Problem posing in mathematics education: a comparative study of textbooks in Ethiopia, South Sudan and Norway[1]

Solomon A. Tesfamicael[1], Øyvind Andersen Lundeby[1], Berie Getie[1], Oduho George Ben Soforon [1],
[1]Norwegian University of Science and Technology (NTNU), Department of Teacher Education, Trondheim, Norway; solomon.atesfamicael@ntnu.no; oyvind.a.lundeby@ntnu.no: beriegk@stud.ntnu.no; ogsoforo@stud.ntnu.no

This paper presents a comparison of textbooks on Problem Posing (PP) activities. The importance of PP activities is motivated in relation to problem solving (PS), a principal component of mathematics education. Using a framework that has nine categories, problem-posing activities are analyzed in six different mathematics textbooks from Ethiopia, South Sudan, and Norway on the topic of algebra. The finding shows that there is sparse or even insignificant number of PP activities in the textbooks, and these PP activities are not comprehensive enough in their types and forms. A deliberate action is needed to embed such activities in mathematics textbooks.

Keywords: Problem Posing, Problem Solving, Textbooks, Curriculum, Comparative Study.

Introduction

Comparative studies in mathematics education document, analyze, contrast or juxtapose similarities and differences across all aspects and levels in the field (Tesfamicael & Lundeby, 2019). Usually, one starts from *a construct* and tries to investigate similarities and differences of that construct in two or more contexts (Artigue & Winsløw, 2010). In this work, problem posing (PP) activities is the construct under comparison, while the mathematics textbooks from Ethiopia, South Sudan, and Norway define the context for comparison. The aim is to improve the quality of education by facilitating the dissemination of different effective frameworks, principles, and models across nations (Tesfamicael & Lundeby, 2019). Especially, the recent reform-oriented mathematics education influence in Norway may serve as a context for reference, not just on the quality of the physical features of the textbook but the contents of the textbook, in this case, PP.

The contexts in comparison

Textbooks have a crucial role in the teaching and learning of mathematics. The textbook curriculum plays the role of mediation between the intended curriculum and the implemented curriculum (Valverde, 2002). Especially the teaching and learning of mathematics has long been heavily

---

[1] Tesfamicael, S. A. Lundeby, Ø. A., Getie, Berie, & Soforon, O. G. B. (2022). Problem posing in mathematics education: a comparative study of textbooks in Ethiopia, South Sudan, and Norway. In Nortvedt, G. A., Buchholtz, N., Fauskanger, J., Hreinsdóttir, F., Hähkiöniemi, M., Jessen, B. E., Kurvits, J., Liljekvist, Y., Misfeldt, M., Naalsund, M., Nilsen, H. K., Pálsdóttir, G., Portaankorva-Koivisto, P., Radišić, J., & Wernberg, A. (Eds.) (2021). Bringing Nordic mathematics education into the future: *Preceedings of Norma 20, The ninth Nordic Conference on Mathematics Education* Oslo, 2021. (Skrifter från Svensk Förening för MatematikDidaktisk Forskning; No. 14). Svensk förening för matematikdidaktisk forskning. pp. 225-232.

associated with textbooks and curriculum materials (Remillard, 2005). In fact, textbooks are the main resources used in mathematics classrooms in many countries (Valverde, 2002; Pepin & Jablonka, 2007). Hence it is meaningful to engage in the improvement of textbook research for improving student learning (Cai & Jiang, 2017).

The contexts that we are engaged in for comparison are textbooks from three different countries, two from Sub-Saharan Africa, Ethiopia and South Sudan, and two from developed countries, Norway. Most schools in Ethiopia and South Sudan use textbooks produced by the respective ministries of education in the countries. This means for a given grade level one textbook fits all in these countries. While in Norway, textbooks are prepared by authors and publishing companies. A variety of textbooks for a given grade level are available for teachers, parents (in the Norwegian case), and schools to choose from.

