Master's thesis

NTNU
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Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

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Designing Delivery Districts and Tactical Routes for Parcel Home Delivery Service

A novel two-stage solution approach and a real-world case study

Master's thesis in Industrial Economics and Technology Management
Supervisor: Anders Gullhav
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Preface

This master's thesis concludes our Master of Science in Industrial Economics and Technology Management at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management.

We want to express our sincere gratitude to our industry partner Posten Norge AS, especially Tarald Langan and Shirin Fallahi, for their helpful discussions and for providing us with available data. Your willingness to discuss our work has been a valuable contribution to the final result. Further, we would like to thank our supervisors, Professor Anders Gullhav and Professor Emeritus Bjørn Nygreen, for their excellent guidance and helpful comments in improving the thesis.

Trondheim, June 11, 2022

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Summary

The Route Partitioning Problem (RPP) is the problem of generating tactical routes for home delivery of parcels by partitioning a geographical area into smaller areas and sequencing these. The main difficulty in designing good solutions to the RPP problem is the need to address stochastic parcel volumes and customer locations. This thesis aims to evaluate whether designing a set of tactical routes to be operated unchanged over a given period of time is a viable alternative to constructing a set of routes every day based on the particular instance of that day.

To solve the RPP, this thesis develops a novel two-stage solution approach. The first stage solves a strategic districting problem by using a location-allocation heuristic algorithm. The algorithm decomposes the districting problem into two phases, a location and an allocation phase, where the allocation phase is solved using the Polygon Partitioning Problem (PPP) implemented as an Integer Programming (IP) model. A solution from the first stage strategic districting problem is a partitioning of a set of postal code areas into non-overlapping polygons that are compact, isolated and avoids geographical barriers in the road network. For each polygon, an expected delivery time within the polygon is calculated for three different demand levels using Monte Carlo simulations. In the second stage, an IP model is implemented to solve the tactical Polygon Routing Problem (PRP). A solution is a tactical routing plan, where each route is a sequence of polygons, which minimizes the total expected delivery time.

Solutions of our two-stage approach are constructed for three different demand levels, identified within Trondheim municipality in Norway, and evaluated based on realizations of the customers' demands. Using Google's Operations Research Tools (OR-tools), the total delivery time of daily routes derived from tactical routes produced by our method is evaluated and compared to the delivery time of a deterministic Vehicle Routing Problem (VRP) solution. Interesting is the impact of different partitionings of the geographical area. Results indicate that using a more granular polygon partitioning when constructing tactical routes generates the lowest average delivery times. Moreover, the tactical routes generated with the most granular partitioning have merely a 2.3% higher average delivery time than the deterministic VRP solutions across all demand levels. The results are encouraging as the logistic cost saving of applying tactical routes is believed to be significantly larger.

To our knowledge, this thesis constitutes the first known attempt of combining strategic districting and tactical routing in a two-stage approach. The computational study has proven that tactical routing does provide a viable alternative to optimizing routes each day from scratch. Nevertheless, our efforts should be construed as an initial investigation, and further analysis of the cost-savings of applying tactical routes needs to be evaluated before being implemented in practice.

This thesis is carried out in collaboration with Posten Norge AS, a Norwegian postal and logistics group.

Sammendrag

Ruteinndelingsproblemet (the Route Partitioning Problem - RPP) i denne oppgaven er problemet med å lage taktiske kjøreruter for hjemlevering av pakker. Problemet innebærer å først dele opp et geografisk område i mindre områder og deretter bestemme rekkefølgen disse områdene skal besøkes i. En sentral utfordring med RPP er uforutsigbare pakkevolumer og kundelokasjoner. Målet med denne oppgaven er å finne ut om det å benytte et sett med taktiske ruter, som skal opereres uendret over en lengre tidsperiode, kan være er et attraktivt alternativ til det å sette sammen helt nye ruter hver dag.

I denne oppgaven utvikler vi en nyskapende to-stegs løsningsmetode for RPP. Det første steget løser et strategisk distriktsproblem ved hjelp av en lokasjon-allokering heuristisk algoritme. Algoritmen dekomponerer distriktsproblemet i to faser, en lokaliserings- og en allokeringsfase, hvor allokeringsfasen løses ved hjelp av Polygoninndelingsproblemet (the Polygon Partitioning Problem - PPP) implementert som en heltallsprogrammeringsmodell (IP). En løsning fra steg en er en inndeling av postnummer områder i ikke-overlappende polygoner som er kompakte, isolerte og unngår geografiske hindringer i veinettet. For hvert polygon beregnes en forventet leveringstid for tre ulike etterspørselsnivåer ved hjelp av Monte Carlo-simuleringer. I steg to implementeres en IP-modell for å løse det taktisk Polygonruteproblemet (the Polygon Routing Problem - PRP). En løsning fra steg to er en taktisk ruteplan, der hver rute er en sekvens av polygoner, som minimerer den totale forventede leveringstiden.

Løsningene til vår to-stegs løsningsmetode er konstruert for tre ulike etterspørselsnivåer for Trondheim kommune i Norge, og er evaluert basert på et antall realiseringer av kunders etterspørsel. Ved bruk av Google sitt optimeringsverktøy (OR-Tools), vil den totale

leveringstiden av daglige ruter basert på våre taktiske ruter evalueres og sammenlignes med leveringstiden til et deterministisk Bilrutingsproblem (Vehicle Routing Problem - VRP). Et interessant aspekt er hvordan ulike inndelinger av det geografiske området påvirker leveringstid. Våre resultater indikerer at bruk av en mer granulær polygoninndeling i gjennomsnitt fører til de laveste leveringstidene. De taktiske rutene generert med den mest granulære inndelingen, fører til en gjennomsnittlig leveringstid som kun er 2.3% høyere enn de deterministiske VRP løsningene på tvers av etterspørselsnivåer. Disse resultatene er motiverende ettersom kostnadsbesparelsene i annen logistikk ved å bruke taktiske ruter antas å være betydelig større.

Oss kjent er denne oppgaven det første forsøket på å kombinere strategisk distriktsinndeling og taktisk ruting i en to-stegs løsningsmetode. Vår beregningsstudie indikerer at taktiske ruter kan være et attraktivt alternativ til det å lage helt nye ruter hver dag. Likevel bør denne oppgaven kun anses som en innledene studie, og videre analyser av kostnadsbesbarelsen ved å ta i bruk taktiske ruter er nødvendig før disse kan anvendes i praksis.

Denne oppgaven er utført i samarbeid med Posten Norge AS, et norsk post- og logistikkonsern.

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Acronyms

CCP Chance-Constrained Programming.

CluVRP Clustered Vehicle Routing Problem.

DP Districting Problem.

LA Location-Allocation.

M-LA Modified Location-Allocation.

OCARP Open Capacitated Arc Routing Problem.

PPP Polygon Partitioning Problem.

PPPHC Polygon Partitioning Problem with Heuristic Constraints.

PRP Polygon Routing Problem.

RPP Route Partitioning Problem.

SPR Stochastic Program with Recourse.

SVRP Stochastic Vehicle Routing Problem.

TSP Traveling Salesman Problem.

VRP Vehicle Routing Problem.

VRPSDC Vehicle Routing Problem with Stochastic Demands and Customers.

VRPTW-ST Vehicle Routing Problem with Hard Time Windows and Stochastic Service Times.

Chapter 1

Introduction

Digitalization is transforming the economics of the Norwegian transportation logistics industry. On the one hand, the volume of traditional letter mail is declining because of cheaper and quicker means of communication. On the other hand, the delivery of packages is rapidly increasing, driven by the growing e-commerce industry, a trend that the COVID-19 pandemic has further intensified. Restructuring and streamlining will be necessary for the Norwegian industry players to cope with these changes in user patterns. Additionally, the deregulation of the Norwegian postal market in 2016 has increased competition. Increased package volumes and competition affect Norwegian industry players, who must deliver a professional, responsive and accurate service at a low cost to maintain competitiveness.

Fast and accurate delivery at a low cost is expected by both consumers and online retailers. It should be easy and cheap for the customer to shop online and get the delivery at the door, in the mailbox, or at a decentralized pickup point. Among these three delivery options, parcel delivery at the door, called home delivery, is an attractive and convenient option for the customer. Especially during the pandemic, this delivery concept had an enormous boost, and Norwegian industry players believe this boost has caused new customer habits with a larger share of home deliveries. However, this is the most time-consuming and costly delivery method for parcel delivery companies. Moreover, considerable uncertainty in the number of parcels to be delivered each day brings a challenge to the planning of dispatching operations, such as sorting of parcels and scheduling of routes, which in turn makes the requirement of fast and accurate delivery at low-cost challenging.

This thesis is motivated by a real-world tactical routing problem faced by Norway's largest postal and logistic company, Posten Norge AS. Posten Norge AS is continuously seeking to improve their parcel delivery process to handle the ever-increasing number of parcels. Today, delivery routes are optimized each day from scratch when demand is known, as daily demand fluctuations and tight resource constraints hinder fixed routes and resource assignments. Posten Norge AS recognizes that this provides little predictability, both at logistic terminals and for the customer. Re-sorting of parcels happens last-minute, and customers must accept inaccurate delivery times. Hence, Posten Norge AS believes there is significant potential for reducing costs in the parcel sorting process and increasing customer satisfaction if delivery routes are determined earlier.

This thesis aims to evaluate whether designing a set of tactical routes to be operated unchanged over a given period of time is a viable alternative to constructing a set of routes every day based on the particular instance of that day. In this thesis we develop a mathematical programming model to help Posten Norge AS with the tactical planning of home delivery routes in Norway. Planning new routes each day to cope with fluctuating demand results in minimal cost routing plans, however, it entails many operational difficulties. Therefore, although at the expense of cheap delivery routes, Posten Norge AS could significantly reduce operational complexity by designing a set of routes in advance and using the preplanned routes as a basis for the daily routes. In this thesis, we propose a novel two-stage approach, consisting of two IP-model formulations and a heuristic algorithm, which partitions a service area into strategic units in the first stage and constructs tactical routes in the second stage by assigning units to vehicles and determining a visiting sequence.

This thesis is organized as follows. Chapter 2 provides the required background information for this project, discussing the industry partner, the parcel delivery market in Norway and the parcel delivery districting and routing problem. Chapter 3 presents relevant academic work on both districting and routing problems. Then, a description of the combined districting and routing problem addressed in this thesis is presented in Chapter 4. Further, Chapter 5 presents the mathematical programming models and solution methods of this project, before the data and computational study are presented in Chapter 6 and Chapter 7, respectively. Finally, concluding remarks and future research are presented in Chapter 8.

Chapter 2

Background

This chapter presents the relevant background information for this thesis. First, the industry partner of the thesis, Posten Norge AS, is presented in Section 2.1. In this section a thorough description of the company's current way of planning delivery routes for their home delivery service is provided. Then, Section 2.2 describes a new alternative planning process that will be concerned in this thesis.

2.1 Posten Norge AS

Posten Norge AS is a Nordic postal and logistics group owned by the Norwegian government. The Nordic region comprises the Group's home market, consisting of 38 terminals and approximately 7 000 distribution and delivery points. The Group meets its customers through two brands, Posten and Bring. The Posten brand concentrates on the consumer market in Norway, while the Bring brand serves the corporate market in the Nordic region.

In 2021, the company delivered 90 million parcels and 389 million letters. While falling letter volumes have been the trend for many years, increased digitalization and e-commerce have accelerated parcel delivery growth and profitability. In the last five years, the volumes of letters have halved while parcels have nearly doubled, much driven by the increasing popularity of parcel home deliveries. In the following subsections, we will briefly describe the home delivery service in Norway currently provided by Posten Norg AS.

2.1.1 The Parcel Home Delivery Service

Never before has the Posten Group delivered as many parcels as in 2021. Self-servicing parcel pickup in shops is the standard for customers in Norway, but a growing share of parcels are delivered by home delivery. Attended home delivery is the traditional home delivery concept where the customer is required to be home at time of the delivery. Other delivery methods include parcels being placed at a specific location near your door or even inside your door. However, in-home delivery requires the customer to possess a digital lock, and parcel delivery outside the door is typically not offered in urban areas due to the high risk of having the parcel stolen.

Home delivery is an attractive and convenient option for the customer, but this is the most expensive and time-consuming delivery method for parcel delivery companies. The delivery person has to stop at each customer's home, walk to the door, find the correct doorbell, and hope that the customer is at home. If nobody is home, the parcel must be returned to the logistic terminal for another delivery attempt later or reallocated for customer pickup at a facility. Unsuccessful delivery attempts cause inefficiencies and increase costs. Other important drivers of high costs are traffic jams and missing parking spots in congested streets. To sum up, home delivery is convenient for customers, but it is the most expensive and time-consuming delivery method for parcel delivery companies.

2.1.2 Parcel Sorting and Route Planning

Postal logistic terminals throughout Norway handle parcel and letter mail volumes. At the terminals, parcels are sorted according to their final destination. After being sorted, all parcels are loaded into vehicles according to the route stop sequence. When the vehicle leaves the terminal, the customer receives a text message with an estimated delivery time. As consumers increasingly turn to e-commerce for their shopping needs, speedy fulfilment and distribution have become the expectation of every online shopping experience. However, for parcel delivery companies, time-definite delivery with short deadlines and significant demand fluctuations make it challenging to get parcels in and out of postal logistic terminals efficiently and cost-effectively.

Today's practice of parcel sorting is executed at two different layers. The first layer is performed by automated sorting systems, which sort a vast number of parcels with high speed and reliability. The first-layer sorting takes place based on given intervals of postal codes. Postal codes are four-digit numbers representing a geographical area of a city, a district, or an urban area. The primary purpose of these postal codes is to streamline the sorting and distribution of mail. This first-layer sorting happens in the morning; however, new parcels with same-day delivery may arrive at the terminal throughout the day. The second layer is a manual re-sorting, in which all parcels are further re-sorted and allocated to specific delivery routes. Due to uncertainty in daily parcel delivery volumes, the delivery routes are planned in the afternoon. Hence, the re-sorting happens in the afternoon when route decisions have been made, right before the vehicles leave the terminal.

Posten Norge AS believes that considerable savings could be made by sorting according to delivery routes directly instead of a first layer sorting based on postal code intervals. This would partially or even entirely remove the manual re-sortation layer. However, with today's delivery route planning practice, it is impossible to make these savings as the delivery routes are only determined last minute.

2.2 Parcel Delivery Districting and Routing

An obvious approach to deal with fluctuating demand is to optimize routes on a daily basis, which is the method currently employed by Posten Norge AS. This approach generates minimal cost routing plans by taking into account the actual demand; however, it has significant drawbacks since routes must be computed in a short amount of time, and all parcels must be re-sorted in the two layer parcel sorting system.

There are two main approaches in the literature that focus on reducing the operational complexity of daily routing and scheduling by initiating ahead of time planning decisions: 1) Generate a priori routes and make daily adaptations, and 2) assign drivers to fixed regions (Konvacs et al., 2014). The first approach, which solves the problem of assigning customers to vehicles and determining a visit sequence, can be treated as a classical Vehicle Routing Problem (VRP). However, because of high fluctuation in demand and tight resources, as is common in parcel delivery, the visiting sequence of exact addresses will often require large daily adaptions. The second approach, called districting, considers customers on an aggregated level (basic units), e.g. on the level of streets, instead of exact addresses. The goal of districting approaches is to group basic units into delivery districts such that each district can be serviced by one vehicle. The daily delivery tour for a given set of customers of a district is then planned either based on specific driver knowledge or with the help of computerized decision support. However, as customer demand fluctuates, strictly fixed service regions will typically produce far from optimal routing solutions.

This thesis considers a parcel delivery districting and routing problem, called the Route Partitioning Problem (RPP), on behalf of Posten Norge AS. In this problem, the two dominant strategies existing in the literature are combined into a two-stage approach. The first stage proposes long-term districting decisions on a strategic level, in which a set of postal code areas serviced by a terminal is partitioned into smaller areas by grouping customers on the level of streets. Then, the second stage designs tactical vehicle routes through the set of areas, such that a visit sequence of areas is determined for each vehicle. The aim of this new planning process, is to reduce operational complexity without significantly increasing route costs.

The strategic problem involves decision-making for a relatively long planning horizon, typically several months, such that the partitioning should accommodate the whole range of possible demand levels. The tactical routing decisions can be more frequently updated. To account for either daily, weekly or monthly demand fluctuations, a set of different tactical routing plans are to be constructed for different demand levels and operated unchanged for a given period of time. Although the actual parcel volume of a given day changes until vehicles leave the terminal, forecasts are usually available in advance. Hence, the appropriate tactical routing plan, matching a given demand prediction, for either the upcoming day, week or month, can be selected in advance from the set of predefined tactical routing plans. This enables parcels to be sorted to a given route ahead of time, decreasing the requirement of manual re-sorting.

Although the customer data examined in this project belongs to Posten Norge AS, the method could be applied to any network of customers. Thus, the findings of this thesis could be relevant to both other delivery problems and competing industry players.

Chapter 3

Literature Review

This chapter presents the literature relevant to our work. First, an explanation of the literature search strategy is given in Section 3.1. Then, a brief overview of different research areas within Postal Logistics is conferred in Section 3.2. Further, research concerning optimization of a priori routes or districts for delivery operations is presented in Section 3.3. Finally, we position our work within the literature and summarize views regarding our contribution in Section 3.4.

3.1 Literature Search Strategy

This literature review is devoted to districting and routing problems on a strategic or tactical level. The search terms to attain relevant literature were classified into five themes as seen in Table 3.1. This classification formed the basis for the search strings, as different conjunction combinations of the terms were applied in Google Scholar. In addition, other scientific papers relevant to the project that did not appear in the literature search were included to ensure the comprehensiveness of the literature search. The primary source for the literature search is Google Scholar, but additional resources from other scientific citation databases like Scopus and Springer Link were also used. Literature was selected by reading abstracts, and relevant papers were studied in more detail by investigating mathematical modeling formulations and solution approaches.

Table 3.1: Grouping of terms used in search strategy

Problem	Application	Objective	Constraints	Solution method
Districting	Postal service	Compactness	Contiguity	Mathematical model
Territory design	Parcel delivery	Travel Time	Compactness	Exact
Zone design	Logistics	Distance	Balance	Heuristic
Vehicle Routing	Tactical level	Minimization	Barriers	Location-allocation

To further distinguish the more relevant literature from the less, some inclusion and quality criteria were identified as presented in Table 3.2. The inclusion criteria recognize key features that align with the scope of this thesis and were used to order the literature by relevance. The inclusion criteria (IC) were divided into primary (IC1 and IC2) and secondary (IC3 and IC4) criteria according to importance. Further, the quality criteria (QC) were used to evaluate the studies that fulfilled the inclusion criteria. QC1 and QC2, contemplate the overall relevance and trustworthiness of the literature. Eventually, almost 40 scientific papers were perceived relevant for our project. 10 of these papers were directly impacting our choice of mathematical modelling formulation and solution approach in one way or another. These papers are further described in Section 3.4.

Table 3.2: Inclusion and quality criteria used for articles

	Criteria
IC1	The study's main focus is districting and/or routing on a strategic or tactical level
IC2	The study cope with fluctuating demand
IC3	The study cover a relevant real-world application area
IC4	The study formulates a mathematical model
QC1	The research objective is clearly stated
QC2	The study is widely referenced

3.2 Optimization in the Postal Service

Postal service optimization is a widely researched subject and has lately received renewed interest. Increased market competition and growing parcel volumes, force the postal companies to constantly improve their networks for letter and freight mail. The goal is usually to reduce transportation and delivery time, or to minimize costs under service quality constraints (Sebastian, 2012). In this section, we argue that the problem at hand is both a strategic and tactical activity.

3.2.1 The Distribution Networks for Letter and Parcel Mail

A distribution network for letter and parcel mail consists of many components. Sebastian (2012) separates a typical distribution network into four subnetworks as illustrated in Figure 3.1.

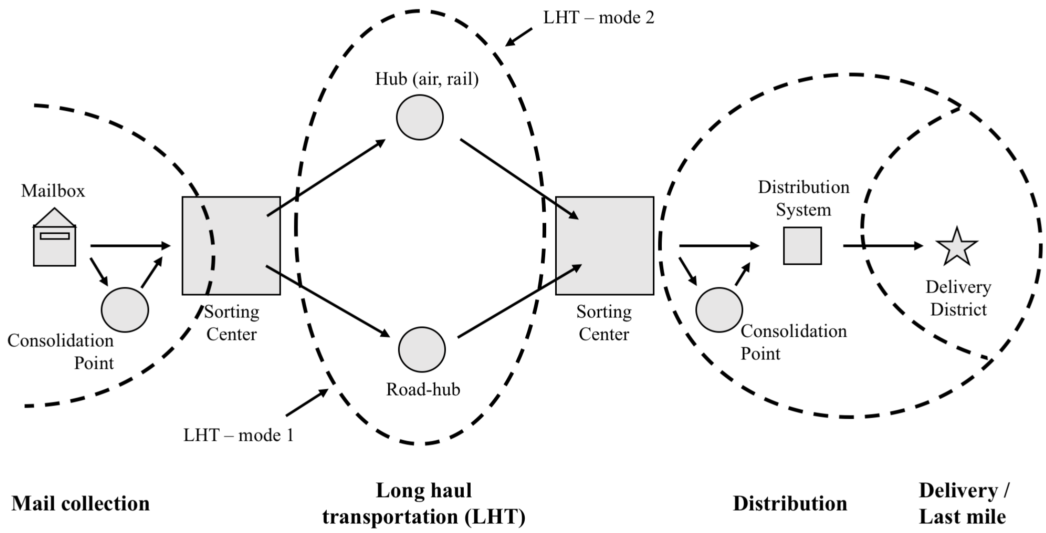


Figure 3.1: The four subnetworks of a typical letter and parcel mail distribution network

Mail collection: In this subnetwork mail is collected from different mail sources, and consolidation points are used in order to transport the mail to sorting centers. *Long Haul Transportation (LHT)*: Exchanges the mail between the sorting centers. The idea is to use consolidation and large vehicles for the long distances. *Distribution*: The distribution networks distribute the mail from sorting centers to a central depot, where the final preparation for the postmen's tours takes place and where the postmen usually start their delivery routes. *Delivery (The Last Mile)*: Postmen visit their assigned delivery districts in order to deliver the mail to customers. In this thesis we do not cover the complete parcel supply chain, but focus on last-mile delivery.

