Journal on Mathematics Education
Volume 14, No. 2, 2023, pp. 189-204

Toward a theoretical framework for task design in mathematics education

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Received: 7 November 2022 | Revised: 6 March 2023 | Accepted: 18 March 2023 | Published Online: 23 March 2022

Abstract

Task design is an important element of effective mathematics teaching and learning. Past research in mathematics education has investigated task design in mathematics education from different perspectives (e.g., cognitive and cultural) and offered a number of (theoretical) frameworks and sets of principles. In this study, through a narrative research in the form of autoethnography, I reflected on my past teaching and research experience and proposed a (theoretical) framework for task design in mathematics education. It contains four main principles: (a) inclusion, (b) cognitive demand, (c) affective and social aspects of learning mathematics, and (d) theoretical perspective(s) toward learning mathematics. This framework could be used as a tool for critically reflecting on current practices in terms of task design in teaching mathematics and research in mathematics education. It may also contribute to ongoing research in mathematics education about task design and enable or enhance opportunities for dialogue between lecturers, teachers, and researchers about how to design rich mathematical tasks for teaching and research purposes.

Keywords: Cognitive Demand, Culturally Responsive Teaching, Inclusion, Real-World Context, Task Design

How to Cite: Radmehr, F. (2023). Toward a theoretical framework for task design in mathematics education. *Journal on Mathematics Education, 14*(2), 189-204. http://doi.org/10.22342/jme.v14i2.pp189-204

Task design is at the heart of effective mathematics teaching and learning and also an important focus in research about student mathematical learning (Watson & Ohtani, 2015). *Task* could be defined as "questions, situations, and instructions that might be used when teaching students. Tasks prompt activity which offers students opportunities to encounter mathematical concepts, ideas, and strategies (Sullivan et al., 2015, p. 83). The term *task design* surfaces in mathematics education in the late 1990s while it was an established term in psychology since 1960s (Ruthven, 2015). It has been "more clearly present" from early 2000s in mathematics education research (Kieran et al., 2015, p. 27) and now a growing body of literature focus on task design in mathematics education (e.g., Cevikbas & Kaiser, 2021; Watson & Ohtani, 2015). The importance of task design could be viewed from different perspectives. For example, Watson and Ohtani (2015) highlighted its significance from cognitive, cultural, and practical perspectives:

From a cognitive perspective, the detail and content of tasks have a significant effect on learning; from a cultural perspective, tasks shape the learners' experience of the subject and their understanding of the nature of mathematical activity; from a practical perspective, tasks are the bedrock of classroom life, the "things to do." (p. 3)

http://doi.org/10.22342/jme.v14i2.pp189-204

Several theoretical frameworks for task design have been proposed in mathematics education literature with different consideration to learning environment, theories, and task genres (for more information see Kieran et al., 2015). One could classify them into *grand theoretical* frames of (mathematics) education (e.g., constructivist), *intermediate-level* frames (i.e., have more specific focus compared to grand theories (e.g., the Anthropological Theory of the Didactics)) and *domain-specific* frames (i.e., focusing on particular reasoning process or particular content) (Kieran et al., 2015). In the current paper, I propose a (theoretical) framework for task design in mathematics education, reflecting on research-informed criteria in task design in mathematics education and my past teaching and research experience, especially my research on task design, here using the revised Bloom's taxonomy (RBT) (Anderson et al., 2001) and networking theories in mathematics education. The proposed framework is empowered by frameworks of all three mentioned types, however, on its own could be classified as a framework laid between *intermediate-level* and *domain-specific* frames. I hope it could be used as a tool by mathematics teachers, lecturers, and educators to critically reflect on mathematics teaching and learning and enable or enhance opportunities to design rich mathematical tasks for teaching and research purposes.

METHODS

The present study is a *narrative research* in the form of *autoethnography* (Creswell & Poth, 2018), where I reflect on my past teaching and research experience as a mathematics educator and a mathematics lecturer. I have taught many undergraduate mathematics courses (e.g., pre-calculus, calculus, multivariable calculus, differential equations and introductory linear algebra) in Iran and New Zealand. Furthermore, as a mathematics educator, I have taught many mathematics education courses in Iran and Norway, from bachelor to PhD level. I am also an active mathematics education researcher who focused on several aspects of mathematics education in my past career that helped me reflect on different important task design principles in this paper. Of the two well-known modes of narrative research, this paper is reported as a thematic approach (Creswell & Poth, 2018), where the themes are principles and sub-principles of task design that came to my attention over the years. Here, I do acknowledge that mathematics teachers/lecturers play an important role in selecting, modifying, designing, redesigning, and evaluating tasks. Moreover, there is often a notable difference between the intended and enacted task (Sullivan et al., 2015). However, this paper focuses more on the former, the design of the task, not its implementation in mathematics classrooms or lectures.

