

1 **Numerical modeling of nonstationary hydrodynamic forces and induced motions**
2 **of a coupled offshore floating installation system**

3 **Shujie Zhao ^a, Xun Meng ^a, Zhen Gao ^b, Huajun Li ^{a*}**

4 ^aOcean University of China, Qingdao 266100, China

5 ^bDepartment of Marine Technology, Norwegian University of Science and Technology,
6 Trondheim, 7491, Norway

7 **Abstract**

8 Offshore installations always involve multi-body systems in which the hydrodynamic and
9 mechanical interactions are complicated for the properties keep changing under certain operation
10 procedures. Float-over deck (FOD) installation is a typical example, where the draft of barge
11 continuously varies during the load transfer operation. Numerical analysis of such continuous
12 operation is crucial to identify the extreme responses, but it's difficult to model the nonstationary
13 properties involved in the hydrodynamic loads and motions and contact loads of the system. In the
14 study, a numerical modeling methodology is developed to calculate these nonstationary
15 hydrodynamic forces and induced responses, considering an active update of the hydrodynamic
16 properties with the time varying body boundary conditions. The effects of time varying body
17 boundary conditions on hydrodynamic forces and induced nonstationary motions and contact loads
18 of a FOD installation system is studied. The results indicate that the dynamic analysis of the
19 continuous load transfer operation could be effectively carried out with one update of hydrodynamic
20 forces at the end of the operation. The barge roll motions and its natural period are the most affected.
21 The proposed method could be used in intensive simulation of offshore multi-body systems with
22 obvious draft variation during operations.

23 **Keywords**

24 Time varying body boundary conditions; Nonstationary responses; Numerical model; Continuous
25 load transfer operation

26 **1. Introduction**

27 Execution of offshore installation is a critical phase among entire EPCI (engineering,

28 procurement, construction and installation) projects due to the high costs and intensive risky
29 activities (Liu and Li, 2017). It generally includes loadout at the quay, transportation from the quay
30 to the reservoir site, and offshore installation operation in open waters. Because of the complexity
31 of wave forces and multi-body interactions, more attention has been given to the dynamic responses
32 of the structures employed in offshore installations. Intensive motion simulation and load
33 calculation need to be carried out to accurately identify the extreme responses for the success of
34 operation.

35 One major challenge for analysis of the kind of offshore activities is the reasonable simulation
36 of time varying characteristics during each installation stage, such as the mating process of a float-
37 over deck (FOD) installation (O'Neill et al., 2000) or the lowering operation of an offshore wind
38 turbine monopile substructure (Li et al., 2015). Here, a ballasting/de-ballasting operation or
39 lowering operation with specific mechanical components are generally performed, where the mass
40 and the wet surface of the floating body are time varied under random waves. Consequently, the
41 system responses like hydrodynamic forces, motions and contact loads show the nonstationary
42 properties that the mean value or variance are changed during the operation. A methodology to
43 calculate the nonstationary responses of the system needs to be developed.

44 As for the changes in mass properties, the Simo theory manual provides an efficient point mass
45 simulation method. Additional efforts are focused on the nonstationary hydrodynamic forces with
46 the time varying body boundary conditions. CFD and the direct time domain approach could be
47 used to deal with the problem. While CFD methods cost significant computational efforts to
48 simulate the environmental conditions when taking into account stochastic features (Orihara and
49 Miyata, 2003). Since the viscous effect of large-volume structures such as the FOD operations is
50 ignorable, CFD approach would not be necessary for hydrodynamic analysis here. For the direct
51 time domain approach, numerical calculation of the boundary value problem at each time step is
52 also computationally intensive (Lin, 1990). Therefore, global dynamic analysis using hybrid
53 frequency-time domain approach (Taghipour et al., 2008) with potential flow theory is widely used
54 to analysis marine operations, and in most cases steady-state hydrodynamic load is a matter of
55 expediency to simplify the analysis process.

56 To carry out the analysis of the continuous operation with the hybrid frequency-time domain

57 approach, there are two different simulation approaches. One is the discrete phase analysis method,
58 in which the nonstationary process is divided into several critical subphases, and the steady-state
59 hydrodynamic forces are considered at each subphase. Koo et al. (2010a, 2010b) carried out
60 numerical simulations and experimental tests for catamaran float-over operations. The pre-mating
61 phase and 0%, 20%, 50%, 80% and 100% load transfer phases were analyzed. Chen et al. (2017)
62 developed a coupled heave-roll-pitch model of FOD system to analyze the complex responses
63 behavior of 0% and 100% load transfer phases. Many software programs like AQWA (Ansys, 2013),
64 Simo (MARINTEK, 2012) and Moses (Nachlinger, 2013) also provides the discrete phase
65 simulations of the nonstationary process.

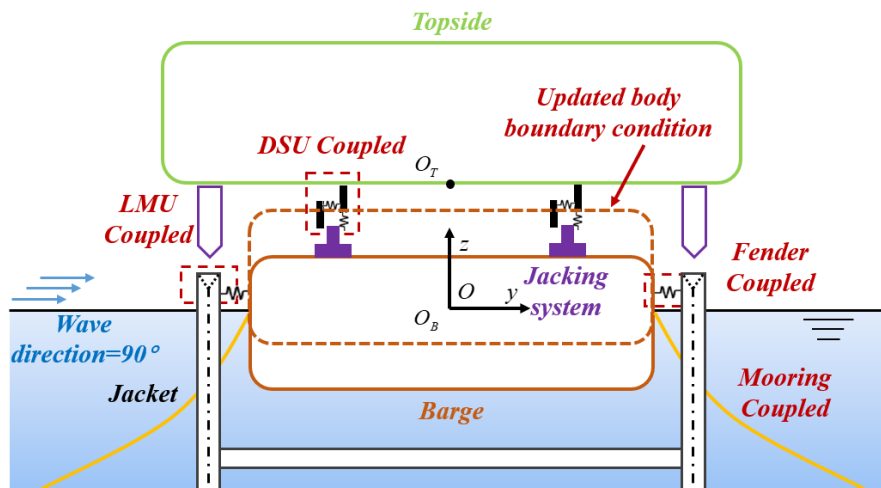
66 Another approach is developed under the assumption that the nonstationary process is
67 represented by stepwise steady-state stages (Li et al., 2015). Bai et al. (2021, 2020a) carried out an
68 experimental investigation of the complex continuous load transfer operation of float-over operation.
69 It indicate that continuous transfer modeling could capture variation property of the loads and
70 provide much more reasonable results of the dynamic float-over operation compared with traditional
71 discrete steady-state modeling (Bai et al., 2020b). The continuous operation modeling would be
72 preferred due to the more realistic simulations of the entire nonstationary process.

73 During the simulations of continuous process, calculation of the nonstationary hydrodynamic
74 forces is crucial. There are few literatures on the action mechanism and realization mode of
75 nonstationary hydrodynamic forces of the complicated offshore floating system in time domain
76 analysis. In the study, a numerical modeling method for assessing of nonstationary hydrodynamic
77 forces and induced motions and contact loads of floating structures in random waves is proposed.
78 Essentially, the method combines the analysis of the operation and the hydrostatic/hydrodynamic
79 loads and motion analysis with time varying body boundary conditions. A case study on float-over
80 installation of a heavy integrated topside is carried out. The effects of time varying body boundary
81 conditions on the hydrostatic restoring forces, wave excitation forces and radiation wave forces are
82 discussed respectively. The corresponding effects on the induced motions and contact loads are also
83 investigated. The paper is organized as follows. Section 2 provides a description of the nonstationary
84 hydrodynamic problems of float-over deck installation. Section 3 elaborates on the developed
85 numerical modeling method to calculate nonstationary hydrodynamic forces and induced responses.

86 Section 4 and Section 5 present the detailed analysis of the effects of time varying body boundary
87 conditions on system responses. Finally, Section 6 summarizes the conclusions.

88 2. Nonstationary hydrodynamic problems of FOD installation

89 Offshore platform mainly consists of a topside and a supporting substructure, which are
90 designed and fabricated separately and integrated together at the site. With the increasing size and
91 weight of the integrated topside, the capacity of the traditional heavy lifting cannot meet the need.
92 The FOD installation technology is preferred for integrated deck operations because of its larger
93 lifting capacity and lower costs. Targeting at the load transfer operation of a single barge FOD
94 installation, a simplified model is illustrated in Fig. 1. All bodies including a barge, a topside and a
95 fixed jacket are considered rigid. The leg mating unit (LMU), the deck support unit (DSU) and the
96 fender components are installed to absorb the impact loads between the bodies.



97

98 Fig. 1. Load transfer operation of FOD installation

98

99 Except for ballast or de-ballast technology, the rapid load transfer technique (Yu et al., 2018)
100 with specific jacking system is used to lower the topsides, where the lowering speed is generally set
101 to 0.25 m/s. During the lowering operation, the draft of float-over barge is decreased as the load of
102 the topside transferring to the substructure, shown as the dotted profile of the floating barge in Fig.
103 1. The equilibrium for the varying draft with loads transfer is coordinated for each time instant on
104 the hydrostatic and gravitational load as well as quasistatic mooring and contact forces (i.e., forces
105 of the LMU, DSU and fender components). The new equilibrium position implies that all
106 hydrodynamic loads due to waves would be estimated again considering the updated body boundary
107 condition at new quasistatic equilibrium. This kind of hydrodynamic problem is defined as a

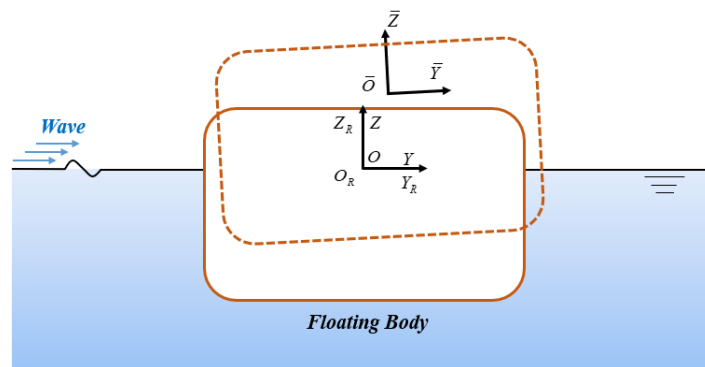
108 nonstationary hydrodynamic problem in the study. It is related to a physical process involving a
 109 continuous update of the body boundary condition for hydrodynamic analysis under stationary
 110 Gaussian random wave processes.

111 The 12 degree-of-freedom (12-DOF) multi-body motion equations of a barge and a topside, as
 112 well as the coupled model of LMU, DSU, fender and jacking components, are developed to carry
 113 out the time domain analysis. These model and components particulars were deliberated by Zhao et
 114 al. (2021), more efforts are paid on the introduction mechanism of nonstationary hydrodynamic
 115 forces and corresponding responses of the multi-body system at rapid load transfer operation.

116 3. Methodology

117 A detailed methodology to obtain the nonstationary hydrostatic/hydrodynamic loads
 118 considering the FOD operation is developed and explained in this section.

