Relations between grade 1 students' number sense and nonverbal and verbal reasoning

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This study investigated relations between number sense and nonverbal and verbal reasoning in 75 Norwegian grade 1 students. Number sense was measured by a digital assessment utilising eight components of the foundational number sense (FoNS) framework in combination with reasoning measures. Analysis revealed that number sense related moderately to verbal reasoning and moderately to strongly to nonverbal reasoning. However, number sense components correlated differently to the two reasoning types. Our results support, extend, and contradict previous research, perhaps due to how number sense was operationalised and measured. We found arithmetic to be the component with the strongest correlation to both nonverbal and verbal reasoning, while estimation did not correlate significantly to any reasoning types. We discuss individual variations and the use of visual and verbal task demands, particularly number lines, to support or estimate.

Keywords: number sense, reasoning, number lines, visual and verbal information, grade 1 students

Introduction

Number sense is ambiguously defined beyond being the ability to flexibly work with numbers and quantity and a predictor of mathematical competence (Andrews & Sayers, 2015). Several research fields focus on number sense, but there remains a need to resolve problems regarding polysemous and synonymous constructs thereof (Whitacre et al., 2020). This has led to varying foci on the perspectives of preverbal, applied, and foundational number sense. Preverbal number sense concerns the innate ability to estimate and compare small quantities without counting (Dehaene, 2001). Foundational number sense (FoNS) builds on preverbal number sense, is typically acquired during the first years of school, and requires instruction. Applied number sense builds on FoNS and consists of the basic mathematics needed in everyday life (Sayers & Andrews, 2015).

Developing number sense is cognitively complex and requires reasoning processes to make mental representations of numbers: to reason is to manipulate and work with information from sounds, signs, and symbols (Geary et al., 2018). Visual information is reasoned nonverbally, while spoken information is reasoned verbally. When combinations of visual and spoken information are given simultaneously, both nonverbal and verbal reasoning are required. Consequently, measures of nonverbal and verbal reasoning as cognitive prerequisites and focus on nonverbal visual and verbal task demands are being included more often in studies of children's abilities with counting and number representations (Whitacre et al., 2020). However, it is unknown to what extent and in what ways nonverbal and verbal reasoning are interrelated and how they are involved with the different task designs in components of number sense. Therefore, this paper's research question is:

How is the number sense of grade 1 students' related to their nonverbal and verbal reasoning?

Previous research

Nonverbal reasoning is suggested to underpin the understanding of numbers and arithmetic (Dehaene, 2001). As such, subitising is assumed to be important for verbal counting skills and arithmetic (Sayers et al., 2016). Number line estimation and mental number line representations have been found to correlate with number sense (e.g., arithmetic), counting, grouping, discrimination, and comparison of quantities, as well as with school achievement tests (Dehaene, 2001; Schneider et al., 2018). Despite the fact that geometrical, nonverbal reasoning is presumed important for number line estimation (Olkun et al., 2019), research findings differ on whether nonverbal or verbal reasoning is predominant for and involved in number sense. Jordan et al. (2013) highlighted both reasoning types as important.

Like Geary et al. (2018), who found support for verbal reasoning in cardinal understanding of numbers, Cross et al. (2019) found that verbal reasoning is involved in number identification, counting, and arithmetic, but not in number line and quantity comparison tasks. This illustrates that both nonverbal and verbal reasoning are important for some number sense components, but only one type is implicit in others. Extended knowledge and clarification about what number sense is and its relation to cognitive reasoning abilities is needed to better identify children who struggle to learn mathematics and to improve teaching methods and content (Geary et al., 2018).

Methods

The relationships between grade 1 students' number sense and their nonverbal and verbal reasoning were measured using a digital assessment operationalising number sense, while two standardised assessments measured nonverbal and verbal reasoning.

Participants

Following informed parental consent, 75 grade 1 students were recruited from two typical neighbourhood schools in Mid-Norway. The 10% who scored the highest and the lowest on the digital number sense assessment made up the subgroups *proficient students* and *developing students*, respectively.

Assessments

Nonverbal reasoning was measured using Raven's Progressive Matrices 2 (RPM). The students manipulated and compared visually presented figures to fill in the missing part (Raven, 2000).

The New Reynell Developmental Language Scales (NRDLS) were used to measure verbal reasoning, which resulted in two separate standard scores: language perception and language production (Letts et al., 2014). This assessment was conducted as a play-based dialogue about animals (Letts et al., 2014). Both the RPM and NRDLS results were standardised for Norwegian samples.