RESULTS AND DISCUSSION

The framework comprises four main principles and several subprinciples that task designers (e.g., mathematics teachers/lecturers and mathematics educators) could consider when designing tasks for teaching and research purposes and for critically reflecting on their current practice. These four main principles are *inclusion*, *cognitive demand*, *affective and social aspects of learning mathematics*, and *theoretical perspective(s) toward learning mathematics* (Figure 1).

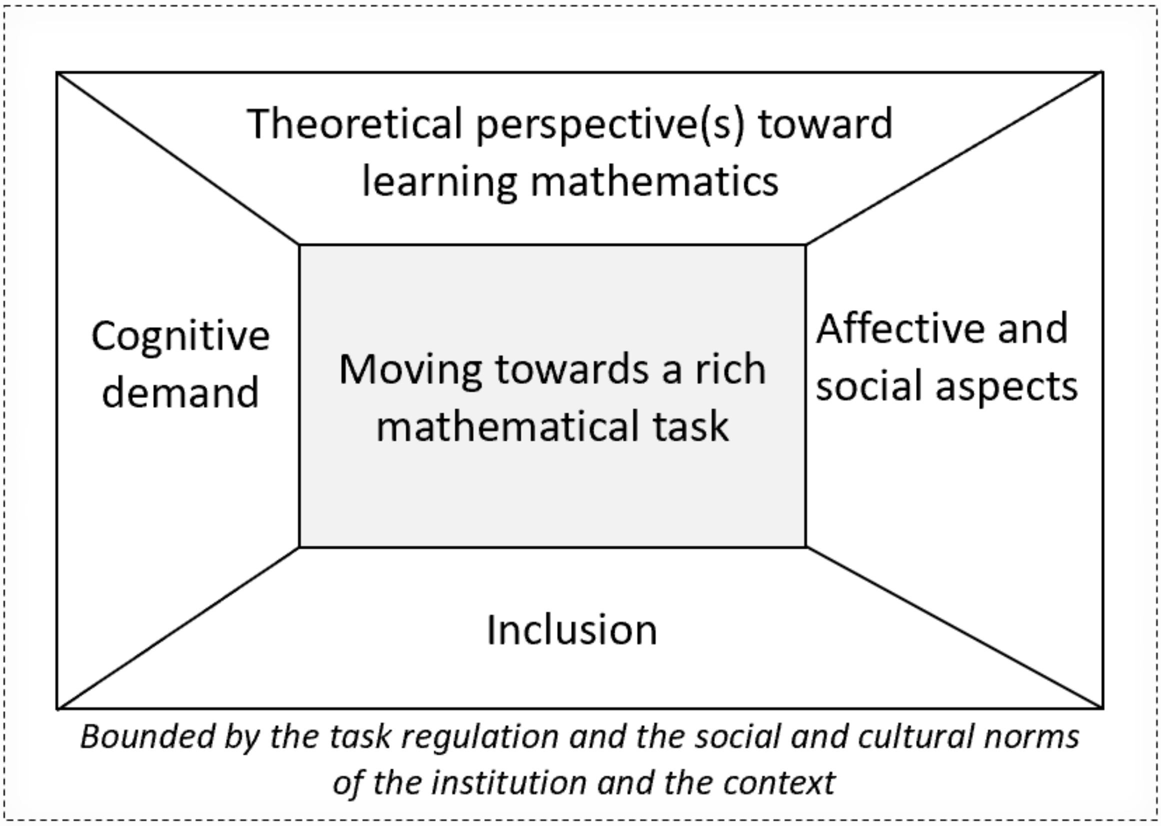

Figure 1. The main principles of task design in mathematics education

I perceive inclusion as the foundation of a rich mathematical task, so I place it on the floor/base of the framework. To achieve meaningful mathematics learning, we need to consider both the (a) cognitive and (b) affective and social aspects of mathematics learning. Therefore, I placed these two principles as the two sides/walls, leaving the top for the theoretical perspective(s) toward learning mathematics principle, which indirectly influences the other three main principles. In my future work, I will develop a practical framework for task design with several suggestions for task types, templates, and examples of how to address these principles.

The institution's social and cultural norms and the context for which the task is designed have impact on to what extent task designers consider these principles. For example, if an institution emphasizes inclusion in terms of culturally responsive teaching (Gay, 2002), more focus would probably be given to this principle during task design. Task regulation is another important factor impacting task design, which refers to a set of rules that a designer considers for a task (Greefrath & Vos, 2021), such as time allocation for students' engagement with the task or whether students could use online resources when engaging with the task. However, task regulation is not within the scope of this framework. The final remark before unpacking these principles is that they are to some extent related. However, I have distinguished between them to make them more concrete, practical, and comprehensible.