119 The coordinate system is defined in Fig. 2, where the XY plane of the global Earth-fixed
 120 coordinate system $O\text{-}XYZ$ coincides with the still water line and the Z-axis points upwards. The
 121 body-fixed coordinate system $\bar{O}\text{-}\bar{X}\bar{Y}\bar{Z}$ follows the motions of the floating body, which is used to
 122 describe the positions of the coupling components, such as the mooring system and DSU relative to
 123 the floating body. In addition, the body-related coordinate system $O_R\text{-}X_R Y_R Z_R$ is defined to
 124 calculate the hydrodynamic coefficients, that follows the horizontal equilibrium position of the
 125 floating body. At the initial stage, the origin of the defined body-fixed and body-related coordinate
 126 system coincides with the global coordinate system.



127

128 Fig. 2. Definition of the coordinate systems

129 The rigid-body motions of a floating body with 6 DOFs in waves can be obtained from
 130 Newton's second law, as shown in Eq. (1), where \mathbf{F}_B denotes floater position- and orientation-

131 dependent buoyancy forces, \mathbf{F}_M denotes mooring forces, \mathbf{F}_C denotes contact forces from LMU,
 132 DSU and fender components, \mathbf{F}_W denotes wave excitation forces, \mathbf{F}_{HS} denotes hydrostatic
 133 restoring forces due to wave induced body motions, \mathbf{F}_R stands for wave radiation forces
 134 corresponding to added mass and potential damping effects, and \mathbf{F}_G is gravity forces of floating
 135 body. For the FOD installation with application of the rapid load transfer technique, the ballasting
 136 operation is not included during the load transfer phase so that the gravity forces of the float-over
 137 barge are considered constant.

$$138 \quad \mathbf{M}\ddot{\mathbf{x}}(t) = \mathbf{F}_B(t) + \mathbf{F}_M(t) + \mathbf{F}_C(t) + \mathbf{F}_W(t) + \mathbf{F}_{HS}(t) + \mathbf{F}_R(t) + \mathbf{F}_G \quad (1)$$

139 During the load transfer operation, the float-over barge will move to a new equilibrium position
 140 to achieve balance at each time instant as the weight of the topside is gradually transferred. For the
 141 developed numerical modeling method, the motions of a floating body of a nonstationary physical
 142 process are considered with two parts, as shown in Eq. (2). One is the quasistatic motions $\mathbf{x}_s(t)$
 143 due to the operations in an assumed calm water, and the other is the dynamic motions $\mathbf{x}_d(t)$ with
 144 consideration of the wave induced hydrodynamic loads and the inertial loads of the body.

$$145 \quad \mathbf{x}(t) = \mathbf{x}_s(t) + \mathbf{x}_d(t) \quad (2)$$

146 Correspondingly, the forces are divided into quasistatic forces and dynamic forces, as shown
 147 in Table 1. The hydrodynamic forces are all rewritten as functions of \mathbf{x}_s , which means that the
 148 forces are calculated based on the boundary condition at \mathbf{x}_s . The linear approach to calculate forces
 149 is also summarized here, where the term $\bar{\mathbf{K}}$, $\bar{\mathbf{C}}$ and $\bar{\mathbf{M}}$ are the generalized stiffness matrix,
 150 generalized damping matrix and generalized mass matrix respectively. Their product with the
 151 corresponding motion is the linear calculated forces. It is worth noting that the linear model is only
 152 valid for small displacements with respect to the initial linearization point. Nonlinearities are
 153 observed for both quasistatic and dynamic forces, as listed in Table 1. In the analysis, the
 154 displacements \mathbf{x}_s are large changed with respect to the initial linearization point $\mathbf{x}_s(0)$; therefore,
 155 the nonlinear relationships between the load and displacement for quasistatic analysis are captured.
 156 While the displacements \mathbf{x}_d are a small value with respect to the new updated quasistatic positions,
 157 the linear model in Table 1 is used for the calculation of dynamic forces.

Table 1 Summary of the forces in the numerical model

Force components	Nature of these forces (quasistatic, dynamic or both)		Linear model for these forces		Nonlinear model for these forces	
$\mathbf{F}_B(t)$	Quasistatic forces	$\mathbf{F}_B(t)$		$\mathbf{F}_B(t) - \bar{\mathbf{K}}\mathbf{x}_s(t)$		Buoyancy force considering the instantaneous wet surface due to quasistatic motions
$\mathbf{F}_M(t)$		$\mathbf{F}_{Ms}(t)$	$\mathbf{F}_{Md}(t)$	$\bar{\mathbf{K}}\mathbf{x}_s(t)$	$\bar{\mathbf{K}}\mathbf{x}_d(t)$	Nonlinear restoring forces of the mooring lines
$\mathbf{F}_C(t)$		$\mathbf{F}_{Cs}(t)$	$\mathbf{F}_{Cd}(t)$	$\bar{\mathbf{K}}\mathbf{x}_s(t)$ $\bar{\mathbf{C}}\dot{\mathbf{x}}_s(t)$	$\bar{\mathbf{K}}\mathbf{x}_d(t)$ $\bar{\mathbf{C}}\dot{\mathbf{x}}_d(t)$	Nonlinear contact forces of stiffness and damping effects
$\mathbf{F}_W(t)$	Dynamic forces	$\mathbf{F}_W(t, \mathbf{x}_s)$		Refer as Eq. (10)		Wave excitation force considering the instantaneous wet surface
$\mathbf{F}_{HS}(t)$		$\mathbf{F}_{HS}(t, \mathbf{x}_s)$		Refer as Eq. (11)		Hydrostatic restoring force considering the hydrostatic pressure integration over the instantaneous wet surface due to dynamic motions
$\mathbf{F}_R(t)$		$\mathbf{F}_R(t, \mathbf{x}_s)$		Refer as Eq. (12)		Wave radiation force with the radiation wave pressure integration over the instantaneous wet surface

159 3.1 Quasistatic analysis of the installation operation

160 Quasistatic analysis could be executed firstly as Eq. (3). The balance of gravity and equilibrium
161 position-dependent forces (the buoyancy, quasistatic mooring forces and quasistatic lift object
162 contact forces) are accounted to determine the changing quasistatic equilibrium position of the
163 floating body. The changed equilibrium due to the operation are represented by the change of drafts
164 and heeling and trimming angles.

$$165 \quad \mathbf{0} = \mathbf{F}_B(t) + \mathbf{F}_{Ms}(t) + \mathbf{F}_{Cs}(t) + \mathbf{F}_G \quad (3)$$

166 As summarized in Table 1, for the mooring system, the restoring forces are modeled
167 considering a nonlinear load-displacement relationship based on a rod theory (Garrett, 1982). The
168 nonlinearity of the contact forces is observed from the nonlinear stiffness and damping properties
169 of components such as DSUs and LMUs. The piecewise linearization of nonlinear stiffness and
170 damping coefficients is used in the simulation. For the position updated buoyancy, nonlinearities
171 are seen from the large varying mean water plane of the floating body during the physical process.

172 To consider the nonlinearities, the changed buoyancy is evaluated through the integration of pressure
 173 p_s on the hull wet surface, shown as Eq. (4).

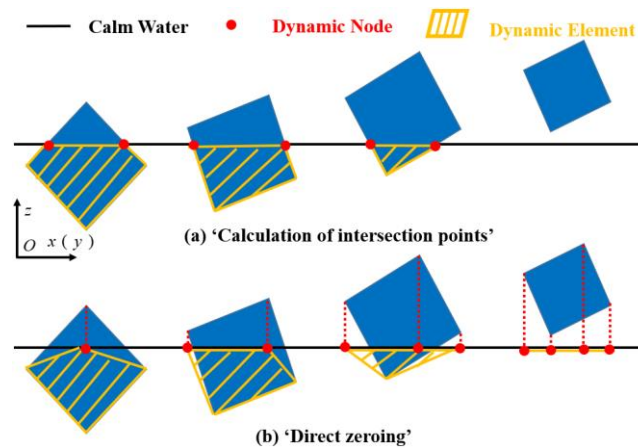
$$174 \quad \mathbf{F}_B(t) = \iint_{S_0(\mathbf{x}_s)} p_s \mathbf{n} ds \quad (4)$$

175 An additional time-updated panel model is required for calculation, which is developed as
 176 following three steps.

- 177 (a). The whole surface of floating bodies is divided by the quadrilateral element.
 178 (b). For a new static equilibrium position, the coordinates of each node are updated as Eq. (5),
 179 where ξ_s and α_s are the quasistatic translational and rotational motion vectors, and \mathbf{r}
 180 represents the radius vector for each node.

$$181 \quad \mathbf{Node}_{i,x_s} = \mathbf{Node}_{i,x_{s,0}} + \xi_s + \alpha_s \times \mathbf{r} \quad (5)$$

182 (c). The last step is to generate a new wet surface mesh. It requires first determining the position
 183 of each element relative to the still water, including fully submerged, fully out of water and in
 184 between. The last situation requires intercepting the part of an element that is subject to hydrostatic
 185 pressure. As shown in Fig. 3 (a), the ‘calculation of intersection point’ is carried out in a correct way,
 186 that requires to calculate the X and Y coordinates of intersection points. While each element is
 187 considered small enough, the calculation can be simplified with a ‘direct zeroing’ procedure as Fig.
 188 3 (b), in which the Z coordinates of each node above the still water is set as zero, and any calculation
 189 of other coordinates is not required.



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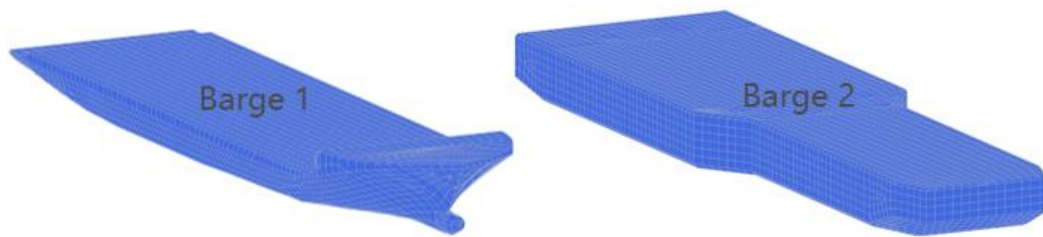
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Fig. 3. Time-updated node and panel model

192

The simplified calculation of intersection points will reduce the computational works, but some

193 errors are also introduced. The accuracy of the procedure to calculate buoyancy needs a verification.
 194 As shown in Fig. 4, the buoyancy of two barges with different quasistatic motions is calculated. The
 195 verification results listed in Table 2, demonstrate the developed ‘direct zeroing’ procedure as Fig. 3
 196 (b) achieves good agreement with the results integrated on the accurate wet surface as Fig. 3 (a).
 197 Besides, because of the largely changed mean water plane of barge 1 at different quasistatic positions,
 198 the linear model calculated results have large differences with nonlinear model. It is important to
 199 capture the nonlinear relationships between buoyancy and displacement when the mean water plane
 200 of the floating body is largely changed.