The definition and scope of last-mile delivery differ, but it is widely agreed that the term refers to all logistics activities related to the distribution of shipments (Boysen et al., 2020). According to this definition, last-mile delivery starts once a shipment has reached a starting point in an area, for instance, a depot after long-haul transportation, and ends when the shipment reaches the final customer's preferred destination point. For this reason, problems related to home delivery of parcels, as of this thesis, are within the area of last-mile delivery.

3.2.2 Different Levels of Planning

The literature separates the Postal Logistics into three levels of planning, the *strategic*, *tactical* and *operational* level (Sebastian, 2012). At the strategic level, the long-term type of decisions are made. These decisions are typically the quantity and quality of the main resources such as locations, facilities and human resources, and the selection of services to be offered. Decisions in the tactical phase are mid-term decisions related to the design of the service network, assuming that the strategic decisions have already been made. These decisions are typically definition of routes. Decisions of the operational phase are short-term decisions often made on-line and in real-time, and typically performed by local management. For example the planning of routes each morning after actual customer demand is known.

Planning of delivery routes before full demand information becomes available is a strategic or tactical decision. The routing plans are often used for a longer period of time, and the main goal is typically robustness with regard to cost, that is, finding a single set of routes that minimizes the expected travel cost for a large number of demand realizations. Tactical planning of delivery routes is the problem regarded in this thesis, and different approaches to solving the problem is described in Section 3.3.

3.3 Approaches to Deal with Fluctuating Demand

Planning new routes each day to cope with fluctuating demand results in minimal cost routing plans, but it entails many operational difficulties. Therefore, alternative approaches have been proposed in the literature. These approaches can be divided into two main groups: approaches that generate *a priori* routes and perform daily adaptations if necessary, and approaches that assign each driver to a fixed service region (Konvacs et al., 2014). The first approach generates a route which specifies an ordering of all possible customers that a particular driver may need to visit, and can be treated as a classical Vehicle Routing Problem (VRP). The second approach in which potential customers are grouped into districts, can be modelled as a Districting Problem (DP).

Both approaches make decisions before full demand information becomes available and the cost implications of applying operational routes derived from the preplanned routes or districts is therefore an important topic in most papers. In the following subsections, we provide an overview of the two approaches, the a priori routing approach and the districting approach, respectively.

3.3.1 A Priori Routing

A rich literature has been developed on various VRPs, commonly distinguished into deterministic (DVRP) and probabilistic/stochastic (PVRP or SVRP) versions of the problem. In a practical context, many aspects of the VRP cannot with certainty be known in advance, such as customer demand, customer locations, travel times and service time (Bouyahia et al., 2018). Stochastic VRP formulations are employed to reflect this real-world uncertainty.

The a priori based optimization methods have been used to solve the Stocastic VRPs. In the context of vehicle routing, a priori based solutions mean that the planner determines one or more routes based on probabilistic information. The a priori route is then used as a basis for the daily delivery routes (Barrett et al., 2008).

3.3.1.1 The Cost of A Priori Routing

The cost of applying routes derived from a priori routes, compared to the cost of routes optimized each day from scratch, depends on different characteristics of the application at hand. A key concern is the level of demand fluctuation incurred in the specific application (Konvacs et al., 2014). Konvacs et al. (2014) argue that designing a priori routes is typically a cost-competitive alternative when demand remains relatively constant from day to day. On the contrary, when large fluctuations in demand prevail, much overhead is generally required to update the routes to match the realizations of a given day, meaning that an a priori approach might not be the most cost-effective. However, Benton et al. (1992) argue that designing a priori routes can be valuable from a practical standpoint, as there are a great deal of hidden costs associated with routes optimized each day from scratch. These hidden costs can include consistency and customer relationship considerations, as well as management overhead.

3.3.1.2 Overview of Applications

In the context of routing, there are several areas which might contain uncertainty. One of the first appearances of a priori routing in the literature was proposed by Bartholdi et al. (1983). In this paper, the authors develop a priori routes for use in meals-on-wheels routing. A routing system based on a travelling salesman heuristic was successfully implemented to handle the efficient daily routing of a varying number of vehicles to delivery points who's location changed daily. Further, Bouyahia et al. (2018) propose a novel SVRP algorithm that takes into consideration both uncertain transport demand and travel time, in the context of chartered buses. Other applications adopting the concept of a priori routing include, e.g., schoolbus routing, municipal waste collection, and daily delivery of dairy goods. In these applications uncertainty can be incorporated in travel times, service times or the presence of customers in the routing plan (Golden et al., 1978).

3.3.1.3 Overview of Optimization Criteria

All vehicle routing problems, regardless of application, typically impose the following design criteria: 1) Each task is served by exactly one vehicle, 2) The total demand of tasks on a route must not exceed the vehicle capacity and 3) each vehicle begins and ends its route at a depot (Stewart Jr. et al., 1983). In the following section, other typical routing criteria are introduced.

Consistency
Route consistency is a side benefit of a priori optimization, as the daily routing adaptations is derived from the same set of a priori routes (Konvacs et al., 2014). As a consequence of consistency, the driver get familiar with his service area, hence increasing the driver's efficiency at performing the required routing operations. Additionally, the customers in

the area will generally be visited by familiar faces, which also increases the customer satisfaction, and allow for the establishment of a driver-customer relationship (Bertsimas, 1992; Vidal et al., 2020).

The realized consistency in the daily routing adaptations, depends upon the selected updating strategy (Konvacs et al., 2014). Different strategies are employed, depending on when the demand information gets available for the driver (Campbell et al., 2008). When demands are revealed upon arrival at the particular customer, a commonly applied strategy is for the driver to visit every single customer in the order defined by the a priori route, however, only customers with demand are serviced. Then if the customer demand exceeds the vehicle capacity, a route failure is said to occur. In such case, an additional expected costs is imposed, as the vehicle must take a detour to the depot for reloading to ensure route feasibility (Bertsimas, 1992). In situations in which the customers with demand are known prior to the start of the tour, a common updating strategy is for the driver to visit all customers in the same order as defined by the a priori route, but skipping the customers with no demand (Laporte et al., 1994; Errico et al., 2016). Depending on the fluctuations in demand, and hence how many customers are skipped from one day to another the route consistency is impacted (Konvacs et al., 2014).

Compactness & Separation

Compact and non-overlapping routes are commonly employed criteria for many routing problems, which are argued to increase driver acceptance of the proposed routing plan, as the routes become more visually appealing. At the same time, such plans are typically associated with a reduction in travel time overall. Further, designing non-overlapping routes, are argued to simplify route coordination, as intra-route changes, due to e.g. unexpected demand, might not affect the remainder of the routes (Lum et al., 2017).

Poot et al. (2002) suggest to optimize route compactness by accounting for a measure of geographical spread, by e.g. minimizing the average distance between all pairs of customers in a route. Separation of the routes is typically attained by minimizing a measure of overlapping routes, by e.g. minimizing the number of customers that are closer to a center or median of another route than the one they are currently assigned to (Poot et al., 2002; Vidal et al., 2020).

Balance

Driver acceptance generally also increases by endeavouring balanced routes, with respect to driver workload. Balanced routes tend to reduce both the chances of overtime and bottlenecks in resource utilization (Lum et al., 2017; Vidal et al., 2020). In the mathematical modelling formulation, balance can be sought by e.g. imposing upper bounds on service duration or the number of customers a given vehicle is allowed to service (Vidal et al., 2020).

3.3.1.4 Overview of Solution Methods

Both exact and heuristic approaches are proposed in the literature to acquire a priori routing solutions. Due to the complexity of the problem, solving a priori routing problems with exact methods is challenging for larger sized problems. Hence, most papers address heuristic approaches (Campbell et al., 2008).

Mathematical Programming Approaches

Laporte et al. (1994) propose an exact method to construct a priori routes for the SVRP, with the aim of acquiring a first-stage tour that minimizes the expected cost of the second-stage tour. The authors formulate a stochastic program and suggest a branch-and-cut approach to construct a Hamiltonian cycle through all customers. The computational results indicated success for merely an instance size of 50 customers or less. The same accounts for the more recent paper addressed by Errico et al. (2016), proposing an exact branch-and-price algorithm for the Vehicle Routing Problem with Hard Time Windows and Stochastic Service Times (VRPTW-ST). The VRPTW-ST is modelled as a two stage Stochastic Program with Recourse (SPR), that in the first stage establishes a set of a priori routes and in the second stage performs recourse actions based on realized demand for service a given day. Again, results on benchmark data, implied that only instances up to 50 customers were possible to solve.

Heuristic Approaches

To reduce complexity of the a priori models, heuristic approaches are commonly concerned. An early paper addressing a cyclic heuristic to solve the SVRP, is proposed by Bertsimas (1992), in which an a priori sequence through all potential customers with a known probability distribution is constructed. The author argues that his heuristic procedure performs satisfactorily from a worst-case perspective, in particular if the demand distribution of the customers remains constant.

Further, Gendreau et al. (1996) propose a Tabu Search heuristic for the Vehicle Routing Problem with Stochastic Demands and Customers (VRPSDC). A set of a priori routes are constructed in the first stage, starting at the depot, visiting all customers, and then returning to the depot when the tour is finished. In the second stage, the a priori routes are followed until a route failure occurs. The authors present computational tests indicating that instance sizes of several hundreds were solved to optimally applying their proposed Tabu Search heuristic. A more recent paper that also provide valuable results for large scale instances, is addressed by Bouyahia et al. (2018), introducing a simulated annealing heuristic for the construction of a priori routes in a SVRP with stochastic travel times.

A proposed method in the literature to reduce the overall complexity of an a priory SVRP tour, is to aggregate the individual customers of the service area into regions, such that the a priori tour turns into a tour through given regions rather than customers (Campbell et al., 2008). Horvat-Marc et al. (2018) address such an approach, solving a routing problem through predefined clusters of aggregated customers; a so-called Clustered Vehicle Routing Problem (CluVRP). The authors propose a novel decomposition-based method to solve the CluVRP, that splits the problem into two easier subproblems. The first subproblem establishes the a priori routes using a genetic algorithm, visiting clusters in a given order. Then using a Simulated Annealing algorithm, the second subproblem finds, for each collection of a priori routes, the sequence of customers in each cluster to visit, such that all customers in a cluster is visited consecutively by the same vehicle.

3.3.2 Districting

In the a priori routing approach customers must be preassigned to routes, along with the sequence of customer visits. The downside of this approach appears when significant change in demand prevails, as much overhead in the daily adaptations will be imposed. In the alternative approach, called distrcting, each driver is assigned to a fixed delivery region instead of a fixed route (Konvacs et al., 2014). Then, for any given day, a daily route can be designed within each district, either manually, based on area-specific knowledge of the driver, or with the help of optimization decision tools (Boysen et al., 2020).

3.3.2.1 The Cost of Districting

The price of the districting first-routing second approach, is obviously the possible increase in routing cost due to the loss of flexibility in constructing optimal routes (Konvacs et al., 2014). In the papers proposed by Christofides (1971) and Hardy (1980), the vehicle travel distance for servicing customers in predefined districts, is compared with routes constructed each day from scratch. The results indicate an increase in travel distance for the district-dependent tours of merely 3.8% in Christofides (1971), and 7.1% in Hardy (1980). When the increase in travel cost, imposed by district requirements is not significant, it can be argued that the positives of implementing a districting approach over a daily re-optimization of routes, surpass the reduction in flexibility; e.g. increase in driver familiarity (Haugland et al., 2007; Carlsson et al., 2013), and reduced size and complexity of the routing problem (Sevaux et al., 2008; Defryn et al., 2017).

3.3.2.2 Applications within Distribution Districting

Numerous real-world applications have been applied to the DP. Kalcsics et al. (2015) identifies four categories of DPs: *Political districting*, *sales districting*, *service districting* and *distribution districting*. In this thesis we focus on districting problems for vehicle routing applications, and more specifically, the area of distribution districting.

Distribution districting typically regards establishing districts where a set of customers have varying day-to-day demand that must be satisfied through a pickup and delivery planning problem (Langevin et al., 1989; Zhong et al., 2007; González-Ramírez et al., 2017). Typically, the customers are assigned to a fixed driver, so that the drivers become familiar with their districts, and hence increase their performance and the customer satisfaction (Zhong et al., 2007). Common for most DPs that facilitate for the organization of delivery and pickup operations, is to seek a compact and contiguous partitioning (Ramirez et al., 2011; Bender et al., 2020). Endeavouring balanced workload with respect to e.g. vehicle travel time or vehicle profit is also common for many problems (Haugland et al., 2007; Lei et al., 2016). These criteria are further discussed below.

3.3.2.3 Overview of Optimization Criteria

Different planning criteria can be applied to ensure that the districting partitioning account for the routing operations to be performed within the district borders. A key criteria that is generally mandatory in most DPs, regardless of application, is the requirement of a complete and exclusive assignment of the districts. That is, every basic unit must be assigned to exactly one district (Laporte et al., 2015).

Along with the endeavorment of a balanced, compact and contiguous partitioning, how to identify district centers is a key modelling criteria. Also, when constructing districts that facilitate for efficient routing, accounting for geographical barriers in the final partitioining have been discussed as a key criteria (Novaes et al., 2010). This section begins with a description of how DPs commonly define the district centers. Further the criteria of balance, compactness and contiguity, respectively, are presented, before the barrier inclusion criteria is specified.

District Center

Identifying a district center is required to evaluate important districting criteria, such as contiguity and compactness. Depending on the problem, these centers can either be predetermined or be subject to the planning process. The number of districts generally corresponds to the number of center points and the number is usually predefined.

Solving a DP in which the location of centers and the allocation of basic units are to be decided simultaneously, induce great complexity and is limited to instances with a few hundred basic units when using exact methods (Kalcsics et al., 2005). Hence, when solving DPs with centers as decision variables, a common approach in the districting literature is to fix the district centers in a heuristic approach (Kalcsics et al., 2005). Location-allocation based heuristic algorithms are the most common ones (Yanık et al., 2016). Successful applications of this location-allocation heuristic applied to DPs can be found in the works of for example George et al. (1997), Fleischmann et al. (1988) and Bender et al. (2020).

Balance

Balance seeks an evenly distributed workload among the delivery vehicles assigned to a given district. Such workload balance is desired as it typically maintain employee satisfaction, and reduce bottlenecks in resource utilization (Vidal et al., 2020). Additionally, attaining a balanced workload partitioning based on the contractual working times of the drivers, enables the potential overtime required to be fairly equalized among the drivers (Bender et al., 2020).

The balance requirement is included in the mathematical model, either as hard constraints (Hess et al., 1965; Zoltners et al., 2005) or incorporated in the objective function (Blais et al., 2003; Assis et al., 2014). When included as hard constraints, equitable districts are accomplished through lower and/or upper bounds on a given measurement. Additionally, some papers impose balance in both ways, utilizing hard constraints and a balance measurement in the objective function (Bergey et al., 2003; Salazar-Aguilar et al., 2013).

Compactness

Compactness indicates that a district is somewhat round-shaped and undistorted, and is endeavoured as it tend to reduce travel distances within each district (Kalcsics et al., 2015; González-Ramírez et al., 2017). Additionally, compact shaped districts favour flexibility in the routing decisions, with respect to the sequence of customer visits, without an excessive increase in distance travelled. This can be advantageous if unforeseen events occur, such as traffic congestion or if a new customer visit is required to e.g. pick up a package becomes known during the tour (Bender et al., 2020). Regarding traffic congestion, Bender et al. (2020) discuss that compact districts reduce the impact of the congestion due to on average shorter edges comprising the vehicle route, especially for tours occurring in urban regions.

Contiguity

District contiguity ensures that any pair of basic units in a given district can be linked through other basic units within the district, i.e. it is possible to travel around the district without leaving its borders. For delivery applications, the hope is that contiguous districts reduce unproductive travel times (Kalcsics et al., 2015). Contiguity appears when an underlying graph represents adjacency between basic units, and assures that each district induce a connected subgraph (Ríos-Mercado, 2019). The graph, wherein nodes correspond to basic units and an edge exists between neighboring basic units, is known as the *contiguity graph*. The contiguity graph is almost always present in papers that account for contiguity, even when it is not explicitly referred to (Ricca et al., 2013).

The main challenge for solving DPs via exact methods is to find an efficient way to deal with contiguity: Solving DPs with exact methods requires an exponential number of contiguity constraints. For this reason, many of the models proposed in the literature do not explicitly model contiguity as a criterion. Instead, the hope is that a compactness-seeking objective suffices to obtain contiguous districts (Hess et al., 1965). Compactness is strictly related to contiguity. However, even if compactness may help in pursuing contiguity, it does not guarantee contiguous districts. Hence, contiguity should be imposed as a hard constraint to avoid manual postprocessing. A simpler heuristic approach to model contiguity is successfully applied in the papers proposed by Ríos-Mercado et al. (2013) and Bender et al. (2020), in which some assignments of basic areas to given center points are restricted to reduce the complexity of the model.

Geographical Barriers

Geographical barriers are commonly referred to in many papers in the literature, however, few papers explicitly account for them in their model formulation. A distribution DP by Novaes et al. (2010) is one of the papers that concern barriers in their construction of a districting plan. In this problem the displacement of vehicles is restricted by physical barriers, more specifically freeways in the road network. Using a Voronoi diagram approach, the delivery districts cannot be designed such that they cover areas located at both sides of the freeway.

Furthermore, a political districting problem by George et al. (1997) and a school redistricting problem by Sutcliffe et al. (1984), account for barriers in the disticting optimization. Both papers represent barriers as a parameter with weighted values. In George et al. (1997) the barriers are represented as e.g. waterways or a mountain ranges, and if a line segment between a basic unit and an electoral district centroid is crossed by a barrier, the distance between this basic unit-district pair is increased to aim at preventing the assignment to occur. In Sutcliffe et al. (1984) a parameter representing the difficulty for a student to travel from a primary school to a transferring secondary school, is aimed to be minimized. The difficulty of travel is measured by travelling obstacles, such as requirements to walk over half a mile, crossing the town center or the possibility for congestion.

3.3.2.4 Overview of Solution Methods

Careful attention is often given to the problem of deciding on efficient vehicle routing operations within predetermined districts. However, significant savings could be made if the routing application at hand was considered at the time the district borders were determined (Cattrysse et al., 1997). Taking the routing decisions into account when the district borders are established, enables the final districting plan to be more robust to minor changes in the daily routing operations. Attaining the same districting plan for various routing scenarios, makes sense from a managerial point of view, because of the fact that even though a different districting solution might prove to be more cost effective for a different scenario, the determination of district borders typically is a long-term decision, whereas the routing decisions often changes from day to day (Cattrysse et al., 1997; Haugland et al., 2007).

Various solution methods are proposed in the literature to solve distribution DPs that facilitate for the organization of routing activities. The earliest contribution is based on geometric algorithms. Later, both mathematical programming approaches and heuristic approaches are proposed. This section presents an overview of the most common approaches to solve the distribution DP.

Computational Geometry Approaches
One of the earliest contributions to solve a districting problem that facilitate for the organization of routing activities, is proposed in the letter and parcel pickup and delivery problem by Langevin et al. (1989). The authors propose an iterative procedure based on computational geometry, in which an urban region is partitioned into approximately rectangular zones organized into concentric rings around a center depot. The final partitioning endeavours minimal length of multiple-vehicle tours, and the vehicles should with high probability be able to service all customers in their zone.

Improvements to the ring-radial model presented in Langevin et al. (1989) is proposed by Galvão et al. (2006). In this paper a Voronoi diagram approach is applied to solve a distribution DP. The Voronoi diagram of a set of points is the division of the plane into polygonal cells. Each polygonal cell is associated with a generator center point, and will contain the area of the plane that lies closer to this point than to any other. Hence, the boundaries between the subregions are line segments, where each borderline separating two areas lie exactly in the middle of the associated generator points (Novaes et al., 2009). Galvão et al. (2006) discuss that their Voronoi diagram approach provides a better representation of the underlying road network, as it relaxes the initial district boundaries that is confined in other geometrically-shaped districting patterns, such as the ring-radial partitioning. The increased freedom when determining the district contours is argued to make the vehicles obtain more equalized utilization levels.