Inclusion

For me, inclusion in task design could be conceptualized as considering students' prior knowledge and experience when designing tasks. In mathematics education, this includes students' prior knowledge and experience about the mathematical concepts underlying the task, their cultural identity, and real-life experiences. Reflecting on this conceptualization of inclusion, I focus on five subprinciples (Figure 2) that are unpacked in the following sections.

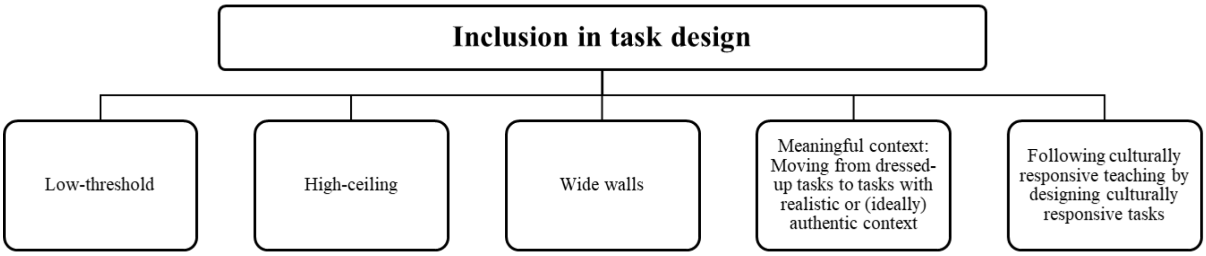

Figure 2. The five subprinciples of inclusion

Low Threshold, High Ceiling, and Wide Walls

Mathematics educators have promoted designing low-threshold and high-ceiling (or low-floor, high-ceiling) tasks (e.g., Gadanidis, 2012; Gjesteland & Vos, 2019). This type of task is accessible to all students, regardless of their prior mathematical knowledge (low-threshold) and also provides opportunities for students with strong mathematical knowledge and skills to challenge themselves (high-ceiling). The NRICH (Norwich, (The) Royal Institution, Cambridge (University) and Homerton (College)) project at the University of Cambridge has a long tradition of promoting this type of task, and many free online mathematical resources are available through the website's project for school mathematics. However, one could argue that it is not enough to provide only one path for students to move from the low-floor to the high-ceiling. Task designers could think about designing tasks in a way that can be solved using different approaches (often referred to as wide walls) that suit students with diverse interests, learning, and problem-solving styles (English, 2017; Gadanidis et al., 2017).

Meaningful Context

Another consideration regarding inclusion is the task's context. Different considerations are given to the meaning of *context* (see Van Den Heuvel-Panhuizen, 2005). Here, by context, I mean "a situation or event in the task, which often is from real-life or from imaginary situations (e.g., fairy tales)" (Vos, 2020, p. 36). I would not claim that mathematical tasks without context have no value; however, we could include more contextual tasks in our teaching/research to make them more attractive to students. I would argue that in each classroom, some students are interested in learning mathematics because they find mathematics interesting and enjoy its inner structure, regardless of its usefulness (as conceptualized by Williams (2012), having exchange value (e.g., helping them to study a specific major in university), or/and use-value (e.g., the mathematics that they learn can be used in their future profession or daily life)). However, this is not the case for many students. Many students at school and university learn mathematics because it is a requirement for completing their education (i.e., exchange value) or are only interested in the mathematical content because it will be used in their future profession or daily life (i.e., use-value) (Den Braber et al., 2019; Harris et al., 2015). Therefore, teachers and lecturers as task designers could focus more on use-value in task design by including contextual tasks to make mathematics teaching more interesting and meaningful for all students. Using context gives meaning to the mathematical objects presented in the task, could increase the task's accessibility, and may help students use their out-of-school/university knowledge to develop their mathematical understanding (Van Den Heuvel-Panhuizen, 2005; Vos, 2020).

Previous studies in mathematics education have provided different classifications of mathematical tasks in terms of their relation to reality (e.g., Hiebert et al., 2003; Vos, 2020). For example, Vos (2020) has recently identified five different categories of mathematical tasks in terms of their relation to reality: *bare task*, *task with mathematical context*, *dressed-up task*, *task with realistic context*, and *task with authentic context*. When it comes to designing contextual tasks, the distinction made by Vos (2020) between dressed-up tasks, tasks with realistic context, and tasks with authentic context is quite important. Dressed-up tasks are those that have a certain context, but the question(s) asked is not justified in the context; therefore, the reason for answering the question(s) is not clear to the students (Vos, 2020). For example, a dressed-up task could be the following: How can a cake in circular form be divided into 16 equal slices? Here, in the task, we do not mention why the cake needs to be divided into 16 equal slices, so for some pupils, the only reason to engage with this task is that the teacher asked them to solve it; therefore, the task might not be meaningful and interesting for some students to engage with.