The construct under comparison

What is PP? (Silver, 1994, s. 19) defined it as follows: "It refers to both the generation of new problems and the re-formulation of given problems. Thus, posing can occur before, during, or after the solution of a problem." (Kilpatrick, 1987) indicated that problem formation is an important companion to PS. Further, it has been advocated by many researchers that PP and PS are central to mathematical thinking, creativity, discourse in mathematics (Cai et al., 2015; Kılıç, 2017), in such a way that both are used as a means of instruction as well as an object of instruction to improve student learning (Kilpatrick, 1987; Silver, 1994; NCTM, 2000; McDonald & Smith, 2020). Furthermore, in recent reform movements PP is a focus on its own (NCTM, 2000), that is, the purpose of the instruction is to formulate or reformulate a problem, not just to solve it. Hence PP has become part of the mathematics curriculum and pedagogy (Cai & Jiang, 2017; English, 2020).

Research question

Cai et al. (2015) synthesized answered and unanswered questions concerning research on PP in mathematics education. Among the 14 unanswered questions, two of them related to PP activities and mathematics curricula are considered in this article. These are "How do the actual textbooks include PP?" and "If curriculum designers intend to integrate PP into textbooks and teaching materials, what are the best ways to do so?" Based on these questions, we aspire to start addressing the following research question. How do the textbooks in Ethiopia, South Sudan, and Norway include PP activities related to Algebra? In addition, we aim to investigate if PP activities are embedded in textbooks from these countries, focused on Algebra, just to limit the scope of our investigation. But first, we present the conceptual framework we have used in the study.

Conceptual framework

Recently, researchers have focused on teaching through PS than on learning about PS (Schoenfeld, 1992; Lester, 1994). Hiebert and colleagues (1996) emphasized that reform in curriculum and instruction should be based on PS. Hence, there is a huge interest on doing research on PS as compared to PP (Cai & Jiang, 2017; Deringöl, 2020). Cai and colleagues (2015) argue that PP is different from PS. In PS the students are asked to solve tasks, whereas in PP students are asked to generate new problems or re-formulate the existing ones. This does not mean that the students are required to solve the problem. Cai & Jiang (2017) reflected that the correlation between PS and PP is one reason why PP activities should be included in textbooks. Even for teachers to encourage students to pose problems, it is unlikely that they will have time and skills to do that unless teachers are offered such an opportunity, at least supplemental materials are needed (Cai et al., 2015).

How and when PP activity happens in relation to PS can be explained using the diagrammatic presentation as given in Figure 1, as an extension of the work of Stoyanova and Ellerton (1996). Box 1 stands for a PP activities induced by teachers, as usually is done in traditional mathematics instruction, or given in a textbook, or it can even be generated by students. Hence it is prior to the PS process. The next phase is the PS process, and one option is doing it using George Polya's (1945) four steps: understanding the problem, making a plan, executing the plan, and looking back. These are reinterpreted as three Boxes (2, 3, and 4) in the diagram.