Mathematical Programming Approaches

Carlsson et al. (2013) propose a mathematical programming approach to solve a distribution DP. With the aim of designing a partitioning with balanced workload among the drivers, they present two branch-and-bound algorithms. The first attains a globally optimal solution by minimizing the worst-case Traveling Salesman Problem (TSP) tour of the two subregions of a 2-partitioning problem. Further, the second algorithm finds a local optimum by solving the more complicated n-partitioning problem, in which each subregion have asymptotically equal worst-case TSP tours, such that a balanced workload partitioning of the region is obtained.

Similarly to Carlsson et al. (2013), Carlsson (2012) solves a special case of the equitable partitioning problem, in which a recursive algorithm is employed to construct a partitioning such that multiple TSP tours of all the points in each subregion, starting and ending at the depots, are asymptotically equal.

Heuristic Approaches

Some of the more recent papers in the literature solving a DP in a vehicle routing context, include both strategic districting decisions and operational routing decisions. Haugland et al. (2007) address this problem, solving a two-stage Stochastic Program with Recourse (SPR) with stochastic customer demands. The districting decisions are heuristically constructed in the first stage, using either a Tabu search or a multistart heuristic. Then, before the vehicles leave the depot, the actual demand is realized, and a deterministic VRP is solved for each district in the second stage.

Lei et al. (2012) and Lei et al. (2016) address similar two-stage SPRs as Haugland et al. (2007), where the districting decisions are made in the first stage, in which some of the customers are modelled as random variables. In the second stage, after the stochastic customers are revealed, the Beardwood–Halton–Hammersley formula is used to approximate a TSP tour over all customers in a district.

Further, Bender et al. (2020) present a novel two-stage districting approach, in which tactical delivery districts are established heuristically in the first stage, before flexibility is added by allowing for adaptations of the districts based on the concrete demand realization of a given day in the second stage. The first stage problem is solved with a location-allocation heuristic that decomposes the problem into two subproblems, which are solved iteratively until convergence is attained. The first subproblem involves location decisions of a given number of district centers, and the second subproblem allocates basic areas and driver types to these centers. The second stage of the problem, deals with operational planning decisions solved on a day-to-day basis. This problem uses the tactical solution, together with the demand realizations of a given day to solve an IP model that reassignes basic areas to different delivery districts if the maximum workload of a district is exceeded.

3.3.2.5 Accounting for Uncertainty

The way of accounting for uncertainty is a key modelling choice when solving DPs that facilitate for efficient routing, as the actual customer demand is typically not known prior to the establishment of the districts. The uncertainty is typically handled with either deterministic or stochastic input parameters (Konvacs et al., 2014).

One approach to account for uncertainty using deterministic input parameters, is to sample customer demand from historical data (Christofides, 1971; Hardy, 1980; Bender et al., 2020). Additionally, deterministic input parameters are often acquired from given probability distribution functions. Both Langevin et al. (1989) and Novaes et al. (1999) apply a discrete distribution function, by distributing the customer in accordance with a Poisson distribution. Further, Carlsson (2012) and Carlsson et al. (2013) account for uncertainty in customer locations by considering independent and identically distributed samples from a given probability density function, in such a way that a worst-case distribution of the locations are attained.

Stochastic input parameters are generally applied in problems solving DPs with Chance-Constrained Programming (CCP) or a Stochastic Program with Recourse (SPR). Novaes et al. (1999) solves a CCP, in which the customer demands and service times are included as random variables, and the probability to exceed the vehicle capacity and the daily cycle time of a vehicle tour is below a given threshold. Using a CCP model to solve a stochastic problem, rather than a SPR, allow to delay the inclusion of corrective actions to a later stage of the modelling. This is beneficial when operational decisions are not accounted for (Novaes et al., 1999). The more recent papers solving a stochastic program, include both strategic districting decisions, in which stochastic input parameters are incorporated, and operational routing decisions when the actual demand is known. This applies to the papers proposed by e.g. Haugland et al. (2007), Lei et al. (2012) and Lei et al. (2016), solving a two-stage SPR, in which all or a subset of the customers requiring service are included as random variables in the first stage when the districting decisions are made.

3.4 Our Contribution

Decision support for distribution district design has a long tradition in operations research. However, to the best of our knowledge, we are a pioneer paper in the literature to combine the districting approach and the a priori routing approach to offer tactical routing solutions for parcel delivery operations.

We propose a two-stage solution approach, to solve the problem refereed to as the Route Partitioning Problem (RPP). The two stages are solved sequentially; the first solving a strategic DP to acquire a polygon partitioning, and the second adopting the cluVRP idea to construct tactical routes through the set of predefined polygons. In contrast to the papers in the literature addressing the cluVRP, our first stage districting approach, allow us to consider multiple elements of the real-world to construct fundamentally strong building blocks for the routing operations to be performed.

This thesis includes criteria that increase the practical usability of our project. To the best of our knowledge, few other existing papers on districting concern real road network distances and barriers. Also, we propose a new measure that we refer to as isolation. The inclusion of road network distances enable a more precise prediction of drivers' travel time. The tactical routing decisions will thus likely be more analogous to an operational vehicle routing decision. Reducing the need for drivers to pass through road obstacles, i.e. barriers, such as freeways, accommodate both driver efficiency and satisfaction. Lastly, to the best of our knowledge, the measure referred to as isolation, has never before been including as a criteria in the literature on DPs. The measure seeks to find a partitioning in which each polygon is isolated in the road network, meaning that a polygon has few possible entry/exit points. Accounting for isolated units make it more likely that a polygon-independent tour is similar to the delivery tour bounded by the polygon borders. This measure is further discussed in the following chapters.

The realistic nature of our project is further accommodated by favouring robust and flexible decision-making. Robust routing solutions are accomplished by accounting for estimated vehicle delivery time already in the strategic districting decisions. In this way, the polygons will be more robust to changes in daily demand. Further, traditional approaches designing delivery districts provide little flexibility, and ignores the problem of highly varying demands through seasons. The few papers that explicitly incorporate flexibility into their model formulations, account for operational routing adaptions based on daily demand realizations. In our proposed method, flexibility is integrated already on the tactical planning level, by allowing for the construction of different tactical routing plans for various demand levels.

Lastly, this thesis contributes with a computational study which asses the performance of our two-stage solution approach on attaining satiable tactical routing solutions for large instances encountered in practice. We provide extensive computational results from a real-world case study provided by Posten Norge AS, in which the impact of specified polygon design determinants are evaluated by comparing the performance of our tactical routes against a benchmark of deterministic VRP solutions.

A systematized outline of the most relevant literature used to get insight on districting and a priori routing problems, and inspiration on solution approaches for this thesis, is provided in Table 3.3. The relevance is determined based on the inclusion and quality criteria presented in Section 3.1. A "✓" is indicated when a given criterion is proposed in the referenced paper. Our contribution is provided at the end of Table 3.3.

Table 3.3: Systematization of literature considering preplanning of daily routes using either an a priori routing or districting approach

Reference	Strategy	Planning level	Daily adaptation/recourse	Objective function	Road network distances	Barriers	Solution method
Errico et al. (2016)	A priori routing	Strategic	✓	Min expected cost			Exact
Bertsimas (1992)	A priori routing	Strategic	✓	Min expected cost			Heuristic
Gendreau et al. (1996)	A priori routing	Strategic	✓	Min expected cost			Heuristic
Horvat-Marc et al. (2018)	A priori routing	Strategic		Max compactness			Heuristic
Bender et al. (2020)	Districting	Tactical	✓	Max compactness	✓		Heuristic
Haugland et al. (2007)	Districting	Strategic	✓	Min expected cost			Heuristic
Lei et al. (2012)	Districting	Strategic	✓	Max compactness			Heuristic
Lei et al. (2016)	Districting	Strategic	✓	Max compactness and balance			Heuristic
González-Ramírez et al. (2017)	Districting	Strategic		Max compactness and balance			Heuristic
Novaes et al. (2010)	Districting	Strategic		Max compactness and balance		✓	Exact
Our contribution	**Districting and a priori routing**	**Strategic and tactical**		**Min expected delivery time**	✓	✓	**Heuristic districting and exact routing**

Chapter 4

Problem Description

This chapter presents the Route Partitioning Problem (RPP). Section 4.1 gives a description of the problem, before the objective is described in Section 4.2. Lastly, Section 4.3 gives an overview of the problem assumptions.

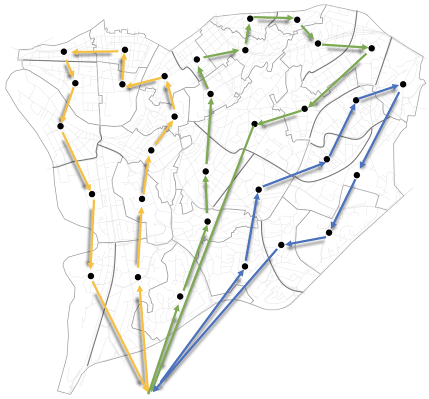
4.1 Problem Definition

As introduced in Chapter 2, the Route Partitioning Problem (RPP) solved for a given service area, is the problem of (1) performing strategic districting decisions by partitioning each postal code area, represented by its road network, into a set of non-overlapping polygons, and (2) determining a set of tactical routes for home delivery of parcels, each route represented by a sequence of polygons, starting and ending at the terminal. Figure 4.1a shows an example of a subset of postal code areas belonging to the same service area, while Figures 4.1b and 4.1c illustrate problem (1), and (2), respectively.

On a strategic level, we have to subdivide the road network into an adequate number of polygons that in the best way accommodate polygon-*dependent*-tours, i.e. a tour where the driver services all customers in a given polygon before travelling to the next. Larger polygons are likely to have less variance and support more precise approximations of expected demand in a polygon. Precise approximations are important for the tactical routes to be valid when actual demands are revealed. On the contrary, the polygons

(a) Postal code areas of a given service area

(b) Partitioning of postal code areas into strategic polygons



(c) Polygons sequenced into tactical routes starting and ending at the terminal

Figure 4.1: Illustration of the two problems of the RPP. Figure 4.1b illustrates the strategic problem, while Figure 4.1c illustrates the tactical problem. In Figure 4.1b, a red line represents a partitioning of a postal code area into polygons. In Figure 4.1c, each colored path through the set of polygons represents a tactical route, starting and ending at a logistic terminal

should leave a fair amount of flexibility in the tactical routing phase to minimize costs. Smaller polygons provide more flexibility as they enable a greater number of possible polygon-sequence combinations. This implies that the final route design, bounded by polygons, is more likely to be within a tolerable gap from the optimal length when smaller polygons are employed. Hence, we face conflicting objectives between which a reasonable compromise must be found.

The partitioning of a postal code area into non-overlapping polygons is conditioned on the expected demand for parcels on each specific road. The expected demand for parcels on each road is the aggregation of the expected demand of customers living at the addresses along the road. Any reasonable polygon must, with high probability, always contain at least one requested delivery. This criterion puts a lower bound on the size of a polygon. Furthermore, polygons should be contiguous and rather isolated in the road network. The greater the degree of isolation, the greater the probability that any optimal polygon-*independent*-tour visits the same customers and completes them all before traveling further. To further support isolation, physical barriers should be taken into account when determining polygon boundaries. A freeway typically have crossing points spaced along the road which are frequently congested. As a practical consequence, one should not design polygons covering areas situated at both sides of the freeway. Finally, it is important that polygons are compact, as compact polygons promotes short intra-polygon distances.

On a tactical planning level, the polygons are further grouped into a sufficient number of delivery routes, constrained by a maximum number of delivery vehicles. A tour should be a contiguous set of polygons, which can be serviced by a driver in a single tour starting and ending at the terminal. Within a tour, the risk of exceeding a driver's maximum workload should be low. The workload of a tour can only be approximated, as the workload of a driver depends on the travel time and service time and hence on the concrete tour, of which is not yet determined at the time of the polygon tour construction.

4.2 Problem Objective

The objective of the RPP is to minimize the total expected delivery time of a set of tactical delivery routes, constructed based on a partitioning of postal code areas into non-overlapping, contiguous, compact and isolated polygons, constrained by the drivers' maximum workload and the number of available vehicles.

4.3 Problem Assumptions

When solving the problem described above, some simplifications are introduced. The assumptions made for the problem studied in this thesis are given below.

(i) For simplicity we assume a homogeneous fleet of vehicles.

(ii) The vehicle capacity is not a constraining resource. That is, there is sufficient space in the vehicle for all parcels on each route. However, each route is constrained by the duration of the driver's work-shifts.

Chapter 5

Modeling and Solving the Route Partitioning Problem (RPP)

In this chapter, we present the mathematical models and the solution methods to solve the Route Partitioning Problem (RPP) in two stages. An overview of the model decisions and a description of the relationship between the models is provided in Section 5.1. Next, Section 5.2-5.3 presents the Integer Programming (IP) model and the solution method to solve the first stage strategic districting problem. Then, Section 7.2.2 describes how the expected delivery time is calculated within a polygon using Monte Carlo simulation. Lastly, Section 5.4 presents an IP model for the second stage tactical routing problem.

5.1 Description of the Two-Stage Model

Figure 5.1 illustrates our proposed two-stage approach for solving the RPP. In Stage 1, postal code areas are partitioned into smaller polygons using the location-allocation heuristic algorithm outlined in Section 5.3. The algorithm iteratively solves a location and an allocation phase until a satisfactory partitioning is obtained. In the location phase, the algorithm seeks to find the optimal location of a given number of central units. The central units are given as input to the allocation phase, in which the optimal allocation of roads to each center is determined by solving the IP model outlined in Section 5.2. Using a Voronoi diagram, roads allocated to each center are uniquely partitioned into disjoint polygon regions as described in Appendix A. The set of polygons constitutes the partitioning of the postal code area. Next, Monte Carlo simulation is employed to estimate the expected delivery time of each polygon for a given demand level. Delivery time estimation using Monte Carlo simulation will be further explained in Chapter 7. The expected delivery time is required as input to Stage 2, in which the IP model outlined in Section 5.4 seeks to construct routes through the set of polygon centers with minimal expected delivery time.

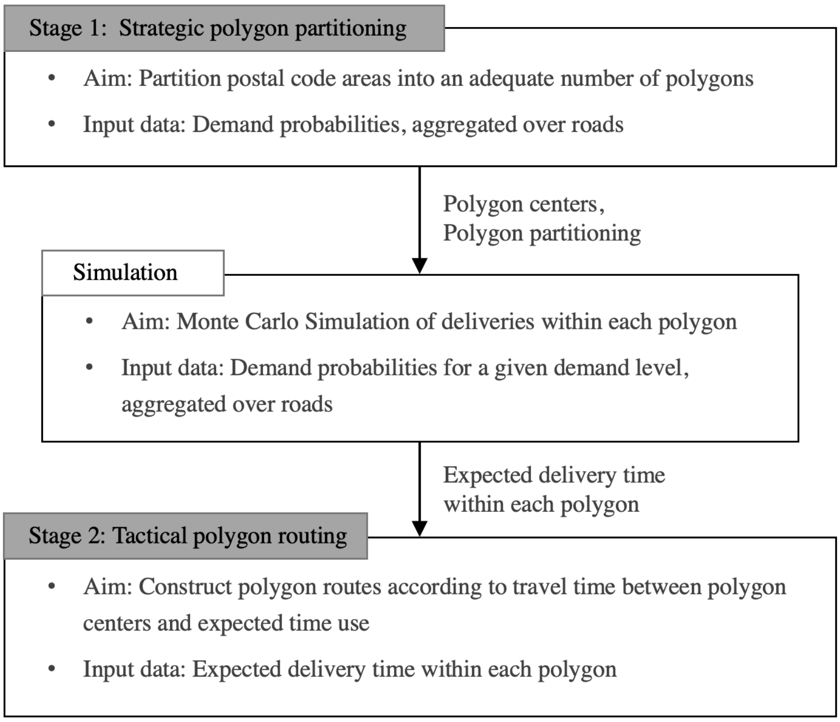


Figure 5.1: Overview of the two-stage solution approach, including aim and input data of each step

5.2 Stage 1: The Polygon Partitioning Problem (PPP)

In this section, the allocation phase of the location-allocation algorithm is formulated as an IP model.

5.2.1 Model Assumptions

- The road network is modeled as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ with vertices $\mathcal{V}$ and directed arcs $\mathcal{A}$. We assume a two-way road network, i.e. each road can be traversed in both directions.
- The travel time between two arcs is defined by the shortest path between the two most distant end-vertices.
- We assume that customer demands are independent, thus, a Poisson distribution can be used to derive the expected customer demand in an area. A Poisson distribution is derived based on the historical demand of customers within a specified limited time period.

5.2.2 Notation

The notation used in this thesis follows conventional modeling standards. Sets are defined as calligraphic, upper-case letters, whereas parameters are defined through upper-case letters. Decision variables and indices are lower-case. Set and parameter identifiers are used through superscript, while indices are applied as subscript.

5.2.2.1 Sets

$\mathcal{V}$	Set of vertices representing street crossings or alleys in the given service region, $\mathcal{V} = \{1, ..., V\}$
$\mathcal{A}$	Set of arcs representing directed road segments, $(i, j) \in \mathcal{A}$, connecting street crossings or alleys $i \in \mathcal{V}$ and $j \in \mathcal{V}$
$\mathcal{A}^S$	Set of central units in the given service region, $\mathcal{A}^S \subset \mathcal{A} \wedge i < j$
$\mathcal{A}^B$	Set of arcs representing "barriers", i.e. road segments of type European motorway, national road or county road, $\mathcal{A}^B \subset \mathcal{A}$

5.2.2.2 Indices

i	Vertex, $i \in \mathcal{V}$
(i, j)	Arc, $(i, j) \in \mathcal{A}$
s	Center unit, $s = (i, j) \in \mathcal{A}^S$

5.2.2.3 Parameters

D_{ijs}	Travel time from each arc (i, j) to center unit s
Q_{ij}	Expected (average) demand associated with arc (i, j)
$\underline{Q}$	The minimum required expected (average) demand of any polygon
M	The maximum number of arcs that can possibly be assigned to any polygon, $M = \lvert\mathcal{A}\rvert/2 - \lvert\mathcal{A}^S\rvert + 1$
α	Penalty cost to limit the number of neighbor arcs assigned to a different polygon
β	Penalty cost for assignment of arcs representing barriers

5.2.2.4 Variables

x_{ijs}	1 if arc (i, j) is assigned to the polygon centered at center unit s, 0 otherwise
f_{ijs}	Amount of flow in arc (i, j) sent on the path to center unit s, needed to guarantee the connectivity of polygons
v_{ijs}	The number of arcs which are connected to arc (i, j) but belongs to center unit s' different than s, needed to measure the degree of polygon isolation

5.2.3 Model

$$\min \quad \sum_{s \in \mathcal{A}^S} \sum_{(i,j) \in \mathcal{A}} D_{ijs} x_{ijs} + \alpha \sum_{s \in \mathcal{A}^S} \sum_{(i,j) \in \mathcal{A}} v_{ijs} + \beta \sum_{s \in \mathcal{A}^S} \sum_{(i,j) \in \mathcal{A}^B} x_{ijs} \tag{5.1}$$

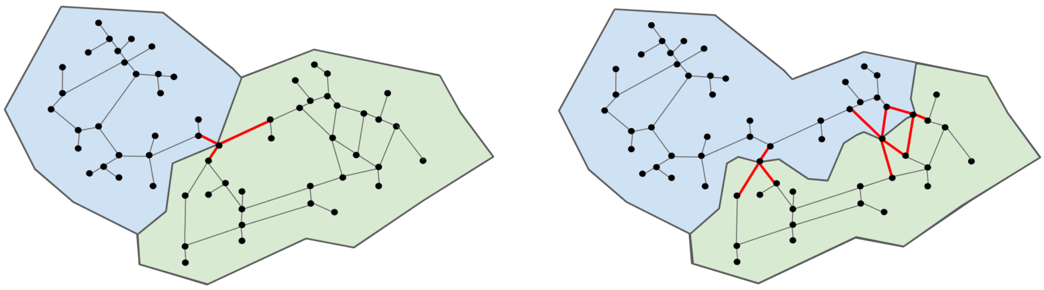
$$
\begin{aligned}
\text{s.t.}\quad & \sum_{s\in\mathcal{A}^{\mathcal{S}}}(x_{ijs}+x_{jis})=1 && (i,j)\in\mathcal{A}\backslash\mathcal{A}^{B} && (5.2)\\
& \sum_{s\in\mathcal{A}^{\mathcal{S}}}(x_{ijs}+x_{jis})\leq 1 && (i,j)\in\mathcal{A}^{B} && (5.3)\\
& \sum_{j:(i,j)\in\mathcal{A}} f_{ijs}-\sum_{j:(j,i)\in\mathcal{A}\backslash\mathcal{A}^{\mathcal{S}}} f_{jis}=\sum_{j:(i,j)\in\mathcal{A}} x_{ijs} && i\in\mathcal{V},s\in\mathcal{A}^{\mathcal{S}} && (5.4)\\
& \sum_{j:(j,i')\in\mathcal{A}} f_{ji's}+\sum_{j:(j,j')\in\mathcal{A}} f_{jj's}=\sum_{(i,j)\in\mathcal{A}} x_{ijs} && s=(i',j')\in\mathcal{A}^{\mathcal{S}} && (5.5)\\
& f_{ijs}\leq (M-1)\cdot x_{ijs} && (i,j)\in\mathcal{A},s\in\mathcal{A}^{\mathcal{S}} && (5.6)\\
& \underline{Q}\leq \sum_{(i,j)\in\mathcal{A}} Q_{ij}\cdot x_{ijs} && s\in\mathcal{A}^{\mathcal{S}} && (5.7)\\
& v_{ijs}\geq \sum_{s'\in\mathcal{A}^{\mathcal{S}}\backslash\{s\}}\Big(\sum_{k:(i,k)\in\mathcal{A}}(x_{iks'}+x_{kis'}) && && \\
& \quad +\sum_{k:(j,k)\in\mathcal{A}}(x_{jks'}+x_{kjs'})\Big)-(1-x_{ijs})\cdot M && (i,j)\in\mathcal{A},s\in\mathcal{A}^{\mathcal{S}} && (5.8)\\
& v_{ijs}\geq 0 && (i,j)\in\mathcal{A},s\in\mathcal{A}^{\mathcal{S}} && (5.9)\\
& f_{ijs}\geq 0 && (i,j)\in\mathcal{A},s\in\mathcal{A}^{\mathcal{S}} && (5.10)\\
& x_{ijs}\in\{0,1\} && (i,j)\in\mathcal{A},s\in\mathcal{A}^{\mathcal{S}} && (5.11)
\end{aligned}
$$

5.2.3.1 Objective Function

The objective function (5.1) includes indicators of compactness, isolation and barrier cost. The first objective maximizes compactness by minimizing the sum of travel times of all arcs to their assigned center. The second objective is sensitive to the coefficient α. The objective with larger values of α seek to find more isolated polygons. The third objective penalizes polygon designs containing barriers. Larger values of the coefficient β discourage the model from assigning roads classified as "barriers" to a polygon.

