Curriculum Implications of Using Ethnomathematics to Promote Student Learning in Ethiopia

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Abstract

This paper examines mathematical enculturation in Ethiopia, based on Alan Bishop's Framework of Mathematical Activities, which he created after studying a range of indigenous cultures. The aim is to assist a culturally responsive mathematics education in Ethiopia with the intention of improving student learning. Data was collected using ethnographic approach. The findings demonstrate the existence of rich mathematical activities aligned to Bishop's six fundamental mathematical activities. Our work has implications for curriculum and instruction in mathematics education ranging from early childhood to higher education. It may also inform mathematics educators and policymakers in other countries.

Keywords: Ethnomathematics, Indigenous mathematics, mathematical activities, curriculum and instruction implications, student learning

Introduction

Background

Ethiopia is an east African oriental nation with its own unique history, and with a culture and traditions dating back more than 3,000 years. Ethiopia is gifted with an exceptional heritage including ceremonies, festivals, celebrations, rituals, and other living expressions, as well as a rich cultural landscape enhanced by the representation of numerous religions including Christianity, Islam, Judaism, and other traditional belief systems. It is among those nations with a rich and diverse cultural heritage, including languages, tradition, art, music, a unique numbering system, calendar, counting system, design features, etc., which it still preserves. It also has unique wildlife, food styles and so on. It is the only country to use a time system with a 12-hour clock counting from dawn to dusk, and from dusk to dawn. It has many UNESCO World Heritage Sites. It is the home to Lake Tana, which feeds into Abay river (the Blue Nile), one of the two major tributaries of the River Nile, contributing 85% of its water to the longest river in the world. In the past, most civilizations were situated around rivers or water sources. Lake Tana and Abay contribute to its historical, cultural and social context, even today. The Grand Ethiopian Renaissance Dam (GERD) (Amharic. ታላቁ የኢትዮጵያ ሕዳሴ ግድብ) is one example (Tesfa, 2013).

One other feature is that Ethiopia was not colonized by western forces, except during the five -year occupation by Italy between the two world wars. Hence, it has not yet been extensively studied in depth, and is little known to the rest of the world. Nevertheless, Ethiopian education is not unique in the world, and is modelled on western systems (Nair &Abera, 2017; Weldeana, 2016). This has created a conception of education as being detached from reality and from the existing societal context. Knowledge is somehow foreign, rather than acknowledging existing social, cultural, and environmental realities, including mathematics education. Its terms and concepts are defined by, and related to, another culture. Being educated in Ethiopia, one can say, means that one is well fitted to serve another system, instead of developing and shaping existing knowledge and skills, using wisdom and knowledge from other parts of the world. There is no problem in learning the universal aspects of mathematics or other subjects in general. However, pupils are ineffective in solving their immediate problems (*mathema*) and are unable to model them using mathematical ideas, procedures, and practices from daily life. One should, therefore, address such gaps using the principles of ethnomathematics, not as the only solution but within a comprehensive set of research and practice-based solutions (Shirley, 2001; Rosa et al., 2016; Greene, 2017; Meaney & Shockey, 2020).

The complaint that students are not able to connect their school knowledge with their daily lives, and are unable to solve problems, or to do mathematical modeling, could be resolved, if the gap between school mathematics and real life-related contexts is reduced. This gap between school mathematics and real-life problem solving has been addressed (Boaler, 1993; 2016; Fosnot & Dolk, 2001). Boaler (1993) states that "if the context of an assessment task is capable of determining, to some extent, mathematical performance and procedure, then the degree of mathematical specificity which can be maintained within and across contexts and, more importantly, the processes which determine this should certainly be examined" (p. 12) In this regard, we argue that indigenous mathematical activities are a rich source for curriculum designers, textbook developers and teachers to explore. In the words of Langdon (1989, p. 179) as quoted by Boaler (1993, p. 16): "students acquire a better understanding of mathematics by discovering that it is already a part of their environment …." Hence, we pose the questions: a) *What type of ethnomathematical activities are embedded in Ethiopian culture? b) How can these activities be integrated into the curriculum, possibly into classroom instruction?*

To answer these questions, an ethnographic approach has been used, such as interviews, documentation, and field notes by the researchers. Bishop's (1988) six mathematical activities framework is used as the main categorizing tool of the data collection and data analysis. This is done for two reasons. First, Bishop's study is generic, in a way it works for many cultures as the study was grounded across cultures. It is the foundations for the development of mathematics in culture, detail comes later. Second, ethnomathematics studies are somehow new in Ethiopia and there seem to be few studies that try to embed cultural activities into the curriculum and classroom. We think that it is beneficial to start with Bishop's work. But we want to remark that, at this stage, our study does not include actual classroom observation, which may be carried out in subsequent studies. We therefore first define the main topic ethnomathematics that helps us to define the context of this article, and then we present the Bishop's mathematical activities as a framework.

Conceptual Framework

What is ethnomathematics?

The term ethnomathematics was first used by D'Ambrosio in 1977, in the narrow sense of the mathematics of indigenous populations (Rosa et al., 2016). D' Ambrosio (1990) later defined ethnomathematics in a broader way, as presented by Rosa & Orey (2011):

> The prefix *ethno* is today accepted as a very broad term that refers to the social-cultural context and therefore includes language, jargon, and codes of behavior, myths, and symbols. The derivation of *mathema* is difficult, but tends to mean to explain, to know, to understand, and to do activities such as ciphering, measuring, and classifying, inferring, and modelling. The suffix *tics* is derived from *techné*, and has the same root as technique. (p. 81)

In this definition, there are several terms that require our attention, and we present them in an order that is meaningful for us. The first is *ethno*. According to D'Ambrosio, this term has a much broader meaning than *ethnic*. It means a culturally identified group sharing knowledge and practice, language and myths (Rosa et al., 2016). Rosa et al. (2016) clarify further that the term refers to a group within the same natural and socio-cultural environment that exhibits compatible behaviors.

The other term is *tics.* It is about the techniques used by the *ethnos*, such as counting, ordering, sorting, measuring, weighing, ciphering, classifying, inferring and modeling. The last term in this definition is *mathema.* Rosa and Orey (2011) define *mathema* as: "mathema means to explain and understand the world in order to transcend, manage and cope with reality so that the members of cultural groups can survive and thrive" (p. 35). Therefore, according to *D'Ambrosio*, ethnomathematics is derived from three Latin words, and it is *tics* of *mathema* in distinct *ethnos*.

In this definition, *sociocultural theory* is included as a broad background. Sociocultural perspectives on learning and development were pioneered in the early 20th century by Vygotsky, who introduced the concept of 'Zone of Proximal Development' (ZPD) to explain how an individual's cognition originates in social interaction. This led to the so-called social turn in mathematics education in the 1990s (Lerman, 2000; Lave & Wenger, 1991). That is, learning is seen as a product of social activity in the cultural context. Ethnomathematics can be seen as a product, or also as a contributor to, such theory in general. Further, Rosa et al. (2016) summarize the six dimensions of ethnomathematics programs as: cognitive, conceptual, educational, epistemological, historical and political, to analyze the socio-cultural roots of mathematical knowledge. In this article, ethnomathematics in relation to learners' social and cultural contexts is considered to be one element in fostering student learning.

Rosa and Orey (2011) also describe ethnomathematics as the cultural aspect of mathematics that presents mathematical concepts of the school curriculum in a way which relates these concepts to the students' cultural and daily experiences, thereby enhancing their abilities to elaborate meaningful connections and deepen their understanding of mathematics. In this definition, we can see two distinct aspects of mathematics: the school mathematics curriculum and the cultural aspect of mathematics. At times, in some *ethnos*, there may not be a clear connection or correlation between these two aspects, hence school mathematics can be foreign to pupils. In order to address this problem, and to enhance the possibility of making meaningful connections and deepening the understanding of mathematics, researchers have investigated the embeddedness of mathematics in the cultural and social contexts in which learners are situated. Hence, ethnomathematics researchers should "investigate ways in which different cultural groups comprehend, articulate, and apply ideas, procedures, and techniques identified as mathematical practices" (Rosa et al., 2016, p.7). Another definition was provided by Begg (2001, p. 71) as:

> Ethnomathematics implies mathematics of cultures … definition of cultures (and sub-cultures) not only refers to ethnic cultures, but also to any sets (or subsets) of people who share common experiences such as languages, beliefs, customs, or history. Using this definition, an ethnic group forms a culture. Similarly, a religious group, a group involved in a sport, a group of people who cannot hear, an artistic community, a gender group, or the residents of a district, all form either a culture or a sub-culture.

