A new strategy for the atmospheric gravity effect in gravimetric geoid determination

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Abstract. Prior to Stokes integration, the gravitational effect of atmospheric masses must be removed from the gravity anomaly Dg. One theory for the atmospheric gravity effect on the geoid is the well-known International Association of Geodesy approach in connection with Stokes' integral formula. Another strategy is the use of a spherical harmonic representation of the topography, i.e. the use of a global topography computed from a set of spherical harmonics. The latter strategy is improved to account for local information. A new formula is derived by combining the local contribution of the atmospheric effect computed from a detailed digital terrain model and the global contribution computed from a spherical harmonic model of the topography. The new formula is tested over Iran and the results are compared with corresponding results from the old formula which only uses the global information. The results show significant differences. The differences between the two formulas reach 17 cm in a test area in Iran.

Key words: Atmospheric effect – Geoid - Stokes formula – Spherical harmonics

1 Introduction

The geoid is frequently determined from gravity data by the well-known Stokes' formula. This formula is the solution of an exterior-type boundary value problem, implying that masses exterior to the geoid, i.e topography and atmospheric masses, are not permitted in the solution. Consequently, the effect of the atmospheric masses must also be removed or reduced prior to Stokes integration, which corresponds to the so-called direct atmospheric effect. Gravity anomalies corrected for this effect (also other effects, e.g. the direct topographical effect) are allowed to be used in the Stokes integral. Thereafter, Stokes' formula computes a so-called co-geoid height with respect to the reference ellipsoid, which is later corrected using the so-called indirect effect (restoration of the topography and the atmosphere) to the geoidal height.

Ecker and Mittermayer (1969) derived a formula for the direct atmospheric effect on gravity, which was later named the International Association of Geodesy approach (IAG). They investigated atmospheric effects on observed ground gravity. They formulated the atmospheric potential of the ellipsoidal shell that was split into a harmonic and a non-harmonic part. The harmonic part was added to the normal field of the solid reference ellipsoid (normal field), while the non-harmonic part of the atmospheric potential was responsible for the atmospheric gravity correction derived as its negative vertical derivative. Ecker and Mittermayer evaluated a table of positive atmospheric corrections to gravity as attractions of a series of homogeneous ellipsoidal shells (all of them being outside the Brillouin sphere) using the Newtonian theory with a point mass of the atmosphere being computed by a developed recursion formula. An exponential function modulated by a higher degree polynomial was used for the evaluation of the atmospheric density. The coefficients in this equation were estimated by fitting this function to the standard atmospheric models (CIRA, US Standard). The obtained corrections vary from 0:869 (mean sea level) to 0.000 mGal (46 km). The IAG approach is described in Moritz (1980). Anderson et al. (1975) computed the global values of the atmospheric corrections to gravity and the geoid. They used a surface spherical harmonic representation of the Earth's topography. The atmospheric density was modeled by the piecewise linear function with the upper limit at a height of 40 km. They assessed the magnitudes to be about 0:87 mGal for the first-order atmospheric correction and about 0.1 mGal for the second-order correction. Sjöberg (1993) emphasized that there could be additional significant direct and indirect atmospheric effects stemming from a more detailed treatment of the Earth’s topography than is made in the IAG approach. The IAG approach assumes that the Earth is approximately spherical (with radius R) with a spherical layering of the atmosphere. Consequently, the approximation from a more likely ellipsoidal layering to a spherically layered atmosphere is assumed negligible and all space above a geocentric sphere through the computation point is assumed to consist of atmosphere. This means that topography is more or less ignored. It should be mentioned that the effect of the ellipticity of the atmosphere is not investigated in this article. Sjöberg (1999) and Sjöberg and Nahavandchi (2000) investigated the direct and indirect atmospheric gravity and geoid effects in Stokes’ original and modified formulas. They derived formulas for the direct atmospheric gravity and geoid effects using a spherical harmonic representation of the topography. It was shown that the atmospheric effect is the first-order effect of elevation and its direct effect on the geoid reached 40 cm (Sjöberg and Nahavandchi 2000). As mentioned above, Anderson et al. (1975) also studied the direct atmospheric effect. They noted that the real atmosphere is neither laterally homogeneous nor regular in shape. In particular, its lower boundary is very irregular and, in- deed, takes on the shape of the Earth’s topography. Consequently, the atmospheric gravity effect is not the same even for points of equal altitude. Anderson et al. came up with the second-order atmospheric effect on gravity and the geoid. The magnitude of the corresponding correction to gravity reached a few hundreds of microGal for gravity and more than 40 cm for the geoid.

