On some methods of downward continuation of mean free-air gravity anomaly

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Abstract

This paper investigates four methods of downward continuation of free-air gravity anomalies to sea-level, an iterative process based on the Poisson's integral, a linear simple formula, a method based on the Pellinen approximation and the one based on the topographc-isostatic potential proposed by Sjöberg. Mean free-air gravity anomalies in $6' \times 10'$ cells from a test area with topographical heights between 71-396 metres have been successfully downward continued to sea-level. The effect of the downward continuation is everywhere positive on the geoid, while it is negative and positive on gravity anomalies. The results show some differences between these methods. The methods based on the Poisson's integral and topographic-isostatic potential have the smallest difference among the methods. A mean difference of 0.97 cm for the downward continuation effect on the geoid is computed between these two methods. In the Poissons' integral, long-wavelength contributions have been evaluated using a global gravity model and in the method based on topographc-isostatic potential, $30' \times 30'$ heights information over the world are used to consider the long-wavelength contributions. They are missing in the other two methods. The correctness of the solution has then been checked by back substitution of the gravity values at the geoid to estimate them at the surface of the Earth. The Poisson's integral uses the gravity anomalies, directly, for the downward continuation of them to the geoid, while the Sjöberg's method implies the height data. Also, because some approximation made in the latter, the iterative process of the Poisson's integral is preferred.

Key words: Downward continuation, Poisson's integral, topographic-isostatic potential, Pellinen approximation

1 Introduction

Geoid determination by Stokes well known formula requests that the gravity anomalies Δg represent boundary values at the geoid, which implies all topographic masses are removed or reduced and gravity anomalies Δg must refer to the geoid. For satisfying the second condition, as the observations (gravity values g) are available on the surface of the Earth, we have to reduce them from the Earth's surface to the geoid. This reduction is called "downward continuation". The main problem with the downward continuation is the masses between the surface level and the geoid and irregularity of the density distribution, which causes the disturbing potential is non-harmonic outside the geoid. However, the reduction problem would only fail, if a fully analytical approach be used. As terrestrail gravity anomalies are only available at discrete points or as mean grid values, therefore a unique solution could be expected. The free-air model has been used in the solving of the Molodenskii's problem (Heiskanen and Moritz, 1967; Moritz, 1980).

Bjerhammar (1962) and (1963) pointed out that it is always possible to downward continue point gravity anomalies (at least for a limited set of observations Δg) to an internal sphere of radius R (the Bjerhammar sphere) embedded in the Earth. In this study, we carry out the downward continuation of mean free-air anomalies. Hence, the smooting (due to avergaing) in the methods studied herein are present.

Heiskanen and Moritz (1967) also proposed an iterative process to downward continue the free-air gravity anomalies from surface level to the geoid. This process uses Poisson's integral, directly, which is more accurate than the linear and planar approximations (see ibid). Assuming that the gravity anomalies Δg are linearly correlated with elevation, Moritz (1980) proposed a method to downward continue gravity anomalies to sea-level based on the Pellinen approximation.

Sjöberg (1998) has developed another way to downward continue the gravity anomalies. His study is based on the external type of topographic-isostatic potential and gravity anomaly and its vertical derivatives, derived from the Airy/Heiskanen model. The effect is estimated on the geoid for the downward continuation of gravity to sea-level in the application of Stokes formula.

We have also to mention that recent investigation by Vaníček et al. (1996) and Sun and Vaníček (1996) and (1998) concentrated on the downward continuation of mean Helmert grvaity. They showed that the downward continuation of mean Helmert's anomaly is a well posed problem with a unique solution in $5' \times 5'$ cells and can be done routinely to any accuracy desired in the geoid computation.

2 Downward continuation of free-air anomaly by the Poisson's integral

The Stokes's integral requires that the disturbing potential T is harmonic on the geoid, which implies that there are no masses outside the geoid. Assume a fictious field of gravity anomalies Δg^* on the geoid, which generate on the ground level the measured free-air anomalies Δg. These two anomalies can be related by the Poisson's formula (excluding the spherical harmonics of degrees zero and one)(Kellogg, 1929; MacMillan, 1930):

$$\Delta g = \frac{t^2(1-t^2)}{4\pi} \iint_\sigma \frac{\Delta g^*}{D^3} d\sigma \,, \tag{1}$$

where

$$t = \frac{R}{r} \;;\; r = R + H_P.$$

In this equation spherical approximation has been used; R is the mean radius of the Earth, H_P is the orthometric height of the surface point P, σ is the unit sphere and

$$D = \sqrt{1 - 2t cos\psi + t^2} = \frac{\ell}{r},$$

where ℓ and ψ are the spatial and spherical distances between the surface point P and the running points under integral. The observed values Δg at level surface are obtained by measurements, and the free-air anomalies Δg^* at sea-level are desired. In this sense, Eq. (1) can be solved in different ways; for example by a linear approximation as:

$$\Delta g_P^* = \Delta g_P - \frac{\partial \Delta g}{\partial H_P} H_P \; . \tag{2}$$

