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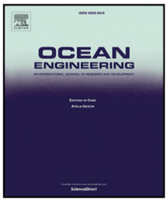

Verification of collision avoidance algorithms in open sea and full visibility using fuzzy logic

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ABSTRACT

Autonomous Surface Vehicles (ASV) have been a subject undergoing intense study since they provide good potential regarding reducing the cost, increasing safety, reliability, efficiency and sustainability. The complexity of the ASVs needs to be tested to gain trust and to prevent fatal consequences. This paper proposes a method using fuzzy logic for evaluating the compliance of Collision-Avoidance Systems (CAS) with regard to the Convention on the International Regulations for Preventing Collisions at Sea (COLREG). For the proof-of-concept purpose, the set of rules for open sea and full visibility is considered; but the framework from this paper can be extended to include narrow channels, traffic separation schemes and restricted visibility by added more fuzzy rules. Fuzzy logic allows handling vague terms and represents linguistic variables and values written in COLREG in a quantitative mathematical form without introducing sharp assumptions or specifications to original COLREG. The paper develops four evaluation systems (1) evaluation of all vessels in pairwise encounters, (2) ownship (OS) evaluation in pairwise encounters for the interest of Class societies, (3) evaluation of all vessels in multiple-vessel encounters, and (4) OS evaluation in multiple-vessel encounters for the interest of Class societies. The designed systems were verified on a set of simulated scenarios. The obtained results were validated against visual assessment of the trajectory, heading and speed plots. The results showed that the evaluation systems provided variables that would be challenging or impossible to obtain by visual assessment.

1. Introduction

Previous studies have shown that the majority of ship collisions were caused by human errors, the percentage of marine accidents and causalities caused by human errors is 75%–96% (Antao and Soares, 2008; Rothblum et al., 2002; Safety4sea, 2018; Insight, 2019). To deal with this risk, collision avoidance rules were formulated as the Convention on the International Regulations for Preventing Collisions (COLREG) by the International Maritime Organization (IMO, 1972). In spite of these sets of rules, collision incidents still happen as 56% of marine accidents and causalities were due to violations of COLREG (Rothblum et al., 2002).

COLREG is written for humans (Benjamin and Curcio, 2004) and this will lead to subjectivity (Naeem et al., 2012). There are several examples showing the flexibility in applying COLREG Rules (Naeem et al., 2012). For example, Rule 14 states to make a starboard alteration, but nothing about how much of an alteration. Another example is that Rule 16 states that the give-way (GW) vessel should "*take early and substantial action*" to avoid a collision (IMO, 1972) but it does not precisely states when is an early action. The conclusion is that COLREG is precise enough, but it needs to be flexible, since how much and what maneuver depends on the situation and especially location. However, its flexibility could be exploited by humans. In another study, Mohovic et al. (2016) presented data to identify gaps in the knowledge and learning of COLREG, based on questionnaires given to nautical Bachelor of science students and experienced captains. The study showed that there are several Rules that are difficult to understand even for professional seafarers and licensed watch officers.

The introduction of autonomous surface vessels (ASVs) will potentially allow reducing the operators' errors due to fatigue or other harsh working conditions (Hoem et al., 2019). ASVs have been a subject undergoing intense study. Autonomy may reduce cost, increasing safety, reliability, efficiency and sustainability. Collision avoidance system (CAS) is one of the most important elements of ASVs. By using a CAS, a potential undesired physical contact can be avoided at a certain

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time in the future by departing a vessel from its planned trajectory. Techniques involved in collision avoidance, could be divided into three categories, i.e. route planning, path planning and reactive collision avoidance (Huang et al., 2020).

On a traditional vessel, the action to avoid a collision should be assessed by a human operator while on a ASV the action will be decided by an onboard computer system. The actions should be compliance with COLREG Rules. It is challenging to incorporate COLREG into a computer (Naeem et al., 2012). One of the dilemma of incorporating them into an ASV is the complexity and vagueness of the Rules (Woerner, 2016). Another challenge is subjective measure when interpreting the Rules since they are written for human operators.

The ASV can be classified into several Levels of Autonomy (LoA), e.g. four levels as in Utne et al. (2017) or ten levels as in Sheridan (1992). For the former, level one is remote operated system, level two management by consent, level three semi autonomous (management by exception), and level four highly autonomous system. The highest level of autonomous systems is able to make decision itself. However, the higher LoA an ASV has, the more complex the software systems become. The increasing complexity of software systems impose a challenge to assure safety-critical systems such that ASVs can be accepted and adopted into the maritime domain. Validation and Verification (V&V) of an ASV is required to obtain trust (Glomsrud et al., 2020) and reduce the chances of fatal consequences, e.g. a collision. It is important to develop improved test schemes capable of dealing with the complexity of safety-critical autonomous systems (Thompson, 2008). Conventional V&V might be insufficient due to the non-deterministic nature of autonomous systems that are capable of learning and adapting continuously (Helle et al., 2016).

V&V has been the focus for automotive and aerospace industries (Koopman and Wagner, 2016; Helle et al., 2016; Lee et al., 2020). In maritime traffic, an important regard for V&V for autonomous vessel is the compliance with COLREG (Ramos et al., 2019). COLREG Rules have been developed for one-to-one vessels, i.e. one pair (Johansen et al., 2016; Woerner and Benjamin, 2018) while the concerning multiple-vessel encounter scenarios are considerably common in practice. The role of a vessel in one-to-one encounter scenario is either stand-on (SO) or give-way (GW). In a multiple-vessel encounter, a conflict might happen since a vessel can be both SO w.r.t. one vessel and GW w.r.t. another vessel (Kjerstad, 2019). In the literature, there has been a lack of protocols of examination of COLREG's compliance for an ASV (Woerner and Benjamin, 2015); and methods for evaluating risk and compliance with COLREG are still being developed.

In 2015, Woerner and Benjamin (2015) argued that there is little literature on scoring protocol compliance in a metric manner. Later (Woerner, 2016) developed an algorithm which evaluated how the vessel responded to avoid a collision in regards to COLREG, with possibilities to perform online and post mission evaluation. The evaluation provided scoring for each active COLREG Rule experienced in the scenario. Woerner et al. (2019) proposed a method for standardized evaluation and certification of collision avoidance systems subjective to COLREG. The paper considered the Rules of overtaking, head-on, crossing, give-way, and stand-on. However, the paper mentioned that quantification of COLREG in terms of angles, velocities and distances and their dependencies on encounter specific factors should be investigated further. Field testings were attempted (Kufoalor et al., 2020) but the number of test scenarios is limited. Therefore, digital twins have been used for simulation-based testing in a dynamic test scenario generation towards automatic assessment COLREG's compliance (Yang et al., 2007; Pedersen et al., 2020; Torben et al., 2022a; Minne, 2017). By using digital twin, it is possible to generate multiple-vessel encounters for automatic testing (Pedersen et al., 2020). In V&V of CAS using scenario-based testing, the challenge is to cover all possibilities. Test case (especially edge-case) scenario generation was studied by using discriminating artificial neural network (Porres et al., 2020), adaptive sampling and unsupervised clustering (Stankiewicz and Mullins, 2019), clustering algorithm (Bolbot et al., 2022), Systems-Theoretic Process Analysis (STPA) (Rokseth et al., 2019), maritime game using machine learning and model checking techniques (Shokri-Manninen et al., 2020), and formal verification approach by integrating the numerical computation with its hybrid system verification (Foster et al., 2020). Torben et al. (2022b) proposed a framework using formal methods which is a mathematically based method taking into account the specification of COLREG, contract-based design and automation of simulation-based testing. However, V&V is only a part of the whole process design and verification rather than a dedicate attempt for V&V.

By this, developing methods to evaluate a CAS for one-to-one and multiple-vessel encounters in regard to COLREG is a great motivation for this paper. Utilizing fuzzy logic to take care of the problem of incorporating COLREG into a computer language is of high interest. The benefits of fuzzy logic is that it reasons more like a human by providing effective responses to complex input and ability to obtain a degree to truth, i.e. a value between 0 and 1, as compared to Boolean logic (Peri and Simon, 2005; Sharma, 2020). Moreover, it can accomplish great results with inexpensive hardware. The system could easily improve performance by adding new features or new fuzzy logic rules. Fuzzy logic is robust, due to elimination of the complicated mathematical modeling process by use of control set rules, which also result in simpler implementation than modern control theory. According to Sharma (2020), fuzzy logic has real-life applications in controlling aircraft, satellites, spaceships, automotive systems, chemical processing, and providing decision-making support for large companies.

Fuzzy logic has been used in a design of controller, e.g. controller for an inverted pendulum robot (Nour et al., 2007) or controller for a fully autonomous robot (Peri and Simon, 2005). In maritime CAS, Kijima and Furukawa (2001) designed a collision avoidance and control of the rudder using fuzzy logic to obtain the desired course change. The input to the collision avoidance fuzzy logic system were the time to the closest point of approach (TCPA) and the closest point of approach (CPA), which measure the collision risk. The fuzzy membership function (FMF) was constructed with triangular shapes. Perera et al. (2011) proposed a fuzzy logic system to obtain decision making for collision avoidance. The algorithm utilized collision distance, collision region, relative speed ratio and relative collision angle as input variables with trapezoidal FMFs and the outputs were the collision risk warning and a fuzzy decision. The paper concluded that the decision making fuzzy logic system performed well for a one-to-one encounter, but for a multiple-vessel encounter the system should be updated.

Based on the previous work, we observe that a systematic evaluation system for COLREG's compliance is a challenging task since COLREG is written for human with vagueness and complexity. In addition, there is a lack of an evaluation system that can provide one overall score. This one metric can be for all vessels involved in an scenario or one interested vessel when maneuvering with other surrounding vessels. The later is more important for Class societies when approving an AVS.

The objective of this paper is to develop an evaluation system for autonomous surface vessels with consideration of COLREG's compliance. This will evaluate in one score, i.e. one metric for all vessels involved or one vessel, in the range of 0%–100% for multiple-vessel encounter scenarios (Fig. 1). Implementing such concept would provide a more complete picture and more realistic scenarios of vessel encounter. The proposed evaluation system adopts fuzzy logic algorithms as the methodology. Four types of evaluation systems are developed:

- EVALSYS1: evaluation of all vessels (both OS and TS) in compliance with COLREG in pairwise encounters.
- EVALSYS2: evaluation of only OS in compliance with COLREG in pairwise encounters. In testing and verification of a control system, Class societies only consider OS.
- EVALSYS3: evaluation of all vessels (both OS and TS) in compliance with COLREG in multiple-vessel encounters.

Table 1
List of notations.

Symbol[a]	Meaning
ASV	Autonomous Surface Vehicle.
CAS	Collision Avoidance System.
COG	Center of Gravity.
COLREG	Convention on the International Regulations for Preventing Collisions at Sea.
EVALSYS	Evaluation System.
OS	Ownship.
TS	Target ship.
GW	Give way.
SO	Stand on.
CPA	the Closest Point of Approach.
DCPA	Distance at the Closest Point of Approach (range between two vessels).
TCPA	Time to the Closest Point of approach.
TSD	Travel in the Same Direction
FMF	Fuzzy Membership Function.
R_i	Role of OS/TS.
x_i, y_i	North and East position of OS/TS.
U_i	OS/TS speed.
ψ_i	OS/TS heading.
$\Delta\chi_i$	Course change
ΔU_i	Speed change
$r_{\mathrm{TS}}^{\mathrm{OS}}$	Range between OS and TS.
β and $\bar{\beta}$	Bearing and relative bearing from OS to TS, in short, bearing and relative bearing, respectively.
α and $\bar{\alpha}$	Bearing and relative bearing from TS to OS, in short, contact angle and relative contact angle, respectively.
C_{GW} and C_{SO}	Compliance of GW and SO vessel, respectively.
$\#SC$	Number of speed and course alterations.

[a] i subscript is referred to as OS or TS.

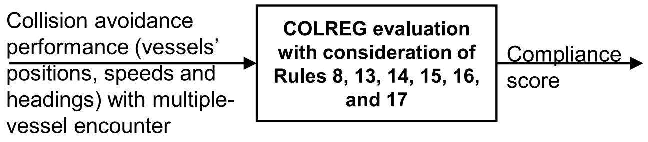


Fig. 1. The objective of the paper.

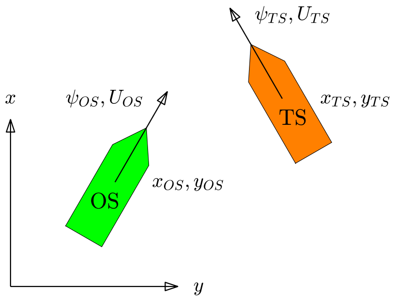


Fig. 2. Notation for own ship and target ship (Benjamin, 2017).

- EVALSYS4: evaluation of only OS in compliance with COLREG in multiple-vessel encounters. This evaluation system is important for Class societies.

This paper is divided into five main sections. Section 2 provides the necessary theory to understand the development of the system, consisting of collision avoidance notation and fuzzy logic theory. Section 3 develops the methods to evaluate the system to obtain one overall score. Section 4 validates the evaluation models by using simulated data and discusses among the results. Finally, Section 5 contains the conclusion.

2. Background theory

2.1. General collision avoidance notation

This section describes general notation used in collision avoidance, inspired by Benjamin (2017). Table 1 summarizes the main notations used in this paper.

Definition 2.1 (*OS and TS, Fig. 2*)**.** An ownship (OS) is defined as the vessel being controlled while a target ship (TS) is any vessel involved in a risky encounter with the OS where COLREG should apply. Fig. 2 illustrates the notation for OS and TS with the position, heading and speed with the following symbol:

- Current position of OS (and TS): $x_{\mathrm{OS}}, y_{\mathrm{OS}}$ (and x_{TS}, y_{TS})
- Current speed and heading OS (and TS): $U_{\mathrm{OS}}, \psi_{\mathrm{OS}}$ (and $U_{\mathrm{TS}}, \psi_{\mathrm{TS}}$)

The relative heading difference between OS and TS can be expressed using the smallest absolute value, according to

$$\Delta(\psi_1, \psi_2) = \left| [\psi_1 - \psi_2]_{180^\circ} \right| \tag{1}$$

where 180° denotes that it is $\in [0, 180^\circ)$. One example is that the difference between a heading of 340° and 10° is 30°, not 330°.

