is that higher trend inflation requires the central bank to respond more aggressively to inflation deviations from the steady state to maintain stability. The explicit goal of using monetary policy to contribute to "high" levels of production might then not be compatible with stabilising inflation expectations due to the need for rapid increases in the policy rate in face of inflation. It will also become clear that introducing a trade-off in monetary policy will necessitate a more significant focus on stabilising variation in inflation, no matter the consequences for production. The fourth implication is that a more open economy is partially insulated from the adverse effects of domestic trend inflation due to a flatter Phillips Curve resulting from openness.

To gain a better understanding of the key drivers of the model dynamics, we summarise the most important equations of the dynamic system as follows

[15] As given by $i_t = 4 + 1.5(\pi_t - 2) + 0.5(y_t - y*)$ where it is assumed an inflation target of 2% and a steady state interest rate of 2%.

$$(46) \quad \pi_{H,t} = \kappa_{\upsilon}(\bar{\pi})\hat{y}_t + \lambda(\bar{\pi})\varphi\hat{x}_t + b_1(\bar{\pi})E_t\{\pi_{H,t+1}\} + b_2(\bar{\pi})\left[(1-\sigma)\hat{y}_t - E_t\{\psi_{t+1}\}\right],$$

$$(47) \quad \hat{\psi}_t = [1-\theta\beta\bar{\pi}^{\epsilon}]\left[\varphi\hat{x}_t + (\sigma_{\upsilon}+\varphi)(1-\sigma)\hat{y}_t\right] + [\theta\beta\bar{\pi}^{\epsilon}]\, E_t\{\hat{\psi}_{t+1}\} + [\theta\beta\epsilon\bar{\pi}^{\epsilon}]\, E_t\{\hat{\pi}_{t+1}\},$$

$$(49) \quad \hat{x} = \left[\frac{\epsilon\theta\bar{\pi}^{\epsilon-1}}{1-\theta\bar{\pi}^{\epsilon-1}}(\bar{\pi}-1)\right]\hat{\pi}_{H,t} + \theta\bar{\pi}^{\epsilon}\hat{x}_{t-1},$$

$$(55) \quad \hat{y} = E_t\hat{y}_{t+1} - \frac{1}{\sigma_{\upsilon}}(i_t - E_t\{\hat{\pi}_{H,t+1}\} - r^n),$$

$$(56) \quad r^n_t = \rho - \sigma_{\upsilon}\Gamma_a(1-\rho_a)a_t + \sigma_{\upsilon}\Gamma_x E_t\{\Delta x_{t+1}\} + \Psi_* E_t\{\Delta y^*_{t+1}\} + \Psi_z(1-\rho_z)z_t,$$

$$(57) \quad i_t = \phi_{\pi}\pi_{H,t} + \phi_y\hat{y}_t,$$

where the complete model can be found in Appendix C. The above system exhibits three key aspects distinguishing it from the canonical New Keynesian model. The first is that all the coefficients of the Phillips curve are non-linear functions of trend inflation, $\bar{\pi}$, so that even though the underlying parameters are constant, the coefficients they make up drift when trend inflation drift. The second is that price dispersion and domestic inflation are functions of each other, creating a feedback loop between the two, where the strength of the feedback is determined by φ. The third is that higher trend inflation makes firms put more weight on future inflation rather than current economic conditions when setting their prices, making inflation less responsive to changes in the interest rate.

In what follows, these model implications will be explored using parameter calibration close to Galí (2015).[16] This will allow us to identify the interplay between openness and trend inflation. A statement about what we expect to observe when data is brought to the model is formed for all four implications. This will bring structure to the empirical analysis and inform the interpretation of the estimation results.

[16] The calibration is $\beta = 0.99$, $\sigma = 2$, $\varphi = 3$, $\epsilon = 9$, $\theta = 0.6$, $\phi_{\pi} = 1.5$, $\phi_y = 0.5$, $\eta = 1$ and $\upsilon = 0.4$. The main difference in calibration from Galí (2015) is that θ and φ are slightly reduced so that the Blanchard & Kahn conditions are satisfied for higher levels of trend inflation, allowing for comparison of different levels.

5.1 Increased volatility and introduction of trade-offs

As a result of price dispersion caused by trend inflation, situations can arise where the divine coincidence does not hold. To illustrate this implication, we simulate the dynamic response to a productivity shock when considering progressively higher levels of trend inflation. Other shocks create similar trade-offs for the central bank, where the dynamic response to a demand shock, monetary policy shock and world production shock are shown in Appendix C, accompanied by a brief explanation of the dynamics.

The mechanism by which price dispersion leads to trade-offs for the central bank, forcing a choice between stabilising inflation and production, is through its impact on productivity. We can easily see that there is a one-to-one relationship between firm productivity and price dispersion by evaluating the log-linearised firm labour demand schedule given by (25)

$$y_t = a_t - x_t + n_t. \tag{58}$$

Both increased, and decreased price dispersion resulting from shocks will thus lead to productivity shifts, creating a wedge between natural and time t production. Using the calibrated parameter values to evaluate the impulse response function to a productivity shock, we observe that price dispersion gradually decreases with higher levels of trend inflation.

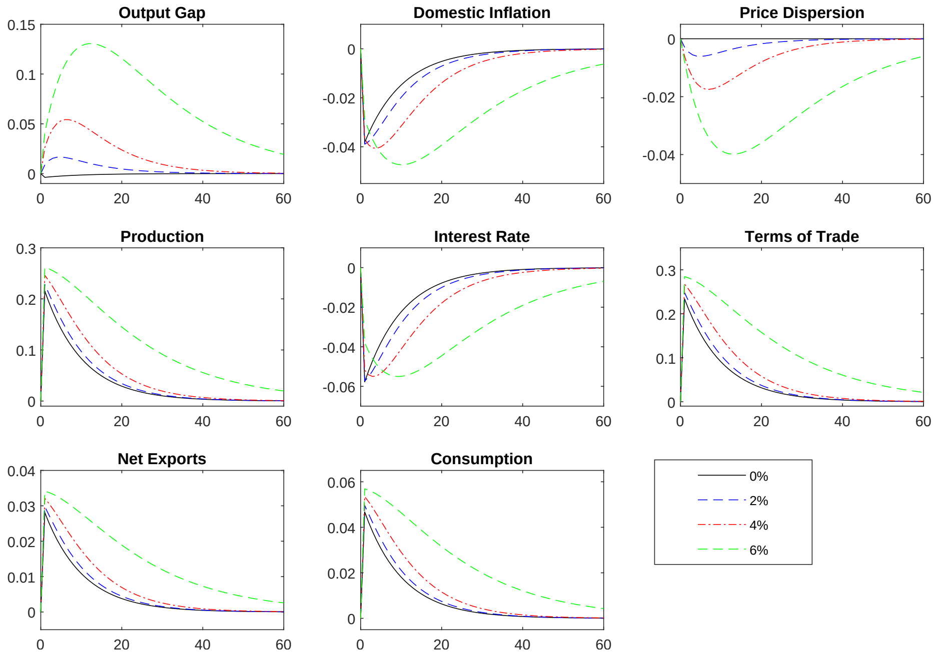

Figure 2: IRF response to a 0.05% productivity shock

As trend inflation increases, we observe that the shock responses are more volatile. Trend inflation, through price dispersion, introduces a trade-off between stabilising output and inflation.

As a baseline for understanding the dynamic response to a productivity shock, we first assess what happens in the 0% trend inflation scenario. In response to increased productivity, the natural level of production increases. So does period t production, but less than natural production, creating a negative production gap. The negative production gap propagates to a negative marginal cost gap, giving us a negative inflation gap. In response to a negative inflation and production gap, the central bank decreases the policy rate to boost the economy, completely neutralising the deviation from a steady state in roughly 20 business cycles. The reduction in nominal interest rates, and most importantly, real interest rates, leads to a depreciation of the domestic currency and an increase in the terms of trade, leading to positive net exports.

In the positive trend inflation scenario, we see that higher trend inflation gives rise to increasingly lower price dispersion due to the feedback loop between inflation and price dispersion. The decrease in price dispersion has three interesting effects. The first can be seen from equation (16), where a reduction of price dispersion reduces the labour need for a given production level, amounting to a de facto productivity increase. The productivity increase is responsible for the gradual increase in production and output gap as trend inflation increases. The second effect goes through the shift in the firm's pricing decision (47), making them less forward-looking in their pricing decision and thus deepening the negative inflation gap. The third is a feedback loop between price dispersion and inflation, where the strength of the feedback is determined by φ as seen in (46). As the green dashed line displays, the feedback loop will make positive and negative inflation gaps more persistent.

The boost to productivity in the face of trend inflation makes period t production higher than the natural production level, creating a positive production gap. The result is a situation where the central bank faces a trade-off between stabilising the production and inflation. Since the central bank puts a larger weight on fluctuations in inflation than production, we know that their response will be to lower the policy rate. Consequently, the rate reduction will neutralise the inflation gap by amplifying the production gap, making the shock more persistent or "sticky."

The effect of being a small open economy enters through σ_v in determining the strength of the relationship between the production gap, inflation gap, and price dispersion. We see that if $\sigma\eta > 1$, an increase in the openness of the economy *reduces* the sensitivity of domestic inflation to both production and price dispersion, effectively flattening the Phillips curve and negating some of the volatility stemming from higher trend inflation. As for the effects of trend inflation on trade, we can observe that higher trend inflation, and the persistent negative inflation gap that follows, lead to a more considerable depreciation of the domestic currency, making the boost to net exports larger.

To summarise, we make the following statements about how trend inflation affects monetary policy in a small open economy

(i) *Higher trend inflation increases the volatility of shocks,*

(ii) *Trend inflation gives rise to monetary policy trade-offs,*

which will be examined when bringing data to the model.

5.2 Monetary policy flexibility and the role of openess

Similarly to the insights given by Ascari and Sbordone (2014), we can see that higher trend inflation necessitates more aggressive monetary policy for the Blanchard & Kahn conditions to be satisfied, making a rational expectations equilibrium possible.[17] In the face of business cycle fluctuations, the central bank will thus have to respond more forcefully to inflation gaps by responding more aggressively with the policy rate. By repeatedly simulating the above model, varying the policy strength parameters ϕ_π and ϕ_y and the level of trend inflation $\bar{\pi}$ we can find the threshold determinacy values for increasingly higher levels of trend inflation.[18]

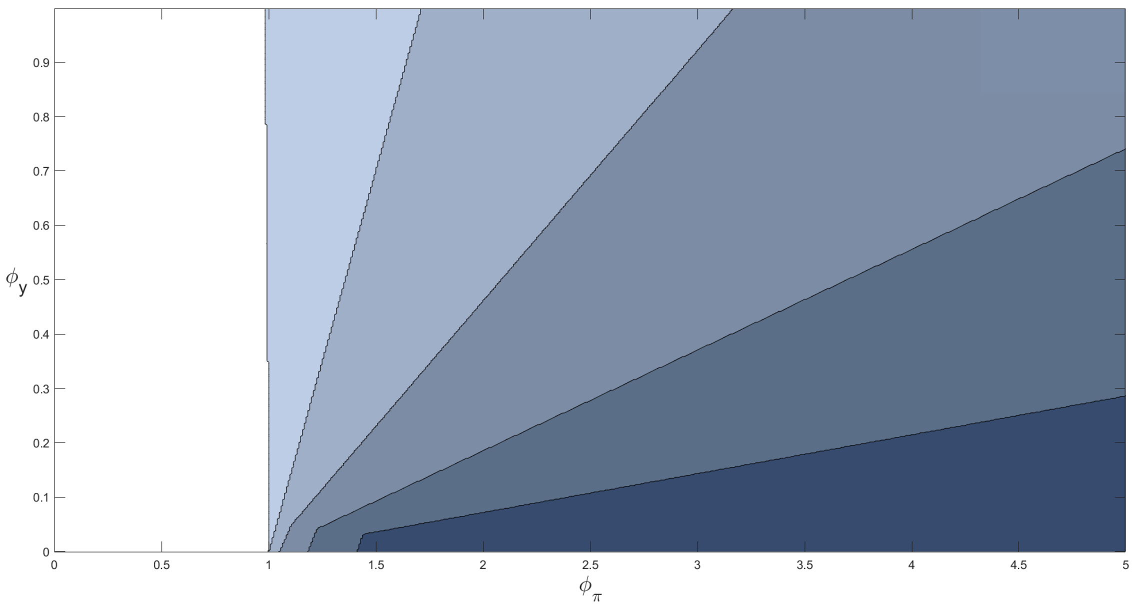

Figure 3: Reduction in determinacy region due to higher trend inflation

As trend inflation increases the combinations of ϕ_π and ϕ_y consistent with a rational expectations equilibrium shrink. All the blue areas in are possible combinations of ϕ_π and ϕ_y in the absence of trend inflation. As trend inflation increases, progressively fewer combinations are possible, as illustrated by the shrinking area. Especially the ability to respond to output gaps through ϕ_y shrinks rapidly.