After Bjerhammar (1962), Heiskanen and Moritz (1967) proposed an iterative process to solve the integral (1), which is more accurate than linear approximation in Eq. (2). An alternative expression for Eq. (1) is (Heiskanen and Moritz, 1967):

$$\Delta g_P^* = \Delta g_P - \frac{t^2(1-t^2)}{4\pi} \iint_\sigma \frac{\Delta g^* - \Delta g_P^*}{D^3} d\sigma \; , \tag{3}$$

which can be used as the recursive formula that lends to an iterative solution. As a first approximation one can set

$$\Delta g^{*^1} = \Delta g \; , \tag{4}$$

i.e. taking the known free-air gravity anomalies at the surface level as the first iteration for all points in the area of interest. This iterative process downward continues the free-air anoamalis from surface level (Δg) to the geoid (Δg^*). If we convolve downward continuation correction on gravity anomaly with Stokes's kernel, the effect on the geoid for the downward continuation of gravity to sea-level is obtained. The physical interpretation of this process is that the free-air gravity anomalies Δg^* at geoid, computed by the iterative method, generate on the level surface gravity anomalies Δg, which are identical with actual grvaity anomalies generated by observations.

The convergence of the iterative process has to be paied enough attention, because the solution of integral equations of first kind (like the Poisson's integral) may be unstable and cause some errors in the sought free-air anomaly at the geoid (Vaníček et al., 1996). The prescribed limit of the convergence of the iterative process is set up to 10 μGal to guarantee the magic level 1 cm for the accuracy of the geoid (Vaníček and Martinec, 1994). The iterative process uses gravity anomalies as a main input data to downward continue them to the geoid. The height data used in the Poisson's integral is in second degree of importance compared with the gravity anomalies.

Fortunately, the Poisson's integration kernel vanishes quickly with growing the distance from the computation point P. It means that it is enough to integrate Eq. (3) over a small area of σ_0 around the computation point, instead of the whole Earth (over σ). But, limiting the area of integration to σ_0 causes an error which is, here, called truncation error. We have tested different radius of integration and found out that a radius of integration $\psi_0 = 1°$ gives small truncation error (see also, Vaníček et al., 1996) . To obtain accurate results for the downward continuation correction, we have also modified the Poisson's kernel by the minimizing the upper limit of the truncation error (see e.g. Molodenskii et al., 1962; Sjöberg, 1984; Vaníček and Sjöberg, 1991). Discribing the Poisson's kernel by

$$K(H,\psi) = \frac{R(r^2-R^2)}{\ell^3} = \sum_{n=0}^{\infty}(2n+1)(\frac{R}{r})^{n+1}P_n(\cos\psi) , \tag{5}$$

the modified Poisson's kernel can be evaluated from

$$K^m(H,\psi,\psi_0) = K(H,\psi) - \sum_{n=0}^{L}\frac{2n+1}{2}s_n(H,\psi_0)P_n(\cos\psi) , \tag{6}$$

where s_n is the unknown coefficients to be computed from the following system of equations (Molodenskii et al., 1962)

$$\sum_{n=0}^{L}\frac{2n+1}{2}s_n(H,\psi_0)e_{in}(\psi_0) = Q_i(H,\psi_0) \qquad i=0,1,\cdots,L \tag{7}$$

where

$$e_{in}(\psi_0) = \int_{\psi_0}^{\pi} P_i(\cos\psi)P_n(\cos\psi)\sin\psi d\psi , \tag{8}$$

$$Q_n(H,\psi_0) = \int_{\psi_0}^{\pi} K(H,\psi)P_n(\cos\psi)\sin\psi d\psi . \tag{9}$$

We have selected L=20 in our computations. The contribution of the rest of the world ($Tg(P)$=truncation error) is quite small (will be shown in numerical investigations) and can be evaluated using a global gravity model as (see, e.g. Vaníček et al., 1996):

$$Tg(P) = \frac{R\gamma}{2r}\sum_{n=2}^{\infty}\sum_{m=-n}^{n}(n-1)\bar{Q}_n(H,\psi_0)T_{nm}Y_{nm}(P) \tag{10}$$

where

$$\bar{Q}_n(H,\psi_0) = \int_{\psi_0}^{\pi} K^m(H,\psi,\psi_0)P_n(\cos\psi)\sin\psi d\psi , \tag{11}$$

and the modified Poisson's kernel in a spectral form is

$$K^m(H,\psi,\psi_0) = \sum_{n=0}^{\infty}\frac{2n+1}{2}\bar{Q}_n(H,\psi_0)P_n(\cos\psi) . \tag{12}$$

γ is the normal gravity, T_{nm} are the potential coefficients taken from a global gravity model and Y_{nm} are the fully normalized spherical harmonics.