Our understanding of ethnomathematics is shaped by these definitions. And it is in line with these thoughts that we intend to study mathematical activities and practices in an Ethiopian context, in order to contribute to building a culturally responsive curriculum in mathematics education in Ethiopia and also to reveal the existence of such *ethno based tics for mathema.* Before we present the main conceptual framework used in the article, let us provide the nuances that must be considered when deciding what is relevant to the learners in a particular context where the learners are found in**.**

Mathematics, Culture and Student Learning

In the past, the content of the mathematics curriculum, and classroom practices used in schools, have been guided by the view that mathematics is a value-free body of knowledge disassociated from its cultural context (Matang & Owens, 2014; Bishop, 1988; Ernest, 1991). The social turn (Lerman, 2014) in mathematics education, however, has questioned this assumption, as a consequence of the impact of socio-cultural theory on learning theories. According to Lave & Wegner (1991), learning is a social phenomenon, i.e., knowledge is acquired in social settings. They argued that learning is not only situated in practice but is also an integral part of generative social practice in the lifeworld. This has had a significant impact on how student learning is perceived in the teaching and learning of mathematics. Boaler (1993) suggests that "mathematics is a part of students' social and cultural lives, and the mathematics classroom has its own social and cultural life" (p. 16). However, this view is not without its critics (Anderson et al., 1996), since not all learning is situation-specific.

Student learning and achievement are crucial measures of an education programme (Cai et al., 2020; OCED 2016), and are affected by many factors. To provide a broader picture, Sammons et al. (2009) situate student outcomes by considering the complexity of educational systems. Students are nested within classes that are nested within schools, where variables within and across these levels can be directly and indirectly related, and where changes occur. This framework also considers the national context, and is connected to the educational system in general, including educational policy at a regional and/or national level (Nilsen & Gustafsson, 2016). Figure 1 provides an adapted version of this framework. The concepts of 'Intended, Implemented and Attained' components of the curriculum, including the textbook curriculum, the component that mediates intention and implementation, (Valverde et al., 2002), are introduced, in order to emphasize the connection between curriculum and instruction. Furthermore, all these levels (national or regional, school, class, and student levels) are embedded in a rich social, cultural and historical context. In this article, we consider the value of context as a rationale for deploying indigenous, 'ethnomathematical' activities. Real world contexts, in which the learner finds herself/himself at home, are meaningful in learning to mathematize (Fosnot & Dolk, 2001; Boaler, 1993).

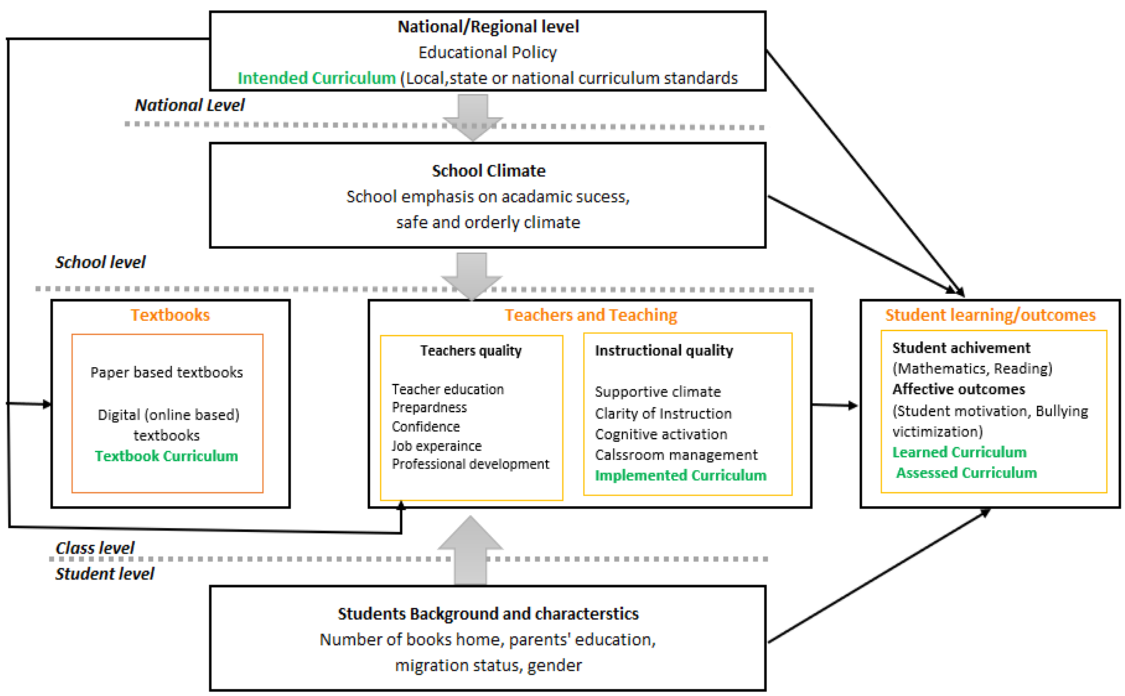


Figure 1 Adapted from the conceptual framework of determinants of student outcomes (Nilsen & Gustafsson, 2016; Sammons, 2009)

Student learning and achievement are, in one way or another, related to the goals and purposes of education systems in a specific society or country. If the goal is to prepare students for life, embedding contexts related to real life that are meaningful for the individual learner is crucial in the teaching and learning of mathematics. But if the goal is to prepare elites to work in abstract mathematics, then teaching mathematics as a set of abstract concepts and procedures will be the norm in classrooms, which is reflected in many curricula. We argue that a balanced view concerning the relevance of real-life context in mathematics education is vital. Though contexts are relevant for fostering student motivation and for helping students to be able to connect mathematics to problem solving related to real life, one should be careful to note that contexts can also be a barrier to understanding the mathematics involved (Boaler, 1993). For example, the famous "Hand shake Problem" is used in many classrooms as a student-friendly context-based problem in western cultures (see the blog by Fay-Zenk, Trio of Friendly Problems[1]). But this task can be problematic for students who are in a culture where handshaking is not the norm. They may greet by kissing on the cheek or pushing their shoulder a few times, and not through a hand shake, which happens only once. This will have effects on the processing and solution of the task. Hence, context is crucial in mathematizing (Fosnot & Dolk, 2001; Boaler, 1993).

The question is; therefore, which context is relevant for learners? Those who design the curriculum, write textbooks and are responsible for implementing the curriculum should be aware of the real life-related contexts of specific groups of learners in their endeavors. School mathematics curricula and instructional methods cannot be designed without considering such opportunities for learning, if the goal of education is to improve student learning or achievement. This is a decision mainly for those who make policy at *national level* and *class level* as shown in Figure 1. To give an example, teachers, learners and parents in our study expressed their worries about the contexts used in a particular mathematics textbook in Ethiopia. The text book includes tasks that are based on Indian culture, which has nothing to do with the Ethiopian culture or context. Both teachers and learners struggle to understand and mathematize in that context. The

[1] www.aimath.org/.../materials/MFZTrioofFriendlyProblemslessononly.pdf

cognitive barriers created by such alien contexts may be serious, depending on the level of abstractness of the mathematical concept or procedure involved. In this paper, we intended to provide a conceptual framework that can assist educators, policymakers and mathematics teachers endeavors in providing relevant cultural context for the learners. This is presented as follows.