201
202 Fig. 4. Discrete panel model of barges

203 Table 2 Calculated buoyancy with different quasistatic motions

Buoyancy (MN) with different quasistatic Heave (m) and Pitch (deg) motions			(0, 0)	(2, 0)	(0, -3)	(2, -3)
Barge 1	Nonlinear calculation as Eq. (4)	Model as Fig. 3 (a)	184.84	100.45	210.15	125.31
		Model as Fig. 3 (b)	185.56	101.03	211.26	126.27
	Linear calculation as Table 1		184.84	93.72	203.04	111.92
Barge 2	Nonlinear calculation as Eq. (4)	Model as Fig. 3 (a)	671.18	510.41	706.42	545.55
		Model as Fig. 3 (b)	671.20	510.42	707.52	546.64
	Linear calculation as Table 1		671.18	510.30	708.16	547.09

204 **3.2 Dynamic analysis of the installation operation**

205 In the second step, a dynamic analysis is conducted with account of the wave induced
 206 hydrodynamic loads acting on the floating body, inertial loads, dynamic forces of the mooring
 207 system and contact forces between the structures as Eq. (6).

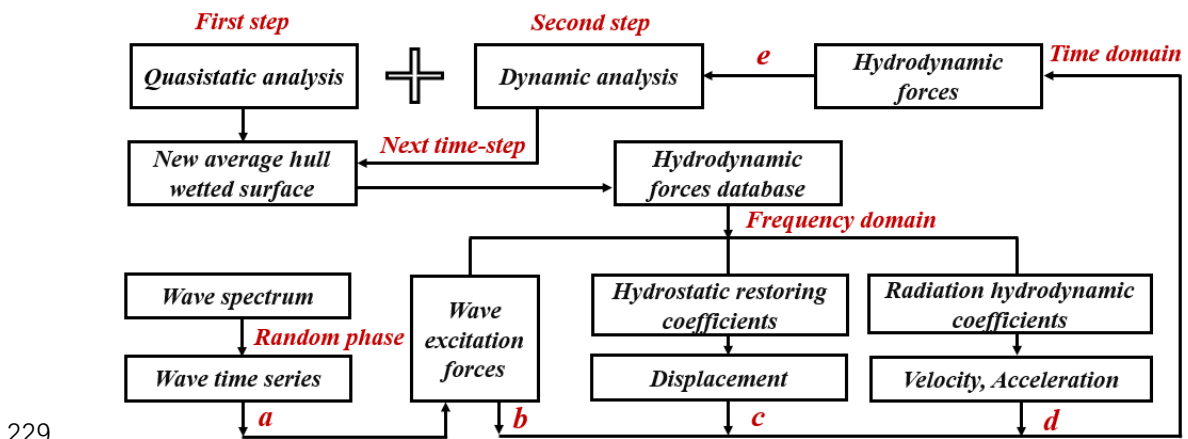
208
$$\mathbf{M}\ddot{\mathbf{x}}_d(t) = \mathbf{F}_l(t) + \mathbf{F}_w(t, \mathbf{x}_s) + \mathbf{F}_{HS}(t, \mathbf{x}_s) + \mathbf{F}_R(t, \mathbf{x}_s) + \mathbf{F}_{Md}(t) + \mathbf{F}_{Cd}(t) \quad (6)$$

209 Here, the quasistatic motions induced inertial force \mathbf{F}_l is included, that could be calculated

210 by interpolation of the determined quasistatic motions defined in Eq. (7). For load transfer operation
 211 employing rapid load transfer technique, it plays an important role to reflect the operation induced
 212 dynamic responses of the system. Whether the simulation is considered in calm water or waves, the
 213 quasistatic motions induced inertial force could be always included. The effects of the forces for
 214 barge motions will be discussed in section 4.3.

215
$$\mathbf{F}_l(t) = -\mathbf{M}\ddot{\mathbf{x}}_s(t) \quad (7)$$

216 Assuming the speed of quasistatic motions is slow with respect to dominant wave periods, the
 217 hydrodynamic loads due to waves can be estimated by interpolation of the wave loads at different
 218 body boundary conditions. To carry out the interpolation, the hydrodynamic analysis is conducted
 219 in the body-related coordinate system. Due to the fact that the changes in horizontal directions do
 220 not cause the changes of wet surface of barge and it is considered as a small value, it is assumed
 221 that the equilibrium position change in the horizontal directions is negligible for hydrodynamic
 222 analysis. Correspondingly, the body-related coordinate system always coincides with the global
 223 coordinate system. The frequency domain hydrodynamic analysis using Wadam (Wadam, 2010)
 224 covers the frequency range from 0.02 rad/s to 2 rad/s with a step of 0.02 rad/s. The hydrodynamic
 225 coefficients, including hydrostatic restoring coefficients, first-order wave excitation forces, added
 226 masses and damping coefficients, are calculated with a step changed drafts of 0.2 m of the floating
 227 body and stored in the database. Then the nonstationary hydrodynamic forces and responses of
 228 system are calculated as Fig. 5, and each sub-step is illustrated as following.



229
 230 Fig. 5. Hydrodynamic forces and induced motions with time varying body boundary conditions

231 (a) Environmental conditions for offshore installation works are normally mild; thus, a linear

232 wave theory is used to obtain the wave loads on floating structures (Faltinsen, 1993). The wave
 233 spectrum $S(\omega)$ describes the wave condition of offshore operation waters. The random wave
 234 elevation time series $\eta(t)$ are generated by superimposing a series of regular wave components
 235 with different amplitudes ζ_j , wave frequencies ω_j and random phases ε_j , as shown in Eq. (8)
 236 and Eq. (9).

$$237 \quad \eta(t) = \text{Re} \sum_{j=1}^N \zeta_j e^{i(\omega_j t + \varepsilon_j)} \quad (8)$$

$$238 \quad \zeta_j = \sqrt{2S(\omega_j)\Delta\omega} \quad (9)$$

239 (b) The wave excitation forces are linearly calculated as Eq. (10), where $f_{Wj}(\omega, \mathbf{x}_s)$
 240 represents the linear wave excitation force transfer functions in the frequency domain at the
 241 quasistatic equilibrium position \mathbf{x}_s .

$$242 \quad \mathbf{F}_W(t, \mathbf{x}_s) = \text{Re} \sum_{j=1}^N \zeta_j f_{Wj}(\omega, \mathbf{x}_s) e^{i(\omega_j t + \varepsilon_j)} \quad (10)$$

243 (c) Similar to wave excitation forces, the linearized hydrostatic restoring forces in Eq. (11)
 244 are calculated, where $\mathbf{K}_{HS}(\mathbf{x}_s)$ represents the restoring coefficient matrix at quasistatic
 245 equilibrium position \mathbf{x}_s .

$$246 \quad \mathbf{F}_{HS}(t, \mathbf{x}_s) = -\mathbf{K}_{HS}(\mathbf{x}_s)\mathbf{x}_d \quad (11)$$

247 (d) The linear radiation forces are proportional to the acceleration and velocity of dynamic
 248 motions \mathbf{x}_d , which is calculated using the Cummins equations (Cummins, 1962) as Eq. (12). The
 249 time τ is defined as the previous time, and time t is the current time. In Eq. (12), $\mathbf{x}_{s,t}$ and $\mathbf{x}_{s,\tau}$
 250 are the time varying quasistatic equilibrium positions at time instances t and τ respectively, and
 251 $\mathbf{A}_\infty(\mathbf{x}_{s,t})$ is the infinite frequency-added mass matrix at $\mathbf{x}_{s,t}$. The convolution term represents the
 252 fluid memory effects, and essentially these are the radiation forces at the current time t due to all
 253 the waves (or the radiated wave velocity potential) generated by the motions of body at all the
 254 previous time instants τ . Since the boundary conditions are different at time τ and t , the
 255 calculation becomes complicated:

256 ● The radiated wave velocity potential $\phi^R(x, y, z, \mathbf{x}_{s,\tau})$ generated under the previous boundary

257 condition $\mathbf{x}_{s,\tau}$ is calculated, but it should be integrated at the current wet surface $\mathbf{x}_{s,t}$ to
 258 obtain the potential damping coefficients $\mathbf{B}(\omega, \mathbf{x}_{s,\tau})$ as Eq. (14). However, the calculated
 259 velocity potential is all integrated under its corresponding panel model for hydrodynamic
 260 analysis in frequency domain, that is not available for integration as Eq. (14). An approximation
 261 method is proposed as Eq. (15), that uses the average of the potential damping coefficients at
 262 the quasistatic positions $\mathbf{x}_{s,t}$ and $\mathbf{x}_{s,\tau}$.

263 ● Then the retardation function $\mathbf{K}(t_R, \mathbf{x}_{s,\tau})$ can be calculated by the established relationship by
 264 (Ogilvie, 1964) as Eq. (13). With the time t_R shifts, the retardation function will be decayed
 265 to near zero, so the retardation function is truncated at 70 s here. And the previous databases
 266 of added mass and damping coefficients are re-calculated as Eq. (13)~Eq. (15) and stored as
 267 retardation function in the database.

$$268 \quad \mathbf{F}_R(t, \mathbf{x}_s) = - \left\{ \mathbf{A}_\infty(\mathbf{x}_{s,t}) \ddot{\mathbf{x}}_d(t) + \int_{-\infty}^t \mathbf{K}(t-\tau, \mathbf{x}_{s,\tau}) \dot{\mathbf{x}}_d(\tau, \mathbf{x}_{s,\tau}) d\tau \right\} \quad (12)$$

$$269 \quad \mathbf{K}(t_R, \mathbf{x}_{s,\tau}) = \frac{2}{\pi} \int_0^\infty \mathbf{B}(\omega, \mathbf{x}_{s,\tau}) \cos(\omega t_R) d\omega \quad (13)$$

$$270 \quad \mathbf{B}(\omega, \mathbf{x}_{s,\tau}) = \text{Im} \left(\omega \rho \iint_{S_0(\mathbf{x}_{s,t})} \phi^R(x, y, z, \mathbf{x}_{s,\tau}) \mathbf{n} ds \right) \quad (14)$$

$$271 \quad \mathbf{B}(\omega, \mathbf{x}_{s,\tau}) = \text{Im} \left(0.5 \omega \rho \left(\iint_{S_0(\mathbf{x}_{s,\tau})} \phi^R(x, y, z, \mathbf{x}_{s,\tau}) \mathbf{n} ds + \iint_{S_0(\mathbf{x}_{s,t})} \phi^R(x, y, z, \mathbf{x}_{s,t}) \mathbf{n} ds \right) \right) \quad (15)$$

272 (e) The mass matrix is updated to the established body-related coordinate system as Eq. (16)
 273 and Eq. (17), where (x_g, y_g, z_g) is the vector of the center of gravity at the initial quasistatic
 274 position, and (x_s, y_s, z_s) is the vector of the quasistatic motions and R_g is the inertia radius.

$$275 \quad \begin{cases} x_{gs} = x_g + x_s \\ y_{gs} = y_g + y_s \\ z_{gs} = z_g + z_s \end{cases} \quad (16)$$