5.2.3.2 Constraints

Constraints (5.2) ensure complete and exclusive assignment of all "required" arcs, i.e. each road segment, that is not a barrier, is assigned to exactly one center unit $s\in\mathcal{A}^{S}$, while Constraints (5.3) ensure that arcs representing barriers are assigned to at most one center unit $s \in \mathcal{A}^S$. Constraints (5.4)-(5.6) enforce connectivity; flow is conserved if and only if the road network within a polygon is contiguous. Furthermore, Constraints (5.4) control the net flow at node $j \in \mathcal{V}$ so that whenever flow traverses an arc from i to j, it must continue along a neighboring arc from j. Constraints (5.6) impose upper bounds on the incoming flow for all arcs, while (5.5) impose upper bounds on the incoming flow for center arcs. Constraints (5.7) set a lower bound on the expected demand in each polygon. Constraints (5.8) enforce the value of auxiliary variables v_{ijs}. These variables equal the total number of neighboring arcs that are assigned to different centers, i.e. entry/exit roads. Figures 5.2a and 5.2b illustrate the effect of incorporating the v_{ijs} variables in the second objective for finding more isolated polygons. Finally, non-negativity constraints for variables are defined in (5.9) and (5.10), while (5.11) are constraints for the binary variables.



(a) High degree of isolation. The sum of v_{ijs} is equal to 2 for each polygon

(b) Low degree of isolation. The sum of v_{ijs} is equal to 8 for each polygon

Figure 5.2: Figures 5.2a and 5.2b illustrate two different partitionings of an area into two polygons. In Figure 5.2a the blue polygon has one arc connected to two arcs in the green polygon, while the green polygon has two arcs connected to one arc in the blue polygon. Hence, each polygon is connected to the other through two possible paths. When penalizing variables v_{ijs} in the objective, the PPP model will favour the partitioning in Figure 5.2a as there are fewer entry/exit paths and thus more isolated polygons, compared to the partitioning in Figure 5.2b

5.3 Stage 1: Solution Approach

The first stage districting problem described in Chapter 4 is complex and challenging to solve for several reasons. Firstly, solving a districting problem in which central units are subject to the optimization process, is limited to instances with a few hundred basic units when using exact methods, see Chapter 3. Hence, in order solve the instances of our case study, exceeding several hundred basic units, this thesis applies a location-allocation heuristic algorithm. Secondly, there are many ways an area can be partitioned, meaning that a vast number of solution combinations exist in the allocation phase, i.e. PPP. Lastly, the exponential number of contiguity constraints is one of the main difficulties when solving districting problems with exact methods, that is, Constraints (5.4)-(5.6) in the model formulation of the PPP.

This section describes our location-allocation algorithm to solve the first stage districting problem. To increase the efficiency of the location-allocation algorithm, we devise an approach to eliminate some solution combinations in the allocation phase, together with an approach to heuristically enforce contiguity.

5.3.1 Overview of Location-Allocation Algorithm

As discussed in Chapter 3, location-allocation based heuristic algorithms are the most common in the literature to solve districting problems where the district partitioning and location of central units are to be decided simultaneously. Our implemented iterative location-allocation algorithm, is illustrated in Figure 5.3. The aim of the algorithm is to find a configuration of centers, which incurs minimum travel time among all polygons. First, an initial configuration of polygon centers must be determined (Step 1 in Figure 5.3). Next, the allocation phase deals with the allocation of roads to the given set of polygon centers (Step 2). Once the allocations are completed, each polygon center is updated in the location phase, by finding the new center among the roads allocated to that polygon (Step 3). The iterative location-allocation algorithm is used until centers do not change or the stopping condition is reached. In the following sections, we address each step of the proposed algorithm in detail.

5.3.2 Initial Polygon Centers

Although it is possible to start the location-allocation algorithm by randomly selecting the polygon centers, a more promising initial configuration reduces the computational effort required to attain better solutions.

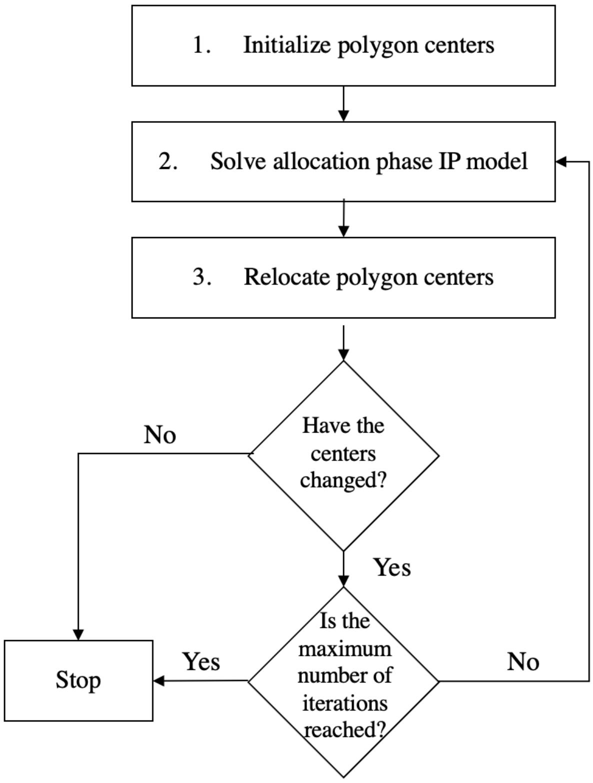


Figure 5.3: The location-allocation heuristic algorithm used to solve the first stage districting problem. The allocation phase is solved by the PPP mathematical programming model

To initialize the polygon centers, we use the following procedure based on the implementation of the well-known p-means heuristic, as proposed by Ríos-Mercado et al. (2020). First, p roads are randomly chosen as initial central unit. Second, each remaining unassigned road is assigned to its closest center. This assignment defines p polygons. Once polygons are formed, new centers for each polygon are selected, based on the most centered road, and the process is repeated. The iterative procedure takes place until no new set of central units is found. This procedure is a very quick heuristic that returns, as output, an initial set of centers. However, it is well-known that the p-means algorithm strongly depends on the initial configuration of centers. Consequently, we propose an extension of the algorithm to a multi-start method, by simply applying the p-means heuristic from multiple random initial solutions. 10 multiple random starts were used, each producing a solution, of which the best overall was applied as the initial set of centers in the location-allocation algorithm.

5.3.3 Allocation Phase

In the allocation phase, the configuration of centers found in the previous iteration is used as an input, $\mathcal{E}^s$, and the PPP is solved to determine the optimal allocation of roads, i.e. arcs. To increase efficiency of the mathematical model, and enable solving larger test instances, we propose methods to eliminate variables, and heuristically impose contiguity. This modified version of the location-allocation algorithm is illustrated in Figure 5.4. We will refer to this location-allocation algorithm as the Modified Location-Allocation (M-LA), while the original algorithm without heuristic contiguity constraints and variable fixation will be referred to as the Location-Allocation (LA).

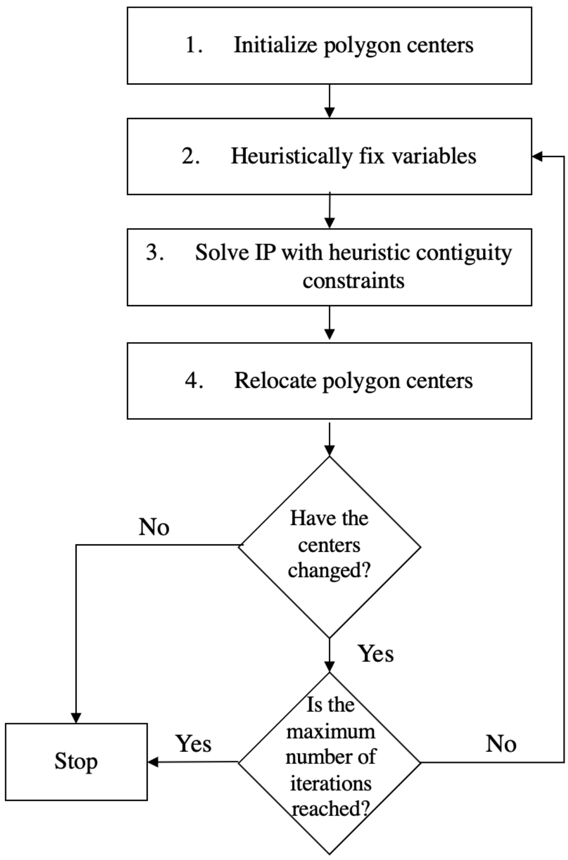


Figure 5.4: Overview of the modified location-allocation algorithm. The allocation phase is solved using the PPP with heuristic contiguity constraints and variable fixation

5.3.3.1 Heuristically Fix Variables

Inspired by Ríos-Mercado et al. (2013), we propose to heuristically eliminate some of the assignment variables x_{ijs}, by forbidding assignments of arcs to centers that are far away. For each center all arcs are sorted by their travel time to that center, in a decreasing order. Suppose an arc (i, j) is located furthest away from center s, and center s is not the center closest to arc (i, j). In that case, we assume that the corresponding assignment variable can be fixed to zero, $x_{ijs} = 0$, as long as the polygon still, with high probability, will contain at least one requested delivery, that is Constraints (5.7) are not violated. This process is repeated until no assignment variable can be fixed to zero.

5.3.3.2 Solve with Heuristic Contiguity Constraints

A heuristic approach to reduce the exponential number of contiguity constraints have been successfully employed by Ríos-Mercado et al. (2013) and Bender et al. (2020). Following this approach, we can solve our mathematical model with heuristic constraints, instead of exact contiguity constraints. The heuristic constraints enforce that if a arc $\{i, j\}$ is assigned to a center s, then one of the immediate neighbors of that arc along some shortest path to the center, must also be assigned to the same center. These immediate neighbors are denoted by set $\mathcal{E}^C_{ijs}$. Hence, constraints (5.4)-(5.6) and (5.10) in the original PPP formulation are replaced by the following constraints:

$$x_{ijs} \leq \sum_{\{k,l\} \in \mathcal{E}^C_{ijs}} x_{kls} \quad \{i, j\} \in \mathcal{E}, s \in \mathcal{E}^S \tag{5.12}$$

These constraints constitute a more restrictive condition than the original constraints of the PPP model, where we merely require that a given basic unit attains some other adjacent basic unit that is also assigned to the same central unit. Therefore, this heuristic approach guarantees contiguity, but eliminates some contiguous polygons. Note that the heuristic formulation allow us to use edges instead of arcs. The directed arcs (i, j) and $(j, i) \in \mathcal{A}$ are then replaced by their undirected counterpart $\{i, j\} \in \mathcal{E}$ where $i < j$, thus reducing the number of variables to half. The full heuristic mathematical modelling formulation is provided in Appendix B.

5.3.4 Location Phase

In the location phase, a new configuration of polygon centers is found from the solution obtained in the allocation phase. Like in the work of, e.g., Fleischmann et al. (1988) and Yanık et al. (2016), we relocate the centers by simply solving a 1-median problem for each polygon. The optimal location of a polygon center is selected as the arc that attains the minimum travel time to all other arcs in the given polygon. The set of the updated central units is then applied to the next iteration of the allocation phase.

5.4 Stage 2: The Polygon Routing Problem (PRP)

In this section, the PRP is formulated as an IP model.

5.4.1 Model Assumptions

- A polygon from the PPP is represented by its central unit.
- A vehicle traveling along a route serves all customers in a polygon before traveling to the next polygon, thus, a polygon can be viewed as a "super-stop".
- We consider a homogeneous fleet of vehicles and the capacity of a vehicle, i.e. the maximum number of parcels, is assumed a non-binding constraint and is therefore disregarded. Posten Norge AS assumes 3 minutes of service time per parcel and a vehicle capacity of 130 parcels, resulting in a service time of 6.5 hours for a fully capacitated vehicle. Hence, the service time alone would exceed a drives work-shift of 5 hours.

5.4.2 Notation

5.4.2.1 Sets

$\mathcal{P}$	Set of vertices $\mathcal{P} = \{0, ..., P+1\}$, where vertices 0 and $P+1$ represent the origin and departure terminal, respectively, while $\mathcal{P} \setminus \{0, P+1\}$ are the central units of the polygons
$\mathcal{N}_p$	Set of neighboring vertices for vertex p, $\mathcal{N}_p \subset \mathcal{P}$

5.4.2.2 Indices

p, r, q	Vertex, $p, r, q \in \mathcal{P}$

5.4.2.3 Parameters

C^D_{pq}	Travel time along the shortest path from central unit p to central unit q, where p and q are neighbors, i.e. $q \in \mathcal{N}_p$
C^E_p	Expected delivery time, i.e. travel and service time, in the polygon with central unit p. Calculated from the Monte Carlo simulation.
K	Number of available vehicles at the terminal
$\overline{C}$	The maximum expected delivery time allowed for each route

5.4.2.4 Variables

x_{pq}	1 if any route includes a path from vertex p to q directly, 0 otherwise
y_p	The cumulative expected delivery time on the route that visits node p up to this visit, before node p is serviced

5.4.3 Model

$$\min \sum_{p\in\mathcal{P}} \sum_{q\in\mathcal{N}_p} (C^D_{pq} + C^E_p) \cdot x_{pq} \tag{5.13}$$

$$\text{s.t.} \quad \sum_{q\in\mathcal{N}_p\setminus\{0\}} x_{pq} = 1 \qquad p \in \mathcal{P}\setminus\{0, P+1\} \tag{5.14}$$

$$\sum_{p\in\mathcal{N}_r\setminus\{P+1\}} x_{pr} - \sum_{q\in\mathcal{N}_r\setminus\{0\}} x_{rq} = 0 \qquad r \in \mathcal{P}\setminus\{0, P+1\} \tag{5.15}$$

$$\sum_{p\in\mathcal{P}\setminus\{0,V+1\}} x_{0p} \leq K \tag{5.16}$$

$$y_q \geq y_p + (C^D_{pq} + C^E_p) \cdot x_{pq} - \overline{C} \cdot (1 - x_{pq}) \qquad p \in \mathcal{P}, q \in \mathcal{N}_p \tag{5.17}$$

$$C^E_p \leq y_p \leq \overline{C} \qquad p \in \mathcal{P} \tag{5.18}$$

$$x_{pq} \in \{0, 1\} \qquad p \in \mathcal{P}, q \in \mathcal{N}_p \tag{5.19}$$

5.4.3.1 Objective Function

The objective function (5.13) minimizes the total expected delivery time of all routes, where the expected delivery time of a route consists of both the travel time between the polygon centers and the expected delivery time within each polygon.

5.4.3.2 Constraints

Constraints (5.14) ensure that each polygon center, is visited exactly once. Constraints (5.15) guarantee correct flow of vehicles through the arcs, by stating that if a vehicle arrives at a polygon center, then it must also depart from this polygon center. Constraints (5.16) ensure that the number of routes does not exceed the number of available vehicles K. Constraints (5.17), together with Constraints (5.18) limit the expected delivery time of a route to $\overline{C}$. Constraints (5.17) also act as sub-tour elimination constraints, avoiding cycling routes that do not pass through the terminal. Finally, (5.19) are binary constraints on the binary variables x_{pq}.

Chapter 6

Data

This chapter presents the data used to solve the Route Partitioning Problem (RPP) on Trondheim municipality's postal code areas. Two primary sources were used: (1) geographical data and (2) historical parcel delivery data. Section 6.1 presents geographical data of Trondheim municipality's postal code areas and the underlying road network. Section 6.2 presents the historical parcel delivery data provided by Posten Norge AS, while Section 6.3 describes the data preprocessing and the generation of Poisson distributions used when solving the RPP for different demand levels. The different data sources are used to derive test instances for our computational study. Test instances are presented in Chapter 7.

6.1 Geographical Data

The RPP is solved for 68 postal code areas within Trondheim municipality, a subset of all the postal code areas in which Posten Norge AS's postal logistic center in Trondheim is responsible for delivering mail to. Figure 6.1 shows an overview of the relevant postal code areas. The geospatial and nonspatial attributes of each postal code area were retrieved from Geonorge in File Geodatabase (FGDB) format.

The geographic features of Trondheim municipality's road network and its speed limits were retrieved from national road databases. The road network was retrieved from

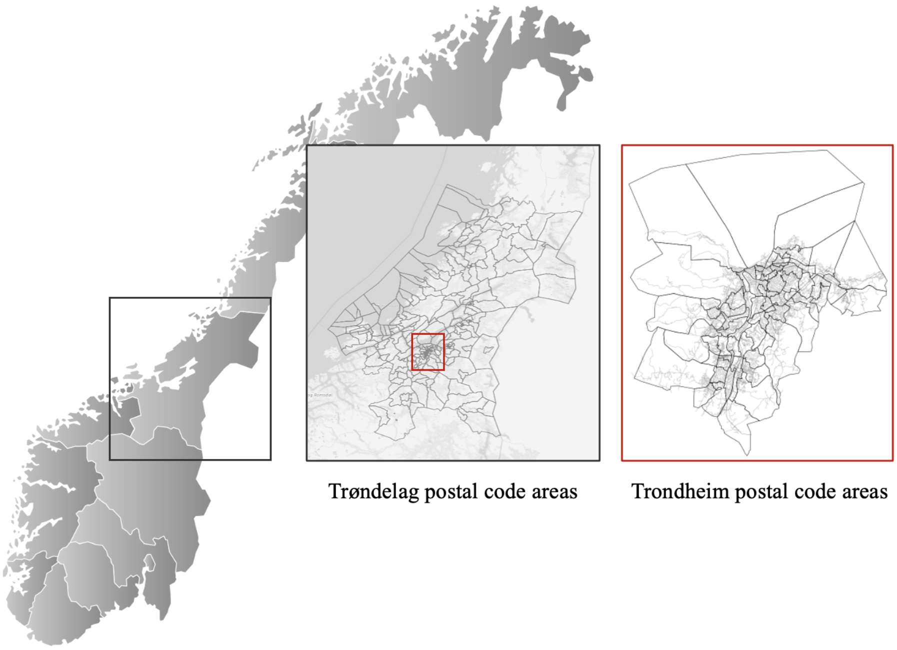


Figure 6.1: The figure shows the area covered by the provided data. Outlined in grey is the total area serviced by the Trondheim Terminal, that is 255 postal code areas, while outlined in red is the 68 postal code areas within Trondheim municipality

Geonorge's Vbase. Vbase is a nationwide database showing all driveable roads longer than 50 meters. Vbase also includes the classification of roads. This is used in the Polygon Partitioning Problem (PPP) model outlined in Chapter 5 when penalizing polygon designs containing barriers. Specifically, highways (European motorways, national and county roads) are considered barriers. Speed limits were retrieved from the Norwegian Public Roads Administration's National Road Data Bank (NVDB). NVDB is a database which contains information on road geometry and topology, accidents and traffic volumes, which is used actively in Norway's road administration and forms the basis for map solutions and route calculators.

The road network data from Geonorge was utilized to construct a NetworkX graph in Python, which was used in the implemented mathematical programming models to solve the RPP. In this graph, an edge represents a road and a node represents a road-intersection or a road-endpoint. Further, driving speed along an edge was determined in line with the speed limits retrieved from NVDB.

