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Roel Leonardus Gyula Nagy

Real options analysis of investment under uncertainty in the future energy system

NTNU
Norwegian University of Science and Technology
Thesis for the Degree of
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Dept. of Industrial Economics and Technology
Management

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“The *Guide* is definitive. Reality is frequently inaccurate."

The Hitchhiker’s Guide to the Galaxy

Abstract

Renewable energy sources, such as solar and wind power, have emerged as promising alternatives to traditional fossil fuels due to their ability to reduce greenhouse gas emissions and mitigate the impacts of climate change. However, the deployment and adoption of renewable energy technologies is often hindered by various forms of uncertainty and policy risk. These factors can create challenges for investors seeking to fund renewable energy projects, as they must consider the potential for changes in regulatory frameworks, technological advancements, and market conditions. This thesis aims to explore the complex interplay between renewable energy investment, uncertainty, and policy risk, with the goal of providing insights that can inform the development of more effective investment strategies and policy design.

In this thesis, I use real options analysis to examine the trade-offs involved in renewable energy investment under uncertainty and policy risk. Real options analysis is a powerful tool that has gained widespread use in the valuation of investment decisions that involve uncertainty, flexibility, and irreversibility. In the third paper included in this thesis, I incorporate Bayesian learning into a real options model. Through the use of Bayesian learning, the decision maker can update their understanding of the risks that may disrupt or terminate a project by incorporating new information. By continuously updating its beliefs about the likelihood of an event terminating the project, Bayesian learning can help a decision-maker make more informed investment decisions under uncertainty.

Paper I examines investment under uncertainty and subsidy withdrawal risk with a capacity size decision, taking both the point of view of a profit-maximizing investor and a welfare-maximizing social planner. The results show that investment is done sooner under a larger subsidy withdrawal risk, but this goes at the cost of a lower investment size. In terms of welfare, a lump-sum subsidy can only increase welfare if the withdrawal risk is low. Paper II revisits the framework of Paper I, but investment is assumed to be done incrementally instead of lumpy. The welfare-optimal policy strongly depends on the time frame of the social planner, as there is a strong trade-off between welfare in the short and long term. Finally, Paper III proposes a framework to examine active learning in a project subject to termination risk, where the learning rate can be chosen and comes with a cost. The decision to invest in learning is driven by uncertainty, and not by expected revenue.

Acknowledgements

This thesis is the result of many people helping and supporting me along the way. I want to express my heartfelt gratitude to all the people who supported me during my research. Without their guidance, encouragement, and assistance, this work would not have been possible. For this help and support, my acknowledgements are due to several people – but, of course, there are many, many others who contributed during this process.

First and foremost, I wish to express my sincere gratitude to my supervisor Verena Hagspiel. Your guidance, support, and encouragement throughout the duration of this journey have been invaluable. Your insights and expertise have been instrumental in shaping this work. Furthermore, you gave me the freedom to work in my own preferred way with both support and understanding, even at times when progress was lacking.

I am greatly appreciative of my co-supervisor, Stein-Erik Fleten. I appreciate that I could always drop by for any question that I had. Your immediate trust in my teaching skills and your continued emphasis of relying on my own knowledge has helped me grow as both a teacher and an academic.

I am grateful to my co-authors of the papers in this thesis. I want to thank Peter Kort for both the guidance in writing my first journal publication and supporting my interest in doing a PhD from the day I expressed interest in academia. Thanks to your efforts, I got in contact with the Norwegian University of Science and Technology (NTNU), which started my PhD journey. I am grateful to Lars Sendstad for being both a good colleague and a great friend. Your always open door invited not only interesting discussions on real options and academia, but also good friendly banter and great laughs. Your listening ear and advice throughout my PhD have helped me keep going, even in times when the PhD was challenging and daunting. As part of my PhD, I had the pleasure of visiting Jacco Thijssen at the University of York. Thank you for hosting me. I am especially grateful for your support and lending me an ear during my 10-day quarantine upon arrival in York. Your insights and help in tackling the mathematical challenges of Bayesian updating have significantly shaped this thesis. As part of my teaching work at NTNU, I had the pleasure of co-supervising Sebastian Sund for his project thesis, which planted the seed for the third paper in this thesis. Your endless curiosity and motivation while working on very complex problems made our meetings fly by.

I would like to extend my appreciation to the Department of Economics and Technology Management and NTNU for providing the resources and facilities necessary to complete this research. My time in Trondheim and at NTNU would not have been the same without the kindness and support from so many colleagues. I want to specifically mention and thank my office mates and fellow '11th floor' PhD candidates throughout my four-and-a-half years in Trondheim: Andreas, Farida, Felipe, Olga, Rodrigo, Semyon, Simon, and Tord. Thank you for joining me on many coffee breaks and listening to me sharing my highs and lows during the PhD (and also my apologies for the times I distracted you while you were focused on work).

Finally, I want to thank my friends and family for their love and support during this journey. I am deeply thankful for all that you have done for me. Your encouragement and understanding have meant the world to me, and words cannot fully capture my gratitude. I am looking forward to visiting and catching up with each and every one of you – you all know how talkative and curious I can be.

Contents

Chapter 1

Introduction

Many countries have set ambitious targets to reduce greenhouse gas emissions in order to limit climate change. By the end of 2021, 166 countries had renewable energy targets (REN21, 2022a:p.2). In early 2022, the European Commission raised its target that, by 2030, 45% of the total final energy consumption is delivered by renewable energy sources (REN21, 2022a:p.36), which has been revised upwards from the originally set target of 27% (REN21, 2019). To reach this target, it is critical to increase the share of renewable energy production in the energy mix (European Commission, 2017). Policy makers use support schemes to guide the transition from fossil fuels to renewable energy sources. 164 countries had renewable energy policies in place by the end of 2021. However, these policy schemes are frequently changed or terminated unexpectedly, affecting investors. Investors need to update their beliefs over time, as other factors, such as technological developments, also change the value of investments.

This thesis uses real options analysis for evaluating investment opportunities and government policy in the energy sector, with a focus on renewables. It examines the effect of uncertainty on investment, accounting for both market and policy risk. The real options analysis framework is chosen since it accounts for the three important elements of real investment in energy. First, it accounts for the value of the investment is subject to uncertainty. The value of an investment is driven by, for example, energy prices, which change over time and are subject to uncertainty. Second, real options analysis accounts for the investment being irreversible. Energy projects often involve the construction of infrastructure, such as power plants or pipelines, that cannot easily be dismantled or repurposed. Third, real options analysis accounts for the investor having the flexibility to choose the timing of its investment. Flexibility is an important factor in energy investment, as energy projects often have long lead times and can be subject to changes in market conditions or regulatory environments.

This thesis has the objective of providing insights on how policy and related risk, such as market uncertainty, technological uncertainty, and other risk factors affecting the profitability of real investments, influence investment decisions in the future energy system. It analyzes how both the firms' investment decisions and the policy maker's policy decisions are affected by uncertainty and learning about this uncertainty. More specifically, we are trying to answer the following research questions. First: how does the availability of subsidies affect the investment strategy of a firm in renewable energy projects? Second: how is this strategy affected by the risk that a subsidy may be withdrawn in the future? Third: how can regulators design sustainable and effective policies by anticipating the endogenous response from investors? Finally, fourth: how does the option to learn about the profitability of a project change the firm's decision to invest in this project?

All three papers included in this thesis study optimal investment strategies under uncertainty. Papers I and II answer the first three research questions, while Paper III answers the fourth research question. Paper I examines the optimal investment timing and size of a project under a lump-sum subsidy, in which the subsidy is subject to termination risk. In addition to the investor's perspective, we also consider the perspective of a policy maker and derive the welfare-maximizing subsidy as well as the subsidy required to reach a capacity target before a deadline. In Paper II, we revisit the setting in Paper I, but instead of looking at a single project, we take the perspective of a firm holding multiple options to invest. We assume investments can be done incrementally over time, instead of viewing investment as a one-time, lumpy decision. We derive the optimal growth strategy for the industry under subsidy termination risk. We also solve the policy maker's welfare maximization problem and determine the welfare-maximizing subsidy size. In Paper III, we study a sequential decision problem in which a firm has the option to invest in a project, the future profitability of which, the firm can learn about prior to investment. We derive the firm's optimal learning choice and examine the drivers behind the decision to invest in learning.

This thesis finalizes the PhD program in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The research presented in this thesis was initiated through the research project *Investment under uncertainty in the future energy system: The role of expectations and learning* (InvestExL), which was financed by the Research council of Norway under the ENERGIX program. This thesis, like the InvestExL program, aims to address the priority area of *Security of supply and output in the Norwegian energy system in the long term* of the ENERGIX program.

The thesis is structured as follows. Chapter 2 provides background information on the energy market and energy policy, including a background on electricity and evaluating investment in energy. The methodology used in my papers is discussed in Chapter 3. In Chapter 4, I discuss the contribution of this thesis as a whole and list the individual contributions of each paper. Finally, Chapter 5 provides a conclusion with recommendations for future research.

Chapter 2

Background

2.1 Energy production and renewable energy policy

With the Paris Agreement, the goal is to limit global warming to 'well below 2, but preferably to 1.5 degrees Celsius (°C)', compared to pre-industrial levels (United Nations Framework Convention on Climate Change, 2015). One of the most important tasks is to adapt our energy production in order to reach this goal. Currently, fossil fuels are dominant in both our worldwide energy and electricity mix as shown in Figures 2.1 and 2.2, respectively (REN21, 2022a). In the task to adapt our energy production, one may be cautiously optimistic as the growth of renewable power capacity was at a record high in 2021, despite the COVID-19 pandemic and rise in global commodity prices (REN21, 2022a:p.35). However, the progress in energy conservation, energy efficiency and the share of renewables in the total energy mix is generally slow (REN21, 2022a:p.21). In its key messages for the 2022 report, REN21 states that the 'global energy transition is not happening', and 'rising energy consumption and a hike in fossil fuel use outpaced growth in renewables in 2021' (REN21, 2022b). In fact, the world is currently not in line with the measures considered necessary to limit global warming to 1.5°C, as the policies currently in place point to a 2.8°C temperature rise by the end of the twenty-first century (UNEP, 2022:p.35). During the UN's annual climate summit COP27 in Egypt in 2022, other alarming reports were discussed, which indicate, for example, that the earth's temperatures in the past eight years were the hottest on record, and that sea levels are rising twice as fast as in 1990 (The Economist, 2022). A structural shift in the global energy system is increasingly urgent (REN21, 2022a:p.21). To achieve this transition, increasing the share of renewable energy production to the overall energy mix has been recognized as critical (European Commission, 2017). Furthermore, policy guidance and support remain indispensable to achieve the goals of limiting global temperature rise to 1.5°C and bringing CO2 emissions to net zero by 2050 (REN21, 2022a:p.58).

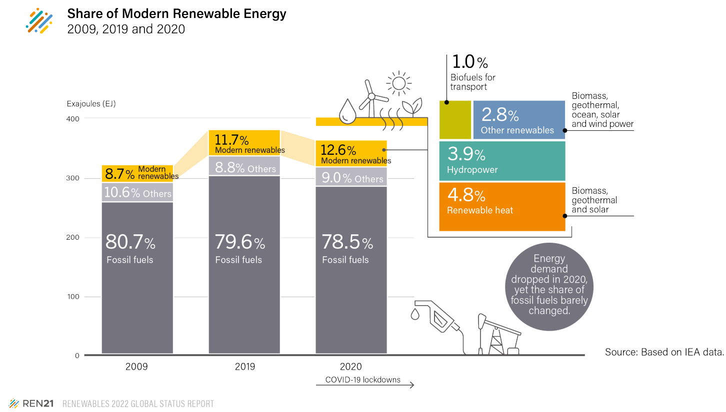

Figure 2.1: Breakdown of the worldwide energy production in 2009, 2019 and 2020 [Source: REN21 (2022a):p.37, based on IEA data].

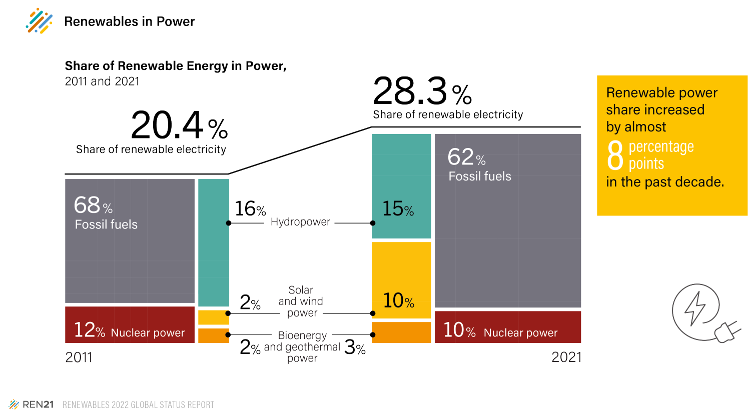

Figure 2.2: Breakdown of the worldwide electricity production in 2011 and 2021 [Source: REN21 (2022a):p.44, based on IEA data].

The energy mix consists of mostly fossil fuels and a slowly increasing share of renewables, as shown in Figure 2.1. Several different sources of renewable energy are used in the energy mix, mainly solar or photo-voltaic (PV), wind, hydroelectric, geothermal and biomass. Despite an increase in renewable energy in 2021, the growth was offset by the overall rise in electricity demand and the drought conditions that significantly decreased global hydropower generation (REN21, 2022a:p.36).

In the production of electricity, shown in Figure 2.2, there is also some use of nuclear power apart from fossil fuels and renewables.

Solar, wind, hydroelectric and geothermal energy are all clean, renewable sources of energy that are widely available. All produce low to no greenhouse gas emissions during operation, and they can be used to generate electricity (EIA, 2022). Solar panels, wind turbines, hydroelectric dams and geothermal systems are also relatively low maintenance, and the operating and management cost of all these energy sources are declining (IRENA, 2022b). Most renewable energy sources have lifespans that are comparable to or longer than those of combined cycle gas turbine (CCGT) power plants, which typically have a lifespan of 30 years (Parsons Brinckerhoff, 2011:p.37). The exception is wind turbines, which have a lifespan of around 20 years (Ziegler *et al.*, 2018). Solar panels typically have a lifespan of 30-35 years (EERE, 2022), hydroelectric dams have a lifespan of 50-100 years (EIA, 2022), and geothermal systems have a lifespan of 20-25 years (EERE, 2011). Furthermore, electricity from solar and onshore wind from most new projects in 2021 have been cost-competitive (Wind Europe, 2020; IRENA, 2022a), as shown in the top bar chart in Figure 2.3. In Figure 2.3, all capacity above zero in this figure represents projects with a lower levelized cost of electricity (LCOE[1]) than the cheapest fossil fuel-fired new generation option, at USD 54/MWh for a CCGT power plant in the United States (IRENA, 2022b:p.35). This means that those projects are cost-competitive. All capacity below the zero line had higher costs than USD 54/MWh.

However, solar, wind, hydroelectric and geothermal energy also each have their disadvantages. The initial cost of installing solar panels, wind turbines, hydroelectric dams, and geothermal system can all be high, even though the costs of each have decreased significantly over time (IRENA, 2022a). For example, offshore wind is not cost-competitive yet (IRENA, 2022b), due to the significant additional installation costs to connect the turbines to the grid compared to onshore wind. The investment cost per MW in an offshore wind farm is about 50% more expensive than in onshore wind (Díaz and Soares, 2020). In case of hydroelectric dams, the construction process can have environmental impacts, such as the loss of habitat for wildlife (EIA, 2022). Hydroelectric dams can also disrupt the natural flow of rivers, which can have negative impacts on ecosystems downstream (EIA, 2022). Another disadvantage of solar and wind energy is that both are dependent on weather conditions. Solar may not be reliable in cloudy or rainy areas, while wind may not be reliable in areas with low wind speeds. In case of wind turbines, some people also object to the appearance of wind turbines, which can be considered as noisy and visually obtrusive (Klick and E. R. Smith, 2010; EIA, 2022). Finally, geothermal systems also require a reliable source of geothermal energy, which may not be available in all areas (Zarrouk and

[1] The LCOE is a metric that estimates the overall cost of generating electricity from a particular source over the lifetime of the generator, taking into account the present value of those costs (Edenhofer *et al.*, 2013). It provides a way to compare the costs of different electricity generation options and can be used to evaluate the economic feasibility of a project.

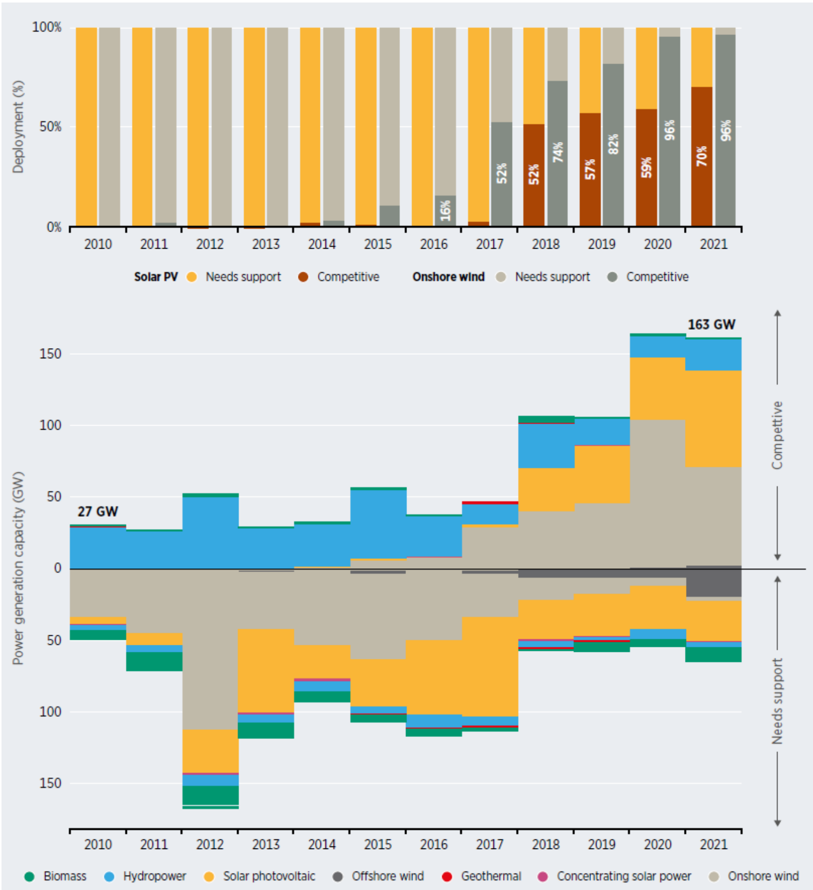

Figure 2.3: Annual and cumulative total new renewable electricity generation capacity added at a lower cost than the cheapest fossil fuel-fired option, 2010-2021 (IRENA©) [Source: IRENA (2022b):p.35, IRENA Renewable Cost Database].

McLean, 2019).

Compared to solar, wind, hydroelectric and geothermal energy, biomass energy has the disadvantage that it is the only source that it produces significant greenhouse gas emissions during operation (EIA, 2022). In fact, some claim the emissions are higher than those of fossil fuels (PFPI, 2011; NRDC, 2022). Biomass also requires the use of land and water resources for growing and processing the organic materials, which have environmental impacts (Trimble *et al.*, 1984). The advantage of biomass is its wide availability and that it can be produced from a variety of organic materials, including wood, agricultural waste, and landfill gas (EIA, 2022).

Despite countries having set renewable energy capacity targets to increase the amount of renewable energy, there are several reasons why these targets have not been reached. For example, there are technical challenges associated with integrating renewable energy into the electricity grid, particularly if the grid is not designed to accommodate variable renewable energy sources like wind and solar (Cochran *et al.*, 2015; Impram *et al.*, 2020).

There are also two fundamental market failures, or so-called market barriers, which cause these targets not to be reached (Jaffe *et al.*, 2005; Edenhofer *et al.*, 2013). First and foremost, there are climate externalities. The socioeconomic cost of greenhouse gas emissions is today still higher than the economic cost (Rennert *et al.*, 2022). Second, there are technological externalities in the energy market. For example, oil and gas have advantages of having more mature infrastructure due to being crucial for the energy supply over a longer period, which is referred to as a 'network externality' (Saltvedt *et al.*, 2022:p.33). By contrast, renewable energy is relatively new and many sources of renewable infrastructure still require large investments in order to be used on a similar scale. Furthermore, uncertainties in environmental benefits and energy market conditions can make it difficult to effectively address long-term issues like climate mitigation and assess investments in renewable energy (Jaffe *et al.*, 2005).

As a result of the technical challenges, market failures and uncertainty, investors demand larger expected returns to take on these risks and investments in renewables are delayed. This leads to renewable energy not being sufficiently included into the energy mix to reach the renewable energy capacity targets.

Countries set policies and implement subsidies to address the technical challenges and market imperfections that slow down the investment in renewable energy. The number of countries with renewable power policies aimed at accelerating investments in renewables increased again in 2021 (REN21, 2022a:p.76). As of 2021, 135 countries had renewable power targets and 156 countries have regulatory renewable power policies in place (REN21, 2022a:p.44).

However, the introduction of such policies can also create new challenges. In recent years, many support schemes have been unexpectedly retracted or revised. The Trump administration attempted to roll back 112 environmental rules in the United States in the period 2017 – 2021 (The New York Times, 2021). Among others, the Trump administration proposed a rule to repeal the Clean

Power Plan, which was a key policy aimed at reducing carbon dioxide emissions from power plants by promoting the use of renewable energy (The New York Times, 2017). In Brazil, subsidies for new solar and wind farms were phased out in 2022 (REN21, 2022a:p.177). The initial announcement of this decision lead to a spike in requests of renewable projects in 2021 (PV-Tech, 2021), as investors hoped to obtain subsidy before the subsidies were terminated. China implemented sudden changes in their feed-in tariff in 2018, making new solar power projects less likely to be eligible for subsidy (The Economist, 2018). Several other countries have also implemented changes to subsidies and tax exemptions related to renewable energy, including removing a tax exemption on companies selling renewable energy in Ukraine (REN21, 2015), adjusting the size of subsidy payments in Belgium, Bulgaria, the Czech Republic, Greece and Spain (Boomsma and Linnerud, 2015), and reducing feed-in-tariffs in Bulgaria, Germany, Greece, Italy and Switzerland in 2014 (REN21, 2015). These changes can be the result of a number of reasons. For example, a change of governments, new information about climate sensitivity, or fiscal pressure. In some cases, governments may not be fully committed to transitioning to renewable energy and may not provide sufficient support or incentives for the development of renewable energy projects. Furthermore, renewable energy projects may face opposition from local communities or environmental groups, which can slow down or prevent their development.

These policy changes lead to reduced profits for investors, hence the possibility of policy changes is a risk that investors need to consider. Evidence from the United Kingdom showed that changes in green energy policy have an immediate impact on investments, as new investments in green energy in UK declined by 56% from 2016 to 2017 as a result of government policy changes (The Guardian, 2018). Furthermore, the investment rate for PV and for onshore wind in the European Union decreased due to a retroactive subsidy change by approximately 45% and 16%, respectively (Sendstad, Hagspiel *et al.*, 2022).

Policy risk is a significant concern for investors considering renewable energy investments (Jones, 2015; Egli, 2020). Policy risk has been identified as a barrier to investment in low-carbon development for developing countries in Africa and Asia (CEPA, 2014). Also in the developed world, investors consider policy risk to be a factor affecting investment. For example, Statkrat, a large European renewable energy producer located in Norway, emphasizes the impact of uncertainty in the future development of national framework conditions and public regulations on its investment decisions (Statkraft, 2021).

This raises the question of how energy policy should be implemented to reach the ambitious policy targets. In case of market imperfections that are temporary, such as a lack of a mature technology, the corresponding measure should be evaluated and adapted frequently, while market imperfections that are stable over time require measures that are predictable and designed for the long term (Stern, 2022; Saltvedt *et al.*, 2022). Predictable flexibility has been a principle in monetary policy in western countries for some time, providing a good and reliable environment for investors (Stern, 2022; Saltvedt *et al.*, 2022). Energy policy should have the same goal, but this is challenging due to energy policy often resulting from short-term compromises and coalitions (Saltvedt *et al.*, 2022). When a policy for, e.g., support for emission-reducing activities is presented and decided upon, the policy makers should be open about the conditions under which the policy will possibly be ended, and under which conditions allocations will take place (Jones, 2015; Saltvedt *et al.*, 2022). For example, investment requires clear and credible signals, and when policy is revised and flexible, it should be in a way that is predictable (Stern, 2022). If a policy maker is flexible in its policies, but investors are unable to understand and predict changes, then this creates policy risk, which will increase the risk premium investors seek for projects that are subject to these policies (Saltvedt *et al.*, 2022). Implementing a 10-year policy is more effective in reducing investors' risk than a short-term (5 years) policy (Yang *et al.*, 2008).

In the next section, I move from the broader energy market to discuss and zoom in on the electricity as a commodity identifying its specific features.

2.2 Electricity

The discussion on electricity in this section is strongly based on Chapters 1 and 5 by Wangensteen (2012).

Electricity has several features that make it a unique commodity, and a market for it cannot be compared to other goods – even other energy sources such as oil or gas. Getting electricity from the producer to the consumer is not a trivial task and goes in several steps. The main steps are generation, transmission, distribution, retail and consumption, as shown in Figure 2.4. The generators produce electricity and form the supply side of the electricity market, while the retailers are the demand side[2]. Transmission carries electricity from one area to another, while distribution delivers the electricity to consumers.

Electricity is a complex good with complexities on both the supply and demand side. On the supply side, there are several complexities to consider. First and foremost, electricity cannot be stored in significant quantities in an economically efficient manner, unlike oil and gas, for example. The storage of electricity on an industrial scale is a fairly recent phenomenon (Nadajarah and Secomandi, 2022), and mainly results from the growth of power generation from renewable sources, which can be varying and somewhat unpredictable. Second, electricity is a good that is immediately and continuously generated and consumed. Third, electricity is a homogeneous good that cannot be traced, meaning a consumed kWh cannot be linked back to an individual producer, putting special requirements on the metering and billing for electricity. Finally, the grid to move electricity around needs to be balanced at all times to avoid breakdowns. These breakdowns can lead

[2]One can also consider the market for household electricity, meaning households are the demand side, looking for electricity contracts, which are offered by the retailers. I will use retail and consumption interchangeably, as examining the household market lies beyond the scope of this thesis.

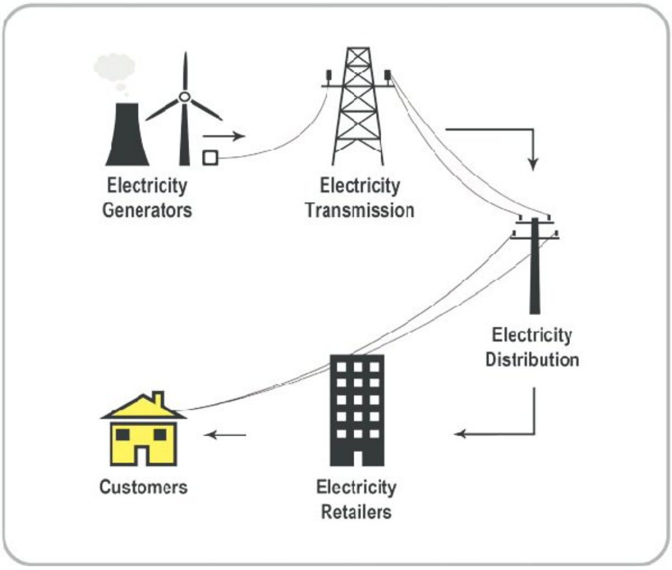

Figure 2.4: The electricity supply chain as shown in Stevenson *et al.* (2016).

to widespread power outages, affecting many people. Italy experienced a blackout in September 2003 that affected 56 million people, caused by an overloaded transmission line. Turkey had a similar incident resulting in a power outage for 70 million people in 2015. More recently, Java, Indonesia's most populous island, had a blackout that affected 120 million people in 2019 due to a disturbance in transmission. The financial impact of such incidents are tremendous.

On the demand side, the complexity arises from both consumption variability and the fact that electricity is essential to the community. Electricity consumption varies both on a day/night-cycle and within a week, but has seasonal patter during a year. Perhaps the most complicating factor in electricity demand, is that electricity is an absolute necessity to everyone. From businesses to individuals, there is little to no acceptance that electricity is unavailable to anyone. Businesses need electricity to function, while many individuals need electricity to access cooking and hot water – especially with gas being less and less frequently used in western households.

Before about 1990, the supply side of the electricity market was heavily regulated and organized according to one of two main styles. The first style is a market consisting of privately owned utilities with public regulation, as was typical for the USA. Investor Owned Utilities (IOUs) dominate the electricity supply. The supply was regulated by public regulatory commissions. The regulatory commissions had an important role in setting tariffs. The second style is a market in which the utilities were publicly owned, as was typical for Europe. In, for example, England, France and Italy, a centralized, state owned utility managed the market, while in Scandinavia, decentralized utilities controlled the market. In some countries, such as Spain and Germany, a combination of the two styles existed, where utilities were mainly privately owned without any direct public regulation, but with some public control in place.

Just before and in the 1990s, the electricity markets in England and Wales (1989), Denmark, Finland, Norway, Sweden (all 1990) were privatized. Later, Spain (1998), and the Netherlands (1999) followed by creating a fully competitive electricity market. These countries privatized their electricity markets partly due to early national initiatives, but also partly due to the initiative of the European Commission, which aimed for a fully open electricity market by 2007. It is important to note the directive for the electricity market in 2003 (2003/54/EC), which emphasizes that each member state has the responsibility of ensuring a high level of security of supply. This concerns both the network security and maintaining sufficient generating capacity.

The power system consists of several layers: generation, distribution, transmission and retail/consumption. In restructuring the electricity market, not every layer was privatized. Distribution can be considered a natural monopoly, as competition in distribution would lead to parallel grids, raising infrastructure and environmental challenges, and a higher cost per unit (Filippini, 1998; Salvanes and Tjøtta, 1998; Gunn and Sharp, 1999; Wangensteen, 2012). Transmission may benefit from competition, but it is most suitable to keep transmission as a regulated monopoly (Gans and King, 2000; Wangensteen, 2012). The characteristics of electricity coupled with poorly defined property rights create a natural monopoly in transmission (Hogan, 1995). Furthermore, for the purpose of operational coordination, it is most suitable to leave the responsibility for the transmission grid to a single company (Wangensteen, 2012:p.77). Electricity generation is not a monopoly, as it is assumed to have no significant cost of scale – meaning a monopoly on electricity generation would not lower the electricity price (Wangensteen, 2012:p.78).

There are several reasons for the privatization of the electricity generation. Privatization leads to avoiding excessive investment, and is more likely to develop profitable projects than expensive and unprofitable projects. Unlike a competitive utility, a public utility is able to develop more capacity than a consumer would be willing to pay for, and may also install the capacity in an expensive and suboptimal way. In a large country such as Norway, a competitive electricity market also results in reasonable geographical variation. Hydroelectric plants are then built where they are most valuable for the power intensive industry. Furthermore, private companies have a natural incentive to reduce costs, which is not necessary for a public company. In a competitive system the price is settled by the market. This results, in theory, in the consumer paying the lowest price. In Norway, when prices were set by a government or municipality, it resulted in a too high price for commercial costumers and too low for residential costumers, resulting in an unjustified implicit subsidy and tax for certain costumers.

However, also a private electricity market has its limitations. It may be that competition is limited (even in a privatized market) or that the producers have more or better information than the consumers, resulting in an electricity price that may be higher than under perfect competition. Another downside to a privatized market, is that electricity prices are extremely high in uncertain and volatile times such as a pandemic or war, making a lasting financial impact on the poorest in society. Households cannot reduce their electricity consumption to zero, as electricity is an absolute necessity, serving as a basis for many fundamental human needs.

In addressing the limitations of the private market, it is crucial to have sufficient generation capacity and encourage competition in segments where this is naturally not sufficiently the case. However, increasing competition is challenging as counter-trading is ineffective (Dijk and Willems, 2011). The potential benefits of additional competition, meaning more lower electricity prices for consumers and lower production costs for producers, are outweighed by the distortions, such as the additional investment cost for the entrant and the shifting of the costs of congestion to final consumers (Dijk and Willems, 2011).

The next section examines the challenges related to investment in both energy and electricity.

2.3 Investment valuation, flexibility and learning

Examining investment opportunities in energy and electricity is complex for several reasons. First, an electricity generation project, such as an offshore wind power park, has uncertain revenues. The uncertainty in the revenues roots from different sources. One source of risk is market uncertainty or demand uncertainty. The returns of an energy project are highly uncertain as they often depend on the spot price, i.e., the price retailers pay on the whole-sale market. The spot price depends on the demand, which fluctuates, leading to demand uncertainty. Another example of uncertainty is the interest rate. Interest rates affect the profitability of the energy investment as the returns of such projects are accumulated over a long time horizon.

Other factors that may affect the profitability of energy investment in unpredictable ways include policy and technological developments. Green energy is frequently supported by feed in tariffs, net metering, renewable quota obligations and (energy) certificates, or subsidies, investment grants and tax benefits (Nadajarah and Secomandi, 2022). These support schemes are temporary, and any green technology should at some point be competitively viable to survive with limited or no policy support. Consequently, these support schemes come with uncertainty about when they are stopped or revised, which is referred to as 'policy risk'. It is important to distinguish between policy uncertainty and policy risk, despite some literature uses them interchangeably. I use policy risk to refer to the possibility that a policy decision or action taken by a government or other organization will have negative consequences, where both the possible actions and the likelihood of each action are known[3]. Policy risk to investors includes actions like regulatory changes that create additional costs or barriers to entry for businesses, or policy decisions that disrupt established economic or social systems. Policy uncertainty, on the other hand, refers to the lack of clarity or predictability surrounding the policy decisions. In other words, both the possible policy maker's actions and the corresponding likelihood of such actions are unknown to the investor. The World Bank, World Economic Forum (WEF) and Organisation for Economic Co-operation and Development (OECD) all have indicated that policy risk induced by governments is a significant deterrent to worldwide investment, particularly for infrastructure (World Bank, 2004; WEF, 2021; OECD, 2015; Baker *et al.*, 2016; World Bank, 2020; World Bank, 2021). Investors require a higher risk premium for such projects to be willing to invest in such projects with uncertain profitability.

Second, investment in energy is difficult to examine as the investment in such projects is irreversible, with little to no options to be put to alternative use after being build. For example, once a wind park is built, it is not possible to reverse this decision and retrieve the full amount invested. Comparing this investment to investment in real estate, the latter can relatively easily be put to alternative use, such as offices being remodelled into apartments.

A third challenge with examining and modelling investment in renewables is to account for an investor's flexibility in its decision. An investor's decision to install a new renewable energy project comes with the flexibility to choose *when* it will start to build. An investor is hardly ever faced with a now-or-never decision, and when examining the value of an option to invest, one needs to include the value of waiting with the decision till a later date. The decision when to build is referred to investment timing or just 'timing' in the literature. A firm considers the trade-off between waiting for better market conditions versus foregone profits by waiting when it decides when to invest. An investor is also able to *learn* about the profitability of the investment as information on future policy and technology becomes available to the investor. Therefore, the value of flexibility in choosing its own timing is not limited to the value of waiting for better market conditions to arrive, but also being able to improve its decisions by actively learning about the future.

In the next chapter, I discuss the methodology used in this thesis that accounts for these challenges related to evaluating investment in energy.

[3]This definition of policy risk is consistent with, e.g., Kang and Létourneau (2016), who use a real options approach to analyze the impact of policy risk on the investment decisions of firms in the emission permit market.

Chapter 3

Methodology and literature

This chapter summarizes the main methods used in the articles included in this thesis. In Section 3.1, I discuss approaches to examine investments, mainly focusing on the real options approach. The applicability of the real options approach is broad, but for the application in this thesis, I focus on real investment related to electricity and energy. I zoom in on policy design and modelling policy risk related to investments in Section 3.2. Finally, the approach to model active learning when holding an option to invest is discussed in Section 3.3.