[17]The Blanchard & Kahn conditions needs to be fulfilled for the system to be *stable*. One could think of the opposite case as "dynamite," where a slight disturbance of the system would lead to exponential deviation from the steady state.

[18]Notably, Ascari and Sbordone (2014) does not discuss the sensitivity of the following results to the level of price stickiness. It is important to highlight that the magnitude of the effect diminishes significantly when the Calvo parameter takes lower values. For the sake of argument, we thus calibrate θ to 0.75 in this section so that the effect of higher trend inflation on monetary policy flexibility is apparent. A Calvo parameter of 0.75 is fairly standard in the literature, and is what is used in both Gali and Monacelli (2005) and Ascari and Sbordone (2014).

In the above illustration, we see the values of ϕ_π and ϕ_y compatible with a stable, steady state for increasingly higher levels of trend inflation. The determinacy region when trend inflation is at 0% is given by all the coloured areas of the above figure. We then see that the area shrinks when trend inflation becomes progressively higher, here given by 2%, 4%, 6% and 8%, respectively. The determinacy region for monetary policy, when trend inflation is at 8%, is thus darkest blue area of the figure.

As shown in Ascari and Sbordone (2014), the mechanism that leads to the reduction of the determinacy region when trend inflation increases is that firms become more forward-looking in their price setting when trend inflation increases, making them less sensitive to current economic conditions. Again, by evaluating equation (47)

$$\hat{\psi}_t = [1 - \theta\beta\bar{\pi}^\epsilon]\,[\varphi\hat{x}_t + (\sigma_\upsilon + \varphi)(1-\sigma)\hat{y}_t] + [\theta\beta\bar{\pi}^\epsilon]\,E_t\hat{\psi}_{t+1} + [\theta\beta\epsilon\bar{\pi}^\epsilon]\,E_t\hat{\pi}_{t+1}, \tag{47}$$

we see that increasing levels of trend inflation make intermediate firms put more weight on future inflation and less on current economic conditions. Since interest rates only affect inflation through the production gap, and firms are less sensitive to the interest rate when there is high trend inflation, the central bank has to increase the policy rate to achieve the same result when trend inflation increases.

Due to fewer values of ϕ_y being consistent with determinacy when trend inflation increases, the central bank has to put increasingly more weight on only stabilising the inflation gap, leaving less room for consideration of fluctuations in production. In the 0% trend inflation scenario, we see that the central bank has the flexibility to put a lot of weight on stabilising the production gap. However, as trend inflation increases, trade-offs of the kind discussed above become increasingly prominent, leaving the central bank with no other choice than only stabilising inflation, no matter the consequences this stabilisation has for the output gap.

We can also observe that a more open economy has more flexibility in monetary policy. We know that when $\sigma\eta > 1$ an increase in the openness of the economy lowers σ_υ, reducing the sensitivity of domestic inflation to the output gap. By assuming that $\phi_y = 0$, we can plot the values of ϕ_π and υ that are consistent with determinacy.

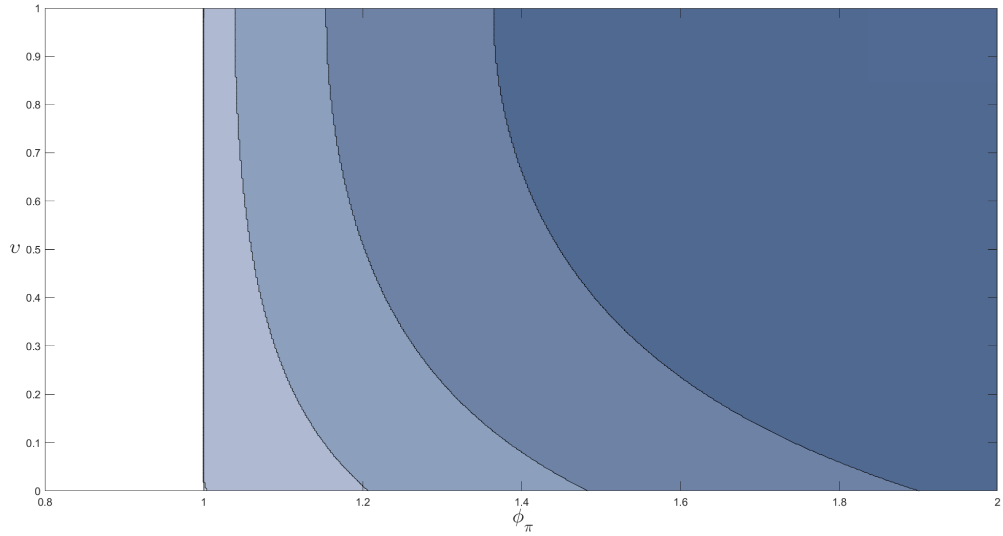

Figure 4: Increased determinancy region when opening up the economy

As trend inflation increases, the central bank has to respond more aggressively to fluctuations in inflation. However, as openness υ increases, the strength of the policy response needed decreases.

As trend inflation increases, we can again observe that a more aggressive monetary policy is required to stabilise the system. In the case of no trend inflation, represented by all the blue areas in the figure combined, we see that all combinations of ϕ_π and υ are consistent with a stable equilibrium as long as the central bank responds to a one percentage increase in the inflation gap with more than a one percentage increase in the policy rate. Through the mechanism described above, we can observe that as trend inflation increases, a higher degree of openness *expands* the determinacy region, giving the central bank more flexibility when determining the policy rate. A small open economy is thus shielded from some of the adverse effects of higher trend inflation because of the decoupling of domestic fluctuations in output and consumption.

To summarise, we make a third and fourth statement about how trend inflation affects monetary policy in a small open economy

(iii) *Higher trend inflation reduces the determinacy region of monetary policy,*

(iv) *openness negates some of the adverse effects of domestic trend inflation*

that also will be examined when bringing data to the model.

6 Estimation

In order to test the above implications and analyse the effects of trend inflation on the Norwegian economy, it is necessary to apply the theoretical model to relevant data specific to Norway. Testing the theory will also allow us to assess the validity and relevance of the model's implications, allowing for a deeper understanding of how trend inflation affects the small open economy and its implications for monetary policy. Therefore, this section will outline how a Bayesian approach is taken to make the model fit observed data by determining the values of structural parameters that best match the theoretical construct with the actual economic processes driving the fluctuations.

To that end, this section will first describe the data used and how it has been transformed to correspond to the model. This involves selecting what data to bring to the model and ensuring that the data is consistent with the model's assumptions and requirements. The data used in estimation is collected from Statistics Norway and Norges Bank and contain information on CPI inflation, GDP, short-term interest rates, consumption, world price level, and the terms of trade. These time series are detrended or demeaned depending on the underlying data generation process.

Once the data has been described, an explanation of the Bayesian estimation approach used to fit the data to the model will follow. Bayesian estimation is a statistical method that combines prior knowledge or beliefs about the parameters with the observed data to update and refine those beliefs. In addition, it allows for the calculation of posterior distributions, which represent the probability distribution of parameter values given the data and prior beliefs.

Lastly, the resulting posterior distributions are presented. They will provide insights into the likely values of the structural parameters that maximise the fit between the model and the actual economic processes driving fluctuations. In addition, these distributions describe a range of possible parameter values along with their associated probabilities, allowing for a nuanced understanding of the relationships and dynamics within the model. Unfortunately, some of the resulting posterior distributions do seem to suffer from poor identification, so the 90% highest posterior density interval (HPDI) is large. The posterior mode seems to be consistent with previous literature, so the estimates do provide some indication of whether the above statements are applicable to the Norwegian case but not necessarily the strength of the effects.

6.1 Data

When applying data to a numerical model, the relationship between the model variables and the data applied must be well specified. Proper specification typically entails some transformation of the raw data, where one separates the trend of a variable from its fluctuations around the steady state. The latter is often the most interesting when applying data to DSGE models as it corresponds to the short-term and stationary variance around the steady-state that this model class tries to capture. To shape our thinking about how data is applied to the model, it is useful to follow the useful guide of Pfeifer (2021) in abstracting from the complexity of our model to represent it in its state-space form[19]

$$x_t = g(x_{t-1}, \varepsilon_t^{struct}), \tag{59}$$

$$y_t^{obs} = h(x_t, \varepsilon_t^{obs}). \tag{60}$$

Here define equation (59) as the state-transition equation, which provides a compact description of how the model behaves over time in response to structural shocks. In this equation, the vector of state variables in time t, denoted as x_t, is a function of the policy function, denoted as g.[20] The policy function captures the relationship between the previous state x_{t-1} and the structural shocks ε_t^{struct}, describing the system's motion. Equation (60) is the observation equation, which represents how the observed variables y^{obs} map into the state variables according to the policy function of the observed variables. Quite intuitively, this implies an assumption that our model framework captures at least some aspects of the observed reality, where the unspecified policy function for the observables,

[19]This way of representing numerical models has its roots in optimal control theory, see for example (Kalman, 1960).

[20]There are many ways to abstract from the full system of linear difference equations to its state-space representation, see for example Sims (2002).

h captures the link between our theoretical construct and actual observations.

When applying data to the numerical model, we thus have to make sure that the data corresponds to the variables in the model in some intuitive way. In practice, this means telling our estimation software how the observed data relates to the variables in the model so that it can compute how x_t maps into y^{obs}. In our case, this creates a need to distinguish between trending and stationary variables as they require different transformations to be used in estimation.

6.1.1 Trending variables

In the context of DSGE modelling, a trending variable is typically a variable that grows over time. The obvious example of such a variable would be GDP, which exhibits some year-on-year growth, usually around 2%. Analysing the long-run dynamics of the trend itself can be interesting but irrelevant to our purpose, as we assume that we are looking at the short-term implications of shocks. This creates a need to separate the trending component of the variable from the stochastic variation around that trend. A typical formulation of such a process is

$$y_t = \alpha_0 + \alpha_1 t + \varepsilon_t,$$

where α_0 is the value of y_t in period 0, α_1 is the time invariant trend, and ε is an i.i.d sequence with $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = \sigma_\varepsilon^2$.[21] The average value of the above expression is a linear function of time, given by $E(Y_t) = \alpha_0 + \alpha_1 t$. To illustrate how the trend can be separated from variance in the context of our model, we look at how data on the gross domestic product for Norway is manipulated to fit our model specification. Data on gross domestic product from 1987Q1 - 2022Q4 is collected from the statistics Norway (SSB) stat-bank and plotted. [22]

[21] See for example (Wooldridge, 2015).

[22] Accessed from https://www.ssb.no/statbank.

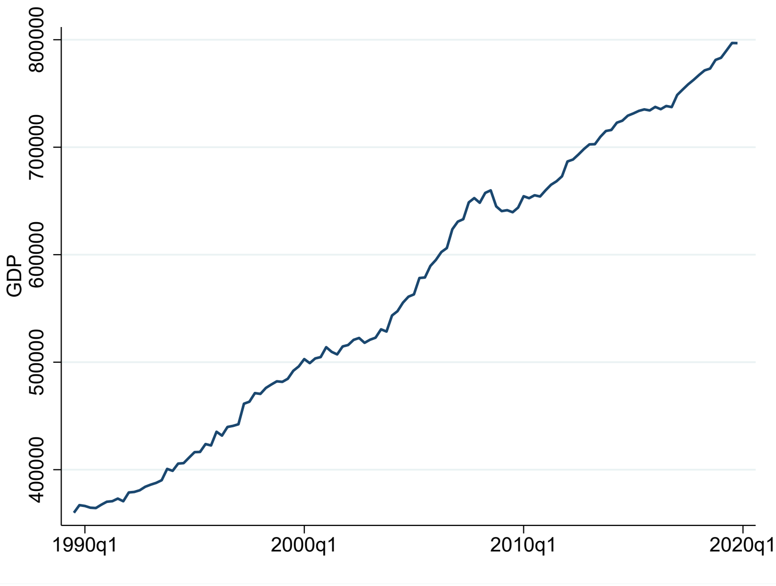

Figure 5: Norwegian gross domestic product from 1986Q1 to 2022Q4.

The logarithmically transformed data on the gross national product.

There are several ways to decompose figure 5 into trend and variance. In this thesis, we follow the approach of Hodrick and Prescott (1997) in using what was later called the HP filter to detrend the growth variables. The logarithmically transformed time series is put through the filter, giving us figure 6.