Bishop's Mathematical Activities as Framework

Bishop (1988) created a huge impact on mathematics education when he introduced the idea of mathematics as a cultural product. By studying cultures and literature from around the world, Bishop found that humans in different cultures have always had a need to solve problems and tasks that they were relevant to their daily lives. He discovered that, to a large extent, different cultures develop similar mathematical systems, because the need for mathematical thinking in daily life around the world is similar across cultures. Bishop's focus on similarities between cultural groups, rather than differences, made it possible to understand more about the roots of mathematical thinking and learning.

Bishop presented new content, structures and processes related to mathematics that built on the relevant beliefs and processes of history, but also generated new ideas. He created an alternative conception of mathematics by suggesting six mathematical activities that collectively constitute mathematics as a tool and way of thinking developed by problem-solvers in all cultures. This view of mathematics makes the subject fundamental and universal, as it has been developed by different humans in different cultures at different times. Mathematics is constantly changing and developing, as are humans and societies, and we are all continually affected by our everyday life worlds. In our increasingly complex societies, there is always a corresponding need for more complex mathematics. Treating mathematics as a cultural product therefore implies that we all encounter and develop the mathematics that is all around us, making the subject fundamental and universal. Bishop indeed recognizes that these six activities might not be completely universal, as societies may exist somewhere that do not have these elements in their culture. Still, the idea of the six activities as widespread, significant and similar in most cultures makes it fair to characterize them as universal.

Bishop's (1988) six key 'universal' activities are, therefore, the foundations for the development of mathematics in culture. These activities are *counting, locating, measuring, designing, playing and explaining*. Critiques of his work asked why there were only six, and not more categories, enabling more areas in mathematics to be covered. Bishop himself did not believe that the particular number of activities is important. He focused on how they collectively conceptualize and define the mathematical field (1988, p. 22). Though the critics have a point, we chose Bishop's framework as a starting point, so that we can contribute to a culturally responsive curriculum in the Ethiopian cultural context. It is, of course possible to develop it further.

The Six Mathematical Activities

Bishop divided the six mathematical activities into three groups. His first two activities are *counting and measuring*. Even though they are distinct, they both have to do with numbers, and relate to numbers in different ways. Counting and communicating quantities of objects is what many will think of first when it comes to mathematics. Numbers are important both in mathematics and in our daily lives. Although cultures have various number systems, type of representations and needs for accuracy, having a way of counting is significant for humans in all cultures. Counting also includes seeking number patterns and doing statistics. When we measure something, we use numbers to communicate its relative size. This can be through quantities such as length, area, volume, weight, time, speed, temperature and so on. In all cultures, people need some form of measurement and have therefore developed suitable tools and units for measuring. But before children can develop an understanding of measuring, they need experience with comparing and ordering.

The next two activities that Bishop describes are *locating* and *designing*. They both have to do with spatial structuring, but give rise to two different kinds of geometric ideas. Locating has to do with understanding our environment, using directions, orientation, navigation and placement. Bishop calls this the "geographical" part of geometry. In all cultures, people need to understand spatial connections in their environment, and to describe how objects are related to one another. These descriptions can be made through words, maps, graphs, diagrams and coordinates. Design, a spatial sense, on the other hand, has to do with shapes, figures, patterns, art and architecture. Spatial sense is important in order to understand the features of two- and three-dimensional shapes, and how they are connected. When working with shapes, it is important to analyze their properties, explore how they fit together and develop an understanding of patterns.

As culture not only concerns our link to the physical environment, but also to each other, Bishop felt the need to define some additional activities that, mathematically, are very important for our social environment. The activities *playing* and *explaining* are therefore concerned with social interaction and creativity. Not all types of play are mathematical, but when we think of dice, puzzles and logics, the mathematical side is obvious. By playing, people in cultures develop their ability to think strategically, to plan, to perform logical reasoning, and to follow rules. When children play, they use numbers, they measure, they compare and they use their creative mathematical reasoning. Sometimes they even change the rules of the game to increase their probability of winning. In all areas of mathematics, it is essential to understand and explain how and why things are connected. By encouraging children to explain and argue, they increase their logical skills and their use of mathematical language.

Bishop makes it clear that these all activities are significant, both separately and in their interactions, for the development of mathematical ideas in any culture.

Research Methods

Data Collection and Sampling

Ethnographic methods were used for this study as it a qualitative design in which the researcher describes and interprets the data (Creswell & Poth, 2018). That is, field work was conducted using observation, interview, and interpretation to provide an in-depth evaluation of the problems, challenges and positive aspects of the issue at hand (Whitehead, 2005). The sampling procedure, validation and implementation of the instruments, method of analysis, the consideration given to ethical standards and code of conduct in terms of informed consent and confidentiality accord with conventional research practice. The use of questionnaires, interviews and document analysis added, so to speak, the necessary qualitative flesh to the bones of quantification. The data collection activities were as follows. The researcher did field work in several parts of the country: Afar region (eastern part), Gondar (North western), Gambela (western), South Omo (southern) and in Addis Ababa (central part). He made observations of historical places, houses, churches, market places; conducted interviews, and made audio and video recordings, based on prepared interview questions at selected sites. The study was carried out, where necessary, by a native speaker with key informants, using unstructured interviews in Gambela with an Opo community native speaker. Unstructured or open-ended questions were used to probe the topic more deeply. In-depth interviews were also carried out with local elders, regional culture and tourism experts and church tourist guides to assess the missing links between school mathematics and indigenous mathematics activities. Document analysis with secondary data has also been carried out, including selected documents on the internet. Archival and statistical data was collected from various administrative sources at national, state and local levels (Addis Ababa , Gondar).

Data Analysis

As the study is guided by Bishop's mathematics activities to explore the possibilities for indigenous mathematics activities in the pre-primary and early grades of Ethiopian education, these constructs and their organizing concepts are used in this article to analyze the data. Table 1 provides the definition of the six constructs and the organizing concepts related to the activities.

Table 1

Bishop's Six Fundamental Activities with their Organizing Concepts

The six universal activities according to Bishop (1988)	**Organizing concepts related to the activities (https://www.csus.edu/indiv/o/oreyd/acp.htm_files/abishop.htm)**
Counting. The use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names.	Quantifiers (each, some, many, none); Adjectival number names; **Finger and body counting**; **Tallying**; **Numbers**; Place value; Zero; Base 10; Operations on numbers; Combinatorics; .Accuracy; Approximation; Errors; Fractions; Decimals; Positive, Negatives; Infinitely large, small; Limit; Number patterns; Powers; Number relationships; Arrow diagrams; Algebraic representation; Events; Probabilities; Frequency representations.
Locating. Exploring one's spatial environment and conceptualizing and symbolizing that environment, with models, diagrams, drawings, words or other means. (p. 182)	**Prepositions**; Route descriptions; Environmental locations; N.S.E.W. Compass bearings; **Up/down**; **Left/right**; **Forwards/Backwards**; **Journeys (distance)**; Straight and Curved lines; Angle as turning Rotations; Systems of location: Polar coordinates, 2D/3D coordinates, Mapping; Latitude / Longitude; Loci; Linkages; Circle; Ellipse; Vector; Spiral

Measuring. Quantifying qualities for the purposes of comparison and ordering, using objects or tokens as measuring devices with associated units or 'measure words.'	Comparative quantifiers (faster, thinner); Ordering; Qualities; **Development of units** (heavy - heaviest - weight); **Accuracy of units**; **Estimation**; **Length**; **Area**; **Volume**; **Time**; **Temperature**; **Weight**; Conventional units; Standard units; System of units (metric); Money; Compound units.
Designing. Creating a shape or design for an object or for any part of one's spatial environment. It may involve making the object, as a 'mental template', or symbolizing it in some conventionalized way.	**Design**; Abstraction; **Shape**; **Form**; **Aesthetics**; Objects compared by properties of form; Large, small; Similarity; Congruence; Properties of shapes; **Common geometric shapes**, figures and solids; Nets; Surfaces; **Tessellations**; **Symmetry**; Proportion; Ratio; Scale-model Enlargements; Rigidity of shapes.
Playing. Devising, and engaging in, games and pastimes, with more or less formalized rules that all players must abide by.	Games; **Fun**; **Puzzles**; Paradoxes; Modelling; Imagined reality; **Rule-bound** activity; Hypothetical reasoning; **Procedures**; **Plans Strategies**; **Cooperative games**; **Competitive games**; Solitaire games; Chance, prediction
Explaining. Finding ways to account for the existence of phenomena, be they religious, animistic or scientific.	**Similarities**; Classifications; Conventions; Hierarchical classifying of objects; Story explanation; **logical connectives**; **Linguistic explanations**: Logical arguments, Proofs; Symbolic explanations: Graphs, Diagrams, Charts, Matrices; Mathematical modelling; Criteria: **internal validity**, **external generalizability**.