We have also subtracted the low degree harmonics Δg_L from the gravity anomalies Δg at the surface of the Earth (see, also Vaníček et al., 1996). Δg_L have been computed from EGM96 global model (Lemoine et al, 1997). We have downward continued this long-wavelength part, separately. Finally, the contributions from these long-wavelength part and corrections due to the truncation error are added to the short-wavelength downward continued part of free-air anomaly determined by the iterative process.

To have a simpler computing procedure, in opposite with the iterative process, Eq. (2) is used instead of the Poisson's integral. To downward continue the gravity anomalies by this simple expression, one need to know the vertical gradient of gravity anomalies. We have assumed that the free-air gravity anomaly changes linearly with elevation according to the Bouguer plate correction $2\pi G\rho h$, and this change is only dependent to the variations in computation point P. G is the gravitational constant and ρ is the density of the terrain. Hence, Eq. (2) can be written to

$$\Delta g_P^* = \Delta g_P - 2\pi G\rho H_P \ . \tag{13}$$

This formula will next be compared with the results of the iterative process.

3 Downward continuation of free-air anomaly by the Pellinen approximation

The vertical gradient of gravity anomaly in Eq. (2) is needed to be evaluated correctly for accurate computation of downward continuation of free-air gravity anomalies. It can be estimated from (Heiskanen and Moritz, 1967)

$$\frac{\partial \Delta g}{\partial r} = \frac{R^2}{2\pi}\iint_\sigma \frac{\Delta g - \Delta g_P}{\ell_0^3} d\sigma \ , \tag{14}$$

where $\ell_0 = 2R\sin\frac{\psi}{2}$. Assuming that there are linear correlation between gravity anomalies and heights, Moritz (1980) came to the following expression for the vertical gradient of gravity anomalies based on the Pellinen approximation:

$$\frac{\partial \Delta g}{\partial r} = 2\pi G\rho \frac{R^2}{2\pi}\iint_\sigma \frac{H - H_P}{\ell_0^3} d\sigma \ . \tag{15}$$

Inserting Eq. (15) into Eq. (2), one can estimate downward continuation effect of Δg to the geoid (Δg^*).

4 Downward continuation of gravity anomaly by a method based on topographic-isostatic compensation potential

Sjöberg (1998) has studied downward continuation of gravity anomalies on the geoid in a different way. His study would mainly apply the downward continuation of the gravity anomaly and its effect on the geoid, directly. Moritz (1980) assumed that the free-air anomaly changes linearly with elevation. This linear correlation derived based on the simplified Airy/Heiskanen compensation potential. In Sjöberg (1998), the effect on the geoid is considered for the downward continuation of free-air gravity to sea-level using original Airy/Heiskanen compensation potential. The gravity anomaly can analytically be continued down to sea-level, e.g by the Taylor expansion

$$\Delta g^* = \Big(1 - H_P\frac{\partial}{\partial r_P} - \frac{1}{2}H_P^2\frac{\partial^2}{\partial r_P^2}\cdots\Big)\Delta g \; . \tag{16}$$

Sjöberg (1998) used the external type of topographic-isostatic potential and gravity anomaly and its vertical derivatives, derived from the Airy/Heiskanen model for isostatic compensation. From the first and the second radial derivatives of the gravity anomaly, the effect on the geoid is estimated for the downward continuation of gravity to sea-level in the application of Stokes's formula. This effect is composed of two parts as

$$\begin{aligned} \delta N_{dwc} &= \frac{R}{4\pi\gamma}\iint_\sigma S(\psi)\delta g_{dwc}d\sigma \\ &= \delta N_{dwc}^1 + \delta N_{dwc}^2 \; , \end{aligned} \tag{17}$$

where $S(\psi)$ is Stokes function, γ is the normal gravity and δg_{dwc} is the downward continuation of free-air gravity anomaly to sea-level. The first part of the correction can be obtained from (Sjöberg, 1998):

$$\delta N_{dwc}^1 = \frac{1}{4\pi}\iint_\sigma S(\psi)(AH_Q^3 + BH_Q^4)d\sigma_Q \tag{18}$$

or in spectral form

$$\delta N_{dwc}^1 = \sum_{n=2}^{M}\frac{1}{n-1}\sum_{m=-n}^{n}[A(H^3)_{nm} + B(H^4)_{nm}]Y_{nm}(P) \; , \tag{19}$$

where

$$(H^\nu)_{nm} = \frac{1}{4\pi}\iint_\sigma H^\nu Y_{nm}d\sigma; \quad \nu = 3, 4 \; , \tag{20}$$

$A = 0.585 \quad m/km^3$

$B = -4.9 \times 10^{-3} \quad m/km^4$

and M is the maximum degree of expansion. We have used M=360 in our computations.