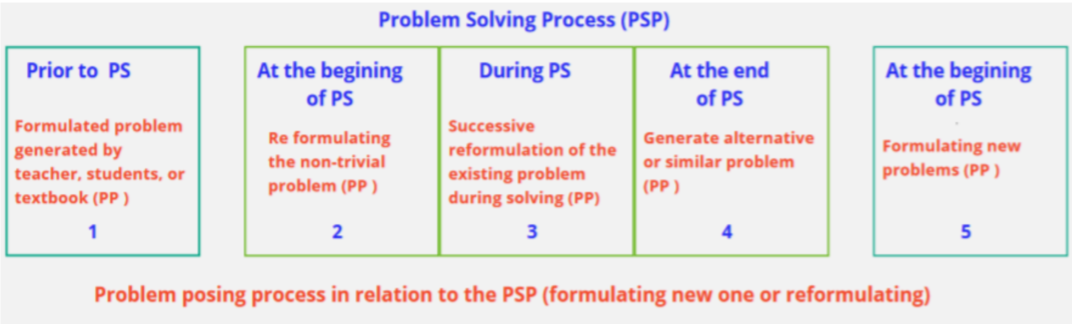

Figure 1:Diagram showing the occurrence of PP relationship to PS

At the beginning of solving the problem, a student can reformulate a given complex/non-trivial problem to a simplified version so that it can easily be solved with known strategies. During PS the expert solver tries to reformulate the existing problem in a successive reformulation and transformation of the problem. Somehow the solver is engaged in a series of posing problems (Box 3). Further, at the end of PS steps, as Polya called it, looking back, the solver can generate alternative or similar problems (Box 4). Formulating a new problem can also happen after a problem is solved. That new problem can demand a new imagination and creativity. It can be based on the previous solved problem, that is why we have post PS Box number 5. As in Box number 1, this can also be independent of the PS process. Since the study is on textbooks, it might not be possible to observe the PS process in boxes 2,3&4, unless it is made explicit. Mathematics textbooks can therefore be designed in such a way that teachers and students have access to PP activities (as described at Box 1 and 5 phases above).

General framework for problem posing types (Stoyanova & Ellerton, 1996)	Categorgy of PP Activitys	Specific Criteria (Adopted from, Cai & Jiang, 2017; Deringöl, 2020)
Free Problem Posing Activity (FPP)	#1	The student is asked to pose a problem without providing any data and without limitation.
Structured Problem Posing Activity (SPP)	#2	A Similar Problem Posing: The student is given a problem. The student is asked to pose another problem similar to this problem.
Semi-Structured Problem Posing Activity (SSPP)	#3	Problem posing in accord with the given **operation** (**symbolic**)

	#4	Problem posing in accord with the picture or using the information in the **picture** (**diagram**).
	#5	Problem posing in accord with the information given in the **table**
	#6	Problem posing in accord with the **graphic**
	#7	Problem posing in accord with **manipulatives** (**physical tools**)
	#8	Problem posing by completing **problem sentences** (or **verbal text**) through writing the missing information or through writing the appropriate question sentences.
	#9	Problem posing through given **context**

Table 1: Framework for PP analysis used in this work

Stoyanova and Ellerton (1996) proposed a framework that can link PP, PS, and curricula. The framework has three components. (i) *Free problem posing (FPP):* Students are asked to pose a problem based on a natural situation. (ii) *Semi-Structured problem-posing (SSPP):* "Students are given an open situation and are invited to explore the structure and to complete it by applying knowledge, skills, concepts and relationships from their previous mathematical experiences (p. 520)". (iii) *Structured problem-posing (SPP):* It occurs when a well-structured problem or problem situation is given, and the task is to construct similar problems (see also Table 1). Cai & Jiang (2017) classified PP tasks into five categories according to what it required students to do, in connection to the information provided in the task. Further, Deringöl (2020) has provided a framework for analysis joining these two frameworks. We have proposed a new way of sub categorizing *SSPP*. These seven sub categories under SSPP are based on the concept of multiple representation as given in Van de Walle (2018, s. 45). As SSPP activities include given conditions, and these conditions could be given in context, via diagram or graph, and so on, providing opportunity for students to make connections among representations as a way of showing mathematical understanding (see Table 1).

Method

In order to set up a reasonable comparison we took topic-based selection of textbooks. To start with, the topic Algebra is considered. It is a tool for generalizing arithmetic and representing patterns in our world, almost similar in all the textbooks from the three countries. This concept is presented in grade 7 and 8 mathematics textbooks in both Ethiopia and South Sudan. That means unit 2 in grade 7 (U2-G7-E), unit 2 & 3 in grade 8 mathematics textbooks (U2-G8-E & U3-G8-E) in Ethiopia; unit 4 in both grades 7 and 8 of South Sudan textbook are included (U4-G7-SS & U4-G8-SS). In Norway, there are a variety of textbooks produced by publishers. It means the sample space for choice of the textbook is broader in contrast to the two countries, where only one textbook for each grade level produced by the respective governments is used. Further, we base the choice of textbook for our research in Norway solely on one criteria. That is, the most recent textbook published in the country. The rationale behind the selection of recent textbooks is that we assume the textbook is influenced by the recent developments in mathematics education. That means the reformed mathematics education movement influence could be visible in the contents of the textbook. Textbooks considered, therefore, are Matemagisk and Maximum, and they are among the actively used and recent textbooks (Tesfamicael & Lundeby, 2019). Chapter 4 in both grade 7 Matemagisk and grade 8 Maximum (C4-G7MM-N & C4-G8MX-N) presents the topic Algebra, and these are included in the data analysis. Many textbook producers and publishers are revising the older versions of their textbooks after the implementation of the new curriculum in Norway in 2020. But these textbooks were not available to be considered in our study.