Table 1 serves as a tool for collecting, organizing and analyzing the data. The data was collected using the six activities of counting, locating, measuring, designing, playing and explaining as a starting point. The organizing concepts under each activity were used to determine the categories, and those in bold refer to the ones used in this research. The categorizing and classification of activities under each mathematical area might seem arbitrary at times, but those organizing concepts serve us a source of reference and comparison with other existing mathematical activities. The researcher actively engaged in identifying and categorizing the activities according to the framework described above, which demands the identification of emerging themes or issues that involve logical and intuitive thinking, to actively follow up and register meaningful, relevant and important data according to Bishop's six activities. The intention is to lay the foundation for embedding culturally available activities into the curriculum and mathematics instruction, based on research, theory and practice at the kindergarten (Pre-primary) and primary school levels.

Findings

In this section, from the various data collected using the methods described above, we present some examples, as the goal of this study is to obtain answers about the usefulness and relevance of indigenous mathematics (ethnomathematics) for instruction and curriculum in the Ethiopian education system. The six activities presented above help us to structure our findings further. The bold items under the column related to organizing concepts are the ones that are reflected in the findings.

Counting

Under counting activities, we choose to present three out of many ways of counting. The first is the counting system used originally in the Geez language (also used in Amharic and Tigrigna languages), which is very old, mainly used by clergies in the Orthodox Coptic church, and national calendars still in use today (See Table 2). The second is Opo Enumeration (Figure 2). The Opo community can communicate numeral value, basic number formation and symbolic representation using local objects and human morphology in day to day activities. The third one is Ari communit Enumeration (Figure 3).

Table 2

Geez Numeration

	1	2	3	4	5	6	7	8	9	10
Geez numerals	፩	፪	፫	፬	፭	፮	፯	፰	፱	፲
	20	30	40	50	60	70	80	90	100	1000
Geez numerals	፳	፴	፵	፶	፷	፸	፹	፺	፻	፻፻

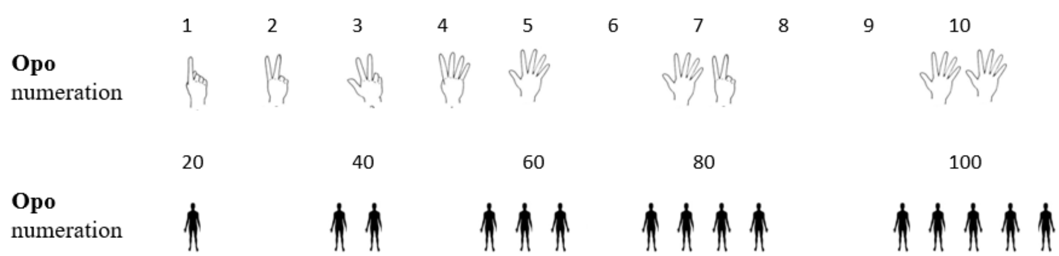


Figure 2 Opo Community Numeration

Figure 3 Ari Community Numeration

The Mathematics Revealed in These Counting Activities

Primarily these are number systems for respective group of people and communities. Building a number system to represent different numbers uniquely. For example, in Geez numeral system, the number 437 is represented as ፬፻፴፯ (in Amharic አራት መቶ ሰላሳ ሰባት). That is, ፬ = four, ፻ = hundreds, ፴ = 30 and ፯=7. The numbers system is an additive grouping system with base 10. If one wants to add ፻፸፭ and ፶፮ , that is 175 + 56, then one has to re-group ones, tens and hundred again. ፭ + ፮ is ፲፩, not ፩፩. That happens if the number system is fully place value based. Re -grouping the tens, we have ፲ + ፸ + ፶ , which is 10 + 70 + 50. That gives us ፻፴ (one hundred and thirty). Including a hundred from the task, the answer will be ፪፻፴፩ (in Amharic ሁለት መቶ ሰላሳ አንድ)[2]. When one writes 200 in Geez, it is ፪፻. which also indicates the existence of a multiplicative grouping system in the Geez numeration. Grouping and re-grouping, which are visible in this old number system, are among the big ideas in place value system in mathematics (Fosnot & Dolk, 2001; Van de Walle et al., 2020).

The Opo numeral system is also additive, similar to the old Egyptian number system, as it uses available contextual symbols like local objects and human morphology including fingers, hands, and the whole body as shown in the figure. It uses one symbol  for representing the number 20. That means 5 of these symbols gives us 100. There is a logical explanation for why 20 is represented by the figure of a whole person. For instance, two hands (10 fingers), two legs (two legs, 10 toes) or the whole body of a person are considered as 20, therefore the number 20

[2] Ge'ez is an ancient Semitic language with its own script, which originated around the 5th century BC. It uses the 'Abugida' writing system and is mainly used in the Ethiopian Orthodox Tewahedo church. https://www.metaappz.com/References/AmharicAndGeezNumbersReferenceTable.aspx

is also considered as the stem of all numbers. The representation used in the Ari community counting system is similar to that of the Opo community, with some important differences. That is, the numbers 8 and 9 are represented in relation to number 10. 8 is ‘Kestani temenis’, means ten minus two, and 9 is ‘Woleken temersi’, meaning ten with one missing. This also suggests that the ancient Ari community practiced the concept of subtraction as soon as it became acquainted with counting.

Locating

Locating is an activity that is related to the surrounding spatial environment. Bishop (1988) affirms that all societies have developed more or less sophisticated ways to code and symbolize their spatial environment. In this article, data from three communities is presented. The first is from the Afar people. In the past, the Afar were mainly livestock owners. One strong tradition of the people is requesting information exchange with each other. The term is called “Dagu Bahe”, meaning, “bring information”. Here, after they have exchanged greetings, each person describes what they have experienced on their way to meet each other. This also includes which clan is on which side, at which place and so on. As most of the land of Afar is one of the hottest places on earth and many places look alike, it seems the community has developed a deeper understanding of direction, east, west, north and south, in relation to the movement and position of the sun and moon, as well as local landscape features, as in other cultures. Hence, they have developed high levels of skill in describing their spatial environment.

The other two sets of data are collected from people of the Ari and Opo communities. As with other communities, both have expressions or names for up/down, left/right, forward/backward, north, south, west and south, curve, ecology, and so on, including facial and hand gestures. It might be helpful to further study these and other practices in depth to discover how they navigate in their environment and relate this to ethno-mathematical practices.

Measuring

The third activity is measuring. In almost all cultures, there are notions of measurement according to the situation in which the community lives. Some have developed sophisticated ways to measure space (distance, area, volume), time, weight, temperature and other constructs, while others have simpler ways to make comparisons. Measurement is all about comparing, directly or indirectly, and almost all cultures perform this activity every day. Human morphology is one of the most common measuring device in most cultures. In the communities of Afar, Ari and Opo, such practices are also common.

In most parts of the highlands of Ethiopia, the following units are used for different measurements (see Table 3). These units are used to measure solid, liquid, gas, distance and so on. As illustrated in Table 4, the Opo community has local units of measurement for different quantities as compared to standard units of measurement. Figure 4 shows some of the units used in the community.