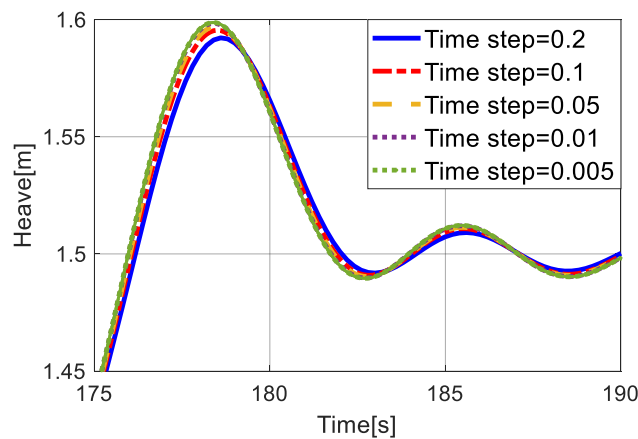
$$\mathbf{M} = m \begin{bmatrix} 1 & 0 & 0 & 0 & z_{gs} & -y_{gs} \\ 0 & 1 & 0 & -z_{gs} & 0 & x_{gs} \\ 0 & 0 & 1 & y_{gs} & -x_{gs} & 0 \\ 0 & -z_{gs} & y_{gs} & R_{gx}^2 + y_{gs}^2 + z_{gs}^2 & R_{gy}^2 - x_{gs} y_{gs} & R_{gz}^2 - x_{gs} z_{gs} \\ z_{gs} & 0 & -x_{gs} & R_{gy}^2 - x_{gs} y_{gs} & R_{gy}^2 + x_{gs}^2 + z_{gs}^2 & R_{gz}^2 - y_{gs} z_{gs} \\ -y_{gs} & x_{gs} & 0 & R_{gz}^2 - x_{gs} z_{gs} & R_{gz}^2 - y_{gs} z_{gs} & R_{gz}^2 + x_{gs}^2 + y_{gs}^2 \end{bmatrix} \quad (17)$$

277 Finally, the system motion and contact forces during the continuous operation could be
278 obtained from the addition of calculated quasistatic and dynamic responses.

279 4. Response analysis of nonstationary responses during the installation operation

280 Since the nonstationary hydrodynamic forces and nonlinear stiffness of LMU and DSU are
281 acting in the studied continuous operation, the convergence simulation is launched with
282 consideration of the whole load transfer operation with varying time steps. The 4th-order Runge-
283 Kutta numerical method is applied to drive the time-domain responses. The random wave condition
284 of Jonswap spectrum with a significant wave height (H_s) of 1.5 m and spectral peak period (T_p) of
285 6.7 s in head sea is employed here. Fig. 6 shows the heave motions of the float-over barge with
286 different time steps. The converged results can be obtained with a time step of 0.01 s, which is
287 adopted for the follow-up time domain simulations. The simulations are carried out on a personal
288 computer (AMD Ryzen 5 4600H@3.0 GHz 4.8 GHz Turbo, 16 GB of RAM). It costs 906 s for a
289 300 s time series simulation with a time step of 0.01 s.

290



291

292

Fig. 6. Heave motions of float-over barge with different time steps

293

4.1 Properties of float-over barge at the initial and last operation positions

294

The changed properties of float-over barge before and after load transfer are summarized in

295 Table 3. The weight of the topside is 10000 t in analyzed FOD installation operation, which is the
 296 difference of total mass of barge before and after load transfer. The offloading weight to
 297 substructures leads to the changes of draft and trim angle of the barge. In terms of the natural
 298 vibration periods, there are small differences of those in heave and pitch motions at the initial and
 299 last operation phase, while large changes are observed in roll motions.

300 Table 3 Properties of float-over barge with varying quasistatic positions

Properties		Before load transfer	After load transfer
Total Mass [t]		68489	58489
Draft [m]		8.55	7.04
Trim [deg]		0	-1.87
Natural period [s]	Heave	9.2	9.1
	Roll	11.4	10.2
	Pitch	10.1	10.0

301 **4.2 Quasistatic motions of float-over barge**

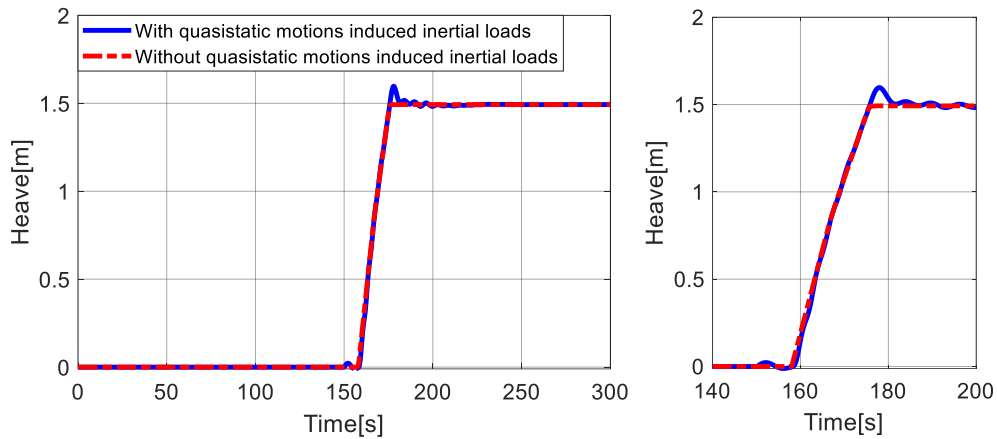
302 Using the verified ‘direct zeroing’ procedure to generate new wet surface mesh, the changed
 303 quasistatic motions of the barge with the representation of changed drafts and trim angles are
 304 calculated. Based on the average of 10 times calculation of quasistatic motions of barge, the
 305 simplified ‘direct zeroing’ process saves 18.1% of the calculation time compared to the ‘ calculation
 306 of intersection points’. The results are listed in Table 4. The load transfer operation begins at 150 s,
 307 and the first 8 s is to reduce the clearance and the vertical height of docking cones. In this period,
 308 the floating condition of the barge is unchanged because the weight of the topside is still fully
 309 supported by the barge. After that, the drafts and trim angles of the barge are reduced with the load
 310 transferring from the barge to the substructure.

311 Table 4 Floating condition of float-over barge during load transfer operation

Time [s]		0~158	160	165	170	175	178~End
Floating Condition	Draft [m]	8.550	8.360	7.886	7.474	7.109	7.039
	Trim [deg]	0.000	-0.212	-0.744	-1.260	-1.767	-1.865

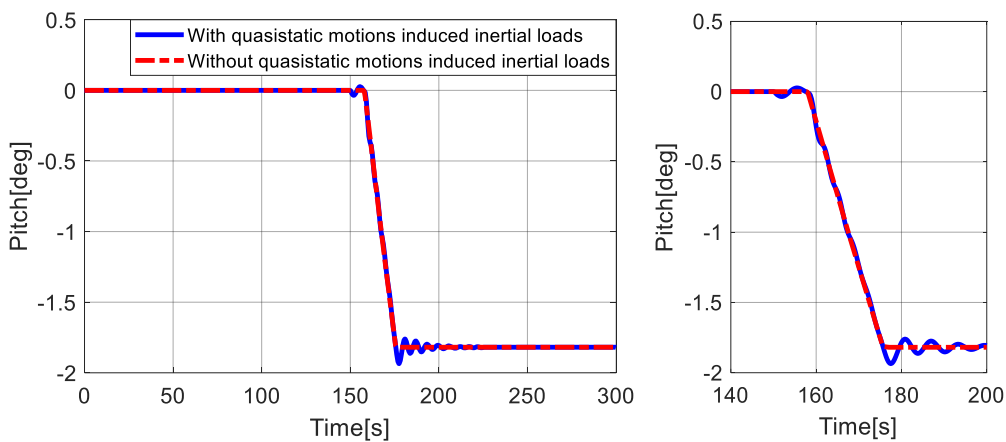
312 **4.3 Effects of quasistatic motions induced inertial force**

313 Once the quasistatic motions are determined, the quasistatic motions induced inertial force
314 could be calculated. The simulations are carried out in calm waters. Fig. 7 shows the results of the
315 effect of barge quasistatic motions induced inertial force. It is observed that both heave and pitch
316 motions are increased at the end of the load transfer operation considering of inertial loads. They
317 should not be neglected in analysis of rapid load transfer operation.



318

319 (a) Heave motion of float-over barge



320

321 (b) Pitch motion of float-over barge

322 Fig. 7. Effects of quasistatic motions induced inertial force

323 **5. Effects of hydrodynamic forces with time varying body boundary conditions**

324 The hydrodynamic forces acting on the floating body include hydrostatic restoring forces, wave
325 excitation forces and radiation wave forces. The effects of the time varying body boundary
326 conditions for different hydrodynamic forces are different from one another. In this section, the
327 nonstationary hydrodynamic forces and induced dynamic motions and contact loads are investigated.

328 **5.1 Simulation settings**

329 Calculation settings of hydrostatic and hydrodynamic coefficients considering different body
 330 boundary conditions are listed in Table 5.

331 Table 5 Definition of cases with respect to different modeling methods for hydro forces

Method	Description
C	The calculation is carried out with constant hydro coefficients at the initial floating position.
NHS	The calculation is carried out with nonstationary hydrostatic restoring forces as Eq. (11) and constant wave excitation and radiation coefficients at the initial floating position.
NHS+NFW	The calculation is carried out with nonstationary hydrostatic restoring forces as Eq. (11) and nonstationary wave excitation forces as Eq. (10), and constant radiation coefficients at the initial floating position.
NHS+NFW+NFR	The calculation is carried out with nonstationary hydrodynamic forces as Eq. (10)~Eq. (12) with continuous updated boundary conditions.

332 Jonswap spectrum with 3 typical peak periods in both head sea (forward along the X axis) and
 333 beam sea (forward along the Y axis) are considered. The main parameters of wave conditions are
 334 listed in Table 6.

335 Table 6 Main parameters of the considered sea states

Condition		C1	C2	C3
T_p [s]		6.7	10	14
H_s [m]	Head sea	1.5		
	Beam sea	0.5		

336 **5.2 Hydrodynamic forces of float-over barge with time varying body boundary conditions**

337 **5.2.1 Hydrostatic restoring forces**

338 Fig. 8 shows the normalized hydrostatic restoring coefficients during the load transfer
 339 operation based on the value of the initial quasistatic equilibrium position. The results show obvious
 340 changes in roll direction compared with the hydrostatic restoring coefficients in heave and pitch
 341 directions.