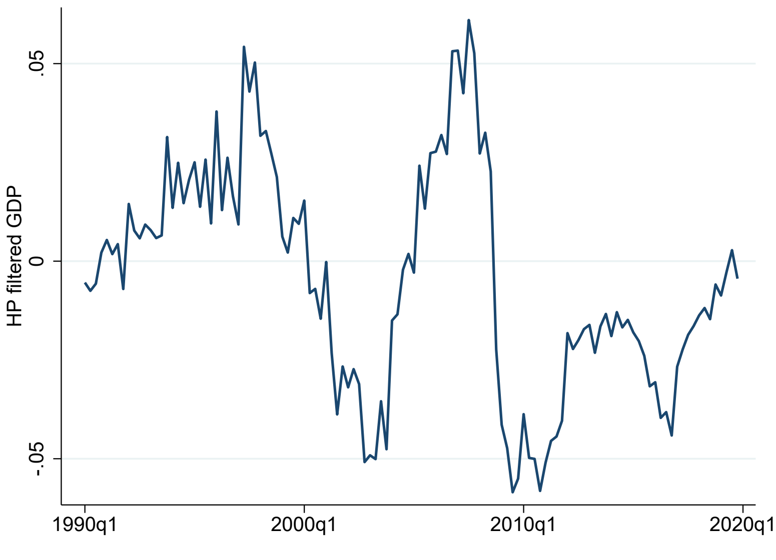

Figure 6: HP filtered gross domestic product

The result of HP-filtering the logarithmically transformed data on gross domestic product. The resulting data can be interpreted as percentage deviations from steady state.

Where the variation component perfectly corresponds with percentage deviations from steady state, $\hat{y}$.

6.1.2 Stationary variables

A stationary variable is one whose joint probability distribution does not change across time. Similar to trending variables, one could also think of stationary variables as having a trending and cyclical component, only that the trend in stationary variables does not exhibit a growth pattern across time. Instead, the trend roughly corresponds to the long-run mean of the variable and typically takes a non-zero value. In the context of our model, we are often interested in observing percentage deviations from a steady state characterised by zero. So when specifying stationary observables for estimation, we need a strategy to detrend the data. A typical approach is to simply demean the data according to

$$\Pi_t^{obs} = log(\Pi_t^{data}) - log(\bar{\Pi}_t) = \hat{\pi}_t, \tag{61}$$

where $\bar{\Pi}_t$ is the steady state, and $\hat{\Pi}_t$ is percentage deviations around that steady state. The challenge when detrending stationary variables is thus finding the long-run steady state, $\bar{\Pi}_t$, to be subtracted from observed inflation. One could take the same approach with trending variables by using an HP filter to separate the trend from the cyclical component, implicitly assuming a time-varying trend. The simple approach is to demean the logarithmically transformed data according to the above formula, which is the approach taken in this thesis. To illustrate how a stationary variable is manipulated to fit our model, we look at demeaned CPI inflation for Norway from 1987Q1-2022Q4.

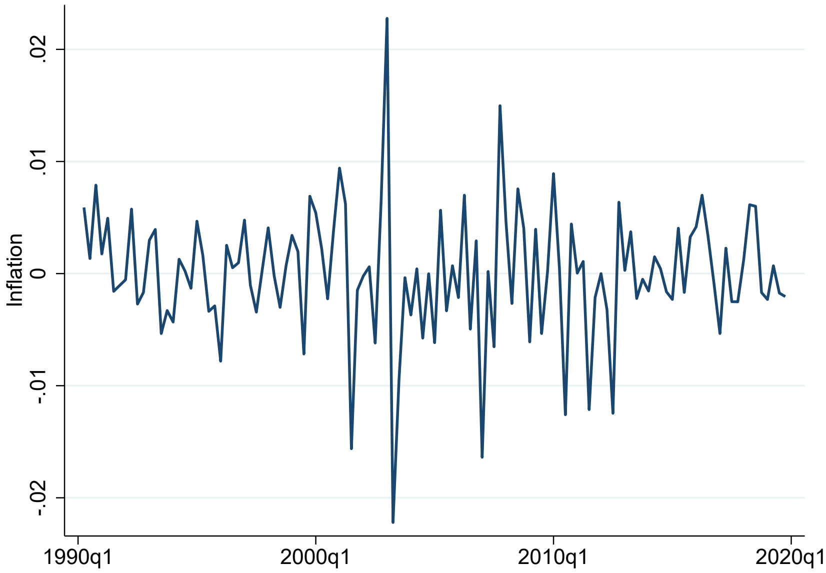

Figure 7: Percentage change in inflation

The result of taking the first differences and demeaning the CPI index. The resulting data can be interpreted as percentage deviations from steady state.

were we can observe that the resulting variable can be interpreted as percentage change in inflation around a zero mean, corresponding to CPI inflation in the model, π_t.

6.2 Priors

A major reason Bayesian estimation has grown in popularity in recent literature is that it allows for the incorporation of prior information about the likely values of our structural parameters to be estimated as a "starting point" for the estimation. This information takes the form of a *prior probability distribution* that specifies regions of the likelihood function consistent with prior economic theory or data, which is to be given more weight in estimation. The prior is simply a description of the type of probability function, like, for instance the normal distribution and its corresponding first and second moments.[23] By assigning more weight to a region of parameter space, Bayesian methods provide a framework for combining prior knowledge with observed data to obtain information of the *posterior distribution* of the parameters. The technical details of how the posterior is

[23] Other distributions used are the gamma, inverse gamma and beta distributions.

extracted are discussed in the next section of the thesis. For now, the priors to be used in estimation and the reasoning for the choices of their moments are presented.

The first group of parameters to be estimated are the structural parameters determining household and firm behaviour.

Parameter	Description	Domain	pdf	Mean	s.d
θ	Calvo parameter	[0,1)	Beta	0.50	0.10
φ	Frish elasticity	[0, ∞)	Gamma	3.00	0.20
σ	Intertemporal substitution	[0, ∞)	Gamma	2.00	0.20
η	Elasticity home/foreign	[0, ∞)	Gamma	1.00	0.10
ϕ_π	Taylor: inflation	[0, ∞)	Gamma	1.50	0.10
$\phi_{\hat{y}}$	Taylor: output	[0, ∞)	Gamma	0.50	0.05

The choice of prior for the Calvo parameter aligns with the typical prior found in the literature (Lubik & Schorfheide, 2005; Zhang & Dai, 2020). To capture the dynamics of the effects of trend inflation, the prior chosen for the Frich elasticity reflects the examples used in Ascari and Sbordone (2014). The priors for the inter-temporal elasticity of substitution and substitution elasticity between domestic and foreign goods are chosen in line with previous literature. The priors for monetary policy are consistent with a typical Taylor rule like the one found in Gali and Monacelli (2005).

In line with Smets and Wouters (2007), the prior distributions for the parameters determining the persistence of the structural shocks are set to be 0.5, and the shocks themselves to be 0.1.

Parameter	Description	Domain	pdf	Mean	s.d
ρ_i	Shock persistence	[0,1)	Beta	0.50	0.20
σ_i	Shock variance	[0, ∞)	Inverse Gamma	0.10	2.00

Not all parameters can be estimated either due to perfect collinearities or because they are not identified in the data. A typical example of such a parameter is the discount factor

β, which is tricky to observe in economic time series. The parameters not estimated are thus calibrated according to Galí (2015) and Ascari and Sbordone (2014).[24]

6.3 Bayesian estimation

Bayesian inference has become increasingly popular in the DSGE literature due to the ability to incorporate existing knowledge of the structural parameters in estimation, often yielding results more with economic theory and observed data than other methods.[25] This section of the thesis will provide a brief overview of why the Bayesian method has gained its popularity and show how it is applied to estimate the structural parameters of the above model, following Herbst and Schorfheide (2016) and the guide of Griffoli (2010).

A natural starting point for a brief review of Bayesian estimation is the *likelihood function* denoted as $\mathcal{L}(Y|\theta)$, that can be understood as the likelihood of observing the data, Y^t, given the structural parameters of the model, θ. In estimation, the likelihood function is used to update our *a priori* beliefs of the parameters $\mathcal{P}(\theta)$,[26] by use of sample information on our endogenous variables, Y^t. The result is a *posterior distribution*, $\mathcal{P}(\theta|Y^t)$, that can be understood as the probability of observing the parameters, θ, conditional on having seen the data, Y. The likelihood function and priors are combined to create the posterior distribution through Bayes theorem

$$p(\theta|Y^T) = \frac{\mathcal{L}(Y^t|\theta)p(\theta)}{p(Y^t)} \propto \mathcal{L}(Y^t|\theta)p(\theta). \tag{62}$$

Here, the posterior distribution is proportional to the product of the likelihood function and the prior distribution. The denominator, $\mathcal{P}(Y^T)$, represents the marginal data density, which is the probability of observing the data conditional on the model. Since it does not depend on the parameters being estimated, it can be treated as a constant. The resulting posterior distribution provides a probabilistic representation of the parameters,

[24] The openess parameter υ is calibrated to 0.4, the discount factor, β is calibrated to 0.99, and the elasticity of consumption ϵ is calibrated to be 9.

[25] There has been a paradigm shift away from the frequentist to the Bayesian approach to estimating DSGE models as computing power has become more abundant. The frequentist approach often yielded absurd parameter estimations, not consistent with economic theory

[26] In the context of our model, this would be the priors specified above.

with the shape of the distribution indicating the uncertainty associated with the parameter estimates. The mode of the posterior distribution represents the most probable value for the parameters, and the distribution around the mode characterises the estimate's uncertainty.

While the researcher has direct control over the choice of the prior distribution, obtaining the likelihood function can be challenging, especially in complex numerical models. To overcome this challenge, the *Kalman filter* is used as an approximation technique for the likelihood function. To employ the Kalman filter to estimate the likelihood function, we have to return to the state space representation

$$x_t = g(x_{t-1}, \varepsilon_t^{struct}), \tag{59}$$

$$y_t^{obs} = h(x_t, \varepsilon_t^{obs}), \tag{60}$$

we now note that some of our states x_t in the state-transition equation are partially unobserved. The likelihood function used in estimation has to consist of only observable variables, and so the unobserved states have to be filtered out by the use of the linear prediction error algorithm.[27] The result of the filtering is the posterior kernel, from which the posterior mode can be estimated.

To uncover the posterior mode, which represents the most probable parameter values given the observed data, a widely employed approach is the use of Markov Chain Monte Carlo (MCMC) algorithms. In particular, the Metropolis-Hastings algorithm is commonly utilised in Bayesian estimation. This algorithm facilitates the exploration of the posterior distribution and enables the identification of its mode, corresponding to the parameter values with the highest probability.

The MCMC algorithm works by constructing a Markov chain that explores the parameter space, guided by the shape of the posterior distribution. Starting from an initial set of parameter values, the algorithm iteratively proposes new parameter values based on a proposal distribution. The proposed values are then accepted or rejected based on a

[27] The technicalities of the Kalman filter are theoretically hard to grasp, and so further detail can be found in for example Hamilton (2020).

criterion that takes into account the likelihood function and the prior distribution.

By repeating this process for a sufficiently large number of iterations, the Markov chain converges to the posterior distribution. The samples obtained from the chain provide a representation of the posterior distribution, with a higher concentration of samples around the mode. The posterior mode can then be estimated as the parameter values corresponding to the highest density of samples.

6.4 Estimation results

The resulting posterior distributions are obtained via the Metropolis-Hastings Monte Carlo algorithm described above. Five chains of 500 000 iterations have been computed, yielding posterior distributions for all the variables like the selection shown here.

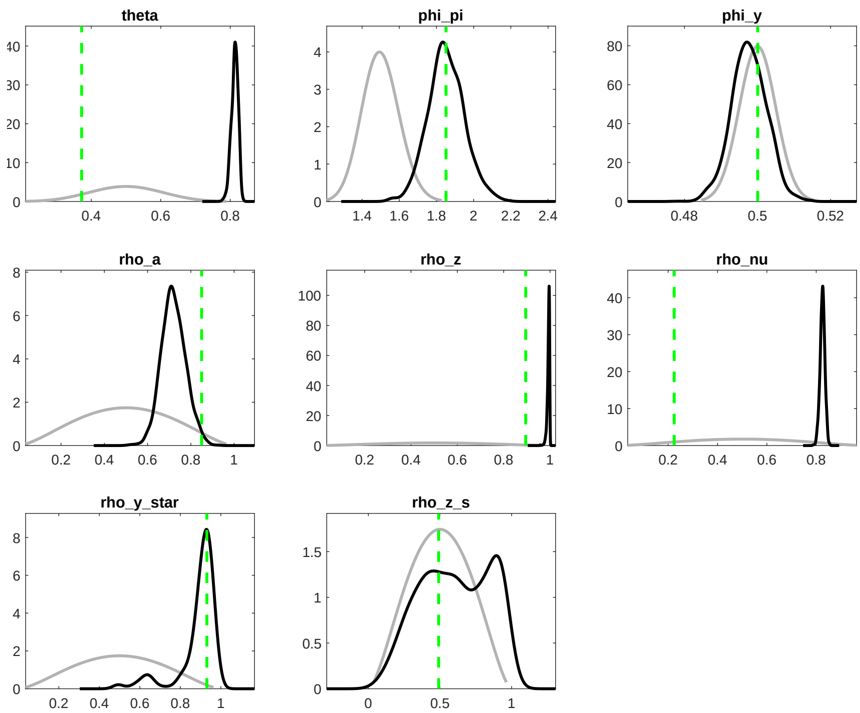

Figure 8: Prior and posterior distributions

This figure depicts the prior and posterior distributions for some of the estimated parameters. The x-axis depicts a range of parameter values and the y-axis depict the likelihood. The grey distribution is the prior, whilst the black distribution is the posterior. The posterior peaks at the most probable posterior value, giving us the posterior mean.