A qualitative research method, content analysis, is used in this study. Cohen and colleagues (2018) explained that qualitative content analysis helps to set up a set of procedures for the rigorous analysis, examination, inference, and verification of contents of original texts. Each of us has analyzed the textbooks from our respective countries using the 9 categories provided in Table 1 above. We agreed to use a subtask as a counting unit of analysis, just to simplify our comparison. We have also shared a google drive file to cross-check with each other's analysis. In addition, we selected some tasks and compared the result at the beginning for the sake of boosting the reliability of the study.

Findings

From the investigation of 7 chapters/units in algebra, in the 6 mathematics textbooks, around 1567 tasks were analyzed. And only 62 tasks were identified or approximated as PP tasks or activities, which is only 3.93% of the total task (See Table 2). Among these PP tasks most of them are categorized under SSPP (16 under category #3, 4 under category # 8, 34 under category # 9 and the rest 0). That is 87% of these PP activities are SSPP. There are 8 tasks that are SPP activities, but 0 tasks under category FPP.

Textbooks	Number of Tasks	Number of PP activities				Percentage of PP activities
		FPP	SPP	SSPP	**Total**	
U2-G7-E	148	0	0	4	**4**	2.7%
U2-G8-E	253	0	4	7	**11**	4.35%
U3-G8-E	219	0	0	0	**0**	0%
U4-G7-SS	72	0	1	4	**5**	6.94%
U4-G8-SS	134	0	0	0	**0**	0%
C4-G7MM-N	219	0	0	5	**5**	2.28%
C4-G8MX-N	531	0	3	34	**37**	6.97%
Total	1576	0	8	54	**62**	3.93%

Table 2: Number of tasks and PP activities in the 6 textbooks

Most of the PP activities are from the Maximum textbook (C4-G8MX-N). This coincides with our expectation that it will be influenced by the recent development in the teaching and learning of mathematics, as this textbook series is published in recent years.

The percentage of PP activities in U4-G7-SS is closer to that of C4-G8MX-N. However, looking closer to the tasks, we have a doubt if PP activities are embedded in U4-G7-SS intentionally. We can provide two reasons. First, four of the five tasks are actually subtasks under a task which are counted as one PP activity in our investigation. Second, there is no PP activity in U4-G8-SS, a textbook from the same country. Somehow this can indicate the absence of intentionality.

Figure 2 shows example of tasks found in Matemagisk (C4-G7MM-N) and Maximum (C4-G8MX-N) in connection to patterns and generalization in algebra. Task 89 is coded as SSPP activity, since it asks the pupils to create a pattern that contains 12 circles in figure 3 and 27 circles in figure 8. This is considered as SSPP activity, as it requires some kind of reformulation of the previous task based on the given conditions, as explained in PSP in Figure 2. Task 5.29 is categorized as SPP activity since it asks a to formulate similar task. These are not FPP activity, since the students are asked to make a similar pattern, that means some kind of limitation is embedded in the formulation of the task.