Several methods to obtain a vessel position in relation to another vessel are the range, bearing angle, contact angle, relative bearing angle, relative contact angle, and relative course (Definitions 2.2–2.7).

Definition 2.2 (*Range, r*)**.** The range (in meters) from OS to TS is the linear distance between OS and TS, denoted as $r_{\mathrm{TS}}^{\mathrm{OS}}$ in Fig. 3. The range from OS to TS is the same as the range from TS to OS, given by

$$r_{\mathrm{TS}}^{\mathrm{OS}} = r_{\mathrm{OS}}^{\mathrm{TS}} = \sqrt{(x_{\mathrm{OS}} - x_{\mathrm{TS}})^2 + (y_{\mathrm{OS}} - y_{\mathrm{TS}})^2} \tag{2}$$

Definition 2.3 (*Bearing, β*)**.** The bearing from OS to TS is the angle from OS North to the linear distance to TS, denoted as $\beta \in [0, 360^\circ)$ in Fig. 3, given by

$$\beta = \mathrm{atan2}(y_{\mathrm{OS}} - y_{\mathrm{TS}}, x_{\mathrm{OS}} - x_{\mathrm{TS}}) \tag{3}$$

Definition 2.4 (*Contact Angle, α*)**.** The contact angle is the angle from TS North to the linear distance to OS. The contact angle is related to the bearing angle by

$$\alpha = \beta + \pi \tag{4}$$

Definition 2.5 (*Relative Bearing, $\bar{\beta}$*)**.** The relative bearing from OS to TS is the angle from OS heading to the linear distance to TS, denoted

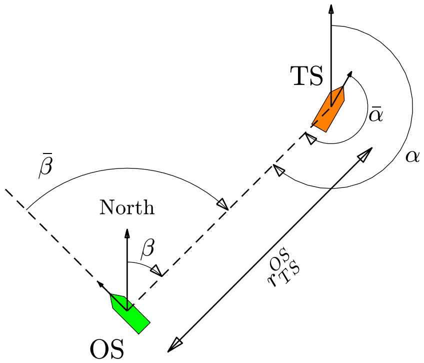


Fig. 3. Range, bearing and relative bearing inspired by Benjamin (2017).

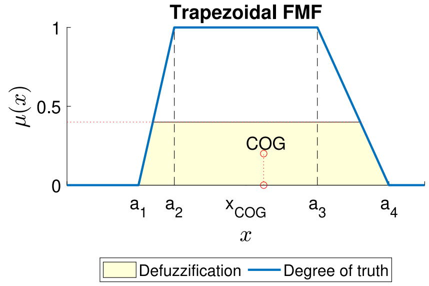


Fig. 4. Example of a trapezoidal FMF.

as $\bar{\beta} \in [0, 360°)$ in Fig. 3, given by

$$\bar{\beta} = \begin{cases} 360° - \text{abs}(\beta - \psi_{\text{OS}}) & \beta - \psi_{\text{OS}} < 0° \\ \beta - \psi_{\text{OS}} - 360° & \beta - \psi_{\text{OS}} \geq 360° \\ \beta - \psi_{\text{OS}} & else \end{cases} \tag{5}$$

Definition 2.6 (*Relative Contact Angle, $\bar{\alpha}$*). The relative contact angle from TS to OS is the angle from TS heading to the linear distance to OS, denoted as $\bar{\alpha} \in [0, 360°)$ in Fig. 3. This is calculated as the same manner as shown in Eq. (5) by swapping $\bar{\beta}$ with $\bar{\alpha}$, β with α and ψ_{OS} with ψ_{TS}.

Definition 2.7 (*Relative Course, $\Delta\chi$*). Relative course in OS perspective is the difference between OS course and TS course, given by

$$\Delta\chi = \text{mod}(\chi_{\text{OS}} - \chi_{\text{TS}}, 360°) \tag{6}$$

where mod is the modulus operator; and χ_{OS} and χ_{TS} are the courses of OS and TS respectively.

Remark 2.1 (*Angles*). The angles are positive clockwise.

Risk is a variable used in collision avoidance, but COLREG does not provide a standard precise measure of this quantity. From the literature review in the Introduction Section, DCPA is a measure that several papers have utilized to measure risk.

Definition 2.8 (*CPA*). The closest point of contact (CPA) is defined as the point on ownship's track where the range of the encounter between the ownship and the target ship is at its minimum.

Definition 2.9 (*TCPA*). $TCPA$ (or t_{cpa}) is the time of CPA. This can be obtained by setting the derivative of the range to 0 and solving for t (Benjamin, 2017), given by

$$t_{cpa} = \frac{-k_1}{2k_2} \tag{7}$$

Definition 2.10 (*DPCA*). $DCPA$ is defined as the range at the time of the closest point of approach. As defined, DCPA can be calculate once $TCPA$ is known, given by

$$DCPA = r(t_{cpa}) = \sqrt{k_2 t_{cpa}^2 + k_1 t_{cpa} + k_0}, \tag{8}$$

where t_{cpa} is found in Eq. (7); and k_0, k_1 and k_2 are some trigonometric functions relating OS and TS heading, position and velocities (Benjamin, 2017), according to

$$\begin{aligned} k_2 = {} & \cos^2(\psi_{\text{OS}})U_{\text{OS}}^2 - 2\cos(\psi_{\text{OS}})U_{\text{OS}}\cos(\psi_{\text{TS}})U_{\text{TS}} \\ & + \cos^2(\psi_{\text{TS}})U_{\text{TS}}^2 + \sin^2(\psi_{\text{OS}})U_{\text{OS}}^2 \\ & - 2\sin(\psi_{\text{OS}})U_{\text{OS}}\sin(\psi_{\text{TS}})U_{\text{TS}} + \sin^2(\psi_{\text{OS}})U_{\text{OS}}^2 \end{aligned} \tag{9}$$

$$\begin{aligned} k_1 = {} & 2\cos(\psi_{\text{OS}})U_{\text{OS}}y_{\text{OS}} - 2\cos(\psi_{\text{OS}})U_{\text{OS}}y_{\text{TS}} \\ & - 2y_{\text{OS}}\cos(\psi_{\text{TS}})U_{\text{TS}} + 2\cos(\psi_{\text{TS}})U_{\text{TS}}y_{\text{TS}} \\ & + 2\sin(\psi_{\text{OS}})U_{\text{OS}}x_{\text{OS}} - 2\sin(\psi_{\text{OS}})U_{\text{OS}}x_{\text{TS}} \\ & - 2x_{\text{OS}}\sin(\psi_{\text{TS}})U_{\text{TS}} + 2\sin(\psi_{\text{TS}})U_{\text{TS}}x_{\text{TS}} \end{aligned} \tag{10}$$

$$k_0 = y_{\text{OS}}^2 - 2y_{\text{OS}}y_{\text{TS}} + y_{\text{TS}}^2 + x_{\text{OS}}^2 - 2x_{\text{OS}}x_{\text{TS}} + x_{\text{TS}}^2 \tag{11}$$

where x_i, y_i are the vessel North and East position; ψ_i is the vessel heading; U_i is the vessel speed; and i is referred to as OS or TS.

Definition 2.11 (*#SC*). Actions to avoid collision considered in this paper are the changes of speed and course. The number of alterations in speed and course (*#SC*) is defined as the number of deviations from original speed and path. *#SC* is obtained by taking the maximum number of speed and course alterations, i.e. $\#SC_i = \max(\#SC_{\Delta\chi_i}, \#SC_{\Delta U_i})$, where i is referred to as OS or TS.

2.2. Fuzzy logic

Fuzzy logic is a branch of Artificial Intelligence that mimics the human ability of reasoning under uncertainty and partial information. With a Boolean logic, a statement takes a binary state, either true or false meaning that there is no such a partial degree of truth. Fuzzy logic intention on the other hand is to model a logical reasoning with imprecise or vague statements (Cintula et al., 2017). The vagueness (or uncertainty) is expressed via fuzzy sets introduced independently by Zadeh (1965).

Definition 2.12 (*Fuzzy Set, Zadeh 1965*). A fuzzy set (class) A in X is characterized by a membership (characteristic) function $\mu_A(x)$ which associates with each point in X a real number in the interval [0 1], with the value of $\mu_A(x)$ at x representing the "*grade of membership*" of x in A, according to

$$A = \{x, \mu_A(x) | x \in X\} \tag{12}$$

where X denotes the universe of discourse and x is an element; and $\mu_A(x)$ is the fuzzy membership function (FMF) which maps each element in the universe of discourse, X, to a membership value between 0 and 1 (a degree of truth).

In this paper, trapezoidal FMFs are mainly used.

Definition 2.13 (*Trapezoidal FMF*). A trapezoidal FMF is defined as a vector of 4 elements $[a_1, a_2, a_3, a_4]$, where $a_1, \ldots, a_4$ are the coordinates of the trapezoidal vertices in x-axis. The coordinates of the 4 vertices in y-axis are $[0, 1, 1, 0]$ defining the degree of truth (Fig. 4). In this paper, the coordinates in y-axis are not shown when defining a trapezoidal FMF for simplicity purpose.

Fig. 5. Simple flowchart of fuzzy logic, inspired by Tizhoosh (2019).

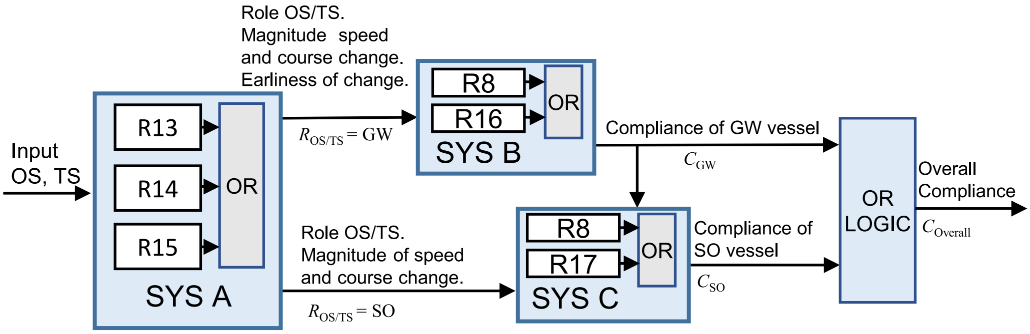


Fig. 6. System diagram of the considered rules and their algorithmization. In this figure the abbreviation SYS stands for Sub-system.

Fuzzy logic provides mathematical tools that bridge between a linguistic value to the universe of discourse (Tizhoosh, 2019). Fuzzy reasoning includes three main processes, i.e. fuzzification, inference and defuzzification (Fig. 5). A variable is linguistic if its values are linguistic, i.e. the values are words or sentences (Zadeh, 1975). For example, *COLREG's compliance* is a linguistic variable if its values are linguistic, e.g. *medium good*, *good*, *medium bad* or *bad* instead of numerical values.

The linguistic variables and linguistic values are modeled mathematically by fuzzy variables and fuzzy values, respectively. Moreover, a fuzzy variable can include multiple fuzzy values.

Fuzzification is the process of partial decomposition of a crisp quantity or characteristic into linguistic values. Based on the designed fuzzy variables, numerical input are mapped into fuzzy subsets through FMFs. The *inference* executes the designed fuzzy rules, also called the "if-then" rules. The final step is the *defuzzification* process, where the fuzzy subsets derived in the fuzzy inference are aggregated to a fuzzy set, through which the defuzzification process calculates a crisp output value. The defuzzified value is obtained by the center of gravity. The defuzzification and corresponding COG are illustrated in Fig. 4. More details on fuzzy logic can be found in Zadeh (1975) and Tizhoosh (2019).

2.3. COLREG

COLREG, applying to all vessels, consists of a set of rules and annexes concerning responsibility, risk of collision, safe speed, look out etc. In addition, it describes what to do in situations when two vessels meet each other, i.e. head-on, crossing, overtaking scenarios. This means that COLREG does not mention how to handle the situation when a vessel is approaching two other vessels, e.g. "head-on" towards one vessel and "crossing" towards the other one. However, from Rule 2 (Responsibility) it is quite clear that the ship operator takes the responsibility in case of a collision in any circumstance meaning that one can even break parts of COLREG to ensure no collision. In the situation with both head-on and crossing, a vessel may come into a situation that it is both stand-on and give-way at the same time. As a consequence, it will need to break at least one rule to solve the situation. Even though COLREG does not mention multi-vessel scenario, the evaluation system needs to consider this situation. This paper considers the set of COLREG Rules applicable for open sea and full visibility. Readers can find detailed information of the Rules in IMO (1972). The considered Rules are summarized in the followings:

- Rule 8 — Action to avoid collision. The action altering course and/or speed should be large enough such that another vessel could observe this action on the radar or visually. The action should also be made in ample time and results in passing at a safe distance.
- Rule 13 — Overtaking. An overtaking vessel shall keep out of the way (GW role) of the overtaken vessel (SO role). A vessel is considered as overtaking when she comes up towards another vessel from a direction more than 22.5° abaft the overtaken vessel's beam.
- Rule 14 — Head-on situation. This happens when two vessels are meeting on reciprocal/nearly reciprocal courses and a risk is present. Both vessels shall GW by altering course to starboard to obtain a portside passing.
- Rule 15 — Crossing situation. When two vessels are crossing and risk is present, the vessel which has the other on her own starboard side shall GW.
- Rule 16 — Action by give-way vessel. This Rule states that the GW vessel shall take early and substantial action to avoid a collision.
- Rule 17 — Action by stand-on vessel. This Rule states that the SO vessel shall keep course and speed, but if the GW vessel does not do appropriate action or the SO vessel experiences that collision cannot be avoided by the GW vessel actions alone, the SO vessel must take actions to avoid a collision.

3. Verification method for collision avoidance

This section develops COLREG-compliance evaluation methods. The first presented evaluation system (EVALSYS1) is developed for a one-to-one (pairwise) vessel encounter. The second system (EVALSYS2) only evaluates OS COLREG's compliance. Finally, based on EVALSYS1 and EVALSYS2, the evaluations of multiple-vessel encounter scenarios (EVALSYS3 and EVALSYS4), i.e. when the number of vessels $\geqslant 2$, are developed.

For this section it is important to separate the COLREG Rules and the fuzzy rules. The goal with the fuzzy logic systems is to reconstruct the COLREG Rules in a fuzzy logic manner. The rules listed in the following subsections are the fuzzy rules implemented in the fuzzy logic system, while Rule 13 is referred to Rule 13 in COLREG.