6.2 Historical Parcel Home Delivery Data

Historical parcel home delivery data provided by Posten Norge AS was used to generate customer demand levels applied when solving the RPP. The data includes the number of home deliveries in Trondheim municipality during weekdays, excluding holidays, from July to December 2021 and the geographic coordinates of each delivery. The provided data of historical home deliveries are visualized in Figure 6.2. As seen from the figure, demand is subject to seasonal and weekday variations, with the days around Black Friday, at the end of November, and the period before Christmas with the largest delivery volumes.

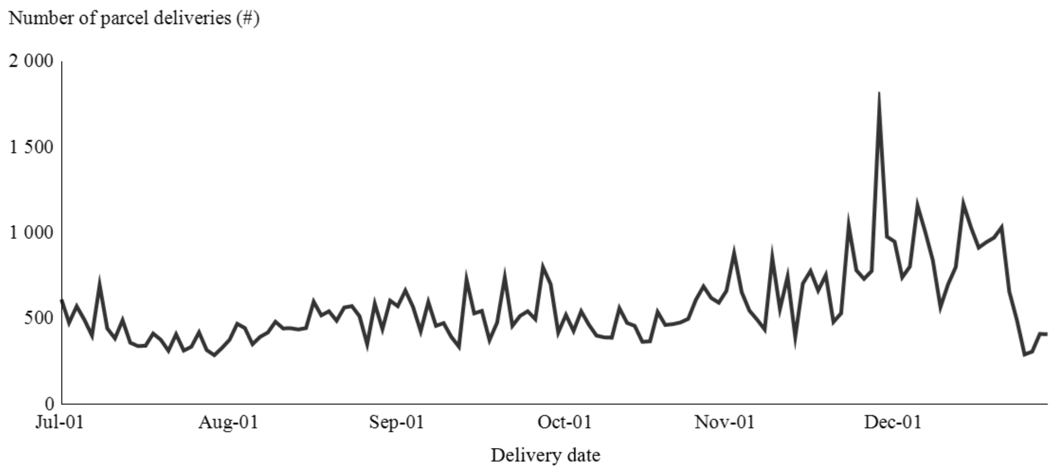


Figure 6.2: Posten Norge AS's historical parcel home deliveries from July to December 2021 within Trondheim municipality

Figure 6.3 shows the historical home delivery data presented in Figure 6.2 sorted by descending number of parcel deliveries. From this, four demand levels, d were defined in collaboration with Posten Norge AS: Low [<402), medium [402, 729), high [729, 1166) and peak [>1166] based on percentiles of the historical data. However, due to the limited data of merely a single day for the peak demand level, this demand level is disregarded. The remaining demand levels; low, medium and high, are used to generate customer demand levels described in Section 6.3.

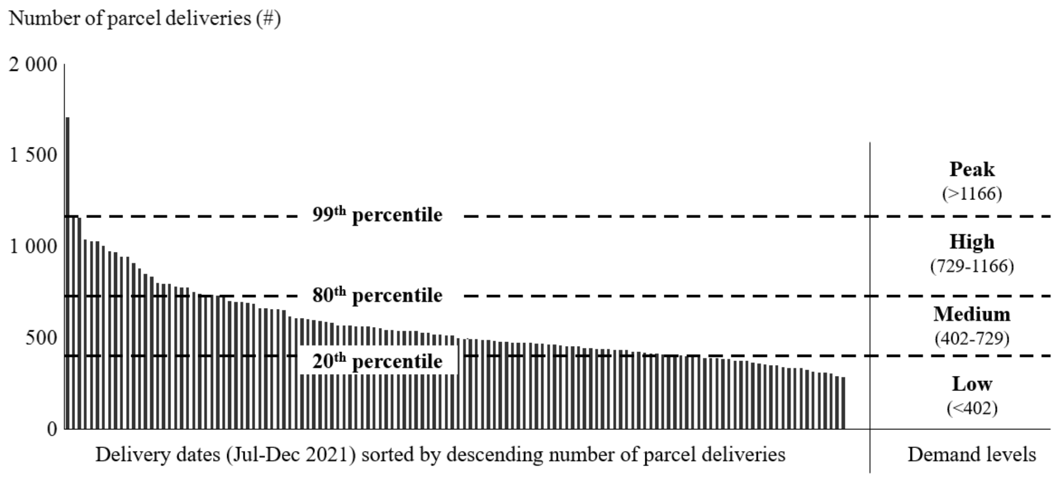


Figure 6.3: Posten Norge AS's historical parcel home deliveries from July to December 2021 within Trondheim municipality sorted by descending number of parcel deliverables. Four demand levels are identified: Low, medium, high and peak

6.3 Data Processing

The geographical and historical delivery data was processed before it could be applied to solve the RPP. Specifically, the coordinates of deliveries presented in Section 6.2 were aggregated to the road network described in Section 6.1. The aggregation was done by projecting the delivery coordinates to their closest road segment, as illustrated in Figure 6.4.

As parcel delivery volumes is subject to seasonal and weekday variations, the PRP model introduced in Chapter 5 is solved for the different demand levels, identified in Figure 6.3. We assume that the demand for parcels along a road, X_{rd}, can be modeled by a Poisson distribution with a mean λ_{rd}, for road segment r and demand level d. For each demand level, the Poisson distribution of demand for parcels along road segment r is modeled as $f(k, \lambda_{rd}) = Pr(X_{rd} = k) = \frac{\lambda_{rd}^k e^{-\lambda_{rd}}}{k!}$, where k is the number of parcels ($k = 0, 1, 2 ...$), e is Euler's number, and ! is the factorial function. In a Poisson distribution, λ_{rd} is equal to the expected value of X_{rd} and also to its variance, $\lambda_{rd} = E(X_{rd}) = Var(X_{rd})$. Historical data was used to calculate λ_{rd} for each demand level d and road segment r as illustrated in Figure 6.4.

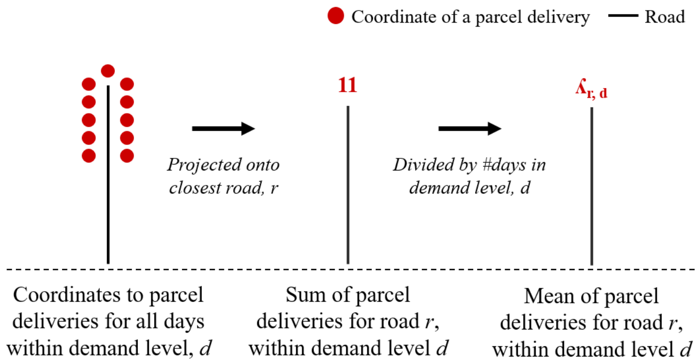


Figure 6.4: Aggregation of parcel delivery data from delivery coordinates to their closest road segment

Chapter 7

Computational Study

This chapter conducts a computational study of the Route Partitioning Problem (RPP) described in Chapter 4. The study quantifies and evaluates the implications of applying home delivery routes derived from tactical routes generated with our proposed two-stage solution approach. The aim is to evaluate whether designing a set of tactical routes to be operated unchanged over a given period of time is a viable alternative to constructing a new set of routes every day based on the particular instance of that day.

Section 7.1 presents the software and hardware specifications used to conduct the computational study, before the analysis setup is introduced in Section 7.2. Further, the computational study consists of two main parts, a technical analysis and an economic analysis. First, Section 7.3 and 7.4 present the technical analysis of our proposed models and solution approach. Second, Section 7.5 conducts the economic analysis, evaluating the performance of our solution approach, by comparing the total delivery time of daily routes derived from tactical routes, against the delivery time of a deterministic Vehicle Routing Problem (VRP) solution.

7.1 Software and Hardware Specifications

The mathematical models presented in Chapter 5 were implemented in Python and solved using the commercial optimization software Gurobi Optimizer, with PyCharm as the integrated development environment (IDE). GeoPandas, an open-source Python software package, handled geospatial data to analyze problem instances and display the solutions. Experiments were performed on a Dell OptiPlex 7780 machine. Further specifications can be found in Table 7.1.

Table 7.1: Specifications of software and hardware used in computational experiments

Computer	Dell OptiPlex 7780
Operating system	Windows 10, version 21H1
CPU	Intel® Core™ i7-10700 CPU 2.90 GHz - 8 core
RAM	16 GB
Disk	M.2 512 GB PCIe NVMe SSD-disk
Python version	3.9.0
Gurobi version	9.5.0
PyCharm version	2021.3

7.2 Analysis Setup

The first part of the computational study, the technical analysis, investigates runtime and model configurations. First, an evaluation of the location-allocation algorithm implemented to solve the strategic districting problem is provided. Preliminary testing results of input parameters are discussed, before we compare the implementation using exact contiguity constraints (LA) with the implementation using heuristic contiguity constraints and variable fixation (M-LA). Due to the limited time of the project, a time limit of five hours per iteration of the location-allocation algorithm seems reasonable for the strategic problem at hand. Second, an evaluation of the model implemented to solve the tactical routing problem, the Polygon Routing Problem (PRP) is provided. A runtime analysis of the model is concerned, and we recognize how large scale instances our proposed solution approach is able to solve within a reasonable time limit of five hours.

In the second part of the computational study, the economic analysis, we aim to evaluate the usability of tactical routes generated with our proposed solution approach. The basic question raised in the introduction, is whether or not a set of routes designed to be operated unchanged over a period of time, provides a viable alternative to constructing a set of routes from scratch every day. To be able to provide at least an empirical answer to this question, we conducted the following experiment. First, tactical routes were constructed for each demand level identified in Chapter 6 and for different partitionings of a geographical area. We then generated ten realizations of customer demands, and compared the delivery time of daily routes derived from tactical routes against the delivery time of a deterministic VRP solution. The daily route solutions are attained using Google's Operations Research Tools (OR-Tool), which is further specified in 7.2.3.

7.2.1 Test Instances

As described in Chapter 6, test instances are constructed from 68 postal code areas in Trondheim municipality. Characteristics of the postal code areas are presented in Table C.1 in Appendix C.

Applying the criteria that each polygon must obtain at least one requested delivery with high probability (95%), each postal code area can be divided into a maximum number of polygons. The demand criteria is imposed using the Poisson distribution from Chapter 6. Table C.1 in Appendix C presents the maximum number of polygons, i.e. polygon centers, that each of the 68 postal code areas can be partitioned into, while preserving the demand criteria. The medium demand level defined in Chapter 6 is used for solving the Polygon Partitioning Problem (PPP), as this demand level is the most frequently occurring, and in the middle with respect to the other demand levels, low and high. As can be noted, not all postal code areas can be partitioned into smaller units to sustain the demand criteria in a medium demand level. Table 7.2 summarizes the number of test instances, i.e. postal code areas, that can be solved for a given number of polygon centers.

Table 7.2: Table showing the number of postal code areas that can be solved for a given number of polygon centers. Naturally, the entire set of postal code areas comprise the one polygon centered partitioning, representing the postal code area itself. No postal code area can be partitioned into more than five polygons

Polygon centers [#]	Postal code areas [#]
1	68
2	43
3	21
4	8
5	4

In this chapter we evaluate and compare solutions of the 68 postal code areas of the Trondheim municipality using different partitioning granularities. The impact of different granularities is tested based on two extremes; a minimum polygon partitioning (111 polygons) and a maximum polygon partitioning (144 polygons). We will refer to the minimum partitioning as P-MIN and the maximum partitioning as P-MAX. In P-MIN all postal code areas that are feasible to partition into two polygons, will be partitioned. For example, a postal code area that can be partitioned into a maximum of five polygons, will be partitioned into two polygons in P-MIN and five polygons in P-MAX.

7.2.2 Estimating Delivery Time within Polygons via Monte Carlo Simulation

When a polygon partitioning is attained, the expected delivery time, i.e. travel time and service time, within each polygon must be estimated. As described in Chapter 5, the expected delivery time is used in the second stage of our two-stage approach, when tactical routes of minimal expected delivery time are to be constructed. The expected delivery time is calculated using Monte Carlo simulations. In this procedure, the roads within a polygon are sampled to create a realization of those roads requiring service on any given day along with their parcel count. Demand vectors are generated, indicating the parcel count of each road. The demand vectors are generated by taking a random demand value for each road drawn from its Poisson probability distribution.

For each realization, the Open Capacitated Arc Routing Problem (OCARP) outlined in Appendix E is solved for each polygon. This model identifies the shortest path within a polygon covering all roads with demand. Simulating a series of 20 realizations allow us to calculate the expected delivery time of each polygon as the average of all realizations.

7.2.3 Generating Operational Routes for Testing

To be able to provide an answer to the question raised in the economic analysis, we must generate realizations of the customer demands and construct operational routes for a given realization. For a given area, a demand realization is a sampling from the Poisson distributions of a selected demand level, low, medium or high. Further, for a given demand realization, solutions are acquired both for deterministic routing of the entire service area, and for routing within areas bounded by our tactical routes.

The deterministic routing solutions of the entire area are employed as benchmark solutions. For each demand level, two different tactical routing solutions through the set of polygons in P-MIN and P-MAX are generated, and for each demand realization, the operational routes attained from the two tactical routing solutions are tested against the benchmark. Throughout this chapter, we will refer to the operational routes derived from tactical routes using the P-MIN and the P-MAX partitioning, as P-MIN-OR and P-MAX-OR, respectively.

Google's Operations Research Tools (OR-Tools), an open source heuristic based software for solving optimization problems, is used to generate the operational routes. The advantage of this software is that it enables us to efficiently obtain operational routes and the corresponding delivery times using Python. The runtime of OR-Tools is fast compared to other techniques, and its solutions are usually near-optimal when compared with exact methods. A transformation of the arc routing model into a VRP is necessary to enable implementation in OR-Tools and was therefore conducted. Based on preliminary testing of solution convergence, a time limit of 30 minutes was selected to generate the benchmark solutions, and a time limit of 15 minutes was selected to construct each route in P-MIN-OR and P-MAX-OR. Additionally, when generating a benchmark for P-MIN-OR and P-MAX-OR, the number of vehicles was constrained to the number of vehicles in the corresponding tactical routing plan. This was conducted to enable a comparison of the routing solutions.

7.2.4 Notation

Below follow clarifications of how results are presented throughout this computational study:

- **Polygon centers** [#] is the selected number of centers in the strategic districting problem, corresponding to the number of polygons a postal code area is partitioned into.
- **Iterations** [#] is the number of iterations elapsed in the location-allocation al-

gorithm when solving the strategic districting problem.

- **Isolation** [#] is the number of neighboring roads that are assigned to different polygon centers, i.e. the sum of all v_{ijs} variables in the PPP model presented in Chapter 5.
- **Barriers** [#] is the number of European motorways, national roads or county roads within a polygon, i.e. the sum of barriers in set $\mathcal{A}_B$ that are assigned to a polygon center in the PPP model.
- **Compactness [sec]** is the total intra-polygon travel time, i.e. the value of the first objective in the PPP model.
- **P-MIN** is a solution of the PRP using the minimum number of polygons.
- **P-MAX** is a solution of the PRP using the maximum number of polygons.
- **Travel time [sec]** is the time it takes to drive from the Posten Norge AS's logistic postal terminal and drive by all parcel recipients before returning to the same terminal.
- **Service time [sec]** is the total time it takes from stopping by all parcel recipients, deliver the parcel at the door of the recipients, before continuing on the route. A service time of 3 minutes per parcel is assumed.
- **Delivery time [sec]** is the sum of the travel time and the service time.
- **P-MIN-OR** operational routes derived from tactical routes based on the minimum number of polygons (P-MIN), generated using Google's OR-Tools.
- **P-MAX-OR** operational routes derived from tactical routes based on the maximum number of polygons (P-MAX), generated using Google's OR-Tools.
- **Benchmark** is a deterministic VRP solution of the entire service area for a selected demand realization generated using Google's OR-Tools.
- **Increase from benchmark [%]** is the percentage increase in delivery time when applying an operational routing solution derived from tactical routes, compared to a benchmark solution.

7.3 Technical Analysis of the Location-Allocation Algorithm

This section evaluates the two different versions of the location-allocation algorithm, LA and M-LA. First, parameter configuration is presented using the LA. Second, an analysis that compares the runtime and performance of both versions is presented.

7.3.1 Preliminary Testing of Input Parameters

The objective function (5.1) of the PPP model presented in Chapter 5, includes the objectives of maximizing isolation and minimizing barriers, weighted by the parameters α and β, respectively. A higher α value generates more isolated polygons, and a higher value of β discourages assigning roads classified as "barriers" to a polygon.

The aim of the parameter tuning is to find the values of α and β that generate the most isolated polygons with the fewest barriers. Furthermore, the trade-off between runtime and solution quality is evaluated. The parameters α and β are tested for 10 values ranging from 0 to 200 000. Different combinations of the values were tested for three different postal code areas using the LA algorithm. Each postal code area was divided into two polygons for preliminary testing.

Table 7.3 provides a subset of all the evaluated parameter combinations. We observe that a configuration of both α and β to a value of 10 000 or higher generates the most isolated polygon partitioning with the fewest barriers. Note that there seems to be a low degree of conflict between the isolation and barrier measures, since a high weighing of isolation, also seems to result in fewer barriers, and conversely, a high weighing of barriers seems to result in more isolated polygons.

Table 7.3 show that a value of α and β equal to 10 000 or higher compared to values of 0, decreases the average number of roads in the isolation and barrier measures for the three postal code areas, from 8 to 0 and from 11 to 9, respectively. Hence, we observe the effect of higher values of α and β. This is visually depicted in Figure 7.1 for postal code area 7021. We note that the zero number of roads in the isolation measure is only possible due to the presence of barriers in the selected postal code areas. In the PPP, roads classified as barriers are not required to be assigned to any polygon. Hence, roads may be neighboring with roads not part of any polygon. In this case the road will not be included in the isolation measure.

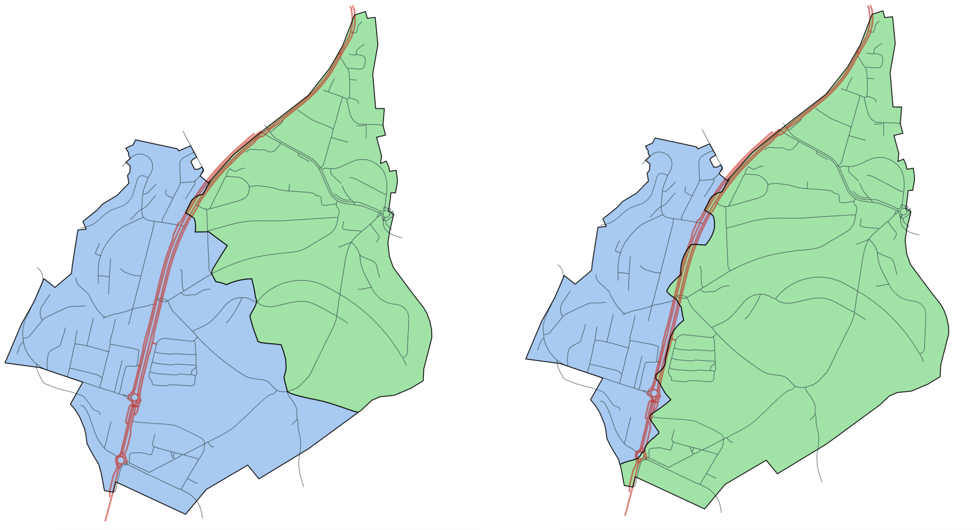
To identify the most favourable set of parameter values, we investigated the average runtime of each α and β configuration that attained both the most isolated partitioning and fewest barriers. The results indicated that a value of 50 000 for both α and β had the lowest average runtime of 0.87 seconds. This configuration was thus selected for all further calculations in this computational study.

Table 7.3: Preliminary testing of the PPP objective parameters, α and β

Isolation (α)	Barriers (β)	7021				7022				7056			
		Isolation [#]	Barriers [#]	Compactness [sec]	Runtime [sec]	Isolation [#]	Barriers [#]	Compactness [sec]	Runtime [sec]	Isolation [#]	Barriers [#]	Compactness [sec]	Runtime [sec]
200 000	200 000	0	5	12 143	2.62	0	4	6 970	0.36	0	17	8 619	0.79
100 000	100 000	0	5	12 143	2.85	0	4	6 970	0.35	0	17	8 619	1.94
50 000	50 000	0	5	12 143	1.41	0	4	6 970	0.39	0	17	8 619	0.81
10 000	10 000	0	5	12 143	3.34	0	4	6 970	0.4	0	17	8 619	0.86
1000	1000	0	5	12 143	6.09	0	4	6 970	0.86	0	17	8 619	1.42
10	10	12	6	9 144	0.86	4	7	6 617	0.60	4	20	8 195	0.55
0	0	16	6	9 142	0.29	4	7	6 617	0.11	4	20	8 195	0.26
50 000	100 000	0	5	12 143	4.05	0	4	6 970	0.39	0	17	8 619	1.34
10 000	100 000	0	5	12 143	4.22	0	4	6 970	0.41	0	17	8 619	1.03
1000	100 000	0	5	12 143	3.87	0	4	6 970	0.39	0	17	8 619	0.99
10	100 000	0	5	12 143	2.48	0	4	6 970	0.36	0	17	8 619	1.07
0	100 000	0	5	12 143	1.01	0	4	6 970	0.16	0	17	8 619	0.54
100 000	50 000	0	5	12 143	3.03	0	4	6 970	0.36	0	17	8 619	0.82
100 000	10 000	0	5	12 143	3.28	0	4	6 970	0.33	0	17	8 619	0.59
100 000	1000	0	5	12 143	4.28	0	4	6 970	0.44	0	17	8 619	0.52
100 000	10	0	6	11 813	3.83	0	4	6 970	2.04	0	19	8 349	0.42
100 000	0	0	6	11 813	7.46	0	4	6 970	2.66	0	19	8 349	0.35

7.3.2 Evaluating Performance of LA and M-LA

Table 7.4 shows the average total runtime, average number of iterations and achieved objective value for LA and M-LA, with an increasing number of polygon centers. For both solution approaches, the average total runtime of the models, as well as the average number of iterations increase as the number of polygon centers increase. Moreover, it is evident that the M-LA, that implements both heuristic contiguity constraints and fixation of variables, reduces the average runtime considerably, with as much as 91.04% compared to the LA. This is also true for the instances that are not solved to optimality, given in Table 7.5. Regardless of the significant deviation in runtime, we note that the average number of iterations increases with the number of centers for both the LA and M-LA.