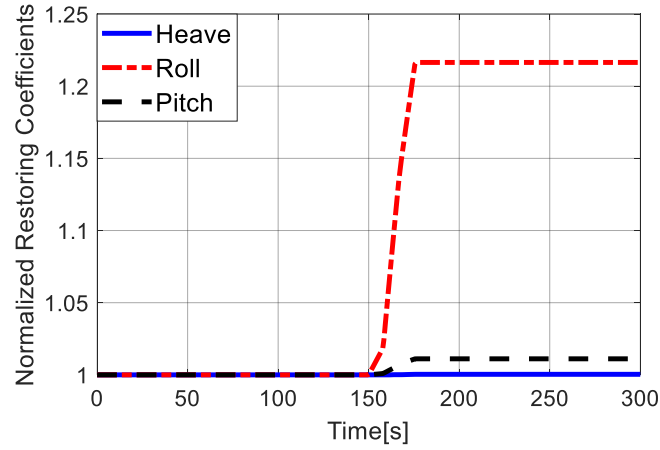


Fig. 8. Normalized hydrostatic restoring coefficients with load transfer operation

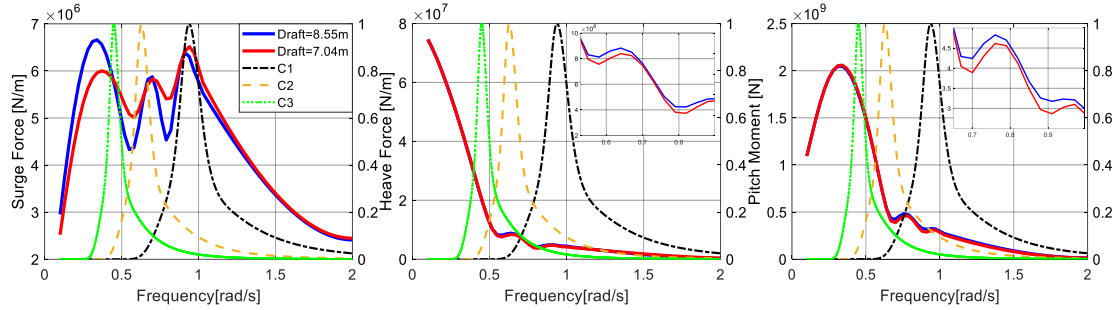
The hydrostatic restoring coefficients of roll and pitch motions are calculated as Eq. (18), where V is displacement of float-over barge, z_v is the center of buoyancy, M_B and M_T are the mass of float-over barge and the topside respectively, z_{gB} and z_{gT} are the center of mass of float-over barge and topside, I_{xx} and I_{yy} are the second moment about X-axis and Y-axis over the waterplane area. During the load transfer operation, the second moment I_{xx} changes little, but the changes in the center of mass and buoyancy lead to a great increase in the hydrostatic restoring coefficient for roll motions. As for pitch motions, I_{yy} is far larger than the others in equation because the barge is similar to a slender structure, there is no significant variation in the hydrostatic restoring coefficient for pitch motions.

$$\begin{aligned}
 K_{HS}(roll, roll) &= \rho g V z_v - (M_B + M_T) g \frac{M_B z_{gB} + M_T z_{gT}}{M_B + M_T} + \rho g I_{xx} \\
 K_{HS}(pitch, pitch) &= \rho g V z_v - (M_B + M_T) g \frac{M_B z_{gB} + M_T z_{gT}}{M_B + M_T} + \rho g I_{yy}
 \end{aligned}
 \tag{18}$$

5.2.2 Wave excitation forces

The wave excitation forces are related to the incident waves and response amplitude operator (RAO) of forces of float-over barge, that are shown in Fig. 9 and Fig. 10 in head sea and beam sea. The results shows that the effects of the time varying body boundary condition are varied with different wave period. For surge wave excitation forces, the different change tendency is observed among the analyzed wave period. For heave, pitch and roll wave excitation forces, it shows that there is a minimum change between two boundary conditions when long period waves (for most is wave period larger than 12 s) are considered. Besides, the results also illustrate that the changes of

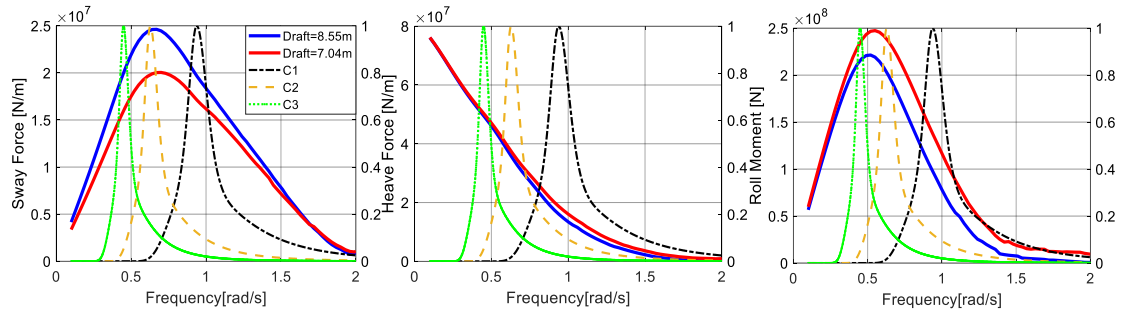
362 wave excitation forces in beam sea are larger than those in head sea. Especially for heave and pitch
 363 forces in head sea, the differences are small with two boundary conditions. The heave and pitch
 364 forces between 0.5 rad/s and 1 rad/s are magnified to see the changes.



365

366

Fig. 9. Wave excitation forces in head sea at the initial and last quasistatic positions



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Fig. 10. Wave excitation forces in beam sea at the initial and last quasistatic positions

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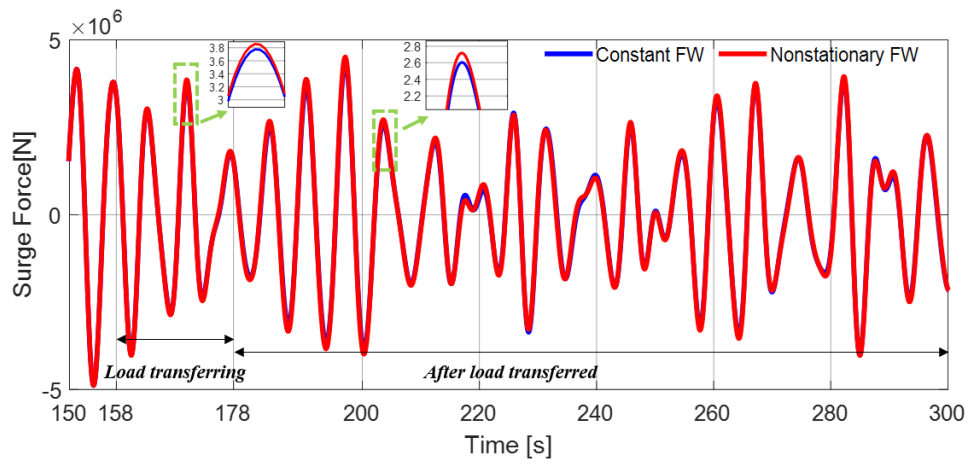
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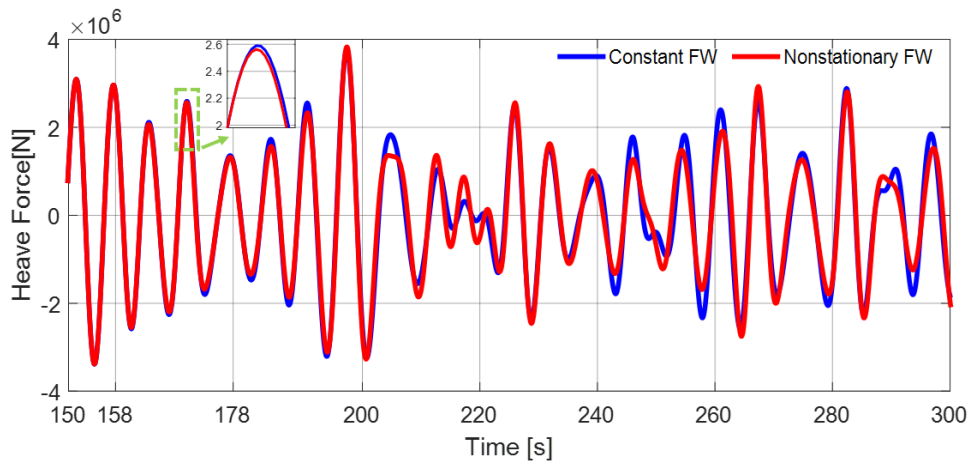
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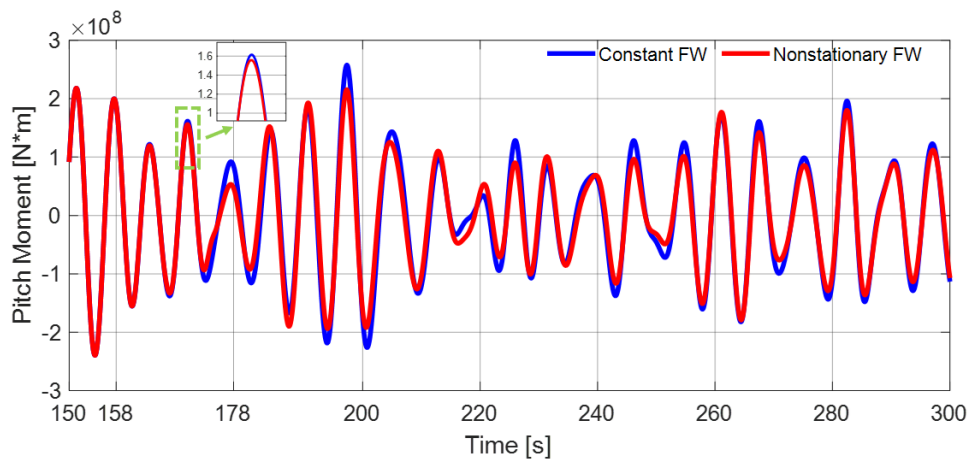
The time series of constant wave excitation forces (using constant wave excitation force coefficients at the initial floating position) and nonstationary wave excitation forces (using time updated wave excitation force coefficients) in head sea and beam sea are shown in Fig. 11 and Fig. 12. The time series are clarified into load transferring process and after load transfer process. It illustrates that wave excitation forces change little during the load transfer operation while the difference becomes large when the load transfer ended. Because the installation is carried out with the application of the rapid load transfer technique, the load transfer operation only lasts approximately 20 s (158 s~178 s) so that the effects of the time varying boundary conditions could not be observed in a limited time history. Although the frequency domain wave excitation forces coefficients change little for heave and pitch motions, the large differences are still observed in time domain.