3.1. Evaluation system for a one-to-one encounter

This section will provide the methodology of the one-to-one vessel encounter evaluation of OS and TS (EVALSYS1). EVALSYS1 is developed as a combination of three sub-systems, i.e. A, B and C, as illustrated in Fig. 6.

Table 2
Input of the evaluation sub-systems.

Variable[a]	Reference	Sub-system
$\Delta\chi$	Eq. (6)	A
$\bar{\beta}$	Eq. (5)	A
$\bar{\alpha}$	Definition 2.6	A
DCPA	Eq. (8)	A
TSD	Section 3.1.1	A
TCPA(ts_i)	Eq. (7)	B
R_i	Output of Sub-system A, Section 3.1.1	B and C
$\|\Delta\chi_i\|$	Section 3.1.2	B and C
$\|\Delta U_i\|$	Section 3.1.2	B and C
$\#SC_i$	Definition 2.11	B and C
C_{GW}	Output of Sub-system B, Section 3.1.2	C

[a] i subscript is referred to as OS or TS.

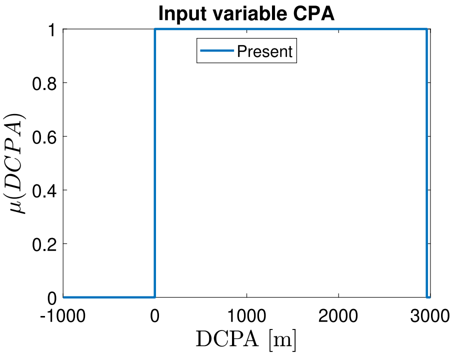


Fig. 7. FMF for determination of risk.

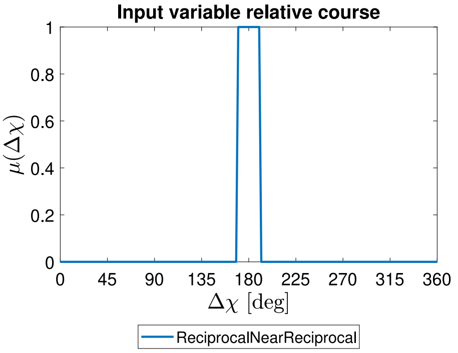


Fig. 8. FMF for relative course for R14.

The goal of Sub-system A is to determine the role of OS and TS whether a vessel is GW or SO. This is calculated based on the relative distance between vessels when the range reaches a pre-determined range and time to CPA (TPCA) reaches a threshold (two vessels become in sight of one another). Any subsequent alteration does not change the type of the encounter situation. Sub-system B determines the compliance of the GW vessel, $\mu(C_{\mathrm{GW}})$; and Sub-system C determines the compliance of the SO vessel, $\mu(C_{\mathrm{SO}})$. The final "OR" logic is a combination of the GW and SO compliance score to obtain an overall compliance score, i.e. $C_{\mathrm{Overall}} = \max(\mu(C_{\mathrm{GW}}), \mu(C_{\mathrm{SO}}))$. This is due to the fact that a vessel should comply to COLREG by executing the right maneuver, i.e. either SO or GW. Sub-systems B and C take the patterns of traffic of both vessels during the entire scenario time-window from entrance to exit as input. Table 2 lists different variables as input to different sub-systems.

3.1.1. *Sub-system A*

Sub-system A covers three COLREG Rules, i.e Rules 13, 14, and 15 constructed by the fuzzy logic systems R13, R14 and R15, respectively, as shown in Fig. 6. The input to Sub-system A are $\Delta\chi$, $\bar{\beta}$, $\bar{\alpha}$, DCPA to estimate risk, and a variable to denote that the vessels Travel in the Same Direction (TSD).

Rule 13. Part (b) specifies "*A vessel shall be deemed to be overtaking when coming up with another vessel from a direction more than* 22.5° *abaft her beam*" (IMO, 1972). R13 receives $\bar{\beta}$ and $\bar{\alpha}$ to determine where each vessel is with respect to the other. From these relative bearings, fuzzy logic will check whether OS is overtaking TS (and vice versa) via an FMF. Fig. 9 illustrates the FMF for the fuzzy value *TSOvertakingOS* of the fuzzy variable $\bar{\beta}$. Similarly, the same FMF but with the fuzzy variable $\bar{\alpha}$ is used for *OSOvertakingTS*.

In addition, Rule 13 part (c) states that if in any doubt if it is an overtaking situation the vessel shall assume an overtaking situation. Thereof, an angle of 2.5° is added as a region of uncertainty on each side of the fuzzy values (*TSOvertakingOS* and *OSOvertakingTS*) to tolerate "*if in any doubt*". As a consequence, a trapezoidal FMF is designed with parameters $[110, 112.5, 247.5, 250]$ to reflect Rule 13.

To reflect the phrase "*coming up with another*" in part (b), a Matlab script is developed by taking into account the variable TSD. The reason is that $\bar{\beta}$ and $\bar{\alpha}$, which only consider OS and TS heading, may provide values in the fuzzy value *TSOvertakingOS* and *OSOvertakingTS* while the vessels are actually traveling apart from each other.

The risk is approximated by the DCPA and provided as input to the FMF given in Fig. 7. The trapezoidal FMF parameters are set to $[0, 0, 2960, 2960]$, where the distance of 2960 meters (1.6 nautical miles) is considered as risk is present.

These values are inspired by the literature review on CPA, where a study on DCPA proposed that a DCPA of 1.6 to 2.5 nautical miles are considered as a safe DCPA at open sea (Vujičić et al., 2017). This limit corresponds to a vessel with length overall (L_{OA}) of 200 meters and at a speed of 15 knots. The vessels considered in simulations in this paper have a L_{OA} of 116 m; therefore, the lower bound given in the study on DCPA is implemented in the evaluation system.

By this the rules are specified in fuzzy rules A_1 and A_2 given in Table 3.

Remark 3.1 (*Tabulated Fuzzy Rules*)**.** In this paper, the fuzzy rules are tabulated in several tables for convenience. An example of reading the fuzzy rule A1 in Table 3 is: "If $\bar{\beta}$ is TSOvertakingOS and TSD is true and Risk is Present then R_{OS} is SO and R_{TS} is GW".

Rule 14. Part (a) of the Rule specifies the head-on happens *when two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision*. DCPA utilizes the same FMF as presented for R13 (Fig. 7).

The variable $\Delta\chi$ is an input fuzzy variable to determine if the courses are *reciprocal or nearly reciprocal*. In addition, to take into account *when a vessel is in any doubt* (in part (c) of the Rule), the parameter is proposed with a range of uncertainty modeled by partial membership for the FMFs. As a consequence, the trapezoidal FMF for the fuzzy value *ReciprocalNearReciprocal* is designed with parameters $[168, 170, 190, 192]$. Fig. 8 illustrates the fuzzy value *ReciprocalNearReciprocal*.

Part (b) of the Rule specifies that Head-on situation *deemed to exist when a vessel sees the other ahead or nearly ahead*. The variables $\bar{\beta}$ (or $\bar{\alpha}$) is used to determine whether TS (or OS) is *Ahead*. For convenience in Matlab implementation, we use *NotAhead* to reflect this. The "NOT" *NotAhead* is utilized to avoid having two subsets for the fuzzy value *Ahead* which eventually increases the number of rules (4 times). To obtain *Ahead*, the fuzzy rules specifies "If $\bar{\alpha}$ is "NOT" *NotAhead*", i.e. $\mu_{Ahead}(\bar{\alpha}) = \mu_{NOT\,NotAhead}(\bar{\alpha}) = 1 - \mu_{NotAhead}(\bar{\alpha})$. The fuzzy value *Ahead* is obtained for OS in the same manner, but with the fuzzy variable $\bar{\beta}$.

Figs. 9 and 10 illustrate the fuzzy value *NotAhead* for the fuzzy variable $\bar{\beta}$ and $\bar{\alpha}$. It is noted that the curves for FMF *NotAhead* are not

Table 3
Rules for Sub-system A.

Rule	If (AND logic)					Then (AND logic)	
#	$\bar{\alpha}$	$\bar{\beta}$	TSD	Risk	$\Delta\chi$	R_{OS}	R_{TS}
A_1	–	TSOvertakingOS	true	Present	–	SO	GW
A_2	OSOvertakingTS	–	true	Present	–	GW	SO
A_3	–	–	–	Present	Reciprocal-NearReciprocal	GW	GW
A_4	NOT NotAhead	NOT NotAhead	–	Present	–	GW	GW
A_5	–	TSStarOfOS	–	Present	–	GW	SO
A_6	–	OSStarOfTS	–	Present	–	SO	GW

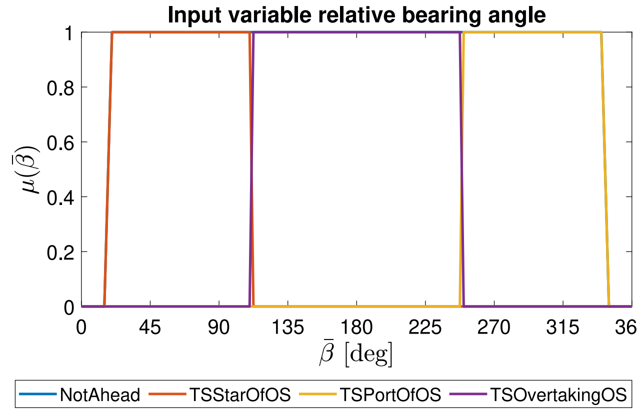


Fig. 9. FMFs for relative bearing angle.

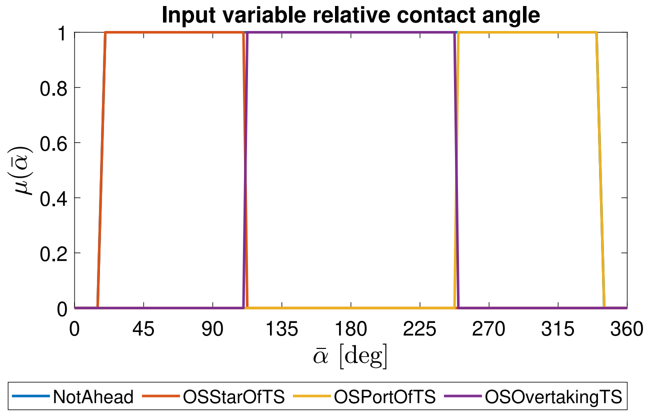


Fig. 10. FMFs for relative contact angle.

visible in Figs. 9 and 10 due to overlapping by other curves. Readers can refer to the trapezoidal FMF for the fuzzy value *NotAhead* with parameters $[15, 20, 340, 345]$ (also designed with a range of uncertainty) for both TS and OS.

By this the fuzzy rules A_3 and A_4 for both vessels are developed and given in Table 3. The fuzzy rules, i.e. A3 and A4, also specify that both vessels shall GW if one or both of the *if-then* fuzzy rules are true, which agrees with Rule 14 in COLREG. Rule 14 specifies what action the vessels should execute to obtain compliance, which is considered in Sub-system B.

Rule 15. This Rule states the action of two vessels when they are crossing and risk is present. The risk of collision is translated to fuzzy logic by the same FMF and same fuzzy variable (input) as given in R13 and R14 (Fig. 7).

The Rule specifies that *the vessel which has the other on her own starboard side shall keep out of the way*. To obtain if a vessel is on starboard side of the other vessel, the fuzzy variables $\bar{\alpha}$ and $\bar{\beta}$ are used. In Figs. 9 and 10, the same FMF parameters are utilized, but with different fuzzy variable and value, i.e. $\bar{\alpha}$ for *OSStarOfTS* and $\bar{\beta}$ for *TSStarOfOS*. The trapezoidal FMFs are designed with the parameters $[15, 20, 110, 112.5]$ (Cho et al., 2022) which has a range to take into account the uncertainty.

The fuzzy rules A_5 and A_6 for the fuzzy logic are designed and presented in Table 3.

Output. Fig. 9 summarizes all fuzzy values for $\bar{\beta}$. It is observed that at any $\bar{\beta}$ angle, the sum will be one. Fig. 10 summarizes all fuzzy values for $\bar{\alpha}$. The fuzzy variable $\bar{\alpha}$ would provide a similar conclusion, but with different fuzzy values. Thereof, the evaluation system will entitle R_i for all possible positions that OS has in relation to TS and vice versa when there is a risk of collision.

The output of R13, R14 and R15 are connected with an "OR" logic in a fuzzy logic manner (Fig. 6). The "OR" fuzzy logic system receives six input, i.e. the two outputs from R13, two outputs from R14 and two outputs from R15 to determine R_i ($R_{\mathrm{OS}}/R_{\mathrm{TS}}$). The output of R13 and R15 could be a degree of truth to *SO* or *GW* or in-between these fuzzy values. For R14 the only role of vessel i is GW.

Table 4
Rules for "OR" logic in Sub-system A.

Rule	If (OR logic)			Then
#	$R_{i,13}$	$R_{i,14}$	$R_{i,15}$	R_i
A_7	GW	GW	GW	GW
A_8	SO	SO	SO	SO

The "OR" logic will take the max value of the input. However, there is an edge case when output of R13 and R15 are both 0.5 and the output of R14 is less than 0.5. The former happens when no rules are fired meaning that both vessels need to GW and SO. The latter happens when both vessels are not in a head-on situation. The consequence of the "OR" logic will lead to the overall output of 0.5 for the Sub-system A, meaning that both vessels need to GW and SO.

To solve this problem, we design the input FMF for the fuzzy "OR" logic system (Fig. 11) different from the output from R13, R14 and R15 (Fig. 12). The *GW* and *SO* FMFs for the fuzzy "OR" logic system are defined as $[0, 0, 0.2533, 0.5]$ and $[0.5, 0.7467, 1, 1]$ respectively, (Fig. 11). These values, 0.2533 and 0.7467, are the maximum values possible to obtain as output (defuzzification) from R13 and R15 for *GW* and *SO* role respectively. For R14 the maximum output value is 0.2533 due to the fact that it only considers the *GW* fuzzy value.

The output variables for the three sub-systems (R13, R14 and R15), see Fig. 6, utilizes the FMFs given in Fig. 12. The defuzzified output values are calculated using the center of gravity. The trapezoidal FMF for the fuzzy value *GW* is defined by the parameters $[0, 0, 0.4, 0.6]$ and *SO* by the parameters $[0.4, 0.6, 1, 1]$ (Fig. 12).