In this paper, the previous formula of Sjöberg and Nahavandchi (2000) for the direct atmospheric effect is improved by accounting for a better treatment of the Earth’s topography. The Sjöberg and Nahavandchi (2000) formula includes only global information (due to the spherical harmonic representation of topography), omitting significant short-wavelength contributions. A new formula for the direct atmospheric gravity and geoid effects is derived by combination of the local contribution and the set of spherical harmonic coefficients of the topography.

2 Direct atmospheric effect computed only from a spherical harmonic representation of topography

Following the approach presented in Sjöberg and Nahavandchi (2000), the atmospheric potential at an arbitrary point P on the topography can be written

$$V^a(P) = \iint_\sigma \int_{r_s}^{\infty} \frac{\rho_a r^2 dr}{\ell_P} d\sigma \tag{1}$$

where ρ_a is the density of the air scaled by the gravitational constant G; $\ell_P = \sqrt{r_P^2 + r^2 - 2r_P r \cos\psi}$; r_P, r and r_s are the geocentric radii of P, the running point in the integration and the Earth's surface ($r_s = R + H$), respectively; ψ is the geocentric angle between r, and r_P; and σ denotes the surface of the unit sphere. It should be noted that if the density is considered constant, it can be put outside the integral in Eq. (1). The atmospheric potential to a first-order approximation with respect to the topographic height H reads finally (Sjöberg and Nahavandchi 2000):

$$V^a(P) \doteq 4\pi\rho_0 R^2 \left\{\frac{1}{\upsilon-2} - \sum_{n=0}^{\infty} \frac{1}{2n+1}\left(\frac{r_P}{r}\right)^n \frac{H_n(P)}{R}\right\} \tag{2}$$

where ρ_0 is the density of the atmosphere at sea level (ρ^0) multiplied by the universal gravitational constant (G), $\upsilon > 2$ is a constant: R is the mean Earth radius, and

$$H_n(P) = \sum_{m=-n}^{n} H_{nm} Y_{nm}(P) \tag{3}$$

are usually called Laplace harmonics of the height function H, where

$$H_{nm} = \frac{1}{4\pi} \iint_{\sigma} H\, Y_{nm} d\sigma \tag{4}$$

are spherical harmonic analysis of the height function *H* and

$$H_P = \sum_{n,m} H_{nm} Y_{nm}(P) \tag{5}$$

are spherical harmonic synthesis of the height function *H*. Here the harmonics Y_{nm}, usually called surface spherical harmonics, obey

$$\frac{1}{4\pi} \iint_{\sigma} Y_{nm} Y_{n'm'} d\sigma = \begin{cases} 1 & \text{if n=n' and m=m'} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Thereafter, the atmospheric gravity and gravity anomaly are derived as (Sjöberg and Nahavandchi 2000):

$$g^a(P) = -\frac{\partial}{\partial r_P} V^a(P) \doteq 2\pi\rho_0 H(P) \tag{7}$$

The result of Eq. (7) is gravitation or gravitational attraction of atmospheric masses at the point *P*. Equation (7) represents gravitational attraction of an infinite layer of thickness *H*(*P*) and constant density considered at a point that is located at its upper boundary. Thus it represents an analogy to the Bouguer plate effect.