In tasks with a realistic context, the question(s) asked is justified by the given context and has use-value in the given context (Vos, 2020). To take the previous example to the next level and make it more meaningful and interesting for students, we could include a scenario of why the cake needs to be divided into 16 equal slices. For example, we could write the task as follows:

Tomorrow, Nora will turn eight, and she has invited 15 of her classmates to her birthday party. Nora's mother bought a cake that is in a circular form. How should Nora's mother cut the cake so that all kids have an equal slice?

Here, we justify why the cake needs to be cut into 16 slices, but some improvements could still be made to the context to make it more real, interesting, and meaningful for students. Tasks with an authentic context, like a realistic context, justify the question(s) raised in the task; however, the context here is genuine and described using authentic resources, such as real data from governmental datasets and photos from real objects (as opposed to a schematic picture or drawing) (Vos, 2020). Returning to the initial example, first, a photo of a real birthday cake can be provided in the task, hopefully similar to the birthday cake that pupils see in the stores in their country (see the next subprincipal regarding culturally responsive teaching). Second, instead of asking how to cut it into 16 equal slices, we could say that Nora has invited 15 of her classmates to her birthday party. Then, the students themselves can decide on the number of slices and whether the slices should be equal in size or not. They could also consider the size of the cake based on the photo, but this was not possible in the previous examples. In reality, there are even more considerations. For example, do they want to include some extra slices for those who want a second slice? Does Nora wish to keep a few extra slices for her parents and (possible) siblings? Or does she even want a slice for herself to enjoy the next morning for breakfast? Then, the task becomes more real than an elementary geometry question. When it comes to an authentic task, more elements are important to consider, depending on the age of the guests. For example, if it was a birthday party for adults, then factors such as diet restrictions because of pre-existing health conditions, such as having high blood sugar or on a diet to lose weight, become important in terms of slice sizes. In summary, if task designers are interested in designing contextual tasks, they could focus more on realistic and (ideally) authentic tasks to make the teaching/research more interesting for all students.

Designing Culturally Responsive Tasks

Following culturally responsive teaching (CRT) by designing culturally responsive tasks is another important consideration when a task designer thinks about inclusion. The cultural characteristics and experiences of students could be used as a means to teach more effectively (Gay, 2002), and in the past few decades, many mathematics education scholars have paid attention to how CRT could be implemented in mathematics classrooms (e.g., Averill et al., 2009; Mogari, 2017). This subprinciple becomes more critical when a task designer adapts or adopts mathematical tasks from other cultural contexts (e.g., when using mathematical modelling tasks developed in other contexts).

Task designers could think about how the cultural identities of (diverse) students could be integrated into the tasks before using them for teaching or research purposes. For example, a culturally responsive task designed by Heays et al. (1994) for the New Zealand context to make a measurement lesson more meaningful for Māori (the indigenous Polynesian people of New Zealand) students should be significantly revised to be meaningful to students of other cultures as well. However, several lessons can be learned from the task in terms of respecting the cultural identity of diverse students, in this case, Māori students. For example, here, Heays et al. (1994) used the verb "know" in the second line, endorsing the belief of Māori people instead of choosing another verb such as "believe" that could indicate this belief might not be true:

In the South Island [of New Zealand] there is a lake whose waters, by day and by night, rise and fall, rise and fall. The Māori people know that the pulsing of the water comes from the beating of a giant's heart, the heart of Matau who was burnt by the brave Matakauri. The waters of Lake Wakatipu rise and fall about every five minutes. If the lake was formed 1000 years ago, how many times has Matau's heart beat since then? If Matau's heart beat once every three minutes, how many times would Lake Wakatipu have risen and fallen over the last 100 years? (Heays et al., 1994, p. 8)

Cognitve Demand

Cognitive demand is one of the important dimensions needed for creating a powerful mathematics classroom (Schoenfeld, 2014). Task designers need to think carefully about how they challenge students intellectually and pay close attention to inclusion when increasing task complexity.