By this the fuzzy rules are given in Table 4.

The time index is removed for simplicity for all variables in Sub-system A. In fact, all variables are calculated at the timestamp when the range and TCPA are equal or below a threshold to evaluate if the vessels are in sight of each other. Range specifies the distance; and TCPA specifies how far away the vessel is in time from the CPA. For example, for a range of 3 nautical miles, a high speed vessel will give a TCPA lower than for a slow vessel, meaning that the timestamp for

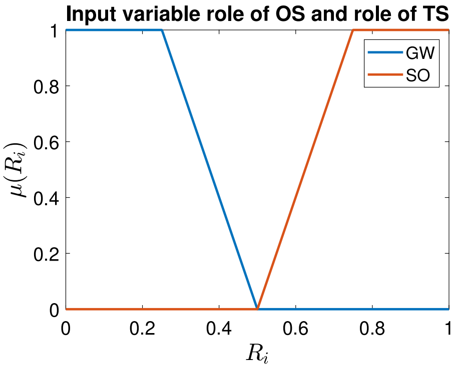


Fig. 11. Input for fuzzy "OR" logic FMF.

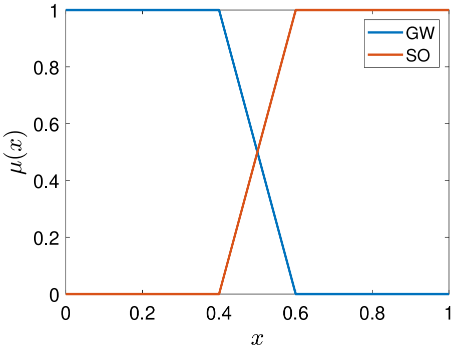


Fig. 12. Output FMF of R13, R14, and R15.

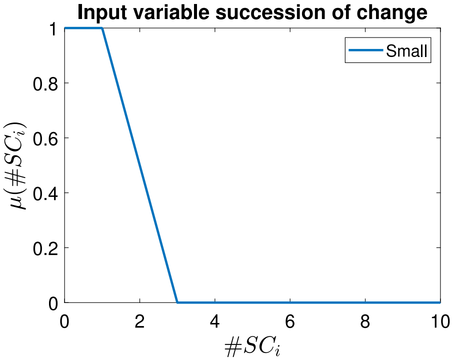


Fig. 13. FMF to assess successive deviation alterations for TS.

evaluation must therefore be earlier. In this paper, the threshold for the range is set to 5 nautical miles; and for the TCPA is 1200 s.

3.1.2. Sub-system B

The fuzzy input variables to Sub-system B are R_i, TCPA(ts_i), $|\Delta\chi_i|$, $|\Delta U_i|$ and $\#SC_i$. The output is the degree of compliance of the GW vessel, i.e. C_{GW}. Sub-system B evaluates Rules 8 and 16.

Rule 8. Part (b) of the Rule states that action to avoid collision should be *large enough* and that *small succession of alterations for course and/or speed shall be avoided.*

This is a good example of vague terms used in COLREG as it does not state how large a substantial action should be. The method to test for succession of small alterations is managed by counting the numbers of continuous heading offsets and propulsion commands.

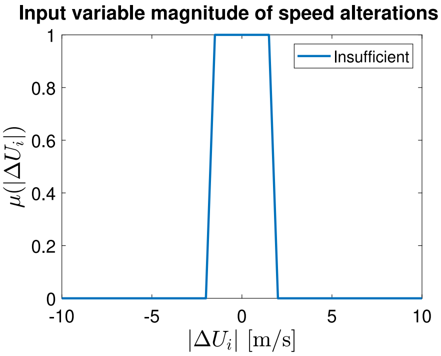


Fig. 14. The magnitude of velocity FMF.

The variable $\#SC_i$ defined in Definition 2.11 is given to the trapezoidal FMF $[0, 0, 1, 3]$ (Fig. 13) to reflect *small succession of alterations for course and/or speed.*

The evaluation of *large enough alterations (substantial)* using fuzzy logic will be presented in Rule 16.

Rule 16. This Rule states that action of a GW vessel to avoid a collision should be *early* and *substantial*. This is another example of vague terms as it does not state when an early action should be nor how large a substantial action should be.

The *earliness* is calculated as TCPA at the first time the vessel deviates form original speed or path (ts_i), i.e. TCPA(ts_i). Vujičić et al. (2017) suggested a DCPA of 1.6–2 nautical miles for a vessel at 15 knots as a safe DCPA. The time a vessel takes to travel 1.6 nautical miles at 15 knots vessel speed is 383 s. Therefore, we take into account 383 s in a proposed trapezoidal FMF $[360, 383, 10000, 10000]$ to assess the *earliness*. Here, the trapezoidal function starts at 360 s to cover to case when the vessel speed is a bit higher (16 knots) for the same DCPA of 1.6 nautical miles.

The *substantial* action to avoid collision is assessed via the magnitude of course and speed changes, denoted as $|\Delta\chi_i|$ and $|\Delta U_i|$, respectively. These are calculated as the largest offset of speed and heading experienced in the continuous heading offsets and propulsion command values sent to the feedback linearizing controller. In real life, there might be a challenge since it is harder to interpret an action with successions of small alterations to avoid collision than an action with a large and consistent alteration.

The variables $|\Delta\chi_i|$ and $|\Delta U_i|$ are assigned to different FMFs since it is easier to interpret a course change than a speed change from another vessel perspective. In addition, Rule 8 part (b) states "*alteration of course alone may be the most effective action*" (IMO, 1972). The *sufficient* action can be in either direction, i.e. negative or positive values. As a consequence, the fuzzy subset of *sufficient* alteration need to be divided into two subsets resulting in extra FMFs and extra rules to cover all possibilities. To avoid this, only *Insufficient* subset is defined and the *Sufficient* subset is establish by $\mu_{\text{Sufficient}}(x) = \mu_{\text{NOT Insufficient}}(x) = 1 - \mu_{\text{Insufficient}}(x)$. The fuzzy variable $|\Delta U_i|$ is given to the fuzzy value *Insufficient* by the trapezoidal FMF $[-2, -1.5, 1.5, 2]$ (Fig. 14); and the fuzzy variable $|\Delta\chi_i|$ in degrees is given to the fuzzy value *Insufficient* by the trapezoidal FMF $[-4, -2, 2, 4]$ (Fig. 15).

The input variables R_i to the Sub-system B is the output from Sub-system A presented in Section 3.1.1. The largest part of the universe of discourse is mapped to full and zero membership which correspond to clearly-classified situations.

A Matlab script is developed to determine if OS passes on portside of TS, denoted as "OSPortOfTS"; and if TS passes on portside of OS, denoted as "TSPortOfOS". This is used for evaluating Rule 14 Head-on situation, where Rule 14 states that both vessels should GW by "*each shall alter her course to starboard so that each shall pass on the portside of the other*" (IMO, 1972). This must therefore be incorporated into Sub-system B to obtain C_{GW} if Rule 14 is applied.

Table 5
Rules for Sub-system B.

Rule #	If (AND logic)								Then
	R_{OS}	R_{TS}	$\|\Delta U_{OS}\|$	$\|\Delta U_{TS}\|$	$\#SC_{OS}$	$\#SC_{TS}$	TCPA(ts_{OS})	TCPA(ts_{TS})	C_{GW}
B_1	GW	NOT GW	NOT Insufficient	–	Small	–	Early	–	Good
B_2	NOT GW	GW	–	NOT Insufficient	–	Small	–	Early	Good
	R_{OS}	R_{TS}	$\|\Delta\chi_{OS}\|$	$\|\Delta\chi_{TS}\|$	$\#SC_{OS}$	$\#SC_{TS}$	TCPA(ts_{OS})	TCPA(ts_{TS})	C_{GW}
B_3	GW	NOT GW	NOT Insufficient	–	Small	–	Early	–	Good
B_4	NOT GW	GW	–	NOT Insufficient	–	Small	–	Early	Good
	R_{OS}	R_{TS}	OSPort-OfTS	TSPort-OfOS	$\|\Delta U_i\|$	$\|\Delta\chi_i\|$	$\#SC_i$	TCPA(ts_i)	C_{GW}
B_5	GW	GW	true	true	NOT Insufficient	–	Small	Early	Good
B_6	GW	GW	true	true	–	NOT Insufficient	Small	Early	Good
	R_{OS}	R_{TS}	OSPort-OfTS	TSPort-OfOS	$\|\Delta\chi_{OS}\|$	$\|\Delta U_{TS}\|$	$\#SC_i$	TCPA(ts_i)	C_{GW}
B_7	GW	GW	true	true	NOT Insufficient	NOT Insufficient	Small	Early	Good
	R_{OS}	R_{TS}	OSPort-OfTS	TSPort-OfOS	$\|\Delta\chi_{TS}\|$	$\|\Delta U_{OS}\|$	$\#SC_i$	TCPA(ts_i)	C_{GW}
B_8	GW	GW	true	true	NOT Insufficient	NOT Insufficient	Small	Early	Good

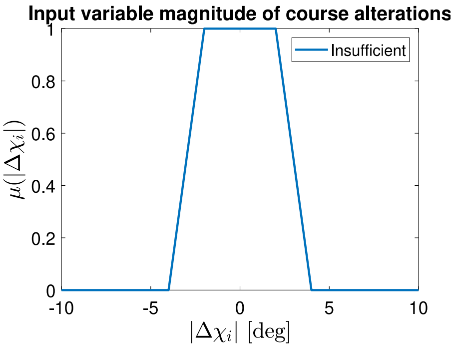


Fig. 15. The magnitude of course FMF.

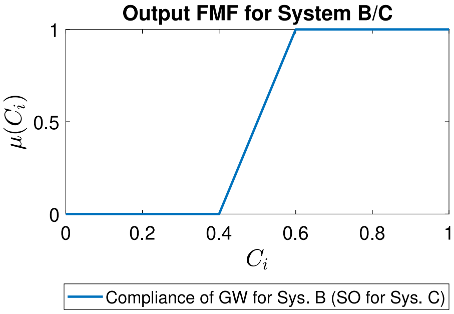


Fig. 16. Output FMF for Sub-system B/C.

The fuzzy logic rules of Sub-system B are given in Table 5.

Output. These parallel rules are connected with an "OR" logic. The output from this "OR" logic, which is also the output of Sub-system B, is the degree of compliance of the GW vessel as stated in output FMF given in Fig. 16. This output FMF is a trapezoidal function with the parameters $[0.4, 0.6, 1, 1]$.

3.1.3. Sub-system C

The input to Sub-system C are R_i, C_{GW}, $|\Delta\chi_i|$, $|\Delta U_i|$, $\#SC_i$, "OSPortOfTS", "TSPortOfOS" and "Crossing". The output is the degree of compliance of the SO vessel, i.e. C_{SO}. Sub-system C evaluates Rules 8 and 17. The fuzzification of Rule 8 was already presented in Section 3.1.2. In this Subsection, the fuzzy logic for Rule 17 is presented.

Rule 17. Part (a) (ii) states that the SO vessel (which should maintain course and speed) may take action to avoid collision if the GW vessel does not take appropriate action in compliance with COLREG.

Part (b) states that when the SO vessel is in a situation where keeping course and speed would lead to a collision, "*she shall take such actions as will best aid to avoid collision*" (IMO, 1972).

According to the above texts, first of all the roles of the vessels need to be assessed. The variable R_i are the fuzzy variables mapped to the fuzzy values given by the FMFs in Fig. 12 to obtain which role they have. Next, the SO vessel needs to be assessed whether it executes sufficient actions when *the GW vessel does not take appropriate action*. This is done by evaluating the change of speed ($|\Delta U_i|$), the change of course ($|\Delta\chi_i|$), and the number of speed and course alterations ($\#SC_i$) of the SO vessel. To reflect this, the fuzzification is done on the variables $|\Delta U_i|$ and $|\Delta\chi_i|$ to assess *Insufficient* change, same as in Sub-system B, i.e. trapezoidal FMFs in Figs. 14 and 15, respectively. The variable $\#SC_i$ is the fuzzy variable mapped to the fuzzy value *Small*, same as Sub-system B (Fig. 13). The fuzzy value C_{GW} uses the "NOT" *Good* to determine if the SO vessel should execute actions to avoid collision.

We need to cover the case where the SO vessel changes the course and speed due to tracking Waypoints (WPs). In this case, the action is not meant to avoid collision. Fuzzy variables are designed to determine if there are any change due to a WP change, i.e. $|\Delta\chi_i|$ and $|\Delta U_i|$. These WP changes would not provide any belongingness to the fuzzy value *Insufficient*.

Part (c) of the Rule states that an SO vessel that has to take actions to avoid collision *cannot alter course to port* when another vessel is on her own portside in a *crossing* situation. A Matlab script is developed to determine if R_i alters her course to *portside* in crossing situations, i.e. the fuzzy variable $\Delta\chi_i$ with the fuzzy value *Portside*.

The fuzzy variable C_{GW}, which is the output from Sub-system B, is the input to Sub-system C. From Fig. 16, the defuzzification gives $C_{GW} = 0.7242$ for non-compliance ($\mu(C_{GW}) = 0.5$) and $C_{GW} = 0.7467$ for full compliance ($\mu(C_{GW}) = 1.0$). Hence the input FMF of Sub-system C for C_{GW} is designed with the parameters $[0.7242, 0.7467, 1, 1]$ (Fig. 17). "Crossing" is a binary variable. More detail on the binary variable is discussed in Section 3.4.

By this the fuzzy rules of Sub-system C are designed in Table 6.

Output. The output of this system is the fuzzy variable C_{SO} which has the same output FMF (fuzzy value) as Sub-system B for the C_{GW}, i.e. $[0.4, 0.6, 1, 1]$ given in Fig. 16. It is noted that to reduce the number of rules, Sub-systems B and C do not contain the non-compliance cases. If one chooses to design the rules for non-compliance cases, the number of rules has to be doubled. Therefore, Sub-systems B and C will output a value 0.5 in the case of the compliance case rules are not applicable, i.e. no rules fired.

3.1.4. Overall compliance

The final "overall" degree of compliance depends on the output of Sub-systems B and C, i.e. C_{GW} and C_{SO}, respectively. The defuzzification will lead to the range of output of Sub-systems B and C to

Table 6
Rules for Sub-system C.