3.1 Real options approach

Traditionally, real investment decisions have been studied using the net present value (NPV) criterion. This decision rule implies that it is optimal for a firm to invest in a project right now if its expected future revenue flows discounted to today exceed the investment cost. This decision rule is only applicable in very niche cases, and, generally, three aspects of real investment cause this decision rule to be sub-optimal. First, the revenue and costs of a project in the future are uncertain, which requires an investor a so-called risk premium before they are willing to commit to an investment. Second, investment is partially or fully irreversible. A firm needs to account that once it has invested a large share of its investment is a sunk cost. Third, an investor has the flexibility when to invest, meaning investment is not a now-or-never decision as the NPV implicitly assumes. Investment in energy has all three characteristics, as outlined in Section 2.3. Therefore, we use real options analysis instead of the NPV criterion to evaluate real investment in energy.

Real options derives its inspiration from financial option theory, in which one derives the value of a financial option, such as a put or a call. A real option gives the holder the right, but not the obligation, to take action – such as starting a project, expanding, abandoning et cetera – at a predetermined cost. Real options analysis is a tool used to evaluate the flexibility and value of strategic investment decisions. It allows companies to make better-informed decisions about whether to pursue a particular course of action by considering the potential future outcomes and the costs and benefits associated with each option. The term ‘real option’ has been coined by Myers (1977), and *the value of waiting* to describe the value of the investor’s flexibility to decide when to invest was already introduced by McDonald and Siegel (1985). Dixit and Pindyck (1994) and Trigeorgis (1996) are two pioneering books on the topic of real options. The key advantage of real options analysis is that it recognizes that investment decisions are often made under conditions of uncertainty, and that the value of an option to wait or to take action depends on the evolution of the underlying uncertainties. As such, real options analysis provides a framework for incorporating the value of flexibility into investment decision making.

As investment in energy is subject to uncertainty, investment opportunities in energy and electricity are often examined using real options analysis. Fernandes *et al.* (2011) discuss how the real options approach can be applied to a variety of energy sector investments, including renewable energy projects, oil and gas exploration, and electricity generation and transmission, and provides a literature review. Starting in the early 2000s, real option theory has been applied to study renewables from the investor’s viewpoint (Davis and Owens, 2003; Siddiqui, Marnay *et al.*, 2007; Kumbaroğlu *et al.*, 2008; Siddiqui and Fleten, 2010; Lee and Shih, 2010), with some applications to specific resources such as wind (Venetsanos *et al.*, 2002; W. Yu *et al.*, 2006; Muñoz *et al.*, 2009; Detemple and Kitapbayev, 2020a; Detemple and Kitapbayev, 2020b) and hydropower (Kjaerland, 2007; Bøckman *et al.*, 2008; Martínez-Ceseña and Mutale, 2011). This literature derived the optimal timing of investment in, switching to, or abandonment of a renewable energy project or technology, and derived the value of the option.

Apart from timing, the size of the investment is also an important factor to consider when examining investment in renewable energy, as it can impact the potential returns on the investment. Larger investments may be associated with higher potential returns, but they may also carry higher risks, such as increased exposure to market fluctuations.

An investment size decision can be categorized in either of two groups: lumpy investment with capacity choice or incremental investment. Lumpy investment with capacity choice is a one-time decision in which the investor has to decide on the investment size. After the firm has invested, it has no option to change the capacity size – not upward nor downward. This approach is suitable for investment at micro or firm level, in which a firm decides on whether to start-up a single project (see, e.g., Dangl (1999) and Huisman and Kort (2015)). Both Dangl (1999) and Huisman and Kort (2015) find that a firm invests more when it invests later. The firm invests more when there is more demand uncertainty, as the firm delays investment and invests when demand is higher. Paper I takes this approach to model investment size, and extends this work by accounting for policy risk.

The alternative approach to model the investment size decision is to assume incremental investment, which is the approach taken in Paper II in this thesis. In incremental investment, the firm has to decide when to install small, marginal capacity extensions. This is occasionally also referred to as stepwise investment (Chronopoulos *et al.*, 2016; Sendstad and Chronopoulos, 2020). Thus, the firm has the option to change the capacity size upward, but not downward. This approach is suitable for investment at macro or industry level, when multiple firms investment in small lumpy projects or when a single firm can start multiple projects (see, e.g., Dixit and Pindyck (1994):Chapter 11, Guthrie (2020) and Zwart (2021)). It is important to note that incremental investment allows for *repeated* capacity expansions. In some cases, the flexibility of the ability to lump investments together in order to take advantage of increasing returns to investment is crucial to the firm (Guthrie, 2020). Real options models should allow for multiple rounds of investment in such cases (Guthrie, 2020).

It is important to also distinguish between lumpy and incremental investment, as both have different implications for optimal investment timing and size. The investment size decreases with a lower market risk in case of lumpy investment (Paper I), while the opposite is true for incremental investment (Paper II). This difference results from the investor's additional flexibility to scale capacity upward later in case of incremental investment, while the investor is fully committed to its investment size when investing in case of lumpy investment.

A recent trend in the real options literature is to also examine policy and welfare. Not much has been done in this topic until Broer and Zwart (2013) and Huisman and Kort (2015) introduced the concept of welfare and total surplus in real options, but now it is growing (see, e.g., Wen *et al.* (2017); Willems and Zwart (2018); Detemple and Kitapbayev (2020a); Azevedo *et al.* (2021); Paxson *et al.* (2022); Lavrutich *et al.* (2023)) due to its practical relevance. This literature shows the limitation of unregulated markets that investment does not naturally occur in a social optimal manner. Suggestions to improve welfare generally involve actions from a policy maker, varying from implementing a subsidy or tax, or using regulation. Their effectiveness may depend on a number of factors, such as, for example, the demand function (Paxson *et al.*, 2022).

In the next section, I explain how real options models are extended to account for policy and policy risk in renewable energy. Furthermore, I also discuss how policy can contribute to improving welfare or reaching policy targets.

3.2 Policy design and policy risk

In light of market imperfections, there is a substantial amount of papers examining incentive regulation of a firm within an uncertain dynamic framework (see, e.g., Brennan and Schwartz (1982); Dobbs (2004); Evans and Guthrie (2005); Evans and Guthrie (2012); Guthrie (2006); Guthrie (2020)). Some related literature incorporates real options analysis into the study of the social planner's or government's decisions (see, e.g., Pennings (2000); Willems and Zwart (2018); Azevedo *et al.* (2021)). One key advantage of real options analysis in policy design is the inclusion of the impact of dynamic considerations. Furthermore, incentive regulation is typically designed to encourage the firm to behave in a way that is beneficial over the long term. The long-term aspect may require the regulator to take into account changes over time, such as how the firm's actions may impact the future evolution of the market. Real option analysis is used to account for the long-term aspect and it also accounts for timing, which plays an important role for both investor and policy maker in their decisions and (discounted) payoff. Paper I and II evaluate a policy and use real options analysis to account for uncertainty. Paper II also contrasts the short-term effects of a policy with its effects on the mid- and long-term.

Over time, the interest in the role of policy on renewable energy investment has grown and the number of papers studying it using a real options approach (see, e.g., Laurikka and Koljonen (2006); W. Yu *et al.* (2006); Fuss *et al.* (2008); Kumbaroğlu *et al.* (2008); Siddiqui and Fleten (2010); Lee and Shih (2010); Boomsma, Meade *et al.* (2012); Barroso and Balibrea-Iniesta (2014); Linnerud *et al.* (2014); Pringles *et al.* (2014); Balibrea-Iniesta *et al.* (2015); Boomsma and Linnerud (2015); Adkins and Paxson (2016); Chronopoulos *et al.* (2016); Gatzert and Vogl (2016); H. Yu *et al.* (2016); Kozlova (2017); Kozlova, Collan *et al.* (2017); Bigerna *et al.* (2019); Kozlova, Fleten *et al.* (2019); Wesseler and Zhao (2019); Azevedo *et al.* (2021)) has increased. This literature considers the effect of different subsidy schemes on investment timing and the value of an option to invest in or switch to a renewable energy project.

Part of the real options literature on renewable energy policy also considers the investor's investment size and the policy maker's interest in reaching the renewable energy capacity targets (see, e.g., Lee and Shih (2010); Boomsma, Meade *et al.* (2012); Boomsma and Linnerud (2015); Chronopoulos *et al.* (2016); Hustveit *et al.* (2017); Bigerna *et al.* (2019); Kozlova, Fleten *et al.* (2019); Azevedo *et al.* (2021); Tsiodra and Chronopoulos (2021)). Lee and Shih (2010) examine the investment capacity in wind power in Taiwan under a (fixed) feed-in tariff (FIT). Interestingly, they find that a FIT for developing wind power technology can negatively affect the cost savings among consumers if the FIT policy is set too high. Boomsma, Meade *et al.* (2012) compare FITs and renewable energy certificate trading in the Nordic region and find that the FIT results in earlier investment while renewable energy certificate trading results in larger investment. Boomsma and Linnerud (2015) compare a feed-in premium (FIP), FIT and tradeable green certificates and find the differences in market risk between the support schemes is less than commonly believed. Chronopoulos *et al.* (2016) consider a FIT with an investor that can do either lumpy or stepwise investment, and conclude the value of the project in case of stepwise investment is larger than under lumpy investment. Hustveit *et al.* (2017) analyze the Swedish-Norwegian tradeable green certificate market and study different capacity targets. They find, among others, that the certificate prices are highly sensitive to changes in electricity consumption and generation. Bigerna *et al.* (2019) consider an Italian investor who decides on the investment timing and size of wind power plant, of which the output is eligible for a feed-in premium (FIP). They conclude an environmental target can only be reached if the subsidy is not too high nor too low, as the former results in a too low investment size and the latter in too late investment time. Kozlova, Fleten *et al.* (2019) compare a capacity mechanism (i.e., providing a certain return on investment), a FIT and FIP scheme for Russian renewable energy investment. They find that the Russian policy is effective in transferring market risks away from the investor and has potential for a unique combination of effectiveness and cost efficiency. Azevedo *et al.* (2021) consider a combination of a subsidy and a tax – where the tax is used to finance the subsidy, i.e. a *zero-cost package* – and find that this can increase welfare. Furthermore, a fixed (variable) subsidy induces earlier (larger) investments. Tsiodra and Chronopoulos (2021) provide a bi-level model model in which both a profit-maximizing investor and a government with a capacity target interact. They find that a firm with a greater risk aversion lowers the amount of installed capacity yet postpones investment. Furthermore, although the optimal subsidy to reach the capacity target increases with uncertainty under risk neutrality, the opposite is true under high levels of risk aversion.

Papers I and II contribute to this literature on renewable energy policy with investment timing and size by examining a lump-sum investment subsidy, which is a popular policy tool to promote renewable energy[1]. Furthermore, to my knowledge, Papers I and II are the first to examine the effect of a lump-sum subsidy subject to policy risk on welfare or total surplus.

One important aspect of policy design is the role of policy with the risk and uncertainty about its termination or changes (Fuss *et al.*, 2008). When a policy is active, there is also an inherent uncertainty to when it is changed or retracted, which is referred to as policy risk. As outlined in Chapter 2, policy risk induced by governments can be a significant barrier for investment (World Bank, 2004; WEF, 2021; OECD, 2015; Baker *et al.*, 2016; World Bank, 2020; World Bank, 2021).

Among the first studies to study policy risk in a real options setting are Hassett and Metcalf (1999) and Dixit and Pindyck (1994):Chapter 9, who applied real options theory to study optimal investment decisions in the presence of policy risk. In both papers, the policy risk that is studied is uncertainty about the lifetime of an available investment tax credit. In Dixit and Pindyck (1994):Chapter 9, the effect of implementing and withdrawing a tax credit policy on investment timing is analyzed. It is assumed that the investor can only invest once (lumpy investment) and invests in a fixed capacity. In both Hassett and Metcalf (1999) and Dixit and Pindyck (1994):Chapter 9, the subsidy can be withdrawn and reenacted multiple times. This risk of subsidy implementation/termination is modelled as a Poisson jump process. Both papers find that investors delay (speed up) investment when a subsidy is more likely to be implemented (withdrawn) in the near future. Papers I and II examine the same investment subsidy as in Hassett and Metcalf (1999) and Dixit and Pindyck (1994):Chapter 9, and extend their analysis by studying

[1] A lump-sum investment subsidy is a general class of investment subsidies including investment tax credits and capital subsidies. Investment tax credits are often implemented with the aim to increase the affordability and profitability of renewable energy production (IRENA, IEA and REN21, 2018:page 70). Worldwide, an estimated amount of 30 to 40 countries used investment or production tax credits to support renewable energy installations over the past decade (IRENA, IEA and REN21, 2018:page 69).

capacity choice, assumed to be either lumpy (Paper I) or incremental (Paper II).

Pawlina and Kort (2005) extend the framework of both Hassett and Metcalf (1999) and Dixit and Pindyck (1994):Chapter 9 in the way policy change is modelled. Pawlina and Kort (2005) assume a tax implementation is the consequence of a booming economy and, thus, is observable. Hassett and Metcalf (1999) and Dixit and Pindyck (1994):Chapter 9 both assume a policy change is the consequence of a random event, such as a change in government. Pawlina and Kort (2005) study the role of uncertain tax implementation on a firm's investment decision. A tax is implemented when the economic environment, which determines the firm's project value, reaches a certain trigger. They find that the uncertainty about the value of this trigger has a non-monotonic effect on the firm's investment threshold. The investor's investment threshold decreases with a larger policy risk for low levels, and increases with policy risk for high levels.

Policy risk related to policy schemes on the output prices, such as FITs, FIPs or certificate prices, may interact with market uncertainty and can affect the level or the volatility of the prices (Boomsma, Meade *et al.*, 2012). In the Swedish-Norwegian green certificate market, policy interventions exacerbate price risks and increase price volatility, which disrupts the investment climate in certificate markets and affects the effectiveness of policy (Ganhammar, 2021). Some literature on carbon and emission pricing examines investment and the link between regulatory or policy risk and prices (see, e.g., Laurikka and Koljonen (2006); Blyth *et al.* (2007); Fuss *et al.* (2008); Yang *et al.* (2008); Gatzert and Vogl (2016); Kang and Létourneau (2016); Hustveit *et al.* (2017)). The increase in price volatility increases the risk premium required to trigger power generation investment (Blyth *et al.*, 2007; Fuss *et al.*, 2008; Yang *et al.*, 2008; Gatzert and Vogl, 2016), and unclear signals of a policy maker impact the investor more than market uncertainty does (Fuss *et al.*, 2008). The message of this literature is twofold. First, energy producers need to adapt and account for uncertainty involved in the process of policy-making (Laurikka and Koljonen, 2006; Fuss *et al.*, 2008; Kang and Létourneau, 2016). Second, policymakers need to learn how their policy signals affect investment behavior to understand the viability and effectiveness of their policies, and should aim to provide long-term regulatory certainty to minimize investment risk (Blyth *et al.*, 2007; Fuss *et al.*, 2008; Yang *et al.*, 2008; Kang and Létourneau, 2016; Hustveit *et al.*, 2017). Transparency and credibility in policy-making can help to reduce policy risk and promote greater investment in emission reduction technologies (Kang and Létourneau, 2016).

In recent years, the real options literature on energy has also examined the policy risk of a regime change, such as random provision, revision or retraction of a subsidy (see, e.g., Boomsma and Linnerud (2015); Adkins and Paxson (2016); Chronopoulos *et al.* (2016); Eryilmaz and Homans (2016); Ritzenhofen and Spinler (2016); Detemple and Kitapbayev (2020a); Sendstad and Chronopoulos (2020)). The effect of this type of policy risk on investment timing and size depends on the type of subsidy and how policy changes are applied. Policy termination risk delays investment if the termination of a subsidy is applied retroactively, but it speeds up investment otherwise (Boomsma and Linnerud, 2015; Sendstad and Chronopoulos, 2020; Sendstad, Hagspiel *et al.*, 2022). Comparing different types of subsidies, policy risk regarding a retractable lump-sum investment subsidy leads to early investment (Hassett and Metcalf, 1999; Dixit and Pindyck, 1994; Adkins and Paxson, 2016). Similarly, Eryilmaz and Homans (2016) find that wind energy in the US is more likely to occur today if the advantageous production tax credit has a higher probability of disappearing in the future. In case of a FIT for wind, increased likelihood of withdrawal reduces payoffs of wind projects and postpones investment in them (Sendstad and Chronopoulos, 2020; Detemple and Kitapbayev, 2020a), while it accelerates investment in alternatives such as gas (Detemple and Kitapbayev, 2020a). Furthermore, policy risk regarding a FIT can prevent investors to undergo large investments and slows down (total) investment (Ritzenhofen and Spinler, 2016; Chronopoulos *et al.*, 2016). This may stagnate investment in renewable energy, consequently leading to a failure in meeting the ambitious targets to cut emissions significantly (Bigerna *et al.*, 2019; Kozlova, Fleten *et al.*, 2019; Sendstad, Hagspiel *et al.*, 2022).

This raises the question how a policy maker can avoid investments being slowed down by policy risk. When a policy is introduced, the policy maker should be transparent about the conditions under which the policy will be terminated. There is ample literature showing that this commitment creates value to investors and is important in reaching policy goals such as a capacity target (Blyth *et al.*, 2007; Fuss *et al.*, 2008; Yang *et al.*, 2008; Kang and Létourneau, 2016; Hustveit *et al.*, 2017; Dalby *et al.*, 2018; Finjord *et al.*, 2018; Sendstad and Chronopoulos, 2020; Sendstad, Hagspiel *et al.*, 2022; Stern, 2022).

Papers I and II contribute to this literature on energy policy by examining a lump-sum investment subsidy under policy withdrawal risk. Specifically, it accounts for both investment timing and size and examines the policy maker's goals such as welfare and a capacity target as well. To the best of my knowledge, the combination of examining welfare under policy risk has not been studied before. Furthermore, the possibility of using a lump-sum subsidy to reach a capacity target while accounting for subsidy withdrawal risk is unexplored.

In the next section, I discuss how an investor can use Bayesian learning to improve decision making under uncertainty.

3.3 Bayesian learning

Bayesian learning is a type of statistical learning that is based on the Bayesian approach to probability. In Bayesian learning, probabilities are used to represent uncertain knowledge or beliefs about the likelihood of an event or hypothesis. The Bayesian approach is based on the principle that probabilities can be updated as new evidence or data becomes available, allowing for a more flexible and adaptive approach to statistical modelling and decision making. There is a wide variety of applications for Bayesian learning, one of which is decision making[2]. Bayesian learning can be used to examine investment under uncertainty by taking into account the probabilities of different outcomes and their associated costs or benefits.

An investor holding the option to invest in a project in energy is faced with several factors that are uncertain, such as changes in policy or the arrival of technological progress. In order to assess these uncertainties over time, an investor needs to update its beliefs on the profitability of an occurrence of such event. Some of the literature on investment under uncertainty accounts for both uncertainty and active learning by combining a real options approach and Bayesian learning (see, e.g., Bergemann and Hege (1998); Herath and Park (2001); Armstrong *et al.* (2004); Thijssen, Huisman *et al.* (2004); Décamps *et al.* (2005); Grenadier and Malenko (2010); Kwon and Lippman (2011); Pertile *et al.* (2014); Harrison and Sunar (2015); Kwon, Xu *et al.* (2016); Thijssen and Bregantini (2017); Dalby *et al.* (2018); Sund *et al.* (2022)). Following Sund *et al.* (2022), I refer to active learning as a conscious activity of the firm to acquire new information. This is in contrast to passive learning, in which the learning is a passive consequence of waiting to exercise the option (Sund *et al.*, 2022). Traditional real option models such as Dixit and Pindyck (1994) account for passive learning and its value to decision making. Active learning also holds value to an investor as it improves the decision making process, despite the value of an option decreases with learning as the uncertainty decreases (Martzoukos and Trigeorgis, 2001; Bellalah, 2001; Harrison and Sunar, 2015).

The literature on investment under uncertainty with Bayesian learning applied to energy is rather scarce (see, e.g. Armstrong *et al.* (2004); Dalby *et al.* (2018). Armstrong *et al.* (2004) examine technical uncertainty in the valuation of oil projects, using real options analysis and Bayesian learning. The option to perform the production enhancement procedure was strongly in the money, but for the oil well in their study, the option to gather more information was not as advantageous when the price of oil increases. Dalby *et al.* (2018) consider a firm that can invest in a renewable energy producing project, which is subject to an expected adjustment of a fixed FIT it is currently backed by. The firm may learn about the arrival rate of the adjustment from a continuous stream of signals. Dalby *et al.* (2018) find that the investors are less likely to invest when the arrival rate of a policy change increases. Investors also prefer a lower FIT with a long expected lifespan. Paper III contributes to this literature by examining how learning affects the optimal investment behavior of a firm that has the option to invest in a project facing a disruptive[3] event over time. This is applicable to, for example, the situation in which a firm holds the option to install a renewable energy project eligible for subsidy, but that a policy maker considers to change or terminate the

[2]Other applications of Bayesian learning include prediction and model selection. Bayesian learning can be used to make predictions about future events based on past data and current beliefs. In Bayesian model selection, different models are compared based on their posterior probability given the data, allowing for the selection of the most appropriate model for a particular problem.

[3]I define disruptive risk as the risk that a sudden and unexpected event occurs that causes the project to lose value. This definition is in line with Kwon (2014).

subsidy.

Active learning has value to investors as it improves decision making, but there are also costs involved in doing active learning. First, there is an implicit cost, as learning takes time. This results in potentially delaying decisions, such as initiating a project. Second, there is a cost to hire employees or researchers who do the active learning. These are referred to as 'information costs' (Merton *et al.*, 1987; Bellalah, 2001). Information costs consist of two parts: (1) the cost of gathering and processing data, and (2) the cost of transmitting information from one party to another (Merton *et al.*, 1987). Costly Bayesian learning is a type of Bayesian learning in which the acquisition of new information or data has a cost associated with it. The decision of whether or not to learn must take into account the trade-off between the potential benefits of the learning and the costs associated with acquiring it.

With the trade-off between costs and benefits in mind, two questions are important in the decision to invest in learning. First, when does the option to learn have value, i.e. when do the benefits of learning exceed the costs associated with it? Second, if it is optimal to learn, what is the optimal learning rate[4]? To the best of my knowledge, there are only a few papers (Bellalah, 2001; Moscarini and L. Smith, 2001; Pertile *et al.*, 2014; Harrison and Sunar, 2015; Thijssen and Bregantini, 2017) that consider Bayesian learning in a real options framework under the assumption that learning has an explicit (monetary) cost[5]. Bellalah (2001) develops a continuous-time model of irreversible investment in the presence of information costs and considers learning costs as an additional premium on top of the risk-free rate. They emphasize the importance of considering information costs when examining the value of an option. Moscarini and L. Smith (2001) examine a firm with the option to invest and the possibility to learn via sequential experimentation against a learning cost. They find that the chosen learning intensity grows with a project's expected payoff. Both Pertile *et al.* (2014) and Thijssen and Bregantini (2017) propose a model for optimal investment and abandonment of a project, where information is provided via sequential experimentation. In both papers, the learning costs are assumed constant. Thijssen and Bregantini (2017) conclude that more noise in the signals lowers the value of the project, and find that its effect on the expected time at which a decision is taken is ambiguous. Harrison and Sunar (2015) also consider a firm that has to decide on when to stop learning and either invest or abandon the project. The learning occurs dynamically, via selecting one of several costly learning nodes. Similar to Moscarini and

[4]Kwon, Xu *et al.* (2016) define the 'rate of learning' as a term that reflects the magnitude of the difference between the prior and posterior probability distributions that change due to Bayesian updating.

[5]There exists literature in which learning costs are modelled as an implicit cost (see, e.g. Keller and Rady (1999)). If learning costs are only implicit, then it is likely the firm will always learn at least for a short period of time as starting to learn requires no significant initial investment. Paper III finds that learning can be unattractive if learning requires an upfront (monetary) investment. For example, the firm foregoes learning if the project has a large annual revenue, even if the firm believes the project is likely to be short-lived.

L. Smith (2001), the firm can adjust the rate of learning over time.

Paper III proposes a framework to examine Bayesian learning in a project subject to termination risk, where the firm can choose the learning rate at a cost. It assumes that the learning cost depends on an a priori chosen learning rate. Unlike the learning rate in Moscarini and L. Smith (2001) and Harrison and Sunar (2015), the firm commits to this learning rate and cannot change it over time. Paper III contributes to the aforementioned literature by deriving the firm's optimal rate of learning when it has the option to choose the learning rate against a cost.

Chapter 4

Contributions

This chapter provides a summary of the overall contribution of this thesis. The thesis consists of three papers, which are enclosed as appendices. The following three sections each summarize a paper. In each section, I also discuss the paper's contribution to investment under uncertainty in energy and energy policy, from the point of view of both academic research and practice. The goal of this thesis as a whole is to understand how policy risk and learning affects investment behavior. In the application, I mostly focus on renewable energy, but the modelling itself is not limited to this application. For example, the models proposed in the papers could also be applied (with only marginal adjustments) to infrastructure or agriculture investments.

I am the main author of all three papers included in this thesis. In all papers included in this thesis, I have contributed to the following: conceptualization, methodology, software (mainly MATLAB®), validation, formal analysis, writing of the original draft, review and editing of the manuscript, and visualization. At the end of the discussion of each paper, I briefly discuss the contribution by my co-authors.

Paper I – Green capacity investment under subsidy withdrawal risk

Authors: Roel L. G. Nagy, Verena Hagspiel and Peter M. Kort
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This paper studies the effect of a lump-sum subsidy subject to the risk of retraction on optimal investment decisions in terms of timing and capacity size installed. It is motivated by both the investor's perspective, who holds an option to invest in a renewable energy project with uncertain profits, and the social planner, who is given a target. The setting we examine in our paper is graphically summarized in Figure 4.1.

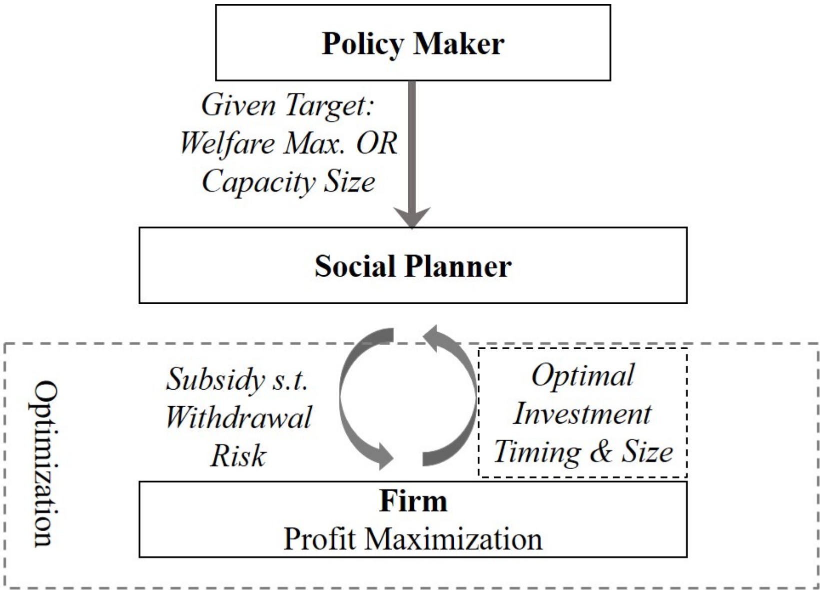

Figure 4.1: Overview of the setting examined in Paper I.

Starting at the top, we study two types of policy targets for the social planner. The social planner's first target is to maximize welfare, in which welfare is the sum of consumer surplus and producer surplus. The second goal is a capacity target, in which the social planner wants the investor to install a capacity of a certain size as soon as possible. In our paper, we examine the interaction between the social planner and the firm, as indicated in the larger dashed block in Figure 4.1. The investor decides both on when to invest in the project and the capacity of the installed project, as indicated in the smaller dashed block in the figure. It faces the trade-off between the value of waiting, inherently present when holding an option, and the risk of subsidy termination.

We examine investment under uncertainty and policy intervention starting with the same modelling framework as Dixit and Pindyck (1994):Chapter 9, who use a real options approach. In both Paper I and Dixit and Pindyck (1994):Chapter 9, there is uncertainty regarding the lifetime of an investment tax credit. This uncertainty is referred to as 'subsidy withdrawal risk', and sometimes abbreviated to 'subsidy risk' or 'withdrawal risk'. This subsidy risk is assumed to follow a Poison jump process. Dixit and Pindyck (1994):Chapter 9 specifically examine investment timing and assumes the size to be fixed. We extend this model to account for an investor's discretion over investment size. We follow Huisman and Kort (2015) in the approach how to incorporate capacity choice. At the time of investment, the firm can also decide on the investment size.

This paper has implications for both the investor and the social planner. We find that the investor will invest sooner when the likelihood of subsidy withdrawal is higher, and also when the subsidy size is larger. As the firm invests sooner, it will also invest in a smaller size. The implications for the social planner depend on whether it aims to reach a capacity target as soon as possible or aims to maximize welfare, i.e. total surplus. In the former case, we find a lump-sum subsidy can speed up investment if the social planner's capacity target is *lower* that the firm's investment size. If the social planner's capacity target is larger than the firm's investment size, then the social planner can only achieve the target by implementing a subsidy that is provided conditional on the firm investing in this size. In the case that the social planner aims to maximize welfare, whether a subsidy is effective depends on the subsidy retraction risk. A subsidy increases welfare compared to a no subsidy baseline if the subsidy retraction risk is low. The social planner is better off not implementing a subsidy in the case of a large subsidy retraction risk, as the firm would invest in a capacity that is too small otherwise.

This paper contributes to the literature by examining, among other things, how the risk of policy change intervenes with the effect of policy measures. To our knowledge, we are the first to conclude that a larger likelihood of an investment subsidy withdrawal damages both welfare and the policy maker's ability to increase renewable energy capacity.

This paper is co-authored by Verena Hagspiel and Peter Kort. Both co-authors have contributed to the conceptualization, methodology, and reviewing and editing of the manuscript.

Paper II – Don't stop me now: Incremental capacity growth under subsidy termination risk

Authors: Roel L. G. Nagy, Stein-Erik Fleten and Lars H. Sendstad
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In this paper, we adapt the framework from Paper I to examine growth in the industry rather than having the single firm viewpoint as in Paper I. We study the industry's capacity in both the short and long term. We also examine how the social planner's optimal policy depends on the time at which the social planner wants to reach a target. We assume the investor to be a monopolist that holds the option to do incremental investment, that is, it has the option to increase its capacity repeatedly with small increments over time. The monopolist's goal is to maximize profit by deciding on when to install the capacity increments. Initially, a lump-sum subsidy is available for each installment, but the subsidy is withdrawn at a random point in the future. The social planner maximizes welfare, i.e. total surplus, and decides on the size of the lump-sum subsidy.

As in Paper I, we examine investment under uncertainty and policy intervention using the modelling framework developed by Hassett and Metcalf (1999) and

Dixit and Pindyck (1994):Chapter 9. In this case, we assume that the firm's investment is incremental, following Dixit and Pindyck (1994):Chapter 11, rather than lumpy as in Paper I. This allows us to study multiple investments and consider capacity growth in the long-run, as opposed to the lumpy investment scenario, in which no capacity growth is considered after the initial investment.

The paper has implications for the effectiveness of subsidies in both the short and long term. In the short term, a subsidy can effectively increase the industry's capacity. The firm installs capacity expansions sooner and, consequently, installs a larger capacity than it without a subsidy. The total investment during the subsidy's lifetime increases with both the subsidy size and the likelihood of subsidy withdrawal. However, this increase in total investment during the subsidy's lifetime comes at the cost of reduced effectiveness after subsidy termination. Investment directly after the subsidy is retracted is lower than in the baseline scenario without a subsidy.

This has implications for a policy maker. The optimal social subsidy size strongly depends on the deadline by which a social planner aims reach its target. In other words, the social optimal subsidy size to maximize welfare in 10 years is significantly different than when maximizing welfare in 30 years. The optimal size of a welfare-maximizing subsidy is larger the further into the future a policy maker aims to maximize welfare, because it takes a long time for the benefits of a capacity increment to outweigh the initial cost of investment. The investment cost of a capacity increment is high and fully paid upfront, while the benefits in terms of the producer and consumer surplus are slowly accumulated over time. Additionally, the optimal size of a welfare-maximizing subsidy depends on the policy maker's discretion to adjust the subsidy size.

This paper makes three contributions to the literature. First, we examine the effect of a subsidy and subsidy termination risk on social welfare, focusing on how a subsidy affects incremental investment compared to the literature on lumpy investment. We find that a firm's capacity is larger with a subsidy than without in the case of incremental investment, but the reverse is true for lumpy investment. This has significant implications for total welfare. If a firm engages in incremental investment, a subsidy can increase welfare in the long run, whereas in the case of lumpy investment, a subsidy can only increase welfare if the risk of subsidy withdrawal is low or zero. Our second contribution is an examination of the long-term effects of a subsidy and what happens after subsidy withdrawal. A subsidy that covers investment costs tends to accelerate investment while it is in place, but this effect diminishes over time after the subsidy is terminated. Finally, our third contribution is the demonstration that a policy maker's time horizon is crucial in determining the welfare-optimal policy.

This paper is co-authored by Stein-Erik Fleten and Lars Sendstad. Both co-authors have contributed to the conceptualization, methodology, and reviewed and edited the manuscript.

Paper III – Investment under a disruptive risk with costly Bayesian learning

Authors: Roel L. G. Nagy, Verena Hagspiel, Sebastian Sund and Jacco J. J. Thijssen
Submitted to an international, peer-reviewed journal.

This paper examines a profit-maximizing investor's sequential decision-making process in which it has the option to invest in a project and can learn about the project's profitability prior to investment. Specifically, the profitability of the project is uncertain and subject to the risk of a disruptive event, the timing of which is uncertain. The decision process is divided into two stages. In the first stage, the firm decides whether and how much to invest in learning. If the firm decides to invest in learning, it learns about the arrival rate of the disruptive event. In the second stage, the firm decides whether and when to stop learning. When it stops learning, it either initiates the project by paying an investment cost, or abandons the option to invest by paying an abandonment cost.

We propose a general model that applies to investment under uncertainty with the possibility to learn about the disruptive risk related to the investment at a cost. The investor has a prior belief about the likelihood of the disruptive event. The learning process is assumed to follow a probability measure, which is continuously updated with new signals. The learning costs consist of a lump-sum payment and a continuous payment over time. If the firm decides to invest in learning, it has to decide whether and when it initiates or abandons the project. We solve this problem by determining an abandonment and investment threshold, which are the values of the prior at which the firm decides to abandon and initiate the project, respectively.

We find that the value of the option to learn and the learning investment itself are influenced by the possible range of disruptive risk. A firm's decision to invest in learning depends on its prior belief about the disruptive risk, but the amount of investment in learning is not greatly affected by this prior belief. In deciding on the optimal learning rate and decision thresholds, the firm must balance the time it takes to make the right decision with the need to make a decision quickly in order to save on learning costs. The paper shows that a firm that learns slowly performs worse in terms of making the right decision and takes longer to decide. On the other hand, a firm that efficiently learns from a signal, invests less in learning, and makes decisions faster is less likely to suffer large losses or miss out on a profitable project.

This paper contributes to two strands in the literature on learning: the first focuses on the question of whether a firm should learn, and the second aims to answer the question of how much a firm should learn. We show that these two questions should be examined simultaneously.

This paper is co-authored by Verena Hagspiel, Jacco Thijssen and Sebastian Sund. All co-authors have contributed to the conceptualization and methodology. Verena Hagspiel contributed to reviewing and editing of the manuscript and supervision. Jacco Thijssen contributed to the software, and review and editing of

the manuscript. Sebastian Sund contributed to the software, validation, formal analysis and writing of the original draft.