The first correction term has mostly short-wavelength nature and contributes locally. It is the dominent part of downward continuation. The second part of the correction, δN_{dwc}^2, expected to have a long-wavelength nature (see Sjöberg, 1998) and a spherical harmonic representation has been used for numerical investigations. It can be written (Sjöberg, 1998):

$$\delta N_{dwc}^2 = -\sum_{n,m}\frac{2}{n-1}C_{nm}Y_{nm} \; , \tag{21}$$

where

$$C_{nm} = \frac{1}{4\pi}\iint_\sigma H\Big(\frac{\Delta g}{\gamma} + 2\frac{\zeta}{r}\Big)Y_{nm}d\sigma \; . \tag{22}$$

$\frac{\Delta g}{\gamma} + 2\frac{\zeta}{r}$ may be computed from:

$$\frac{\Delta g}{\gamma} + 2\frac{\zeta}{r} = \sum_{n,m}(n+1)T_{nm}\left(\frac{R}{r}\right)^{n+2} Y_{nm} \,, \tag{23}$$

where T_{nm} are the potential coefficients and ζ is the height anomaly. Formula (21) can also be written:

$$\delta N^2_{dwc} = -\frac{1}{2\pi}\iint_\sigma S(\psi)H\left(\frac{\Delta g}{\gamma} + 2\frac{\zeta}{r}\right)d\sigma \,. \tag{24}$$

Then the total effect on geoid from downward continuation of gravity anomalies to sea-level, can be computed from the summation of the local contribution δN^1_{dwc} and the global contribution δN^2_{dwc}. In the formula (17), the direct gravity anomaly effect, caused by the reduction of the terrain, has also been applied to the gravity anomalies at the surface level. In iterative process this direct effect has not been applied. It has also to be mentioned that in the main part of the downward continuation (δN^1_{dwc}) in formula (17), the input data needed for the computation is the height data. It is in opposite with the iterative process by the Poisson's integral, which the gravity anomalies are the main input data.

Equation (18) is an integral formula with the Stokes's kernel. One is supposed to evaluate this integral over the whole Earth, which is impractical. Therefore, a modification procedure, the same as for the Poisson's kernel, has been performed and the modified Stokes's kernel used in further computations. The inner zone integration area for the short-wavelength contributions is set to $\psi_0 = 6°$. The long-wavelength contributions of the rest of the world are computed from a global height data set using Eq. (19).

5 Numerical investigations

In this section, we have numerically investigated the downward continuation of free-air gravity anomaly by the Poisson's integral based on the iterative procedure (Eqs. 3-4), the simple formula (Eq. 13), the method based on the Pellinen approximation (see Eq. 15) and the one developed by Sjöberg based on the topographic-isostatic compensation potential (see Eqs. 17-24). In the last approach $\frac{\Delta g}{\gamma} + 2\frac{\zeta}{r}$ has been computed according to Eq. (23) in $30' \times 30'$ cells all over the whole Earth, which has then been used in Eq. (24). A test area of size $1° \times 1°$ is chosen in Sweden. It is limited by latitudes 57° N and 58° N and longitudes 13° E and 14° E, located in the north-west of Sweden. The mean free-air anomalies at the surface of the Earth are in $6' \times 10'$ cells. They vary between -32.17 mGal and 40.89 mGal. The mean heights, used in this test area, are in $2.5' \times 2.5'$ cells (GETECH, 1995a) and range from 70.92 to 395.60 metres. To reduce the effect of leakage of the data coverage for the integration cap (along the edge of the test area), we have increased the integration area 6° in each direction, so that the area for which the downward continuation would actually be computed is $13° \times 13°$. But, to escape from the edge effect, the original $1° \times 1°$ test area is used at the end. The potential coefficients used in this study are taken from EGM96 model .

First, the downward continuation correction by the Poisson's integral is determined. To do this, we define the modified Poisson's kernel and then compute the truncation error in the test area. Figure 1 represents the correction due to the truncation error on geoid. This effect reaches to 3.64 mm. It should be mentioned that the effect of the truncation error on gravity anomalies ranges from -0.18 mGal to 0.26 mGal. To minimize this contribution from the rest of the world, low degree and order field is subtracted from the observed free-air anomalies and downward continued, separately. This long-wavelength contributions have also been added to the contributions from the iterative procedure (short-wavelength part). Downward continuation correction by the simple formula and the Pellinen approximation as well as by the method based on the topographic-isostatic potential are also determined. In the last method, second part of