Rule #	If (AND logic)								Then
	R_{OS}	R_{TS}	C_{GW}	$\Delta\chi_{\mathrm{OS}}$	$\Delta\chi_{\mathrm{TS}}$	$\lvert\Delta U_{\mathrm{OS}}\rvert$	$\lvert\Delta U_{\mathrm{TS}}\rvert$		C_{SO}
C_1	GW	SO	Good	–	Insufficient	–	Insufficient		Good
C_2	SO	GW	Good	Insufficient	–	Insufficient	–		Good
	R_{OS}	R_{TS}	C_{GW}	Crossing	OSPort-OfTS	$\Delta\chi_{\mathrm{TS}}$	$\lvert\Delta U_{\mathrm{TS}}\rvert$	$\#SC_{\mathrm{TS}}$	C_{SO}
C_3	GW	SO	Not Good	true	true	NOT Portside	NOT Insufficient	small	Good
	R_{OS}	R_{TS}	C_{GW}	Crossing	OSPort-OfTS	$\Delta\chi_{\mathrm{TS}}$	$\lvert\Delta\chi_{\mathrm{TS}}\rvert$	$\#SC_{\mathrm{TS}}$	C_{SO}
C_4	GW	SO	Not Good	true	true	NOT Portside	NOT Insufficient	small	Good
	R_{OS}	R_{TS}	C_{GW}	Crossing	TSPort-OfOS	$\Delta\chi_{\mathrm{OS}}$	$\lvert\Delta\chi_{\mathrm{OS}}\rvert$	$\#SC_{\mathrm{OS}}$	C_{SO}
C_5	SO	GW	Not Good	true	true	NOT Portside	NOT Insufficient	small	Good
	R_{OS}	R_{TS}	C_{GW}	Crossing	TSPort-OfOS	$\Delta\chi_{\mathrm{OS}}$	$\lvert\Delta U_{\mathrm{OS}}\rvert$	$\#SC_{\mathrm{OS}}$	C_{SO}
C_6	SO	GW	Not Good	true	true	NOT Portside	NOT Insufficient	small	Good
	R_{OS}	R_{TS}	C_{GW}	Crossing	$\lvert\Delta U_{\mathrm{TS}}\rvert$	$\lvert\Delta\chi_{\mathrm{TS}}\rvert$	$\#SC_{\mathrm{TS}}$		C_{SO}
C_7	GW	SO	Not Good	NOT true	NOT Insufficient	–	small		Good
C_8	GW	SO	Not Good	NOT true	–	NOT Insufficient	small		Good
	R_{OS}	R_{TS}	C_{GW}	Crossing	$\lvert\Delta U_{\mathrm{OS}}\rvert$	$\lvert\Delta\chi_{\mathrm{OS}}\rvert$	$\#SC_{\mathrm{OS}}$		C_{SO}
C_9	SO	GW	Not Good	NOT true	NOT Insufficient	–	small		Good
C_{10}	SO	GW	Not Good	NOT true	–	NOT Insufficient	small		Good

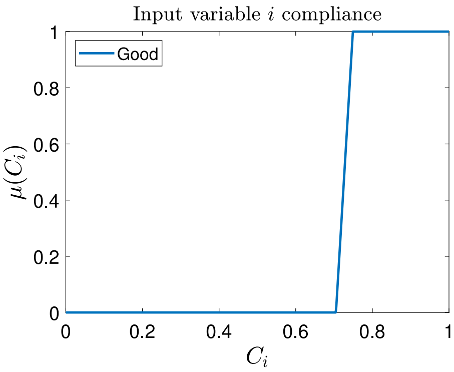


Fig. 17. GW compliance FMF.

be $\in [0.7242, 0.7467]$. This is then scaled to the range $[0, 1]$ to show the *degree of compliance*. This means that if $C_{\mathrm{GW}} = 0.7242$, *degree of compliance* $= 0$; and if $C_{\mathrm{GW}} = 0.7467$, *degree of compliance* $= 1$. The same applies for C_{SO}.

Here, C_{GW} and C_{SO} variables are connected to a "OR" fuzzy logic system through two fuzzy variables designed with the fuzzy value similar to the fuzzy value *Good* in Fig. 17. By this the fuzzy rule is developed as:

OA_1 : If C_{GW} is *Good* or C_{SO} is *Good* then C_{Overall} is *Good*

This is the same as $C_{\mathrm{Overall}} = \max(\mu(C_{\mathrm{GW}}),\ \mu(C_{\mathrm{SO}}))$. The overall compliance is taken as the degree of membership to *Good* compliance without the defuzzification phase.

3.2. OS compliance evaluation

In this subsection, the evaluation system of OS control system is also developed, i.e. EVALSYS2, where this evaluation system is similar to and based on the design of the evaluation system given in EVALSYS1. Thereof, the methodology of EVALSYS2 design is less detailed.

Sub-system A will remain the exact same, with the same "OR" fuzzy logic for combining R13, R14, and R15.

Sub-system B has some modifications, where B_5–B_8 are deleted as they verify that both vessels GW according to COLREG Rule 14. There are two rules added and tabulated in Table 7.

By this B_5^{OS} and B_6^{OS} only focus on OS compliance in a head-on situation, i.e. Rule 14 from COLREG. Even though B_2 and B_4 provide the C_{GW} for TS, these are needed to verify if the GW vessel does not GW in Sub-system C.

For **Sub-system C**, the rules that are applicable are C_1, C_5, C_6, C_9 and C_{10}. The other fuzzy rules evaluate TS actions, which are not relevant for OS evaluation.

The overall compliance evaluation system utilizes the same rule with scaling the defuzzified output value. Since OA_1 could provide an overall compliance including the C_{GW} by TS, an additional MATLAB script is designed to choose the overall compliance by only OS. This means overall compliance is C_{GW} when OS was the GW vessel or the overall compliance is the C_{SO} when OS was the SO vessel. The overall compliance for the EVALSYS2 is denoted C_{OS}.

3.3. Evaluation system for multiple-vessel encounter

The multiple-vessel encounter evaluation system, denoted as EVALSYS3, is developed based on EVALSYS1. The extension is the final "AND" logic to combine the levels of compliance resulting from evaluation of multiple pairs of OS and TSes (Fig. 18). If OS vessel has a risk of collision with multiple TSes, say TS_i and TS_j, each pair of vessels involving OS is evaluated independently using one-to-one vessel evaluation system (EVALSYS1). Finally, the overall compliance of the multiple-vessel scenario is calculated through a combination of the individual evaluations. This is due to the fact that COLREG are designed for pairwise cases.

The input for the system are the same variables as in EVALSYS1, but with different TSes. The multiple-vessel encounter system could have as many systems as desired, e.g. evaluation of one OS and four TSes this would make 4 systems.

The input for the "AND" fuzzy logic system are the overall compliance from the pairwise evaluation of EVALSYS1, i.e. $\mu(C_{\mathrm{Overall}}(\mathrm{OS}, \mathrm{TS}_i))$. The overall compliance for a multiple-vessel encounter is calculated as according to

$$OverallCompliance = \min(\mu(C_{\mathrm{Overall}}(\mathrm{OS}, \mathrm{TS}_i))) \qquad i = 1, \dots, N \tag{13}$$

where $N > 1$. From Eq. (13), the final score is not defuzzified and would produce a more intuitive value in the range of $[0, 1]$.

The fourth evaluation system denoted as EVALSYS4 is to evaluate multiple-vessel encounter for OS compliance, is designed similarly, but with an extension of EVALSYS2.

Table 7
Additional rules for Sub-system B (EVALSYS2).

Rule	If (AND logic)							Then
#	R_{OS}	R_{TS}	TSPort-OfOS	$\lvert\Delta\chi_{\mathrm{OS}}\rvert$	$\lvert\Delta U_{\mathrm{OS}}\rvert$	$\#SC_{\mathrm{OS}}$	$\mathrm{TCPA}(ts_{\mathrm{OS}})$	C_{GW}
$\mathrm{B}_5^{\mathrm{OS}}$	GW	GW	true	NOT Insufficient	–	Small	Early	Good
$\mathrm{B}_6^{\mathrm{OS}}$	GW	GW	true	–	NOT Insufficient	Small	Early	Good

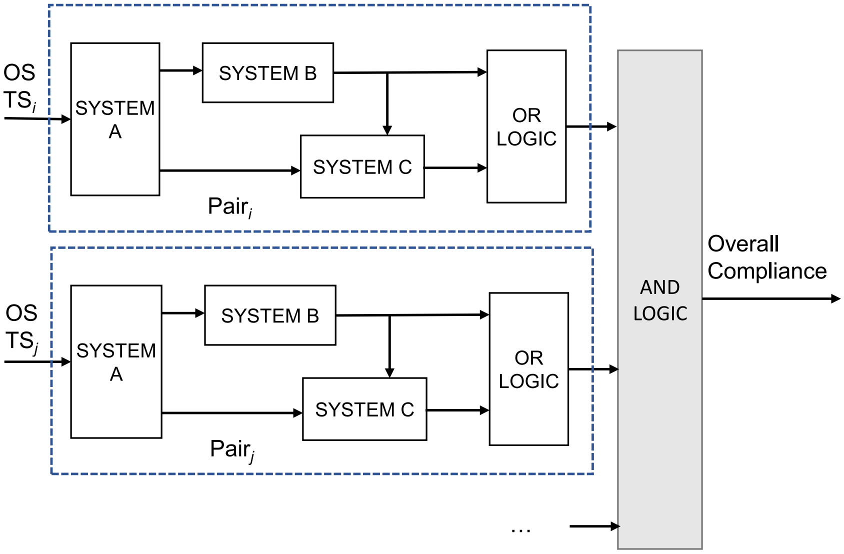


Fig. 18. Extension to compliance evaluation in multiple-vessel scenarios. In this figure, the prefix "Sub-" in the "Sub-system" is removed for simplicity.

3.4. Comments on the developed systems

This subsection discusses the evaluation system design by comparing one fuzzy rule in EVALSYS1 to its equivalent original COLREG Rule.

We take Rule 17 for discussion. The COLREG Rule and its corresponding fuzzy logic rule are repeated for convenience.

Part (c) of Rule 17 states that "*A power-driven vessel which takes action in a crossing situation. In accordance with subparagraph (a) (ii) of this Rule to avoid collision with another power-driven vessel shall, if the circumstances of the case admit, not alter course to port for a vessel on her own port side*" (IMO, 1972).

The corresponding fuzzy rule is C_5 (Section 3.1.3):

C_5 : If R_{OS} is *SO* and R_{TS} is *GW* and C_{GW} is NOT *Good* and $\lvert\Delta\chi_{\mathrm{OS}}\rvert$ is NOT *Insufficient* and $\#SC_{\mathrm{OS}}$ is *Small* and Crossing is *true* and TSPortOfOS is *true* and $\Delta\chi_{\mathrm{OS}}$ is NOT *Portside* then C_{SO} is *Good*

Rule C_5 transforms crossing situation since it has the fuzzy variable "Crossing is *true*". More specifically, the role of OS is SO ($R_{\mathrm{OS}} = SO$) and the role of TS is GW ($R_{\mathrm{TS}} = GW$).

Subparagraph (a) (ii) of Rule 17 states that if TS does not GW and she is on the portside of OS then SO vessel (OS vessel) may take actions to avoid collision. This is transformed into fuzzy variables "C_{GW} is NOT *Good*" and "TSPortOfOS is *true*".

Part (c) of Rule 17 states that OS should take action *not alter course to port*. This is transformed to the fuzzy variables $\lvert\Delta\chi_{\mathrm{OS}}\rvert$ is "NOT" *Insufficient* and $\Delta\chi_{\mathrm{OS}}$ is "NOT" *Portside*.

Some binary variables are developed in MATLAB scripts incorporate into fuzzy logic. These binary variable would provide full belongingness to the fuzzy value. One binary variable which has been developed for Sub-system A, is TSD to determine if the vessels are traveling in same direction. Two binary variables developed for Sub-system B are OSPortOfTS and TSPortOfOS to determine where a vessel passes in relation to the other vessel for Rule 14. Other boolean variables developed for Sub-system C are course altering to portside, crossing, and alterations due to a WP change. Generally it is stated in the literature that COLREG are vague, they are not specific enough, but the designed binary variables are specific enough to utilized binary variables and incorporate them into the fuzzy logic evaluation system.

The variables R_i, C_i and C_{Overall} are developed for evaluation, while the others are calculated from the position, heading, speed, WP, offset heading and offset velocity.

3.5. Parameters for FMFs

The evaluation system depends considerably on the parameters of the FMFs. Fuzzy logic can create subjectivity since it converts the vagueness of COLREGs to computer language. In this section we discuss the methods to choose these parameters to reduce the subjectivity, given by

- *Domain knowledge*. The knowledge of these parameters can be obtained by reviewing the previous work and surveying the literature. Another source of domain knowledge can be drawn from experienced experts in ship classification societies, e.g. Det Norske Veritas (DNV), Lloyd's Register (LR), American Bureau of Shipping (ABS), etc. From these domain knowledge sources, the classification societies have the position to establish guidelines and standards ensuring a consistent evaluation.
- *Data knowledge*. The available data from maritime traffic can help to build the parameters by learning from the historical data set. In shipping industry, automatic identification system (AIS) is used for information transmission among vessels containing sailing status introduced by the International Maritime Organization (IMO). Unsupervised machine learning can be applied on these data to cluster the common behaviors of ship-collision avoidance. The shape of the FMFs can also be adapted to the available data, e.g. an exponential FMF can be used instead of a trapezoidal one. However, in this paper, we use the trapezoidal FMF to show the concept of the evaluation system; and further work should be carried out to investigate other shapes of FMFs.

Table 8
Summary of the parameter design for FMFs.

Measures	FMF input	FMF	Method
Change of course	$\Delta\chi$	$[-4, -2, 2, 4]$	Learning from AIS data
Change of speed	ΔU	$[-2, -1.5, 1.5, 2]$	Learning from AIS data
Overtaking	$\bar{\beta}$ (for TS) and $\bar{\alpha}$ (for OS)	$[110, 112.5, 247.5, 250]$	Rule 13. Part (b)
Starboard crossing	$\bar{\beta}$ (for TS) and $\bar{\alpha}$ (for OS)	$[15, 20, 110, 112.5]$	Cho et al. (2022)
Reciprocal or Near Reciprocal	$\bar{\beta}$ (for TS) and $\bar{\alpha}$ (for OS)	$[168, 170, 190, 192]$	Cho et al. (2022)
Risk of collision	DCPA	$[0, 0, 2960, 2960]$	Vujičić et al. (2017)
Earliness of collision avoidance action	TCPA	$[360, 383, 10000, 10000]$	Vujičić et al. (2017)

- *Evolving fuzzy rules (drift with time)*. The amount of available data will increase overtime making it possible to adjust the parameters of FMFs (Tizhoosh, 2019). The idea is similar to retraining of deep neural networks to adapt to the new available data.
- *Sensitivity study*. As the evaluation depends on the parameters of FMFs, the effect of variation of these parameters needs to be studied. If parameters are sensitive to the evaluation result, the choice of these parameters need to be studied carefully, and vice versa.