Figure 8 depicts a visual comparison between the prior and posterior distributions.The green dashed line represent the posterior mode, whilst the peak of the black line represent the posterior mean. It is the ladder, and its associated credible interval, that is used when examining the Bayesian impulse response function presented below. The observed prior and posterior distributions exemplify some common challenges that can arise when employing full information estimation for model parameters. For instance, when inspecting the posterior density of parameter ϕ_y, it becomes evident that the posterior distribution closely resembles the prior distribution. This similarity suggests that either the available data lacks sufficient information for accurate parameter identification or the model specification inadequately captures real-world dynamics. Consequently, the posterior distribution becomes highly reliant on the prior.

Another issue highlighted by the aforementioned distributions involves obtaining distributions with peculiarly shaped peaks. For instance, in the case of ρ_z, it appears that the likelihood function exhibits multiple peaks. Consequently, the prior dominates and determines the posterior mode. The use of prior distributions based on previous literature mitigates the need for precise identification of certain parameters. This implies that small deviations between the prior and posterior distributions are not necessarily a major concern. An overview of all the estimates and their associated credible intervals can be found in Appendix C.

6.5 Bayesian IRF curves

By utilising the estimated parameter values and their corresponding uncertainties, it is possible to generate Bayesian impulse response functions that depict the system's dynamic response to estimated shocks.In the figures below, the black line represents the mean impulse response, while the grey shading indicates the uncertainty associated with the mean. From the graphs, it is evident that there is uncertainty regarding the shock size, which makes it difficult to draw definitive conclusions about the magnitude of the effect. It still provides us with an indication of the change in the dynamics of the system as trend inflation increases. To compare the estimated results with the theoretical implications the dynamic response to a productivity shock is shown.

Similar to the calibrated model, we observe that a productivity shock results in progressively negative price dispersion, primarily due to the negative variance in the inflation gap, as depicted below.

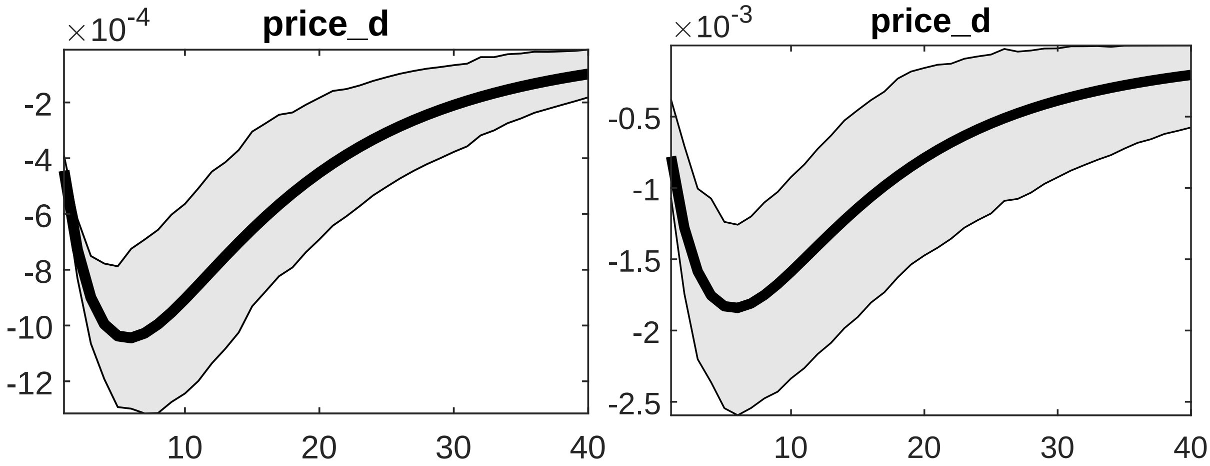

Figure 9: Price dispersion for 2% and 4% trend inflation

Due to the negative inflation gap that arise as a consequence of the productivity shock, price dispersion becomes negative. Due to the feedback between dispersion and inflation, it becomes more negative for higher levels of trend inflation.

The negative price dispersion acts as a productivity boost, making it so that as trend inflation increases the output gap has an increasing period of being positive after the initial negative impact of the productivity shock. Interestingly, in the scenario with 2% trend inflation, the negative price dispersion acts as a mitigating factor for the volatility in the output gap caused by the positive productivity shock. However, in the scenario with 4% trend inflation, we observe a notable transition from a negative production gap to a substantial positive output gap.

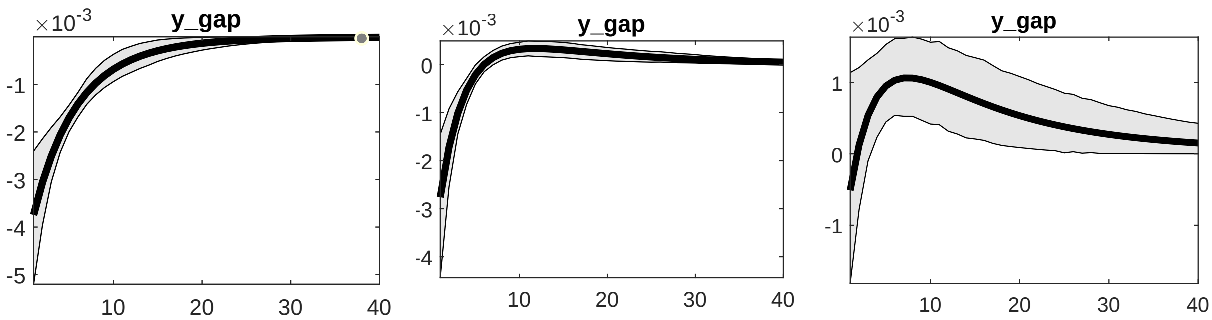

Figure 10: Bayesian IRF of production gap for 0% 2% and 4% trend inflation

Due to the decreasing price dispersion the output gap turns increasingly positive for higher levels of trend inflation.

Across all three scenarios, we note that the initial effect of negative output gaps resulting from the productivity shock leads to a corresponding negative inflation gap. The escalating level of price dispersion contributes to an intensification of the negative inflation gap, rendering it more volatile in response to the productivity shock. Consequently, higher trend inflation results in a progressively negative inflation gap.

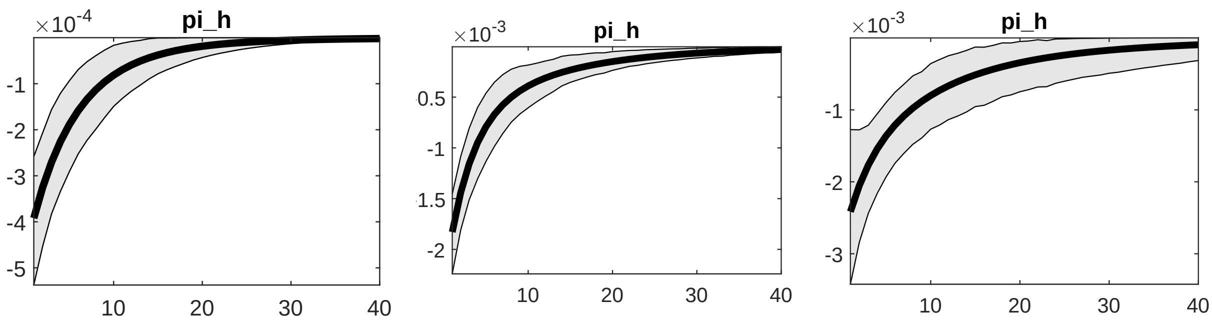

Figure 11: Bayesian IRF of price dispersion 0%, 2% and 4% trend inflation

Due to the initial impact of the negative output gap, in tandem with the feedback loop between inflation and price dispersion we see that the variation in inflation increase for higher level of trend inflation.

By analysing the estimated response to a productivity shock a couple of interesting observations can be made. Firstly, it is noteworthy that the variance in the response of the output gap to the productivity shock initially diminishes with higher trend inflation. The reason for this is that higher productivity and lower price dispersion act upon together in such a way that they mitigate each other. Secondly, the volatility in domestic inflation response to a productivity shock grows progressively higher with increasing levels of trend inflation, primarily due to the negative price dispersion. The response of the central bank is thus to respond to the shock with increasing strength to tame the progressively higher inflation.

Consequently, the estimation results suggest the relevance of both implication (i) and (ii) in comprehending the potential economic costs associated with higher trend inflation in the Norwegian context, albeit with a minor caveat. Interestingly, in certain scenarios, higher trend inflation appears to contribute to stabilising the output gap, suggesting that elevated trend inflation does not invariably result in significant trade-offs in monetary policy.

6.6 Determinancy region

To explore the loss of monetary policy flexibility as trend inflation increases we use the estimated parameters for the 2% trend inflation scenario to create a determinancy plot analogous to that in section 5.2.

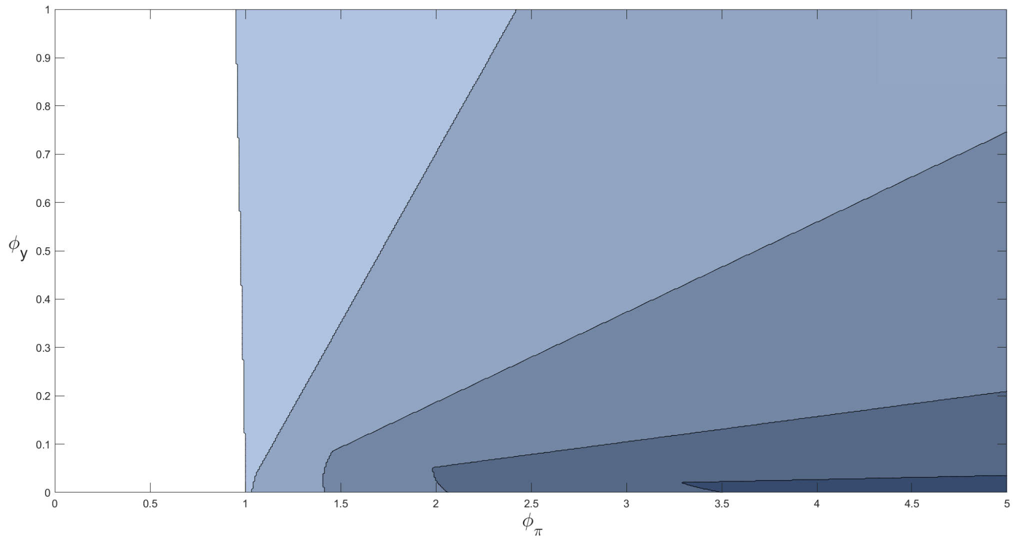

Figure 12: Bayesian determinancy

As trend inflation increases, the central bank has to respond more aggressively to deviations from the inflation target, represented by ϕ_π. However, as openness υ increases, the strength of the policy response needed decreases. The reduction in policy flexibility for higher levels of trend inflation are even more drastic when the parameters are estimated.

Figure (12) illustrates a more pronounced decline in monetary policy flexibility when utilising the parameters estimated from Norwegian data. The larger loss in monetary policy flexibility can be attributed to the higher value of θ, which, akin to trend inflation, increases the forward-looking behaviour of intermediate goods firms when adjusting their prices. Furthermore, apart from the diminishing capacity to address fluctuations in the output gap, it is evident that the central bank must adopt a more assertive response to changes in the inflation gap.

The Norwegian central bank explicitly aims to maintain forward-looking and flexible monetary policy to foster high and stable output. The substantial decline in monetary policy flexibility with rising trend inflation underscores the incompatibility between higher trend inflation and the objective of sustaining high and stable output.

To examine the effect of the degree of openness of the domestic economy to the world economy, the values of ϕ_π and υ compatible with a rational expectations equilibrium are plotted.