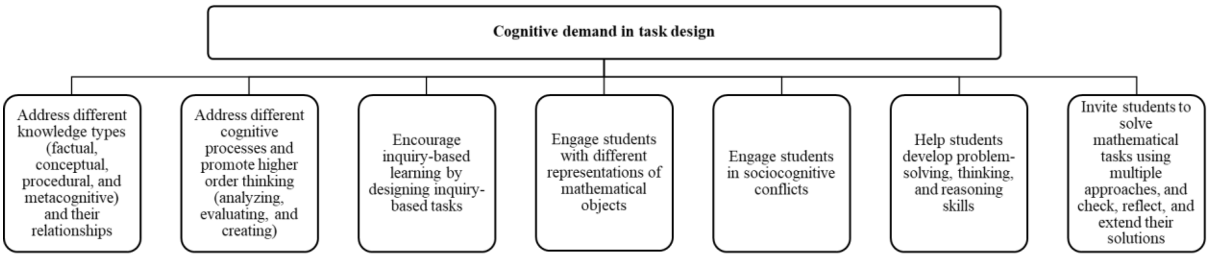

Figure 3. The seven subprinciples of cognitive demand

Schoenfeld (2014) highlighted, "There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea" (p. 407). Here, I focus on seven subprinciples regarding cognitive demand (Figure 3), two of which are influenced by the RBT (Anderson et al., 2001).

Address Different Knowledge Types and Their Relationships

The RBT (Anderson et al., 2001) is a robust framework for task design, especially when it comes to cognitive demand. As a two-dimensional framework, the RBT encourages task designers to think about how to design tasks to address different knowledge types and activate different cognitive processes. In more detail, the knowledge dimension is divided into four types: factual, conceptual, procedural, and metacognitive knowledge. These four types are further divided into 11 subtypes. Radmehr and Drake (2017a) have unpacked these 11 subtypes in the context of integral calculus; however, this contextualization could help task designers in mathematics to familiarize themselves with these 11 subtypes in a more general way and think about how they can address them in task design. Therefore, one of the subprinciples of cognitive demand is to design tasks that address all RBT knowledge types and (ideally) subtypes. Here, it is worth mentioning that metacognitive knowledge was not included in the Bloom's taxonomy. Even now, more than 40 years after its introduction by Flavell (1979), this type of knowledge is still not well addressed in task design, especially at the upper secondary and tertiary levels. For instance, Radmehr and Drake (2017b, 2019) recently reported that upper secondary and tertiary students' metacognitive experiences and skills need further development in the context of learning integral calculus.

Address Different Cognitive Processes and Promote Higher-Order Thinking

Focusing on the cognitive process dimension, the RBT offers six cognitive processes: *remembering, understanding, applying, analyzing, evaluating,* and *creating.* These cognitive processes are further divided into 19 subcategories (Anderson et al., 2001). The handbook of RBT (Anderson et al., 2001) provides several approaches to design questions to address these 19 subcategories in mathematics and other subjects. Recently, mathematics educators (Radmehr & Drake, 2018; Radmehr & Vos, 2020) have provided several examples of how to design mathematical questions/tasks to address these cognitive processes. Therefore, when it comes to the second subprinciple of cognitive demand, I would encourage task designers to address different RBT cognitive processes to help them better elicit students' mathematical thinking and identify where students need further support. The RBT has been successfully operationalized to elicit students' mathematical thinking related to integral calculus (Radmehr & Drake, 2017a, 2019) and combinatorics (Salavatinejad et al., 2021) at the upper secondary and tertiary levels.

When a new mathematical object/procedure emerges in the mathematical discourse, students typically participate in the discourse *ritually* and imitate the knowledgeable person's performance (e.g., teacher) (Sfard, 2017, 2020). Tasks that are usually used at this stage of learning focus on lower cognitive processes (i.e., remembering, understanding, and applying) to help students understand the mathematical object/procedure. However, I would argue that to facilitate the *deritualization* process and help students participate *exploratively* in the discourse (see, e.g., Sfard, 2020), students need to engage with tasks that address higher-order thinking (HOT). HOT tasks address analyzing, evaluating, and creating (Radmehr & Vos, 2020) and are closely related to the type of tasks that are promoted in the next subprincipal.

Encourage Inquiry-Based Learning by Designing Inquiry-Based Tasks

Mathematics educators have promoted using inquiry-based learning (IBL) to improve the teaching and learning of mathematics at school (e.g., Jaworski, 2014) and university levels (e.g., Jaworski, 2020; Laursen & Rasmussen, 2019). One of the core elements of this student-centred teaching approach is to help students develop a meaningful understanding of mathematics through inquiry in mathematics with their peers via engaging with inquiry-based (IB) tasks. IB tasks are also associated with HOT (Laursen & Rasmussen, 2019) and invite students "to solve problems, conjecture, experiment, explore, create, and communicate, all critical skills and habits of mind in which mathematicians and scientists engage regularly" (The Mathematical Association of America, 2018, p. 22). Therefore, as task designers, we could promote IBL by designing IB tasks and creating opportunities for students' collaboration and inquiry in mathematics inside and/or outside the classroom/lecture.