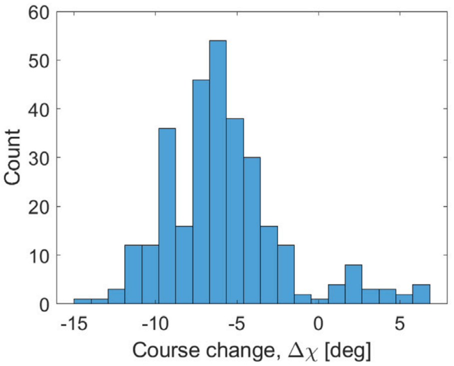
In this paper, we aim to demonstrate the design of the parameters based on the literature review and the historical AIS data rather than a detailed design due to the low Technology Readiness Level (TRL) of the present work. Table 8 summarizes the design of parameters based on the two methods, namely domain knowledge and data knowledge. The design of parameters for Overtaking, Starboard crossing, Reciprocal, Risk of collision, and Earliness of collision avoidance action FMFs is conducted via domain knowledge (literature review). The design of parameters for course change and speed change FMFs will be demonstrated by learning from historical AIS data as the following.

The AIS data sets are collected from two independent regions of Trøndelag and Møre and the region of Skagerrak between Norway and Denmark with the duration from the 1st of Jan 2018 to the 31st of Dec 2019 excluding June and July 2019. Fig. 19 shows the histogram of course and speed changes to avoid collisions. It is observed that in Fig. 19a the most probable course change is from 2 to 10 degrees to starboard side. This might not be considered as an apparent change which is 20 degrees according to Woerner (2016). One possible reason is that the majority of actions to avoid collision is "early" in the data set; therefore a small change of the course is sufficient. However, this needs to be further investigated by learning from more data set. In this paper, for demonstration, the FMF is chosen to be $[-4, -2, 2, 4]$ to agree with the AIS data. For the speed change, it is less evident that the speed change action was used to avoid collision as shown in the speed change histogram (Fig. 19b). This is inline with COLREG Rule 8 (c) that an alteration of course alone may be the most effective action to avoid a collision if there is sufficient sea-room. However, in this paper if we assume that the speed change is the only action to avoid collision for the evaluation system, the FMF is chosen to be $[-2, -1.5, 1.5, 2]$.

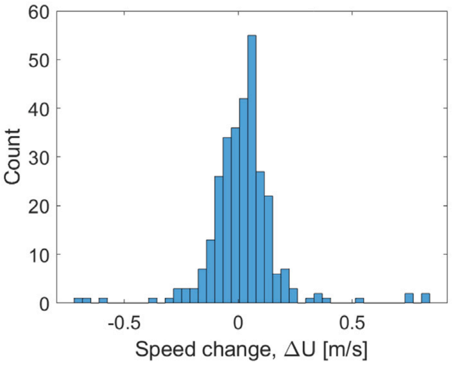
4. Results and discussion

This section provides results and discussions of the developed evaluation systems. The evaluated scenarios are the data extracted from simulated scenarios provided by Kjerstad (2019). Several one-to-one vessel encounters and some multiple-vessel encounters are presented in this section. The multiple-vessel encounter evaluation systems are validated in this section for 2-pair scenario. However, the designed method can be scaled easily to a number of TSes > 2.

The simulated scenarios assume no environmental forces applied to the vessel encounters; therefore, the course and heading are equal.



(a) Course change where a negative number (counter-clockwise) corresponds to a starboard alteration.



(b) Speed change

Fig. 19. Histogram of course change and speed change to avoid collision from historical AIS data.

4.1. One-to-one encounter

This section evaluates the design for the one-to-one encounter, i.e. EVALSYS1 and 2. The testing is divided into three scenarios, i.e. overtaking (O1 and O2), head-on (H1 and H2), and crossing (Cr1 and Cr2) focusing on Rules 8 and 13–17. The scenarios are summarized in Table 9. Table 10 shows the results for one-to-one scenarios where the first part is input, and the last two parts are evaluation results for EVALSYS1 and EVALSYS2. All fuzzy variables utilized in the evaluation system will be provided. These variables are similar between EVALSYS1 and 2. The difference is that EVALSYS2 does not utilize all variables, e.g. TCPA(ts_{TS}), to evaluate earliness in a head-on situation since TS is not important for OS evaluation.

The MATLAB scripts developed to obtain the binary variables, e.g. TSD, are not provided in the results as these variables are intuitively obtained by the plots.

There might be a difference in the course/heading change between the heading plots in time domain and $|\Delta\chi_i|$ in the result table (Table 10). This is because $|\Delta\chi_i|$ is calculated from the commanded heading, while the heading plot in time domain is the actual heading.

Table 9
Summary of one-to-one scenarios.

Scenario	Rules	OS	TS	Notes
Overtaking O1	8, 13, 16	GW	SO	Both vessels follow R13
Overtaking O2	8, 13, 16, 17	SO	should GW	TS violates R16 by not GW. OS follows R17
Head-on H1	8, 14, 16	GW	GW	Both vessels follow R14
Head-on H2	8, 14, 16	GW	should GW	TS violates R14 by turning to port
Crossing Cr1	8, 15, 16, 17	GW	SO	Both vessels follow R15
Crossing Cr2	8, 15, 16, 17	GW	should SO	TS does not SO

Table 10
Results for one-to-one scenarios.

	Variable	Unit	O1	O2	H1	H2	Cr1	Cr2
Input	R_{OS}	–	GW	SO	GW	GW	GW	GW
	R_{TS}	–	SO	GW	GW	GW	SO	SO
	$\bar{\beta}$	degree	0	180	360	360	45	45
	$\bar{\alpha}$	degree	180	0	0	0	315	315
	$\lvert\Delta\chi_{\mathrm{OS}}\rvert$	degree	48.0	60.1	16.9	52.8	79.4	77.5
	$\lvert\Delta\chi_{\mathrm{TS}}\rvert$	degree	0	0	16.9	0	37.7	0
	$\lvert\Delta U_{\mathrm{OS}}\rvert$	m/s	0	0	0	0	0.90	2
	$\lvert\Delta U_{\mathrm{TS}}\rvert$	m/s	0	0	0	0	0.53	0
	$\#SC_{\mathrm{OS}}$	–	1	1	1	1	2	2
	$\#SC_{\mathrm{TS}}$	–	0	0	1	0	4	0
	DCPA	m	0.0	0.0	0.0	0.0	0.0	1.47
	$\Delta\chi$	degree	360	360	180	180	270	270
	TCPA(ts_{OS})	s	699.9	679.9	659.9	659.9	679.9	699.9
	TCPA(ts_{TS})	s	–	–	659.9	–	679.9	
Results	C_{GW}	EVALSYS1	1	0	1	0	0.90994	0.90994
	C_{SO}		1	1	0	0	0	0
	C_{Overall}		1	1	1	0	0.90994	0.90994
	C_{GW}	EVALSYS2	1	–	1	1	0.90994	0.90994
	C_{SO}		–	1	–	–	–	–
	C_{OS}		1	1	1	1	0.90994	0.90994

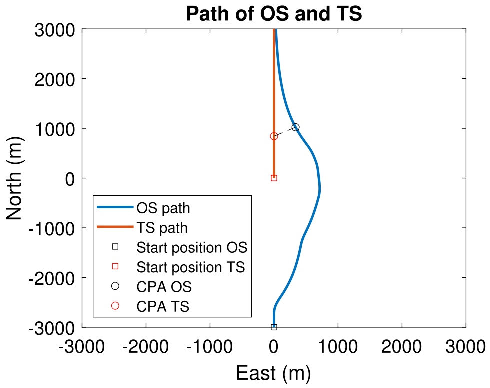


Fig. 20. Path of OS and TS, Overtaking O1.

The vessel tends to return to its original path but the actual heading will follow with some delay.

The heading plots in time domain are provided with $\psi_i \in [-180, 180]$. The detailed results are presented in the follow subsections.

4.1.1. *Overtaking*

This section will provide two scenarios where Rule 13 applies, i.e. an overtaking situation.

Overtaking O1. The trajectory plot (Fig. 20) indicates that at the beginning of the scenario, OS is overtaking TS since OS is coming up towards TS with a direction of more than 22.5° abaft TS beam. This means that OS is the GW vessel and TS is the SO vessel.

TS does not execute any alterations; and OS changes her heading. This is observed in the speed and heading plots in time domain (Fig. 21). This means that OS does GW and TS does SO. It is noted that in reality a heading change entitles a speed change. However, the speed of OS (Fig. 21) is unchanged even though its heading changes. This is due to a simulation simplification.

Column O1 in Table 10 shows the results for EVALSYS1 and 2. As seen $R_{\mathrm{OS}} = \mathrm{GW}$ and $R_{\mathrm{TS}} = \mathrm{SO}$, as expected when OS overtakes TS. OS does GW by $|\Delta\chi_{\mathrm{OS}}| = 48.0°$ with one change, i.e. $\#SC_{\mathrm{OS}} = 1$, and TS does SO by $|\Delta U_{\mathrm{TS}}| = 0$ m/s and $|\Delta\chi_{\mathrm{TS}}| = 0°$. Therefore, $C_{\mathrm{GW}} = 1$ and $C_{\mathrm{SO}} = 1$ with EVALSYS1 evaluation, which results in 100% compliance, i.e. $C_{\mathrm{Overall}} = 1$.

For EVALSYS2, $C_{\mathrm{OS}} = 1$, i.e. compliance of OS, as expected since OS does GW and act according to Rules 13 and 16. C_{SO} is not applicable since this is TS role and thereof not evaluated by EVALSYS2 system.

Overtaking O2. In this scenario, TS is coming up towards OS with a direction of more than 22.5° abaft OS' beam (Fig. 22). TS is overtaking OS so TS should be the GW vessel; and OS is the SO vessel.

From the plot in time domain (Fig. 29), TS does not alter the course nor the speed and hence violates Rule 16, but OS makes a heading change and acts accordingly to Rule 17.

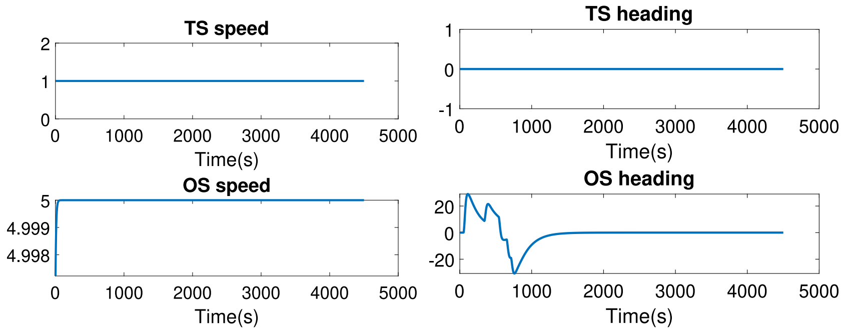
The evaluation result shown in column O2 in Table 10 states that $R_{\mathrm{OS}} = \mathrm{SO}$ and $R_{\mathrm{TS}} = \mathrm{GW}$. This is as expected since TS is overtaking OS by Rule 13. EVALSYS1 results in $C_{\mathrm{GW}} = 0$ as TS does not do any action to GW, i.e. $|\Delta\chi_{\mathrm{TS}}| = 0°$ and $|\Delta U_{\mathrm{TS}}| = 0$ m/s. SO compliance $C_{\mathrm{SO}} = 1$ as the SO vessel, acts according to Rule 17 by making actions when the GW vessel does not, given by $|\Delta\chi_{\mathrm{OS}}| = 60.1°$. By this $C_{\mathrm{Overall}} = 1$, i.e. full compliance from EVALSYS1 evaluation.

EVALSYS2 of this overtaking scenario results in $C_{\mathrm{OS}} = 1$. This is expected as OS does action to avoid collision since TS does not GW and act according to Rule 17.

4.1.2. *Head-on*

This section provides evaluation results for two different head-on scenarios following Rule 14. The first scenario, i.e. H1, contains a situation where both OS and TS act to avoid collision; and the second scenario, i.e. H2, only OS acts to avoid collision.

Head-on H1. The North-East trajectory (Fig. 23) shows that both vessels avoid collision by altering course (heading) to starboard



(a) Speed [m/s] (b) Heading [°]

Fig. 21. Speed and heading for OS and TS, Overtaking O1.

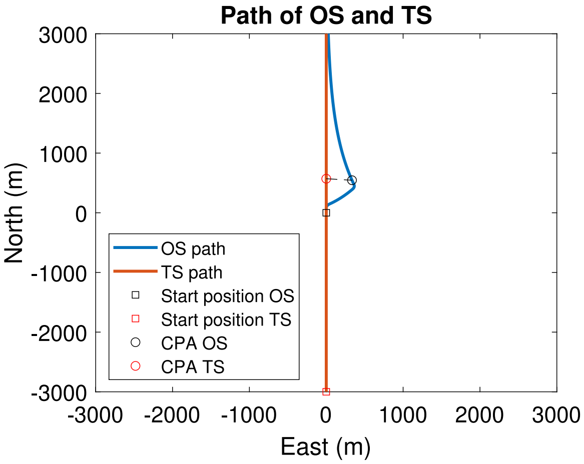


Fig. 22. Path of OS and TS, Overtaking O2.