(a) α and β equal to 0 **(b)** α and β equal to 50 000

Figure 7.1: Partitionings of postal code area 7021 into two polygons, i.e. blue and green area, for different values of input parameters α and β. Roads marked in red are considered barriers

The reduction in runtime, can be reasoned in the significant decrease in number of variables and constraints for the M-LA compared to the LA. Variables and constraints after presolve for each test instance are provided in Table G.1 of Appendix G. The LA attained on average 4 979 variables and 4 045 constraints, compared to the M-LA with on average 1 461 variables and 1 593 constraints, imposing a decrease of 70.66% and 60.62%, respectively.

Further, a key observation from Table 7.4 is that the models' objective values, i.e. the average compactness, average isolation and average barriers, do not deviate much for the two solution approaches. The average increase of the three objectives when using the M-LA compared to LA are 0.76%, 11.51% and 28.05%, respectively. This suggests that the solution quality attained with the M-LA is not significantly reduced.

The results discussed in this section imply that a general conclusion can be drawn regarding the performance of the two versions of the location-allocation algorithm. Foremost, the LA and M-LA achieve both similar and satisfactory solutions concerning total compactness and degree of isolation and barriers in the final partitionings. However, M-LA has significantly reduced runtime due to considerably fewer variables and constraints.

Table 7.4: Table shows average total runtime, number of iterations and average runtime per iteration of postal code areas solved for a given number of polygon centers, for the LA and M-LA. A time limit of 18 000 seconds per iteration of the location-allocation algorithm is employed. Averages are only calculated for the postal code area partitionings that attained a solution in both LA and M-LA. The LA did not acquire a solution for five of the instances

Polygon centers [#]	Postal code areas [#]	LA					M-LA				
		Iterations [Avg. #]	Total runtime [Avg. sec]	Compactness [Avg. sec]	Isolation [Avg.#]	Barriers [Avg. #]	Iterations [Avg. #]	Total runtime [Avg. sec]	Compactness [Avg. sec]	Isolation [Avg. #]	Barriers [Avg. #]
2	43	1.70	3 530.15	12 224.37	8.98	5.86	1.72	8.53	12 235.07	9.81	7.86
3	21	2.33	28 254.68	12 665.52	18.10	5.90	2.35	111.25	12 682.01	20.24	8.67
4	6	3.00	48 958.22	17 124.50	26.00	8.67	2.75	3 130.00	17 887.91	31.67	10.17
5	1	3.25	54 900.10	26 789.00	30.00	35.00	3.50	23 433.25	27 016.03	30.00	37.00
Average		2.02	15 405.55	12 974.08	13.41	6.52	2.02	632.62	13 053.15	15.03	8.71

Table 7.5: Table shows the number and percentage of time limits (TL) reached in the final iteration of LA and M-LA, using a TL of 18 000 seconds

Polygon centers [#]	Postal code areas [#]	LA			M-LA		
		TL [sum]	TL [%]	Gap [Avg. %]	TL [sum]	TL [%]	Gap [Avg. %]
2	43	3	7	27.41	0	0	0
3	21	8	38	32.89	0	0	0
4	8	7	88	47.30	0	0	0
5	4	4	100	68.48	1	25	24.30

7.3.3 Variance Analysis

As specified in Chapter 4, precise delivery time approximations are important for the tactical routes to be valid when actual demand is revealed, and to reduce the chances of route failures in upcoming operational routing decisions. Moreover, we further discussed a hypothesis that larger polygons typically attain the lowest delivery time variance.

Based on twenty simulations of different demand realizations (see Chapter 5), the expected delivery time and standard deviation can be calculated for each polygon in a partitioning. This is used to calculate the average expected delivery time and standard deviation for different partitionings of the 68 postal code areas in Table 7.6. Numbers are reported for each demand level, low, medium and high. The results confirm our hypothesis, as P-MIN attains the lowest average standard deviation across all three demand levels. Hence, employing the P-MAX partitioning results in the most fluctuating delivery times on average. Interesting is how this result affects the tactical routes constructed through the set of all polygons in both P-MIN and P-MAX. This remark is concerned in Section 7.5.

Table 7.6: Average delivery time and standard deviation of 20 Monte Carlo simulations per polygon per demand level, where polygons are generated according to the minimum and maximum number of partitionings of a postal code area, P-MIN and P-MAX

		Low demand		**Medium demand**		**High demand**	
		P-MIN	*P-MAX*	*P-MIN*	*P-MAX*	*P-MIN*	*P-MAX*
Total time	*Mean expected delivery time [sec]*	550.8	416.5	815.2	623.4	1 370.3	1 043.5
	Mean SD delivery time [sec]	340.4	307.9	375.5	346.8	480.9	448.0
	Mean SD [% of mean]	61.8	73.9	46.1	55.6%	35.1	42.9
Travel time	*Mean expected travel time [sec]*	104.2	75.1	146.5	107.8	207.2	152.7
	Mean SD travel time [sec]	64.3	54.8	68.7	60.6	71.9	63.0
	Mean SD [% of mean]	61.7	72.9	46.9	56.2	34.7	41.3
Service time	*Mean expected service time [sec]*	446.6	341.4	668.6	515.6	1 163.0	890.8
	Mean SD service time [sec]	285.2	260.2	320.7	296.8	427.4	399.5
	Mean SD [% of mean]	63.9	76.2	48.0	57.6	36.7	44.8

7.4 Technical Analysis of the PRP

This section evaluates the implemented IP model to solve the PRP, by performing a runtime analysis of the model. From the P-MAX partitioning, we derived subsets of polygons and generated tactical routes for each subset to evaluate the runtime and solution quality of the PRP model. Table 7.7 presents the runtime results, for an increasing number of polygons using the expected delivery time of a medium demand level. Additionally, the upper and lower bounds are reported, with the corresponding optimality gaps. As can be observed, the model reached the time limit of 5 hours for test instances with more than 90 polygons. However, the largest test instance of 144 polygons obtained an optimality gap of merely 2.00%. This optimality gap is considered acceptable, and thus we can conclude that an exact method is sufficient to solve the test instances in our case study.

We note from Table 7.7 that the PRP attains few variables and constraints, relative to the number of polygon centers, i.e. nodes, in the routing problem. This is reasoned in a neighboring graph, in which the polygon centers are only connected to their immediate neighbors, i.e. polygons that share a road segment, and the logistic terminal. In the P-MAX partitioning, each node has on average only 8 neighbors. This is why we are able to acquire satiable solutions for large scale instances.

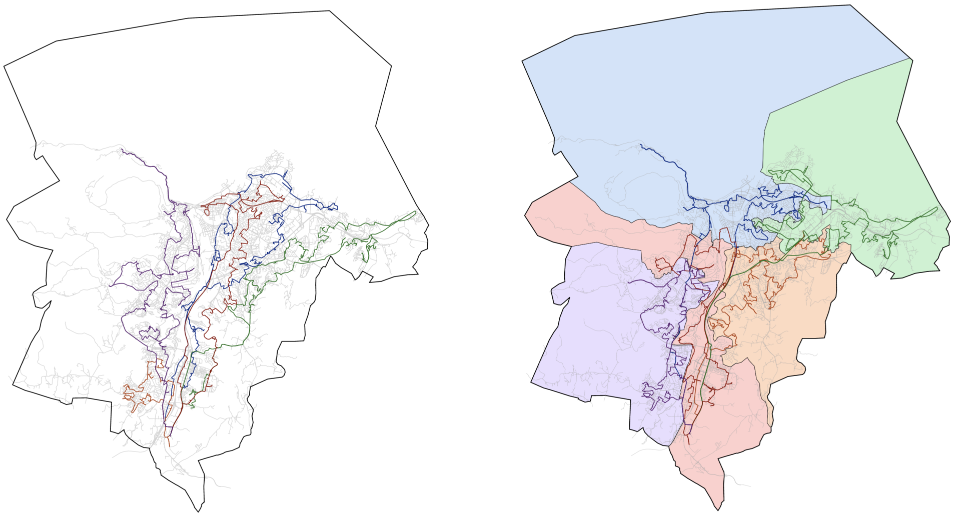
Table 7.7: The number of constraints and variables after presolve when solving the PRP for subsets of the P-MAX partitioning with expected delivery times according to the medium demand level. "TL" denotes that the time limit of 18 000 seconds is reached. Gurobi tolerance gap is 0.01%

Polygons [#]	Constraints [#]	Variables [#]	LB	UB	Gap [%]	Runtime [sec]
15	93	73 (58 binary)	14 279.23	14 279.23	0	0.20
30	229	198 (168 binary)	25 916.68	25 916.68	0	1.35
45	344	298 (253 binary)	44 020.89	44 020.89	0	104.73
60	473	412 (352 binary)	53 468.66	53 469.35	0	434.46
75	608	532 (457 binary)	67 258.86	67 261.06	0	34 062.28
90	727	727 (548 binary)	80 974.91	82 238.25	1.56	TL
105	864	761 (656 binary)	94 362.61	95 504.29	1.21	TL
120	993	875 (755 binary)	107 118.54	108 775.26	1.55	TL
135	1 128	996 (861 binary)	120 521.75	122 350.31	1.52	TL
144	1 207	1 066 (922 binary)	132 455.37	135 106.47	2.00	TL

7.5 Economic Analysis

This section analyzes the usability of the tactical routes generated with our proposed solution approach. Tactical routes were constructed for each demand level and for different partitionings of the geographical area, P-MIN and P-MAX. The tactical routing plans are illustrated in Figures H.1-H.3 in Appendix H. Using Google's OR Tools the delivery time of daily routes derived from our tactical routes, i.e. P-MIN-OR and P-MAX-OR, are compared against the delivery time of a deterministic VRP solution, i.e. benchmark. Figures 7.2a and 7.2b provide an example of a benchmark solution and operational routes bounded by tactical routes for a given demand realization.

Figures 7.3-7.5 show the total delivery time of P-MIN-OR and P-MAX-OR for 10 different demand realizations of low, medium and high demand, compared against the benchmark solution. As seen from Figure 7.3, the benchmark outperforms P-MIN-OR and P-MAX-OR across all the 10 simulations at a low demand level. We note that P-MAX-OR outperforms P-MIN-OR for all simulations. Further, for a normal and high demand level in Figures 7.4 and 7.5, we observe that the same trends repeat, that is, the benchmark outperforms P-MIN-OR and P-MAX-OR across all simulations, and P-MAX-OR outperforms P-MIN-OR for most simulations. An interesting remark is that the average delivery time of P-MIN-OR and P-MAX-OR are more similar at higher demand levels, than at the low demand level.



(a) Home delivery routes planned from scratch, i.e. a benchmark solution

(b) Home delivery routes derived from tactical routes

Figure 7.2: Each colored path in Figure 7.2a and 7.2b indicates one tour of a driver. Each colored area of Figure 7.2b indicates one tour in the tactical routing plan. While routes in Figure 7.2a can cross each others paths, e.g. red and blue tour, the routes in Figure 7.2b are bounded by the tactical routing plan and are non-overlapping

Key metrics from the 10 simulations of each demand level is provided in Table 7.8. Across all demand levels, the P-MIN-OR and P-MAX-OR incur on average 2.89% and 2.34% higher total delivery times compared to the benchmark. This is equivalent to an average of 58 and 47 minutes longer daily delivery times when applying the routes from P-MAX-OR and P-MIN-OR, respectively.

As discussed in Section 7.3.3 larger polygons attain lower variance. However, we note from Table 7.8 that when the polygons are combined into routes, the routes attain relatively low standard deviations in average delivery time across all demand levels, and no significant difference is observed between P-MIN-OR and P-MAX-OR. That is, a trend indicating lower variance for routes derived from a partitioning with larger polygons do not prevail, even though the variance of each polygon in isolation is proved to be lower. This result seems reasonable as the variance of a single polygon will even out when more polygons are combined into routes.

To conclude, the results of this section indicate that the operational routes derived from tactical routes perform slightly worse than benchmark. However, it should be noted that the total delivery time difference is relatively small. Thus, other logistical advantages of applying tactical routes, such as sorting efficiencies, may outweigh the decreased delivery time of optimizing routes from scratch every day. As P-MAX-OR outperforms P-MIN-OR across all demand levels, employing a more flexible partitioning with smaller polygons in the route construction phase is seemingly the most competitive to a deterministic VRP. Applying the routes of P-MAX-OR over routes that are daily optimized from scratch, can hence be perceived as a viable approach, incurring insignificant increases in daily delivery time, and at the same time preserving the advantages obtained from constructing routes a priori.

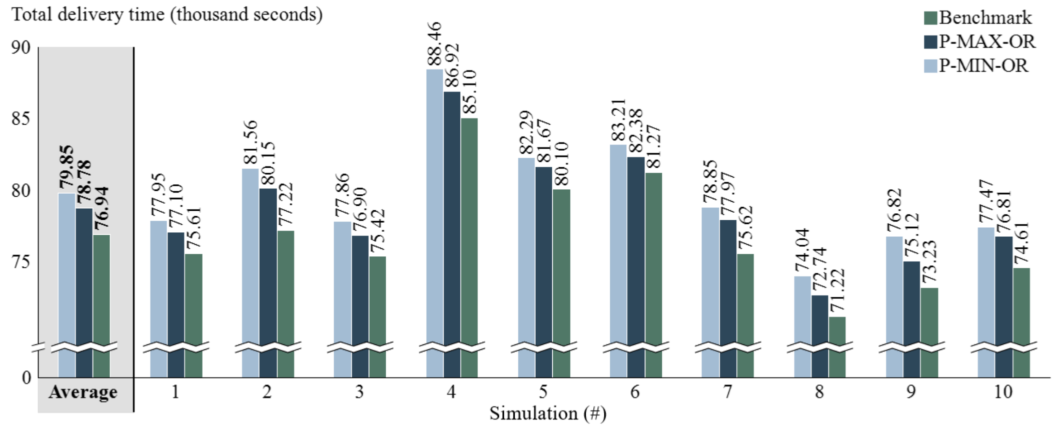


Figure 7.3: The total delivery time of simulations, i.e. operational routing, of 10 different demand realizations of a low demand level

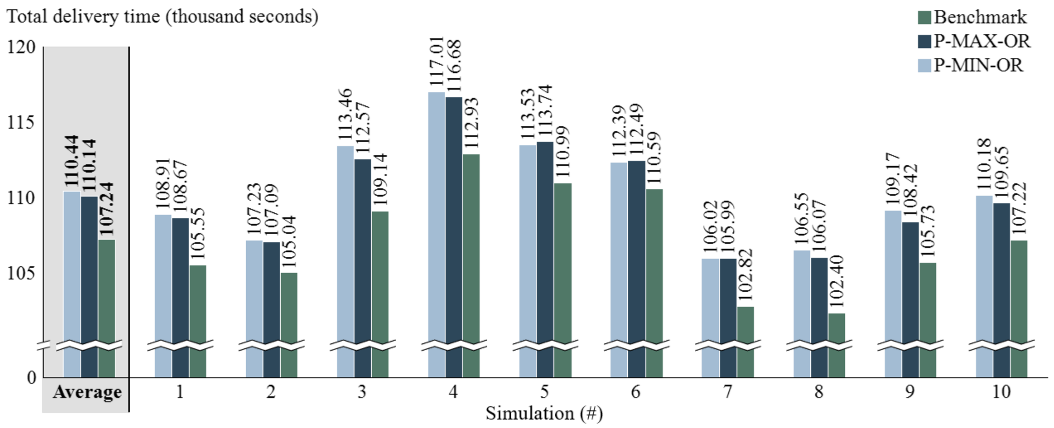


Figure 7.4: Total delivery time of simulations, i.e. operational routing, of 10 different demand realizations of a medium demand level

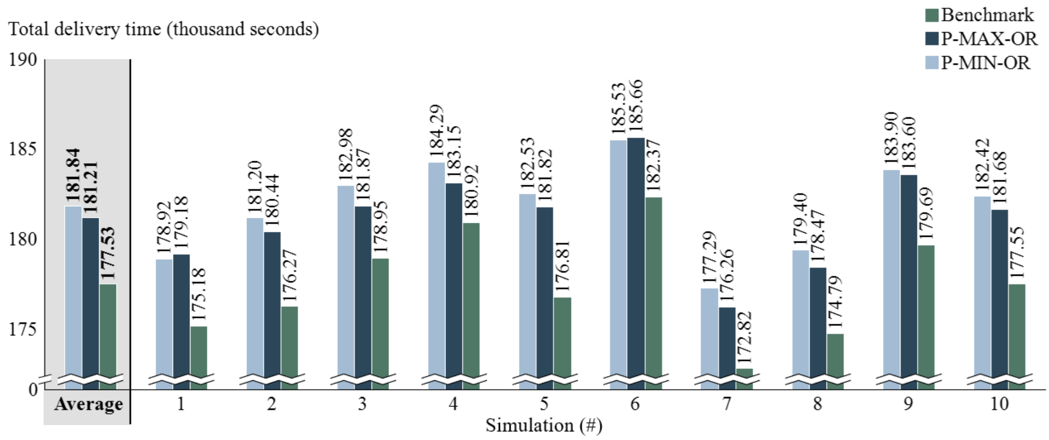


Figure 7.5: The total delivery time of simulations, i.e. operational routing, of 10 different demand realizations of a high demand level

Table 7.8: Average delivery time of P-MIN-OR, P-MAX-OR and the benchmark solution. The averages are given for 10 different demand realizations of low, medium and high demand levels, respectively, and compared to the benchmark average

		Average delivery time [hours and min]	SD [hours and min]	Increase from benchmark [hours and min]	Increase from benchmark [%]
Low	P-MIN-OR	22 h 11 m	1 h 8 m	0 h 48 m	3.74%
	P-MAX-OR	21 h 53 m	1 h 8 m	0 h 30 m	2.34%
	Benchmark	21 h 23 m	1 h 9 m	-	-
Medium	P-MIN-OR	30 h 41 m	1 h 0 m	0 h 53 m	2.96%
	P-MAX-OR	30 h 36 m	1 h 0 m	0 h 48 m	2.68%
	Benchmark	29 h 48 m	0 h 59 m	-	-
High	P-MIN-OR	50 h 31 m	0 h 44 m	1 h 13 m	2.47%
	P-MAX-OR	50 h 20 m	0 h 46 m	1 h 2 m	2.10%
	Benchmark	49 h 18 m	0 h 50 m	-	-
ALL	P-MIN-OR	34 h 28 m	12 h 7 m	0 h 58 m	2.89%
	P-MAX-OR	34 h 16 m	12 h 9 m	0 h 47 m	2.34%
	Benchmark	33 h 30 m	11 h 57 m	-	-

7.6 Project Limitations

This section presents limitations of the solution approach used to solve the Route Partitioning Problem (RPP). Firstly, in this project, the postal code areas lay the basis for the polygon partitionings. This assumption limits the possible partitionings of the service area. Hence, a partitioning better accommodating for the construction of tactical routes, could possibly be acquired by disregarding the postal code areas. However, this would probably entail the need to implement faster approaches to solve the districting problem within reasonable time, since the runtime of our implemented location-allocation algorithm rapidly increases with number of polygon centers and basic units.

Another potential limitation regards our benchmark solution. Since this project employs Google's OR-Tools to construct the benchmark, instead of Posten Norge AS's in-house route optimization software, the routes might not be fully analogous to those applied in practice. Using solutions from Posten Norge AS's software as benchmark, would probably give a better indication and quantification of the potential of applying tactical routes in practice. We also note that Google's OR Tools delivers heuristic solutions, meaning that we cannot guarantee that our optimal solutions are in fact optimal compared to a solution of an exact method.

Another limitation of our study is that the constraints imposing a requirement of 95% probability for at least one delivery for a given polygon, is not tested for other threshold values. Lower thresholds will enable a higher number of possible partitionings of each postal code area, which could favour greater degree of flexibility, even though the probability of no deliveries in a given polygon increases. Investigating the affect of this trade-off on our proposed tactical routes, could be interesting as we have verified that flexible partitionings provide routes that are more analogous to the routes derived from a deterministic VRP.