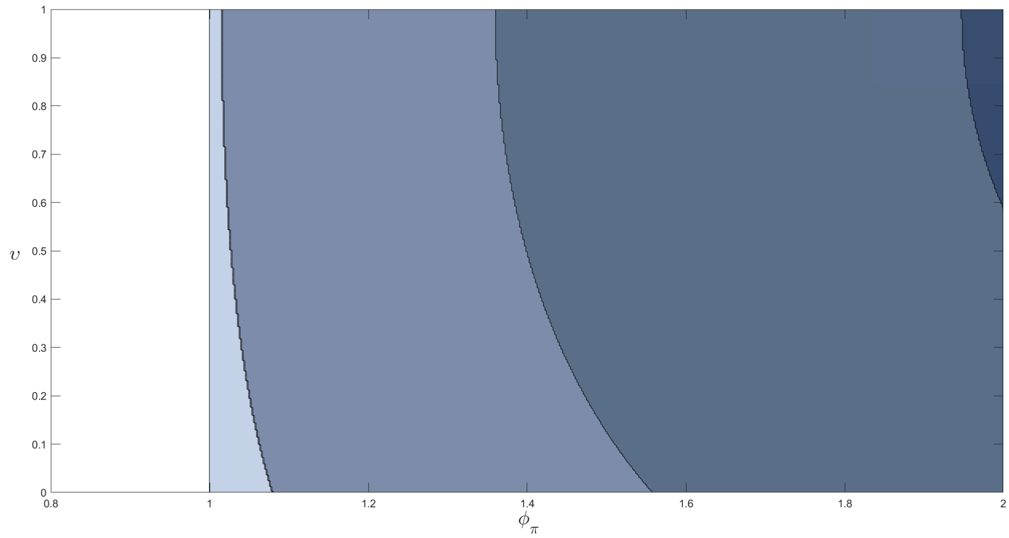

Figure 13: Increased determinancy region when opening up the economy

As trend inflation increases, the central bank has to respond more aggressively to deviations from the inflation target, represented by ϕ_π. However, as openness υ increases, the strength of the policy response needed decreases. The decrease is smaller when using the estimated parameters due to σ and η being lower.

Upon employing the estimated parameter values, we observe a marginal rise in monetary policy flexibility with an increase in openness. However, it is noteworthy that this effect is more subdued compared to the corresponding figure in section 5. The underlying reason for the diminished impact of openness is the lower estimated values of both the intertemporal substitution elasticity σ and the elasticity of substitution between domestic and foreign goods η in comparison to their calibrated counterparts. Consequently, the influence of openness υ on σ_υ is relatively reduced.

Based on the empirical evidence, it appears that both implication (iii) and (iv) hold relevance in comprehending the implications of trend inflation on Norwegian monetary policy. The loss of the ability to effectively stabilise the output gap becomes increasingly pronounced at higher levels of trend inflation. Moreover, it is evident that the Norwegian economy cannot remain insulated from the adverse effects of trend inflation, primarily owing to its low elasticity of substitution across borders and over different time periods.

7 Discussion and conclusion

The first part of the research question in this thesis ask how one can explore the effect of higher trend inflation on monetary policy within a New Keynesian DSGE model. This has been achieved by incorporating trend inflation dynamics from Ascari and Sbordone (2014) into the small open economy model of Gali and Monacelli (2005). The results of this exploration is four theoretical implications, namely that higher trend inflation increases the volatility of shocks, gives rise to monetary policy trade-offs and reduces the determinacy region for monetary policy. It also found that an economy which is more exposed to the world economy will suffer less from the adverse effects of trend inflation.

The second part of the research question asks what the implications of trend inflation is for Norwegian monetary policy. By using Bayesian estimation to connect the theoretical framework to Norwegian data, the thesis finds that all the theoretical implications are relevant to understanding how higher trend inflation can influence Norwegian monetary policy.

Seen through the lens of today's inflationary situation, this thesis thus emphasises that the central bank should focus on reducing inflation and inflation expectations back in line with the 2% inflation target. It is evident that it is difficult to find the right balance between the necessary tightening of monetary policy to reduce inflation and not tightening more than necessary, to avoid a recession. Recent evidence suggests that central banks worldwide have found the right balance for tightening, making a recession-free "soft landing" plausible.[28] However, one can still see how costly it has been to tame inflation, so it is easy to argue that avoiding such trade-offs in the future, partly due to higher trend inflation than 2%, is desirable.

The second reason this thesis argues for the importance of keeping inflation expectations at a low level is to retain monetary policy's ability to contribute to high levels of output and employment. It seems reasonable that firms in a high-inflation environment will put a larger weight on hedging against future inflation instead of responding to the current

[28] Lots of investors think inflation is under control. Not so fast. The Economist. Accessed from https://www.economist.com/briefing/2023/02/16/lots-of-investors-think-inflation-is-under-control-not-so-fast. February 16, 2023, on June 14, 2023.

real economy. Such an adaptation can reduce the central bank's ability to respond to deviations from the natural output in favour of only being able to perform its primary task of keeping the growth in prices low and stable.

The theoretical framework presented here is restricted by its relatively simple specification. Possible future research could thus be to explore modifications of the model with the aim of better capturing the effect of trend inflation in a small open economy. Examples of such extensions would be the incorporation of factors such as incomplete pass-through of foreign prices, household habit formation and sticky wages. Furthermore, using the framework as is to further explore the interplay between trend inflation and negative supply shock would be an interesting avenue for future research.

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A Appendix A

A.1 Optimal expenditure allocation

For ease of notation I show optimal expenditure allocation for a closed economy. The maths to find optimal allocation will be the same for both home and foreign goods in the open economy. For any given level of consumption the household have to choose the vector of intermediate goods that optimises total consumption, C_t. We get the following optimisation problem for a household that minimises consumption expenditures

$$\mathcal{L} = \int_0^1 P(i)_t C(i)_t di + P_t \left(C_t - \left[\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right] \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{A.1.1}$$

The Lagrange multiplier is replaced with P_t from the onset due to its interpretation as the shadow-price for a unit of consumption, which we know to be P_t. Minimizing with respect to C_t yields

$$\frac{\partial L}{\partial c_t(i)} = P_t(i) + P_t \left(\frac{\epsilon}{\epsilon - 1} \right) \left[\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}-1} \left(\frac{\epsilon}{\epsilon - 1} \right) (c_t(i))^{\frac{\epsilon}{\epsilon-1}-1}. \tag{A.1.2}$$

Using the definition of the Dixit Stigliz aggregation technology the above equation simplifies to the demand schedule for an intermediate good

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \tag{A.1.3}$$

The same can be done for goods from the home economy, country j and total imports, giving us demand functions (4) and (6).

A.2 Aggregation

Substituting the demand into the aggregation technology gives us

$$C_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 \left(\left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \right)^{\frac{\epsilon-1}{\epsilon}} di, \tag{A.2.1}$$

that when rearranged gives us aggregation of prices

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}. \tag{A.2.2}$$

Evaluating the budget constraint will then give us

$$\int_0^1 C_t(i) P_t(i) di = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t P_t(i) di. \tag{A.2.3}$$

That when rearranged gives us the intuitive insight that prices and consumption aggregate according to

$$\int_0^1 C_t(i) P_t(i) di = P_t C_t. \tag{A.2.4}$$

The same can be done for all levels of goods, both domestic and imported, giving us the budget constraint (7).

A.3 Household decision functions

Before formulating the maximization problem for the representative household, let's establish some assumptions regarding saving. Firstly, we assume that the discount factor, denoted as Q_t, can be represented by the price of an arrow security, namely $V_{t,t+1}$. This security is purchased at time t and has a probability ϱ of yielding a one unit of currency payoff at time $t+1$, with the remaining probability $(1-\varrho)$ resulting in a zero payoff. We thus define $Q_{t,t+1} \equiv E_t \frac{V_{t,t+1}}{\varrho_{t,t+1}}$. The budget constraint is given by

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 \left(\int_0^1 P_{j,t}(i)C_{j,t}(i)di \right) dj + E_t Q_{t,t+1} D_{t,t+1} \leq D_t + W_t N_t. \quad \text{(A.3.1)}$$

By aggregating as shown above the budget constraint can be written much simpler as

$$P_t C_t + E_t Q_{t,t+1} D_{t,t+1} \leq D_t + W_t N_t. \quad \text{(A.3.2)}$$

Using the utility function we can specify the following Lagrangian

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t - \lambda_t (P_t C_t + Q_{t,t+1} D_{t,t+1} - D_{t-1} - W_t N_t) \quad \text{(A.3.3)}$$

Taking the partial derivative with respect to C_t, N, t and D_t yields

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow \beta^t C_t^{\sigma} = \lambda_t P_t, \quad \text{(A.3.4)}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow \beta^t N^{\varphi} = \lambda_t W_t, \quad \text{(A.3.5)}$$

$$\frac{\partial \mathcal{L}}{\partial D_t} = 0 \Rightarrow -\lambda_t Q_{t,t+1} + \lambda_{t+1} = 0. \quad \text{(A.3.6)}$$

By solving (A.3.4) for λ_t and inserting into (A.3.6) we get the consumption Euler equation

$$Q_{t,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \frac{P_t}{P_{t+1}} \right]; \quad \text{(A.3.7)}$$

Similarly, by solving (A.3.4) for λ_t and inserting the resulting expression into (A.3.5) we get intertemporal allocation

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}. \tag{A.3.8}$$

Which concludes the derivation of the household decision functions

A.4 Optimal price setting

By inserting for the demand schedule into (20), and optimising with respect to the optimal reset price, $\tilde{P}_{H,t}$, we get

$$\sum_{k=0}^{\infty} \theta^k E_t \left[\beta^k \left(\frac{C_{t+k}}{c_t} \right)^{-\sigma} \frac{P_{H,t}}{P_{H,t+k}} \left((1-\epsilon) \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} C_{t,t+k} + MC_{t+k|t}\epsilon \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon-1} C_{t+k} \frac{1}{P_{H,t+k}} \right) \right] \tag{A.4.1}$$

That when rearranged can be expressed as

$$\sum_{k=0}^{\infty} = \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t} \left(\tilde{P}_{H,t} - \frac{\epsilon}{\epsilon - 1} MC_{t+k|t} \right) \right] = 0, \tag{A.4.2}$$

which when inserted for Y_{t+k} and $Q_{t,t+k}$ and divided over $P_{H,t}$ after a lot of algebra yields the firm optimal reset price schedule

$$\frac{\tilde{P}_{H,t}}{P_{H,t}} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j C_{t+k}^{1-\sigma} \Pi_{H,t,t+j}^{\epsilon} MC_{t+j,t}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j C_{t+k}^{1-\sigma} \Pi_{H,t,t+j}^{\epsilon-1}}. \tag{A.4.3}$$

Corresponding to (21) in the thesis

B Appendix B

B.1 The Euler equation

To log-linearize the consumption Euler equation we rewrite (8) as

$$1 = E_t\left[e^{i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho}\right], \tag{B.1.1}$$

where we have defined $i_t \equiv -logQ_t$, $\rho \equiv -log\beta$ and $\pi_{t+1} \equiv log(\frac{p_t}{p_{t+1}}$. In steady state we assume constant inflation $\bar{\pi}$ and constant growth γ, yielding a steady state characterised by

$$i = \rho + \pi + \sigma\gamma. \tag{B.1.2}$$

A first order Taylor expansion of $e^{i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho}$ around that steady state yields[29]

$$1 = E_t[e^{i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho}] \approx E_t[1 + (\rho - \rho)(i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi)]; \tag{B.1.3}$$

$$1 = 1 - \rho + i_t - \sigma E_t\{\Delta c_{t+1}\} - E_t\{\pi_{t+1}\} \tag{B.1.4}$$

Giving us the linearised Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - \rho). \tag{B.1.5}$$

B.2 Consumer price index

Log linearising the consumer index yields

$$\begin{aligned} P_t \approx \left((1-\upsilon)P^{1-\eta} + \upsilon P^{1-\eta}\right)^{\frac{1}{1-\eta}} + \frac{1}{1-\eta}\left((1-\upsilon)P^{1-\eta} + \upsilon P^{1-\eta}\right)^{\frac{1}{1-\eta}-1} \\ \left[((1-\upsilon)(1-\eta)P^{-\eta} + \upsilon P^{\eta-1})(P_{H,t} - P) + \upsilon(1-\eta)P^{-\eta}(P_{F,t} - P)\right], \end{aligned} \tag{B.2.1}$$

that when manipulated so that it can be interpreted as percentage deviations from steady

[29] We use the fact that $-\rho = -i + \sigma\gamma + \pi$ in the derivation

state can expressed as

$$\frac{P_t - P}{P} \approx (1-\upsilon)\frac{P_{H,t} - P}{P} + \upsilon\frac{P_{F,t} - P}{P}. \tag{B.2.2}$$

Substituting (30) into the above expression yields

$$p_t = p_{H,t} + \upsilon s_t, \tag{B.2.3}$$

Which is the linearised consumer price index

B.3 International risk sharing

By assuming that all foreign households have the same utility function as the domestic households facing the exact same savings-opportunities as the domestic household so that $Q_{t,t+1}$ is the same, we get a condition identical to (8) for all countries, here for country j.

$$Q_{t,t+1} = \beta E_t\left[\left(\frac{C^j_{t+1}}{C^j_t}\right)^{-\sigma}\left(\frac{P^j_t}{P^j_{t+1}}\right)\left(\frac{\wp_t}{\wp_{t+1}}\right)\right]; \tag{B.3.1}$$

Dividing this over the domestic Euler equation yields

$$1 = \frac{\beta E_t\left[Q^{-1}_{t,t+1}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{Z_{t+1}}{Z_t}\right)\frac{P_t}{P_{t+1}}\right]}{\beta E_t\left[Q^{-1}_{t,t+1}\left(\frac{C^j_{t+1}}{C^j_t}\right)^{-\sigma}\left(\frac{P^j_t}{P^j_{t+1}}\right)\left(\frac{\wp_t}{\wp_{t+1}}\right)\right]}, \tag{B.3.2}$$

That after some algebra can be written as

$$1 = E_t\left\{\left(\frac{c^j_{t+1}}{c_{t+1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\left(\frac{C^j_{t+1}}{C^j_t}\right)^{\sigma}\frac{\wp_{t+1}P^j_{t+1}}{\wp_t p^j_t}\right\}. \tag{B.3.3}$$

Inserting for the real exchange rate and demand definitions into the above expression yields

$$c_t^{-\sigma} = E_t \left\{ \left(\frac{C_{t+1}^j}{C_{t+1}} \right) C_t^{j-\sigma} \frac{\xi_{t+1}}{\xi_t} \right\} \tag{B.3.4}$$

Which simplifies into (13) when assuming that $\int_0^1 C_t^j = C^*$

$$C_t = \vartheta C_t^* Z_t^{\frac{1}{\sigma}} \wp_t^{\frac{1}{\sigma}}, \tag{B.3.5}$$

where $\vartheta \equiv E_t \left\{ \frac{C_{t+1}}{C_{t+1}^j \xi_{t+1}^{\frac{1}{\sigma}}} \right\}$, and can be understood as a measure of initial net asset positions, assumed to be symmetrical across the world.