Also

$$\Delta g^a(P) = -\left(\frac{\partial}{\partial r_P} + \frac{2}{r_P}\right) V^a(P) \doteq 4\pi\rho_0 \sum_{n=0}^{\infty} \frac{n+2}{2n+1} H_n(P) \tag{8}$$

The right-hand side of Eq. (8) represents both the so-called direct atmospheric effect on gravity and the (secondary) indirect atmospheric effect on gravity (Heiskanen and Moritz 1967, p. 142). The latter is the second term on the right-hand side of the equation. It accounts, strictly speaking, for a change (shift) of the actual equipotential surface passing through the point *P* as a consequence of the direct atmospheric effect on potential (all equipotential surfaces including the geoid are shifted due to the change of the geopotential). The first term on the right-hand side of the Eq. (8) is the direct gravity effect. The second term is the effect on gravity due to the change of level surface in Stokes's formula from the geoid to the atmospheric to-geoid. The second term can also be regarded as the so called second secondary indirect effect on gravity. The series expansions in Eqs. (7) and (8) are limited to the first-order terms in the topographic height *H*(*P*). The direct gravity anomaly correction, i.e. the correction on the gravity anomaly by removing the atmosphere, is obviously $-\Delta g^a$. The direct geoid correction is the correction to the geoid for removing the atmosphere. It should be noted that the effect of downward continuation of the gravity anomaly to the geoid due to the atmospheric masses is not studied here. However, this effect has been computed in Nahavandchi (accepted). The direct geoid effect becomes

$$N^a_{\mathrm{dir}}(P) = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi)\, \Delta g^a d\sigma \tag{9}$$

where $S(\psi)$ is the Stokes function and γ is the normal gravity at sea level. Finally, expanding the Stokes integral into the spectral form, the atmospheric geoid effect was derived as (neglecting zero and first-degree effects) (Sjöberg and Nahavandchi 2000)

$$N^a_{\mathrm{dir}}(P) \doteq \frac{R}{\gamma} \sum_{n=2}^{\infty} \frac{\Delta g^a_n(P)}{n-1} \tag{10}$$

or

$$N_{\mathrm{dir}}^{a}(P) \doteq \frac{4\pi\rho_0 R}{\gamma}\sum_{n=2}^{\infty}\frac{n+2}{(2n+1)(n-1)}H_n(P) \tag{11}$$

Sjöberg and Nahavandchi (2000) computed Eq. (11) using the height coefficients up to degree and order 360. The direct effect reached values of up to 40 cm over Antarctica and the Himalayas. These results agree with the so-called second-order effect determined by Anderson et al. (1975).

3 A new formula for direct atmospheric effect computations

A more detailed treatment of the Earth's topography may lead to significant improvement in computing the direct atmospheric effects. The IAG approach assumes a spherical Earth and spherically layered atmosphere. However, the formulas for the direct atmospheric effect derived by Sjöberg and Nahavandchi (2000) represented the topography using spherical harmonic coefficients. It is obvious that spherical harmonics provide global information and that short-wavelength contributions are missing. To solve this problem, a new formula for the direct atmospheric effects is derived. To achieve this, we first rewrite Eq. (8) as follows:

$$\Delta g^{a}(P) \doteq 4\pi\rho_0\sum_{n=0}^{\infty}\frac{1}{2}\left(\frac{3}{2n+1}+1\right)H_n(P) \tag{12}$$

Inserting

$$H_n(P) = \frac{2n+1}{4\pi}\iint_{\sigma} HP_n(\cos\psi)\,d\sigma \tag{13}$$

and considering that

$$\sum_{n=0}^{\infty}P_n(\cos\psi) = \frac{R}{\ell_0} \tag{14}$$

where ℓ_0 is the distance function for two points at the surface of the reference sphere of radius R, we arrive at

$$\Delta g^{a}(P) \doteq 2\pi\rho_0\left[H_P + \frac{3R}{4\pi}\iint_{\sigma}\frac{H}{\ell_0}\,d\sigma\right] \tag{15}$$