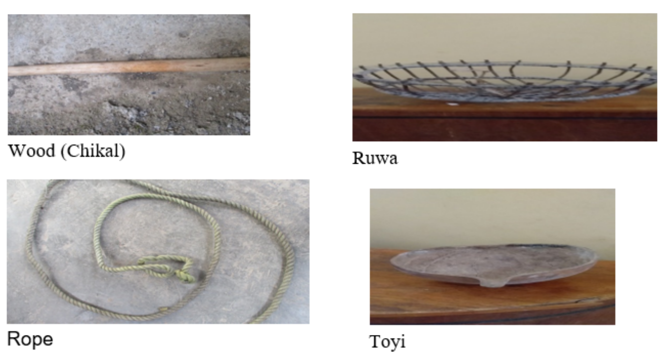


Figure 4 Opo community rope is a unit of measurement for length. Similarly, weight, size, width, volume, distance and area is measured by toyi, rope, ruwa, day travel and pace respectively in the Opo ethnic group

Table 3

Ancient Ethiopian Units of Measurement

Local Unit Name	Quantity Name	Definition	Estimated amount compared to recent unit
Kunna	Grain	Made of bamboo used to measure cereals	5-10kg
Medeb	Meat, tomato, etc	be large or small, and is mainly used for vegetables	5kg
Esir	Butter and cabbage	A "bundle" often used for cabbage and chat (a mild stimulant)	2kg
Tassa	Grain	A large serving can (often for cereals, pulses and liquids)	1kg
Kubaya	Grain	A mug, often for cereals, pulses and liquids	0.5kg
Birchiko	All liquids	Made up of clay or a glass often for pulses and liquids	0.25kg
Sini	coffee	A small ceramic or clay cup often used for coffee, pulses (e.g. oilseeds) and spices	10gm
Sahin	Corps	Made up of clay	50gm
sinzir	Clothes	To tear and sewing clothes	20cm
kind	Land	To measure plot of land	50cm
Gemed	Land	To measure different spaces	50cm-10m
Timad	Land	Pair of ox plough whole day	

Table 4

Opo Community Unit of Measurement

Measurement of quantities	Local Unit of measurement	Standard measurement
Length	Rope	50 cm
Mass	Toyi	20kg
Size	Rope	50cm
Width	Rope	50cm
Volume	Ruwa	2.5 kg
Distance	One day travel	50km
Area	Pace	50cm

Another important example in a culturally specific context is the consumption of meat on special occasions. After an ox is slaughtered, the parts are sliced into pieces. Then people divide this into 12 equal parts (12 also has a significant cultural meaning). One part is called “Medeb”, (see Figure 5), which is the unit of division here. A family can share ½ or ¼ of a “Medeb”. The whole process is called “Kircha.”

Figure 5 Ethiopian cultural partition of an ox -'Kircha' (Seleshe, S., et al. 2014)

The Mathematics Revealed in Measuring Activities

Measurement is all about comparison of objects (directly or indirectly). More appropriately, as defined by Van de Walle et al. (2020, p. 507), “[it] is a number that indicates a comparison between the attribute of the object (or situation or event) being measured and the same attribute of a given unit of measure.” Thus, measuring activities follow the central idea that comparison is done between an attribute of the item (or situation) with a unit that has the same attribute. Another big idea is the development of the concept of estimation in measuring different kinds of stuff. “Medeb” is used as a unit for measuring stuff like tomatoes, meat and the like. Here, one needs to ensure that all the groups are divided equally, without using precise weighing instruments. Even the different parts of the ox should be shared fairly too. Having participated in such an ordeal, one always prefers one “Medeb” over another, as it looks better than the others!

Measurement demands comparative judgement and it can be developed using different approaches. Lehrer (2003) recommends a developmental approach, using concepts such as *unit-attribute relations, iteration, tiling, identical units, standardization, proportionality, additivity, origin (zero-point)* as the foundations for understanding measurement. Cultural measurement activities involving these concepts can be used as a starting point to improve student learning about measurement.

Designing

Designing is another activity that exists in almost every local community. The houses in various communities in Ethiopia follow different shapes and forms. According to respondents, Afar, Ari and Dasanech huts have different structures. Afar and Dasanech women need 20 and 12 wooden poles respectively to build their huts. During interviews, the respondents from Afar and Dasanech, sketched the hut shapes on the ground. The researcher drew the following pictures from Afar and Dasanech respondents' demonstrations (See Figure 6, a and c). Furthermore, the formation of that local houses follows traditional procedures, which are very efficient in the respective weather conditions in which these communities live (See Figure 6, b and d).

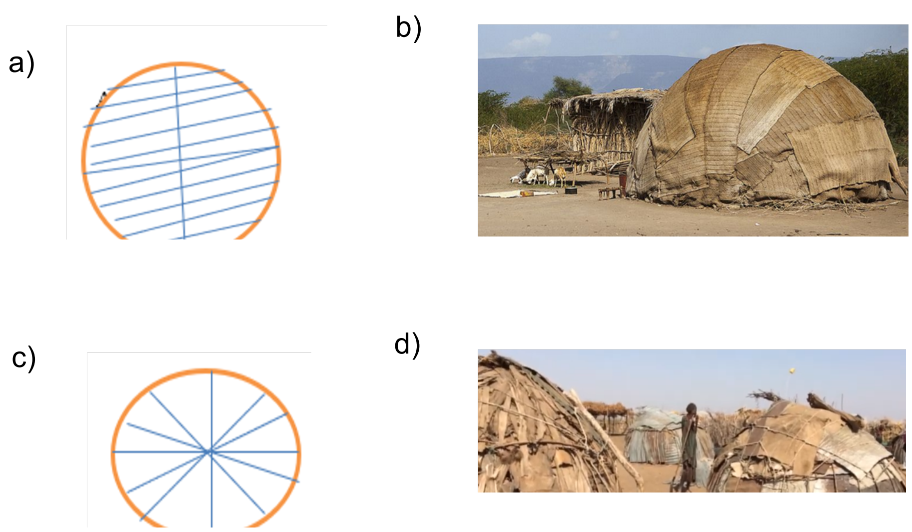


Figure 6 a) Drawing for Afar home b) Afar dome-shaped homes Picture from Eric Lafforgue at flickr (www.ericlafforgue.com) c) Drawing for Dasanech home d) Afar dome-shaped home Picture from SLAWEK – Photo gallery (gotoslawek.org).

To give another example, in the Orthodox church of Ethiopia, shapes, forms, and figures have been one way of teaching the doctrines of the church to the majority of the people who were not able to read and understand the clergy language Geez at that time. Different shapes and forms were used to represent different values, persons and secrets of the church. Consider the following drawing, Figure 7, which may be about 1000 years old. In this drawing, we see that the shape of the face of the people is oval shape and half faced. The people who are perceived as "saint" are represented by a full face and those who are half faced represent those who do evil things (non-believers).

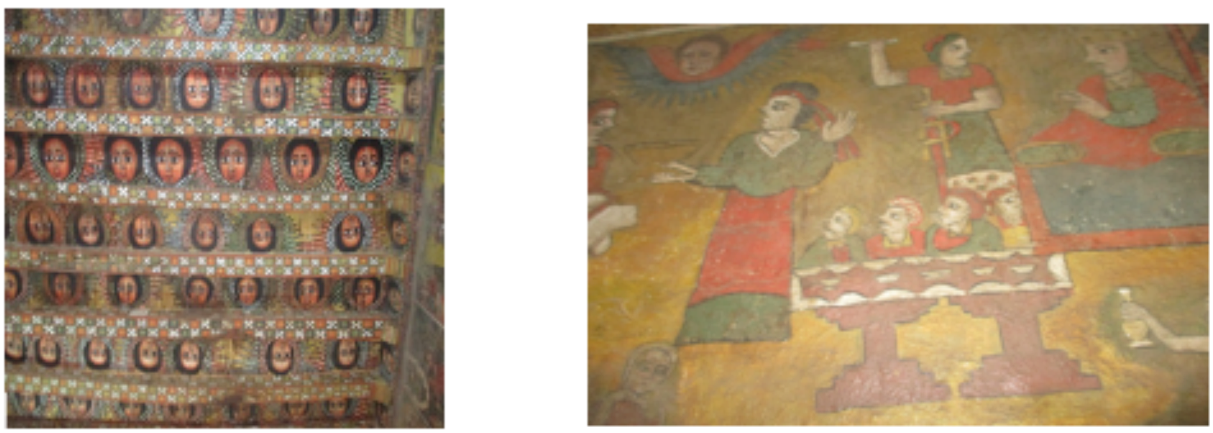


Figure 7 Full Face (oval shape) Pictures for Believers and Half Face Pictures for Non-Believers.