B.4 Optimal price setting

To log-linearize the intermediate firm optimal pricing condition, we first take the logarithmic transformation (21)

$$\tilde{p}_t(i) = \hat{\psi}_t - \hat{\phi}_t, \tag{B.4.1}$$

To find $\hat{\psi}_t$ we take the linear approximation of (23)[30]

$$\begin{aligned} \psi_t \approx & \psi + \left[MC\bar{Y}^{1-\sigma} \right] \hat{MC} + \left[(1-\sigma) MC\bar{Y}^{1-\sigma} \right] \hat{Y} \\ & + \left[\theta\beta\epsilon\bar{\pi}^{\epsilon}\psi \right] E_t\hat{\pi}_{t+1} + \left[\theta\beta\bar{\pi}^{\epsilon}\psi \right] E_t\hat{\psi}_{t+1}, \end{aligned} \tag{B.4.2}$$

that after dividing over ψ becomes

[30]Here I make use of the fact that $(z_t - z) * \frac{z}{z} = Z\hat{z}_t = (z_t - z)$

$$
\begin{aligned}
\hat{\psi}_t \approx & \left[\frac{MC\bar{Y}^{1-\sigma}}{\psi}\right]\hat{MC} + \left[(1-\sigma)\frac{MC\bar{Y}^{1-\sigma}}{\psi}\right]\hat{Y} \\
& + \left[\theta\beta\epsilon\bar{\pi}^{\epsilon}\right]E_t\hat{\pi}_{t+1} + \left[\theta\beta\bar{\pi}^{\epsilon}\right]E_t\hat{\psi}_{t+1}.
\end{aligned}
\tag{B.4.3}
$$

Next, we need to find an expression for ψ in steady state. By inserting for steady state marginal cost MC, steady state production $\bar{Y}$, steady state trend inflation $\bar{\pi}$ and assuming that $\psi_t = \psi_{t+1} = \psi$ in (23) we get

$$
\begin{aligned}
\psi &= MC\bar{Y}^{1-\sigma} + \beta\theta\bar{\pi}^{\epsilon}\psi; \\
\psi(1-\beta\theta\bar{\pi^{\epsilon}}) &= MCY^{1-\sigma}; \\
\psi &= \frac{MC\bar{Y}^{1-\sigma}}{1-\beta\theta\bar{\pi}^{1-\epsilon}}.
\end{aligned}
$$

Inserting this into the above expression, and realising that $MC\bar{Y}^{1-\sigma}$ cancels out gives us

$$
\begin{aligned}
\hat{\psi} =& \left[1-\beta\theta\bar{\pi}^{\epsilon}\right]\left[\hat{MC} + (1-\sigma)\hat{Y}\right] \\
& + \left[\theta\beta\epsilon\bar{\pi}^{\epsilon}\right]E_t\hat{\pi}_{t+1} + \left[\theta\beta\bar{\pi}^{\epsilon}\right]E_t\hat{\psi}_{t+1}.
\end{aligned}
\tag{B.4.4}
$$

That is one third of the linear approximation done. We continue by linearizing equation (24) in the same fashion

$$
\begin{aligned}
\hat{\phi}_t =& \left[(1-\sigma)\frac{\bar{Y}^{1-\sigma}}{\phi}\right]\hat{Y}_t + \left[(\epsilon-1)\theta\beta\phi\bar{\pi}^{\epsilon-1}\right]E_t\hat{\pi}_{t+1} \\
& + \left[\theta\beta\phi\bar{\pi}^{\epsilon-1}\right]E_t\hat{\phi}_{t+1}.
\end{aligned}
\tag{B.4.6}
$$

Inserting for ϕ in steady state yields

$$
\hat{\phi} = \left[1-\beta\theta\bar{\pi}^{1-\epsilon}\right](1-\sigma)\hat{Y}_t + \left[\theta\beta\bar{\pi}^{\epsilon-1}\right]\left[(\epsilon-1)E_t\hat{\pi}_{t+1} + E_t\hat{\phi}_{t+1}\right]. \tag{B.4.7}
$$

To find the log linearised relative reset price we log linearise (19)

$$0 = \left[\theta(\epsilon-1)\bar{\pi}^{\epsilon-1}\right]\hat{\pi}_t + \left[(1-\theta)(1-\epsilon)p(i)^{1-\epsilon}\right]\hat{p}(i). \tag{B.4.8}$$

Inserting for steady state $p(i)$ gives us

$$\hat{p}(i) = \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_t, \tag{B.4.8}$$

B.4.1 The phillips curve in terms of marginal costs

Now that we have a linear expression of the optimal pricing decision we can rewrite it to give us a relationship between the marginal cost gap and inflation. We start by substituting (18) into (36) to get an expression of $\hat{\phi}_t$:

$$\hat{\phi}_t = \hat{\psi}_t - \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_t. \tag{B.4.9}$$

Inserting this into the expression for $\hat{\psi}$ above gives us

$$\begin{aligned}\hat{\psi}_t &= \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_t + \left[1-\beta\theta\bar{\pi}^{1-\epsilon}\right](1-\sigma)\hat{Y}_t \\ &+ \left[\theta\beta\bar{\pi}^{\epsilon-1}\right]\left[E_t\hat{\psi}_{t+1} - \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}E_t\hat{\pi}_{t+1} + (\epsilon-1)E_t\hat{\pi}_{t+1}\right].\end{aligned} \tag{B.4.10}$$

We then insert for ψ and get

$$\begin{aligned}&\left[1-\beta\theta\bar{\pi}^{\epsilon}\right]\left[\hat{MC} + (1-\sigma)\hat{Y}\right] - \left[\theta\beta\epsilon\bar{\pi}^{\epsilon}\right]E_t\hat{\pi}_{t+1} \\ &+ \left[\theta\beta\bar{\pi}^{\epsilon}\right]E_t\hat{\psi}_{t+1} = \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_t + \left[1-\beta\theta\bar{\pi}^{1-\epsilon}\right](1-\sigma)\hat{Y}_t \\ &+ \left[\theta\beta\bar{\pi}^{\epsilon-1}\right]\left[E_t\hat{\psi}_{t+1} - \frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}E_t\hat{\pi}_{t+1} + (\epsilon-1)E_t\hat{\pi}_{t+1}\right],\end{aligned} \tag{B.4.11}$$

that after some rearranging gives us

$$\frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}\hat{\pi}_t = \left[1-\beta\theta\bar{\pi}^{1-\epsilon}\right](\sigma-1)\hat{Y}_t + \left[1-\beta\theta\bar{\pi}^{\epsilon}\right]\left[\hat{MC}+(1-\sigma)\hat{Y}\right] + \left[(\theta\beta\bar{\pi}^{\epsilon})-(\theta\beta\bar{\pi}^{\epsilon-1})\right]E_t\hat{\psi}_{t+1} - \left[\theta\beta\epsilon\bar{\pi}^{\epsilon}\right]E_t\hat{\pi}_{t+1} - \left[\theta\beta\bar{\pi}^{\epsilon-1}\right]\left[\frac{\theta\bar{\pi}^{\epsilon-1}}{(1-\theta\bar{\pi}^{\epsilon-1})}E_t\hat{\pi}_{t+1}+(\epsilon-1)E_t\hat{\pi}_{t+1}\right]. \tag{B.4.12}$$

At last yielding an expression for period t inflation as a function of the marginal cost gap, future inflation, production gap and future marginal costs

$$\hat{\pi}_t = \frac{(1-\beta\theta\bar{\pi}^{\epsilon-1})(1-\theta\bar{\pi}^{\epsilon-1})}{\theta\bar{\pi}^{\epsilon-1}}\hat{mc}_t + \beta\left[1+\epsilon(\bar{\pi}-1)(1-\theta\bar{\pi}^{\epsilon-1})\right]E_t\hat{\pi}_{t+1} + \beta[1-\bar{\pi}][1-\theta\bar{\pi}^{\epsilon-1}][(1-\sigma)\hat{Y}_t - E_t\hat{\psi}_{t+1}], \tag{B.4.13}$$

where we see that period t inflation is a function of the marginal cost gap $\hat{mc}$, future expected inflation $E_t\hat{\pi}_{t+1}$ and the discounted present value of future marginal costs $\hat{\psi}_{t+1}$.

B.5 Linearising marginal costs

To find the log-linearised expression for real marginal costs we start by linearising (17)

$$mc_t^r = (w_t - p_t) + (p_t - p_{H,t}) - a_t; \tag{B.5.1}$$

That when inserting for the definition of the real wage from (28) and the definition of the terms of trade from (30) gives us

$$mc_t^r = \sigma c_t + \varphi n_t + \upsilon s_t - a_t. \tag{B.5.2}$$

Using the definition of domestic consumption from (35) to insert for c_t and the log linear labour demand schedule (25)[31] for n_t we get

$$mc_t = \sigma(c^* + \frac{1-\upsilon}{\sigma}s_t) + \varphi(x_t + y - a) + \upsilon s_t - a_t; \tag{B.5.3}$$

$$mc_t = \sigma y^* + \varphi y + \varphi x_t - (1-\varphi)a_t + s_t. \tag{B.5.4}$$

At last we insert for s_t from the equation to be derived below and get

$$mc_t = \varphi x_t + (\sigma_\upsilon + \varphi)y_t + (\sigma - \sigma_\upsilon)y^* - (1-\varphi)a_t. \tag{B.5.5}$$

Which is the real marginal costs used in to derive the New keynesian Phillips curve.

B.6 Market clearing and the IS curve

Market clearing implies that domestic output must equal domestic consumption of domestic goods plus foreign consumption of domestic goods. This can be expressed as

$$Y_t(i) = C_{H,t}(i) + \int_0^1 C^j_{H,t}(i)dj \tag{B.6.1}$$

Where $C_{H,t}(i)$ is domestic consumption of domestically produced variety (i), and $\int_0^1 C^j_{H,t}(i)dj$ is total consumption of domestically produced variety (i). Going up the layers of demand one can express domestic consumption of domestic output as

$$C_{H,t}(i) = (1-\upsilon)\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{B.6.2}$$

Similarly one can do the same for any foreign country, but now evaluating the domestic economy as foreign, giving us

[31] Which is simply $y_t = n_t - x_t + a_t a_t$

$$C_{H,t}(i)^j = \upsilon \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left(\frac{P_{H,t}}{\wp_{j,t} P^j_{F,t}}\right)^{-\epsilon f} \left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\eta} C^j_t \tag{B.6.3}$$

Combining these three together yields

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left[(1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \int_0^1 \upsilon \left(\frac{P_{H,t}}{\wp_{j,t} P^j_{F,t}}\right)^{-\epsilon f} \left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\eta} C^j_t dj\right] \tag{B.6.4}$$

Using the definition of the aggregation technology $\left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$ we can aggregate domestic demand

$$Y_t = \left[\left(\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left[(1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \int_0^1 \upsilon \left(\frac{P_{H,t}}{\wp_{j,t} P^j_{F,t}}\right)^{-\epsilon f} \left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\eta} C^j_t dj\right]\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{B.6.5}$$

From the above expression it can be shown that the assumption of a continuum from $0-1$ makes it so that $\frac{\epsilon-1}{\epsilon}$ and $\frac{\epsilon}{\epsilon-1}$ will cancel out. By definition we also know that $\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right) di = 1$ leaving us with the much simpler expression

$$Y_t = (1-\upsilon)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \int_0^1 \upsilon \left(\frac{P_{H,t}}{\wp_{j,t} P^j_{F,t}}\right)^{-\epsilon f} \left(\frac{P^j_{F,t}}{P^j_t}\right)^{-\eta} C^j_t dj \tag{B.6.6}$$