Chapter 5

Concluding remarks and future research

The papers in this thesis consider investment under uncertainty in the future energy system. The proposed methodology accounts for uncertainty, the investor's flexibility, and the value of information. It allows for the optimization of a profit-maximizing investor's investment decisions under uncertainty associated with both policy and market conditions. Furthermore, I also consider the perspective of a social planner. Papers I and II both examine the investor's optimal decision under a subsidy with an uncertain lifetime, and they also derive policy advice for a social planner determining the subsidy size. Paper III shows the value of information and active learning for an investor when the profitability of a project is uncertain.

This thesis aims to answer four questions related to investment under uncertainty and policy. The first question is to understand how a subsidy affects investment capacity. Papers I and II conclude that the availability of a subsidy speeds up investment. Whether a subsidy attracts a larger investment size depends on the investor's flexibility to increase capacity. If the investor has the flexibility to increase its capacity repeatedly, as in the incremental investment problem in Paper II, a subsidy leads to a larger capacity during its lifetime. In the case where the firm's investment is a lumpy and a one-time decision, as in Paper I, a subsidy leads to a *lower* capacity.

The second question in this thesis is to examine the role of subsidy retraction risk on the aforementioned effects of the subsidy. Both Papers I and II conclude that the risk of subsidy retraction speeds up investment even more. However, the effect on the investment size is different in each paper. If the firm's investment is lumpy, as in Paper I, the investment size is smaller when the subsidy retraction risk is higher. In the case of incremental investment, as in Paper II, the investment size is larger during the subsidy lifetime. However, this is at the cost of less investment after the subsidy has been retracted.

The third question this thesis addresses is the optimal social policy design. In Paper I, we find that subsidies can increase welfare only if they are perceived as low risk. In terms of reaching a social planner's capacity target, subsidies are effective in reaching a low capacity target (i.e., the capacity target is lower than the firm's optimal investment size without subsidy) sooner, but they are ineffective in increasing capacity if investment is lumpy. In Paper II, we assume the firm's investment is incremental and find that a social planner can increase capacity in the short run, but this effect fades after subsidy withdrawal. We find that the optimal policy design strongly depends on the time by which the policy maker aims to maximize welfare or capacity. The further in the future this goal lies, the larger the optimal subsidy.

The fourth and final main research question that is answered is how the option to learn about the profitability of a project changes the firm's decision to invest or abandon it. The option to learn is most valuable when it is unclear whether the project is low profit or high profit. In the case of such 'intermediate' profit, the firm learns first to decide whether to invest or abandon later. A firm that is able to learn more efficiently, is able to both improve its selection in an investment project – meaning it is less likely to invest in projects with low profit and more likely to invest in projects with high profit – and spend less time and money on learning.

This thesis focuses on deriving the optimal profit-maximizing investment decision and the optimal social policy under uncertainty, accounting for different sources of risk such as policy risk and demand uncertainty. For both the industry and a policy maker, it is important to understand these factors. However, different sources of risk interact, and more work needs to be done in understanding these interactions. For example, a policy maker may change its policy when technology or demand uncertainty changes over time. Therefore, technological risk and policy risk are not independent, and the investor's aim to hedge itself against both risks may be more complex.

The articles in this thesis also take the perspective of a social planner. They specifically study the situation in which the social planner's flexibility is limited to setting a policy only once or only capable of using a single policy tool. However, it may be interesting to contrast this with a scenario in which a social planner has the flexibility to combine it with another policy tool. This is especially relevant as policy makers get more information about the industry over time and should be able to adjust their policy based on new insights on the market and technology.

Another direction that could result in relevant insights for the industry and policy makers is studying the effect of competition combined with private or limited information. In this thesis, I focus on understanding the role of a large investor in the industry. However, large investors still compete for projects, and this competition affects an investor's decision.

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Paper I

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Green capacity investment under subsidy withdrawal risk☆

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ABSTRACT

Subsidies initially installed to stimulate green capacity investments tend to be withdrawn after some time. This paper analyzes the effect on investment of this phenomenon in a dynamic framework with demand uncertainty. We find that increasing the probability of subsidy withdrawal incentivizes the firm to accelerate investment at the expense of a smaller investment size. A similar effect is found when subsidy size as such is increased. When subsidy withdrawal risk is zero or very limited, installing a subsidy could increase welfare. In general we get that the larger the subsidy withdrawal probability, the smaller the welfare maximizing subsidy rate is. Therefore, a policy maker aiming to maximize welfare should try to reduce subsidy withdrawal risk.

1. Introduction

In an attempt to limit climate change, many countries have set ambitious targets to reduce greenhouse gas emissions during the past two decades. Increasing the share of renewable energy production to the overall energy mix is recognized as critical in reaching those targets [European Commission, 2017]. As of 2017, 179 countries had renewable energy targets, where, in particular, 90 countries had targets to generate more than 50% of their electricity from renewables no later than by 2050 [REN21, 2018b]. The European Commission, for example, has set a recent new target according to the "2030 framework for climate and energy policies", which is to achieve 32% of total energy consumption for the entire European Union in 2030 to be delivered by renewable energy sources. Another example is China that has just reached an accumulated wind capacity of 217 gigawatts (GW) in 2019 [World Wind Energy Association, 2019], and aims to increase total renewable power capacity to 680 GW by 2020 [REN21, 2018b].

Many countries have introduced support schemes aimed at accelerating investments in renewable energy over the past two decades, in order to reach these ambitious targets. Governments therewith, want to ensure competitiveness of renewable energy production and encourage investment. As of 2017, 128 countries had power regulatory incentives and mandates [REN21, 2018b]. China, for example, implemented the world's largest emissions trading scheme in 2017 [REN21, 2018b].

However, many support schemes have been retracted or revised suddenly and unexpectedly over the last years. For example, Ukraine removed a tax exemption on companies selling renewable energy [REN21, 2015]. Furthermore, the size of subsidy payments was retroactively adjusted in Belgium, Bulgaria, the Czech Republic, Greece and Spain [Boomsma and Linnerud, 2015], and the feed-in-tariffs were reduced in Bulgaria, Germany, Greece, Italy and Switzerland in 2014 [REN21, 2015]. China implemented sudden changes in their feed-in tariff in 2018, making new solar power projects less likely to be eligible for subsidy [The Economist, 2018].

One of the main reasons for subsidy policy change results from technological progress. Initially, a subsidy is implemented to ensure competitiveness of renewable energy production, but when technology advances such that the technique is profitable on itself, the subsidy is no longer needed and can be withdrawn. Another reason for subsidy withdrawal can be that the original renewable energy capacity target has been reached or that the budget has been depleted. Norway and Sweden created a joint electricity certificate market in 2012 to boost renewable electricity production in both countries. Norway will no longer provide electricity certificates to facilities that start operating after 31 December 2021, because the goal of having a green energy production of 28.4 TWh by 2020 has been reached [Energy Facts Norway, 2015]. Alternatively, a policy can be withdrawn or altered due to a depleted

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budget, as was the case in Italy for their solar photovoltaic (PV) support in 2013 [Karneyeva and Wüstenhagen, 2017]. However, in some locations green technologies are still unable to survive without subsidies [Institute for Energy Research, 2017]. For these countries, the question what consequences subsidy withdrawal has for renewable energy production and renewable energy investment, will be a relevant question in the near future.

In countries where policy changes already occurred, it had a severe impact on the profitability of renewable energy projects and investment behavior. In Spain an unforeseen subsidy retraction caused a 40% drop in profitability for investors [Del Rio and Mir-Artigues, 2012]. Spain's largest power group, Iberdrola, reported a 91% decline in net profits from wind after subsidies were reduced [Financial Times, 2014]. Similarly, subsidy cuts in the UK for solar PV damaged investor confidence and could also delay the point at which solar could be cost competitive [The Guardian, 2015]. Del Rio and Mir-Artigues [2012] mentions that when policy costs are high, the social acceptance of the policy decreases, increasing the pressure to implement (retroactive) changes to the policy. This increases policy instability, creating uncertainty and risks for investors, who, in return, want higher risk premiums. This all increases costs and reduces profitability.

This paper aims to determine how the optimal investment decisions related to renewable energy projects depend on the availability of a subsidy, the size of the subsidy and the withdrawal risk of the subsidy. A social planner wants to know how social welfare is affected by the subsidy, its size and the withdrawal risk. Studying the effect on social welfare is the standard approach in the public economics literature. However, it is not necessarily the standard approach in public decision-making, where it is of main importance to set the right goals and targets [Stern, 2018]. We, therefore, also look at the question how the ability to reach a capacity target within a certain time-frame is affected by the subsidy, subsidy size and subsidy withdrawal risk.

We consider a firm that has the option to invest in a renewable energy project. It has to decide on both the time to invest, as well as the size of the capacity it wants to install. We consider a dynamic framework with demand uncertainty. The cost of installing capacity of a certain size depends on the size of the capacity as well as the availability of support. Support is provided in the form of a lump-sum investment subsidy, which represents a general class of investment subsidies including investment tax credits and capital subsidies. Investment tax credits constitute the most widespread policy instrument for renewable energy globally,[1] and is often implemented with the aim to increase the affordability and profitability of renewable energy production [REN21, 2018a, page 70]. We study the effect of policy uncertainty in the form of retraction of a currently provided subsidy.

We first derive the optimal investment decisions of a profit-maximizing firm facing subsidy retraction risk. We find that increasing the subsidy size speeds up investment but this goes at the expense of a decreased optimal investment size. Increasing subsidy retraction risk for a given subsidy size has the same effect. Surprisingly, the firm's optimal investment size when there is no subsidy provided is larger than the optimal investment size when subsidy is provided but there is risk of future retraction.[2]

We then take the viewpoint of a policy maker, where we analyze the effect of the subsidy on the resulting investment decision of the firm. We find that a subsidy could increase welfare. Numerical experiments suggest that a subsidy increases welfare when subsidy withdrawal risk is sufficiently small. A policy maker aiming to maximize welfare should minimize subsidy withdrawal risk, since welfare decreases with a larger subsidy withdrawal risk. We also derive the impact of a subsidy on the ability to reach certain policy targets. When a proposed capacity target is smaller than the firm's optimal investment size, a subsidy can be used to speed up investment, thereby raising the probability that the target is reached in time.

Our paper contributes to different strands of literature. First, we contribute to the literature on incentive regulation of a firm within an uncertain dynamic framework (see, e.g., Brennan and Schwartz, 1982, Dobbs, 2004, Evans and Guthrie, 2005, 2012, Guthrie, 2006, 2020, Willems and Zwart, 2018 and Azevedo et al., 2020). Azevedo et al. [2020] consider revenue neutral tax-subsidy package on the firm's timing and capacity decision under demand uncertainty without regulatory uncertainty. Within the aforementioned strand of literature, regulatory uncertainty is considered by Teisberg [1993], Dixit and Pindyck [1994, Chapter 9], and Hassett and Metcalf [1999], where the latter two also consider the effect of subsidy size on investment timing. Motivated by recent frequent occurrences of changes in regulatory policies in the green energy industry, we contribute to this literature by focusing on the effect of policy risk in the form of potential subsidy withdrawal. In addition we determine the optimal subsidy size looking at different aims, such as welfare maximization and capacity targets, and we study the role of policy risk in determining the optimal subsidy size.

Our paper also contributes to an increasing strand of literature that studies the effect of subsidies on green investment (e.g., Pizer, 2002; Eichner and Runkel, 2014; Nesta et al., 2014; Abrell et al., 2019; Bigerna et al., 2019). Pennings [2000] and Danielova and Sarkar [2011] focus on the combination of subsidy and tax rate reduction. Unlike the aforementioned papers, we also analyze how the risk of policy change intervenes with the effect of policy measures. Some of the literature focuses on carbon pricing and studies how policy uncertainty affects the volatility of the prices (see, e.g., Blyth et al., 2007, Fuss et al., 2008, Yang et al., 2008, and Kang and Létourneau, 2016). The carbon pricing literature generally concludes that more policy uncertainty results in larger volatility in prices and, therefore, delays investment.

Some recent literature related to renewable energy accounts for policy uncertainty related to random provision, revision or retraction of a subsidy, such as, for example, Boomsma et al. [2012], Boomsma and Linnerud [2015], Adkins and Paxson [2016], Eryilmaz and Homans [2016], Ritzenhofen and Spinler [2016] and Chronopoulos et al. [2016]. These papers focus on how uncertainty in the availability of a certain type of subsidy affects investment behavior. The effect of uncertainty in availability of a subsidy on investment behavior strongly depends on the type of subsidy in place as well as the level of uncertainty. We contribute to this literature by studying a lump-sum investment subsidy, the most widespread policy instrument for renewable energy globally [REN21, 2018a, page 70], and study the role of subsidy size and the risk of potential subsidy withdrawal on investment. Furthermore, we do not solely focus on the firm's investment behavior, but also study the effect of policy risk on the goals of the social planner and welfare. To our knowledge, we are the first to conclude that a larger likelihood of an investment subsidy withdrawal damages both welfare and the policy maker's ability to increase renewable energy capacity.

The remainder of this paper is organized as follows. Section 2 presents the model and characterizes the optimal investment decisions both from a profit-maximizing firm and social welfare point of view. In Section 3, we study the optimal investment decision of a firm in more detail by providing comparative statics. Numerical experiments are performed in Section 4. Section 5 focuses on the effect of both the subsidy size and the likelihood of subsidy withdrawal on reaching certain environmental targets as well as welfare. In Section 6 we discuss the role of the type of subsidy we study on our results. Section 7 concludes.

[1] Worldwide, an estimated amount of 30 to 40 countries used investment or production tax credits to support renewable energy installations over the past decade [REN21, 2018a, page 69].

[2] On the macro level it could still be the case that more firms invest when a subsidy is provided, and that - despite that the average installed capacity per firm is smaller - the total renewable energy capacity on the market increases. See, for example, Hassett and Metcalf [1999], in which it is obtained that providing a lump-sum subsidy increases the total market capacity when many firms are faced with the option to invest in a project of fixed size.

2. Model

We propose a theoretical framework that studies a firm's optimal investment decision under uncertain subsidy support. We consider a risk-neutral, profit-maximizing firm that holds the option to invest in a renewable energy project with an uncertain future revenue stream. The firm has to determine the optimal timing of the investment and the size of the capacity to be acquired. We assume that the firm produces up to capacity, and cannot scale up capacity in the future. Renewable energy projects, such as wind parks, are location- and firm-specific due to governmental concessions needed to obtain the investment option. In most concession-based contracts for renewable energy generation capacity, the investment is a one-time lumpy decision.

We assume the firm to be sufficiently large so that it exerts market power. This is supported by the fact that a series of studies has indicated that the electricity market is highly concentrated. In the United States, a government report by the United States General Accounting Office [2005] states that the four federal Power Marketing Administrations (PMAs) exert market power from the federal hydroelectric dams and projects. Signs of market power on the US electricity market are also reported on a state level.[3] In Europe, signs of market power are reported on a national level, for example in Italy,[4] England and Wales,[5] and the Nordic countries.[6] We refer to Karthikeyan et al. [2013] for a thorough review on market power in the electricity market in different countries. The output price at time t, $P(t)$, is given by:

$$P(t) = X(t)(1-\eta K), X(0) = x \tag{2.1}$$

where K is the firm's production capacity, and $\eta > 0$ is a constant.[7]

The output price $P(t)$ depends on an exogenous shock $X(t)$, which is assumed to follow a geometric Brownian motion process given by:

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t) \tag{2.2}$$

where μ is the drift rate, σ the uncertainty parameter and $dW(t)$ the increment of a Wiener process. The inverse demand function (2.1) is a special case of the one used by Dixit and Pindyck [1994, Chapter 9], which assumes $P = XD(K)$ with an unspecified demand function $D(K)$, and is frequently used in the literature (see, e.g., Pindyck, 1988, He and Pindyck, 1992, and Huisman and Kort, 2015).

The cost of one unit of investment is set equal to δ. Hence, installing a production capacity of size K yields an investment cost of δK when no subsidy is in effect. Subsidy provides a one time discount at rate θ on the investment cost, so that the investment costs are then equal to $(1-\theta)\delta K$.

Initially, the lump-sum subsidy[8] is assumed to be available, but due to technological development (or a restriction from the budget constraint or a change in government), the firm expects the subsidy to be withdrawn. We model the firm's perceived risk of subsidy retraction by an exponential jump with parameter λ. This implies that the firm's perceived probability that the subsidy will be retracted in the next time interval dt is equal to λdt.

The optimization problem for the profit-maximizing firm is then given by an optimal stopping problem in which it aims to find the optimal time τ to invest in a capacity of optimal size K:

$$F(x,\theta) = \sup_{\{\tau,K\}} \mathbb{E}\left[\int_\tau^\infty P(t)Ke^{-rt}dt - \left(1-\theta\cdot 1_{\xi(\tau)}\right)\delta K e^{-r\tau} \,|\, X(0) = x, \xi(0) = 1\right] \tag{2.3}$$

with

$$\xi(t) = \begin{cases} 0 & \text{if subsidy retraction has occurred at time } t \text{ or earlier} \\ 1 & \text{otherwise} \end{cases} \tag{2.4}$$

When investing, the firm pays a lump-sum investment cost and obtains the revenue stream $P(t)K$ from time τ on. r is the risk-free rate, where we assume $r > \mu$. In case $r \le \mu$, the problem is trivial as it would always be optimal to wait with investment.

Obviously, it is optimal for the firm to invest when the output price $P(t)$ is large enough, where (2.1) learns that $P(t)$ is proportional to $X(t)$. It follows that the investment rule is of a threshold type. In particular, there exists a threshold value of $X(t)$ at which the firm is indifferent between investing and waiting with investment.[9] It is intuitively clear that when the price is below a certain threshold level, denoted by X_1, the firm will not invest, independently of whether the subsidy is available or not. Furthermore, when the price is high enough, i.e. above a threshold $X_0 > X_1$, the firm will always invest, independent of the availability of the subsidy. For $X(t)$ in the interval $[X_1, X_0]$, the firm will only invest when the subsidy is active, and it will not do so when the subsidy has been withdrawn. Therefore, X_1 (X_0) is the value of the geometric Brownian motion at which the firm is indifferent between investing and not investing, while the policy is (not) in effect. Fig. 1 summarizes the above.

The thresholds X_0 and X_1 are directly linked to the investment timing. When there is (no) subsidy available, investment is done when the geometric Brownian motion defined in eq. (2.2) hits the value X_1 (X_0) for the first time from below. As a result, there exists a one-to-one mapping between the investment threshold and the investment time. Throughout this paper, we will refer to X_0 and X_1 both as the investment thresholds and the timing of investment.

Assuming the initial value of the geometric Brownian motion process, x, meets the requirement[10] $x < X_1$, then there are two cases that can occur regarding the timing of the investment. In the first case, the firm invests when the geometric Brownian motion hits the threshold X_1 for the first time while the subsidy has not been retracted. Alternatively, the subsidy is retracted before the GBM hits the threshold X_1 and the firm invests when the process hits the threshold X_0 for the first time. Let s denote the time at which the policy maker withdraws the subsidy. The firm's expected investment time follows from the investment thresholds and the withdrawal time of the subsidy, and is equal to:

$$\text{Expected time to investment} = \mathrm{P}[s>\tau_1]\cdot\mathbb{E}[\tau_1] + (1-\mathrm{P}[s>\tau_1])\cdot\mathbb{E}[\tau_0] \tag{2.5}$$

[3] A government report by the United States General Accounting Office [2002] on the California power market concluded prices did not follow patterns consistent with prices under competitive conditions. Furthermore, Woerman [2019] estimates the impact of market power on the Texas electricity market, and finds that a 10% increase in demand causes markups to more than double, showing that producers do have market power.

[4] European Commission [2011] reports that the Italian energy market is highly concentrated, and also Bosco et al. [2010], Bigerna et al. [2016] and Sapio and Spagnolo [2016] find empirical evidence of market power on the Italian energy market.

[5] David and Wen [2001] found that two dominant suppliers in the England and Wales pool, which is a highly concentrated market, decrease capacity to increase profits during peak periods.

[6] Lundin and Tangerås [2020] empirically reject the hypothesis of perfect competition on Nord Pool, the day-ahead market of the Nordic power exchange, during the period 2011–2013. Tangerås and Mauritzen [2018] test the hypothesis of perfect competition in some areas in Sweden in the period 2010–2013 and reject this hypothesis. Fleten and Lie [2013] conclude that Norway's largest hydro power producer has an incentive to reduce thermal production in order to increase the market spot price.

[7] Note that output price is always positive, as the production capacity K is endogenous. Therefore, the firm will choose the production capacity such that it will be less than $\frac{1}{\eta}$. By applying Eq. (2.1), we implicitly assume that the production quantity is constant. In the short and medium term, renewable energy generation is highly variable due to a large dependency on, among others, weather conditions. However, in the long run production is more predictable and less variable. As the decision to install a renewable energy project, as well as policy decisions have a long term focus, we refrain from focusing on fluctuations in productions on the short and medium term. See, for example, Boomsma et al. [2012], Dalby et al. [2018] and Bigerna et al. [2019] for similar assumptions. A reader interested in how production flexibility affects a firm's investment timing and size can for example look at Hagspiel et al. [2016].

[8] We also use subsidy to refer to the lump-sum subsidy.

[9] See, for example, Dixit and Pindyck [1994] or Huisman and Kort [2015].

[10] If $x \ge X_1$, it is optimal for the firm to invest immediately, and the problem is trivial.

in which $\mathbb{P}[s>\tau_1]$ is the probability that the subsidy withdrawal occurs after threshold X_1 is hit, and $\mathbb{E}[\tau_1]$ ($\mathbb{E}[\tau_0]$) is the expected first hitting time of threshold X_1 (X_0).[11]

To determine the optimal investment decision, the first step is to derive the value the firm obtains by investing. Denoting the value of the firm at the moment of investment by V_0 if the subsidy has already been retracted, and by V_1 in case the subsidy is still in effect, we get[12]

$$V_0(X,K)=\frac{X(1-\eta K)K}{r-\mu}-\delta K \tag{2.6}$$

$$V_1(X,K)=\frac{X(1-\eta K)K}{r-\mu}-(1-\theta)\delta K \tag{2.7}$$

Using the value functions (2.6) and (2.7) the optimal investment size for a given value of X can be straightforwardly derived. The result is presented in Corollary 1.

Corollary 1. *Let $K_1(X)$ ($K_0(X)$) denote the optimal investment size while the policy is (not) in effect. When the firm decides to invest at X, the optimal investment size is equal to:*

$$K_0(X)=\frac{1}{2\eta}\left(1-\frac{\delta(r-\mu)}{X}\right) \tag{2.8}$$

$$K_1(X)=\frac{1}{2\eta}\left(1-\frac{(1-\theta)\delta(r-\mu)}{X}\right) \tag{2.9}$$

The proofs of all corollaries and propositions can be found in Appendix A.

Using similar steps as in Dixit and Pindyck [1994] and Huisman and Kort [2015], the value of the investment option with and without the subsidy can be derived. These are stated in Proposition 1.

Proposition 1. *Let $F_1(X,K)$ ($F_0(X,K)$) denote the value of the option to invest at X while the policy is (not) in effect. When the firm decides to invest at X, it invests in capacity K. The value of the option to invest at X after the subsidy has been retracted is equal to:*

$$F_0(X,K)=\begin{cases}\dfrac{X(1-\eta K)K}{r-\mu}-\delta K & \text{if } X\in[X_0,\infty)\\ A_0X^{\beta_{01}} & \text{otherwise}\end{cases} \tag{2.10}$$

where A_0 is a (positive) constant and β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2+(\mu-\frac{1}{2}\sigma^2)\beta-r=0, \beta_{01}>1$.

The value of the option to invest at X while the subsidy is available is equal to:

$$F_1(X,K)=\begin{cases}\dfrac{X(1-\eta K)K}{r-\mu}-(1-\theta)\delta K & \text{if } X\in[X_1,\infty)\\ A_1X^{\beta_{11}}+A_0X^{\beta_{01}} & \text{otherwise}\end{cases} \tag{2.11}$$

where A_1 is a (positive) constant and β_{11} is the positive solution to $\frac{1}{2}\sigma^2\beta^2+(\mu-\frac{1}{2}\sigma^2)\beta-(r+\lambda)=0, \beta_{11}>\beta_{01}>1$.

When the subsidy is (not) available, it is optimal to invest when $X\geq X_1$ ($X\geq X_0$), yielding Eq. (2.7) (Eq. (2.6)) as the value of the investment option. The firm does not invest, thus waits, when the current output price is too low, i.e. when $X<X_1$ ($X<X_0$) if the subsidy is (not) available. If the subsidy is still present, the value of the investment option consists of two parts: the value of holding the option to invest while the subsidy is available and the option to invest after the subsidy has been retracted. When the subsidy is retracted, the former value is lost as the subsidy will not be re-enacted again in the future.

After the subsidy has been abolished, policy uncertainty will not influence the investment decision anymore. The problem to be solved in such a situation is already analyzed in Huisman and Kort [2015]. Proposition 2 presents the optimal investment decision in this case.

Proposition 2. *When the subsidy is abolished, the optimal investment threshold satisfies:*

$$X_0=\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu) \tag{2.12}$$

whereas the corresponding investment size[13] is given by:

$$K_0^*=[\eta(\beta_{01}+1)]^{-1} \tag{2.13}$$

Proposition 3 presents the firm's optimal investment decision when the subsidy is still available.

Proposition 3. *If the investment subsidy has not been retracted yet, the optimal investment threshold X_1 is implicitly given by:*

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}}-\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu}+(1-\theta)\delta K_1^*=0 \tag{2.14}$$

in which K_1^* is the optimal capacity under subsidy when investing at $X=X_1$, i.e. eq. (2.9) evaluated at $X=X_1$.

In the special case in which there is no subsidy retraction risk, eq. (2.14) can be solved explicitly. Corollary 2 presents the optimal investment decisions under a lump-sum subsidy without retraction risk.

Corollary 2. *In case of a subsidy with no subsidy retraction risk* (i.e. $\lambda=0$), *the optimal investment timing and size are given by:*

$$X_1=\frac{\beta_{01}+1}{\beta_{01}-1}\cdot(1-\theta)\delta(r-\mu) \tag{2.15}$$

and

$$K_1^*=[\eta(\beta_{01}+1)]^{-1} \tag{2.16}$$

Comparing the investment decision under a subsidy and the one without subsidy, we observe that the optimal investment sizes are the same ($K_1^*=K_0^*$), but the timing threshold with subsidy is actually smaller than the one without subsidy ($X_1=(1-\theta)X_0<X_0$). The reason behind this is that lower investment costs allow for investment at lower output prices, i.e. earlier. The decrease in investment costs has two effects on the optimal size. First, there is a direct effect. The lower the investment costs, the more the firm likes to invest for a given level of X. Second, there is an indirect effect via the timing. As investment is done sooner, i.e. at a lower output price, the firm can only justify a smaller investment size. The two effects cancel out when the firm invests at the optimal time.

Now, we consider the problem from the perspective of a social planner with the objective to maximize social welfare. The social planner maximizes the total surplus (TS), which consists of the sum of the consumer (CS) and producer surplus (PS)[14] minus the subsidy costs of $\theta\delta K$. We assume the social planner uses the same discount rate r as the firm, following, for example, Huisman and Kort [2015] and Bigerna et al. [2019]. A discussion on alternative assumptions regarding the social planner's discount rate is included in Section 6.

[11] Explicit derivation of the expected time to investment is shown in Appendix C.2.
[12] We write X instead of $X(t)$ for convenience.

[13] For convenience of notation, we use $K_0^*=K_0(X_0)$ and $K_1^*=K_1(X_1)$.
[14] The producer surplus is defined as the value of the firm's project.

The total surplus when investing at X with capacity K is equal to[15]

$$\mathrm{TS}(X,K) = \frac{X(2-\eta K)K}{2(r-\mu)} - \delta K \tag{2.17}$$

Note that the total surplus does not directly depend on the subsidy. This is the result of the fact that the subsidy is solely a welfare-transfer with a zero-sum contribution to total surplus. In other words, each unit of currency used for the subsidy represents on the one hand a cost for the social planner and on the other hand a gain for the producer. Therefore, the net direct impact of the subsidy on total surplus is zero. A subsidy can however impact total surplus indirectly, via influencing the firm's investment decision.

We can determine the socially optimal timing and capacity using similar steps as before. Proposition 4 states the first-best social optimum.

Proposition 4. *The socially optimal capacity for a given level of X is equal to*:

$$K_S(X) = \frac{1}{\eta}\left(1 - \frac{\delta(r-\mu)}{X}\right) \tag{2.18}$$

The total surplus (*TS*) *is then given by*:

$$\mathrm{TS}(X,K) = \begin{cases} \frac{X(2-\eta K)K}{2(r-\mu)} - \delta K & \text{if } X \in [X_S, \infty) \\ A_S X^{\beta_{01}} & \text{otherwise} \end{cases} \tag{2.19}$$

in which A_S is a (*positive*) *constant, and X_S is the social planner's optimal timing threshold. At this threshold, the social planner is indifferent between investing and not investing. The optimal timing maximizing the total surplus is given by*:

$$X_S = \frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu) \tag{2.20}$$

The socially optimal capacity, K_S^*, is given by:

$$K_S^* = 2[\eta(\beta_{01}+1)]^{-1} \tag{2.21}$$

We find that the investment timing of the social planner and the firm are identical when there is no subsidy (i.e. $X_S = X_0$). Regarding the size of investment, we conclude that it is socially optimal to invest twice as much as the profit-maximizing firm (i.e. $K_S^* = 2K_0^*$). The reason is that the social planner is more eager to invest than the private firm, as the social planner also accounts for consumer surplus. This means that the social planner either invests sooner and adapts size accordingly, or invests more and adapts timing accordingly. We conclude that within our framework the social planner wants to invest more than the profit-maximizing firm. Thus, to obtain the first-best solution, the social planner should stimulate firm investment in such a way that the firm will invest more without changing the investment time. The next section investigates whether introducing a subsidy can achieve this.

3. Investment and subsidy

This section analyzes the effect of an investment subsidy and the probability that the subsidy will be retracted, on the firm's optimal investment decision. The following proposition states how the optimal investment decision is affected by subsidy retraction risk.

Proposition 5. *The optimal investment timing and size are affected by the subsidy retraction risk λ in the following way*:

$$\frac{dX_1}{d\lambda} < 0, \frac{dK_1}{d\lambda} < 0 \tag{3.1}$$

if and only if

$$\frac{(1-\theta)\delta(r-\mu)}{X_1} \geq \frac{(\beta_{01}-1)(\beta_{11}-1)}{\beta_{01}\beta_{11} + \sqrt{\beta_{01}^2 + \beta_{11}^2 - 1}} \tag{3.2}$$

where β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$, and β_{11} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r+\lambda) = 0$.

Proposition 5 states that a higher subsidy retraction risk decreases both the optimal investment threshold[16] and the optimal investment size. A firm speeds up investment under a higher subsidy retraction risk in order to make use of the subsidy now, as it is less likely it will be available in the future. Investing at a lower threshold implies that the firm invests when the output price is lower, which leads to a smaller optimal investment size. There is no direct effect of subsidy retraction risk on optimal investment size, but only an indirect effect via the timing, as can be straightforwardly concluded from expression (2.9). The intuition behind this is that the investment subsidy only affects the investment payoff at the moment of the investment, so that the optimal investment size does not depend on whether the subsidy will be withdrawn very soon after investing or remains for a long period of time.

Inequality (3.2) states that when the ratio of costs and the price shock at the moment of investment are above a threshold, then the results in (3.1) hold. Extensive numerical results suggest that this condition is in fact satisfied for any lump-sum subsidy.

The result that a higher probability of retraction of a subsidy speeds up investment is in accordance with findings of Hassett and Metcalf [1999] and Dixit and Pindyck [1994]. Chronopoulos et al. [2016] however find that subsidy retraction risk delays investment for high levels of subsidy retraction risk. This is because Chronopoulos et al. [2016] study a subsidy in the form of a price premium. This keeps on having an effect after the investment has been undertaken, because in case of a price premium a higher retraction probability reduces the expected net present value of the investment. The latter does not happen in our case, because the lump-sum subsidy just affects the investment payoff at the moment of the investment, implying that a retraction of the subsidy occurring at a later date has no effect.

We find that the investment size decreases with subsidy retraction risk. In Chronopoulos et al. [2016] this also holds for low levels of subsidy retraction risk. However, when subsidy retraction risk is high, the fact that the effect of increasing the subsidy retraction risk will delay investment, has the implication that a larger withdrawal risk increases the firm's investment size in Chronopoulos et al. [2016].

Proposition 6 presents the influence of the size of the subsidy on the optimal investment decision.

Proposition 6. *The effects of the subsidy size θ on the optimal investment threshold and the investment size are given by*:

$$\frac{dX_1}{d\theta} < 0, \frac{dK_1}{d\theta} < 0 \tag{3.3}$$

if and only if condition (3.2) holds.

Proposition 6 shows that a larger size of the subsidy speeds up investment and decreases the investment size. Increasing the subsidy size has two different effects on the optimal investment decision. First, providing a larger subsidy gives some incentive to invest more for a given output price. Second, as the lower costs make the investment profitable at lower output prices, it gives also some incentive to invest

[15] See Huisman and Kort [2015] for the details of the derivation of the total surplus.

[16] It can be shown that $X_0 > X_1$ holds for any level of subsidy withdrawal risk λ as long as condition (3.2) is met. From Proposition 2 and Corollary 2, it follows that $X_0 > X_1$ when $\lambda = 0$. By Proposition 3, we have that X_1 decreases if λ increases when condition (3.2) is met.

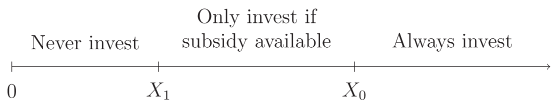

Fig. 1. Optimal investment strategy at different output prices.

earlier, and as result of the dependency between timing and size, invest in a smaller capacity. We find that the second effect always dominates the first, leading to the result in Proposition 6.

From a policy maker's point of view it might be interesting to analyze under which of the following two scenarios the firm's investment is larger: (1) a small subsidy subject to a low probability of retraction, or (2) a larger subsidy subject to a larger probability of retraction. From Propositions 5 and 6, it follows that both a larger retraction risk and a larger subsidy in fact decrease both the investment threshold and the investment size. Therefore, the investment size under the second scenario will be smaller than in the first. However, the firm will have invested sooner under the second scenario compared to under the first.

4. Quantitative analysis

This section contains a numerical analysis of an investment opportunity in a hydro power plant. The parameter values, displayed in Table 1, are taken from Fleten et al. [2016] and Finjord et al. [2018]. The data set in Fleten et al. [2016] consists of 214 licenses to build small hydro power plants granted by the Norwegian Water Resources and Energy Directorate (NVE).

Fig. 2 presents the investment timing thresholds X_0 and X_1, and the investment sizes K_0^* and K_1^* as functions of the subsidy retraction risk λ, using the parameter values in Table 1. Fig. 2 is in accordance with the results presented in Proposition 5 in the sense that investment timing X_1 and size K_1^* decrease with subsidy retraction risk λ. Furthermore, as X_0 and K_0^* are the investment threshold and capacity size *after* retraction of the lump-sum subsidy, these do not depend on λ.

More importantly, Fig. 2 shows that the optimal investment size when there is no subsidy available (K_0^*) is in fact larger than the optimal investment size when the subsidy is available (K_1^*) but exposed to retraction risk (i.e. $\lambda > 0$). This means that when there is a risk of subsidy retraction, the firm's optimal investment size at the corresponding investment threshold is *larger* without subsidy than it is with subsidy, but it is equal if there is no subsidy retraction risk. There are three underlying opposing effects of receiving subsidy that influence the firm's optimal investment decision and lead to the aforementioned observation. The first two effects, the direct effect of subsidy on investment size (increasing the optimal size) and the indirect effect of subsidy on investment size via timing (decreasing the optimal size), cancel each other out, as discussed when presenting Corollary 2. The third effect is that retraction risk speeds up investment, as the firm prefers to obtain the subsidy over not obtaining subsidy. Speeding up in fact means investing at a lower threshold where the output price is smaller. This causes the optimal investment size under subsidy to be smaller than without subsidy.