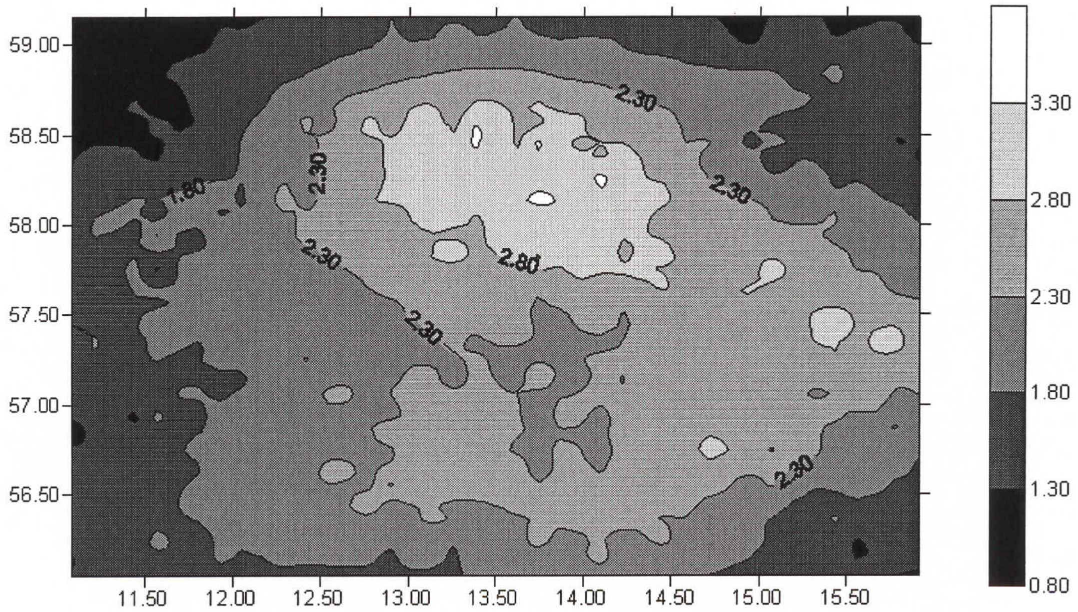

Figure 1: The correction due to the runcation error on geoid in the modified Poisson's integral. Contour interval is 0.5 mm.

the correction (δN^2_{dwc}) has a global nature and is evaluated from Eqs. (21). The main part of the correction (δN^1_{dwc}), described by Eq. (18), has to be integrated over the whole Earth, which is impractical. To investigate this, we have performed this effect in two integration areas 6° and 15°, and compared the results. The statistics of differences are presented in Table 1, which indicate the effect of the farzone contributions. This was obvious because of the shape of the Stokes's kernel. Hence, we decided to modify Stokes's kernel based on the minimization of the upper limit of the truncation error. The integration area is selected to be 6° around the computation point. The heights data used in this inner zone area are GETECH $2.5' \times 2.5'$ DTM (GETECH, 1995a). The long-wavelength part can be evaluated from a spectral form (Eq. 19). The harmonic coefficients of heights $(H^3)_{nm}$ and $(H^4)_{nm}$ are determined from Eq. (20). For this, a $30' \times 30'$ Digital Terrain Model (DTM) was generated using the GETECH $5' \times 5'$ DTM (GETECH, 1995b). This $30' \times 30'$ DTM is averaged using area weighting. Since the interest is in continental elevation coefficients, the heights below sea level are all set to zero. The coefficients were computed to degree and order 360. Finally, the low frequency part (δN^2_{dwc}) is added to the results from the innerzone area.

Table 1: The statistics of the differences of downward continuation correction on geoid by the method based on the topographic-isostatic potential in 6° and 15° integration area. Units in cm

	$\psi_0 = 6°$-$\psi_0 = 15°$
Min	0.21
Max	5.02
Mean	2.60
SD	0.95

The statistics of the results of downward continuation of mean free-air anomaly by different methods are shown in Table 2. It shows that the effect on geoid from downward continuation gravity anomaly to sea-level is everywhere positive. In the next step, the statistics of differences

Table 2: The statistics of downward continuation correction on geoid computed by different methods. Units in cm

	Poisson's integral	topographic-isostatic	Pellinen approximation	Simple method
Min	14.27	16.78	23.72	22.09
Max	19.87	18.50	17.65	24.15
Mean	17.06	17.63	20.72	23.30
SD	1.45	0.53	1.59	0.54

the iterative process from other methods for each point of interest are shown in Table 3. The results of Table 3 show that the downward continuation of mean free-air anomaly by the Poisson's integral and the method based on the topographic-isostatic potential have the smallest differences among the other methods. Maximum of differences has been computed to 3.51 cm. This

Table 3: The statistics of differences of downward continuation correction on geoid between the Poisson's integral and the other methods. Units in cm

	Poisson - topographic/isostatic potential	Poisson-Pellinen	Poisson -simple method
Min	-2.77	3.22	4.27
Max	3.51	4.28	8.01
Mean	0.97	3.66	6.24
SD	1.03	0.22	1.13

difference may be caused by the differences in the definition of these two methods. In the former method the downward continuation is only applied to the surface gravity anomalies, but in the latter, the direct gravity anomaly effect, caused by the reduction of the terrain masses, has also been included in the solution. The topographic-isostatic model includes several approximations/assumptions which might be the second reason for the differences (see, Sjöberg, 1998). Also, the effect of the truncation error is considered in the iterative process, while it is not applied in the method based on the topographic-isostatic potential. Results of Table 3 also show that there are some differences between the iterative process, the simple formula and the method based on the Pellinen approximation. A maximum difference of 8.01 cm has been computed in the test area between the first two methods. This difference can be caused by the linear approximation made in the latter method. Also, in the simple formula, we have assumed that the vertical gradient of gravity is only dependent to the variation of gravity anomaly in the computation point, and the effect of the other points are neglected. They are considered in the former method.