If one closely examines such drawings, mathematical concepts like circles, triangles, quadrants, rectangles, trapeziums, lines, parallel lines, symmetry, transformation (reflection, rotation, translation) and so are all embedded within them. In many homes, among the different ethnic people in Ethiopia, women prepare materials that will help them serve food. Some of them are present even in modern houses. Figure 8 illustrates a collection of such materials with different designs. These materials are full of different geometric shapes and symmetries.

Figure 8 Traditional Ethiopian Straw Basket Wavering (*https://no.pinterest.com/pin/233202086928587372/*).

Finally, all the Ethiopian cultures have different forms of dress, and these colorful, beautiful and rich clothes can also be sources of context to teach geometric concepts. The dresses in Figure 9 contain geometric contents: patterns, symmetry, diamond, triangle, rectangle, polygon, circle, parallel lines, symmetry, transformation, reflection, rotation, tessellation and so on. According to Van Heile's theory (1984), there are different stages or levels of development in geometric thinking, involving a five-level hierarchy of understanding spatial ideas: visualization, analysis, informal deduction, deduction and rigor. Visualizing, classifying and sorting the properties of the geometric shapes and forms can be done in early grades or pre-primary stages. These culturally-specific items of clothing can be sources for such activities.

Figure 9 Different Ethiopian Traditional Dresses (Top: new Habesha kemis from Naomi Habesha Kemis, Gondar dress, Oromo kids' dress; Bottom: range of colored Afar dresses (https://no.pinterest.com/pin/342836590382056939/)

Playing

Two different games or contexts which are common among most kids are presented under the category playing. These are “Gebeta” and “Boacha Chewata.”

Gebeta

This game is one of the oldest games emanating from the ancient Ethiopians. It is very difficult to find literature about the Gebeta game, which was found by archaeologists in Eretria in a place called Matara, then part of Ethiopia. Archeologists have found Gebeta playing materials, from which they estimated that the game was played in Ethiopia around 6^{th} to 7^{th} centuries BC. Likewise, during the regime of the Emperor Tewodros, a British citizen who was a close friend of the Emperor, Walter Palawidon, reported that the Gebeta game was practiced by the youths who were living in Begamider Ras Alula campus. Documents also show that Gebeta was played by Royals and monarchs in their palaces during leisure time.

The Gebeta game was conducted between two opponents based on stated rules. The game was played with either 12 or 18 holes on a wooden board, based on the interest of individuals who wanted to play. Gebeta playing materials were also prepared from sand and mud, or other local materials. Recently, standard playing materials have also been made from wood (see Figure 10).

Figure 10 Gebeta Playing Pebbles and Gebeta Playground

For formal competition, since it is an integral part of the game, the pebble's size and shape is crucial. The playing pebble is a round or spherical shape that can be made of marble, plant seeds or other local materials. The playing hole diameter is 6 cm and the depth of the hole is 2 cm. The circumference of the pebble is between 3 and 4 cm. For the 12 hole Gebeta game, each hole contains 4 pebbles, making a total of 48, but 6 pebbles are kept in reserve, making a total of 54 pebbles. Similarly, for the 18-hole game, each hole contains 3 pebbles making a total of 54 plus 6 reserve pebbles = 60 pebbles. Rules of the Gebeta Game are described as follows:

- The game can be started by some kind of random selection strategy, like tossing a coin either head or tails (as kids we used many other strategies like counting figures and counting who gets the last number… and so on). 30 seconds are allowed for each player to think about each pebble in the formal game.
- The first player of the game starts by picking all the pebbles in a hole that belongs to him/her and puts one pebble into each hole in a clockwise direction. When the last pebble in the hand has been placed, the player picks up all the pebbles in that hole and repeats the same procedure until the last pebble lands in an empty hole.
- The second player continues the same procedure, beginning with the holes that belong to him/her and goes around until the last pebble is placed in an empty hole.
- The player should observe attentively and pick when the pebbles build from an empty hole, that is, 0, to 1, 2, 3, and 4. When the hole contains four pebbles, the player picks it and places it by his/her side since one hole or 'home to be owned' is finished.
- Both players continue to play until all the pebbles are used to make full homes with 4 pebbles and the one with more holes (homes) wins the game.

- A winning strategy means that one is able to put the last pebble on the opponent's hole, which has three pebbles. In that case, the player takes the four and builds one more home for him/herself.

Affordance of Gebeta for the Teaching and Learning of Mathematics

Gebeta is mainly about counting. Addition and subtraction strategies are involved, as one counts the accumulated pebbles in a whole and distributes each pebble around each whole until no pebble is left. The concept of zero is represented by empty holes and empty hands when no pebble is left to be distributed further. Actually, winning homes in the Gebeta game is predicated on a base five counting system. That is, one builds from 0, 1, 2, 3, and 4 in the hole. Another aspect that is built into the nature of the game, as with chess, is that it fosters critical thinking. One should also be able to estimate, locate and design solutions based on the development of each game. Unlike chess, however, Gebeta can be played by almost every child (Tesfamicael et al., 2021). It is also possible to give students in higher grade the task of finding out how many different ways exist to win the game. It is even possible to simulate it with a computer program (Meaney & Shockey, 2020). Playing the Gebeta game requires counting, measuring, designing, and locating skills, as well as the preparation of playing materials, and the board features holes in the shape of half spheres, symmetry of the holes, the size of the holes, shape of pebbles, and so on.

The ancient Ethiopian people who invented Gebeta as an entertainment after intensive work were used to estimating, counting, measuring, locating and designing practical activities, and were therefore well acquainted with the prerequisite concepts, because they realized that a person who has no counting skills cannot play Gebeta (Tesfamicael et al., 2021). Similarly, a person who has no estimation skills is unable to prepare a piece of wood for the board, which requires gouging either 12 or 18 holes. In the same vein, a person who has no geometrical skills cannot prepare the shapes of pebbles and holes. Hence, in ancient times, the concepts of counting, measuring, locating, and designing, explaining and playing were regularly practiced in the community during day-to-day activities, which clearly demonstrates that ancient Ethiopians had strong foundations in indigenous mathematics. They were well aware of the importance of indigenous mathematical concepts for numerical analysis of things, setting rules and laws and critical thinking skills.

Gebeta is an activity that has clearly defined rules and regulations. In the game, there is always competition to win, rules to follow and enjoyment to experience. Gebeta game activity helps teachers to create a better teaching learning environment. The competitive element also stimulates students' participation and gives them confidence, motivating them to get the best score in a game or even to be the best in the class. This could be an effective tool for motivating students to learn mathematics and develop their logical thinking to solve mathematical problems.