Table 1
Parameter values used in the numerical example.

Notation	Parameter	Value
μ	Electricity price trend	2%
σ	Electricity price volatility	5%
r	Risk-free interest rate	6%
δ	Investment cost per unit of capacity	350 /MWh
η	Slope of the linear demand curve	0.01

Based on Fig. 2, we generate some important policy advice regarding green investment projects. Investors in green investment projects usually have long-term goals and high investment costs. Given that a subsidy has been implemented and the policy maker wants the firm to invest as much as possible, the optimal situation for the policy maker would be that the firm perceives no subsidy retraction risk (i.e. $\lambda = 0$).

To study a situation where the policy risk is large, we set $\lambda = 1$. This means that the firm expects the subsidy to be retracted in about one year. The investment timing thresholds X_0 and X_1, and the investment sizes K_0^* and K_1^* are shown as functions of subsidy size θ in Fig. 3. In accordance with Proposition 6, both timing and size decrease when increasing subsidy size.

To study the effect of subsidy size θ and interpret Fig. 3, it is important to distinguish between two different cases. Firstly, the simple case, in which the firm is in the stopping region at the start of the planning horizon, i.e. the starting value of the GBM X, x, is larger than the investment threshold X_1. Then the firm invests immediately, at the price $P(x)$ and the optimal capacity is equal to $K_1(x)$, i.e. expression (2.9) evaluated at $X = x$. When the government pays for almost all investment costs, that is, the subsidy size θ is close to one, the investment quantity is close to $\frac{1}{2\eta}$, which represents the optimal capacity if investment costs would be equal to zero. That is, the firm maximizes total revenues. Secondly, the firm is in the waiting region at the start of the planning horizon, i.e. $x < X_1$. In this case, the firm waits with investment until the threshold X_1 is hit (or X_0 if the subsidy is withdrawn before investment) and invests in K_1 (K_0) as shown in the right-hand graph in Fig. 3.

Fig. 3 helps to analyze the situation in which a government aims to speed up investment of the waiting firm by threatening to remove the subsidy soon. Whether the firm will invest immediately under large subsidy withdrawal risk, depends on the size of the subsidy and the current output price level. When the government has implemented a large subsidy (i.e. θ close to one), threatening to take away the subsidy soon results in firms investing immediately to still receive the large investment cost subsidy. However, it could happen that then, if the current output price is low, firms will invest in a small capacity.

However, when the subsidy size is relatively small, the approach to make the firm invest immediately by threatening to remove the subsidy soon is not always effective. For example, consider a subsidy size of $\theta = 0.15$. Fig. 3 shows that the optimal timing threshold while the subsidy is available, X_1, is equal to 19.15. Increasing the subsidy withdrawal risk even further than $\lambda = 1$ makes the threshold eventually converge to a value of approximately 18.29 (see Fig. 2). Therefore, when the current value of the demand intercept is smaller than 18.29, trying to let the firm invest immediately by threatening to remove the subsidy, is ineffective as it is never optimal to invest immediately, independent of the subsidy withdrawal risk.

Finally, we study the effect of demand volatility on the investment size and investment threshold. Fig. 4 presents the investment timing threshold X_1 and the investment size K_1^* as functions of the subsidy retraction risk λ for different levels of demand volatility σ, using the parameter values in Table 1. Fig. 5 shows the investment timing threshold X_1 and the investment size K_1^* as functions of the subsidy retraction risk θ for different levels of demand volatility. We observe the standard real options result that a larger demand volatility delays investment and increases investment size (see, e.g., Dangl, 1999 and Huisman and Kort, 2015). However, this effect does not eliminate the effects of subsidy withdrawal risk and subsidy size as shown in Proposition 5 and 6. Even when demand volatility σ is large, both the investment threshold and the investment size decrease with subsidy withdrawal risk and subsidy size.

5. Capacity target and total surplus

We now study how a policy maker can influence and steer the decisions of the firm towards a socially optimal (first-best) decision. In the

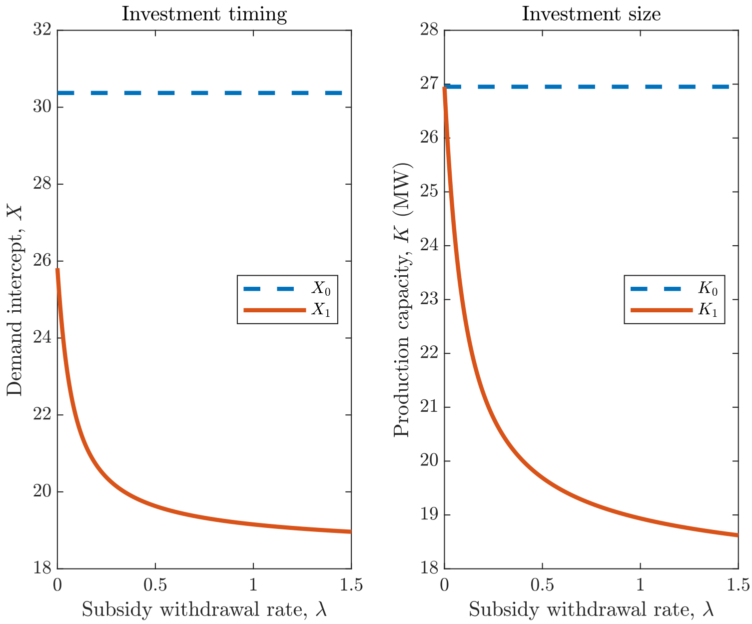

Fig. 2. Investment timing (left) and size (right) as functions of the subsidy withdrawal rate λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\theta = 0.15$.]

following we consider two different types of objectives for the social planner. In Section 5.1, we assume that the policy maker strives to achieve a predetermined capacity target as soon as possible. This is especially relevant considering renewable energy capacity targets. In Section 5.2, we consider a social planner that has the aim to increase total surplus.

5.1. Capacity target

We first focus on the case where the social planner has the aim to reach a certain capacity target $\overline{K}$ as soon as possible. Fig. 6 illustrates the optimal subsidy size required to reach a certain capacity target (left panel) and the resulting investment timing (right panel) as a function of subsidy retraction risk λ.[17]

In case the target is lower than the firm's optimal investment without subsidy (i.e. $\overline{K} < K_0^*$), the social planner can use the policy instrument to speed up the firm's investment. In this scenario a subsidy can be used to reach the capacity target earlier, as illustrated in Fig. 6. The smaller the capacity target, the sooner investment will take place, which is accelerated by offering a larger subsidy. When the subsidy withdrawal risk increases, the subsidy required to reach a certain capacity target decreases. The optimal investment threshold, however, increases as a result of the smaller subsidy size.

Until now we have seen subsidies that are used to speed up investment and, as a side effect, it decreases the firm's optimal investment size. A different matter arises when the capacity target is larger than the firm's optimal investment size if no subsidy is provided. The only way to reach such a target is to implement a conditional subsidy in the sense that such a subsidy is only provided at the moment that the firm invests in a capacity size corresponding to the target.

[17] Note that when $\lambda = 0$, the firm's optimal investment size does not depend on the subsidy size (see equation (2.16)), and thus the social planner cannot influence the firm's optimal size decision. Therefore, the lines in Fig. 6 start for positive λ and not for $\lambda = 0$.

5.2. Total surplus

In this section, we study the question whether a policy maker can increase total economic surplus[18] by use of a subsidy, with a focus on the role of subsidy retraction risk. To analyze the effect of subsidy retraction risk and subsidy size on the total surplus (TS), we study the relative difference between economic surplus generated by the first-best solution and welfare under the investment decision made by the firm. This relative difference is called the relative welfare loss (RWL), and depends on the likelihood of subsidy withdrawal λ and the subsidy size θ. In case there is no subsidy in effect, we can show that the RWL is always equal to:

$$\mathrm{RWL}(X_0, K_0^*) = \frac{\mathrm{TS}(X_S, K_S^*) - \mathrm{TS}(X_0, K_0^*)}{\mathrm{TS}(X_S, K_S^*)} = \frac{1}{4} \tag{5.1}$$

See Appendix C.1 for the derivation details.

This implies that a subsidy only has value in terms of increasing total surplus if it can decrease RWL below 25%. We find that the first-best outcome can in fact not be obtained with a lump-sum subsidy. To achieve the first-best outcome, we learn from Proposition 4 that the subsidy should be such that it should let the firm double the size of the investment without affecting the investment timing. However, providing a subsidy would result in an investment size being less than or equal to the size without subsidy. We conclude that steering the firm towards the first best outcome by providing a subsidy is not possible.

We present further results illustrated by the numerical example with the same parameter values as in Table 1. Fig. 7 plots the total surplus as a function of subsidy retraction risk λ. For any given subsidy level, we find

[18] In this paper, we focus on the question whether a subsidy can increase total economic surplus, assuming no government inefficiencies or market distortions caused by the financing of the subsidy. We use welfare to describe the total economic surplus. Note that in practice, policy makers may need to account for inefficiencies in government spending as well as the costs of obtaining the budget to implement a subsidy. For example, if the subsidy is financed from a distortionary tax, these effects are the consequence of implementing the subsidy.

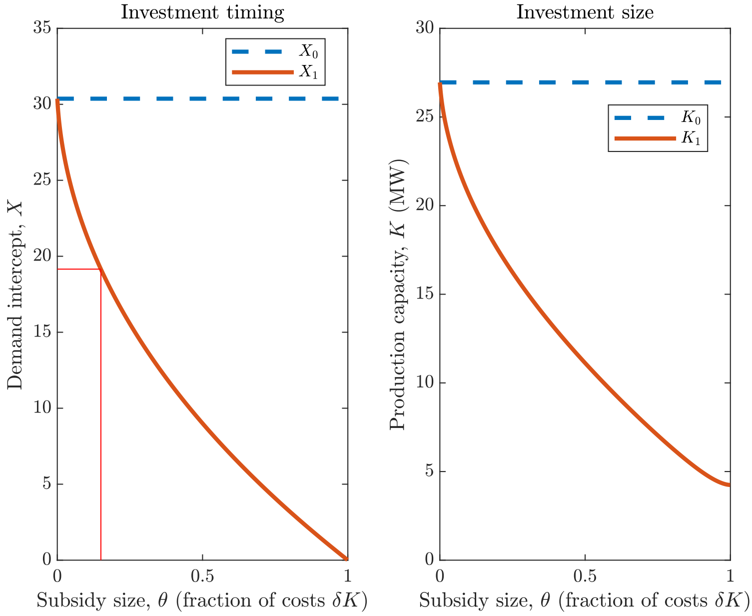

Fig. 3. Investment timing (left) and size (right) as functions of the subsidy size θ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\lambda = 1$.]

that the higher is the perceived risk of subsidy retraction, the lower the total surplus becomes. The reason is the following. First note that, taking it from a welfare perspective, already under zero retraction risk the firm invests too early in a too low capacity. Fig. 2 learns that the larger the perceived risk of subsidy retraction, the sooner the firm invests in less. So in this way under a subsidy retraction risk the firm's investment decision departs even further away from socially optimal investment. Hence, we conclude that no subsidy retraction risk is optimal in terms of total surplus and a policy maker maximizing total surplus should try to eliminate this risk. Fig. 7 in fact shows that already very small increases in subsidy retraction risk drastically decrease total surplus.

Next, we turn our analysis to the socially optimal subsidy size θ. Fig. 8 plots the total surplus as a function of subsidy size θ. We obtain that providing subsidy can increase welfare as illustrated in both the left and

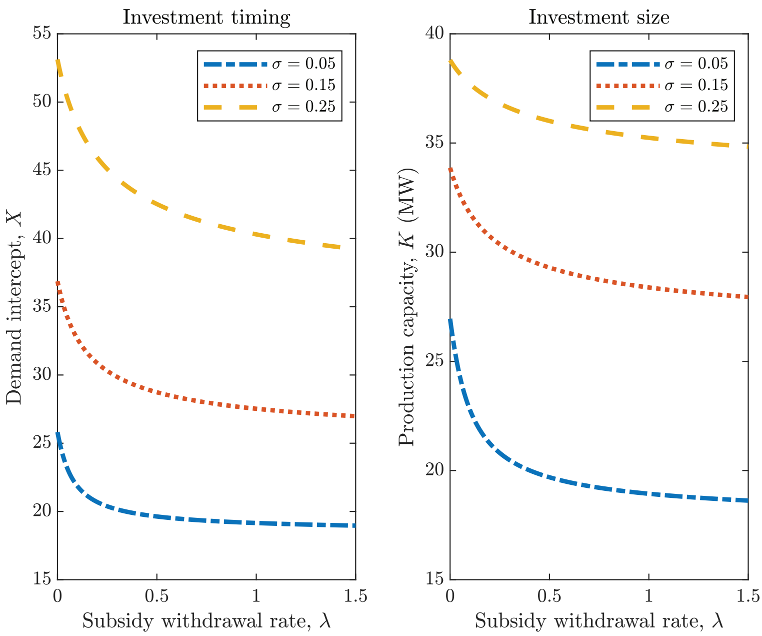

Fig. 4. Investment timing (left) and size (right) as functions of the subsidy withdrawal rate λ. [Parameter values: $\mu = 0.02$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\theta = 0.15$.]

middle panel of Fig. 8. The left panel of Fig. 8 shows that in case of no subsidy retraction risk the total surplus is highest when $\theta = 0.156$, i.e. the lump-sum subsidy is equal to 15.6% of the firm's total investment costs. At $\theta = 0.156$, the total surplus is equal to 429.79, while the first-best outcome leads to a total surplus of 543.25. This results in a RWL of 20.9% opposed to the 25% when the subsidy is not provided. By implementing the subsidy, the relative welfare loss decreases by approximately 16.4%. The increase in welfare is the result of the fact that, under no withdrawal risk, the firm invests earlier and in the same size. This increases both the discounted consumer surplus and the discounted producer surplus, and these increases outweigh the costs of providing the subsidy. This result holds when there is no policy risk. We now study how policy risk affects this result.

The middle panel of Fig. 8 shows the total surplus if there is a low subsidy retraction risk. If we introduce only a small probability of subsidy withdrawal by setting $\lambda = 0.0001$, the optimal subsidy size is slightly smaller and equal to $\theta^* = 0.135$ compared to when there is no risk of subsidy retraction ($\theta^* = 0.156$). Introducing a probability of a subsidy retraction, results in that the investment is done sooner and, therefore, with a smaller capacity. Decreasing the subsidy size makes the firm postpone investment. When it invests, it, therefore, invests in a larger size. Thus, decreasing the subsidy size counters the effect of the increased probability of subsidy retraction. Comparing the middle panel with the left panel in Fig. 8, we observe that for any given subsidy size the total surplus decreases when there is subsidy retraction risk.

Assuming a slightly larger subsidy withdrawal risk by setting $\lambda = 0.001$, it in fact becomes optimal not to introduce a subsidy at all. This is because the firm has a strong incentive to invest early, but therefore, in a small capacity. The investment is done too early and at a too small scale from a welfare-maximizing point of view. Therefore, when policy risk is large, it is best for social welfare not to offer a subsidy at all.

6. Discussions

Next, we discuss the effect of alternative assumptions on our results. We discuss the effect of different types of subsidies, the effect of the firm having the option to expand, and the effect of the social planner's discount rate in this section. A detailed analysis of the effect of a different demand function is included in Appendix B, in which we assume an isoelastic demand function.

Firstly, we compare our results under a lump-sum subsidy with the (expected) results under two different types of subsidies: the feed-in tariff (FIT) and feed-in premium (FIP). Feed-in policies (i.e. tariffs and premiums) are still widely used. By the end of 2019, they were in place in 113 jurisdictions at the national, state or provincial levels [REN21, 2020]. The main difference between a lump-sum subsidy on the one hand and the FIT and FIP on the other hand is that the lump-sum subsidy is a one-time transfer at the time of investment, while both the FIT and FIP payments happen during the project life-time. This difference is also the key explanatory factor in the difference in conclusions.

We find that, under a lump-sum subsidy, an increase in the subsidy withdrawal risk, lowers the firm's investment threshold and decreases its investment size. Chronopoulos et al. [2016] studies investment under subsidy withdrawal risk under a FIP and draws the same conclusion when the risk of subsidy withdrawal is low. This is the result of a firm wanting to obtain subsidy and it is being threatened the subsidy may disappear in the near future. When the risk of subsidy withdrawal is high, this effect disappears for the FIP, but not for the lump-sum subsidy. In case of a FIP, a firm increases its investment threshold and increases its investment size when the subsidy withdrawal risk of withdrawal increases. The firm's gain from a feed-in premium is obtained from production, hence a firm only invests when either the output price is high or when the expected lifetime of the feed-in premium is substantial. This is different from the lump-sum subsidy, for which the gain is fully obtained at the moment of investment.

Boomsma et al. [2012] studies the effect of FITs on investment. Assuming there is no risk of subsidy withdrawal, Boomsma et al. [2012] conclude that FITs encourage earlier investment. The firm invests earlier under a FIT as it is protected from risk on the market. When accounting for the risk of subsidy withdrawal, the firm faces a trade-off similar to the scenario in which the subsidy available is a FIP. We would expect both the investment threshold and investment size to go down (up)

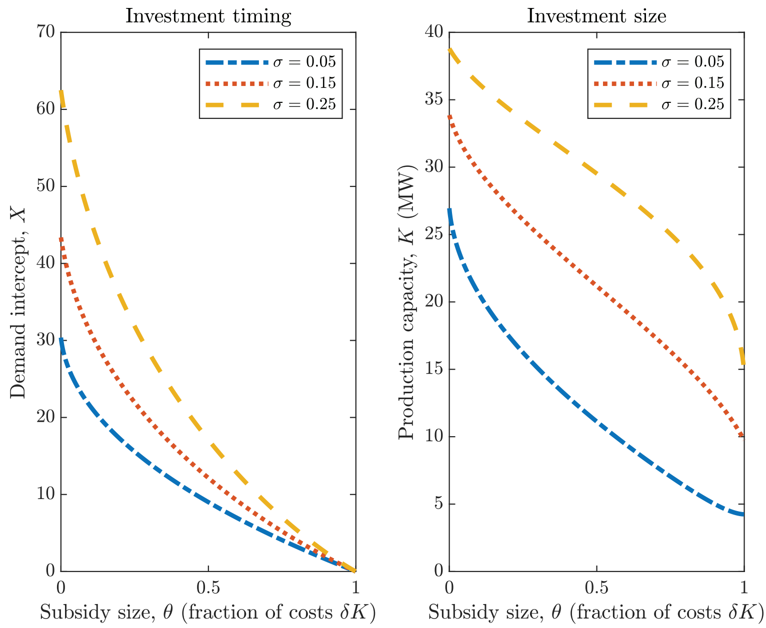

Fig. 5. Investment timing (left) and size (right) as functions of the subsidy size θ. [Parameter values: $\mu = 0.02$, $r = 0.06$, $\eta = 0.01$, $\delta = 350$ and $\lambda = 1$.]

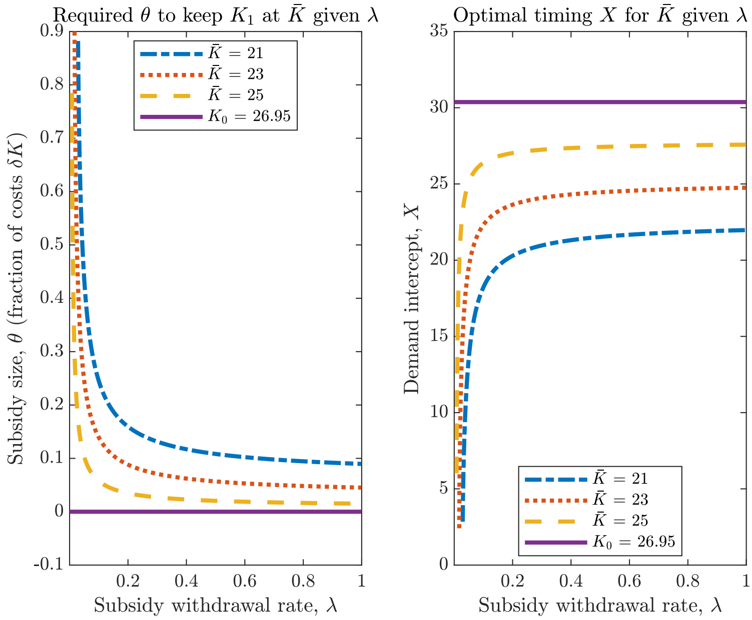

Fig. 6. Subsidy size (left) and optimal investment timing (right) as functions of subsidy withdrawal rate λ for different capacity targets. [Parameter values: $\mu = 0.02, \sigma = 0.05, r = 0.06, \eta = 0.01, \delta = 350$.]

with retraction risk when the risk of retraction is low (high). The trade-off consists of two opposing effects. Firstly, the firm has an incentive to invest sooner in order to still obtain the subsidy. The firm would then also invest in a smaller size. Secondly, it wants to keep its revenue high also in the case when the FIT is retracted. Hence, it has the incentive to increase its investment threshold to make sure output prices are sufficiently high. In this case, the firm would increase its investment size.

Secondly, we discuss the case in which the firm has the option to expand the renewable energy capacity by investment in new locations after. This means it faces a sequential investment decision. In the case of sequential investment, a firm can invest early to take advantage of the available subsidy, while still being able to scale up investment later if output prices are high. This provides it with more flexibility. We expect that this leads to the firm investing sooner to obtain subsidy and also investing more in the long-run if output prices are high.

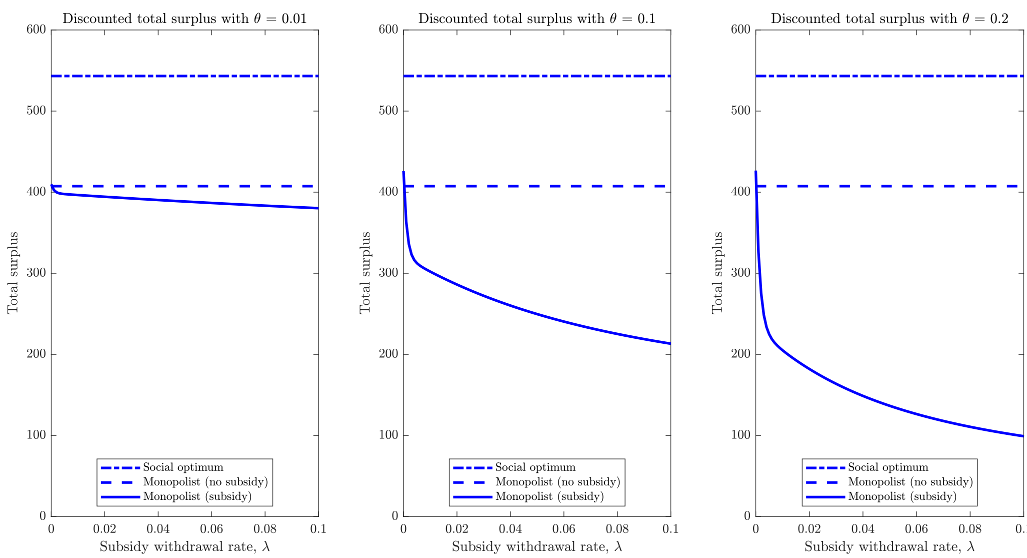

Fig. 7. Total surplus as a function of the subsidy retraction risk λ for different subsidy sizes θ. [Parameter values: $\mu = 0.02, \sigma = 0.05, r = 0.06, \eta = 0.01$ and $\delta = 350$.]

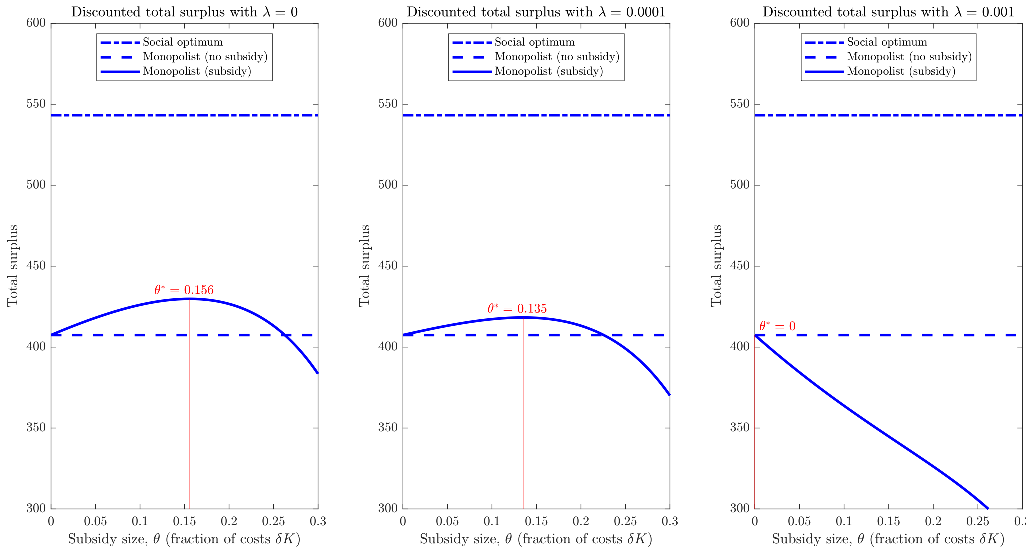

Fig. 8. Total surplus as a function of the subsidy size θ for different levels of subsidy withdrawal risk λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\eta = 0.01$ and $\delta = 350$.]

Lastly, we discuss the effect of a difference between the social planner's and the firm's discount rate. The firm's investment size and quantity are affected in the same way by a lump-sum subsidy under withdrawal risk as discussed in Sections 3 and 4: both a higher withdrawal risk and a higher subsidy size speed up investment and decrease the investment size. In case the social planner maximizes total surplus and has a higher discount rate than the firm, it prefers that the firm invests sooner than the firm would without subsidy. Therefore, the larger the social planner's discount rate, the larger its optimal subsidy.

7. Conclusions

This paper studies the effect of a lump-sum subsidy subject to risk of retraction on optimal investment decisions in terms of timing and capacity size installed. We find that increasing the likelihood of subsidy withdrawal gives the firm an incentive to invest sooner to still obtain the subsidy. As the firm invests sooner, it also invests in a smaller size. The same effect, i.e. investing sooner in a smaller size, is obtained by increasing the subsidy size under positive subsidy withdrawal risk.

Since the firm does not take into account the consumer surplus when investing, it has less incentives to invest than a social planner maximizing total surplus. When demand is linear, a profit-maximizing firm invests at the right time but in a too small capacity. When demand is isoelastic, the firm does invest in the same capacity as the social planner, but the profit-maximizing firm invests later. We find that in both cases a lump-sum subsidy can increase welfare when there is no subsidy retraction risk, but it harms welfare when there is substantial subsidy retraction risk. Therefore, a social planner maximizing welfare should try to minimize the subsidy retraction risk. If subsidy retraction risk increases, the socially optimal subsidy size decreases, and welfare decreases rapidly as the firm invests in a much too small size from a socially optimal point of view.

In case the policy maker aims to reach a capacity target that is smaller than the firm's optimal investment size without subsidy, implementing a lump-sum subsidy can speed up the firm's investment. If the policy maker sets a capacity target that is larger than the firm's optimal investment size, the only way to achieve the target is to implement a subsidy that is provided conditional on the firm investing in the right capacity size.

Our model can be extended for the case in which the firm is able to receive signals on future government decisions, so that it can update its beliefs about the possibility of a subsidy retraction. Pawlina and Kort [2005] propose a model with consistent authority behavior, which takes into account that the government will only intervene at a certain price level, but they only consider the investment timing decision and not the investment size decision. Dalby et al. [2018] provide a model in which firms receive signals and can learn about the timing of subsidy revision. However, their model does not account for a firm's investment timing and capacity size decisions.

Appendix A. Proofs of theorems and propositions

A.1. Proof of corollary 1

Proof of Corollary 1. This proof shows that the expression for $K_1(X)$ (expression (2.9)) holds for $X > X_1$. The proof that Eq. (2.8) is correct for $X > X_0$ follows the same steps.

The optimal investment size $K = K_1^*$ maximizes $V_1(K,X)$ for $X > X_1$. Since $\frac{d^2V_1}{dK^2} = -\frac{2\eta X}{r-\mu} < 0$ for $X > 0$, it holds that $V_1(K,X)$ is concave in K as $X > X_1 > 0$. Therefore the first order condition, $\frac{dV_1}{dK} = 0$, can be applied here.

$$\frac{dV_1}{dK} = 0 \Leftrightarrow \frac{X(1-2\eta K)}{r-\mu} - (1-\theta)\delta = 0 \quad (A.1)$$

$$\Leftrightarrow K_1(X) = \frac{1}{2\eta}\left(1 - \frac{(1-\theta)\delta(r-\mu)}{X}\right) \quad (A.2)$$

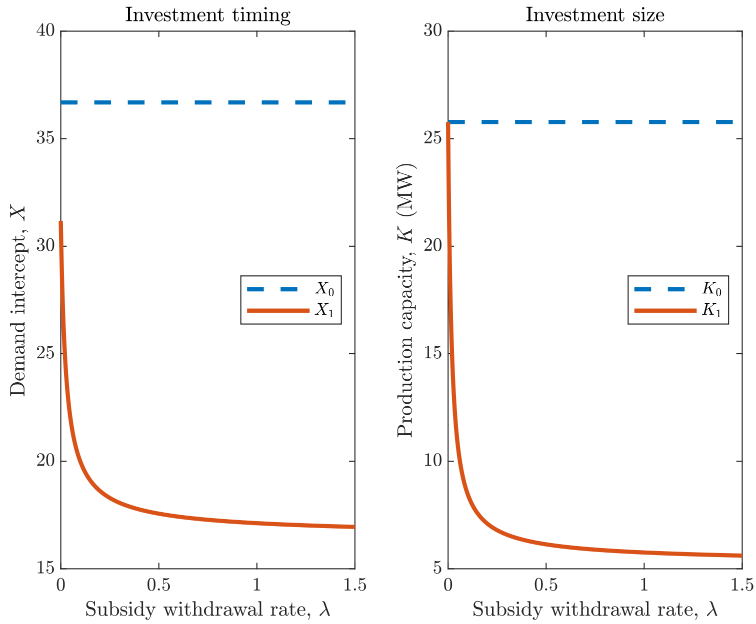

Fig. 9. Investment timing (left) and size (right) under iso-elastic demand as functions of the subsidy retraction risk λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$, $\gamma = 0.4$ and $\theta = 0.15$.]

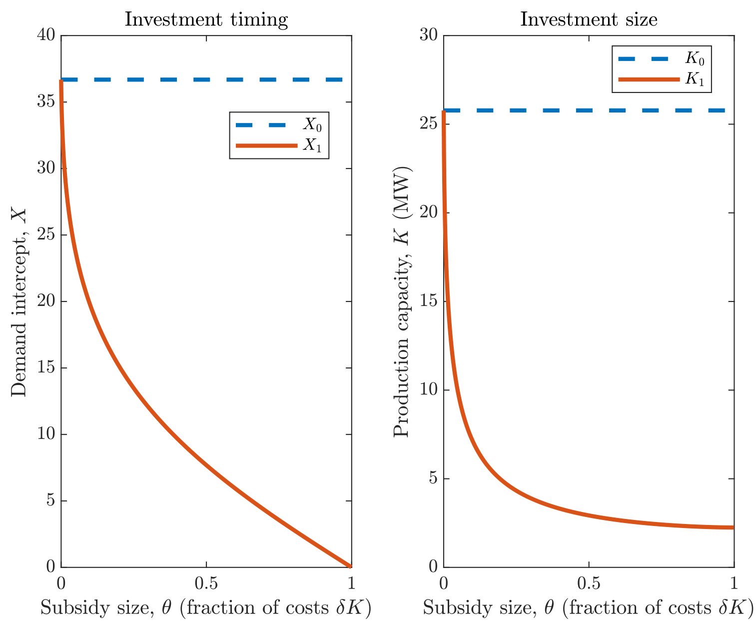

Fig. 10. Investment timing (left) and size (right) under iso-elastic demand as functions of the subsidy size θ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$, $\gamma = 0.4$ and $\lambda = 1$.]

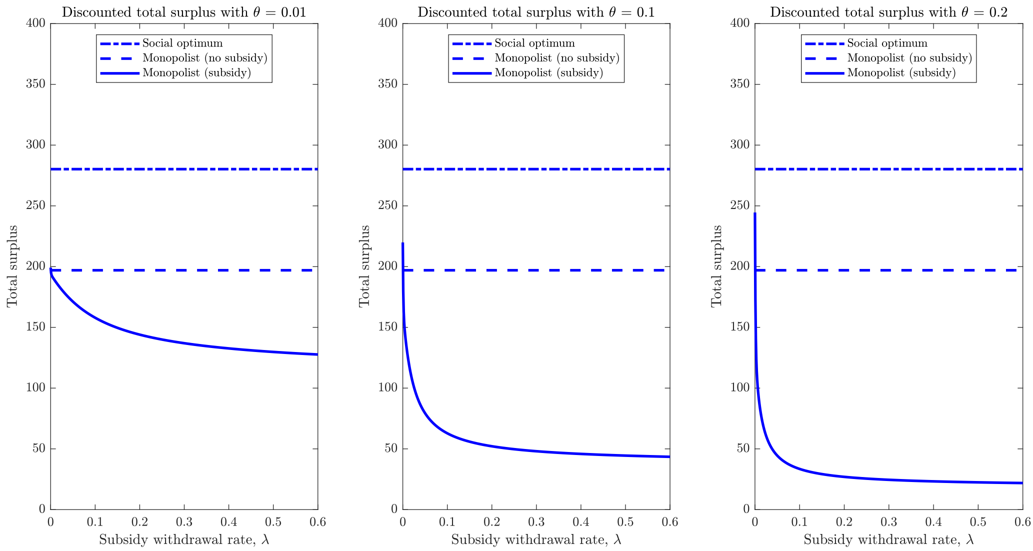

Fig. 11. Total surplus under iso-elastic demand as a function of the subsidy retraction risk λ for different subsidy sizes θ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$ and $\gamma = 0.4$.]

A.2. Proof of proposition 1

Proof of Proposition 1. *Firstly, looking at the value of the investment option without the subsidy, we can follow* Huisman and Kort [2015] *as there is no subsidy uncertainty in this case. When $X > X_0$, it is optimal to invest, and we have*:

$$V_0(X,K) = \frac{X(1-\eta K)K}{r-\mu} - \delta K \tag{A.3}$$

When $X < X_0$, it is optimal to wait with investing. It can be shown that the following holds for $V_0(X)$, the value of the investment at level X when the policy has been withdrawn (see, e.g., Dixit and Pindyck, 1994):

$$\frac{1}{2}\sigma^2 X^2 V_0''(X) + \mu X V_0'(X) - r V_0(X) = 0 \tag{A.4}$$

Solving this ordinary differential equation yields $V_0(X) = A_0 X^{\beta_{01}} + B_0 X^{\beta_{02}}$. In this expression, A_0 and B_0 are constants that remain to be determined. β_{01} (β_{02}) is the positive (negative) solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. Since $V_0(0) = 0$ and $\beta_{02} < 0$, it follows that $B_0 = 0$, hence:

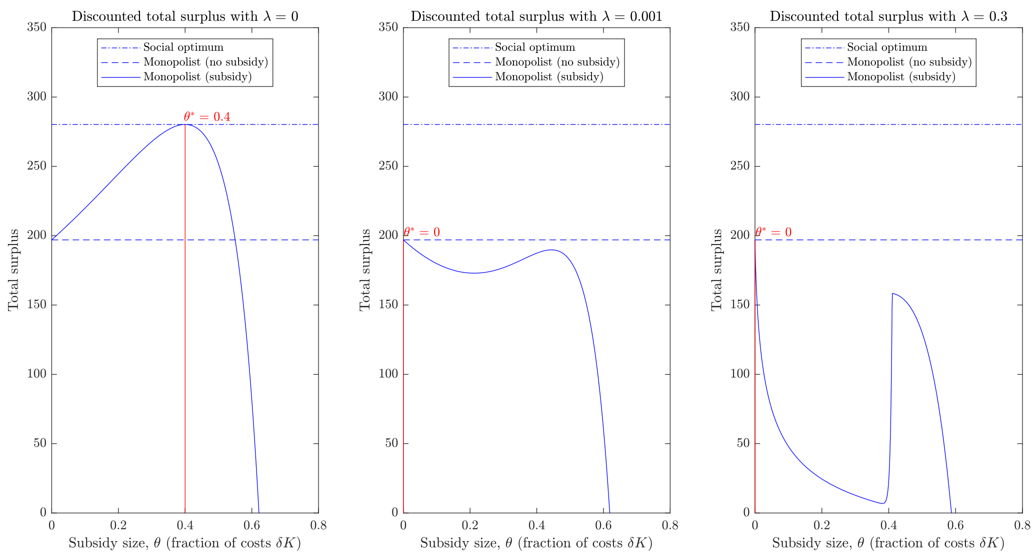

Fig. 12. Total surplus under iso-elastic demand as a function of the subsidy size θ for different levels of subsidy withdrawal risk λ. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$ and $\gamma = 0.4$.]