The methods based on the Poisson's integral and the Pellinen approximation are mostly in good agreement with each other. A mean of differences 3.66 cm has been estimated in the test area. There are some reasons for these differences. Firstly, a linear approximation is used to evaluate the latter method. Also, in the method based on the Pellinen approximation, some long-wavelength contributions might be missing due to the limitation of integration area. The other reason is that free-air gravity anomalies may not simply change linearly with elevation according to the Bouguer plate using Pellinen approximation. Such a linear correlation with elevation was derived by Moritz (1968) based on a topographic-isostatic potential using a simplified Airy/Heiskanen compensation, with a density layer at an internal sphere. Sjöberg (1998) uses original Airy/Heiskanen compensation potential for the downward continuation of gravity anomaly to sea-level. This explains the differences between the methods based on the Pellinen approximation and the topographic-isostatic potential (see Table 2). It has to be mentioned that there are not any convergence problem in the iterative process in this study. By 5 iterations the results converge to an accuracy better than 0.01 mGal, which it makes sure that the downward continuation is accurate enough. However, these procedures have to be tested in other test areas with more rugged topography.

The question, if the cell size (in this study $6' \times 10'$) of the gravity anomalies is enough dense to guarantee the correctness of the iterative process, still remains. Subsequently, this question arises that, if we have treated the Poisson's integral correctly, by replacing it simply with the summations. Otherwise, it has to be treated more carefully (see e.g. Vaníček et al., 1996). To study this, the iterative process is recomputed with the gravity anomalies in $10' \times 12'$ cells in the same test area. The statistics of the differences between the results computed with $6' \times 10'$ and $10' \times 12'$ cells are shown in Table 4. Mean of differences are estimated to 0.60 cm with a maximum difference of 2.02 cm. It means that the $6' \times 10'$ free-air anomalies is enough dense in this test area. Therefore, Poisson's integral can safely be discretized by summations. The effect of the cell size in more rugged area has still to be tested. It has to be mentioned that the effect of downward continuation on gravity anomalies shows a very short-wavelength nature. Figure 2 shows this nature in the test area. However, convolving this effect with Stokes's integral and estimating the effect on geoid (which is our final goal), smooth the results. Contribution to the geoid from the downward continuation of mean free-air gravity anomalies by iterative procedure is shown in Figure 3.

Table 4: The statistics of differences between the effect of downward continuation on geoid by Poisson's integral in $6' \times 10'$ and $10' \times 12'$ cells. Units are in cm.

