TWG1: Transitions to, across and from University Mathematics

Thomas Hausberger[1] and Heidi Strømskag[2]

[1]IMAG, University of Montpellier, France, thomas.hausberger@umontpellier.fr; [2]Norwegian University of Science and Technology, Trondheim, heidi.stromskag@ntnu.no

INTRODUCTION

A specific thematic working group dedicated to the topic of transitions first appeared at the INDRUM conference in 2018. To attend to the growing number of papers, the programme committee introduced this theme—in line with increasing research on transitions in mathematics education research (Gueudet et al., 2016)—as a new TWG transversal to mathematical domains, alongside students' and teachers' practices. Although it disappeared in 2020, it continued to thrive through a dedicated chapter in the ERME volume on INDRUM research from the first two conferences (Hochmuth et al., 2021). The INDRUM2020 keyword "transition to and across university mathematics" was then modified in the INDRUM2022 call for papers to encompass a larger spectrum of transitions and TWG1 was named accordingly.

In fact, the school to university transition (Klein's first discontinuity) is still dominant: it is the focus of 5/8 papers and 3/4 posters which were assigned to our group. Dually, 2 papers and 1 poster deal with the transition from university to secondary education (Klein's second discontinuity). A single paper considers the "across university" transition (with a focus on the teaching of a mathematical concept throughout the Bachelor in the Abstract Algebra track). To complete the perspective, papers assigned to other TWGs but mentioning transitions as a keyword shall also be counted; hence there are 2 additional papers on the school-university transition in TWG3 (focusing on proof), 2 in TWG6 (on students' learning), as well as 3 papers on Klein's second discontinuity attached to the new TWG5 on teacher education. It is worth noting that papers which investigate the case of engineering students do not use the lens of transitions, so that the transition from university to the workplace remains under-researched except in the context of pre-service teacher education.

Altogether, the theme of transitions overlaps with several TWGs and the core of the idea of transition that grants the unity to our TWG is still an open research question. Moreover, various facets of transitions may be studied using a diversity of theoretical/methodological frameworks. In what follows, we restrict our account to the 8 papers and 1 poster which have been presented and discussed during the group sessions, hence the figures in parentheses. We thus note the following facets: epistemological (7), cognitive (2), affective-emotional (1), socio-cultural, institutional (7); and the frameworks used: the Anthropological Theory of the Didactic, ATD (6), Commognition (1), concept image/concept definition (1), person-environment fit (1), and mathematical content analysis (3).

As Hochmuth et al. (2021) already pointed out, a large number of authors use the institutional perspective of ATD to study transitions, which led us to group those papers in the first parallel presentation session. By contrast, a diversity of perspectives (facets of transitions and theoretical/methodological tools) were offered in the second presentation session. After an in-depth discussion of each paper, discussions opened up to examine the topic of transitions in the light of all the papers and finally envisage opportunities for collaborations and avenues for further research. We will begin with an account of the contributions and then highlight some of the main points raised during our discussions. We conclude with a few ideas on the topic of transitions that may inspire future research.

HIGHLIGHTS FROM THE CONTRIBUTIONS

We asked authors to produce a highlight of their research in the form of a question/problem and its answer. In this section, we use these highlights—which were communicated in the group report at the conference—as a means to summarize striking features of the contributions and introduce readers to these works.

The school-university transition

Sarah Khellaf and Jana Peters raise the following questions: In what way can praxeological analysis inform the creation of study materials for first-year mathematics (teacher) students, that aim to make apparent to them differences between the institutions of school mathematics and university mathematics? What type of empirical questions about the implementation of these tasks could be asked and answered in the framework of ATD? As an answer, a task-design rationale has been explained in the paper. The reference model discussed can be used to identify 'unusual' (personal) praxeologies in student solutions. These can be compared with known dominant epistemological models from school and university, to generate hypotheses about their possible origin.

Tobias Mai and Rolf Biehler put the following problem in the foreground: School textbooks tend to introduce vectors as a mixture of the notions of n-tuples, translations, and sets of arrows—there is a need to explicitly and mathematically work out and integrate these settings in order to analyse and untangle interwoven approaches in school textbooks. As an answer, in the reference model presented in the paper, all three approaches to vectors are explained and finally discussed regarding their isomorphy. In the end, the most ostensive (illustrative) approach via arrows turns out to be the most complex approach of the three.

Jelena Pleština and Željka Milin Šipuš ask: How do polynomial-related praxeologies develop and differentiate through secondary school and a first-year bachelor programme in mathematics? In secondary school textbooks, the algebraic and analytical approaches to polynomials induce two disjoint praxeological organizations. In a first-year bachelor programme, specific and reduced praxis blocks align with the general logos blocks of praxeologies whose object of knowledge is the notion of polynomial. As a consequence, the relation first-year undergraduate students have to formal polynomials is marked by almost empty logos blocks.

Sarah Schlüter and Michael Liebendörfer explore: Which strategies do students use to cope with difficulties in "borderline cases" when their concept image seems to contradict the definition? Even if students apply the definition correctly, they do not trust the formal argumentation and tend to rely on intuition and their concept image. In addition to strategies based on informal reasoning, they manage to argue on a meta-level themselves, for instance by using transfers to similar "borderline cases".

Katharina Kirsten and Gilbert Greefrath ask: What are the characteristics of university students who choose on-campus or distance learning courses? Students with weaker connections to mathematics (e.g., in terms of self-efficacy and final math grade) and a higher digital readiness are more likely to choose a distance learning course. By contrast, students with strong math prerequisites tend to choose an on-campus course. Learning types based on self-regulation and peer learning do not play a significant role in course decision—at least in preparatory courses.