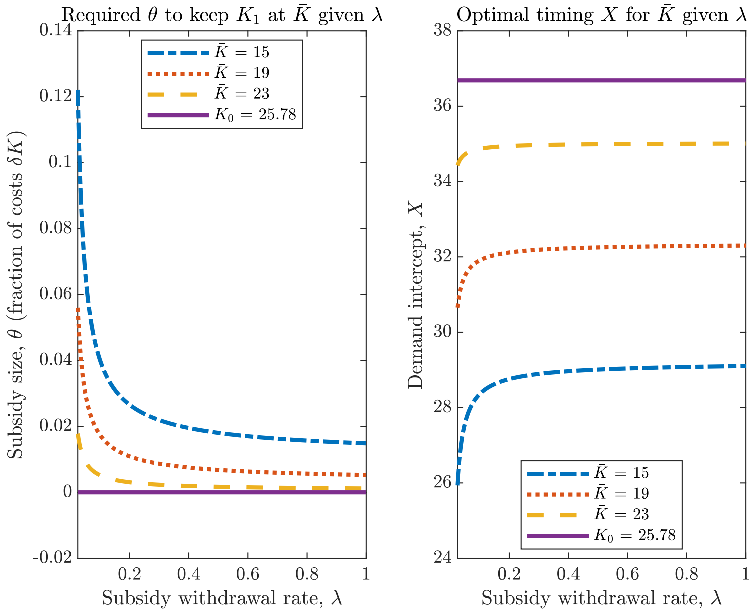

Fig. 13. Subsidy size (left) and optimal investment timing (right) under iso-elastic demand as functions of the subsidy withdrawal rate λ for different capacity targets. [Parameter values: $\mu = 0.02$, $\sigma = 0.05$, $r = 0.06$, $\delta_1 = 150$, $\delta_2 = 200$ and $\gamma = 0.4$.]

$$V_0(X) = A_0X^{\beta_{01}} \tag{A.5}$$

Combining expressions (A.3) and (A.5) yields the expression (2.10) for V_0.

Secondly, we derive expression (2.11) for V_1. When $X > X_1$, it is optimal to invest and the value of the option to invest when the subsidy is in effect is equal to:

$$V_1(X,K) = \frac{X(1-\eta K)K}{r-\mu} - (1-\theta)\delta K \tag{A.6}$$

For $X < X_1$, it holds that it is best to wait. The investment option while the policy is active satisfies the following ordinary differential equation:

$$\frac{1}{2}\sigma^2X^2V_1''(X) + \mu XV_1'(X) - rV_1(X) + \lambda(V_0(X) - V_1(X)) = 0 \tag{A.7}$$

The main difference with Eq. (A.4) is the addition of the term $\lambda(V_0(X) - V_1(X))$, which has been added as the value of the option to invest can drop from V_1 to V_0 if the subsidy is retracted while we wait. Since $X < X_1$ means $X < X_0$, we have $V_0(X) = A_0X^{\beta_{01}}$ for $X < X_1$. Solving the homogeneous part of the above ordinary differential equation yields solution $V_1^H(X) = A_1X^{\beta_{11}} + B_1X^{\beta_{12}}$. β_{11} (β_{12}) is the positive (negative) solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r+\lambda) = 0$.

To find a particular solution to the ordinary differential equation in (A.7), one can try $V_1^P(X) = C_1X^{\beta_{01}}$, as the in-homogeneous part is $A_0X^{\beta_{01}}$. From this it follows that $C_1 = A_0$. Combining the homogeneous and particular solution gives $V_1(X) = A_1X^{\beta_{11}} + B_1X^{\beta_{12}} + A_0X^{\beta_{01}}$. However, as $V_1(0) = 0$ and $\beta_{12} < 0$, it follows that $B_1 = 0$.

This results in the following expression for $V_1(X)$:

$$V_1(X) = A_1X^{\beta_{11}} + A_0X^{\beta_{01}} \tag{A.8}$$

where A_1 and A_0 are constants that needs be determined. As before, β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. Combining expressions (A.6) and (A.8) yields expression (2.11) for V_1.

A.3. Proof of proposition 2

Proof of Proposition 2. *The constant A_0 and thresholds X_0 satisfy the value matching and smooth pasting condition for V_0. The value matching equation for V_0 is (A.9), which guarantees that the value for $V_0(X_0, K_0^*)$ is uniquely defined.*

$$A_0X_0^{\beta_{01}} = \frac{X_0(1-\eta K_0^*)K_0^*}{r-\mu} - \delta K_0^* \tag{A.9}$$

Apart from value matching condition, there is also a smooth pasting condition for V_0. Eq. (A.10) guarantees that $\frac{dV_0}{dX}$ has a unique value at $X = X_0$.

$$A_0\beta_{01}X_0^{\beta_{01}-1} = \frac{(1-\eta K_0^*)K_0^*}{r-\mu} \tag{A.10}$$

Multiplying (A.9) by β_{01} and subtracting X_0 times (A.10) from it yields:

$$0 = (\beta_{01}-1)\cdot\frac{X_0(1-\eta K_0^*)K_0^*}{r-\mu} - \beta_{01}\delta K_0^* \tag{A.11}$$

$$\Leftrightarrow X_0(1-\eta K_0^*) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\delta(r-\mu) \tag{A.12}$$

Plugging the expression for the optimal capacity K_0^* (see expression (2.8)) into (A.12) and rewriting this equation results in:

$$X_0 = \frac{\beta_{01}+1}{\beta_{01}-1} \cdot \delta(r-\mu) \tag{A.13}$$

Substituting the expression (A.13) for X_0 into (2.8) yields an expression for the optimal capacity when the subsidy is not available.

$$K_0(X_0) = [\eta(\beta_{01}+1)]^{-1} \tag{A.14}$$

A.4. Proof of proposition 3

Proof of Proposition 3. *The constant A_1 and threshold X_1 satisfy the value matching and smooth pasting conditions for V_1. The value matching equation is (A.15), which guarantees that the value for $V_1(X_1,K_1^*)$ is uniquely defined.*

$$A_1 X_1^{\beta_{11}} + A_0 X_1^{\beta_{01}} = \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} - (1-\theta)\delta K_1^* \tag{A.15}$$

Apart from value matching condition, there is also smooth pasting condition (A.16), which guarantees that $\frac{dV_1}{dX}$ has a unique value at $X = X_1$.

$$A_1 \beta_{11} X_1^{\beta_{11}-1} + A_0 \beta_{01} X_1^{\beta_{01}-1} = \frac{(1-\eta K_1^*)K_1^*}{r-\mu} \tag{A.16}$$

Subtracting $\frac{X_1}{\beta_{11}}$ times Eq. (A.16) from (A.15) yields:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot A_0 X_1^{\beta_{01}} = \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} - (1-\theta)\delta K_1^* \tag{A.17}$$

Rearranging terms in (A.17) leads to:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot A_0 X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} + (1-\theta)\delta K_1^* = 0 \tag{A.18}$$

In the above, an expression for A_0 can be derived by rewriting Eq. (A.10) and subsequently substituting the derived expressions for X_0 and K_0^*:

$$A_0 = \frac{\delta}{\eta\left(\beta_{01}^2-1\right)} \cdot X_0^{-\beta_{01}} \tag{A.19}$$

A.5. Proof of proposition 4

Proof of Proposition 4. To derive the optimal capacity from a social welfare point of view, we take the first order condition of TS with respect to K, similar to deriving the optimal capacity for the profit-maximizing firm, see the proof in Appendix A.1.

We take the same steps as the proof in Appendix A.2 when determining the expression for V_0 to derive the value of the option to invest for the social planner.

The threshold for the social planner X_S satisfies the value matching and smooth pasting conditions. The value matching equation is:

$$A_S X_S^{\beta_{01}} = \frac{X_S(2-\eta K_S(X_S))K_S(X_S)}{2(r-\mu)} - \delta K_S(X_S) \tag{A.20}$$

and the smooth pasting condition is:

$$A_S \beta_{01} X_S^{\beta_{01}-1} = \frac{(2-\eta K_S(X_S))K_S(X_S)}{2(r-\mu)} \tag{A.21}$$

The interpretation of the value matching and smooth pasting conditions are the same as the value matching and smooth pasting conditions for the profit-maximizer, which are discussed in Section 2.

The threshold X_S can be derived using the same steps as in Appendix A.3 and even yields:

$$X_S = \frac{\beta_{01}+1}{\beta_{01}-1} \cdot \delta(r-\mu) \tag{A.22}$$

A.6. Proof of proposition 5

Proof of Proposition 5. *We start by proving the first statement of this proposition:*

$$\frac{dX_1}{d\lambda} < 0 \Leftrightarrow X_1 \leq \frac{\beta_{01}\beta_{11} + \sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)} \cdot (1-\theta)\delta(r-\mu) \tag{A.23}$$

To derive the effect of subsidy retraction risk λ on timing threshold X_1, we only have to look at the direct effect of λ on X_1, as there is no indirect effect via investment size, since $\frac{\partial K_1^*}{\partial \lambda} = 0$. Therefore:

$$\frac{dX_1}{d\lambda} = \frac{\partial X_1}{\partial \lambda} \tag{A.24}$$

Let implicit Eq. (2.14) be denoted by f. To derive $\frac{\partial X_1}{\partial \lambda}$, we apply total differentiation to f:

$$0 = \frac{df}{d\lambda} = \frac{\partial f}{\partial \lambda} + \frac{\partial f}{\partial X} \cdot \frac{\partial X_1}{\partial \lambda} \Leftrightarrow \frac{\partial X_1}{\partial \lambda} = -\frac{\left(\frac{\partial f}{\partial \lambda}\right)}{\left(\frac{\partial f}{\partial X}\right)} \tag{A.25}$$

We are going to show that $\frac{\partial f}{\partial \lambda} < 0$ always holds, and $\frac{\partial f}{\partial X} < 0$ if and only if condition (3.2) holds.

To derive $\frac{\partial f}{\partial \lambda}$, we can use that $\frac{\partial K_1}{\partial \lambda} = 0$. This gives:

$$\begin{aligned} \frac{\partial f}{\partial \lambda} &= \frac{\beta_{11} \cdot \frac{d\beta_{11}}{d\lambda} - (\beta_{11}-\beta_{01}) \frac{d\beta_{11}}{d\lambda}}{\beta_{11}^2} \cdot A_0 X_1^{\beta_{01}} \\ &- \frac{\beta_{11} \cdot \frac{d\beta_{11}}{d\lambda} - (\beta_{11}-1) \frac{d\beta_{11}}{d\lambda}}{\beta_{11}^2} \cdot \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} \\ &= \frac{1}{\beta_{11}^2} \cdot \frac{d\beta_{11}}{d\lambda} \left(\beta_{01} A_0 X_1^{\beta_{01}} - \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} \right) \end{aligned} \tag{A.26}$$

where

$$\frac{d\beta_{11}}{d\lambda} = \frac{1}{\sigma^2(\beta_{11}-\frac{1}{2}) + \mu} > 0$$

We rewrite the smooth pasting condition (see Eq. (A.16)) as:

$$\beta_{01} A_0 X_1^{\beta_{01}} = \frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu} - \beta_{11} A_1 X_1^{\beta_{11}} \tag{A.27}$$

and plug (A.27) into (A.26). This gives:

$$\frac{\partial f}{\partial \lambda} = -\frac{1}{\beta_{11}} \cdot \frac{d\beta_{11}}{d\lambda} \cdot A_1 X_1^{\beta_{11}} < 0 \tag{A.28}$$

To prove $\frac{\partial f}{\partial X} < 0$ if and only if condition (3.2) holds, we start with taking the partial derivative of f with respect to X. Note that we also need to account for the derivative of the optimal investment size under subsidy with respect to the timing evaluated at the optimal timing threshold, which we denote by $\frac{dK_1^*}{dX}$. Taking the partial derivative of f with respect to X gives the following, after using that $\frac{X_1(1-2\eta K_1^*)}{r-\mu} = (1-\theta)\delta$ can be derived from the expression for K_1^* (substituting $X = X_1$ into (2.9)), and rearranging terms:

$$\begin{aligned}\frac{\partial f}{\partial X} &= \frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\beta_{01}A_0X_1^{\beta_{01}-1}-\frac{\beta_{11}-1}{\beta_{11}}\cdot\\ &\left(\frac{(1-\eta K_1^*)K_1^*}{r-\mu}+\frac{X_1(1-2\eta K_1^*)}{r-\mu}\cdot\frac{dK_1}{dX}\right)+(1-\theta)\delta\cdot\frac{dK_1^*}{dX}\end{aligned} \tag{A.29}$$

$$\begin{aligned}&=\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\beta_{01}A_0X_1^{\beta_{01}-1}-\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{(1-\eta K_1^*)K_1^*}{r-\mu}\\ &+\frac{1}{\beta_{11}}\cdot(1-\theta)\delta\cdot\frac{dK_1^*}{dX}\end{aligned} \tag{A.30}$$

$$\begin{aligned}&=\frac{\beta_{01}}{X_1}\left(\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}}-\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu}\right)\\ &+(\beta_{01}-1)\cdot\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{(1-\eta K_1^*)K_1^*}{r-\mu}+\frac{1}{\beta_{11}}\cdot(1-\theta)\delta\cdot\frac{dK_1^*}{dX}\end{aligned} \tag{A.31}$$

The term $\frac{dK_1^*}{dX}$ is the derivative of $K_1(X)$ with respect to X evaluated at X_1 and can be rewritten into terms of X_1 and K_1^* as follows:

$$\frac{dK_1^*}{dX}=\left.\frac{dK_1}{dX}\right|_{X=X_1}=\frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1^2}=\frac{1}{X_1}\left(\frac{1}{2\eta}-K_1^*\right) \tag{A.32}$$

Note that the term between the brackets in the first line of Eq. (A.31) can be substituted out by using the implicit Eq. (2.14), i.e. $\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0X_1^{\beta_{01}}-\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta K_1^*)K_1^*}{r-\mu}=-(1-\theta)\delta K_1^*$. Using this fact, combined with Eq. (A.32) for $\frac{dK_1^*}{dX}$ and Eq. (2.9) evaluated at $X = X_1$ for K_1^* reduces (A.31) after some algebra to:

$$\frac{\partial f}{\partial X}=\frac{1}{4\eta X_1^2(r-\mu)}\cdot g(X_1) \tag{A.33}$$

where

$$\begin{aligned}g(X_1) &= (\beta_{01}-1)(\beta_{11}-1)X_1^2-2\beta_{01}\beta_{11}(1-\theta)\delta(r-\mu)X_1+\\ &(\beta_{01}+1)(\beta_{11}+1)(1-\theta)^2\delta^2(r-\mu)^2.\end{aligned} \tag{A.34}$$

Since $\frac{1}{4\eta X_1^2(r-\mu)}>0$, we conclude that $\frac{\partial f}{\partial X}<0$ if and only if $g(X_1) < 0$. It is straightforward that g is a parabola that opens upward for $\beta_{11} \ge \beta_{01} > 1$. The two zeros are at:

$$X_{g,L}=\frac{\beta_{01}\beta_{11}-\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \tag{A.35}$$

$$X_{g,R}=\frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \tag{A.36}$$

Since $\frac{dX_1}{d\lambda}\le 0$ if and only if $g(X_1) < 0$, we can conclude that $\frac{dX_1}{d\lambda}\le 0$ if and only if $X_1 \in (X_{g,L}, X_{g,R})$ always holds. Since $X_1 \le X_{g,R}$ is the condition (3.2), only a lower bound on X_1, X_{min}, meeting the requirement $X_{g,L} \le X_{min}$ needs to be shown.

A lower bound on X_1 is found by assuming all value is lost after subsidy withdrawal, i.e. $A_0 = 0$. Then solving implicit (2.14), we find $X_{min}=\frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu)$. To show $X_{g,L} \le X_{min}$, we rewrite it as follows:

$$X_{g,L}\le X_{min}\Leftrightarrow\frac{\beta_{01}\beta_{11}-\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\le\frac{\beta_{11}+1}{\beta_{11}-1} \tag{A.37}$$

$$\Leftrightarrow\beta_{01}\beta_{11}-\sqrt{\beta_{01}^2+\beta_{11}^2-1}\le(\beta_{11}+1)(\beta_{01}-1) \tag{A.38}$$

$$\Leftrightarrow\beta_{11}-\beta_{01}+1\le\sqrt{\beta_{01}^2+\beta_{11}^2-1} \tag{A.39}$$

Since $\beta_{11} - \beta_{01} + 1 > 0$, we can square both sides, and the inequality still holds. Therefore:

$$X_{g,L}\le X_{min}\Leftrightarrow(\beta_{11}-\beta_{01}+1)^2\le\beta_{01}^2+\beta_{11}^2-1 \tag{A.40}$$

$$\Leftrightarrow-2\beta_{01}\beta_{11}+2\beta_{11}-2\beta_{01}+2\le 0 \tag{A.41}$$

$$\Leftrightarrow 2(\beta_{11}+1)(1-\beta_{01})\le 0 \tag{A.42}$$

which holds since $\beta_{11} \ge \beta_{01} \ge 1$.

Next, we prove the second part of Proposition 5:

$$\frac{dK_1}{d\lambda}<0\Leftrightarrow X_1\le\frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \tag{A.43}$$

We apply total differentiation to K_1^* to get:

$$\frac{dK_1}{d\lambda}=\frac{\partial K_1}{\partial\lambda}+\frac{\partial K_1}{\partial X}\cdot\frac{\partial X_1}{\partial\lambda} \tag{A.44}$$

Since $K_1(X)=\frac{1}{2\eta}\left(1-\frac{(1-\theta)\delta(r-\mu)}{X}\right)$, we have $\frac{\partial K_1}{\partial\lambda}=0$ and $\frac{\partial K_1}{\partial X}=\frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X^2}>0$.

As shown previously, if and only if $X_1\le\frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu)$, we conclude:

$$\frac{\partial X_1}{\partial\lambda}\le 0 \tag{A.45}$$

Therefore, if and only if $X_1\le\frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu)$, we have that

$$\frac{dK_1}{d\lambda}=0+\frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1^2}\cdot\frac{\partial X_1}{\partial\lambda}\le 0 \tag{A.46}$$

A.7. Proof of proposition 6

Proof of Proposition 6. *We start the proof by showing that*

$$\frac{dX_1}{d\theta}<0\Leftrightarrow X_1\le\frac{\beta_{01}\beta_{11}+\sqrt{\beta_{01}^2+\beta_{11}^2-1}}{(\beta_{01}-1)(\beta_{11}-1)}\cdot(1-\theta)\delta(r-\mu) \tag{A.47}$$

Taking the total differential of X_1 with respect to θ yields:

$$\frac{dX_1}{d\theta}=\frac{\partial X_1}{\partial\theta}+\frac{\partial X_1}{\partial K}\cdot\frac{\partial K_1}{\partial\theta} \tag{A.48}$$

We can directly derive $\frac{\partial K_1}{\partial\theta}$ from the closed-form expression of K_1^*, Eq. (2.9), yielding:

$$\frac{\partial K_1}{\partial\theta}=\frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1}>0 \tag{A.49}$$

Furthermore, after rewriting (Eq. (2.9)) to

$$X_1(K)=\frac{(1-\theta)\delta(r-\mu)}{1-2\eta K} \tag{A.50}$$

and using (Eq. (2.9)) evaluated at $X = X_1$ for K_1^*, it follows that:

$$\frac{\partial X_1}{\partial K} = \frac{2\eta X_1^2}{(1-\theta)\delta(r-\mu)} > 0 \tag{A.51}$$

Thus, the indirect effect of subsidy size on timing is captured by:

$$\frac{\partial X_1}{\partial K}\cdot\frac{\partial K_1}{\partial \theta} = \frac{2\eta X_1^2}{(1-\theta)\delta(r-\mu)}\cdot\frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} = \frac{X_1}{1-\theta} > 0 \tag{A.52}$$

Therefore, $\frac{dX_1}{d\theta} < 0$ if and only if

$$\frac{\partial X_1}{\partial \theta} < -\frac{\partial X_1}{\partial K}\cdot\frac{\partial K_1}{\partial \theta} = -\frac{X_1}{1-\theta} \tag{A.53}$$

Let f be the implicit Eq. (2.14). To derive the $\frac{\partial X_1}{\partial \theta}$, we apply total differentiation to f:

$$0 = \frac{df}{d\theta} = \frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial X}\cdot\frac{\partial X_1}{\partial \theta} \Leftrightarrow \frac{\partial X_1}{\partial \theta} = -\frac{\left(\frac{\partial f}{\partial \theta}\right)}{\left(\frac{\partial f}{\partial X}\right)} \tag{A.54}$$

A larger subsidy size decreases the investment threshold if and only if:

$$\frac{dX_1}{d\theta} = -\frac{\left(\frac{\partial f}{\partial \theta}\right)}{\left(\frac{\partial f}{\partial X}\right)} + \frac{X_1}{1-\theta} < 0 \tag{A.55}$$

We will show that $\frac{\partial f}{\partial \theta} < 0$ always holds, and, therewith, condition (A.55) can only hold if $\frac{\partial f}{\partial X} < 0$. $\frac{\partial f}{\partial \theta}$ is derived via partial differentiation on implicit equation f:

$$\frac{\partial f}{\partial \theta} = -\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-2\eta K_1^*)}{r-\mu}\cdot\frac{\partial K_1}{\partial \theta} + (1-\theta)\delta\cdot\frac{\partial K_1}{\partial \theta} - \delta K_1^* \tag{A.56}$$

From the first order condition with respect to capacity, it can be shown that $\frac{X_1(1-2\eta K_1^*)}{r-\mu} = (1-\theta)\delta$. Therefore, we can derive the following:

$$\frac{\partial f}{\partial \theta} = \left(-\frac{\beta_{11}-1}{\beta_{11}} + 1\right)(1-\theta)\delta\cdot\frac{\partial K_1}{\partial \theta} - \delta K_1^* \tag{A.57}$$

$$= \left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1} - 1\right)\frac{\delta}{2\eta} \tag{A.58}$$

We first note $\frac{\partial f}{\partial \theta}$ is monotonically decreasing in X_1 for $X_1 > 0$. As shown in the proof in Appendix A.6, $X_1 \geq \frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu)$ holds. Therefore, we can show that $\frac{\partial f}{\partial \theta} < 0$:

$$\frac{\partial f}{\partial \theta} = \left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1} - 1\right)\frac{\delta}{2\eta} \tag{A.59}$$

$$\leq \left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{\left(\frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu)\right)} - 1\right)\frac{\delta}{2\eta} \tag{A.60}$$

$$= -\frac{1}{\beta_{11}}\cdot\frac{\delta}{2\eta} \tag{A.61}$$

$$< 0 \tag{A.62}$$

Assuming (3.2) holds, we have that $\frac{\partial f}{\partial X} < 0$, as shown in Proposition 5. Then, (A.55) can be rewritten as:

$$\frac{\partial f}{\partial \theta} - \frac{X_1}{1-\theta}\cdot\frac{\partial f}{\partial X} < 0 \tag{A.63}$$

Plugging in Eqs. (A.58) for $\frac{\partial f}{\partial \theta}$ and (A.33) for $\frac{\partial f}{\partial X}$ into (A.63), condition (A.63) can be rewritten to:

$$\frac{\partial f}{\partial \theta} - \frac{X_1}{1-\theta}\cdot\frac{\partial f}{\partial X} < 0 \Leftrightarrow \frac{1}{4\eta(1-\theta)(\beta_{01}-1)\beta_{11}X_1(r-\mu)}\cdot h(X_1) < 0 \tag{A.64}$$

where

$$h(X_1) = -(\beta_{11}-1)X_1^2 + 2\beta_{11}(1-\theta)\delta(r-\mu)X_1 - (\beta_{11}+1)(1-\theta)^2\delta^2(r-\mu)^2 \tag{A.65}$$

Since $\frac{1}{4\eta(1-\theta)(\beta_{01}-1)\beta_{11}X_1(r-\mu)} > 0$, we have that $\frac{dX}{d\theta} < 0$ if and only if $h(X_1) < 0$. h is a parabola that opens downward with the following two zeros:

$$X_{h,L} = (1-\theta)\delta(r-\mu) \tag{A.66}$$

$$X_{h,R} = \frac{\beta_{11}+1}{\beta_{11}-1}\cdot(1-\theta)\delta(r-\mu) \tag{A.67}$$

We have shown that $X_1 > X_{h,R}$ in the proof of Proposition 5 as $X_{h,R}$ is the lower bound on X_1 by assuming all value is lost after subsidy withdrawal. Therefore, $h(X_1) < 0$ and we conclude that $\frac{dX}{d\theta} < 0$.

Deriving the conditions for $\frac{dK_1}{d\theta} < 0$ if condition (3.2) holds, can be shown by starting with total differentiation:

$$\frac{dK_1}{d\theta} = \frac{\partial K_1}{\partial \theta} + \frac{\partial K_1}{\partial X}\cdot\frac{\partial X_1}{\partial \theta} \tag{A.68}$$

As previously derived:

$$\frac{\partial K_1}{\partial \theta} = \frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} > 0 \tag{A.69}$$

$$\frac{\partial K_1}{\partial X} = \frac{1}{2\eta}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1^2} > 0 \tag{A.70}$$

$$\frac{\partial X_1}{\partial \theta} = -\frac{\left(\frac{\beta_{11}+1}{\beta_{11}}\cdot\frac{(1-\theta)\delta(r-\mu)}{X_1} - 1\right)\frac{\delta}{2\eta}}{\frac{\partial f}{\partial X}} \tag{A.71}$$

We can rewrite expression (A.68) to:

$$\frac{dK_1}{d\theta} = \left(1 + \frac{1-\theta}{X_1}\cdot\frac{\partial X_1}{\partial \theta}\right)\cdot\frac{1}{2\eta}\cdot\frac{\delta(r-\mu)}{X_1} \tag{A.72}$$

When $\frac{dX_1}{d\theta} < 0$ holds, it follows that $\frac{\partial X_1}{\partial \theta} < -\frac{1-\theta}{X_1}$ from (A.53), hence $\frac{dK_1}{d\theta} < 0$.

Appendix B. Robustness under iso-elastic demand

This appendix performs a robustness analysis on the results of Sections 3 and 5 by replacing the linear demand curve (2.1) with an iso-elastic curve. In case of iso-elastic demand, the output price at time t, $P(t)$, is given by:

$$P(t) = X(t)K^{-\gamma}, \tag{B.1}$$

where K is the firm's installed capacity, and $\gamma \in (0, 1)$ is the elasticity parameter. X follows the GBM, defined in (Eq. (2.2)).

For this analysis we make two additional assumptions (see also Huisman and Kort, 2015). Firstly, the costs of investing in a capacity of size K are $\delta_1 K + \delta_2$, where $\delta_2 > 0$ is the fixed cost component. Secondly, we assume $\beta_{01}\gamma > 1$, where β_{01} is defined as before. Under these assumptions the firm's optimal investment decision again consists of a threshold that determines the timing without subsidy, X_0, and an investment size without subsidy, K_0^*.

The firm's optimization problem is given by:

$$F(x,\theta) = \sup_{\{\tau,K\}} \mathbb{E}\Big[\int_\tau^\infty P(t)Ke^{-rt}dt - \Big(1-\theta\cdot 1_{\xi(\tau)}\Big)\cdot(\delta_1 K + \delta_2)e^{-r\tau} \,|\, X(0) = x, \xi(0) = 1\Big] \tag{B.2}$$

with $P(t)$ as in (B.1) and

$$\xi(t) = \begin{cases} 0 & \text{if subsidy retraction has occurred at time } t \text{ or earlier} \\ 1 & \text{otherwise} \end{cases} \tag{B.3}$$

We take the same steps as in Section 2 to solve the optimization problem in (B.2). To do so, we can derive the firm's optimal investment decision when the subsidy has been abolished. The result is stated in Proposition 7.

Proposition 7. *When the subsidy is abolished, the optimal investment threshold is given by:*

$$X_0 = \frac{(K_0^*)^\gamma}{1-\gamma}\cdot\delta_1(r-\mu) \tag{B.4}$$

whereas the corresponding investment size is given by:

$$K_0^* = \frac{\beta_{01}(1-\gamma)}{\beta_{01}\cdot\gamma-1}\cdot\frac{\delta_2}{\delta_1} \tag{B.5}$$

where β_{01} is the positive solution to the fundamental quadratic, as defined in Proposition 1.

Proof. The proof takes the same steps as the proof of Proposition 2 in Appendix A.3 and is therefore omitted.

Proposition 8 presents the firm's optimal investment decision under isoelastic demand when the subsidy is still available, but subject to subsidy retraction risk.

Proposition 8. *If the investment subsidy has not been retracted yet, the optimal investment threshold X_1 is implicitly given by:*

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0 X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1\cdot(K_1^*)^{1-\gamma}}{r-\mu} + (1-\theta)\cdot(\delta_1 K_1^* + \delta_2) = 0 \tag{B.6}$$

in which K_1^* is the optimal capacity under subsidy when investing at $X = X_1$, i.e.:

$$K_1^* = \left(\frac{(1-\gamma)X_1}{(1-\theta)\delta_1(r-\mu)}\right)^{\frac{1}{\gamma}} \tag{B.7}$$

Proof. The proof takes the same steps as the proof of Proposition 3 in Appendix A.4 and is therefore omitted.

Proposition 9 contains the socially optimal investment decision.

Proposition 9. *The social planner maximizes the total surplus, which under iso-elastic demand, is given by:*

$$\mathrm{TS}(X,K) = \frac{XK^{1-\gamma}}{r-\mu} - (\delta_1 K + \delta_2) \tag{B.8}$$

The optimal timing maximizing the total surplus, X_S, is equal to:

$$X_S = (K_S^*)^\gamma\cdot\delta_1(r-\mu) \tag{B.9}$$

The socially optimal capacity, K_S^*, is given by:

$$K_S^* = \frac{\beta_{01}(1-\gamma)}{\beta_{01}\cdot\gamma-1}\cdot\frac{\delta_2}{\delta_1} \tag{B.10}$$

Proof. The proof is omitted as it takes the same steps as the proof of Proposition 4 in Appendix A.5.

We find that the investment threshold of the social planner is lower than the one of the firm when there is no subsidy (i.e. $X_S = (1-\gamma)X_0$). The firm without subsidy and the social planner do optimally invest in the same size (i.e. $K_S^* = K_0^*$). Like with linear demand, also here the social planner is more eager to invest than the firm. However, where in case of linear demand this results in more investment than the firm at the same optimal time, under iso-elastic demand the social planner invests sooner than the firm in the same investment size.

In order to study the robustness of our results in Sections 3 and 5, we provide a numerical example. We consider the following parameter values as in Table 2.

Table 2
Parameter values used in the iso-elastic demand scenario.

Notation	Parameter	Value
μ	Electricity price trend	2%
σ	Electricity price volatility	5%
r	Risk-free interest rate	6%
δ_1	Variable investment cost	150 €/MWh
δ_2	Fixed investment cost	200 €
γ	Demand elasticity	0.4

First, we discuss the results that are robust under iso-elastic demand. In short, the effect of subsidy withdrawal risk and subsidy size on the optimal investment timing and size as well as the effect of subsidy withdrawal risk effect on total surplus remains the same. Furthermore, it also holds that under iso-elastic demand a lump-sum subsidy can only speed up investment and decrease the investment size as a result. The only difference in results between linear and iso-elastic demand, is the optimal subsidy size to maximize welfare under subsidy withdrawal risk.

The effect of subsidy withdrawal risk and subsidy size on the firm's optimal investment decisions are shown in Figs. 9 and 10. Fig. 9 shows the optimal investment timing threshold with subsidy (X_1) and without subsidy (X_0) as functions of subsidy retraction risk λ, as well as the optimal investment size with subsidy (K_1^*) and without subsidy (K_0^*) as functions of subsidy retraction risk λ. Firstly, the results in Fig. 9 are conform Corollary 2. In case of no subsidy retraction risk (i.e. $\lambda = 0$) we have $X_0 < X_1$ and $K_0^* = K_1^*$, which was also the case for linear demand. Secondly, Fig. 9 is conform Proposition 5: the larger the subsidy retraction risk, the lower the optimal investment threshold and the lower the optimal capacity. In Fig. 10, the optimal investment decisions subject to large subsidy retraction risk are shown, where $\lambda = 1$. It shows that increasing subsidy size decreases both the investment threshold and the optimal investment size. These results are similar to Fig. 3.

Next, we present two figures representing total surplus as a function of the welfare retraction probability and the subsidy size. Fig. 11 shows that increasing subsidy retraction risk harms welfare, independent of the size of the subsidy, which coincides with the conclusion drawn from Fig. 7 in the linear demand case. The only difference between the linear demand and iso-elastic demand case is that it could still be optimal in the linear demand case to implement a subsidy in case of a positive but not too significant subsidy withdrawal risk, which is not the case under iso-elastic demand (see Fig. 12).

As can be seen in Fig. 11, a larger subsidy withdrawal risk decreases total surplus and no subsidy withdrawal risk is optimal; both results are identical to the results in Section 5. The sensitivity of the total surplus with respect to the subsidy size parameter is slightly different from before, as the investment cost structure between the linear demand case in Section 5 and the iso-elastic demand case differ. As before, the maximum total surplus is largest when the subsidy withdrawal risk is zero. In this case, the maximum total surplus is attained by setting subsidy size $\theta = 0.4$. It is optimal not the implement a subsidy when the subsidy withdrawal risk is positive, as can be seen from both the middle and right panel of Fig. 11. From the middle and right panel of Fig. 11, it shows that the subsidy size θ has a non-monotonic effect on the total surplus. This is the result of two different effects that work in opposite direction. Firstly, increasing subsidy size lowers the firm's optimal investment threshold (left panel of Fig. 10) causing the expected time to investment to decrease and, therefore, increases the total surplus. Secondly, a larger subsidy size lowers the firm's optimal investment size (right panel of Fig. 10) decreasing consumer surplus, which has a negative effect on total surplus. The upward jump in the total surplus just after $\theta = 0.4$ is caused by the fact that the firm's expected time to invest drops to zero. As the subsidy becomes larger, there is a point at which the firm's optimal investment threshold is equal to or smaller than the starting value of the GBM, which is assumed to be equal to 10. This means that investment is done immediately, which is beneficial for total surplus.

In Fig. 13, we study the role of subsidy withdrawal risk on the social planner's ability to reach certain capacity targets as soon as possible. Similarly to the results under linear demand shown in Fig. 6, a lump-sum subsidy can only speed up investment at the cost of a lower investment size.