Andrews and Sayers (2015) defined number sense as consisting of eight components; of these, the digital number sense assessment (Saksvik-Raanes & Solstad, personal communication, September 2020), which consists of 69 tasks, assessed seven of the components. See Table 1 for the operationalisation of Andrews and Sayers' (2015) FoNS framework. Subitising was also included because of its importance to FoNS (Sayers et al., 2016). The digital number sense assessment distributes number sense tasks over three sets, two of which were solved by all participating students and used in the study. Tasks within the representing number component were placed in the third set, which not all students did. Because of this, the representing number component was not included.

Table 1 shows the included components and the number of tasks and exemplifies content within each component. Each task included verbal (sound file), visual, or both verbal and visual instruction. To respond, students typically dragged and dropped, organised objects on the screen, or tapped the appropriate multiple-choice item response.

Table 1. Type of component, content, and number of items in the number sense assessment

Component	Content	N
Number identification	Recognise number symbols, vocabulary, and meaning	8
Systematic counting	Ordinality. Counting to twenty and back (arbitrary starting point)	8
Number and quantity	Cardinality. 1-1 correspondence between symbol and quantity	10
Quantity discrimination	Compare quantities. Vocabulary: larger, smaller, more/less than	8
Estimation	Estimate the size of a set and the position on a number line	6
Arithmetic competence	Transforming small sets by using addition or subtraction	15
Number patterns	Continue or complete a number sequence	3
Subitising	Perceive quantity without counting. Perceptual/conceptual. Timed	11

Table 1 shows included components from Saksvik-Raanes & Solstad's (personal communication, September 2020) number sense assessment.

Procedures

Each student was tested in two sessions that occurred within two weeks of one another in autumn 2020. Data collection was completed within seven weeks. The first session, where RPM and NRDLS were conducted, was a 45-minute, one-on-one session with the first author and the student. Number sense was assessed via digital assessment in a 20–30-minute second session, wherein students were divided into groups of eight.

Analytical procedures

Descriptive statistics and pair-wise correlation analysis of standard scores from the RPM assessment, as well as raw scores from the digital number sense assessment, were conducted in SPSS. Pearson's (2019) efficacy reporting framework clarified the strength of the relations, classifying r-values < 0.20 as weak, $r < 0.50$ as moderate, and $r < 0.80$ as large.

Strengths and limitations

Measures of cognitive reasoning in combination with the FoNS framework provided opportunities to investigate unknown number sense relationships. Including seven of Andrews and Sayers' (2015) FoNS framework components provided a broad operationalisation of number sense. Adding subitising further strengthened the study. Task design within each component varied, elucidating visual and verbal contributions to the students' understanding when content was the same. However, not including the FoNS representing number was a limitation.

Investigating language perception and production made it possible to discuss understanding and production as different and overlapping aspects of language related to number sense. While the sample size of 75 participants limited the statistical inference possibilities, especially regarding the small number of developing and proficient students, including grade 1 students at all ability levels was a strength.

Results and analysis

Of 69 possible points, the 75 students averaged 43.5 (*SD* = 10.5) on number sense. Developing students averaged 23.5 (SD = 4.4) points, and proficient students averaged 59.1 (SD = 1.7) points.

The average nonverbal reasoning score of all 75 students was 93.0 (*SD* = 9.5); developing students' average nonverbal reasoning score was 85.9 (SD = 9.3), and proficient students' average nonverbal reasoning score was 103.8 (SD = 6.4). Regarding verbal reasoning, the 75 students' verbal reasoning average was 99.4 (*SD* = 14.1). A tendency towards lower verbal reasoning in the developing students (*m* = 93.2, SD = 15.3) and higher verbal reasoning in proficient students (*m* = 110.8, SD = 10.9) was observed. Standard deviation showed that within group variation was larger for developing students than for proficient students in all three measures of number sense, nonverbal and verbal reasoning.

Regarding verbal reasoning, the 75 students' average language perception was 100.4 (SD = 17.9); language production was 98.7 (SD = 13.1). We observed lower language perception in the developing students (*m* = 86.8, SD = 23.9) and higher language perception in the proficient students (*m* = 116.6, SD = 15.1), which was also the pattern observed for language production in the proficient students (*m* = 104.9, SD = 12.4) and the developing students (*m* = 86.0, SD = 17.6).

Number sense's relations to nonverbal and verbal reasoning

Figure 1 displays the relationships between number sense variation and nonverbal and verbal reasoning for all 75 students.