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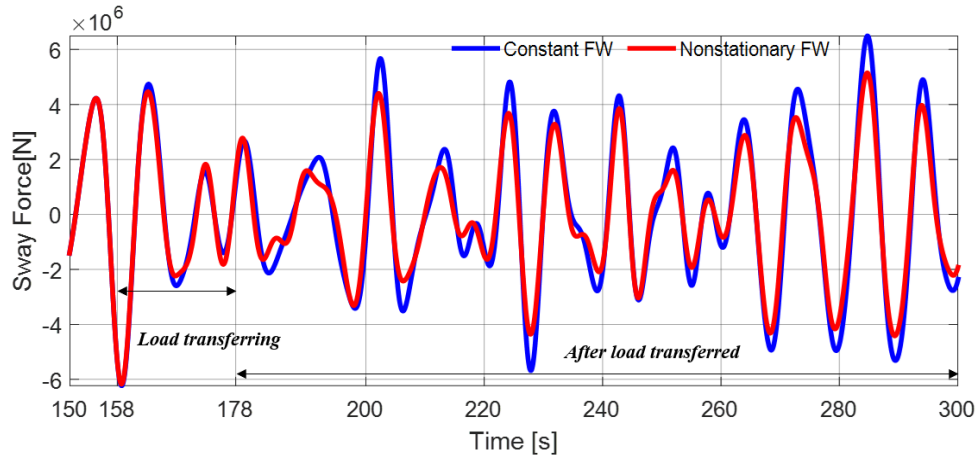
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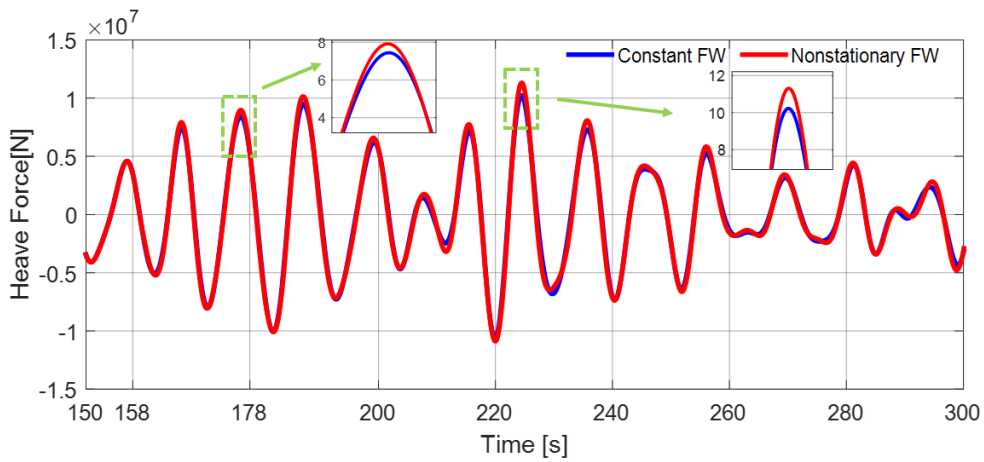
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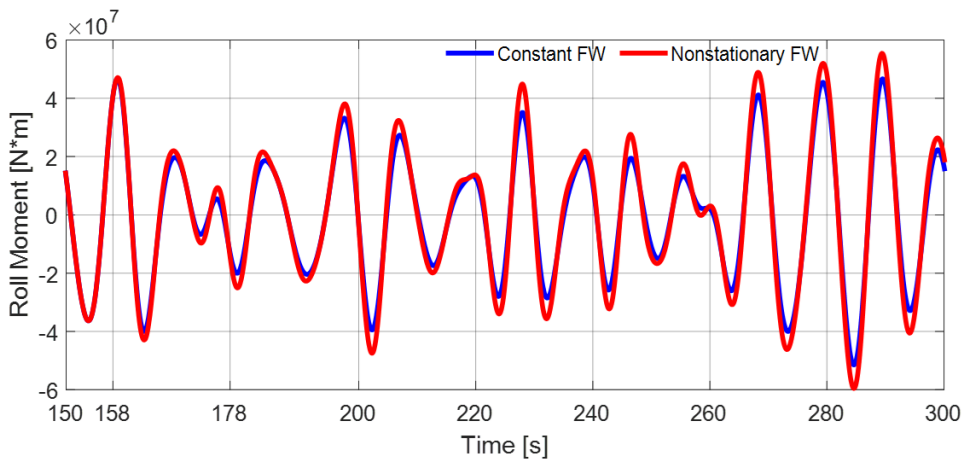
Fig. 11. Wave excitation forces in head sea ($H_s=1.5$ m, $T_p=6.7$ s)



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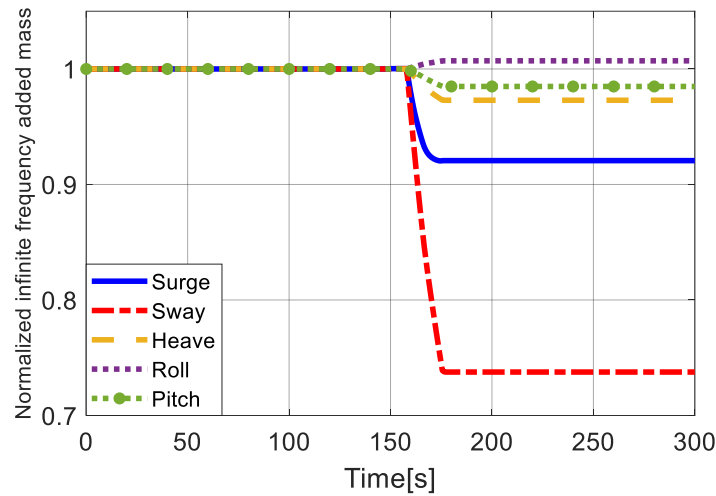
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387 Fig. 12. Wave excitation forces in beam sea ($H_s=0.5$ m, $T_p=10$ s)

388 **5.2.3 Radiation wave forces**

389 The radiation forces are represented by the infinite frequency added mass and retardation
 390 functions in time domain. Fig. 13 shows the infinite frequency added mass during the load transfer
 391 operation, which is normalized based on the value of the initial quasistatic equilibrium position. The
 392 changed body boundary condition affects the infinite frequency added mass for surge and sway

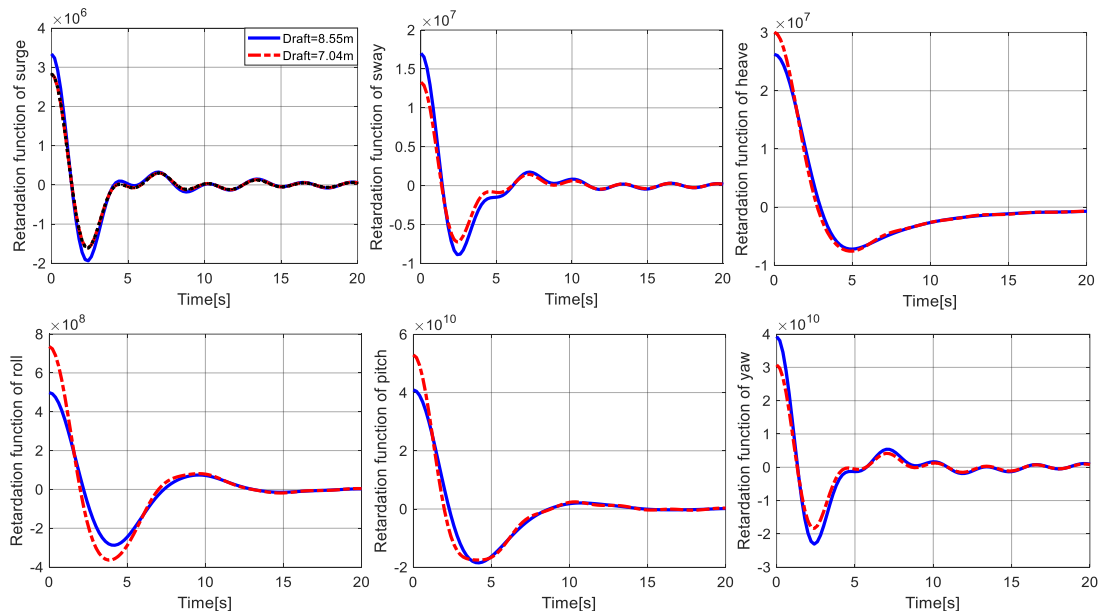
393 motions most, and only the infinite frequency added mass for roll motions is increased. It is different
 394 for those in other freedoms.



395

396 Fig. 13. Normalized infinite frequency added mass with the load transfer operation

397 In Fig. 14, the retardation functions at different drafts of float-over barge are compared. For
 398 surge and sway motions, the retardation functions have the same decrease tendency as the infinite
 399 frequency added mass. This will lead the increase of surge and sway motions. Different from the
 400 decreases of infinite frequency added mass for heave and pitch motions, the retardation functions
 401 are increased considering the changed body boundary condition.



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Fig. 14. Retardation functions at different drafts

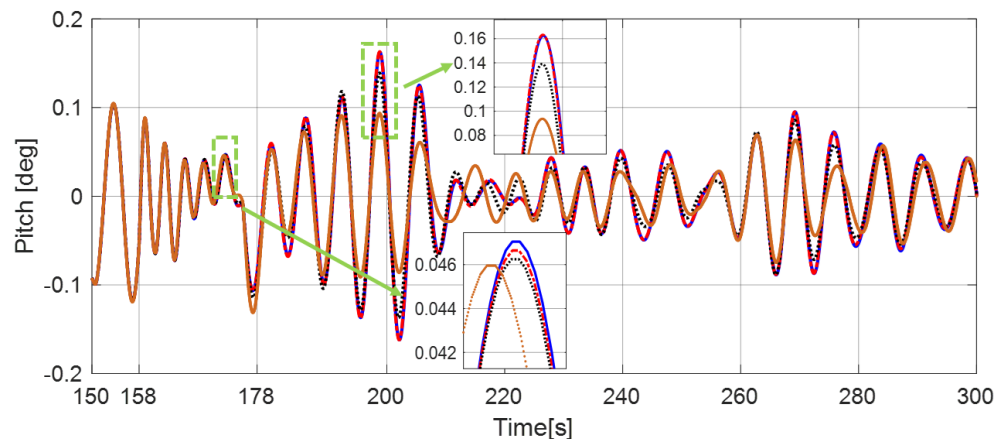
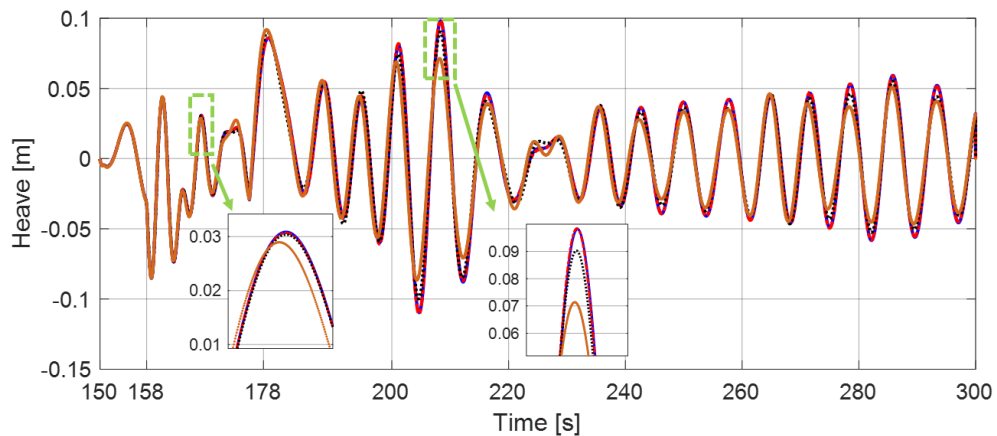
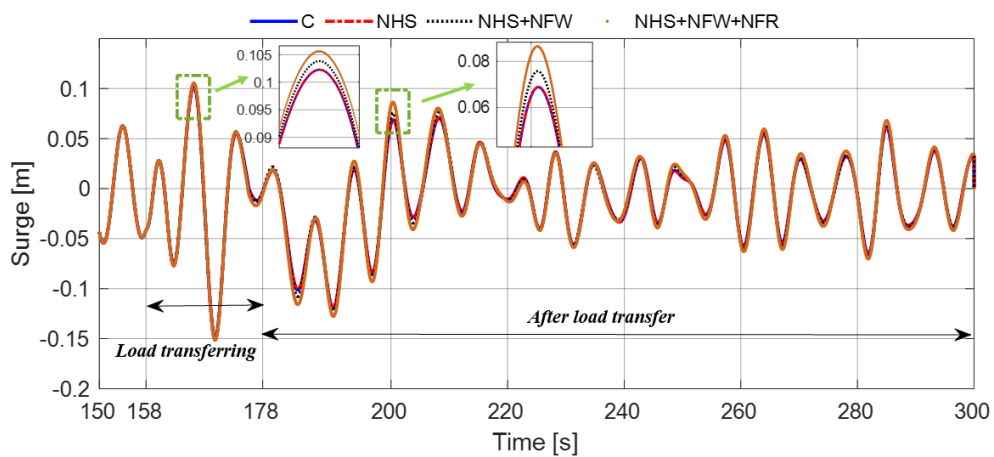
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5.3 Motion comparison