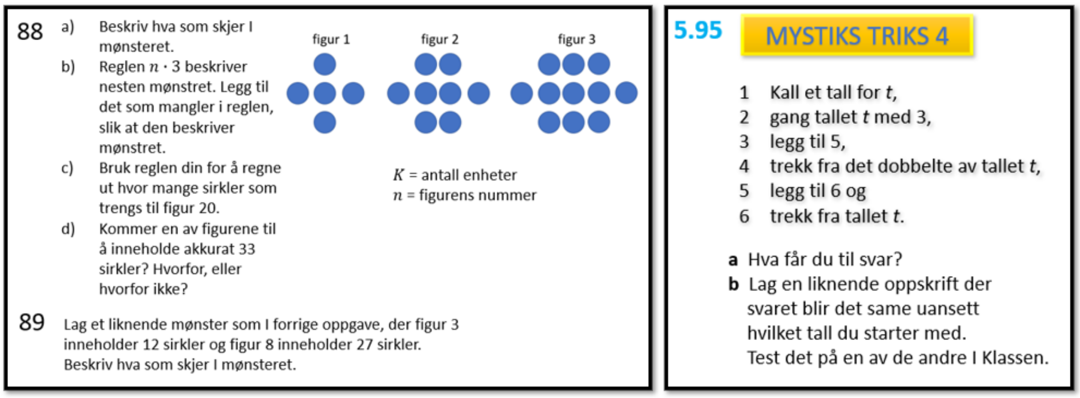

Figure 2: Task 89 in Matemagisk categorized as SSPP, Category #3 (Left). Task 5.95 b in Maximum 8, Grunnbok categorized as SPP (Right) (All figures recreated by the authors)

Discussion

The findings show that there is little PP activity in relation to the number of tasks provided by the textbooks in the area of Algebra in these textbooks from the three countries. This was a surprising result for us. Except Maximum, the textbook from Norway (C-G8MX-N), there are few PP activities or even not present in the case of U3-G8-E in Ethiopia and U4-G8-SS South Sudan textbooks (see Figure 2). What could be the cause of such minimum or absent PP activity in these textbooks? Could it be the intended curriculum of the countries have no such mention or emphasis? Or is it due to the topic we selected, Algebra? Or is it just assumed that teachers would pose the tasks for the students and maybe teachers will ask the students to pose tasks to each other? All these need deeper investigation. Actually, in our preliminary investigation after our data collection, we found that the new Norwegian curriculum includes pupils to pose problems and uses the phrase "stiller matematiske spørsmål," translated as asking mathematical questions (LK20, 2020). But does this refer to PP activities? This needs further investigation, since asking questions might not be the same as problem posing.

The textbooks that include PP activities related to Algebra are mostly under the SSPP category. They do not provide the opportunity to students to make FPP activities, while there are 8 tasks (0.5% of the total tasks) of the category SPP, which is also a negligible amount. Hence these textbooks tend to embed the categories of the type SSPP activities mostly. Even the two textbooks in Norway differ in how they integrate the PP activities: Matemagisk contained only SSPP, while Maximum provided both SPP and SSPP activities. Deringöl (2020) reported only one task that is of the category FPP. This might limit the creativity and intuition of the learners. In addition, similar to the findings of Cai and Jiang (2017) on Chinese and US elementary mathematics textbooks, the PP activities across the grade levels of Ethiopia and South Sudan varies.

In the process of analyzing the tasks, identifying and categorizing a task as a PP activity was somehow problematic for us. This might be due to many reasons. For example, if the curriculum of a particular nation does not mention PP (implicitly or explicitly), then textbook producers might not consider including such tasks. In that case, looking for something that was not intended to be there in the first place, is meaningless. Such study is more appropriate when PP activities are considered at all levels of the curriculum (intended, textbook, implemented, and attained).

Conclusion

In this preliminary investigation of these 7 textbooks from 3 different countries, we found sparse PP activities among the tasks related to Algebra. These *PP activities are restricted in form.* That is, most PP activities are of type SSPP, and FPP activities are nowhere in all the 7 books. This needs a shift in including PP tasks in curriculum at all levels (Valverde et al. 2002). As discussed in the beginning, to improve student learning opportunity, and students critical thinking and reasoning, PP has been supported by several research (Cai & Jiang, 2017; Cai et al., 2015). If students and teachers are going to engage in such activity, the textbooks in the respective countries should provide opportunities for it. A deliberate action is needed. Especially, in the case of Ethiopia and South Sudan textbooks, where teachers adhere using only one textbook for each grade level, including PP actives is critical if students are expected to engage posing problems. One can argue that in Norway, teachers have more flexibility in choosing textbooks, and hence tasks and activities, and therefore the problem may not be as severe as compared to Ethiopia and South Sudan. Nonetheless, researchers have also indicated that most mathematics teachers, including those in Norway are heavily depending on textbooks (Pepin & Jablonka, 2007).