where the notation below is used

$$H_P = \sum_{n=0}^{\infty}H_n(P) \tag{16}$$

We finally obtain

$$\Delta g^{a}(P) \doteq 2\pi\rho_0 H_P + 6\pi\rho_0\bar{H}_P \tag{17}$$

where

$$\bar{H}_P = \sum_{n=0}^{\infty}\frac{1}{2n+1}H_n(P) \tag{18}$$

Equation (17) can also be derived from Eq. (8) directly by considering the expansion in Eq. (16) only. The first term on the right-hand side of Eq. (17) includes height of computation points and it can be computed using a local detailed digital terrain model (DTM). This part of the formula is here considered as the local contribution. The second term on the right- hand side of Eq. (17) is obviously a representation of topography including a smoothing factor and it can be computed using the set of spherical harmonics. Further, Eq. (10) or (11) can be used to compute the direct atmospheric geoid effect. These formulas obviously treat the direct atmospheric effect in a more precise way as the heights of computation points are also included in the formulas. It should be mentioned that the effect of atmosphere on the downward continuation of surface data to the geoid is not considered here. However, this effect has been computed in Nahavandchi (accepted). The correction term of Eq. (17) for removing the effect of atmospheric masses from the gravity anomaly Δg must be used prior to Stokes integration (other correction terms to the gravity anomaly, the most important ones being the topographical corrections, must also be accounted for).

4 Numerical investigations

In order to investigate the atmospheric effects, the Sjöberg and Nahavandchi (2000) formula for the direct atmospheric gravity effect [Eq. (8)] and the new formula for the atmospheric effect [Eq. (17)] are evaluated over Iran. The topography over Iran, depicted in Fig. 1, varies from 0 to 5671 m. The height coefficients H_{nm} are determined from Eq. (4). For this, a $30' \times 30'$ DTM is generated using the Geophysical Exploration Technol- ogy (GETECH) $5' \times 5'$ DTM (GETECH, 1995). This $30' \times 30'$ DTM is averaged using area weighting. Since the interest is in continental elevation coefficients and we are trying to evaluate the effect of the masses above the geoid, negative heights over seas are all set to zero. Parameter definitions are as follows: $\rho_0 = G\rho^0$, where $G = 6.673 \times 10^{-11}$ m³ kg⁻¹ s⁻² and $\rho^0 = 1.23$ kg/m³, R=6371 km, and γ = 981 Gal. The coefficients H_{nm} are computed to degree and order 360 so that the corresponding cell size is $30' \times 30'$. A $1\ km \times 1\ km$ DTM over Iran is used to compute the height of computation points in the new formula. The computation points are the same as the location of the $1\ km \times 1\ km$ DTM over Iran. The local contributions in the new formula are the heights of the computation points (H_P), obtained from the local DTM. For the global contributions, the new formula uses the spherical harmonic representation of $\overline{H}_P$ at a computation point.

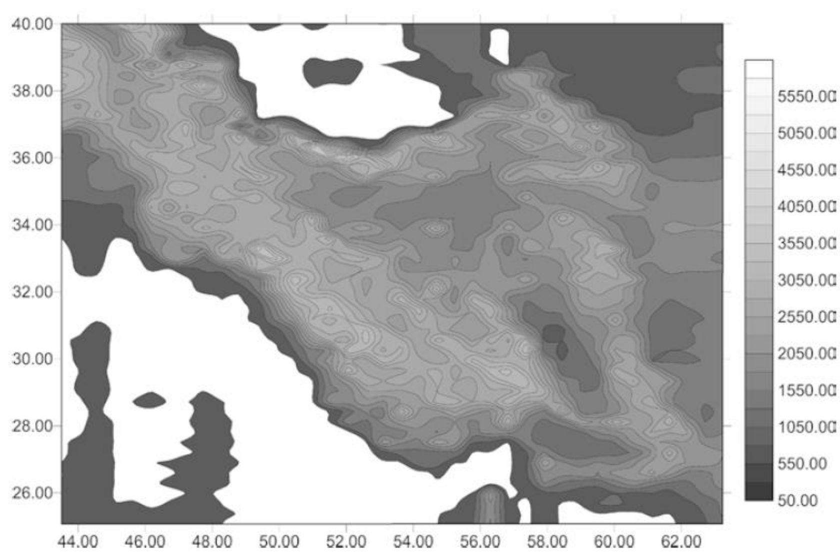

Fig. 1. Presentation of topography over Iran. Units: m

Figure 2 shows the direct atmospheric effect on gravity computed by the Sjöberg and Nahavandchi (2000) formula over Iran. It reaches 0.195 mGal over the Damavand mountainous area with a maximum elevation of 5671m. This effect is also computed with the new formula and is depicted in Fig. 3; it reaches