	$6' \times 10'$ -$10' \times 12'$
Min	-1.21
Max	2.02
Mean	0.60
SD	0.85

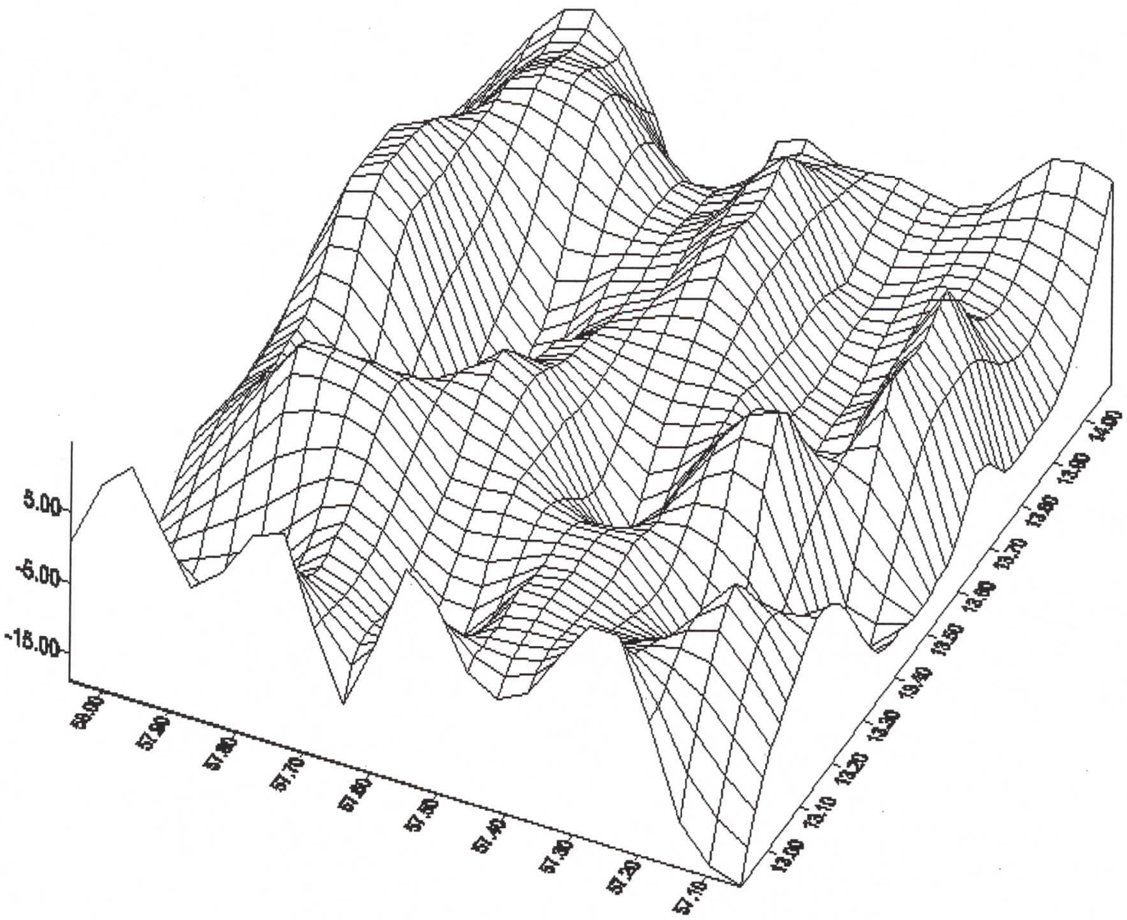

Figure 2: Downward continuation of free-air gravity anomalies by Poisson's integral in mGal.

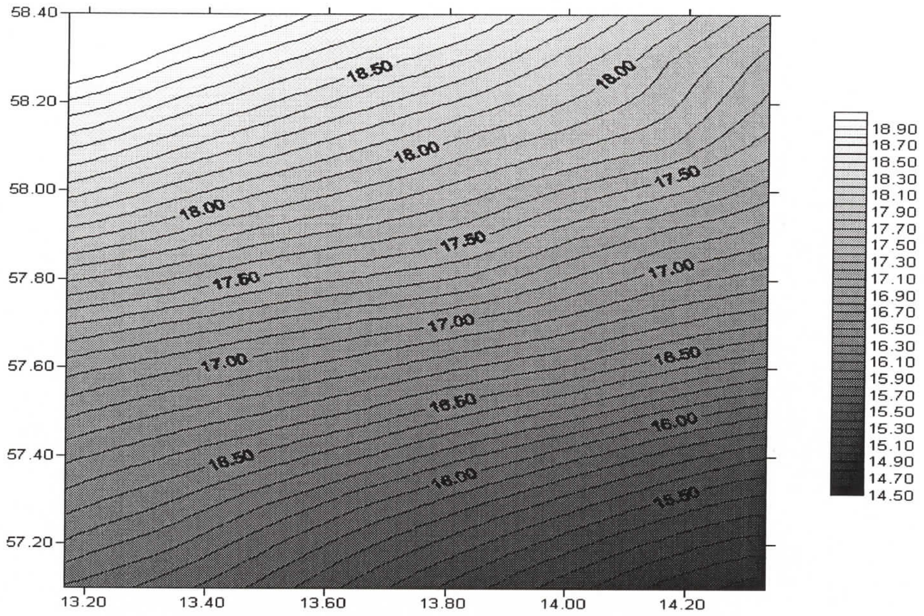

Figure 3: The effect on geoid from the downward continuation of free-air gravity anomalies. Contour interval is 0.1 cm.

The edge effect, which is due to the incompleteness of data used in the boundary of the area of interest, is the other problem which is here studied. We have performed the iterative process in a smaller area being immerged in the $13^\circ \times 13^\circ$ computing area. The results of downward continuation for these two areas are next subtracted. The differences are depicted in Figure 4. This figure presents clearly the boundary effect in the $6^\circ \times 5^\circ$ area. Figure 4 also shows that the area of integration could be safely chosen around 1° in each direction (see also Sun and Vaníček, 1998).

In another experiment to check the correctness of the solution by Poisson's integral, we have back substituted the downward continued values at sea-level to compute the values at the surface of the Earth. The results have been compared with the original values at the surface of the Earth. Statistics of the differences are shown in Table 5. The results of the iterative process agree with the original values at the surface of the Earth with an accurcay of better than 0.07 μGal.

6 Conclusions

We have numerically investigated four methods for the downward continuation of free-air anomalies to the sea-level. The first method is an iterative process based on the Poisson's integral. A simple formula based on linear approximation and a method based on the Pellinen approximation is the other two methods. The last one is based on the topographc-isostatic potential derived from Airy/Heiskanen model.