Appendix C. Additional derivations

C.1. Derivation of constant relative welfare loss under no subsidy

Let X_S and K_S^* denote the socially optimal timing and capacity, and let X_0 and K_0^* be the firm's optimal timing and capacity without any subsidy. Using that $X_S = X_0$ and $K_S^* = 2K_0^*$, and expressions (2.12) and (2.13) for X_0 and K_0^*, the relative welfare loss is equal to:

$$\begin{aligned}
\mathrm{RWL} = \frac{\mathrm{TS}(X_S,K_S^*)-\mathrm{TS}(X_0,K_0^*)}{\mathrm{TS}(X_S,K_S^*)} &= \frac{\frac{X_S(2-\eta K_S^*)K_S^*}{2(r-\mu)}-\delta K_S^*-\left(\frac{X_0(2-\eta K_0^*)K_0^*}{2(r-\mu)}-\delta K_0^*\right)}{\frac{X_S(2-\eta K_S^*)K_S^*}{2(r-\mu)}-\delta K_S^*}\\
&= \frac{\frac{4X_0(1-\eta K_0^*)K_0^*}{2(r-\mu)}-2\delta K_0^*-\frac{X_0(1-\eta K_0^*)K_0^*+X_0K_0^*}{2(r-\mu)}+\delta K_0^*}{\frac{4X_0(1-\eta K_0^*)K_0^*}{2(r-\mu)}-2\delta K_0^*}\\
&= \frac{3X_0(1-\eta K_0^*)-X_0-2\delta(r-\mu)}{4X_0(1-\eta K_0^*)-4\delta(r-\mu)}\\
&= \frac{\frac{3}{2}(X_0+\delta(r-\mu))-(X_0+2\delta(r-\mu))}{2(X_0+\delta(r-\mu))-4\delta(r-\mu)}\\
&= \frac{1}{4}
\end{aligned}$$

C.2. Stochastic discount factor and expected time to investment

When analyzing the effect of a subsidy on welfare, we need to take into account that a subsidy speeds up investment, and thus investment is done at a different time under subsidy than without the subsidy. As we compare welfare outcomes under different times, we need to discount both the welfare with and without subsidy properly. This subsection shows that the discount factor for investment without subsidy is equal to:

$$S_0 = \left(\frac{x}{X_0}\right)^{\beta_{01}} \tag{C.1}$$

We also derive that when investment is influenced by a subsidy subject to subsidy retraction risk, the discount factor is equal to:

$$S_1 = \mathrm{P}[s>\tau_1]\cdot\left(\frac{x}{X_1}\right)^{\beta_{01}} + (1-\mathrm{P}[s>\tau_1])\cdot\left(\frac{x}{X_0}\right)^{\beta_{01}} \tag{C.2}$$

where

$$\mathrm{P}[s>\tau_1] = \exp\left\{\frac{(X_1-x)}{\sigma}\left(\frac{\mu}{\sigma}-\sqrt{\frac{\mu^2}{\sigma^2}+2\lambda}\right)\right\} \tag{C.3}$$

The discount factor for discounting investment without subsidy risk has been derived in Dixit and Pindyck [1994] and has been addressed in Huisman and Kort [2015].

To derive the stochastic discount factor for investment under subsidy subject to subsidy retraction risk, we need to derive the expected time to investment. We define the first hitting times of the thresholds as follows:

$$\tau_0 = \min\{t : X(t)\geq X_0\} \tag{C.4}$$

$$\tau_1 = \min\{t : X(t)\geq X_1\} \tag{C.5}$$

Furthermore, let s be the time at which the exponential jump process with parameter λ has its first jump. Then the expected time to investment τ^* can be written as follows:

$$\tau^* = \mathrm{P}[s>\tau_1]\cdot\mathbb{E}[\exp(-r\cdot\tau_1)] + (1-\mathrm{P}[s>\tau_1])\cdot\mathbb{E}[\exp(-r\cdot\tau_0)] \tag{C.6}$$

The first part of the sum takes the scenario in which the first exponential jump occurs after the first hitting time of investment threshold X_1. In that case, the first hitting time of X_1 is relevant for our solution. The second part of the sum takes the scenario in which the first exponential jump occurs before the first hitting time of investment threshold X_1. Then, the policy is withdrawn before we invest and we are no longer interested in the first time the GBM process reaches X_1, but the first hitting time of threshold X_0 is the relevant stochastic variable.

In Eq. (C.6), the analytic expressions for $\mathbb{E}[\tau_0]$ and $\mathbb{E}[\tau_1]$ are known from, e.g., Dixit and Pindyck [1994, p. 315–316]:

$$\mathbb{E}[\exp(-r\cdot\tau_0)] = \left(\frac{x}{X_0}\right)^{\beta_{01}} \tag{C.7}$$

$$\mathbb{E}[\exp(-r\cdot\tau_1)] = \left(\frac{x}{X_1}\right)^{\beta_{01}} \tag{C.8}$$

$\mathbb{P}[s > \tau_1]$ is the probability that the exponential jump occurs after the first time the GBM process X hits the threshold X_1. Thus, we compare two first passage times of two independent random processes. In general, this problem is solved by solving the following integral:

$$\int_0^\infty e^{-\lambda t} f_{\tau_1}(t)dt \tag{C.9}$$

where $f_{\tau 1}(t)$ is the density function of the hitting time of the GBM. Valenti et al. [2007] state that the distribution of time τ_1 for a GBM process X starting at x (see Eq. (2.2)) to reach threshold X_1 is given by the inverse Gaussian:

$$f(X_1,x)=\frac{X_1-x}{\sqrt{2\pi\sigma^2\tau_1^3}}\cdot e^{\frac{-(X_1-x-\mu\tau_1)^2}{2\sigma^2\tau_1}} \tag{C.10}$$

To simplify the derivation, we rewrite (C.10) into the standard form of an inverse Gaussian pdf:

$$f(\tau_1;X_1,x)=\frac{X_1-x}{\sqrt{2\pi\sigma^2\tau_1^3}}\cdot\exp\left\{\frac{-(X_1-x-\mu\tau_1)^2}{2\sigma^2\tau_1}\right\} \tag{C.11}$$

$$=\sqrt{\frac{\left(\frac{X_1-x}{\sigma}\right)^2}{2\pi\tau_1^3}}\cdot\exp\left\{-\frac{(X_1-x)^2}{\sigma^2}\cdot\frac{\left(\tau_1-\frac{X_1-x}{\mu}\right)^2}{2\tau_1\left(\frac{X_1-x}{\mu}\right)^2}\right\} \tag{C.12}$$

Expression (C.12) is an inverse Gaussian pdf with parameters $\widehat{\lambda}$ and $\widehat{\mu}$, where $\widehat{\lambda}=\left(\frac{X_1-x}{\sigma}\right)^2$ and $\widehat{\mu}=\frac{X_1-x}{\mu}$. So, from now on, we use

$$f(\tau_1)=\sqrt{\frac{\widehat{\lambda}}{2\pi\tau_1^3}}\cdot\exp\left\{-\widehat{\lambda}\cdot\frac{(\tau_1-\widehat{\mu})^2}{2\tau_1\cdot\widehat{\mu}^2}\right\} \tag{C.13}$$

for the pdf of the first hitting time.

Now the integral can be solved as follows:

$$\int_0^\infty \exp(-\lambda t)f_{\tau_1}(t)dt$$
$$=\int_0^\infty \exp(-\lambda t)\sqrt{\frac{\widehat{\lambda}}{2\pi t^3}}\cdot\exp\left\{-\widehat{\lambda}\cdot\frac{(t-\widehat{\mu})^2}{2t\widehat{\mu}^2}\right\}dt \tag{C.14}$$

$$=\exp\left\{\frac{\widehat{\lambda}}{\widehat{\mu}}\left(1-\sqrt{1+\frac{2\lambda\widehat{\mu}^2}{\widehat{\lambda}}}\right)\right\} \tag{C.15}$$

Plugging in the expressions for $\widehat{\lambda}$ and $\widehat{\mu}$ into (C.15), we get:

$$\exp\left\{\frac{\widehat{\lambda}}{\widehat{\mu}}\left(1-\sqrt{1+\frac{2\lambda\widehat{\mu}^2}{\widehat{\lambda}}}\right)\right\}$$
$$=\exp\left\{\frac{\mu(X_1-x)}{\sigma^2}\left(1-\sqrt{1+2\lambda\cdot\frac{\sigma^2}{\mu^2}}\right)\right\} \tag{C.16}$$

$$=\exp\left\{\frac{X_1-x}{\sigma}\left(\frac{\mu}{\sigma}-\sqrt{\frac{\mu^2}{\sigma^2}+2\lambda}\right)\right\} \tag{C.17}$$

The probability the exponential jump occurs after threshold X_1 is hit, is equal to the expression (C.17), in which x is the starting value of the GBM.

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Paper II

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Don't stop me now: Incremental capacity growth under subsidy termination risk

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ABSTRACT

Once a subsidy scheme is close to reaching its goal or loses political support, it may be terminated. An important question for policy makers is how to minimize the negative impact of the risk of subsidy termination on industrial investment. We assume the social planner aims to increase capacity and welfare and uses a subsidy, which has an uncertain lifetime, for the purpose. We examine a monopolist supplying an uncertain demand, faced with the option to expand capacity by irreversibly investing in small increments. We find that the firm installs capacity expansions sooner and, consequently, installs a larger capacity than a firm without a subsidy. A firm's total investment during the subsidy's lifetime increases with both the subsidy size and the likelihood of subsidy withdrawal. However, this happens at the cost of less investment directly after the subsidy has been retracted. The optimal subsidy size strongly depends on the point in time at which the social planner aims to maximize the welfare — the further into the future, the larger the welfare optimal subsidy. Furthermore, the welfare optimal subsidy size strongly depends on the social planner's discretion over adjustments to the subsidy size.

1. Introduction

Subsidies are commonly used to mitigate market imperfections and consequently increase welfare. Alternatively, subsidies can be used to encourage investment to develop a technology that fulfills a social need and is not yet economically viable. As subsidies are used to reach a specific goal, they are usually terminated at some point. The profitability of investors' projects depends largely on a subsidy's lifetime; thus, it is important that investors account for the risks related to the termination of a subsidy. This is a challenge for many industries, as politicians typically cannot commit to long-term policies due to short election cycles. For a policy maker, it is important to account for an industry's response to the risk of subsidy termination as a firm's investment decisions are key to reaching the policy maker's targets. Examples of such transitions in which subsidy and subsidy termination play a role are renewable energy,[1] hydrogen[2] and agriculture.[3] Ganhammar (2021) provides evidence that regulatory uncertainty may disrupt the effect of energy policy in the Swedish–Norwegian certificate market because regulatory interventions increase the volatility in certificate prices.

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[1] The annual installed wind capacity in the United States (US) in the period 1997–2005 strongly depended on the production tax credit. Investment increased in the years before the tax credit expired, and has been low in the following years (The Economist, 2013). Stokes (2015) and Stokes and Warshaw (2017) point out the important role of public opinion on renewable energy policy in the US. Stokes and Warshaw (2017) address the withdrawal risk caused by public opinion: "Since 2011[,] several [US] states have weakened their renewable energy policies. Public opinion will probably be crucial for determining whether states expand or contract their renewable energy policies in the future".

[2] Already in the early 2000s, Van Benthem et al. (2006) mention that there was a broad consensus on hydrogen investment projects being eligible for government support, citing George W. Bush's State of the Union address in 2003 (Bush, 2003) and the then European Commission Chairman Romano Prodi (CORDIS Europa, 2004). More recently, the European Commission released the Hydrogen Strategy for a Climate-neutral Europe in 2020 (European Commission, 2020), as part of its European Green Deal. It mentions the availability of European Union (EU) funding as well 14 Member States having included hydrogen in their national infrastructure policy frameworks.

[3] The amount of subsidies in agriculture has declined compared that from the late 90 s to the period 2009–2011 in Europe (The Economist, 2012), while the EU has recently debated on limiting spending on agriculture (The Economist, 2019). In the UK, farmers are concerned about the consequences of missing out on the £3bn of annual subsidy under the EU's common agricultural policy after the UK has left the EU (The Economist, 2020). The uncertainty regarding the farmers' income affects their investment decisions; Musshoff and Hirschauer (2008) show that low volatility in the total gross margin differences encourages farmers to invest more in a new technology.

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A popular subsidy scheme in energy and renewables is the investment tax credit. An investment tax credit allows the investment to be fully or partially credited against the tax obligations or income of the investor (REN21, 2022, page 231). Investment tax credits constitute the most widespread policy instrument for renewable energy globally,[4] and are often implemented with the aim to increase the affordability and profitability of renewable energy production (IRENA et al., 2018, page 70). Recently, the United States used an investment tax credit, combined with the Renewable Fuel Standard 2 (RFS2) and California's Low Carbon Fuel Standard (LCFS), to increase production of Hydrotreated Vegetable Oil (HVO) (REN21, 2022, page 106). The popularity of investment tax credits may be explained by Bunn and Muñoz (2016), who show that "reducing capital cost through grants (e.g. capital allowances, capacity payments and/or fiscal benefits) is more effective [in attracting new investment in renewables] than through energy prices (e.g. green certificates)".

We examine the impact of an investment tax credit on industrial capacity growth. We consider a market with uncertain demand and a supply side, comprising a single, risk-neutral, profit-maximizing firm that holds the option to invest in irreversible capacity expansions.[5] No stock can be created. The investments are eligible for a subsidy, and face the risk of a potential future subsidy retraction. We examine the effect of the risk of subsidy termination on the firm's investment decision. The cost of investment is dependent on the availability of a subsidy. We consider an investment cost subsidy in the form of a percentage coverage of the investment cost. This represents a general class of subsidies including investment tax credits and capital subsidies. The subsidy is implemented by a social planner, who aims to reach a capacity target or maximize welfare, and decides on the subsidy size. We assume the subsidy is merely a welfare transfer, i.e., any subsidy payment to the firm is a cost to society, which means that the net cost of implementing the subsidy is zero.

This setting is applicable to, for example, a country's renewable energy capacity, in which many projects gradually increase the country's or industry's total capacity. In this study, we seek to answer the following open research questions: (i) How is the rate of capacity expansion affected by an investment cost subsidy under the prospect of policy termination and how does the rate of expansion change after subsidy termination? (ii) How does the prospect of policy termination affect the social planner's ability to increase total surplus, and (iii) how should the social planner set their subsidy size optimally to maximize total surplus?

In answering the first question, we find that a monopolist faced with an investment cost subsidy subject to withdrawal risk expands sooner while the subsidy is available and, consequently, installs a larger aggregate capacity during the subsidy's lifetime than a monopolist without the subsidy. Once the subsidy is withdrawn, the monopolist stops investment until demand – and consequently output prices – has grown sufficiently to attract investment without subsidy. This means that a policy maker aiming to reach a capacity target must implement a subsidy that is sufficiently large such that the target is reached during the subsidy's lifetime. If the target is not reached during the subsidy's lifetime, the target will be reached at the same time as in the scenario in which a subsidy is never implemented.

When we examine our second question, we find that the social planner can increase welfare by implementing a subsidy even when the subsidy is subject to withdrawal risk. While the subsidy is in effect, an optimally set subsidy can positively impact welfare in the long run. The welfare increases as the subsidy attracts more investment, which increases the consumer surplus that accumulates over time. However, in the short term, the welfare under a subsidy is lower than that without due to the cost of subsidizing investment.

Third, we find that the optimal subsidy size increases with the monopolist's capacity and decreases with the risk of subsidy withdrawal. The optimal subsidy strongly depends on whether the social planner can adjust the subsidy size throughout the lifetime of the subsidy as well as the time at which the social planner aims to maximize the surplus. We find that a social planner sets a larger subsidy if they aim to maximize welfare far in the future, to the detriment of short-term welfare.

Numerous studies analyze the effects of support schemes in renewables on investment and/or welfare. The topic of policy making for renewable energy is especially interesting as investors may need a progressively higher level of support over time, due to the high risk and low return of renewable energy projects (Muñoz and Bunn, 2013). Furthermore, Gan et al. (2007) state that the policy instruments in the US and Europe in the early 2000s provide insufficient incentive for the long-term development of new, green technologies, which are important in reaching long-term policy goals. As support schemes are considered crucial for inducing investments in energy, it is also important, from a policy maker's viewpoint, to implement efficient and effective policies. Both theoretical (Kydland and Prescott, 1977; Nordhaus, 2007; Gerlagh et al., 2009; Stern, 2018; Keen, 2020; Stern et al., 2022) and empirical (Stern, 2006; García-Álvarez et al., 2018; Rossi et al., 2019; Liski and Vehviläinen, 2020) studies attempt to determine the optimal government policy or subsidy design — with a strong focus on attracting investments in renewable energy or reaching targets in battling climate change. Liski and Vehviläinen (2020) propose a policy design leading to energy producers' incurring most of the cost of the subsidy that supports clean technologies instead of the consumers. Real options are also often applied to investment in the energy sector, see Fernandes et al. (2011) for a literature review. Kozlova (2017) provides an extensive literature review of renewable energy investment under uncertainty using real options, which also considers the literature on energy policy. We contribute to this literature in that we study an uncertain market accounting for policy risk. We focus on the long-term perspective and show that the social planner's time at which a policy target should be reached is key in determining what the optimal policy is.

Real options theory is also frequently applied to studying investment problems under (market) uncertainty to determine the optimal subsidy and/or tax (e.g., Pennings (2000), Yu et al. (2007), Danielova and Sarkar (2011), Boomsma et al. (2012), Rocha Armada et al. (2012), Sarkar (2012), Feil et al. (2013), Barbosa et al. (2016), Ritzenhofen and Spinler (2016), Azevedo et al. (2021), Tsiodra and Chronopoulos (2021), and Hu et al. (2022)), and sometimes accounting for policy change (Dixit and Pindyck (1994, Chapter 9), Hassett and Metcalf (1999), Boomsma and Linnerud (2015), Chronopoulos et al. (2016) and Nagy et al. (2021)). Dixit and Pindyck (1994, Chapter 9), Hassett and Metcalf (1999), and Nagy et al. (2021) all examine the case of a monopolist facing a one-time investment decision under a lump-sum subsidy subject to withdrawal risk. They conclude that the firm invests sooner under the subsidy if the likelihood of subsidy withdrawal is larger. Nagy et al. (2021) includes capacity choice and concludes that a firm opts for earlier investment, but also at a lower investment size. Boomsma and Linnerud (2015) derive a similar conclusion if a support scheme is non-retroactively terminated, but also find that "the prospect of [support scheme] termination will slow down investments if it is retroactively applied".

A specific branch of literature examines incremental investment, stepwise investment, or capacity expansion as a real options model; see, for example, Dixit and Pindyck (1994, Chapter 11), Bar-Ilan and Strange (1999), Panteghini (2005), Chronopoulos et al. (2016), and Gryglewicz and Hartman-Glaser (2020). In this literature, an industry or firm invests more than once, which means that the capacity

[4] An estimated amount of 30 to 40 countries used investment or production tax credits to support renewable energy installations over the past decade (IRENA et al., 2018, page 69).

[5] This casts us in a setting of real options, where each investment increment can be seen as an American call option on marginal production capacity.

can be adjusted upward over time. Dixit and Pindyck (1994, Chapter 11) and Bar-Ilan and Strange (1999) assume production to follow a Cobb–Douglas function, and that the industry maximizes its own total profit, implicitly assuming that it acts as a monopolist. Bar-Ilan and Strange (1999) find that demand uncertainty affects an industry's investment size differently when investment is incremental compared to when it is lumpy. This implies that one cannot directly derive the results for incremental investment under policy from models that study lumpy investment under policy uncertainty. Panteghini (2005) finds that for a two-stage investment project, a tax does not provide any incentives for the firm to change its behavior, i.e., the taxation is neutral. Gryglewicz and Hartman-Glaser (2020) examine the role of incentive costs in the value and exercise of an option in a model in which incremental capital is assumed to be stochastic. The decision maker decides on the optimal investment rate, which determines the incremental capital trend. The costs of accumulating capital affects the relationship between managerial hazard and the option value and exercise.

Closely related to our study is Chronopoulos et al. (2016), who examine the effect of the subsidy withdrawal risk of a price premium on a monopolist's investment timing and size, where investment is either lumpy or stepwise. When investment occurs in two steps, they find that the firm invests in a larger aggregate capacity than when the investment is lumpy, as the firm has more flexibility to adjust its capacity over time. They mention the assumption of the electricity price being independent of the size of the project as a limitation of their work.[6] We extend the analysis by Chronopoulos et al. (2016) by relaxing this assumption as well as examining the effect of policy on a social planner's targets. The combination of an option to implement multiple capacity expansions and a subsidy subject to withdrawal risk has scarcely been examined in the literature, and we revisit this setting. Furthermore, unlike Chronopoulos et al. (2016) and previously mentioned literature, we examine total surplus as a welfare measure to study the policy maker's point of view.

In short, this study contributes to the literature in three ways. First, we examine how subsidies affect incremental investment in contrast to the literature on lumpy investment, and how this impacts social welfare. By assuming the industry invests incrementally instead of lumpy, we take a long-term perspective instead of looking at a one-time decision. We show that a subsidy can increase total welfare in a dynamic monopoly and attain a first-best solution, even if the lifetime of the subsidy is uncertain. Our second contribution lies in providing new insights by examining the long-term effects of a subsidy as well as what happens after subsidy withdrawal. An investment cost subsidy is an effective tool for accelerating investment; however, this effect tapers off over time. Third, we show that the policy maker's time horizon plays a crucial role in determining the welfare optimal policy.

The remainder of this paper is structured as follows: The model is presented in Section 2. In Section 3, we derive the optimal investment decisions with and without subsidy withdrawal risk. We also study the optimal investment from a social planner's perspective, as well as derive the optimal subsidy. Section 4 provides a numerical case study, while Section 5 concludes.

[6] Chronopoulos et al. (2016) emphasize that the limitation of their assumption is especially visible when considering installation of large projects. We assume market power, as one can expect that aggregate capacity and price are strongly linked in any industry. For the energy industry specifically, there are several examples of countries in which market power lead to prices being affected. See Karthikeyan et al. (2013) for a thorough review on market power in the electricity market in different countries, or Nagy et al. (2021) for a detailed reflection on market power on the energy market.

2. Model

We propose a theoretical framework that examines a single firm's optimal investment decision under uncertain subsidy support. The firm aims to maximize its profits. We assume that the monopolist currently produces K units and can invest in small, fixed-size projects. The future revenue stream from the production is uncertain. The monopolist's total capacity increases gradually as it installs its projects.

The output price is denoted by $P(X, K)$ and given by

$$P(X, K) = X(1 - \eta K), \tag{1}$$

where η is a positive constant.[7] The output price depends on both K, the monopolist's total production capacity, and $X(t)$, which represents exogenous shocks. The exogenous shocks are assumed to follow a geometric Brownian motion process given by

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t),\ X(0) = x, \tag{2}$$

where μ is the drift rate, σ the uncertainty parameter, and $dW(t)$ the increment of a Wiener process.

The cost of installing one unit of capacity is set equal to κ. The size of the next expansion is given by dK. Hence, assuming the current capacity to be K, increasing the production capacity leads to a new capacity of $K + dK$ at an investment cost of $\kappa \cdot dK$ when no subsidy is in effect. A subsidy provides a discount at a rate, θ, on the investment cost; thus, the investment costs are then equal to $(1-\theta)\kappa \cdot dK$.

Initially, the subsidy is assumed to be available; however, it can be withdrawn due to a random event, such as the depletion of the public budget or a change in government. We assume that the monopolist's perceived likelihood of subsidy retraction follows an exponential jump process with parameter λ. This implies that the monopolist's perceived probability of subsidy retraction in the next time interval, dt, is equal to λdt.

Next, we derive the monopolist's objective that maximizes its profit. Without loss of generality, we assume a current production capacity of $K(0) = k$. The monopolist chooses when to install its expansions, i, $i \in \mathbb{N}$, which means that it chooses the investment times, τ_i, where $\tau_i \leq \tau_j$ for all $i \leq j$. We denote the capacity after the ith expansion by K_i:

$$K_i = K_{i-1} + dK = k + i \cdot dK. \tag{3}$$

The monopolist maximizes the producer surplus (PS). Its objective is given by

$$\begin{aligned} V &= \sup_{\tau_1,\tau_2,\ldots} PS(X, K) \\ &= \sup_{\tau_1,\tau_2,\ldots} \sum_{i=1}^{\infty} \mathbb{E}\left[\int_{\tau_{i-1}}^{\tau_i} P(X(t), K_i) \cdot K_i \cdot e^{-rt} dt \right. \\ &\quad \left. - (1 - \theta \cdot \mathbb{1}_{\xi(\tau_i)})\kappa \cdot dK \cdot e^{-r\tau_i} \,\middle|\, X(\tau_{i-1}), \xi(\tau_{i-1}) \right], \end{aligned} \tag{4}$$

where $\tau_0 = 0$ indicates the start of the planning horizon, and $\mathbb{1}_{\xi(t)}$ is an indicator function that assumes a value of 1 if the subsidy is still available at time t and zero otherwise. As the subsidy is available at the start of the planning horizon, we have $\xi(0) = 1$.

We show that the problem in which the monopolist maximizes their total profits as defined in (4) is equivalent to that in which

[7] The inverse demand function in (1) is a special case of the one used by Dixit and Pindyck (1994, Chapter 11), which assumes $P = XD(K)$, with an unspecified demand function, $D(K)$, and is frequently used in the literature (see, e.g., Pindyck (1988), He and Pindyck (1992), and Huisman and Kort (2015)).

they maximize the added value of each extra unit of capacity. The monopolist's objective in (4) can be rewritten to.[8]

$$V = \mathbb{E}\left[\int_0^\infty P(X(t),k)\cdot k\cdot e^{-rt}dt \,\middle|\, X(0)=x, \xi(0)=1\right] + \sum_{i=1}^{\infty}\sup_{\tau_i}\left\{ \mathbb{E}\left[\int_{\tau_i}^{\infty}\left(P(X(t),K_i)\cdot dK + \Delta P_i(X(t))\cdot K_{i-1}\right)\cdot e^{-rt}dt - (1-\theta\cdot \mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot e^{-r\tau_i} \,\middle|\, X(\tau_{i-1}),\xi(\tau_{i-1})\right]\right\}, \quad (5)$$

where $\Delta P_i(X(t))$ is the price change from increasing the capacity for the ith time when the value of the demand shock is given by $X(t)$, i.e., $\Delta P_i(X(t)) = P(X(t),K_i) - P(X(t),K_{i-1})$. The equivalence of the objective functions in (4) and (5) holds for any demand function satisfying the Markov property.

Using the rewritten form of the objective in (5), we solve the monopolist's problem of maximizing their total profit by solving multiple, independent, optimization problems that maximize the added value of each capacity expansion. This approach is preferred as it avoids dealing with dependencies between different capacity expansions and, hence, is easier than directly solving (4). The optimal times for the capacity expansions are derived in the next section.

In the remainder of this section, we derive an expression for the objective of the social planner, who maximizes total surplus (TS). The social planner cannot invest directly in the market itself, but decides on the size of the subsidy that is available to the monopolist. By doing so, the social planner can try to align the monopolist's investment with the welfare optimal investment. The total surplus comprises the sum of the producer and consumer (CS) surpluses, i.e.,

$$TS = PS + CS. \quad (6)$$

The consumer surplus is defined as the difference between the price consumers are willing to pay and the price they actually pay.

The social planner's objective under the demand function (1) is given by,[9]

$$TS(X,K) = \mathbb{E}\left[\int_0^\infty X(t)\cdot\left(1-\frac{1}{2}\eta k\right)\cdot k\cdot e^{-rt}dt \,\middle|\, X(0)=x\right] + \sum_{i=1}^{\infty}\mathbb{E}\left[\int_{\tau_i}^{\infty}\left(X(t)\cdot(1-\eta K_i)+\eta X(t)\cdot dK\right)\cdot dK\cdot e^{-rt}dt - \kappa\cdot dK\cdot e^{-r\tau_i} \,\middle|\, X(\tau_{i-1})\right]. \quad (7)$$

The maximization of the total surplus can be rewritten as the sum of the maximizations of the added value of each independent extra unit of capacity, as stated in (7). Therefore, we can solve the problem of the maximization of the total surplus by solving multiple, independent, maximization problems, which are easier to solve.

3. Investment and subsidy

In this section, we derive the optimal capacity expansion threshold as well as the welfare optimal policy. We derive the optimal investment threshold for both the monopolist and the social planner in Section 3.1. For the monopolist, we examine both the scenarios with and without subsidy. In Section 3.2, we consider the position of the social planner deciding on the subsidy. The social planner sets their subsidy such that it maximizes total surplus, considering that the monopolist decides on when to invest.

[8] The derivation and discussion of the firm's objective function in (5) can be found in Appendix A.1

[9] The derivation of the consumer surplus and the social planner's objective in (7) and a discussion of the social planner's objective can be found in Appendix A.2.

3.1. Optimal investment

We first derive the optimal investment threshold for the monopolist when there is no subsidy in place. For this, we maximize the marginal revenue of the expansion.

Let V_1 (V_0) denote the value of the option to expand capacity with(out) the subsidy. The value of the monopolist's investment without subsidy under the demand function (1) is given by

$$V_0(X_0,K) = \frac{x(1-\eta k)k}{r-\mu} + \sum_{i=1}^{\infty}\left(\frac{x}{X_0^i}\right)^{\beta_{01}}\cdot\left(\frac{X_0^i\left(1-\eta(2K_i-dK)\right)}{r-\mu}-\kappa\right)dK, \quad (8)$$

where X_0^i denotes the monopolist's optimal timing threshold without a subsidy, to implement the ith capacity expansion. For convenience, we denote X_0 as the vector containing all X_0^i. Moreover, β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2+(\mu-\frac{1}{2}\sigma^2)\beta-r=0$. β_{01} can be interpreted as a measure of the wedge between the monopolist's optimal investment threshold and the investment costs. $\beta_{01}>1$ holds and the value of β_{01} depends on the market uncertainty, σ, market growth rate, μ, and the discount rate, r.

We derive the optimal threshold at which to implement the ith capacity expansion without a subsidy using the same approach as Dixit and Pindyck (1994, Chapter 11). The expression for the optimal expansion threshold without a subsidy is given by Proposition 1.

Proposition 1. *The optimal investment threshold without a subsidy is given by*

$$X_0^i(K_i) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{(r-\mu)\kappa}{1-2\eta K_i}. \quad (9)$$

The proofs of all corollaries and propositions can be found in Appendix B.

Next, considering the scenario in which a subsidy is available, the value of the monopolist's investment is given by

$$V_1(X_1,K) = \frac{x(1-\eta k)k}{r-\mu} + \sum_{i=1}^{\infty}\left(\frac{x}{X_1^i}\right)^{\beta_{11}}\cdot\left(\frac{X_1^i\left(1-\eta(2K_i-dK)\right)}{r-\mu}-(1-\theta)\kappa\right)dK, \quad (10)$$

where X_1^i denotes the monopolist's optimal timing threshold, under a subsidy, for the ith capacity expansion. For convenience, we denote X_1 and X_0 as the vector containing all X_1^i and X_0^i respectively. Furthermore, β_{11} is the positive solution to $\frac{1}{2}\sigma^2\beta^2+(\mu-\frac{1}{2}\sigma^2)\beta-(r+\lambda)=0$. β_{11} can be interpreted as a measure of the wedge between the monopolist's optimal investment threshold and the investment costs when the subsidy is available. The equation is similar to the expression for β_{01}, with the only difference being that β_{11} depends on λ. We have that $\beta_{11}>\beta_{01}$ as the likelihood of subsidy withdrawal decreases the wedge because there is a risk that investment costs significantly increase.

An implicit expression for the optimal expansion threshold under subsidy is given by Proposition 2.

Proposition 2. *The optimal investment threshold for the ith capacity expansion under a subsidy, X_1^i, is given by*

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\frac{dA_{01}(K_i)}{dK}\cdot(X_1^i)^{\beta_{01}}+\frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1^i(1-2\eta K_i)}{r-\mu}-(1-\theta)\kappa=0. \quad (11)$$

To the best of our knowledge, the implicit Eq. (11) does not have an analytical solution. In Section 4, we numerically solve this expression.

We can derive how the optimal investment threshold changes with respect to the policy parameters. The following corollary states how the optimal investment decision is affected by subsidy retraction risk.

Corollary 1. *The optimal investment timing, X_1^i, is negatively affected by the subsidy retraction risk, λ.*

From Corollary 1, one may be tempted to conclude that the best situation for a social planner interested in maximizing investment during the subsidy's lifetime is a situation in which the subsidy withdrawal risk is large.[10] However, the larger withdrawal risk does not only lower the firm's investment threshold under the subsidy, but it also shortens the expected lifespan of the subsidy. The shorter the lifespan of the subsidy, the less time there is for the capacity to grow. Therefore, the total impact of a higher subsidy withdrawal risk on the monopolist's capacity is ambiguous. We examine this impact, as well as the situation after subsidy withdrawal, in detail in Section 4.

Corollary 2. *The optimal investment timing, X_1^i, is negatively affected by the subsidy size, θ.*

The result that a larger investment cost subsidy accelerates investment is a well-known one in different settings; for example, in the case of lumpy investment both with policy uncertainty (Dixit and Pindyck, 1994; Hassett and Metcalf, 1999; Nagy et al., 2021) and without policy uncertainty (Pennings, 2000; Rocha Armada et al., 2012; Azevedo et al., 2021).

An important advice for a social planner interested in maximizing investment during a subsidy's lifetime follows from Corollary 2. Such a social planner should set the subsidy as large as possible, as this incentivizes capacity growth. Although this maximizes the investment during the subsidy's lifetime, it is also important for a policy maker to know the impact of their policy after withdrawal.

Next, we solve the investment problem from the perspective of the social planner, who maximizes total surplus. The maximization of the total surplus can be rewritten into smaller optimization problems in which the added value of an each capacity expansion is maximized, as shown in Eq. (7).

Let V_S denote the value of the option to expand capacity for the social planner. The total surplus of the investment under the demand function (1) is given by

$$V_S(X_S, K) = \frac{x(2-\eta k)k}{2(r-\mu)} + \sum_{i=1}^{\infty} \left(\frac{x}{X_S^i}\right)^{\beta_{01}} \cdot \left(\frac{X_S^i\left(2-\eta(2K_i - dK)\right)}{2(r-\mu)} - \kappa\right) dK, \tag{12}$$

where X_S^i denotes the social planner's optimal timing threshold to implement the ith capacity expansion. The optimal social threshold at which to expand capacity is stated in Proposition 3.

Proposition 3. *The optimal investment threshold for a social planner is given by*

$$X_S^i(K_i) = \frac{\beta_{01}}{\beta_{01}-1} \cdot \frac{(r-\mu)\kappa}{1-\eta K_i}. \tag{13}$$

We examine the difference between the social planner's and monopolist's investments by comparing the optimal social investment threshold in (13) with the firm's optimal threshold without a subsidy in (9). The social planner increases the capacity earlier than the monopolist, as a larger aggregate capacity positively impacts the consumer surplus. The monopolist, meanwhile, keeps the output price high by maintaining the capacity lower than is optimal level from a social planner's viewpoint. The social planner uses the subsidy to align the monopolist's decision with the welfare optimal investment. The optimal subsidy from the social planner's viewpoint is examined in the next section.

[10] Due to Donald Trump's hard stance against renewables (The New York Times, 2019; Center for American Progress, 2020; Forbes, 2020), favoring coal and fossil fuels (The Economist, 2018), we call this the Trumpian strategy.

3.2. Optimal subsidy

This subsection examines how the subsidy should be set to maximize the total surplus, given the monopolist's investment decisions. First, we study the situation in which the social planner can alter the subsidy size after each investment until the subsidy is terminated. We refer to this as the *flexible* subsidy. Next, we assume that the social planner can only set the subsidy at the beginning, and that it remains at that size until the subsidy is abolished. This subsidy is referred to as the *fixed* subsidy.