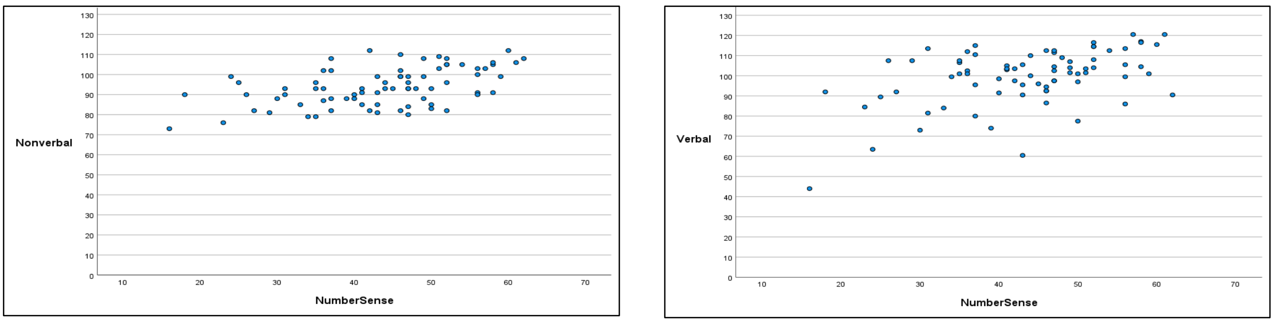

Figure 1. Number sense variation's relations to nonverbal and verbal reasoning

There was a moderate to strong correlation between number sense and nonverbal reasoning ($r = 0.503$, $p < 0.01$), and a moderate correlation between number sense and verbal reasoning ($r = 0.463$, $p < 0.01$). Nonverbal and verbal reasoning correlated significantly, and a thorough look into their aspects was sought. Investigating verbal reasoning showed that number sense correlated moderately to language perception ($r = 0.397$, $p < 0.01$) and moderately to strongly with language production ($r = 0.447$, $p < 0.01$). Due to the large variations in number sense and nonverbal and verbal reasoning within and between groups of students, we explored the different number sense components' correlations to nonverbal and verbal reasoning.

Number sense components' relations to nonverbal and verbal reasoning

The results support previous research indicating relations between number sense and both nonverbal and verbal reasoning (see also Cross et al., 2019; Olkun et al., 2019). Table 2 nuances the relations between the components making up number sense and their relations to reasoning.

Table 2. Correlations between number sense components and nonverbal and verbal reasoning

Reasoning	N	NI	SC	NQ	QD	ES	AC	NP	SU
Nonverbal	75	0.229 *	0.230 *	0.328**	0.238 *	0.121	0.504**	0.373**	0.433**
Verbal	75	0.238 *	0.435**	0.344**	0.314**	0.089	0.449**	0.228 *	0.273 *

Statistical significance of the correlation coefficients (presented in r-values) for number identification (NI), systematic counting (SC), number and quantity (NQ), quantity discrimination (QD), estimation (ES), arithmetic competence (AC), number pattern (NP), and subitising (SU) in all 75 students are marked * at $p < 0.05$, and ** at $p < 0.01$.

Focusing on each number sense component, we found that arithmetic correlated the strongest to nonverbal reasoning; number identification correlated slightly stronger to verbal reasoning; and systematic counting correlated the strongest to verbal reasoning. As such, we partly support the findings of Cross et al. (2019), who found that number identification, counting, and arithmetic mainly involved verbal reasoning. Our findings might be discussed in light of Jordan et al.'s (2013) findings that nonverbally mastering number line estimation strongly contributed to competence in arithmetic procedures.

Arithmetic competence, number patterns, and subitising correlated stronger to nonverbal reasoning than to verbal reasoning, supporting previous studies highlighting nonverbal reasoning in arithmetic (Dehaene, 2001; Sayers et al., 2016). Comparing the number sense components, arithmetic was the component with the strongest correlation to nonverbal reasoning, followed by subitising and number patterns. Estimation showed no significant correlation to either nonverbal or verbal reasoning. Our findings extend and contradict those of Cross et al. (2019), who did not find the importance of verbal reasoning in number line and quantity comparison tasks. Subitising is considered a type of nonverbal reasoning (Dehaene, 2001; Sayers et al., 2016). Still, arithmetic competence correlated stronger to nonverbal reasoning than subitising, while subitising and arithmetic competence correlated strongly, supporting previous findings of nonverbal reasoning being closely related to arithmetic competence (Olkun et al., 2019; Sayers et al., 2016).