Furthermore, the tactical routes are determined for a limited geographical area, meaning that we cannot state with certainty that tactical routing in general introduce a viable alternative to a deterministic VRP. Lastly, the RPP do not account for recourse policies, of which prevent us to accommodate for adaptations of the tactical routes based on daily demand realizations. Hence, if resource capacity for a given day is exceeded, our proposed tactical routes will not suffice. The limitations and implications touched upon in this section, will be further elaborated on in Chapter 8.

Chapter 8

Concluding Remarks

In this thesis, we have studied the Route Partitioning Problem (RPP), the problem of generating tactical routes for home delivery of parcels by partitioning a geographical area into smaller areas and sequencing these. A two-stage solution approach have been implemented, and solved using two Integer Programming (IP) models and a location-allocation heuristic algorithm.

The thesis contributes to the literature in three ways. Firstly, an extensive literature review identified limited use of strategic districting combined with tactical routing. Further, we have identified that few of the existing papers on districting include multiple elements of the real world into their formulations, which reduce the practical credibility of the proposed solutions. Secondly, a novel two-stage approach was proposed and constitutes the first known attempt to combine strategic districting and tactical routing. Thirdly, the thesis performed an empirical computational study based on real-world data, which quantified the delivery time difference between applying tactical routes compared to optimizing routes each day from scratch. Our computational study identified opportunities for improvement in Posten Norge AS's current routing strategy. Specifically, the results highlighted the benefits of including flexibility in the route construction phase, using a more granular districting partitioning.

The aim of the computational study was to evaluate whether designing a set of tactical routes to be operated unchanged over a given period of time is a viable alternative to constructing a set of routes every day based on the particular instance of that day. To provide an empirical answer to this question, realizations of the customer demands were generated, and operational routes were derived from tactical routes and compared to a deterministic VRP. The results from the computational study show that the implemented approach yield promising results. On average, the tactical routing solution with the most granular partitioning incurred 2.3% longer total delivery times than a deterministic VRP. This slight increase in total delivery times is encouraging as the cost-benefits of applying tactical routes can lead the way to other operational improvements such as sorting and delivery predictability. This indicates that tactical routes generated with our solution approach does provide a viable alternative to optimizing routes each day from scratch. However, further research is needed to conclude on the quality of the proposed approach.

8.1 Future Research

This section highlights research opportunities that can contribute to further research of the Route Partitioning Problem (RPP). Firstly, Section 8.1.1 discusses how the quality and robustness of the tactical routes generated from the proposed solution approach can be improved. Specifically, more detailed data of the underlying road network, evaluation of other geographical areas and criteria extension of the mathematical programming model are discussed. Section 8.1.2 evaluates how an extension of the Route Partitioning Problem (RPP) can be valuable to generate more comprehensive and better support for postal logistic companies.

8.1.1 Further Development of Tactical Routes

To decrease total delivery time and increase the real-world performance of our proposed approach, more comprehensive data and practical considerations in the road network data should be included, both when solving the PPP and the PRP. For instance, traffic-specific information can be incorporated to identify points of congestion that indicate barriers. Additionally, denoting, e.g. roundabouts and one-way streets might increase the practical usability of our proposed tactical routes.

The computational study in this thesis indicates promising results for the tactical routes generated with our proposed solution approach. However, further testing needs to be conducted on different geographical areas before it can be concluded that tactical routing provides a viable alternative in general. Specifically, the model needs to be evaluated with geographical areas of varying sizes, parcel volumes, demand levels and delivery densities. Moreover, the tactical routes should be evaluated against other commercial route planners employed in the industry. Further testing will provide insightful information on the robustness of our proposed solution method.

Additional criteria when solving the PRP could be valuable to investigate to improve the practical usage of tactical routes. Merely considering the minimization of total routing costs as the main objective, including minimum travel time and a minimal number of vehicles, will alone not be sufficient to capture all applicable criteria to accommodate practical use cases in the best way possible (Vidal et al., 2020). Firstly, balancing the drivers' workloads could be implemented to minimize possible overtime relative to the drivers' contractual working times and ensure a higher degree of resource robustness regarding parcel volume fluctuations. Secondly, criteria to ensure consistency in the routing plans could be implemented. By ensuring consistency in the routing plans by endeavouring a minimal deviation of polygon-driver assignments from one tactical plan to another, drivers will get familiar with the peculiarities of their assigned routes over time which might lead to higher delivery efficiencies. Moreover, favouring consistency may result in the driver getting to know its customers in a service area, leading to improved customer satisfaction and personalization. Lastly, to increase the acceptance of the proposed routing plans, the intuitive appeal of the routes should be favoured by seeking a compact representation of the routes.

8.1.2 Extensions of Problem Scope

A failure occurs when the travel and service time of operational routes derived from the tactical routes exceeds the drivers' working-shift. As our project does not consider the optimizing of operational routes, a given customer demand realization might prove a tactical routing plan infeasible for a given day. As addressed in Chapter 3, it is common in the literature that preplanned routes are updated on the current day to better match the actual demand realization. The development of a suitable recourse policy would be an essential extension of the problem scope to improve the practical usability of the RPP solutions. Since customers are considered on an aggregated level in our tactical routes, rather than exact addresses, it should be relatively easy to extend the approach to include recourse actions. A recourse action could simply be to move a polygon from one route to another to attain feasibility. This approach merely requires some routes to be manually re-sorted each day, as opposed to the entire service region.

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Appendix A

Polygon Visualization Using Voronoi Diagrams

As noted in Chapter 5, the Voronoi diagram is used to illustrate regional borders of the partitioning obtained when solving the location-allocation algorithm. Figures A.1-A.3 illustrate the three consecutive steps of obtaining Voronoi regions of a partitioning. To generate the Voronoi diagram, we employ a Python package, called Geovoronoi. To be able to use the Python package, we must first split each road/line segment into a set of points. The interval between these points was set to five meters. The Voronoi diagram of the set of points is then the division of the plane into polygonal cells, as illustrated in Figure A.1. Each polygonal cell associated with a point, contain the area of the plane that lies closer to this point than to any other. Hence, the boundaries between the cells are line segments, where each borderline separating two areas lie exactly in the middle of the associated points. Secondly, we aggregated these points and their corresponding Voronoi cells into road segments, as shown in Figure A.2. Finally, the roads were aggregated into their assigned polygon, resulting in regional borders as illustrated in Figure A.3.

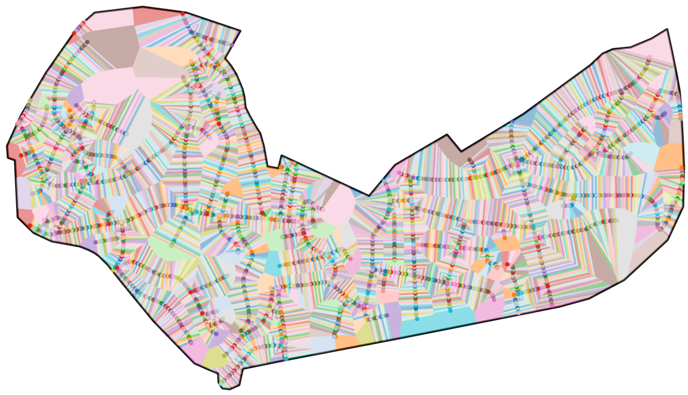


Figure A.1: The figure illustrates how roads in a postal code area are split into a set of points, and how their associated Voronoi cell is generated. The set of cells comprise a Voronoi diagram

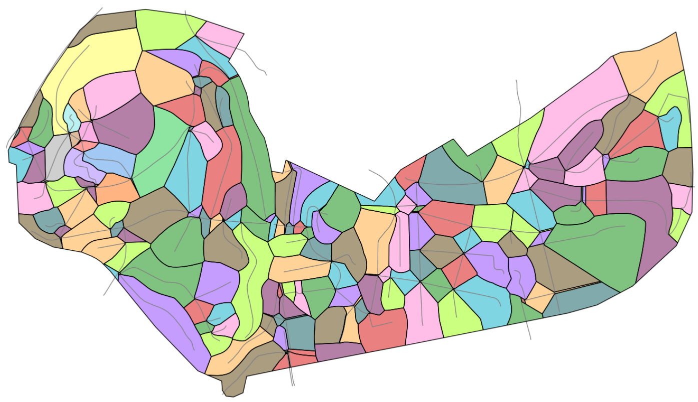


Figure A.2: The figure illustrates how Voronoi cells from Figure A.1 are aggregated according to road segments

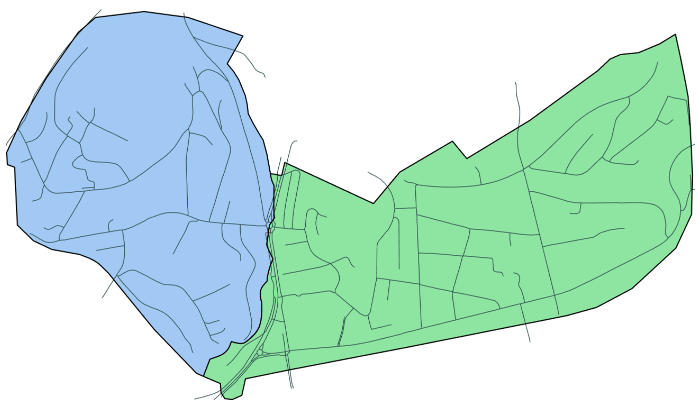


Figure A.3: The figure illustrates how the road segments and their corresponding polygonal cells from Figure A.2 are aggregated into polygons. The aggregation is performed according to the central unit which each road is assigned to in the allocation phase of location-allocation algorithm. The final result provides a polygon partitioning with regional borders

Appendix B

The Polygon Partitioning Problem with Heuristic Constraints (PPPHC)

The following model is a heuristic approach to solve the PPP model presented in Chapter 5. To increase efficiency and enable solving larger test instances, flow constraints (5.4)-(5.6) in the PPP are replaced by heuristic contiguity constraints, arcs are replaced by edges and some assignment variables are heuristically fixed.

B.1 Notation

Sets are defined as calligraphic, upper-case letters, whereas parameters are defined through upper-case letters. Decision variables and indices are lower-case. Set and parameter identifiers are used through superscript, while indices are applied as subscript.

B.1.1 Sets

$\mathcal{V}$	Set of vertices representing street crossings or alleys in the given service region, $\mathcal{V} = \{1, ..., V\}$
$\mathcal{E}$	Set of edges representing road segments, $\{i, j\} \in \mathcal{E} \wedge i < j$, connecting street crossings or alleys $i \in \mathcal{V}$ and $j \in \mathcal{V}$
$\mathcal{E}^S$	Set of central units in the given service region, $\mathcal{E}^S \subset \mathcal{E}$

$\mathcal{E}^B$	Set of edges representing "barriers", i.e. road segments of type European motorway, national road or county road, $\mathcal{E}^B \subset \mathcal{E}$
$\mathcal{E}^N_{ij}$	Set of neighbor edges of $\{i,j\} \in \mathcal{E}$, $\mathcal{E}^N_{ij} \subset \mathcal{E}$
$\mathcal{E}^C_{ijs}$	Set of immediate neighbor edges of $\{i,j\} \in \mathcal{E}$ that are along some shortest path from $\{i,j\}$ to center unit s, $\mathcal{E}^C_{ijs} \subseteq \mathcal{E}^N_{ij}$
$\mathcal{E}^F_s$	Set of edges that cannot be assigned to center s, $\mathcal{E}^F_s \subset \mathcal{E}$

B.1.2 Indices

i	Vertex, $i \in \mathcal{V}$
$\{i,j\}$	Edge, $\{i,j\} \in \mathcal{E}$
s	Center unit, $s = \{i,j\} \in \mathcal{E}^S$

B.1.3 Parameters

D_{ijs}	Distance from each edge $\{i,j\}$ to center unit s
Q_{ij}	Expected (average) demand associated with edge $\{i,j\}$
$\underline{Q}$	The minimum required expected (average) demand of any polygon
M	The maximum number of edges that can possibly be assigned to any polygon, $M = \lvert\mathcal{E}\rvert - \lvert\mathcal{E}^S\rvert + 1$
α	Penalty cost to limit the number of neighbor edges assigned to a different polygon
β	Penalty cost for assignment of edges representing barriers

B.1.4 Variables

x_{ijs}	1 if edge $\{i,j\}$ is assigned to the polygon centered at center unit s, 0 otherwise
v_{ijs}	The number of edges which are connected to edge $\{i,j\}$ but belongs to center unit s' different than s, needed to measure the degree of polygon isolation

B.2 Model

$$\min \sum_{s\in\mathcal{E}^{\mathcal{S}}} \sum_{\{i,j\}\in\mathcal{E}} D_{ijs}x_{ijs} + \alpha \sum_{s\in\mathcal{E}^{\mathcal{S}}} \sum_{\{i,j\}\in\mathcal{E}} v_{ijs} + \beta \sum_{s\in\mathcal{E}^{\mathcal{S}}} \sum_{\{i,j\}\in\mathcal{E}^{B}} x_{ijs} \tag{B.1}$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{E}^{\mathcal{S}}} x_{ijs} = 1 \qquad \{i,j\}\in\mathcal{E}\backslash\mathcal{E}^{B} \tag{B.2}$$

$$\sum_{s\in\mathcal{E}^{\mathcal{S}}} x_{ijs} \leq 1 \qquad \{i,j\}\in\mathcal{E}^{B} \tag{B.3}$$

$$x_{ijs} = 0 \qquad \{i,j\}\in\mathcal{E}^{F}_{s}, s\in\mathcal{E}^{S} \tag{B.4}$$

$$x_{ijs} \leq \sum_{\{k,l\}\in\mathcal{E}^{C}_{ijs}} x_{kls} \qquad \{i,j\}\in\mathcal{E}, s\in\mathcal{E}^{S} \tag{B.5}$$

$$\underline{Q} \leq \sum_{\{i,j\}\in\mathcal{E}} Q_{ij}\cdot x_{ijs} \qquad s\in\mathcal{E}^{\mathcal{S}} \tag{B.6}$$

$$v_{ijs} \geq \sum_{s'\in\mathcal{E}^{\mathcal{S}}\backslash\{s\}} \sum_{\{k,l\}\in\mathcal{E}^{N}_{ij}} x_{kls'} - (1-x_{ijs})\cdot M \qquad \{i,j\}\in\mathcal{E}, s\in\mathcal{E}^{\mathcal{S}} \tag{B.7}$$

$$v_{ijs} \geq 0 \qquad \{i,j\}\in\mathcal{E}, s\in\mathcal{E}^{\mathcal{S}} \tag{B.8}$$

$$x_{ijs} \in \{0,1\} \qquad \{i,j\}\in\mathcal{E}, s\in\mathcal{E}^{\mathcal{S}} \tag{B.9}$$

B.2.1 Objective Function

Equivalently to the PPP model in Chapter 5, the objective function (B.1) includes indicators of compactness, isolation and crossing of barriers.

B.2.2 Constraints

Constraints (B.2) ensure complete and exclusive assignment of all "required" edges, while constraints (B.3) ensure that edges representing barriers are assigned to at most one center unit $s \in \mathcal{A}^{S}$. Constraints (B.4) heuristically fix variables by forbidding assignments of network edges that are far away from the center. Moreover, (B.5) are heuristic contiguity constraints, meaning that if a network edge $\{i,j\}$ is assigned to a center s then one of its immediate neighbors along some shortest path to the center, must also be assigned to the center. Furthermore, constraints (B.6) impose the lower bound on expected demand in each polygon. Constraints (B.7) set the value of auxiliary variables v_{ijs}. These variables equal the total number of neighboring arcs that are assigned to different centers, i.e. entry/exit roads. Finally, non-negativity constraints for variables are defined in (B.8), while (B.9) are constraints for the binary variables.

Appendix C

Overview of Postal Code Areas

Table C.1 provides an overview of the postal code areas used as test instances for this thesis, with corresponding area-characteristics and the maximum number of polygons a postal code area can be partitioned into.

Table C.1: Overview of test instances used for computational studies, with corresponding characteristics

Postal code area	Surface area [km^2]	Road-segments [#]	Avg. deliveries per volume scenario [#]			Maximum polygon centers [#]	Min polygon area [km^2]	Max polygon area [km^2]
			Low	Medium	High			
7010	58.99	166	2.06	3.10	3.69	1	58.99	58.99
7011	0.26	110	2.19	2.68	3.08	1	0.26	0.26
7012	0.51	173	4.03	6.21	10.15	2	0.18	0.32
7013	0.28	66	0.52	0.95	1.69	1	0.28	0.28
7014	0.36	135	4.81	7.86	14.69	2	0.17	0.19
7015	0.27	83	1.81	2.59	3.92	1	0.27	0.27
7016	0.26	77	1.77	2.24	4.00	1	0.26	0.26
7017	0.60	124	2.13	3.25	4.54	1	0.60	0.60
7018	37.72	240	5.06	7.76	11.38	2	0.95	36.76
7019	0.90	137	1.58	3.08	3.69	1	0.90	0.90
7020	24.25	526	13.32	19.63	32.69	5	0.31	12.12
7021	1.51	319	6.84	10.87	17.08	3	0.39	0.69

Table C.1 continued from previous page

7022	1.26	228	7.00	12.25	18.38	3	0.33	0.54
7023	1.03	209	6.81	10.49	18.23	3	0.31	0.38
7024	10.87	385	4.84	7.13	12.15	2	0.71	10.16
7025	1.05	80	2.10	3.84	7.15	1	1.05	1.05
7026	8.17	484	7.10	10.14	17.62	3	0.52	5.46
7027	1.42	350	5.87	8.59	16.46	2	0.54	0.88
7028	0.89	152	1.45	1.92	4.31	1	0.89	0.89
7029	1.09	122	2.48	4.33	7.00	1	1.09	1.09
7030	1.59	360	9.03	16.16	23.77	5	0.18	0.68
7031	1.25	270	5.26	8.11	12.46	2	0.52	0.73
7032	0.56	119	3.77	5.76	9.31	1	0.56	0.56
7033	1.45	346	9.87	12.38	24.15	4	0.31	0.46
7034	0.36	71	0.29	0.24	0.54	1	0.36	0.36
7035	0.24	48	2.23	3.05	3.77	1	0.24	0.24
7036	10.34	529	11.32	15.02	27.85	5	0.27	8.38
7037	2.23	235	4.35	6.29	11.77	2	0.90	1.33
7038	1.86	165	4.68	7.79	12.92	2	0.62	1.24
7040	20.02	213	5.55	8.30	14.31	2	9.70	10.32
7041	2.35	314	6.00	10.86	13.69	3	0.50	1.08
7042	0.58	72	3.19	4.27	7.69	1	0.58	0.58
7043	0.35	106	4.61	7.48	11.85	2	0.12	0.23
7044	1.00	195	3.97	4.94	4.69	1	1.00	1.00
7045	1.10	326	8.03	11.44	19.31	3	0.33	0.43
7046	1.52	422	8.48	12.14	24.31	4	0.21	0.56
7047	1.73	277	3.90	6.33	9.54	2	0.44	1.29
7048	1.63	294	7.45	13.84	25.31	4	0.21	0.89
7049	5.76	386	6.42	9.02	14.54	3	0.38	4.51
7050	0.94	289	5.65	9.59	15.54	3	0.26	0.36
7051	0.33	103	2.87	4.52	6.54	1	0.33	0.33
7052	1.57	476	12.48	19.00	33.08	5	0.14	0.52
7053	6.85	333	5.45	7.86	13.77	2	2.93	3.92
7054	5.99	419	5.58	8.87	17.23	3	0.77	3.68
7055	11.98	454	7.19	9.87	17.69	3	2.58	6.11
7056	2.02	219	7.84	10.97	19.54	3	0.31	1.09
7058	1.43	172	7.03	11.25	18.54	3	0.28	0.69
7059	1.29	140	4.71	7.40	13.92	2	0.40	0.89
7066	0.18	38	4.45	6.30	10.08	2	0.07	0.11
7067	0.52	149	4.71	5.86	7.23	1	0.52	0.52
7068	0.25	95	3.00	4.76	6.92	1	0.25	0.25

Table C.1 continued from previous page

7069	0.49	100	1.87	2.05	4.00	1	0.49	0.49
7071	0.69	140	3.35	7.14	11.08	2	0.33	0.36
7072	1.46	367	8.23	10.67	18.08	1	1.46	1.46
7075	3.26	725	6.42	9.08	13.23	2	0.86	2.41
7078	0.70	214	3.71	4.57	9.00	1	0.70	0.70
7079	1.20	205	4.87	7.02	10.62	2	0.49	0.71
7080	2.81	291	3.26	4.13	7.77	1	2.81	2.81
7081	1.66	125	5.55	7.92	14.77	2	0.58	1.08
7082	2.06	233	3.84	7.14	10.00	2	0.35	1.71
7088	1.15	263	6.35	9.24	16.69	3	0.32	0.42
7089	14.87	346	7.71	11.70	24.46	2	1.80	13.08
7091	2.50	503	10.03	14.49	24.31	4	0.28	1.19
7092	7.90	331	5.29	8.67	16.38	2	0.82	7.08
7093	6.70	262	5.42	6.25	11.15	1	6.70	6.70
7097	0.80	96	2.06	2.92	5.38	1	0.80	0.80
7098	0.29	115	1.94	3.87	4.23	1	0.29	0.29
7099	1.10	253	4.32	6.79	12.54	2	0.40	0.69

Appendix D

Heatmaps of Parcel Delivery Within Trondheim Municipality

Figures D.1-D.3 illustrate the average number of parcel deliveries in each of the 68 postal code areas in Trondheim municipality for a low, medium and high demand level, respectively.