By clever manipulation and the use of the definition of the real exchange rate (11) and the bilateral terms of trade (10) the foreign part of the above expression can be written in terms of domestic consumption, yielding

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\epsilon} C_t \left[(1-\upsilon) + \upsilon \int_0^1 (S^j_t S_j t)^{\epsilon f - \eta} \xi_t^{\eta - \frac{1}{\sigma}} dj\right] \tag{B.6.7}$$

By log linearising the above expression and inserting for the linearised definition of the real exchange rate and doing a lot of algebra gives us the following expression

$$y_t = c_t + \frac{\upsilon\Theta}{\sigma} s_t \tag{B.6.8}$$

where $\Theta \equiv \sigma\eta + (1-\upsilon)(\sigma\eta - 1)$ is a parameter consisting of the degree of openness and the substitutability between home and foreign goods. A similar condition must hold for all other countries so that one can aggregate through all j to find the world market clearing condition $c^* = y^*$. The latter is achieved by realising that $\int_0^1 s_t^j dj = 0$. By inserting for (35) into the above expression we get

$$y_t = y_t^* + \frac{1}{\sigma_\upsilon} s_t \tag{B.6.9}$$

Inserting from c_t from (29) gives us

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\upsilon\Theta}{\sigma}\Delta s_{t+1} \tag{B.6.10}$$

Which when inserting for s_t from above, using our definitions of σ_υ and Θ, and doing a lot of algebra yields

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \upsilon\Theta E_t\{\Delta y_{t+1}^*\} \tag{B.6.11}$$

By defining the real rate of interest as $r \equiv i_t - E_t\{\pi_{H,t+1}\}$, inserting it into the above expression and evaluating the resulting expression in the frictionless steady state we get an expression for natural output

$$y_t^n = E_t\{y_{t+1}^n\} - \frac{1}{\sigma_\upsilon}(r^n - \rho) + \upsilon\Theta E_t\{\Delta y_{t+1}^*\} \tag{B.6.12}$$

By the same logic of finding the marginal costs gap we can subtract the natural output from its period t counterpart to find an expression for the output gap

$$\hat{y}_t = E_t\{\hat{y_{t+1}}\} - \frac{1}{\sigma_v}(i_t - E_t\{\pi_{H,t+1}\} - r^n) \tag{B.6.14}$$

To find the natural rate we start by evaluating the real marginal costs for frictionless output as

$$mc = -\mu = (\sigma_v + \phi)y_t^n + (\sigma - \sigma_v)y_t^* - (1 - \phi)a_t \tag{B.6.15}$$

Solving for natural output yields

$$y_t^n = \frac{1+\varphi}{\sigma_v + \varphi}a_t - \frac{\sigma - \sigma_v}{\sigma_v + \varphi}y_t^* - \frac{1}{\sigma_v + \varphi}x_t \tag{B.6.16}$$

Combining this with (11.70) and (11.84) yields the natural rate of interest as given by

$$r_t^n \equiv \rho - \sigma_v \Gamma_a (1 - \rho_a)a_t + \Psi_* E_t\{\Delta y_{t+1}^*\} + \frac{\sigma_v}{\sigma_v + \varphi}\Delta x + \Psi_z(1 - \rho_z)z_t. \tag{B.6.17}$$

C Appendix C

The complete system of linear difference equations consists of the following relations

$$(46) \quad \pi_{H,t} = \kappa_\upsilon(\bar{\pi})\hat{y}_t + \lambda(\bar{\pi})\varphi\hat{x}_t + b_1(\bar{\pi})E_t\{\pi_{H,t+1}\} + b_2(\bar{\pi})\left[(1-\sigma)\hat{y}_t - E_t\{\psi_{t+1}\}\right],$$

$$(47) \quad \hat{\psi}_t = [1-\theta\beta\bar{\pi}^\epsilon]\left[\varphi\hat{x}_t + (\sigma_\upsilon + \varphi)(1-\sigma)\hat{y}_t\right] + [\theta\beta\bar{\pi}^\epsilon]\, E_t\hat{\psi}_{t+1} + [\theta\beta\epsilon\bar{\pi}^\epsilon]\, E_t\hat{\pi}_{t+1},$$

$$(49) \quad \hat{x} = \left[\frac{\epsilon\theta\bar{\pi}^{\epsilon-1}}{1-\theta\bar{\pi}^{\epsilon-1}}(\bar{\pi}-1)\right]\hat{\pi}_{H,t} + \theta\bar{\pi}^\epsilon\hat{s}_{t-1},$$

$$(55) \quad \hat{y} = E_t\hat{y}_{t+1} - \frac{1}{\sigma_\upsilon}(i_t - E_t\{\hat{\pi}_{H,t+1}\} - r^n) + en,$$

$$(56) \quad r_t^n \equiv \rho - \sigma_\upsilon\Gamma_a(1-\rho_a)a_t + \Psi_* E_t\{\Delta y_{t+1}^*\} + \Psi_z(1-\rho_z)z_t,$$

$$(57) \quad i_t = \phi_\pi\pi_{H,t} + \phi_y\hat{y}_t + nu,$$

$$(44) \quad y^n = \Gamma_a a + \Gamma_z z + \Gamma^* y^* - \Gamma_x x,$$

$$(39) \quad \pi_t = \pi_{H,t} + \upsilon\Delta s_t,$$

$$(16) \quad y = a_t - x_t + n_t,$$

$$(28) \quad w - p = \sigma c + \varphi n,$$

$$(35) \quad c_t = y_t^* + \left(\frac{1-\upsilon}{\sigma}\right)s_t + \frac{1}{\sigma}z_t,$$

$$(34) \quad q_t = (1-\upsilon)s_t,$$

$$(B.6.8) \quad nx_t = y_t = c_t + \frac{\upsilon\Theta}{\sigma}s_t,$$

$$(C.1.1) \quad s_t^n = \sigma_\upsilon(y^n - y^*) - (1-\upsilon)\sigma_\upsilon * \frac{1}{\upsilon}$$

$$(C.1.2) \quad \hat{y} = y_t - y_t^n,$$

$$(C.1.3) \quad p_{H,t} = p_{H,t-1} + \pi_{H,t},$$

$$(C.1.4) \quad \Delta q_t = q_t - q_{t-1},$$

$$(C.1.5) \quad y^* = \rho_{y^*}y_{t-1}^* + \varepsilon_{y^*}$$

$$(C.1.6) \quad a = \rho_a a_{t-1} + \varepsilon_a$$

$$(C.1.7) \quad z = \rho_z z_{t-1} + \varepsilon_z$$

$$(C.1.8) \quad nu = \rho_i nu_{t-1} + \varepsilon_{nu}$$

$$(C.1.9) \quad en = \rho_y en_{t-1} + \varepsilon_{en}$$

Where shock variables are added so that we can examine the dynamic response to exogenous shocks in addition to allowing for the estimation of the model parameters. The shocks are assumed to be auto-regressive so that there is an initial shock to the system and then a gradual fading over time. The parameter ρ_i determine the persistence of the shock.

C.1 Impulse response functions

The dynamic response to a shock in preferences, monetary policy and world output, similar to the one presented in 4.1, are simulated and displayed. The graphs will be accompanied by a brief explanation of the dynamic response.

C.1.1 Preference shock

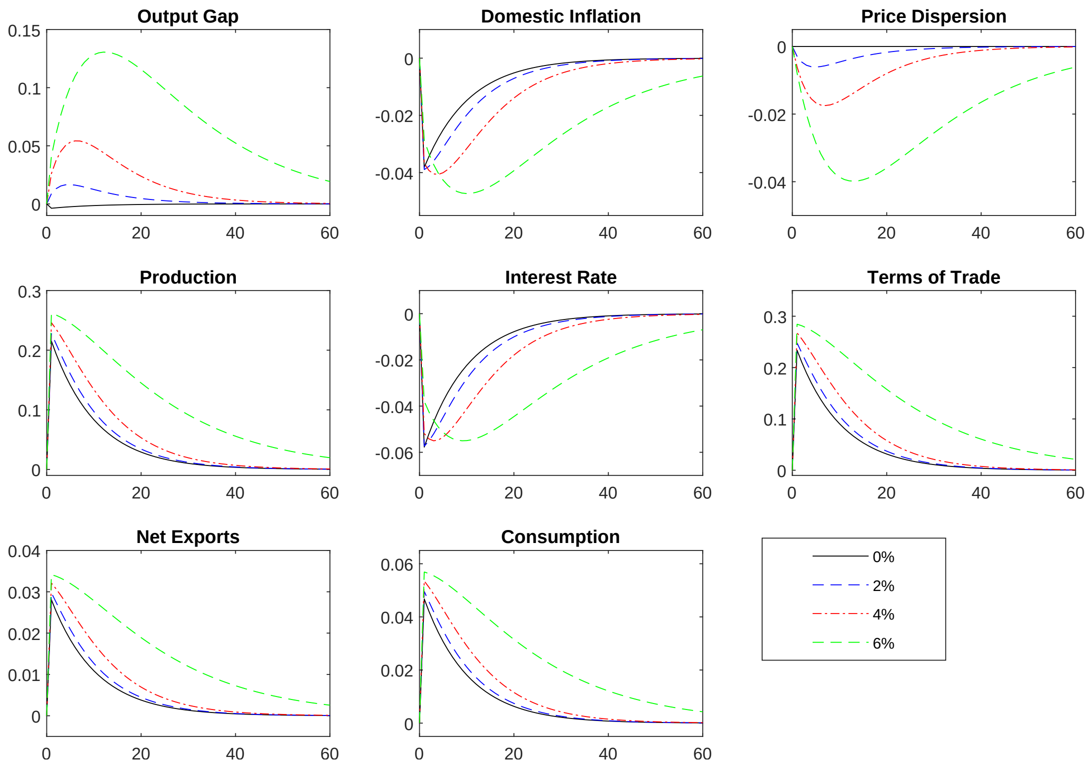

Figure 14: Dynamic response to a preference shock

In response to the preference shock we can see a sudden shift in consumption and domestic production, initially creating a negative production gap, leading to a negative inflation gap. For higher levels of trend inflation we see that the negative inflation gap leads to a negative price dispersion gap, acting as a de facto productivity increase, creating more volatility in the system. We get a trade-off between stabilising inflation and production in addition to firms being more forward looking, making the shock more persistent. Also notable is the feedback loop between price dispersion and domestic inflation. The prolonged negative interest rates and domestic inflation actually lead to a boost to the terms of trade, and thus the net exports.

C.1.2 Monetary policy shock

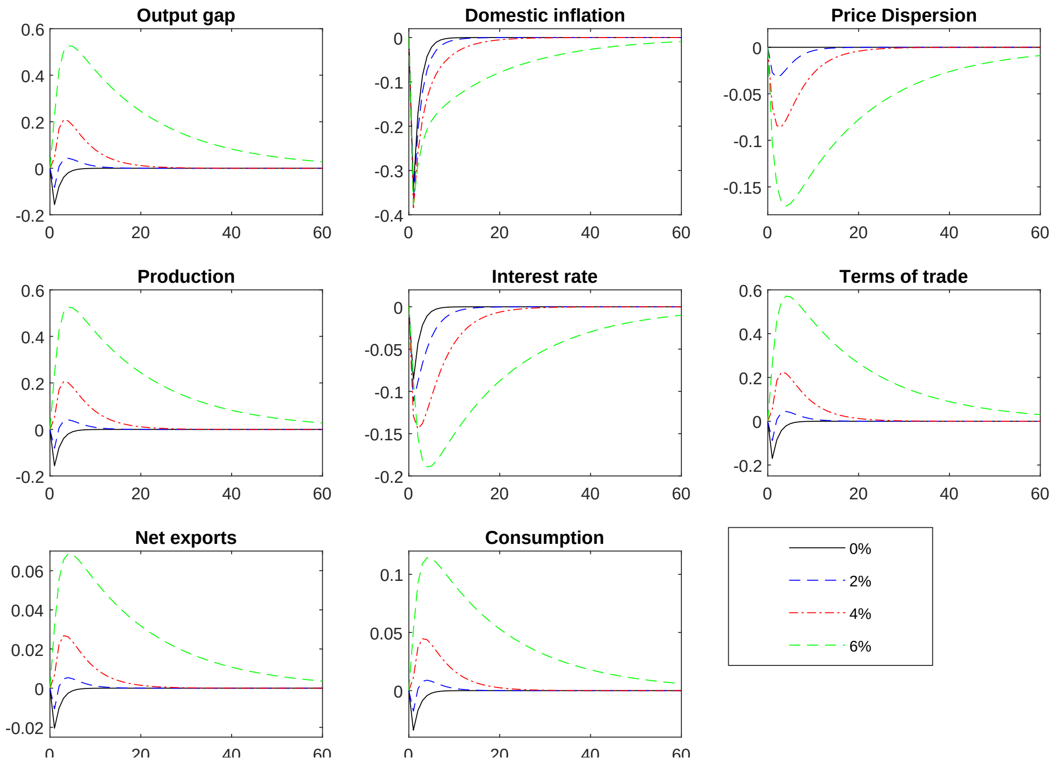

Figure 15: Dynamic response to a monetary policy shock

The response to a monetary policy shock is quite interesting. The postive interst rate shock leads to a intitial reduction in both the output gap and the inflation gap, but due to the ensuing negative price dispersion acting as a productivity booster we get a positive output gap and a negative inflation gap. In response to the negative inflation gap the central bank lowers back down its interest rate, further worsening the output gap, but stabilsing the inflation gap. In the zero trend inflation scenario we see that an increase in interest rates worsens the terms of trade due to appreciation of the domestic currency, leading to negative net exports and consumption. As trend inflation increases, however, this effect is reversed due to the prolonged period of negative interest rates.