We start with the welfare optimal flexible subsidy and use $\theta_\lambda^*(K)$ to denote the welfare optimal subsidy size for a given subsidy withdrawal level, λ, and a current capacity of K. The following proposition states how the social planner who maximizes total surplus should set their flexible subsidy.

Proposition 4. *To maximize surplus, the social planner should set their subsidy size equal to*

$$\theta_\lambda^*(K) = 1 - \frac{1}{\beta_{11}(\beta_{01}-1)} \cdot \left[\beta_{01}(\beta_{11}-1) \cdot \frac{1-2\eta K}{1-\eta K} - (\beta_{11}-\beta_{01}) \cdot \left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}}\right], \tag{14}$$

where $K < \frac{1}{2\eta}$ is the monopolist's current capacity, while β_{01} and β_{11} are the positive solutions to the equations, $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$ and $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r+\lambda) = 0$, respectively.

If there is no subsidy withdrawal risk (i.e., the subsidy is never withdrawn, $\lambda = 0$), the expression for the optimal subsidy size simplifies. The social planner who maximizes surplus sets their subsidy size equal to

$$\theta_{\lambda=0}^*(K) = \frac{\eta K}{1-\eta K}. \tag{15}$$

Eq. (15) implies that the social planner should increase their subsidy to keep additional investment attractive for the monopolist as the capacity grows. The monopolist has a strong incentive to keep prices high by maintaining supply low. However, unlike the monopolist, the social planner has an incentive to increase capacity as the consumer surplus does increase with capacity. The subsidy is used as a tool to decrease investment costs to such a level that the monopolist has an incentive to increase capacity even when output prices are low.

Furthermore, Eq. (15) shows that the optimal subsidy rate is increasing in the market power parameter, η. A firm with considerable market power invests only very little to keep prices high. The social planner wants to attract more investment as it will increase consumer surplus; thus, a significant subsidy is used to incentivize the firm to invest more.

Interestingly, the welfare optimal subsidy under a non-zero subsidy withdrawal risk depends on the demand uncertainty, σ, while the welfare optimal subsidy under a zero subsidy withdrawal risk does not. The social planner can optimally set the subsidy size at each point in time and only needs to account for the firm's market power today if the subsidy is available forever. However, if the subsidy may be terminated in the future, they must consider what may happen after subsidy withdrawal. As the firm's future investment depends on the demand uncertainty, it also impacts the optimal policy.

Corollary 3 shows the effect of the monopolist's capacity on the welfare optimal subsidy size for any level of subsidy retraction risk.

Corollary 3. *The welfare optimal subsidy size, $\theta_\lambda^*(K)$, is positively affected by the monopolist's capacity, K.*

The social planner should install a larger subsidy when the monopolist's capacity is larger. This holds even when there is subsidy withdrawal risk. Corollary 3 implies that a social planner only needs a small subsidy to align the monopolist's investment with the welfare

Table 1
Parameter values used.

Notation	Parameter	Value
r	Risk-free interest rate	5%
μ	Price trend	2%
σ	Price volatility	10%
x	demand shock at $t = 0$	10
η	Slope of linear demand function	0.005
dK	Size of the capacity expansion	1 unit/year
κ	Variable investment cost	300 €/dK

optimal investment in an emerging market, but needs a large subsidy to perfectly align the monopolist's investment when the monopolist has already installed a large capacity.

Corollary 4 discusses the effect of the subsidy retraction risk on the welfare optimal subsidy size for any level of the monopolist's capacity.

Corollary 4. *The welfare optimal subsidy size, $\theta^*_\lambda(K)$, is negatively affected by the subsidy retraction risk, λ.*

It follows from Corollary 4 that a social planner should install a smaller subsidy when the withdrawal risk is larger. The social planner aligns the timing of the monopolist's investment and the optimal social investment. The gap in timing between the two investments decreases when the subsidy withdrawal risk is larger. The monopolist installs an additional unit of capacity sooner as they are afraid to lose the subsidy if they wait longer.

The welfare optimal policy depends on the firm's capacity level and must be updated after each investment. We now relax the assumption that the social planner can change the subsidy size after each increment of the firm, and assume that the social planner sets a fixed subsidy size at the start of the time horizon. We derive the optimal fixed subsidy size via simulation. For a given λ, the welfare optimal subsidy size, $\tilde{\theta}^*_\lambda$, is the subsidy size at which the average total surplus over all simulations is maximized.

4. Numerical study

In this section, we discuss the effect of a subsidy and the likelihood of its withdrawal on the decision to expand capacity and illustrate the relevant dynamics in a numerical example.[11] The data used in the numerical example, displayed in Table 1, are meant for illustrative purposes.

In Section 4.1, we first illustrate Propositions 1–3 for the non-subsidized firm's, the subsidized firm's and the social planner's optimal investment threshold in our numerical example. Next, we illustrate the findings of Corollaries 1 and 2. Finally, we examine the capacity growth under the optimal decision of a non-subsidized firm and compare it to a subsidized firm. In Section 4.2, we compare the welfare optimal subsidy size assuming the social planner aims to maximize the total surplus at fixed point of time in the future. We also compare the welfare under this policy to the welfare optimal policy when the social planner has an infinite time horizon (see Proposition 4).

4.1. Industry: Investment and capacity growth

First, we are interested in the rate at which the monopolist expands their production capacity during the lifetime of the subsidy for different withdrawal risk levels and subsidy sizes. We revisit the analytical results illustrating how a non-subsidized firm (Proposition 1), a subsidized firm (Proposition 2) and a social planner (Proposition 3) optimally invest via a numerical example. In Fig. 1(a), we illustrate the result of Corollary 1 and plot the optimal investment threshold, X_1 (see Proposition 2), as a function of the current capacity, K, for different levels of the subsidy termination intensity, λ, while keeping the subsidy size, θ, fixed. The effect of different subsidy sizes while keeping the withdrawal risk, λ, fixed, is examined in Fig. 1(b). Fig. 1(b) illustrates the result of Corollary 2. For comparison, we also plot X_0 (see Proposition 1), which is the optimal investment threshold without a subsidy and without subsidy termination risk, as well as X_S (see Proposition 3), the optimal social investment threshold, in both figures.

We observe that the monopolist invests sooner with a subsidy than without it for a given capacity, as the investment cost is lower with a subsidy. In Fig. 1(b), we see that the larger the subsidy, the lower the firm's investment threshold — consistent with the result stated in Corollary 2. As investment is cheaper, the firm is inclined to invest at a lower threshold, which means earlier investment, ceteris paribus. In Fig. 1(a), we also observe that the firm's investment threshold decreases with the subsidy withdrawal risk — consistent with the result stated in Corollary 1.

In addition to the investment threshold for a given level of capacity, these figures also have a second interpretation, related to the monopolist's capacity. The supremum of the demand shock, X, over time provides us with the monopolist's current capacity.[12] Our results indicate that the monopolist installs a larger aggregate capacity for as long as the subsidy is alive if the subsidy withdrawal risk is larger. This results from the fact that increasing the capacity is cheaper under a subsidy and that the firm fears that this subsidy will disappear sooner. We also conclude that the monopolist's optimal capacity for a given demand shock level is higher under a subsidy than without. The monopolist has an incentive to increase capacity early to guarantee that the capacity expansion is subsidized.

This is in stark contrast with both Chronopoulos et al. (2016) and Nagy et al. (2021). Chronopoulos et al. (2016) examine the retraction risk of a price premium and find that a greater likelihood of a permanent subsidy retraction increases the incentive to invest, but *lowers* the installed capacity. Nagy et al. (2021) examine a single firm having the option to undertake a lumpy investment subject to an investment cost subsidy, and find that the subsidized firm invests in a *smaller* capacity than a firm without a subsidy. In the case of a capacity expansion decision, as in this study, the firm still has the flexibility to extend capacity after investment, which leads to the difference in the results.

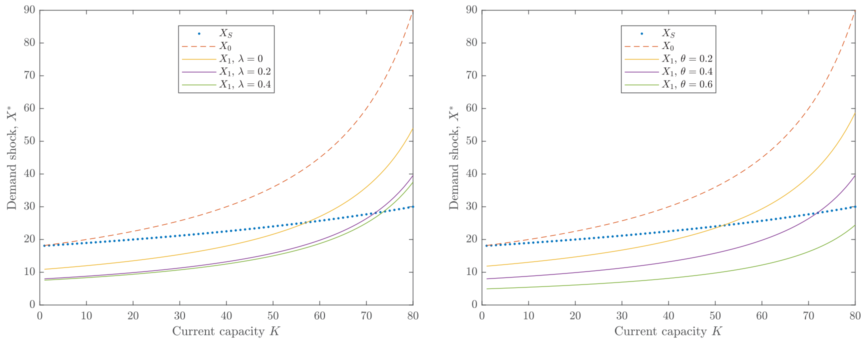
A policy maker who aims to increase a firm's capacity under a subsidy can increase the monopolist's investment by threatening to withdraw the subsidy soon. However, if the policy maker threatens to withdraw the subsidy soon but keeps the subsidy alive much longer than planned, the firm may perceive the actual subsidy withdrawal risk differently from what has been communicated by the policy maker. A future threat of withdrawing the subsidy becomes less effective, as the firm learns from experience that the subsidy will be available longer than is announced.

From the social planner's viewpoint, we observe that the sensitivity of the optimal social investment threshold, X_S, with respect to the current capacity is lower than that of the monopolist's threshold. This results from a difference in objectives between the monopolist and the social planner, as the latter includes the consumer surplus. The consumer surplus increases with capacity as long as the demand shock, X, has a positive value. Therefore, the social planner already has an incentive to install a larger aggregate capacity at lower output prices compared to the monopolist.

Comparing the monopolist's investment to the optimal social investment, we conclude that the former's threshold without a subsidy

[11] We use MATLAB R2021a for all numerical procedures as well as for the production of functional plots.

[12] Note that the monopolist's capacity is capped at $\frac{1}{2\eta}$, as the marginal revenue is non-positive for a capacity at that level or larger, meaning that no firm is willing to invest. With the parameter values in Table 1, the maximum capacity equals 100.



(a) Subsidy size $\theta = 0.4$.

(b) Subsidy termination risk $\lambda = 0.2$.

Fig. 1. Investment timing as a function of the current production capacity, K, for different levels of subsidy termination risk, λ (left), and for different subsidy sizes, θ (right), compared to the optimal social decision, X_S, and firm's decision without subsidy, X_0. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$.].

is equal to the latter's threshold when the capacity is low. A social planner interested in aligning the monopolist's investment threshold with the optimal social threshold can use a subsidy for the purpose when capacity is larger. The larger the capacity, the larger the subsidy required to align the thresholds, as can be seen from Fig. 1(a).

A policy maker may be interested in the question of whether the monopolist is more sensitive to a change in subsidy size than to one in subsidy withdrawal risk. We find that the monopolist decreases their investment threshold more from an increase in subsidy size than from an increase in subsidy withdrawal risk by the same percentage change. The effect of the former is direct, as it lowers the investment cost immediately, hence it is more effective. The effect of the latter is indirect, as the threat of subsidy withdrawal increases the probability of investment costs in the future being higher; however, it does not change the net present value (NPV) of investing today.

Next, we consider capacity growth over a longer period of time, and after subsidy withdrawal. We perform 10,000 simulations[13] to establish how the monopolist invests over time, and how this depends on subsidy withdrawal risk, subsidy size, and the time of subsidy withdrawal. Fig. 2 shows an example of two simulation runs, labeled A and B respectively, of the demand shock, X (Fig. 2(a)), and the corresponding monopolist's capacity over time (Fig. 2(b)).

Any simulation run can be broken down into three parts, although for some runs only the first two stages are reached: (1) a firm's total capacity grows while the subsidy is available; (2) the capacity stagnates after subsidy withdrawal; and (3) once the output prices reach a sufficiently high level, the capacity grows while the subsidy is unavailable. These three parts result from the monopolist's increasing the capacity sooner under a subsidy than without. The monopolist's investment threshold rises steeply at the time the subsidy is withdrawn, as its marginal cost of investment rises. Consequently, they do not invest directly after subsidy withdrawal, and delay investment until the output prices are significantly larger. In Fig. 2, simulation run A does not reach sufficiently high output prices to attract investment after subsidy withdrawal; thus, it only has the first two stages.

In Fig. 3, we show the average capacity over time of 10,000 simulations for different levels of subsidy withdrawal risk, λ. We compare the capacity growth against a baseline without a subsidy.

We observe that the monopolist's capacity under a subsidy is larger than that without a subsidy. A subsidy that is provided forever, i.e., there is no withdrawal risk, yields the most investment. In the case of a subsidy subject to withdrawal risk, the positive effect of the subsidy on the capacity is most pronounced during the lifetime of the subsidy and remains for some time after subsidy withdrawal; however, it fades after some time.

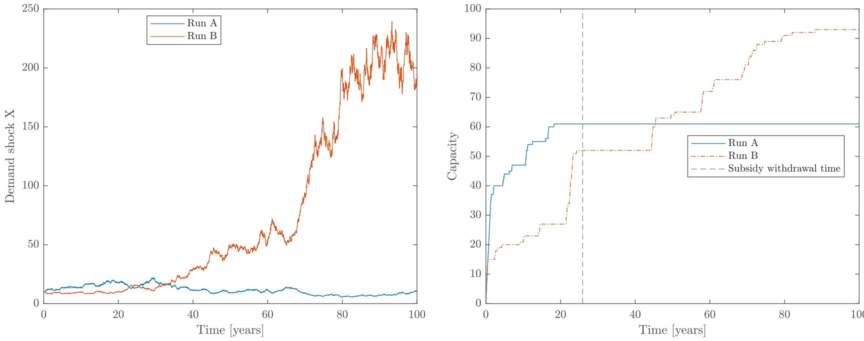
Next, we discuss the role of subsidy withdrawal risk on the monopolist's total capacity over time. The monopolist increases their capacity more during the subsidy's lifetime when the subsidy withdrawal risk is higher. As the monopolist anticipates the future withdrawal of the subsidy and the resulting increase in the investment cost, they move the investment that they would usually undertake at the mid-term (10–20 years) to the short term (less than 10 years). Consequently, the short-term capacity under a subsidy is higher when the expected life span of the subsidy is shorter. However, the threat of the subsidy being unavailable at the mid-term results in little to no expected investment at the midterm. Hence, the capacity at the mid-term under a subsidy with a longer life span is higher than that under a subsidy with a shorter life span. This effect also remains for the long term, until the time at which the effect of the subsidy has completely faded, after approximately 40 to 50 years.

We show the average capacity over time of 10,000 simulations for different subsidy sizes, θ, in Fig. 4. We again compare the capacity growth against a baseline without a subsidy.

In contrast to the effect of a lower subsidy withdrawal risk, the positive impact of a larger subsidy on investment capacity does last for long and takes longer to fade away. The monopolist increases their capacity more during the subsidy's lifetime when the subsidy is larger in size. However, the monopolist's total capacity grows at a lower rate once the subsidy is withdrawn.

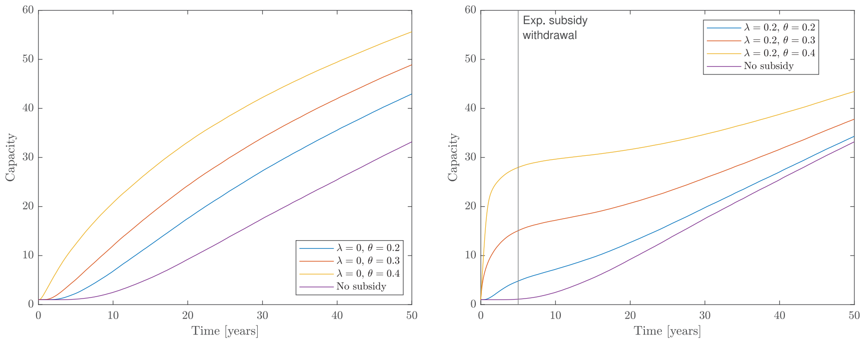
We also examine the effect of a subsidy on investment *after* its withdrawal. The monopolist does not increase their capacity for quite some time directly after the subsidy withdrawal, as shown in the example runs in Fig. 2. The larger the subsidy or the larger the likelihood of a subsidy retraction, the longer the period without investment after a subsidy withdrawal. Both a larger subsidy and a larger subsidy withdrawal risk increase the monopolist's investment during the subsidy's lifetime. Consequently, the monopolist's capacity is at a higher level at the time of the subsidy withdrawal. Once the subsidy is withdrawn, the investment costs for the monopolist rise, while the output prices

[13] The simulation of the geometric Brownian motion in (2) is performed using $X(t_i) = X(t_{i-1}) \cdot e^{(\mu-\frac{\sigma^2}{2})\cdot dt+\sigma\cdot W_i\sqrt{dt}}$ for all moments in time, t_i. W_i is a draw from the standard normal distribution, and t_i, t_{i-1} are two consecutive moments in time with step size, dt. We use antithetic variables for the simulation of the geometric Brownian motion; thus, for a simulation with draws, W_1, W_2, …, a run with $-W_1, -W_2, \ldots$ is performed. The time of subsidy withdrawal, τ_S, is randomly regenerated via the inverse cumulative distribution function (cdf) of a Poisson jump: $\tau_S = -\frac{\log(1-Z)}{\lambda}$, where Z is a draw from the standard normal distribution. We drew 5,000 simulations of the subsidy withdrawal and used the same withdrawal time for the two runs that are linked via the use of antithetic variables.



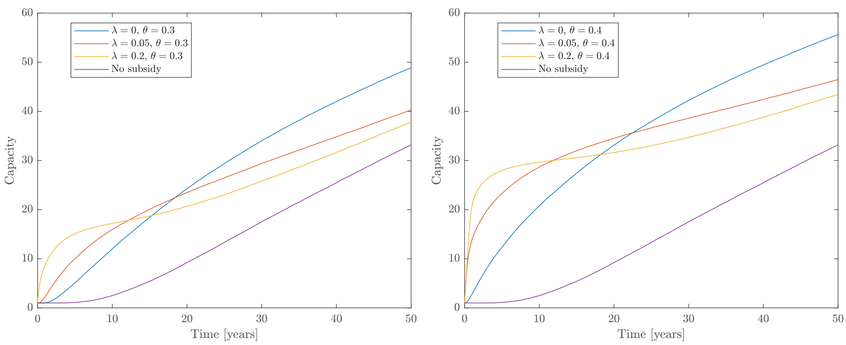
(a) Simulated demand shock X over time (b) Monopolist's capacity over time

Fig. 2. Two example runs of the simulated demand shock, X (left), and the firm's capacity (right). [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$, $\lambda = 0.2$, $\theta = 0.4$.].



(a) No subsidy withdrawal risk ($\lambda = 0$). (b) Subsidy withdrawal risk $\lambda = 0.2$.

Fig. 3. Expected firm's total capacity over time for different levels of subsidy withdrawal risk. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $dK = 1$, $x = 10$.].



(a) $\theta = 0.3$. (b) $\theta = 0.4$.

Fig. 4. Expected firm's total capacity over time for different subsidy sizes. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $dK = 1$, $x = 10$.].

remain at approximately the same level as at the end of the subsidy's lifetime. The higher the monopolist's capacity at the time of subsidy withdrawal, the longer it takes the output prices to grow to a level that attracts investment without a subsidy.[14]

These results have several implications for the policy maker. First, a permanent subsidy is the only way to make a permanent impact on capacity, as the effects of a subsidy scheme fade away over time. Second, a policy maker aiming to reach a (long-term) capacity target must implement a subsidy that is sufficiently large to reach the target during the subsidy's lifetime. The subsidy has no contribution to reaching the goal on time otherwise, while the social planner still pays for the subsidy. If the target is not reached during the subsidy's lifetime, there will be a dry spell of investment and the target will be reached at the same time as in the scenario in which the subsidy is never implemented. Third, a policy maker who is only interested in maximizing the investment while the subsidy is in effect can do this by making the subsidy available for only a short period of time. This happens at the cost of less investment shortly after the subsidy withdrawal and results in less investment in the long run compared to a subsidy of the same size that is available longer.

4.2. Policy: Optimal subsidy and total surplus

We define the welfare optimal subsidy size as the subsidy size that maximizes the total surplus. We consider two different types of investment cost subsidies: a fixed subsidy and a flexible subsidy. With the former, we assume that the policy maker sets the subsidy size equal to a constant, and does not change this over time. In the case of a flexible subsidy, the policy maker can adjust the subsidy size for as long as the subsidy is alive. We aim to answer the following two questions: First, what is the optimal subsidy size in the case of a fixed subsidy.[15]? Second, is it possible for a policy maker to improve welfare via a flexible or fixed investment cost subsidy?

We start by answering the first question of the optimal subsidy size in the case of a fixed subsidy. We find the welfare optimal fixed subsidy size via an interval search maximizing the average total surplus over 1000 simulations. It is important to consider the time, T, at which the social planner wants to maximize the total surplus. In Fig. 5, we plot the welfare optimal fixed subsidy size as a function of the time, T, at which the social planner aims to maximize the total surplus, assuming no policy withdrawal risk (i.e., $\lambda = 0$). From this figure, we can determine how a social planner should choose their subsidy size given a certain horizon at which the total surplus should be maximized. For example, if a social planner aims to maximize the total surplus after 60 years, the optimal fixed subsidy is 60% ($\tilde{\theta}^*_{\lambda=0} = 0.6$) if the initial price is $x = 20$ and 13% ($\tilde{\theta}^*_{\lambda=0} = 0.13$) if the initial price is $x = 20$.

The welfare optimal fixed subsidy size strongly depends on the time at which the social planner maximizes the total surplus as well as the initial output price. The further into the future the social planner aims to obtain the maximum surplus, the larger the optimal subsidy. The trade-off faced by the social planner is whether it is worth incurring high costs for the investment today to accumulate more of both consumer and producer surpluses over time. A social planner with a short-term focus should not implement a subsidy as it takes time for the consumer and producer surpluses to grow and offset the high costs of investment. The optimal fixed subsidy size also increases with the initial output price. The larger the initial output price, the more valuable new investments are from a social welfare viewpoint.

Next, we examine the total surplus under different policies, comparing it with that with a no-subsidy baseline. We compare the fixed

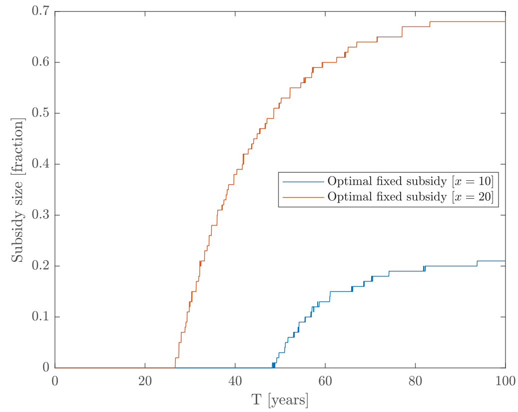


Fig. 5. Optimal fixed subsidy maximizing total surplus at different time horizons for different starting prices, x.

subsidy that maximizes the surplus at time T with the flexible subsidy that maximizes the surplus over an infinite time horizon, where the latter is equivalent to a social planner who maximizes total surplus, investing as discussed in Proposition 3. Fig. 6 shows the accumulated total surplus over time for different subsidy withdrawal risks when the social planner aims to maximize the total surplus after 100 years using a fixed subsidy.

Straightforwardly, the flexible subsidy outperforms the fixed subsidy and the no-subsidy scenario over the long term, as the social planner can adapt the subsidy size over time. However, the fixed subsidy still yields better total welfare results than the no-subsidy baseline. The difference between the fixed-subsidy and no-subsidy scenarios is largest when there is no subsidy withdrawal risk.

In Figs. 7(a) and 7(b), we show what happens to the accumulated total surplus when the social planner aims to maximize surplus after 50 years with a fixed subsidy instead of after 100 years (as shown in Figs. 6(a) and 6(b)).[16]

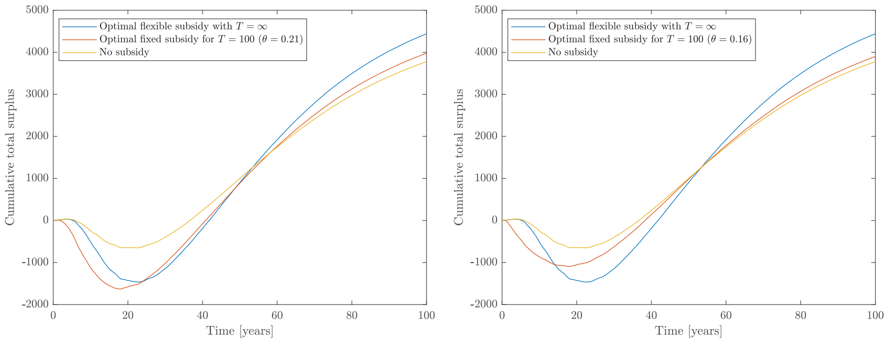
Comparing Figs. 6 and 7, we conclude that the total surplus under a fixed subsidy moves closer to the no-subsidy curve and further away from the optimal flexible subsidy when the social planner with the fixed subsidy has a more myopic mindset. The optimal subsidy is also significantly smaller when the social planner is more myopic and the subsidy is subject to subsidy withdrawal risk — similar to the scenario of no-subsidy withdrawal risk shown in Fig. 5. Interestingly, both the fixed and the flexible subsidies perform poorly in the short term compared to the no-subsidy scenario. The reason for this is that the subsidy attracts investment that leads to significant costs in the short term. These costs are only offset by the consumer surplus that is gained over a long time period. Thus, a subsidy only has value for total welfare in the long term. If the social planner aims to maximize the total surplus today, it is better *not* to implement a subsidy.

Surprisingly, the optimal fixed subsidy is larger under a subsidy withdrawal risk, $\lambda = 0.2$, than under no-subsidy withdrawal risk, ($\lambda = 0$). This results from the relatively short-time focus of the social planner in combination with the subsidy being available forever when $\lambda = 0$. Therefore, the firm has little incentive to invest now, while the social planner wants to see investment relatively soon. It means that attracting investment now via the subsidy is rather costly, while the cost cannot be earned back in 50 years. However, the subsidy withdrawal risk of $\lambda = 0.2$ provides the firm with a natural incentive

[14] A detailed analysis of the distribution of the times of the first investment after a subsidy withdrawal is show in Appendix C.

[15] Note that when the subsidy is flexible, the welfare optimal subsidy size is given by Proposition 4 We provide a numerical example in Appendix D.

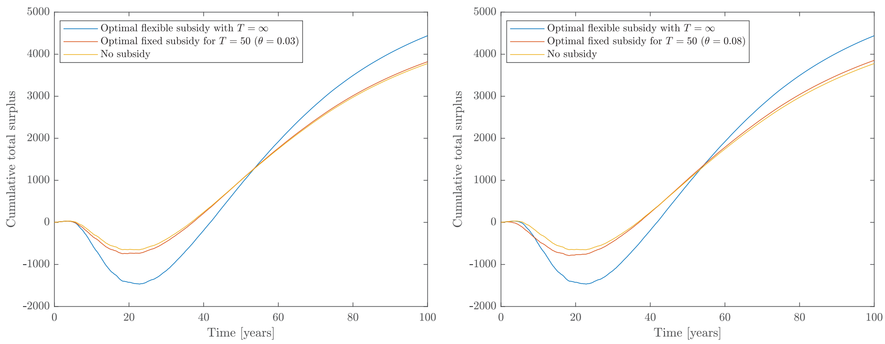
[16] In Appendix E, we perform the same analysis as in Figs. 6 and 7 with an initial demand intercept $x = 20$ instead of $x = 10$.



(a) $\lambda = 0$.

(b) $\lambda = 0.2$.

Fig. 6. Cumulative total surplus over time for different levels of subsidy termination risk, λ, with a social planner maximizing total surplus at $T = 100$. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$.].



(a) $\lambda = 0$, $T = 50$.

(b) $\lambda = 0.2$, $T = 50$.

Fig. 7. Cumulative total surplus over time for different levels of subsidy termination risk, λ, with a social planner who maximizes total surplus at $T = 50$. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$.].

to invest relatively early, even when the subsidy is small. This means that the social planner requires less subsidy to attract investment now, leading to a higher surplus at $T = 50$.

5. Conclusion and policy implications

This study examines the effect of an investment cost subsidy subject to withdrawal risk on a monopolist's series of infinitesimal investments. The social planner aims to increase capacity or maximize welfare, and does so by implementing a subsidy. The size of the subsidy is decided upon by the social planner and is assumed to be either variable or fixed throughout the entire lifetime of the subsidy. The timing of the subsidy termination is assumed to be random, with a known probability distribution. The monopolist determines when to invest. The investment is irreversible and incremental. We examine both the problem of the profit-maximizing firm and that of the social planner who aims to maximize welfare.

Examining the firm's problem, we find that a firm invests sooner when the likelihood of subsidy withdrawal or the subsidy size is larger. Compared to a scenario in which no subsidy is implemented, the monopolist is having a ball and invests more during the lifetime of the subsidy. This result starkly contrasts with the investment of a monopolist that instead has a one-time (lumpy) investment. Once the subsidy is withdrawn, the monopolist stops with investment until the prices have grown sufficiently to attract investment without a subsidy. This means that a policy maker can use a subsidy to attract investment in the short term; however, this effect of the subsidy tapers off over time. Furthermore, for a subsidy to be effective in letting the industry's capacity reach a capacity target faster than an industry without subsidy, the subsidy must be sufficiently large such that the target is reached during the subsidy's lifetime. If the target is not reached during the subsidy's lifetime, the target will be reached at the same time as in the scenario in which a subsidy is never implemented, meaning that the subsidy has no contribution to reaching the goal on time.

Considering the social planner's problem of welfare maximization, we find that both flexible and fixed subsidies increase total welfare in the long run if optimally set. When a firm accounts for the risk of a subsidy being withdrawn in the future, the policy maker can use a flexible subsidy as a tool to perfectly align the monopolist's investment with the optimal social investment. The optimal flexible subsidy size increases with the monopolist's capacity and decreases with subsidy withdrawal risk. Although the social planner can use a flexible or fixed subsidy to increase welfare in the long run, the total surplus in the short term under a subsidy is generally lower than that without a subsidy. Investment is very costly, while it takes time to accumulate consumer and producer surpluses to offset the investment cost. This also leads to welfare in the midterm being lower for the welfare optimal flexible subsidy with a long-term horizon than for the welfare optimal fixed subsidy with a mid-term horizon. The optimal fixed subsidy is extremely sensitive to the social planner's time horizon.

Its size decreases if a social planner is more myopic. A social planner with long-term goals should implement a large subsidy, and this policy is most effective if the subsidy withdrawal risk is low. Generally, the optimal fixed subsidy size decreases with the subsidy retraction risk. The exception is when prices are low, in which case an increase in subsidy retraction risk can lead to an increase in the optimal fixed subsidy.

For future research, it may be interesting to study the role of competition. Huisman and Kort (2015) examine a duopoly in which two firms each can do a lumpy investment, and find that the market leader invests sooner than a monopolist due to the competition. A similar effect can be expected in the presence of a subsidy subject to withdrawal risk: a firm subject to competition and subsidy withdrawal risk may invest sooner than a monopolist subject to subsidy withdrawal risk alone. This effect is most likely amplified if one assumes the social planner is subject to a budget constraint and may withdraw the subsidy when the budget for the subsidy is depleted.

Furthermore, one may want to include technological developments as well as multiple policy interventions to examine long-term policy impact. In our study, we focused on the long-term impact of a single policy. However, to do a forecast of the future and explore whether long-term policy targets will be reached, one needs to understand how technologies will develop over time, and how policy interventions on the mid-term can steer the market for the long-term.

CRediT authorship contribution statement

Roel L.G. Nagy: Conceptualization, Methodology, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing, Visualization. **Stein-Erik Fleten:** Conceptualization, Methodology, Writing – review & editing, Supervision, Funding acquisition. **Lars H. Sendstad:** Conceptualization, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Miscellaneous derivations

A.1. Derivation and discussion of the monopolist's objective

Let $\Delta P_i(X)$ be the price change from increasing the capacity for the ith time, i.e., $\Delta P_i(X) = P(X\cdot, K_i) - P(X\cdot, K_{i-1})$. The objective can be rewritten as follows:

$$V = \sup_{\tau_1,\tau_2,\ldots} \mathbb{E}\left[\sum_{i=0}^{\infty}\int_{\tau_i}^{\tau_{i+1}} P(X,K_i)\cdot K_i \cdot e^{-rt}dt - \sum_{i=1}^{\infty}(1-\theta\cdot \mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot e^{-r\tau_i}\middle| X(\tau_i),\xi(\tau_i)\right] \tag{A.1}$$

$$= \sup_{\tau_1,\tau_2,\ldots}\mathbb{E}\left[\int_0^{\infty} P(X,k)\cdot k\cdot e^{-rt}dt + \sum_{i=1}^{\infty}\int_{\tau_i}^{\infty}\left(\Delta P_i(X)\cdot K_{i-1} + P(X,K_i)\cdot dK\right)\cdot e^{-rt}dt - \sum_{i=1}^{\infty}(1-\theta\cdot\mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot e^{-r\tau_i}\middle| X(0)=x,\xi(0)=1\right] \tag{A.2}$$

$$= \mathbb{E}\left[\int_0^{\infty}P(X,k)\cdot k\cdot e^{-rt}dt \middle| X(0)=x,\xi(0)=1\right] + \sum_{i=1}^{\infty}\sup_{\tau_i}\left\{\mathbb{E}\left[\int_{\tau_i}^{\infty}\left(\Delta P_i(X)\cdot K_{i-1}+P(X,K_i)\cdot dK\right)\cdot e^{-rt}dt -(1-\theta\cdot\mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK\cdot e^{-r\tau_i}\middle| X(0)=x,\xi(0)=1\right]\right\}. \tag{A.3}$$

Eq. (A.3) shows that when capacity is increased, there are only three relevant factors (which are within the sup-operator of Eq. (A.3)):

(a) the additional revenue from the capacity expansion, captured by the term, $P(X(t),K_i)\cdot dK$;
(b) the price change decreasing the marginal revenue of every unit of the current capacity, captured by the term, $\Delta P_i(X(t))\cdot K_{i-1}$; and
(c) the investment cost of expanding the capacity, dependent on the availability of the subsidy and captured by the term, $(1-\theta\cdot \mathbb{1}_{\xi(\tau_i)})\kappa\cdot dK$.

As long as the costs explained in factors (b) and (c) together outweigh the benefits from (a), it is optimal for the monopolist to delay increasing its capacity.