Numerical investigations have been done in a test area of size $1^\circ \times 1^\circ$. To reduce the effect of leakage of the data coverage in the edges of the test area, the integration area is increased

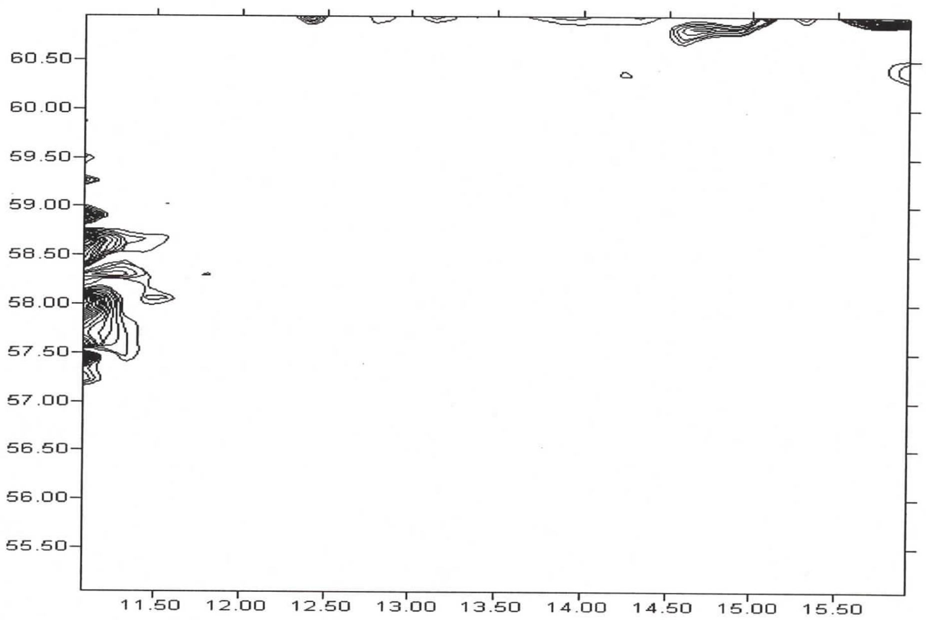

Figure 4: The boundary effect in $6^\circ \times 5^\circ$ ared due to the incompletness of the data. Contour interval is 0.1 mGal.

Table 5: The statistics of differences between back substituted values by the Poisson's integral with the original values at the surface of the Earth. Units are in μGals.

	back substituted values-original values
Min	0.002
Max	0.07
Mean	0.04
SD	0.002

with 6° in each direction. Finally, to escape from the edge effects, we have only used the original 1° × 1° test area. Choosing 1° computation area around the point of interest guarantees that the edge effect does not cause any problem.

Using the Poisson's kernel in the iterative process may cause some problems due to this fact that it has to be integrated over the whole Earth, which is impractical. We have overcome this problem by using two integration areas, inner and outer zones. A 1° integration cap is appropriate for the inner zone area. The effect of the outer zone area (truncation error) has been found to be within 4 mm. This effect is evaluated from EGM96 gravity model. The correction due to the truncation error is finally added to the short-wavelength contributions coming from the iterative process. To minimize the effect of the outer zone, we have subtracted the low degree and order field (to degree and order L=20) from the observed mean free-air anomalies at surface of the Earth. This contribution is downward continued, separately, and added to the downward continuation of the short-wavelength contributions, carried out by the iterative process. It is demonstrated that mean free-air anomalies in $6' \times 10'$ cells (the cell size used in Sweden) can be successfully downward continued from the Earth's surface to the geoid. However, in the smaller cell sizes this might not have a unique solution. The smoothing induced by averaging also helps to the solvability of the downward continuation. This is the case in practice when the mean gravity anomalies are used instead of point gravity anomalies.

In the method based on the topographic-isostatic potential, the main part of the correction (δN^1_{dwc}) has to be evaluated over the whole Earth, by convolution with the Stokes's kernel. But, because of the behaviour of Stokes's kernel, we split the effect into two parts; low frequency part computed from the convolution with modified Stokes's kernel, and high frequency part computed from a spectral form. We have modified the Stokes's kernel according to the Molodenskii et al. (1962) procedure. The contribution of the rest of the world has been evaluated from a $30' \times 30'$ height data set in a spectral form. The results of the Poisson's integral and this method have the smallest differences among the methods computed. The method based on the topographic-isostatic potential considers the direct effect in downward continuation of gravity anomalies. However, this effect is not applied in the other methods. The other source of the the differences is several approximations made in the method based on the topographic-isostatic potential.

The short- and long-wavelength contributions are considered in the iterative process based on the Poisson's integral and the method based on the topographic-isostatic potential. This might be the reason for the better agreement of these two methods among the other methods. We have also shown that the effect of downward continuation on gravity anomaly has short-wavelength nature. However, this effect on geoid behaves smoother. The results also show that the density of $6' \times 10'$ for the mean gravity anomalies are enough in this study.

Disregarding the simple method which is not accurate enough, the topographic-isostatic model has the smallest computing labour, while it is the largest in the Poisson's integral.

Finally, for the check of the correctness of the iterative solution, we have back substituted the downward continued values at the sea-level, to compute them at the surface of the Earth. The results show that the accuracy of the solution is within 0.07 μGal.

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