Task design may give further information to the relations observed. Jordan et al. (2013) assumed that both nonverbal and verbal reasoning are important to master number line estimation tasks, but Cross et al. (2019) did not find verbal reasoning important for number line tasks. Number lines were included in arithmetic competence, number and quantity, subitising, and estimation but with different purposes: to estimate in estimation, and to support in the other components. Arithmetic consisted of some number line tasks and correlated strongly to both nonverbal and verbal reasoning, but more strongly to nonverbal reasoning. Subitising also correlated stronger to nonverbal reasoning than to verbal reasoning, but the opposite pattern was observed for number and quantity. Estimation exclusively consisted of number line tasks and had no significant correlations to either nonverbal or verbal reasoning. Estimation, arithmetic, and number and quantity tasks gave verbal instructions, while subitising did not.

Figure 2 shows three examples of nonverbal and verbal information provided to the students in the number sense assessment and exemplifies that task demand made students engage in both nonverbal and verbal reasoning. Tasks A, B, and C are examples from the arithmetic component.

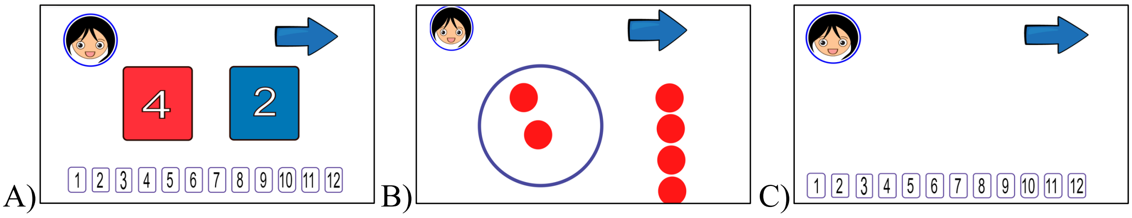

Figure 2. Examples of task instructions given in the arithmetic competence component

Task A provided visual and verbal information. The verbal instruction ”four plus two” and number symbols as nonverbal visual support were given to the students. They were then told to tap their answer on the number line. The number line might help execute the task mentally. Task B provided the verbal instruction ”Move enough balls so that there are four balls in the circle” without number line support. Tasks A and B differed in abstraction level of representation of numbers and the adding operation. Task C provided a picture of a number line with the verbal cue ”What does two plus three make?” and the verbal instruction to tap their answer on the number line. No visual representations of the numbers or the operation were provided.

Due to task demand, estimation is perhaps more appropriate to compare with Cross et al.’s (2019) study, as tasks in both studies gave quantities represented as numbers to estimate on number lines. Still, our findings contradict, as Cross et al. (2019) found number line tasks to strongly correlate to nonverbal reasoning, while estimation had little correlation to nonverbal reasoning in this study. The contradictory findings of number line tasks may be due to Cross et al. (2019) providing some numbers written and others verbally, while this study provided only written numbers.

Discussion

Our findings support, extend, and contradict previous findings of relations between number sense and nonverbal and verbal reasoning. Contradictory results may reflect different number sense and number line (estimation) constructs, as well as different focus on nonverbal visual and verbal demand in task design or as cognitive prerequisites for number sense. We operationalised verbal reasoning as both language perception and language production, while Cross et al. (2019) operationalised verbal reasoning only as language perception. Inclusion criteria for participation also vary across studies: in Cross et al.’s (2019) study, the participants had language difficulties. As we observed large variations within and between subgroups, we need to investigate whether their finding of no correlation between nonverbal reasoning and quantity comparison tasks can be observed in this study’s developing students.

Insight into interrelations between number sense and nonverbal and verbal reasoning might improve teaching methods and content in grade 1 mathematics instruction. The correlations we found suggest that working with subitising small quantities, considering spatial aspects of number sense, and developing mental number lines will probably ensure FoNS (Andrews & Sayers, 2015; Olkun et al., 2019; Sayers et al., 2016). They implicate classroom activities that strengthen connections between nonverbal and verbal representations of numbers and quantities and support interaction of students’ number and space systems to improve the development of mental representations and a mental number line to achieve a robust number sense (Geary et al., 2018; Olkun et al., 2019). Further consideration of correlations between number sense components in developing and proficient students in a longitudinal predictive perspective is needed. Consideration of estimation and subitising as nonverbal prerequisites or preverbal number sense instead of components of foundational number sense is needed to develop the research field (Whitacre et al., 2020), the teaching content, and methods.

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