Engage Students with Different Representations of Mathematical Objects

As task designers, we could think about creating opportunities for students to engage with different representations of mathematical objects to facilitate students' mathematical learning. This subprinciple is indirectly covered in the first two subprinciples, but I consider it a separate subprinciple because of its importance. The importance of creating opportunities for students to engage with different representations of mathematical objects has been frequently highlighted in the literature (e.g., Dreher & Kuntze, 2015) because working with different representations is a crucial element in supporting students' conceptual understanding of mathematics (NCTM, 2000). The NCTM (2000, p. 67) highlighted the following:

Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling.

Engage Students in Sociocognitive Conflicts

Past research in educational research and mathematics education from the constructivist view toward learning has suggested creating sociocognitive conflicts as a useful technique to address students' misconceptions and develop meaningful understanding (Foster, 2011, 2012; Neugebauer et al., 2016). The term sociocognitive conflict has emerged as a way to highlight the importance of peer communication in creating and resolving cognitive conflicts (Foster, 2011) by decentralizing the focus from one's perspective that might have shortcomings and only related to certain aspects of the knowledge/reality to possible other perspectives (Kazak et al., 2015). It is grounded on the viewpoint that "learning is not a mere product of imitation, but it can result from sociocognitive construction, that is elaboration of new cognitive schemas or new knowledge, on the basis of the articulation of different points of view" (Buchs & Butera, 2004, p. 80). As task designers, we could think about the various ways to question students' prior knowledge and beliefs by confronting them with contradictory information or anomalous data (Limón, 2001), which would help them work together to create and resolve a cognitive conflict through reconstructing and reconfiguring their schemas to accommodate the new information (Foster, 2012).

Help Students Develop Problem-Solving, Thinking, and Reasoning Skills

I would argue that as task designers, although we focus on finding ways in which we can help students develop their meaningful understanding of mathematics, we could also think about how we can create opportunities for students to develop their problem-solving, thinking (e.g., critical thinking, lateral thinking), and reasoning skills. Recent research in mathematics education (e.g., Klymchuk, 2017; Radmehr et al., 2021; Rezvanifard et al., 2022) has suggested that approaches such as puzzle-based learning (Klymchuk, 2017; Thomas et al., 2013) not only could contribute to developing students' mathematical understanding, but also contribute to improving students' problem solving, thinking, and reasoning skills. For instance, Klymchuck (2017) highlighted, "Interesting puzzles, paradoxes and sophisms can engage students' emotions, creativity and curiosity and also enhance their conceptual understanding, critical thinking skills, problem-solving strategies and lateral thinking …" (p. 1106).

Invite Students to Solve Mathematical Tasks using Multiple Approaches and Check, Reflect, and Extend Their Solutions

Inviting students to solve mathematical tasks using multiple approaches, often referred to as multiple solution tasks (MSTs), has been found to be beneficial for improving mathematics teaching and learning. For instance, it has been reported as one of the ways to foster connectedness of students' mathematical knowledge and positively impact students' creativity and flexibility, contributing to developing students' mathematical understanding and problem-solving skills. It has also been found to be a valuable tool for examining students' mathematical knowledge and creativity (see, e.g., Levav-Waynberg & Leikin, 2012a, 2012b).

The well-known mathematical problem-solving models (e.g., Polya, 1949; Schoenfeld, 1985) highlight the importance of checking, reflecting, and extending the solution to mathematical problems to develop problem-solving skills and their association with developing students' metacognition at different levels of schooling (e.g., Zemira & Bracha, 2014). Furthermore, these three activities are useful for developing mathematical thinking skills (Mason et al., 2010). Mason et al. (2010) highlighted the following:

When you reach a reasonably satisfactory resolution or when you are about to give up, it is essential to review your work… [I]t is a time for looking back at what has happened in order to improve and extend your thinking skills, and for trying to set your resolution in a more general context. It involves both looking back, to CHECK what you have done and to REFLECT on key events, and looking forward to EXTEND the processes and the results to a wider context. (p. 36)

Therefore, as task designers, we could first think about designing tasks in a way that can be solved using multiple approaches, which is in line with the wide-wall approach. Second, we could dedicate part of the task to explicitly asking students to solve it using multiple approaches. Furthermore, we could think about the ways to encourage students to check, reflect on, and extend their solutions for mathematical problems to other contexts.