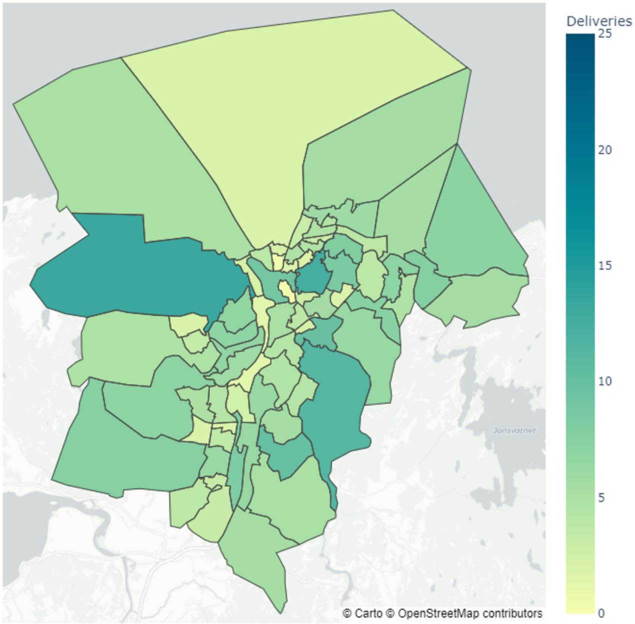


Figure D.1: Daily average deliveries for a low demand level in each postal code area, that is averages are calculated from days defined as low demand. This comprise 48 days in the historical data provided by Posten Norge AS. The total average deliveries per day is 337 parcels for the entire area

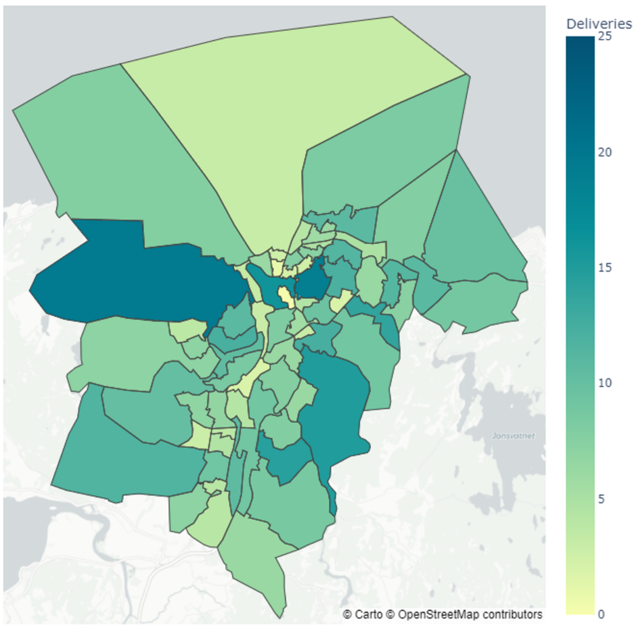


Figure D.2: Daily average deliveries for a medium demand level in each postal code area, that is averages are calculated from days defined as medium demand. This comprise 68 days in the historical data provided by Posten Norge AS. The total average deliveries per day is 518 parcels for the entire area

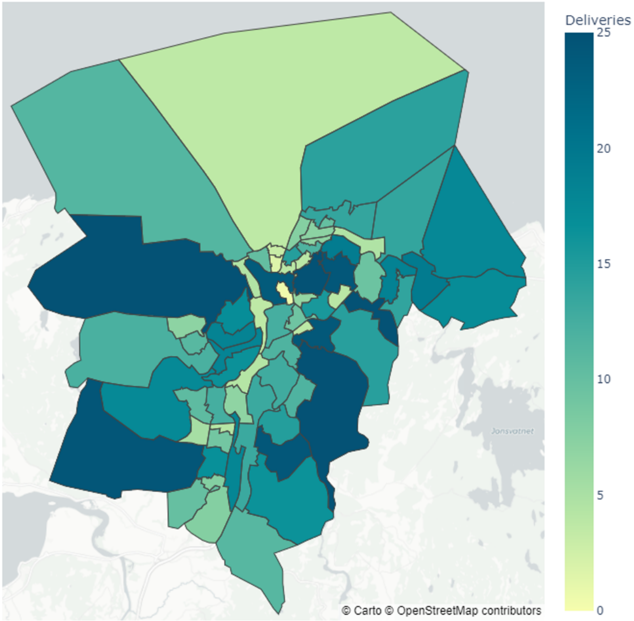


Figure D.3: Daily average deliveries for a high demand level in each postal code area, that is averages are calculated from days defined as high demand. This comprise 13 days in the historical data provided by Posten Norge AS. The total average deliveries per day is 877 parcels for the entire area

Appendix E

The Open Capacitated Arc Routing Problem (OCARP)

The Open Capacitated Arc Routing Problem (OCARP) is used to calculate expected delivery time within a polygon. Using the Open Capacitated Arc Routing Problem (OCARP) to simulate a series of customer demand realizations, the expected delivery time of each polygon is calculated as the average delivery time of all simulations.

For a given realization of customer demands, the Open Capacitated Arc Routing Problem (OCARP) identifies the shortest path within a polygon that serves all edges with positive demand while minimizing travel time. A path in the Open Capacitated Arc Routing Problem (OCARP) starts at the center edge of the polygon, outputted from the first stage districting problem, and ends when all demand is served.

E.1 Notation

The notation used for the model follows conventional modelling standards. Sets are defined as calligraphic, upper-case letters, whereas parameters are defined through upper-case letters. Decision variables and indices are lower-case. Set and parameter identifiers are used through superscript, while indices are applied as subscript.

E.1.1 Sets

$\mathcal{V}$	Set of nodes, $\mathcal{V} = \{0, ..., V\}$
$\mathcal{E}^R$	Set of required edges (edges with demand + edge from start to end node)
$\mathcal{A}$	Set of arcs, $(i, j) \in \mathcal{A}$
$\mathcal{S}$	Set of center edge for flow, i.e. "dummy edge" from start to end node, $\{i', j'\} \in \mathcal{S} \subset \mathcal{E}^R$

E.1.2 Indices

v	node, $v \in \mathcal{V}$
a	arc, $a = (i, j) \in \mathcal{A}$

E.1.3 Parameters

D_{ij}	Estimated driving time/distance on arc (i, j)
M	The maximum number of arcs that can possibly be traversed, $M = \lvert\mathcal{A}\rvert$

E.1.4 Variables

x_{ij}	1 if arc $(i, j) \in \mathcal{A}$ is traversed, 0 otherwise
α_i	Indegree of node i
β_i	Outdegree of node i
y_{ij}	1 if edge $\{i, j\}$ is serviced, 0 otherwise, $\{i, j\} \in \mathcal{E}^R, i < j$
f_{ij}	Amount of flow sent through arc $(i, j) \in \mathcal{A}$

E.2 Model

$$\min \sum_{i \in \mathcal{V}} \sum_{j:(i,j) \in \mathcal{A}} D_{ij} \cdot x_{ij} \tag{E.1}$$

$$
\begin{aligned}
\text{s.t.} \quad & -\beta_i \leq \sum_{j:(i,j)\in\mathcal{A}} (x_{ji} - x_{ij}) \leq \alpha_i && i \in \mathcal{V} && \text{(E.2)}\\
& \sum_{i\in\mathcal{V}} \beta_i \leq 1 && && \text{(E.3)}\\
& \sum_{i\in\mathcal{V}} \alpha_i \leq 1 && && \text{(E.4)}\\
& y_{ij} = 1 && \{i,j\} \in E^R, i < j && \text{(E.5)}\\
& x_{ij} + x_{ji} \leq y_{ij} && && \text{(E.6)}\\
& \sum_{j:(i,j)\in A} f_{ij} - \sum_{j:(j,i)\in A\setminus E^R} f_{ji} = \sum_{j:(i,j)\in A} x_{ij} && i \in V && \text{(E.7)}\\
& \sum_{j:(j,i')\in A} f_{ji'} + \sum_{j:(j,j')\in A} f_{jj'} = \sum_{(i,j)\in A} x_{ij} && \{i',j'\} \in \mathcal{S} && \text{(E.8)}\\
& f_{ij} \leq (M-1)\cdot x_{ij} && (i,j) \in A\setminus\mathcal{S} && \text{(E.9)}\\
& f_{ij} \geq 0 && (i,j) \in A && \text{(E.10)}\\
& x_{ij} \in \{0,1\} && (i,j) \in A && \text{(E.11)}\\
& y_{ij} \in \{0,1\} && \{i,j\} \in E^R && \text{(E.12)}\\
& \alpha_i, \beta_i \in \{0,1\} && i \in \mathcal{V} && \text{(E.13)}
\end{aligned}
$$

E.2.0.1 Objective Function

The objective function (E.1) minimizes the approximated travel time of delivering a set of parcels in the polygon.

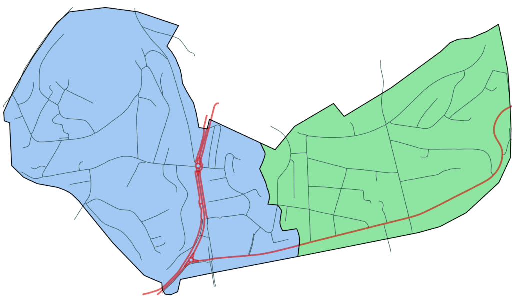
E.2.0.2 Constraints

Constraints (E.2)-(E.4) ensure that the vehicle will traverse the edges as an open tour; the visited nodes will have indegree equal to their outdegree, except at most two nodes, which can have a difference of one between indegree and outdegree. Constraints (E.5) guarantee that all required edges should be serviced exactly once, while (E.6) ensure that if the vehicle services an edge, it also has to traverse the edge. (E.7)-(E.9) are flow conservation constraints. Finally, (E.10) are non-negativity constraints, while (E.11)-(E.13) are the integrality constraints for the binary variables.

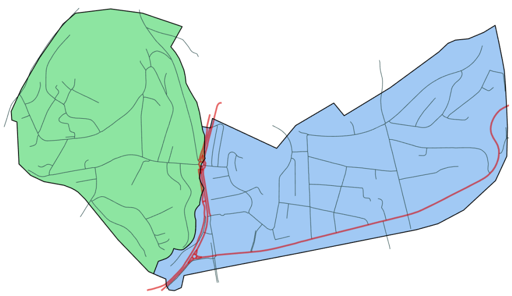
Appendix F

Parameter Tuning of the PPP Objective Weights

Figures F.1 and F.2 illustrate partitionings with two polygon centers of postal code area 7022 and 7056, respectively, using different parameter values of α and β. Compactness, isolation and barrier measures for these partitionings are presented in Chapter 7.



(a) α and β equal to 0



(b) α and β equal to 50 000

Figure F.1: Partitionings of postal code area 7022 into two polygons, i.e. blue and green area, for different values of input parameters α and β. Roads marked in red are considered barriers

(a) α and β equal to 0

(b) α and β equal to 50 000

Figure F.2: Partitionings of postal code area 7056 into two polygons, i.e. blue and green area, for different values of input parameters α and β. Roads marked in red are considered barriers

Appendix G

Comparison of LA and M-LA Allocation Phase Models

Table G.1 reports the number of variables and constraints in the allocation phase models of LA and M-LA, i.e. the PPP and the PPP with heuristic contiguity constraints and variable fixation, respectively.

Table G.1: Number of rows and columns after Gurobi's presolve for every possible polygon center of each postal code area, solved with location-allocation with exact contiguity constraints (LA) versus with heuristic contiguity constraints and variable fixation (M-LA)

		LA		M-LA	
Postal code area	**Polygon centers** [#]	**Rows** [#]	**Columns** [#]	**Rows** [#]	**Columns** [#]
7012	2	1551	1864	466	429
7014	2	1163	1402	288	286
7018	2	1986	2375	771	688
7020	2	4453	5344	2186	2130
7020	3	6489	8077	2234	2025
7020	4	8462	10739	4186	3864
7020	5	10466	13467	4717	3870
7021	2	2809	3353	658	577
7021	3	4064	5028	1850	1693

Table G.1 continued from previous page

7022	2	1928	2310	393	347
7022	3	2783	3459	1017	922
7023	2	1750	2117	269	240
7023	3	2548	3198	1011	932
7024	2	3191	3846	707	655
7026	2	4213	5043	2046	1978
7026	3	6067	7545	2974	2808
7027	2	2996	3552	867	774
7030	2	3396	4054	1154	1040
7030	3	4906	6067	2084	1885
7030	4	6405	8065	3158	3040
7030	5	7843	9968	3725	3439
7031	2	2351	2820	753	685
7033	2	2980	3567	984	881
7033	3	4363	5412	1770	1624
7033	4	5679	7182	2700	2503
7036	2	4577	5464	2060	1808
7036	3	6647	8227	3173	3016
7036	4	8623	10874	3424	3180
7036	5	10632	13562	5166	4826
7037	2	2091	2514	940	838
7038	2	1349	1615	384	359
7040	2	1808	2169	377	344
7041	2	2730	3283	445	387
7041	3	3943	4920	1487	1351
7043	2	881	1063	200	194
7045	2	2992	3563	1191	1052
7045	3	4341	5355	1865	1693
7046	2	3767	4460	1132	1006
7046	3	5459	6692	2253	2048
7046	4	6977	8721	3096	2854
7047	2	2448	2933	873	791
7048	2	2610	3115	314	312
7048	3	3773	4668	653	629
7048	4	4914	6195	1964	1804
7049	2	3276	3901	1101	984
7049	3	4678	5802	2299	2113
7050	2	2625	3126	750	689
7050	3	3789	4675	1872	1716

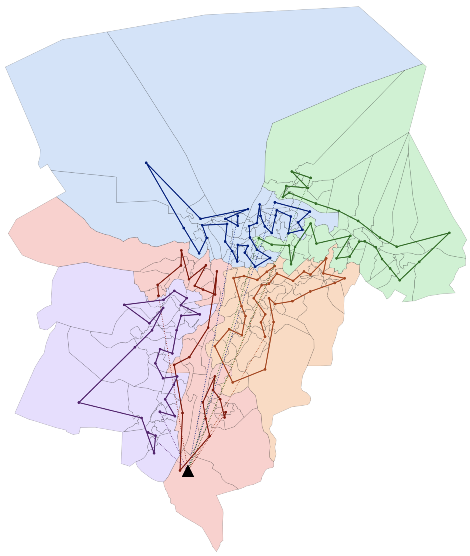
Table G.1 continued from previous page

7052	2	4481	5324	1547	1362
7052	3	6419	7867	2173	1970
7052	4	8400	10479	3655	3355
7052	5	10373	13092	4762	4490
7053	2	3066	3646	1015	914
7054	2	3635	4340	1721	1543
7054	3	5275	6528	2469	2481
7055	2	4057	4873	1154	1018
7055	3	5865	7307	2797	2578
7056	2	1922	2308	659	586
7056	3	2774	3456	1320	1212
7058	2	1434	1722	359	331
7058	3	2079	2591	517	481
7059	2	1103	1315	216	207
7066	2	302	372	29	33
7071	2	1120	1348	184	189
7075	2	6775	8087	638	588
7079	2	1883	2222	553	506
7081	2	1023	1235	363	342
7082	2	2103	2511	776	710
7088	2	2301	2781	1001	901
7088	3	3314	4158	1593	1450
7089	2	2572	3091	280	281
7091	2	4665	5540	1988	1799
7091	3	6749	8303	3231	2942
7091	4	8833	11059	4234	3847
7092	2	2862	3409	850	770
7099	2	2298	2708	982	887

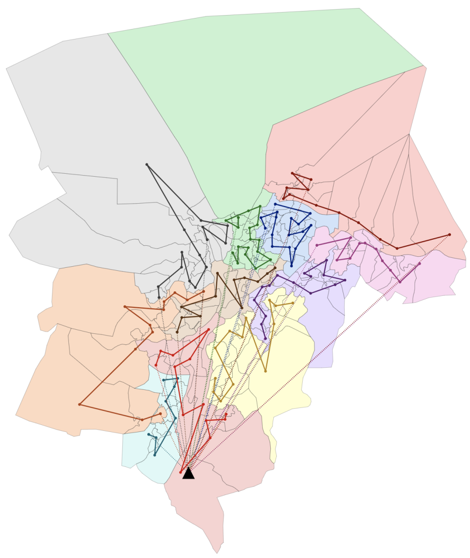
Appendix H

Tactical Routing Solutions from the PRP Model

Figures in this chapter illustrate tactical routing plan solutions from the PRP for different demand levels and polygon partitionings of the geographical area of Trondheim municipality. All routes in a routing plan starts and ends at the Trondheim Terminal, and visits a number of polygon centers.

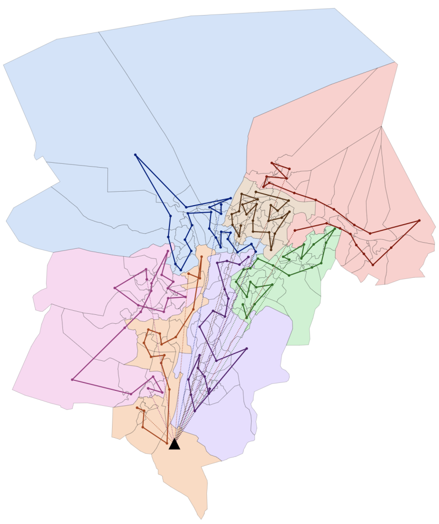


(a) Tactical routes through the polygon centers of a P-MAX partitioning

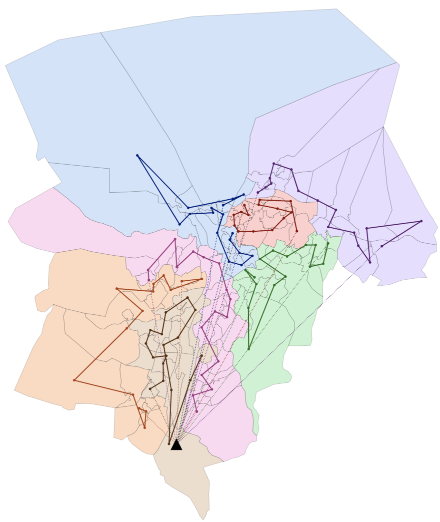


(b) Tactical routes through the polygon centers of a P-MIN partitioning

Figure H.1: Tactical routes from the PRP model in a low demand level. Figures H.1a and H.1b show the tactical routes with the maximum and minimum number of polygons, respectively. There are 5 routes, each represented by a colored path

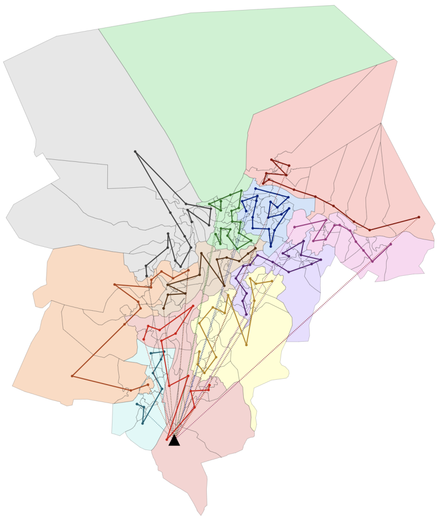


(a) Tactical routes through the polygon centers of a P-MAX partitioning

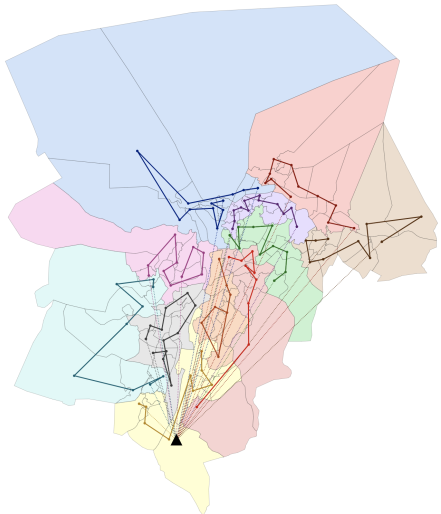


(b) Tactical routes through the polygon centers of a P-MIN partitioning

Figure H.2: Tactical routes from the PRP model in a medium demand level. Figures H.2a and H.2b show the tactical routes with the maximum and minimum number of polygons, respectively. There are 7 routes, each represented by a colored path



(a) Tactical routes through the polygon centers of a P-MAX partitioning



(b) Tactical routes through the polygon centers of a P-MIN partitioning

Figure H.3: Tactical routes from the PRP model in a low demand level. Figures H.3a and H.3b show the tactical routes with the maximum and minimum number of polygons, respectively. There are 11 routes, each represented by a colored path

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