0.347 mGal. The local contributions are the reason for the differences. The difference between these two approaches is shown in Fig. 4; it reaches 0.176 mGal. The difference between the two methods is large, reaching the magnitude of the direct effect computed by the old formula [Eq. (8)]. This magnitude of difference was expected, as the old formula only used a spherical harmonic representation of the topography, whereas the new formula uses both the spherical harmonics and a dense DTM in this study. It is obvious that a denser DTM than the one used in this study ($1\ km \times 1\ km$) will reveal better local irregularities.

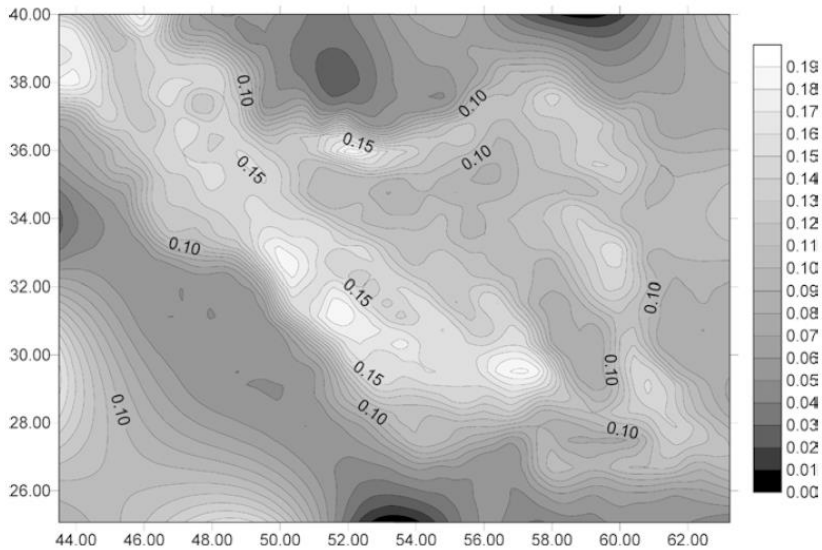

Fig. 2. The direct atmospheric gravity effect with the old formula [Eq. (8)]. Units: mGal

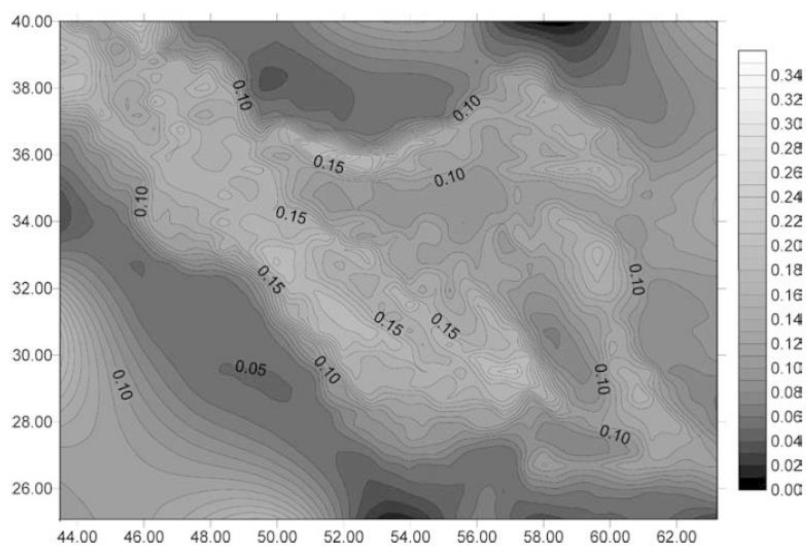

Fig. 3. The direct atmospheric gravity effect with the new formula [Eq. (17)]. Units: mGal