Affective and Social Aspects of Learning Mathematics

As task designers, although we consider the task's cognitive demand and inclusion, we could also think about how we can address the affective and social aspects of learning mathematics in task design (Figure 4). I would start by highlighting that we could explore students' perceptions of different task types (e.g., Radmehr et al., 2021; Nedaei et al., 2019) and reflect on the obtained knowledge in task design. In other words, if a learning goal can be achieved to the same degree using different task types (e.g., using a traditional closed-ended problem-solving task, a problem-posing task, or a mathematical modelling task), we could choose the type(s) that previous research has suggested students find more interesting and entertaining. Similarly, we could choose task type(s) in which past research has indicated that students' engagement with the task positively impacts their attitudes toward learning mathematics and their appreciation of mathematics (e.g., mathematical modelling tasks; see Di Martino, 2019).

Focusing on the social aspect of learning mathematics, task designers could think about designing tasks in a way that provides many opportunities for students to develop their communication skills (e.g., Stein, 2007), such as listening, arguing, and reporting skills (verbally and written). Stein (2007) highlighted, "Mathematics should be taught in a way that mirrors the nature of the discipline […] have students use mathematical discourse to make conjectures, talk, question and agree or disagree about problems in order to discover important mathematical concepts" (p. 285). Developing communication skills is related to a construct known as social norms (see Stephan, 2020; Yackel & Cobb, 1996) that could be defined as the expectations of the teacher and student from one another in the course of academic discussion (Stephan, 2020).

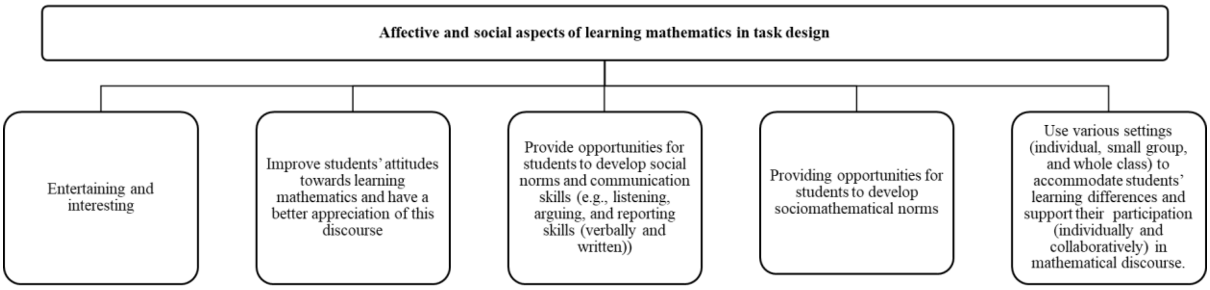

Figure 4. The five subprinciples of affective and social aspects

While developing social norms is important, it is also worth highlighting the importance of socio-mathematical norms in task design in mathematics education. Stephan (2020), reflecting on past research and the importance of the quality of mathematical contribution in classrooms, highlighted: "while inquiry social norms are mandatory for creating student-centered mathematics classrooms, they are insufficient for supporting mathematical growth" (p. 803). There are differences between social and socio-mathematical norms; Social norms apply to all subjects and are not specifically related to mathematics, whereas socio-mathematical norms are "the normative criteria by which students within classroom communities create and justify their mathematical work" (Stephan, 2020, p. 802). For instance, a social norm in classrooms is teachers expect students to explain how they solve questions/problems and describe their ways of thinking. However, what could be considered an acceptable mathematical explanation is a socio-mathematical norm (Yackel & Cobb, 1996). Other examples of socio-mathematical norms are the negotiation of the criteria between the teacher and students, what could be considered a different, sophisticated, or efficient mathematical solution (Stephan, 2020). Therefore, task designers could provide more opportunities for students and teachers to negotiate these criteria and develop them further.

The final subprinciple here is to consider various settings (individual, small group, and whole class) in task design (Olsen et al., 2021) to accommodate students' learning differences and support their participation (individually and collaboratively) in mathematical discourse.

Theoretical Perspective(s) toward Learning Mathematics

In a way, this principle is integrated with the others, but it is still important to be discussed on its own. When it comes to conducting research, mathematics educators are often very much influenced when designing tasks based on their theoretical perspective(s) toward learning mathematics. For example, if the FAMT framework (Stewart & Thomas, 2009) is used in a study—which is an attempt to integrate APOS (action, process, object, and schema) theory (see Arnon et al., 2014) with Tall's three worlds of mathematics (Tall, 2008)—the tasks to explore students' learning of a mathematical concept(s) will be designed in a way that provides opportunities for students to demonstrate action, process, and object thinking in each of the three mathematical worlds (i.e., conceptual-embodied, proceptual-symbolic, and axiomatic-formal). However, when it comes to teaching mathematics or mathematics education, many mathematics teachers, lecturers, and even educators only unconsciously design tasks based on how they perceive meaningful learning. So here, I would argue that task designers could reflect on their perceptions of the meaningful understanding of mathematics when designing tasks and think about whether the tasks that they have designed provide enough opportunities for students to develop the intended learning (e.g., from the FAMT perspective, being able to flexibly move between three worlds of mathematics and develop process and object thinking in each mathematical world). I end this section by providing Figure 5, which presents the main principles and their sub-principles.