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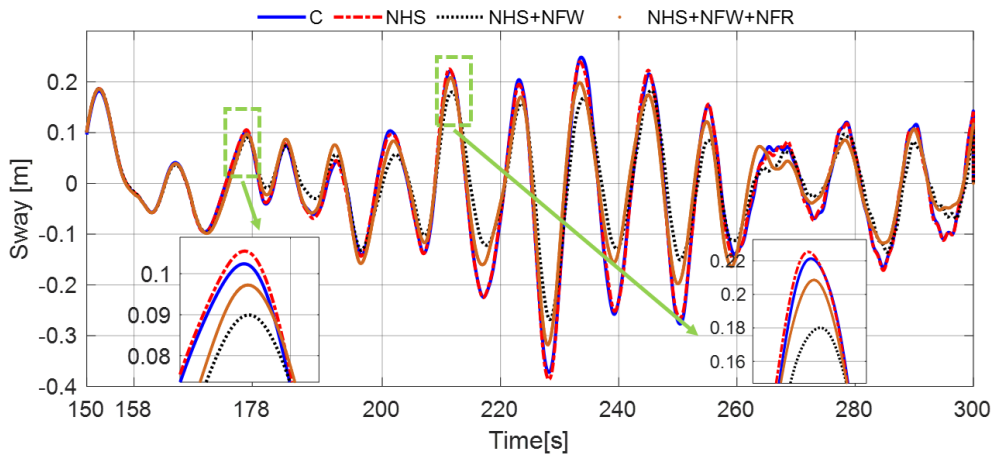
Fig. 15 and Fig. 16 show the dynamic motions of the barge with different hydrodynamic

407 modeling methods in head sea and beam sea. The magnified results of all motions of the barge
 408 during the load transfer operation and after load transfer show the same change tendency under the
 409 effects of the nonstationary hydrodynamic forces. Same as the changes in wave excitation forces,
 410 all the motions of the barge also change little during the load transfer operation. But when the load
 411 transfer ended, a larger difference is observed for all motions of the barge. The roll motion is the
 412 most affected with the nonstationary hydrodynamic forces, including both motion amplitudes and
 413 periods.

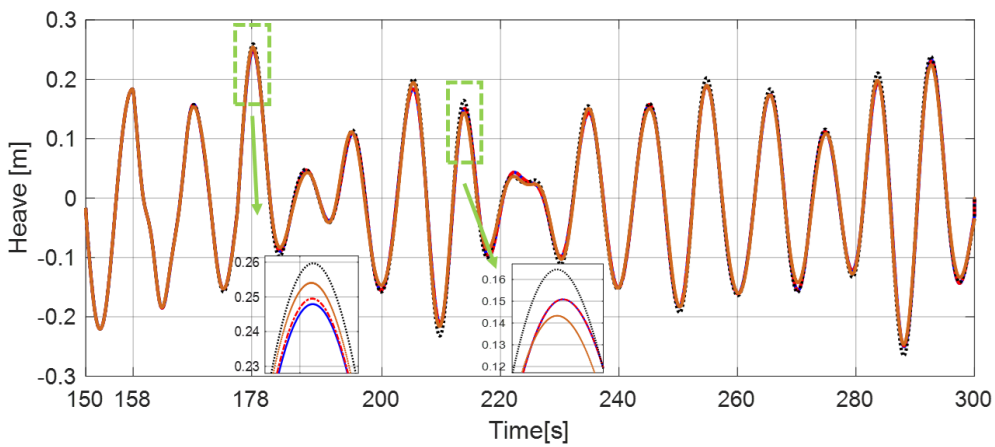


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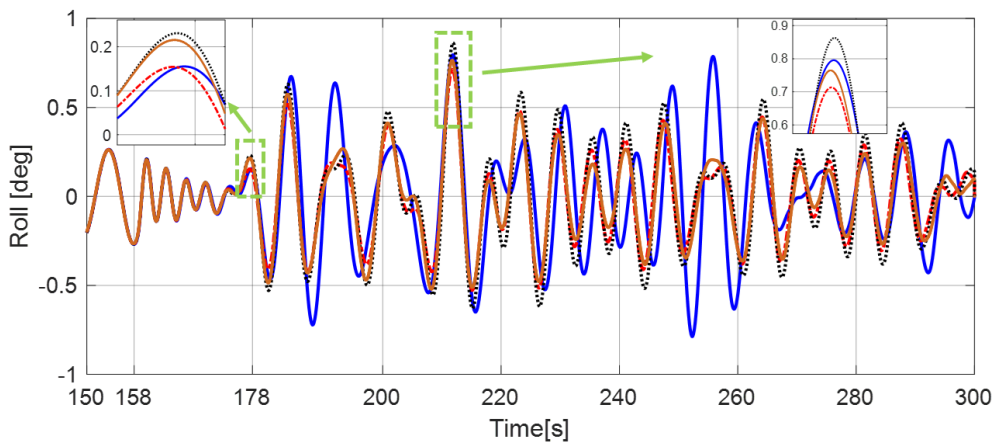
Fig. 15. Dynamic motions of float-over barge in head sea ($H_s=1.5$ m, $T_p=6.7$ s)



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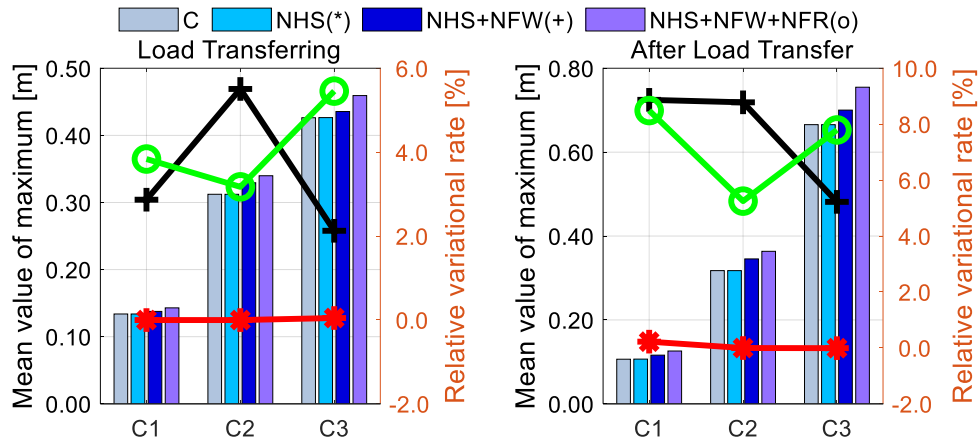
421

Fig. 16. Dynamic motions of float-over barge in beam sea ($H_s=0.5$ m, $T_p=10$ s)

422

To consider the variability of stochastic waves, 20 times realization of irregular waves are
 423 conducted for sea states in Table 6. For each realization, the same wave time series are ensured for
 424 the calculation model set in Table 5. The statistical results of barge motions during the load transfer
 425 operation and after load transfer in head sea and beam sea are shown in Fig. 17 and Fig. 18. The
 426 mean value of the maximum of 20 simulations are used to represent the system statistical results.

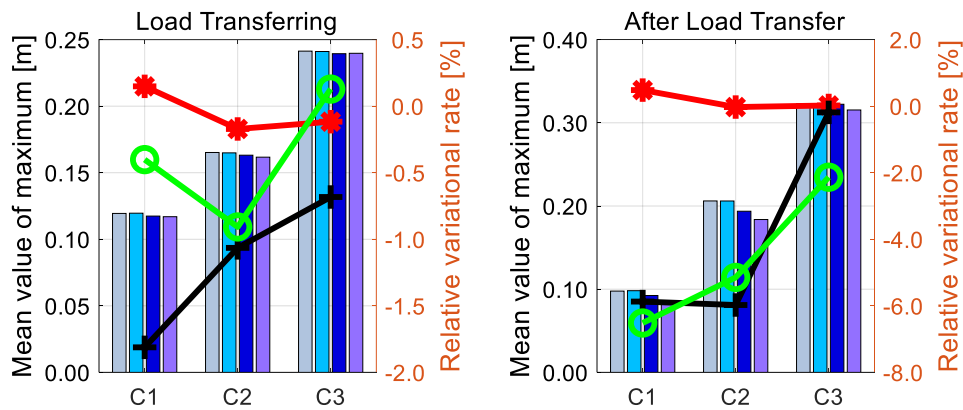
427 The relative variations between different hydrodynamic modeling methods are given in right axis
 428 in figure. It further explains the different effects of nonstationary hydrostatic restoring forces,
 429 nonstationary wave excitation forces and nonstationary radiation wave forces respectively.



430

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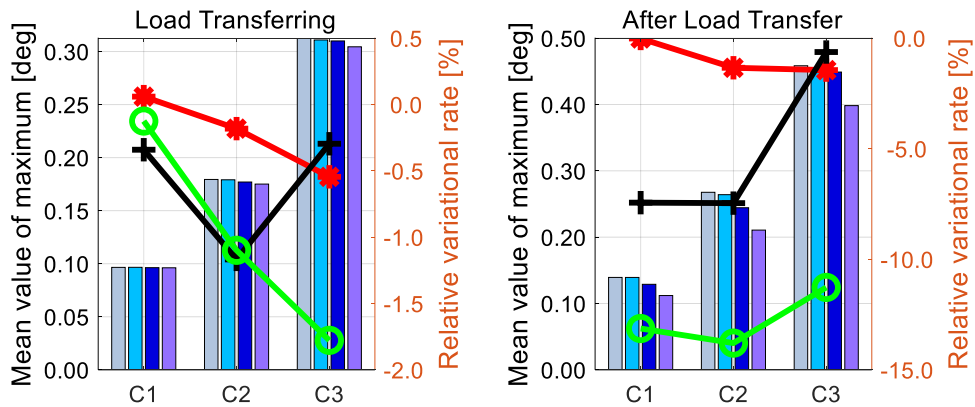
(a) Surge motions