In our next step, data from the curriculums of other Nordic countries, teacher guides, and several mathematical topics will be included for deeper understanding.

Acknowledgment

The study was partially funded by the Norwegian Program for Capacity Development in Higher Education and Research for Development (NORHED).

References

Artigue, M., & Winsløw, C. (2010). International comparative studies on mathematics education: a viewpoint from the anthropological theory of didactics. *Récherches en Didactiques des Mathématiques, 30(1)*, 47–82.

Cai, J., & Jiang, C. (2017). An Analysis of Problem-Posing Tasks in Chinese and US Elementary Mathematics Textbooks. *International Journal of Science and Mathematics Education, 15,* 1521–1540.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-Posing Research in Mathematics Education: Some Answered and Unanswered Questions. In F. M. Singer, N. F. Ellerton, & J. C. (Eds.), *Mathematical Problem Posing* (pp. 3–34). New York: Springer .

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education.* London: Routledge.

Deringöl, Y. (2020). Problem Posing Activities in Primary School Mathematics Textbooks. *Elementary Education Online, 19(3),* 1619-1646.

English, L. D. (2020). Teaching and learning through mathematical problem posing: Commentary. *International Journal of Educational Research, 102*, 101451.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem Solving as a Basis for Reform in Curriculum and Instruction: The Case of Mathematics. *Educational Researcher, 25(4),*12–21.

Kılıç, Ç. (2017). A New Problem-Posing Approach Based on Problem-Solving Strategy: Analyzing Pre-service Primary School Teachers' Performance. *Educational Sciences: Theory & Practice, 17(3),* 771–789.

Kilpatrick, J. (1987). Formulating the problem: Where do good problems come from?Lawrence Erlbaum Associates. In A. H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 123–147). Lawrence Erlbaum Associates.

Lester, F. K. (1994). Musings about Mathematical Problem Solving Research. *Journal for Research in Mathematics Education (25th anniversary special issue), 25(6),* 660-675.

McDonald, P. A., & Smith, J. M. (2020). Improving mathematical learning in Scotland's Curriculum for Excellence through problem posing: An integrative review. *The Curriculum Journal, 31(3),* 398–435.

National Counicil of Teachers of Mathematics (NCTM)(2000). *Principles and Standards for School Mathematics.* VA: Reston.

Pepin, B., & Jablonka, E. (2007). Comparative Studies in mathematics education – comparing the incomparable? *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*, (pp. 2399–2402).

Polya, G. (1945). *How to Solve It. A New Aspect of Mathematical Method.* Princeton University Press.

Remillard, J. T. (2005). Examining Key Concepts in Research on Teachers' Use of Mathematics Curricula. *Review of Educational Research, 75(2),* 211–246.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 334–370). Macmillan Publishing Co, Inc.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14(1),* 19–28.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. C. (Ed.), *Technology in mathematics education* (pp. 518-525). Melbourne, Australia: Mathematics Education Research Group of Australasia. .

Tesfamicael, S. A., & Lundeby, Ø. A. (2019). A Comparative Study of Norwegian and Ethiopian Textbooks: The Case of Relations and Functions Using Anthropological Theory of Didactics (ATD). *Universal Journal of Educational Research, 7(3)*,754–765.

Valverde, G. (2002). *According to the Book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* London: Kluwer Academic Publishers.

Van, W. J. A., Karp, K. S., & Bay-Williams J. M. (2018). *Elementary and middle school mathematics: Teaching developmentally*. Pearson Education limited.