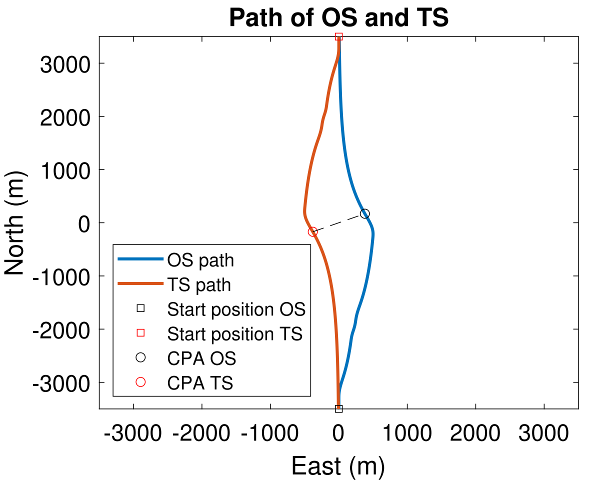


Fig. 23. Path of OS and TS, Head-on H1.

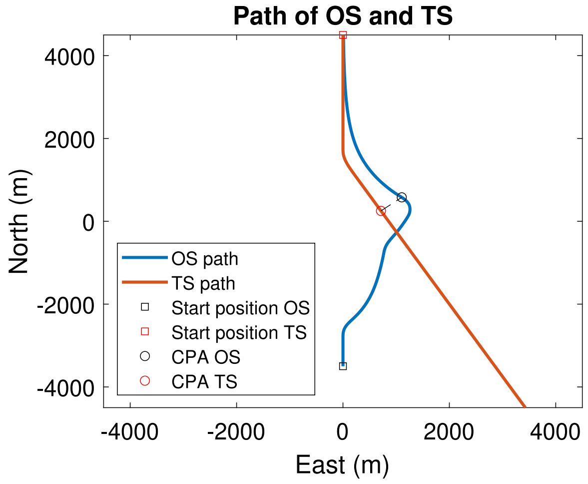


Fig. 24. Path of OS and TS, Head-on H2.

(Fig. 30) to obtain a portside passing. Therefore, both vessels act according to Rule 14.

The result is tabulated in column H1 in Table 10. The evaluation results in for EVALSYS1 $C_{\mathrm{GW}} = 1$, $C_{\mathrm{SO}} = 0$ and $C_{\mathrm{Overall}} = 1$ which indicates that in the head-on situation both vessels does GW by altering course to starboard to obtain a portside passing (Rule 14) with a substantial and early action (Rules 8 and 16), given by TCPA(ts_i) and $|\Delta\chi_i|$. In a head-on situation it is expected that $C_{\mathrm{SO}} = 0$ as Rule 14 states that both vessels should GW.

Head-on H2. At the beginning of the scenario, both vessels are in head-on situation (Fig. 24). By Rule 14 both vessels should then GW by altering course to starboard, but TS alters course to portside due to a WP tracking and by this she violates Rule 14 (Fig. 31). OS alters heading to starboard to obtain a portside passing (Fig. 31).

The evaluation result is tabulated in column H2 in Table 10. The result shows that $C_{\mathrm{SO}} = 0$ as expected since both vessels should GW in a head-on situation, also proven by $R_i = \mathrm{GW}$. For EVALSYS1, $C_{\mathrm{GW}} = 0$ and $C_{\mathrm{Overall}} = 0$ since TS does not comply with Rule 14.

EVALSYS2 results in $C_{\mathrm{OS}} = 1$, i.e. OS is 100% compliance by COLREG. This is due to the fact that OS does GW and acts according to Rules 8, 14 and 16 by altering course to starboard early and substantially, i.e. $|\Delta\chi_{\mathrm{OS}}| = 52.8°$, $\mathrm{TCPA}(ts_{\mathrm{OS}}) = 659.9$ sec and $\#SC_{\mathrm{OS}} = 1$ respectively, to obtain a portside passing.

4.1.3. *Crossing*

This section will provide two scenarios with a crossing situation to verify the design for Rule 15, i.e. Cr1 and Cr2.

Crossing Cr1. The trajectory plot (Fig. 25) shows that this is a crossing situation since OS has TS on her starboard side and should therefore GW; and TS should SO (Rule 15).

Both vessels do change the speed and heading (Fig. 32). Therefore, TS violates Rules 15 and 17 as she should have SO.

The evaluation result is tabulated in column Cr1 in Table 10. OS does GW by changing the course and speed, i.e. $|\Delta\chi_{\mathrm{OS}}| = 79.4°$

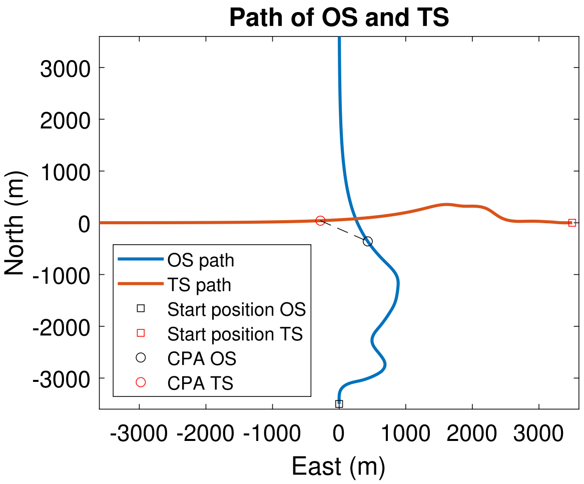


Fig. 25. Path of OS and TS, Crossing Cr1.

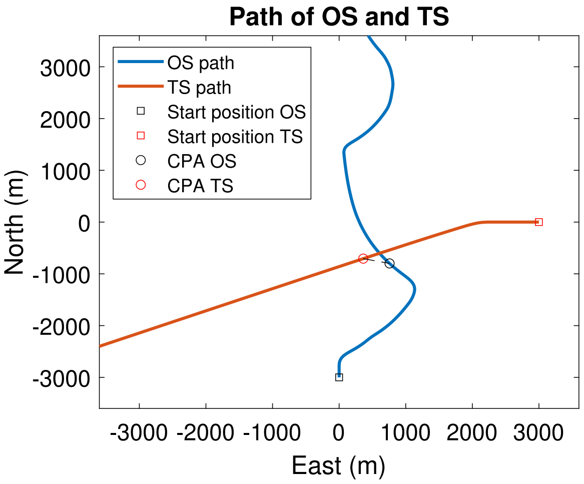


Fig. 26. Path of OS and TS, Crossing Cr2.

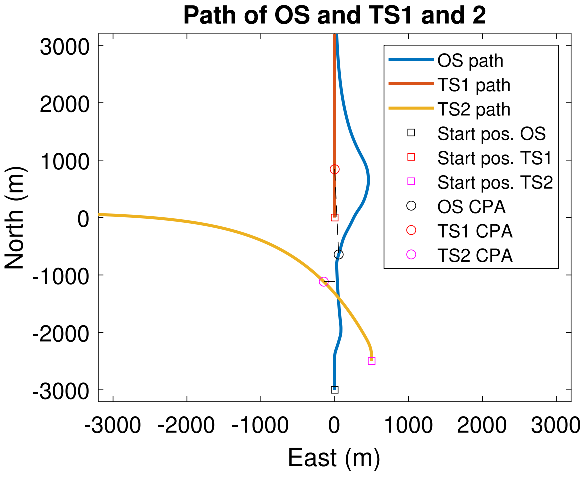


Fig. 27. Path of OS and TSes, Overtaking.

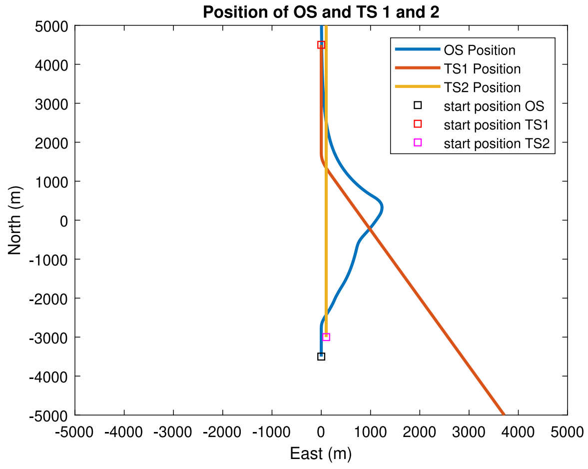


Fig. 28. Path of OS and TSes, Head-on and overtaking.

and $|\Delta U_{\mathrm{OS}}| = 0.90444$ m/s. From the FMF (Fig. 15), $|\Delta\chi_{\mathrm{OS}}|$ has full belongingness to "NOT" *Insufficient*. From the FMF (Fig. 14), $|\Delta U_{\mathrm{OS}}|$ has full belongingness to *Insufficient*. The insufficient speed change will be overwritten by the sufficient course change due to the "OR" logic. However, C_{GW} is not 100% due to $\#SC_{\mathrm{OS}} = 2$ which does not provide full belongingness to the fuzzy value *Small* (Fig. 13). The compliance score $C_{\mathrm{GW}} \neq 1$ provides a degree of belongingness to a fuzzy value between *Good* and "NOT" *Good*. From visually analyzing the plots it is more or less impossible to state that $C_{\mathrm{GW}} \neq 1$.

Since C_{GW} is not 100%, the SO vessel (TS in this case) could do actions to avoid a collision. TS does actions by changing heading and speed, However, $C_{\mathrm{SO}} = 0$ due to $\#SC_{\mathrm{TS}} = 4$ which does not provide any belongings to the fuzzy value *Small*, see Fig. 13.

EVALSYS2 results in $C_{\mathrm{OS}} = 0.90994$ due to $\#SC_{\mathrm{OS}} = 2$. C_{SO} is not applicable since the SO role is by TS, which is not concerned in EVALSYS2.

Assuming that TS did comply to Rule 17, EVALSYS1 would have resulted in $C_{\mathrm{Overall}} = 1$ and EVALSYS2 will still provide 90.994% compliance.

Crossing Cr2. The trajectory plot (Fig. 26) shows that this is a crossing situation since OS has TS on her starboard side and should therefore GW; and TS should SO (Rule 15).

TS does not alter the speed, but changes the heading (Fig. 33) due to a WP change. By this TS violates Rule 15 as she should SO. OS acts according to Rule 16 by altering speed and heading to GW (Fig. 33).

The evaluation result is tabulated in column Cr2 in Table 10. OS does GW by $|\Delta U_{\mathrm{OS}}| = 2$ m/s and $|\Delta\chi_{\mathrm{OS}}| = 77.5°$, where both actions provides full belongingness to "NOT" *Insufficient* (Figs. 14 and 15). $C_{\mathrm{GW}} \neq 1$ due to $\#SC_{\mathrm{OS}} = 2$ and by this does not provide full belongingness to the fuzzy value *Small* (see Fig. 13). TS which is the SO vessel, does not comply to SO since she changes heading due to a WP tracking. She could change heading or speed due to an action to avoid a collision since $C_{\mathrm{GW}} \neq 1$, but the change was due to a WP change. Therefore $C_{\mathrm{SO}} = 0$ by the evaluation with EVALSYS1.

4.2. Multiple encounter scenarios

This section provides the results from evaluation on two different multiple-vessel encounters where there are two TSes involved in a risky encounter with OS.

Table 11
Multiple-vessel encounters, Overtaking.

	Variable	Unit	Pair 1	Pair 2
Input	R_{OS}	–	GW	GW
	R_{TS}	–	SO	SO
	$\bar{\beta}$	degree	0	45
	$\bar{\alpha}$	degree	180	225
	$\lvert\Delta\chi_{\mathrm{OS}}\rvert$	degree	38.7	38.7
	$\lvert\Delta\chi_{\mathrm{TS}}\rvert$	degree	0	0
	$\lvert\Delta U_{\mathrm{OS}}\rvert$	m/s	2.5	2.5
	$\lvert\Delta U_{\mathrm{TS}}\rvert$	m/s	0	0
	$\#SC_{\mathrm{OS}}$	–	3	3
	$\#SC_{\mathrm{TS}}$	–	0	0
	DCPA	m	0.0	1118.0
	$\Delta\chi$	degree	360	0
	TCPA(ts_{OS})	s	709.9	513.2
	TCPA(ts_{TS})	s	–	–
Result	C_{GW}	EVALSYS1	0	0
	C_{SO}		0	0
	C_{Overall}		0	0
	C_{Overall}	EVALSYS3	0	
	C_{GW}	EVALSYS2	0	0
	C_{SO}		0	0
	C_{OS}		0	0
	C_{OS}	EVALSYS4	0	

Table 12
Multiple-vessel encounters, Head-on and overtaking.

	Variable	Unit	Pair 1	Pair 2
Input	R_{OS}	–	GW	GW
	R_{TS}	–	GW	SO
	$\bar{\beta}$	degree	349.9	11.3
	$\bar{\alpha}$	degree	360.0	191.3
	$\lvert\Delta\chi_{\mathrm{OS}}\rvert$	degree	31.8	31.8
	$\lvert\Delta\chi_{\mathrm{TS}}\rvert$	degree	0	10^{-3}
	$\lvert\Delta U_{\mathrm{OS}}\rvert$	m/s	2.4	2.4
	$\lvert\Delta U_{\mathrm{TS}}\rvert$	m/s	0	0
	$\#SC_{\mathrm{OS}}$	–	5	5
	$\#SC_{\mathrm{TS}}$	–	0	10
	DCPA	m	653.8	1005.0
	$\Delta\chi$	degree	169.9	0
	TCPA(ts_{OS})	s	759.9	845.8
	TCPA(ts_{TS})	s	–	–
Result	C_{GW}	EVALSYS1	0	0
	C_{SO}		0	0
	C_{Overall}		0	0
	C_{Overall}	EVALSYS3	0	
	C_{GW}	EVALSYS2	0	0
	C_{SO}		0	0
	C_{OS}		0	0
	C_{OS}	EVALSYS4	0	

4.2.1. *Overtaking*

This section provides a scenario where only OS does actions to avoid collision. The trajectory plot (Fig. 27) indicates that OS is overtaking both TSes, and should therefore GW with respect to TS1 and TS2. TS1 and TS2 should SO to OS.

From Fig. 34, OS executes actions by changing the heading. TS1 does not change speed nor heading; and TS2 does a heading and speed change but due to a WP tracking.

In Table 11 the variables utilized in the evaluation systems are presented. From the table, for both pairs $\#SC_{\mathrm{OS}} = 3$ which provides zero belongings to the fuzzy value *Small*, (Fig. 13). Even though $R_{\mathrm{OS}} =$ GW and does GW by $\lvert\Delta\chi_{\mathrm{OS}}\rvert = 38.7°$ and $\lvert\Delta U_{\mathrm{OS}}\rvert = 2.5$ m/s, for both pairs, the GW compliance, $C_{\mathrm{GW}} = 0$ due to $\#SC_{\mathrm{OS}} = 3$.