432

433

(b) Heave motions



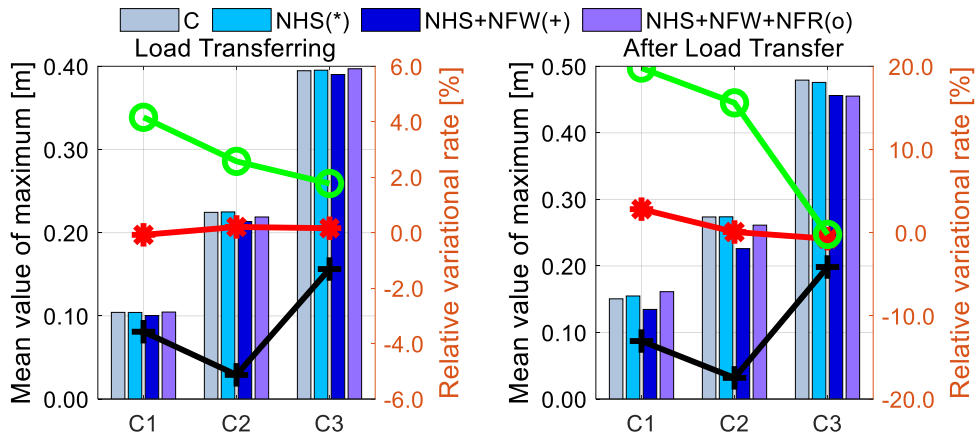
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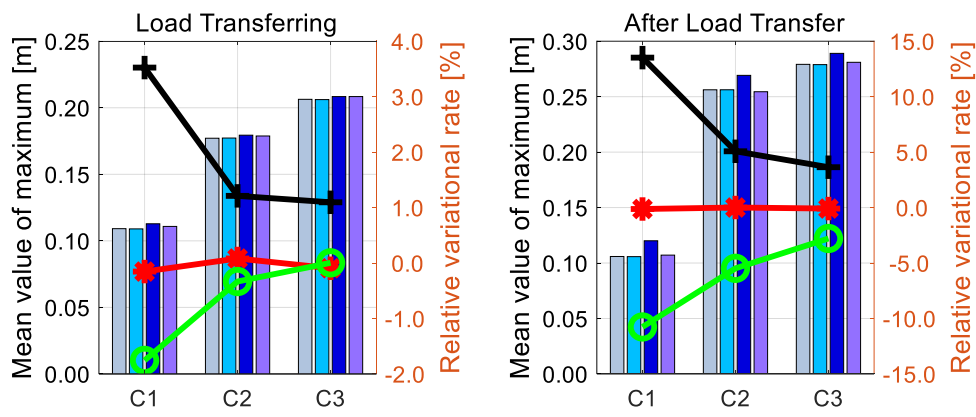
(c) Pitch motions

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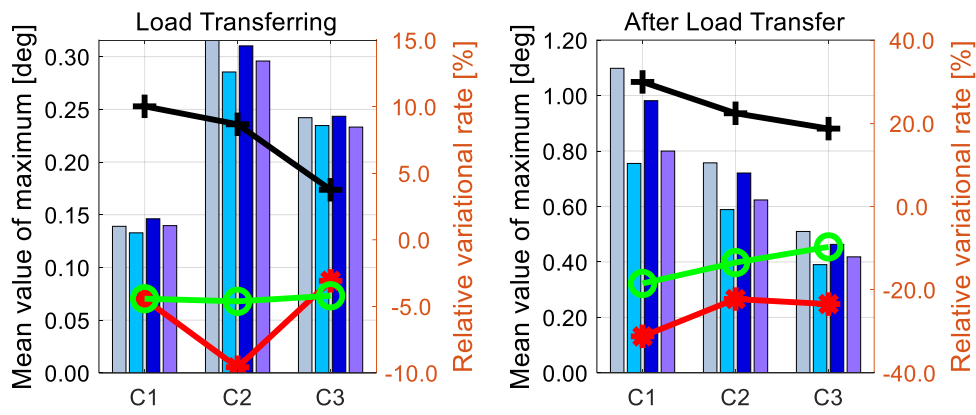
Fig. 17. Statistical responses in head sea



(a) Sway motions



(b) Heave motions



(c) Roll motions

Fig. 18. Statistical responses in beam sea

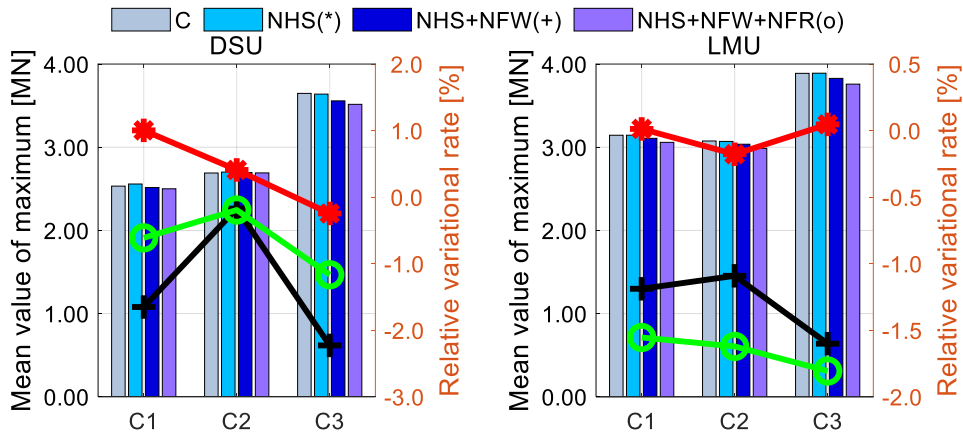
From Fig. 17 and Fig. 18, the statistical results of the motions of float-over barge are agree with the time history results shown as Fig 15 and Fig 16. It is noteworthy that all motions have a significant change under the effects of nonstationary forces after the operation, while the largest variation of motions during load transfer process is approximately within 5% except for roll motion.

448 The relative variation rate of roll motions could reach 10% at load transfer process and 30% after
449 load transfer process. In summary, the effects of nonstationary hydrodynamic forces are both
450 observed for the analyzed load transfer operation and after load transfer operation. Based on the
451 methodology proposed, the reasonable dynamic analysis could be simply carried out with one
452 update of hydrodynamic forces at the end of the rapid load transfer operation.

453 Besides, the nonstationary hydrostatic restoring forces only take effects on the roll motions,
454 and the roll motions are also most affected by the nonstationary wave excitation and radiation forces
455 among all wave induced motions. The attention is also paid to the effects of nonstationary wave
456 excitation forces on motions, that is shown as mark '+' in figures. Previously, it has been concluded
457 that there is a minimum change for heave, pitch and roll wave excitation forces when a long period
458 waves are considered. The heave, pitch and roll motions of barge also show a same change.

459 **5.4 Contact forces**

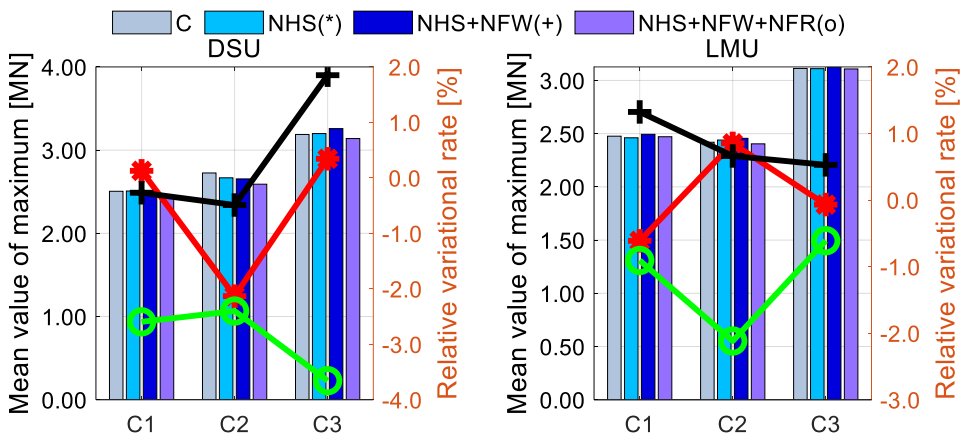
460 There are 6 sets of DSUs and LMUs used for the FOD installation, and they play the most
461 important role in reducing the impact loads acting on the barge and topside. The statistical vertical
462 contact forces of one represented DSU and LMU that support the maximum weight of the topside
463 are illustrated in Fig. 19~Fig. 20. When the load transfer is ended, there are no more dynamic contact
464 forces between bodies so only the results during the operation are shown in figures. Like the motion
465 responses, the statistical results of the contact forces show same change tendency as motions. In
466 head sea, the contact forces of the DSU and LMU decrease under the effects of nonstationary wave
467 excitation forces and radiation wave forces, and the heave and pitch motions of the barge in Fig.17
468 (b) and Fig.17 (c) are also decreased. In beam sea, the nonstationary wave excitation forces lead an
469 increase on heave and roll motions, and the nonstationary radiation wave forces lead a decrease of
470 same motions. Correspondingly, the contact forces of DSU and LMU as Fig. 19 and Fig. 20 show
471 the same changes as motions in beam sea.



472

473

Fig. 19. Statistical vertical forces of DSU and LMU in head sea



474

475

Fig. 20. Statistical vertical forces of DSU and LMU in beam sea

476 6. Conclusion

477 The present study develops a numerical modeling method to calculate nonstationary
 478 hydrodynamic forces and induced responses for simulation of continuous offshore installations,
 479 where a continuous update of the body boundary conditions for hydrodynamic analysis is occurred.
 480 Targeting the load transfer operation for FOD installation, the proposed method is used to calculate
 481 the quasistatic motions of the barge first, and then the effects of time varying body boundary
 482 conditions on the different hydrodynamic forces are discussed. The corresponding effects on the
 483 induced motions and the contact loads are also investigated. Some conclusions can be drawn from
 484 the study.

485 (a) In quasistatic analysis, a panel model with improved efficiency is developed to calculate
 486 the time varying buoyancy of the floating body with a nonlinear method based on the integration of
 487 hydrostatic pressure considering the instantaneous position/inclination of the barge. The results

488 show that it is important to capture the nonlinear relationships between buoyancy and displacement
489 when the mean water plane of the floating body is largely changed.

490 (b) The quasistatic motions induced inertial force are defined representing the operation
491 induced dynamic forces. Larger motions are observed at the end of the load transfer operation, that
492 indicates it plays an important role for analysis.

493 (c) The hydrodynamic forces, barge motions and contact forces show same change tendency
494 under the effects of time varying body boundary conditions. However, the changes of these
495 nonstationary responses are little during the load transfer operation. When the load transfer ended,
496 larger effects on the hydrodynamic forces, motions and contact forces are observed. It could be
497 recommended that the dynamic analysis of the rapid load transfer operation for FOD installation
498 can be effectively carried out with one update of hydrodynamic forces at the end of the operation.

499 (d) For different wave period, there is a minimum effect of time varying body boundary
500 conditions on heave, pitch and roll wave excitation forces and motions with condition of long period
501 waves.

502 (e) The results show that the barge roll motions including its natural period and amplitude are
503 the most affected with consideration of the time varying body boundary conditions, which should
504 be carefully considered in offshore operation.

505 The developed methodology is stimulated by the demand from the huge platform
506 integration/decommission as well as the offshore wind turbine installation activities, where there
507 are wet surface variations with the rapid load transfer at critical mating operation stage. For the sake
508 of simplicity, the drift motions caused by the second-order wave forces are ignored. Further research
509 work on it and the intensive experimental validations needs to be done in the future.

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