C.1.3 World output shock

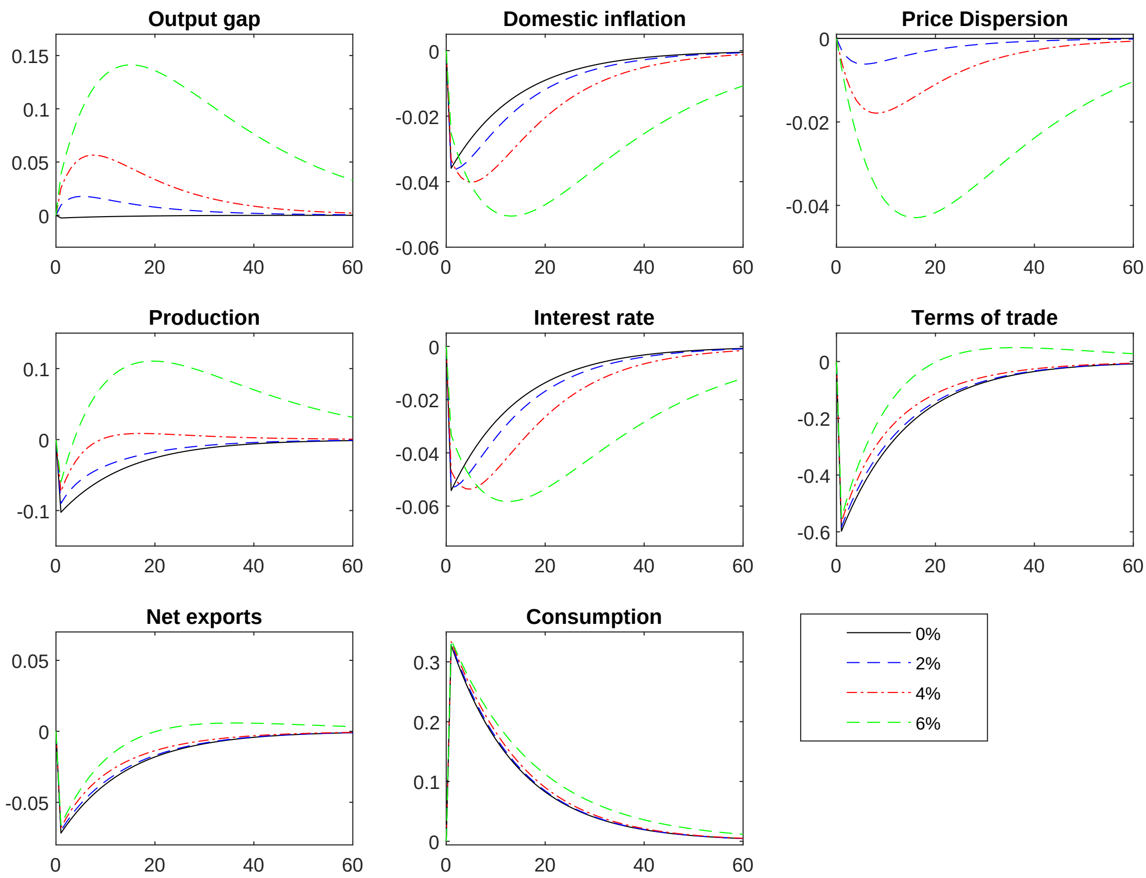

Figure 16: Caption

A shock to world output has two effects. The first is that it enters directly into the consumption equation, and so we will see a large increase in domestic consumption as a result of increased world output. The second effect goes through a decrease in the natural rate of interest, leading to a reduction in the domestic production gap. Following the reducton, domestic inflation also drops leading to price disperson. This creates a dynamic where the central bank lowers the interest rate, further stoking the fire under the production gap, leading to a more persistent shock. Terms of trade and net exports increase as the domestic price level decrease.

D Appendix D

D.1 Posterior distribution for 0% trend inflation

Parameter	Prior	Posterior	90% HDP		prior	pstdev
η	1.000	0.6761	0.5695	0.8016	invg	0.500
φ	3 .000	2.9420	2.6047	3.2510	gamm	0.2000
σ	2 .000	1.5786	1.4408	1.6908	gamm	0.1000
θ	0 .500	0.9418	0.9312	0.9528	beta	0.1000
ϕ_π	1 .500	1.5067	1.3624	1.6910	gamm	0.1000
ϕ_y	0 .500	0.4978	0.4895	0.5067	beta	0.0050
ρ_a	0 .500	0.8286	0.7429	0.8967	beta	0.2000
ρ_z	0 .500	0.8430	0.7807	0.9093	beta	0.2000
ρ_{nu}	0 .500	0.9795	0.9698	0.9885	beta	0.2000
ρ_{y^*}	0 .500	0.9192	0.8688	0.9671	beta	0.2000
ρ_s	0 .500	0.5278	0.1833	0.8889	beta	0.2000
σ_a	0.100	0.0137	0.0122	0.0151	invg	2.0000
σ_z	0 .100	0.0663	0.0568	0.0759	invg	2.0000
σ_{nu}	0 .100	0.0255	0.0217	0.0292	invg	2.0000
σ_{y^*}	0 .100	0.0352	0.0292	0.0416	invg	2.0000
σ_s	0 .100	0.0341	0.0232	0.0434	invg	2.0000
p^*	0 .100	0.0608	0.0538	0.0672	invg	2.0 000

D.2 Posterior distribution for 2% trend inflation

Parameter	Prior	Posterior	90% HDP		prior	pstdev
η	1.000	0.7424	0.6394	0.8524	invg	0.5000
φ	3.000	2.7234	2.4476	3.0158	gamm	0.2000
σ	2.000	1.5385	1.4283	1.6310	gamm	0.1000
θ	0.500	0.8126	0.7969	0.8281	beta	0.1000
ϕ_π	1.500	1.8518	1.7009	2.0166	gamm	0.1000
ϕ_y	0.500	0.4976	0.4900	0.5054	beta	0.0050
ρ_a	0.500	0.7192	0.6343	0.8141	beta	0.2000
ρ_z	0.500	0.9930	0.9866	0.9994	beta	0.2000
$\rho_n u$	0.500	0.8252	0.8102	0.8422	beta	0.2000
ρ_{y^*}	0.500	0.8862	0.7845	0.9860	beta	0.2000
ρ_a	0.500	0.6029	0.2636	0.9571	beta	0.2000
σ_a	0.100	0.0130	0.0118	0.0141	invg	2.0000
σ_z	0.100	0.0671	0.0571	0.0751	invg	2.0000
σ_{nu}	0.100	0.0300	0.0253	0.0336	invg	2.0000
σ_{y^*}	0.100	0.0358	0.0285	0.0437	invg	2.0000
σ_s	0.100	0.0363	0.0258	0.0501	invg	2.0000
p^*	0.100	0.0609	0.0548	0.0668	invg	2.0000

D.3 Posterior distribution for 4% trend inflation

Parameter	Prior	Posterior	90% HDP		prior	pstdev
η	1.000	0.8178	0.6899	0.9400	invg	0.5000
φ	3.000	2.7896	2.4741	3.1189	gamm	0.2000
σ	2.000	1.5268	1.4301	1.6186	gamm	0.1000
θ	0.500	0.7243	0.7006	0.7486	beta	0.1000
ϕ_π	1.500	1.9480	1.7975	2.0996	gamm	0.1000
ϕ_y	0.500	0.4986	0.4901	0.5068	beta	0.0050
ρ_a	0.500	0.7716	0.6535	0.9815	beta	0.2000
ρ_z	0.500	0.9861	0.9735	0.9990	beta	0.2000
ρ_{nu}	0.500	0.7445	0.7207	0.7684	beta	0.2000
ρ_{y^*}	0.500	0.7974	0.4672	0.9813	beta	0.2000
ρ_s	0.500	0.6891	0.3129	0.9822	beta	0.2000
σ_a	0.100	0.0127	0.0118	0.0137	invg	2.0000
σ_z	0.100	0.0689	0.0592	0.0790	invg	2.0000
$\sigma_n u$	0.100	0.0301	0.0258	0.0338	invg	2.0000
σ_{y^*}	0.100	0.0345	0.0248	0.0444	invg	2.0000
σ_s	0.100	0.0386	0.0248	0.0525	invg	2.0000
p^*	0.100	0.0612	0.0542	0.0673	invg	2.0000

E Appendix E

Norwegian data on consumption, GDP and CPI from 1990Q1-2019Q4 are all gathered from statistics Norway (SSB). Data on domestic interest rates are proxied by NIBOR, and data on the terms of trade are proxied by i44, both collected from the Norwegian Bank. The US price level is used as a representation of the world price level, and is collected from FRED.

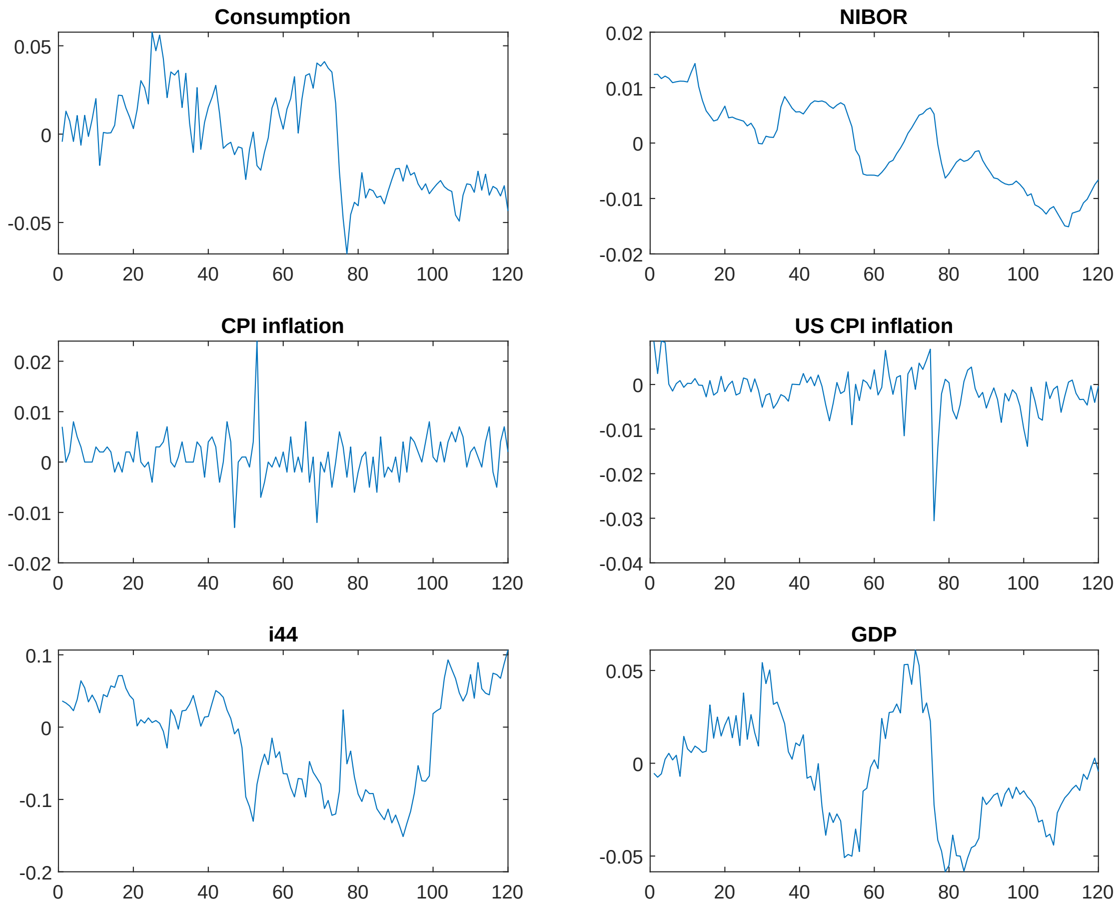

Figure 17: Plots of the data. 1990Q1-2019Q4.

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