Boacha Chewata

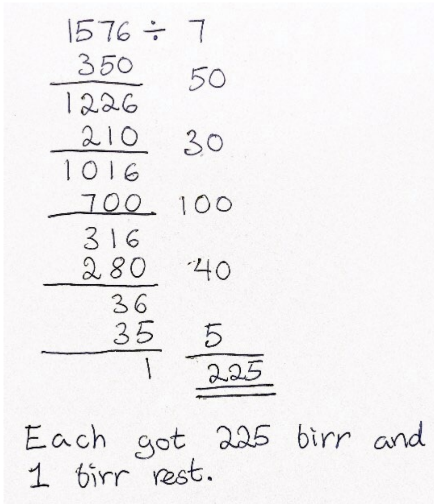
This game can be played by two or more people. It is better if an even number of participants are playing, but there is a way to deal with having an odd number of players. The person who has ended up in the middle will be left out of some of the parts of the game as a neutral participant. First, players collect stones and make two “sofa” shaped target, with 50 meters or so of space between them, for example. The game is played by throwing flat rocks. The flatter the rock, the better it is for playing. So, everyone has to look for or prepare that rock. Then points are scored according to the distance. If one lands his/her rock on the sofa, then he/she gets 4 points, those who touch the sofa get 2 points and those who did not get a point if their rock is closer to the sofa than the paired opponent (it can be played in groups or individually). After one reaches 12 points, the fun part begins, the loser carries the one with the greater score seated on his or her back and they continue to play, both throwing their rocks. In addition, the loser has to carry the winner to the opposite sofa. That is, the winner gets free transport!

Mathematics Revealed in Boacha

Boacha is about counting by one, two, and four. Additionally, multiplication and counting are all involved together with spatial knowledge and location: at the top, by the side, at the back of the sofa. Those rocks that land at the back do not get a score. There are different ways to win. If one person, or a team, is able to put three rocks on the sofa, then they win, and so on. Collaboration, discourse, problem solving, and critical thinking are involved in this game also. It is highly motivational, as it involves winning and losing. Algorithmic thinking is also involved, as one should iterate many times to come up with the winning number. Spatial knowledge and estimation are involved, as the distance between the two sofas can be set at 60, 80 or more meters, or the local equivalent.

Explaining

Under this activity, we consider an activity that happens in most parts of the highland regions of Ethiopia. It is a traditional kids' play activity that happens in September, especially in relation to the national holiday celebration called "Meskel Festival." Kids get together and go round homes singing "*Hoya hoye*." By singing this traditional song with lyrics, they persuade families to give them money. Now, let us assume that seven kids join together and engage in the yearly "hoya hoye" and receive 1576 birr.   They can decide to share the money at the end, or they can share it on the way as they go around.



If the strategy used to share the money is provided as such, how can one explain the way the kids divide the 1576 birr equally among each other and receive 225 birr for each with 1 birr left over.

The Mathematics Involved in This Story

In this activity, the context of money, coupled with the cultural history of the learners is involved. This context can be used to teach kids to mathematize. Firstly, they do addition, as they collect money from each home. Secondly, they have to share equally. Hence, the concept of division with equal sharing is also present. We want to draw attention to the following strategy, or division algorithm, that can go with the activity, called the Nils-Johan division algorithm (Botten, 2016). The logical explanation that goes with this division algorithm is that 7 kids have in total received 1576 birr in that year's *hoya hoye*. Firstly, they shared 350 birr (50 each), and next they shared 210 birr, (30 birr each). Later, they shared a total of 700 birr (100 each). They then shared 280 birr, (40 birr each) and lastly, they had 36 birr, took 5 birr each and had 1 birr left over. The question can then be, can this algorithm be used every year when they participate in *Hoya hoye*? Here, the context is used to teach strategies for division, which most students find hard to learn.

Curriculum Implications: The Six Activities as a Basis for the Curriculum

Bishop suggests (1988, p. 96) that the six intercultural mathematical ideas should be the basis for structuring curricula. He argues that this makes it possible to develop mathematical ideas with children in a way that easily and powerfully connects to their culture. Bishop (2002) finds it problematic that children too often see mathematics in preschool and primary school as a set of boring and meaningless routines that they have to learn, rather than as satisfactory and meaningful experiences relevant to their lives. The reason for this gap between what children think of as mathematics, and what they should think, is, according to Bishop, related to mathematics not being seen as a cultural product. By structuring the curriculum around the six mathematical activities, it will be clearer that the mathematical situations in our daily lives, and those in more formal teaching situations, are strongly connected. Bishop's ideas have influenced many curricula around the world (Bishop, 2002).

In Norway, the pedagogical practice of preschools is described in the *Framework Plan for Kindergartens* (Ministry of Education and Research, 2017). In this plan, mathematics has its own chapter, "Quantities, spaces and shape", that explicitly describes expectations of what institutions and pedagogues should provide in the way of mathematical opportunities in the preschool environment. In both Norwegian and Swedish preschool curricula (Carlsen et al., 2011; Helenius et al., 2014), the mathematical objectives highlighted in the Framework Plan, although not formally acknowledged, can be traced back to Bishop's six mathematical activities (Bishop, 1988). The Framework Plan focuses on children's opportunities to play with, experiment, problem-solve and discover mathematical ideas. According to Devlin (2012), this brings mathematics in preschool closer to the kind of work done by mathematicians. Bishop's universal activities remind us of the broad array of mathematical competencies children develop in their early years. These different competencies together constitute deep mathematical understanding. Even though curricula are a political instrument, and are continually being changed, the fundamental activities will still always be the same. Any curriculum should therefore build on these activities (Nakken & Thiel, 2014; Helenius et al., 2014).

In Ethiopia too, Bishop's six mathematical activities can contribute to the design of a curriculum that can motivate and improve student learning, as suggested by the data gathered in this study. This data indicates that there is a huge potential for indigenous mathematical activities to enrich school mathematics, if used in the classroom in a meaningful way. Of course, further detailed work is necessary. It is worth noting that the goal is not to substitute ethnomathematics for all of school mathematics, but is, rather, to improve student learning and achievement by making mathematics more relevant, and by increasing the motivation for students to mathematize using real life contexts (Fosnot & Dolk, 2001; Rosa & Orey, 2011).

Taking as an example the Ethiopian curriculum for grade one mathematics, we find the following:

> Students should be able to [acquire] basic skills in **handling money**, use concrete examples to show understanding of halves and quarters, **use non- formal measures to find length, weight and capacity of everyday objects**, …, acquire basic knowledge of **Ethiopian currency**, acquire basic knowledge of **time**, …, complete simple **patterns of color, shape and number**. (Ministry of Education, 2008, p. iv)

Already, most of Bishop's activities are aligned with these statements. The problem with school mathematics is that it becomes increasingly abstract for learners, with a focus on algorithmic calculation, fostering instrumental understanding instead of relational understanding (Skemp, 1978). Teacher training programs should, therefore, ensure that teachers are trained to foster students' learning and deep mathematical understanding, as described in the intended curriculum. Hence, there is a need for revision of teacher training materials and curricula, and professional development programs for in-service teachers.

Conclusions

Improving student learning and achievement provides a rationale for involving ethnomathematics activities in mathematics teaching and learning. This can be done via real life mathematical activities embedded in the cultural and social contexts of the students. Here, Bishop's six fundamental mathematical activities, which are found in almost every culture, including in Ethiopia, are used as a framework and as a starting point. Data was gathered through ethnographic approaches and selected examples aligned to the chosen framework were presented. Our findings show that there are many potential activities that can be carefully embedded in the curriculum and instructional practices, as ethnomathematics involves mathematical concepts and procedures in a cultural context. We recommend that a deeper study, involving educators, anthropologists, and mathematics educators be done in the area. In that case, broader approaches should also be considered, including, for instance, Ethnocomputing, Ethnomodelling, the Trivium Curriculum and the like (Rosa & Orey, 2016). The recent cycle of curriculum and textbook preparation by the Ministry of Education in Ethiopia used a participatory model, which involved teachers, educators, subject specialists, and policy makers in a promising way. These participants may wish to consider the use of such ethnomathematics activities to foster student learning. Finally, we hope that this study may inform educators, mathematics educators, and policy makers in other countries, where ethnomathematics activities are insufficiently integrated into their respective educational systems, to foster students' ability to mathematize, solve real life problems, and undertake modelling tasks.