Finally, the poster by Ana Katalenić, Aleksandra Čižmešija and Željka Milin Šipuš tackles the following question: How can the discourses on asymptotes change and develop in the transition from upper secondary to university education? As a result, discourses can develop from colloquial narratives supported by iconic representation through working on techniques of evaluating function values and finding asymptotes, towards the formal definition using distance between points on a curve and the line and expressions with a function limit.

Other transitions

Thomas Hausberger and Julie Jovignot investigate: How can students' difficulties in acquiring a structural sense be understood in terms of institutional gaps in the Abstract Algebra track throughout the bachelor programme in mathematics? As a result, the study of structuralist levels of structuralist praxeologies and the values of their didactic variables in relation to the dialectic of contextualisation and decontextualisation points towards a huge gap at the 3rd year of the bachelor programme in France. It seems to be reinforced by the compartmentalisation of knowledge in small teaching units that hinders the vitality of the dialectic.

Heidi Strømskag and Yves Chevallard examine: What transformations has the notion of concavity of functions undergone during the didactic transposition process from the knowledge taught at university to the knowledge to be taught in upper secondary school? Praxeological analyses of a university textbook and a Grade 12 textbook show that while in the university presentation, the graphical notion of concavity is mathematised, in the school presentation, it remains non-mathematised: concavity is to be seen on the graph of the function—where the theorem proved at university becomes now a mathematically unfounded definition of concavity.

Finally, Max Hoffmann and Rolf Biehler study the following question: What prior knowledge do student teachers have on the geometric concept of congruence before taking a geometry course at university? As a fact, this multi-faceted concept is treated rather "one-dimensionally" in German schools. Taking at university the resulting pre-formal and superficial prior knowledge not into account to focus on formal aspects is likely to perpetuate Klein's second discontinuity. There is a risk that prior mathematical knowledge from school will coexist with the academic mathematics learned, rather than being studied, corrected, and updated.

HIGHLIGHTS FROM THE DISCUSSIONS

Common themes emerging

Definitions in mathematics were debated in relation to their role in acquiring concepts, solving problems, and proving theorems. The notion of borderline/challenging cases was treated as such examples play an important role in complementing an incomplete predominant concept image. Polynomials in school mathematics, in abstract algebra, and in analysis appeared as examples in the considerations. Another aspect that came up was that of theorems and examples used as definitions in school mathematics, most notably in textbooks—possibly with the intention of making the knowledge at stake available to a larger group of students—, a transformation that likely simplifies and distorts the mathematical knowledge.

Attention was also given to the various transitions that occur in the education of teachers and that teacher education should address. Studying mathematics in view of teaching it requires developing other, new relations to mathematics compared to relations a mathematics student must develop. The *topos* changes, for instance, a mathematics teachers will have to choose examples and design tasks related to particular mathematics content in order to create opportunities for others to study it.

Theoretical frameworks and methodologies

The concept of *praxeology*—an analytic tool provided by ATD to model any human activity in terms of *praxis* (the type of tasks and the technique to solve them) and *logos* (the way to explain the technique and the theory to justify the explanation)—was used in six papers. Praxeological analysis was discussed on a general basis and linked to the notion of *reference epistemological model* (REM) to be used, for example, in didactic design, trying to remedy ruptures identified in dominant epistemological models, as well as overcoming didactic phenomena caused by such dominant models.

In ATD, Klein's double discontinuity can be expressed in terms of transpositive processes: When one goes from a level n to a level $n + p$ in a curriculum (e.g., from secondary school to university), one generally faces an increasing rate of *mathematisation*, and conversely, in the opposite direction, there is generally a *demathematisation* of the mathematical content. This was discussed and related to the formalization developed by Winsløw and Grønbæk (2014).

On methodology, the problem of standardized methods for elaboration of REMs was raised and related to three dimensions of the questioning of any object: its structure, its functioning, and its utility. Networking of theories, ATD and Commognition or ATD and Stoffdidaktik (subject matter didactics), was mentioned as a promising research methodology to cross perspectives and promote collaborations but not really discussed in depth due to lack of time. Finally, an understanding was reached that when communicating research to non-specialists of ATD (especially in oral presentations), it is appropriate to avoid excessive formalism.

CONCLUSION

With a focus on transitions, researchers are aiming at the investigation of didactical phenomena in terms of continuities/discontinuities/ruptures. They may be pursuing different goals: their endeavour may be to identify difficulties related to epistemological/cognitive/institutional discontinuities, to suggest ways to smoothen ruptures or assess existing measures (to respond to institutional and societal demands), to contribute to teacher education (since most researchers are teacher educators), to refine theoretical constructs (such as models of transitions), or to study the effects of the didactic transposition.

Avenues for further research are wide. At the level of the school-university transition, collaboration among researchers should entitle a shift from small-scale local studies (centred on a concept or a single institutional context) to wider perspectives and contexts, including comparative or longitudinal studies. Research on ruptures across university studies, in particular towards advanced mathematics, is still rare. With the intensification of research on Klein's second discontinuity, we expect reports on curricular innovation to account for strategies developed to tackle institutional constraints and to provide means to cooperate with mathematicians. Finally, transitions from university to the workplace for other careers than teachers (e.g., engineers) need also greater attention. INDRUM looks forward to receiving contributions in these directions at the next conference.

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