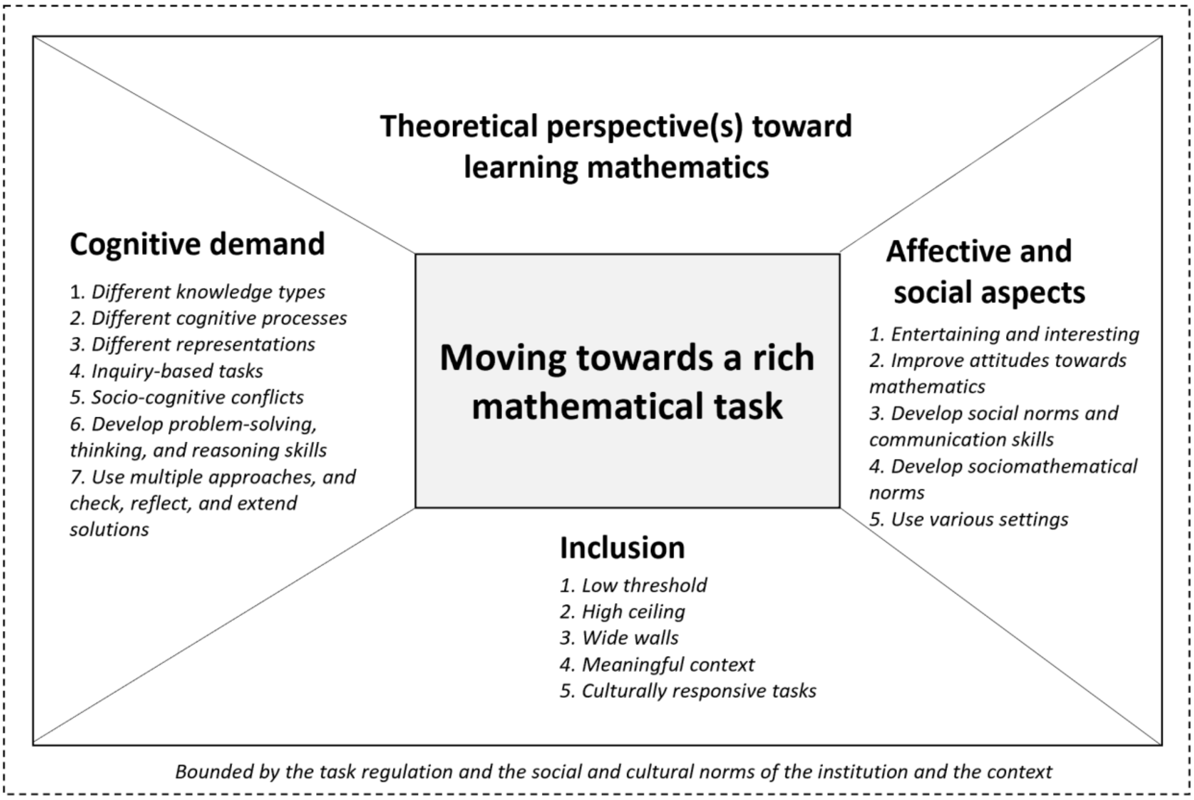

Figure 5. The main principles and subprinciples of task design in mathematics education

CONCLUSION

To conclude, it is practical and possible to consider all the four main principles during task design; however, it might be challenging to consider all the subprinciples (Figure 5) when doing so, especially when beginning to use this framework, when one might not have mastered the subprinciples yet. If a task designer found it challenging or neither practical nor necessary to consider all the subprinciples in a single task, then one could think about how to develop a lesson(s) with different tasks that overall address all the subprinciples. Second, as the title of the current paper signals, I am not claiming this framework is inclusive and addresses all the principles of task design in mathematics education; it is simply a reflection on my past teaching and research experiences as a mathematics educator and a mathematics lecturer; however, I believe the way the four main principles are conceptualized in this paper could help mathematics teachers/lecturers and mathematics educators improve the teaching and learning of mathematics (and possibly mathematics education in the teacher education courses); indeed, educators could use this framework for task design or as a tool to critically reflect on their current practice.

Declarations

Funding Statement: The article processing charge (APC) is covered by the Norwegian University of Science and Technology (NTNU).

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