References

Anderson, J. R., Reder, L. M. & Simon, H. A. (1996). Situated learning and education. *Educational Researcher, 25*(4), 5–11. DOI:10.2307/1176775

Begg, A. (2001). Ethnomathematics: why, and what else? *ZDM Zentralblattfür Didaktik der Mathematik, 33*(3), 71-74.

Bishop, A. J. (1988). *Mathematical Enculturation: A Cultural Perspective on Mathematics Education*. Dordrecht: Kluwer Academic Publishers.

Bishop, A. J. (2002). Mathematical Acculturation, Cultural Conflicts, and Transition. In G. de Abreu & N. C. Presmeg (Eds.), *Transitions Between Contexts of Mathematical Practices* (pp. 193–212). Dordrecht: Springer. https://doi.org/10.1007/0-306-47674-6_10

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more "real"? *For the Learning of Mathematics*, *13*, 12–17.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. Jossey-Bass.

Botten, G. (2016). *Matematikk med mening - mening for alle.* Bergen: Caspar Forlag.

Cai, J., Morris, A., Hohensee, C., Hwang, S., & Robison, V. (2020). Maximizing the Quality of Learning Opportunities for Every Student. *Journal for Research in Mathematics Education. 51*(1)*,* 12-25. https://www.jstor.org/stable/10.5951/jresematheduc.2019.0005

Carlsen, M., Wathne U. & Blomgren, G. (2011). *Matematikk for førskolelærere.* Kristiansand: Høyskoleforlaget.

Creswell, J. W. & Poth C.N. (2018). *Qualitative Inquiry and Research Design: Choosing Among Five Approaches.* SAGE Publications.

D'Ambrosio, U. (1990). The history of Mathematics and Ethnomathematics: How a native culture intervenes in the process of learning science. *Impact of Science on Society, 40*(4), 369-378.

D'Ambrosio. U. (2001). What is ethnomathematics and how can it help children in schools? *Teaching Children Mathematics*, *7*(6), 308-310.

D'Ambrosio, U. (2006). *Ethnomathematics: Link between Traditions and Modernity*. Rotterdam: Sense Publishers.

Devlin, K. J. (2012). *Introduction to Mathematical Thinking*. Palo Alto, CA: Keith Devlin.

Ernest, P. (1991). *The philosophy of mathematics education*. London: Falmer Press.

Fosnot, C. & Dolk, M. (2001). *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction.* Portsmouth: Heinemann.

Greene, C. (2017). Decolonizing Mathematics through Cultural Knowledge: Construction of the Nehiyawak Mîkiwâhp (Cree Tipi). *Journal of Mathematics and Culture*.

Helenius, O., Johansson, M., Lange, T., Meaney, T., Riesbeck, E., & Wernberg, A. (2014). Preschool Teachers' Awareness of Mathematics. In O. Helenius, A. Engström, T. Meaney, P. Nilsson, E. Norén, J. Sayers, & M. Österholm (Eds.), *Evaluation and comparison of mathematical achievement: Dimensions and perspectives: Proceedings from Madif9, February 4-5, 2014.* Göteborg: Svensk Förening för matematikdidaktisk forskning.

Langdon, N. (1989). Cultural Starting Points in Keitel, C, Damerow, P., Bishop, A , Gerdes, P (Eds) *Mathematics, education and society* United Nations Educational Scientific: Paris.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate Peripheral Participation.* Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511815355

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), A *Research Companion to Principles and Standards for School Mathematics* (pp. 179–192). Reston, VA: National Council of Teachers of Mathematics.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19-44). Westport, CT: Ablex.

Lerman, S. (2014). Learning and knowing mathematics. In P. Andrews & T. Rowland (Eds.), *MasterClass in mathematics education: International perspectives on teaching and learning* (pp. 15-26). London: Bloomsbury.

Lesh, R., Post, T. R. & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janiver (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Lawrence Erlbaum.

Matang, R. A. & Owens, K. (2014). The role of indigenous traditional counting systems in children's development of numerical cognition: results from a study in Papua New Guinea. *Mathematics Education Research Journal, 26*, 531–553.

Meaney, T. & Shockey, T. (2020). The Influence of digital technologies on the relationship between mathematics and culture. *Journal of Mathematics and Culture, 14*(2), 132–151.

Ministry of Education (MOE). (2018). Ethiopian Education Development Roadmap (2018-30). Ministry of Education: Addis Ababa.

Ministry of Education (MOE) (2008). Mathematics Syllabus Grade 1. Ministry of Education: Addis Ababa.

Nair, S. B. & Abera, T. (2017). Indigenous knowledge in Ethiopian School Curriculum: *Social Sciences, 18*(2), 89-95.

Nakken, A. Hj. & Thiel, O. (2019). *Matematikkens kjerne*. Bergen: Fagbokforlaget.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

Nilsen, T. & Gustafsson, J. E. (Eds.). (2016). *Teacher quality, instructional quality and student outcomes*. Berlin: Springer.

Orey, D. & Rosa, M. (2006). Ethnomathematics: cultural assertions and challenges towards pedagogical action. *The journal of Mathematics and culture, 1*(1), 57-78

Organization for Economic Cooperation and Development (OECD). (2016). *PISA2015 Results (Volume I). Excellence and Equity in Education.* Paris: OECD. https://doi.org/10.1787/9789264266490-en.

Rosa, M. & Orey, D. C. (2011). Ethnomathematics: the cultural aspect of mathematics. *Revista Latinoamericana de Etnomatemática*, *4*(2), 32-54.

Rosa, M., D'Ambrosio, U., Orey, D. C., Shirley, L., Alangui, W. V., Palhares, P. & Gavarrete, M. E. (2016). *Current and Future Perspectives of Ethnomathematics as a Program.* New York: Springer.

Sammons, P. (2009). The dynamics of educational effectiveness: A contribution to policy, practice and theory in contemporary schools. *School Effectiveness and School Improvement, 20* (1), 123–129.

Seleshe, S., Jo C., Lee, M. (2014). Meat Consumption Culture in Ethiopia. *Korean J Food Sci Anim Resour, 34*(1), 7-13.

Shirley, L. (2001). Ethnomathematics as a fundamental of instructional methodology. *Zentralblatt Für Didaktik Der Mathematik, 33*, 85–87. https://doi.org/10.1007/BF02655699

Skemp, R. (1978). Relational Understanding and Instrumental Understanding. *The Arithmetic Teacher*, *26*(3), 9-15.

Tesfa, B. C. (2013). *Benefit of Grand Ethiopian Renaissance Dam Project (GERDP) for Sudan and Egypt*. Discussion Paper. EIPSA Communicating Article: Energy, Water, Environment & Economic, Online.

Tesfamicael, S. A., Nakkeen, A. H., Tumebo, T., & Grey, P. (2021). Ethnomathematics in Ethiopia using glocal approach: the case of 'Gebeta' playing. Accepted for the 14th International Congress on Mathematical Education, Shanghai, 11th –18th July 2021. (In Press)

Valverde, G., Bianchi, L. J., Wolfe, R., Schmidt, W. H. & Houang, R. T. (2002). *According to the Book: Using TIMSS to Investigate the translation of policy into practice through the world of textbooks*. London: Kluwer Academic Publishers.

Van Hiele, P. (1984). The child's Thought and Geometry. In D. Fuys, D. Geddes, R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele*, (pp. 243–252). Brooklyn: Brooklyn College, School of Education. (ERIC Document Reproduction Service No. 287 697).

Van de Walle, J., Karp, K. S. & Bay-Williams J. M. (2020). *Elementary and middle school mathematics: Teaching developmentally*. Pearson Education Limited.

Whitehead, T. L. (2005). *Basic Classical Ethnographic Research Methods.* Ethnographically Informed Community and Cultural Assessment Research Systems Working Paper Series.

Weldeana, H. N. (2016). Ethnomathematics in Ethiopia: Futile or Fertile for Mathematics Education? *Momona Ethiopian Journal of Science*, *8*(2), 146-167. https://doi.org/10.4314/mejs.v8i2.4