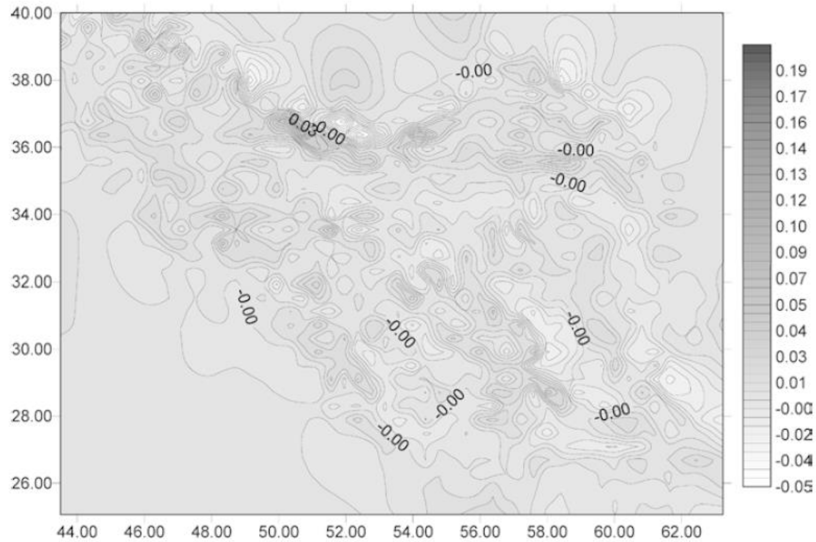

Fig. 4. The differences on direct atmospheric gravity effect between the old [Eq. (8)] and new [Eq. (17)] formulas. Units: mGal

In order to gain further insight into the differences between the old and new formulas, the direct atmospheric effect on the geoid is also computed [see Eq. (11)] using both methods. A test area in Iran limited by latitudes 54 and 56° N and longitudes 33 and 35° E is chosen. The elevation in this area varies from 600 to 2290 m. The direct atmospheric geoid effects are computed by the new and old formulas in this test area. The direct atmospheric effect on the geoid reaches 16 cm using the old formula [Eq. (8)], while it reaches 33 cm using the new formula [Eq. (17)]. Preliminary results show that the use of the new direct atmospheric correction in gravimetric geoidal height computations yields a better agreement with the global positioning system (GPS)–leveling-derived geoidal heights, which are used to demonstrate such improvements, than the results of gravimetric geoidal heights for the same GPS stations but using the old formula.

5 Conclusions

The classical IAG formula assumes that the Earth is approximately spherical (with radius R) with a spherical layering of the atmosphere, and the topography is also more or less ignored. The Sjöberg and Nahavandchi (2000) formula treats the Earth as a sphere of radius R with a variable topography of height H on top of the sphere, such that $r_s = R + H$. Sjöberg and Nahavandchi finally derive a formula for the direct atmospheric effect using a spherical harmonic representation of the topography. It is obvious that only the global information is considered in their formula (considering the resolution of the global heights used in that study). The present study improves on the Sjöberg and Nahavandchi (2000) formula. The direct atmospheric effect in gravimetric geoid determination is reformulated and a new formula is derived which uses the global topography and the local DTM for the computations. This implies that the previous formulas may have some numerical problems in representing all significant contributions. The local contributions in the present study are the heights of computations points derived from a dense DTM. The results of a comparison between the old and the new formulas for the direct atmospheric effect show some significant differences. These reach to 0.176 mGal for gravity, which is quite significant. On the geoid, the differences reach 17 cm. The reason for these differences is the better treatment of the Earth's topography using the new formula for the direct atmospheric effect. The use of the local DTM in the new formula significantly improves the atmospheric effects in geoid computations.

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