The evaluation results for each pair are tabulated in the lower part of Table 11. From the table, the overall compliance for EVALSYS1 is zero for pair 1 ($C_{\mathrm{Overall,\ pair1}} = 0$). The GW compliance for OS, $C_{\mathrm{GW,\ pair1}} = 0$ due to $\#SC_{\mathrm{OS}} = 3$, therefore TS1 should have taken actions to avoid collision. The variables $\lvert\Delta\chi_{\mathrm{TS,\ pair1}}\rvert = 0°$ and $\lvert\Delta U_{\mathrm{TS,\ pair1}}\rvert = 0$ m/s mean that TS1 did not act according to Rule 17. Therefore, $C_{\mathrm{SO,\ pair1}} = 0$.

The reason for $C_{\mathrm{Overall,\ pair2}} = 0$ is similar. $C_{\mathrm{GW,\ pair2}} = 0$ and TS2 should have taken actions to avoid a collision since the GW vessel does not provide any C_{GW}, but TS2 does not act according to Rule 17 ($\lvert\Delta\chi_{\mathrm{TS,\ pair2}}\rvert = 0°$ and $\lvert\Delta U_{\mathrm{TS,\ pair2}}\rvert = 0$ m/s). The overall compliance for EVALSYS3 is calculated by the minimum of $C_{\mathrm{Overall,\ pair1}}$ and $C_{\mathrm{Overall,\ pair2}}$. Therefore, $C^{\mathrm{EVALSYS3}}_{\mathrm{Overall}} = 0$.

Similarly $C^{\mathrm{EVALSYS4}}_{\mathrm{OS}} = 0$ is calculated from the minimum of $C_{\mathrm{OS,\ pair1}}$ and $C_{\mathrm{OS,\ pair2}}$.

4.2.2. *Head-on and overtaking*

This section provides a scenario where only OS does actions to avoid collision. The trajectory plot (Fig. 28) indicates that (pair 1) OS is in a head-on situation with TS1 (Rule 14) and (pair 2) OS is in an overtaking situation with TS2 (Rule 13). By these Rules OS should GW in both encounters; TS1 should GW with regard to OS; and TS2 should SO with regard to OS.

In Fig. 35 the speed and heading plots for OS, TS1 and TS2 are provided. TS1 alters heading, but this is due to a WP tracking. TS2 does SO even with a small heading change of 10^{-3} degrees. OS does GW by altering heading and speed.

In Table 12 the variables utilized in the evaluation systems are presented. $R_{\mathrm{OS}} =$ GW and does GW by $\lvert\Delta\chi_{\mathrm{OS}}\rvert = 31.8°$ and $\lvert\Delta U_{\mathrm{OS}}\rvert = 2.4$ m/s. Even though OS does GW actions, the GW compliance, $C_{\mathrm{GW}} = 0$ due to $\#SC_{\mathrm{OS}} = 5$.

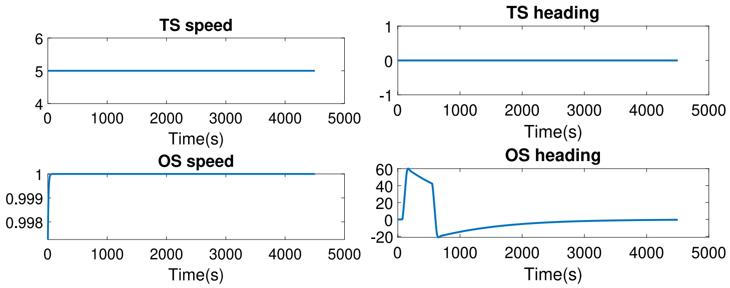
The evaluation results for each pair are tabulated in the lower part of Table 12. For EVALSYS1, TS1 should GW as $R_{\mathrm{TS}} =$ GW (pair 1). TS2 should have taken actions to avoid a collision by Rule 17 since $C_{\mathrm{GW}} = 0$ (pair 2). However, both TS1 and TS2 do not execute any action due to $\lvert\Delta\chi_{\mathrm{TS,\ pair1}}\rvert = 0°$, $\lvert\Delta U_{\mathrm{TS,\ pair1}}\rvert = 0$ m/s, $\lvert\Delta\chi_{\mathrm{TS,\ pair2}}\rvert = 0°$ and $\lvert\Delta U_{\mathrm{TS,\ pair2}}\rvert = 0$ m/s. Therefore, $C_{\mathrm{Overall,\ pair1}} = C_{\mathrm{Overall,\ pair2}} = 0$. As a consequence, $C^{\mathrm{EVALSYS3}}_{\mathrm{Overall}} = 0$. For EVALSYS2 and EVALSYS4, the compliance of OS are zeros due to $C_{\mathrm{GW}} = 0$.

5. Conclusion

The main motivation for this work is the lack of evaluating a CAS in the literature for multiple-vessel encounters with regard to COLREG in one metric. Therefore, the objective of this paper is to develop the evaluation systems for autonomous surface vessels with consideration of COLREG's compliance. Fuzzy logic was used to develop the evaluation system which can consider both one-to-one and multiple-vessel encounters. The evaluation system also considered the score for all the involved ships or only ownship. The systems were validated in several scenarios, where the evaluation results are compared with the interpretation of the COLREG's compliance.

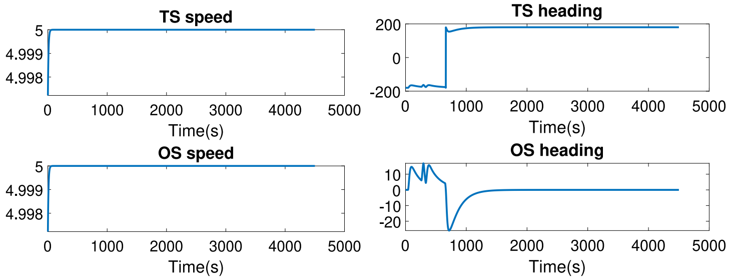
The significant findings in this paper can be summarized as follows:

1. The proposed system is a systematic evaluation which can provide variables that would be challenging or impossible to obtain by visual assessment,
2. Fuzzy logic can be used to incorporate COLREG evaluation into a computer even though COLREG is expressed vaguely, for human reasoning and might be exploited by human operators,
3. The evaluation system can be scaled easily to realize the multiple-vessel encounters which is a challenge when using COLREG for collision avoidance,



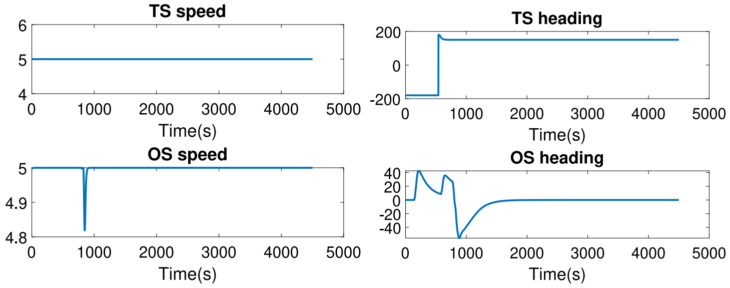
(a) Speed [m/s] (b) Heading [°]

Fig. 29. Speed and heading for OS and TS, Overtaking O2.



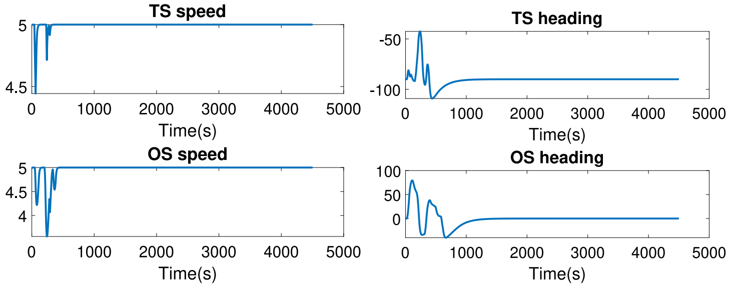
(a) Speed [m/s] (b) Heading [°]

Fig. 30. Speed and heading for OS and TS, Head-on H1.



(a) Speed [m/s] (b) Heading [°]

Fig. 31. Speed and heading for OS and TS, Head-on H2.



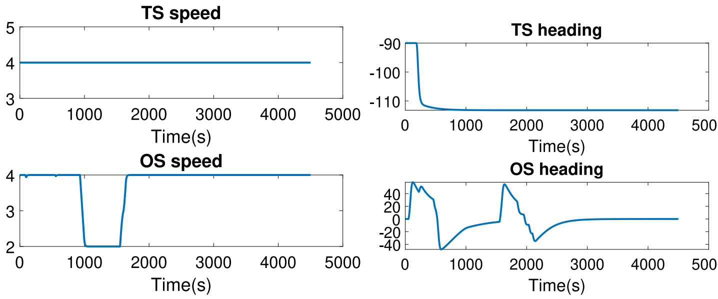
(a) Speed [m/s] (b) Heading [°]

Fig. 32. Speed and heading for OS and TS, Crossing Cr1.

4. The evaluation provides one metric score which is the ultimate goal for V&V a CAS, especially for autonomous vessel approval by class societies.

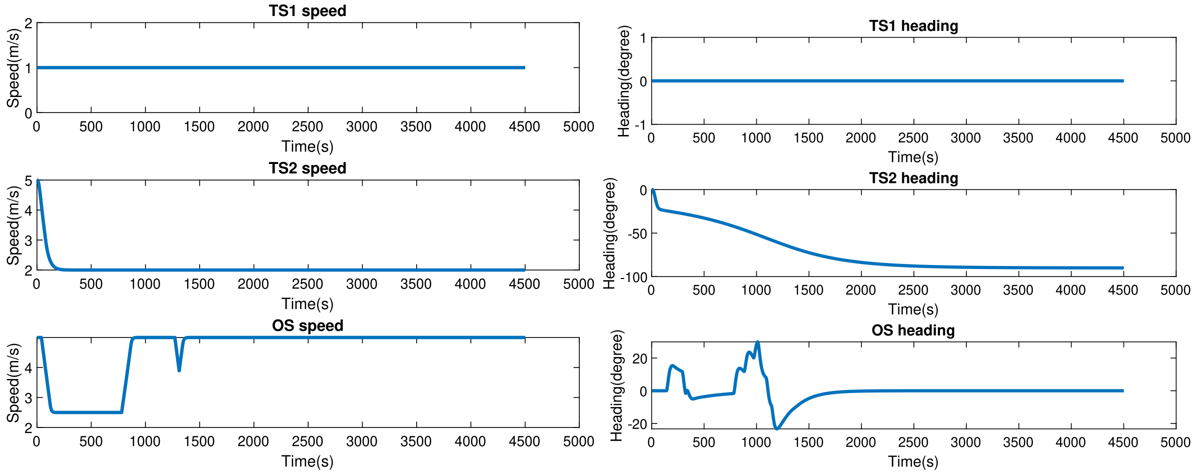
Based on the findings presented above, the study of the evaluation system can be extended to widen its applicability. The focus of the paper was to build the framework for the evaluation system rather than detailed design of the evaluation system. Some recommendations for future work are listed below:

1. This paper utilized the trapezoidal FMF. The design of FMFs could be investigated with different geometric and parameters.



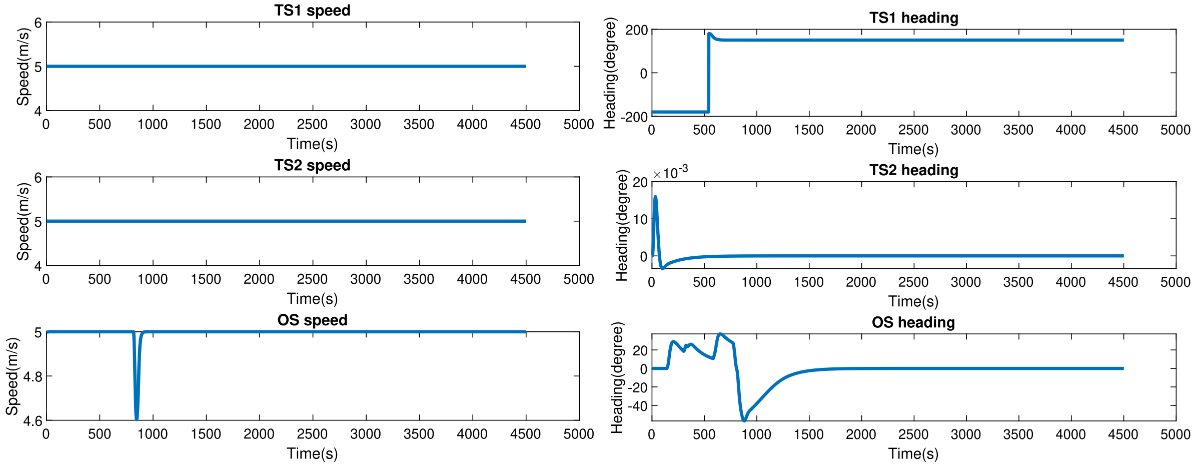
(a) Speed [m/s] (b) Heading [°]

Fig. 33. Speed and heading for OS and TSes, Crossing Cr2.



(a) Speed [m/s] (b) Heading [°]

Fig. 34. Speed and heading for OS and TSes, Overtaking.



(a) Speed [m/s] (b) Heading [°]

Fig. 35. Speed and heading for OS and TSes, Head-on and overtaking.

This can be done by interviewing experts or by learning from the collected data, e.g. AIS (automatic identification system),

2. This paper evaluated simulation data, therefore the changes of speed and heading for collision avoidance were easily identified. However, it might be difficult to identify them in a real collision situation. A realistic method to determine path deviation, speed deviation and to count the numbers of deviations can be developed in future work by e.g. using change-point detection to estimate such information,
3. The paper focused on Rule 8 and 13–17 (full visibility and open areas). Other rules can be implemented in future work on top of these rules, e.g. Rule 6 safe speed, Rule 9 narrow channels, Rule 10 traffic separation schemes, and Rule 19 conduct of vessels in restricted visibility.

CRediT authorship contribution statement

Dong Trong Nguyen: Main supervisor, Writing – review & editing. **Marius Trodahl:** Writing – original draft, Methodology, Software, Validation. **Tom Arne Pedersen:** Co-supervisor, Reviewing. **Azzeddine Bakdi:** Co-supervisor, Reviewing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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Appendix

A.1. Time-series

See Figs. 29–35.

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