A.2. Derivation and discussion of the social planner's objective

The consumer surplus is calculated by taking the expectation and integral over the instantaneous consumer surplus (ICS) (see, e.g., Huisman and Kort (2015)). The instantaneous consumer surplus is given by

$$ICS(X,K) = \int_{P(X,K)}^{X} D(P)dP \tag{A.4}$$

$$= \frac{1}{2}\eta X K^2, \tag{A.5}$$

where $D(P)$ is the demand function, i.e., the inverse of (1). The consumer surplus can be derived as follows:

$$CS(X,K) = \mathbb{E}\left[\sum_{i=1}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\eta X(t)\cdot K_{i-1}^2\cdot e^{-rt}dt\middle| X(0)=x\right] \tag{A.6}$$

$$= \mathbb{E}\left[\sum_{i=1}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\eta X(t)\cdot\left(k^2+2k(i-1)dK + (i-1)^2dK^2\right)\cdot e^{-rt}dt\middle| X(0)=x\right] \tag{A.7}$$

$$= \mathbb{E}\left[\int_0^{\infty}\frac{1}{2}\eta X(t)\cdot k^2\cdot e^{-rt}dt\middle| X(0)=x\right] + \mathbb{E}\left[\sum_{i=1}^{\infty}\int_{\tau_{i-1}}^{\tau_i}\frac{1}{2}\eta X(t)\cdot\left(2k(i-1) + (i-1)^2dK\right)dK\cdot e^{-rt}dt\middle| X(0)=x\right] \tag{A.8}$$

$$= \mathbb{E}\left[\int_0^{\infty}\frac{1}{2}\eta X(t)\cdot k^2\cdot e^{-rt}dt\middle| X(0)=x\right] + \sum_{i=1}^{\infty}\mathbb{E}\left[\int_{\tau_{i-1}}^{\infty}\frac{1}{2}\eta X(t)\cdot\left(2k + (2i-1)dK\right)dK\cdot e^{-rt}dt\middle| X(0)=x\right]. \tag{A.9}$$

The producer surplus under any demand function is derived in Appendix A.1. The producer surplus under the demand function given by (1) is given by

$$\sup_{\tau_1,\tau_2,\dots} PS(X,K) = \mathbb{E}\left[\int_0^\infty X(t)\cdot(1-\eta k)\cdot k\cdot e^{-rt}dt \,\middle|\, X(0)=x\right] + \sum_{i=1}^{\infty}\sup_{\tau_i}\left\{ \mathbb{E}\left[\int_{\tau_i}^{\infty}\left(-\eta X(t)\cdot dK\cdot K_{i-1} + X(t)\cdot(1-\eta K_i)\cdot dK\right)\cdot e^{-rt}dt - \kappa\cdot dK\cdot e^{-r\tau_i}\,\middle|\, X(0)=x\right]\right\}. \tag{A.10}$$

Then, we add the expressions for the consumer surplus and the producer surplus to find the total surplus:

$$TS(X,K) = \mathbb{E}\left[\int_0^\infty X(t)\cdot\left(1-\frac{1}{2}\eta k\right)\cdot k\cdot e^{-rt}dt\,\middle|\, X(0)=x\right] + \sum_{i=1}^{\infty}\mathbb{E}\left[\int_{\tau_i}^{\infty}\left(X(t)\cdot(1-\eta K_i)+\eta X(t)\cdot dK\right)\cdot dK\cdot e^{-rt}dt - \kappa\cdot dK\cdot e^{-r\tau_i}\,\middle|\, X(\tau_{i-1})\right]. \tag{A.11}$$

In Eq. (A.11), the term in the first line, $X(t)\cdot(1-\frac{1}{2}\eta k)\cdot k$, captures the total surplus if capacity remains at capacity $K=k$ forever. When increasing the capacity, there are three elements (all in the second line of Eq. (A.11)) that change the total surplus:

(i) the producer obtains an additional profit from the additional unit of capacity, dK, that is sold against the price, $X(t)\cdot(1-\eta K_i)$;
(ii) the consumer surplus increases as supply increases, while the producer's marginal revenue for their current production decreases as supply increases. The increase in consumer surplus dominates the negative effect on the producer surplus. Both effects are captured in the term, $\eta X(t)\cdot dK^2$;
(iii) when the producer increases their capacity, they pay the investment cost, $\kappa\cdot dK$.

A social planner maximizing total surplus will increase their capacity when the effects in (i) and (ii) jointly outweigh the investment cost of increasing the capacity in (iii). Compared to the profit-maximizing monopolist's considerations outlined in the discussion of Eq. (A.3), effects (i) and (iii) for the social planner are the same as (a) and (c) for the producer. The social planner and the monopolist have different optimal decisions due to the difference in the effect of the increase of supply discussed in (ii) and (b), respectively. For a firm, increasing the supply has a negative effect on the value of the current production (i.e., production at the level before the capacity increase) as it decreases the output price. For the social planner, the negative effect is offset by an increase in consumer surplus.

A.3. Derivation of the optimal subsidy without subsidy withdrawal risk

Solving the monopolist's optimal investment threshold for a subsidy of size θ without any withdrawal risk, $(\lambda=0)$, yields

$$X_1|_{\lambda=0}(K) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{(r-\mu)(1-\theta)\kappa}{1-2\eta K}. \tag{A.12}$$

The social planner's optimal investment threshold for maximizing total surplus is given by

$$X_S(K) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{(r-\mu)\kappa}{1-\eta K}. \tag{A.13}$$

Solving $X_1|_{\lambda=0}(K) = X_S(K)$ for θ yields the stated expression.

Alternatively, one can derive the stated expression by substituting $\lambda=0$ into (14), using the fact that $\beta_{11}=\beta_{01}$ when $\lambda=0$.

Appendix B. Proofs of propositions and corollaries

B.1. Proof of Proposition 1

Using Itô calculus and the Bellman equation, it follows that

$$\frac{1}{2}\sigma^2X^2\cdot\frac{d^2V_0(X,K)}{dX^2}+\mu X\cdot\frac{dV_0(X,K)}{dX}-rV_0(X,K)=0 \tag{B.1}$$

should hold for the value of the option to expand capacity without the subsidy for the current value, X, of the demand shock process and K for the capacity. In this ordinary differential equation (ODE), r is the discount rate. The solution to this ODE is given by

$$V_0(X,K) = A_{01}(K)\cdot X^{\beta_{01}} + \frac{X(1-\eta K)K}{r-\mu}, \tag{B.2}$$

where $A_{01}(K)$ is a positive expression to be determined. The marginal revenue of the option with respect to capacity is given by

$$\frac{dV_0(X,K)}{dK} = \frac{dA_{01}(K)}{dK}\cdot X^{\beta_{01}} + \frac{X(1-2\eta K)}{r-\mu}. \tag{B.3}$$

We follow the approach by Dixit and Pindyck (1994) and apply the value matching and smooth pasting conditions to the objective (B.3) to derive the optimal investment threshold. The value matching condition tells us that when the monopolist decides to expand, their marginal revenue equals marginal costs. The smooth pasting guarantees that the expression we value match is smooth with respect to the timing threshold, X. The value matching and smooth pasting conditions for the investment threshold without subsidy are given by

$$\frac{dA_{01}(K)}{dK}\cdot\left(X_0^i\right)^{\beta_{01}} + \frac{X_0^i(1-2\eta K_i)}{r-\mu} = \kappa, \tag{B.4}$$

$$\beta_{01}\cdot\frac{dA_{01}(K)}{dK}\cdot\left(X_0^i\right)^{\beta_{01}-1} + \frac{1-2\eta K_i}{r-\mu} = 0. \tag{B.5}$$

Dixit and Pindyck (1994) solve this system of equations and conclude that the optimal investment threshold without subsidy is given by

$$X_0^i(K_i) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{(r-\mu)\kappa}{1-2\eta K_i}. \tag{B.6}$$

The expression $A_{01}(K)$ has to satisfy

$$\frac{dA_{01}(K_i)}{dK} = -\left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1}\cdot\left(\frac{1-2\eta K_i}{\beta_{01}(r-\mu)}\right)^{\beta_{01}}. \tag{B.7}$$

By integration,[17], we obtain

$$A_{01}(K_i) = \left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1}\cdot\frac{1-2\eta K_i}{2\eta(\beta_{01}+1)}\cdot\left(\frac{1-2\eta K_i}{\beta_{01}(r-\mu)}\right)^{\beta_{01}} \tag{B.8}$$

$$= \frac{\kappa(1-2\eta K_i)}{2\eta(\beta_{01}-1)(\beta_{01}+1)}\cdot\left(\frac{(\beta_{01}-1)(1-2\eta K_i)}{\beta_{01}\kappa(r-\mu)}\right)^{\beta_{01}}. \tag{B.9}$$

B.2. Proof of Proposition 2

We apply Ito's lemma and use the Bellman equation to derive the value of the option to expand capacity under a subsidy. The equation is given by

$$\frac{1}{2}\sigma^2X^2\cdot\frac{d^2V_1(X,K)}{dX^2}+\mu X\cdot\frac{dV_1(X,K)}{dX}-rV_1(X,K)+ \lim_{dt\to 0}\mathbb{P}[\text{Subsidy withdrawal occurs in time interval } dt]\cdot\frac{1}{dt}\cdot(V_0(X,K)-V_1(X,K))=0. \tag{B.10}$$

In this case, we also have to account for the risk of policy withdrawal. We obtain

$$\frac{1}{2}\sigma^2X^2\cdot\frac{d^2V_1(X,K)}{dX^2}+\mu X\cdot\frac{dV_1(X,K)}{dX}-rV_1(X,K) + \lambda(V_0(X,K)-V_1(X,K))=0. \tag{B.11}$$

[17] Following Dixit and Pindyck (1994, p. 365–366) we set the integration constant to be equal to zero.

The solution to (B.11) is given by

$$V_1(X,K) = A_{11}(K)\cdot X^{\beta_{11}} + A_{01}(K)\cdot X^{\beta_{01}} + \frac{X(1-\eta K)K}{r-\mu}, \tag{B.12}$$

where $A_{11}(K)$ is a positive expression to be determined.

Similar to the case without a subsidy, the optimal investment threshold follows from solving the system comprising the value matching and smooth pasting conditions. We thus obtain the following two equations:

$$\frac{dA_{11}(K)}{dK}\cdot (X_1^i)^{\beta_{11}} + \frac{dA_{01}(K_i)}{dK}\cdot (X_1^i)^{\beta_{01}} + \frac{X_1^i(1-2\eta K_i)}{r-\mu} = (1-\theta)\kappa, \tag{B.13}$$

$$\beta_{11}\cdot\frac{dA_{11}(K)}{dK}\cdot (X_1^i)^{\beta_{11}-1} + \beta_{01}\cdot\frac{dA_{01}(K_i)}{dK}\cdot (X_1^i)^{\beta_{01}-1} + \frac{1-2\eta K_i}{r-\mu} = 0. \tag{B.14}$$

Combining these two equations results in an implicit equation for our investment threshold, X_1^i:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\frac{dA_{01}(K_i)}{dK}\cdot (X_1^i)^{\beta_{01}} + \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1^i(1-2\eta K_i)}{r-\mu} - (1-\theta)\kappa = 0. \tag{B.15}$$

B.3. Proof of Corollary 1

We refer to the implicit Eq. (11) as $f(X_1)$:

$$f(X_1) \equiv \frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\frac{dA_{01}(K)}{dK}\cdot X_1^{\beta_{01}} + \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-2\eta K)}{r-\mu} - (1-\theta)\kappa = 0. \tag{B.16}$$

By total differentiation, we derive the following:

$$0 = \frac{df}{d\lambda} = \frac{\partial f}{\partial\lambda} + \frac{\partial f}{\partial X}\cdot\frac{\partial X}{\partial\lambda} \iff \frac{\partial X}{\partial\lambda} = -\frac{(\frac{\partial f}{\partial\lambda})}{(\frac{\partial f}{\partial X})}. \tag{B.17}$$

We show that $\frac{df}{d\lambda} < 0$ by showing that both $\frac{\partial f}{\partial\lambda} > 0$ and $\frac{\partial f}{\partial X} > 0$. First, we prove $\frac{\partial f}{\partial\lambda} > 0$. By directly differentiating (B.16) with respect to λ, we derive the following:

$$\frac{\partial f}{\partial\lambda} = \frac{1}{\beta_{11}^2}\cdot\frac{d\beta_{11}}{d\lambda}\cdot\left(\beta_{01}\cdot\frac{dA_{01}(K)}{dK}\cdot X_1^{\beta_{01}} + \frac{X_1(1-2\eta K)}{r-\mu}\right) \tag{B.18}$$

$$= -\frac{1}{\beta_{11}}\cdot\frac{d\beta_{11}}{d\lambda}\cdot\frac{dA_{11}(K)}{dK}\cdot X_1^{\beta_{11}}, \tag{B.19}$$

where $\frac{d\beta_{11}}{d\lambda} = \frac{1}{\sigma(\beta_{11}-1)+\mu} > 0$. $\frac{\partial f}{\partial\lambda} > 0$ follows from $\frac{dA_{11}(K)}{dK} < 0$.

Second, it remains to be proven that $\frac{\partial f}{\partial X} > 0$ for any λ. The expression for $\frac{\partial f}{\partial X}$ is given by

$$\frac{\partial f}{\partial X} = \frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot\frac{dA_{01}(K)}{dK}\cdot\beta_{01}\cdot X_1^{\beta_{01}-1} + \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{1-2\eta K}{r-\mu}, \tag{B.20}$$

where $\frac{dA_{01}(K)}{dK} = -\left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1}\cdot\left(\frac{1-2\eta K}{\beta_{01}(r-\mu)}\right)^{\beta_{01}} < 0$. We rewrite the condition, $\frac{\partial f}{\partial X} > 0$, using the expressions for $\frac{\partial f}{\partial X}$ and $\frac{dA_{01}(K)}{dK}$ to

$$(\beta_{11}-\beta_{01})\cdot\left(\frac{\beta_{01}-1}{\beta_{01}}\cdot\frac{1-2\eta K}{\kappa(r-\mu)}\cdot X_1\right)^{\beta_{01}-1} < \beta_{11}-1. \tag{B.21}$$

By recognizing the expression for the investment threshold without a subsidy, X_0, on the left hand side, we can rewrite this as

$$(\beta_{11}-\beta_{01})\cdot\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1} < \beta_{11}-1. \tag{B.22}$$

Note that this expression holds for $\lambda = 0$, as then $\beta_{11} = \beta_{01} > 1$ and $X_1 = (1-\theta)X_0 < X_0$. Therefore, $\frac{\partial f}{\partial\lambda} < 0$ at $\lambda = 0$. For $\lambda > 0$ (hence $\beta_{11} > \beta_{01}$), we can rewrite the condition to

$$\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1} < \frac{\beta_{11}-1}{\beta_{11}-\beta_{01}}. \tag{B.23}$$

This condition always holds for positive λ as then, both $\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1} < 1$, while $\frac{\beta_{11}-1}{\beta_{11}-\beta_{01}} > 1$. To see $\left(\frac{X_1}{X_0}\right)^{\beta_{01}-1} < 1$, note that at $\lambda = 0$, we have $X_1 < X_0$ and $\frac{df}{d\lambda} < 0$. Therefore, at some small positive λ, we see that X_1 is lower, hence $X_1 < X_0$ still holds and condition (B.23) holds, leading to $\frac{df}{d\lambda} < 0$ at that positive value of λ.

B.4. Proof of Corollary 2

Similarly to the proof of Corollary 1 (Appendix B.3), the derivative of the optimal investment threshold with respect to subsidy size, θ, can be written as

$$\frac{\partial X}{\partial\theta} = -\frac{(\frac{\partial f}{\partial\theta})}{(\frac{\partial f}{\partial X})}. \tag{B.24}$$

We directly derive $\frac{\partial f}{\partial\theta}$ by differentiation of the implicit Eq. (B.16):

$$\frac{\partial f}{\partial\theta} = \kappa. \tag{B.25}$$

Therefore,

$$\frac{\partial X}{\partial\theta} = -\frac{\kappa}{(\frac{\partial f}{\partial X})}, \tag{B.26}$$

and

$$\frac{\partial X}{\partial\theta} < 0 \iff \frac{\partial f}{\partial X} > 0. \tag{B.27}$$

Proving $\frac{\partial f}{\partial X} > 0$ for any θ can be done in the same way as proving $\frac{\partial f}{\partial X} > 0$ for any λ; see the second half of the proof of Corollary 1 in Appendix B.3.

B.5. Proof of Proposition 3

Repeating the steps in the proof of Proposition 1 (Appendix B.1), it follows that the value of the social planner's option satisfies the following ODE:

$$\frac{1}{2}\sigma^2X^2\cdot\frac{d^2V_S(X,K)}{dX^2} + \mu X\cdot\frac{dV_S(X,K)}{dX} - rV_S(X,K) = 0 \tag{B.28}$$

The marginal added surplus of the option with respect to capacity is given by

$$\frac{dV_S(X,K)}{dK} = \frac{dA_S(K)}{dK}\cdot X^{\beta_{01}} + \frac{X(1-\eta K)}{r-\mu}, \tag{B.29}$$

in which V_S is the value of the social planner's option, and $A_S(K)$ is some positive function.

We apply the value matching and smooth pasting conditions to the objective (B.29) to derive the optimal social investment threshold. The value matching and smooth pasting conditions for the optimal social investment threshold (denoted by X_S^i) are given by

$$\frac{dA_S(K_i)}{dK}\cdot (X_S^i)^{\beta_{01}} + \frac{X_S^i(1-\eta K_i)}{r-\mu} = \kappa, \tag{B.30}$$

$$\beta_{01}\cdot\frac{dA_S(K_i)}{dK}\cdot (X_S^i)^{\beta_{01}-1} + \frac{1-\eta K_i}{r-\mu} = 0. \tag{B.31}$$

We find that the optimal investment threshold without a subsidy is given by

$$X_S^i(K_i) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{(r-\mu)\kappa}{1-\eta K_i}. \tag{B.32}$$

The expression, $A_S(K)$, has to satisfy the following:

$$\frac{dA_S(K_i)}{dK} = -\left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1}\cdot\left(\frac{1-\eta K_i}{\beta_{01}(r-\mu)}\right)^{\beta_{01}}. \tag{B.33}$$

As before, we integrate to obtain

$$A_S(K_i) = \left(\frac{\beta_{01}-1}{\kappa}\right)^{\beta_{01}-1}\cdot\frac{1-\eta K_i}{\eta(\beta_{01}+1)}\cdot\left(\frac{1-\eta K_i}{\beta_{01}(r-\mu)}\right)^{\beta_{01}} \tag{B.34}$$

Table C.2
The percentage of simulations in which no investment occurred after (before) a subsidy withdrawal.

	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$
$\lambda = 0.05$	14.81 (44.72)	18.73 (20.77)	25.01 (0.28)
$\lambda = 0.1$	10.91 (53.97)	14.38 (15.42)	20.13 (0.50)
$\lambda = 0.2$	9.65 (60.19)	12.85 (2.26)	18.33 (0.82)

$$= \frac{\kappa(1-\eta K_i)}{\eta(\beta_{01}-1)(\beta_{01}+1)} \cdot \left(\frac{(\beta_{01}-1)(1-\eta K_i)}{\beta_{01}\kappa(r-\mu)}\right)^{\beta_{01}}. \quad \text{(B.35)}$$

B.6. Proof of Proposition 4

Solving the monopolist's optimal investment threshold for a given subsidy of size θ and any level of withdrawal risk follows from the implicit Eq. (11). Substituting the social planner's optimal investment threshold for maximizing total surplus, defined in (13), into (11) and solving for θ yields

$$\theta^*_\lambda(K) = 1 - \frac{1}{\kappa} \cdot \left(\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot \frac{dA_{01}(K)}{dK} \cdot X_S^{\beta_{01}} + \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_S(1-2\eta K)}{r-\mu}\right). \quad \text{(B.36)}$$

Plugging in the optimal social investment threshold, X_S, yields

$$\theta^*_\lambda(K) = 1 - \frac{1}{\beta_{11}(\beta_{01}-1)} \cdot \left[\beta_{01}(\beta_{11}-1) \cdot \frac{1-2\eta K}{1-\eta K} - (\beta_{11}-\beta_{01}) \cdot \left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}}\right]. \quad \text{(B.37)}$$

B.7. Proof of Corollary 3

Taking the derivative with respect to K of the optimal subsidy size, $\theta^*(K)$, defined in (14) yields

$$\frac{d\theta^*}{dK} = \frac{\beta_{01}}{\beta_{11}} \cdot \frac{\beta_{11}-1}{\beta_{01}-1} \cdot \frac{\eta}{(1-\eta K)^2} \cdot \left[1 - \frac{\beta_{11}-\beta_{01}}{\beta_{11}-1} \cdot \left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}-1}\right]. \quad \text{(B.38)}$$

As $\beta_{11} > \beta_{01} > 1$ and $\left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}-1} \in (0,1]$, we have that $\frac{d\theta^*}{dK} > 0$.

B.8. Proof of Corollary 4

Taking the derivative with respect to λ of the optimal subsidy size, $\theta^*(K)$, defined in (14) yields

$$\frac{d\theta^*}{d\lambda} = \frac{d\beta_{11}}{d\lambda} \cdot \frac{\beta_{01}}{\beta_{01}-1} \cdot \frac{1}{\beta_{11}^2} \cdot \frac{1-2\eta K}{1-\eta K} \cdot \left[\left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}-1} - 1\right]. \quad \text{(B.39)}$$

As $\frac{d\beta_{11}}{d\lambda} = \frac{1}{\sigma(\beta_{11}-1)+\mu} > 0$ and $\left(\frac{1-2\eta K}{1-\eta K}\right)^{\beta_{01}-1} \in (0,1]$, we have that $\frac{d\theta^*}{d\lambda} \leq 0$.

Appendix C. Statistics on capacity growth after subsidy withdrawal

In Fig. 8, we show the histograms of the number of years it takes the monopolist to increase their capacity for the first time after the subsidy has been withdrawn.

Table C.2 shows both the percentage of simulations that yield no investment after a subsidy withdrawal and the percentage of simulations in which no investment occurs during the subsidy's lifetime. We simulate a total period of 100 years. When the subsidy is small, i.e., $\theta = 0.2$, less than 15% of the simulations always result in no investment after a subsidy retraction. However, when the subsidy is large, i.e., $\theta = 0.4$, approximately 18% to 25% of the simulations yield no investment after a subsidy withdrawal.

We observe that the larger the subsidy, the more likely that investment occurs during the subsidy's lifetime. However, the likelihood of no investment after subsidy withdrawal also increases with subsidy size. The likelihood of no investment after a subsidy withdrawal also increases with subsidy withdrawal risk.

The effect of the likelihood of a subsidy withdrawal has a non-monotonic effect on investment during a subsidy's lifetime due to two opposing effects. First, the firm's incentive to invest now increases when the likelihood of a subsidy withdrawal is larger. However, the time during which the firm can invest under a subsidy has also become shorter. When the subsidy is small, the likelihood of no investment during the subsidy's lifetime increases with the subsidy withdrawal rate, λ. The reward for the firm from investing during the subsidy's lifetime is small and the second effect dominates the first. The likelihood of no investment during the subsidy's lifetime decreases with subsidy withdrawal risk when the subsidy is large. The first effect dominates the second, as the reward for the firm from investing during the subsidy's lifetime is large.

Appendix D. Welfare optimal flexible study

The optimal subsidy size when the subsidy size is flexible is given by Proposition 4. In this appendix, we provide a numerical example and show that our results for the subsidy size are consistent with Corollaries 3 and 4. Furthermore, we break down the total surplus from the simulations of the welfare optimal flexible subsidy.

D.1. Welfare optimal flexible study size

The optimal subsidy size that maximizes total surplus as a function of the firm's capacity, K, θ^*_λ, for different levels of subsidy withdrawal risk, λ, is plotted in Fig. 9.

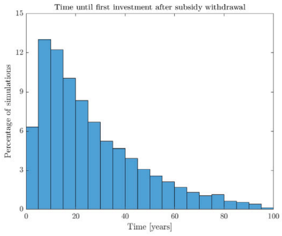
We observe that the optimal subsidy size increases in the monopolist's capacity, which is consistent with Corollary 3. The gap between the monopolist's and social planner's optimal investment thresholds is larger when the current capacity is large. On the one hand, the monopolist has less incentive to increase their capacity when the current capacity is already large, due to one additional unit of capacity yielding a low marginal revenue. On the other hand, a social planner's optimal threshold is somewhat invariant to the current capacity level (see X_S in Fig. 1). A larger subsidy is required to close this gap.

The optimal subsidy size decreases with subsidy withdrawal risk, in line with Corollary 4. The gap between the social planner's optimal threshold and the monopolist's decreases when the likelihood of subsidy withdrawal increases. The monopolist increases their capacity sooner under the pressure of losing the subsidy in the future. Despite the fact that the policy maker's and monopolist's thresholds are better aligned under a larger subsidy withdrawal, this does not per se mean the policy maker's long-term targets are reached faster. Due to the larger subsidy withdrawal risk, the subsidy is also very likely to be withdrawn sooner, meaning that the encouraging effect of the subsidy are also in effect for a shorter period of time.

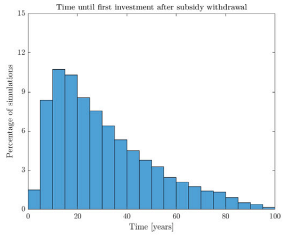
D.2. Statistics on consumer and producer surpluses

This Appendix breaks down the total surplus from welfare optimal flexible subsidies using simulations, identical to the simulations outlined in Section 3.1.

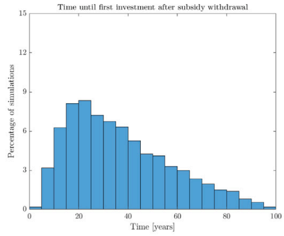
In Fig. 10, the total surplus under the decisions by the monopolist without a subsidy is shown on the left, while the total surplus under the optimal flexible subsidy is shown on the right. The total surplus is broken down into producer and consumer surpluses in both figures. Under the welfare optimal flexible subsidy, the consumer surplus is larger than under the firm's decisions, while the producer surplus is approximately zero. When the monopolist makes the decision, they maximize producer surplus, and the consumer surplus is much smaller



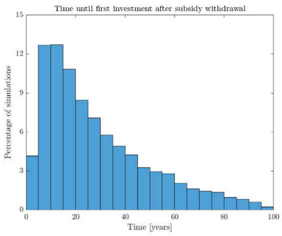
(a) $\lambda = 0.05$, $\theta = 0.2$.



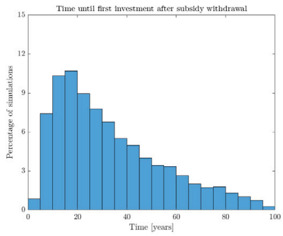
(b) $\lambda = 0.05$, $\theta = 0.3$.



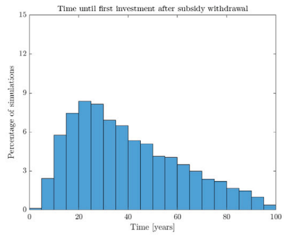
(c) $\lambda = 0.05$, $\theta = 0.4$.



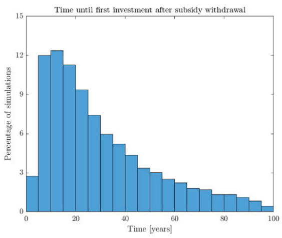
(d) $\lambda = 0.1$, $\theta = 0.2$.



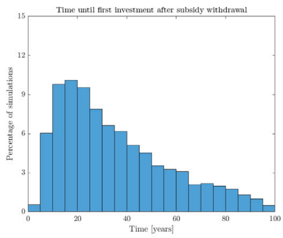
(e) $\lambda = 0.1$, $\theta = 0.3$.



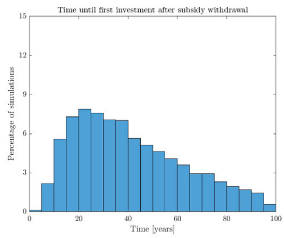
(f) $\lambda = 0.1$, $\theta = 0.4$.



(g) $\lambda = 0.2$, $\theta = 0.2$.



(h) $\lambda = 0.2$, $\theta = 0.3$.



(i) $\lambda = 0.2$, $\theta = 0.4$.

Fig. 8. Histograms of the time until the first investment after a subsidy withdrawal for different levels of subsidy termination risk, λ, and subsidy size, θ. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$.].

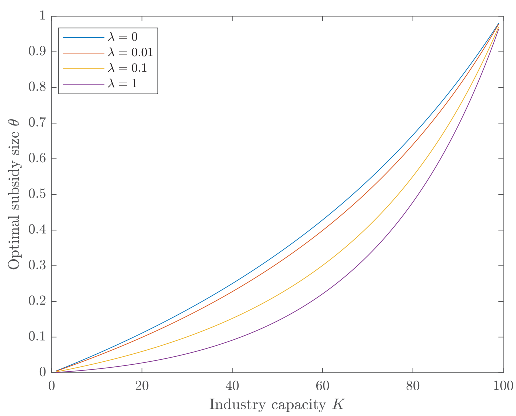


Fig. 9. Optimal subsidy size, θ, as a function of the firm's total capacity for different subsidy retraction risk, λ. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 10$].

than the producer surplus; the firm increases capacity at a lower rate than the social planner to keep output prices higher than desirable from an optimal social viewpoint.

The consumer and producer surplus under the decisions by the monopolist for an optimal subsidy is shown in Fig. 11(b), and the gain in total surplus compared to the no-subsidy case is shown in Fig. 11(a), both for a subsidy withdrawal risk $\lambda = 0.2$. In most of the simulations, the total surplus increases due to a subsidy. The firm invests sooner under a subsidy, hence the consumer surplus increases compared to the case of no-subsidy. The subsidy also increases the producer surplus; however, the increase in producer surplus is mainly financed from the subsidy, hence the social planner's subsidy payouts increase at approximately the same rate. As the firm invests sooner under a subsidy, the consumer surplus increases compared to the no-subsidy scenario.

However, note too that the subsidy is not successful in increasing the total surplus in all simulations. In some of the simulations, the total surplus decreases due to the subsidy while in many no changes to the total surplus occur. As the subsidy causes the firm to increase capacity sooner, there are cases in which the prices decline quickly after the firm has increased capacity. This may lead to significant losses to the already-installed units of capacity, leading to a negative producer surplus. It also keeps out new investors, causing the monopolist's capacity to be low, hence the consumer surplus is also low.

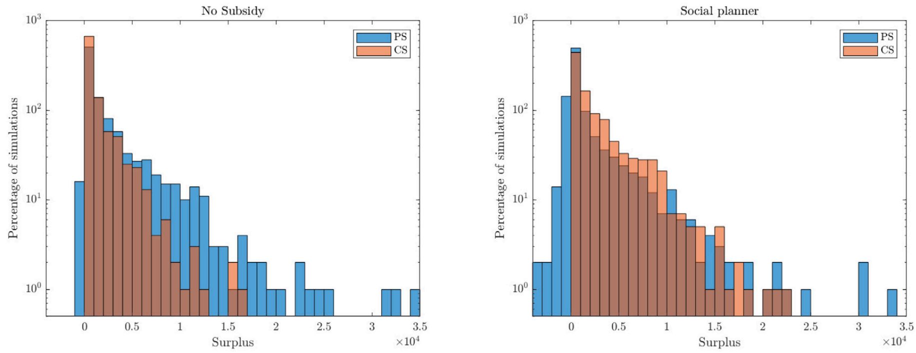


Fig. 10. Distribution of producer and consumer surpluses in the simulations when investment decisions are made by the monopolist without a subsidy (left) and by the social planner (right). [General parameter values: $\mu = 0.01$, $\sigma = 0.05$, $r = 0.03$, $\eta = 0.01$, $\kappa = 300$, $dK = 1$, $x = 10$.].

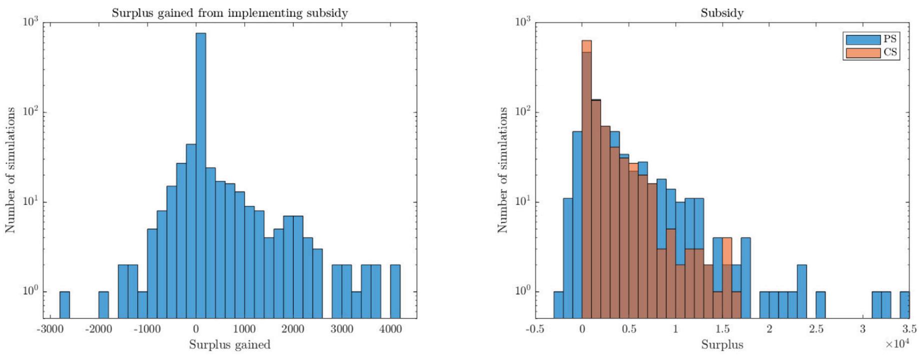
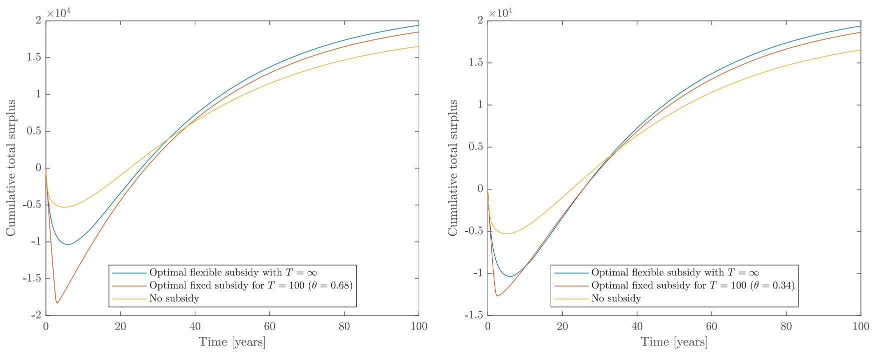


Fig. 11. The gain in total surplus compared to the case when the monopolist not subsidized (left) and the producer and consumer surplus in the simulations by a subsidized monopolist (right), with $\lambda = 0.2$ and $\theta = \theta^*_\lambda$. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.01$, $\kappa = 300$, $dK = 1$, $x = 10$.].



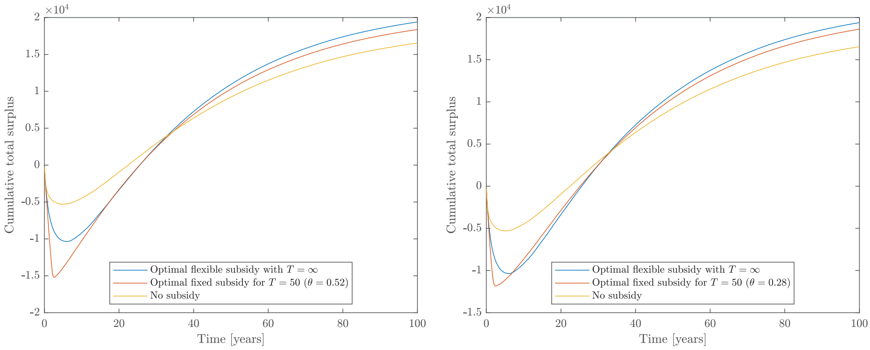
(a) $\lambda = 0$. (b) $\lambda = 0.2$.

Fig. 12. Cumulative total surplus over time for different levels of subsidy termination risk, λ, with a social planner maximizing total surplus at $T = 100$. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 20$.].

Appendix E. Sensitivity analysis for total surplus and initial demand shock x

In this Appendix, we assume $x = 20$ instead of $x = 10$, and examine the total surplus over time in figures, similar to Figs. 6 and 7. The total surplus in all the scenarios has increased due to the higher prices. The trajectories of the no-subsidy and the optimal flexible subsidy cases are approximately identical to their counterparts in Figs. 6 and 7. In this Appendix, we mainly focus on the effects for the optimal fixed subsidy.

In Fig. 12, we plot the cumulative total surplus over time in three different scenarios - no subsidy, optimal fixed subsidy maximizing surplus at $T = 100$, and the optimal flexible subsidy. The optimal fixed subsidy is significantly larger compared to their counterparts in



(a) $\lambda = 0$. (b) $\lambda = 0.2$.

Fig. 13. Cumulative total surplus over time for different levels of subsidy termination risk, λ, with a social planner maximizing total surplus at $T = 50$. [General parameter values: $\mu = 0.02$, $\sigma = 0.10$, $r = 0.05$, $\eta = 0.005$, $\kappa = 300$, $dK = 1$, $x = 20$.].

Figs. 6 and 7 due to a higher initial demand intercept, x. The optimal subsidy increases as the value of investment from a social perspective (i.e., the consumer surplus) increases significantly with the higher prices. Considering the role of the time horizon, T, we still see that the optimal subsidy decreases the more myopic a policy maker is. The argument remains the same: A more myopic social planner does not care about the surplus accounted for over a very long time period, but is more affected by the high investment costs incurred early.

The main difference is the role of the subsidy withdrawal, λ, for the social planner with time horizon $T = 50$ in Fig. 13 compared to Fig. 7. In Fig. 13, the optimal subsidy when $\lambda = 0.2$ is smaller than when $\lambda = 0$, while with the lower output price in Fig. 7, it is the other way around. In both cases, the increase in subsidy withdrawal risk leads to a small decrease in total surplus after both 50 and 100 years.

Appendix F. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.enpol.2022.113309.

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